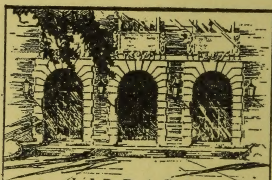

ENGINEERS' AND MECHANICS' POCKET-BOOK.

CONTAINING

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DIMENSIONS OF STEAMERS, MILLS, ETC.;

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ETC., ETC., ETC.

Thirty-second Edition, Revised and Enlarged.

BY CHAS. H. HASWELL,

CIVIL, MARINE, AND MECHANICAL ENGINEER, MEMBER OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS, CORRESPONDING MEMBER OF THE AMERICAN INSTITUTE OF ARCHITECTS, ASSOCIATE OF THE INSTITUTION OF NAVAL ARCHITECTS, ENGLAND, ETC.

An examination of facts is the foundation of science.

NEW YORK:
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1876.

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Civil, Marine, and Mechanical Engineer and Surveyor.

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INSCRIBED

TO

CORNELIUS VANDERBILT, Esq.,

AS A TRIBUTE TO HIS

UNEQUALED ENTERPRISE AND PATRONAGE

OF MECHANIC ARTS,

FROM AN EARLY ACQUAINTANCE AND FRIEND,

THE AUTHOR.

PREFACE

To the Twenty-first Edition.

THE First Edition of this work, consisting of 284 pages, was submitted to the Engineers and Mechanics of the United States by one of their number in 1843, who designed it for a convenient reference to Rules, Results, and Tables connected with the discharge of their various duties.

At the period of its first publication, the want of a work of this description had long been felt, and this was undertaken to meet the requirement, and, from the adaptation of its rules to the Metals, Woods, and Manufactures of the United States, it has supplied that want to an extent beyond what was anticipated.

This Edition was commenced in 1856, is the result of much labor, and, in addition to the design of the original work, it has been essayed to embrace very general information upon Arithmetical, Physical, and Mechanical subjects.

The Tables comprising the Weights of Metals, of Balls, Pipes, etc., were computed expressly for this work, from specific gravities of the different materials taken for the purpose.

The proportions of the parts of Steam-engines and of Boilers will be found to differ in some instances from English authorities; but as they are based upon the actual results of successful experience, the author is of the conviction that an adherence to them will insure both success and satisfaction.

To the Young Engineer and Mechanic it is recommended to cultivate a knowledge of Physical Laws and to note results, without which, eminence in his profession can never be securely attained; and if this work shall assist him in the attainment of these objects, one great purpose of the author will be fully accomplished.

INDEX.

A*

H.

EXPLANATIONS OF CHARACTERS

Used in Calculations, etc., etc.

$=$ *Equal to*, as 12 inches $=1$ foot, or $8\times8=16\times4$.

$+$ *Plus*, or *More*, signifies addition; as $4+6+5=15$.

$-$ *Minus*, or *Less*, signifies subtraction; as $15-5=10$.

$\times$ *Multiplied by*, or *Into*, signifies multiplication; as $8\times9=72$. $a\times d$, $a.d$, or ad, also signify that a is to be multiplied by d.

$\div$ *Divided by*, signifies division; as $72\div9=8$.

: *Is to*, :: *So is*, : *To*, signifies *Proportion*, as 2 : 4 :: 8 : 16; that is, as 2 *is to* 4, *so is* 8 *to* 16.

—— The *Vinculum*, or *Bar*, signifies that the numbers, etc., over which it is placed, are to be taken together; as $\overline{8-2}+6=12$, or $3\times\overline{5+3}=24$.

. *Decimal point*, signifies, when prefixed to a number, that that number has some power of 10 for its denominator; as .1 is $\frac{1}{10}$, .15 is $\frac{15}{100}$, etc.

$\backsim$ *Difference*, signifies, when placed between two quantities, that their difference is to be taken, it being unknown which is the greater.

$^{\circ}\ '\ ''\ '''$ signify *Degrees*, *Minutes*, *Seconds*, and *Thirds*.

$\angle$ signifies *Angle*.

$\perp$ signifies *Perpendicular*.

Δ signifies *Triangle*.

$\square$ signifies *Square*, as $\square$ inches; and $\boxtimes$ cube, as cubic inches.

$>\urcorner$, $<\llcorner$ signify *Inequality*, or *greater* or *less than*, and are put between two quantities; as $a\urcorner b$ reads a greater than b, and $a\llcorner b$ reads a less than b.

$\therefore$ signifies *Therefore* or *Hence*.

$\because$ signifies *Because*.

( ) [ ] *Parentheses* and *Brackets* signify that all the figures, etc., within them are to be operated upon as if they were only one; thus, $(3+2)\times5=25$; $[8-2]\times5=30$.

p or π is used to express the ratio of the circumference of a circle to its diameter $=3.1416$.

a' a'' a''' signify a *prime*, a *second*, a *third*, etc.

$\pm$ $\mp$ signify that the formula is to be adapted to two distinct cases

$\sqrt{\ }$ *Radical sign*, which, prefixed to any number or symbol, signifies that the *square root* of that number, etc., is required; as $\sqrt{9}$, or $\sqrt{a+b}$. The degree of the root is indicated by the number placed over the sign, which is termed the *index* of the *root* or *radical*; as $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, etc.

NOTES.—The degrees of temperature used are those of Fahrenheit.

g is the common expression for gravity $=32.166$, $2g=64.33$, $\sqrt{2g}=8.02$ *feet*.

$\frac{1}{2}$, $\frac{1}{3}$, etc., set *superior* to a number, signify the square or cube root, etc., of the number; as $2^{\frac{1}{2}}$ signifies the square root of 2.

$\frac{4}{2}$, $\frac{4}{3}$, $\frac{6}{3}$, etc., set *superior* to a number, signify the square or cube root of the 4th power, etc.

$^{1.7}$, $^{3.6}$, etc., set *superior* to a number, signify the tenth root of the 17th power, etc.

$_1$, $_2$, added to or set *inferior* to a symbol, reads *sub* 1 or *sub* 2, and is used to designate corresponding values of the same element, as h, h_1, h_2, etc.

^{2}, ^{3}, ^{4}, added or set *superior* to a symbol, signifies that that number, etc., is to be *squared*, *cubed*, etc.; thus, 4^2 means that 4 is to be multiplied by 4; 4^3, that it is to be *cubed*, as 4^3 is $=4\times4\times4=64$. The *power*, or number of times a number is to be multiplied by itself, is shown by the number added, as ^{2}, ^{3}, ^{4}, ^{5}, etc.

⊠ signifies *Dead flat*, or the location of the frame of a vessel at its greatest transverse section.

′ ″ set *superior* to a figure or figures, signify *Feet* and *Inches*.

NOTATION.

1=I.	7=VII.	40=XL.
2=II.	8=VIII	50=L.
3=III.	9=IX.	60=LX.
4=IV.	10=X.	70=LXX.
5=V.	20=XX.	80=LXXX.
6=VI.	30=XXX.	90=XC.

100=C.	10,000=$\overline{\text{X}}$, or CCIↃↃ.
500=D, or IↃ.	50,000=$\overline{\text{L}}$, or IↃↃↃ.
1000=M, or CIↃ.	60,000=$\overline{\text{LX}}$.
2000=MM.	100,000=$\overline{\text{C}}$, or CCCIↃↃↃ.
5000=$\overline{\text{V}}$, or IↃↃ.	1,000,000=$\overline{\text{M}}$, or CCCCIↃↃↃↃ.
6000=$\overline{\text{VI}}$.	2,000,000=$\overline{\text{MM}}$.

As often as a character is repeated, so many times is its value repeated.

A less character before a greater diminishes its value, as IV=I−V, or 1 subtracted from 5=4.

A less character after a greater increases its value, as XI=X+I, or 1 added to 10=11.

For every Ↄ annexed, the sum is increased 10 times.

For every C and Ↄ, placed one at each end, the sum becomes 10 times as many.

A bar, thus ¯, over any number, increases it 1000 times.

Illustrations.—1840, MDCCCXL. 18560, $\overline{\text{XVIII}}$DLX.

ALGEBRAIC SYMBOLS AND FORMULÆ.

Where l	*represents the*	*length,*	c	*represents the*	*chord,*
b	"	*breadth,*	a	"	*area,*
d	"	*depth,*	r	"	*radius,*
h	"	*height,*	v	"	*versed sine,*
h'	"	*h prime,*	$h_{,}$	"	*h sub.*

$\frac{l+b}{d}$ = sum of the length and the breadth divided by the depth.

$\frac{l\,b}{d}$ = product of the length and the breadth divided by the depth.

$\frac{l-b}{d}$ = difference of the length and the breadth divided by the depth.

$l^2\,b^3$ = product of the square of the length and the cube of the breadth.

$\frac{\sqrt{l}}{\sqrt[3]{b}}$ = square root of the length divided by the cube root of the breadth.

$\frac{\sqrt{l+b}}{d}$ = square root of the sum of the length and the breadth divided by the depth.

$\sqrt[3]{\frac{h' \backsim h_{,}}{\sqrt{2g}}}$ = cube root of the difference of *h prime* and *h sub*, divided by the square root of $2g$.

$c \mp v = c$ greater or less than v. Here there are expressed two values: first, the difference between c and v; second, the sum of c and v.

In this and like expressions, the upper symbol takes preference of the lower.

$\sqrt{a+(c-r)^2}=x$. Add the square of the difference between the chord and radius to the area, and extract the square root; the result will be equal to x.

It is frequently advantageous to begin the interpretation of a formula at the right hand, as in the above case.

$l\sqrt{\frac{(x+y)^2}{y^2}-1}=z$. Divide the square of the sum of x and y by the square of y; subtract *unity* from the quotient; extract the square root of the result; multiply it by the length, and the product will be equal to z.

$\frac{2(\sin.\ 75^\circ)^2}{1+(\sin.\ 75^\circ)^2}$. Divide twice the square of the sine of the angle of 75° by the square of the sine of the angle of 75° added to *unity*.

$$\frac{2a}{(S\sqrt{2g})^2}\left\{S\sqrt{2g}\left(\sqrt{h}-\sqrt{h'}\right)+2.303\ c.\ log.:\frac{S\sqrt{2gh}-b}{S\sqrt{2gh'}-b}\right\}=t.$$

Multiply S by the $\sqrt{\ }$ of $2g$, and this product by the difference between the square roots of h and *h prime*; add this to 2.303 times the common logarithm of the quotient arising from dividing the product of S into $\sqrt{2gh}$ diminished by b, by the product of S into $\sqrt{2gh\ prime}$ diminished by b, and multiply this sum by the quotient of $2a$ divided by the square of the product of S into $\sqrt{2g}$, which will be equal to t.

$2a-3$ cos. $98° = 2a+3\times$ cos. $98°$ = the sum of twice a added to three times the cosine of 98°.

The cosine of any angle greater than 90° and less than 270° is always — or negative.

$39.127-.09982$ cos. $2L=l. = 39.127+\overline{\text{cos. } 2L\times .09982}=l.$

Assume L=46°. Here, cos. 2×46°, being equal to 92° and less than 270°, becomes —; therefore —.09982 and —cos. 90° become +.

CHRONOLOGICAL ERAS AND CYCLES FOR 1876.

The year 1876, *or the* 101*st year of the Independence of the United States of America, corresponds to*

The year 7384–85 of the Byzantine Era;
" 6589 of the Julian Period;
" 5636–37 of the Jewish Era;
" 2652 of the Olympiads, or the fourth year of the 663d Olympiad, commencing in July, 1875, the era of the Olympiads being placed at 775.5 years before Christ, or near the beginning of July of the 3938th year of the Julian Period;
" 2629 since the foundation of Rome, according to Varro;
" 2188 of the Grecian Era, or the Era of the Seleucidæ;
" 1592 of the Era of Diocletian.

The year 1293 of the Mohammedan Era, or the Era of the Hegira, begins on the 7th of February, 1876.

The first day of January of the year 1876 is the 2,406,255th day since the commencement of the Julian Period.

Dominical Letter	B	Lunar Cycle or Golden Number	15
Epact	4	Solar Cycle	9

Chronology.

B.C.

4004. Creation of the World (according to Julius Africanus, Sept. 1st, 5508; Samaritan Pentateuch, 4700; Septuagint, 5872; Josephus, 4658; Talmudists, 5344; Scaliger, 3950; Petavius, 3984; Hales, 5411).

2348. Deluge (according to Hales, 3154).
2203. Chinese Monarchy.
2090. First Egyptian Pyramid.
1180. Troy destroyed.
1111. Mariner's Compass discovered.
753. Foundation of Rome.
576. Money coined at Rome.
605. Geometry, Maps, etc., first introduced.
289. First Sun-dial.
219. Hannibal crossed the Alps.
219. Surveying first introduced.
155. Time first measured by water.
51. Cæsar invaded Britain.

A.D.

214. Grist Mills introduced.
667. Glass discovered.
991. Arabic numerals introduced.
1066. Battle of Hastings.
1180. Mariner's Compass introduced in Europe.
1383. Cannon introduced.
1492. America discovered.
1627. Barometer and Thermometer invtd.
1752. New Style, introduced into Britain; Sept. 3 reckoned Sept. 14.
1769. James Watt—First design and patent of a steam-engine having a separate vessel of condensation.
1789. French Revolution.

A.D.

1772. Oliver Evans—Designed the non-condensing engine. 1792. Applied for a patent for it. 1801. Constructed and operated it.
1790. Water lines first introduced in the models of vessels in the U. S.
1797. John Fitch—Propelled a yawl boat by the application of steam to side wheels, and also to a screw propeller, upon the Collect pond, New York.
1807. Robert Fulton — First passenger Steam-boat.
1827. First Rail Road in U. S., from Quincy to Neponset, Mass.

UNITED STATES MEASURES AND WEIGHTS.

According to Act of 1866.

For Equivalents of Old Measures and Weights to New, see page 630.

Measures of Length.

Denominations and Values.		Equivalents in use.
Myriameter...	10 000 meters.	6.2137 *miles.*
Kilometer	1 000 meters.	.62137 *mile*, or 3280 *feet* and 10 *ins.*
Hectometer...	100 meters.	328 *feet* and 1 *inch.*
Dekameter ...	10 meters.	393.7 *inches.*
Meter	1 meter.	39.37 *inches.*
Decimeter	$\frac{1}{10}$th of a meter.	3.937 *inches.*
Centimeter ...	$\frac{1}{100}$th of a meter.	.3937 *inch.*
Millimeter....	$\frac{1}{1000}$th of a meter.	.0394 *inch.*

Measures of Surface.

Denominations and Values.		Equivalents in use.
Hectare	10 000 square meters.	2.471 *acres.*
Are	100 square meters.	119.6 *square yards.*
Centare	1 square meter.	1550 *square inches.*

Measures of Volume.

Denominations and Values.			Equivalents in use.	
Names.	No. of Liters.	Cubic Measure.	Dry Measure.	Liquid or Wine Measure.
Kiloliter or Stere	1000	1 cubic meter	1.308 *cubic yards.*	264.17 *gallons.*
Hectoliter.	100	$\frac{1}{10}$ cubic meter	2 *bush.* and 3.35 *pecks.*	26.417 *gallons.*
Dekaliter .	10	10 cubic decimeters	9.08 *quarts.*	2.6417 *gallons.*
Liter	1	1 cubic decimeter	.908 *quart.*	1.0567 *quarts.*
Deciliter ..	$\frac{1}{10}$	$\frac{1}{10}$ cubic decimeter	6.1022 *cubic inches.*	.845 *gill.*
Centilliter.	$\frac{1}{100}$	10 cub. centimeters	.6102 *cubic inch.*	.338 *fluid oz.*
Milliliter..	$\frac{1}{1000}$	1 cubic centimeter	.061 *cubic inch.*	.27 *fluid drm.*

Weights.

Denominations and Values.			Equivalents in use.
Names.	Number of Grams.	Weight of Volume of Water at its Maximum Density.	Avoirdupois Weight.
Millier or Tonneau.	1 000 000	1 cubic meter.	2204.6 *pounds.*
Quintal...........	100 000	1 hectoliter.	220.46 *pounds.*
Myriagram	10 000	10 liters.	22.046 *pounds.*
Kilogram or Kilo..	1 000	1 liter.	2.2046 *pounds.*
Hectogram	100	1 deciliter.	3.5274 *ounces.*
Dekagram	10	10 cubic centimeters.	.3527 *ounce.*
Gram.............	1	1 cubic centimeter.	15.432 *grains.*
Decigram	$\frac{1}{10}$	$\frac{1}{10}$th of a cubic centimeter.	1.5432 *grains.*
Centigram	$\frac{1}{100}$	10 cubic millimeters.	.1543 *grain.*
Milligram.........	$\frac{1}{1000}$	1 cubic millimeter.	.0154 *grain.*

For Measuring Surfaces the square Dekametre is used under the term of ARE; the Hectare, or 100 ares, is equal to about 2 acres.

The Unit of Capacity is the cubic Decimetre or LITRE, and the series of measures is formed in the same way as in the case of the table of lengths.

The cubic Metre is the unit of measure for solid bodies, and is termed STERE.

The Unit of Weight is the GRAMME, which is the weight of one cubic centimetre of pure water weighed in a vacuum at the temperature of 4° Centigrade, or 39°.2 Fahrenheit, which is about its temperature of maximum density.

In practice, the term cubic Centimetre, abbreviated C. C., is used instead of Millilitre, and cubic Metre instead of Kilolitre.

According to Previous and Existing Laws.

MEASURES OF LENGTH.

The Standard of measure is a brass rod, which, at the temperature of 32°, is the standard yard.

Lineal.

12	inches	=1 foot.
3	feet	=1 yard.
5.5	yards	=1 rod.
40	rods	=1 furlong.
8	furlongs	=1 mile.

Inches	Feet.	Yards.	Rods.	Furl.
36=	3.			
198=	16.5=	5.5.		
7920=	660 =	220 =	40.	
63360=	5280 =	1760 =	320=	8.

The inch is sometimes divided into 3 *barley corns*, or 12 *lines*.
A hair's breadth is the .02083 (48th part) of an inch.

1 yard is................................ .000568 of a mile.
1 inch is................................ .0000158 of a mile.

Gunter's Chain.

7.92 inches=1 link.
100 links =1 chain, 4 rods, or 22 yards.
80 chains =1 mile.

Ropes and Cables.

6 feet=1 fathom. | 120 fathoms=1 cable's length.

Geographical and Nautical.

1 degree of a great circle of the earth= 69.77 Statute miles.
1 mile =2046.58 yards.

Log Lines.

Estimating a mile at 6139.75 feet, and using a 30″ glass,
1 knot =51.1629 feet, or 51 feet 1.95 inches.
1 fathom= 5.11629 feet, or 5 feet 1.395 inches.

If a 28″ glass is used, and 8 divisions, then
1 knot=47 feet 9.024 inches. | 1 fathom=5 feet 11.627 inches.

The line should be about 150 fathoms long, having 10 fathoms between the chip and first knot for stray line.

NOTE.—Bowditch gives 6120 feet in a sea mile, which, if taken as the length, with a 28″ glass, will make the divisions 47.6 feet and 5.95 feet.

Cloth.

1 nail =2.25 inches=.0625 of a yard.
1 quarter =4 nails.
5 quarters=1 ell.

Pendulums.

6 points=1 line. | 12 lines=1 inch.

Shoemakers'.

No. 1 is 4.125 inches in length, and every succeeding number is .333 of an inch.

There are 28 numbers or divisions, in two series of numbers, viz., from 1 to 13, and 1 to 15.

Miscellaneous.

1 palm=3 inches.	1 span =9 inches.
1 hand=4 inches.	1 metre=3.2809 feet.

MEASURE OF TIME.

60 seconds =1 minute.	
60 minutes=1 degree.	3600″=60′. °
360 degrees =1 circle.	1296000=21600=360.

Sidereal day=23 h., 56 m., 4.092 sec., in solar or mean time.
Solar day, mean=24 h., 3 m., 56.555 sec., in sidereal time.
Sidereal year, or revolution of the earth, 365.25635 solar days.
Solar, Equinoctial, or Calendar year, 365.24224 solar days.

1 day=.002739 of a year. | 1 minute=.000694 of a day.
30°=1 sign.

MEASURES OF SURFACE.

144 square inches=1 square foot.
9 square feet =1 square yard.
100 " " =1 square (*Architect's Measure*).

Land.

			Yards	Rods.	Roods.
30.25	square yards	=1 square rod.	1210		
40	square rods	=1 square rood.			
4	square roods	=1 acre.	4840=160.		
10	square chains	=1 acre.			
640	acres	=1 square mile.	3097600=102400=2560.		

208.710326 feet, 69.570109 yards, or 220 by 198 feet square=1 acre.

Paper.

24 sheets=1 quire. | 20 quires=1 ream.

Drawing Paper.

Cap	13	×16	inches.	Columbia	23×	33.75	inches.
Demy	15.5	×18.5	"	Atlas	26×	33	"
Medium	18	×22	"	Theorem	28×	34	"
Royal	19	×24	"	Doub.Elephant	26×	40	"
Super-royal	19	×27	"	Antiquarian	31×	52	"
Imperial	21.25	×29	"	Emperor	40×	60	"
Elephant	22.25	×27.75	"	Uncle Sam	48×	120	"

Tracing Paper.

Double Crown.......	20 × 30 inches.	Grand Royal	18 × 24 inches.
Double D. Crown...	30 × 40 "	Grand Aigle........	27 × 40 "
Double D. D. Crown	40 × 60 "	Vellum Writing,	18 to 28 in wide.

Miscellaneous.

1 sheet = 4 pages.	1 duodecimo = 24 pages.
1 quarto = 8 "	1 eighteenmo = 36 "
1 octavo = 16 "	1 bundle = 2 reams.

Roll of Parchment = 60 sheets.

MEASURES OF VOLUME.

The Standard gallon measures 231 cubic inches, and contains 8.3388822 avoirdupois pounds, or 58373. troy grains of distilled water, at the temperature of its maximum density 39°.83, the barometer at 30 inches.

The Standard bushel is the *Winchester*, which contains 2150.42 cubic inches, or 77.627413 lbs. avoirdupois of distilled water at its maximum density.

Its dimensions are 18.5 inches diameter inside, 19.5 inches outside, and 8 inches deep; and when heaped, the cone must not be less than 6 inches high, equal 2747.715 cubic inches for a true cone.

Liquid.

	Gills.	Pints.
4 gills = 1 pint.		
2 pints = 1 quart.	8.	
4 quarts = 1 gallon.	32 =	8.

Dry.

	Pints.	Quarts	Gallons.
2 pints = 1 quart.			
4 quarts = 1 gallon.	8.		
2 gallons = 1 peck.	16 =	8.	
4 pecks = 1 bushel.	64 =	32 =	8.

Cubic.

	Inches.
1728 cubic inches = 1 foot.	
27 cubic feet = 1 yard.	46656.

NOTE.—A cubic foot contains 2200 cylindrical inches, 3300 spherical inches, or 6600 conical inches.

Fluid.

	Minims.	Drachms	Ounces.
60 minims = 1 drachm.			
8 drachms = 1 ounce.	480.		
16 ounces = 1 pint.	7680 =	128.	
8 pints = 1 gallon.	61240 =	1024 =	128.

Miscellaneous.

1 cubic foot	7.4805 gallons.
1 bushel	9.30918 gallons.
1 chaldron=36 bushels, or	57.244 cubic feet.
1 cord of wood	128 cubic feet.
1 perch of stone	24.75 cubic feet.

1 quarter =8 bushels.
1 sack flour=5 "

1 load hay or straw=36 trusses.
1 M quills =1200 quills.

	Galls.
Butt of Sherry	108
Pipe of Port	115
Pipe of Teneriffe	100
Butt of Malaga	105
Puncheon of Scotch Whisky	110 to 130
Puncheon of Brandy	110 to 120
Puncheon of Rum	100 to 110
Hogshead of Brandy	55 to 60
Pipe of Madeira	92
Hogshead of Claret	46

A Hogshead is one half, a Quarter cask is one fourth, and an Octave is one eighth of a Pipe, Butt, or Puncheon.

MEASURES OF WEIGHT.

The Standard avoirdupois pound is the weight of 27.7015 cubic inches of distilled water weighed in air, at 39°.83, the barometer at 30 inches.

A cubic inch of such water weighs 252.6937 grains.

Avoirdupois.

16 drachms=1 ounce.
16 ounces =1 pound.
112 pounds =1 cwt.
20 cwt. =1 ton.

Drachms.	Ounces.	Pounds.
256.		
28672=	1792.	
573440=	35840=	2240.

1 pound=14 oz. 11 dwts. 16 grs. troy, or 7000 grains.
1 ounce =18 dwts. 5.5 grains troy, or 437.5 grains.

Troy.

24 grains =1 dwt.
20 dwt. =1 ounce.
12 ounces =1 pound.

Grains.	Dwt.
480.	
5760=	240.

7000	troy grains	=	1 lb. avoirdupois.
437.5	troy grains	=	1 oz. "
175	troy pounds	=	144 lbs. "
175	troy ounces	=	192 oz. "
1	troy pound	=	.822857 lb.
1	avoirdupois pound	=	1.215278 lbs. troy.

Apothecaries.

20 grains =1 scruple.
3 scruples =1 drachm.
8 drachms =1 ounce.
12 ounces =1 pound.

Grains.	Scruples.	Drachms.
60.		
480 =	24.	
5760 =	288 =	96.

45 drops =1 tea spoonful or a fluid drachm.
2 table spoonsful=1 ounce.

The pound, ounce, and grain are the same as in Troy Weight.

Diamond.

1 carat = 4 grains.	16 parts = .8 troy grains.
1 grain = 16 parts.	4 grains = 3.2 "

Miscellaneous.

1 stone	=14 lbs.		
1 cubic foot of ordinary anthracite coal from 50 to 55 lbs.			
1 cubic foot of ordinary bituminous coal from 45 to 55 lbs.			
1 cubic foot of Cumberland coal	= 53	lbs.	
1 cubic foot of cannel coal	= 50.3	"	
1 cubic foot charcoal	= 18.5	"	(hard wood).
1 cubic foot charcoal	= 18	"	(pine wood).
1 cord Virginia pine	=2700	"	
1 cord Southern pine	=3300	"	

Coals are usually purchased at the conventional rate of 28 bushels (5 pecks) to a ton=43.56 cubic feet.

MEASURES OF VALUE.

10 mills = 1 cent.	10 dimes = 1 dollar.
10 cents = 1 dime.	10 dollars = 1 eagle.

The *Standard* of gold and silver is 900 parts of pure metal and 100 of alloy in 1000 parts of coin.

The *Fineness* expresses the quantity of pure metal in 1000 parts.

The *Remedy of the Mint* is the allowance for deviation from the exact standard fineness and weight of coins.

The nickel cent contains 88 parts of copper and 12 of nickel.

The new bronze cent contains 95 parts of copper and 5 of tin and zinc.

Pure Gold 23.22 grains=$1 00. Hence the value of an ounce is $20 67.183+.

Pure Silver 371.25 grains=$1 00. Hence the value of an ounce=$1 29.29+.

Silver coins of less value than one dollar are 385.8 grains in fineness.

Standard Gold, $18 60.465+ per ounce.
Standard Silver, $1 16.3636+ per ounce.
Trade Dollar, 420.9 grains in fineness.

Double Eagle =516 troy grs.	Half Dol. (silv.) = 192.9 + troy grs.
Eagle =258 "	5 Cent (nickel) = 77.16 "
Dollar (gold) = 25.8 "	3 Cent " = 30 "
Dollar (silver) = 412.5 "	Cent (bronze) = 48 "

The British standards are: *Gold*, $\frac{22}{24}$ of a pound,* equal to 11 parts pure gold and 1 of alloy; *Silver*, $\frac{222}{240}$ of a pound, equal to 37 parts pure silver and 3 of alloy.

A Troy ounce of standard gold is coined into £3 17*s.* 10*d.* 2*f.*, and an ounce of standard silver into 5*s.* 6*d.*

Copper is coined in the proportion of 2 shillings to the pound avoirdupois.

The *Remedy of the Mint* is,
Gold, 12 grains per lb. in weight; Silver, 1 dwt. per lb. in weight.
" $\frac{1}{16}$ of a carat in fineness; " 1 dwt. per lb. in fineness.
Copper, $\frac{1}{40}$ of the weight, both in weight and fineness.

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* A pound is assumed to be divided into 24 equal parts or carats, hence the proportion is equal to 22 carats.

Weight and Mint Values of Foreign Gold and Silver Coins.

By Laws of Congress, August, 1834, January, 1838, February 25, 1853, Regulations of the Mint, November 10, 1858, and Reports of Director of 1863–71.

Country.	Piece.	Weight. Ounces.	Fineness. Old Piece.	Fineness. New Piece.	Value. $ c.
Argentine Republic	Doubloon, "Provincias de la Plata," 1813–32				{ 14 66 15 5
	Peso 1813–32				{ 92 95 05
	" 1838, 39				1 06
Australia..........	Sovereign............. 1855 ..	.256.5		916	4 85 7
	Pound .... 1852 ..	.281		916.5	5 32 37
Austria and Lombardy...........	Ducat..........................	.112	986		2 28 28
	Souverain......................	.363	900		6 75 35
	Crown, new	.357		900	6 64 19
	Rix Dollar	.902	833		1 02 27
	Florin, before 1858............	.451	833		51 14
	" new....................	.397		900	48 63
	20 Kreutzers	.215	582		17 00
	Scudo..........................	.836	902		1 02 64
	Dollar, Union..................	.596		900	73 1
	Lira...........................		902		17 00
Belgium..........	25 Francs......................	.254		899	4 72 03
	5 "	.803		897	98 04
Brazil............	Moidore..... 4000 reis.........	.261	914		4 92
	6400 ".........	.461	915		8 72
	20 mil " 1854–56..	.575	917 5		10 90 57
	960 "				1 06
	640 "				70
	1200 " 1837.....		891		1 05
	2000 " 1851.....	.820		918.5	1 03 53
Bolivia...........	Doubloon, 1827–36..............	.867	870		15 59 25
	Dollar, down to 1848...........	.871	900.5		1 06 07
	" new....................	.891		900	98 1
	Half Dollar, down to 1828......				53 03
	Quarter " " 1828......				26 06
	Half Dollar, from 1830.........	.432	667		39 22
	Quarter " " 1830.........				19 05
Canada	20 Cents	.150		925	18 87
	25 "	.187 5		925	23 6
Chili.............	Doubloon, from 1819–40.........	.867		870	{ 15 59 26 15 66
	Dollar, from 1817-51...........	.864		908	1 06 79
	10 Pesos, since 1855...........	.492		900	9 15 35
	Peso, since 1854–56............	.801		900.5	98 17
	Half Peso......................				49 01
China.............	Dollar, Eng.	.866		901	1 06 2
Denmark..........	2 Rix Dollars..................	.927	877		1 10 65
	10 Thalers	.427	895		7 90 01
Central America...	2 E (two escudos)..............	.209		853.5	3 68 75
	average of dates..........	.205		850	3 60
	4 E............................	.434	851		7 62
	Doubloon, down to 1833	.869	833		14 96
	Half E (dollar)................				83 05
	8 R (dollar), 1840–42				{ 97 1 00
	8 R (dollar)...................	.836		850	1 00 19
Ecuador...........	4 E, 2 E, and 1 E, 1835–36......	.433	844		{ 7 55 46 3 77 73 1 88 86
	2 R (quarter dollar), 1833–47 ...			675	20

Table of Weight and Mint Values—*Continued.*

Country	Piece	Weight Ounces.	Fineness. Old Piece.	Fineness. New Piece.	Value. $ c.
England	Sovereign, 1816–1851	.256	915.5		4 84 08
	" 1851 and since	.256.7		916.5	4 86 34
	" average	.256.2		916	4 85 1
	Shilling, 1816 and since	.182.5		924.5	22 96
	" average	.178		925	22 41
East Indies and Japan	Mohur, 15 rupees	.374	916		7 08 18
	Rupee	.374	916		46 62
	Itzebu (rectangular)	.279	991		37 63
	" new	.279		890	33 80
	Cobang, old	.362	568		4 44
	" new	.289		572	3 57 6
France	40 Francs				7 72
	20 " Average value of 20-	.207.5		899	3 84 69
	10 " franc piece, $3 84 5.	.103.7		899.5	1 93
	5 "	.800	900	898 5	98
	1 "				19 06
	20 " new	.207.5		899.5	3 85 83
Germany	10 Thalers (Brunswick)	.427	895		7 90 01
	" (Hanover)	.427	895		7 90
	Crown	.357		900	6 64 2
	Ducat	.112	986		2 28 28
	Thaler; new	.595		900	72 89
	Florin or Guilder, old and new	.340	900	900	41 65
	Double Thaler, or 3.5 Guilder	1.192		900	1 46
	Kromer Thaler	.946	875		1 12 06
	36 Grote				37 05
Greece	20 Drachms	.185	900		3 44 29
	5 "	.719	900		88 08
Italy	20 Lire	.207	898		3 84 26
Mexico	Doubloon (8 escudos), new	.867.5		870.5	15 61 05
	" (Caliacan & Chihuahua.)	.867.5		866	15 52 98
	8 R (peso), new	.867.5		903	1 06 22
	" general average	.866		901	1 06 2
Morocco	Bontqui (40 reals)				1 99 52
Netherlands	10 Guilders	.215	899		3 99 7
	1 Guilder, before 1841	.346	896		42 02
	10 Cents (one tenth guilder)	.034.6	539		4 02
	2.5 Guilders, 1841 and after	.804		944	1 03 31
	1 Guilder, " "			944	41 04
Norway	2 Rigsdaler	.927	877		1 10 65
Portugal	Coroa (crown), 1838	.308	912		5 80 66
	1000 Reis	.950	912		1 18
	100 "	.035	912		11 08
Peru	Doubloons, 1826 to 1837	.867	868		15 55 67 15 62
	20 Pesos, 1855 and since	1.055		898	19 21 3
	1 Peso, old	.866		901	1 06 2
	Half Peso				49
	" 1835–1838	.433	650		38 31
	8 R, 1855	.766 .772	909		94 95 05
Prussia	Crown, new	.357		900	6 64 19
	Thaler, before 1857	.712	750		72 68
	" new	.595		900	72 89
New Granada and Venezuela	8 Escudos (Doub'n), 1823 (Bogota)	.868	870		15 61 1
	8 Escudos, 1823 (Popayan)	.867	858		15 37 75
	8 Escudos, 1737 to 1843	.867	868		15 56
	25.8064 G, 1849 and since	.826		894	15 31
	16.400 M (Popayan)	.525		891.5	9 67 51
	Dollar (8 reals), 1835–36				1 07 05
	" " 1839				68
	" " 1857	.803		896	97 92

Table of Weight and Mint Values—*Continued.*

Country.	Piece.	Weight Ounces.	Fineness. Old Piece.	Fineness. New Piece.	Value. $ c.
Naples	6 Ducati, new	.245		996	5 04 43
	Scudo (120 grains)	.844	830		95 34
	"	.887		833	1 00 05
Rome	2.5 Scudi, new	.140		900	2 60 47
	1 Scudo (100 bajochi)	.864		900	1 05 84
Russia	5 Roubles	.210		916	3 97 64
	1 Rouble (100 Copecks)	.667		875	7[illegible] 44
Sardinia	5 Lira	.800	900		98
	Lira (equal to 1 franc)				19 06
Spain	Half Doubloon, down to 1824	.433	865		7 75
	100 Reals	.268		896	4 96 39
	20 Reals (dollar)				1 01 05
	80 "	.215	869.5		3 86 44
Sweden	Ducat	.111	975		2 23 72
	Rix Dollar	1.092		750	1 11 48
Switzerland	2 Francs	.323		899	39 52
Tunis	25 Piastres	.161	900		2 99 54
	5 "	.511	898.5		62 49
Turkey	100 " since 1845	.231		915	4 36 93
	20 "	.770		830	86 98
	1 "	.038		830	4 39
Tuscany	Zecchino (sequin)	.112		999	2 31 29
	Florin (100 quattrini)	.220		925	27 7

Weights and British Value of Foreign Gold and Silver Coins not included in the foregoing Table.

Country.	Piece.	Weight Grains.	Fineness in Carats and Dwts.	Fineness U. S Standard.	Fine Metal. Grains.	Standard Weight.	Value in United States. $ c. m.
Baden	Ducat	47.5	*23.6875	.987	46.9	51.17	2 00 7
East Indies	Sicca Rupee	179.5	†2[illegible]5.	.953	175.8	1[illegible]0.	
	Company's Rupee.	180.	220.	.892	165.	178.4	
France	Napoleon 20 franc.	99.5	21.5625	.898	87.40	97.5[illegible]	3 85
Hanover	Ducat	53.75	23.8125	.992	53.33	58.18	2 29 7
	Florin	50.	18.875	.787	39.32	42.9	1 69 3
Hamburg	Rix Dollar	450.	212.	.860	397.5	429.7	
Holland	Ducat	53.75	23.5625	.982	52.77	57.57	2 27 3
	Florin	162.	217.5	.881	146.8	158.7	
Milan	Sequin	53.75	23.75	.989	53.19	58.03	2 29 9
	Lira	96.	132.	.5[illegible]5	5[illegible].8	57.1	
Naples	Oncetta, 1818	58.25	23.875	.995	57.95	63.21	2 49 5
	Ducat	351.	202.	.820	295.4	319.4	
Netherlands	Florin, 1816	166.	214.5	.869	148.4	160.4	
Prussia	Frederick, 1800	103.	21.5	.8[illegible]4	92.27	100.66	3 97 3
	Rix Dollar, Conv'n	433.	199.	.807	359.	388 1	
Russia	Ducat, 1796	54.	23.625	.984	53.16	57.99	2 28 9
	Imperial, 1801	185.25	23.5625	.982	181.87	198.41	7 83 2
	10 Copecks, 1802	32.5	209.	.848	28.3	30.6	
Spain	Doubloon, 1772	416.5	21.4375	.893	372.03	405.85	16 02 2
	Pistole, 1801	104.25	20.75	.864	90.13	98 33	3 88 1
	Dollar	416.	214.	.868	370.9	401.	
Sweden	Ducat	53.	23.5	.979	51.9	56.61	2 23 5
	Rix Dollar	449.	207.666	.873	388.5	420.	
Turkey	Zecchin	24.					1 40
Tuscany	Lira, 1803	56.	229.	.954	53.4	57.8	
Vienna	Ducato	342.	197.	.821	280.7	303.5	
United States	Eagle	258.	21.6	.900	232.2	258.	10 00
	Dollar	412.5	216.	.900	357.03	412.5	1 00

* 22 carats is the standard of gold, and 1 carat is divided into 4 grains.
† 222 dwts. is the standard of silver.

FOREIGN MEASURES AND WEIGHTS.

MEASURES OF LENGTH.

BRITISH. The Imperial standard yard is referred to a natural standard, which is the length of a pendulum vibrating seconds in vacuo in London, at the level of the sea; measured on a brass rod, at the temperature of 62°.

Admiralty knot=6080 feet.

FRENCH. *Old System*. (U. S. inches.)

1 line =12 points ...= 0.08881	1 toise=6 feet=76.735 inches.
1 inch =12 lines.....= 1.06577	1 league=2280.33 toises (com'n).
1 foot =12 inches ...=12.78916	1 league=2000 toises (post).

New System. (U. S. inches.) Prior to Law of 1866.

1 millimetre= .0393707 inches.	1 decametre = 32.80899 feet.
1 centimetre= .3937079 "	1 hectometre = 328.0899 "
1 decimetre =3.9370797 "	1 kilometre =1093.633 yards.
1 *metre** =39.370797 "	1 myriametre= 6.213825 miles.

NOTE.—In the new French system, the values of the base of each measure—viz., Metre, Litre, Stere, Are, and Gramme—are decreased or increased by the following words prefixed to them. Thus,

Milli expresses the 1000th part.	Deca expresses 10 times the value.
Centi " 100th "	Hecto " 100 "
Deci " 10th "	Kilo " 1000 "

Myrio expresses 10000 times the value.

Table of Lengths of Foreign Lineal Measure.

Place.	Measure.	U.S. Inch.	Place.	Measure.	U.S. Inch.
Abyssinia	Pic, geometrical	30.37	China	Chik or Covid	13.125
Aleppo and Asia	Pic	26.63	"	" Engineer's	12.71
Amsterdam	Foot	11.144	"	" commercial	14.1
Antwerp	Fuss	11.275	Damascus	Pic	22.93
Arabia	Guz	25.	Dantzic	Fuss	11.3
Austria	Fuss	12.445	Denmark	Fod	12.357
Baden	"	11.81	Dresden	Fuss	11.15
Bavaria	"	11.48	Egypt	Derah	25.49
Belgium[1]	Elle	39.371	Florence	Braccio	22.98
Bengal	Cubit or Guz	18.	Frankfort	Fuss, Surveyor's	14.01
Berlin	Fuss	12.357	Geneva	Pied	23.028
Birmah	Cubit	18.	Genoa	Piede Manuale	13.488
Bohemia[2]	Fuss	12.445	Gibraltar[5]	Foot	12.
Bombay	Hath	18.	Greece	Cubit	18.
Brazil[3]	Cubit	25.98	Guinea	Jacktan	144.
Bremen	Fuss	11.38	Hamburg	Fuss	11.279
Brunswick	Schuh or Fuss	11.23	Hanover	"	11.49
Calcutta	Cubit	18.	Ionian Isles[8]	Foot	12.
Canary Isles[4]	Foot	11.128	Japan	Ink or Tattamy	74.824
Candia	Pic or Ell	25.089	"	Fan	12.+
Ceylon	Covid	18.504	Java	Foot	12.357
Constantinople	Pic	26.89	Leipsic	Fuss	11.148

* According to Captain Kater's comparison, and the one adopted by the U. S. Ordnance Corps=39.3707971 inches, or 3.280899 feet.

Table of Lengths of Foreign Lineal Measures.

Continued.

Place.	Measure.	U. S. Inch.	Place.	Measure.	U. S. Inch.
Madras	Covid	18.6	Sardinia	Oncia	1.686
Malta	Pié	11.167	"	Liprando	20.23
Mauritius[9]	Foot	12.	Saxony	Fuss	11.148
Mexico[10]	Pié	11.128	Siam	Ken	39.
Milan[11]	Foot	15.62	Sicily	Palmo	9.53
Mocha	Guz	25.	Smyrna	Pic	26.48
Modena	Piede	20.592	Spain	Foot	11.128
Morocco	Cubit or Canna.	21.	"	Toesas	66.768
Moscow	Foot	13.18	"	Palmo Mayor	8.34
Naples	Palmo	10.381	"	Vara	33.384
Norway	Fod	12.353	Sweden	Fot	11.657
Parma	Pié	21.441	Switzerland	Fuss (Berne)	11.81
Persia	Arish	38.27	"	" (Geneva)	23.028
Poland	Foot	14.032	Tripoli	Pic or Dreah	21.75
Portugal	Palmo da junta.	7.882	Turin	Fuss	13.488
"	Foot	13.33	Turkey	Pic great	27.9
Prussia	Fuss	12.357	Tuscany	Foot	11.94
Rhineland	Foot	12.357	Utrecht	"	10.74
Riga	"	10.79	Venice	Pié	13.68
Rome	Pié, commercial	11.592	"	Braccio Grosso	26.9
" *	Palmo	9.8	"	Braccio	39.371
Russia	Verschok	1.75	Vienna	Fuss	12.45
"	Foot	13.75	Warsaw	Foot (Cracow)	14.03
"	Archine	28.	Zürich	"	11.812

Table of Lengths of Foreign Road Measures.

Place.	Measure.	U. S. Yards.	Place.	Measure.	U. S. Yards.
Arabia	Mile	2146.	Leghorn	Miglio	1809.
Austria	Meile (post)	8297.	Leipsic	Meile (post)	7432.
Baden	Stunden	4860.	Lithuania	"	9781.
Belgium[1]	Kilometre	1093.63	Malta	Canna	2.29
"	Meile	2132.	Mecklenburg	Meile	8238.
Bengal	Coss	2000.	Mexico[10]	Legua	4638.
Birmah	Dain	4277.	Milan[11]	Miglio	1093.63
Bohemia[2]	League (16 to 1°)	7587.	Mocha	Mile	2146.
Brazil[3]	" (18 to 1°)	6750.	Naples	Miglio	2025.
Bremen	Meile	6865.	Netherlands	Mijle	1093.63
Brunswick	"	11816.	Norway	Mile	12182.
Calcutta	Coss	2160.	Persia	Parasang	6076.
Ceylon[9]	Mile	1760.	Poland	Mile (long)	8100.
China	Li	608.5	Portugal	Mitha	2250.
Denmark	Miil	8238.	"	Vara	3.609
Dresden	Post-meile	7432.	Prussia	Mile (post)	8238.
Egypt	Feddan	1.47	Rome	Kilometre	1093.63
England	Mile	1760.	"	Mile	2025.
Flanders	Mijle	1093.63	Russia	Verst	1166.7
Florence	Miglio	1809.	"	Sashine	2.33
France†	Kilometre	1093.6	Sardinia	Miglio	2435.
Genoa	Mile (post)	8527.	Saxony	Meile (post)	7432.
Germany	Mile (15 to 1°)	8101.	Siam	Roënung	4333.
Greece	Stadium	1083.33	Spain	League, legal	4638.
Guinea	Jacktan	4.	"	" common	6026.24
Hamburg	Meile	8238.	,,	Milla	1522.
Hanover	"	8114.	Sweden	Mile	11660.
Hungary	"	9139.	Switzerland	Meile	8548.
India	Wussa	24.89	Turkey	Berri	1828.
Italy	Mile	2025.	Tuscany	Miglio	1809.
Japan	Ink	2.038	Venice	Miglio	1900.

* Carara, Palmo, 9.6 ins. † 1.60931 miles=1 kilometre.

MEASURES OF SURFACE.

FRENCH. *Old System.*

1 square inch =1.13587 U. S. inches.
1 toise =6.3946 U. S. feet.
1 arpent (Paris) =900 square toises=4089 square yards.
1 arpent (woodland)=100 square royal perches=6108.24 square yards.

New System.

1 *are*=1 square decametre=1076.4309 square feet.
=100 square metres=119.6033 square yards.

1 decare=10 ares. | 1 hectare=100 ares=2.4711 acres.

1 square metre=1550.0599 square inches, or 10.7643 sq. feet.

1 centiare=10 7643 square feet. | 1 deciare=11.9603 square yards.

Table of Lengths of Foreign Measures of Surface.

Place.	Measure.	Square Yards.	Place.	Measure.	Square Yards.
Amsterdam	Morgen	9722.	Ionian Isles	Misura	1445.
Austria	Joch	6884.	Modena	Biolca	3392.
Baden	Viertel	1076.4	Naples	Moggi	4165.
"	Morgen	4305.6	Portugal	Geira	7004.
Berlin	" (small)	3054.	Prussia	Morgen	3054.
Bremen	"	3070.	Rome	Pezza	3160.
Brunswick	"	2990.	Russia	Dessatina	13067.
Canary Isles[4]	Fanegada	2420.	Spain	Fanegada (max)	7682.
Ceylon	Acre	4840.	"	El Area	119.6
Denmark	Skieppe	329.75	Sweden	Tunnland	5872.
Egypt	Feddan al ris'h	2674.	Switzerland	Juchart (tillage)	425.9
England	Acre	4840.	Turin	Giornata	4546 7
Geneva	Arpent	6179.	Tuscany	Quadrato	407.2
Hamburg	Scheffel	5026.34	Vienna	Joch	6884.
Hanover	Morgen	3131.5	Zürich	Juchart	425.9

MEASURES OF VOLUME.

BRITISH. The *Imperial gallon* measures 277.274 cubic inches, containing 10 lbs. avoirdupois of distilled water, weighed in air, at the temperature of 62°, the barometer at 30 inches. 6.2355 gallons in a cubic foot.

Imperial bushel=2218.192 cubic inches.

* *Heaped bushel*=19.5 inches diameter, cone 6 inches high =2815.4872 cubic inches.

For Grain—8 bushels =1 quarter; 1 quarter =10.2694 cubic feet.

Coal, or heaped measure—3 bushels =1 sack; 12 sacks = 1 chaldron.

1 chaldron =58.656 cubic feet, and weighs 3136 pounds.

FRENCH. *Old System.*—1 Boisseau = 13.01 litres = 793.963 cubic inches, or 3.437 gallons.

1 pinte =0.931 litres, or 56.816 cubic inches.
1 cubic inch, 1.06577^3= 1.20157 U. S. inches.
1 cubic foot..............=2091.8667 U. S. inches.
13.08516 hectolitres...=1 chaldron.

* When heaped in the form of a true cone.

French. *New System.*—Decilitre = 6.1027 U. S. cubic inches.
Litre = 1 cubic decimetre, or 61.0271 cubic inches = 1.05675 U. S. quarts.

Decalitre................ = 610.271 cubic inches.
Kilolitre................ = 35.3166 cubic feet.
Decistere................ = 3.53166 cubic feet.
Stere (a cubic metre) = 35.3166 cubic feet = 61027 0963 cubic in.
Decastere......... = 353.166 "

Note.—For the *Square* and *Cubic Measures* of other countries, take the length of the measure in table, page 29–30, and square or cube it as required.

Table of Volume of Foreign Liquid Measures.

Place.	Measure.	Cub. Inch
Amsterdam	Anker	2331.
"	Wine Stekan	1183.6
Antwerp	Stoop	168.
Arabia	Gudda	554.5
Austria	Mass	86.3
Bavaria	Eimer	3914.3
Berlin	Anker	2285.7
Bombay	Parrah	6721.1
Brazil[3]	Medida	165.5
Bremen	Stübchen	195.9
Brunswick	"	227.
Canary Isles[4]	Arroba	949.
Candia	Mistate	681.
Ceylon	Parrah	1558.4
China	Tau	332.7
Cognac	Brandy velte	4454.6
Cologne	Viertel	363.1
Constantinople	Almud	319.4
Denmark	Anker	2299.
"	Pot	58.9
Dresden	Eimer	4627.6
Egypt	Ardeb	1358.
Frankfort	Viertel	437.53
Florence	Wine Barile	2781.8
"	Oil "	2225.6
Germany (Baden)	Stütse	915.1
Geneva	Setier	2760.
Genoa	Wine Barile	4528.6
"	Pinte	90.6
Greece	Kila	2030.1
Hamburg	Stübchen	220.9
"	Ohm	8836.
Hanover	"	9400.6
"	Stübchen	237.2
Havana	Arroba	947.
"	Wine Arroba	2781.
Holland[6]	Kan	61.027
Hungary	Eimer	3454.4
Ionian Isles[8]	Dicotoli	34.6
Java[7]	Kanne	111.
Leghorn	Oil Barile	2225.6
"	Wine "	2781.8
Leipsic	Eimer	4627.6
Lisbon	Almude	1009.5
Lucerne	Ohm	3162.8
Madras	Marcal	749.8
Madeira	Alquiere	504.69
"	Almude	1009.38
Malaga	Arroba	965.3
Malta	Caffiso	1270.
Marseilles	Millerolle	3922.4
Milan[11]	Pinte	61.03
Mocha	Gudda	554.5
Modena	Fiasco	127.
Nantes	Wine Barique	14638.9
Naples	" Barile	2544.
"	Oil Stajo	617.6
Norway	Kanna	1276.5
Oporto	Almude	1530.71
Persia	Artaba	4013.
Poland	Garnice	97.05
Prussia	Anker	2096.
"	Eimer	4192.
"	Ohm	8384.
Riga	Anker	2387.33
Rome	Wine Barile	2560.
"	Oil "	3506.8
"	Boceale	111.2
Rotterdam	Ohm	9236.3
Russia	Vedro	750.1
Sardinia	Barile	4528.6
Saxony	Eimer	4627.6
Siam	Sesti	739.4
Sicily	Oil Caffiso	662.
"	Salma (Mes'a)	5232.4
Smyrna	Almud	319.4
Spain	Arroba	980.7
"	Oil Arroba	770.6
"	Quartillos	30.65
Sweden	Kanna	158.
Syria	Almud	319.4
Switzerland	Eimer (Berne)	2547.4
Tripoli	Barile	3956.
Trieste	Eimer	3452.6
Turkey	Almud	319.4
Tunis	Oil Barile	1157.
Tuscany	" "	2225.6
"	Fiasco	139.1
Venice[11]	Pinta	61.3
Vienna	Eimer	3454.
"	Mass	86.3
Zürich	"	99.82

Note.—In Bengal and Calcutta the measures are by weight.

Table of Foreign Dry Measures.

Place	Measure.	Cubic Inch.	Place.	Measure.	Cubic In.
Abyssinia	Ardeb	277.	Lisbon	Fanega	3300.
Africa	"	277.	Leipsic	Scheffel	6340.3
Alexandria	Rebele	9592.2	Madeira	Alquiere	684.
Austria	Metze	3752.7	Malaga	Fanega	3438.8
Algiers	Zarni	1220.5	Modena	Sacco	8597.7
Amsterdam	Mudde	6786.	Malta	Salma	17676.8
Asia	Sesti	739.39	Milan[11]	Soma	6103.
Azores	Alquiere	731.	Majorca	Quarten	4296.8
"	Sack	4947.	Madras	Marcal	749.9
Barbary	Temer	1637.7	Norway	Spann	4469.6
Bavaria	Scheffel	13569.	Naples	Tomolo	3122.
Brazil[3]	Alquiere	2240.	Netherlands	Mudde	6103.
Brunswick	Himt	1897.9	Oporto	Alquiere	1041.7
Belgium[1]	Litron	61.027	Persia	Artaba	4013.
Berlin	Scheffel	3180.	Poland	Zorrec	3120.
Bombay	Parah	6721.12	Prussia	Scheffel	3354.
Bremen	Scheffel	4520.	Parma	Stajo	3134.7
Cadiz	Fanega	3438.8	Rome	Rubbio	17968.3
Canada[12]	Minot	2381.5	"	Quarta	4492.1
Candia	Carga	9288.	Riga	Loop	3978.
China	Tau	443.	Rotterdam	Saik	6[illegible]61.
Constantinople	Killow	20.3.	Russia	Tschetwerik	1600.
Corsica	Stajo	6014.	Sardinia	Mina	7366.6
Dresden	Scheffel	6340.3	Spain	Cahiz	41266.
Dantzic	"	3354.	"	Fanega	3438.8
Denmark	Tonne	8487.6	Sicily	Salma, gros	21010.
Egypt	Ardeb	10869.2	"	" general.	16[illegible]4.8
Florence	Stajo	1487.1	Smyrna	Killow	2023.
Frankfort	Malta	6[illegible]02.4	Sweden	Tunna	8[illegible]40.
Geneva	Coupe	4739.	Siam	Sesti	739.4
Genoa	Miria	7366.6	Saxony	Scheffel	6340.3
Greece	Kila	2030.	Scotland	Firlot	21[illegible]7.
Germany	Malter (Baden)	9154.	Switzerland	Maas (Berne)	854.9
Holland[6]	Kop	61.027	Tripoli	Temen	1637.7
Hanover	Himt	1897.9	Tuscany	Stajo	1487.1
Hamburg	Scheffel	6429.5	Turkey	Killoio	2023.
Ionian Isles[6]	Chilo	2218.2	Zürich	Mutt	47[illegible]8.
Leghorn	Stajo	1487.1	Venice[11]	Soma	6103.
Lisbon	Alquiere	825.2	Vienna	Metzen	3753.

NOTES.—In Arabia the Tomand measures 168 lbs. avoirdupois of rice.
In Bengal and Calcutta the measures are by weight.

MEASURES OF WEIGHT.

BRITISH. 1 troy grain = .003961 cubic inches of distilled water.
1 troy pound = 22.815689 cubic inches of water.
1 avoir. drachm = 27.34375 troy grains.

1 clove = 7 pounds.	1 truss straw = 36 pounds.
1 sack wool = 364 "	1 sack flour = 28.2 "

1 quarter flour = 4 pounds 5 oz. 8.25 dr.

FRENCH. *Old System.*

1 grain ... = 0.8188 grains troy.	1 once = 1.0780 oz. avoirdupois.
1 gross ... = 58.9548 "	1 livre = 1.0780 lbs. "

New System.

Milligramme = .01543 troy gr's.	*Gramme* ...... = 15.43316 tr. gr.
Centigramme = .15433 "	Decagramme = 154.33159 "
Decigramme = 1.54331 "	Hectogramme = 1543.3159 "

1 kilogramme .... = 2.204737 lbs. avoirdupois.
1 myriagramme .... = 22.04737 "
1 millier .... = 1000 kilogrammes = 1 ton sea weight.

453.5688 grammes = .4535688 kilogramme = 1 pound avoirdupois.
372.2223 " = .3732223 " = 1 pound troy

Table of Value of Foreign Weights.

Place.	Weight.	Pounds Avoirdupois.	Place.	Weight.	Pounds Avoirdupois.
Abyssinia	Liter	.6857	Germany	Unze	.0657
Africa	Rottoli	.6857	Greece	Pound	.8811
"	Wakea	.0571	Guinea	Benda	.1417
Aleppo	Batman	16.974	Hamburg	Pfund	1.0685
"	Oke	2.829	Hanover	"	1.0731
Amsterdam	Pound (old)	1.009	Holland[6]	Ponden	2.2057
"	" (Flem.)	2.2	Japan[7]	Catty	1.3
Alexandria	Rottoli	.93	Java	Catty	1.3333
Algiers	"	1.19	Leghorn	Libbra	.7486
Arabia	Maund	3.	Leipsic	Pfund (comm'n)	1.0309
Asia	Catty	2.583	Madeira	"	1.0119
" (Ottoman)	Oke	2.843	Madras	Vis	3.125
Austria	Pfund	1.235	Malta	Rottoli	1.333
"	Mark	.6195	Milan[11]	Libbra	2.2046
Barbary	Rottoli	1.09	Mocha	Maund	3.
Batavia	Catty	1.3333	Modena	Libbra	.7046
Bavaria	Pfund	1.2346	Morea	Pound	1.1014
Belgium[1]	Livre	2.2047	Morocco	Pound (comm'l)	1.19
Bengal	Seer (Factory)	1.8667	"	" (market)	1.785
"	"	2.0533	Munich	Pfund	1.2366
"	Maund (Fact'ry)	74.667	Naples	Rottoli	1.9643
"	"	82.123	"	Rotolo (piccolo)	1.0607
Berlin	Pfund	.311	Norway	Skalpund	.9376
Birmah	Vis	3.3333	"	Mark	.465
Bombay	Seer	.7	Parma	Libbra	.7197
Brazil[3]	Mark	.5533	Persia	Rattel (shirez)	2.1136
Bremen	Pfund	1.0986	"	Dirhem	.0214
Bologna	Pound	1.2531	Portugal	Pound	1.0119
Brunswick	Pfund	1.03	Prussia	Pfund	1.0311
Cairo	Rottoli	1.008	Rome	Libbra	.7477
Calcutta	Seer (Factory)	1.8667	Rotterdam		1.0895
"	"	2.0533	Russia	Funt	.9026
"	Maund (Fact'ry)	82.123	Sardinia	Rottolo	1.0483
"	" (")	74.667	Saxony	Pfund	1.0309
Canary Isles[4]	Libra	1.0148	Shiraz	Batman	12.6816
Candia	Rottoli	1.1650	Siam	Catty	2.583
Ceylon	Candy	500.	Sicily	Libbra	.7
China	Tael or ounce	.0833	"	Rottolo (grosso)	1.925
"	Catty	1.3333	Smyrna	Oke	2.829
Cologne	Mark	.5156	"	Cantaro	127.3
Constantinople	Oke	2.8	Spain[10]	Libra	1.0164
Corsica	Kilogramme	2.2047	Sumatra	Catty	1.333
Cyprus	Rottoli	5.2439	Sweden	Skalpund	.9376
Damascus	Oke	2.8290	Switzerland	Pfund	1.1514
Denmark	Pund	1.1029	Tripoli	Rottol	1.097
Dresden	Pfund	1.0309	Tunis	"	1.11
East Indies	Sicca or Tola	.0257	Turin	Libbra	.813
Egypt	Rotl	1.008	Turkey	Rottolo	1.2729
Florence	Libbra	.7486	"	Almud (oil)	22.6[illegible]5
Frankfort	Pfund	1.0314	"	Oke	2.8286
Geneva	Pfund	1.2143	Tuscany	Libbra	.7486
Genoa	Rottolo	1.0483	Venice[11]	Pound	2.204[illegible]
Germany (Bad.)	Mark	.5155	Vienna	Pfund	1.235
" (")	Pfund	1.1029	Warsaw	"	.890[illegible]

Notes to the preceding Tables.

1. The measures and weights of Belgium are the same as those of France and Holland.
2. The measures and weights of Bohemia are the same as those of Austria.
3. The measures and weights of Brazil, with some additions, are the same as those of Portugal.
4. The measures and weights of the Canary Isles are the same as those of Spain, with some variations.
5. The measures and weights of Gibraltar are the same as those of England.
6. The measures and weights of Holland are the same as those of France and Belgium.
7. The weights of Japan are nearly the same as those of China.
8. Since 1817 the measures and weights of the Ionian Isles are the same as those of England, with Italian designations.
9. The measures and weights of Mauritius are the same as those of England and France.
10. The measures and weights of Mexico are the same as those of Spain, with some additions difficult to obtain.
11. The measures and weights of Milan and Venice are the same, and are those of France.
12. The measures and weights of Canada, and all the British Possessions in North America, are the same as those of Great Britain, but the U. S. gallon and bushel are most in use.

SCRIPTURE AND ANCIENT MEASURES.

Scripture Long Measures.

	Inches.		Feet.	Inches.
Digit .... =	0.912	Cubit ....=	1	9.888
Palm .... =	3.648	Fathom ....=	7	3.552
Span .... =	10.944			

Egyptian Long Measures.

	Feet.	Inches.		Feet.	Inches.
Nahud cubit	1	5.71	Royal cubit	1	8.66

Grecian Long Measures.

	Feet.	Inches.		Feet.	Inches.
Digit ....=		0.7554	Stadium ...... =	604	4.5
Pous (foot)......=	1	0.0875	Mile .... =	4835	
Cubit ....=	1	1.5984⅜			

Attic or Olympic foot ....= 12.108 inches.
Pythic or natural foot ....= 9.768 "
Ancient Greek (16 Egyptian fingers)......11.81 "
Keramion or Metretes .... 8.488 gallons.

Jewish Long Measures.

	Feet.		Feet.
Cubit ....=	1.824	Mile .....(4000 cubits) .... =	7296
Sabbath day's journey =	3648.	Day's journey........	33.164 miles.

Roman Long Measures.

	Inches.		Feet.	Inches.
Digit .... =	.72575	Cubit ....=	1	5.406
Uncia (inch) .... =	.967	Passus ....=	4	10.02
Pes (foot)........ =	11.604	Mile (millarium) =	4842	

Roman Weight.

	Pounds.
Ancient libra	.7094

1

Ancient Weights.

	Troy grains.		Troy grains
Attic obolus................	8.2* / 9.1†	Egyptian mina......	8.326*
		Ptolemaic "	8.985*
" drachma	51.9* / 54.6† / 69. ‡	Alexandrian "	9.992*
		Denarius (Roman)....	51.9* / 62.5†
Lesser mina................	3.892	" (Nero)	54. ‡
Greater mina......	$\frac{1}{10}$ of drachma.	Ounce....................	415.1* / 437.2† / 431.2‡
Talent = 60 minæ = 56 lbs. avoirdupois.		Pound..........	12 Roman ounces.
Drachm	= 146.5 (Troy grains.)		

Miscellaneous.

	Feet.		Feet.
Arabian foot............	= 1.095	Hebrew foot................	= 1.212
Babylonian foot.........	= 1.140	" cubit...............	= 1.817
Egyptian finger.........	= .06145	" sacred cubit......	= 2.002

GEOGRAPHICAL MEASURES AND DISTANCES.

To Reduce Longitude into Time.

RULE.—Multiply the degrees, minutes, and seconds by 4, and the product is the time.

EXAMPLE.—Required the time corresponding to 50° 31′.

$$\begin{array}{r} 50^\circ\ 31' \\ 4 \\ \hline 3h.\ \ 22'\ \ 4''. \end{array}$$

To Reduce Time into Longitude.

RULE.—Reduce the hours to minutes and seconds, divide by 4, and the quotient is the longitude.

Or, multiply them by 15.

EXAMPLE.—Required the longitude corresponding to 5*h.* 8′ 11.2″.

h. *m.* *s.* *m.* *s.*
5 8 11.2. = 308 11.2‴, which ÷ 4 = 77° 2′ 45.5″.

Or, multiplying by 15 :

5*h.* 8*m.* 11.2*s.* × 15 = 77° 2′ 45.5″.

Table of Departures for a Distance run of 1 Mile.

Course.	Departure.	Course.	Departure.	Course.	Departure.
3.5 points.	.773	4.5 points.	.634	5.5 points.	.471
4. "	.707	5. "	.556	6. "	.383

Thus, if a vessel holds a course of 4 points, that is, without leeway, for the distance of 1 mile, she will make .707 of a mile to windward.

Or, a vessel sailing E.N.E. upon a course of 6 points for 100 miles will make 38.3 miles (100 × .383) longitude.

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* Christiani. † Arbuthnot. ‡ Paucton.

Table showing the Degrees, Minutes, and Seconds of each Point of the Mariner's Compass with the Meridian.

NORTH.	SOUTH.	Points.	° ′ ″	Sin. A.*	Cos. A.*	Tan A.*
		.25	2 48 45	.0489	.9988	.0491
N.	S.	.5	5 37 30	.098	.9952	.0985
		.75	8 26 15	.1467	.9891	.1484
		1.	11 15	.195	.9808	.1989
N. by E.	S.E. by E. ..	1.25	14 3 45	.2429	.97	.2504
N. by W. ...	S. by W.	1.5	16 52 30	.2903	.9569	.3034
		1.75	19 41 15	.3368	.9415	.3578
		2.	22 30	.3827	.9239	.4142
N.N.E.	S.S.E.	2.25	25 18 45	.4275	.90	.4729
N.N.W.	S.S.W.	2.5	28 7 30	.4714	.8819	.5345
		2.75	30 56 15	.5141	.8577	.5994
		3.	33 45	.5556	.8315	.6682
N.E. by N...	S.E. by S. ..	3.25	36 33 45	.5957	.8032	.7416
N.W. by N. .	S.W. by S. ..	3.5	39 22 30	.6344	.773	.8207
		3.75	42 11 15	.6715	.7409	.9063
		4.	45	.7071	.7071	1.
N.E.	S.E.	4.25	47 48 45	.7404	.6715	1.103
N.W.	S.W.	4.5	50 37 30	.773	.6344	1.218
		4.75	53 26 15	.8032	.5957	1.348
		5.	56 15	.8315	.5556	1.497
N.E. by E...	S.E. by E. ..	5.25	59 3 45	.8577	.5141	1.668
N.W. by W..	S.W. by W..	5.5	61 52 30	.8819	.4714	1.871
		5.75	64 41 15	.904	.4275	2.114
		6.	67 30	.9239	.3827	2.414
E.N.E.	E.S.E.	6.25	70 18 45	.9415	.3368	2.795
W.N.W. ...	W.S.W.	6.5	73 7 30	.9569	.2903	3.296
		6.75	75 56 15	.97	.2429	3.941
		7.	78 45	.9808	.195	5.027
E. by N.	E. by S.	7.25	81 33 45	.9891	.1467	6.741
W. by N. ...	W. by S.	7.5	84 22 30	.9952	.098	10.153
		7.75	87 11 15	.9988	.0489	20.555
East or West	East or West	8.	90	1.	.0000	∞

* A, representing course or points from the meridian.

Table of the Visible Distance of Objects in Statute Miles.

Height in Feet.	Distance in Miles.	Height in Feet.	Distance in Miles.	Height in Feet.	Distance in Miles.	Height in Feet.	Distance in Miles.
*.582	1.	11	4.36	30	7.18	150	16.05
1	1.31	12	4.54	35	7.76	200	18.54
2	1.85	13	4.71	40	8.3	300	22.7
3	2.27	14	4.9	45	8.8	400	26.2
4	2.62	15	5.07	50	9.37	500	29.3
5	2.93	16	5.24	55	9.72	1000	41.45
6	3.21	17	5.4	60	10.14	2000	58.61
7	3.47	18	5.56	70	10.97	3000	71.79
8	3.7	19	5.72	80	11.72	4000	82.9
9	3.93	20	5.86	90	12.43	5000	92.68
10	4.15	25	6.55	100	13.1	1 mile.	95.23

* For a Statute mile the curvature = 6.99 inches.

The difference in two levels is as the square of their distance.

ILLUSTRATION.—If the height is required for 2 miles,

$1^2 : 2^2 :: 6.99 : 27.96$ *inches;*

and if for 100 miles,

$1^2 : 100^2 :: 6.99 :: 1.103+$ *miles.*

The difference in two distances is as the square root of their heights.

ILLUSTRATION.—If the distance is required for 3 feet,

$\sqrt{.582}=.763 : \sqrt{1} :: \sqrt{3}=1.732 : 2.27$ *miles;*

and if for 8 feet,

$\sqrt{.582}=.763 : \sqrt{1} :: \sqrt{8}=2.828 : 3.70$ *miles.*

Table of the Visible Distance of Objects in Geographical or Nautical Miles.

Height in Feet.	Distance in Miles	Height in Feet.	Distance in Miles.	Height in Feet.	Distance in Miles	Height in Feet.	Distance in Miles.
* .663	1.	11	4.08	30	6.74	150	15.07
1.	1.23	12	4.26	35	7.28	200	17.4
2.	1.74	13	4.43	40	7.78	300	21.32
3.	2.13	14	4.6	45	8.25	400	24.64
4.	2.46	15	4.77	50	8.7	500	27.52
5.	2.75	16	4.92	55	9.13	1000	38.92
6.	3.01	17	5.07	60	9.53	2000	55.04
7.	3.25	18	5.22	70	10.29	3000	67.41
8.	3.48	19	5.36	80	11.01	4000	77.84
9.	3.69	20	5.5	90	11.68	5000	87.03
10.	3.89	25	6.15	100	12.31	1 mile.	89.43

* For a Geographical or Nautical mile, the curvature = 7.962 inches.

ILLUSTRATION.—If a man at the foretop-gallant mast-head of a ship, 100 feet from the water, sees another and a large ship "hull to," how far are the ships apart?

A large ship's bulwarks are at least 20 feet from the water.

Then, by table, 100 feet................ =12.31
20 " .... = 5.50
Distance...... 17.81 *miles.*

NOTE.—The .0760 part should be added for horizontal refraction.

When an observation for distance is taken from an elevation, as from a light-house or a vessel's mast, of an object that intervenes between the observer and the horizon, or contrariwise, the observer being at a horizon to the elevated object; the distance of the observer from the intervening object can be determined by ascertaining or estimating its distance from the horizon or elevation, as the case may be, and subtracting it from the whole distance between the observer and the point from which the observation is taken, and the remainder will give the distance of the object from the observer.

In this case, however, the distance of the intervening object can not be computed unless the height of it is known or may be estimated.

ILLUSTRATION.—The top of the smoke-pipe of a steamer, assumed to be 50 feet above the surface of the water, is in range with the horizon from an elevation of 100 feet; what is the distance to the steamer?

100 feet................ =12.31
50 " .... = 8.70
3.61 *miles.*

Lengths of a Degree of Longitude on the parallels of Latitude, for each Degree of Latitude from the Equator to the Pole.

Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.
1°	59.99	19°	56.73	37°	47.92	55°	34.41	73°	17.54
2	59.96	20	56.38	38	47.28	56	33.45	74	16.54
3	59.92	21	56.01	39	46.63	57	32.68	75	15.53
4	59.85	22	55.63	40	45.96	58	31.79	76	14.52
5	59.77	23	55.23	41	45.28	59	30.9	77	13.5
6	59.67	24	54.81	42	44.59	60	30.	78	12.48
7	59.55	25	54.38	43	43.88	61	29.09	79	11.45
8	59.42	26	53.93	44	43.16	62	28.17	80	10.42
9	59.26	27	53.46	45	42.43	63	27.74	81	9.38
10	59.09	28	52.97	46	41.68	64	26.3	82	8.35
11	58.89	29	52 48	47	40.92	65	25.36	83	7.31
12	58.69	30	51.96	48	40.15	66	24.4	84	6.27
13	58.46	31	51.43	49	39.36	67	23.41	85	5.23
14	58.22	32	50.88	50	38.57	68	22.48	86	4.18
15	57.95	33	50.32	51	37.76	69	21.5	87	3.14
16	57.67	34	49.74	52	36.94	70	20.52	88	2.
17	57.38	35	49.15	53	36.11	71	19.53	89	1.05
18	57.06	36	48.54	54	35.27	72	18.54	90	.00

NOTE.—Degrees of longitude are to each other in length as the Cosines of their latitudes.

SOUNDING.

To Reduce a Sounding to Low Water.

$$\frac{h}{2}\left(1 \mp \cos. \frac{180t'}{t}\right) = h';$$

h representing vertical rise of tide, and h′ sounding or depth at low water, in feet; t time between high and low water, and t′ time from time of sounding to low water, in hours.

$$-\cos. \text{ when } \frac{180t'}{t} < 90^\circ, \text{ and } + \cos. \text{ when } > 90^\circ.$$

EXAMPLE.—Low water occurring at 3.45, and high water at 10.15 P.M., a sounding taken at 5.30 P.M. was 18.25 feet; what was the depth at low water, the vertical rise being 10 feet?

$h = 10$ *feet*; $t' = 5h.\ 30m. - 3h.\ 45m. = 1h.\ 45m. = 1.75$ *hours*.
$t = 10h.\ 15m. - 3h.\ 45m. = 6h.\ 30m. = 6.5$ *hours*.

Then $\frac{10}{2}\left(1 \mp \cos. \frac{180 \times 1.75}{6.5}\right) = 5(1 - 48^\circ\ 27'\ 24'') = 5 \times (1 - .663186) = 1.68407$ *feet*.

Sounding............	18.25 *feet*.
Reduction...........	1.68407 "
	16.56593 *feet*.

VULGAR FRACTIONS.

A Fraction, or broken number, is one or more parts of a Unit.

Illustration.—12 inches are 1 foot.

Here, 1 *foot* is the unit, and 12 *inches* its parts; 3 *inches*, therefore, are the *one fourth* of a foot, for 3 is the quarter or fourth of 12.

A *Vulgar Fraction* is a fraction expressed by *two numbers* placed one above the other, with a line between them; as, 50 cents is the $\frac{1}{2}$ of a dollar.

The upper number is termed the *Numerator*, because it shows the number of parts used.

The lower number is termed the *Denominator*, because it denominates, or gives name to the fraction.

The *Terms* of a fraction express both numerator and denominator; as, 6 and 9 are the terms of $\frac{6}{9}$.

A *Proper* fraction has the numerator equal to, or less than the denominator; as, $\frac{1}{4}$, etc.

An *Improper* fraction is the reverse of a proper one; as, $\frac{2}{1}$, etc.

A *Mixed* fraction is a compound of a whole number and a fraction; as, $5\frac{7}{8}$, etc.

A *Compound* fraction is the fraction of a fraction; as, $\frac{1}{2}$ of $\frac{3}{4}$, etc.

A *Complex* fraction is one that has a fraction for its numerator or denominator, or both; as, $\frac{\frac{1}{2}}{6}$, or $\frac{\frac{5}{3}}{4}$, or $\frac{\frac{1}{2}}{\frac{3}{4}}$, or $\frac{3\frac{1}{2}}{6}$, etc.

Note.—A Fraction denotes division, and its value is equal to the quotient obtained by dividing the numerator by the denominator; thus, $\frac{12}{4}$ is equal to 3, and $\frac{21}{5}$ is equal to $4\frac{1}{5}$.

REDUCTION OF VULGAR FRACTIONS.

To Ascertain the greatest Number that will divide Two or more Numbers without a Remainder.

Rule.—Divide the greater number by the less; then divide the divisor by the remainder; and so on, dividing always the last divisor by the last remainder, until there is no remainder, and the last divisor is the greatest common measure required.

Example.—What is the greatest common measure of 1908 and 936?

```
936) 1908 (2
     1872
       36) 936 (26
           72
           216.  Hence 36 = greatest common measure.
```

To Ascertain the least Common Multiple of Two or more Numbers.

Rule.—Divide the given numbers by any number that will divide the greatest number of them without a remainder, and set the quotients with the undivided numbers in a line beneath.

Divide the second line as before, and so on, until there are no two numbers that can be divided; then the continued product of the divisors and last quotients will give the multiple required.

Example.—What is the least common multiple of 40, 50, and 25?

```
5) 40 . 50 . 25
5)  8 . 10 .  5
2)  8 .  2 .  1
    4 .  1 .  1.  Then 5×5×2×4×1×1=200.
```

To Reduce Fractions to their lowest Terms.

RULE.—Divide the terms by any number or series of numbers that will divide them without a remainder, or by their greatest common measure.

EXAMPLE.—Reduce $\frac{720}{960}$ of a foot to its lowest terms.

$$\frac{720}{960} \div 10 = \frac{72}{96} \div 8 = \frac{9}{12} \div 3 = \frac{3}{4}, \text{ or 9 } \textit{inches.}$$

To Reduce a Mixed Fraction to its Equivalent, an Improper Fraction.

NOTE.—Mixed and Improper fractions are the same; thus, $5\frac{1}{2} = \frac{11}{2}$.

RULE.—Multiply the whole number by the denominator of the fraction and to the product add the numerator; then set that sum above the denominator.

EXAMPLE.—Reduce $23\frac{2}{6}$ to a fraction.

$$\frac{23 \times 6 + 2}{6} = \frac{140}{6}$$

EX. 2.—Reduce $\frac{123}{6}$ inches to its value in feet.

$$123 \div 6 = 20\frac{3}{6}; \text{ that is, 1 } \textit{foot } 8\tfrac{1}{2} \textit{ inches.}$$

To Reduce a Whole Number to an Equivalent Fraction having a given Denominator.

RULE.—Multiply the whole number by the given denominator, and set the product over the said denominator.

EXAMPLE.—Reduce 8 to a fraction the denominator of which shall be 9.

$$8 \times 9 = 72; \text{ then } \frac{72}{9} \text{ the } \textit{result.}$$

To Reduce a Compound Fraction to an Equivalent Simple one.

RULE.—Multiply all the numerators together for a numerator, and all the denominators together for a denominator.

NOTE.—When there are terms that are common, they may be omitted.

EXAMPLE.—Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$ to a simple fraction.

$$\frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}. \quad \text{Or, } \frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{4}, \textit{ by canceling the } 2\text{'s } \textit{and } 3\text{'s.}$$

EX. 2.—Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of a pound to a simple fraction.

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}.$$

To Reduce Fractions of different Denominations to Equivalent ones having a Common Denominator.

RULE.—Multiply each numerator by all the denominators except its own for the new numerators; and multiply all the denominators together for a common denominator.

NOTE.—In this, as in all other operations, whole numbers, mixed, or compound fractions, must first be reduced to the form of simple fractions.

2. When many of the denominators are the same, or are multiples of each other, ascertain the least common multiple of the denominators, and then multiply the terms of each fraction by the quotient of the least common multiple divided by its denominator.

EXAMPLE.—Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

$$\left.\begin{array}{l} 1 \times 3 \times 4 = 12 \\ 2 \times 2 \times 4 = 16 \\ 3 \times 2 \times 3 = 18 \end{array}\right\} = \frac{12}{24} = \frac{16}{24} = \frac{18}{24}.$$
$$\overline{2 \times 3 \times 4 = 24}$$

The operation may be performed mentally thus:

Reduce $\frac{1}{8}$, $\frac{3}{2}$, $\frac{6}{8}$, and $\frac{5}{2}$ to a common denominator.

$$\frac{3}{2} = \frac{12}{8}. \qquad \frac{1}{8} = \frac{1}{8}. \qquad \frac{6}{8} = \frac{6}{8}. \qquad \frac{5}{2} = \frac{20}{8}.$$

To Reduce Complex Fractions to Simple ones.

RULE.—Reduce the two parts both to simple fractions; then multiply the numerator of each by the denominator of the other.

EXAMPLE.—Simplify the complex fraction $\frac{2\frac{2}{3}}{4\frac{4}{5}}$.

$$2\tfrac{2}{3} = \tfrac{8}{3} \qquad 8 \times 5 = 40$$
$$4\tfrac{4}{5} = \tfrac{24}{5} \qquad 3 \times 24 = 72 \qquad \tfrac{40}{72} = \tfrac{5}{9}.$$

ADDITION OF VULGAR FRACTIONS.

RULE.—If the fractions have a common denominator, add all the numerators together, and place the sum over the denominators.

NOTE.—If the fractions have not a common denominator, they must be reduced to one. Also, compound and complex must be reduced to simple fractions.

EXAMPLE.—Add $\frac{1}{4}$ and $\frac{3}{4}$ together.

$$\tfrac{1}{4} + \tfrac{3}{4} = \tfrac{4}{4} = 1.$$

Ex. 2.—Add $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{6}{10}$ to $2\frac{1}{8}$ of $\frac{3}{4}$.

$$\tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{6}{10} = \tfrac{18}{80}.$$
$$2\tfrac{1}{8} \text{ of } \tfrac{3}{4} = \tfrac{17}{8} \times \tfrac{3}{4} = \tfrac{51}{32}. \quad \text{Then, } \tfrac{18}{80} + \tfrac{51}{32} = 1\tfrac{131}{160}.$$

SUBTRACTION OF VULGAR FRACTIONS.

RULE.—Prepare the fractions the same as for other operations, when necessary; then subtract the one numerator from the other, and set the remainder over the common denominator.

EXAMPLE.—What is the difference between $\frac{5}{6}$ and $\frac{1}{6}$?

$$\tfrac{5}{6} - \tfrac{1}{6} = \tfrac{4}{6}.$$

Ex. 2.—Subtract $\frac{3}{9}$ from $\frac{6}{8}$.

$$\left.\begin{array}{l} 6 \times 9 = 54 \\ 3 \times 8 = 24 \\ 8 \times 9 = 72 \end{array}\right\} = \tfrac{54}{72} - \tfrac{24}{72} = \tfrac{30}{72}.$$

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.—Prepare the fractions as previously required; multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

EXAMPLE.—What is the product of $\frac{3}{4}$ and $\frac{3}{9}$?

$$\tfrac{3}{4} \times \tfrac{3}{9} = \tfrac{9}{36} = \tfrac{1}{4}.$$

Ex. 2.—What is the product of 6 and $\frac{2}{3}$ of 5?

$$\tfrac{6}{\cdot} \times \tfrac{2}{3} \text{ of } 5 = \tfrac{6}{\cdot} \times \tfrac{10}{3} = \tfrac{60}{3} = 20.$$

DIVISION OF VULGAR FRACTIONS.

RULE.—Prepare the fractions as before; then divide the numerator by the numerator, and the denominator by the denominator, if they will exactly divide; but if not, invert the terms of the divisor, and multiply the dividend by it, as in multiplication.

EXAMPLE.—Divide $\frac{25}{9}$ by $\frac{5}{3}$.

$$\tfrac{25}{9} \div \tfrac{5}{3} = \tfrac{5}{3} = 1\tfrac{2}{3}.$$

APPLICATION OF REDUCTION OF VULGAR FRACTIONS.

To Ascertain the Value of a Fraction in Parts of a whole Number.

Rule.—Multiply the whole number by the numerator, and divide by the denominator; then, if any thing remains, multiply it by the parts in the next inferior denomination, and divide by the denominator, as before, and so on as far as necessary; so shall the quotients placed in order be the value of the fraction required.

Example.—What is the value of $\frac{1}{2}$ of $\frac{2}{3}$ of 9?

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{2}{6}, \text{ and } \frac{2}{6} \times \frac{9}{1} = \frac{18}{6} = 3.$$

Ex. 2.—Reduce $\frac{3}{4}$ of a pound to an avoirdupois ounce.

3
1
4) 3 (0 *lbs.*
16 *ounces in a lb.*
4) 48 (12 *ounces.*

Ex. 2.—Reduce $\frac{3}{10}$ of a day to hours.

$$\frac{3}{10} \times \frac{24}{1} = \frac{72}{10} = 7\frac{2}{10} \text{ hours.}$$

To Reduce a Fraction from one Denomination to another.

Rule.—Multiply the number of the required denomination contained in the given denomination by the numerator *if the reduction is to be to a less name*, but by the denominator *if to a greater*.

Example.—Reduce $\frac{1}{4}$ of a dollar to the fraction of a cent.

$$\frac{1}{4} \times 100 = \frac{100}{4} = \frac{25}{1}.$$

Ex. 2.—Reduce $\frac{1}{6}$ of an avoirdupois pound to the fraction of an ounce.

$$\frac{1}{6} \times 16 = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}.$$

Ex. 3.—Reduce $\frac{2}{7}$ of a cwt. to the fraction of a lb.

$$\frac{2}{7} \times 4 \times 28 = \frac{224}{7} = \frac{32}{1}.$$

Ex. 4.—Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of a mile to the fraction of a foot.

$$\frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} \times 5280 = \frac{31680}{12} = \frac{2640}{1}.$$

Ex. 5.—Reduce $\frac{1}{2}$ of a square inch to the fraction of a square yard.

$$\frac{1}{2} \times 1296 = \frac{1}{2592} = \frac{2592}{1}.$$

For Rule of Three in Vulgar Fractions, see page 46.

DECIMAL FRACTIONS.

A Decimal Fraction is that which has for its denominator a unit (1), with as many ciphers annexed as the numerator has places; it is usually expressed by setting down the numerator only, with a point on the left of it. Thus, $\frac{4}{10}$ is .4; $\frac{85}{100}$ is .85; $\frac{0075}{10000}$ is .0075; and $\frac{125}{100000}$ is .00125. When there is a deficiency of figures in the numerator, prefix ciphers to make up as many places as there are ciphers in the denominator.

Mixed numbers consist of a whole number and a fraction; as, 3.25, which is the same as $3\frac{25}{100}$, or $\frac{325}{100}$.

Ciphers on the right hand make no alteration in their value; for .4, .40, .400 are decimals of the same value, each being $\frac{4}{10}$, or $\frac{2}{5}$.

ADDITION OF DECIMALS.

Rule.—Set the numbers under each other according to the value of their places, as in whole numbers, in which position the decimal points will stand directly under each other; then begin at the right hand, add up all the columns of numbers as in integers, and place the point directly below all the other points.

Example.—Add together 25.125 and 293.7325.

```
  25.125
 293.7325
 --------
 318.8575 sum.
```

SUBTRACTION OF DECIMAL FRACTIONS.

Rule.—Place the numbers under each other as in addition; then subtract as in whole numbers, and point off the decimals as in the last rule.

Example.—Subtract 15.15 from 89.1759.

```
 89.1759
 15.15
 -------
 74.0259 remainder.
```

MULTIPLICATION OF DECIMALS.

Rule.—Place the factors, and multiply them together the same as if they were whole numbers; then point off in the product just as many places of decimals as there are decimals in both the factors. But if there are not so many figures in the product, supply the deficiency by prefixing ciphers.

Example.—Multiply 1.56 by .75.

```
   1.56
    .75
   ----
    780
  1092
 ------
 1.1700 product.
```

BY CONTRACTION.

To Contract the Operation so as to retain only as many Decimal places in the Product as may be thought necessary.

Rule.—Set the unit's place of the multiplier under the figure of the multiplicand, the place of which is the same as is to be retained for the last in the product, and dispose of the rest of the figures in the contrary order to what they are usually placed in; then, in multiplying, reject all the figures that are more to the right hand than each multiplying figure, and set down the products, so that their right-hand figures may fall in a column directly below each other, and increase the first figure in every line with what would have arisen from the figures omitted; thus, add 1 for every result from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, 4 from 35 to 44, etc., etc., and the sum of all the lines will be the product as required.

Example.—Multiply 13.57493 by 46.20517, and retain only four places of decimals in the product.

```
 13.574 93
 71 502.64
 ---------
 54 299 72
  8 144 96+2 for 18
     271 50+2  "  18
       6 79+4  "  35
         14+1  "   5
          9+2  "  21
 ---------
 627.23 20
```

Note.—When the exact result is required, increase the last figure with what would have arisen from all the figures omitted.

DIVISION OF DECIMALS.

RULE.—Divide as in whole numbers, and point off in the quotient as many places for decimals as the decimal places in the dividend exceed those in the divisor; but if there are not so many places, supply the deficiency by prefixing ciphers.

EXAMPLE.—Divide 53 by 6.75.

6.75) 53.00000 (=7.851+.

Here 5 ciphers were annexed to the dividend to extend the division.

BY CONTRACTION.

RULE.—Take only as many figures of the divisor as will be equal to the number of figures, both integers and decimals, to be in the quotient, and ascertain how many times they may be contained in the first figures of the dividend, as usual.

Let each remainder be a new dividend; and for every such dividend leave out one figure more on the right-hand side of the divisor, carrying for the figures cut off as in Contraction of Multiplication.

NOTE.—When there are not so many figures in the divisor as there are required to be in the quotient, continue the first operation until the number of figures in the divisor are equal to those remaining to be found in the quotient, after which begin the contraction.

EXAMPLE.—Divide 2508.92806 by 92.41035, so as to have only four places of decimals in the quotient.

```
92.4103|5) 2508.928|06 (27.1498        4 608
           1848 207+1                  3 696
           ----------                  -----
            660 721                      912
            646 872+2                    832+4
            ---------                    ---
             13 849                       80
              9 241                       74+2
             ------                       --
              4 608                        6
```

REDUCTION OF DECIMALS.

To Reduce a Vulgar Fraction to its Equivalent Decimal.

RULE.—Divide the numerator by the denominator, annexing ciphers to the numerator as far as may be necessary.

EXAMPLE.—Reduce $\frac{4}{5}$ to a decimal.

```
5) 4.0
   ---
    .8
```

To Ascertain the Value of a Decimal in Terms of an Inferior Denomination.

RULE.—Multiply the decimal by the number of parts in the next lower denomination, and cut off as many places for a remainder, to the right hand, as there are places in the given decimal.

Multiply that remainder by the parts in the next lower denomination, again cutting off for a remainder, and so on through all the parts of the integer.

EXAMPLE.—What is the value of .875 dollars?

```
          .875
           100
        ------
Cents,  87,500
            10
        ------
Mills,   5.000 = 87 cents 5 mills.
```

EX. 2.—What is the volume of .140 cubic feet in inches?

```
   .140
   1728 cubic inches in a cubic foot.
-------
241.920 cubic inches.
```

EX. 3.—What is the value of .00129 of a foot? .01548 *inches.*

To Reduce Decimals to Equivalent Decimals of higher Denominations.

RULE.—Divide by the number of parts in the next higher denomination, continuing the operation as far as required.

EXAMPLE.—Reduce 1 inch to the decimal of a foot.

$$12\,|\,1{,}00000 \quad .08333 + \textit{foot.}$$

EX. 2.—Reduce 14″ 12‴ to the decimal of a minute.

```
      14″ 12‴
      60
60 | 852.‴
60 |  14.2″
   |   .23666′ + minute.
```

When there are several numbers to be reduced all to the decimal of the highest.

RULE.—Reduce them all to the lowest denomination, and proceed as for one denomination.

EXAMPLE.—Reduce 5 feet 10 inches and 3 barleycorns to the decimal of a yard.

```
     Feet.  In.  Bc.
       5    10    3
      12
      70
       3
 3 | 213.
12 |  71.
 3 |   5.9166
   |   1.9722 + yards.
```

RULE OF THREE IN DECIMALS.

RULE.—Prepare the terms by reducing the vulgar fractions to decimals, compound numbers to decimals of the highest denomination, the first and third terms to the same name; then proceed as in whole numbers. See Rule, page 48.

EXAMPLE.—If $\frac{1}{2}$ a ton of iron cost $\frac{3}{4}$ of a dollar, what will .625 of a ton cost?

$\frac{1}{2} = .5$, $\frac{3}{4} = .75$ .5 : .75 : : .625

```
       .75
      .625
.5) .46875
     .9375 dollars.
```

DUODECIMALS.

In Duodecimals, or Cross Multiplication, the dimensions are taken in feet, inches, and twelfths of an inch.

RULE.—Set the dimensions to be multiplied together one under the other, feet under feet, inches under inches, etc.

Multiply each term of the multiplicand, beginning at the lowest, by the feet in the multiplier, and set the result of each immediately under its corresponding term, carrying 1 for every 12 from one term to the other. In like manner, multiply all the multiplicand by the inches of the multiplier, and then by the twelfth parts, setting the result of each term one place farther to the right hand for every multiplier. The sum of the products is the result.

EXAMPLE.—Multiply 1 foot 3 inches by 1 foot 1 inch.

Feet.	Ins.	
1	3	
1	1	
1	3	
	1	3
1 *foot.*	4 *ins.*	3 *twelfths of an inch.*

PROOF.—1 foot 3 inches is 15 inches, and 1 foot 1 inch is 13 inches; and $15 \times 13 = 195$ square inches.

Ex. 2.—How many square inches are there in a board 35 feet $4\frac{1}{2}$ inches long and 12 feet $3\frac{1}{3}$ inches wide?

Feet.	Ins.	Twelfths.		
35	4	6		
12	3	4		
424	6	0		
8	10	1	6	
	11	9	6	0
434	3	11	0	0

Ex. 3.—Multiply 20 feet $6\frac{1}{2}$ inches by 40 feet 6 inches.

By duodecimals, *Ans.* 831 feet 11 inches 3 twelfths equal 831 square feet and 135 square inches.

By decimals ....40 feet 6 ins. = 40.5
20 " $6\frac{1}{2}$ " = 20.541666, etc.
831.937499 *feet.*
144
134.999856 *square ins.*

Value of Duodecimals in Square Feet and Inches.

	Sq. Ft.	Sq. Ins.		Sq. Ft.	Sq. Ins.
1 Foot .... =	1	or 144	$\frac{1}{12}$ of 1 twelfth =	$\frac{1}{1728}$	or .083333, etc.
1 Inch .... =	$\frac{1}{12}$	" 12	$\frac{1}{12}$ of $\frac{1}{12}$ of " =	$\frac{1}{20736}$	" .006944, etc.
1 Twelfth .... =	$\frac{1}{144}$	" 1			

ILLUSTRATION.—What number of square inches are there in a floor 100 feet 6 inches broad and 25 feet 6 inches and 6 twelfths long?

2566 *feet* 11 *ins.* 3 *twelfths* = 2566 *feet* 135 *ins.*

MEAN PROPORTION.

MEAN PROPORTION is the proportion to two given numbers or terms.

RULE.—Multiply the two numbers or terms together, and extract the square root of their product.

EXAMPLE.—What is the mean proportionate velocity to 16 and 81?

$16 \times 81 = 1296$, and $\sqrt{1296} = 36$ *mean velocity.*

RULE OF THREE.

The RULE OF THREE teaches how to compute a fourth proportional to three given numbers.

It is either DIRECT or INVERSE.

It is Direct when more requires more, or less requires less; thus, if 3 barrels of flour cost $18, what will 10 barrels cost?

In this case the Proportion is *Direct*, and the stating must be,

As 3 : 10 : : 18 : 60.

It is Inverse when more requires less, or less requires more; thus, if 6 men build a certain quantity of wall in 10 days, in how many days will 8 men build the like quantity? Or, if 3 men dig 100 feet of trench in 7 days, in how many days will 2 men perform the same work?

Here the Proportion is *Inverse*, and the stating must be,

As 8 : 6 : : 10 : 7.5.
2 : 3 : : 7 : 10.5.

The fourth term is always ascertained by multiplying the 2d and 3d terms together, and dividing the product by the 1st term.

Of the three given numbers necessary for the stating, two of them contain the supposition, and the third a demand.

RULE.—State the question by setting down in a straight line the three necessary numbers in the following manner:

Let the 3d term be that of *supposition*, of the same denomination as the answer, or 4th term is to be, making the *demanding* number the 2d term, and the other number the 1st term when the question is in *Direct Proportion*, but contrariwise if in *Inverse Proportion;* that is, let the *demanding* number be the 1st term.

Multiply the 2d and 3d terms together, and divide by the 1st, and the product will be the answer, or 4th term sought, of the same denomination as the 2d term.

NOTE.—If the first and third terms are of different denominations, reduce them to the same. If, after division; there be any remainder, reduce it to the next lower denomination, divide by the divisor as before, and the quotient will be of this last denomination.

Sometimes two or more statings are necessary, which may always be known by the nature of the question.

EXAMPLE.—If 20 tons of iron cost $225, what will 500 tons cost?

Tons. Tons. Dolls.
20 : 500 : : 225
500
2|0) 11250|0
5625 *dollars.*

Ex. 2.—If 15 men raise 100 tons of iron ore in 12 days, how many men will raise a like quantity in 5 days?

Days. Days. Men. Men.
As 5 : 12 : : 15 : 36

Ex. 3.—A wall that is to be built to the height of 36 feet, was raised 9 feet high by 16 men in 6 days; how many men could finish it in 4 days at the same rate of working?

Days. Days. Men. Men.
4 : 6 : : 16 : 24

Then, if 9 feet requires 24 men, what will 27 feet require?

9 : 27 : : 24 : 72 *men.*

COMPOUND PROPORTION.

COMPOUND PROPORTION is the rule by means of which such questions as would require two or more statings in simple proportion (Rule of Three) can be resolved in one.

As the rule, however, is but little used, and not easily acquired, it is deemed preferable to omit it here, and to show the operation by two or more statings in Simple Proportion.

EXAMPLE.—How many men can dig a trench 135 feet long in 8 days, when 16 men can dig 54 feet in 6 days?

Feet. Feet. Men. Men.
First .... As 54 : 135 : : 16 : 40
Days. Days. Men. Men.
Second .... As 8 : 6 : : 40 : 30

Ex. 2.—If a man travel 130 miles in 3 days of twelve hours each, in how many days of 10 hours each would he require to travel 360 miles?

Miles. Miles. Days. Days.
First............ As 130 : 360 : : 3 : 8.307
Hours. Hours. Days. Days.
Second........... As 10 : 12 : : 8.307 : 9.9684

Ex. 3.—If 12 men in 15 days of 12 hours build a wall 30 feet long, 6 wide, and 3 deep, in how many days of 8 hours will 60 men build a wall 300 feet long, 8 wide, and 6 deep?
120 *days.*

Or, by Cancelation,

RULE.—On the right of a vertical line put the number of the same denomination as that of the required answer.

Examine each simple proportion separately, and if its terms demand *a greater answer than the 3d term*, put the *larger number on the right*, and the *lesser on the left of the line;* but if its terms demand a *less answer than the 3d term*, *put the smaller number on the right and the larger on the left of the line.*

Then *Cancel* the numbers divisible by a common divisor, and evolve the 4th term or answer required.

Take the preceding, example first: 3d term, or term of supposition of the same denomination as the required answer. 16 *men.*

135 *feet* require more men than 54 *feet.*
8 *days* " less " 6 *days.*

Statement.

	16
54	135
8	6

$2 \times 5 \times 3 = 30$ *men.*

Result by Cancelation.

	~~16~~ 2
2 ~~54~~	~~135~~ 5
~~8~~	~~6~~ 3

Ex. 3.—3d term, 15 *days.*

60 *men* require less days than 12 *men.*
8 *hours* " more " 12 *hours.*
300 *feet* " " " 30 *feet.*
8 " " " " 6 "
6 " " " " 3 "

Statement.

	15
60	12
8	12
30	300
6	8
3	6

$3 \times 4 \times 10 = 120$ *days.*

Result by Cancelation.

	~~15~~
4 ~~60~~	~~12~~ 3
~~8~~	~~12~~ 4
~~30~~	~~300~~ 10
~~6~~	~~8~~
~~3~~	~~6~~

INVOLUTION.

INVOLUTION is the multiplying any number into itself a certain number of times. The products obtained are called *Powers.* The number is called the *Root*, or first power.

When a number is multiplied by itself once, the product is the *square* of that number; twice, the *cube;* three times, the *biquadrate*, etc. Thus, of the number 5.

5 *is the Root*, or 1st *power.*
$5 \times 5 = 25$ " *Square*, or 2d *power*, and is expressed 5^2.
$5 \times 5 \times 5 = 125$ " *Cube*, or 3d *power*, and is expressed 5^3.
$5 \times 5 \times 5 \times 5 = 625$ " *Biquadrate*, or 4th *power*, and is expressed 5^4.

The lesser figure set superior to the number denotes the power, and is termed the *Index* or *Exponent.*

EXAMPLE.—What is the cube of 9? 729.

Ex. 2.—What is the cube of $\frac{3}{4}$? $\frac{27}{64}$.

Ex. 3.—What is the 4th power of 1.5? 5.0625.

EVOLUTION.

EVOLUTION is ascertaining the *Root* of any number.

The sign $\sqrt{\ }$ placed before any number indicates that the *square root* of that number is required or shown.

The same character expresses any other root by placing the index above it.

Thus, $\sqrt{25} = 5$, and $4 + 2 = \sqrt{36}$.
And, $\sqrt[3]{27} = 3$, and $\sqrt[3]{64} = 4$.

Roots which only approximate are called Surd Roots.

TO EXTRACT THE SQUARE ROOT.

RULE.—Point off the given number from units' place into periods of two figures each.

Ascertain the greatest square in the left-hand period, and place its root in the quotient; subtract the square number from this period, and to the remainder bring down the next period for a dividend.

Double this root for a divisor; ascertain how maay times it is contained in the dividend, exclusive of the right-hand figure, which, when multiplied by the number to be put to the right hand of this divisor, the product will be equal to, or the next less than the dividend; place the result in the quotient, and also at the right hand of the divisor.

Multiply the divisor by the last quotient figure, and subtract the product from the dividend; bring down the next period, and proceed as before.

NOTE.—Mixed decimals must be pointed off both ways from units.

EXAMPLE.—What is the square root of 2?

```
   1| 2.000000 (1.414, +.
   1| 1
  ------
  24| 100
   4|  96
 --------
 281|  400
   1|  281
 ---------
2824| 11900
   4| 11296
 ---------
2828|   604
```

Ex. 2.—What is the square root of 144?

```
 1| 144 (12
 1| 1
 -------
22| 044
  |  44
 -------
     00
```

SQUARE ROOTS OF VULGAR FRACTIONS.

RULE.—Reduce the fractions to their lowest terms, and that fraction to a decimal, and proceed as in whole numbers and decimals.

NOTE.—When the terms of the fractions are squares, take the root of each and set one above the other; as, $\frac{5}{6}$ is the square root of $\frac{25}{36}$.

EXAMPLE.—What is the square root of $\frac{9}{12}$? .8660254.

To Ascertain the 4th Root of a Number.

RULE.—Extract the square root twice, and *for the* 8*th root* thrice, etc., etc.

TO EXTRACT THE CUBE ROOT.

RULE.—From the table of roots (page 210) take the nearest cube to the given number, and call it the assumed cube.

Then, as the given number added to twice the assumed cube, is to the assumed cube added to twice the given number, so is the root of the assumed cube to the required root, *nearly;* and by using in like manner the root thus found as an assumed cube, and proceeding as above, another root will be found still nearer; and in the same manner as far as may be deemed necessary.

EXAMPLE.—What is the cube root of 10517.9?

Nearest cube, page 210; 10648, root 22.

```
10648.     10517.9
    2            2
-------    -------
21296      21035.8
10517.9    10648.
-------    -------
31813.9 :  31683.8 : : 22 : 21.9 +.
```

To Ascertain or To Extract the Square or Cube Roots of Roots, Whole Numbers, and of Integers and Decimals, see Table of Squares and Cubes, and Rules, p. 210–243.

To Extract any Root whatever.

Let P represent the number. | Let A represent the assumed power, r its root.
n " the index of the power. | R " the required root of P.

Then, as the sum of $n+1 \times A$ and $n-1 \times P$ is to the sum of $n+1 \times P$ and $n-1 \times A$, so is the assumed root r to the required root R.

EXAMPLE.—What is the cube root of 1500?

The nearest cube, page 210, is 1331, root 11.

$$P=1500,\ n=3,\ A=1331,\ r=11;$$
$$\text{then, } n+1 \times A=5324,\ n+1 \times P=6000$$
$$n-1 \times P=3000,\ n-1 \times A=2662$$
$$\overline{8324} \quad : \quad \overline{8662} :: 11 : 11.446+.$$

To Ascertain the Root of an Even Power greater than those given in the Table of Square and Cube Roots.

RULE.—Extract the square or cube root of it, which will reduce it to half the given power; then the square or cube root of that power reduces it to half the same power; and so on until the required root is obtained.

ILLUSTRATION.—Suppose a 12th power is given; the square root of that reduces it to a 6th power, and the square root of a 6th power to a cube.

EXAMPLE.—What is the biquadrate, or 4th root, of 2560000?

$$\sqrt{2560000}=1600, \text{ and } \sqrt{1600}=40.$$

PROPERTIES OF NUMBERS.

1. A *Prime Number* is that which can only be measured (divided without a remainder) by 1 or unity.
2. A *Composite Number* is that which can be measured by some number greater than unity.
3. A *Perfect Number* is that which is equal to the sum of all its divisors or aliquot parts; as $6=\frac{6}{6}, \frac{6}{3}, \frac{6}{2}$.
4. If the sum of the digits constituting any number be divisible by 3 or 9, the whole is divisible by them.
5. A square number can not terminate with an *odd* number of ciphers.
6. No square number can terminate with two equal digits, except two ciphers or two fours.
7. No number the last digit of which is 2, 3, 7, or 8, is a square number.

Table of the first Nine Powers of the first Nine Numbers.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th	9th.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is a series of numbers increasing or decreasing by a constant number or difference; as, 1, 3, 5, 7, 9, 15, 12, 9, 6, 3. The numbers which form the series are called *Terms;* the first and last are called the *Extremes*, and the others the *Means.*

When any three of the following elements are given, the remaining two can be ascertained, viz.: The *First* term, the *Last* term, the *Number* of terms, the *Common Difference*, and the *Sum* of all the terms.

To Ascertain the Last Term, When the First Term, the Common Difference, and the Number of Terms are given.

RULE.—Multiply the number of terms less 1, by the common difference, and to the product add the first term.

EXAMPLE.—A man traveled for 12 days, going 3 miles the first day, 8 the second, and so on; how far did he travel the last day?

$$12 - 1 \times 5 = 55, \text{ and } 55 + 3 = 58 \textit{ miles.}$$

To Ascertain the Common Difference, When the Number of Terms and the Extremes are given.

RULE.—Divide the difference of the extremes by 1 less than the number of terms.

EXAMPLE.—The extremes are 3 and 15, and the number of terms 7; what is the common difference?

$$15 - 3 \div (7 - 1) = \frac{12}{6} = 2.$$

To Ascertain the Sum of all the Terms, When the Extremes and Number of Terms are given.

RULE.—Multiply the number of terms by half the sum of the extremes.

EXAMPLE.—How many times does the hammer of a clock strike in 12 hours?

$$12 \times (\overline{12 + 1} \div 2) = 78 \textit{ times.}$$

To Ascertain the Number of Terms, When the Common Difference and the Extremes are given.

RULE.—Divide the difference of the extremes by the common difference, and add 1 to the quotient.

EXAMPLE.—A man traveled 3 miles the first day, 5 the second, 7 the third, and so on, till he went 57 miles in one day; how many days had he traveled at the close of the last day?

$$57 - 3 \div 2 = 27, \text{ and } 27 + 1 = 28 \textit{ days.}$$

To Compute two Arithmetical Means between two given Extremes.

RULE.—Subtract the less extreme from the greater, and divide the difference by 3, the quotient will be the common difference, which being added to the less extreme, or taken from the greater, will give the means.

EXAMPLE.—Ascertain two arithmetical means between 4 and 16.

$$16 - 4 \div 3 = 4 \textit{ com. dif.}$$
$$4 + 4 = 8 \textit{ one mean.}$$
$$16 - 4 = 12 \textit{ second mean.}$$

To Compute any Number of Arithmetical Means between two Extremes.

RULE.—Subtract the less extreme from the greater, and divide the difference by 1 more than the number of means required to be ascertained, and then proceed as in the foregoing rule.

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is any series of numbers continually increasing by a constant multiplier, or decreasing by a constant divisor, as 1, 2, 4, 8, 16, and 15, 7.5, 3.75.

The constant multiplier or divisor is the *Ratio.*

When any three of the following elements are given, the remaining two can be ascertained, viz.: The *First* term, the *Last* term, the *Number* of Terms, the *Ratio*, and the *Sum* of all the Terms.

To Compute the Last Term, When the First Term and the Ratio are Equal.

RULE.—Write a few of the leading terms of the series and place their indices over them, beginning with a unit. Add together the most convenient indices to make the index to the term required.

Multiply the terms of the series of these indices together, take the product will be the term required.

Or, multiply the first term by the ratio raised to a power, den ted by the number of terms less 1.

EXAMPLE.—The first term is 2, the ratio 2, and the number of terms 13; what is the last term?

Indices, 1 2 3 4 5
Terms, 2, 4, 8, 16, 32.

Then $5+5+3=13=$ *sum of indices*, and $32\times32\times8=8192=$ *last term.*
Or, $2\times2^{13-1}=8192$.

Ex. 2. The price of 12 horses being 4 cents for the first, 16 for the second, and 64 for the third; what is the price of the last horse? $167,772.16.

When the First Term and the Ratio are Different.

RULE.—Write a few of the leading terms of the series, and place their indices over them, beginning with a cipher. Add together the most convenient indices to make an index less by 1 than the term sought.

Multiply the terms of these series belonging to these indices together, and take the product for a dividend.

Raise the first term to a power, the index of which is 1 less than the number of terms multiplied; take the result for a divisor; proceed with their division, and the quotient will give the term required.

EXAMPLE.—The first term is 1, the ratio 2, and the number of terms 23; what is the last term?

Indices, 0 1 2 3 4 5
Terms, 1, 2, 4, 8, 16, 32.

Then $5+5+5+5+2=22=$ *sum of indices*, and $32\times32\times32\times32\times4=4194304$, and $4194304\div$ the 5th power $(6-1)$ of $1=1=4194304$.
Or, $1\times2^{23-1}=4194304$.

Ex. 2. If one cent had been put out at interest in 1630, what would it have amounted to in the year 1834, if it had doubled its value every 12 years?

$1834-1630=204$, which $\div 12=17$, and $17+1=18=$ *number of terms.*

Indices, 0 1 2 3 4 7
Terms, 1, 2, 4, 8, 16, 128.

Then $7+4+3+2+1=17$, and $128\times16\times8\times4\times2\times1=131072$, and $131072\div1$, the 4th power $(5-1)$ of $1=\$1.310.72$.

Ex. 3. If a man were to work 20 days, for 4 cents for the first day, 12 for the second, and 36 for the third, and so on, what would be the amount of his pay upon the last day?

Indices, 0 1 2 3 4 5 6
Terms, 4, 12, 36, 108, 324, 972, 2916.

Then $6+5+4+3+1=19=$ *sum of indices*, and $2916\times972\times324\times108\times12=1190155742208$, and this sum $\div$ the 4th power $(5-1)$ of the first term $=256$, and $1190155742208\div256=4649045868$ *cents.*

To Compute the Sum of the Series, When the First Term, the Ratio, and the Number of Terms are given.

RULE.—Raise the ratio to a power the index of which is equal to the number of terms, from which subtract 1; then divide the remainder by the ratio less 1, and multiply the quotient by the first term.

EXAMPLE.—The first term is 2, the ratio 2, and the number of terms 13; what is the sum of the series?
$2^{13}-1=8192-1=8191$, and $8191\div(2-1)=1=8191$, and $8191\times2=16382$.

Ex. 2.—If a man were to buy 12 horses, giving 2 cents for the first horse, 6 cents for the second, and so on, what would they cost him? $5.314.40.

When the Last Term is given.

RULE.—Multiply the last term by the ratio, and from the product subtract the first term; then divide the remainder by the ratio less 1.

EXAMPLE.—The first term is 1, the ratio 2, and the last term 131072; what is the sum of the series?

$$131072\times2-1=262143, \text{ and } 262143\div\overline{2-1}=262143.$$

To Compute the Ratio, When the First Term, the Last Term, and the Number of Terms are given.

RULE.—Divide the last term by the first, and the quotient will be equal to the ratio raised to the power denoted by 1 less than the number of terms; then extract the root of this quotient.

EXAMPLE.—The last term, or greatest extreme of a geometrical progression, is .46, the first, or least term, .005, and the number of terms 40; what is the ratio?
$\frac{.46}{.005}=92$, or the 39th power $(40-1)$ of the ratio; then log. $92^{40-1}=$ log. of $92=1.963788$, which $\div 39=.0503535$, and the number corresponding to the log. of $.0503535=.112293$.

To Compute the Number of Terms, When the Ratio, the First, and the Last Terms are given.

RULE.—Divide the logarithm of the quotient of the product of the ratio and the last term, divided by the first term, by the logarithm of the ratio.

EXAMPLE.—The ratio is 2, and the first and last terms are 1 and 131072; what is the number of the terms?

$$\text{log. } \frac{2\times131072}{1}=\text{log. } 262144 - 5.41854, \text{ and } 5.41854\div\text{log. of } 2=\frac{5.41854}{.30103}=18.$$

Table of Geometrical Progression,

Whereby any questions of Geometrical Progression and of Double Ratio may be solved by Inspection, the number of terms not exceeding 56.

1	1	15	16384	29	268435456	43	4398046511104
2	2	16	32768	30	536870912	44	8796093022208
3	4	17	65536	31	1073741824	45	17592186044416
4	8	18	131072	32	2147483648	46	35184372088832
5	16	19	262144	33	4294967296	47	70368744177664
6	32	20	524288	34	8589934592	48	140737488355328
7	64	21	1048576	35	17179869184	49	281474976710656
8	128	22	2097152	36	34359738368	50	562949953421312
9	256	23	4194304	37	68719476736	51	1125899906842624
10	512	24	8388608	38	137438953472	52	2251799813685248
11	1024	25	16777216	39	274877906944	53	4503599627370496
12	2048	26	33554432	40	549755813888	54	9007199254740992
13	4096	27	67108864	41	1099511627776	55	18014398509481984
14	8192	28	134217728	42	2199023255552	56	36008797018963968

ILLUSTRATIONS.—The 12th power of $2=4096$, and the 7th root of $128=2$.

PERMUTATION.

PERMUTATION is a rule for ascertaining how many different ways any given number of numbers of things may be varied in their position.

The permutation of the 3 letters *abc*, taken *all together*, are 6; taken *two* and *two*, are 6; and taken *singly*, are 3.

RULE.—Multiply all the terms continually together, and the last product will give the result required.

EXAMPLE.—How many variations will the nine digits admit of?

$$1\times2\times3\times4\times5\times6\times7\times8\times9=362880.$$

When only part of the Numbers or Elements are taken at once.

RULE.—Take a series of numbers, beginning with the number of things given, decreasing by 1, until the number of terms equals the number of things or quantities to be taken at a time, and the product of all the terms will give the sum required.

EXAMPLE.—How many changes can be rung with 4 bells (taken 4 and 4 together) out of 6? $6\times5\times4\times3=360.$

When several of the Elements are alike

RULE.—Ascertain the permutations of all the numbers or things, and of all that can be made of each separate kind or division; divide the number of the permutations of the whole by the product of the several partial permutations, and the quotient will give the number of permutations.

EXAMPLE.—How many permutations can be made out of the letters of the word persevere (9 letters, having 4 e's and 2 r's)?

$$1\times2\times3\times4\times5\times6\times7\times8\times9=362880;$$

$$1\times2\times3\times4=24;\ 1\times2=2,\text{ and }24\times2=48;\text{ and }362880\div48=7560.$$

Table of Permutations,

Whereby any questions of Permutation from 1 *may be solved by Inspection, the number of terms not exceeding* 20.

1	1	5	120	9	362880.	13	6227020800	17	355687428096000
2	2	6	720	10	3628800	14	87178291200	18	6402373705728000
3	6	7	5040	11	39916800	15	1307674368000	19	121645100408832000
4	24	8	40320	12	479001600	16	20922789888000	20	2432902008176640000

POSITION.

POSITION is of two kinds, SINGLE and DOUBLE, and is determined by the number of SUPPOSITIONS.

Single Position.

RULE.—Take any number, and proceed with it as if it were the correct one; then, as the result is to the given sum, so is the supposed number to the number required.

EXAMPLE.—A commander of a vessel, after sending away in boats $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{4}$ of his crew, had left 300; what number had he in command?

Suppose he had............ 600.

$\frac{1}{3}$ of 600 is 200

$\frac{1}{6}$ of 600 is 100

$\frac{1}{4}$ of 600 is 150   450

150 : 300 : : 600 : 1200 *men.*

EX. 2.—A person being asked his age, replied, if $\frac{3}{4}$ of my age be multiplied by 2, and that product added to half the years I have lived, the sum will be 75. How old was he? 37.5 *years.*

Double Position.

RULE.—Take any two numbers, and proceed with each according to the conditions of the question; multiply the results or *errors* by the contrary supposition; that is, the first position by the last error, and the last position by the first error.

If the errors are too great, mark them +; and if too little, —.

Then, if the errors are *alike*, divide the *difference* of the products by the *difference* of the errors; but if they are *unlike*, divide the *sum* of the products by the *sum* of the errors.

EXAMPLE.—F asked G how much his boat cost; he replied that if it cost him 6 times as much as it did, and $30 more, it would have cost him $300. What was the price of the boat?

Suppose it cost............	60		30	
	6	*times.*	6	*times.*
	360		180	
and	30	*more*	and 30	*more.*
	390		210	
	300		300	
	90 +		90 —	
	30	*2d position.*	60	*1st position.*
90	2700		5400	
90	5400			
180)	8100	(45 *dollars.*		

EX. 2.—What is the length of a fish when the head is 9 inches long, the tail as long as its head and half its body, and the body as long as both the head and tail.

6 *feet.*

FELLOWSHIP.

FELLOWSHIP is a method of ascertaining gains or losses of individuals engaged in joint operations.

Single Fellowship.

RULE.—As the whole stock is to the whole gain or loss, so is each share to the gain or loss on that share.

EXAMPLE.—Two men drew a prize in a lottery of $9500. A paid $3, and B $2 for the ticket; how much is each share?

5 : 9500 : : 3 : 5700, A's *share.*
5 : 9500 : : 2 : 3800, B's *share.*

Double Fellowship,

Or Fellowship with Time.

RULE.—Multiply each share by the time of its interest in the Fellowship; then, as the sum of the products is to the product of each interest, so is the whole gain or loss to each share of the gain or loss.

EXAMPLE.—A ship's company take a prize of $10,000, which they divide according to their rate of pay and time of service on board. The officers have been on board 6 months, and the crew 3 months; the pay of the lieutenants is $100, ensigns $50, and crew $10 per month; and there are 2 lieutenants, 4 ensigns, and 50 men; what is each one's share?

2 lieutenants	$100 = 200 × 6 = 1200
4 ensigns	50 = 200 × 6 = 1200
50 men	10 = 500 × 3 = 1500
	3900

Lieutenants.........	3900 : 1200 : : 10,000 : 3076.92 ÷ 2 =	1538.46 *dolls.*
Ensigns.............	3900 : 1200 : : 10,000 : 3076.92 ÷ 4 =	769.23 "
Men	3900 : 1500 : : 10,000 : 3846.16 ÷ 50 =	76.92 "

ALLIGATION.

ALLIGATION is a method of finding the mean rate or quality of different materials when mixed together.

To Compute the Mean Price of the Mixture.

RULE.—Multiply each quantity by its rate, divide the sum of the products by the sum of the quantities, and the quotient will be the rate of the composition.

EXAMPLE.—If 10 lbs. of copper at 20 cents per lb., 1 lb. of tin at 5 cents, and 1 lb. of lead at 4 cents, be mixed together, what is the value of the composition?

$$10\times20=200$$
$$1\times 5=\ \ 5$$
$$1\times 4=\ \ 4$$
$$\overline{12}\ )\ \overline{209}\ (17.416 \textit{ cents.}$$

To Ascertain what Quantity of each Article must be taken, When the Prices and Mean Price are given.

RULE.—Connect with a line each price that is *less* than the mean rate with one or more that is *greater*.

Write the difference between the mixture rate and that of each of the simples opposite the price with which it is *connected;* then the sum of the differences against any price will express the quantity to be taken of that price.

EXAMPLE.—How much gunpowder, at 72, 54, and 48 cents per pound, will compose a mixture worth 60 cents a pound?

$$60\begin{cases}48 \\ 54 \\ 72\end{cases}\qquad \begin{array}{r}12, \text{ at } 48 \textit{ cents.}\\ 12, \text{ at } 54 \textit{ cents.}\\ 12+6=18, \text{ at } 72 \textit{ cents.}\end{array}$$

Then $12\times48+12\times54+18\times72=2520$, and $2520\div\overline{12+12+12+6}=60$ *cents.*

NOTE.—Should it be required to mix a definite quantity of any one article, the quantities of each, determined by the above rule, must be increased or decreased in the proportion they bear to the defined quantity.

Thus, had it been required to mix 18 pounds at 48 cents, the result would be 18 at 48, 18 at 54, and 27 at 72 cents per pound.

When the whole Composition is limited.

RULE.—As the sum of the relative quantities, as ascertained by the above rule, is to the whole quantity required, so is each quantity so ascertained to the required quantity of each.

EXAMPLE.—Required 100 pounds of the above mixture.

Then $12+12+18=42$.

Then 42 : 100 : : 12 : 28.571.
42 : 100 : : 12 : 28.571.
42 : 100 : : 18 : 42.857.

SIMPLE INTEREST.

To Compute the Interest on any Given Sum for a Period of One or more Years.

RULE.—Multiply the given sum or *principal* by the rate per cent. and the number of years; point off two figures to the right of the product, and the result will give the interest in dollars and cents for 1 year.

EXAMPLE.—What is the interest upon $1050 for 5 years at 7 per cent.?

$1050\times7\times5=36750$, and $367.50=\$367.50$.

When the Time is less than One Year.

RULE.—Proceed as before, multiplying by the number of months or days, and dividing by the following units; viz. 12 for months, and 365 or 366, as the case may be, for days.

EXAMPLE.—What is the interest upon $1050 for 5 months and 30 days at 7 per cent.?

5 months and 30 days = 183 *days.*

$$\frac{1050\times7\times183}{365}=3685, \text{ and } 36.85=\$36.85.$$

The interest upon any sum at 6 per cent. = 1 per cent. for 2 months.

The interest at 5 per cent. is $\frac{1}{6}$th less than at 6 per cent.

The interest at 7 per cent. is $\frac{1}{6}$th greater than at 6 per cent.

The operation of computing interest may be performed thus:

Taking the preceding example — 2 months = 1 per cent. = 10.50
2 " = 1 " 10.50
1 " = $\frac{1}{2}$ " 5.25
30 days = 1 month = 5.25
31.50
Add $\frac{1}{6}$th for 7 per cent. = 5.25
$36.75

NOTE.—The difference between this amount and the preceding arises from 183 days being taken in the one case, and half a year, or 182.5 days, in the other.

COMPOUND INTEREST.

If any Principal be multiplied by the amount (in the following table) opposite the years, and under the rate per cent., the sum will be the amount of that principal at compound interest for the time and rate taken.

EXAMPLE.—What is the amount of $500 for 10 years at 6 per cent.?

Tabular amount .... 1.79084, and 1.79084×500 = 895.42 *dollars.*

Table showing the Value of $1, etc., for any Number of Years not exceeding 24, at the Rates of 5, 6, and 7 per Ct. per Annum Compound Interest.

Years.	5 Per Cent.	6 Per Cent.	7 Per Cent.	Years.	5 Per Cent.	6 Per Cent.	7 Per Cent.
1	1.05	1.06	1.07	13	1.88564	2.13292	2.40985
2	1.1025	1.1236	1.1449	14	1.97993	2.26090	2.57853
3	1.15762	1.19101	1.22504	15	2.07892	2.39655	2.75903
4	1.2155	1.26247	1.3108	16	2.18287	2.54035	2.95216
5	1.27628	1.33822	1.40255	17	2.29201	2.69277	3.15881
6	1.34	1.41851	1.50073	18	2.40661	2.85433	3.37994
7	1.4071	1.50363	1.60578	19	2.52695	3.02559	3.61654
8	1.47745	1.59384	1.71819	20	2.65329	3.20713	3.8697
9	1.55132	1.68947	1.83846	21	2.78596	3.39956	4.14057
10	1.62889	1.79084	1.96715	22	2.92526	3.60353	4.43041
11	1.71033	1.89829	2.10485	23	3.07152	3.81974	4.74054
12	1.79585	2.01219	2.25219	24	3.22509	4.04873	5.07238

REBATE.

REBATE is a deduction or *Discount* upon money paid before it is due.

To Compute the Rebate upon any Sum.

RULE.—Multiply the amount by the rate per cent. and by the time, and divide the product by the sum of the product of the rate per cent. and the time added to 100.

EXAMPLE.—What is the rebate upon $12,075 for 3 years, 5 months, and 15 days, at 6 per cent.?

3 years 5 months and 15 days = 3.4574 *years.*

$$\frac{12075\times6\times3.4574}{100+(6\times3.4574)}=\frac{250488.63}{120.7444}=2074.53=\$2074.53.$$

INTEREST AND DISCOUNT.

To Ascertain the Principal, the Time, Rate per Cent., and Interest being given.

RULE.—Divide the given interest by the interest of $1, etc., for the given rate and time.

EXAMPLE.—What sum of money at 6 per cent. will in 14 months produce $14?

$14 \div .07 = 200$ *dollars.*

To Ascertain the Rate per Cent., the Principal, Interest, and Time being given.

RULE.—Divide the given interest by the interest of the given sum, for the time, at 1 per cent.

EXAMPLE.—If $32.66 was the discount from a note of $400 for 14 months, what was that per cent.?

The interest on 400 for 14 months at 1 per cent. = 4.66.

Then $32.66 \div 4.66 = 7$ *per cent.*

To Ascertain the Time, the Principal, Rate per Cent., and Interest being given.

RULE.—Divide the given interest by the interest of the sum, at the rate per cent. for one year.

EXAMPLE.—In what time will $108 produce $11.34, at 7 per cent.?

The interest on 108 for one year is 7.56.

$11.34 \div 7.56 = 1.5$ *years.*

EQUATION OF PAYMENTS.

RULE.—Multiply each sum by its time of payment in days, and divide the sum of the products by the sum of the payments.

EXAMPLE.—A owes B $300 in 15 days, $60 in 12 days, and $350 in 20 days; when is the whole due?

$$\begin{array}{l} 300 \times 15 = 4500 \\ \;\;60 \times 12 = \;\;720 \\ 350 \times 20 = 7000 \\ \overline{710} \quad) \; \overline{12220} \; (17 + \textit{days.} \end{array}$$

ANNUITIES.

To Ascertain the Amount of Annuity, the Time, and Rate of Interest being given.

RULE.—Raise the *ratio* to a power denoted by the time, from which subtract 1; divide the remainder by the *ratio* less 1, and the quotient, multiplied by the annuity, will give the amount.

NOTE.—$1 added to the given rate per cent. is the ratio, and the preceding table in Compound Interest is a table of ratios.

EXAMPLE.—What is the amount of an annual pension of $100, interest 5 per cent., which has remained unpaid for four years?

1.05 ratio; then $1.05^4 - 1 = 1.21550625 - 1 = .21550625$, and $.21550625 \div (1.05 - 1)$ $.05 = 4.310125$, which $\times 100 = \$431.0125$.

To Ascertain the Present Worth of an Annuity, the Time, and Rate being given.

RULE.—Ascertain the value of it for the whole time; and this amount divided by the ratio, involved to the time, will give the worth.

EXAMPLE.—What is the present worth of a pension or salary of $500, to continue 10 years at 6 per cent. compound interest?

$500, by the last rule, is worth $6590.3975, which, divided by 1.06^{10} (by table, page 58, is 1.79084) = $3680.05.

Or, multiply the tabular amount in the following table, by the given annuity, and the product will be the present worth.

Table showing the Present Worth of an Annuity at 5, 6, and 7 per Cent. Compound Interest for any Number of Years under 34.

Years.	5 Per Cent.	6 Per Cent.	7 Per Cent.	Years.	5 Per Cent.	6 Per Cent.	7 Per Cent.
1	.95238	.94339	.9345	18	11.68958	10.8276	10.0591
2	1.85941	1.83339	1.808	19	12.08532	11.15811	10.3356
3	2.72325	2.67301	2.6243	20	12.46221	11.46992	10.594
4	3.54595	3.4651	3.3872	21	12.82115	11.76407	10.8355
5	4.32948	4.21236	4.1001	22	13.163	12.04158	11.0612
6	5.07569	4 91732	4.7665	23	13.48807	12.30338	11.2722
7	5.78637	5.58238	5.3892	24	13.79864	12.55035	11.4693
8	6.46321	6.20979	5.9712	25	14.09394	12.78335	11.6536
9	7.10782	6.80169	6.5152	26	14.37518	13.00316	11.8258
10	7.72173	7.36008	7.0235	27	14.64303	13.21053	11.9867
11	8.30641	7.88687	7.4986	28	14.89813	13.40616	12.1371
12	8.86325	8.38384	7.9426	29	15.14107	13.59072	12.2777
13	9.39357	8.85268	8.3576	30	15.37245	13.76483	12.409
14	9.89864	9.29498	8.7454	31	15.59281	13.92908	12 5318
15	10.37966	9.71225	9.1079	32	15.80268	14.08398	12.6465
16	10.83778	10.10589	9.4466	33	16.00255	14.22917	12.7538
17	11.27407	10.47726	9.7632	34	16.1929	14.36613	12.854

ILLUSTRATION.—As above; 10 years at 6 per cent. = 7.36008, and 7.36008 × 500 = 3680.04 *dollars.*

When Annuities do not commence till a certain period of time, they are said to be in *Reversion.*

To Ascertain the Present Worth of an Annuity in Reversion.

RULE.—Take two amounts under the rate in the above table, viz., that opposite the sum of the two given times and that of the time of reversion; multiply their difference by the annuity, and the product is the present worth.

EXAMPLE.—What is the present worth of a reversion of a lease of $40 per annum, to continue for 6 years, but not to commence until the end of 2 years, allowing 6 per cent. to the purchaser?

6 + 2 = 8 years = ... 6.20979
2 " = ... 1.83339
4.37640 × 40 = 175.05.6 *dollars.*

For *Half-yearly* and *Quarterly* payments, multiply the annuity for the given time by the amount in the following table:

Rate per Cent.	Half-yearly.	Quarterly.	Rate per Cent.	Half-yearly.	Quarterly.
3	1.007445	1.011181	$5\frac{1}{2}$	1.013567	1.020395
$3\frac{1}{2}$	1.008675	1.013031	6	1.014781	1.022257
4	1.009902	1.014877	$6\frac{1}{2}$	1.015993	1.024055
$4\frac{1}{2}$	1.011126	1.016720	7	1,017204	1.025880
5	1.012348	1.018559			

EXAMPLE.—What will an annuity of $50, payable yearly, amount to in 4 years, at 5 per cent., and what if payable half yearly?

By table, page 58, $1.05^4 = 1.2155$.

1.2155 − 1 ÷ (1.05 − 1) = 4.31, and 4.31 × 50 = 215.50 *dollars for yearly payment,*
and 215.50 × 1.012348, as by above table = 218.16 " " *half-yearly* "

PERPETUITIES.

Perpetuities are such Annuities as continue forever.

To Ascertain the Value of a Perpetual Annuity.

Rule.—Divide the annuity by the rate per cent., and multiply the quotient by the unit in the preceding table.

Example.—What is the present worth of an annuity for $100, payable semi-annually, at 5 per cent.?

100 ÷ .05 = 2.000, and 2,000 × 1.012348, from preceding table = 2.024.70 *dollars*.

For *Perpetuities in Reversion*, subtract the present worth of the annuity for the time of reversion from the worth of the annuity, to commence immediately.

Example.—What is the present worth of an estate of $50 per annum, at 5 per cent., to commence in 4 years?

50 ÷ .05.. = 1000

$50, for 4 years, at 5 per cent. = 3.54595 (from table) × 50 = 177.2975

822.7025 *dollars*,

which in 4 years, at 5 per cent. compound interest, would produce $1000.

COMBINATION.

Combination is a rule for ascertaining how often a less number of numbers or things can be chosen varied from a greater, or how many different collections may be formed without regard to the order of each collection.

The combinations of any number of things signify the different collections which may be formed of their quantities, without regard to the *order* of their arrangement.

Thus the 3 letters, *a*, *b*, *c*, taken *all together*, form but one combination, *abc*. Taken *two* and *two*, they form 3 combinations, as *ab*, *ac*, *bc*.

Rule.—Multiply together the natural series 1, 2, 3, etc., up to the number to be taken at a time. Take a series of as many terms, decreasing by 1, from the number out of which the combination is to be made, ascertain their continued product, and divide this last product by the former.

Example.—How many combinations may be made of 7 letters out of 12?

$$\frac{1\times 2\times 3\times 4\times 5\times 6\times 7}{12\times 11\times 10\times 9\times 8\times 7\times 6}=\frac{5040}{3991680}, \text{ and } \frac{3991680}{5040}=792.$$

When two Numbers or Things are Combined.

Rule.—Multiply together the natural series 1, 2, 3, etc., to a less term than the number of the combinations; ascertain their continued product, and proceed as before.

Note.—The class of the combination is determined by the number of elements or things to be taken; if two are taken the combination is of the 2d class, and so on.

Example.—There are 3 cards in a box, out of which 2 are to be drawn in a required order. Here there are 2 terms; hence 2 — 1 = 1, and $\frac{1}{3\times 2}=\frac{1}{6}=6\div 1=6$.

Combination without Repetitions.

Rule.—From the number of terms of the series subtract the number of the class of the combination, less 1; multiply this remainder by the successive increasing terms of the series, up to the last term of the series; then divide this product by the number of permutations of the terms, denoted by the class of the combination.

Example.—How many combinations can be made of 4 letters out of 10, excluding any repetition of them in any second combination?

10 — (4 — 1) = 7 = *number of terms — number of class, less* 1.

7 × 8 × 9 × 10 = 5040 = *prod. of remainder, and the successive terms up to the last term.*

1 × 2 × 3 × 4 = 24 = *permutations of the class of the combination.* Then $\frac{5040}{24}=210$.

Combinations with Repetitions.

In this case the repetition of a term is considered a new combination. Thus 1, 2, admits of but one combination, if not repeated; if repeated, however, it admits of three combinations, as 1, 1; 1, 2; 2, 2.

RULE.—To the number of the terms of the series add the number of the class of the combination, less 1; multiply the sum by the successive decreasing terms of the series, down to the last term of the series; then divide this product by the number of permutations of the terms, denoted by the class of the combination.

EXAMPLE.—How many different combinations of numbers of 6 figures can be made out of 11?

$11+(6-1)=16=$ *sum of number of terms, and the number of class, less* 1.
$16\times15\times14\times13\times12\times11=5765760=$ *prod. of sum, and successive terms to last term.*
$1\times2\times3\times4\times5\times6=720=$ *permutations of the class of the combination.* Then $\frac{5765760}{720}=8008.$

Variations with Repetitions.

Every different arrangement of individual number or things, including repetitions, is termed a Variation.

The *Class* of the Variation is denoted by the number of individual things taken at a time.

RULE.—Raise the number denoting the individual things to a power, the exponent of which is the number expressing the class of the variation.

EXAMPLE.—How many variations with 4 repetitions can be made out of 5 figures?

$$5^4=625.$$

PROBABILITY.

The Probability of any event is the *ratio* of the favorable cases to all the cases which are similarly circumstanced with regard to the occurrence. Thus, from a receptacle containing 1 white and 2 black balls, the probability of drawing a white ball, by the abstraction of 1, is $\frac{1}{2}$; the probability of throwing ace with a die is $\frac{1}{6}$: in other words, the odds are 2 to 1 against the first, and 5 to 1 against the second.

If $m+n=$ the whole number of chances, m *represents the number which are favorable and* n *the unfavorable.* $\frac{m}{m+n}=$ *the probability of the event.*

ILLUSTRATION.—If a cent is thrown twice into the air, the probability of its falling with its head up, twice in succession, is as 1 to 4. Thus, it may fall:

1. The head up twice in succession.
2. The head up the 1st time and the wreath the 2d time.
3. The wreath up the 1st time and the head the 2d time.
4. The wreath up twice in succession.

These are the only results possible, and being all similarly circumstanced as to probability, the probability of each case is as 1 to 4, or the odds are as 3 to 1.

The probability of either the head or wreath being up twice in succession is as 1 to 1, or the chances are even, because the 1st and 4th cases favor such a result; the probability of the head once and the wreath once in any order is as 1 to 2, because the 2d and 3d cases favor such a result; and the probability of the head or the wreath once is as 3 to 4, or the odds are as 3 to 1, because the 1st, 2d, and 3d, or the 2d, 3d, and 4th cases, favor such a result.

NOTE.—1 to 2 is an equal chance, for 1 out of 2 chances $=1$ to 1, being an equal chance; again, 1 to 5 is 4 to 1, for 1 out of 5 chances is 1 to 4.

ILLUS. 2.—Suppose with two bags, one containing 5 white balls and 2 black, and the other 7 white and 3 black. The number of cases possible in one drawing from each bag is $(5+2)\times(7+3)=7\times10=70$, because every ball in one bag may be drawn alike to one in the other.

The number of cases which favor the drawing of a white ball from both bags is $5\times7=35$, for every one of the 5 white balls in one bag may be drawn in combination with every one of the 7 in the other. For a like cause, the number of cases which form the drawing of a white ball from the 1st bag and a black one from the 2d is $5\times3=15$; a black ball from the 1st bag and a white ball from the 2d is $7\times2=14$; and a black ball from both is $3\times2=5$.

Therefore the probability of drawing is as

$\frac{5\times7}{70}\times\frac{35}{70}=\frac{1}{2}=1$ to 1, *a white ball from both bags.*

$\frac{5\times3}{70}=\frac{15}{70}=\frac{3}{14}=11$ to 3, *a white ball from the 1st, and a black from the 2d.*

$\frac{7\times2}{70}=\frac{14}{70}=\frac{1}{5}=4$ to 1, *a black ball from the 1st, and a white from the 2d.*

$\frac{3\times2}{70}=\frac{6}{70}=\frac{3}{35}=32$ to 3, *a black ball from both.*

$\frac{5\times3+2\times7}{70}=\frac{29}{70}=41$ to 29, *a white ball from one, and a black from the other,* for both the 2d and third cases favor this result; hence $\frac{1}{5}+\frac{3}{14}=\frac{29}{70}$.

$\frac{5\times7+5\times3+2\times7}{70}=\frac{64}{70}=\frac{32}{35}=33$ to 32, *at least one white ball* for the 1st, 2d, and 3d cases form this result; hence $\frac{1}{2}+\frac{3}{14}+\frac{1}{5}=\frac{32}{35}$.

Again, if the number of white and black balls in each bag are the same, say 5 white and 2 black, $5+2\times5+2=49$, then the probability of drawing is as

$\frac{5\times5}{49}=\frac{25}{49}=25$ to 24, *a white ball from both.*

$\frac{5\times2}{49}=\frac{10}{49}=39$ to 10, *a white ball from the 1st, and a black from the 2d.*

$\frac{2\times5}{49}=\frac{10}{49}=39$ to 10, *a black ball from the 1st, and a white from the 2d.*

$\frac{2\times2}{49}=\frac{4}{49}=45$ to 4, *a black ball from both.*

ILLUS. 3.—When two dice are thrown, the probability that the sum of the numbers on the upper sides is any given number, say 7, is as follows:

As every one of the six numbers on one of the dice may come up alike to or in combination with the other, the number of throws is $6\times6=36$.

The number 7 may be a combination of $\left\{\begin{array}{l}1 \text{ and } 6\\2 \text{ `` } 5\\3 \text{ `` } 4\end{array}\right\}$; and as these numbers may be upon either dice, there are $3\times2=6$ throws in favor of the combination of 7; hence the probability of throwing 7 is $\frac{6}{36}=\frac{1}{6}$, or as 5 to 1.

ILLUS. 4.—The probability of a player's partner at Whist holding a given card is as follows:

The number of cards held by the other 3 players is $3\times13=39$; the probability, therefore, that it is held by the partner is $\frac{1}{39}$, but it may be one of the 13 cards which he holds; hence the probability is $1\times13=\frac{13}{39}=\frac{1}{3}$, or as 2 to 1.

ILLUS. 5.—The probability of a player's partner at Whist holding two given cards is as follows:

The number of combinations of 39 things, taken 2 and 2 together, is $\frac{1\times2}{39\times38}=\frac{2}{741}$; therefore the probability that these 2 cards are any given 2 cards in the partner's hand is $\frac{1}{\frac{39\times38}{1\times2}}=\frac{1}{39\times19}=\frac{1}{741}=740$ to 1; but they may be any 2 cards in the partner's hand; therefore, since the number of combinations of 13 cards, taken 2 and 2 together, is $\frac{1\times2}{13\times12}=\frac{2}{156}=78$, the probability required is $\frac{78}{741}=\frac{2}{19}$, or as 17 to 2.

Similarly, the probability that he holds any 3 given cards is as $\frac{22}{703}$, or as 681 to 22.

The probabilities at a game of Whist upon the following points are:

7 to 5, *that one hand has two honors, and two hands one;*
13 to 2, *that two hands have each two honors;*
17 to 2, *that each hand holds an honor;*
5 to 1, *that one hand has three honors, and one hand one;*
94 to 1, *that the four honors are held by one hand.*

ILLUS. 5.—If 5 half dollars are thrown into the air, the probability of any of the possible combinations of their falling is as follows:

$$\left(\frac{1}{2}+\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^5 + \frac{5}{1}\left(\frac{1}{2}\right)^5 + \frac{5\times4}{1\times2}\left(\frac{1}{2}\right)^5 + \frac{5\times4}{5\times4}\left(\frac{1}{2}\right)^5 + \frac{5}{1}\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5.$$

Hence the probabilities are:

$\left(\frac{1}{2}\right)^5 = .03125 = 1$ to 31 — *5 heads;*

$5\times\left(\frac{1}{2}\right)^5 = .15625 = 5$ to 27 — *4 heads and 1 tail;*

$10\times\left(\frac{1}{2}\right)^5 = .3125 = 5$ to 11 — *3 heads and 2 tails;*

$10\times\left(\frac{1}{2}\right)^5 = .3125 = 5$ to 11 — *2 heads and 3 tails;*

$5\times\left(\frac{1}{2}\right)^5 = .15625 = 5$ to 27 — *1 head and 4 tails;*

$\left(\frac{1}{2}\right)^5 = .03125 = 1$ to 31 — *5 tails.*

All Wagers are founded upon the principle of the product of the event, and the contingent gain being equal to the amount at stake.

ILLUSTRATION.—Suppose 3 horses, A, B, and C, are started for a race, and I have wagered 12 to 5 against A, 11 to 6 against B, and 10 to 7 against C.

If A wins, I win $6+7-12=1$.
" B " I " $5+7-11=1$.
" C " I " $5+6-10=1$.

Hence I win 1, whichever horse wins, from having taken the field against each horse at the odds named.

The odds given in favor of $\begin{cases}\text{A are 5 to 12}\\ \text{B " 6 " 11}\\ \text{C " 7 " 10}\end{cases}$; the corresponding probability is $\begin{cases}\frac{5}{17}\text{ in favor of A,}\\ \frac{6}{17}\text{ " B,}\\ \frac{7}{17}\text{ " C,}\end{cases}$

and $\frac{5}{17}+\frac{6}{17}+\frac{7}{17}=\frac{18}{17}=1.06=1.06$ *to* 1 *in favor of the taker of the odds.*

The odds given upon the first seven favorite horses entered for the Oaks Stakes of 1828 were so great that the probability in favor of the taker of the odds when reduced was as follows:

The odds were { 1. 5 to 2   2. 5 to 2   3. 4 to 1   4. 7 to 1   5. 14 to 1   6. 14 to 1   7. 15 to 1

$$=\frac{2}{7}+\frac{2}{7}+\frac{1}{5}+\frac{1}{8}+\frac{1}{15}+\frac{1}{15}+\frac{1}{16}=\frac{4}{7}+\frac{5}{15}+\frac{3}{16}=\frac{4}{7}+\frac{1}{3}+\frac{3}{16}=\begin{cases}4\times3\times16=192\\ 1\times7\times16=112\\ 3\times7\times\ 3=\ 63\\ \overline{7\times3\times16\quad 336}\end{cases}$$

$=\frac{367}{336}=1.092=1.092$ *to* 1, *in favor of the taker of the odds*, yet neither of the horses upon which these odds were given won.

ILLUS. 2.—If the odds are 3 to 1 against a horse running a race, and 6 to 1 against another horse winning a second race, the probability of the 1st horse winning is $\frac{1}{4}$, and of the other $\frac{1}{7}$. Therefore the probability of both the races being won is $\frac{1}{28}$, and the odds against it 27 to 1, or 1000 to 37.037. The odds upon such an event were given in 1828 at 1000 to 60, or 16.67 to 1.

Table showing the Odds between Results or Chances, and between any Number and the Whole Number, at the various Odds against each, also the Value of each Chance in parts of 100.

Odds against each.	Value of Chance.	Odds against each.	Value of Chance.	Odds against each.	Value of Chance.
Even	50.	3½ to 1	22.22	9½ to 1	9.52
11 to 10	47.62	4 " 1	20.	10 " 1	9.09
6 " 5	45.45	4½ " 1	18.18	12 " 1	7.7
5 " 4	44.44	5 " 1	16.66	15 " 1	6.25
5½ " 4	42.1	5½ " 1	15.38	18 " 1	5.26
6 " 4	40.	6 " 1	14.28	20 " 1	4.76
6½ " 4	38.1	6½ " 1	13.33	25 " 1	3.84
7 " 4	36.36	7 " 1	12.5	30 " 1	3.22
7½ " 4	34.78	7½ " 1	11.76	40 " 1	2.44
2 " 1	33.33	8 " 1	11.11	50 " 1	1.96
2½ " 1	28.57	8½ " 1	10.52	60 " 1	1.64
3 " 1	25.	9 " 1	10.	100 " 1	.99

OPERATION.—Divide 100, or the unit, as the case may be, by the sum of the odds, and multiply the quotient by the lesser chance or odds.

ILLUSTRATION.—6 to 4. $6 + 4 = 10$, and $\frac{100}{10} \times 4 = 40 =$ *value of chance.*

CHRONOLOGY.

A *Solar day* is measured by the rotation of the earth upon its axis with respect to the Sun.

The motion of the Earth, on account of the ellipticity of its orbit and of the perturbations produced by the planets, is subject to an acceleration and retardation. To correct this fluctuation, time-pieces are adjusted to an average or mean solar day (*mean time*), which is divided into hours, minutes, and seconds.

In *Astronomical* computation and in *Nautical* time the day commences at M., and in the former it is counted throughout the 24 hours.

In *Civil* computation the day commences at midnight, or A.M., and is divided into two portions of 12 hours each.

A *Solar Year*, termed also an *Equinoctial*, *Tropical*, *Civil*, or *Calendar Year*, is the time in which the Sun returns from one Vernal Equinox to another; and its average time, termed a *Mean Solar Year*, is 365.24224 solar days, or 365 *days*, 5 *hours*, 48 *minutes*, and 49.536 *seconds*.

A *Year* is divided into 12 calendar months, or 365 days.

A *Calendar Month* varies from 28 to 31 days.

A *Mean Lunar Month*,* or lunation of the moon, is 29 days, 12 hours, 44 minutes, 2 seconds, and 5.24 thirds.

A *Bissextile* or *Leap Year* consists of 366 days; the correction of one year in four is termed the *Julian;* hence a mean *Julian* year is 365.25 days.

In the year 1582 the error of the Julian computation of a year had amounted to a period of 10 days, which, by order of Pope Gregory VIII., was suppressed in the Calendar, and the 5th of October reckoned as the 15th.

The error of the Julian computation, .00776 days, is about 1 day in 128.79 years, and the adoption of this period as a basis of intercalation is termed the *Gregorian Calendar*, or *New Style*,† the *Julian Calendar* being termed the *Old Style*.

The error of the Gregorian year (365.2425 days) amounts to 1 day in 3571.4286 years.

The New Style was adopted in England in 1752 by reckoning the 3d of September as the 14th.

By an English law, the years 1800, 1900, 2100, 2200, 2300, and 2500, and any other 100th year, excepting only every 400th year, commencing at 2000, are not to be reckoned Bissextile years.

* Ferguson.

† Now adopted in every Christian country except Russia.

The *Dominical* or *Sunday Letter* is one of the first seven letters of the alphabet, and is used for the purpose of determining the day of the week corresponding to any given date. In the *Ecclesiastical Calendar* the letter A is placed opposite to the 1st day of the year, January 1st; B to the second; and so on through the seven letters; then the letter which falls opposite to the first Sunday in the year will also fall opposite to every following Sunday in that year.

In the *Ecclesiastical Year* the intercalary day is reckoned upon the 24th of February; hence the 24th and 25th days are denoted by the same letter, the dominical letter being set back one place.

In the *Civil Year* the intercalary day is added at the end of February, the change of letter taking place at the 1st of March.

To Compute the Dominical Letter.

RULE.—Divide the number of centuries and the years of the given century each by 4, and the years again by 7; multiply the remainders respectively by 2, 2, and 4; add together the three products, and increase their sum by 1; then divide the whole sum by 7, and the remainder will be the ordinal number of the dominical letter required.

NOTE.—If 0 remain, it will be the 7th, or G.

EXAMPLE.—What will be the dominical letter for the year 1942?

Centuries $= 19$, which $\div 4 = 4$, and 3 rem.; years $= 42$, which $\div 4 = 10$, and 2 rem.; and 42 again by $7 = 6$, hence the remainders $= 3.2.0$.

Then, $3 \times 2 = 6$; $2 \times 2 = 4$, and $0 \times 4 = 0$, and $6+4+0+1 = 11$, and $11 \div 7 = 4$ *remainder, the ordinal number of the required letter being* D.

NOTE.—In bissextile years two dominical letters are used, one before and the other after the intercalary day.

A *Dominical Cycle* is a period of 400 years, when the same order of dominical letters and days of the week will return.

A *Cycle of the Sun*, or the *Sunday Cycle*, is the 28 years before the same order of dominical letters return to the same days of the month, and it is considered as having commenced 9 years before the era of the Julian Calendar.

To Compute the Cycle of the Sun.

RULE.—Add 9 to the given year; divide the sum by 28; the quotient is the number of cycles that have elapsed, and the remainder is the number or years of the cycle.

NOTE.—The use of this computation is the determination of the dominical letter for any given year of the Julian Calendar for each of the 28 years of a cycle.

By the adoption of the *Gregorian Calendar*, the order of the letters is necessarily interrupted by the suppression of the century bissextile years in 1900, 2100, 2200, etc., and a table of dominical letters must necessarily be reconstructed for the following century.

The *Lunar Cycle*, or *Golden Number*, is a period of 19 years, after which the new moons fall on the same days of the month of the Julian year, within 1.5 hours.
The year of the birth of Jesus Christ is reckoned the first of the Lunar Cycle.

To Compute the Lunar Cycle, or Golden Number.

RULE.—Add 1 to the given year; divide the sum by 19, and the remainder is the golden number.

NOTE.—If 0 remain, it is 19.

EXAMPLE.—What is the golden number for 1866?

$1866 + 1 \div 19 = 98$, and the remainder $= 5 =$ *the golden number.*

NOTE.—There are two objections to the permanency of this lunar period. In the first place, the Julian year, which is the basis of the computation, does not correctly represent either the mean solar or the Gregorian year. In the second place, a period of 6939.75 days is nearly 1.5 hours in excess of 235 mean astronomical lunations; and hence the correct time of the new moon will in successive cycles occur so much earlier, and will retrogade a day in every 308 years.

The *Epact* for any year is a number designed to represent the age of the moon on the 1st day of January of that year.

To Compute the Epact.

RULE.—Multiply the golden number less 1 by 11, divide the product by 30, and the remainder is the result required.

NOTE.—If 0 remain, it is 29.

EXAMPLE.—Required the epact for 1865?

The golden number $= 4$.

$4 - 1 \times 11 = 33$, and $33 \div 30 = 1$ and 3 *remainder;* hence $3 =$ *epact.*

To Compute the Roman Indiction.

RULE.—Add 3 to the given year; divide the sum by 15, and the remainder is the indiction.

NOTE.—If 0 remain, the Indiction is 15.

The *Dionysian Period* is a period of 532 years, the product of the lunar and solar cycles (19×28), and it was designed for the purpose of including all the varieties of the new moons and dominical letters, so that after every 532 years they were expected to recur in the same order. The measure of the lunar cycle, however, not being exact, and the Sunday cycle being interrupted at the centenary years that are not bissextile, this period is altogether in disuse.

$129 +$ (given year $- 1800$) $=$ year of the Dionysian, extending to 2203.

The *Number of Direction* is the number of days that Easter-day occurs after the 21st of March.

Easter-day is the first Sunday after the first full moon which occurs upon or next after the 21st of March; and if the full moon occurs upon a Sunday, then Easter-day is the Sunday after, and it is ascertained by adding the number of direction to the 21st of March. It is therefore March $N + 21$, or April $N - 10$.

ILLUSTRATION.—If the number of direction is 19, then for March, $19 + 21 = 40$, and $40 - 31 = 9 = $ *9th of April;*

again for April, $19 - 10 = 9 =$ *9th of April.*

NOTE.—The moon upon which Easter immediately depends is termed the *Paschal Moon.*

Full Moon is the 14th day of the moon, that is, 13 days after the preceding day of the new moon.

Perpetual Table for Ascertaining the Number of Direction, the Epact and Dominical Letter being given.

Epact.	Dominical Letter.							Epact.	Dominical Letter.						
	A	B	C	D	E	F	G		A	B	C	D	E	F	G
	No.	No.	No.	No.	No.	No.	No.		No.	No.	No.	No.	No.	No.	No.
0	26	27	28	29	30	24	25	15	12	15	14	15	9	10	11
1	26	27	28	29	23	24	25	16	12	13	14	8	9	10	11
2	26	27	28	22	23	24	25	17	12	13	7	8	9	10	11
3	26	27	21	22	23	24	25	18	12	6	7	8	9	10	11
4	26	20	21	22	23	24	25	19	5	6	7	8	9	10	11
5	19	20	21	22	23	24	25	20	5	6	7	8	9	10	4
6	19	20	21	22	23	24	18	21	5	6	7	8	9	3	4
7	19	20	21	22	23	17	18	22	5	6	7	8	2	3	4
8	19	20	21	22	16	17	18	23	5	6	7	1	2	3	4
9	19	20	21	15	16	17	18	24	33	34	35	29	30	31	32
10	19	20	14	15	16	17	18	25	33	34	35	29	30	31	32
11	19	13	14	15	16	17	18	26	33	34	28	29	30	31	32
12	12	13	14	15	16	17	18	27	33	27	28	39	30	31	32
13	12	13	14	15	16	17	11	28	26	27	28	29	30	31	32
14	12	13	14	15	16	10	11	29	26	27	28	29	30	31	25

Perpetual Table for Ascertaining Easter-day, the Epact and Dominical Letter being given.

Epact.	Dominical Letter. A	B	C	D	E	F	G
	Apr.	Apr.	Apr.	Apr.	Apr.	Apr.	Apr.
0	16	17	18	19	20	14	15
1	16	17	18	19	13	14	15
2	16	17	18	12	13	14	15
3	16	17	11	12	13	14	15
4	16	10	11	12	13	14	15
5	9	10	11	12	13	14	15
6	9	10	11	12	13	14	8
7	9	10	11	12	13	7	8
8	9	10	11	12	6	7	8
9	9	10	11	5	6	7	8
10	9	10	4	5	6	7	8
11	9	3	4	5	6	7	8
12	2	3	4	5	6	7	8
13	2	3	4	5	6	7	1
						Mar.	
14	2	3	4	5	6	31	1
					Mar.		
15	2	3	4	5	30	31	1

Epact.	Dominical Letter. A	B	C	D	E	F	G
				Mar.			
16	2	3	4	29	30	31	1
			Mar.				
17	2	3	28	29	30	31	1
		Mar.					
18	2	27	28	29	30	31	1
	Mar.						
19	26	27	28	29	30	31	1
							Mar.
20	26	27	28	29	30	31	25
21	26	27	28	29	30	24	25
22	26	27	28	29	23	24	25
23	26	27	28	22	23	24	25
	Apr.	Apr.	Apr.	Apr.	Apr.	Apr.	Apr.
24	23	24	25	19	20	21	22
25	23	24	25	19	20	21	22
26	23	24	18	19	20	21	22
27	23	17	18	19	20	21	22
28	16	17	18	19	20	21	22
29	16	17	18	19	20	21	15

The *Roman Indiction* is a period of 15 years, in use by the Romans. The precise time of its adoption is not known beyond the fact that the year 313 A.D. was a first year of a Cycle of Indiction.

The *Julian Period* is a cycle of 7980 years, the product of the Lunar and Solar Cycles and the Indiction ($19 \times 28 \times 15$), and it commences at 4714 years B.C.

6513 + (given year — 1800) = year of the Julian Period, extending to 3267.

NOTE.—If the year of the Julian Period is divided by 19, 28, 15, or 32, the remainders will respectively give the *Lunar* and *Solar Cycles*, the *Indiction*, and the *Year of the Dionysian.*

Dates of the Day of the Week, corresponding to the Day determined by the preceding Table.

Thus, if Monday is the day determined by the year given, the following dates are the Mondays in that year :

February, March, November.	February,* August.	May.	January, October.	January,* April, July.	September, December.	June.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOTE.—In leap-year, January and February must be taken in the columns marked *.

To Compute the Moon's Age.

RULE.—To the day of the month add the *epact* and *number* of the month, then subtract 29 days 12 hours and 44 minutes (the period of a mean lunation) as often as the sum exceeds this period, and the result will give the moon's age.

NOTE.—This is an approximate rule, serviceable only when the lunations of the moon are not known with precision.

Numbers of the Month.

	d.	h.		d.	h.		d.	h.		d.	h.
January	0	0	April	1	16	July	4	2	October	7	11
February	1	17	May	2	3	August	5	13	November	8	23
March		4	June	3	14	September	7		December	9	10

EXAMPLE.—Required the age of the Moon on the 5th of February, 1859.

Given day 5
Epact 26
Number of month .. 1.17

} 32*d.* 17*h.*, from which subtract 29.12. = 2 *days* 5 *hours.*

Table of Epacts, Dominical Letters, and an Almanac, from 1773 to 1901.

Years.	Days.	Dom. Letters.	Epact.	Years.	Days.	Dom. Letters.	Epact.	Years.	Days.	Dom. Letters.	Epact.
1773	Tuesday.	C	6	1816	Friday.*	GF	1	1859	Tuesday.	B	26
1774	Wednesd.	B	17	1817	Saturday.	E	12	1860	Thursday.*	AG	7
1775	Thursday.	A	28	1818	Sunday.	D	23	1861	Friday.	F	18
1776	Friday.*	GF	9	1819	Monday.	C	4	1862	Saturday.	E	29
1777	Saturday.	E	20	1820	Wednesd.*	BA	15	1863	Sunday.	D	11
1778	Sunday.	D	1	1821	Thursday.	G	26	1864	Tuesday.*	CB	22
1779	Monday.	C	12	1822	Friday.	F	7	1865	Wednesd.	A	3
1780	Wednesd.*	BA	23	1823	Saturday.	E	18	1866	Thursday.	G	14
1781	Thursday.	G	4	1824	Monday.*	DC	29	1867	Friday.	F	25
1782	Friday.	F	15	1825	Tuesday.	B	11	1868	Sunday.*	ED	6
1783	Saturday.	E	26	1826	Wednesd.	A	22	1869	Monday.	C	17
1784	Monday.*	DC	7	1827	Thursday.	G	3	1870	Tuesday.	B	28
1785	Tuesday.	B	18	1828	Saturday.*	FE	14	1871	Wednesd.	A	9
1786	Wednesd.	A	29	1829	Sunday.	D	25	1872	Friday.*	GF	20
1787	Thursday.	G	11	1830	Monday.	C	6	1873	Saturday.	E	1
1788	Saturday.*	FE	22	1831	Tuesday.	B	17	1874	Sunday.	D	12
1789	Sunday.	D	3	1832	Thursday.*	AG	28	1875	Monday.	C	23
1790	Monday.	C	14	1833	Friday.	F	9	1876	Wednesd.*	BA	4
1791	Tuesday.	B	25	1834	Saturday.	E	20	1877	Thursday.	G	15
1792	Thursday.*	AG	6	1835	Sunday.	D	1	1878	Friday.	F	26
1793	Friday.	F	17	1836	Tuesday.*	CB	12	1879	Saturday.	E	7
1794	Saturday.	E	28	1837	Wednesd.	A	23	1880	Monday.*	DC	18
1795	Sunday.	D	9	1838	Thursday.	G	4	1881	Tuesday.	B	29
1796	Tuesday.*	CB	20	1839	Friday.	F	15	1882	Wednesd.	A	11
1797	Wednesd.	A	1	1840	Sunday.*	ED	26	1883	Thursday.	G	22
1798	Thursday.	G	12	1841	Monday.	C	7	1884	Saturday.*	FE	3
1799	Friday.	F	23	1842	Tuesday.	B	18	1885	Sunday.	D	14
1800	Saturday.	E	4	1843	Wednesd.	A	29	1886	Monday.	C	25
1801	Sunday.	D	15	1844	Friday.*	GF	11	1887	Tuesday.	B	6
1802	Monday.	C	26	1845	Saturday.	E	22	1888	Thursday.*	AG	17
1803	Tuesday.	B	7	1846	Sunday.	D	3	1889	Friday.	F	28
1804	Thursday.*	AG	18	1847	Monday.	C	14	1890	Saturday.	E	9
1805	Friday.	F	20	1848	Wednesd.*	BA	25	1891	Sunday.	D	20
1806	Saturday.	E	11	1849	Thursday.	G	6	1892	Tuesday.*	CB	1
1807	Sunday.	D	22	1850	Friday.	F	17	1893	Wednesd.	A	12
1808	Tuesday.*	CB	3	1851	Saturday.	E	28	1894	Thursday.	G	23
1809	Wednesd.	A	14	1852	Monday.*	DC	9	1895	Friday.	F	4
1810	Thursday.	G	25	1853	Tuesday.	B	20	1896	Sunday.*	ED	15
1811	Friday.	F	6	1854	Wednesd.	A	1	1897	Monday.	C	26
1812	Sunday.*	ED	17	1855	Thursday.	G	12	1898	Tuesday.	B	7
1813	Monday.	C	28	1856	Saturday.*	FE	23	1899	Wednesd.	A	18
1814	Tuesday.	B	9	1857	Sunday.	D	4	1900	Thursday.	G	29
1815	Wednesd.	A	20	1858	Monday.	C	15	1901	Friday.	F	10

USE OF THE TABLE.—To ascertain the day of the week on which any given day of the month falls in any year from 1773 to 1901.

ILLUSTRATION.—The great fire occurred in New York on the 16th of December, 1835: what was the day of the week?

Opposite 1835 is Sunday; and by the preceding table, under December, it is ascertained that the 13th was Sunday: *consequently, the 16th was Wednesday.*

Table showing the Age of the Moon on the Day preceding the first Day of a Month for the Years 1876 to 1880, at 12 M., at New York.

[By S. H. Wright, A.M., Ph.D.]

Month.	1876.	1877.	1878.	1879.	1880.
	D. H. M.	D. H. M.	D. H. M.	D. H. M.	D. H. M.
January	3 21 47	15 22 37	26 18 46	7 19 17	18 5 34
February........	5 2 57	17 3 14	28 2 32	9 5	19 18 13
March	4 10 23	15 7 8	26 8 27	7 12 58	19 5 53
April	5 20 41	16 14 6	27 21 24	8 19 56	20 16 16
May	6 10	16 23 12	27 19 47	9 3	21 1 40
June............	8 1 38	18 11 24	29 3 57	10 10 44	22 10 19
July............	8 18 34	19 2 15	4 15	10 20 18	22 18 48
August	10 11 49	20 18 39	1 19 12	12 7 45	24 3 32
September	12 4 20	22 11 30	3 11 8	13 20 55	25 13 16
October	12 19 7	23 4 3	4 3 2	14 11 9	26 0 15
November.......	14 7 16	24 14 16	5 17 45	16 1 44	26 12 4
December.......	14 16 31	25 8 12	6 7 35	16 17 41	28 0 28

January, 1881, 3*h.*

Table showing the Times or Periods of New Moons at New York for the Years 1876 to 1880.

Month.	1876.	1877.	1878.	1879.	1880.
	D. H. M.	D. H. M.	D. H. M.	D. H. M.	D. H. M.
Jan....	26 9 3 A.M.	14 8 46 A.M.	3 9 28 A.M.	22 7 A.M.	11 5 47 P.M.
Feb....	25 1 37 "	13 4 52 "	2 3 33 "	20 11 2 P.M.	10 6 24 A.M.
March .	25 3 19 P.M.	14 9 54 P.M.	3 2 36 P.M.	22 4 4 "	10 7 44 P.M.
April ..	24 2 A.M.	13 0 48 "	2 4 13 "	21 9 A.M.	9 10 20 A.M.
May ...	23 10 22 "	13 0 36 A.M.	{2 8 3 A.M. {31 9 4 P.M.	21 1 16 "	9 1 41 "
June ..	21 5 26 P.M.	11 9 45 "	30 7 45 A.M.	19 3 42 P.M.	7 5 12 P.M.
July ...	21 0 11 A.M.	10 5 21 P.M.	29 4 48 P.M.	19 4 15 A.M.	7 8 28 A.M.
August	19 7 40 "	9 0 30 A.M.	28 0 52 A.M.	17 3 5 P.M.	5 10 44 P.M.
Sept...	17 4 53 P.M.	7 7 57 "	26 8 58 "	16 0 51 A.M.	4 11 45 A.M.
Oct....	17 4 44 A.M.	6 4 44 P.M.	25 6 15 P.M.	15 10 16 "	3 11 56 P.M.
Nov ...	15 7 29 P.M.	5 3 48 A.M.	24 4 25 A.M.	13 6 19 P.M.	2 11 32 A.M.
Dec....	15 1 23 "	4 5 14 P.M.	23 4 42 P.M.	13 6 26 A.M.	{1 10 13 P.M. {31 9 A.M.

January, 1881, 30*d.* 0*h.* 36*m.* A.M.

To Compute the Age of the Moon at Meridian on any Day in the Years 1876 to 1880.

Rule.—Look in the table for the year, and under it, in the line opposite to the given month, is given a period which, when added to the day of the month, will give the age of the moon, or the period since it was new, on the meridian of New York.

When, however, the sum of this period and the day of the month exceeds the period of the actual lunation, subtract the period of the lunation therefrom; and if the remainder yet exceeds the lunation, subtract a like period, and the remainder will give the age in days, hours, and minutes.

Note.—*When the Period of an actual Lunation can not be determined*, subtract 29 *days*, 12 *hours*, 44 *min.*, which is the period of a mean lunation, and proceed as for an actual lunation.

EXAMPLE.—Required the age of the moon in New York on the 22d of May, 1879, at 12 M.

Under 1879, and opposite to May, in the table preceding (p. 70), is 9*d.* 3*h.*, to which add 22 days = 31 *days* 3 *hours;* from which subtract the period of a mean lunation = (31*d.* 3*h.*) — (29*d.* 12*h.* 44*m.*) = 1*d.* 14*h.* 16*m.*, *the age required.*

By Actual Lunation, as by the following Rule, the age is as follows:

(Time of new moon for May, 1879, 21*d.* 1*h.* 16*m.* A.M.) + 30 *days* in the preceding month, April = (51*d.* 1*h.* 16*m.*); from which subtract the time of the preceding new moon (table, p. 70), (21*d.* 9*h.* A.M.) = 29*d.* 16*h.* 16*m.*, the moon's actual age when new in May. Hence (51*d.* 3*h*) — (29*d.* 16*h.* 16*m.*) = 1*d.* 10*h.* 44*m.*, the moon's age at 12 M. on 22d May.

To Compute the Period of Lunation.

RULE.—From the sum of the time of the new moon at which a required lunation ends, and the number of days in the preceding month, subtract the time of the preceding new moon, and the result will give the period required.

NOTE.—If the minuend is in P.M., and subtrahend in A.M., add 12 hours to the result; and if the minuend is in A.M., and the subtrahend in P.M., subtract 12 hours from the result.

A Mean Lunation is 29.5305387 *days* = 29 *days* 12 *hours* 44 *min.* 2 *sec.* and 5.24 *primes;* but a *True Lunation* varies with every moon; hence, when the sum of the epact and day of the month exceeds 29*d.* 12*h.* 44*m.*, the age of the moon will not be ascertained with precision unless the lunation under computation happens to be the exact length of the mean. But all epacts and days, the sum of which is less than 29*d.* 12*h.* 44*m.*, can be ascertained by the use of the table and rule with exactness.

Thus, on the 9th of January, 1864, the moon's age at New York was 29*d.* 20*h.* 33*m.* — 29*d.* 12*h.* 44*m.* = 7*h.* 49*m.;* but 9*h.* 11*m.* was the exact age, and *that* lunation was 29*d.* 11*h.* 22*m.* Again, on the 8th of January, the day preceding, the moon's age was 28*d.* 20*h.* 33*m.* at 12 M., the exact time given by a like table and rule.

EXAMPLE.—Required the period of lunation ending 20th February, 1879.

Time of new moon 20*d.* 11*h.* 2*m.* P.M. (table, p. 70) + 31 *days* in the preceding month, January = (51*d.* 11*h.* 2*m.*) — (22*d.* 7*h.* A.M.), time of preceding new moon (table, p. 70) = 29*d.* 4*h.* 2*m.;* to which add 12 hours for period from P.M. to A M. = 29*d.* 16*h.* 2*m.*, *the period required.*

To Compute the Age of the Moon at Mean Noon at any other Location than at New York, or that given in the Table.

RULE.—Ascertain the age as per preceding rule, and add or subtract the difference of longitude or time, according as the place may be west or east of it, to or from the time determined by the rule.

EXAMPLE.—What will be the age of the moon at Cincinnati, Ohio, on the 1st of January, 1879?

Difference of longitude = 41 *min.* 58 *sec.;* hence, by table and rule page 70, age of the moon at New York on that day at 12 M. is 7 *days* 19 *hours* 17 *min.;* to which add 41 *min.* 58 *sec.* = 7 *days* 19 *hours* 58 *min.* 58 *sec.*

NOTE.—The time of the moon's Southing, or of its age, when ascertained for New York, will answer approximately for any part of the Atlantic coast, as it crosses the different meridians at the same *relative* time that it does at New York, and does not, therefore, need a correction for any difference of longitude when the longitude is not very remote.

The moon's Southing, as usually given in the United States Almanacs, both Civil and Nautical, is computed for Washington; but as the time-table (p. 90) is computed for the meridian of New York, this location is given in the preceding tables, for the purpose of maintaining a uniformity of expression.

When the time of new moon is ascertained for a location, and it is required to ascertain it for any other, *add* the difference of longitude or time of the place, if east, and *subtract* it if it is west of it.

To Compute the Time of High-water next after the Moon's Transit, or Southing, at Different Locations, without the aid of a Nautical Almanac.

Rule.—From the table (p. 70) note the period of time under the year, and opposite to the month for which the time of the high-water is required.

To this number add the day of the month, and subtract the period of the actual lunation, if it can be determined, and if not, 29*d.* 12*h.* 44*m.* (a mean lunation) from the sum, when it exceeds the period of the lunation used.

Opposite to this, the age of the moon, in the left-hand column of the following table, note the hours and minutes in the adjoining column, which add to the time of high-water of the given place, on the days of the full and change of the moon, or *the establishment of the Port* (for which see following table), and subtract 12*h.* 26*m.*, or 24*h.* 52*m.* (*a lunar day*), whenever the result exceeds either of these times, and the remainder will give the time of high-water required.

Example.—Required the time of high-water at New York, January 30th, 1879, next after the moon's transit.

In table, opposite to January and under 1879 is 7*d.* 19*h.* 7*m.*, which being added to 30, the day of the month = 37*d.* 19*h.* 7*m.* — 29*d.* 14*h.* 7*m.* (the actual lunation ending with the new moon in January, 1879) = 8*d.* 5*h.* = the moon's age on that day at 12 M.

Opposite to 8*d.* in the following table (p. 73) is 6*h.* 44*m.* P.M., and the difference between that and 8.5*d.* or 12*h.* = 25*m.*; hence 8*d.* 5*h.* — 8*d.* = 5*h.*, and as 12*h.*: 25*m.* : : 5*h.* : 10.4*m.*, which added to 6*h.* 44*m.* = 6*h.* 54 + *m.*; which added to 8*h.* 13*m.* (the establishment of the Port, or the time of high-water at the full and change of the moon at Governor's Island, New York) = 15*h.* 7*m.*, from which subtract 12*h.* 26*m.* = 2*h.* 41*m.* P.M.

When the moon souths at an hour which, when added to the establishment of the Port, exceeds 12 *hours after* M., then the first tide that succeeds this southing of the moon occurs in the A.M. of the ensuing day.

When the time of the tide preceding this is required; from the sum of the moon's southing, and establishment of the Port, subtract 12*h.* 26*m.*, and the remainder will give the time of high-water on that day.

Illustration.—On the 9th of June, 1864, the time of the moon's southing at New York, was 4*h.* 12*m.* P.M., to which add 8*h.* 13*m.* for the establishment of the Port, and the sum is 12*h.* 25*m.*, from which subtract 12*h.*, and the difference of 25 is 25*m.* A.M. of the next day, the 10th.

When the time of high-water, following the meridian passage of the moon, exceeds 12 P.M., it is in the A.M. of the following day.

Thus, if the moon souths at New York, 4*h.* 12*m.* P.M.
Establishment of the Port, 8 13

12 25 = 12 *hours*, 25 *min. from* M. *of one day, or* 25 *min.* A.M. *of the following day.*

When the time of high-water preceding the moon's southing is required, subtract from the time obtained as above 12*h.* 26*m.*, the half of a lunar day, or contrariwise if the half of a lunar day exceeds the time.

Thus, 12*h.* 26*m.*
Time as above, 12 25

— 1, or = 1 *min.* before M. of the day.

Note.—The time for a tide being ascertained, that of the next succeeding is ascertained by the addition of 12 hours 26 minutes.

To Compute the Time of High-water by the Aid of the American Nautical Almanac.

Rule.—Ascertain the time of transit of the moon for Greenwich preceding the time of the high-water required.

For any other location (west of Greenwich), multiply the time in the column "diff. for one hour" by the longitude of the location west of Greenwich, expressed in hours, and add the product to the time of transit.

NOTE.—It is frequently necessary to take the transit for the preceding astronomical day, as the latter does not end until noon of the day under computation.

EXAMPLE.—Required the time of high-water at New York on the 25th of August, 1864.

Longitude of New York from Greenwich = 4*h.* 56*m.* 3 *sec.*, which, multiplied by 2.17 *min.*, the difference for 1 hour = 10.71 *min.* for the correction to be added to the time of transit, to obtain the time of transit at New York.

Time of transit, 18*h.* 38.8*m.*; then 18*h.* 38.8*m.* + 10.71*m.* = 18 *hours* 49.5 *min.*

Time of transit at New York,	24*d.* 18*h.* 50*m.*
Establishment of the Port,	8 13
	25*d.* 3*h.* 3*m.* = *time of high-water.*

NOTE.—The time of the 25th at 3*h.* 3*m.* Astronomical computation = 25th at 3*h.* 3*m.* P.M. Civil Time.

To Compute the Time of High-water at the Full and Change of the Moon, the Time of High-water and the Age of the Moon on any Day being given.

RULE.—Note the age of the moon, and opposite to it, in the last column of the following table, take the time, which subtract from the time of high-water at this age of the moon, added to 12*h.* 26*m.*, or 24*h.* 52*m.* as the case may require (when the sum to be subtracted is the greatest), and the remainder is the time required.

EXAMPLE.—What is the time of high-water at the full and change of the moon at New York?

The time of high-water at Governor's Island on the 25th of Jan., 1864, was 9*h.* 20*m.* A.M. civil time. The age of the moon at 12 M. on that day was 16*d.* 8*h.* 59*m.*

Opposite to 16 *days*, in the following table, is 13*h.* 1*m.*, and the difference between 16*d.* and 16*d.* 12*h.* = (16.5) is 25*m.*; hence, if 12*h.* = 25*m.* 16*d.* 8*h.* 59*m.* — 16*d.* = 8*h.* 59*m.* = 18.7, or 19*m.*, which, added to 13*h.* 1*m.* = 13*h.* 20*m.*

Then 9*h.* 20*m.* + 12*h.* 26*m.* (as the sum to be subtracted is greater than the time) — 13*h.* 20*m.* = 21*h.* 46*m.* — 13*h.* 20*m.* = 8 *hours*, 26 *min.*

This is a difference of but 13 minutes from the establishment of the Port.

Table showing the Time after apparent Noon before the Moon next passes the Meridian, the Age at Noon being given.

(S. H. WRIGHT, A.M., Ph.D.)

Age of Moon.	Moon at Merid'n.	Age of Moon.	Moon at Merid'n.	Age of Moon.	Moon at Meridian.	Age of Moon.	Moon at Meridian.	Age of Moon.	Moon at Meridian.
Days.	H. M. P.M.	Days.	H. M. P.M.	Days.	H. M. P.M.	Days.	H. M. A.M.	Days.	H. M. A.M.
.0	0	6	5 03	12	10 06	18	15 08	24	20 11
.5	25	6.5	5 28	12.5	10 31	18.5	15 34	24.5	20 37
1	50	7	5 53	13	10 56	19	15 59	25	21 02
1.5	1 16	7.5	6 19	13.5	11 21	19.5	16 24	25.5	21 27
2	1 41	8	6 44	14	11 47	20	16 49	26	21 52
					A.M.				
2.5	2 06	8.5	7 09	14.5	12 12	20.5	17 15	26.5	22 17
3	2 31	9	7 34	15	12 37	21	17 40	27	22 43
3.5	2 57	9.5	7 59	15.5	13 02	21.5	18 05	27.5	23 08
4	3 22	10	8 25	16	13 28	22	18 30	28	23 33
4.5	3 47	10.5	8 50	16.5	13 53	22.5	18 56	28.5	23 58
5	4 12	11	9 15	17	14 28	23	19 21	29	24 24
5.5	4 38	11.5	9 40	17.5	14 43	23.5	19 46	29.5	24 48

Tide-Table for the Coast of the United States,

Showing Time of High-water at the Full and Change of the Moon, termed the Establishment of the Port, being the Mean Interval between Time of Moon's Transit and Time of High-water.

[By Prof. A. D. Bache, U. S. Coast Survey.]

Places and Time.	h.	m.	Rise and Fall. Spring. Feet.	Neap. Feet.
COAST FROM PORTLAND TO NEW YORK.				
Portland .... Me.	11	25	9.9	7.6
Portsmouth ... N. H.	11	23	9.9	7.2
Newburyport.. Mass.	11	22	9.1	6.6
Salem .... "	11	13	10.6	7.6
Boston Light.. "	11	12	10.9	8.1
Boston† .... "	11	27	10.3	8.5
Nantucket .... "	12	24	3.6	2.6
Edgartown.... "	12	16	2.5	1.6
Holmes's Hole. "	11	43	1.8	1.3
Tarpaulin Cove "	8	4	2.8	1.8
Wood's Hole, n. side.	7	50	4.7	3.1
Wood's Hole, s. side.	8	34	2.0	1.2
Bird Isl'd Light, Mass.	7	50	5.3	3.5
N. Bedford Entrance (Dumpling Rock).	7	57	4.6	2.8
Newport .... R. I.	7	45	4.6	3.1
Point Judith ... "	7	32	3.7	2.6
Montauk Point. N.Y.	8	20	2.4	1.8
Sandy Hook.... N. J.	7	29	5.6	4.0
New York‡.....N.Y.	8	13	5.4	3.4
LONG ISLAND SOUND.				
Watch Hill .... R. I.	9	0	3.1	2.4
Stonington ...... Ct.	9	7	3.2	2.2
Little Gull Isl'd. N.Y.	9	38	2.9	2.3
New London..... Ct.	9	28	3.1	2.1
New Haven ..... "	11	16	6.2	5.2
Bridgeport ...... "	11	11	8.0	4.7
Oyster Bay.... N.Y.	11	7	9.2	5.4
Sands's Point .. "	11	13	8.9	6.4
New Rochelle .. "	11	22	8.6	6.6
Throg's Neck... "	11	20	9.2	6.1
COAST OF NEW JERSEY.				
Cold Spr'g Inlet. N. J.	7	32	5.4	3.6
Cape May Landing "	8	19	6.0	4.3
DELAWARE BAY AND RIVER.				
Delaware Breakwater	8	0	4.5	3.0
Higbee's (Cape May).	8	33	6.2	3.9
Egg Isl'd Light. N. J.	9	4	7.0	5.1
Mahon's River.. Del.	9	52	6.9	5.0
Newcastle ...... "	11	53	6.9	6.6
Philadelphia .... Pa.	13	41	6.8	5.1

Places and Time.	h.	m.	Rise and Fall. Spring. Feet.	Neap. Feet.
CHESAPEAKE BAY AND RIVERS.				
Old Pt. Comfort§. Va.	8	17	3.0	2.0
Point Lookout .. Md.	12	58	1.9	0.7
Annapolis ...... "	17	4	1.0	0.8
Bodkin Light... "	18	8	1.3	0.8
Baltimore...... "	18	50	1.5	0.9
James R. (City Pt.) Va.	14	37	3.0	2.5
Richmond ...... "	16	58	3.4	2.3
COASTS OF N. AND S. CAROLINA, GEORGIA, AND FLORIDA.				
Hatteras Inlet . N. C.	7	4	2.2	1.8
Beaufort ...... "	7	26	3.3	2.2
Smithv. (C. Fear) "	7	19	5.5	3.8
Charleston‖ (C. H. Wharf) ... S. C.	7	26	6.0	4.1
Fort Pulaski..... Ga.	7	20	8.0	5.9
Savannah (Dry Dock Wharf) ..... Ga.	8	13	7.6	5.5
St. Augustine... Fla.	8	21	4.9	3.6
Cape Florida ... "	8	34	1.8	1.2
Sand Key ...... "	8	40	2.0	0.6
Key West ...... "	9	22	1.6	1.0
Tampa Bay..... "	11	21	1.8	1.0
Cedar Keys (Dépôt Key)........ Fla.	13	15	3.2	1.6
WESTERN COAST.				
San Diego...... Cal.	9	38	5.0	2.3
San Pedro...... "	9	39	4.7	2.2
Cuyler's Harbor. "	9	25	5.1	2.8
San Luis Obispo. "	10	8	4.8	2.4
Monterey ...... "	10	22	4.3	2.5
South Farallone. "	10	37	4.4	2.8
San Francisco .. "	12	6	4.3	2.8
Mare Island (San Francisco Bay) ..	13	40	5.2	4.1
Benicia .... Cal.	14	10	5.1	3.7
Ravenswood.... "	12	36	7.3	4.9
Bodega .... "	11	17	4.7	2.7
Humboldt Bay . "	12	2	5.5	3.5
Astoria .... Or.	12	42	7.4	4.6
Nee-ah Harb'r, Wash.	12	33	7.4	4.8
Port Townshend "	3	49	5.5	4.0
Semi-ah-moo Bay "	4	50	6.6	4.8

Note.—The mean interval has been increased 12 hours 26 minutes (half a mean lunar day) for some of the ports in Delaware River and Chesapeake Bay, so as to give the succession of times from the mouth; hence 12 hours 26 minutes is to be subtracted from the establishments which are greater than that time, in order to give the interval required.

Bench Marks referred to in preceding Table.

† Boston.—The top of the wall or quay, at the entrance to the dry-dock in the Charlestown navy-yard, 14.76 feet above mean low-water.

‡ New York.—The lower edge of a straight line, cut in a stone wall, at the head of the wooden wharf on Governor's Island, 14.51 feet above mean low-water.

§ Old Point Comfort, Va.—A line cut in the wall of the light-house, one foot from the ground, on the southwest side, 11. feet above mean low-water.

‖ Charleston, S. C.—The outer and lower edge of embrasure of gun No. 3, at Castle Pinckney, 10.13 feet above mean low-water.

Establishment of the Port for several Places not included in the preceding Table.

Place.	Time.	Rise and Fall.	Place.	Time.	Rise and Fall
	H. M	Feet.		H. M.	Ft.
Albany .... N. Y.	3 30	1	Cape Henry .... Va.	7 51	6
Amboy .... N. J.	8 15	5	Eastport .... Me.	11 30	15
Bay of Fundy .... N. S.	12	60	Egg Harbor .... N. J.	9 34	5
Blue Hill Bay	11	12	Halifax .... N. S.	7 30	9
Campo Bello .... Me.	11	25	Hell Gate .... N. Y.	9 35	6
Cape Ann .... "	11 30	11	Kingston .... Jam.	2 30	2
Cape Cod .... Mass.	11 30	6	Providence .... R. I.	8 25	5
Cape Hatteras .... N. C.	9 1	5	St. Johns .... N. S.	12	30

Rise and Fall of Tides at several Places in the Gulf of Mexico.

Places.	Mean.	Spring.	Neap.	Places.	Mean.	Spring.	Neap.
	Feet.	Feet.	Feet.		Feet.	Feet.	Feet.
St. George's Island..Fla.	1.1	1.8	.6	Isle Dernière.......La.	1.4	1.2	.7
Pensacola.......... "	1.0	1.5	.4	Entrance to Lake Calcasieu ....La.	1.5	1.1	.6
Fort Morgan (Mobile Bay)...........Ala.	1.0	1.5	.4	Galveston ....Texas	1.1	1.6	.8
Cat Island ....Miss.	1.3	1.9	.6	Aransas Pass .... "	1.1	1.8	.6
Southwest Pass .....La.	1.1	1.4	.5	Brazos Santiago.. "	.9	1.2	.5

Establishment of the Port for several Places in Europe, etc.

Port.	Country.	Time.	Port.	Country.	Time.
		h. m.			*h. m.*
Amsterdam	Netherlands ...	3	Funchal	Madeira	11 30
Antwerp.........	" ...	4 25	Gravesend	England.......	1 14
Beachy Head	England.......	11 50	Greenock	W. C. of Scotl'd	8
Belfast	Ireland........	10 43	Havre-de-Grâce ..	France	9 51
Bordeaux........	France	6 50	Holyhead	Wales.........	10 11
Bologne	"	11 25	Hull	England.......	6 29
Bremen	Netherlands ...	6	Lisbon	Portugal	2 30
Brest Harbor	" ...	3 47	Liverpool	England.......	11 16
Bristol	England	7 21	London Bridge ...	River Thames .	2 7
Cadiz	Spain	1 40	Nassau	New Providence	7 30
Calais	France	11 49	Newcastle	England.......	1 22
Calf of Man......	St. Geo. Channel	11 5	Pembroke Dk.-y'd	Wales.........	6 12
Cape St. Vincent .	Spain	2 30	Quebec	Canada	8
Chatham	England.......	1 2	Portsmouth D.-y'd	England.......	11 41
Cherbourg	France	7 49	Rye Bay.........	"	11 20
Clear Cape	Ireland........	4 0	Sierra Leone	Africa.........	8 15
Cork Harbor	"	5 1	Southampton	England.......	11 40
Cowes	Isle of Wight ..	10 46	Thames R., mouth	"	12
Dover Pier	England.......	11 12	Waterford Harbor	Ireland........	6 6
Dublin Bar	Ireland........	11 12	Woolwich	England.......	2 15

To Approximate to the Time which has elapsed from Low or High Water, by knowing the Rise or Fall of the Tide in the Interval.

If the proportion of the rise and fall in a given time were the same in the different ports, this could easily be shown in a single table, giving the proportional rise and fall. The proportion, however, is not the same in different ports, nor in the same port for tides of different heights.

The following table shows the relation between the heights above low-water for each half hour, for New York and for Old Point Comfort, and for spring and neap tides, at each place. Units express the total rise of high-water above low-water, and the figures opposite to each half hour denote the proportional fall of the tide from high-water onward to low water.

Table to Ascertain the Rise and Fall of a Tide for any Given Time from High or Low Water,

Giving the Height of the Tide above Low-water for each Half Hour before or after High-water, the Total Range being taken as Equal to 1.

Time before or after High-water.	New York		Old Pt. Comfort.		Time before or after High-water	New York.		Old Pt Comfort.	
	Spring.	Neap.	Spring.	Neap.		Spring	Neap.	Spring.	Neap.
h. *m.*					*h.* *m.*				
–	1.	1.	1.	1.	3 30	.49	.31	.49	.44
30	.98	.98	.98	.98	4	.39	.19	.37	.34
1	.94	.93	.95	.94	4 30	.28	.10	.26	.22
1 30	.89	.86	.88	.87	5	.18	.02	.17	.13
2	.80	.72	.80	.78	5 30	.09	–	.08	.05
2 30	.72	.59	.70	.68	6	.05	–	.03	.01
3	.60	.45	.59	.57	6 30		–	–	–

Spring tides occur about 2 days after the full and change of the moon, and *Neaps* two days after the first and last quarter.

Illustration.—At New York, 3 hours after high-water, a spring tide has fallen .6 (.60) of the whole fall. Suppose the whole rise and fall of that day to be 5.4 feet, then 3 hours after high-water the tide will have fallen 3.24 feet, or 3 feet 3 inches nearly. Conversely, if a spring tide has fallen 3 feet 3 inches, we know that high-water has passed about 3 hours.

Tides of the Gulf of Mexico.

On the coast of Florida, from Cape Florida around to St. George's Island, near Cape San Blas, the tides are of the ordinary kind, but with a large daily inequality. From St. George's Island, Apalachicola entrance, to Dernière Isle, the tides are usually of the single-day class, ebbing and flowing but once in 24 (lunar) hours. At Calcasieu entrance the double tides reappear, and except for some days about the period of the moon's greatest declination, the tides are double at Galveston, Texas. At Aransas and Brazos Santiago the single-day tides are as perfectly well marked as at St. George's, Pensacola, Fort Morgan, Cat Island, and the mouths of the Mississippi. For some 3 to 5 days, however, about the time when the moon's declination is nothing, there are generally two tides at all these places in the 24 hours, the rise and fall being quite small.

The highest high and lowest low waters occur when the greatest declination of the moon happens at full or change. The least tides when the moon's declination is nothing at the first or last quarter.

Tides of the Pacific Coast.

On the Pacific coast there is, as a general rule, one large and one small tide during each day, the heights of two successive high-waters occurring, one A.M., and the other P.M. of the same 24 hours, and the intervals from the next preceding transit of the moon are very different. These inequalities depend upon the moon's declination. When the moon's declination is nothing, they disappear, and when it is the greatest. either north or south, they are the greatest. The inequalities for low-water are not the same as for high, though they disappear, and have the greatest value at nearly the same time.

When the moon's declination is north, the highest of the two high tides of the 24 hours occurs at San Francisco, about eleven and a half hours after the moon's southing (transit); and when the declination is south, the lowest of the two high tides occurs about this interval.

The lowest of the two low-waters of the day is the one which follows next the highest high-water.

To Convert Chemical Formulæ into a Mathematical Expression.

RULE.—Multiply together the equivalent and the exponent of each substance, and the product will give the proportion in the compound by weight. Divide 1000 by the sum of their products, and multiply this quotient by each of these products, and the products will give the respective proportion of each part by weight in 1000.

EXAMPLE.—The chemical formulæ for alcohol is $C_4H_6O_2$. Required their proportional parts by weight in 1000?

$$\left.\begin{array}{lll} C_4 \text{ Carbon} & = 6.1 \times 4 = & 24.4 \\ H_6 \text{ Hydrogen} = & 1 \times 6 = & 6 \\ O_2 \text{ Oxygen} = & 8 \times 2 = & 16 \end{array}\right\} \times 21.55 \left\{\begin{array}{l} 525.82 \\ 129.3 \\ 344.8 \end{array}\right\} \text{by weight.}$$

$$1000 \div 46.4 = 21.55 \quad 999.92$$

Elementary Bodies, with their Symbols and Equivalents.

Body.	Symb.	Equiv.	Body.	Symb.	Equiv.	Body.	Symb.	Equiv.
Aluminium	Al	13.7	Hydrogen	H	1.	Potassium	K	39.2
Antimony	Sb	64.6	Iodine	I	126.5	Rhodium	R	52.2
Arsenic	As	37.7	Iridium	Ir	98.5	Ruthenium	Ru	52.1
Barium	Ba	68.6	Iron	Fe	28.	Selenium	Se	40.
Bismuth	Bi	71.5	Lantanium	Ln	48.	Silicon	Si	22.
Boron	B	11.	Lead	Pb	103.7	Silver	Ag	108.3
Bromine	Br	78.4	Lithium	L	7.	Sodium	Na	23.5
Cadmium	Cd	55.8	Magnesium	Mg	12.7	Strontium	Sr	43.8
Calcium	Ca	20.5	Manganese	Mn	26.	Sulphur	S	16.1
Carbon	C	6.1	Mercury	Hg	200.	Tellurium	Te	64.2
Cerium	Ce	46.	Molybdenum	Mo	47.9	Terbium	Tb	—
Chlorine	Cl	35.5	Nickel	Ni	29.5	Thorium	Th	60.
Chromium	Cr	26.2	Niobium	Nr	—	Tin	Sn	58.9
Cobalt	Co	29.5	Nitrogen	N	14.2	Titanium	Ti	24.5
Columbium	Ta	184.8	Norium	No	—	Tungsten	W	92.
Copper	Cu	31.7	Osmium	Os	99.7	Uranium	U	60.
Didymium	D	48.	Oxygen	O	8.	Vanadium	V	68.5
Erbium	E	—	Palladium	Pd	53.3	Yttrium	Y	32.
Fluorine	F	18.7	Pelopium	Pe	—	Zinc	Zn	32.3
Glucinum	G	6.9	Phosphorus	P	15.9	Zirconium	Zr	34.
Gold	Au	196.6	Platinum	Pt	98.8			

Analysis of certain Organic Substances by Weight.

	Carbon.	Hydrogen.	Oxygen.	Nitrogen.		Carbon.	Hydrogen.	Oxygen.	Nitrogen.
Sugar	42.2	6.6	51.2	—	Hordein	44.2	6.4	47.6	1.8
Starch	44.2	6.7	49.1	—	Veratrin	66.7	8.5	19.6	5.
Gum	42.7	6.4	50.9	—	Cinchonin	77.8	7.4	5.9	8.9
Lignin	52.5	5.7	41.8	—	Quinine	75.8	7.5	8.6	8.1
Tannin	52.6	3.8	43.6	—	Brucine	70.9	6.7	17.4	5.
Indigo	73.3	2.5	10.4	13.8	Strychnine	76.4	6.7	11.1	5.8
Camphor	73.4	10.7	14.6	.3	Narcotine	65.	5.5	27.	2.5
Caoutchouc	87.2	12.8	—	—	Morphine	72.3	6.4	16.3	5.
Albumen	52.9	7.5	23.9	15.7	Oil, Spermaceti	78.	11.8	10.2	—
Fibrin	53.4	7.	19.7	19.9	Castor	74.	10.3	15.7	—
Casein	59.8	7.4	11.4	21.4	Linseed	76.	11.3	12.7	—
Urea	18.9	9.7	26.2	45.2	Alcohol	52.7	12.9	34.4	—
Gelatine	47.9	7.9	27.2	17.	Atmospheric air	—	—	77.	23.

Food.

HUMAN AND ANIMAL SUSTENANCE.

Least Quantity of Food required to sustain Life.

	Carbon. Grs.	Nitrogen. Grs.
Adult Man,	4300	200
Adult Woman,	3900	180
Mean,	4100	190

These quantities and proportions are contained in about 2 lbs. 2 oz. ordinary bakers' bread.

A man, for his daily sustenance, requires about 1220 grs. nitrogenous matter, and bread contains 8.1 per cent. of it.

Therefore 2 *lbs.* 2 *oz.* = 14875 *grains* × 8.1 = 1205 *grains.*

Nutritive Values of Food in Grains per Pound.

Food.	Carbon.	Nitrogen.	Food.	Carbon.	Nitrogen.
Beef	1.854	184	Mutton	1.900	189
Barley Meal	2.563	68	New Milk	599	44
Bakers' Bread	1.975	88	Oat Meal	2.831	136
Buttermilk	387	44	Pearl Barley	2.660	91
Bullock's Liver	934	204	Potatoes	769	22
Beer and Porter	274	1	Parsnips	554	12
Carrots	508	14	Rye Meal	2.693	86
Cheddar Cheese	3.344	306	Rice	2.732	68
Cocoa	3.934	140	Red Herrings	1.435	217
Dry Bacon	5.987	95	Split Peas	2.698	248
Fat Pork	4.113	106	Sugar	2.955	—
Flour, Seconds	2.700	116	Skimmed Milk	438	43
Fresh Butter	6.456	—	Skim Cheese	1.947	483
Green Vegetables	420	14	Suet	4.710	—
Green Bacon	5.426	76	Salt Butter	4.585	—
Indian Meal	3.016	120	Turnips	263	13
Lard	4.819	—	Whey	154	13
Molasses	2.395	—	Whitefish	871	195

Nutritive Equivalents. Computed from the Amount of Nitrogen in the Substances when Dried. Human Milk at 1.

Rice	.81	Cheese	3.31
Potatoes	.84	Eel	4.34
Corn	1	Mussel	5.28
Rye	1.06	Liver, Ox	5.70
Radish	1.06	Pigeon	7.56
Wheat	1.19	Mutton	7.73
Barley	1.25	Salmon	7.76
Oats	1.38	Lamb	8.33
Bread, Black	1.66	Egg, White	8.45
Bread, White	1.42	Lobster	8.59
Peas	2.39	Skate	8.59
Lentils	2.76	Veal	8.73
Haricots	2.83	Beef	8.80
Beans	3.20	Pork	8.93
Milk, Cows'	2.37	Turbot	8.98
Egg, Yolk	3.05	Ham	9.10
Oysters	3.05	Herring	9.14

Quantities of Different Foods required to furnish 1220 *Grains of Nitrogenous Matter.*

	Pounds.		Pounds.		Pounds.
Cheese	.4	Corn Meal	1.6	Barley Meal	2.9
Pease	.7	Wheat Flour	1.7	Milk	4.2
Meat, lean	.9	Bacon, fat	1.8	Potatoes	8.3
Fish, White	1	Bread	2.1	Parsnips	15.9
Meat, fat	1.3	Rye Meal	2.3	Turnips	15.9
Oatmeal	1.5	Rice	2.8	Beer or Porter	158.6

DIGESTION.

Time required for Digestion of several Articles of Food.

(Beaumont, M.D.)

	H.	M.
Apples, sweet and mellow	1	50
sour and mellow	2	
sour and hard	2	50
Barley, boiled	2	
Beans, boiled	2	30
Beans and Green Corn, boiled	3	45
Beef, roasted rare	3	
roasted dry	3	30
Steak, broiled	3	
boiled	2	45
boil'd, with mustard, etc.	3	30
tendon, boiled	5	30
tendon, fried	4	
old salted, boiled	4	15
Beets, boiled	3	45
Bread, Corn, baked	3	15
Wheat, baked, fresh	3	30
Butter, melted	3	30
Cabbage, crude	2	30
crude, vinegar	2	
crude, vin'r, boil'd	4	
	4	30
Carrots, boiled	3	15
Cartilage, boiled	4	15
Cheese, old and strong	3	30
Chickens, fricasseed	2	45
Custard, baked	2	45
Duck, roasted	4	
	4	30
Dumplings, Apple, boiled	3	
Eggs, boiled hard	3	30
boiled soft	3	
fried	3	30
uncooked	2	
whipped, raw	1	30
Fish, Cod or Flounder, fried	3	30
Cod, cured, boiled	2	
Salmon, salt'd and boil'd	4	
Trout, boiled or fried	1	30
Fowls, boiled or roasted	4	
Goose, roasted	3	
Gelatine, boiled	2	30
Heart, Animal, fried	4	
Lamb, boiled	2	30
Liver, Beef's, boiled	2	
Meat and Vegetables, hashed.	2	30
Milk, boiled or fresh	2	
	2	15
Mutton, roasted	3	15
broiled or boiled	3	
Oysters, raw	2	55
roasted	3	15
stewed	3	30
Parsnips, boiled	2	30
Pigs, Sucking, roasted	2	30
Feet, soured, boiled	1	
Pork, fat and lean, roasted	5	15
recently salted, boiled.	4	30
" " fried	4	15
" " broiled	3	15
" " raw	3	
Potatoes, boiled	3	30
baked	3	20
roasted	2	30
Rice, boiled	1	
Sago, boiled	1	45
Sausage, Pork, broiled	3	20
Soup, Barley	1	30
Beef and Vegetables	4	
Chicken	3	
Mutton or Oyster	3	30
Sponge-cake, baked	2	30
Suet, Beef, boiled	5	30
Mutton, boiled	4	30
Tapioca, boiled	2	
Tripe, soured	1	
Turkey, roasted, Wild	2	18
Turkey, roasted, Domestic	2	30
boiled	2	25
Turnips, boiled	3	30
Veal, roasted	4	
fried	4	50
Brains, boiled	1	45
Venison Steak, broiled	1	35

ANALYSIS OF VARIOUS FOODS.

Food.	Water.	Nitrogenous Matter.	Fat.	Saline Matter.	Non-Nitrogenous Matter.	Lactine.
Beef, roast	54	27.6	15.45	2.95		
lean	72	19.3	3.6	5.1		
fat	51	14.8	29.8	4.4		
Mutton, fat	53	12.4	31.1	3.5		
Poultry	74	21	3.8	1.2		
Veal	63	16.5	15.8	4.7		
Pork, fat	39	9.8	48.9	2.3		
Bacon, dry	15	8.8	73.3	2.9		
Liver, Calf's	72.33	20.55	5.58	1.54		
Tripe	68	13.2	16.4	2.4		
Fish, white flesh	78	18.1	2.9	1		
Salmon	77	16.1	5.5	1.4		
Eels	75	9.9	13.8	1.3		
Lobster, flesh	76.6	19.17	1.17	1.8	1.26	
Oysters	80.39	14.01	1.52	2.7	1.38	
Egg	74	14	10.5	1.5		
Milk, Cow's	86	4.1	3.9	.8		5.2
Cheese	36.8	33.5	24.3	5.4		

Food.	Water.	Nitrogenous Matter.	Fat.	Saline Matter.	Non-Nitrogenous Matter.	Sugar.	Cellulose.	Ash, etc.
Wheat Flour	15	10.8	2	1.7	63.1	4.2	3.5	1.7
Bran	13	18	6		60			3
Bread*	37	8.1	1.6	2.3	45.4	3.6		2
Oats	21	14.4	5.5		48.2		7.6	3.3
Barley Meal	15	6.3	2.4	2	69.4	4.9		
Rye	15	12.5	2.3	1.8	62.6		3.2	2.6
Corn Meal	14	11.1	8.1	1.7	58		5.9	1.2
Rice	13	6.3	.7	.5	78.1	.4		1
Buckwheat	13	13.1	3	.4	64.5		3.5	2.5
Beans, White	9.9	25.5	2.8		55.7		2.9	3.2
Peas	15	23	2.1	2.5	50.2	2	3.1	2.1
Potatoes	75	2.1	.2	.7	16.8	3.2	1.4	1
Carrots	83	1.3	.2	1	7.4	6.1		1
Parsnips	82	1.1	.5	1	9.6	8.8		1
Turnips	91	1.2		.6	4.3	2.1		.8
Butter and Fats	15		83	2				

* Water absorbed by flour varies from 40 to 60 per cent. of the weight of the flour, the best quality absorbing the most. 100 lbs. flour yield 130 lbs. bread.

Water in Various Foods. (Per Cent.)

Sugar	5	Butter and Fats	15	Poultry	74	Cabbage	91
Rice	13	Molasses	23	Egg	78	Ale and Beer	91
Oatmeal	15	Beef and Mutton, lean	72	Buttermilk	88	Coffee and Tea	100

Alimentary Principles.

The primary division of Food is into Organic and Inorganic.

The Organic is sub-divided into Nitrogenous and Non-Nitrogenous: the Inorganic is composed of water and various saline principles. The former elements are destined for the growth and maintenance of the body, and are termed the "plastic elements of nutrition." The latter are designed for undergoing oxidation, and thus become the source of heat, and are termed "elements of respiration," or "Calorifaciant."

Although Fat is non-nitrogenous, it is so mixed with nitrogenous matter that it becomes a nutrient as well as a calorifaciant.

Calorific Powers of Different Foods.

NOTE.—Every pound of water raised 1° F. is equivalent to 772 lbs. lifted 1 foot in height.

Calorific Power and Mechanical Energy of 10 *Grains of the following Foods, in their Normal Condition, when completely Oxidized in the Animal Body.*

Food.	Water raised 1° in lbs.	Pounds raised 1 foot in height.	Food.	Water raised 1° in lbs.	Pounds raised 1 foot in height.
Dry Flesh.....	13.12	10.13	Arrow Root..	10.06	7.77
Albumen.....	12.85	9.92	Butter.......	18.68	14.42
Lump Sugar..	8.61	6.65	Beef Fat.....	20.91	16.14

Sugar in Various Products.

(Per Cent.)

Sugar, crude......	95	Oatmeal..........	5.4	Potatoes..........	3.2
Molasses..........	77	Milk..............	5.2	Turnips...........	3.1
Buttermilk.......	6.4	Barley Meal.......	4.9	Peas..............	2
Carrots..........	6.1	Rye Flour.........	3.7	Corn Meal.........	.4
Parsnips.........	5.8	Wheat Bread......	3.6	Rice..............	.4

Volume of Oxygen required to Oxidize 100 *parts of the following Foods as consumed in the Body:*

Grape Sugar.................	106	Albumen....................	150
Starch......................	120	Fat........................	293

Hence, assuming capacity for oxidation as a measure, albumen has half the value of fat as a food-producing element, and a greater value than either starch or sugar.

Relative Value of Various Foods as Productive of Force.

When Oxidized in the Body.

Cabbage..........	1	Veal, lean........	2.8	Pea Meal.........	9
Carrots...........	1.2	Mackerel.........	3.8	Wheat Flour.....	9.1
Egg, White........	1.4	Ham, lean.......	4	Arrowroot........	9.3
Milk..............	1.5	Bread, crumb....	5.1	Oatmeal.........	9.3
Apples............	1.5	Egg, hard boiled..	5.4	Cheese..........	10.4
Ale...............	1.8	Egg, Yolk........	7.9	Cocoa...........	16.3
Fish..............	1.9	Sugar...........	8	Butter..........	17.3
Potatoes..........	2.4	Isinglass........	8.7	Fat of Beef.......	21.6
Porter............	2.6	Rice.............	8.9	Cod-liver Oil.....	21.7

Analysis of Various Fruits.

Fruit.	Water.	Sugar.	Acid.	Albumen.	Pectous Substances.	Seeds, Skin, etc.	Ash.
Apple..	85	7.6	1	.2	2.8	2.9	.5
Pear...	84	7.4	.1	.3	3.3	4.6	.3
Grape..	80	13.7	1	.8	.6	3.5	.4

MISCELLANEOUS NOTES.

SHOT.

Diameter and Number of Pellets in an Ounce of Lead Shot, American Standard.—[Le Roy & Tatham.]

No.	Diameter in Inches.	Pellets.	No.	Diameter in Inches.	Pellets.	No.	Diameter in Inches.	Pellets.
TT	.21	32	1	.16	69	7	.10	278
T	.20	38	2	.15	82	8	.9	375
BBB	.19	44	3	.14	98	9	.8	560
BB	.18	49	4	.13	121	10	.7	822
B	.17	58	5	.12	149	11	.6	982
			6	.11	209	12	.5	1778

Weather-foretelling Plants.—[Hanneman.]

If Rain is imminent.—Chickweed,* *Stellaria media;* its flowers droop and do not open. Crowfoot anemone, *Anemone ranunculoides;* its blossoms close. Bladder Ketmia, *Hibiscus trionum;* its blossoms do not open. Thistle, *Carduus acaulis;* its flowers close. Clover, *Trifolium pratense*, and its allied kinds, and Whitlow grass, *Draba verna;* they droop their leaves. Nipple-wort, *Lampsana communis;* its blossoms will not close for the night. Yellow Bedstraw, *Galium verum;* it swells, and exhales strongly; and Birch, *Betula alba*, exhales and scents the air.

Indications of Rain.—Marigold, *Calendula pluvialis;* when its flowers do not open by 7 A.M. Hog Thistle, *Sonchus arvensis* and *oleraceus;* when its blossoms open.

Rain of short duration.—Chickweed, *Stellaria media;* if its leaves open but partially.

If cloudy.—Wind flower, or Wood Anemone, *Anemone memorasa;* its flowers droop.

Termination of Rain.—Clover, *Trifolium pratense;* if it contracts its leaves. Birdweed and Pimpernel, *Convolvulus* and *Anagallis arvensis;* if they spread their leaves.

Uniform Weather.—Marigold, *Calendula pluvialis;* if its flowers open early in the A.M. and remain open until 4 P.M.

Clear Weather.—Wind-flower, or Wood Anemone, *Anemone memorasa;* if it bears its flowers erect. Hog Thistle, *Sonchus arvensis* and *oleraceus;* if the heads of its blossoms close at and remain closed during the night.

* The Chickweed spreads its leaves at 9 A.M., and they remain open until noon.

Locomotive Traction and Resistance.

Formula to ascertain the Traction of a Locomotive.

$$\frac{d^2\, l\, p}{D} = T;$$

d representing diameter of cylinder in inches; l, length of stroke of piston; and D, *diameter of wheel in feet or inches; p, the mean pressure in pounds per square inch; and* T, *the traction in pounds at rails.*

Formula to ascertain the Resistance of a Locomotive and Train upon a Level Railway.

$$\frac{S^2}{171} S = R;$$

S *representing speed of train in miles per hour; and* R, *resistance of each ton* (2240) *of the gross weight in pounds.*

Estimated Consumption of Bituminous Coal per actual Horses' Power.

Type of Engine.	Coal in lbs. per hour.
Improved Compound	2.
Ordinary, with surface condenser and superheater	3.5
Ordinary, Injection	4.5
Non-condensing	6.

Expansion and Contraction of Building Stones.

[Lieut. W. H. C. BARTLETT, U. S. E.]

Expansion or Contraction for each Degree of Temperature.

	For One Inch.		For One Inch.
Granite	.000004825	Sandstone	.000009532
Marble	.000005668	Whitepine	.00000255

Resistance of Stones, etc., to the Effects of Freezing.

Various experiments show that the power of stones, etc., to resist the effects of freezing is a fair exponent of that to resist compression.

To Preserve Meat.

Meat of any kind may be preserved in a temperature of from 80° to 100°, for a period of ten days, after it has been soaked in a solution of 1 pint of salt dissolved in 4 gallons of cold water and ½ gallon of a solution of bisulphate of calcium.

By repeating this process the preservation may be extended by the addition of a solution of gelatin or the white of an egg to the salt and water.

Silk.—A thread of silk is the 2500th of an inch in diameter.

Spider's Thread.—Four miles of a spider's thread weighs one grain.

Soap Bubble.—The film of a soap bubble is the 2500000th of an inch in thickness.

Gold Leaf is the 280000th part of an inch in thickness.

Air and Ventilation.

An average-sized man will exhale from his lungs and body from .6 to .7 of a cubic foot of carbonic acid per hour. A lighted oil lamp or two candles will furnish the same volume.

Assuming, then, that there are 4 volumes of carbonic acid in 10 000 volumes of air, and that a man in a room with a lighted lamp or two candles furnishes from 1.2 to 1.4 cubic feet of acid per hour, there will be required to maintain the air at the required condition for health for one man, the allowable pollution of it being 6 volumes in 10 000, fully 3000 cubic feet of fresh air. By experiments made in Paris it was shown that there was required from 2400 to 3120 cubic feet per hour.

Result of Observations of the Vitiation of the Air.

[Angus Smith, M.D.]

Atmosphere	3.2 to 3 4	Theatres, average	8. to 32
City Parks	3.2 to 3.8	Offices "	17. to 22
" Streets	3.8 to 4.4	Workshops "	20. to 30
" " in a fog	6. to 6.8	Mines "	78. to 250

[See continuation, p. 629.]

Latitude and Longitude of Principal Places and Public Observatories.

Compiled from the Records of the U. S. Coast Survey and Topographical Engineer Corps, the Imperial Gazetteer, and Bowditch's Navigator.

Longitude computed from the Meridian of Greenwich.

L. *represents Light-house;* Ch., *Church;* S. H., *State-house;* C. H., *Custom-house;* C. S., *Coast Survey;* and Obs., *Observatory.*
N. and S., *the divisions of the Latitude;* and E. and W., *the courses East and West of Greenwich.*

Place.	Latitude.			Longitude.		
NORTH AND SOUTH AMERICA.	N.			W.		
	°	′	″	°	′	″
Acapulco........Mex.	16	50	19	99	49	09
Albany.........N. Y.	42	39	50	73	44	49
Annapolis........Md.	38	58	42	76	29	06
Ann Arbor .....Mich.	42	16	48	83	43	03
Antigua, E. Pt...W. I.	17	05		61	45	
Auburn.........N. Y.	42	55		76	28	
Augusta..........Ga.	33	28		81	54	
Augusta..........Me.	44	18	43	69	50	
Austin..........Tex.	30	13	30	97	39	
Baltimore, Mon't..Md.	39	17	48	76	36	39
Bangor, M. Ch....Me.	44	48	20	68	45	42
Baton Rouge......La.	30	26		91	18	
Benicia..........Cal.	38	03	21	122	07	13
Beaufort, Arsen'l, S. C.	32	25	57	80	41	23
Bellevue, Am. Fur Co. Post	38	08	24	95	47	46
Boston, S. H.....Mass.	42	21	30	71	03	30
Boston, L....... "	42	19	36	70	53	06
Balize ....La.	29	08	05	89	01	04
Brazos Santiago..Tex.	26	06		97	12	
Bridgeport .....Conn.	41	10	30	73	11	04
Bristol.........R. I.	41	40	11	71	16	05
Brooklyn, N. Yd..N. Y.	40	42		73	58	30
Brunswick ....Me.	43	54	29	69	57	24
Buffalo, L.......N. Y.	42	50		78	59	
Burlington......N. J.	40	04	52	74	52	37
Burlington........Vt.	44	27		78	10	
Bath, W. S. Ch....Me.	43	54	55	69	48	40
Barnegat, L. ....N. J.	39	46		74	06	
Beaufort........N. C.	34	43	05	76	39	28
Barbadoes, S. Pt..W. I.	13	03		59	37	
		S.				
Buenos Ayres..Brazil	34	36	08	58	22	
		N.				
Cambridge, Obs..Mass.	42	22	48	71	07	40
Calais, C. S. Sta'n, Me.	45	11	05	67	16	30
Camden.........S. C.	34	17		80	33	
Canandaigua....N. Y.	42	54	09	77	17	
Cape Ann, S. L..Mass.	42	38	11	70	34	10
Cape Cod, L. P. L.. "	42	2		70	09	48
Cape Flat'ry, L..W. T.	48	23	15	124	43	54
Cape Hancock, Colo. R.	46	16	35	124	01	45
Cape Hatteras, L. N. C.	35	15	02	75	30	54
Cape May, L.....N. J.	38	55	48	74	57	18
Cape Race.......N. S.	46	39	24	53	04	3
Cape Henlopen, L. Del.	38	46	06	75	04	07
Cape Fear ......N. C.	33	48		77	57	
Cape Carnaveral, Fla.	28	27	30	80	33	
Cape Florida, L... "	25	39	54	80	09	02
Caraccas .. Maracaibo	10	30		67	01	30
		S.				
Cape St. Roque, Brazil	5	28		35	17	
Cape Horn, S. Pt. Hermit's Island	55	50		67	16	

Table of Latitude and Longitude—(*Continued*).

Place.	Latitude.	Longitude.
NORTH AND SOUTH AMERICA.	S. ° ′ ″	W. ° ′ ″
Callao, Flag Staff, Peru	12 4	77 13
	N.	
Cape Sable......N. S.	43 24	65 36
Cape Sable, C. S..Flo.	25 6 53	81 15
Cape Charles.....Va.	37 7 18	75 57 54
Cape Henry, L.... "	36 55 30	76 0 2
Cape Breton..... "	45 57	59 48 5
Castine..........Me.	44 22 30	68 45
Cedar Keys, Depot Isl.	29 7 27	82 56 12
Charleston, C.Ch..S.C.	32 46 44	79 55 39
Chagres, Centre of Plateau..........N. G.	9 20	80 1 21
Cheboygan, L...Mich.	45 40 9	84 24 37
Chicago, R. C. Ch..Ill.	41 53 48	87 37 47
Cincinnati, Obs..Ohio	39 5 54	84 29 31
Charlestown, B. Hill Monument ...Mass.	42 22 36	71 3 18
Carthagena......N. G.	10 26	75 38
Cleveland, L.....Ohio	41 31	81 51
Columbia, S. H...S. C.	33 59 57	81 1 54
Columbus.......Ohio	39 57	83 3
Concord, S. H...N. H.	43 12 29	71 29
Corpus Christi...Tex.	27 47 18	97 27 2
Council Bluffs, Neb. T.	41 30	95 48
Crescent City, L..Cal.	41 44 34	124 11 22
Campeachy..Yucatan	19 49	90 33
Dayton..........Ohio	39 44	84 11
Des Moines.......Iowa	41 35	93 40
Detroit, St. P. Ch., Mich.	42 19 46	83 2 23
Dover............Del.	39 10	75 30
Dover..........N. H.	43 13	70 54
Dominica, N. P'r, W.I.	15 38	61 26
Dubuque.........Iowa	42 29 55	90 39 57
Eastport, Un. Ch..Me.	44 54 10	66 58 59
Edenton, C. H...N. C.	36 3 27	76 35 48
Erie, L.........Penn.	42 8 43	80 4 12
Fredericksb'g, E. Ch., Va.	38 18 6	77 27 17
Falls St. Anth'y, Minn.	44 58 40	93 10 30
Fire Island, L...N. Y.	40 37 54	73 12 48
Fort Gibson, Ind. Ter.	35 47 35	95 15 10
Fort Laramie..Neb. T.	42 12 10	104 47 43
Fort Leavenworth, Ks.	39 21 14	94 44
Frankfort.........Ky.	38 14	84 40
Frederick........Md.	39 24	77 18
Frederickton....N. B.	46 3	66 38 15
Galveston, Cath'l, Tex.	29 18 17	94 46 59
Gloucester, E. P. L., Mass.	42 34 47	70 39 33
Guadaloupe, S. W. Pt., W. I.	15 57	61 44
Georgetown......Ber.	32 22 2	64 37 6
	S.	
Guayaquil......Quito	2 13	79 53
	N.	
Grand Cayman, E. Pier..........W. I.	19 20	81 10
Havana, Moro...Cuba	23 9	82 21 23
Hole in the Wall, L., Bahamas.	25 51 5	77 10 6
Halifax, Obs., D.Y'rd, N. S.	44 39 4	63 35
NORTH AND SOUTH AMERICA.	N. ° ′ ″	W. ° ′ ″
Harrisburg.....Penn.	40 16	76 50
Hartford, S. H..Conn.	41 45 59	72 40 45
Holmes Hole, Ch., Mass.	41 27 13	70 35 59
Huntsville.......Ala.	34 36	86 57
Hudson........N. Y.	42 14	73 46
Indianapolis.....Ind.	39 55	86 5
Jackson.........Miss.	32 23	90 8
Jalapa..........Mex.	19 30 8	96 54 30
Jefferson City....Mo.	38 36	92 8
Key West, L......Flo.	24 33	81 47 18
Kingston, C. H..C. W.	44 8	76 28 37
Kingston.....Jamaica	17 58	76 46
Knoxville......Tenn.	35 59	83 54
Laguayra ..Maracaibo	10 36	67 2
	S.	
Lima...........Peru	12 3	77 6
	N.	
Lancaster......Penn.	40 2 36	76 20 33
Lexington........Ky.	38 6	84 18
Little Rock......Ark.	34 40	92 12
Lockport........N.Y.	43 11	78 46
Los Angelos......Cal.	34 3 15	118 10 44
Louisville........Ky.	38 3	85 30
Lowell...St. A.'s Ch., Mass.	42 38 46	71 19 2
Matamoras......Tex	25 52 50	97 27 50
Machias Bay.....Me.	44 33	67 22
Madison, Dome..Wis.	43 4 31	89 23 26
Marblehead, L...Mass.	42 30 14	70 50 39
Matagorda, C. S. Station...........Tex.	28 41 29	95 57 29
Mexico..........Mex.	19 25 45	99 5 6
Macon, Arsn'l....Ga.	32 50 24	83 37 39
Milwaukee .....Mich.	43 2 24	87 54 4
Montgomery, S.H. Ala.	32 22 46	86 17 48
Mobile, E. Ch.... "	30 41 26	88 1 29
Montreal........C. E.	45 31	73 32 56
Monterey, C. S. Station...........Cal.	36 37 36	122 49 31
Martinico, S. Point, W. I.	14 27	60 55
Montserrat, W. E. Pi'r, W. I.	16 48	62 12
Maracaibo, Maracaibo	10 39	71 45
	S.	
Monte Video, Rat Isl'd, Brazil	34 53	56 13
	N.	
Mona Island, E. Pier, W. I.	18 7	67 47
Matanzas.......Cuba	23 3	81 40
Nantucket, S. Tower, Mass.	41 16 54	70 5 36
Nashville, U...Tenn.	36 9 33	86 49 3
Nassau, L.......N. P.	25 5 2	77 21 2
Natchez........Miss.	31 34	91 24 42
Nebraska, Junction of Forks of Platte Riv.	41 5 5	101 21 24
New Bedford....Mass.	41 38 10	70 55 16
Newbern........N. C.	35 20	77 5
Newburgh.......N. J.	41 31	74 1
Newburyport, E. L., Mass.	42 48 25	70 48 40

Table of Latitude and Longitude—(*Continued*).

Place.	Latitude.			Longitude.		
NORTH AND SOUTH AMERICA.	N.			W.		
	°	′	″	°	′	″
Newcastle, E.Ch.. Del.	39	39	36	75	33	27
New Haven, Col, Conn.	41	18	26	72	55	25
New London, P. Ch., Conn.	41	21	16	72	5	29
New Orleans, M't..La.	29	57	46	90	2	30
Newport, L. .....R. I.	41	29	12	71	18	29
NEW YORK, C. H. N.Y.	40	42	43	74		3
Norfolk, F. Bank..Va.	36	50	50	76	18	47
Norwich ......Conn.	41	33		72	7	
Nantucket, L....Mass.	41	23	24	70	2	24
Ocracoke, L.....N. C.	35	6	28	75	58	51
Ogdensburg, L...N.Y.	44	45		75	30	
Olympia....Wash. T.	47	3		122	55	
Ottawa ....C. W.	45	23		75	42	4
Old Point Comfort, L.. Va.	37	0	2	76	18	6
Panama, Cath'l, N. G.	8	57	9	79	27	17
Pensacola, Sq're. Flo.	30	24	33	87	12	4
Perote..........Mex.	19	28	57	97	8	15
Philadelphia, S. H., Penn.	39	56	53	75	8	42
Pittsburg ......Penn.	40	32		80	2	
Petersburg, C. H.. Va.	37	13	47	77	23	55
Plattsburg......N. Y.	44	42		73	26	
Plymouth, C. S. Stat'n, Mass.	41	57	23	70	39	47
Point Hudson..W. T.	48	7	3	122	44	33
Portland, C. H....Me.	43	39	28	70	14	58
Providence, U. Ch. R. I.	41	49	26	71	23	59
Portsmouth, N. L. N. H.	43	4	14	70	42	12
Puebla de los Angelos, Mex.	19		15	98	2	21
Porto Rico, N. E. Pier, W. I.	18	24		65	39	
Port au Prince..W. I.	18	33		72	16	3
Porto Cabello, M'caibo	10	28		68	7	
Porto Belio .....N. G.	9	34		79	40	
Prairie du Chien, Wis.	43	2		91	8	35
Quebec, Citadel..C. E.	46	49	12	71	12	15
Raleigh, Square..N. C.	35	46	50	78	37	50
Richmond, Cap. ..Va.	37	32	16	77	25	43
Rochester, Rochr. H., N. Y.	43	8	17	77	51	
	S.					
Rio Janeiro, Sugar L'f	22	56		43	9	
	N.					
Sackett's Harb'r, N.Y.	43	55		75	57	
Savannah, Exch...Ga.	32	4	52	81	5	15
Sacramento......Cal.	38	34	41	121	27	44
St. Augustine....Flo.	29	48	30	81	35	
St. Louis ....Mo.	38	37	28	90	15	16
St. Paul........Minn.	44	52	46	95	4	54
Salem, Spire....Mass.	42	31	12	70	53	36
Saltillo ....Mex.	25	26	22	101	1	45
Salt Lake City...Utah	40	46	8	112	6	8
San Antonio.....Tex.	29	25	22	98	29	15
San Diego, C.S.O., Cal.	32	41	58	117	13	22
Sandusky, L....Ohio	41	32	30	82	42	15
Sandy Hook, L...N. J.	40	27	42	73	59	48
San Francisco, Presidio............Cal.	37	47	36	122	26	48
San Francisco, C. S. Station ....Cal.	37	48		122	23	19
San Louis Obispo. "	35	10	38	120	43	31
San Pedro........ "	33	43	20	118	16	3
Santa Fé.....N. Mex.	35	41	6	106	1	22
Schenectady ....N. Y.	42	48		73	55	
Syracuse .... "	43	3		76	9	16
Springfield, L......Ill.	39	48		89	33	
Stonington, L...Conn.	41	19	36	71	54	
Sweet Water River, Mouth of...Neb. T.	42	27	18	107	45	27
St. Christopher, N. Pt., W. I.	17	24		C2	50	
St. Eustatia, Town, W. I.	17	29		63		
St. Josephs ....Mo.	23	3	13	109	40	44
St. Bartholomew, S. Point..........W. I.	17	53	30	62	56	54
St. Martins, Fort. "	18	5		63	3	
St. Croix, Obs.... "	17	44	30	64	40	42
St. John's........ "	18	18		64	42	
St. Thomas, Fort Ch'n, W. I.	18	21		64	55	18
St. Domingo ....W. I.	18	29		69	52	
St. Jago de Cuba, Entrance........W. I.	19	58		75	52	
St. Vincent's, S. Point, W. I.	13	9		61	14	
Turk's Island, N. Pt. G. Turk......W. I.	21	32		71	10	
Tobago, N.E. P'r. "	11	20		60	27	
Trinidad, Fort .. "	10	39		61	32	
Tampa Bay, E. Key. Flo.	27	36		82	45	15
Tallahassee...... "	30	28		84	36	
Tampico, Bar....Mex.	22	15	30	97	51	51
Taunton, T. C. Ch., Mass.	41	54	11	71	5	55
Toronto.........C. W.	43	39	35	79	23	21
Trenton, P. Ch...N. J.	40	13	10	74	45	30
Troy ....Un'y...N. Y.	42	43	44	73	40	41
Tuscaloosa ....Ala.	33	12		87	42	
Utica, Dut. Ch...N. Y.	43	6	49	75	13	
Vandalia ....Ill.	38	50		89	2	
Vera Cruz ......Mex.	19	11	52	96	8	36
Victoria.........Tex.	28	46	57	97	1	
Vincennes ....Ind.	38	43		87	25	
		S.				
Valparaiso, Fort, Chili	33	2		71	41	
		N.				
WASHINGTON, Capitol	38	53	20	77	0	15
West Point, Obs. M. A., N. Y.	41	23	26	73	57	1
Wheeling ....Va.	40	7		80	42	
Wilmington, C. H., N. C.	34	14	3	77	56	47
Wilmington, T. H., Del.	39	44	27	75	32	42
Worcester, Ant. H., Mass.	42	16	17	71	48	13
York ....Penn.	39	58		76	40	
Yorktown........Va.	37	13		76	34	

Table of Latitude and Longitude—(*Continued*).

Place.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N.	E.
	° ′ ″	° ′ ″
Antwerp	51 13	4 24
Alexandria, L.	31 12	29 53
Archangel	64 32	40 33
Athens	37 58	23 44
Aleppo	36 11	37 10
Algiers, L.	36 47	3 4
Amsterdam	52 22	4 53
Borneo, Roads	5	115
	S.	
Batavia, Obs.	6 8	106 50
	N.	
Bussorah	30 30	48
	S.	
Botany Bay, Cape Roads	34 2	151 13
	N.	
Barcelona	41 23	2 11
Bombay, Flag Staff	18 56	72 54
		W.
Bristol	51 27	2 35
		E.
Bremen	53 5	8 49
Berlin, Obs.	52 30 16	13 23 45
Brussels, Obs.	50 51 10	4 22
	S.	
Bencoolen, Fort, Sumatra	3 48	102 19
	N.	W.
Cape Clear	51 26	9 29
		E.
Calais	50 58	1 51
Constantinople, St. Sophia	41 1	28 59
	S.	
Cape St. Mary, Madagascar	25 39	45 7
	N.	
Canton	23 7	113 14
Cronstadt	59 59	29 47
Copenhagen	55 41	12 34
	S.	
Cape of G. Hope, Obs.	33 56 3	18 28 45
	N.	W.
Cadiz	36 32	6 18
		E.
Calcutta	22 34	88 20
Christiana	59 55	10 43
Corinth	37 54	22 52
Cairo	30 3	31 18
Candia	35 31	25 8
Ceylon, Pt. Pedro	9 49	80 23
	S.	
Congo River	6 8	12 9
	N.	W.
Dublin	53 23 12	6 20 30
		E.
Dover	51 8	1 19
		W.
Edinburgh	55 57	3 12
	S.	
Falkland Islands, St. Helena, Obs.	15 55	5 45
	N.	
Fayal, S. E. Point	38 30	28 42
	S.	E.
Feejee Group, Ovolau, Obs.	17 41	178 53

Place.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N.	E.
	° ′ ″	° ′ ″
Florence	43 46	11 16
		W.
Funchal, Madeira	32 38	16 55
GREENWICH	51 28 38	—
		E.
Geneva	46 11 59	6 9 15
		W.
Gallego Island	1 42	104 5
Glasgow	55 52	4 16
Gibraltar	36 7	5 22
		E.
Genoa	44 24	8 53
Honolulu	21 19	157 52
	S.	
Hood's Island, Marq's	9 26	138 57
	N.	
Hamburg	53 33	9 58
Havre	49 29	6
Jeddo	35 40	140
Jerusalem	31 48	37 20
		W.
Liverpool, Obs.	53 24 47	3
		E.
Leyden	52 9 28	4 29 15
Leghorn, L.	43 32	10 18
		W.
Lisbon	38 42	9 9
		E.
Leipsic	51 20 20	12 22
Moscow	55 40	35 33
Malta, Valetta	35 54	14 30
Messina, L.	38 12	15 35
		W.
Madrid	40 25	3 42
Malaga	36 43	4 26
		E.
Mocha	13 20	43 12
Muscat	23 37	58 35
Marseilles	43 18	5 22
Majorca, Castle	39 34	2 23
Manilla	14 36	121 2
Madras	14 4 9	80 15 45
	S.	
New Zealand, N. Cape	34 24	173 1
New Hebrides, Table Island	15 28	167 7
	N.	
Niphon, Cape Idron, Japan	34 36 3	138 50 35
Naples, L.	40 50	14 16
	S.	W.
Navigators' Islands, Opoun, E. Pier	14 9	169 2
Owhyhee	20 23	155 54
	N.	E.
Odessa	46 28	30 44
Pekin	39 54	116 28
Palermo, L.	38 8	13 22
Paris, Obs.	48 50 13	2 20
	S.	
Prince of Wales Isl'd, Torres Strait	10 46	142 12
	N.	W.
Porto Praya, Cape Verd Islands	14 54	23 3

Table of Latitude and Longitude—(*Continued*).

Place.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	S.	E.
	° ′ ″	° ′ ″
Port Jackson	38 51 32	151 18
	N.	
Rome, St. Peter's....	41 54	12 27
Rotterdam..........	5 54	4 29
	S.	W.
Scilly Islands.......	16 30	155 10
	N.	E.
Sevastopol..........	44 37	33 30
Smyrna	38 26	27 7
Siam...............	14 55	100
Surat, Castle........	21 11	72 47
		W.
Santa Cruz.....Ten.	28 28	16 16
		E.
Singapore	1 17	103 50
	S.	
Sydney	33 52 42	151 23
	N.	W.
Seville	36 59	5 58

Place.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N.	W.
	° ′ ″	° ′ ″
Senegal, Fort.......	16 1	16 32
	S.	
Sierra Leone........	8 30	13 18
	N.	E.
Suez	29 59	32 34
	S.	
St. Helena....	15 55	5 45
	N.	
Stockholm, Obs.	59 20 31	18 6
St. Petersburg	59 56	30 19
Toulon.............	43 07	5 22
Tripoli	34 54	13 11
Tunis, City.........	36 47	10 6
Tangier............	35 47	5 54
Venice	40 50	14 26
Vienna.............	48 13	16 23
Warsaw, Obs.	52 13 5	21 2 9
	S.	
Zanzibar Island..Sp.	6 28	39 33

Public and Private Observatories.

Longitude given in Time.

Place.	Latitude.	Longitude.
	N.	W.
	° ′ ″ ‴	h. m. s.
Albany, Dudley.	42 39 49.55	4 54 59.52
		E.
Berlin..........	52 30 16.7	53 35.5
		W.
Birr Castle, Earl of Rosse	53 5 47	31 40.9
		E.
Brussels........	50 51 10.7	17 28.9
		W.
Cambridge, U. S.	42 22 49	4 44 32
		E.
Cambridge	52 12 51.6	22.75
	S.	
Cape of G. Hope.	33 56 3	1 13 55
	N.	
Copenhagen, University	55 40 53	50 19.8
		W.
Dublin..........	53 23 13	25 22
Edinburgh	55 57 23.2	12 43.6
		E.
Florence........	43 46 41.4	45 3.6
Geneva.........	46 11 59.4	24 37.7
		W.
Georgetown, U.S.	38 54 26.1	5 8 18.15
Greenwich......	51 28 38	—
		E.
Hamburg	53 33 5	39 54.1
Leipsic	51 20 20.1	49 28.5
Leyden.........	52 9 28.2	17 57.5

Place.	Latitude.	Longitude.
	N.	W.
	° ′ ″ ‴	h. m. s.
Liverpool.......	53 24 47.8	12 .11
		E.
Madras.........	13 4 8.1	5 20 57.3
Marseilles	43 17 50	21 29
Moscow	55 45 19.8	2 30 16.96
Munich, Bogenhausen	48 8 45	46 26.5
Naples, Capo di Monte........	40 51 46.6	56 58.86
Palermo........	38 6 44	53 24.17
Paris...........	48 50 13	9 20.63
		W.
Portsmouth.....	50 48 3	4 23.9
		E.
Quebec	46 48 30	4 44 49.02
Rome, College...	41 53 52.2	49 54.7
Stockholm......	59 20 31	1 12 14.8
St. Petersburg .. Academy	59 56 29.7	2 1 13.5
		W.
Santiago de Chili	33 26 24.8	4 42 18.9
Washington	38 53 39	5 8 12
		E.
Unkrechtsberg, Olmutz.......	49 35 40	1 9 .1
		W.
L.M. Rutherfurd, New York	40 43 48.53	4 55 55.73
	S.	E.
Sydney.........	33 51 41.1	10 4 59.86

Table showing the Difference in the Time at the following Places.

Longitude computed both from New York and Greenwich.

Place.	New York. h. m. s.	Greenwich. h. m. s.
Acapulco	1 43 17S.	6 39 17S.
Albany	1 1F.	4 54 59
Alexandria, Egy't	6 55 32	1 59 32F.
Algiers	5 8 16	12 16
Amsterdam	5 15 32	19 32
Antwerp	5 13 30	17 36
Auburn	9 52S.	5 5 52S.
Augusta .....Ga.	41 36	5 37 36
Austin	1 34 36	6 30 36
Baltimore	10 27	5 6 27
Bangor	20 57F.	4 35 3
Barbadoes	57 32	3 58 28
Bath	16 45	4 30 15
Baton Rouge	1 9 12S.	6 5 12
Berlin	5 49 35F.	53 35F.
Beaufort....N. C.	10 38S.	5 6 38S.
Boston......S. H.	11 46F.	4 44 14
Bombay	9 47 36	4 51 36F.
Bremen	5 31 16	35 16
Bridgeport	3 16	4 52 44S.
Brunswick	16 10	4 39 50
Buffalo, L.	19 56S.	5 15 56
Burlington..N. J.	3 30	4 50 30
Buenos Ayres	1 2 32F.	3 53 28
Brooklyn, N. Yard	6	4 55 54
Cadiz	4 30 48	25 12
Callao	12 52S.	5 8 52
Calais,.......Me.	26 53F.	4 29 7
Calcutta	10 49 20	5 53 20F.
Canton	12 28 56	7 32 56
Cape Race	1 23 44	3 32 16S.
Cairo......Egypt	7 1 12	2 5 12F.
Cape May	3 54S.	4 59 54S.
Cape Horn	36 56F.	4 19 4
Chicago	54 31S.	5 50 31
Cincinnati	41 58	5 37 58
Charleston	23 43	5 19 43
Cleveland	31 24	5 27 24
Columbus...Ohio	36 12	5 32 12
Concord....N. H.	10 4F.	4 45 56
Charlestown	11 47	4 44 13
Columbia....S. C.	28 8S.	5 24 8
Corpus Christi	1 33 48	6 29 48
Cape of Good Hope	6 9 55F.	1 13 55F.
Constantinople	6 51 56	1 55 56
Copenhagen	5 46 16	50 16
Dayton	40 44S.	5 36 44S.
Detroit	36 10	5 32 10
Dubuque	1 6 40	6 2 40
Dublin	4 30 38F.	25 22
Dover......N. H.	12 24	4 43 36
Dover.......Del.	6 S.	5 2
Eastport	28 4F.	4 27 56
Edinburgh	4 43 12	12 48
Erie	24 17S.	5 20 17
Florence	5 41 4F.	45 4F.
Fort Leavenworth	1 22 56S.	6 18 56S.
Fredericksb'g, Va.	13 49	5 9 49
Frederickt'n, N. B.	29 27F.	4 26 33
Frankfort....Ky.	42 40S.	5 38 40
Funchal	3 48 20F.	1 7 40
Galveston	1 23 8S.	6 19 8
Genoa	5 31 32F.	35 32F.
Geneva	5 20 37	24 37
Georgetown, Ber..	37 32	4 18 28S.
Gibraltar	4 34 32	21 28
Glasgow	4 38 56	17 4
GREENWICH	4 56	—
Halifax.....N. B.	41 40	4 14 20
Harrisburg	11 20S.	5 7 20
Hamburg	5 35 52F.	39 52F.
Hartford	5 17	4 50 43S.
Havana	33 25S.	5 29 25
Havre	4 55 36F.	24
Hudson	56	4 55 4
Huntsville	51 48S.	5 47 48
Indianapolis	48 20	5 44 20
Jackson	1 4 32	6 0 32
Jeddo	14 16 F.	9 20 F.
Jerusalem	7 25 20	2 29 20
Jefferson City	1 12 32S.	6 8 32S.
Kingston...C. W.	9 54S.	5 5 54
Kingston ...Jam.	11 4	5 7 4
Knoxville	39 36	5 35 36
Leghorn	5 37 12F.	41 12F.
Lima	12 24S.	5 8 24S.
Lisbon	4 19 24F.	36 36
Liverpool	4 44	12
Little Rock	1 12 48S.	6 8 48
Lexington	41 12	5 37 12
Louisville	46	5 42
Lowell	10 44F.	4 45 16
Macon	38 30S.	5 34 30
Madrid	4 41 52F.	14 8
Malaga	4 38 16	17 44
Malta	5 54	58 F.
Manilla	13 0 8	8 4 8
Marseilles	5 17 28	21 28
Matanzas	30 40S.	5 26 40S.
Matagorda	1 27 49	6 23 49
Matamoras	1 33 51	6 29 51
Mexico	1 40 20	6 36 20
Milwaukee	55 36	5 51 36
Mocha	7 48 48F.	2 52 48F.
Mobile	56 6S.	5 52 6S.
Montreal	1 49F.	4 54 11
Monterey....Cal.	3 11 18S.	8 7 18
Montgomery	49 11	5 45 11
Moscow	7 18 12F.	2 22 12F.
Monte Video	1 11 8	3 44 52S.
Naples	5 53 4	57 4F.
Natchez	1 9 38S.	6 5 38S.
Nassau, L.	13 24	5 9 24
Nantucket, S. Ch.	15 38F.	4 40 22
Nashville	51 16S.	5 47 16
New Orleans	1 4 10	6 0 10
New London	7 38F.	4 48 22
Newport	10 56	4 45 14
New Bedford	12 19	4 43 41

Table—(*Continued*).

Place.	New York.	Greenwich.	Place.	New York.	Greenwich.
	h. m. s.	*h. m. s.*		*h. m. s.*	*h. m. s.*
New York......	—	4 56 S.	Siam...........	11 36 F.	6 40 F.
			Schenectady.....	20	4 55 40S.
Newburg........	4S.	4 56 4	Springfield....Ill.	1 2 12S.	5 58 12
Newcastle.......	6 14	5 2 14	Sandy Hook.....	1F.	4 55 59
New Haven......	4 18F.	4 51 42	Stonington	8 24	4 47 36
Norfolk	9 15S.	5 5 15	St. Croix	37 17	4 18 43
Norwich.........	7 32F.	4 48 28	St. Thomas	36 19	4 19 41
Odessa..........	6 58 56	2 2 56F.	St. John's...W. I.	37 12	4 18 48
Odgensburg	6 S.	5 2 S.	St. Domingo.....	16 32	4 39 28
Old Point Comfort	9 12	5 5 12	St. Jago de Cuba .	7 28S.	5 3 28
Ottawa	6 48	5 2 48	Smyrna	6 44 28F.	1 48 28F.
Owhyhee........	5 27 36	10 23 36	Suez............	7 6 16	2 10 16
Paris	5 5 20F.	9 20F.	St. Petersburg ...	6 57 16	2 1 16
Panama.........	21 48S.	5 17 48S.	Stockholm.......	6 8 24	1 12 24
Pekin...........	12 41 52F.	7 45 52F.	St. Helena.......	4 33	23 S.
Petersburg	13 36S.	5 9 36S.	St. Augustine....	30 20S.	5 26 20
Palermo.........	5 49 28F.	53 28F.	St. Louis	1 5 1	6 1 1
Pittsburg........	24 8S.	5 20 8S.	St. Paul.........	1 16 20	6 12 20
Philadelphia.....	4 34	5 0 34	St. Joseph....Mo.	2 22 43	7 18 43
Plattsburg.......	2 16F.	4 53 44	Syracuse	8 37	5 4 37
Portland	15 1	4 40 59	Tampa Bay......	35 1	5 31 1
Porto Rico.......	33 24	4 22 36	Tallahassee......	42 24	5 38 24
Portsmouth, N. H.	13 12	4 42 49	Tampico Bar	1 35 27	6 31 27
Pensacola	52 48S.	5 48 48	Taunton	11 37F.	4 44 23
Providence	10 24F.	4 45 36	Toronto.........	21 33S.	5 17 33
Porto Praya	3 22	1 34	Toulon..........	5 17 38F.	21 28F.
Prairie du Chien.	1 8 34S.	6 4 34	Tripoli..........	5 48 44	52 44
Quebec..........	11 11F.	4 44 49	Troy............	1 17	4 54 43S.
Raleigh	18 31S.	5 14 31	Trenton.........	3 2S.	4 59 2
Rio Janeiro......	2 3 24F.	2 56 36	Tunis...........	5 36 24F.	40 24F.
Richmond	13 43S.	5 9 43	Tuscaloosa.......	54 48S.	5 50 48S.
Rotterdam.......	5 13 56F.	17 56F.	Turk's Island....	11 20F.	4 44 40
Rome	5 45 48	49 48	Utica	4 52S.	5 0 52
Rochester	15 24S.	5 11 24S.	Valparaiso.......	9 16F.	4 46 44
Sackett's Harbor.	7 48	5 3 48	Vandalia........	1 0 8S.	5 56 8
Sacramento......	3 9 51	8 5 51	Vera Cruz.......	1 28 34	6 24 34
Santa Cruz, Ten'fe	3 50 56F.	1 5 4	Vincennes.......	53 40	5 49 40
Savannah........	28 21S.	5 24 21	Venice..........	5 53 4F.	57 4F.
San Diego.......	2 52 53	7 48 53	Vienna..........	6 1 32	1 5 32
Sandusky........	34 49	5 30 49	Victoria.....Tex.	1 32 4S.	6 28 4S.
San Antonio	1 37 57	6 33 57	Warsaw.........	6 20 9F.	1 24 9F.
Salt Lake City ...	2 32 24	7 28 24	Washington, Cap.	12 1S.	5 8 12S.
Salem	12 25F.	4 43 35	Wilmington, Del.	6 11	5 2 11
San Francisco....	3 13 47S.	8 9 47	Wilmington, N. C.	15 47	5 11 47
Santa Fé........	2 8 5	7 4 5	Wheeling........	26 48	5 22 48
Singapore	11 52 F.	6 56 F.	West Point......	12F.	4 55 48
Sydney	5 9 32S.	10 5 32S.	Worcester.......	8 47	4 47 13
Seville	4 32 8F.	23 52	York............	10 40S.	5 6 40
Sierra Leone.....	4 2 48	53 12	Yorktown.......	10 16	5 6 16

F *representing Fast, and* S *Slow.*

To Ascertain the Difference of Time between New York and Greenwich and any Place not given in the Table.

Reduce the longitude of the place to time, and if it is W. of the assumed meridian it is *Slow;* if E., it is *Fast.*

If the difference for New York is required, and it exceeds 4*h.* 56*m.*, subtract this sum, and the remainder will give the difference of time, S.; and if it does not exceed it, subtract the difference from it, and the remainder will give the difference of time, F.

Distances by Railroad in Miles between the following Cities of the U. S.–1876.

ILLUSTRATION OF THE TABLE.—New York to Washington is 228 miles.

BY RIVERS EXCLUSIVELY.

Pittsburg	to Cincinnati....	500 miles.		
Cincinnati	" Louisville.....	150 "		
Louisville	" Cairo.........	400 "		
Cairo	" St. Louis......	180 "		
St. Louis	" Cincinnati....	730 "		
Cairo	" Memphis.....	240 "		
"	" New Orleans..	1040 "		
Memphis	" Vicksburg....	400 "		
"	" Port Hudson..	650 "		
"	" New Orleans..	800 "		
Louisville	" " ..	1450 "		
Natchez	" " ..	320 "		

St. Louis	to Denver.......	622 miles.
"	" Galveston.....	870 "
New York	" Brashear......	1486 "
"	" Vicksburg....	1337 "
"	" Omaha.......	1402 "
"	" Galveston.....	1900 "
"	" San Francisco..	3315 "
Boston	" Halifax.......	641 "
"	" St. John, N. B..	451 "

NOTE.—For additional distances, see page 95.

* By rail and steamer.

	Buffalo	Pittsburg	Cincinnati	Chicago	St. Louis	Galveston	New Orleans	Mobile	Montgomery	Savannah	Charleston	Norfolk	Richmond	Washington	Baltimore	Philadelphia	New York	Albany	Boston	Portland
Boston																				115
Albany																			201	315
New York																		143	228	343
Philadelphia																	90	233	318	433
Baltimore																98	188	331	416	531
Washington															40	138	228	371	456	571
Richmond														136	176	274	364	507	592	707
Norfolk													104	240	280	378	468	611	696	811
Charleston												452	458	595	635	727	823	966	1051	1166
Savannah											104	556	562	802	842	934	1030	1173	1254	1373
Montgomery										378	482	804	705	711	751	849	939	1082	1167	1282
Mobile									168	546	650	972	873	879	919	1017	1107	1250	1335	1450
New Orleans								140	308	686	790	912	1013	1019	1059	1157	1247	1390	1475	1590
Galveston							320*	460	628	1006	1110	1222	1333	1339	1379	1477	1567	1710	1795	1910
St. Louis						870	695	623	761	1131	1103	1079	975	893	919	973	1071	836	1036	1414
Chicago					281	451	913	920	752	741	713	981	877	795	795	818	913	729	929	1044
Cincinnati				294	340	1145	881	772	604	589	561	660	556	553	579	667	736	724	924	1100
Pittsburg			313	468	619	1458	1194	1206	917	970	866	513	409	327	350	354	444	566	766	787
Buffalo		269	427	505	539	1572	1308	1318	1031	1170	1079	867	521	439	399	350	423	297	497	612
Nashville	1052	608	295	547	319	933	613	473	305	582	614	802	722	848	888	986	1076	1119	1304	1419

Nearest Traveling Distances in Statute Miles between the Principal Cities of Europe.

[Nystrom.]

Illustration of the Table.—London to Liverpool is 212 miles.

	Copenhagen	Stockholm	St. Petersburg	Moscow	Odessa	Constantinople	Warsaw	Trieste	Rome	Munich	Vienna	Turin	Berne	Berlin	Hamburg	Antwerp	Lisbon	Madrid	Paris	Liverpool	London
Liverpool																					212
Paris																				509	297
Madrid																			625	1002	799
Lisbon																		265	890	1267	1055
Antwerp																	1142	877	252	544	332
Hamburg																391	1560	1295	580	936	724
Berlin															175	509	1515	1250	625	984	772
Berne														608	605	541	1150	885	442	951	739
Turin													193	803	901	736	895	630	445	954	742
Vienna												662	574	500	679	751	1724	1459	790	1295	1083
Munich											270	508	281	454	600	513	1431	1166	551	1045	833
Rome										507	745	328	521	963	1109	1022	1223	958	773	1373	1161
Trieste									407	342	338	456	584	796	942	855	1351	1086	1107	1399	1187
Warsaw								798	1069	563	460	1024	836	536	711	1045	2051	1786	1297	1589	1377
Constantinople							1200	1000	1050	1679	1409	1456	1584	1909	2084	2160	2350	2086	2412	2704	2492
Odessa						750	805	1400	1907	1369	1099	1856	2000	1336	1511	1836	2750	2485	1900	2450	2238
Moscow					910	1660	792	2751	3158	2683	2413	3191	2964	2489	2664	3173	4004	3739	3114	3717	3505
St. Petersburg				403	1200	1950	733	1506	1774	1271	1162	1742	1544	969	1144	1478	2484	2219	1594	1953	1741
Stockholm			440	835	1953	2775	1148	1633	1928	1291	1370	1799	1606	866	691	1082	2251	1986	1334	1626	1413
Copenhagen		435	875	1270	1772	2340	967	1198	1493	856	935	1364	1171	431	256	647	1816	1551	899	1191	979
Lübeck	234	609	1109	1504	1514	2123	709	743	1146	630	718	849	643	214	39	352	1530	1334	710	896	634

Sailing Distances in Geographical or Nautical Miles between the Principal Ports of the United States, etc.

ILLUSTRATION OF THE TABLE.—Halifax to Boston is 383 miles.

Pensacola	to Tortugas...	472	miles.
New Orleans	" Havana....	601	"
"	" Tampico...	717	"
"	" Galveston..	379	"
"	" Matamoras.	580	"
"	" Vera Cruz .	782	"
New York	to Matamoras...	1969	"
"	" Greytown	2090	"
"	" Bermudas....	680	"
"	" Nassau	962	"
"	" Panama......	2047	"
"	" Aspinwall.....	2000	"

Panama to San Francisco......3240 miles.

	Vera Cruz	New Orleans	Havana	Key West	Savannah	Charleston	Cape Hatteras	Richmond	Norfolk	Washington	Baltimore	Cape Henry	Philadelphia	Cape Henlopen	New York	Sandy Hook	Nantucket Light	Boston	Halifax
Boston																			383
Nantucket Light																		103	365
Sandy Hook																	257	338	572
New York																18	275	356	590
Cape Henlopen															168	150	326	407	641
Philadelphia														94	262	244	420	501	735
Cape Henry													246	152	275	257	430	511	740
Baltimore												166	412	318	441	423	596	677	906
Washington											183	186	432	338	461	443	616	697	926
Norfolk										190	158	33	279	185	308	290	463	544	773
Richmond									101	275	223	129	375	281	404	386	559	640	869
Cape Hatteras								253	157	310	290	124	316	222	348	330	489	570	796
Charleston							265	527	431	584	564	398	597	503	623	605	761	842	1066
Savannah						98	341	603	507	660	640	474	673	579	699	681	837	918	1142
Key West					564	594	764	1026	930	1083	1063	897	1096	1002	1122	1104	1260	1341	1552
Havana				71	699	639	809	1071	975	1128	1108	942	1141	1047	1167	1149	1305	1386	1597
New Orleans			602	573	1137	1167	1337	1599	1503	1656	1636	1470	1669	1575	1695	1677	1833	1914	2125
Vera Cruz		782	877	859	1413	1443	1613	1875	1779	1932	1912	1746	1945	1851	1971	1953	2109	2190	2401
Porto Bello	1558	1410	972	1142	1650	1643	1682	1928	1832	1985	1965	1799	1990	1896	20[illegible]5	1987	2057	2138	2293

Sailing Distances in Geographical or Nautical Miles between the Principal Ports of the World.

ILLUSTRATION OF THE TABLE.—Canton to San Francisco is 6090 miles.

New York	to Canton *via* Pacific R. R.	10,500	miles.
London	" " " "	13,500	"
"	" " " Suez Canal	10,000	"
New York	" " " Panama	11,500	"
"	" " " Suez Canal	11,500	"
"	" Melbourne	12,720	"
San Francisco	" Oahu, Sandwich Islands	2,080	"
Oahu	" Canton	4,910	"

	St. Petersburg	Port Jackson	New York	New Orleans	Marseilles	London	Liverpool	Lisbon	Havre	Hamburg	Gibraltar	Copenhagen	Constantinople	Cape Horn	Cape of Good Hope	Canton	Batavia
Canton																	1900
Cape of Good Hope																7000	5200
Cape Horn															3610	8840†	8820* / 8720†
Constantinople														8400	7040	14020*	12250*
Copenhagen													3673	8150	6900	13940*	12140*
Gibraltar												1953	1920	6470	5123	12140*	10330*
Hamburg											1657	492	3577	7810	6560	13600*	11800*
Havre										492	1189	830	3109	7460	6130	13140*	11340*
Lisbon									1040	1498	293	1835	2213	6020	4700	11700*	9900*
Liverpool								1010	550	992	1312	1329	3224	7400	6080	13100*	11310*
London							689	1196	190	417	1358	682	3278	7510	6260	13300*	11500*
Marseilles						2036	1846	973	1896	2336	680	2676	1470	7150	5800	12800*	11000*
New Orleans					5165	4730	4480	4370	4570	5030	4485	5370	6400	7600	7260	16440†	12470*
New York				1699	3850	3234	2986	3148	3068	3540	3170	3873	5100	7232	6790	16070†	12000*
Port Jackson			12900†	13260†	11800*	12250*	12100	10690*	12120*	12550*	11110*	12890*	13020*	5663	5990	4680	3910
St. Petersburg		13620	4100	6100	3400	1412	2060	2565	1560	1222	2653	730	4573	8880	7630	14670*	12870*
San Francisco	15250†	6454	13610†	13980	13500†	13880†	13800†	12400†	13840†	14200†	12850†	14520†	14770†	6380	9990†	6090	7650

* Around Cape of Good Hope. † Around Cape Horn. ‡ New South Wales.

Sailing Distances between various Ports of England, the Canadas, the United States, etc., etc.

	Miles		Miles.
Halifax to Liverpool	2,463	New York to Panama, *via* C. Horn	11,329
Boston to Halifax	383	New Orleans to Minatitlan	816
" " Liverpool (*via* Halifax)	2,856	" " Puerto Cabello, Honduras	945
Philadelphia to Liverpool	3,147	Bermudas to Nassau	804
Cape Bonavista to Cape Spear	76	Panama to David.......Chiriqui	276
Cape Spear to Cape Race	55	" " San Juan del Sud	570
Cape Race to Liverpool	1,992	" " Gulf of Fonseca	739
" " Halifax	457	" " Acapulco	1,416
" " Boston	835	" " Manzanilla	1,724
" " New York	1,004	" " San Diego	2,897
" " Philadelphia	1,155	" " Monterey	3,198
" " Cape Pine	19	" " San Francisco	3,240
St. Johns, N. F., to Quebec	891	San Francisco to San Juan del Sud	2,685
" " Boston	890	" " Gulf of Fonseca	2,591
" " Liverpool	1,956	" " Acapulco	1,841
" " Galway	1,677	" " Manzanilla	1,543
" " Bristol	1,936	" " San Diego	474
" " Greenock	1,848	" " Monterey	105
" " St. Peter's L't.	183	" " Humboldt	200
" " Cape Spear	5	" " Columbia R. Bar	530
" " Cape Race	60	" " Vancouver	638
" " C. Bonavista	72	" " Portland	650
New York to Minatitlan	1,962	" " Port Townshend	732
" " Puerto Cabello, Honduras)	2,114	" " Victoria	715

Sailing Distances between various Ports and New York and London.

Not included in the preceding Table.

	Miles	Miles.		Miles.	Miles.		Miles	Miles.
Alexandria	4893	2,980	Cork	2,782	560	Pensacola	1623	4,654
Amsterdam	3291	262	Cowes	3,092	200	Philadelphia	227	3,404
Barbadoes	1855	3,812	Funchal	2,760	1,303	Quebec	1360	3,080
Batavia	8972	11.492	Galway	2,720	721	Queenstown	2780	551
Bermudas	682	3,142	Glasgow	2,913	807	Rio Janeiro	4970	5,076
Bombay	8522	10,703	Greenock	2,835	789	St. Helena	5096	3,433
Boston	340	3,084	Halifax	560	2,706	St. Johns	1064	2,214
Bremen	3428	498	Havana	1,161	4,197	Southampton	3103	211
Bristol	2979	591	Hobart Town	9,187	11,368	Swan River	8480	10,661
Buenos Ayres	6010	6.162	Kingst'n, Jam.	1,456	4,305	Teneriffe	2909	1,522
Cadiz	3125	1.115	Lima	10,050	10,149	Tortugas	1151	4,182
Calcutta	9350	11,531	Madras	8,707	10,888	Venice	4953	2,950
Cape Race	1004	2,249	Norfolk	390	3,447	Washington	530	3,612

Distances between several Cities of the U. S.,

Not included in Table on page 91.

	Miles.		Miles.
New Orleans to Cairo	548	St. Louis to Nashville	319
" " Jackson	183	Louisville " Montgomery	490
Memphis " Grenada	100	" " Memphis	377
" " Little Rock	134	" " Nashville	185
St. Louis " Cairo	147	Montgomery " Pensacola	163
" " Omaha	479	St. Louis " Denison	585

Table showing the least Water in the Channels of certain Harbors, Rivers, and Anchorages on the Coast of the United States. [U. S. Coast Survey.]

ATLANTIC COAST.

Harbors, etc.	Locations.	Feet.	Feet.
Kennebec River.....	Up to Hanniwell's Point.....................	25.5	33.6
Portland........Me.	Breakwater to anchorage....................	16	24.9
	Channel off town..........................	27	35.9
Portsmouth....N. H.	Narrows to the city........................	45	53.5
Newburyport	Over Bar...................................	7	14.8
Ipswich	Over Bar...................................	7.5	16.1
Annisquam	Over Bar...................................	6.5	15.5
Gloucester..........	Channel to S. E. Harbor....................	30	38.9
	Up into inner Harbor.......................	24	32.9
Salem.........Mass.	Southern Ship Channel......................	28	37.2
	Inside of Salem Neck.......................	19	28.2
Boston ....Mass.	Channel, Lovell's and Gallop's Islands......	28.5	38.5
	Channel, Governor's and Castle Islands.....	18	28
Plymouth	Up to anchorage	14	24.2
	Anchorage in the Cow Yard..................	24	34.2
Barnstable Harbor...	Over Bar...................................	7.7	17
Newport........R. I.	Anchorage S. and W. of Goat Island.........	33	36.9
	Wharves inside of Goat Island	21	24.9
	Newport to Prudence Island.................	31	34.9
	Mount Hope Bay.............................	42	45.9
New York	Gedney's Channel...........................	23	27.8
	Swash Channel..............................	17	21.8
	South Channel..............................	21	25.8
	Main Channel...............................	31	35.8
	Ship Channel, after passing S.W. Spit buoy.	23	27.8
Arthur's Kill........	Anchorage Perth Amboy......................	22	26.9
	*Woodbridge to Rossville...................	13.5	18.6
	†Rossville to Chelsea	14	19.1
	‡Chelsea, Western Channel, to Elizabethport.	13	18.1
	Elizabethport to Shooter's Island	6.5	10.9
Kill von Kull	Shooter's Island to Bergen Point L. H.......	10	14.3
	Bergen Point L. H. to New Brighton.........	27	31.3
Newark Bay	§Bergen Pt. L. H. to mouth of Hackensack R.	7	11.6
Hudson River.......	Castle Garden to Manhattanville............	32	36
	Manhattanville to Yonkers..................	27	30.8
	Yonkers to Piermont........................	39	42.6
	‖Piermont Ferry to Sing Sing	24.5	28
	Sing Sing to Haverstraw....................	26	29.1
	Haverstraw to Peekskill	27	30.1
Delaware Bay.......	¶Main Channel, passing Delaw're Breakwat'r	61	64.5
	Off Brandywine L. H.	43	46.5
	Main Channel to Bombay Hook L.	27.5	33.4
	Main Channel, Liston's Point	20	25.9
Delaware River	Main Channel to Reedy Island	20	26
	Main Channel, Reedy Island L. H............	24.5	30.5
	Opposite Delaware City.....................	30	36
	Up to Marcus Hook	20.5	27
	Opposite Chester...........................	24.5	30.7
	Bar off Hog Island	18.5	24.7
	Greenwich Point to Philadelphia............	21.5	27.5
Chesapeake Bay.....	Capes at entrance to Hampton Roads	30	32.5

* Two bars, each a quarter of a mile, have a less depth than 18 feet.

† A small shoal, with 12 feet, lies in the middle of the Kill, opposite the wharf at Blazing Star; and another, with 10 feet, a quarter of a mile to the northward; but deeper water is found on the east side of both.

‡ A shoal, with 4 feet, obstructs the eastern channel, half way between Chelsea and its junction with the main channel.

§ From Bergen Point Light, half way to Newark Bay Light-house, 17 feet may be carried.

‖ A shoal of 12.5 feet occurs about a mile below Sing Sing.

¶ Soundings varying between 10 and 15 fathoms.

Table—(*Continued*).

ATLANTIC COAST.

Harbors, etc.	Locations.	Feet.	Feet.
Chesapeake Bay	Anchorage, Hampton Roads................	59	61.5
	Hampton Roads to Sewall's Point............	25	27.5
	S. of Sewall's Point (one mile and a half)....	21	23.5
	Up to Norfolk..........................	23	25.5
	Hampton Roads to James River............	27	29.5
Potomac River			
York River...... Va.	Tail of York Spit to Yorktown..............	33	35.5
James River ..... Va.	White Shoal Bar.........................	16	18.5
	Up to Jamestown Island Bar................	19	21.5
	Channel to one mile above Deep Water L. H.	23	25.5
	Jamestown Island Bar......................	15	17.5
	Harrison's Bar...........................	13.5	16.3
	* Trent's Reach	8.5	11.7
	* Warwick Bar..........................	12.5	15.7
	* Richmond Bar	7	9.9
Elizabeth River. . Va.	Norfolk to Navy-yard......................	25.5	28
Hatteras Inlet. . N. C.	Over Bar..................................	14	17.6
	† Over Bulkhead into Pamlico Sound.........	7	9
Ocracoke Inlet	Over Bar	10	12.4
	Anchorage, Wallico's Channel..............	19	21.4
Albemarle Sound....	‡ Light-boat off Caroon's Point...............	7	
	‡ Up the Sound to Martin's Point............	5.5	
North River.... N. C.	‡ At entrance, and seven miles up from Albemarle Sound............................	6.7	
Beaufort .... N. C.	Main Channel	15	17.8
	Through the Slue	7	9.8
Cape Fear	New Inlet Bar............................	8	12.5
Georgetown .... S. C.	Entrance to Winyah Bay	7	10.8
	Anchorage inside of North Island...........	27	30.8
	Up to Georgetown	9	12.6
Bull's Bay	Over Bar..................................	13	17.8
	Anchorage...............................	21	25.8
Charleston ..... S. C.	Main Bar..................................	11	16.1
	North Channel...........................	10	15.1
	Maffit's Channel	11	16.1
Stono Inlet	Over Bar..................................	6.5	12.5
North Edisto	S. or Main Channel.......................	12	17.8
St. Helena Sound....	S. Channel	17	22.9
	S. Edisto.................................	14	19.9
Port Royal..........	S. E. Channel............................	19.5	25.9
Tybee	Bar near Tybee Island....................	19	26
	Tybee Roads.............................	31	38
Savannah..........	Channel up to city........................	11	17.5
Ossabaw Sound	S. Channel to Vernon River	12	18.6
	S. Channel to Ogeechee River	13	19.6
Sapelo Sound	Over Bar	18	25
Doboy Bar (Inlet) ...	Entrance over Bar.........................	15.5	22.1
	Anchorage in Sound.......................	24	30.6
St. Simon's Sound ...	Over Bar..................................	15	21.8
	Entrance to Sound........................	33	44.8
	Turtle River to Blythe Island	21	27.8
	To Brunswick over Bar	9	15.8
	To Brunswick, Channel	13	19.8
St. Mary's	Over Bar	11	16.8
St. Johns River. . Flo.	Over Bar..................................	7	11.5

* The effect of neap and spring tides is very small. The depth is affected much more sensibly by the stage of the river above.

† The tide diminishes rapidly after entering the Inlet.

‡ There are no lunar tides in Albemarle, Currituck, and Pamlico Sounds.

Table—(*Continued*).

ATLANTIC COAST.

Harbors, etc.	Locations.	Feet.	Feet.
St. Johns River . . Flo.	Channel up to Jacksonville	23	25.1
St. Augustine	Over Bar	7	11.2
Florida Reef	Cape Florida L. H. W. S. W. ¾ W.	20	21.5
	Turtle Harbor entrance	26	27.5
	Inside the Reefs, Hawk Channel	11	12.5
	Key Sambo Channel	34	35.3
Key West	Main Channel to middle buoy on Shoals	27	28.3
	Shoals to anchorage	30	31.3
	Sand Key Channel	27	28.3
	W. Channel	30	31.3
Tortugas	N. W. Channel	45	46.2
	S. W. Channel	54	55.2
Tampa Bay	Over Bar	19	20.4
	Channel Egmont and Passage Keys	17	18.4
Waccasassa Bay	Channel to anchorage	8	10.6
Cedar Keys	Main Channel over Bar	9	11.5
St. Mark's	Over Bar	9	11.5
	Up to Fort St. Mark's	7	9.5
St. Gecrge's Sound	E. entrance over Bar	15.5	17.1
	Anchorage	19	20.6
Apalachicola	*Over Bar	13	14.1
	Up to anchorage	10	11.1
St. Andrew's Bay	*Main Channel, over Bar	13	14
Pensacola	*Over Bar	22.5	23.5
	Bar to Navy-yard	27	28
	Wharf at Pensacola	21	22
Mobile Bay and River	*Over outer Bar	21	22
	Main Channel to Fort Morgan	36	37
	To Upper Fleet	12	13
Mississippi Sound	*Grant's Pass to Pascagoula Wharf	7.5	8.7
	Horn Island Pass, over Bar	15	16.2
	Anchorage, Horn Island	19	20.2
	Up to Pascagoula Wharf	8	9.2
Ship Island Harbor	*Channel	19	20.3
	N. W. Channel	19.5	20.8
	Anchorage	18	19.3
Cat Island Harbor	*Ship Channel	16	17.3
	S. Pass	14	15.3
	Shell Bank Channel	15.2	16.5
Mississippi Delta	*Pass à l'Outre, N. Channel	9.5	10.6
	S. Channel	12	13.1
Northeast Pass	*Over Bar, N. entrance	9.5	10.6
Southeast Pass	*Entering	10	11.1
South Pass	*Channel	8	9.1
Southwest Pass	Channel	15 5	16.6
Barrataria Bay	*Bar of Grand Pass	7.5	8.7
	Grand Passage to Independence Island	15	16.2
Dernière, or Last Isl'd	*Chan'l inside, and N. cf Ship Isl'd Shoal L. S.	27	28.4
	Channel N. of Ship Island Shoal	14	15.4
Atchafalaya Bay	Entrance to Cut-off Channel buoy	8	9.6
	On Bulkhead	6.5	8 1
	Mouth of Atchafalaya River	48	49.6
Vermilion Bay	Mid-channel off L. H.	42	43.6
Calcasieu River	*Entrance over Bar	5.5	7.4
Sabine Pass	*Across the Bar	7.5	9
Galveston Bay	*Entrance over Bar	12	13.1
San Luis Pass	*Over Bar	8	9.1
Brazos River	*Over Bar	8	9.1
Matagorda Bay	*Entrance over Bar	9	10.1
Aransas Pass	*Aransas Pass	9	10.1
Rio Grande	Channel	4	4.9

* The highest tides occur at the moon's greatest declination.

Table—(*Continued*).

PACIFIC COAST.

Harbors, etc.	Locations.	Feet.	Feet.
San Diego Bay	Entrance	27.4	31.3
	Abreast of La Plaza	18	21.9
San Pedro	Point Pedro and Dead Man's Island	18	21.9
Point Duma	Anchorage	54	58
San Buenaventura	Anchorage	36	40
Santa Cruz Island	Anchorage, Prisoner's Harbor	75	79
Santa Barbara	Anchorage inside of Kelp	18	22
San Miguel Island	Cuyler's Harbor	37	41.1
Coxo Harbor	Anchorage	30	34
San Luis Obispo	Anchorage	33	36.8
San Simeon	Harbor anchorage	24	27.8
Monterey Harbor	Anchorage	42	45.9
	Near shore	30	33.9
Santa Cruz Harbor	Anchorage	27	31.6
San Francisco Bay	From Four-fathom Bank to S. shore	28	32.1
	Rincon Point	66	70.1
	Market Street Wharf	54	58.1
	Cunningham's Wharf	36	40.1
San Francisco Harbor	On the Bar	33	37.1
Mare Island Straits	Mid-channel	25	30.5
	Mid-channel, Navy-yard, and Vallejo	25	30,5
Ballenas Bay	Inside of Breakers, Duxbury Reef	24	28.1
Sir Frs. Drake's Bay	Inside the Point	17	21.1
Tomales Bay	Over Bar	10	14.1
Bodega Bay	Inside of Reef, off Point	36	40.1
Coast	Haven's anchorage	48	52.3
Albion River	Anchorage	48	52.4
Mendocino City	Anchorage	30	34.4
Shelter Cove	Anchorage	22	26.6
Humboldt Bay	Channel	20	24.7
Crescent City Harbor	Anchorage off city	21	26.2
Ewing Harbor	Anchorage	46	51.7
Koos Bay	Over Bar	11	16.7
Umpqua River	On Bar, opposite Mid-channel	13	19
Columbia River	N. Channel to Baker's Bay	24	30.5
	*Entrance into S. Channel	19	25 5
	On Bar of S. Channel	16	22.5
Shoalwater Bay	On Bar	18	24.5
	S. Channel	25	31.5
Grenville Harbor	Anchorage	22	28.5
Neé-ah Harbor	Anchorage	36	42.5
False Dungeness	Anchorage	54	60.4
New Dungeness	Anchorage	45	51.4
Smith's Isl'd, N. side.	Anchorage near Kelp	25	31.4
Bellingham Bay	Anchorage	18	24.4
Port Townshend	Anchorage	48	54.4
Port Ludlow	Anchorage	36	42.4
Port Gamble	Anchorage	18	24.4
Seattle	Anchorage	20	28
Blakely Harbor	Anchorage	46	54
Steilacoom Harbor	Anchorage	18	29.5
Olympia Harbor	Mid-channel	11	22.5

* 21 feet may be carried in at mean low water.

Proportion of Alcohol in 100 Parts of the following Liquors.—(Brande.)

Small Beer...1. and	1.08	Vin de Grave......	12.08	Colares............	19.75
Cider........5.2 and	9.8	Champagne.......	12.61	Lisbon.............	18.94
Porter.......3.5 and	5.26	" Burgundy	14.57	Malaga............	17.2
Brown Stout. 5.5 and	6.8	Hermitage, red....	12.32	Cape Muscat.......	18.25
Ale........6.87 and	10.	" white..	17.43	Teneriffe...........	19.79
Perry..............	7.26	Amontillado.......	12.63	Lachryma..........	19.7
Rhenish............	7.58	Barsac............	13.86	Currant Wine......	20.55
Moselle............	8.7	Sauterne..........	14.22	Madeira...........	22.27
Johannisberger....	8.71	White Port........	15.	" Sercial.....	27.4
Elder Wine........	8.79	Bordeaux.........	15.1	Marsala...........	25.09
Claret ordinaire....	8.99	Shiraz............	15.52	Raisin Wine.......	25.12
Tokay............	9.33	Malmsey..........	16.4	Cape Madeira......	29.51
Rudesheimer.......	10.72	Sherry............	17.17	Gin...............	51.6
Marcobrunner.....	11.6	" old.........	23.86	Brandy............	53.39
Gooseberry Wine..	11.84	Alba Flora........	17.26	Rum..............	53.68
Frontignac........	12.89	Constantia, red....	18.92	Irish Whisky......	53.9
Hockheimer.......	12.03	Port..............	23.	Scotch Whisky.....	54.32

TABLE SHOWING THE DILUTION PER CENT. NECESSARY TO REDUCE SPIRITUOUS LIQUORS.

Water to be added to 100 volumes of spirit when of the following strength:

Strength Required.	90	85	80	75	70	65	60	55	50
Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
85	5.9								
80	12.5	6.3							
75	20	13.3	6.7						
70	28.6	21.4	14.3	7.1					
65	38.5	30.8	23.1	15.4	7.7				
60	50	41.7	33.3	25	16.7	8.3			
55	63.6	54.5	45.5	36.4	27.4	18.2	9.1		
50	80	70	60	50	40	30	20	10	
40	125	112.5	100	87.5	75	62.5	50	37.5	25
30	200	183.3	166.7	150	133.3	116.7	100	83.3	66.7

Illustration.—100 volumes of spirituous liquor having 90 per cent. of spirit contains: alcohol, 90, water, 10=100.

To reduce it to 30 per cent. there is required 200 volumes of water.

$$\text{Hence } 200+10=210, \text{ and } \frac{90}{210}=\frac{30}{70}=\frac{30 \text{ spirit,}}{70 \text{ water,}} \text{ or } 30 \textit{ per cent.}$$

Acids.

Acetic Acid (Vinegar), the acid of *Malt beer*, etc., etc.

Lactic Acid, the acid of *Millet beer* and *Cider*.

Tartaric Acid, the acid of *Grape wine*.

Order of Acidity of several Wines.

1. Moselle.
2. Rhine.
3. Burgundy.
4. Madeira.
5. Claret.
6. Champagne.
7. Port.
8. Sherry.

Nutritious Properties of different Vegetables and Oil-cake, compared with each other in Quantities.

Oil-cake............	1.	Bran, wheat..2.75 and	3.	Old Potatoes........	20.
Peas and Beans.....	1.5	Corn................	3.	Carrots.............	17.5
Rice................	1.6	Barley..............	3.	Cabbages...........	18.
Wheat, grain.......	2.5	Pea straw...........	3.	Wheat straw........	26.
" flour........	2.	Clover hay..........	4.	Barley "	26.
Oats................	2.5	Hay.................	5.	Oat "	27.5
Rye................	2.5	Potatoes............	14.	Turnips............	30.

Illustration.—1 lb. of oil-cake is equal to 18 lbs. of cabbage.

Divisions of the Compound Substances of the cultivated Crops of Vegetables.

Vegetable.	Divisions. Saccharine or Sugary.	Oleaginous or Oily.	Albuminous or Fleshy.	Sulphur, Phosphorus, Iron, etc.
	Per Cent.	Per Cent.	Per Cent.	Per Cent.
Wheat Flour*	55	2½	10 to 19	20 to 30
Barley	60	2.4	12 to 15	30
Oats	60	5½	14 to 19	40
Rye	60	—	—	—
Corn	70	5½	12	15
Rice	75	—	7	—
Beans	40	2.4	24 to 28	30
Peas	50	2.4	24	30
Potatoes	18	.2	2	8 to 15
Mangel Wurzel	11	—	2	—
Turnips	9	.2	1½	5 to 8
Bran	—	3½	16	—
Wheat Straw	—	2.4	—	50
Oat "	—	4	—	60
Clover Hay	—	3.4	—	90

Mineral Constituents absorbed or removed from an Acre of Soil by several Crops.—(Johnson.)

	Wheat, 25 bushels.	Barley, 40 bushels.	Turnips, 20 tons.	Hay, 1.5 tons.
	Lbs.	Lbs.	Lbs.	Lbs.
Potassa	29.6	17.5	47.1	38.2
Soda	3.	5.2	8.2	12.
Lime	12.9	17.	29.9	44.5
Magnesia	10.6	9.2	19.7	7.1
Oxide of Iron	2.6	2.1	7.1	.6
Phosphoric Acid	20.6	25.8	46.3	15.1
Sulphuric Acid	10.6	2.7	13.3	9.2
Chlorine	2.	16.	3.6	4.1
Silica	118.1	129.5	247.8	78.2
Alumina	—	2.4	—	—
Total	210.	213.	423.	209.

Yield of Oil from Several Seeds.

	Per Cent.		Per Cent.		Per Cent.
Poppy	56 to 63	Castor	25	Hemp	14 to 25
Cress	56 to 58	Sunflower	15	Linseed	11 to 22

Average Quantity of Tannin in several Substances. —(Morfit.)

	Per Cent.		Per Cent.
Catechu, Bombay	55.	Sassafras, root bark	58.
" Bengal	44.	Sumac, Sicily and Malaga	16.
Kino	75.	" Virginia	10.
Nutgalls, Aleppo	65.	" Carolina	5.
" Chinese	69.	Willow, inner bark	16.
Oak, old, inner bark......21. and	14.2	" weeping	16.
" young, "	15.2	Sycamore bark	16.
" " entire bark	6.	Tan shrub "	13.
" " spring-cut bark	22.	Cherry-tree	24.
" " root bark	8.9	Tormentil root	46.
Chestnut, American rose, bark	8.	Corneus Sanguinea	44.
" horse, "	2.	Alder bark	36.

* 100 lbs. of wheat flour contains: Fine flour . . . 40 | Common flour . . 80 | Middlings . . . 10 | Blue flour . . 5 | Bran, etc. . . 25

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Analysis of different Articles of Food, with Reference only to their Properties for giving Heat and Strength.

In 100 *Parts.*

Substances.	Carbon.	Nitrogen.	Substances.	Carbon.	Nitrogen.	Substances.	Carbon.	Nitrogen.
Beef, meat	11	3	Cheese, Chest'r	41.04	4.13	Oatmeal	44	1.95
Liver, Calf's	15.68	3.93	Beans	42	4.5	Bread, stale	28	1 07
Cod-fish, salted	16	5.02	Peas	44	3.66	Potatoes	11	.33
Sardines	29	6	Wheat	41	3	Carrots	5.5	.31
Mackerel	19.26	3.74	Flour	38.5	1.64	Wine	4	.015
Eels	30.05	2	Rye Flour	41	1.75	Alcohol	52	
Eggs	13.5	1.9	Barley	40	1.9	Beer, strong	4.5	.08
Milk, Cow's	8	.66	Corn	44	1.7	Oil, Olive	98	
Oysters	7.18	2.13	Buckwheat	42.5	2.2	Butter	83	.64
Lobster	10.06	2.93	Rice	41	1.8	Coffee	9	1.1

NOTE.—Multiply the figures representing the nitrogen by 6.5, and the equivalent amount of nitrogenous matter is obtained.

Standard Weights of Grain per Bushel.

	Lbs.		Lbs.		Lbs.		Lbs.
Wheat	60	Corn and Rye	56	Oats	32	Barley	48

Relative Value of Foods compared with 100 lbs. of Good Hay.

	Lbs.		Lbs.		Lbs.
Clover, green	400	Cornstalks, dried	400	Oats	57
Corn, green	275	Carrots	276	Corn	59
Wheat straw	374	Rye	54	Sunflower seeds	62
Rye straw	442	Wheat	45	Linseed cake	69
Oat straw	195	Barley	54	Wheat bran	105

Manures.

Relative Fertilizing Properties of various Manures.

Peruvian Guano	1.	Horse	.048	Farm-yard	.0298
Human, mixed	.069	Swine	.044	Cow	.0259

Or, 1. lb. guano = 14½ human, 21 horse, 22½ swine, 33½ farm-yard, and 38½ cow.

ZINC—SHEETS.

Thickness and Weight per Square Foot.

Inch.		Inch.		Inch.	
.0311	= 10 oz.	.0534	= 14 oz.	.0686	= 18 oz.
.0457	= 12 oz.	.0611	= 16 oz.	.0761	= 20 oz.

WINDOW GLASS.

Thickness and Weight per Square Foot.

No.	Thickness.	Weight.	No.	Thickness.	Weight.	No.	Thickness.	Weight.
	Inch.	Oz.		Inch.	Oz.		Inch.	Oz.
12	.059	12	17	.083	17	26	.125	26
13	.063	13	19	.091	19	32	.154	32
15	.071	15	21	.1	21	36	.167	36
16	.077	16	24	.111	24	42	.2	42

Dimensions of a Barrel.

Diameter of head, 17 ins.; bung, 19 ins.; length, 28 ins.; volume, 7689 cub. ins.

Pyramids of Egypt.

Base, square	745 feet.	Height	450.75 feet.
Inclined height	568.25 feet.	Weight	6 848 000 tons.

Capacity of the Principal Churches and Opera Houses.

Estimating a person to occupy an area of 19.7 *inches square.*

Churches.

St. Peter's	54 000	St. John, Lateran	22 900
Milan Cathedral	37 000	Notre Dame, Paris	21 000
St. Paul's, Rome	32 000	Pisa Cathedral	13 000
St. Paul's, London	25 600	St. Stephen, Vienna	12 400
St. Petronio, Bologna	24 400	St. Dominic's, Bologna	12 000
Florence Cathedral	24 300	St. Peter's, Bologna	11 400
Antwerp Cathedral	24 000	Cathedral of Sienna	11 000
St. Sophia's, Constantinople	23 000	St. Mark's, Venice	7 000

Opera Houses and Theatres.

Carlo Felice, Genoa	2560	Academy of Paris	2092
Opera House, Munich	2370	Teatro del Liceo, Barcelona	4000
Alexander, St. Petersburg	2332	Covent Garden, London	2684
San Carlos, Naples	2240	Opera House, Berlin	1636
Imperial, St. Petersburg	2160	New York Academy	2526
La Scala, Milan	2113	" " Stadt	3000

Area and Population of the Earth.

Divisions.	Area.	Population.	Pop. to Sq. Mile.
	Square Miles.		
America	14 491 000	70 677 000	5
Europe	3 760 000	295 954 000	79
Asia	16 313 000	558 562 000	35
Africa	10 936 000	71 604 000	6
Oceanica	4 500 000	23 261 000	5
Total	50 000 000	1 020 058 000	20.4

About $\frac{1}{30}$th of the whole population are born every year, and nearly an equal number die in the same time; making about one birth and one death per second.

The latest, and apparently the fairest, estimate of the world's population, makes it 1 150 000 000, divided as follows:

Whites	640 000 000	Mulattoes	315 000 000
Copper-colored	22 000 000	Blacks	173 000 000
Pagans	676 000 000	Mohammedans	140 000 000
Christians	320 000 000	Jews	14 000 000

The 320 000 000 Christians are divided as follows:

Church of Rome	170 000 000	Greek and East Churches	60 000 000
Protestants	90 000 000		

American Works of Magnitude.

Croton Aqueduct, N. Y.—Has a capacity of 100 000 000 to 118 000 000 gallons per day, and from Dam to Receiving Reservoir is 38.134 miles in length.

Illinois Central Railroad.—Length, Chicago to Cairo 365 miles, Centralia to Dunleith 344 miles, total 709 miles.

National Road.—Over the Cumberland Mountains to Illinois Town, 659⅝ miles in length, and 80 feet in width. Macadamized for a width of 30 feet.

Suspension Bridge, Niagara River.—Wire, span 1042 feet, 10 ins.

Stones.—Of the U. S. Treasury, Washington, are heavier than any in the Pyramids of Egypt.

Washington Aqueduct, D. C. (Captain M. C. Meigs, U. S. Engineers).—Conduit, cylinder of masonry 9 feet in diameter. Stone arch over Cabin John's Creek, 220 feet span, 57¼ feet rise.

Iron pipe bridge over Rock Creek, 200 feet span, 20 feet rise. Arch of 2 lateral courses of cast-iron pipe, 4 feet internal diameter, and 1½ ins. thick. These pipes conveying the water not only sustain themselves over the great span, but support a street road and railway.

Length of Gun Barrels.—(C. T. Coathupe.)

The length of the barrel of a gun, to shoot well, measured from the vent-hole, should be not less than 43 times the diameter of its bore, nor more than 47.

Mason and Dixon's Line.

39° 12′ N. mean latitude. 68.895 miles.

Descent of Western Rivers.

The slope of rivers flowing into the Mississippi from the East is about 3 ins. per mile; and from the West, 6 ins.

The mean descent of the Ohio River from Pittsburg to the Mississippi, 975 miles, is about 5.2 ins. per mile; and that of the Mississippi to the Gulf of Mexico, 1180 miles, about 2.8 ins.

Consumption of Atmospheric Air.—(Coathupe.)

The average daily volume of carbonic acid gas given off by the respiration of an adult human being, amounts to 4.08 per cent. of the air respired.

In 24 hours, the respiration of one healthy adult produces 10.7 cubic feet of carbonic acid gas, and removes from the atmosphere exactly the same volume of oxygen.

One wax candle (three in a pound) destroys, during its combustion, as much oxygen per hour as the respiration of one adult.

The total volume of air that can be required for the respiration of an adult human being in 24 hours, even if no portion of that which has been once respired were to be inspired again, does not exceed 266.7 cubic feet.

A lighted taper, when confined within a given volume of atmospheric air, will become extinguished as soon as it has converted 3 per cent. of the given volume of air into carbonic acid.

Days of the Roman Calendar.

The *Calends* were the first 6 days of a month, the *Nones* the following 9 days, and the *Ides* the remaining days.

In March, May, July, and October, the *Ides* fell upon the 15th and the *Nones* began upon the 7th. In the other months the *Ides* commenced upon the 13th and the *Nones* upon the 5th.

Periods of Gestation.

Elephant, 1.9 years; buffalo and camel, 1 year; horse and ass, 11 months; cow, 9 months; sheep, 5 months; lion, 5 months; dog, 9 weeks; cat, 8 weeks; sow, 16 weeks; guinea-pig, 3 weeks.

Periods of Incubation.

Swan, 42 days; parrot, 40 days; goose and pheasant, 35 days; hens of all gallinaceous birds, 21 days; pigeon, 14 days; canary, 14 days; duck, turkey, and peafowl, 28 days. The temperature of hatching is 104°.

Alimentary Principles.

1. Water; 2. Sugar; 3. Gum; 4. Starch; 5. Pectine; 6. Acetic Acid; 7. Alcohol; 8. Oil or Fat.—*Vegetable and Animal:* 9. Albumen; 10 Fibrine; 11. Caseine; 12. Gluten; 13. Gelatine; 14. Chloride of Sodium.

These alimentary principles, by their mixture or union, form our ordinary foods, which, by way of distinction, may be denominated *compound aliments;* thus, meat is composed of fibrine, albumen, gelatine, fat, etc.; wheat consists of starch, gluten, sugar, gum, etc.

Comparison of Tonnages under Old and New Laws.

	Description of Vessel.	Old Law.	New Law. Flush Deck.	New Law. Houses, etc.	New Law. Total.	Diff. + or − of old Meas't.
		Tons.	Tons.	Tons.	Tons.	Per Ct.
	Full-built Ship ..187 × 42 × 20.7 ft.	1353	1518	107	1625	20 +
	Clipper.........220.5 × 42.5 × 17.7 "	1768	1280	45	1325	25 −
	Half Clipper....213 × 42.5 × 28 "	1831	1687	100	1787	2½ −
PR.	Sea Steamer ....337 × 41.3 × 26.1 "	2803	2554	355	2909	4 +
SW.	" " ....280.6 × 46 × 32.8 "	2818	2419	225	2644	6 −
SW.	River Steamboat 329.9 × 35.4 × 10.4 "	1185	675	590	1265	7 +
SW.	" " ..393 × 51 × 10.2 "	2118	1383	1262	2645	20 +
PR.	Steam-tug ...... 81.6 × 17.4 × 7.8 "	102	55.4	–	55.4	46 −
	Coast'g Schooner.127 × 31 × 10.6 "	363	256	93	349	4 −
	Yacht........... 72 × 20 × 7.6 "	96	42	3	45	53 −
	Fishing Smack .. 36.6 × 13.6 × 3.3 "	12.2	–	–	7.6	40 −
	Canal-boat...... 94.6 × 16.6 × 8 "	118	109	–	109.	7½ −

Weight of Square Rolled Iron,

From 1/16 Inch to 9½ Inches.

ONE FOOT IN LENGTH.

Side.	Weight.	Side.	Weight.	Side.	Weight.	Side.	Weight.
Ins	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.1/16	.013	.⅞	11.883	.⅞	50.756	.⅞	116.671
.⅛	.053	2.	13.52	4.	54.084	6.	121.664
.3/16	.118	.⅛	15.263	.⅛	57.517	.¼	132.04
.¼	.211	.¼	17.112	.¼	61.055	.½	142.816
.⅜	.475	.⅜	19.066	.⅜	64.7	.¾	154.012
.½	.845	.½	21.12	.½	68.448	7.	165.632
.⅝	1.32	.⅝	23.292	.⅝	72.305	.¼	177.672
.¾	1.901	.¾	25.56	.¾	76.264	.½	190.136
.⅞	2.588	.⅞	27.939	.⅞	80.333	.¾	203.024
1.	3.38	3.	30.416	5.	84.48	8.	216.336
.⅛	4.278	.⅛	33.01	.⅛	88.784	.¼	230.068
.¼	5.28	.¼	35.704	.¼	93.168	.½	244.22
.⅜	6.39	.⅜	38.503	.⅜	97.657	.¾	258.8
.½	7.604	.½	41.408	.½	102.24	9.	273.792
.⅝	8.926	.⅝	44.418	.⅝	106.953	.¼	289.22
.¾	10.352	.¾	47.534	.¾	111.756	.½	305.056

ILLUSTRATION.—What is the weight of a bar 1½ ins. by 12 inches in length? In column 1st, find 1½; opposite to it is 7.604 lbs, which is 7 lbs. and .604 of a lb. If the lesser denomination of ounces is required, the result is obtained as follows:

Multiply the remainder by 16, point off the decimals, and the figures remaining on the left of the point give the number of ounces.

Thus, .604 of a lb. = .604 × 16 = .9.664 = 7 *lbs.* 9.664 *ounces.*

To Ascertain the Weight for less than a Foot in Length.

OPERATION.—What is the weight of a bar 6¼ inches square and 9¼ inches long? In column 6th, opposite to 6¼, is 132.04, which is the weight for a foot in length.

6.25 × 12 inches				= 132.04
6.	"	is .5		= 66.02
3.	"	is .5	of 6 =	33.01
.25	"	is .5	of .5 =	2.7508
9.75				= 101.7808 *pounds.*

Weight of Round Rolled Iron,

From 1/16 Inch to 12 Inches in Diameter.

ONE FOOT IN LENGTH.

Diam.	Weight.	Diameter	Weight	Diameter.	Weight	Diameter	Weight.
Ins.	Lbs	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.1/16	.01	.1/4	13.44	.5/8	56.788	.1/2	149.328
.1/8	.041	.3/8	14.975	.3/4	59.9	.3/4	159.456
.3/16	.093	.1/2	16.588	.7/8	63.094	8.	169.856
.1/4	.165	.5/8	18.293	5.	66.35	.1/4	180.696
.3/8	.373	.3/4	20.076	.1/8	69.731	.1/2	191.808
.1/2	.663	.7/8	21.944	.1/4	73.172	.3/4	203.26
.5/8	1.043	3.	23.888	.3/8	76.7	9.	215.04
.3/4	1.493	.1/8	25.926	.1/2	80.304	.1/4	227.152
.7/8	2.032	.1/4	28.04	.5/8	84.001	.1/2	239.6
1.	2.654	.3/8	30.24	.3/4	87.776	.3/4	252.376
.1/8	3.359	.1/2	32.512	.7/8	91.634	10.	265.4
.1/4	4.147	.5/8	34.886	6.	95.552	.1/4	278.924
.3/8	5.019	.3/4	37.332	.1/4	103.704	.1/2	292.688
.1/2	5.972	.7/8	39.864	.3/8	107.86	.3/4	306.8
.5/8	7.01	4.	42.464	.1/2	112.16	11.	321.216
.3/4	8.128	.1/8	45.174	.5/8	116.484	.1/4	336.004
.7/8	9.333	.1/4	47.952	.3/4	120.96	.1/2	351.104
2.	10.616	.3/8	50.815	7.	130.048	.3/4	366 536
.1/8	11.988	.1/2	53.76	.1/4	139.544	12.	382.208

Weight of Flat Rolled Iron,

From 1/2 × 1/8 Inch to 5 3/4 × 6 Inches.

ONE FOOT IN LENGTH.

Thickn	Weight.	Thickn.	Weight	Thickness.	Weight	Thickness	Weight.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.1/2		1.		1.1/4		1.1/2	
.1/8	.211	.1/8	.422	.5/8	2.64	.1/2	2.535
.1/4	.422	.1/4	.845	.3/4	3.168	.5/8	3.168
.3/8	.634	.3/8	1.267	.7/8	3.696	.3/4	3.802
.5/8		.1/2	1.69	1.	4.224	.7/8	4.435
.1/8	.264	.5/8	2:112	1.1/8	4.752	1.	5.069
.1/4	.528	.3/4	2.534	1.3/8		1.1/8	5.703
.3/8	.792	.7/8	2.956	.1/8	.58	1.1/4	6.337
.1/2	1.056	1.1/8		.1/4	1.161	1.3/8	6.97
.3/4		.1/8	.475	.3/8	1.742	1.5/8	
.1/8	.316	.1/4	.95	.1/2	2.325	.1/8	.686
.1/4	.633	.3/8	1.425	.5/8	2.904	.1/4	1.372
.3/8	.95	.1/2	1.901	.3/4	3.484	.3/8	2.059
.1/2	1.265	.5/8	2.375	.7/8	4.065	.1/2	2.746
.5/8	1.584	.3/4	2.85	1.	4.646	.5/8	3.432
.7/8		.7/8	3.326	1.1/8	5.227	.3/4	4.119
.1/8	.369	1.	3.802	1.1/4	5.808	.7/8	4.805
.1/4	.738	1.1/4		1.3/8	6.389	1.	5.492
.3/8	1.108	.1/8	.528	1.1/2		1.1/8	6.178
.1/2	1.477	.1/4	1.056	.1/8	.633	1.1/4	6.864
.5/8	1.846	.3/8	1.584	.1/4	1.266	1.3/8	7.551
.3/4	2.217	.1/2	2.112	.3/8	1.9	1.1/2	8.237

Table—(*Continued*).

Thickn.	Weight.
Ins.	Lbs.
1.¾	
.⅛	.739
.¼	1.479
.⅜	2.218
.½	2.957
.⅝	3.696
.¾	4.435
.⅞	5.178
1.	5.914
1.⅛	6.653
1.¼	7.393
1.⅜	8.132
1.½	8.871
1.⅝	9.61
1.⅞	
.⅛	.792
.¼	1.584
.⅜	2.376
.½	3.168
.⅝	3.96
.¾	4.752
.⅞	5.544
1.	6.336
1.⅛	7.129
1.¼	7.921
1.⅜	8.713
1.½	9.505
1.⅝	10.297
1.¾	11.089
2.	
.⅛	.845
.¼	1.689
.⅜	2.534
.½	3.379
.⅝	4.224
.¾	5.069
.⅞	5.914
1.	6.758
1.⅛	7.604
1.¼	8.448
1.⅜	9.294
1.½	10.138
1.⅝	10.983
1.¾	11.828
1.⅞	12.673
2.⅛	
.⅛	.898
.¼	1.795
.⅜	2.693
.½	3.591
.⅝	4.488
.¾	5.386
.⅞	6.283
1.	7.181
1.⅛	8.079
1.¼	8.977
1.⅜	9.874
1.½	10.772
1.⅝	11.67
1.¾	12.567
1.⅞	13.465
2.	14.362
2.¼	
.⅛	.95
.¼	1.9
.⅜	2.851
.½	3.802
.⅝	4.752
.¾	5.703
.⅞	6.653
1.	7.604
1.⅛	8.554
1.¼	9.505
1.⅜	10.455
1.½	11.406
1.⅝	12.356
1.¾	13.307
1.⅞	14.257
2.	15.208
2.⅛	16.158
2.⅜	
.⅛	1.003
.¼	2.006
.⅜	3.009
.½	4.013
.⅝	5.016
.¾	6.019
.⅞	7.022
1.	8.025
1.⅛	9.028
1.¼	10.032
1.⅜	11.035
1.½	12.038
1.⅝	13.042
1.¾	14.045
1.⅞	15.048
2.	16.051
2.⅛	17.054
2.¼	18.057
2.½	
.⅛	1.056
.¼	2.112
.⅜	3.168
.½	4.224
.⅝	5.28
.¾	6.336
.⅞	7.392
1.	8.448
1.⅛	9.504
1.¼	10.56
1.⅜	11.616
1.½	12.672
1.⅝	13.728
1.¾	14.784
1.⅞	15.84
2.	16.896
2.⅛	17.952
2.¼	19.008
2.⅜	20.064
2.⅝	
.⅛	1.109
.¼	2.218
.⅜	3.327
.½	4.436
.⅝	5.545
.¾	6.654
.⅞	7.763
1.	8.872
1.⅛	9.981
1.¼	11.09
1.⅜	12.199
1.½	13.308
1.⅝	14.417
1.¾	15.526
1.⅞	16.635
2.	17.744
2.⅛	18.853
2.¼	19.962
2.⅜	21.071
2.½	22.18
2.¾	
.⅛	1.162
.¼	2.323
.⅜	3.485
.½	4.647
.⅝	5.808
.¾	6.97
.⅞	8.132
1.	9.294
1.⅛	10.455
1.¼	11.617
1.⅜	12.779
1.½	13.94
1.⅝	15.102
1.¾	16.264
1.⅞	17.425
2.	18.587
2 ⅛	19.749
2.¼	20.91
2.⅜	22.072
2.½	23.234
2.⅝	24.395
2.⅞	
.⅛	1.215
.¼	2.429
.⅜	3.644
.½	4.858
.⅝	6.072
.¾	7.287
.⅞	8.502
1.	9.716
1.⅛	10.931
1.¼	12.145
1.⅜	13.36
1.½	14.574
1.⅝	15.789
1.¾	17.003
1.⅞	18.218
2.	19.432
2.⅛	20.647
2.¼	21.861
2.⅜	23.076
2.½	24.29
2.⅝	25.505
2.¾	26.719
3.	
.⅛	1.267
.¼	2.535
.⅜	3.802
.½	5.069
.⅝	6.337
.¾	7.604
.⅞	8.871
1.	10.138
1.⅛	11.406
1.¼	12.673
1.⅜	13.94
1.½	15.208
1.⅝	16.475
1.¾	17.742
1.⅞	19.01
2.	20.277
2.¼	22·811
2.½	25.346
2.¾	27.881
3.¼	
.⅛	1.373
.¼	2.746
.⅜	4.119
.½	5.492

Table—(*Continued*).

Thickn. Ins.	Weight. Lbs.
3.¼	
.⅝	6.865
.¾	8.237
.⅞	9.61
1.	10.983
1.⅛	12.356
1.¼	13.73
1.⅜	15.102
1.½	16.475
1.⅝	17.848
1.¾	19.221
1.⅞	20.594
2.	21.967
2.¼	24.712
2.½	27.458
2.¾	30.204
3.	32.95
3.½	
.⅛	1.479
.¼	2.957
.⅜	4.436
.½	5.914
.⅝	7.393
.¾	8.871
.⅞	10.35
.1	11.828
.1⅛	13.307
.1¼	14.785
.1⅜	16.264
.1½	17.742
.1⅝	19.221
.1¾	20.699
.1⅞	22.178
.2	23.656
.2¼	26.613
.2½	29.57
.2¾	32.527
.3	35.485
.3¼	38.441
3.¾	
.⅛	1.584
.¼	3.168
.⅜	4.752
.½	6.336
.⅝	7.921
.¾	9.505
.⅞	11.089
1.	12.673
1.⅛	14.257
1.¼	15.841
1.⅜	17.425
1.½	19.009
1.⅝	20.594

Thickn. Ins.	Weight. Lbs.
3.¾	
1.¾	22.178
1.⅞	23.762
2.	25.346
2.¼	28.514
2.½	31.682
2.¾	34.851
3.	38.019
3.¼	41.187
3.½	44.355
4.	
.⅛	1.69
.¼	3.38
.½	6.759
.¾	10.138
1.	13.518
1.¼	16.897
1.½	20.277
1.¾	23.656
2.	27.036
2.¼	30.415
2.½	33.795
2.¾	37.174
3.	40.554
3.¼	43.933
3.½	47.313
3.¾	50.692
4.¼	
.⅛	1.795
.¼	3.591
.½	7.181
.¾	10.772
1.	14.364
1.¼	17.953
1.½	21.544
1.¾	25.135
2.	28.725
2.¼	32.316
2.½	35.907
2.¾	39.497
3.	43.088
3.¼	46.679
3.½	50.269
3.¾	53.86
4.	57.45
4.½	
.¼	3.802
.½	7.604
.¾	11.406
1.	15.208
1.¼	19.01
1.½	22.812
1.¾	26.614

Thickn. Ins.	Weight Lbs.
4.½	
2.	30.415
2.¼	34.217
2.½	38.019
2.¾	41.82
3.	45.623
3.¼	49.425
3.½	53.226
3.¾	57.028
4.	60.83
4.¼	64.632
4.¾	
.¼	4.013
.½	8.026
.¾	12.036
1.	16.052
1.¼	20.066
1.½	24.079
1.¾	28.092
2.	32.105
2.¼	36.118
2.½	40.131
2.¾	44.144
3.	48.157
3.¼	52.17
3.½	56.184
8.¾	60.197
4.	64.21
4.¼	68.223
4.½	72.235
5.	
.¼	4.224
.½	8.449
.¾	12.673
1.	16.897
1.¼	21.122
1.½	25.346
1.¾	29.57
2.	33.795
2.¼	38.019
2.½	42.243
2.¾	46.468
3.	50.692
3.¼	54.916
3.½	59.14
3.¾	63.365
4.	67.589
4.¼	71.813
4.½	76.038
4.¾	80.262
5.¼	
.¼	4.436

Thickn. Ins.	Weight. Lbs.
5.¼	
.½	8.871
.¾	13.307
1.	17.742
1.¼	22.178
1.½	26.613
1.¾	31.049
2.	35.484
2.¼	39.92
2.½	44.355
2.¾	48.791
3.	53.226
3.¼	57.662
3.½	62.097
3.¾	66.533
4.	70.968
4.¼	75.404
4.½	79.839
4.¾	84.275
5.	88.71
5.½	
.¼	4.647
.½	9.294
.¾	13.94
1.	18.587
1.¼	23.234
1.½	27.881
1.¾	32.527
2.	37.174
2.¼	41.821
2.½	46.468
2.¾	51.114
3.	55.761
3.¼	60.408
3.½	65.055
3.¾	69.701
4.	74.348
4.¼	78.995
4.½	83.642
4.¾	88.288
5.	92.935
5.¼	97.582
5.¾	
.½	9.716
.¾	14.574
1.	19.432
1.¼	24.29
1.½	29.148
1.¾	34.006
2.	38.864
2.¼	43.722
2.½	48.58
2.¾	53.43[illegible]

Table—(*Continued*).

Thickn.	Weight.	Thickn.	Weight.	Thickn.	Weight	Thickn.	Weight.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
5.¾		5.¾		5.¾		5.¾	
3.	58.296	3.¾	72.87	4.½	87.443	5.¼	102.017
3.¼	63.154	4.	77.728	4.¾	92.301	5.½	106.876
3.½	68.012	4.¼	82.585	5.	97.159	6.	116.592

EXAMPLES.—What is the weight of a bar of iron 5.¼ ins. in breadth by .¾ in. in thickness?

In column 7, page 108, find 5.¼; and below it, in column 5, .¾; and opposite to that is 13.307, which is 13 lbs. and .307 of a pound.

For parts of a pound and of a foot, operate according to the rule laid down for table, page 105.

Weight and Volume of Cast Iron and Lead Balls.

From 1 *to* 20 *Inches in Diameter.*

Diameter.	Volume.	CAST IRON.	LEAD.
Ins.	Cubic Ins.	Lbs	Lbs.
1.	.5235	.1365	.2147
1.½	1.7671	.4607	.7248
2.	4.1887	1.092	1.718
2.½	8.1812	2.1328	3.3554
3.	14.1371	3.6855	5.7982
3.½	22.4492	5.8525	9.2073
4.	33.5103	8.7361	13.744
4.½	47.7129	12.4387	19.569
5.	65.4498	17.0628	26.843
5.½	87.1137	22.7206	35.729
6.	113.0973	29.4845	46.385
6.½	143.7932	37.4528	58.976
7.	179.5943	46.8203	73.659
7.½	220.8932	57.587	90.598
8.	268.0825	69.8892	109.952
8.½	321.555	83.8396	131.883
9.	381.7034	99.5103	156.553
9.½	448.9204	117.0338	184.121
10.	523.5987	136.5025	214.749
11.	696.9098	181.7648	285.832
12.	904.7784	235.8763	371.096
13.	1150.346	299.623	471.806
14.	1436.754	374.5629	589.273
15.	1767.145	460.6959	724.781
16.	2144.66	559.1142	879.616
17.	2572.44	670.7168	1055.066
18.	3053.627	796.0825	1252.422
19.	3591.363	936.2708	1472.97
20.	4188.79	1092.02	1717.995

Weight of Cast Iron Pipes of different Thicknesses,

From 1 *Inch to* 36 *Inches in Diameter.*

ONE FOOT IN LENGTH.

Diam	Thickn.	Weight.	Diam.	Thickn.	Weight.	Diam.	Thickn.	Weight.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
1.	.¼	3.06	6.	.¾	49.6	11.½	.½	58.82
	.⅜	5.05		.⅞	58.96		.⅝	74.28
1.¼	.¼	3.67	6.½	.½	34.32		.¾	90.06
	.⅜	6.		.⅝	43.68		.⅞	106.14
1.½	.⅜	6.89		.¾	53.3		1.	122.62
	.½	9.8		.⅞	63.18	12.	.½	61.26
1.¾	.⅜	7.8	7.	.½	36.66		.⅝	77.36
	.½	11.04		.⅝	46.8		.¾	93.7
2.	.⅜	8.74		.¾	56.96		.⅞	110.48
	.½	12.23		.⅞	67.6		1.	127.42
2.¼	.⅜	9.65		1.	78.39	12.½	.½	63.7
	.½	13.48	7.½	.½	39.22		.⅝	80.4
2.½	.⅜	10.57		.⅝	49.92		.¾	97.4
	.½	14.66		.¾	60.48		.⅞	114.72
	.⅝	19.05		.⅞	71.76		1.	132.35
2.¾	.⅜	11.54		1.	83.28	13.	.½	66.14
	.½	15.91	8.	.½	41.64		.⅝	83.46
	.⅝	20.59		.⅝	52.68		.¾	101.08
3.	.⅜	12.28		.¾	64.27		.⅞	118.97
	.½	17.15		.⅞	76.12		1.	137.28
	.⅝	22.15		1.	88.2	13.½	.½	68364
	.¾	27.56	8.½	.½	44.11		.⅝	86.55
3.¼	.½	18.4		.⅝	56.16		.¾	104.76
	.⅝	23.72		.¾	68.		.⅞	123.3
	.¾	29.64		.⅞	80.5		1.	142.16
3.½	.½	19.66		1.	93.28	14.	.½	71.07
	.⅝	25.27	9.	.½	46.5		.⅝	89.61
	.¾	31.2		.⅝	58.92		.¾	108.46
3.¾	.½	20.9		.¾	71.7		.⅞	127.6
	.⅝	26.83		.⅞	84.7		1.	147.03
	.¾	33.07		1.	97.98	14.½	.½	73.72
4.	.½	22.05	9.½	.½	48.98		.⅝	92.66
	.⅝	28.28		.⅝	62.02		.¾	112.1
	.¾	34.94		.¾	75.32		.⅞	131.86
4.¼	.½	23.35		.⅞	88.98		1.	151.92
	.⅝	29.85		1.	102.9	15.	.½	75.96
	.¾	36.73	10.	.½	51.46		.⅝	95.72
4.½	.½	24.49		.⅝	65.08		.¾	115.78
	.⅝	31.4		.¾	78.99		.⅞	136.15
	.¾	38.58		.⅞	93.24		1.	156.82
4.¾	.½	25.7		1.	108.84	15.½	.½	78.4
	.⅝	32.91	10.½	.½	53.88		.⅝	98.78
	.¾	40.43		.⅝	68.14		.¾	119.48
5.	.½	26.94		.¾	82.68		.⅞	140.4
	.⅝	34.34		.⅞	97.44		1.	161.82
	.¾	42.28		1.	112.68	16.	.½	80.87
5.½	.½	29.4	11.	.½	56.34		.⅝	101.82
	.⅝	37.44		.⅝	71.19		.¾	123.14
	.¾	45.94		.¾	86.4		.⅞	144.76
6.	.½	31.82		.⅞	101.83		1.	166.6
	.⅝	40.56		1.	117.6	16.½	.½	83.3

Table—(*Continued*).

Diam.	Thickn.	Weight.	Diam.	Thickn.	Weight.	Diam.	Thickn.	Weight.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
16.½	.⅝	104.82	22.	.⅝	138.6	30.	1.	303.86
	.¾	126.79		.¾	167.24		1.⅛	343.2
	.⅞	149.02		.⅞	196.46	31.	.¾	233.4
	1.	171.6		1.	225.38		.⅞	273.4
17.	.½	85.73	23.	.⅝	144.77		1.	313.68
	.⅝	107.96		.¾	174.62		1.⅛	354.24
	.¾	130.48		.⅞	204.78	32.	.¾	240.76
	.⅞	153.3		1.	235.28		.⅞	281.94
	1.	176.58	24.	.⅝	150.85		1.	323.49
17.½	.½	88.23		.¾	181.92		1.⅛	365.29
	.⅝	111.06		.⅞	213.28	33.	.¾	248.1
	.¾	134.16		1.	245.08		.⅞	290.5
	.⅞	157.59	25.	.⅝	156.97		1.	333.24
	1.	181.33		.¾	189.28		1.⅛	376.26
18.	.⅝	114.1		.⅞	221.94		1.¼	420.77
	.¾	137.84		1.	254.86	34.	.¾	255.45
	.⅞	161.9	26.	.¾	196.62		.⅞	298.88
	1.	186.24		.⅞	230.56		1.	342.88
19.	.⅝	120.24		1.	264.66		1.⅛	387.13
	.¾	145.2	27.	.¾	204.04		1.¼	431.76
	.⅞	170.47		.⅞	239.08	35.	.¾	262.7
	1.	195.92		1.	274.56		.⅞	307.62
20.	.⅝	126.33	28.	.¾	211.32		1.	352.86
	.¾	152.53		.⅞	247.62		1.⅛	398.1
	.⅞	179.02		1.	284.28		1.¼	443.96
	1.	205.8	29.	.¾	218.7	36.	.¾	270.18
21.	.⅝	132.5		.⅞	256.2		.⅞	316.36
	.¾	159.84		1.	294.02		1.	362.86
	.⅞	187.6	30.	.¾	226.2		1.⅛	409.34
	1.	215.52		.⅞	264.79		1.¼	456.46

NOTE.—These weights do not include any allowance for spigot and faucet ends.

CAST IRON.

To Compute the Weight of a Cast Iron Bar or Rod.

Find the weight of a wrought iron bar or rod of the same dimensions in the preceding tables or by computation, and from the weight deduct the $\frac{2}{27}$th part; or,

As .1000 : .9257 : : the weight of a wrought bar or rod : to the weight required. Thus, what is the weight of a piece of cast iron $4 \times 3.\frac{3}{4} \times 12$ inches?

In table, page 108, the weight of a piece of wrought iron of these dimensions is 50.692 lbs.

Then 1000 : .9257 : : 50.692 : 46.93 *lbs.*

To Compute the Weight of a piece of Cast or Wrought Iron of any Dimension or Form.

By the rules given in Mensuration of Solids (page 270), ascertain the number of cubic inches in the piece, then multiply by the weight of a cubic inch, and the product will give the weight in pounds.

EXAMPLE.—What is the weight of a cube of wrought iron 10 inches square by 15 inches in length?

$10 \times 10 \times 15 = 1500$ *cubic inches.*
.2816 weight of a cubic inch.*
422.4 *pounds.*

2. What is the weight of a cast iron ball 15 inches in diameter?

By table, page 109, 15 ins. = 176.7149 *cubic inches.*
.2607 weight of a cubic inch.*
460.6957 *pounds.*

COPPER.

To Compute the Weight of Copper.

RULE.—Ascertain the number of cubic inches in the piece; multiply them by .32118,* and the product will give the weight in pounds.

EXAMPLE.—What is the weight of a copper plate ½ an inch thick by 16 inches square?

$16^2 = 256$
.5 for ½ an inch.
$128. \times .32418 = 41.495$ *pounds.*

BRAZIER'S SHEETS are 30×60 inches, and from 12 to 100 lbs. per square foot.

SHEATHING COPPER is 14×48 inches, and from 14 to 34 oz. per square foot.

LEAD.

To Compute the Weight of Lead.

RULE.—Ascertain the number of cubic inches in the piece; multiply the sum by .41015,* and the product will give the weight in pounds.

EXAMPLE.—What is the weight of a leaden pipe 12 feet long, 3.75 inches in diameter, and 1 inch thick?

By Rule in Mensuration of Surfaces, to ascertain the Area of Cylindrical Rings.

Area of $(3.75 + 1 + 1) = 25.967$
" " 3.75 = 11.044
Difference, 14.923, *or area of ring.*
144 = 12 feet.
$2148.912 \times .41015 = 881.376$ *pounds.*

BRASS.

To Compute the Weight of ordinary Brass Castings.

RULE.—Ascertain the number of cubic inches in the piece, multiply them by .3112,* and the product will give the weight in pounds.

* The weights of a cubic inch as here given are for the ordinary metals; when, however, the specific gravity of the metal under consideration is accurately known, the weight of a cubic inch of it should be substituted for the units here given.

Weights of Wrought Iron, Steel, Copper, and Brass Plates.

SOFT ROLLED.

Thickness determined by American Gauge

No. of Gauge.	Thickness of each Number.	Plates—per Square Foot. Wrought Iron.	Steel.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46	18.4575	18.7036	20.838	19.688
000	.40964	16.4368	16.6559	18.5567	17.5326
00	.3648	14.6376	14.8328	16.5254	15.6134
0	.32486	13.0351	13.2088	14.7162	13.904
1	.2893	11.6082	11.7629	13.1053	12.382
2	.25763	10.3374	10.4752	11.6706	11.0266
3	.22942	9.2055	9.3283	10.3927	9.8192
4	.20431	8.1979	8.3073	9.2552	8.7445
5	.18194	7.3004	7.3977	8.2419	7.787
6	.16202	6.5011	6.5878	7.3395	6.9345
7	.14428	5.7892	5.8664	6.5359	6.1752
8	.12849	5.1557	5.2244	5.8206	5.4994
9	.11443	4.5915	4.6527	5.1837	4.8976
10	.10189	4.0884	4.1428	4.6156	4.3609
11	.090742	3.641	3.6896	4.1106	3.8838
12	.080808	3.2424	3.2856	3.6606	3.4586
13	.071961	2.8874	2.9259	3.2598	3.0799
14	.064084	2.5714	2.6057	2.903	2.7428
15	.057068	2.2899	2.3204	2.5852	2.4425
16	.05082	2.0392	2.0664	2.3021	2.1751
17	.045257	1.8159	1.8402	2.0501	1.937
18	.040303	1.6172	1.6387	1.8257	1.725
19	.03589	1.44	1.4593	1.6258	1.5361
20	.031961	1.2824	1.2995	1.4478	1.3679
21	.028462	1.142	1.1573	1.2893	1.2182
22	.025347	1.017	1.0306	1.1482	1.0849
23	.022571	.9057	.9177	1.0225	.96604
24	.0201	.8065	.8173	.91053	.86028
25	.0179	.7182	.7278	.81087	.76612
26	.01594	.6396	.6481	.72208	.68223
27	.014195	.5696	.5772	.64303	.60755
28	.012641	.5072	.514	.57264	.54103
29	.011257	.4517	.4577	.50994	.4818
30	.010025	.4023	.4076	.45413	.42907
31	.008928	.3582	.363	.40444	.38212
32	.00795	.319	.3232	.36014	.34026
33	.00708	.2841	.2879	.32072	.30302
34	.006304	.2529	.2563	.28557	.26981
35	.005614	.2253	.2283	.25431	.24028
36	.005	.2006	.2033	.2265	.214
37	.004453	.1787	.181	.20172	.19059
38	.003965	.1591	.1612	.17961	.1697
39	.003531	.1417	.1436	.15995	15113
40	.003144	.1261	.1278	.14242	13456
Specific Gravities.......		7.704	7.806	8.698	8.218
Weights of a Cubic Foot.		481.25	487.75	543.6	513.6
" " Inch.		.2787	.2823	.3146	.2972

Weights of Wrought Iron, Steel, Copper, and Brass Wire.

Diameters and Thickness determined by American Gauge.

No. of Gauge.	Diam of each Number.	WIRE—per Lineal Foot.			
		Wrought Iron	Steel.	Copper	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46	.56074	.56603	.640513	.605176
000	.40964	.444683	.448879	.507946	.479908
00	.3648	.352659	.355986	.40283	.380666
0	.32486	.279665	.282303	.319451	.301816
1	.2893	.221789	.223891	.253342	.239353
2	.25763	.175888	.177548	.200911	.189818
3	.22942	.13948	.140796	.159323	.150522
4	.20431	.110616	.11166	.126353	.119376
5	.18194	.08772	.088548	.1002	.094666
6	.16202	.069565	.070221	.079462	.075075
7	.14428	.055165	.055685	.063013	.059545
8	.12849	.043751	.044164	.049976	.047219
9	.11443	.034699	.035026	.039636	.037437
10	.10189	.027512	.027772	.031426	.029687
11	.090742	.02182	.022026	.024924	.023549
12	.080808	.017304	.017468	.019766	.018676
13	.071961	.013722	.013851	.015674	.014809
14	.064084	.010886	.010989	.012435	.011746
15	.057068	.008631	.008712	.009859	.009315
16	.05082	.006845	.006909	.007819	.007587
17	.045257	.005427	.005478	.006199	.005857
18	.040303	.004304	.004344	.004916	.004645
19	.03589	.003413	.003445	.003899	.003684
20	.031961	.002708	.002734	.003094	.00292
21	.028462	.002147	.002167	.002452	.002317
22	.025347	.001703	.001719	.001945	.001838
23	.022571	.00135	.001363	.001542	.001457
24	.0201	.001071	.001081	.001223	.001155
25	.0179	.0008491	.0008571	.0009699	.0009163
26	.01594	.0006734	.0006797	.0007692	.0007267
27	.014195	.000534	.0005391	.0006099	.0005763
28	.012641	.0004235	.0004275	.0004837	.000457
29	.011257	.0003358	.0003389	.0003835	.0003624
30	.010025	.0002663	.0002688	.0003042	.0002874
31	.008928	.0002113	.0002132	.0002413	.000228
32	.00795	.0001675	.0001691	.0001913	.0001808
33	.00708	.0001328	.0001341	.0001517	.0001434
34	.006304	.0001053	.0001063	.0001204	.0001137
35	.005614	.00008366	.00008445	.0000956	.00009015
36	.005	.00006625	.00006687	.0000757	.0000715
37	.004453	.00005255	.00005304	.00006003	.00005671
38	.003965	.00004166	.00004205	.00004758	.00004496
39	.003531	.00003305	.00003336	.00003775	.00003566
40	.003144	.0000262	.00002644	.00002992	.00002827

	Wrought Iron	Steel	Copper	Brass
Specific Gravities......	7.774	7.847	8.88	8.386
Weights of a Cub. Foot	485.87	490.45	554.988	524.16
" " Inch.	.2812	.2838	.3212	.3033

The Specific Gravities to determine these weights and the calculations were made by the author for Messrs. J. R. Browne & Sharpe, Providence, R. I.

Weights of Wrought Iron, Steel, Copper, and Brass Plates.

Thickness determined by Birmingham Gauge.

No. of Gauge.	Thickness of each Number.	Plates—per Square Foot.			
		Iron.	Steel.	Copper	Brass.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.454	18.2167	18.4596	20.5662	19.4312
000	.425	17.0531	17.2805	19.2525	18.19
00	.38	15.2475	15.4508	17.214	16.264
0	.34	13.6425	13.8244	15.402	14.552
1	.3	12.0375	12.198	13.59	12.84
2	.284	11.3955	11.5474	12.8652	12.1552
3	.259	10.3924	10.5309	11.7327	11.0852
4	.238	9.5497	9.6771	10.7814	10.1864
5	.22	8.8275	8.9452	9.966	9.416
6	.203	8.1454	8.254	9.1959	8.6884
7	.18	7.2225	7.3188	8.154	7.704
8	.165	6.6206	6.7089	7.4745	7.062
9	.148	5.9385	6.0177	6.7044	6.3344
10	.134	5.3767	5.4484	6.0702	5.7352
11	.12	4.815	4.8792	5.436	5.136
12	.109	4.3736	4.4319	4.9377	4.6652
13	.095	3.8119	3.8627	4.3035	4.066
14	.083	3.3304	3.3748	3.7599	3.5524
15	.072	2.889	2.9275	3.2616	3.0816
16	.065	2.6081	2.6429	2.9445	2.782
17	.058	2.3272	2.3583	2.6274	2.4824
18	.049	1.9661	1.9923	2.2197	2.0972
19	.042	1.6852	1.7077	1.9026	1.7976
20	.035	1.4044	1.4231	1.5855	1.498
21	.032	1.284	1.3011	1.4496	1.3696
22	.028	1.1235	1.1385	1.2684	1.1984
23	.025	1.0031	1.0165	1.1325	1.07
24	.022	.8827	.8945	.9966	.9416
25	.02	.8025	.8132	.906	.856
26	.018	.7222	.7319	.8154	.7704
27	.016	.642	.6506	.7248	.6848
28	.014	.5617	.5692	.6342	.5992
29	.013	.5216	.5286	.5889	.5564
30	.012	.4815	.4879	.5436	.5136
31	.01	.4012	.4066	.453	.428
32	.009	.3611	.3659	.4077	.3852
33	.008	.321	.3253	.3624	.3424
34	.007	.2809	.2846	.3171	.2996
35	.005	.2006	.2033	.2265	.214
36	.004	.1605	.1626	.1812	.1712

Wire—per Lineal Foot.

Diameter determined by Birmingham Gauge.

0000	.454	.546207	.55136	.623913	.589286
000	.425	.478656	.483172	.546752	.516407
00	.38	.38266	.38627	.437099	.41284
0	.34	.30634	.30923	.349921	.3305
1	.3	.2385	.24075	.27243	.25731
2	.284	.213738	.215755	.244146	.230596
3	.259	.177765	.179442	.203054	.191785

Table—(*Continued*).

No. of Gauge.	Diameter of each Number.	Wire—per Lineal Foot.			
		Wrought Iron.	Steel.	Copper.	Brass.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
4	.238	.150107	.151523	.171461	.161945
5	.22	.12826	.12947	.146507	.138376
6	.203	.109204	.110234	.12474	.117817
7	.18	.08586	.086667	.098075	.092632
8	.165	.072146	.072827	.08241	.077836
9	.148	.058046	.058593	.066303	.062624
10	.134	.047583	.048032	.054353	.051336
11	.12	.03816	.03852	.043589	.04117
12	.109	.031485	.031782	.035964	.033968
13	.095	.023916	.024142	.027319	.025802
14	.083	.018256	.018428	.020853	.019696
15	.072	.013738	.013867	.015692	.014821
16	.065	.011196	.011302	.012789	.012079
17	.058	.008915	.008999	.010183	.009618
18	.049	.006363	.006423	.007268	.006864
19	.042	.004675	.004719	.00534	.005043
20	.035	.003246	.003277	.003708	.003502
21	.032	.002714	.002739	.0031	.002928
22	.028	.002078	.002097	.002373	.002241
23	.025	.001656	.001672	.001892	.001787
24	.022	.001283	.001295	.001465	.001384
25	.02	.00106	.001070	.001211	.001144
26	.018	.0008586	.0008667	.0009807	.0009263
27	.016	.0006784	.0006848	.0007749	.0007319
28	.014	.0005194	.0005243	.0005933	.0005604
29	.013	.0004479	.0004521	.0005116	.0004832
30	.012	.0003816	.0003852	.0004359	.0004117
31	.01	.000265	.0002675	.0003027	.0002859
32	.009	.0002147	.0002167	.0002452	.0002316
33	.008	.0001696	.0001712	.0001937	.000183
34	.007	.0001299	.0001311	.0001483	.0001401
35	.005	.00006625	.00006688	.00007568	.00007148
36	.004	.0000424	.0000428	.00004843	.00004574

Thickness of Sheet Brass, Silver, Gold, etc.

By Birmingham Gauge for these Metals.

No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.
	Inch.		Inch.		Inch.		Inch.		Inch.		Inch.
1	.004	7	.015	13	.036	19	.064	25	.095	31	.133
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.013	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

Braziers' and Sheathing Copper.

Braziers' Sheets, 2×4 feet from 5 to 25 lbs., 2½×5 feet from 9 to 150 lbs., and 3×5 feet and 4×6 feet, from 16 to 300 lbs. per sheet.

Sheathing Copper, 14×48 inches, and from 14 to 34 oz. per square foot.

Yellow Metal, 14×48 inches, and from 16 to 34 oz. per square foot.

Comparative Thicknesses of Wire Gauges.

American.				Birmingham.			
No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
0000	7/16 +	5	3/16 −	0000	7/16 +	7	3/16 −
00	3/8 −	8	1/8 +	00	3/8 +	11	1/8 −
0	5/16 +	14	1/16 +	1	5/16 −	16	1/16 +
2	1/4 +	20	1/32 +	3	1/4 +	21	1/32 +

Weight of Wrought Angle Iron,

From 1.1/4 to 4.1/2 Inches.

ONE FOOT IN LENGTH.

Thickness measured in the Middle of each Side.

L Equal Sides.

Sides.	Thickness.	Weight.
Ins.	Ins.	Lbs.
1.25×1.25	.3/16	1.5
1.5 ×1.5	.3/16	2.
1.75×1.75	.1/4	3.
2. ×2.	.1/4	3.5
2.25×2.25	.5/16	4.5
2.5 ×2.5	.5/16	5.
3. ×3.	.3/8	7.
3.5 ×3.5	.7/16	9.
4. ×4.	.1/2	12.5
4.5 ×4.5	.1/2	14.
4.5 ×4.5	.9/16	16.

L Unequal Sides.

Sides.	Thickness.	Weight.
3. ×2.5	.3/8	6.25
3.5 ×3.	.7/16	7.75
3.5 ×3.	.7/16	9.6

L Unequal Sides.

Sides.	Thickness.	Weight.
Ins.	Ins.	Ins.
4. ×3.	.1/2	11.
4. ×3.5	.1/2	11.5
4. ×3.5	.1/2	11.75
4.5 ×3.	.1/2	11.75
5. ×3.	.1/2	12.65
5. ×3.	.9/16	13.7
5.5 ×3.5	.1/2	14.5
5.5 ×3.5	.9/16	15.6
6. ×3.5	.5/8	18.
6. ×4.5	.5/8	20.
T 2. ×2.375*	.3/8	5.5
2.5 ×2.875	.3/8	6.5
3.5 ×3.5	.7/16	10.5
4. × .7/16 ×3.5 × .3/4		13.
4. ×3.5	.3/4	13.5

* This column gives the depth of the web added to the thickness of the base or flange.

Weight of Copper Rods or Bolts,

From 1/8 to 4 Inches in Diameter.

ONE FOOT IN LENGTH.

Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.
Inch.	Lbs.	Inch.	Lbs.	Inch	Lbs.	Inch.	Lbs.
.1/8	.0473	.13/16	1.9982	1.1/2	6.8109	2.3/4	22.8913
.3/16	.1064	.7/8	2.3176	.9/16	7.3898	.7/8	25.0188
.1/4	.1892	.15/16	2.6605	.5/8	7.9931	3.	27.2435
.5/16	.2956	1.	3.027	.3/4	9 .2702	.1/8	29.5594
.3/8	.4256	1.1/16	3.417	.7/8	10.642	.1/4	31.9722
.7/16	.5794	.1/8	3.8312	2.	12.1082	.3/8	34.4815
.1/2	.7567	.3/16	4.2688	.1/8	13.6677	.1/2	37.0808
.9/16	.9578	.1/4	4.7228	.1/4	15.3251	.5/8	39.7774
.5/8	1.1824	.5/16	5.214	.3/8	17.075	.3/4	42.568
.11/16	1.4307	.3/8	5.7228	.1/2	18.9161	.7/8	45.455
.3/4	1.7027	.7/16	6.2547	.5/8	20.8562	4.	48.433

Weight of a Square Foot of Cast and Wrought Iron, Copper, Lead, Brass, and Zinc.

From 1/16 to 1 Inch in Thickness.

Thickn.	Cast Iron.	Wrought Iron.	Copper.	Lead.	Brass.	Zinc.
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.1/16	2.346	2.517	2.89	3.691	2.675	2.34
.1/8	4.693	5.035	5.781	7.382	5.35	4.68
.3/16	7.039	7.552	8.672	11.074	8.025	7.02
.1/4	9.386	10.07	11.562	14.765	10.7	9.36
.5/16	11.733	12.588	14.453	18.456	13.375	11.7
.3/8	14.079	15.106	17.344	22.148	16.05	14.04
.7/16	16.426	17.623	20.234	25.839	18.725	16.34
.1/2	18.773	20.141	23.125	29.53	21.4	18.72
.9/16	21.119	22.659	26.016	33.222	24.075	
.5/8	23.466	25.176	28.906	36.913	26.75	
.11/16	25.812	27.694	31.797	40.604	29.425	
.3/4	28.159	30.211	34.688	44.296	32.1	
.13/16	30.505	32.729	37.578	47.987		
.7/8	32.852	35.247	40.469	51.678		
.15/16	35.199	37.764	43.359	55.37		
1.	37.545	40.282	46.25	59.061		

NOTE.—The Wrought Iron is that of hard rolled Pennsylvania plates, and the Copper that of hard rolled plates from the works of Messrs. Phelps, Dodge & Co., Conn.

Weight of Riveted Iron and Copper Pipes,

From 5 to 30 Inches in Diameter, from 1/8 to 5/16 in Thickness

ONE FOOT IN LENGTH.

Diam.	Thickn.	Iron.	Copper.	Diam.	Thickn.	Iron	Copper.
Inch.	Inch.	Lbs.	Lbs.	Inch.	Inch.	Lbs.	Lbs.
5.	1/8	7.12	8.14	9.	1/4	25.01	28.58
	3/16	10.68	12.21		1/4	26.33	30.09
	1/4	14.25	16.28	10.	1/4	27.75	31.71
5.1/2	1/8	7.78	8.89	10.1/2	1/4	29.19	33.22
	3/16	11.66	13.33	11.	1/4	30.49	34.85
	1/4	15.56	17.78	12.	1/4	33.13	37.86
6.	1/8	8.44	9.64	13.	1/4	35.88	41.
	3/16	12.65	14.46	14.	1/4	38.52	44.02
	1/4	16.88	19.29	15.	1/4	41.26	47.15
6.1/2	1/8	9.1	10.4		5/16	51.57	58.94
	3/16	13.65	15.6	16.	1/4	43.9	50.17
	1/4	18.2	20.8		5/16	54.87	62.71
7.	1/8	9.78	11.18	17.	1/4	46.53	53.18
	3/16	14.68	16.78		5/16	58.17	66.48
	1/4	19.57	22.37	18.	1/4	49.17	56.2
7.1/2	1/8	10.49	11.99		5/16	61.47	70.25
	3/16	15.73	17.98	20.	5/16	68.07	77.79
	1/4	20.89	23.87	24.	5/16	81.33	92.95
8.	3/16	16.7	19.08	25.	5/16	84.57	96.65
	1/4	22.26	25.44	28.	5/16	94.56	107.95
8.1/2	1/4	23.59	26.96	30.	5/16	101.14	115.59

The above weights include the laps of the sheets for riveting and calking.
The weights of the rivets are not added, as the number per lineal foot of pipe depends upon the distance they are placed apart, and their diameter and length upon the thickness of the metal of the pipe.

Table of Standard Dimensions of Wrought Iron Welded Tubes.

Nominal Diam.	External Diam.	Thickness.	Internal Diam.	Internal Circumf.	External Circum.	Length of Pipe per Sq. Foot of internal surface.	Length of Pipe per Sq. Foot of external surface.	Internal Area.	Weight per Foot.	No. of Threads per Inch of Screw.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins	Feet.	Feet.	Ins.	Lbs.	
1/8	.40	.068	.27	.85	1.27	14.15	9.44	.057	.24	27
1/4	.54	.088	.36	1.14	1.7	10.5	7.075	.104	.42	18
3/8	.67	.091	.49	1.55	2.12	7.67	5.657	.192	.56	18
1/2	.84	.109	.62	1.96	2.65	6.13	4.502	.305	.84	14
3/4	1.05	.113	.82	2.59	3.3	4.64	3.637	.533	1.13	14
1	1.31	.134	1.05	3.29	4.13	3.66	2.903	.863	1.67	11½
1¼	1.66	.14	1.38	4.33	5.21	2.77	2.301	1.496	2.26	11½
1½	1.9	.145	1.61	5.06	5.97	2.37	2.01	2.038	2.69	11½
2	2.37	.154	2.07	6.49	7.46	1.85	1.611	3.355	3.67	11½
2½	2.87	.204	2.47	7.75	9.03	1.55	1.328	4.783	5.77	8
3	3.5	.217	3.07	9.64	11.	1.24	1.091	7.388	7.55	8
3½	4.	.226	3.55	11.15	12.57	1.08	0.955	9.887	9.05	8
4	4.5	.237	4.07	12.69	14.14	.95	0.849	12.73	10.73	8
4½	5.	.247	4.51	14.15	15.71	.85	0.765	15.939	12.49	8
5	5.56	.259	5.04	15.85	17.47	.78	0.629	19.99	14.56	8
6	6.62	.28	6.06	19.05	20.81	.63	0.577	28.889	18.77	8
7	7.62	.301	7.02	22.06	23.95	.54	0.505	38.737	23.41	8
8	8.62	.322	7.98	25.08	27.1	.48	0.444	50.039	28.35	8
9	9.69	.344	9.	28.28	30.43	.42	0.394	63.633	34.08	8
10	10.75	.366	10.02	31.47	33.77	.38	0.355	78.838	40.64	8

Diameter and Weight of Lap-welded Iron Boiler Tubes.—[Prosser's *Patent.*]

External Diameter	Thickness. Wire Gauge.	Average Weight.	Price pr. Foot	External Diameter.	Thickness. Wire Gauge.	Average Weight.	Price pr. Foot.
Ins.	No.	Lbs. pr. Ft.	Cents.	Ins.	No.	Lbs. per Ft.	Cents.
1¼	16	1		3	11	3.5	
1½	15	1.16		3¼	11	4	
1¾	14	1.63		4	8	6.4	
2	13	2		5	7	9.1	
2¼	12	2.16		6	6	12.3	
2½	12	2.56		7	6	15.2	
2 11/16	11	2.2		8	7	16	

Weight of Composition Sheathing Nails.

No.	Length.	Number in a Pound.	No.	Length.	Number in a Pound.	No.	Length	Number in a Pound
	Ins.			Ins.			Ins.	
1	¾	290	6	1	190	10	1⅝	101
2	⅞	260	7	1⅛	184	11	1¾	74
3	1	212	8	1¼	168	12	2	64
4	1⅛	201	9	1½	110	13	2¼	59
5	1¼	199						

To Ascertain the Weight of Wrought Iron, Copper, or Brass Tubes and Pipes per Lineal Foot.

From 1/2 an Inch in Internal Diameter to 6 Inches.

Diam.	Area of Plate.	Diam.	Area of Plate.	Diam.	Area of Plate.	Diam.	Area of Plate.
Ins.	Sq. Feet.	Ins.	Sq. Feet.	Ins.	Sq. Feet.	Ins.	Sq. Feet.
1/2	.1309	1 5/16	.3436	2 3/4	.7199	4 1/2	1.1781
9/16	.1473	1 3/8	.36	2 7/8	.7526	4 5/8	1.2108
5/8	.1636	1 7/16	.3764	3	.7854	4 3/4	1.2435
11/16	.18	1 1/2	.3927	3 1/8	.8181	4 7/8	1.2763
3/4	.1964	1 5/8	.4254	3 1/4	.8508	5	1.309
13/16	.2127	1 3/4	.4581	3 3/8	.8836	5 1/8	1.3417
7/8	.2291	1 7/8	.4909	3 1/2	.9163	5 1/4	1.3744
15/16	.2454	2	.5236	3 5/8	.949	5 3/8	1.4072
1	.2618	2 1/8	.5543	3 3/4	7.9818	5 1/2	1.4399
1 1/16	.2782	2 1/4	.587	4	1.0472	5 5/8	1.4726
1 1/8	.2945	2 3/8	.6198	4 1/8	1.0799	5 3/4	1.5053
1 3/16	.3105	2 1/2	.6545	4 1/4	1.1126	5 7/8	1.5381
1 1/4	.3272	2 5/8	.6872	4 3/8	1.1454	6	1.5708

Application of the Table.

When the Thickness of the Metal is given in the Divisions of an Inch.

To the internal diameter of the tube or pipe add the thickness of the metal; take the area of a plate in square feet, from the table for a diameter equal to the sum of the diameter and thickness of the tube or pipe, and multiply it by the weight of a square foot of the metal for the given thickness (see tables, page 118), and again by its length in feet.

ILLUSTRATION.—Required the weight of 10 feet of copper tube 1 inch in circumference and 1/8th of an inch in thickness.

1+1/8=1 1/8=.2945 *square feet for* 1 *foot of length.*

Weight of 1 square foot of copper 1/8th of an inch in thickness, per table, page 118, =5.781 *lbs.*; then .2945×.5781=1.7025 *lbs.*

When the Thickness of the Metal is given in Numbers of a Wire Gauge.

To the internal diameter of the tube or pipe add the thickness of the number from table, pages 113–115; multiply the sum by 3.1416, divide the product by 12, and the quotient will give the area of the plate in square feet. Then proceed as before given.

ILLUSTRATION.—Required the weight of 10 feet of copper pipe 2 inches in diameter, and No. 2 American wire gauge in thickness.

2 + .25763 × 3.1416 ÷ 12 = 2.25763 × 3.1416 ÷ 12 = .591 *square feet;* then .591 × 11.6706 (weight from table) = 6.897 *lbs.*

Table showing the Thickness and Weight of Galvanized Sheet Iron.

Dimensions of Sheet, 2 Feet in Width by from 6 to 9 Feet in Length.

Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.
No.	Oz.	No.	Oz.	No.	Oz	No.	Oz.
30	10	26	15	22	21	18	37
29	11	25	16	21	24	17	43
28	12	24	17	20	28	16	48
27	14	23	19	19	33	14	60

Table of Dimensions and Weights of Seamless Brass and Copper Tubes.—[*American Tube Works.*]

External Diam.	Extreme Length.	Weight per Foot.	Wire Gauge. Eng.	External Diam.	Extreme Length.	Weight per Foot.	Wire Gauge. Eng.
Ins.	Feet.	Lbs.	No.	Ins.	Feet.	Lbs.	No.
5/8	8	.375	18	2	15	2.05	12 & 14
3/4	11	.5	17	2 1/8	12	2.5	12 & 14
7/8	7	.625	17	2 1/4	13	2.375	12 & 14
1	8	.75	16	2 3/8	12	2.5	12 & 14
1 1/4	13	1.25	12 & 14	2 1/2	12	2.66	12 & 14
1 1/2	13	1.5	12 & 14	2 5/8	12	3.	12 & 14
1 5/8	12	1.625	12 & 14	3	12	3.33	12 & 14
1 3/4	13	1.75	12 & 14	3 1/4	10	3.875	12 & 14
1 13/16	12	1.813	12 & 14	3 1/2	9	4.25	12
1 7/8	12	1.875	12 & 14	4	7 1/2	5.	12

Marks and Weight of English Tin-plates.

Brand.	Plates per Box.	Length and Breadth.	Net Weight per Box.
	No.	Ins.	Lbs.
1 C or 1 Com.	225	13 3/4 by 10	112
2 C	225	13 1/4 " 9 3/4	105
3 C	225	12 3/4 " 9 1/2	98
H C	225	13 3/4 " 10	119
H X	225	13 3/4 " 10	157
1 X	225	13 3/4 " 10	140
2 X	225	13 1/4 " 9 3/4	133
3 X	225	12 3/4 " 9 1/2	126
1 XX	225	13 3/4 " 10	161
1 XXX	225	13 3/4 " 10	182
1 XXXX	225	13 3/4 " 10	203
1 XXXXX	225	13 3/4 " 10	224
1 XXXXXX	225	13 3/4 " 10	245
DC	100	16 3/4 " 12 1/2	98
DX	100	16 3/4 " 12 1/2	126
DXX	100	16 3/4 " 12 1/2	147
DXXX	100	16 3/4 " 12 1/2	168
DXXXX	100	16 3/4 " 12 1/2	189
SDC	200	15 " 11	168
SDX	200	15 " 11	188
SDXX	200	15 " 11	209
SDXXX	200	15 " 11	230
SDXXXX	200	15 " 11	251
SDXXXXX	200	15 " 11	272
SDXXXXXX	200	15 " 11	293
Leaded IC	112	20 " 14	112
" IX	112	20 " 14	140
ICW	225	13 3/4 " 10	112
IXW	225	13 3/4 " 10	140
CSDW	200	15 " 11	168
CIIW	100	16 3/4 " 12 1/2	105
XIIW	100	16 3/4 " 12 1/2	126
TT	450	13 3/4 " 10	112
XTT	450	13 3/4 " 10	126

When the plates are 14 by 20 inches, there are 112 in a box.

L

Weight of Lead and Tin Pipe per Foot.

From ⅜ to 5 Inches in Diameter.

WATER-PIPE.

Internal Diam.	Thickness.	Weight.	Internal Diam.	Thickness.	Weight.	Internal Diam.	Thickness.	Weight.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
.⅜	.06	.0424	1.	.10	1.5	2.	.22	7.
.⅜	.08	.625	1.	.11	2.	2.	.27	9.
.⅜	.12	1.	1.	.14	2.5	2.½	.3/16	8.
.⅜	.16	1.25	1.	.17	3.25	.½	.¼	11.
.⅜	.19	1.5	1.	.21	4.	.½	.5/16	14.
.½	.07	.0545	1.	.24	4.75	.½	.⅜	17.
.½	.09	.75	1.¼	.10	2.	3.	.3/16	9.
.½	.11	1.	.¼	.12	2.5	3.	.¼	12.
.½	.13	1.25	.¼	.14	3.	3.	.5/16	16.
.½	.16	1.75	.¼	.16	3.75	3.	.⅜	20.
.½	.19	2.	.¼	.19	4.75	3.½	.3/16	12.5
.⅝	.08	.0727	.¼	.25	6.	.½	.¼	15.
.⅝	.09	1.	1.½	.14	3.5	.½	.5/16	18.5
.⅝	.13	1.5	.½	.17	4.25	.½	.⅜	22.
.⅝	.16	2.	.½	.19	5.	4.	.3/16	12.
.⅝	.20	2.5	.½	.23	6.5	4.	.¼	16.
.⅝	.22	2.75	.½	.27	8.	4.	.5/16	21.
.¾	.08	.0969	1.¾	.13	4.	4.	.⅜	25.
.¾	.10	1.25	.¾	.17	5.	4.½	.3/16	14.
.¾	.12	1.75	.¾	.21	6.5	.½	.¼	18.
.¾	.16	2.25	.¾	.27	8.5	5.	.¼	20.
.¾	.20	3.	2.	.15	4.75	5.	.⅜	31.
.¾	.23	3.5	2.	.18	6.			

WASTE-PIPE.

Internal Diam	Weight.	Internal Diam.	Weight.	Internal Diam.	Weight.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
1.½	2.	4.	5	4.½	8
2.	3.	4.	6	5.	8
3.	3.5	4.	8	5.	10
3.	5.	4.½	6	5.	12

BLOCK-TIN PIPE.

.⅜	.3594	.⅝	.5	1.¼	1.25
.⅜	.375	.⅝	.625	.¼	1.5
.⅜	.5	.¾	.625	1.½	2.
.½	.375	.¾	.75	.½	2.5
.½	.5	1.	.9375	2.	3.
.½	.625	1.	1.125	2.½	3.75

Capacity of Cistern in Gallons.

For each 10 Inches in Depth.

Diam.	Gallons.	Diam.	Gallons.	Diam.	Gallons.	Diam.	Gallons.	Diam.	Gallons.
Feet.		Feet.		Feet.		Feet.		Feet.	
2.	19.5	4.5	99.14	7.	239.88	9.5	461.4	14.	959.6
2.5	30.6	5.	122.4	7.5	275.4	10.	489.6	15.	1101.6
3.	44.6	5.5	148.1	8.	313.33	11.	592.4	20.	1958.4
3.5	59.97	6.	176.25	8.5	353.72	12.	705.	25.	3059.9
4.	78.33	6.5	206.85	9.	396.56	13.	827.4	30.	4406.4

Dimensions and Weights of Bolts and Nuts.

SQUARE AND HEXAGONAL.

Diam. of Bolt.	Depth of Nut.	Width of Square Nut.	* Diam. of Hexag'l Nut.	† Width of Head.	Volume.			
					Square Nut.	Hexagonal Nut.	Hexagonal Head.	Bolt per In. of Length.
Ins.	Ins.	Ins.	Ins.	Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.
.1/8	.15	.2	.1/4	.2	.00416	.00425	.0045	.01227
.3/16	.2	.3	.3/8	.3	.01248	.01276	.0152	.62761
.1/4	.25	.45	.1/2	.4	.03835	.02836	.036	.04908
.5/16	.35	.55	.5/8	.1/2	.07903	.06235	.07	.07669
.3/8	.4	.6	.3/4	.6	.09984	.10209	.1215	.1104
.7/16	.5	.75	.7/8	.7	.2061	.17368	.1929	.1503
.1/2	.55	.85	1.	.3/4	.28941	.25584	.2531	.1963
.9/16	.6	.95	1.1/8	.85	.3924	.34449	.3658	.2485
.5/8	.7	1.1	1.1/4	.95	.6323	.49625	.5076	.3067
.11/16	.75	1.2	1.3/8	1.05	.8016	.64328	.6822	.3712
.3/4	.8	1.3	1.1/2	1.1/8	.9986	.81664	.8543	.4417
.13/16	.9	1.4	1.5/8	1.1/4	1.2977	1.0782	1.143	.5184
.7/8	.95	1.5	1.3/4	1.35	1.5663	1.3199	1.435	.6013
1.	1.1	1.75	2.	1.1/2	2.5048	1.996	2.025	.7854
1.1/8	1.25	1.95	2.1/4	1.7	3.5106	2.8701	2.926	.994
1.1/4	1.37	2.15	2.1/2	1.7/8	4.6518	2.8846	3.955	1.227
1.3/8	1.5	2.4	2.3/4	2.1	6.414	5.1474	5.457	1.484
1.1/2	1.65	2.6	3.	2.1/4	8.2384	6.737	6.834	1.767
1.5/8	1.8	2.8	3.1/4	2.45	10.381	8.6267	8.778	2.073
1.3/4	1.9	3.	3.1/2	2.5/8	12.53	10.559	10.853	2.405
1.7/8	2.05	3.25	3.3/4	2.8	15.993	13.058	13.23	2.761
2.	2.2	3.45	4.	3.	19.275	15.97	16.2	3.141
2.1/8	2.35	3.7	4.1/4	3.3/16	23.838	19.257	19.43	3.546
2.1/4	2.5	3.9	4.1/2	3.3/8	28.085	22.966	23.066	3.976
2.3/8	2.6	4.1	4.3/4	3.9/16	32.188	26.613	27.128	4.43
2.1/2	2.75	4.3	5.	3.3/4	37.351	31.19	31.641	4.908
2.5/8	2.9	4.55	5.1/4	3.15/16	44.345	36.263	36.628	5.411
2.3/4	3.	4.75	5.1/2	4.1/8	49.871	41.17	42.114	5.939
2.7/8	3.15	4.95	5.3/4	4.3/16	56.736	47.249	48.123	6.491
3.	3.3	5.2	6.	4.1/2	65.908	54.105	54.675	7.068
3.1/4	3.6	5.65	6.1/2	4.7/8	85.059	69.003	69.514	8.295
3.1/2	3.85	6.1	7.	5.1/4	106.218	85.582	86.822	9.621
3.3/4	4.1	6.5	7.1/2	5.5/8	127.945	104.626	106.787	11.044
4.	4.4	6.95	8.	6.	157.241	127.75	129.6	12.566
4.1/4	4.65	7.35	8.1/2	6.3/8	185.24	152.411	155.45	14.186
4.1/2	4.95	7.8	9.	6.3/4	222.43	181.893	184.528	15.904
4.3/4	5.2	8.25	9.1/2	7.1/8	261.781	212.901	217.023	17.72
5.	5.5	8.65	10.	7.1/2	303.531	249.507	253.125	19.635
5.1/4	5.75	9.1	10.1/2	7.7/8	351.687	287.589	293.024	21.647
5.1/2	6.05	9.5	11.	8.1/4	402.277	332.097	336.909	23.758
5.3/4	6.3	9.95	11.1/2	8.5/8	441.224	377.872	384.971	25.967
6.	6.6	10.4	12.	9.	527.248	431.152	437.4	28.274

* Extreme diameter of nut.

† Square or hexagonal, and the depth of it should be .8 of the diameter of the bolt.

When the weight of a bolt and nut is required, Ascertain the volume for the bolt from the inside of the head to its point; add to this the volume obtained from the table for the diameter of bolt and description of nut given; multiply the sum by the units in page 163 for the weight of a cubic inch of the metal of which the bolt and nut is made, and the quotient is the weight in pounds.

ILLUSTRATION.—A wrought iron bolt and nut (hexagonal nut) is 1 inch in diameter and 10 inches in length from inside of head to end.

NOTE.—The length of a bolt and nut is taken from the inside of the head to the inside of the nut, or its greatest capacity when in position.

In a computation of the weight, it is necessary to measure the extreme length of the bolt, viz., from the inside of the head to the point.

Volume for head...............	1.5 × .8 = depth of head = 1.8	*cub. ins.*
" " 1 inch of bolt.......	.7854, which × 10 = 7.854	"
" of nut	1.996	"
	11.650	"

which × .2816 (page 161), the weight of a cubic inch of wrought iron bolt = 3.28 *lbs.*

Table showing the Number of Threads to an Inch in V-thread Screws.

Diam.	Thre'ds.	Diam.	Thr'ds.	Diam.	Thr'ds.	Diam.	Thr'ds.	Diam.	Thr'ds.	Diam.	Thr'ds.
Ins.	No.	Ins.	No.	Ins.	No.	Ins.	No.	Ins.	No.	Ins.	No.
¼	20	¾	10	1½	6	2½	4	3¾	3	5	2¾
5/16	18	⅞	9	1⅝	5	2¾	3½	4	3	5¼	2⅝
⅜	16	1	8	1¾	5	3	3½	4¼	2⅞	5½	2⅝
7/16	14	1⅛	7	1⅞	4½	3¼	3¼	4½	2⅞	5¾	2½
½	12	1¼	7	2	4½	3½	3¼	4¾	2¾	6	2½
⅝	11	1⅜	6	2¼	4						

The depth of the threads should be half of their pitch.

NOTE.—The diameter of a screw, to work in the teeth of a wheel, should be such that the angle of the threads does not exceed 10°.

Screw Threads, Bolt Heads and Nuts,

As determined and recommended by Committee of Franklin Institute of Philadelphia, 1864.

NUMBER OF THREADS PER INCH. ANGLE 60°.

Diam. of Bolt.	Thre'ds.	Diam. of Bolt.	Thre'ds.	Diam. of Bolt.	Thre'ds.	Diam. of Bolt.	Thre'ds.	Diam. of Bolt.	Thre'ds.
Ins.	No.	Ins.	No.	Ins.	No.	Ins.	No.	Ins.	No.
¼	20	¾	10	1⅝	5½	3	3½	4¾	2⅝
5/16	18	⅞	9	1¾	5	3¼	3½	5	2½
⅜	16	1	8	1⅞	5	3½	3¼	5¼	2½
7/16	14	1⅛	7	2	4½	3¾	3	5½	2⅜
½	13	1¼	7	2¼	4½	4	3	5¾	2⅜
9/16	12	1⅜	6	2½	4	4¼	2⅞	6	2¼
⅝	11	1½	6	2¾	4	4½	2¾		

Dimensions of Heads and Nuts.

Rough Bolt.—The width between the parallel sides of *both head* and *nut* 1½ times the diameter of the bolt, to which is to be added ⅛th of an inch.

The depth of the head .5 its width. The depth of the nut equal the diameter of the bolt.

Finished Bolt.—The width between the parallel sides of both head and nut 1/16th of an inch less than for a rough bolt.

The depth of the nut 1/16th of an inch less than the diameter of the bolt.

Dimensions and Weights of Bolts and Nuts,

Square and Hexagonal, by the preceding Rules.

Diam. of Bolt.	Width of Sq. Nut and Head.	Diam. of Hexago'l Nut and Head.	Volume. Square Nut.	Volume. Hexagonal Nut.	Volume. Hexagonal Head	Volume. Square Head.	Depth of Head.	Volume of Bolt per Inch of Length.
Ins.	Ins.	Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.	Ins.	Cub. Ins.
1	1⅝	1.878	1.855	1.508	1.864	2.145	$\frac{13}{16}$	.7854
2	2⅛	3.613	13.248	10.681	13.253	15.529	1$\frac{9}{16}$	3.142
3	4⅝	5.346	42.968	34.536	42.966	49.466	2$\frac{5}{16}$	7.068
4	6⅛	7.08	99.8	80.079	99.794	114.89	3$\frac{1}{16}$	12.566
5	7⅝	8.814	192.53	154.33	192.53	221.66	3$\frac{13}{16}$	19.635
6	9⅛	10.548	329.95	264.3	329.98	379.9	4$\frac{9}{16}$	28.274

Comparison of Weights between Bolts and Nuts, of the Proportion given in the preceding Table, and between those determined by the above Rules.

VOLUME OF HEXAGONAL HEAD AND NUT.

Dimensions.	1 Inch.	2 Inch.	3 Inch.	4 Inch.	5 Inch.	6 Inch.
	Cub. Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.	Cub. Ins.
Ordinary...........	4.02	32.17	108.78	257.35	502.6	868.55
Proposed...........	3.37	23.93	77.5	179.87	346.86	594.28

The difference varying from 1.2 to 1.46 *per centum* in favor of the proposed dimensions for equal diameters of bolts.

Ship and Railroad Spikes.

DIMENSIONS AND NUMBER PER POUND.—[*P. C. Page*, Mass.]

Ship Spikes.

¼ In. Sq. Length.	¼ In. Sq. No. in Pound.	$\frac{5}{16}$ In. Sq. Length.	$\frac{5}{16}$ In. Sq. No. in Pound.	$\frac{7}{16}$ In. Sq. Length.	$\frac{7}{16}$ In. Sq. No. in Pound.	½ In. Sq. Length.	½ In. Sq. No. in Pound.	$\frac{9}{16}$ In. Sq. Length.	$\frac{9}{16}$ In. Sq. No. in Pound	⅝ In. Sq. Length	⅝ In. Sq. No. in Pound.	¾ I. Sq. Length.	¾ I. Sq. No. in Pound.
Ins.		Ins.		Ins.		Ins.		Ins.		Ins.		Ins.	
3	19.	3	10.	4	5.4	5	3.4	6	2.2	8	1.4	10	.8
3½	15.8	3½	9.6	4½	5.	5½	3.1	6½	2.	9	1.2	15	.6
4	13.2	4	8.0	5	4.6	6	3.	7	1.9	10	1.1	–	–
4½	12.2	4½	6.0	5½	4.2	6½	2.8	7½	1.8	11	1.	–	–
5	10.2	5	5.8	6	4.	7	2.6	8	1.7	–	–	–	–
–	–	6	5.2	6½	3.2	7½	2.4	8½	1.6	–	–	–	–
–	–	–	–	–	–	8	2.2	9	1.5	–	–	–	–
–	–	–	–	–	–	–	–	10	1.4	–	–	–	–

Railroad Spikes......... ½ in. square × 5½ ins. 2. per lb.
" " .... $\frac{9}{16}$ " " × 5½ " 1.6 "

Thickness of Gas Pipes.

Diam.	Thickn.	Diam.	Thickn.	Diam.	Thickn.
1½ to 3	¼	8 to 10	½	14 to 15	¾
4 " 6	⅜	12 " 13	⅝	16 " 48	⅞

Slating.

A *Square* of slate or slating is 100 superficial feet.

The *Lap* of slates varies from 2 to 4 inches. The standard is assumed to be 3 inches.

The *Pitch* of a slate roof should not be less than 1 in height to 4 of length.

To Compute the Surface of a Slate when laid, and the Number of Squares of Slating.

Subtract the lap from the length of the slate, and half the remainder will give the length of the surface exposed, which, when multiplied by the width of the slate, will give the surface required, and for which the party requiring the slating only pays.

Divide 14400 (the area of a square in inches) by the surface thus obtained, and the quotient will give the number of slates required for a square.

ILLUSTRATION.—A slate is 24×12 inches, and the lap is 3 inches.
$24-3=21$, and $21 \div 2=10.5$, which $\times 12=126$ *inches;* and $14400 \div 126=114.29$ *slates.*

Dimensions of Slates.—[AMERICAN.]

Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
14×7	14×10	16×10	18×11	20×11	22×12	24×13
14×8	16× 8	18× 9	18×12	20×12	22×13	24×14
14×9	16× 9	18×10	20×10	22×11	24×12	24×16

WELSH.

	Ins.		Ins.		Ins.
Doubles....	13× 7	Ladies....	12× 8	Viscountess ..	18×10
Small "	11× 7		14× 8	Countess	20×10
Plantations.	12×10		16× 8	Marchioness .	22×12
	13×10		16×10	Duchess	24×12
	14×12				

The thickness of slates ranges from $\frac{3}{16}$ to $\frac{5}{16}$ of an inch, and their weight varies from 2.6 to 4.53 lbs. per square foot.

Earth Digging.

Number of Cubic Feet of various Earths in a Ton.

Loose Earth 24	Clay..................18.6	Clay with Gravel.14.4
Coarse Sand.....18.6	Earth with Gravel.17.8	Common Soil....15.6

The volume of Earth and Sand in bank exceeds that in embankment in the following proportions:

Sand .... $\frac{1}{7}$ | Clay..................... $\frac{1}{9}$ | Gravel............... $\frac{1}{11}$

and the volume of Rock in embankment quarried in large fragments exceeds that in bank fully one half.

Hay.

270 cubic feet of new meadow hay, and 216 and 243 from large or old stacks, will weigh a ton.

297 to 324 cubic feet of dry clover weigh a ton.

Patent Spikes and Horseshoes.

[H. BURDEN, *Troy, N. Y.*]

Boat Spikes.		Ship Spikes.		Hook Head.		Horseshoes.	
Length.	No. in Pound.	Length.	No. in Pd.	Length.	No. in Pd.	Le'gth.	No. in Pd.
Ins.		Ins.		Ins.		Ins.	
3	17.5	4	8.	4 × ⅜	5.55	1	.84
3½	14.68	4½	6.5	4½ × 7/16	4.14	2	.75
4	12.57	5	4.37	5 × ½	2.52	3	.65
4½	9.2	5½	4.3	5½ × ½	2.41	4	.56
5	7.2	6	4.2	5½ × 9/16	1.87	5	.39
5½	6.3	6½	3.77	6 × 9/16	1.72	–	–
6	4.97	7	2.75	6 × ⅝	1.38	–	–
6½	4.78	7½	2.5	7 × 9/16	1.4	–	–
7	3.62	8	1.74	8 × ⅝	1.1	–	–
7½	3.37	8½	1.63	–	–	–	–
8	2.95	9	1.55	–	–	–	–
8½	2.9	10	1.15	–	–	–	–
9	2.1	–	–	–	–	–	–
10	1.98	–	–	–	–	–	–

Length of Horseshoe Nails.

No. 5	1½ Inch	No. 7	1⅞ Inch	No. 9	2¼ Inch
" 6	1¾ "	" 8	2 "	" 10	2½ "

Lengths of Iron Nails, and Number in a Pound.

Size	Lgth.	No.	Size.	Lgth.	No.	Size.	Lgth.	No.	Size.	Lgth.	No.	Size.	Lgth.	No.
	Ins.			Ins.			Ins.			Ins.			Ins.	
3*d.*	1¼	420	5*d.*	1¾	220	8*d.*	2½	100	12*d.*	3¼	52	30*d.*	4	24
4	1½	270	6	2	175	10	3	65	20	3½	28	40	4¼	20

Hills in an Area of an Acre.

Ft. apart.	No.	Feet apart.	No.	Feet apart.	No.	Ft. apart.	No.
1	43560	5	1742	9	538	16	171
1½	19360	5½	1440	9½	482	17	151
2	10890	6	1210	10	435	18	135
2½	6969	6½	1031	10½	361	20	108
3	4840	7	889	12	302	25	69
3½	3556	7½	775	13	258	30	48
4	2722	8	680	14	223	35	35
4½	2151	8½	692	15	193	40	27

Transportation of Horses and Cattle.

The Space required on board of a Transport is:

For Horses.................................... 30 inches by 9 feet.
" Beeves.................................... 32 " 9 "

The Provender required *per diem* is:

For Horses...Hay, 15 lbs.; Oats, 6 quarts; Water, 4 gallons.
" Beeves... " 18 " — " 6 "

Number of several Seeds in a Bushel, and Number per Square Foot upon an Area of an Acre.

	No.	Sq. Foot.		No.	Sq. Foot.
Timothy	41823360	960	Rye	888390	20.4
Clover......	16400960	376	Wheat	556290	12.8

Snow Line, or Line of Perpetual Congelation.

Lat.	Feet.	Lat.	Feet.	Lat.	Feet.	Lat.	Feet.	Lat.	Feet.
5°	15207	25°	12557	40°	9000	55°	5030	75°	1016
15°	14760	35°	10287	45°	7670	65°	2230	85°	117

Limits of Vegetation in the Temperate Zone.

The Vine ceases to grow at about 2300 feet above the level of the sea, Indian Corn at 2800, Oak at 3350, Walnut at 3600, Ash at 4800, Yellow Pine at 6200, and Fir at 6700.

Lengths of Bridges.

Bridges.	Feet.	Bridges.	Feet.	Bridges.	Feet.
Avignon..........	1710	London..........	950	Potomac	5300
Badajoz	1874	Lyons...........	1560	Riga	2600
Belfast	2500	Menai...........	1050	Strasbourg	3390
Blackfriars	995	Pont Neuf.......	996	Vauxhall........	860
Boston	3483	Pont St. Esprit...	3060	Westminster	1223

St. Lawrence River, 9144 feet.

Lengths of Spans of Bridges.

Bridges.	Feet.	Bridges.	Feet.	Bridges.	Feet.
Britannia	460	Niag'a at the Falls	1268	Schuylkill	340
Conway	400	" at Queens-		Southwark.......	240
Menai............	580	town...........	1040	Wheeling	1010

Lengths of Rivers.

Rivers.	Miles.	Rivers.	Miles.	Rivers.	Miles.
EUROPE.		Jordan	176	Mississippi	1350
Danube	1800	Kiang...........	3290	Missouri and Mis-	
Gaudiana	500	Tigris...........	1160	sissippi........	4300
Po	420	Yeneisy & Selenga	3580	Ohio & Alleghany	1480
Rhine	840	AFRICA.		Potomac.........	420
Rhone............	510	Gambia	1000	Red.............	1520
Seine.............	450	Niger	2400	Rio Bravo	2300
Shannon	250	Nile	3240	St. Lawrence	1450
Tay	180	NORTH AMERICA.		Susquehanna	620
Thames	250	Arkansas........	2070	Tennessee	790
Tiber	190	Colorado	1050	SOUTH AMERICA.	
Vistula...........	700	Columbia........	1100	Amazon and Beni.	4000
Volga, Russia.....	2500	Connecticut		Essequibo........	520
Wye	140	Delaware........	420	Magdalena	900
ASIA.		Hudson and Mo-		Orinoco	1600
Amoor	2500	hawk		Platte	2700
Euphrates	1900	Kansas..........	1400	Rio Madeira	2300
Ganges...........	1850	La Platte........	850	Rio Negro	1650
Hoang Ho........	3040	Mackenzie's	2800	Uruguay	1100

Sea Depths.

	Feet.		Feet.		Feet.
Baltic Sea	120	W. of Cape of Good Hope	16000	Off Cape Carnaveral	2400
Adriatic	130	W. of St. Helena	27000	" Charleston	4200
English Channel	300	Tortugas to Cuba	4200	" Cape Hatteras	3120
Straits of Gibraltar	100	Gulf of Florida	3720	" Cape Henry	4200
Eastward of "	3000	Off Cape Florida	1950	" Sandy Hook	2400
Coast of Spain	6000				

Estimated depth of the Atlantic.............................26 000 feet.
" " Pacific.................................29 000 "
250 miles off Cape Cod, no bottom at 7800 feet.

Course of the Atlantic Telegraph from Ireland to Newfoundland.
Longitude, 15°, 2 446 feet; 20°, 9 258; 30°, 12 000; 40°, 9 000; 47°, 13 000; 50°, 6 600.

Ages of Animals, etc.

Bear, 20 years; Cat, 15; Cow, 20; Camel, 100; Deer, 20; Eagle, 100; Elephant, 400; Fox, 15; Hare, Rabbit, and Squirrel, 7; Horse, 30; Lion, 70; Porpoise, 30; Raven, 100; Rhinoceros, 20; Sheep, 10; Swine, 20; Tortoise, 100 to —; Swan, 300; Whale, estimated 1000; and Wolf, 20.

Rain.

Annual Fall at different Places.

Location.	Ins.	Location.	Ins.	Location.	Ins.
Alabama	30.17	England	31. / 35.	Madeira	22. / 49.
Albany	41.35				
Alleghany	46.66	Fairfield	32.93	Manchester	36.14 / 43.
Antigua	45.	Ft. Crawford, Wis.	29.54		
Auburn	30.17	Ft. Gibson, Ark.	30.64	Mississippi	45.
Baltimore	39.9	Ft. Snelling, Iowa.	30.32	Mobile, 1842	54.94
Barbadoes	55.87	Fortr. Monroe, Va.	52.53	Newburg	40.5
Bath, Me.	34.58	Gordon Castle, Sc'd	29.3	New York	36.
Belfast	39.46	Glasgow	21.3 / 31.	Ohio, mean	36.
Bombay	110.			Petersburg (Eng.).	16.
Boston	39.23	Granada	105. / 126.	Philadelphia	49.
Buffalo	27.27			Poughkeepsie	32.06
Burlington, Vt.	32.	Great Britain	31.88 / 36.	Plymouth (Engl.).	44.
Calcutta	81.			Providence	36.74
Cape St. François.	150.	Greenock	61.8	Rochester	29.
Charleston, S. C.	54.	Hudson	39.32	Rome	39.
Demarara	91.2	Key West	31.39	Savannah	55.
" 1849	132.21	Khassaya, Calc'tta	610.	Schenectady	47.77
Dover (Engl.)	37.52	Lewiston	23.15	Sierra Leone	84.
Dublin	30.87	Liverpool	34.12	State of N. Y., mean	33.79
Dumfries	36.92	London	20.68 / 24.	Utica	39.3
East Hampton	38.52			Vera Cruz	62.
Edinburgh	24.5 / 29.	Louisiana	51.85	West Point	48.7
		Michigan, mean	33.5	Washington	34 62

Globe, mean depth....................................36.
Cape of Good Hope in 1846 .... in 3 days, 6.2 ins.
At Khassaya, in 6 rainy months......... 550 ins.; in 1 day, 25.5 "

Volume of Rain Fall.

Rain fall in inches, × 2323200 = cubic feet per square mile.
" " × 17.3787 = millions of gallons per mile.
" " × 3630 = cubic feet per acre.

The average fall of rain for the southern and eastern counties of Great Britain is about 34 inches; but in the western and hilly counties it is from 48 to 50 inches. The mean quantity of water in a cubic foot of air in that climate is 3.789 grains.

Heights of obtained Elevations, and various Places and Points above the Sea.

Locations.	Feet.	Locations.	Feet.	Locations.	Feet.
Balloon (Gay Lussac)	22900	Isthmus of Darien	645	Mexico, city of	7525
Brazil, Quito and Mexico plains	6000 to 8000	Laguna, Teneriffe	2000	Paris, city	115
		Lake Erie	568	Quito	13500
Dent's Bridge, Alps	11000	" Huron	598	St. Bernard's Mon'y	8040
Gibraltar	1400	" Ontario	234	Volcano, Cotopaxi	18868
Geneva Lake	1036	" Superior	647	Volcano, Mt. Etna	11000
Humboldt's highest elevation	19400	London, city	64	" " Hecla	5000
		Madrid	2200	" Vesuvius	3600

Heights of Mountains above the Level of the Sea.

Mountains.	Feet	Mountains.	Feet.	Mountains.	Feet.
EUROPE.		ASIA.		AMERICA.	
Barthelemy, France	7365	Ararat	12700	Alleghanies	3500
Ben Nevis	4380	Caucasus	16433	Blue Mount, Jam'a.	8000
Etna	10[illegible]26	Dhawalagheri	28077	Catskill	3804
Guadarama, Spain	8520	Geta, Java	8500	Chimborazo	21441
Hecla	5000	Himalaya	25[illegible]59	Cotopaxi	18900
Ida	4960	Mount Libanus	9523	Great Peak, New Mexico	19788
Mount Cenis	6780	Olympus	8000	Mount Elias	18087
" Blanc	15572	Petcha	15000	" Washington	6225
Nephin, Ireland	2634	AFRICA.		Nevado de Sorata	25248
Olympus	6510	Atlas	13000	Orizaba	17371
Parnassus	6000	Compass, Cape of Good Hope	10000	Passages of the Cordilleras	15225 / 13525
Plynlimmon, Wales	2463	Dianai Peak, St. Helena	2700	Popocatapetl	17716
St. Bernard	8172	Geesh	15000	Potosi	18000
" Gothard	11000	Ruivo, Madeira	5160	Sierra Nevada	15700
Sea Fell, England	3266	Teneriffe Peak	12300	Tahiti	108[illegible]5
The Cylinder, Pyr.	10[illegible]30				
Vesuvius	3731				

Crows' Nest, Highlands, N.Y. .... 1370 feet.

Heights of Columns, Towers, Domes, Spires, etc.

Locations.		Feet.
COLUMNS.		
Alexander	St. Petersb.	175
Bunker Hill	Mass.	221
Chimney, St. Rollox	Glasgow	455½
" Muspratt's	Liverpool	406
City	London	202
July	Paris	157
Napoleon	Paris	132
Nelson's	Dublin	134
Nelson's	London	171
Place Vendôme	Paris	136
Pompey's Pillar	Egypt	114
Trajan	Rome	145
Washington	Wash'gton	
York	London	138
TOWERS AND DOMES.		
Babel		680
Balbec		500
Capitol	Wash'gton	287½
" Diam. Dome,	"	124¾
Cathedral	Antwerp	476
"	Cologne	501
"	Cremona	392
"	Escurial	200
"	Florence	384

Locations.		Feet.
Cathedral	Milan	438
"	Petersb'rg.	363
Leaning	Pisa	188
Porcelain	China	200
St. Paul's	London	366
Strasbourg		486
St. Mark's	Venice	328
Utrecht		464
SPIRES.		
Cathedral, new	New York.	325
Grace Church	"	216
Salisbury		450
St. John's	New York.	210
St. Paul's	"	200
St. Mary's	Lübeck	404
St. Peter's	Rome	391
St. Stephen's	Vienna	465
Trinity Church*	New York.	286
Balustrade of Notre Dame	Paris	216
Hôtel des Invalides.	"	344
Pyramid of Cheops	Egypt	520
Pyram. of Sakkara	Egypt	356
St. Peter's	Rome	518

* From high-water level, 336 feet.

Cascades and Waterfalls.

Location.	Feet.	Location.	Feet.	Location.	Feet.
Arve, Savoy	1600	Missouri	50, 80, 94	Passaic	74
Cascade, Alps	2400	Montmorency	250	Potomac	74
Cataracts of the Nile	40	Niagara	164	Ribbon, Yosemite Valley	3300
Mohawk	68			Yosemite Valley	2600

Diameters of Domes.

Domes.	Feet.	Domes.	Feet.	Domes.	Feet.
Capitol, Washington	124¾	St. Paul's	112	St. Peter's	139

Lengths of Tunnels.

Tunnels.	Feet.	Tunnels.	Feet	Tunnels.	Feet.
Blaizy	13455	Nerthe	15153	Riquivel	18623
Blue Ridge	4280	Nochistongo	21659	Thames & Medw.	11880

Mont Cenis, 7.5 *miles* 242 *yards.*

Weights of Bells.

Bells.	Pounds.	Bells.	Pounds.	Bells.	Pounds.
Pekin	130000	Oxford, "Great Tom," Eng.	18000	St. Peter's, Rome	18600
Fire Alarm, 33d St.	21612	Olmutz, Bohemia	40000	Vienna	40200
Linden, Germ'y	10854	Rouen, France	40000	Westminster, "Big Ben," England	30350
Lewiston, Me.	10233	St. Paul's, Eng.	11470	Worcester "	6600
Montreal, C. E.	28560	St. Ivan's, Moscow.	127830	York "	6384
Moscow, Russia	432000				

Areas of Oceans.

Oceans.	Sq Miles.	Oceans.	Sq. Miles.	Oceans.	Sq. Miles.
Antarctic	30,000,000	Baltic	175,000	Indian	17,000,000
Arctic	8,400	Black Sea	950,000	Mediterranean	1,006,000
Atlantic	25,000,000	Caspian	160,000	Pacific	50,000,000

Northern Lakes of the United States.

Lakes.	Length.	Breadth.	Mean Depth.	Height above the Sea.	Area.
	Miles.	Miles.	Feet.	Feet.	Sq. Miles.
Erie	250	80	200	555	6000
Huron	200	160	120	574	20000
Michigan	360	109	900	587	20000
Ontario	180	65	500	282	6000
Superior	355	160	988	627	32000

Sheet Lead.

Sheet Lead is designated by the weight of a square foot of it, and it usually ranges from 2½ to 10 lbs. per square foot.

Bricks.

The variations in the dimensions of bricks by the various manufacturers, and the different degrees of intensity of their burning, render a table of the *exact* dimensions of the different classes of bricks altogether impracticable.

As an exponent, however, of the ranges of their dimensions, the following averages are given:

Description.	Inches.	Description.	Inches.
Baltimore front....	8¼×4⅛×2⅜	Maine	7½×3⅜×2⅜
Philadelphia "	8¼×4⅛×2⅜	Milwaukee.........	8½×4⅛×2⅜
Wilmington "	8¼×4⅛×2⅜	North River........	8 ×3½×2¼
Croton "	8½×4 ×2¼	Ordinary	7¾×3⅝×2¼
Colabaugh	8¼×3⅝×2⅜	Ordinary	8 ×4⅛×2½

Stourbridge fire-brick.......................... 9⅛×4⅝×2⅜ inches.
American (N. Y.)............................... 8⅞×4½×2⅝ "

In consequence of the variations in the dimensions of bricks, and the thickness of the layer of mortar or cement in which they may be laid, it is impracticable to give any rule of general application for the volume of laid brick-work. It becomes necessary, therefore, when it is required to ascertain the volume of bricks in masonry, to proceed as follows:

To Compute the Volume of Bricks and the Number in a Cubic Foot of Masonry.

Rule.—To the face dimensions of the particular bricks used, add one half the thickness of the mortar or the cement in which they are laid, and compute the area; divide the width of the wall by the number of bricks of which it is composed; multiply this area by the quotient thus obtained, and the product will give the volume of the mass of a brick and its mortar in inches.

Divide 1728 by this volume, and the quotient will give the number of bricks in a cubic foot.

Example.—The width of a wall is to be 12¾ inches, and the front of it laid with Philadelphia bricks in courses ¼ of an inch in depth; how many bricks will there be in face and backing in a cubic foot?

Philadelphia front brick, 8¼×2⅜ ins. face.

$8.25+\overline{.25\times2}\div2=8.25+.25=8.5$ = *length of brick and joint;*

$2.375+\overline{.25\times2}\div2=2.375+.25=2.625$ = *width of brick and joint.*

Then $8.5\times2.625=22.3125$ *inches* = *area of face;* $12.75\div3$ (number of bricks in width of wall) $=4.25$ *inches.*

Hence $22.3125\times4.25=94.83$ *cubic inches;* and $1728\div94.83=18.22$ *bricks.*

Lime and Laths.

A Cask of Lime = 240 lbs., will make from 7.8 to 8.15 cubic feet of stiff paste.

A Cask of Cement = 300* lbs., will make from 3.7 to 3.75 cubic feet of stiff paste.

See Limes, Cements, and Mortars, pages 499 to 508.

Laths are 1¼ to 1½ inches by four feet in length, are usually set ¼ of an inch apart, and a bundle contains 100.

* 300 lbs. net is the standard; it usually overruns 8 lbs.

ANCHORS AND KEDGES.

To Compute the Weight of a Bower Anchor for a Vessel of a given Character and Rate.

RULE.—Multiply the square of her extreme breadth by the unit of the character and rate in the following table, and the product will give the weight in pounds, exclusive of the stock.

EXAMPLE.—The extreme breadth of a side-wheel and bark-rigged steamer is 40 feet.

$$40^2 \times 3 = 1600 \times 3 = 4800 \text{ lbs.}$$

The *weight of Anchor and Kedge* is given exclusive of that of its stock

Bower and Sheet Anchors should be alike in weight.

Stream Anchors should be ¼ the weight of the best bower.

Kedges.—When 1 is used, $\frac{1}{8}$ the weight of the Bower
" 2 are " $\frac{1}{6}, \frac{1}{10},$ " " "
" 3 " " $\frac{1}{6}, \frac{1}{8}, \frac{1}{10},$ " " "
" 4 " " $\frac{1}{7}, \frac{1}{8}, \frac{1}{10}, \frac{1}{14},$ " " "
" 5 " " $\frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{14},$ " " "

Table showing the Units to determine the Weights of Anchors, also the Number required to each Class and Rate.

NAVAL AND MERCHANT SERVICE.

Class of Vessel.	Unit.	Number allowed. Bowers.	Sheet.	Stream.	Kedges
SAILING VESSELS.					
Ship of the Line	3.5	2	2	1	5
Frigate	3.5	2	2	1	4
Razeed Frigate	3.	2	2	1	4
Ship	3.	2	1	1	3
	2.8	2	–	1	2
Sloop of War	2.8	2	2	1	3
Bark	2.8	2	1	1	3
	2.6	2	–	1	2
Vessel, full rig, 550 to 300 Tons	2.6	2	1	1	3
	2.3	2	–	1	2
" " 300 to 200 "	2.4	2	1	1	2
	2.1	2	–	1	1
" " 200 to 100 "	2.2	2	1	1	2
	1.9	2	–	1	1
" " less than 100 "	2.	1	1	–	1
	1.8	1	1	–	1
PROPELLER STEAMERS.					
Frigate, Ship, or Sloop of War, ship or bark rigged	3.	2	2	1	4
	2.8	2	1	1	2
Sloop, lighter-rigged	2.5	2	2	1	4
	2.4	2	–	1	2
Vessel, light rig, 1200 to 900 Tons	2.3	2	2	1	3
	2.2	2	1	1	2

Table—(*Continued*).

Class of Vessel.	Unit.	Number allowed.			
		Bowers.	Sheet.	Stream.	Kedges
Vessel, light rig, 900 to 600 Tons	2.3	2	1	1	2
	2.2	2	–	1	2
" " 600 to 500 "	2.	2	1	1	2
	2.	2	–	1	1
" " less than 500 "	2.	2	–	1	2
" without any rig	1.8	1	1	–	1
SIDE-WHEEL STEAMERS.					
Ship or Bark	3. 2.8	2	2	1	4
Brig or Brigantine......................	2.2 2.	2	1	1	3
Vessel 700 to 500 Tons...............	2.	2	1	1	2
	1.8	2	–	1	2
" 500 to 350 "	2.	2	–	1	2
	1.8	2	–	1	2
" 350 to 200 "	2.	2	–	1	2
	1.8	2	–	1	1
" less than 200 "	2.	2	–	1	1
	1.8	2	–	1	1
Steam-boat without any rig, hull much above water..................	2.	2	–	1	2
	1.9	2	–	1	1
IRON-CLADS.					
Hull much above water...............	2.2	2	1	1	4
" alike to a Monitor..............	1.	2	–	–	2
BOATS.					
Any description	1.2	1	–	–	–

NOTE.—*The Tonnage as above given is computed by the old U. S. Measurement.*

2. For a Comparison between the tonnages of all classes of vessels under the old and new law, see page 105.

To Compute the Diameter of a Chain Cable corresponding to a Given Weight of Anchor.

RULE.—Cut off the two right-hand figures of the anchor's weight in pounds; multiply the square root of the remainder by 4; and the product, subtracting 3 when the weight of the anchor exceeds 8000, 2 when it is between 8000 and 7000, and 1 when it is between 7000 and 4000, will give the diameter of the chain in sixteenths of inches.

EXAMPLE.—The weight of an anchor is 2500 lbs.
$\sqrt{25.00} \times 4 = 5 \times 4 = 20$, and $20 - 0$ (weight less than 4000) $= 20$ *sixteenths* $= 1\frac{1}{4}$ *inches.*

NOTE.—The diameter of a chain Messenger should be ⅔ that of the chain Cable to which it is to be applied.

2. The tensile proof of chains in the English Merchant service is for a diameter of chain of 1 inch and under, about 44800 lbs. (20 tons) per square inch of area of a half link, and for diameters exceeding this it is reduced gradually to 42560 (19 tons) per square inch of area.

3. When proved chains are used, they may be one sixteenth of an inch less in diameter, from 1 to 1¾ inches in diameter, and one eighth of an inch less in those above 1¾ inches.

4. The proof in the U. S. naval service is about 37500 lbs. per square inch of area of link for diameters of 1¼ inch and less, and 34500 lbs. for the larger diameters.

5. The British Admiralty proof is 630 lbs. per square of diameter of link in eighths of an inch.

ANCHORS.

From Experiments of a Joint Committee of Representatives of Ship-owners and the Admiralty of Great Britain.

An anchor of the ordinary or Admiralty pattern, the Trotman or Porter's improved (pivot fluke), the Honiball, Porter's, Aylin's, Rodgers's, Mitcheson's, and Lennox's, each weighing, inclusive of stock, 27000 lbs., withstood without injury a proof strain of 45000 lbs.

Comparative Resistance to Dragging.

Dry Ground.

Rodgers's dragged the Admiralty anchor at both long and short stay, and Aylin's at long stay; at short stay Rodgers's and Aylin's gave equal resistance.

Mitcheson's dragged Aylin's at both long and short stay, and Aylin's dragged the Admiralty's at short stay, they giving equal resistance at long stay.

Ground under Water.

Trotman's dragged Aylin's, Honiball's Mitcheson's and Lennox's; Aylin's dragged Rodgers's; Mitcheson's dragged Rodgers's; and Rodgers's and Lennox's dragged the Admiralty's.

The breaking weights between a Porter and Admiralty anchor, as tested at the Woolwich Dock-yard, were as 43 to 14.

ROPES, HAWSERS, AND CABLES.

Ropes of hemp fibres are laid with three or four strands of twisted fibres, and run up to a circumference of 12 inches.

Hawsers are laid with three strands of rope, or with four rope strands.

Cables are laid with three strands of rope only.

Tarred ropes, hawsers, etc., have 25 *per centum* less strength than white ropes; this is in consequence of the injury the fibres receive from the high temperature of the tar = 290°.

Tarred hemp and Manila ropes are of about equal strength. Manila ropes have from 25 to 30 *per centum* less strength than white ropes.

Hawsers and Cables, from having a less proportionate number of fibres, and from the increased irregularity of the resistance of the fibres, have less strength than ropes, the difference varying from 35 to 45 *per centum*, being greatest with the least circumference.

Ropes of four strands up to 8 inches are fully 16 *per centum* stronger than those having but three strands.

Hawsers and cables of three strands up to 12 inches are fully 10 *per centum* stronger than those having four strands.

The absorption of tar in weight by the several ropes is as follows:

Bolt rope..........	18 *per centum*	Cables	21 *per centum*
Shrouding...	15 to 18 "	Spun yarn...	25 to 30 "

White ropes are more durable than tarred.

The greater the degree of twisting given to the fibres of a rope, etc., the less its strength, as the exterior alone resists the greater portion of the strain.

To Compute the Strain that can be borne with safety by new Ropes, Hawsers, and Cables.

Deduced from the experiments of the Russian Government upon the relative strength of different Circumferences of Ropes, Hawsers, etc.

The U. S. Navy test is 4200 *lbs. for a White rope of three strands of best Riga hemp, of* 1¾ *inches in circumference* (=17000 *lbs. per square inch*), *but in the following table* 14000 *lbs. is taken as the unit of strain that can be borne with safety.*

Rule.—Square the circumference of the rope, hawser, etc., and multiply it by the following units for ordinary ropes, etc.

Table showing the Units for computing the safe strain that may be borne by Ropes, Hawsers, and Cables.

Description.	Ropes. White.		Ropes. Tarred.		Hawsers. White	Hawsers. Tarred	Cables. White	Cables. Tarred
	3 strands	4 strands	3 str'ds	4 str'ds	3 str'ds	3 str'ds.	3 str'ds	3 str'ds
Inches Circumference.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
White rope, 2.5 to 6 ins.	1140	1330	–	–	600	–	–	–
" " 6 " 8 "	1090	1260	–	–	570	–	510	–
" " 8 " 12 "	1045	880	–	–	530	–	530	–
" " 12 " 18 "	–	–	–	–	550	–	550	–
" " 18 " 26 "	–	–	–	–	–	–	560	–
Tarred " 2.5 " 5 "	–	–	855	1005	–	460	–	–
" " 5 " 8 "	–	–	825	940	–	480	–	–
" " 8 " 12 "	–	–	780	820	–	505	–	505
" " 12 " 18 "	–	–	–	–	–	–	–	525
" " 18 " 26 "	–	–	–	–	–	–	–	550
Manila 2.5 " 6 "	810	950	–	–	440	–	–	–
" " 6 " 12 "	760	835	–	–	465	–	510	–
" " 12 " 18 "	–	–	–	–	–	–	535	–
" " 18 " 26 "	–	–	–	–	–	–	560	–

When it is required to ascertain the weight or strain that can be borne by ropes, etc., in general use, The above Units should be reduced *one third*, in order to meet the reduction of their strength by chafing and exposure to the weather.

Example.—What is the weight that can be borne *with safety* by a Manila rope of 3 strands having a circumference of 6 inches?

$$6^2 \times 760 = 36 \times 27360 \text{ lbs.}$$

Ex. 2. What is the weight that can be borne by a tarred hawser 10 inches in circumference, in *general use?*

$$10^2 \times \left(560 - \frac{575}{3}\right) = 100 \times 336.67 = 33667 \textit{ lbs.}$$

To Compute the Circumference of a Rope, Hawser, or Cable for a Given Strain.

Rule.—Divide the strain in pounds by the appropriate units in the above table, and the square root of the product will give the circumference of the rope, etc., in inches.

Example.—The stress to be borne in safety is 165550 lbs.; what should be the circumference of a tarred cable to withstand it?

$$165552 \div 550 = 301, \text{ and } \sqrt{301} = 17.35, \text{ say } 17\tfrac{3}{8} \textit{ ins.}$$

Ex. 2.—What should be the circumference of a Manila cable to withstand a strain, in general use, of 149336 lbs.?

Assuming the circumference to exceed 18 ins., the unit = 560.

$$149336 \div \left(560 - \frac{560}{3}\right) = 149336 \div 373.34 = 400, \text{ and } \sqrt{400} = 20 \textit{ ins.}$$

WIRE ROPE.

Wire rope of the same strength as new *Hemp rope* will run on the same-sized sheaves; but the greater the diameter of the sheaves, the longer it will wear. Short bends should be avoided, and the wear increases with the speed. It is better to increase load rather than speed. The adhesion is the same as that of hemp rope.

Wire rope should not be coiled or uncoiled like hemp rope, but should be wound upon a reel.

When substituting wire rope for hemp rope, it is well to allow for the former the same weight per foot which experience has approved of for the latter. As a general rule, one wire rope will outlast three hemp ropes. To guard against rust, stationary rope should be oiled once a year with linseed-oil, or kept well painted or tarred. Running rope, while in use, requires no protection.

Where great pliability is required, the centre or core of wire rope is made of hemp, and small-sized rope is generally made with hemp centres.

Running rope is made of fine wire, and standing rope of coarse wire.

Wire rope made from charcoal-made iron is fully one fourth stronger than the ordinary rope.

The standing rigging of a vessel when composed of wire rope is one fourth less in weight than when of hemp.

Results of an Experiment with Galvanized Wire.

A strand of 2-inch wire rope broke with a strain of 13564 lbs., and a piece of a like rope, when galvanized, withstood a strain of 14796 lbs. before breaking.

Table showing the Diameter, Length, and Weight of Chains, and the Circumference and Weight of Cables corresponding to a Given Weight of Bower Anchor.

Weight of Anchor.	Diam. of Chain.	Length of Chain. Bower	Sheet	Stream.	Cables. Hemp or Manilla.	Stream of Manilla.	Weight per Fath. Close-linked.	Stud.
Lbs.	Ins.	Fath.	Fath.	Fath.	Circumf.	Circumf.	Lbs.	Lbs
50	3/16	60	45	45	2.5	2.21	3.5	–
75	1/4	60	45	45	3.	2.75	5.5	–
100	1/4	75	60	60	3.	2.75	5.5	–
130	5/16	75	60	60	4.	3.	6.25	–
160	3/8	75	60	60	4.	3.	9.5	–
200 250	3/8	75	60	60	4.5	3.25	9.5	–
300 350	7/16	75	60	60	5.	3.75	13.5	–
400	1/2	75	60	60	5.5	4.5	17.	–
450 500	9/16	75	60	60	6.	5.25	21.	–
550 600	5/8	75	60	60	6.75	5.5	26.	–
650	5/8	75	60	60	6.75	5.5	26.	–
700	11/16	75	60	60	7.5	6.	30.	–
800	3/4	90	75	60	8.	6.	37.	34
900	13/16	90	75	60	8.5	6.5	42.	40
1000	7/8	90	75	60	9.	8.	48.	44
1200	15/16	105	90	75	10.	8.75	55.	51
1400	1	105	90	75	10.5	9.5	63.	59
1600	1 1/16	120	105	75	11.	9.5	70.	66
1800	1 1/8	120	105	75	12.	10.5	79.	75
2000	1 1/8	135	105	75	12.	10.5	79.	75
2250	1 3/16	135	105	75	12.5	10.5	88.	82
2500	1 1/4	135	120	90	13.	11.	98.	91
2750	1 1/4	135	120	90	13.5	11.5	98.	91
3000 3250	1 3/8 1 3/8	150	120	90	15.	12.	118.	113
3500	1 7/16	150	135	90	15.5	12.5	–	120
3750	1 1/2	150	135	90	16.	13.	–	132
4000	1 9/16	150	135	90	16.5	13.	–	145
4300	1 9/16	150	135	90	16.5	13.5	–	145
4600 4900	1 5/8 1 5/8	150	135	90	17.	13.5	–	156
5200 5500	1 11/16 1 3/4	165	150	105	17.5	14.	–	162 175
6000	1 13/16	165	150	105	19.	15.	–	189
6500	1 7/8	165	150	105	19.5	15.	–	205
7000	2	165	150	105	20.5	16.	–	240
7500 8000	2 1/16 2 1/16	180	165	120	21.5	17.	–	–
8500	2 1/8	180	165	120	22.	17.5	–	–
9000	2 3/16	180	165	120	22.5	18.	–	–
9500	2 1/4	180	165	120	23.	18.5	–	–
10000	2 5/16	180	165	120	24.	19	–	–

Table of the Maximum Breaking Strain of Wrought-Iron Chain Rigging.

CLOSE-LINKED.

Diam of Iron.	Strain.	Diam. of Iron.	Strain.	Diam. of Iron.	Strain.	Diam. of Iron	Strain.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
3/16	2464	1/2	15680	13/16	44800	1 3/8	100800
1/4	3920	9/16	22400	7/8	51520	1 3/4	120960
5/16	6720	5/8	26880	15/16	58240	1 1/2	143360
3/8	8960	11/16	31360	1	62720	1 5/8	168000
7/16	13440	3/4	38080	1 1/8	82880	1 3/4	201580

NOTE.—The minimum breaking strain is about 9 per cent. less than this.

Close-linked chain is heavier than stud-linked.

The strength of iron chain rigging for *general use*, compared with *tarred* rope, also for general use, is as 3.57* to 1 for each part of a link.

To Compute the Circumference of a Link of Chain of equal Strength of a Tarred Rope.

RULE.—Divide the strain in pounds by 4000,† and the square root of the product will give the circumference of one part of the link of chain in inches.

EXAMPLE.—The stress to be borne is 50000 lbs.; what is the circumference of one part of a link of chain rigging of equivalent resistance?

$50000 \div 4000 = 12.5$, and $\sqrt{12.5} = 3.535$ *ins.*

The diameter of 3.535 ins. circumference $= 1\frac{1}{8}$ *ins.*

CHAINS, CABLES, AND ANCHORS.

In the Table, page 138, the weight of the best Bower anchor is made the exponent of the requirement of dimensions of Cables, etc., and not the Tonnages of the vessel, as hitherto.

The adoption of a new and essentially different admeasurement of tonnages sets aside the propriety of a reference to the tonnage of a vessel under the old measurement; added to which, the beam of a vessel, in connection with her rig and extent of hull above water, is made the sole basis of the computation of the weight of an anchor.

The number and weight of anchors, and the length of chains here given, exceed the usual practice of our Merchant service, but the propriety, if not the necessity of the weights and lengths given, is no less apparent.

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* Chain links $1\frac{1}{8}$ ins. in diameter will bear an average *maximum* strain of 43500 lbs., or a *minimum* of 37500 lbs. per square inch of section, from which is to be deducted $\frac{1}{3}$ for general use $= 25000$ lbs.

White rope of three strands will bear 14000 lbs. per square inch, from which is to be deducted $\frac{1}{4}$, to reduce it to the resistance of tarred rope $= 10500$ lbs., and also $\frac{1}{3}$ for general use $= 7000$ lbs.; hence $\frac{25000}{7000} = 3.57$.

† The constant of 4000 represents for both parts of a link of iron chain the varying units in the table, page 137, for ropes, etc., the occasion of the variance in the latter case arising from the difference of the strength of a rope, etc., whether of three or four strands, or whether rope or hawser laid, or in its circumference, its proportionate strength being inversely as its circumference.

Table of the Relative Dimensions of Wire Rope (Coarse and Fine laid), and of Ropes, Hawsers, and Cables, with their Breaking Strain.

R—J A. Roebling. N—Newall & Co. AG—Admiralty, and Garnock, Bibby & Co.

(COARSE LAID.)

Trade Number.	Manufacture.	Diameter.	Circumference.	Weight per Foot	Breaking Weight or Strain.	Proof Weight or Strain.	Circumference of equal Resistance for General Use. Tarred Ropes. Three Strands.	Ropes. Four Strands.	Haw'rs. Three Strands.	Cables. Three Strands.
No.		Ins	Ins	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Ins.	Ins.
27	R	.25	.78	–	1120	–	1¼	–	–	–
26	R	.26	.88	–	1620	–	1½	–	–	–
25	R	.3	.94	–	2060	–	1¾	–	–	–
–	N	.32	1.	.16	4480	–	2 7/16	2¼	3 5/16	–
24	R	.35	1.11	–	2760	–	–	–	–	–
–	N	.36	1.⅛	.19	5018	–	2 9/16	2⅜	3½	–
23	R	.39	1.23	–	3300	–	–	–	–	–
–	N	.4	1.¼	.21	5600	–	2 11/16	2½	3 11/16	–
22	R	.41	1.31	–	4260	–	–	–	–	–
–	N	.44	1.⅜	.23	6182	–	2⅞	2⅝	3⅞	–
–	N	.48	1.½	.25	6720	–	3	2¾	4 1/16	–
–	AG	–	1.½	.34	6720	5040	3	2¾	4 1/16	–
21	R	.49	1.53	–	5660	–	–	–	–	–
20	R	.52	1.⅝	–	8180	–	–	–	–	–
–	N	.52	1.⅝	.34	8960	–	3 7/16	3⅛	4 13/16	–
–	N	.56	1.¾	.42	11200	–	3 3/16	3½	5 3/16	–
–	AG	–	1.¾	.42	11200	7280	3 13/16	3½	5 3/16	–
–	N	.64	2.	.58	15680	–	4 9/16	4 3/16	6	–
19	R	.6	1.9	–	11600	–	–	–	5¼	–
18	R	.68	2.⅛	–	15200	–	–	–	–	–
–	AG	–	2.	.58	15680	9632	4 9/16	4 3/16	6	–
–	N	.72	2.¼	.75	20160	–	5⅛	4¾	6⅞	–
–	AG	–	2.¼	.75	–	11870	5⅛	4¾	6⅞	–
17	R	.75	2.4	–	17600	–	–	–	–	–
–	AG	–	2.½	.92	19400	14124	5¾	5⅜	7 9/16	–
–	N	.8	2.½	.92	24640	–	5¾	5⅜	7 9/16	–
–	N	.88	2.¾	1.08	29120	–	6¼	5⅞	8¼	8¼
–	AG	–	2.¾	1.08	–	16464	6¼	5⅞	8¼	8¼
16	R	.875	2.68	–	24600	–	–	–	–	–
–	N	.95	3.	1.25	33600	–	6¾	6⅛	8⅝	8⅝
15	R	1.	2.98	–	32000	–	–	–	–	–
–	AG	–	3.	1.25	–	19152	6⅝	6⅝	8⅝	8⅝
–	AG	–	3.¼	1.42	–	23744	7¼	6¾	9 3/16	9 3/16
–	N	1.03	3.¼	1.42	38080	–	7¼	6¾	9 3/16	9 3/16
14	R	1.⅛	3.¼	–	40000	–	–	–	–	–
–	AG	–	3.½	1.67	40880	26208	7 13/16	7 5/16	10	10
–	N	1.11	3.½	1.66	44800	–	7 13/16	7 5/16	10	10
13	R	1.¼	3.⅝	–	50000	–	–	–	–	–
–	AG	–	3.¾	2.	–	30240	8 13/16	8½	10 15/16	10 15/16
–	N	1.19	3.¾	2.	53760	–	8 13/16	8½	10 15/16	10 15/16
12	R	1.⅜	4.	–	60000	–	–	–	–	–
–	AG	–	4.	2.33	–	34272	9½	9¼	11 13/16	11 13/16
–	N	1.27	4.	2.33	62720	–	9½	9¼	11 13/16	11 13/16
11	R	1.4	4.45	–	72000	–	–	–	–	–

Table—(*Continued*).

(COARSE-LAID.)

Trade Number.	Manufacture.	Diameter.	Circumference.	Weight per Foot.	Breaking Weight or Strain.	Proof Weight or Strain.	Circumference of equal Resistance for General Use. TARRED. Ropes. Three Strands.	Ropes. Four Strands.	Haw'rs. Three Strands.	Cables. Three Strands.
No.		Ins.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Ins.	Ins.
–	AG	–	4.¼	2.67	–	38752	9⅞	9 9/16	12¼	12⅛
–	N	1.35	4.¼	2.5	67200	–	9⅞	9 9/16	12¼	12¼
–	N	1.43	4.⅜	2.66	71680	–	10⅛	9⅞	–	12 7/16
–	AG	–	4.½	3.	–	42232	10 7/16	10 3/16	–	12¾
–	N	1.51	4.⅝	3.	80640	–	10¾	10½	–	13⅛
–	AG	–	4.¾	3.33	–	48944	11	10¾	–	13½
–	AG	–	5.	3.66	–	54656	–	11⅞	11 9/16	–
–	N	1.59	5.	3.66	98560	–	–	11⅞	11 9/16	–
–	AG	–	5.¼	4.	108400	63392	–	12½	12¼	–
–	N	1.75	5.½	4.41	118720	–	–	–	–	–
–	AG	–	5.½	4.33	–	72240	–	–	–	–
–	–	–	5.¾	–	130530	–	–	–	–	–
–	N	1.91	6.	5.25	141120	–	–	–	–	–
–	AG	–	6.	5.	–	80640	–	–	–	–
–	–	–	6.½	–	165555	–	–	–	–	–
–	–	–	7.	–	192080	–	–	–	–	–
–	–	–	7.½	–	215048	–	–	–	–	–
–	–	–	8.	–	256880	–	–	–	–	–
					FINE-LAID.					
10¾	R	.5	1.37	–	7500	–	–	3⅛	2⅞	–
10½	R	.9/16	1.68	–	9680	–	–	3 9/16	3¼	4⅞
10¼	R	.⅝	2.12	–	11600	–	–	3 15/16	3⅝	5¼
10	R	.¾	2.45	–	17280	–	–	4 13/16	4⅜	6⅜
9	R	.⅞	2.56	–	22800	–	–	5½	5	7¼
8	R	1.	2.98	–	32800	–	–	7¼	6¼	8¾
7	R	1.⅛	3.36	–	40400	–	–	8 3/16	7	9½
6	R	1.¼	3.91	–	54400	–	–	8 13/16	8 1/16	11
5	R	1.½	4.5	–	70000	–	–	10	9¾	12½
4	R	1.⅝	4.9	–	87200	–	–	11 3/16	10 15/16	–
3	R	1.¾	5.44	–	108000	–	–	12½	12⅛	–
2	R	1.⅞	6.2	–	130000	–	–	–	–	–
1	R	2.¼	6.62	–	148000	–	–	–	–	–

In the above table the determination of the circumference of the rope, etc., is based upon the Breaking Weight or Tensile resistance of the wire being reduced by one fourth, and the *units* or the ultimate resistance of the rope, etc., are reduced one third.

In the U. S. Navy the relative dimensions of Hemp Cable and of Wire Rope are as follows:

Circumference in Inches.

Hemp...	3,	4,	5,	5½,	6,	6¾,	7¼,	8,	9,	10,	10½,	11,	12,
Wire....	1⅝,	2⅛,	2¾,	3,	3¼,	3⅝,	4,	4⅜,	4⅞,	5½,	5¾,	6,	6½.

NOTE.—The difference between the dimensions of the wire rope here given and in the preceding table, of one fourth in area, is in consequence of the high estimate of strength given to the hemp rope made in the U. S. Service.

The circumferences given are for Tarred ropes, etc., alone; if, therefore, the circumferences for White and Manila ropes are required, proceed as follows:

To Compute the Circumference of a White or Manila Rope, Hawser, or Cable compared with one of Tarred Hemp.

RULE.—Multiply the square of the circumference of the given rope by the unit for the circumference, from the table, page 136; divide the product by the unit for the circumference of the rope, etc., required, and the square root of the product will give the circumference required.

NOTE.—If the circumference is required for a rope in general use, reduce the units in the table one third.

ILLUSTRATION.—Required the circumferences for a white rope and a Manila hawser, for general use—equivalent to a tarred rope of 3 strands, and 9½ inches in circumference.

Units of tarred rope of 9½ ins. = 780 − ⅓ = 520.
" white rope of about 9½ " = 1045 − ⅓ = 697.
" Manila hawser of about 9½ " = 760 − ⅓ = 507.

Then $9.5^2 \times 520 = 46930$, which $\div 697 = 67.33$, and $\sqrt{67.33} = 8.2$, say 8¼ *ins.* for the white rope.

Again, $9.5^2 \times 520 = 46930$, which $\div 507 = 92.56$, and $\sqrt{92.56} = 9.62$, say 9⅝ *ins.* for the Manila hawser.

Proof and Breaking Strain of Chain Cables.

Diameter of Chain.	Proved.	Breaking Strain.	Diameter of Chain.	Proved.	Breaking Strain.
Ins.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.
⅞	16750	33500	1⅝	56675	113350
1	21700	43400	1¾	65750	131500
1⅛	27500	55000	1⅞	75650	151300
1¼	33300	66600	2	86100	172200
1⅜	40450	80900	2⅛	97375	194750
1½	48150	96300	2¼	109090	218180

The proof of British Navy Chain is ⅝ the breaking strain.

To Compute the Circumference of the Shrouds of Vessels, and to Ascertain their Number of Shrouds.

Vessels.	Unit.	Number of Pairs Shrouds. Fore.	Main.	Mizzen.	Vessels.	Unit.	Number of Pairs Shrouds. Fore.	Main.	Mizzen.
SAILING VESSELS.					Over 800 Tons ..	.82	6	7	5
Ship	.64	6	7	5	Under 800 " ..	.76	6	7	–
Bark	.63	5	5	4	Under 400 " ..	.6	4	3	–
Brig	.55	4	5	–	SIDE-WHEEL STEAMERS.				
Schooner	.32	3	4	–	First Rate	.96	9	10	6
Sloop	.5 to .3	4 to 2	–	–	Over 1400 Tons ..	.81	6	7	5
SCREW STEAMERS.					Under 1400 " ..	.76	6	7	5
First Rate	.96	10	10	6	Over 800 " ..	.7	5	6	4
Over 1500 Tons ..	.95	9	10	6	Under 800 " ..	.65	5	5	–
Under 1500 " ..	.81	8	9	6	Under 400 " ..	.55	4	3	–

NOTE.—The extreme Unit and Number of pair of Shrouds are given in each case.

RULE.—Multiply the mean extreme length of the fore and main mast (measuring from the keelson) in feet by the unit in the preceding table, and the square root of the product will give the circumference of the fore and main shrouds in inches.

EXAMPLE.—What should be the circumference of the fore and main shrouds of a screw steamer of 1600 tons, the mean length of her fore and main masts being 110 feet?

$$110 \times .95 = 104.5, \text{ and } \sqrt{104.5} = 10.22, \text{ say } 10\tfrac{1}{4} \textit{ ins.}$$

NOTE.—When a mast does not step upon the keelson, assume its length to extend to it.

2. These units are somewhat too large for the circumference of the mizzen shrouds. Thus, by the above rule, the circumference of the mizzen shrouds of a first-class Frigate would be 9 inches, whereas 8 inches is the proper circumference.

To Compute the Weight of Ropes, Hawsers, and Cables.

RULE.—Square the circumference, and multiply it by the appropriate unit in the following table, and the product will give the weight per foot in pounds:

	ROPES.	HAWSERS.	CABLES.
3-strand Hemp	.032	.031	.031
3-strand tarred Hemp	.042	.041	.041
3-strand Manila	.032	.031	.031
4-strand Hemp	.033	—	—
4-strand tarred Hemp	.048	—	—
4-strand Manila	.035	.034	.034

The units for Thread Ropes is the same as that for Ropes of like material.

EXAMPLE.—What is the weight of a coil of 10-inch Manila rope of four strands of 120 fathoms?

$$10^2 \times .035 = 3.5, \text{ and } 120 \times 6 \times 3.5 = 2520 \textit{ lbs.}$$

Weight of Men and Women.

The average weight of 20,000 men and women, weighed at Boston, 1864, was—men, 141½ lbs.; women, 124½ lbs.

Weight of Horses.—(U. S.)

The weight of horses ranges from 800 to 1200 lbs.

WEIGHT OF CATTLE.

To Compute the Dressed Weight of Cattle.

RULE.—*Measure as follows*:

1. The girt close behind the shoulders.

2. The length from the fore-part of the shoulder-blade along the back to the bone at the tail, in a vertical line with the buttocks.

Then multiply the square of the girt in feet by 5 times the length in feet, and divide the product by 1.5; the quotient will give the dressed weight of the quarters.

EXAMPLE.—The girt of a beeve is 6.5 feet, and the length, measured as above, is 5.25 feet.

$$\frac{6.5^2 \times \overline{5.25 \times 5}}{1.5} = 42.25 \times 26.25 = \frac{1109.0625}{1.5} = 739.375 \ lbs.$$

NOTE.—With very fat cattle divide by 1.425, and with very lean by 1.575.
2. The quarters of a beeve exceed by a little half the weight of the living animal.
3. The hide weighs about the eighteenth part, and the tallow the twelfth part.

WEIGHT AND DIMENSIONS OF SHOT AND SHELLS.

The weights of these may be ascertained by rules for the Mensuration of Solids; also, by inspection in the tables, pages 543 and 544.

To Compute the Weight of a Cast-Iron Shot from its Diameter.

A cast-iron shot of 4 inches in diameter weighs 8.736 lbs.

Therefore, $\frac{8.73}{64}$ of the cube of the diameter is the weight of a shot of any diameter; for the weights of spheres are as the cubes of their diameters.

RULE.—Multiply the cube of the diameter in inches by .1365,* and the product is the weight.

EXAMPLE.—What is the weight of a cast-iron shot 10 inches in diameter?

$$10^3 \times .1365 = 136.5 \ lbs.$$

To Compute the Diameter from the Weight.

RULE.—Divide the weight in pounds by .1365, and the cube root of the quotient is the diameter.

EXAMPLE.—What is the diameter of a cast-iron shot, its weight being 99.5 lbs.?

$$99.5 \div .1365 = 729, \text{ and } \sqrt[3]{729} = 9 \ ins.$$

To Compute the Weight or Diameter of a Leaden Shot.

A lead shot 4 inches in diameter weighs 13.744 lbs.

Therefore, $\frac{13.744}{64}$ of the cube of the diameter is the weight of a shot of any diameter.

RULE.—Multiply the cube of the diameter in inches by .2147, and the product is the weight. Or, divide the weight in pounds by .2147, and the cube root of the quotient is the diameter.

EXAMPLE.—What is the weight of a lead shot 10 inches in diameter?

$$10^3 \times .2147 = 214.7 \ lbs.$$

To Compute the Weight of a Cast-Iron Shell.

RULE.—Multiply the difference of the cubes of the exterior and interior diameter in inches by .1365.

EXAMPLE.—What is the weight of a cast-iron shell having diameters of 10 and 8.5 inches.

$$10^3 - 8.5^3 = 1000 - 614.125 = 385.875, \text{ which} \times .1365 = 52.672 \ lbs.$$

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* .1365 represents a cubic inch of cast iron = .2607 lbs., and .1474 a cubic inch of wrought iron = .2816 lbs.

PILING OF SHOT AND SHELLS.

To Compute the Number of Shot in a Triangular Pile.

RULE.—Multiply continually together the number of shot in one side of the bottom course, and that number increased by 1; and again by 2, and one sixth of the product will give the number.

EXAMPLE.—What is the number of shot in a triangular pile, each side of the base containing 30 shot?

$$\frac{30\times\overline{30+1}\times\overline{30+2}}{6}=\frac{29760}{6}=4960 \textit{ shot.}$$

To Compute the Number of Shot in a Square Pile.

RULE.—Multiply continually together the number in one side of the bottom course, that number increased by 1, double the same number increased by 1, and one sixth of the product will give the number.

EXAMPLE.—How many balls are there in a square pile of 30 courses?

$$\frac{30\times\overline{30+1}\times\overline{30\times2+1}}{6}=\frac{56730}{6}=9455 \textit{ shot.}$$

To Compute the Number of Shot in an Oblong Pile.

RULE.—From 3 times the number in the length of the base course subtract one less than the number in the breadth of it; multiply the remainder by the number in the breadth, and again by the breadth, increased by 1, and one sixth of the product will give the number.

EXAMPLE.—Required the number of balls in an oblong pile, the numbers in the base course being 16 and 7?

$$\frac{16\times3-\overline{7-1}\times7\times\overline{7+1}}{6}=\frac{2352}{6}=392 \textit{ shot.}$$

To Compute the Number of Shot in an Incomplete Pile.

RULE.—From the number in the pile, considered as complete, subtract the number conceived to be in that portion of the pile which is wanting, and the remainder will give the number.

FRAUDULENT BALANCES.

To Detect them.

After an equilibrium has been established between the weight and the article weighed, transpose them, and the weight will preponderate if the article weighed is lighter than the weight, and contrariwise if it is heavier.

To Ascertain the True Weight.

RULE.—Ascertain the weight which will produce equilibrium after the article to be weighed and the weight have been transposed; reduce these weights to the same denomination, multiply them together, and the square root of their product will give the true weight.

EXAMPLE.—If the first weight is 32 lbs., and the second, or weight of equilibrium after transposition, is 24 lbs. 8 oz., what is the true weight?

24 lbs. 8 oz. = 24.5 lbs.

Then $32 \times 24.5 = 784$, and $\sqrt{784} = 28$ lbs.

Or, when a represents longest arm, b " shortest arm, | A greatest weight, and B least weight.

Then $Wa = Ab$, and $Wb = Bx$; multiplying these two equations, $W^2ab = ABab$, or $W^2 = AB$, and $W = \sqrt{AB}$.

ILLUSTRATION.—A = 32; B = 24.5; W = 28. Assume the length of the longest arm = 10.

Then 32 : 28 : : 10 : 8.75.

Hence $a = 10$, $b = 8.75$, or $28^2 = 32 \times 24.5$, and $28 = \sqrt{32 \times 24.5}$.

To Ascertain the Weight of a Bar, Beam, etc., by the Aid of a known Weight, as the Body of a Man, etc.

OPERATION.—Balance the bar, etc., over a suitable fulcrum, and note the distance between it and the end of its longest arm. Suspend the known weight from the longest arm, and move the bar, etc., upon the fulcrum, so that the bar with the attached weight will be in equilibrio; subtract the distance between the two positions of the fulcrum from the longest arm first obtained; multiply this remainder by the weight suspended, divide the product by the distance between the fulcrums, and the quotient will give the weight required.

EXAMPLE.—A piece of tapered timber 24 feet in length is balanced over a fulcrum when 13 feet from the less end; but when the body of a man weighing 210 lbs. is suspended from the extreme of the longest arm, the piece and the weight are balanced when the fulcrum is 12 feet from this end. What is the weight of the timber?

13 − 12 = 1, and 13 − 1 = 12 *feet*.

Then, $12 \times 210 \div 1 = 2520$ *lbs*.

BOARD AND TIMBER MEASURE.

BOARD MEASURE.

In *Board Measure*, all boards are assumed to be 1 inch in thickness.

To Compute the Measure or Surface in Square Feet.

When all the Dimensions are in Feet.

RULE.—Multiply the length by the breadth, and the product will give the surface required.

When either of the Dimensions are given in Inches.

RULE.—Multiply as above, and divide the product by 12.

When all the Dimensions are in Inches.

RULE.—Multiply as before, and divide the product by 144.

EXAMPLE.—What are the number of square feet in a board 15 feet in length and 16 inches in width?

$15 \times 16 = 240$, and $240 \div 12 = 20$ *feet*.

TIMBER MEASURE.

To Compute the Volume of Round Timber.

When all the Dimensions are in Feet.

RULE.—Multiply the length by the square of one quarter of the mean girth, and the product will give the volume in cubic feet.

When the Length is given in Feet, and the Girth in Inches.

RULE.—Multiply as above, and divide by 144.

When all the Dimensions are in Inches.

RULE.—Multiply as before, and divide by 1728.

Or, $\frac{l \times c^2}{16} \div 144$, *l representing the length in feet, and c half the sum of the circumference of the two ends in inches.*

EXAMPLE.—The girths of a piece of timber are 31.5 and 62.9 inches, and its length 50 feet; required its volume or measure.

$$50 \times \left(\frac{31.5 + 62.9}{2} \div 4\right)^2 = 50. \times 11.8^2 = 6962, \text{ and } \frac{6962}{144} = 48.347 \textit{ feet.}$$

Or, $$50 \times \frac{\overline{31.5 + 62.9 \div 2}^2}{16} \div 144 = \frac{111392}{16} \div 144 = 48.347 \textit{ feet.}$$

Sawed or *Hewed* Timber is measured by the cubic foot.

To Compute the Volume of Square Timber.

When all the Dimensions are in Feet.

RULE.—Multiply the product of the breadth by the depth, by the length, and the product will give the volume in cubic feet.

When either of the Dimensions are given in Inches.

RULE.—Multiply as above, and divide the product by 12.

When any two of the Dimensions are given in Inches.

RULE.—Multiply as before, and divide by 144.

EXAMPLE.—A piece of timber is 15 inches square, and 20 feet in length; required its volume in cubic feet.

$$15 \times 15 \times 29 = 4500, \text{ and } 4500 \div 144 = 31.25 \textit{ cubic feet.}$$

SPARS AND POLES.

Pine and *Spruce Spars*, from 10 to 4½ inches in diameter inclusive, are to be measured by taking their diameter, clear of bark, at one third of their length from the abut or large end.

Spars are usually purchased by the inch diameter; all under 4 inches are termed *Poles*.

Spars of 7 inches and less should have 5 feet in length for every inch of diameter, and those above 7 inches should have 4 feet in length for every inch of diameter.

HYDROMETERS.

The *U. S. Hydrometer* (Tralle's) ranges from 0 (water) to 100 (pure spirit); it has not any subdivision or standard termed "Proof," but 50, upon the stem of the instrument, at a temperature of 60°, is the basis upon which the computations of duties are made.

In connection with this instrument, a Table of Corrections, for differences in the temperature of spirits, becomes necessary; and one is furnished by the Treasury Department, from which all computations of the value of a spirit are made.

ILLUSTRATION.—A cask contains 100 gallons of whisky at 70°, and the hydrometer sinks in the spirit to 25 upon its stem.

Then, by table, under 70°, and opposite to 25, is 22.99, showing that there are 22.99 gallons of pure spirit in the 100.

The *Commercial Hydrometer* (Gendar's) has a "Proof" at 60°, which is equal to 50 upon the U. S. Instrument and its gradations, run up to 100 with it, and down to 10 below proof, at 0 upon the U. S. Instrument; or the 0 of the Commercial Instrument is at 50 upon the U. S. Instrument, from which it progresses numerically each way, each of its divisions being equal to two of the latter.

In testing spirits, the Commercial standard of value is fixed at proof; hence any difference, whether higher or lower, is added or subtracted, as the case may be, to or from the value assigned to proof.

A scale of Corrections for temperature being necessary, one is furnished with the Thermometer.

Application of the Thermometer.—The elevation of the mercury indicates the correction to be added or subtracted, to or from the indication upon the stem of the hydrometer.

When the elevation is above 60°, subtract the correction; and when below, add it.

ILLUSTRATION.—A hydrometer in a spirit indicates upon its stem 50 below proof, and the thermometer indicates 4 above 60° in the appropriate column.

Then $50 - 4 = 46 =$ *strength below proof.*

To Compute the Strength of a Spirit, or the Volume of its pure Spirit, by a Commercial Hydrometer, and convert it to the Indication of a U. S. Hydrometer.

When the Spirit is above Proof. RULE.—Add 100 to the indication, and divide the sum by 2.

When the Spirit is below Proof. RULE.—Subtract the indication from 100, and divide the remainder by 2.

EXAMPLE.—A spirit is 11 above proof by a Commercial Hydrometer; what proportion of pure spirit does it contain?

$$\overline{11 + 100} \div 2 = 55.5 \textit{ per cent.}$$

To Compute the Strength, etc., by a U. S. Hydrometer.

When the Spirit is above Proof. RULE.—Multiply the indication by 2, and subtract 100.

When the Spirit is below Proof. RULE.—Multiply the indication by 2, and subtract it from 100.

EXAMPLE.—A spirit is 55.5 by a U. S. Hydrometer; what is its per centage above proof?

$$\overline{55.5 \times 2} - 100 = 11 \textit{ per cent.}$$

The Commercial practice of reducing indications of a hydrometer is as follows:

Multiply the number of gallons of spirit by the per centage or number of degrees above or below proof, divide by 100, and the quotient will give the number of gallons to be added or subtracted, as the case may be.

ILLUSTRATION.—50 gallons of whisky are 11 per cent. above proof.

Then $50 \times 11 \div 100 = 5.5$, which, added to $50 = 55.5$ *gallons.*

U. S. ENSIGN, PENNANTS, AND FLAGS.

ENSIGN (*Head, Depth*, or *Hoist*).—Ten nineteenths of its length.

Field.—Thirteen horizontal stripes of equal breadth, alternately red and white, beginning with red.

Union.—A blue field in the upper quarter, next the head, .4 of the length of the field, and 7 stripes in depth, with white stars ranged in equidistant, horizontal, and vertical lines, equal in number to the number of states of the Union.

PENNANT (Narrow).—*Head.*—6.24 inches to a length of 70 feet; 5.76 inches to a length of 55 feet; 5.24 inches to a length of 40 feet; 4.8 inches to a length of 30 feet; and 4.2 inches to a length of 25 feet.

Night.—3.6 inches to a length of 20 feet. *Boat.*—3 inches to a length of 9 feet, and 2.42 inches to a length of 6 feet. *Union.*—A blue field at the head, one fourth the length, with 13 white stars in a horizontal line. *Field.*—A red and white stripe tapered to a point, red uppermost, each of the same breadth at any part of the length.

Night and Boat Pennants.—Union to have but 7 stars.

JACK.—Alike to the Union of an Ensign.

VICE-ADMIRAL'S FLAG—A Rectangle. Blue. Three five-pointed stars set as an equilateral triangle 18 inches from centres, the upper star 18 inches from the head and 27 inches from the tabling.

REAR-ADMIRAL'S FLAG—A Rectangle. *Field* Blue, Red, or White.

Stars.—Two set vertically 18 inches from centres, the upper star set 18 inches from the head and 18 inches from the tabling. White when the flag is Blue or Red, and Blue when it is White.

Boat and Night Flags.—The distances between the stars to be proportionably less than above.

COMMODORE'S PENNANT (Broad).—Blue, Red, or White, with stars ranged in equidistant vertical and horizontal lines, equal in number to the States of the Union, to be white in the blue and red pennants, and blue in the white.

Swallow-tailed, the angle at the tail to be bisected by a line drawn at a right angle from the centre of the depth or hoist, and at a distance from the head of three fifths of the length of the pennant; the lower side is to be rectangular with the head or hoist; the upper side is to be tapered, running the width of the pennant at the tails .1 the hoist. The stars to be ranged in the field alone in equidistant vertical lines, and horizontally to taper with the pennant. *Head.*—.6 their length.

SIGNALS (Numbers).—*Head.*—.8 their length. REPEATERS.—*Head.*—Eleven twentieths of their length. QUARANTINE FLAG—A Rectangle. *Field* yellow. *Head.*—Nine elevenths of their length.

DIVISIONAL MARK.—A Triangle. *Field* of three stripes or divisions, but of two colors only; the centre one being of a color different from the others, to be in the form of a wedge, the bar being one third of the head, and the point extending to the extremity of the fly.

1st Division—blue, white, and blue;
2d " red, white, and red;
3d Division—white, blue, and white;
Service office—white, yellow, and white.

When the *Lengths* are 6.4, 5.6, 4.8, and 4 feet, the *Heads* are 8, 7, 6, and 5 feet.

SECRETARY OF THE NAVY'S FLAG—Blue. *Head.*—10.25 feet; fly 14.4 feet, with a white foul anchor 3 feet in length, set vertically in the centre.

Storm Flag, same, with a head of but 5.4 feet, and a fly of 7.6 feet.

Dimensions and Capacities of the Principal Dry Docks and Railways in the United States, South America, and Egypt.

Locations.		Length.	Breadth.*	Draft of Water.	Capacity.†	Remarks.
		Feet.	Feet.	Feet.	Tons.	
Baltimore	Md.	220	60	18	1500	Railway.
Bermuda		381	83.9	30	7000	Iron Float'g Dock.
Boston	Mass.	354	65.5	19	—	Railways.
"	"	360	66	17		Dry Dock.
Brooklyn	N.Y.	350	66	25	7000	U. S. Navy. Stone.
"	"	476	120	24	—	Dry Dock.
Charleston	S. C.	145	50	10	—	—
Charlestown	Mass.	379	60	25	7000	U. S. Navy. Stone.
Fair Haven	Conn.	125	—	9	450	Railway.
Gowanus Bay	N.Y.	270	58	8.5	700	Railway.
Green Point	"	150	28‡	8	700	Railway.
Hunter's Point	"	400	80	7	1600	Railway.
" "	"	340	50	6	1000	Railway.
Jersey City	N. J.	270	65	12	1800	Sectional.
Mobile	Ala.	245	65	12	3000	Balance.
New London	Conn.	120	35	10	500	Railway.
New Orleans	La.	180	30	6.25	400	Railway.
New York	N.Y.	450	95	25	8500	Sectional.§
"	"	360	88	21	8000	Balance.
"	"	180	35	13	1000	Hydrostatic.
Norfolk	Va.	320	60	25	7000	U. S. Navy. Stone.
"	"	200	65	6	700	Railway.
"	"	—	—	—	1500	Railway.
Philadelphia	Pa.	350	60	22.5	5300	U. S. Navy. Sect'l.
"	"	275	66	11.5	2000	Sectional.
Portland	Me.	425	100	25	8000	Dry Dock.
Portsmouth	N. H.	350	60	23	5300	U. S. Navy. Bal'ce.
Red Hook	N. Y.	500	120	24	—	Dry Dock.
Mare Island	Cal.	350	60	16	5300	U. S. Navy. Sect'l.
Savannah	Geo.	350	90	18	3500	Dry Dock.
Washington	D. C.	240	65	8	—	U. S. Navy. Railw.
"	"	195	28*	9	700	Railway.
Wilmington	Del.	215	60	12	800	Railway.
"	N. C.	200	50	5.5	900	Railway.
St. Thomas	W. I.	300	72	20	6000	Balance.
Cartagena	Spain	324	78	25	6000	Floating.
Callao	Peru	290	72	16	6000	Balance.
Havana	Cuba	300	60	25	2000	Balance.
Maranham	Brazil	290	72	—	3000	Dry Dock.
Montevideo	S. A.	265	56	—	2500	Railway.
Suez	Egypt	413	95	30	—	Dry Dock.

The largest docks, etc., at the different locations are only noted.

* Including the water-wheel guards of a side-wheel steamer. † In Tons of weight.
‡ Of Hull, and not including water-wheel guards. § Located at Hoboken, N. J.

Weights and Distances of Principal Race and Trotting Courses of the United States, etc.

RACE COURSES.

Weights.—*American Jockey Club, Jerome Park, N. Y.; Maryland Jockey Club, Baltimore, Md.; Monmouth Park Association, Long Branch, N. J.; Saratoga Association, Saratoga, N. Y.*—2 years, 75 lbs. (after 1st September, 90 lbs.); 3 years, 90; 4 years, 103; 5 years, 114. In races exclusively for 2 years, 100 lbs.; and for 3 years, 110.

Saratoga and *Monmouth Park Associations* and *Maryland Jockey Club.*—5 years and over, 118 lbs. *American Jockey Club.*—6 years and over, 118 lbs.

Kentucky Association, Lexington, Ky., Louisville Jockey Club, Louisville, Ky., and *Nashville Blood-horse Association, Nashville, Tenn.*—2 years, 86 lbs., in stakes 90; 3 years, 90, in stakes 100; 4 years, 104; 5 years, 110.

Kentucky Association.—6 years and over, 114 lbs. *Nashville Association.*—6 years and over, 115 lbs.

Savannah Jockey Club, Savannah, Central Georgia Jockey Club, Ga., and *South Carolina Jockey Club, Charleston, S. C.*—2 years, 75 lbs.; 3 years, 90; 4 years, 104. In races exclusively for 2 years, except in heats, 90 lbs.; and for 3 years, 100.

Savannah and *Central Georgia Jockey Clubs.*—5 years, 110 lbs.; 6 years and over, 115. *South Carolina Jockey Club.*—5 years, 112 lbs.; 6 years, 118; 7 years and over, 120; and 3 single mile heats in 5, 50.

Louisiana Jockey Club, New Orleans.—2 years, 75 lbs. (84 lbs. in stakes); 3 years, 90; 4 years, 104; 5 years, 114; 6 years, 120; 7 years and over, 124.

Ottawa Turf Club, Canada.—2 years, *catch;* 3 years, 90 lbs.; 4 years, 104; 5 years, 112; 6 years, 118.

Albion Jockey Club, Nashville, Tenn.—2 years, 90 lbs.; 3 years, 95; 4 years, 104; 5 years, 110; 6 years, 115.

Feather.—By weight, 75 lbs.; by custom, a Jockey who is not weighed.

Welter.—40 lbs. added to weight for age, except by South Carolina and Louisville Jockey Clubs, where it is 28 lbs.

Distances.—*Maryland, Louisiana, Louisville, Savannah,* and *Central Georgia Jockey Clubs, Monmouth Park* and *Saratoga Associations.*—1 mile, 40 yards; 2 miles, 50; 3 miles, 60; 4 miles, 70.

Albion Jockey Club.—1 mile, 50 yards; 2 miles, 65; 3 miles, 80; 4 miles, 100; and 3 single mile heats in 5, 50.

Kentucky Association and *South Carolina Jockey Club.*—1 mile, 50 yards; 2 miles, 60; 3 miles, 80; 4 miles, 100.

Nashville Blood-horse Association.—1 mile, 50 yards; 2 miles, 65; 3 miles, 80; 4 miles, 100; and 3 single mile heats in 5, 50.

Ottawa Turf Club.—1 mile, 50 yards; 1½ miles, 70; 2 miles, 80; 3 miles, 100; 4 miles, 120; and 3 single mile heats in 5, 60.

Time between Heats.—*American, Savannah,* and *Central Georgia Jockey Clubs, Saratoga* and *Monmouth Park Associations.*—1 mile, 20 minutes; 2 miles, 25; 3 miles, 35; 4 miles, 40. *Nashville Blood-horse Association* and *South Carolina* and *Albion Jockey Clubs.*—1 mile, 20 minutes; 2 miles, 30; 3 miles, 40; 4 miles, 45; and 3 single mile heats in 5, 25. *Kentucky Association* and *Louisville Jockey Club.*—1 mile, 20 minutes; 2 miles, 30; 3 miles, 40; 4 miles, 45. *Maryland Jockey Club.*—1 mile, 20 minutes; 2 miles, 30; 3 miles, 35; 4 miles, 40. *Louisiana Jockey Club.*—1 mile, 20 minutes; 2 miles, 30; 3 miles, 35; and 3 single mile heats in 5, 25. *Ottawa Turf Club.*—1 mile, 20 minutes; 2 miles, 25; 3 miles, 30; 4 miles, 35.

Age.—Age of all horses dates from 1st of January.

GENERAL RULES.—In racing, a horse is not allowed to start with 5 lbs. overweight. If a rider returns 2 lbs. short of his weight, he loses the heat; and if 3 lbs., he is distanced. Fillies and Geldings are allowed 3 lbs. in all cases.

No *distance* in a *Dash*, and none in a match unless specified.

No two horses or riders from one stable in a heat race, and no two riders except by permission of the Judge.

Only one horse to start from one stable, except in a *Dash*.

No article is allowed to make weight from which a liquid can be wrung.

One pound is allowed for weight of curb or a double bridle.

Bridle is not included in weight to be carried.

In *England*, a Yearling's course is 2 furlongs; 2 years, 6 furlongs; 3 years, 1 mile;

4 years, 2 miles; 5 years and upward, 4 miles. A rider carrying more than 2 lbs. over his assigned weight is distanced. A Feather is 66 lbs., and no horse can start with a less weight. No *distance* in a third heat.

TROTTING COURSES.

Weights.—*National Association.*—Wagon or Sulky, 150 lbs., exclusive of harness; Saddle, 145 lbs., inclusive of saddle and whip.

If 20 lbs. over weight, it is to be announced from the Judges' stand.

Distances.—1 mile heats, 80 yards; 2 miles, 150; 3 miles, 220; 3 single mile heats in 5, 100.

Time between Heats.—1 mile, 20 minutes; 2 miles, 30; 3 miles, 35; 4 miles, 40; 3 single mile heats in 5, 25.

Whips.—Saddle, 2 feet 10 ins.; Sulky, 4 feet 8 ins.; Wagon, 5 feet 10 ins.; Double Team, 8 feet; Tandem and Four-in-hand, unlimited. Snappers, 3 ins. in addition to length of whip.

Lengths of English Race-courses.

Course.	Miles.	Course	Miles	Course	Miles
NEWMARKET.		Summer Course	2.	Derby and Oaks	1.5
		Two-year old, new	.702	T Y C, new	.75
Abingdon Mile	994	Yearling	.277	Metropolitan	2.25
Across the Flat	1.292	DONCASTER.		GOODWOOD.	
Ancaster Mile	1.01			Cup Course	2.5
Beacon	4.206	Circular	1.915	LIVERPOOL, new	1.5
last 3 miles	3.034	Fitzwilliam	1.	NEW CASTLE	1.796
Cambridgeshire	1.136	Red House	.711	OXFORD	2.
Cesarewitch	2.266	St. Leger	1.825	YORK.	
Ditch In	2.06	Cup Course	2.634	Stakes Course	1.75
Ditch Mile	1.136	T Y C	.996	Two-mile	1.923
Round	3.579	EPSOM.		T Y C	.644
Rowley Mile	1.009				
Suffolk Stakes	1.501	Craven	1.25		

Dimensions of Canal Locks.—(U. S.)

Canal	Length	Breadth.	Depth	L'gth Canal.
	Feet	Feet	Feet.	Miles
Albemarle and Chesapeake	220	40	6	14
Black River, Crooked Lake, Chenango, Chemung, and Genesee Valley	90	15	4	77, 8, 97, 33, 113.75
Chesapeake and Delaware*	220	24	9	14
Champlain	110	18	4	66.75
Cayuga and Seneca	110	18	7	24.75
Delaware and Raritan	220	24	7	43
Dismal Swamp	90	17.5	5.5	44
Erie	110	18	7	352
Falls of the Ohio, Ky.	350	80	2 to 60	—
Oneida	90	15	4	7
" Improvement	120	30.5	4.5	19
Oswego	110	18	4	38
Welland	150	26.5	10.5	28

The length of vessel that can be transported is somewhat less than the lengths of the locks.

* Height from under side of Summit Bridge to surface of water, 76 feet 10 inches.

VETERINARY.

Horses.—*Cathartic Ball.*—Cape Aloes, 6 to 10 drs.; Castile Soap, 1 dr.; Spirit of Wine, 1 dr.; Sirup to form a ball. If Calomel is required, add from 20 to 50 grains. During its operation, feed upon mashes and give plenty of water.

Cattle.—*Cathartic.*—Cape Aloes, 4 drs. to 1 oz.; Epsom Salts, 4 to 6 oz.; powdered Ginger, 3 drs. Mix, and give in a quart of gruel. For Calves, one third of this will be sufficient.

Dogs.—*Cathartic.*—Cape Aloes, ½ a dr. to 1 oz.; Calomel, 2 to 3 grs.; Oil of Caraway, 6 drops; Sirup to form a ball. Repeat every 5 hours till it operates.

Horses and Cattle.—*Tonic.*—Sulphate of Copper, 1 oz. to 12 drs.; Sugar, ½ an oz. Mix, and divide into 8 powders, and give one or two daily in food.

Cordial.—Powdered Opium, 1 dr.; powdered Ginger, 2 drs.; Allspice, powdered, 3 drs.; Caraway Seeds, powdered, 4 drs. Make into a ball with sirup, or give as a drench in gruel.

Cordial Astringent Drench for Diarrhœa, Purging, or Scouring.—Tincture of Opium, ½ an oz.; Allspice, 2½ drs.; powdered Caraway, ½ an oz.; Catechu Powder, 2 drs.; strong Ale or Gruel, 1 pint. Give every morning till the purging ceases. For Sheep this will make 4 doses.

Alterative.—Ethiop's Mineral, ½ an oz.; Cream of Tartar, 1 oz.; Nitre, 2 drs. Divide into from 16 to 24 dozes, one morning and evening in all cutaneous diseases.

Diuretic Ball.—Hard Soap and Turpentine, each 4 drs.; Oil of Juniper, 20 drops, powdered Resin to form a ball.

For Dropsy, Water Farcy, Broken Wind, or Febrile Diseases, add to the above Allspice and Ginger, each 2 drs. Divide into 4 balls, and give one morning and evening.

Alterative or Condition Powder.—Resin and Nitre, each 2 oz.; levigated Antimony, 1 oz. Mix for 8 or 10 doses, and give one morning and evening. When given to Cattle, add Glauber Salts, 1 lb.

Fever Ball.—Cape Aloes, 2 oz.; Nitre, 4 oz.; Sirup to form a mass. Divide into 12 balls, and give one morning and evening till the bowels are relaxed; then give an Alterative Powder or Worm Ball.

Tar or Hoof Ointment.—Tar and Tallow, each 1 lb.; Turpentine, ½ a lb. Melt.

Dogs.—*Emetic.*—2 to 4 grs. of Tartar Emetic in a meat ball, or a teaspoonful or two of common salt. Give twice a week if required.

Distemper Powder.—Antimonial Powder, 2, 3, or 4 grs.; Nitre, 5, 10, or 15 grs.; powdered Ipecacuanha, 2, 3, or 4 grs. Make into a ball, and give two or three times a day. If there is much cough, add from ½ a gr. to 1 gr. of Digitalis, and every 3 or 4 days give an Emetic.

Mange Ointment.—Powdered Aloes, 2 drs.; White Hellebore, 4 drs.; Sulphur, 4 oz.; Lard, 6 oz.

Red Mange, add 1 oz. of Mercurial Ointment, and apply a muzzle.

NOTE.—Physic, except in urgent cases, should be given in the morning, and upon an empty stomach; and, if required to be repeated, there should be an interval of several days between each dose.

To Ascertain a Horse's Age.—A foal of six months has six grinders in each jaw, three in each side, and also six nippers or front teeth, with a cavity in each.

At the age of one year, the cavities in the front teeth begin to decrease, and he has four grinders upon each side, one of the permanent and the remainder of the milk set.

At the age of two years, he loses the first milk grinders above and below, and the front teeth have their cavities filled up alike to the teeth of horses of eight years of age.

At the age of three years, or two and a half, he casts his two front uppers, and in a short time after, the two next.

At four, the grinders are six upon each side; and, about four and a half, his nippers are permanent by the replacing of the remaining two corner teeth; the tushes then appear, and he is no longer a colt.

At five, a horse has his tushes, and there is a black-colored cavity in the centre of all his lower nippers.

At six, this black cavity is obliterated in the two front lower nippers.

At seven, the cavities of the next two are filled up, and the tushes blunted; and at eight, that of the two corner teeth. The horse may now be said to be aged. The cavities in the nippers of the upper jaw are not obliterated till the horse is about ten years old, after which time the tushes become round, and the nippers project and change their surface.

GRAVITIES OF BODIES.

GRAVITY acts equally on all bodies at equal distances from the earth's centre; its force diminishes as the distance increases, and increases as the distance diminishes.

The gravitating forces of bodies are to each other,

1. Directly as their masses.
2. Inversely as the squares of their distances.

The gravity of a body, or its weight above the earth's surface, decreases as the square of its distance from the earth's centre in semi-diameters of the earth.

ILLUSTRATION.—If a body weighs 900 lbs. at the surface of the earth, what will it weigh 2000 miles above the surface?

The earth's semi-diameter is 3993 miles (say 4000).

Then $2000+4000=6000$, or 1.5 *semi-diameters*,

and $900 \div 1.5^2 = \frac{900}{2.25} = 400$ *lbs.*

Inversely, if a body weighs 400 lbs. at 2000 miles above the earth's surface, what will it weigh at the surface?

$400 \times 1.5^2 = 900$ *lbs.*

2. A body at the earth's surface weighs 360 lbs.; how high must it be elevated to weigh 40 lbs.?

$\frac{360}{40} = 9$ *semi-diameters*, if gravity acted directly; but as it is inversely, as the square of the distance.

Then $\sqrt{9} = 3$ *semi-diameters* $= 3 \times 4000 = 12000$ *miles.*

3. At what height must a body be raised to lose half its weight?

As $\sqrt{1} : \sqrt{2} :: 4000 : 5656$ = *as the square root of one semi-diameter is to the square root of two semi-diameters, so is one semi-diameter to the distance required.*

Hence $5656 - 4000 = 1656$ = *distance from the earth's surface.*

The diameters of two Globes being equal, and their densities different, the weight of a body on their surfaces will be as their densities.

Their densities being equal and their diameters different, the weight of them will be as their diameters.

The diameters and densities being different, the weight will be as their product.

ILLUSTRATION.—If a body weighs 10 lbs. at the surface of the earth, what will it weigh at the surface of the sun, their densities being 392 and 100, and their diameters 8000 and 883000 miles?

$\overline{883000 \times 100} \div \overline{8000 \times 392} = 28.157$ = *quotient of product of diameter of the sun and its density, and product of diameter of the earth and its density.*

Then $28.157 \times 10 = 281.57$ *lbs.*

NOTE.—The gravity of a body is .00346 less at the Equator than at the Poles.

SPECIFIC GRAVITIES.

The Specific Gravity of a body is the proportion it bears to the weight of another body of known density.

If a body float on a fluid, the part immersed is to the whole body as the specific gravity of the body is to the specific gravity of the fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to the difference between its own weight and the weight of its bulk of the fluid, less the resistance of the fluid to its passage.

Water is well adapted for the standard of gravity; and as a cubic foot of it weighs 1000 ounces avoirdupois, its weight is taken as the unit, *viz.*, 1000.

To Ascertain the Specific Gravity of a Body heavier than Water.

RULE.—Weigh it both in and out of water, and note the difference; then, as the weight lost in water is to the whole weight, so is 1000 to the specific gravity of the body. Or, $\frac{W \times 1000}{W - w} = G$, *w representing the weight in water, and* G *the specific gravity.*

EXAMPLE.—What is the specific gravity of a stone which weighs in air 15 lbs., in water 10 lbs.?

$15 - 10 = 5$; then $5 : 15 :: 1000 : 3000$ *spec. grav.*

To Ascertain the Specific Gravity of a Body lighter than Water.

RULE.—Annex to the lighter body another that is heavier than water, or the fluid used; weigh the piece added and the compound mass separately, both in and out of water, or the fluid; ascertain how much each loses in water, or the fluid, by subtracting its weight in water, or the fluid, from its weight in air, and subtract the less of these differences from the greater; then,

As the last remainder is to the weight of the light body in air, so is 1000 to the specific gravity of the body.

EXAMPLE.—What is the specific gravity of a piece of wood that weighs 20 lbs. in air; annexed to it is a piece of metal that weighs 24 lbs. in air and 21 lbs. in water, and the two pieces in water weigh 8 lbs.?

$20 + 24 - 8 = 44 - 8 = 36 =$ *loss of compound mass in water;*
$24 - 21 \qquad = \underline{3} =$ *loss of heavy body in water.*
$33 : 20 :: 1000 : 606 = 24$ *spec. grav.*

To Ascertain the Specific Gravity of a Fluid.

RULE.—Take a body of known specific gravity, weigh it in and out of the fluid; then, as the weight of the body is to the loss of weight, so is the specific gravity of the body to that of the fluid.

EXAMPLE.—What is the specific gravity of a fluid in which a piece of copper (*spec. grav.* = 9000) weighs 70 lbs. in, and 80 lbs. out of it?

$80 : 80 - 70 = 10 :: 9000 : 1125$ *spec. grav.*

To Compute the Proportions of two Ingredients in a Compound, or to discover Adulteration in Metals.

RULE.—Take the differences of each specific gravity of the ingredients and the specific gravity of the compound, then multiply the gravity of the one by the difference of the other; and, as the sum of the products is to the respective products, so is the specific gravity of the body to the proportions of the ingredients

EXAMPLE.—A compound of gold (*spec. grav.* = 18.888) and silver (*spec. grav.* = 10.535) has a specific gravity of 14; what is the proportion of each metal?

$$18.888 - 14 = 4.888 \times 10.535 = 51.495$$
$$14. - 10.535 = 3.465 \times 18.888 = 65.447$$
$$65.447 + 51.495 : 65.447 :: 14 : 7.835 \textit{ gold},$$
$$65.447 + 51.495 : 51.495 :: 14 : 6.165 \textit{ silver}.$$

To Compute the Weights of the Ingredients, that of the Compound being given.

RULE.—As the specific gravity of the compound is to the weight of the compound, so are each of the proportions to the weight of its material.

EXAMPLE.—The weight, as above, being 28 lbs., what are the weights of the ingredients?

$$14 : 28 :: \begin{cases} 7.835 : 15.67 \textit{ gold}, \\ 6.165 : 12.33 \textit{ silver}. \end{cases}$$

Proof of Spirituous Liquors.

A cubic inch of *proof spirits* weighs 234 grains; then, if an immersed cubic inch of any heavy body weighs 234 grains less in spirits than air, it shows that the spirit in which it was weighed is *proof.*

If it lose less of its weight, the spirit is above proof; and if it lose more, it is below proof.

ILLUSTRATION.—A cubic inch of glass weighing 700 grains weighs 500 grains when weighed in a certain spirit; what is the proof of it?

$$700 - 500 = 200 = \textit{grains} = \textit{weight lost in the spirit}.$$

Then 200 : 234 :: 1. : 1.17 = *ratio of proof of spirits compared to proof spirits,* or 1. = .17 *above proof.*

NOTE.—For Hydrometers and Rules for ascertaining the Proof of Spirits, see page 148; and for a very full treatise on Specific Gravities and on Floatation, see Jamieson's Mechanics of Fluids. Lond., 1837.

Solids.

RULE.—Divide the specific gravity of the substance by 16, and the quotient will give the weight of a cubic foot of it in pounds.

Substances.	Specific Gravity.	Weight of a Cub. Inch.
METALS.		
Aluminum	2560	.0926
Antimony	6712	.2428
Arsenic	5763	.2084
Barium	470	017
Bismuth	9823	.3553
Brass, copper 84, " tin 16	8832	.3194
" copper 67, " zinc 33	7820	.2828
" plate	8380	.3031
" wire	8214	.2972
Bronze, gun metal	8700	.3147
Boron	2000	.0723
Bromine	3000	.1085
Cadmium	8650	.3129
Calcium	1580	.057
Chromium	5000	.2134
Cinnabar	8098	.2929

Substances.	Specific Gravity.	Weight of a Cub. Inch.
METALS.		
Cobalt	8600	.3111
Columbium	6300	.217
Gold, pure, cast	19258	.6965
" hammered	19361	.7003
" 22 carats fine	17486	.6325
" 20 " "	15709	.5682
Copper, cast	8788	.3179
" plates	8698	.3146
" wire	8880	.3212
Iridium	18680	.6756
" hammered	23000	.8319
Iron, cast	7207	.2607
" " gun metal	7308	.264
" hot blast	7065	.2555
" cold "	7218	.2611
" wrought bars	7788	.2817
" " wire	7774	.2811
" rolled plates	7704	.2787

Table—(*Continued*).

Substances.	Specific Gravity	Weight of a Cub. Inch.
METALS.		
Lead, cast	11352	.4106
" rolled	11388	.4119
Lithium	590	.0213
Manganese	8000	.2894
Magnesium	1750	.0633
Mercury — 40°	15632	.5661
" + 32°	135 8	.4918
" 60°	13580	.4 12
" 212°	13370	.4835
Molybdenum	8600	.3111
Nickel	8800	.3183
" cast	8279	.2994
Osmium	10000	.3613
Palladium	11350	.4105
Platinum, hammered	20337	.7356
" native	16000	.5787
" rolled	22069	.7982
Potassium, 59°	865	.0313
Red-lead	8940	.324
Rhodium	10650	.3852
Ruthenium	8600	.3111
Selenium	4500	.1627
Silicium		
Silver, pure, cast	10474	.3788
" " hammered	10511	.3802
Sodium	970	.0351
Steel, plates	7806	.2823
" soft	7833	.2833
" tempered and hardened	7818	.2828
" wire	7847	.2838
Strontium	2540	.0918
Tin, Cornish, hammered	7390	.2673
" " pure	7291	.2637
Tellurium	6110	.221
Thalium	11850	.4286
Titanium	5300	.1917
Tungsten	17000	.6149
Uranium	18330	.6629
Wolfram	7119	.2575
Zinc, cast	6861	.2482
" rolled	7191	.26
WOODS (*Dry*).		Cubic Foot.
Alder	800	50.
Apple	793	49.562
Ash	845	52.812
	6 0	43.125
Bamboo	400	25.
Bay	822	51.375
Beech	852	53.25
"	690	43.125
Birch	567	35.437
Box, Brazilian	1031	64.437
" Dutch	912	57.
" French	1328	83.
Bullet-wood	928	58.
Butternut	376	23.5
Campeachy	913	57.062
Cedar	561	35.062
" Indian	1315	82.157

Substances.	Specific Gravity	Weight of a Cub Foot.
WOODS (*Dry*).		
Charcoal, pine	441	27.562
" fresh burned	380	23.75
" oak	1573	98.312
" soft wood	280	17.5
" triturated	1380	86.25
Cherry	715	44.687
Chestnut, sweet	610	38.125
Citron	726	45.375
Cocoa	1040	65.
Cork	240	15.
Cypress, Spanish	644	40.25
Dog-wood	756	47.25
Ebony, American	1331	83.187
" Indian	1209	75.562
Elder	695	43.437
Elm	570	35.625
	671	41.937
Filbert	600	37.5
Fir (Norway Spruce)	512	32.
Gum, blue	843	5 .687
" water	1000	62.5
Hackmatack	592	37.
Hazel	860	53.75
Hawthorn	9 0	56.875
Hemlock	368	23.
Hickory, pig-nut	792	49.5
" shell-bark	690	43.125
Holly	760	47.5
Jasmine	770	48.125
Juniper	566	35.375
Lance-wood	720	45.
Larch	544	34.
	560	35.
Lemon	703	43.937
Lignum-vitæ	1333	83.312
Lime	8 4	50.25
Linden	604	37.75
Locust	728	45.5
Logwood	913	57.062
Mahogany	720	45.
	1063	66.437
" Honduras	560	35.
" Spanish	852	53.25
Maple	750	46.875
" bird's-eye	576	36.
Mastic	849	53.062
Mulberry	561	35.062
	897	56.062
Oak, African	823	51.437
" Canadian	872	5 .5
" Dantzic	759	47.437
" English	932	58.25
" green	1146	71.625
" heart, 60 years	1170	73.125
" live, green	1260	78.75
" " seasoned	1068	66.75
" white	860	53.75
Orange	705	44.062
Pear	661	41.312
Persimmon	710	44.375
Plum	785	49.062

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Table—(*Continued*).

Substances.	Specific Gravity	Weight of a Cub Foot
WOODS (*Dry*).		
Pine, pitch	660	41.25
" red	590	36.875
" white	554	34.625
" yellow	461	28.812
Pomegranate	1354	84 625
Poon	580	36.25
Poplar	383	23.937
" white	529	33.062
Quince	705	44.062
Rose-wood	728	45.5
Sassafras	482	30.125
Satin-wood	885	55.312
Spruce	500	31.25
Sycamore	623	38.937
Tamarack	383	23.937
Teak (African oak)	657	41.062
	745	46.562
Walnut	671	41.937
" black	500	31.25
Willow	486	30.375
	585	36.562
Yew, Dutch	788	49 25
" Spanish	807	50.437
(*Well Seasoned.**)		
Ash	722	45.125
Beech	624	39.
Cherry	606	37.875
Cypress	441	27.562
Hickory, red	838	52 375
Mahogany, St. Domingo	720	45.
Pine, white	473	29.562
" yellow	541	33.812
Poplar	587	36 687
White Oak, upland	687	42.937
" James River	759	42.437
STONES, EARTHS, ETC.		
Agate	2590	—
Alabaster, white	2730	170.625
" yellow	2699	168.687
Alum	1714	107.125
Amber	1078	67.375
Ambergris	866	—
Asbestos, starry	3073	192.062
Asphaltum	905	56.562
	1650	103.125
Barytes, sulphate	4000	250.
	4865	304.062
Basalts	2740	171.25
	2864	179.
Borax	1714	107.125
Brick	1900	118 75
	1367	85.437
" fire	2201	137.562
" work in cement	1800	112.50
" " " mortar	1600	100.
	2000	125.
Carbon	3500	218.75

Substances.	Specific Gravity	Weight of a Cub Foot
STONES, EARTHS, ETC.		
Cement, Portland	1300	81.25
" Roman	1560	97.25
Chalk	1520	95.
	2784	174.
Chrysolite	2782	—
Clay	1930	120.625
" with gravel	2480	155.
Coal, Anthracite	1436	89.75
	1640	102.5
" Borneo	1290	80.625
" Cannel	1238	77.375
	1318	82.375
" Caking	1277	79.812
" Cherry	1276	79.75
" Chili	1290	80.625
" Derbyshire	1292	80.75
" Lancaster	1273	79.562
" Maryland	1355	84.687
" Newcastle	1270	79.375
" Rive de Gier	1300	81.25
" Scotch	1259	78.687
	1300	81.25
" Splint	1302	81.375
" Wales, mean	1315	82.187
Coke	1000	62.5
" Nat'l, Va	746	46.64
Concrete, mean	2000	125.
Copal	1045	65.312
Coral, red	2700	—
" white	2550	—
Cornelian	2613	—
Diamond, Oriental	3521	—
" Brazilian	3444	—
Earth,† common soil	2194	137.125
" loose	1500	93.75
" moist sand	2050	128.125
" mould, fresh	2050	128.125
" rammed	1600	100.
" rough sand	1920	120.
" with gravel	2020	126.25
Emery	4000	250.
Flint, black	2582	161.375
" white	2594	162.125
Fluorine	1320	82.5
Glass, bottle	2732	170.75
" Crown	2487	155.437
" flint	2933	183.312
	3200	196.
" green	2642	165.125
" optical	3450	215.625
" white	2892	180.75
" window	2642	165.125
Garnet	4180	—
" black	3750	—
Granite, Egyptian red	2654	165.875
" Patapsco	2640	165.
" Quincy	2652	165.75
" Scotch	2625	164.062
" Susquehanna	2704	169.

* Ordnance Manual, 1841.
† Spec. grav. of the earth is variously estimated at from 5450 to 5600.

Table—*(Continued)*.

Substances.	Specific Gravity.	Weight of a Cub. Foot.
STONES, EARTHS, ETC.		
Gravel, common	1749	109.312
Grindstone	2143	133.937
Gypsum, opaque	2168	135.5
Hone, white, razor	2876	179.75
Hornblende	3540	221.25
Iodine	4940	—
Jet	1300	—
Lime, hydraulic	2745	171.562
" quick	804	50.25
Limestone, green	3180	198.75
" white	3156	197.25
Magnesia, carbonate	2400	150.
Marble, Adelaide	2715	169.687
" African	2708	169.25
" Biscayan, black	2695	168.437
" Carara	2716	169.75
" common	2686	167.875
" Egyptian	2668	166.75
" French	2649	165.562
" Italian, white	2708	169.25
" Parian	2838	177.375
" Vermont, white	2650	165.57
Marl, mean	1750	109.375
Mica	2800	175.
Mortar	1384	86.5
	1750	109.375
Millstone	2484	155.25
Mud	1630	101.875
Nitre	1900	118.75
Opal	2114	—
Oyster-shell	2092	130.75
Paving-stone	2416	151.
Pearl, Oriental	2650	—
Peat	600	37.5
	1329	83.062
Phosphorus	1770	110.625
Plaster of Paris	1176	73.5
Plumbago	2100	131.25
Porphyry, red	2765	172.812
Porcelain, China	2300	143.75
Pumice-stone	915	57.187
Quartz	2660	166.25
Rotten-stone	1981	123.812
Red-lead	8940	558.75
Resin	1089	68.062
Rock, crystal	2735	170.937
Ruby	4283	—
Salt, common	2130	133,125
Saltpetre	2090	130.625
Sand, coarse	1800	112.5
" common	1670	104.375
" damp and loose	1392	87.
" dried and loose	1560	97.5
" dry	1420	88.75
" mortar, Ft. Richm'd	1650	103.66
" " Brooklyn	1716	107.25
" silicious	1701	106.33
Sapphire	3994	—
Shale	2600	162.5
Slate	2900	181.25
	2672	167.
Slate, purple	2784	174.
Smalt	2440	152.5
Stone, Bath .... Engl.	1961	122.562
" Blue Hill	2640	165.
" Bluestone (basalt)	2625	164.062
" Breakneck .. N.Y.	2704	169.
" Bristol .... Engl.	2510	156.875
" Caen, Normandy.	2076	129.75
" Common	2520	157.5
" Craigleth ... Engl.	2316	144.75
" Kentish rag "	2651	165.687
" Kip's Bay .. N.Y.	2759	172.
" Norfolk (Parliament House)	2304	144.
" Portland ... Engl.	2368	148.
" Sandstone, mean.	2200	137.5
" " Sydney	2237	139.812
" Staten Isl'd, N.Y.	2976	186.
" Sullivan Co. "	2688	168.
Schorl	3170	198.125
Spar, calcareous	2735	170.937
" Feld, blue	2693	168.312
" " green	2704	169.
" Fluor	3400	212.5
Stalactite	2415	150.937
Sulphur, native	2033	127.062
Talc, mean	2500	156.25
Talc, black	2900	181.25
Tile	1815	113.437
Topaz, Oriental	4011	—
Trap	2720	170.
Turquoise	2750	—
MISCELLANEOUS.		
Asphaltum	905	56.562
	1650	103.125
Atmospheric Air	.001205	.07529
Beeswax	965	60.312
Butter	942	58.875
Camphor	988	61.75
Caoutchouc	903	56.437
Egg	1090	—
Fat of Beef	923	57.687
" Hogs	936	58.5
" Mutton	923	57.687
Gamboge	1222	—
Gum Arabic	1452	90.75
Gunpowder, loose	900	56.25
" shaken	1000	62.5
" solid	1550	96.875
	1800	112.5
Gutta-percha	980	61.25
Horn	1689	105.562
Ice, at 32°	920	57.5
Indigo	1009	63.062
Isinglass	1111	69.437
Ivory	1825	114.062
Lard	947	59.187
Mastic	1074	67.125
Myrrh	1360	85.
Opium	1336	83.5

Table—(*Continued*).

Substances.	Specific Gravity.	Weight of a Cub. Foot.	Substances.	Specific Gravity.	Weight of a Cub. Foot.
MISCELLANEOUS.			LIQUIDS.		
Soap, Castile	1071	56.937	Beer	1034	64.625
Spermaceti	943	58.937	Bitumen, liquid	848	53.
Starch	950	59-375	Blood (human)	1054	65.875
Sugar	1606	100.375	Brandy, ⅚ or 5 of spirit.	924	57.75
" .66 {	1326	82.875	Cider	10[illegible]8	63.625
	972	60.25	Ether, acetic	866	54.125
Tallow	941	58.812	" muriatic	845	52.812
Wax {	964	60.25	" sulphuric	715	44.687
	970	60.625	Honey	1450	90.625
LIQUIDS.			Milk	1032	64.5
			Oil, Anise-seed	985	61.625
Acid, Acetic	1062	66.375	" Codfish	923	57.687
" Benzoic	667	41.687	" Cotton-seed	–	–
" Citric	1034	64.625	" Linseed	940	58.75
" Concentrated	1521	95.062	" Naphtha	848	53.
" Fluoric	1500	93.75	" Olive	915	57 187
" Muriatic	1[illegible]00	75.	" Palm	969	60.562
" Nitric	1217	76.062	" Petroleum	878	54.875
" Phosphoric	1558	97.375	" Rape	914	57.125
" " solid	2800	175.	" Sunflower	926	57.875
" Sulphuric	1849	115.562	" Turpentine	870	54.375
Alcohol, pure, 60°	794	49.622	" Whale	9[illegible]3	57.687
" 95 per cent.	816	51.	Spirit, rectified	824	51.5
" 80 "	8[illegible]3	53.937	Tar	1015	63.437
" 50 "	934	58.375	Vinegar	1080	67.5
" 40 "	951	59.437	Water, Dead Sea	1240	77.5
" 25 "	970	60 625	" 60°	9[illegible]9	62.449
" 10 "	986	61.625	" 212°	957	59.812
" 5 "	992	62.	" distilled, 39°†	998	62.379
" proof spirit,* 50 per cent. . 60°	934	58.375	" Mediterranean	1029	64.312
			" rain	1000	62.5
" proof spirit, 50 per cent. . 80°	875	54.687	" sea	1026	64.125
			Wine, Burgundy	992	6[illegible].
Ammonia, 27.9 per cent.	891	55.687	" Champagne	997	64.375
Aquafortis, double	1300	81.25	" Madeira	1038	62.[illegible]12
" single	1200	75.	" Port	997	62.312

Compression of the following fluids under a pressure of 15 lbs. per square inch:

Alcohol	.0000216	Mercury	.00000265
Ether	.00006158	Water	.00004663

Elastic Fluids.

1‡ *Cubic Foot of Atmospheric Air weighs* 527.04 *Troy Grains. Its assumed Gravity of* 1 *is the Unit for Elastic Fluids.*

Atmospheric air, 34°	1.	Gas, coal {	.4
Ammonia	.589		.752
Azote	.976	Hydrogen	.07
Carbonic acid	1.52	Hydrochloric acid	1.278
" oxyd	.972	Hydrocyanic "	.942
Carbureted hydrogen	.559	Muriatic acid	1.247
Chlorine	2.47	Nitrogen	.972
Chloro-carbonic	3.389	Nitric oxyd	1.094
Cyanogen	1.815	Nitrous acid	2.638

* Specific gravity of proof spirit according to Ure's Table for Sykes's Hydrometer, 920
† 1 cubic inch = .252.69 Troy grains.
‡ Equal to .07529143 lbs. avoirdupois.

Table—(*Continued*).

Nitrous oxyd	1. 27	Vapor of bromine	5.1
Oxygen	1.102	" chloric ether	3.44
Phosphureted hydrogen	1.77	" ether	2.586
Sulphureted "	1.17	" hydrochloric ether	2.2 5
Sulphurous acid	2.21	" iodine	8 675
Steam,* 21 .°	.4883	" nitric acid	3.75
Smoke, of bituminous coal	.102	" spirits of turpentine	4 763
" coke	.105	" sulphuric acid	2 7
" wood	.09	" " ether	2.586
Vapor of alcohol	1 613	" sulphur	2 214
" bisulphuret of carbon	2.64	" water	.623

Weights and Volumes of various Substances in Ordinary Use.

Substances.	Cubic Foot.	Cub. Inch
METALS.	Lbs.	Lbs.
Brass .. {copper 67. zinc 33.}	488.75	.2829
" gun metal	543.75	.3147
" sheets	513.6	.297
" wire	524.16	.3 33
Copper, cast	547.25	.3179
" plates	54 6 5	.3167
Iron, cast	450 437	.2607
" gun metal	466.5	.27
" heavy forging	4 9 5	.2775
" plates	481.5	.2787
" wrought bars	486.75	.2816
Lead, cast	709.5	.41 6
" rolled	711 75	.4119
Mercury, 60°	848.7487	.491174
Steel, plates	487.75	.2823
" soft	489.562	.2833
Tin	455.687	.2637
Zinc, cast	4 8.812	.2482
" rolled	449.437	.2601
WOODS.		Cub. Feet in a Ton.
Ash	52.812	42.414
Bay	51.575	43.601
Cork	15.	149.333
Cedar	35.062	6 .886
Chestnut	38.125	58.754
Hickory, pig-nut	49 5	45.252
" shell-bark	43.125	51. 4
Lignum-vitæ	83 312	26.886
Logwood	57.062	39.255
Mahogany, Hondur's {	35. 65 437	64. 33.714
Oak, Canadian	54.5	41.101
" English	58 25	38.455
" live, seasoned	66 75	33.558
" white, dry	53 75	41.674
" " upland	42.937	52 169
Pine, pitch	41.25	54.303
" red	36.875	60 745
" white	34.625	64 693
" well seasoned	29.562	75.773

Substances.	Cubic Foot	Cubic Feet in a Ton
WOODS.	Lbs.	
Pine, yellow	33.812	66.248
Spruce	31 25	71.68
Walnut, black, dry	31. 5	71.68
Willow	36 562	61. 65
" dry	30.375	73.744
MISCELLANEOUS.		
Air	.075291	–
Basalt, mean	175.	12.8
Brick, fire	137.562	16.284
" mean	102.	21.961
Coal, anthracite .. {	89.75 102.5	24. 58 21 854
" bitumin., mean	80.	28.
" Cannel	94.875	23.609
" Cumberland	84.687	26.451
" Welsh, mean	81.25	27.569
Coke	62 5	35.84
Cotton, bale, mean	14.5	154 48
" " pressed {	20. 25.	114. 89.6
Earth, clay	120.6 5	18.569
" common soil	137.125	16. 35
" " gravel	109.312	20.49
" dry, sand	120.	18 667
" loose	93 75	23.893
" moist, sand	128.1 5	17 482
" mold	128. 25	17.482
" mud	101.875	21.987
" with gravel	126. 5	17.742
Granite, Quincy	165.75	13.514
" Susqueh'na.	169.	13.254
Hay, bale	9.525	23 517
" pressed	25.	89 6
India rubber	56.437	39.69
" vulcanized	–	–
Limestone	197.25	11.355
Marble, mean	167.875	13.343
Mortar, dry, mean	97.98	22.862
Water, fresh	62.5	35.84
" salt	64 125	34.931
Steam	.036747	–

* Weight of a cubic foot, 257.353 Troy grains.

Application of the Tables.

When the Weight of a Substance is required. RULE.—Ascertain the volume of the substance in cubic feet; multiply it by the unit in the second column of tables, and divide the product by 16; the quotient will give the weight in pounds.

When the Volume is given or ascertained in Inches. RULE.—Multiply it by the unit in the third column of the tables, and the product will be the weight in pounds.

EXAMPLE.—What is the weight of a cube of Italian marble, the sides being 3 feet?

$3^3 \times 2708 = 73116$ *oz.*, which $\div 16 = 4569.75$ *lbs.*

Or of a sphere of cast iron 2 inches in diameter?

$2^3 \times .5236 \times .26$ *weight of a cubic inch* $= 1.089$ *lbs.*

Comparative Weight of Timber in a Green and Seasoned State.

Timber.	Weight of a Cub. Foot		Timber.	Weight of a Cub. Ft.	
	Green	Seasoned		Green.	Seasoned.
	Lbs Oz	Lbs. Oz.		Lbs Oz.	Lbs. Oz.
American Pine.......	44.12	30.11	Cedar..............	32.	28. 4
Ash	58. 3	50.	English Oak........	71.10	43. 8
Beech...............	60.	53. 6	Riga Fir...........	48.12	35. 8

To Compute the Capacity of a Balloon.

RULE.—From the specific gravity of the air in grains per cubic foot subtract that of the gas with which it is inflated; multiply the remainder by the volume of the balloon in cubic feet; divide the product by 7000, and from the quotient subtract the weight of the balloon and its attachments.

EXAMPLE.—The diameter of a balloon is 26.6 feet, its weight is 100 lbs., and the specific gravity of the gas with which it is inflated is .06 (air being assumed at 1); what is its capacity?

$$\frac{527.04 - 31.62 \times \overline{26.6^3 \times .5236}}{7000} - 100 = \frac{495.42 \times 9854.726}{7000} - 100 = 597.461 \textit{ lbs.}$$

To Compute the Diameter of a Balloon, the Weight to be raised being given.

By inversion of the preceding rule,

$\sqrt[3]{\frac{W \times 700 \div \overline{s - s'}}{.5236}} = d$, s and s' *representing the weight of air and gas in grains per cubic foot, and d the diameter of the balloon in feet.*

EXAMPLE.—Given the elements in the preceding case.

Then $\frac{\sqrt[3]{597.46 + 100 \times 7000 \div \overline{527.04 - 31.62}}}{.5236} = \sqrt[3]{18821.09} = 26.6$ *feet.*

To Compute the Weight of Cast Metal by the Weight of the Pattern.

When the Pattern is of White Pine.

RULE.—Multiply the weight of the pattern in pounds by the following multiplier, and the product will give the weight of the casting: Iron, 14; Brass, 15; Lead, 22; Tin, 14; Zinc, 13.5.

When there are Circular Cores or Prints.—Multiply the square of the diameter of the core or print by its length in inches, the product by .0175, and the result is the weight of the pattern of the core or print to be deducted from the weight of the pattern.

It is customary, in the making of patterns for castings, to allow for shrinkage per lineal foot of pattern:

Iron and Lead $\frac{1}{8}$th of an inch, Brass and Zinc $\frac{3}{16}$ths, and Tin $\frac{1}{12}$th.

GEOMETRY.

Definitions.—A *Point* has position, but not magnitude.

A *Line* is length without breadth, and is either *Right*, *Curved*, or *Mixed*.

A *Right Line* is the shortest distance between two points.

A *Curved Line* is one that continually changes its direction.

A *Mixed Line* is composed of a right and a curved line.

A *Superficies* has length and breadth only, and is plane or curved.

A *Solid* has length, breadth, and thickness.

An *Angle* is the opening of two lines having different directions, and is either *Right*, *Acute*, or *Obtuse*.

A *Right Angle* is made by a line perpendicular to another falling upon it.

An *Acute Angle* is less than a right angle.

An *Obtuse Angle* is greater than a right angle.

A *Triangle* is a figure of three sides.

An *Equilateral Triangle* has all its sides equal.

An *Isosceles Triangle* has two of its sides equal.

A *Scalene Triangle* has all its sides unequal.

A *Right-angled Triangle* has one right angle.

An *Obtuse-angled Triangle* has one obtuse angle.

An *Acute-angled Triangle* has all its angles acute.

A *Quadrangle* or *Quadrilateral* is a figure of four sides, and has the following particular designations, viz.:

A *Parallelogram*, having its opposite sides parallel.

A *Square*, having length and breadth equal.

A *Rectangle*, a parallelogram having a right angle.

A *Rhombus* or *Lozenge*, having equal sides, but its angles not right angles.

A *Rhomboid*, a parallelogram, its angles not being right angles.

A *Trapezium*, having unequal sides.

A *Trapezoid*, having only one pair of opposite sides parallel.

Note.—*A Triangle is sometimes called a Trigon, and a Square a Tetragon.*

Polygons are plane figures having more than four sides, and are either *Regular* or *Irregular*, according as their sides and angles are equal or unequal, and they are named from the number of their sides or angles. Thus:

A Pentagon	has	five sides.	A Nonagon	has	nine sides.
A Hexagon	"	six "	A Decagon	"	ten "
A Heptagon	"	seven "	An Undecagon	"	eleven "
An Octagon	"	eight "	A Dodecagon	"	twelve "

A Circle is a plane figure bounded by a curved line, called the *Circumference* or *Periphery*.

A *Diameter* is a right line passing through the centre of a circle or sphere, and terminated at each end by the periphery or surface.

An *Arc* is any part of the circumference of a circle.

A *Chord* is a right line joining the extremities of an arc.

A *Segment* of a circle is any part bounded by an arc and its chord.

The *Radius* of a circle is a line drawn from the centre to the circumference.

A *Sector* is any part of a circle bounded by an arc and its two radii.

A *Semicircle* is half a circle.

A *Quadrant* is a quarter of a circle.

A *Zone* is a part of a circle included between two parallel chords.

A *Lune* is the space between the intersecting arcs of two eccentric circles.

A *Gnomon* is the space included between the lines forming two similar parallelograms, of which the smaller is inscribed within the larger, so as to have one angle in each common to both.

A *Secant* is the line running from the centre of the circle to the extremity of the tangent of the arc.

A *Cosecant* is the secant of the complement of an arc, or the line running from the centre of the circle to the extremity of the cotangent of the arc.

A *Sine* of an arc is a line running from one extremity of an arc perpendicular to a diameter passing through the other extremity, and the sine of an angle is the sine of the arc that measures that angle.

The *Versed Sine* of an arc or angle is the part of the diameter intercepted between the sine and the arc.

The *Cosine* of an arc or angle is the part of the diameter intercepted between the sine and the centre.

The *Coversed Sine* of an arc or angle is the part of the secondary radius intercepted between the cosine and the circumference

A *Tangent* is a right line that touches a circle without cutting it.

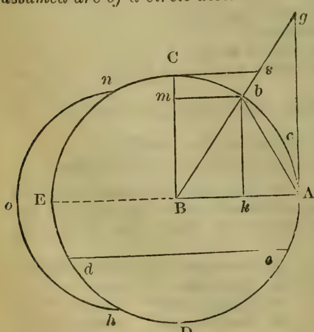
A *Cotangent* is the tangent of the complement of the arc.

The *Circumference* of every circle is supposed to be divided into 360 equal parts termed *Degrees;* each degree into 60 *Minutes*, and each minute into 60 *Seconds*, and so on.

The *Complement* of an angle is what remains after subtracting the angle from 90 degrees.

The *Supplement* of an angle is what remains after subtracting the angle from 180 degrees.

To exemplify these definitions, let A c b, *in the following diagram, be an assumed arc of a circle described with the radius* B A:



A *c b*, an Arc of the circle A C E D.
A *b*, the Chord of that arc.
B A, an Initial radius.
B C, a Secondary radius.
e D *d*, a Segment of the circle.
A B *b*, a Sector.
A D E, a Semicircle.
C B E, a Quadrant.
A *e d* E, a Zone.
n o h, a Lune.
B *g*, the Secant of the arc A *c b;* written Sec.
b k, the Sine of the arc A *c b;* written Sin.
A *k*, the Versed Sine of the arc A*cb;* written Versin.
B *k* or *m b*, the Cosine of the arc A *c b*.
A *g*, the Tangent of " "
C B *b*, the Complement, and *b* B E, the Supplement of the arc A *c b*.
C *s*, the *Cotangent* of the arc; written Cot.
B *s*, the *Cosecant* of the arc; written Cosec.
m C, the *Coversed sine* of the arc, or, by convention, of the angle A B *b;* written Coversin.

The *Vertex* of a figure is its top or upper point. In Conic Sections it is the point through which the generating line of the conical surface always passes.

The *Altitude*, or height of a figure, is a perpendicular let fall from its vertex to the opposite side, called the base.

The *Measure* of an angle is an arc of a circle contained between the two lines that form the angle, and is estimated by the number of degrees in the arc.

A *Segment* is a part cut off by a plane, parallel to the base.
A *Frustrum* is the part remaining after the segment is cut off.
The *Perimeter* of a figure is the sum of all its sides.
A *Problem* is something proposed to be done.
A *Postulate* is something required.
A *Theorem* is something proposed to be demonstrated.
A *Lemma* is something premised, to render what follows more easy.
A *Corollary* is a truth consequent upon a preceding demonstration.
A *Scholium* is a remark upon something going before it.

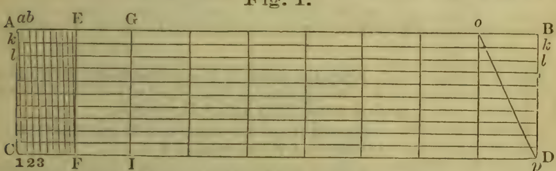
Lengths of the following Elements, Radius = 1.

	Angle 45°.	Angle 60°.		Angle 45°.	Angle 60°.
Sine	.707107	.866025	Cosecant	1.414214	1.1547
Cosine	.707107	.5	Tangent	1.	1.73205
Versed Sine	.292893	.5	Cotangent	1.	.577349
Coversed Sine	.292893	.133975	Chord	.765366	1.
Secant	1.414214	2.	Arc	.785398	1.0472

SCALES.

To Construct a Diagonal Scale upon any Line, as A B—Fig. 1.

(1.)



Divide the line into as many divisions as there are hundreds of feet, spaces of ten feet, feet, or inches required.

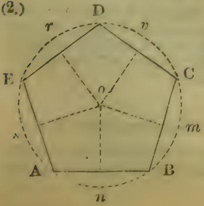
Draw perpendiculars from every division to a parallel line, C D. Divide them and one of the divisions, A E, C F, into spaces of ten if for feet and hundredths, and twelve if for inches; draw the lines A 1, *a* 2, *b* 3, etc., and they will complete the scale.

Thus: The line A B representing ten feet; A to E, E to G, etc., will measure one foot; A to *a*, C to 1, 1 to 2, etc., will measure 1-10th of a foot. The several lines A 1, *a* 2, etc., will measure upon the lines *k k*, *l l*, etc., 1-100th of a foot; and *o p* will measure upon *k k*, *l l*, etc., divisions of 1-10th of a foot.

POLYGONS.

To Circumscribe a Pentagon about a Given Circle—Fig. 2.

(2.)



Divide the circumference of a circle into five points, defining them *s r v m n*.

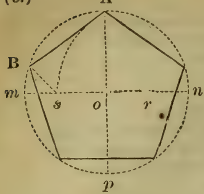
From the centre *o*, draw *o r*, *o v*, etc.

Through *n m*, etc., draw A B, B C, etc., perpendicular to *o n*, *o m*, etc., and complete the figure.

Note.—Any other polygon may be made in a similar manner by drawing tangents to the points, first defining them in the circle.

To Inscribe a Pentagon in a Given Circle—Fig. 3.

(3.)

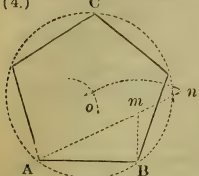


Draw the diameters, A p and m n, at right angles to each other; bisect o n in r, and with r A describe A s; from A with A s describe s B.

Join A B, and the distance is equal to one side of the pentagon required.

To Describe a Pentagon upon a Given Line—Fig. 4.

(4.)

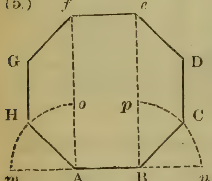


Draw B m perpendicular to A B, and equal to one half of it; extend A m until m n is equal to B m.

From A and B, with the radius B n, describe arcs cutting each other in o; then from o, with the radius o B, describe the circle A C B, and the line A B is equal to one side of the pentagon upon the circle described.

To Describe an Octagon upon a Given Line A B—Fig. 5.

(5.)



From the points A B erect indefinite perpendiculars A f, B e; produce A B to m and n, and bisect the angles m A o and n B p with A H and B C.

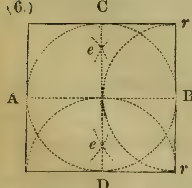
Make A H and B C equal to A B, and draw H G, C D, parallel to A f, and equal to A B.

From G and D, as centres with a radius equal to A B, describe arcs cutting A f, B e, in f and e. Join G f, f e, and e D, and the figure is complete.

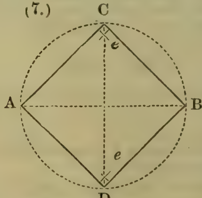
SQUARES.

To Describe a Square about a Given Circle—Fig. 6.

(6.)



(7.)



Draw the line A B through the centre of the circle.

Take any radius, as A e; describe the arcs A e e, B e e; join e e, continuing the line, to C D.

Describe B r and D r; draw and extend B r and D r, and the sides A and C, drawn parallel to them, will describe the square.

To Inscribe a Square in a Given Circle—Fig. 7.

Draw the line A B through the centre of the circle; take any radius, as A e, and describe the arcs A e e, B e e; join e e, continuing the line to C and D; join A C, A D, etc., and the square is inscribed.

To Describe an Octagon in a Given Square--Fig. 8.

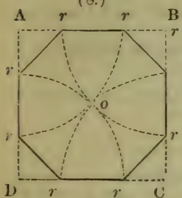
Let A B C D be the given square.

Describe A *o*, B *o*, etc.; join the intersections *r r r r*, etc., and the figure formed is the octagon required.

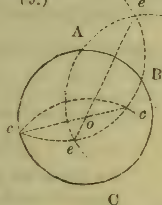
CIRCLES.

To Describe a Circle that shall pass through any three Given Points, as A B C--Fig. 9.

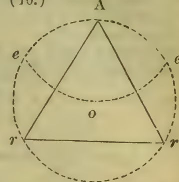
(8.)



(9.)



(10.)



Upon the points A and B, with any opening of a dividers, describe two arcs to intersect each other, as at *e e*.

On the points A C describe two more to intersect each other in the points *c c*.

Draw the lines *e e* and *c c*, and where these two lines intersect *o*, place one leg of the dividers, and extend the other to A, B, or C, and it will describe a circle through the three given points.

To Inscribe an Equilateral Triangle in a Given Circle--Fig. 10.

From any point A, with A *o* equal to the radius of the circle, describe *e o e;* from *e* and *e* describe *e r*, *e r;* join A *r*, *r r*, and *r* A, and the triangle is formed.

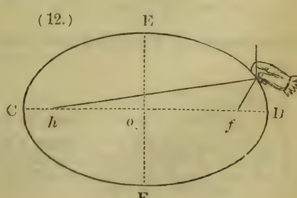
NOTE.—All figures of 10 or 20 sides are readily determined from the side of a pentagon, being halved or quartered; and in like manner, all figures of 6, 12, or 24 sides are readily determined from the radius of a circle, being equal to the side of a hexagon.

ELLIPSES.

To Describe an Ellipse to any Length and Breadth given--Fig. 11.

(11.)

(12.)



Let the longest diameter given be equal to *a b*, and the shortest to *c o*.

Bisect *a b* equally at right angles in *s*.

Make *a r* equal to *c o;* divide *r b* into three equal parts, set off two of those parts from *s* to *r* and from *s* to *u;* then, with the distance *u r*, make the two equilateral triangles *u c r* and *u o r;* these angles are the centres, and the sides being continued are the lines of direction for the several arcs of the ellipse *a c b d*.

NOTE.—When it is required to work an Architrave, etc., of this form: by the use of the four centres *u*, *c*, *r*, *o*, and the lines of direction *h c*, *o f*, *o i*, and *g c*, describe another line around the former, and at any distance required, as *h i f g*.

To Describe an Ellipse to any Length and Breadth required, another Way--Fig. 12.

Let the longest diameter be C D, and the shortest E F.

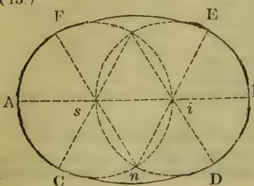
Take the distance C *o* or *o* D, and with it, from the points E and F, describe the arcs *h* and *f* upon the diameter C D.

Insert pins at *h* and at *f*, and put a string around them of such a length that it will just reach to E or F.

Introduce a pencil, and bearing upon the string, carry it around the centre *o*, and it will describe the ellipse required.*

To Describe an Ellipse of any Given Length, without regard to Breadth--Fig. 13.

(13.)



Let A B be the given length.

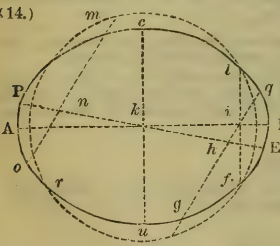
Divide it into three equal parts, as A *s i* B.

With the radius A *s*, describe the circle A F *o i n* C; and from *i*, B D *n s o* E.

With *n* F and *o* C draw F E and C D, and the ellipse is described.

To Ascertain the Centre and two Diameters of an Ellipse --Fig. 14.

(14.)



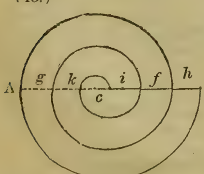
Let A B *c u* be the ellipse.

Draw at pleasure two lines, *q g*, *m o*, parallel to each other; bisect them in the points *h n*, and draw the line P E; bisect it in *k*, and upon *k*, as a centre, describe a circle at pleasure, as *f l r*, cutting the figure in the points *f l*.

Draw the right line *f l;* bisect it in *i*, and through the points *i k* draw the greatest diameter A B, and through the centre, *k*, draw the least diameter *c u*, parallel to the line *f l*.

To Draw a Spiral Line about a Given Point--Fig. 15.

(15.)



Let *c* be the centre.

Draw A *h*, and divide it into twice the number of parts that there are to be revolutions of the line. Upon *c* describe *k i*, *g f*, A *h*, and upon *i* describe *k f*, *g f*, etc.

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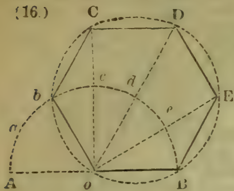
* It is a property of the ellipse that the sum of two lines drawn from the *foci* to meet in any point in the curve is equal to the transverse diameter, and from this the correctness of the above construction is evident.

POLYGONS.

To Describe a Regular Polygon of any required Number of Sides--Fig. 16.

(16.)



From the point *o*, with the distance *o* B, describe the semicircle B *b* A, which divide into as many equal parts, A *a*, *a b*, *b c*, etc., as the polygon is to have sides.

Let a hexagon be required.

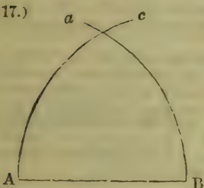
From *o* to the second point of six divisions draw *o b;* and through the other points, *c*, *d*, and *e*, draw *o* C, *o* D, etc.

Apply the distance *o* B, from B to E, from E to D, from D to C, etc. Join these points, as *b* C, C D, etc., and the figure is described.

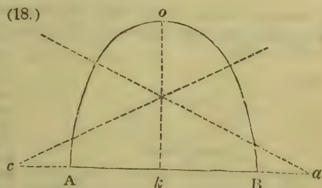
ARCS.

To Describe a Gothic Arc--Fig. 17.

(17.)



(18.)



Take the line A B.

At the points A and B draw the arcs B *a* and A *c*, and it will describe the arc required.

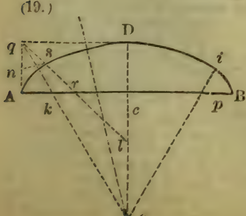
To Describe an Elliptic Arc on the Conjugate Diameter--Fig. 18.

Draw the diameter A B, and in the middle, at *k*, erect the perpendicular *k o*, equal to the height of the arc.

Divide the perpendicular *k o* into two equal parts at *e;* continue the line A B on both sides at pleasure, and from the point *k*, with the distance *k o*, cut A B in *c* and *d;* through *c e*, *d e*, draw *c e f* and *d e g* at pleasure; *d* and *c* are centres for the arcs A *g* and B *f*, and *e* the centre for the arc *g o f*, which will form the arc required.

To Describe an Elliptic Arc, the Chord and Height being given--Fig. 19.

(19.)



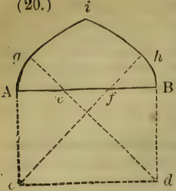
Bisect A B at *c;* erect the perpendicular A *q*, and draw the line *q* D equal and parallel to A *c*.

Bisect A *c* and A *q* in *r* and *n;* make *c l* equal to *c* D, and draw the line *l r q;* draw also the line *n s* D; bisect *s* D with a line at right angles, and cutting the line *c* D in *o;* draw the line *o q;* make *c p* equal to *c k*, and draw the line *o p i*.

Then from *o* as a centre, with the radius *o* D, describe the arc *s* D *i;* and from *k* and *p* as centres, with the radius A *k*, describe the arcs A *s* and B *i*, which will complete the arc required.

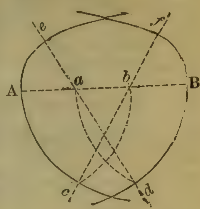
To Describe a Gothic Arc—Figs. 20 and 21.

(20.)



Divide the line A B (*Fig.* 20) into three equal parts, *e f*; from the points A and B let fall the perpendiculars A *c* and B *d*, equal in length to two of the divisions of the line A B; draw the lines *c h* and *d g*; from the points *e f*, with the length of *f* B, describe the arcs A *g* and B *h*, and from the points *c* and *d* describe the arcs *g i* and *i h*, and it will complete the arc required.

(21.)

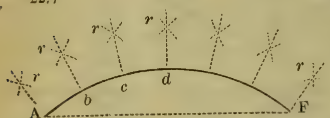


Or, divide the line A B (*Fig.* 21) into three equal parts at *a* and *b*, and on the points A, *a*, *b*, and B, with the distance of two divisions, make four arcs intersecting at *c d*.

Through the points *c*, *d*, and the divisions *a*, *b*, draw the lines *c f* and *d e*, and on the points *a* and *b* describe the arcs A *e* and B *f*, and on the points *c d* the arcs *f g* and *e g*, and it will complete the arc required.

To draw from or to the Circumference of a Circle, Lines leading to the Centre, When the Centre is inaccessible—Fig. 22.

(22.)

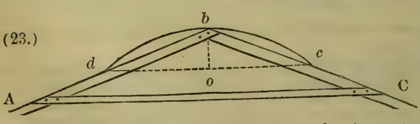


Divide the whole or any given portion of the circumference into the desired number of parts; then, with any radius less than the distance of two divisions, describe arcs cutting each other, as A *r*, *b r*, *c r*, *d r*, etc.; draw the lines *b r*, *c r*, etc., and they will lead to the centre, as required.

To draw the end lines, as A *r*, F *r*. From *b* describe the arc *r*, and with the radius *b* 1, from A or F as centres, cut the arcs A 1, etc., at *r*, or *r*, and the lines A *r*, F *r*, will lead to the centre, as required.

To Describe an Arc, or Segment of a Circle, of a large Radius—Fig. 23.

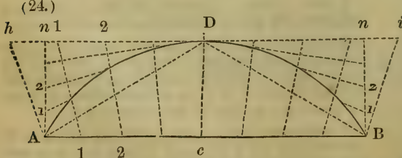
(23.)



Construct of suitable material a triangle, as A *b* C; make A *b*, *b* C, each equal in length to the chord of the arc *d c*, and in height twice that of the arc *b o*. At each end of the chord *d*, *c*, insert a pin, and at *b* attach a tracer (as a pencil); move the triangle against the pins as guides, and the tracer will describe the arc required.

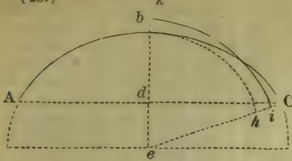
Or, draw the chord A *c* B (*Fig.* 24); also draw the line *h* D *i* parallel with the chord, and at a distance equal to the height of the segment; bisect the chord in *c*, and erect the perpendicular *c* D: join A D, D B; draw A *h* perpendicular to A D, and B *i* perpendicular to B D; erect also the perpendiculars A *n*, B *n*; divide A B and *h i* into any number of equal parts; draw the lines 1 1, 2 2, and divide the lines A *n*, B *n*, each into half the number of equal parts in A B; draw lines to D from each division in the lines A *n*, B *n*, and at the points of intersection with the former lines describe a curve, which will be the arc or segment required.

(24.)



To Ascertain the Length of an Elliptic Curve which is less than half of the entire Figure--Fig. 25.

(25.)



Let the curve of which the length is required be A *b* C.

Extend the versed sine *b d* to meet the centre of the curve in *e*.

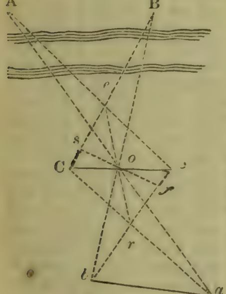
Draw the line *e* C, and from *e*, with the distance *e b*, describe *b h;* bisect *h* C in *i*, and from *e*, with the radius *e i*, describe *k i*, and it is equal to half the arc A *b* C.

To Ascertain the Length when the Curve is greater than half the entire Figure.

Ascertain by the above problem the curve of the less portion of the figure; subtract it from the circumference of the ellipse, and the remainder will be the length of the curve required.

To Ascertain the Distance between two inaccessible Objects, as A, B--Fig. 26.

(26.)

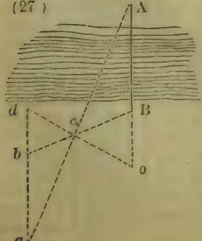


From any point C draw a line C *c*, and bisect it in *o;* take any point *e* in the prolongation of A *c*, and draw the line *e r*, making *o e* equal to *o r*.

In like manner, take any point *s* in the prolongation of B C, and make *o f* equal to *o s*. Produce A *o* and C *r* till they meet in *a*, and also B *o* and *c r* till they meet in *b;* then *a b* equal A B, or the distance between the objects as required.

To Ascertain the Distance of an inaccessible Object on a Level Plane--Fig. 27.

(27.)



Let it be required to ascertain the distance between A and B, A being inaccessible.

Produce the line in the direction of A B to any point, as *o;* draw the line *o d* at any angle to the line A B; bisect the line *o d*, through which draw the line B *b*, making *c b* equal to B *c;* draw the line *d b a;* also through *c*, in the direction *c* A, draw the line A *c a*, intersecting the line *d b a;* then *b a* equal A B, the distance required.

Areas of Circles, from 1/64 to 150.—[*Advancing by an Eighth.*]

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
1/64	.000192	5.	19.635	12.	113.098	19.	283.529
1/32	.000767	.1/8	20.629	.1/8	115.466	.1/8	287.272
1/16	.003068	.1/4	21.6476	.1/4	117.859	.1/4	291.04
1/8	.012272	.3/8	22.6907	.3/8	120.277	.3/8	294.832
3/16	.027612	.1/2	23.7583	.1/2	122.719	.1/2	298.648
1/4	.049087	.5/8	24.8505	.5/8	125.185	.5/8	302.489
5/16	.076699	.3/4	25.9673	.3/4	127.677	.3/4	306.355
3/8	.110447	.7/8	27.1086	.7/8	130.192	.7/8	310.245
7/16	.15033	6.	28.2744	13.	132.733	20.	314.16
1/2	.19635	.1/8	29.4648	.1/8	135.297	.1/8	318.099
9/16	.248505	.1/4	30.6797	.1/4	137.887	.1/4	322.063
5/8	.306796	.3/8	31.9191	.3/8	140.501	.3/8	326.051
11/16	.371224	.1/2	33.1831	.1/2	143.139	.1/2	330.064
3/4	.441787	.5/8	34.4717	.5/8	145.802	.5/8	334.102
13/16	.518487	.3/4	35.7848	.3/4	148.49	.3/4	338.164
7/8	.601322	.7/8	37.1224	.7/8	151.202	.7/8	342.25
15/16	.690292	7.	38.4846	14.	153.938	21.	346.361
1.	.7854	.1/8	39.8713	.1/8	156.7	.1/8	350.497
.1/8	.99402	.1/4	41.2826	.1/4	159.485	.1/4	354.657
.1/4	1.2272	.3/8	42.7184	.3/8	162.296	.3/8	358.842
.3/8	1.4849	.1/2	44.1787	.1/2	165.13	.1/2	363.051
.1/2	1.7671	.5/8	45.6636	.5/8	167.99	.5/8	367.285
.5/8	2.0739	.3/4	47.1731	.3/4	170.874	.3/4	371.543
.3/4	2.4053	.7/8	48.7071	.7/8	173.782	.7/8	375.826
.7/8	2.7612	8.	50.2656	15.	176.715	22.	380.134
2.	3.1416	.1/8	51.8487	.1/8	179.673	.1/8	384.466
.1/8	3.5466	.1/4	53.4563	.1/4	182.655	.1/4	388.822
.1/4	3.9761	.3/8	55.0884	.3/8	185.661	.3/8	393.203
.3/8	4.4301	.1/2	56.7451	.1/2	188.692	.1/2	397.609
.1/2	4.9087	.5/8	58.4264	.5/8	191.748	.5/8	402.038
.5/8	5.4119	.3/4	60.1322	.3/4	194.828	.3/4	406.494
.3/4	5.9396	.7/8	61.8625	.7/8	197.933	.7/8	410.973
.7/8	6.4918	9.	63.6174	16.	201.062	23.	415.477
3.	7.0686	.1/8	65.3968	.1/8	204.216	.1/8	420.004
.1/8	7.6699	.1/4	67.2008	.1/4	207.395	.1/4	424.558
.1/4	8.2958	.3/8	69.0293	.3/8	210.598	.3/8	429.135
.3/8	8.9462	.1/2	70.8823	.1/2	213.825	.1/2	433.737
.1/2	9.6211	.5/8	72.7599	.5/8	217.077	.5/8	438.364
.5/8	10.3206	.3/4	74.6621	.3/4	220.354	.3/4	443.015
.3/4	11.0447	.7/8	76.5888	.7/8	223.655	.7/8	447.69
.7/8	11.7933	10.	78.54	17.	226.981	24.	452.39
4.	12.5664	.1/8	80.5158	.1/8	230.331	.1/8	457.115
.1/8	13.3641	.1/4	82.5161	.1/4	233.706	.1/4	461.864
.1/4	14.1863	.3/8	84.5409	.3/8	237.105	.3/8	466.638
.3/8	15.033	.1/2	86.5903	.1/2	240.529	.1/2	471.436
.1/2	15.9043	.5/8	88.6643	.5/8	243.977	.5/8	476.259
.5/8	16.8002	.3/4	90.7628	.3/4	247.45	.3/4	481.107
.3/4	17.7206	.7/8	92.8858	.7/8	250.948	.7/8	485.979
.7/8	18.6655	11.	95.0334	18.	254.47	25.	490.875
		.1/8	97.2055	.1/8	258.016	.1/8	495.796
		.1/4	99.4022	.1/4	261.587	.1/4	500.742
		.3/8	101.6234	.3/8	265.183	.3/8	505.712
		.1/2	103.8691	.1/2	268.803	.1/2	510.706
		.5/8	106.1394	.5/8	272.448	.5/8	515.726
		.3/4	108.4343	.3/4	276.117	.3/4	520.769
		.7/8	110.7537	.7/8	279.811	.7/8	525.838

Table—(*Continued*).

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
26.	530.93	33.	855.301	40.	1256.64	47.	1734.95
.⅛	536.048	.⅛	861.792	.⅛	1264.51	.⅛	1744.19
.¼	541.19	.¼	868.309	.¼	1272.4	.¼	1753.45
.⅜	546.356	.⅜	874.85	.⅜	1280.31	.⅜	1762.74
.½	551.547	.½	881.415	.½	1288.25	.½	1772.06
.⅝	556.763	.⅝	888.005	.⅝	1296.22	.⅝	1781.4
.¾	562.003	.¾	894.62	.¾	1304.21	.¾	1790.76
.⅞	567.267	.⅞	901.259	.⅞	1312.22	.⅞	1800.15
27.	572.557	34.	907.922	41.	1320.26	48.	1809.56
.⅛	577.87	.⅛	914.611	.⅛	1328.32	.⅛	1819.
.¼	583.209	.¼	921.323	.¼	1336.41	.¼	1828.46
.⅜	588.571	.⅜	928.061	.⅜	1344.52	.⅜	1837.95
.½	593.959	.½	934.822	.½	1352.66	.½	1847.46
.⅝	599.371	.⅝	941.609	.⅝	1360.82	.⅝	1856.99
.¾	604.807	.¾	948.42	.¾	1369.	.¾	1866.55
.⅞	610.268	.⅞	955.255	⅞	1377.21	.⅞	1876.14
28.	615.754	35.	962.115	42.	1385.45	49.	1885.75
.⅛	621.264	.⅛	969.	.⅛	1393.7	.⅛	1895.38
.¼	626.798	.¼	975.909	.¼	1401.99	.¼	1905.04
.⅜	632.357	.⅜	982.842	.⅜	1410.3	.⅜	1914.72
.½	637.941	.½	989.8	.½	1418.63	.½	1924.43
.⅝	643.549	.⅝	996.783	.⅝	1426.99	.⅝	1934.16
.¾	649.182	.¾	1003.79	.¾	1435.37	.¾	1943.91
.⅞	654.84	.⅞	1010.822	.⅞	1443.77	.⅞	1953.69
29.	660.521	36.	1017.878	43.	1452.2	50.	1963.5
.⅛	666.228	.⅛	1024.96	.⅛	1460.66	.⅛	1973.33
.¼	671.959	.¼	1032.065	.¼	1469.14	.¼	1983.18
.⅜	677.714	.⅜	1039.195	.⅜	1477.64	.⅜	1993.06
.½	683.494	.½	1046.349	.½	1486.17	.½	2002.97
.⅝	689.299	.⅝	1053.528	.⅝	1494.73	.⅝	2012.89
.¾	695.128	.¾	1060.732	.¾	1503.3	.¾	2022.85
.⅞	700.982	.⅞	1067.96	.⅞	1511.91	.⅞	2032.82
30.	706.86	37.	1075.213	44.	1520.53	51.	2042.83
.⅛	712.763	.⅛	1082.49	.⅛	1529.19	.⅛	2052.85
.¼	718.69	.¼	1089.792	.¼	1537.86	.¼	2062.9
.⅜	724.642	.⅜	1097.118	.⅜	1546.56	.⅜	2072.98
.½	730.618	.½	1104.469	.½	1555.29	.½	2083.08
.⅝	736.619	.⅝	1111.844	.⅝	1564.04	.⅝	2093.2
.¾	742.645	.¾	1119.244	.¾	1572.81	.¾	2103.35
.⅞	748.695	.⅞	1126.669	.⅞	1581.61	.⅞	2113.52
31.	754.769	38.	1134.118	45.	1590.43	52.	2123.72
.⅛	760.869	.⅛	1141.591	.⅛	1599.28	.⅛	2133.94
.¼	766.992	.¼	1149.089	.¼	1608.16	.¼	2144.19
.⅜	773.14	.⅜	1156.612	.⅜	1617.05	.⅜	2154.46
.½	779.313	.½	1164.159	.½	1625.97	.½	2164.76
.⅝	785.51	.⅝	1171.731	.⅝	1634.92	.⅝	2175.08
.¾	791.732	.¾	1179.327	.¾	1643.89	.¾	2185.42
.⅞	797.979	.⅞	1186.948	.⅞	1652.89	.⅞	2195.79
32.	804.25	39.	1194.593	46.	1661.91	53.	2206.19
.⅛	810.545	.⅛	1202.263	.⅛	1670.95	.⅛	2216.61
.¼	816.865	.¼	1209.958	.¼	1680.02	.¼	2227.05
.⅜	823.21	.⅜	1217.677	.⅜	1689.11	.⅜	2237.52
.½	829.579	.½	1225.42	.½	1698.23	.½	2248.01
.⅝	835.972	.⅝	1233.188	.⅝	1707.37	.⅝	2258.53
.¾	842.391	.¾	1240.981	.¾	1716.54	.¾	2269.07
.⅞	848.833	.⅞	1248.798	.⅞	1725.73	.⅞	2279.64

Table—(*Continued*).

Diam	Area.	Diam	Area	Diam.	Area	Diam	Area.
54.	2290.23	61.	2922.47	68.	3631.69	75.	4417.87
.⅛	2300.84	.⅛	2934.46	.⅛	3645.05	.⅛	4432.61
.¼	2311.48	.¼	2946.48	.¼	3658.44	.¼	4447.38
.⅜	2322.15	.⅜	2958.52	.⅜	3671.86	.⅜	4462.16
.½	2332.83	.½	2970.58	.½	3685.29	.½	4476.98
.⅝	2343.55	.⅝	2982.67	.⅝	3698.76	.⅝	4491.81
.¾	2354.29	.¾	2994.78	.¾	3712.24	.¾	4506.67
.⅞	2365.05	.⅞	3006.92	.⅞	3725.75	.⅞	4521.56
55.	2375.83	62.	3019.08	69.	3739.29	76.	4536.47
.⅛	2386.65	.⅛	3031.26	.⅛	3752.85	.⅛	4551.41
.¼	2397.48	.¼	3043.47	.¼	3766.43	.¼	4566.36
.⅜	2408.34	.⅜	3055.71	.⅜	3780.04	.⅜	4581.35
.½	2419.23	.½	3067.97	.½	3793.68	.½	4596.36
.⅝	2430.14	.⅝	3080.25	.⅝	3807.34	.⅝	4611.39
.¾	2441.07	.¾	3092.56	.¾	3821.02	.¾	4626.45
.⅞	2452.03	.⅞	3104.89	.⅞	3834.73	.⅞	4641.53
56.	2463.01	63.	3117.25	70.	3848.46	77.	4656.64
.⅛	2474.02	.⅛	3129.64	.⅛	3862.22	.⅛	4671.77
.¼	2485.05	.¼	3142.04	.¼	3876.	.¼	4686.92
.⅜	2496.11	.⅜	3154.47	.⅜	3889.8	.⅜	4702.1
.½	2507.19	.½	3166.93	.½	3903.63	.½	4717.31
.⅝	2518.3	.⅝	3179.41	.⅝	3917.49	.⅝	4732.54
.¾	2529.43	.¾	3191.91	.¾	3931.37	.¾	4747.79
.⅞	2540.58	.⅞	3204.44	.⅞	3945.27	.⅞	4763.07
57.	2551.76	64.	3217.	71.	3959.2	78.	4778.37
.⅛	2562.97	.⅛	3229.58	.⅛	3973.15	.⅛	4793.7
.¼	2574.2	.¼	3242.18	.¼	3987.13	.¼	4809.05
.⅜	2585.45	.⅜	3254.81	.⅜	4001.13	.⅜	4824.43
.½	2596.73	.½	3267.46	.½	4015.16	.½	4839.83
.⅝	2608.03	.⅝	3280.14	.⅝	4029.21	.⅝	4855.26
.¾	2619.36	.¾	3292.84	.¾	4043.29	.¾	4870.71
.⅞	2630.71	.⅞	3305.56	.⅞	4057.39	.⅞	4886.18
58.	2642.09	65.	3318.31	72.	4071.51	79.	4901.68
.⅛	2653.49	.⅛	3331.09	.⅛	4085.66	.⅛	4917.21
.¼	2664.91	.¼	3343.89	.¼	4099.84	.¼	4932.75
.⅜	2676.36	.⅜	3356.71	.⅜	4114.04	.⅜	4948.33
.½	2687.84	.½	3369.56	.½	4128.26	.½	4963.92
.⅝	2699.33	.⅝	3382.44	.⅝	4142.51	.⅝	4979.55
.¾	2710.86	.¾	3395.33	.¾	4156.78	.¾	4995.19
.⅞	2722.41	.⅞	3408.26	.⅞	4171.08	.⅞	5010.86
59.	2733.98	66.	3421.2	73.	4185.4	80.	5026.56
.⅛	2745.57	.⅛	3434.17	.⅛	4199.74	.⅛	5042.28
.¼	2757.2	.¼	3447.17	.¼	4214.11	.¼	5058.03
.⅜	2768.84	.⅜	3460.19	.⅜	4228.51	.⅜	5073.79
.½	2780.51	.½	3473.24	.½	4242.93	.½	5089.59
.⅝	2792.21	.⅝	3486.3	.⅝	4257.37	.⅝	5105.41
.¾	2803.93	.¾	3499.4	.¾	4271.84	.¾	5121.25
.⅞	2815.67	.⅞	3512.52	.⅞	4286.33	.⅞	5137.12
60.	2827.44	67.	3525.66	74.	4300.85	81.	5153.01
.⅛	2839.23	.⅛	3538.83	.⅛	4315.39	.⅛	5168.93
.¼	2851.05	.¼	3552.02	.¼	4329.96	.¼	5184.87
.⅜	2862.89	.⅜	3565.24	.⅜	4344.55	.⅜	5200.83
.½	2874.76	.½	3578.48	.½	4359.17	.½	5216.82
.⅝	2886.65	.⅝	3591.74	.⅝	4373.81	.⅝	5232.84
.¾	2898.57	.¾	3605.04	.¾	4388.47	.¾	5248.88
.⅞	2910.51	.⅞	3618.35	.⅞	4403.16	.⅞	5264.94

Table—(*Continued*).—[*Advancing by an Eighth and a Quarter.*]

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
82.	5281.03	89.	6221.15	96.	7238.25	106.	8824.75
.⅛	5297.14	.⅛	6238.64	.⅛	7257.11	.¼	8866.43
.¼	5313.28	.¼	6256.15	.¼	7275.99	.½	8908.2
.⅜	5329.44	.⅜	6273.69	.⅜	7294.91	.¾	8950.07
.½	5345.63	.½	6291.25	.½	7313.84	107.	8992.04
.⅝	5361.84	.⅝	6308.84	.⅝	7332.8	.¼	9034.11
.¾	5378.08	.¾	6326.45	.¾	7351.79	.½	9076.28
.⅞	5394.34	.⅞	6344.08	.⅞	7370.79	.¾	9118.54
83.	5410.62	90.	6361.74	97.	7389.83	108.	9160.91
.⅛	5426.93	.⅛	6379.42	.⅛	7408.89	.¼	9203.37
.¼	5443.26	.¼	6397.13	.¼	7427.97	.½	9245.93
.⅜	5459.62	.⅜	6414.86	.⅜	7447.08	.¾	9288.58
.½	5476.01	.½	6432.62	.½	7466.21	109.	9331.34
.⅝	5492.41	.⅝	6450.4	.⅝	7485.37	.¼	9374.19
.¾	5508.84	.¾	6468.21	.¾	7504.55	.½	9417.14
.⅞	5525.3	.⅞	6486.04	.⅞	7523.75	.¾	9460.19
84.	5541.78	91.	6503.9	98.	7542.98	110.	9503.34
.⅛	5558.29	.⅛	6521.78	.⅛	7562.24	.¼	9546.59
.¼	5574.82	.¼	6539.68	.¼	7581.52	.½	9589.93
.⅜	5591.37	.⅜	6557.61	.⅜	7600.82	.¾	9633.37
.½	5607.95	.½	6575.56	.½	7620.15	111.	9676.91
.⅝	5624.56	.⅝	6593.54	.⅝	7639.5	.¼	9720.55
.¾	5641.18	.¾	6611.55	.¾	7658.88	.½	9764.29
.⅞	5657.84	.⅞	6629.57	.⅞	7678.28	.¾	9808.12
85.	5674.51	92.	6647.63	99.	7697.71	112.	9852.06
.⅛	5691.22	.⅛	6665.7	.⅛	7717.16	.¼	9896.09
.¼	5707.94	.¼	6683.8	.¼	7736.63	.½	9940.22
.⅜	5724.69	.⅜	6701.93	.⅜	7756.13	.¾	9984.45
.½	5741.47	.½	6720.08	.½	7775.66	113.	10028.77
.⅝	5758.27	.⅝	6738.25	.⅝	7795.21	.¼	10073.2
.¾	5775.1	.¾	6756.45	.¾	7814.78	.½	10117.72
.⅞	5791.94	.⅞	6774.68	.⅞	7834.38	.¾	10162.34
86.	5808.82	93.	6792.92	100.	7854.	114.	10207.06
.⅛	5825.72	.⅛	6811.2	.¼	7893.32	.¼	10251.88
.¼	5842.64	.¼	6829.49	.½	7932.74	.½	10296.79
.⅜	5859.59	.⅜	6847.82	.¾	7972.25	.¾	10341.8
.½	5876.56	.½	6866.16	101.	8011.87	115.	10386.91
.⅝	5893.55	.⅝	6884.53	.¼	8051.58	.¼	10432.12
.¾	5910.58	.¾	6902.93	.½	8091.39	.½	10477.43
.⅞	5927.62	.⅞	6921.35	.¾	8131.3	.¾	10522.84
87.	5944.69	94.	6939.79	102.	8171.3	116.	10568.34
.⅛	5961.79	.⅛	6958.26	.¼	8211.41	.¼	10613.94
.¼	5978.91	.¼	6976.76	.½	8251.61	.½	10659.65
.⅜	5996.05	.⅜	6995.28	.¾	8291.91	.¾	10705.44
.½	6013.22	.½	7013.82	103.	8332.31	117.	10751.34
.⅝	6030.41	.⅝	7032.39	.¼	8372.81	.¼	10797.34
.¾	6047.63	.¾	7050.98	.½	8413.4	.½	10843.43
.⅞	6064.87	.⅞	7069.59	.¾	8454.09	.¾	10889.62
88.	6082.14	95.	7088.24	104.	8494.89	118.	10935.91
.⅛	6099.43	.⅛	7106.9	.¼	8535.78	.¼	10982.3
.¼	6116.74	.¼	7125.59	.½	8576.76	.½	11028.78
.⅜	6134.08	.⅜	7144.31	.¾	8617.85	.¾	11075.37
.½	6151.45	.½	7163.04	105.	8659.03	119.	11122.05
.⅝	6168.84	.⅝	7181.81	.¼	8700.32	.¼	11168.83
.¾	6186.25	.¾	7200.6	.½	8741.7	.½	11215.71
.⅞	6203.69	.⅞	7219.41	.¾	8783.18	.¾	11262.69

Table—(*Continued*).—[*Advancing by a Quarter and a Half.*]

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
120.	11309.76	.½	12173.9	133.	13892.94	142.	15836.31
.¼	11356.93	.¾	12222.84	.½	13997.6	.½	15948.53
.½	11404.2	125.	12271.87	134.	14102.64	143.	16060.64
.¾	11451.57	.½	12370.25	.½	14208.08	.½	16173.15
121.	11499.04	126.	12469.01	135.	14313.91	144.	16286.05
.¼	11546.61	.½	12568.17	.½	14420.14	.½	16399.35
.½	11594.27	127.	12667.72	136.	14526.76	145.	16513.03
.¾	11642.03	.½	12767.66	.½	14633.77	.½	16627.11
122.	11689.89	128.	12867.99	137.	14741.17	146.	16741.59
.¼	11737.85	.½	12968.72	.½	14848.97	.½	16856.45
.½	11785.91	129.	13069.84	138.	14957.16	147.	16971.71
.¾	11834.06	.½	13171.35	.½	15065.74	.½	17087.36
123.	11882.32	130.	13273.26	139.	15174.71	148.	17203.4
.¼	11930.67	.½	13375.56	.½	15284.08	.½	17319.84
.½	11979.12	131.	13478.25	140.	15393.84	149.	17436.67
.¾	12027.66	.½	13581.33	.½	15503.99	.½	17553.89
124.	12076.31	132.	13684.81	141.	15614.54	150.	17671.5
.¼	12125.05	.½	13788.68	.½	15725.48	.½	17789.51

To Compute the Area of a Diameter greater than any in the preceding Table.

Rule.—Divide the dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular area for this diameter, multiply it by the square of the divisor, and the product will give the area required.

Example.—What is the area for a diameter of 1050?

$1050 \div 7 = 150$; tab. area, $150 = 17671.5$, which $\times 7^2 = 865903.5$, *area required.*

To Compute the Area of a Diameter in Feet and Inches, etc., by the preceding Table.

Rule.—Reduce the dimension to inches or eighths, as the case may be, and take the area in that term from the table for that number.

Divide this number by 64 (the square of 8) if it is eighths, and the quotient will give the area in inches, and divide again by 144 (the square of 12) if it is in inches, and the quotient will give the area in feet.

Example.—What is the area of 1 foot 6⅜ inches?

1 foot 6⅜ ins. = 18⅜ *ins.* = 147 *eighths.* Area of $147 = 16971.71$, which $\div 64 = 265.18$ *inches;* and by $144 = 1.84$ *feet.*

To Compute the Area of an Integer and a Fraction not given in the Table.

Rule.—Double, treble, or quadruple the dimension given, until the fraction is increased to a whole number, or to one of those in the table, as ⅛, ¼, etc., provided it is practicable to do so.

Take the area for this diameter; and if it is double of that for which the area is required, take one fourth of it; if treble, take one sixteenth of it, etc., etc.

Example.—Required the area for a circle of 2.3/16 inches.

2.3/16 × 2 = 4.⅜, area for which = 15.0331, which ÷ 4 = 3.758 *ins.*

Note.—For a Table of Areas by Feet and Inches, see *Haswell's* Mechanics' Tables.

Circumferences of Circles, from $\frac{1}{64}$ to 150.

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
1/64	.04909	5.	15.708	12.	37.6992	19.	59.6904
1/32	.09818	.1/8	16.1007	.1/8	38.0919	.1/8	60.0831
1/16	.19635	.1/4	16.4934	.1/4	38.4846	.1/4	60.4758
1/8	.3927	.3/8	16.8861	.3/8	38.8773	.3/8	60.8685
3/16	.589	.1/2	17.2788	.1/2	39.27	.1/2	61.2612
1/4	.7854	.5/8	17.6715	.5/8	39.6627	.5/8	61.6539
5/16	.98175	.3/4	18.0642	.3/4	40.0554	.3/4	62.0466
3/8	1.1781	.7/8	18.4569	.7/8	40.4481	.7/8	62.4393
7/16	1.37445	6.	18.8496	13.	40.8408	20.	62.832
1/2	1.5708	.1/8	19.2423	.1/8	41.2335	.1/8	63.2247
9/16	1.76715	.1/4	19.635	.1/4	41.6262	.1/4	63.6174
5/8	1.9635	.3/8	20.0277	.3/8	42.0189	.3/8	64.0101
11/16	2.15985	.1/2	20.4204	.1/2	42.4116	.1/2	64.4028
3/4	2.3562	.5/8	20.8131	.5/8	42.8043	.5/8	64.7955
13/16	2.55255	.3/4	21.2058	.3/4	43.197	.3/4	65.1882
7/8	2.7489	.7/8	21.5985	.7/8	43.5897	.7/8	65.5809
15/16	2.94525	7.	21.9912	14.	43.9824	21.	65.9736
1.	3.1416	.1/8	22.3839	.1/8	44.3751	.1/8	66.3663
.1/8	3.5343	.1/4	22.7766	.1/4	44.7678	.1/4	66.759
.1/4	3.927	.3/8	23.1693	.3/8	45.1605	.3/8	67.1517
.3/8	4.3197	.1/2	23.562	.1/2	45.5532	.1/2	67.5444
.1/2	4.7124	.5/8	23.9547	.5/8	45.9459	.5/8	67.9371
.5/8	5.1051	.3/4	24.3474	.3/4	46.3386	.3/4	68.3298
.3/4	5.4978	.7/8	24.7401	.7/8	46.7313	.7/8	68.7225
.7/8	5.8905	8.	25.1328	15.	47.124	22.	69.1152
2.	6.2832	.1/8	25.5255	.1/8	47.5167	.1/8	69.5079
.1/8	6.6759	.1/4	25.9182	.1/4	47.9094	.1/4	69.9006
.1/4	7.0686	.3/8	26.3109	.3/8	48.3021	.3/8	70.2933
.3/8	7.4613	.1/2	26.7036	.1/2	48.6948	.1/2	70.686
.1/2	7.854	.5/8	27.0963	.5/8	49.0875	.5/8	71.0787
.5/8	8.2467	.3/4	27.489	.3/4	49.4802	.3/4	71.4714
.3/4	8.6394	.7/8	27.8817	.7/8	49.8729	.7/8	71.8641
.7/8	9.0321	9.	28.2744	16.	50.2656	23.	72.2568
3.	9.4248	.1/8	28.6671	.1/8	50.6583	.1/8	72.6495
.1/8	9.8175	.1/4	29.0598	.1/4	51.051	.1/4	73.0422
.1/4	10.2102	.3/8	29.4525	.3/8	51.4437	.3/8	73.4349
.3/8	10.6029	.1/2	29.8452	.1/2	51.8364	.1/2	73.8276
.1/2	10.9956	.5/8	30.2379	.5/8	52.2291	.5/8	74.2203
.5/8	11.3883	.3/4	30.6306	.3/4	52.6218	.3/4	74.613
.3/4	11.781	.7/8	31.0233	.7/8	53.0145	.7/8	75.0057
.7/8	12.1737	10.	31.416	17.	53.4072	24.	75.3984
4.	12.5664	.1/8	31.8087	.1/8	53.7999	.1/8	75.7911
.1/8	12.9591	.1/4	32.2014	.1/4	54.1926	.1/4	76.1838
1/4	13.3518	.3/8	32.5941	.3/8	54.5853	.3/8	76.5765
.3/8	13.7445	.1/2	32.9868	.1/2	54.978	.1/2	76.9692
.1/2	14.1372	.5/8	33.3795	.5/8	55.3707	.5/8	77.3619
.5/8	14.5299	.3/4	33.7722	.3/4	55.7634	.3/4	77.7546
.3/4	14.9226	.7/8	34.1649	.7/8	56.1561	.7/8	78.1473
.7/8	15.3153	11.	34.5576	18.	56.5488	25.	78.54
		.1/8	34.9503	.1/8	56.9415	.1/8	78.9327
		.1/4	35.343	.1/4	57.3342	.1/4	79.3254
		.3/8	35.7357	.3/8	57.7269	.3/8	79.7181
		.1/2	36.1284	.1/2	58.1196	.1/2	80.1108
		.5/8	36.5211	.5/8	58.5123	.5/8	80.5035
		.3/4	36.9138	.3/4	58.905	.3/4	80.8962
		.7/8	37.3065	.7/8	59.2977	.7/8	81.2889

Table—(*Continued*).—[*Advancing by an Eighth.*]

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
26.	81.6816	33.	103.673	40.	125.664	47.	147.655
.⅛	82.0743	.⅛	104.065	.⅛	126.057	.⅛	148.048
.¼	82.467	.¼	104.458	.¼	126.449	.¼	148.441
.⅜	82.8597	.⅜	104.851	.⅜	126.842	.⅜	148.833
.½	83.2524	.½	105.244	.½	127.235	.½	149.226
.⅝	83.6451	.⅝	105.636	.⅝	127.627	.⅝	149.619
.¾	84.0378	.¾	106.029	.¾	128.02	.¾	150.011
.⅞	84.4305	.⅞	106.422	.⅞	128.413	.⅞	150.404
27.	84.8232	34.	106.814	41.	128.806	48.	150.797
.⅛	85.2159	.⅛	107.207	.⅛	129.198	.⅛	151.189
.¼	85.6086	.¼	107.6	.¼	129.591	.¼	151.582
.⅜	86.0013	.⅜	107.992	.⅜	129.984	.⅜	151.975
.½	86.394	.½	108.385	.½	130.376	.½	152.368
.⅝	86.7867	.⅝	108.778	.⅝	130.769	.⅝	152.76
.¾	87.1794	.¾	109.171	.¾	131.162	.¾	153.153
.⅞	87.5721	.⅞	109.563	.⅞	131.554	.⅞	153.546
28.	87.9648	35.	109.956	42.	131.947	49.	153.938
.⅛	88.3575	.⅛	110.349	.⅛	132.34	.⅛	154.331
.¼	88.7502	.¼	110.741	.¼	132.733	.¼	154.724
.⅜	89.1429	.⅜	111.134	.⅜	133.125	.⅜	155.116
.½	89.5356	.½	111.527	.½	133.518	.½	155.509
.⅝	89.9283	.⅝	111.919	.⅝	133.911	.⅝	155.902
.¾	90.321	.¾	112.312	.¾	134.303	.¾	156.295
.⅞	90.7137	.⅞	112.705	.⅞	134.696	.⅞	156.687
29.	91.1064	36.	113.098	43.	135.089	50.	157.08
.⅛	91.4991	.⅛	113.49	.⅛	135.481	.⅛	157.473
.¼	91.8918	.¼	113.883	.¼	135.874	.¼	157.865
.⅜	92.2845	.⅜	114.276	.⅜	136.267	.⅜	158.258
.½	92.6772	.½	114.668	.½	136.66	.½	158.651
.⅝	93.0699	.⅝	115.061	.⅝	137.052	.⅝	159.043
.¾	93.4626	.¾	115.454	.¾	137.445	.¾	159.436
.⅞	93.8553	.⅞	115.846	.⅞	137.838	.⅞	159.829
30.	94.248	37.	116.239	44.	138.23	51.	160.222
.⅛	94.6407	.⅛	116.632	.⅛	138.623	.⅛	160.614
.¼	95.0334	.¼	117.025	.¼	139.016	.¼	161.007
.⅜	95.4261	.⅜	117.417	.⅜	139.408	.⅜	161.4
.½	95.8188	.½	117.81	.½	139.801	.½	161.792
.⅝	96.2115	.⅝	118.203	.⅝	140.194	.⅝	162.185
.¾	96.6042	.¾	118.595	.¾	140.587	.¾	162.578
.⅞	96.9969	.⅞	118.988	.⅞	140.979	.⅞	162.97
31.	97.3896	38.	119.381	45.	141.372	52.	163.363
.⅛	97.7823	.⅛	119.773	.⅛	141.765	.⅛	163.756
.¼	98.175	.¼	120.166	.¼	142.157	.¼	164.149
.⅜	98.5677	.⅜	120.559	.⅜	142.55	.⅜	164.541
.½	98.9604	.½	120.952	.½	142.943	.½	164.934
.⅝	99.3531	.⅝	121.344	.⅝	143.335	.⅝	165.327
.¾	99.7458	.¾	121.737	.¾	143.728	.¾	165.719
.⅞	100.1385	.⅞	122.13	.⅞	144.121	.⅞	166.112
32.	100.5312	39.	122.522	46.	144.514	53.	166.505
.⅛	100.9239	.⅛	122.915	.⅛	144.906	.⅛	166.897
.¼	101.3166	.¼	123.308	.¼	145.299	.¼	167.29
.⅜	101.7093	.⅜	123.7	.⅜	145.692	.⅜	167.683
.½	102.102	.½	124.093	.½	146.084	.½	168.076
.⅝	102.4947	.⅝	124.486	.⅝	146.477	.⅝	168.468
.¾	102.8874	.¾	124.879	.¾	146.87	.¾	168.861
.⅞	103.2801	.⅞	125.271	.⅞	147.262	.⅞	169.254

Table—(*Continued*).—[*Advancing by an Eighth.*]

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
54.	169.646	61.	191.638	68.	213.629	75.	235.62
.⅛	170.039	.⅛	192.03	.⅛	214.021	.⅛	236.013
.¼	170.432	.¼	192.423	.¼	214.414	.¼	236.405
.⅜	170.824	.⅜	192.816	.⅜	214.807	.⅜	236.798
.½	171.217	.½	193.208	.½	215.2	.½	237.191
.⅝	171.61	.⅝	193.601	.⅝	215.592	.⅝	237.583
.¾	172.003	.¾	193.994	.¾	215.985	.¾	237.976
.⅞	172.395	.⅞	194.386	.⅞	216.378	.⅞	238.369
55.	172.788	62.	194.779	69.	216.77	76.	238.762
.⅛	173.181	.⅛	195.172	.⅛	217.163	.⅛	239.154
.¼	173.573	.¼	195.565	.¼	217.556	.¼	239.547
.⅜	173.966	.⅜	195.957	.⅜	217.948	.⅜	239.94
.½	174.359	.½	196.35	.½	218.341	.½	240.332
.⅝	174.751	.⅝	196.743	.⅝	218.734	.⅝	240.725
.¾	175.144	.¾	197.135	.¾	219.127	.¾	241.118
.⅞	175.537	.⅞	197.528	.⅞	219.519	.⅞	241.51
56.	175.93	63.	197.921	70.	219.912	77.	241.903
.⅛	176.322	.⅛	198.313	.⅛	220.305	.⅛	242.296
.¼	176.715	.¼	198.706	.¼	220.697	.¼	242.689
.⅜	177.108	.⅜	199.099	.⅜	221.09	.⅜	243.081
.½	177.5	.½	199.492	.½	221.483	.½	243.474
.⅝	177.893	.⅝	199.884	.⅝	221.875	.⅝	243.867
.¾	178.286	.¾	200.277	.¾	222.268	.¾	244.259
.⅞	178.678	.⅞	200.67	.⅞	222.661	.⅞	244.652
57.	179.071	64.	201.062	71.	223.054	78.	245.045
.⅛	179.464	.⅛	201.455	.⅛	223.446	.⅛	245.437
.¼	179.857	.¼	201.848	.¼	223.839	.¼	245.83
.⅜	180.249	.⅜	202.24	.⅜	224.232	.⅜	246.223
.½	180.642	.½	202.633	.½	224.624	.½	246.616
.⅝	181.035	.⅝	203.026	.⅝	225.017	.⅝	247.008
.¾	181.427	.¾	203.419	.¾	225.41	.¾	247.401
.⅞	181.82	.⅞	203.811	.⅞	225.802	.⅞	247.794
58.	182.213	65.	204.204	72.	226.195	79.	248.186
.⅛	182.605	.⅛	204.597	.⅛	226.588	.⅛	248.579
.¼	182.998	.¼	204.989	.¼	226.981	.¼	248.972
.⅜	183.391	.⅜	205.382	.⅜	227.373	.⅜	249.364
.½	183.784	.½	205.775	.½	227.766	.½	249.757
.⅝	184.176	.⅝	206.167	.⅝	228.159	.⅝	250.15
.¾	184.569	.¾	206.56	.¾	228.551	.¾	250.543
.⅞	184.962	.⅞	206.953	.⅞	228.944	.⅞	250.935
59.	185.354	66.	207.346	73.	229.337	80.	251.328
.⅛	185.747	.⅛	207.738	.⅛	229.729	.⅛	251.721
.¼	186.14	.¼	208.131	.¼	230.122	.¼	252.113
.⅜	186.532	.⅜	208.524	.⅜	230.515	.⅜	252.506
.½	186.925	.½	208.916	.½	230.908	.½	252.899
.⅝	187.318	.⅝	209.309	.⅝	231.3	.⅝	253.291
.¾	187.711	.¾	209.702	.¾	231.693	.¾	253.684
.⅞	188.103	.⅞	210.094	.⅞	232.086	.⅞	254.077
60.	188.496	67.	210.487	74.	232.478	81.	254.47
.⅛	188.889	.⅛	210.88	.⅛	232.871	.⅛	254.862
.¼	189.281	.¼	211.273	.¼	233.264	.¼	255.255
.⅜	189.674	.⅜	211.665	.⅜	233.656	.⅜	255.648
.½	190.067	.½	212.058	.½	234.049	.½	256.04
.⅝	190.459	.⅝	212.451	.⅝	234.442	.⅝	256.433
.¾	190.852	.¾	212.843	.¾	234.835	.¾	256.826
.⅞	191.245	.⅞	213.236	.⅞	235.227	.⅞	257.218

Table—(*Continued*).—[*Advancing by an Eighth and a Quarter.*]

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
82.	257.611	89.	279.602	96.	301.594	106.	333.01
.1/8	258.004	.1/8	279.995	.1/8	301.986	.1/4	333.795
.1/4	258.397	.1/4	280.388	.1/4	302.379	.1/2	334.58
.3/8	258.789	.3/8	280.78	.3/8	302.772	.3/4	335.366
.1/2	259.182	.1/2	281.173	.1/2	303.164	107.	336.151
.5/8	259.575	.5/8	281.566	.5/8	303.557	.1/4	336.937
.3/4	259.967	.3/4	281.959	.3/4	303.95	.1/2	337.722
.7/8	260.36	.7/8	282.351	.7/8	304.342	.3/4	338.507
83.	260.753	90.	282.744	97.	304.735	108.	339.293
.1/8	261.145	.1/8	283.137	.1/8	305.128	.1/4	340.078
.1/4	261.538	.1/4	283.529	.1/4	305.521	.1/2	340.864
.3/8	261.931	.3/8	283.922	.3/8	305.913	.3/4	341.649
.1/2	262.324	.1/2	284.315	.1/2	306.306	109.	342 434
.5/8	262.716	.5/8	284.707	.5/8	306.699	.1/4	343.22
.3/4	263.109	.3/4	285.1	.3/4	307.091	.1/2	344.005
.7/8	263.502	.7/8	285.493	.7/8	307.484	.3/4	344.791
84.	263.894	91.	285.886	98.	307.877	110.	345.576
.1/8	264.287	.1/8	286.278	.1/8	308.27	.1/4	346.361
.1/4	264.68	.1/4	286.671	.1/4	308.662	.1/2	347.147
.3/8	265.072	.3/8	287.064	.3/8	309.055	.3/4	347.932
.1/2	265.465	.1/2	287.456	.1/2	309.448	111.	348.718
.5/8	265.858	.5/8	287.849	.5/8	309.84	.1/4	349.503
.3/4	266.251	.3/4	288.242	.3/4	310.233	.1/2	350.288
.7/8	266.643	.7/8	288.634	.7/8	310.626	.3/4	351.074
85.	267.036	92.	289.027	99.	311.018	112.	351.859
.1/8	267.429	.1/8	289.42	.1/8	311.411	.1/4	352.645
.1/4	267.821	.1/4	289.813	.1/4	311.804	.1/2	353.43
.3/8	268.214	.3/8	290.205	.3/8	312.196	.3/4	354.215
.1/2	268.607	.1/2	290.598	.1/2	312.589	113.	355.001
.5/8	268.999	.5/8	290.991	.5/8	312.982	.1/4	355.786
.3/4	269.392	.3/4	291.383	.3/4	313.375	.1/2	356.572
.7/8	269.785	.7/8	291.776	.7/8	313.767	.3/4	357.357
86.	270.178	93.	292.169	100.	314.16	114.	358.142
.1/8	270.57	.1/8	292.562	.1/4	314.945	.1/4	358.928
.1/4	270.963	.1/4	292.954	.1/2	315.731	.1/2	359.713
.3/8	271.356	.3/8	293.347	.3/4	316.516	.3/4	360.499
.1/2	271.748	.1/2	293.74	101.	317.302	115.	361.284
.5/8	272.141	.5/8	294.132	.1/4	318.087	.1/4	362.069
.3/4	272.534	.3/4	294.525	.1/2	318.872	.1/2	362.855
.7/8	272.926	.7/8	294.918	.3/4	319.658	.3/4	363.64
87.	273.319	94.	295.31	102.	320.443	116.	364.426
.1/8	273.712	.1/8	295.703	.1/4	321.229	.1/4	365.211
.1/4	274.105	.1/4	296.096	.1/2	322.014	.1/2	365.996
.3/8	274.497	.3/8	296.488	.3/4	322.799	.3/4	366.782
.1/2	274.89	.1/2	296.881	103.	323.585	117.	367.567
.5/8	275.283	.5/8	297.274	.1/4	324.37	.1/4	368.353
.3/4	275.675	.3/4	297.667	.1/2	325.156	.1/2	369.138
.7/8	276.068	.7/8	298.059	.3/4	325.941	.3/4	369.923
88.	276.461	95.	298.452	104.	326.726	118.	370.709
.1/8	276.853	.1/8	298.845	.1/4	327.512	.1/4	371.494
.1/4	277.246	.1/4	299.237	.1/2	328.297	.1/2	372.28
.3/8	277.629	.3/8	299.63	.3/4	329.083	.3/4	373.065
.1/2	278.032	.1/2	300.023	105.	329.868	119.	373.85
.5/8	278.424	.5/8	300.415	.1/4	330.653	.1/4	374.636
.3/4	278.817	.3/4	300.808	.1/2	331.439	.1/2	375.421
.7/8	279.21	.7/8	301.201	.3/4	332.224	.3/4	376.207

Table—(*Continued*).—[*Advancing by a Quarter and a Half.*]

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
120.	376.992	.½	391.129	133.	417.833	142.	446.107
.¼	377.777	.¾	391.915	.½	419.404	.½	447.678
.½	378.563	125.	392.7	134.	420.974	143.	449.249
.¾	379.348	.½	394.271	.½	422.545	.½	450.82
121.	380.134	126.	395.842	135.	424.116	144.	452.39
.¼	380.919	.½	397.412	.½	425.687	.½	453.961
.½	381.704	127.	398.983	136.	427.258	145.	455.532
.¾	382.49	.½	400.554	.½	428.828	.½	457.103
122.	383.275	128.	402.125	137.	430.399	146.	458.674
.¼	384.061	.½	403.696	.½	431.97	.½	460.244
.½	384.846	129.	405.266	138.	433.541	147.	461.815
.¾	385.631	.½	406.837	.½	435.112	.½	463.386
123.	386.417	130.	408.408	139.	436.682	148.	464.957
.¼	387.202	.½	409.979	.½	438.253	.½	466.528
.½	387.988	131.	411.55	140.	439.824	149.	468.098
.¾	388.773	.½	413.12	.½	441.395	.½	469.669
124.	389.558	132.	414.691	141.	442.966	150.	471.24
.¼	390.344	.½	416.262	.½	444.536	.½	472.811

To Compute the Circumference of a Diameter greater than any in the preceding Table.

RULE.—Divide the dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular circumference for this dimension, multiply it by 2, 3, 4, 5, etc., according as it was divided, and the product will give the circumference required.

EXAMPLE.—What is the circumference for a diameter of 1050?

1050 ÷ 7 = 150; tab. circum., 150 = 471.239, which × 7 = 3299.073, *circum. required.*

To Compute the Circumference for an Integer and Fraction not given in the Table.

RULE.—Double, treble, or quadruple the dimension given, until the fraction is increased to a whole number or to one of those in the table, as ⅛, ¼, etc., provided it is practicable to do so.

Take the circumference for this diameter; and if it is double of that for which the circumference is required, take one half of it; if treble, take one third of it; and if quadruple, one fourth of it.

EXAMPLE.—Required the circumference of 2.21875 inches.

2.21875 × 2 = 4.4375 = 4$\frac{7}{16}$, which × 2 = 8.⅞; tab. circum. = 27.8817, which ÷ 4 = 6.9704 *ins.*

To Compute the Circumference of a Diameter in Feet and Inches, etc., by the preceding Table.

RULE.—Reduce the dimension to inches or eighths, as the case may be, and take the circumference in that term from the table for that number.

Divide this number by 8 if it is in eighths, and by 12 if in inches, and the quotient will give the area in feet.

EXAMPLE.—Required the circumference of a circle of 1 foot 6⅜ inches.

1 *foot* 6⅜ ins. = 18⅜ ins. = 147 eighths. Circum. of 147 = 461.815, which ÷ 8 = 57.727 *inches;* and by 12 = 4.81 *feet.*

Areas and Circumferences, from $\frac{1}{10}$ to 100.

[*Advancing by Tenths.*]

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.1	.007854	.31416	.4	22.9023	16.9646
.2	.031416	.62832	.5	22.7583	17.2788
.3	.070686	.94248	.6	24.6301	17.593
.4	.125664	1.2566	.7	25.5176	17.9071
.5	.19635	1.5708	.8	26.4209	18.2213
.6	.282744	1.885	.9	27.3398	18.5354
.7	.384846	2.1991	6.	28.2744	18.8496
.8	.502656	2.5133	.1	29.2247	19.1638
.9	.636174	2.8274	.2	30.1908	19.4779
1.	.7854	3.1416	.3	31.1725	19.7921
.1	.9503	3.4558	.4	32.17	20.1062
.2	1.131	3.7699	.5	33.1831	20.4204
.3	1.3273	4.0841	.6	34.212	20.7346
.4	1.5394	4.3982	.7	35.2566	21.0487
.5	1.7671	4.7124	.8	36.3169	21.3629
.6	2.0106	5.0266	.9	37.2929	21.677
.7	2.2698	5.3407	7.	38.4846	21.9912
.8	2.5447	5.6549	.1	39.592	22.3054
.9	2.8353	5.969	.2	40.7151	22.6195
2.	3.1416	6.2832	.3	41.854	22.9337
.1	3.4636	6.5974	.4	43.0085	23.2478
.2	3.8013	6.9115	.5	44.1787	23.562
.3	4.1548	7.2257	.6	45.3647	23.8762
.4	4.5239	7.5398	.7	46.5664	24.1903
.5	4.9087	7.854	.8	47.7837	24.5045
.6	5.3093	8.1682	.9	49.0168	24.8186
.7	5.7256	8.4823	8.	50.2656	25.1328
.8	6.1575	8.7965	.1	51.5301	25.447
.9	6.6052	9.1106	.2	52.8103	25.7611
3.	7.0686	9.4248	.3	54.1062	26.0753
.1	7.5477	9.739	.4	55.4178	26.3894
.2	8.0425	10.0531	.5	56.7451	26.7036
.3	8.553	10.3673	.6	58.0882	27.0178
.4	9.0792	10.6814	.7	59.4469	27.3319
.5	9.6211	10.9956	.8	60.8214	27.6461
.6	10.1788	11.3098	.9	62.2115	27.9602
.7	10.7521	11.6239	9.	63.6174	28.2744
.8	11.3412	11.9381	.1	65.039	28.5886
.9	11.9459	12.2522	.2	66.4763	28.9027
4.	12.5664	12.5664	.3	67.9292	29.2169
.1	13.2026	12.8806	.4	69.3979	29.531
.2	13.8545	13.1947	.5	70.8823	29.8452
.3	14.522	13.5089	.6	72.3825	30.1594
.4	15.2053	13.823	.7	73.8983	30.4735
.5	15.9043	14.1372	.8	75.4298	30.7877
.6	16.6191	14.4514	.9	76.9771	31.1018
.7	17.3495	14.7655	10.	78.54	31.416
.8	18.0956	15.0797	.1	80.1187	31.7302
.9	18.8575	15.3938	.2	81.713	32.0443
5.	19.635	15.708	.3	83.3231	32.3585
.1	20.4283	16.0222	.4	84.9489	32.6726
.2	21.2372	16.3363	.5	86.5903	32.9868
.3	22.0619	16.6505	.6	88.2475	33.301

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	89.9204	33.6151	.2	206.1204	50.8939
.8	91.6091	33.9293	.3	208.6729	51.2081
.9	93.3134	34.2434	.4	211.2412	51.5222
11.	95.0334	34.5576	.5	213.8251	51.8364
.1	96.7691	34.8718	.6	216.4248	52.1505
.2	98.5206	35.1859	.7	219.0402	52.4647
.3	100.2877	35.5001	.8	221.6713	52.7789
.4	102.0706	35.8142	.9	224.3181	53.093
.5	103.8691	36.1284	17.	226.9806	53.4072
.6	105.6834	36.4426	.1	229.6588	53.7214
.7	107.5134	36.7567	.2	232.3527	54.0355
.8	109.3591	37.0709	.3	235.0624	54.3497
.9	111.2205	37.385	.4	237.7877	54.6638
12.	113.0976	37.6992	.5	240.5287	54.978
.1	114.9904	38.0134	.6	243.2855	55.2922
.2	116.8989	38.3275	.7	246.058	55.6063
.3	118.8232	38.6417	.8	248.8461	55.9205
.4	120.7631	38.9558	.9	251.65	56.2346
.5	122.7187	39.27	18.	254.4696	56.5488
.6	124.6901	39.5842	.1	257.3049	56.863
.7	126.6772	39.8983	.2	260.1559	57.1771
.8	128.6799	40.2125	.3	263.0226	57.4913
.9	130.6984	40.5266	.4	265.905	57.8054
13.	132.7326	40.8408	.5	268.8031	58.1196
.1	134.7825	41.155	.6	271.717	58.4338
.2	136.8481	41.4691	.7	274.6465	58.7479
.3	138.9294	41.7833	.8	277.5918	59.0621
.4	141.0264	42.0974	.9	280.5527	59.3762
.5	143.1391	42.4116	19.	283.5294	59.6904
.6	145.2676	42.7258	.1	286.5218	60.0046
.7	147.4117	43.0399	.2	289.5299	60.3187
.8	149.5716	43.3541	.3	292.5536	60.6329
.9	151.7471	43.6682	.4	295.5931	60.947
14.	153.9384	43.9824	.5	298.6483	61.2612
.1	156.1454	44.2966	.6	301.7193	61.5754
.2	158.3681	44.6107	.7	304.806	61.8895
.3	160.6064	44.9249	.8	307.9082	62.2037
.4	162.8605	45.239	.9	311.0263	62.5178
.5	165.1303	45.5532	20.	314.16	62.832
.6	167.4159	45.8674	.1	317.3094	63.1462
.7	169.7171	46.1815	.2	320.4746	63.4603
.8	172.034	46.4957	.3	323.6555	63.7745
.9	174.3667	46.8098	.4	326.8521	64.0886
15.	176.715	47.124	.5	330.0643	64.4028
.1	179.0791	47.4382	.6	333.2923	64.717
.2	181.4588	47.7523	.7	336.536	65.0311
.3	183.8543	48.0665	.8	339.7955	65.3453
.4	186.2655	48.3806	.9	343.0706	65.6594
.5	188.6924	48.6948	21.	346.3614	65.9736
.6	191.1349	49.009	.1	349.6679	66.2878
.7	193.5932	49.3231	.2	352.9902	66.6019
.8	196.0673	49.6373	.3	356.3281	66.9161
.9	198.557	49.9514	.4	359.6818	67.2302
16.	201.0624	50.2656	.5	363.0511	67.5444
.1	203.5835	50.5797	.6	366.4362	67.8586

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	369.837	68.1727	.2	581.0703	85.4515
.8	373.2535	68.4869	.3	585.3508	85.7657
.9	376.6857	68.801	.4	589.6469	86.0798
22.	380.1336	69.1152	.5	593.9587	86.394
.1	383.5972	69.4294	.6	598.2863	86.7082
.2	387.0765	69.7435	.7	602.6296	87.0223
.3	390.5716	70.0577	.8	606.9885	87.3365
.4	394.0823	70.3718	.9	611.3632	87.6506
.5	397.6087	70.686	28.	615.7536	87.9648
.6	401.1509	71.0002	.1	620.1597	88.79
.7	404.7088	71.3143	.2	624.5815	88.5931
.8	408.2823	71.6285	.3	629.019	88.9073
.9	411.8716	71.9426	.4	633.4722	89.2214
23.	415.4766	72.2568	.5	637.9411	89.5356
.1	419.0973	72.571	.6	642.4258	89.8498
.2	422.7337	72.8851	.7	646.9261	90.1639
.3	426.3858	73.1993	.8	651.4422	90.4781
.4	430.0536	73.5134	.9	655.9739	90.7922
.5	433.7371	73.8276	29.	660.5214	91.1064
.6	437.4364	74.1418	.1	665.0846	91.4206
.7	441.1513	74.4559	.2	669.6635	91.7347
.8	444.882	74.7701	.3	674.258	92.0489
.9	448.6283	75.0842	.4	678.8683	92.363
24.	452.3904	75.3984	.5	683.4943	92.6772
.1	456.1682	75.7126	.6	688.1361	92.9914
.2	459.9617	76.0267	.7	692.7935	93.3055
.3	463.7708	76.3409	.8	697.4666	93.6197
.4	467.5957	76.655	.9	702.1555	93.9338
.5	471.4363	76.9692	30.	706.86	94.248
.6	475.2927	77.2834	.1	711.5803	94.5622
.7	479.1647	77.5975	.2	716.3162	94.8763
.8	483.0524	77.9117	.3	721.0679	95.1905
.9	486.9559	78.2258	.4	725.8353	95.5046
25.	490.875	78.54	.5	730.6183	95.8188
.1	494.8099	78.8542	.6	735.4171	96.133
.2	498.7604	78.1683	.7	740.2316	96.4471
.3	502.7267	79.4825	.8	745.0619	96.7613
.4	506.7087	79.7966	.9	749.9078	97.0754
.5	510.7063	80.1108	31.	754.7694	97.3896
.6	514.7196	80.425	.1	759.6467	97.7038
.7	518.7488	80.7391	.2	764.5398	98.0179
.8	522.7937	81.0533	.3	769.4485	98.3321
.9	526.8542	81.3674	.4	774.373	98.6462
26.	530.9304	81.6816	.5	779.3131	98.9604
.1	535.0223	81.9958	.6	784.269	99.2746
.2	539.13	82.3099	.7	789.2406	99.5887
.3	543.2533	82.6241	.8	794.2279	99.9029
.4	547.3924	82.9382	.9	799.2309	100.217
.5	551.5471	83.2524	32.	804.2496	100.5312
.6	555.7176	83.5666	.1	809.284	100.8454
.7	559.9038	83.8807	.2	814.3341	101.1595
.8	564.1057	84.1949	.3	819.4	101.4737
.9	568.3233	84.509	.4	824.4815	101.7878
27.	572.5566	84.8232	.5	829.5787	102.102
.1	576.8056	85.1374	.6	834.6917	102.4162

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	839.8204	102.7303	.2	1146.0871	120.0091
.8	844.9647	103.0445	.3	1152.0954	120.3233
.9	850.1248	103.3586	.4	1158.1194	120.6374
33.	855.3006	103.6728	.5	1164.1591	120.9516
.1	860.4921	103.987	.6	1170.2146	121.2658
.2	865.6993	104.3011	.7	1176.2857	121.5799
.3	870.9222	104.6153	.8	1182.3726	121.8941
.4	876.1608	104.9294	.9	1188.4751	122.2082
.5	881.4151	105.2436	39.	1194.5934	122.5224
.6	886.6852	105.5578	.1	1200.7274	122.8366
.7	891.9709	105.8719	.2	1206.8771	123.1507
.8	897.2724	106.1861	.3	1213.0424	123.4649
.9	902.5895	106.5002	.4	1219.2235	123.779
34.	907.9224	106.8144	.5	1225.4203	124.0932
.1	913.271	107.1286	.6	1231.6329	124.4074
.2	918.6353	107.4427	.7	1237.8611	124.7215
.3	924.0152	107.7569	.8	1244.105	125.0357
.4	929.4109	108.071	.9	1250.3647	125.3498
.5	934.8223	108.3852	40.	1256.64	125.664
.6	940.2495	108.6994	.1	1262.9311	125.9782
.7	945.6923	109.0135	.2	1269.2378	126.2923
.8	951.1508	109.3277	.3	1275.5603	126.6065
.9	956.6251	109.6418	.4	1281.8985	126.9206
35.	962.115	109.956	.5	1288.2523	127.2348
.1	967.6207	110.2702	.6	1294.6219	127.549
.2	973.142	110.5843	.7	1301.0072	127.8631
.3	978.6791	110.8985	.8	1307.4083	128.1773
.4	984.2319	111.2126	.9	1313.825	128.4914
.5	989.8003	111.5268	41.	1320.2574	128.8056
.6	995.3845	111.841	.1	1326.7055	129.1198
.7	1000.9844	112.1551	.2	1333.1694	129.4339
.8	1006.6001	112.4693	.3	1339.6489	129.7481
.9	1012.2314	112.7834	.4	1346.1442	130.0622
36.	1017.8784	113.0976	.5	1352.6551	130.3764
.1	1023.5411	113.4118	.6	1359.1818	130.6906
.2	1029.2196	113.7259	.7	1365.7242	131.0047
.3	1034.9137	114.0401	.8	1372.2823	131.3189
.4	1040.6236	114.3542	.9	1378.8561	131.633
.5	1046.3491	114.6684	42.	1385.4456	131.9472
.6	1052.0904	114.9826	.1	1392.0508	132.2614
.7	1057.8474	115.2967	.2	1398.6717	132.5755
.8	1063.6201	115.6109	.3	1405.3084	132.8897
.9	1069.4085	115.925	.4	1411.9607	133.2038
37.	1075.2126	116.2392	.5	1418.6287	133.518
.1	1081.0324	116.5534	.6	1425.3125	133.8322
.2	1086.8679	116.8675	.7	1432.012	134.1463
.3	1092.7192	117.1817	.8	1438.7271	134.4605
.4	1098.5861	117.4958	.9	1445.458	134.7746
.5	1104.4687	117.81	43.	1452.2046	135.0888
.6	1110.3671	118.1242	.1	1458.9669	135.403
.7	1116.2812	118.4383	.2	1465.7449	135.7171
.8	1122.2109	118.7525	.3	1472.5386	136.0313
.9	1128.1564	119.0666	.4	1479.348	136.3454
38.	1134.1176	119.3808	.5	1486.1731	136.6596
.1	1140.0945	119.695	.6	1493.014	136.9738

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	1499.8705	137.2879	.2	1901.1707	154.5667
.8	1506.7428	137.6021	.3	1908.9068	154.8809
.9	1513.6307	137.9162	.4	1916.6587	155.195
44.	1520.5344	138.2304	.5	1924.4263	155.5092
.1	1527.4538	138.5446	.6	1932.2097	155.8234
.2	1534.3889	138.8587	.7	1940.0087	156.1375
.3	1541.3396	139.1729	.8	1947.8234	156.4517
.4	1548.3061	139.487	.9	1955.6539	156.7658
.5	1555.2883	139.8012	50.	1963.5	157.08
.6	1562.2863	140.1154	.1	1971.3619	157.3942
.7	1569.2999	140.4295	.2	1979.2394	157.7083
.8	1576.3292	140.7437	.3	1987.1327	158.0225
.9	1583.3743	141.0578	.4	1995.0417	158.3366
45.	1590.435	141.372	.5	2002.9663	158.6509
.1	1597.5115	141.6862	.6	2010.9067	158.965
.2	1604.6036	142.0003	.7	2018.8628	159.2791
.3	1611.7115	142.3145	.8	2026.8347	159.5933
.4	1618.8351	142.6286	.9	2034.8222	159.9074
.5	1625.9743	142.9428	51.	2042.8254	160.2216
.6	1633.1293	143.257	.1	2050.8443	160.5358
.7	1640.3	143.5711	.2	2058.879	160.8499
.8	1647.4865	143.8853	.3	2066.9293	161.1641
.9	1654.6886	144.1994	.4	2074.9954	161.4782
46.	1661.9064	144.5136	.5	2083.0771	161.7924
.1	1669.1399	144.8278	.6	2091.1746	162.1066
.2	1676.3892	145.1419	.7	2099.2878	162.4207
.3	1683.6541	145.4561	.8	2107.4167	162.7349
.4	1690.9348	145.7702	.9	2115.5613	163.049
.5	1698.2311	146.0844	52.	2123.7216	163.3632
.6	1705.5432	146.3986	.1	2131.8976	163.6774
.7	1712.871	146.7127	.2	2140.0893	163.9915
.8	1720 2145	147.0269	.3	2148.2968	164.3057
.9	1727.5737	147.341	.4	2156.5199	164.6198
47.	1734.9486	147.6552	.5	2164.7587	164.934
.1	1742.3392	147.9694	.6	2173.0133	165.2482
.2	1749.7455	148.2835	.7	2181.2836	165.5623
.3	1757.1676	148.5977	.8	2189.5695	165.8765
.4	1764.6053	148.9118	.9	2197.8712	166.1906
.5	1772.0587	149.226	53.	2206.1886	166.5048
.6	1779.5279	149.5402	.1	2214.5217	166.819
.7	1787.0128	149.8543	.2	2222.8705	167.1331
.8	1794.5133	150.1685	.3	2231.235	167.4473
.9	1802.0296	150.4826	.4	2239.6152	167.7614
48.	1809.5616	150.7968	.5	2248.0111	168.0756
.1	1817.1093	151.111	.6	2256.4228	168.3898
.2	1824.6727	151.4251	.7	2264.8501	168.7039
.3	1832.2518	151.7393	.8	2273.2932	169.0181
.4	1839.8466	152.0534	.9	2281.7519	169.3322
.5	1847.4571	152.3676	54.	2290.2264	169.6464
.6	1855.0834	152.6818	.1	2298.7166	169.9606
.7	1862.7253	152.9959	.2	2307.2225	170.2747
.8	1870.383	153.3101	.3	2315.744	170.5889
.9	1878.0563	153.6242	.4	2324.2813	170.903
49.	1885.7454	153.9384	.5	2332.8343	171.2172
.1	1893.4502	154.2526	.6	2341.4031	171.5314

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	2349.9875	171.8455	.2	2846.321	189.1243
.8	2358.5876	172.1597	.3	2855.7851	189.4385
.9	2367.2035	172.4738	.4	2865.2649	189.7526
55.	2375.835	172.788	.5	2874.7603	190.0668
.1	2384.4823	173.1022	.6	2884.2715	190.381
.2	2393.1452	173.4163	.7	2893.7984	190.6951
.3	2401.8239	173.7305	.8	2903.3411	191.0093
.4	2410.5183	174.0446	.9	2912.8994	191.3234
.5	2419.2283	174.3588	61.	2922.4734	191.6376
.6	2427.9541	174.673	.1	2932.0631	191.9518
.7	2436.6957	174.9871	.2	2941.6686	192.2659
.8	2445.4529	175.3013	.3	2951.2897	192.5801
.9	2454.2258	175.6154	.4	2960.9266	192.8942
56.	2463.0144	175.9296	.5	2970.5791	193.2084
.1	2471.8187	176.2438	.6	2980.2474	193.5226
.2	2480.6388	176.5579	.7	2989.9314	193.8367
.3	2489.4745	176.8721	.8	2999.6311	194.1509
.4	2498.326	177.1862	.9	3009.3465	194.465
.5	2507.1931	177.5004	62.	3019.0776	194.7792
.6	2516.076	177.8146	.1	3028.8244	195.0934
.7	2524.9736	178.1287	.2	3038.5869	195.4075
.8	2533.8889	178.4429	.3	3048.3652	195.7217
.9	2542.8189	178.757	.4	3058.1591	196.0358
57.	2551.7646	179.0712	.5	3067.9687	196.35
.1	2560.726	179.3854	.6	3077.7941	196.6642
.2	2569.7031	179.6995	.7	3087.6341	196.9783
.3	2578.696	180.0137	.8	3097.4919	197.2925
.4	2587.7045	180.3278	.9	3107.3644	197.6066
.5	2596.7287	180.642	63.	3117.2526	197.9208
.6	2605.7687	180.9562	.1	3127.1565	198.235
.7	2614.8244	181.2703	.2	3137.0761	198.5491
.8	2623.8957	181.5845	.3	3147.0114	198.8633
.9	2632.9828	181.8986	.4	3156.9624	199.1774
58.	2642.0856	182.2128	.5	3166.9291	199.4916
.1	2651.2041	182.527	.6	3176.9116	199.8058
.2	2660.3383	182.8411	.7	3186.9097	200.1199
.3	2669.4882	183.1553	.8	3196.9236	200.4341
.4	2678.6538	183.4694	.9	3206.9531	200.7482
.5	2687.8351	183.7836	64.	3216.9984	201.0624
.6	2697.0322	184.0978	.1	3227.0594	201.3766
.7	2706.2449	184.4119	.2	3237.1361	201.6907
.8	2715.4734	184.7261	.3	3247.2284	202.0049
.9	2724.7175	185.0402	.4	3257.3365	202.319
59.	2733.9774	185.3544	.5	3267.4603	202.6332
.1	2743.253	185.6686	.6	3277.5999	202.9474
.2	2752.5443	185.9827	.7	3287.7551	203.2615
.3	2761.8512	186.2969	.8	3297.9261	203.5757
.4	2771.1739	186.611	.9	3308.1127	203.8898
.5	2780.5123	186.9252	65.	3318.315	204.204
.6	2789.8665	187.2394	.1	3328.5331	204.5182
.7	2799.2363	187.5535	.2	3338.7668	204.8323
.8	2808.6218	187.8677	.3	3349.0163	205.1465
.9	2818.0231	188.1818	.4	3359.2815	205.4606
60.	2827.44	188.496	.5	3369.5623	205.7748
.1	2836.8727	188.8102	.6	3379.8589	206.089

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	3390.1712	206.4031	.2	3981.5382	223.6819
.8	3400.4993	206.7173	.3	3992.7301	223.9961
.9	3410.843	207.0314	.4	4003.9378	224.3102
66.	3421.2024	207.3456	.5	4015.1611	224.6244
.1	3431.5775	207.6598	.6	4026.4002	224.9386
.2	3441.9684	207.9739	.7	4037.655	225.2527
.3	3452.3749	208.2881	.8	4048.9255	225.5669
.4	3462.7972	208.6022	.9	4060.2117	225.881
.5	3473.2351	208.9164	72.	4071.5136	226.1952
.6	3483.6888	209.2306	.1	4082.8312	226.5094
.7	3494.1582	209.5447	.2	4094.1645	226.8235
.8	3504.6433	209.8589	.3	4105.5136	227.1377
.9	3515.1441	210.173	.4	4116.8783	227.4518
67.	3525.6606	210.4872	.5	4128.2587	227.766
.1	3536.1928	210.8014	.6	4139.655	228.0802
.2	3546.7407	211.1155	.7	4151.0668	228.3943
.3	3557.3044	211.4297	.8	4162.4943	228.7085
.4	3567.8837	211.7438	.9	4173.9376	229.0226
.5	3578.4787	212.058	73.	4185.3966	229.3368
.6	3589.0895	212.3722	.1	4196.8713	229.651
.7	3599.716	212.6863	.2	4208.3617	229.9651
.8	3610.3581	213.0005	.3	4219.8678	230.2793
.9	3621.016	213.3146	.4	4231.3896	230.5934
68.	3631.6896	213.6288	.5	4242.9271	230.9076
.1	3642.3789	213.943	.6	4254.4804	231.2218
.2	3653.0839	214.2571	.7	4266.0493	231.5359
.3	3663.805	214.5713	.8	4277.634	231.8501
.4	3674.541	214.8454	.9	4289.2343	232.1642
.5	3685.2931	215.1996	74.	4300.8504	232.4784
.6	3696.061	215.5138	.1	4312.4822	232.7926
.7	3706.8445	215.8279	.2	4324.1297	233.1067
.8	3717.6438	216.1421	.3	4335.7928	233.4209
.9	3728.4587	216.4562	.4	4347.4717	233.735
69.	3739.2894	216.7704	.5	4359.1663	234.0492
.1	3750.1358	217.0846	.6	4370.8767	234.3634
.2	3760.9979	217.3987	.7	4382.6027	234.6775
.3	3771.8756	217.7129	.8	4394.3444	234.9917
.4	3782.7691	218.027	.9	4406.1019	235.3058
.5	3793.6783	218.3412	75.	4417.875	235.62
.6	3804.6033	218.6554	.1	4429.6639	235.9342
.7	3815.5439	218.9695	.2	4441.4684	236.2483
.8	3826.5002	219.2837	.3	4453.2887	236.5625
.9	3847.4722	219.5978	.4	4465.1247	236.8766
70.	3848.46	219.912	.5	4476.9763	237.1908
.1	3859.4635	220.2262	.6	4488.8437	237.505
.2	3870.4826	220.5403	.7	4500.7268	237.8191
.3	3881.5175	220.8545	.8	4512.6257	238.1333
.4	3892.5681	221.1686	.9	4524.5402	238.4474
.5	3903.6343	221.4828	76.	4536.4704	238.7616
.6	3914.7163	221.797	.1	4548.4163	239.0758
.7	3925.814	222.1111	.2	4560.378	239.3899
.8	3936.9275	222.4253	.3	4572.3553	239.7041
.9	3948.9566	222.7394	.4	4584.3484	240.0182
71.	3959.2014	223.0536	.5	4596.3571	240.3324
.1	3970.3619	223.3678	.6	4608.3816	240.6466

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	4620.4218	240.9607	.2	5306.8221	258.2395
.8	4632.4777	241.2749	.3	5319.742	258.5537
.9	4644.5493	241.589	.4	5332.6775	258.8678
77.	4656.6366	241.9032	.5	5345.6287	259.182
.1	4668.7396	242.2174	.6	5358.5957	259.4962
.2	4680.8583	242.5315	.7	5371.5784	259.8103
.3	4692.9928	242.8457	.8	5384.5767	260.1245
.4	4705.1429	243.1598	.9	5397.5908	260.4386
.5	4717.3087	243.474	83.	5410.6206	260.7528
.6	4729.4903	243.7882	.1	5423.6661	261.067
.7	4741.6876	244.1023	.2	5436.7273	261.3811
.8	4753.9005	244.4165	.3	5449.8042	261.6953
.9	4766.1292	244.7306	.4	5462.8968	262.0094
78.	4778.3736	245.0448	.5	5476.0051	262.3236
.1	4790.6337	245.359	.6	5489.1292	262.6378
.2	4802.9095	245.6731	.7	5502.2689	262.9519
.3	4815.201	245.9873	.8	5515.4244	263.2661
.4	4827.5082	246.3014	.9	5528.5955	263.5802
.5	4839.8311	246.6156	84.	5541.7824	263.8944
.6	4852.1698	246.9298	.1	5554.985	264.2086
.7	4864.5241	247.2439	.2	5568.2033	264.5227
.8	4876.8942	247.5581	.3	5581.4372	264.8369
.9	4889.2799	247.8722	.4	5594.6869	265.151
79.	4901.6814	248.1864	.5	5607.9523	265.4652
.1	4914.0986	248.5006	.6	5621.2335	265.7794
.2	4926.5315	248.8147	.7	5634.5303	266.0935
.3	4938.98	249.1289	.8	5647.8428	266.4077
.4	4951.4443	249.443	.9	5661.1711	266.7218
.5	4963.9243	249.7572	85.	5674.515	267.036
.6	4976.4201	250.0714	.1	5687.8747	267.3502
.7	4988.9315	250.3855	.2	5701.25	267.6643
.8	5001.4586	250.6997	.3	5714.6411	267.9785
.9	5014.0015	251.0138	.4	5728.0479	268.2926
80.	5026.56	251.328	.5	5741.4703	268.6068
.1	5039.1343	251.6422	.6	5754.9085	268.921
.2	5051.7242	251.9563	.7	5768.3624	269.2351
.3	5064.3299	252.2705	.8	5781.8321	269.5493
.4	5076.9513	252.5846	.9	5795.3174	269.8634
.5	5089.5883	252.8988	86.	5808.8184	270.1776
.6	5102.2411	253.213	.1	5822.3351	270.4918
.7	5114.9096	253.5271	.2	5835.8676	270.8059
.8	5127.5939	253.8413	.3	5849.4157	271.1201
.9	5140.2938	254.1554	.4	5862.9796	271.4342
81.	5153.0094	254.4696	.5	5876.5591	271.7484
.1	5165.7407	254.7838	.6	5890.1544	272.0626
.2	5178.4878	255.0979	.7	5903.7654	272.3767
.3	5191.2505	255.4121	.8	5917.3921	272.6909
.4	5204.0289	255.7262	.9	5931.0345	273.005
.5	5216.8231	256.0404	87.	5944.6926	273.3192
.6	5229.633	256.3546	.1	5958.3644	273.6334
.7	5242.4586	256.6687	.2	5972.0559	273.9475
.8	5255.2999	256.9829	.3	5985.7612	274.2617
.9	5268.1569	257.297	.4	5999.4821	274.5758
82.	5281.0296	257.6112	.5	6013.2187	274.89
.1	5293.918	257.9254	.6	6026.9711	275.2042

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	6040.7392	275.5183	.2	6822.1729	292.7971
.8	6054.5229	275.8325	.3	6836.8206	293.1113
.9	6068.3224	275.1466	.4	6851.484	293.4254
88.	6082.1376	276.4608	.5	6866.1631	293.7396
.1	6095.9685	276.775	.6	6880.858	294.0538
.2	6109.8151	277.0891	.7	6895.5685	294.3679
.3	6123.6774	277.4033	.8	6910.2948	294.6821
.4	6137.5554	277.7174	.9	6925.0367	294.9962
.5	6151.4491	278.0316	94.	6939.7944	295.3104
.6	6165.3586	278.3458	.1	6954.5678	295.6246
.7	6179.2837	278.6599	.2	6969.3569	295.9387
.8	6193.2246	278.9741	.3	6984.1616	296.2529
.9	6207.1811	279.2882	.4	6998.9821	296.567
89.	6221.1534	279.6024	.5	7013.8183	296.8812
.1	6235.1414	279.9166	.6	7028.6703	297.1954
.2	6249.1451	280.2307	.7	7043.5379	297.5095
.3	6263.1644	280.5449	.8	7058.4212	297.8237
.4	6277.1995	280.859	.9	7073.3203	298.1378
.5	6291.2503	281.1732	95.	7088.235	298.452
.6	6305.3169	281.4874	.1	7103.1655	298.7662
.7	6319.3991	281.8015	.2	7118.1116	299.0803
.8	6333.497	282.1157	.3	7133.0735	299.3945
.9	6347.6107	282.4298	.4	7148.0511	299.7086
90.	6361.74	282.744	.5	7163.0443	300.0228
.1	6375.8851	283.0582	.6	7178.0533	300.337
.2	6390.0458	283.3723	.7	7193.078	300.6511
.3	6404.2223	283.6865	.8	7208.1185	300.9653
.4	6418.4144	284.0006	.9	7223.1746	301.2794
.5	6432.6223	284.3148	96.	7238.2464	301.5936
.6	6446.8459	284.629	.1	7253.3339	301.9078
.7	6461.0852	284.9431	.2	7268.4372	302.2219
.8	6475.3403	285.2573	.3	7283.5561	302.5361
.9	6489.611	285.5714	.4	7298.6908	302.8502
91.	6503.8974	285.8856	.5	7313.8411	303.1644
.1	6518.1995	286.1998	.6	7329.0072	303.4786
.2	6532.5174	286.5139	.7	7344.189	303.7927
.3	6546.8909	286.8281	.8	7359.3865	304.1069
.4	6561.2002	287.1422	.9	7374.5997	304.421
.5	6575.5651	287.4564	97.	7389.8286	304.7352
.6	6589.9458	287.7706	.1	7405.0732	305.0494
.7	6604.3422	288.0847	.2	7420.3335	305.3635
.8	6618.7543	288.3989	.3	7435.6096	305.6777
.9	6633.1821	288.713	.4	7450.9013	305.9918
92.	6647.6256	289.0272	.5	7466.2087	306.306
.1	6662.0848	289.3414	.6	7481.5319	306.6202
.2	6676.5598	289.6555	.7	7496.8708	306.9343
.3	6691.0504	289.9697	.8	7512.2253	307.2485
.4	6705.5567	290.2838	.9	7527.5956	307.5626
.5	6720.0787	290.598	98.	7542.9816	307.8768
.6	6734.6165	290.9121	.1	7558.3833	308.191
.7	6749.17	291.2263	.2	7573.8007	308.5051
.8	6763.7391	291.5405	.3	7589.2338	308.8193
.9	6778.324	291.8546	.4	7604.6826	309.1334
93.	6792.9246	292.1688	.5	7620.1471	389.4476
.1	6807.5409	292.483	.6	7635.6274	309.7618

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
.7	7651.1233	310.0759	.4	7760.0347	312.275
.8	7666.635	310.3901	.5	7775.6563	312.5892
.9	7682.1623	310.7042	.6	7791.2937	312.9034
99.	7697.7054	311.0184	.7	7806.9467	313.2175
.1	7713.2642	311.3326	.8	7822.6154	313.5317
.2	7728.8337	311.6467	.9	7838.2999	313.8458
.3	7744.4288	311.9609	100.	7854.	314.16

To Compute the Area or Circumference of a Diameter greater than any in the preceding Table.

See Rules, pages 176 and 181.

Or, *If the Diameter exceeds* 100 *and is less than* 1001,

Remove the decimal point, and take out the area or circumference as for a Whole Number by removing the decimal point, if for the area, two places to the right; and if for the circumference, one place.

Illustration.—The area of 96.7 is 7344.189; hence for 967 it is 734418.9; and the circumference of 96.7 is 303.7927, and for 967 it is 3037.927.

Areas and Circumferences of Circles.

From 1 to 50 Feet [*advancing by an Inch*], or from 1 to 50 Inches [*advancing by a Twelfth*].

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
1 *ft.*	.7854	3.1416	3 *ft.*	7.0686	9.4248
1	.9217	3.4034	1	7.4668	9.6866
2	1.069	3.6652	2	7.8758	9.9484
3	1.2272	3.927	3	8.2958	10.2102
4	1.3963	4.1888	4	8.7267	10.472
5	1.5763	4.4506	5	9.1685	10.7338
6	1.7671	4.7124	6	9.6211	10.9956
7	1.969	4.9742	7	10.0848	11.2574
8	2.1817	5.236	8	10.5593	11.5192
9	2.4053	5.4978	9	11.0447	11.781
10	2.6398	5.7596	10	11.541	12.0428
11	2.8853	6.0214	11	12.0483	12.3046
2 *ft.*	3.1416	6.2832	4 *ft.*	12.5664	12.5664
1	3.4088	6.545	1	13.0955	12.8282
2	3.687	6.8068	2	13.6354	13.09
3	3.9761	7.0686	3	14.1863	13.3518
4	4.2761	7.3304	4	14.7481	13.6136
5	4.5869	7.5922	5	15.3208	13.8754
6	4.9087	7.854	6	15.9043	14.1372
7	5.2415	8.1158	7	16.4989	14.499
8	5.5852	8.3776	8	17.1043	14.6608
9	5.9396	8.6394	9	17.7206	14.9226
10	6.305	8.9012	10	18.3478	15.1844
11	6.6814	9.163	11	18.9859	15.4462

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
5 *ft.*	19.635	15.708	6	70.8823	29.8452
1	20.2949	15.9698	7	72.1314	30.107
2	20.9658	16.2316	8	73.3913	30.3688
3	21.6476	16.4934	9	74.6621	30.6306
4	22.3403	16.7552	10	75.9439	30.8924
5	23.0439	17.017	11	77.2365	31.1542
6	23.7583	17.2788	10 *ft.*	78.54	31.416
7	24.4837	17.5406	1	79.8545	31.6778
8	25.201	17.8024	2	81.1798	31.9396
9	25.9673	18.0642	3	82.5161	32.2014
10	26.7254	18.326	4	83.8633	32.4632
11	27.4944	18.5878	5	85.2214	32.725
6 *ft.*	28.2744	18.8496	6	86.5903	32.9868
1	29.0653	19.1114	7	87.9703	33.2486
2	29.867	19.3732	8	89.3611	33.5104
3	30.6797	19.635	9	90.7628	33.7722
4	31.5033	19.8968	10	92.1754	34.034
5	32.3378	20.1586	11	93.599	34.2958
6	33.1831	20.4204	11 *ft.*	95.0334	34.5576
7	34.0394	20.6822	1	96.4787	34.8194
8	34.9067	20.944	2	97.935	35.0812
9	35.7848	21.2058	3	99.4022	35.343
10	36.6738	21.4676	4	100.8803	35.6048
11	37.5738	21.7294	5	102.3693	35.8666
7 *ft.*	38.4846	21.9912	6	103.8691	36.1284
1	39.4064	22.253	7	105.38	36.3902
2	40.339	22.5148	8	106.9017	36.652
3	41.2826	22.7766	9	108.4343	36.9138
4	42.2371	23.0384	10	109.9778	37.1756
5	43.2025	23.3002	11	111.5323	37.4374
6	44.1787	23.562	12 *ft.*	113.0976	37.6992
7	45.1659	23.8238	1	114.6739	37.961
8	46.1641	24.0856	2	116.261	38.2228
9	47.1731	24.3474	3	117.8591	38.4846
10	48.193	24.6092	4	119.468	38.7464
11	49.2238	24.871	5	121.088	39.0082
8 *ft.*	50.2656	25.1328	6	122.7187	39.27
1	51.3183	25.3946	7	124.3605	39.5318
2	52.3818	25.6564	8	126.0131	39.7936
3	53.4563	25.9182	9	127.6766	40.0554
4	54.5417	26.18	10	129.351	40.3172
5	55.638	26.4418	11	131.0366	40.579
6	56.7451	26.7036	13 *ft.*	132.7326	40.8408
7	57.8632	26.9654	1	134.4398	41.1026
8	58.9923	27.2272	2	136.1578	41.3644
9	60.1322	27.489	3	137.8868	41.6262
10	61.283	27.7508	4	139.6267	41.888
11	62.4448	28.0126	5	141.3774	42.1498
9 *ft.*	63.6174	28.2744	6	143.1391	42.4116
1	64.801	28.5362	7	144.9117	42.6734
2	65.9954	28.798	8	146.6953	42.9352
3	67.2008	29.0598	9	148.4897	43.197
4	68.417	29.3216	10	150.295	43.4588
5	69.6442	29.5834	11	152.1113	43.7206

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
14 *ft.*	153.9384	43.9824	6	268.8031	58.1196
1	155.7764	44.2442	7	271.2302	58.3814
2	157.6254	44.506	8	273.6683	58.6432
3	159.4853	44.7678	9	276.1172	58.905
4	161.3561	45.0296	10	278.577	59.1668
5	163.2378	45.2914	11	281.0477	59.4286
6	165.1303	45.5532	19 *ft.*	283.5294	59.6904
7	167.0338	45.815	1	286.0219	59.9522
8	168.9483	46.0768	2	288.5255	60.214
9	170.8736	46.3386	3	291.0398	60.4758
10	172.8098	46.6004	4	293.5651	60.7376
11	174.7569	46.8622	5	296.1012	60.9994
15 *ft.*	176.715	47.124	6	298.6483	61.2612
1	178.684	47.3858	7	301.2064	61.523
2	180.6638	47.6476	8	303.7753	61.7848
3	182.6546	47.9094	9	306.3551	62.0466
4	184.6563	48.1712	10	308.9458	62.3084
5	186.6689	48.433	11	311.5475	62.5702
6	188.6924	48.6948	20 *ft.*	314.16	62.832
7	190.7267	48.9566	1	316.7834	63.0938
8	192.7721	49.2184	2	319.4178	63.3556
9	194.8283	49.4802	3	322.0631	63.6174
10	196.8954	49.742	4	324.7193	63.8792
11	198.9734	50.0038	5	327.3864	64.141
16 *ft.*	201.0624	50.2656	6	330.0643	64.4028
1	203.1622	50.5274	7	332.7532	64.6646
2	205.273	50.7892	8	335.4531	64.9264
3	207.3947	51.051	9	338.1638	65.1882
4	209.5273	51.3128	10	340.8854	65.45
5	211.6707	51.5746	11	343.618	65.7118
6	213.8252	51.8364	21 *ft.*	346.3614	65.9736
7	215.9904	52.0982	1	349.1157	66.2354
8	218.1667	52.36	2	351.881	66.4972
9	220.3538	52.6218	3	354.6572	66.759
10	222.5518	52.8836	4	357.4442	67.0208
11	224.7607	53.1454	5	360.2422	67.2826
17 *ft.*	226.9806	53.4072	6	363.0511	67.5444
1	229.2113	53.669	7	365.8709	67.8062
2	231.453	53.9308	8	368.7017	68.068
3	233.7056	54.1926	9	371.5433	68.3298
4	235.9691	54.4544	10	374.3958	68.5916
5	238.2434	54.7162	11	377.2592	68.8534
6	240.5287	54.978	22 *ft.*	380.1336	69.1152
7	242.8249	55.2398	1	383.0188	69.377
8	245.1321	55.5016	2	385.915	69.6388
9	247.4501	55.7634	3	388.8221	69.9006
10	249.779	56.0252	4	391.74	70.1624
11	252.1188	56.287	5	394.6689	70.4242
18 *ft.*	254.4696	56.5488	6	397.6087	70.686
1	256.8312	56.8106	7	400.5594	70.9478
2	259.2038	57.0724	8	403.5211	71.2096
3	261.5873	57.3342	9	406.4936	71.4714
4	263.9817	57.596	10	409.477	71.7332
5	266.3869	57.8578	11	412.4713	71.995

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
23 *ft.*	415.4766	72.2568	6	593.9587	86.394
1	418.4927	72.5186	7	597.5639	86.6558
2	421.5198	72.7804	8	601.18	86.9176
3	424.5578	73.0422	9	604.8071	87.1794
4	427.6067	73.304	10	608.445	87.4412
5	430.6664	73.5658	11	612.0938	87.703
6	433.7371	73.8276	28 *ft.*	615.7536	87.9648
7	436.8187	74.0894	1	619.4242	88.2266
8	439.917	74.3512	2	623.1058	88.4884
9	443.0147	74.613	3	626.7983	88.7502
10	446.129	74.8748	4	630.5016	89.012
11	449.2542	75.1366	5	634.2159	89.2738
24 *ft.*	452.3904	75.3984	6	637.9411	89.5356
1	455.5374	75.6602	7	641.6772	89.7974
2	458.6954	75.922	8	645.4243	90.0592
3	461.8643	76.1838	9	649.1822	90.321
4	465.044	76.4456	10	652.951	90.5828
5	468.2347	76.7074	11	656.7307	90.8446
6	471.4363	76.9692	29 *ft.*	660.5214	91.1064
7	474.6488	77.231	1	664.3229	91.3682
8	477.8723	77.4928	2	668.1354	91.63
9	481.1066	77.7546	3	671.9588	91.8918
10	484.3518	78.0164	4	675.7931	92.1536
11	487.6076	78.2782	5	679.6382	92.4154
25 *ft.*	490.875	78.54	6	683.4943	92.6772
1	494.1529	78.8018	7	687.3613	92.939
2	497.4418	79.0636	8	691.2393	93.2008
3	500.7416	79.3254	9	695.1281	93.4626
4	504.0523	79.5872	10	699.0278	93.7244
5	507.3738	79.849	11	702.9384	93.9862
6	510.7063	80.1108	30 *ft.*	706.86	94.248
7	514.0413	80.3726	1	710.7924	94.5098
8	517.404	80.6344	2	714.7358	94.7716
9	520.7693	80.8962	3	718.6901	95.0334
10	524.1454	81.158	4	722.6553	95.2952
11	527.5324	81.4198	5	726.6313	95.557
26 *ft.*	530.9304	81.6816	6	730.6183	95.8188
1	534.3313	81.9434	7	734.6162	96.0806
2	537.759	82.2052	8	738.6251	96.3424
3	541.1897	82.467	9	742.6448	96.6042
4	544.6313	82.7288	10	746.6754	96.866
5	548.0837	82.9906	11	750.7164	97.1278
6	551.5471	83.2524	31 *ft.*	754.7694	97.3896
7	555.0214	83.5142	1	758.8327	97.6514
8	558.5066	83.776	2	762.907	97.9132
9	562.0028	84.0378	3	766.9922	98.175
10	565.5098	84.2996	4	771.0883	98.4368
11	569.0277	84.5614	5	775.1952	98.6986
27 *ft.*	572.5566	84.8232	6	779.3131	98.9604
1	576.0963	85.085	7	783.4419	99.2222
2	579.6467	85.3468	8	787.5817	99.484
3	583.2086	85.6086	9	791.7323	99.7458
4	586.781	85.8704	10	795.8938	100.0076
5	590.3644	86.1322	11	800.0662	100.2694

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet		Feet.	Feet.
32 *ft.*	804.2496	100.5312	6	1046.3491	114.6684
1	808.4439	100.793	7	1051.1324	114.9302
2	812.649	101.0548	8	1055.9266	115.192
3	816.8651	101.3166	9	1060.7318	115.4538
4	821.092	101.5784	10	1065.5478	115.7156
5	825.3299	101.8402	11	1070.3747	115.9774
6	829.5787	102.102	37 *ft.*	1075.2126	116.2392
7	833.8384	102.3638	1	1080.0613	116.501
8	838.1091	102.6256	2	1084.921	116.7628
9	842.3906	102.8874	3	1089.7916	117.0246
10	846.683	103.1492	4	1094.6731	117.2864
11	850.9863	103.411	5	1099.5654	117.5482
33 *ft.*	855.3006	103.6728	6	1104.4687	117.81
1	859.6257	103.9346	7	1109.3839	118.0718
2	863.9618	104.1964	8	1114.308	118.3336
3	868.3088	104.4582	9	1119.2441	118.5954
4	872.6667	104.72	10	1124.191	118.8572
5	877.0354	104.9818	11	1129.1489	119.119
6	881.4151	105.2436	38 *ft.*	1134.1176	119.3808
7	885.8057	105.5054	1	1139.0972	119.6426
8	890.2073	105.7672	2	1144.0878	119.9044
9	894.6197	106.029	3	1149.0893	120.1662
10	899.043	106.2908	4	1154.1017	120.428
11	903.4772	106.5526	5	1159.1249	120.6898
34 *ft.*	907.9224	106.8144	6	1164.1591	120.9516
1	912.3784	107.0762	7	1169.2042	121.2134
2	916.8454	107.338	8	1174.2603	121.4758
3	921.3233	107.5998	9	1179.3272	121.737
4	925.812	107.8616	10	1184.405	121.9988
5	930.3117	108.1234	11	1189.4937	122.2606
6	934.8223	108.3852	39 *ft.*	1194.5934	122.5224
7	939.3439	108.647	1	1199.7039	122.7848
8	943.8763	108.9088	2	1204.8254	123.046
9	948.4196	109.1706	3	1209.9578	123.3078
10	952.9738	109.4324	4	1215.101	123.5696
11	957.5392	109.6942	5	1220.2552	123.8314
35 *ft.*	962.115	109.956	6	1225.4203	124.0932
1	966.7019	110.2178	7	1230.5963	124.355
2	971.2998	110.4796	8	1235.7833	124.6168
3	975.9086	110.7414	9	1240.9811	124.8786
4	980.5287	111.0032	10	1246.1898	125.1404
5	985.1588	111.265	11	1251.4094	125.4022
6	989.8005	111.5268	40 *ft.*	1256.64	125.664
7	994.4527	111.7886	1	1261.8814	125.9258
8	999.116	112.0504	2	1267.1338	126.1876
9	1003.7903	112.3122	3	1272.3971	126.4494
10	1008.4754	112.574	4	1277.6712	126.7112
11	1013.1714	112.8358	5	1282.9563	126.973
36 *ft.*	1017.8784	113.0976	6	1288.2523	127.2348
1	1022.5962	113.3594	7	1293.5592	127.4966
2	1027.325	113.6212	8	1298.877	127.7584
3	1032.0647	113.883	9	1304.2058	128.0202
4	1036.8153	114.1448	10	1309.5454	128.282
5	1041.5767	114.4066	11	1314.8959	128.5438

Table—(*Continued*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
41 *ft.*	1320.2574	128.8056	6	1625.9743	142.9428
1	1325.6297	129.0674	7	1631.9357	143.2046
2	1331.013	129.3292	8	1637.9081	143.4664
3	1336.4072	129.591	9	1643.8913	143.7282
4	1341.8123	129.8528	10	1649.8854	143.99
5	1347.2282	130.1146	11	1655.8904	144.2518
6	1352.6551	130.3764	46 *ft.*	1661.9064	144.5136
7	1358.0929	130.6382	1	1667.9332	144.7754
8	1363.5416	130.9	2	1673.971	145.0372
9	1369.0013	131.1618	3	1680.0197	145.299
10	1374.4718	131.4236	4	1686.0792	145.5608
11	1379.9532	131.6854	5	1692.1497	145.8226
42 *ft.*	1385.4456	131.9472	6	1698.2311	146.0844
1	1390.9488	132.209	7	1704.3334	146.3462
2	1396.463	132.4708	8	1710.4267	146.608
3	1401.9881	132.7326	9	1716.5408	146.8698
4	1407.5241	132.9944	10	1722.6658	147.1316
5	1413.0709	133.2562	11	1728.8017	147.3934
6	1418.6287	133.518	47 *ft.*	1734.9486	147.6552
7	1424.1974	133.7798	1	1741.1063	147.917
8	1429.777	134.0416	2	1747.275	148.1788
9	1435.3676	134.3034	3	1753.4546	148.4406
10	1440.969	134.5652	4	1759.6451	148.7024
11	1446.5813	134.827	5	1765.8464	148.9642
43 *ft.*	1452.2046	135.0888	6	1772.0587	149.226
1	1457.8387	135.3506	7	1778.2819	149.4878
2	1463.4838	135.6124	8	1784.516	149.7496
3	1469.1398	135.8742	9	1790.7611	150.0114
4	1474.8066	136.136	10	1797.017	150.2732
5	1480.4844	136.3978	11	1803.2838	150.535
6	1486.1731	136.6596	48 *ft.*	1809.5616	150.7968
7	1491.8717	136.9214	1	1815.8502	151.0586
8	1497.5833	137.1832	2	1822.1498	151.3204
9	1503.3047	137.445	3	1828.4603	151.5822
10	1509.037	137.7068	4	1834.7817	151.844
11	1514.7802	137.9786	5	1841.1139	152.1058
44 *ft.*	1520.5344	138.2304	6	1847.4571	152.3676
1	1526.2994	138.4922	7	1853.8112	152.6294
2	1532.0754	138.754	8	1860.1763	152.8912
3	1537.8623	139.0158	9	1866.5522	153.153
4	1543.66	139.2776	10	1872.939	153.4148
5	1549.4687	139.5394	11	1879.3367	153.6766
6	1555.2883	139.8012	49 *ft.*	1885.7454	153.9384
7	1561.1188	140.063	1	1892.1649	154.2002
8	1566.9603	140.3248	2	1898.5954	154.462
9	1572.8126	140.5866	3	1905.0368	154.7238
10	1578.6756	140.8484	4	1911.4897	154.9856
11	1584.5499	141.1102	5	1917.9522	155.2474
45 *ft.*	1590.435	141.372	6	1924.4263	155.5092
1	1596.3309	141.6338	7	1930.9113	155.771
2	1602.2378	141.8956	8	1937.4073	156.0328
3	1608.1556	142.1574	9	1943.9142	156.2946
4	1614.0843	142.4192	10	1950.4318	156.5564
5	1620.0238	142.681	11	1956.9604	156.8182
			50 *ft.*	1963.5	157.08

Table of the Sides of Squares—equal in Area to a Circle of any Diameter.

FROM 1 TO 100.

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
1.	.8862	14.	12.4072	27.	23.9281	40.	35.4491
.¼	1.1078	.¼	12.6287	.¼	24.1497	.¼	35.6706
.½	1.3293	.½	12.8503	.½	24.3712	.½	35.8922
.¾	1.5509	.¾	13.0718	.¾	24.5928	.¾	36.1137
2.	1.7724	15.	13.2934	28.	24.8144	41.	36.3353
.¼	1.994	.¼	13.515	.¼	25.0359	.¼	36.5569
.½	2.2156	.½	13.7365	.½	25.2575	.½	36.7784
.¾	2.4371	.¾	13.9581	.¾	25.479	.¾	37.
3.	2.6587	16.	14.1796	29.	25.7006	42.	37.2215
.¼	2.8802	.¼	14.4012	.¼	25.9221	.¼	37.4431
.½	3.1018	.½	14.6227	.½	26.1437	.½	37.6646
.¾	3.3233	.¾	14.8443	.¾	26.3653	.¾	37.8862
4.	3.5449	17.	15.0659	30.	26.5868	43.	38.1078
.¼	3.7665	.¼	15.2874	.¼	26.8084	.¼	38.3293
.½	3.988	.½	15.509	.½	27.0299	.½	38.5509
.¾	4.2096	.¾	15.7305	.¾	27.2515	.¾	38.7724
5.	4.4311	18.	15.9521	31.	27.473	44.	38.994
.¼	4.6527	.¼	16.1736	.¼	27.6946	.¼	39.2155
.½	4.8742	.½	16.3952	.½	27.9161	.½	39.4371
.¾	5.0958	.¾	16.6168	.¾	28.1377	.¾	39.6587
6.	5.3174	19.	16.8383	32.	28.3593	45.	39.8802
.¼	5.5389	.¼	17.0599	.¼	28.5808	.¼	40.1018
.½	5.7605	.½	17.2814	.½	28.8024	.½	40.3233
.¾	5.982	.¾	17.503	.¾	29.0239	.¾	40.5449
7.	6.2036	20.	17.7245	33.	29.2455	46.	40.7664
.¼	6.4251	.¼	17.9461	.¼	29.467	.¼	40.988
.½	6.6467	.½	12.1677	.½	29.6886	.½	41.2096
.¾	6.8683	.¾	18.3892	.¾	29.9102	.¾	41.4311
8.	7.0898	21.	18.6108	34.	30.1317	47.	41.6527
.¼	7.3114	.¼	18.8323	.¼	30.3533	.¼	41.8742
.½	7.5329	.½	19.0539	.½	30.5748	.½	42.0958
.¾	7.7545	.¾	19.2754	.¾	30.7964	.¾	42.3173
9.	7.976	22.	19.497	35.	31.0179	48.	42.5839
.¼	8.1976	.¼	19.7185	.¼	31.2395	.¼	42.7604
.½	8.4192	.½	19.9401	.½	31.4611	.½	42.982
.¾	8.6407	.¾	20.1617	.¾	31.6826	.¾	43.2036
10.	8.8623	23.	20.3832	36.	31.9042	49.	43.4251
.¼	9.0838	.¼	20.6048	.¼	32.1257	.¼	43.6467
.½	9.3054	.½	20.8263	.½	32.3473	.½	43.8682
.¾	9.5269	.¾	21.0479	.¾	32.5688	.¾	44.0898
11.	9.7485	24.	21.2694	37.	32.7904	50.	44.3113
.¼	9.97	.¼	21.491	.¼	33.0112	.¼	44.5329
.½	10.1916	.½	21.7126	.½	33.2335	.½	44.7545
.¾	10.4132	.¾	21.9341	.¾	33.4551	.¾	44.976
12.	10.6347	25.	22.1557	38.	33.6766	51.	45.1976
.¼	10.8563	.¼	22.3772	.¼	33.8982	.¼	45.4191
.½	11.0778	.½	22.5988	.½	34.1197	.½	45.6407
.¾	11.2994	.¾	22.8203	.¾	34.3413	.¾	45.8622
13.	11.5209	26.	23.0419	39.	34.5628	52.	46.0838
.¼	11.7425	.¼	23.2634	.¼	34.7884	.¼	46.3054
.½	11.9641	.½	23.485	.½	35.006	.½	46.5269
.¾	12.1856	.¾	23.7066	.¾	35.2275	.¾	46.7485

Table—(*Continued*).

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
53.	46.97	65.	57.6047	77.	68.2395	89.	78.8742
.¼	47.1916	.¼	57.8263	.¼	68.461	.¼	79.0957
.½	47.4131	.½	58.0479	.½	68.6826	.½	79.3173
.¾	47.6347	.¾	58.2694	.¾	68.9041	.¾	79.5389
54.	47.8562	66.	58.491	78.	69.1257	90.	79.7604
.¼	48.0778	.¼	58.7125	.¼	69.3473	.¼	79.982
.½	48.2994	.½	58.9341	.½	69.5688	.½	80.2035
.¾	48.5209	.¾	59.1556	.¾	69.7904	.¾	80.4251
55.	48.7425	67.	59.3772	79.	70.0119	91.	80.6467
.¼	48.964	.¼	59.5988	.¼	70.2335	.¼	80.8682
.½	49.1856	.½	59.8203	.½	70.455	.½	81.0898
.¾	49.4071	.¾	60.0419	.¾	70.6766	.¾	81.3113
56.	49.6287	68.	60.2634	80.	70.8981	92.	81.5329
.¼	49.8503	.¼	60.485	.¼	71.1197	.¼	81.7544
.½	50.0718	.½	60.7065	.½	71.3413	.½	81.976
.¾	50.2934	.¾	60.9281	.¾	71.5628	.¾	82.1975
57.	50.5149	69.	61.1497	81.	71.7844	93.	82.4191
.¼	50.7365	.¼	61.3712	.¼	72.0059	.¼	82.6407
.½	50.958	.½	61.5928	.½	72.2275	.½	82.8622
.¾	51.1796	.¾	61.8143	.¾	72.4491	.¾	83.0838
58.	51.4012	70.	62.0359	82.	72.6706	94.	83.3053
.¼	51.6227	.¼	62.2574	.¼	72.8921	.¼	83.5269
.½	51.8443	.½	62.479	.½	73.1137	.½	83.7484
.¾	52.0658	.¾	62.7006	.¾	73.3353	.¾	83.970
59.	52.2874	71.	62.9221	83.	73.5568	95.	84.1916
.¼	52.5089	.¼	63.1437	.¼	73.7784	.¼	84.4131
.½	52.7305	.½	63.3652	.½	73.9999	.½	84.6347
.¾	52.9521	.¾	63.5868	.¾	74.2215	.¾	84.8562
60.	53.1736	72.	63.8083	84.	74.4431	96.	85.0778
.¼	53.3952	.¼	64.0299	.¼	74.6647	.¼	85.2993
.½	53.6167	.½	64.2514	.½	74.8862	.½	85.5209
.¾	53.8383	.¾	64.4730	.¾	75.1077	.¾	85.7425
61.	54.0598	73.	64.6946	85.	75.3293	97.	85.9646
.¼	54.2814	.¼	64.9161	.¼	75.5508	.¼	86.185
.½	54.503	.½	65.1377	.½	75.7724	.½	86.4071
.¾	54.7245	.¾	65.3592	.¾	75.9934	.¾	86.6289
62.	54.9461	74.	65.5808	86.	76.2155	98.	86.8502
.¼	55.1676	.¼	65.8023	.¼	76.4371	.¼	87.0718
.½	55.3892	.½	66.0239	.½	76.6586	.½	87.2933
.¾	55.6107	.¾	66.2455	.¾	76.8802	.¾	87.5449
63.	55.8323	75.	66.467	87.	77.1017	99.	87.7364
.¼	56.0538	.¼	66.6886	.¼	77.3233	.¼	87.958
.½	56.2754	.½	66.9104	.½	77.5449	.½	88.1796
.¾	56.497	.¾	67.1317	.¾	77.7664	.¾	87.4011
64.	56.7185	76.	67.3532	88.	77.988	100.	88.6227
.¼	56.9401	.¼	67.5748	.¼	78.2095	.¼	88.8442
.½	57.1616	.½	67.7964	.½	78.4316	.½	89.0658
.¾	57.3832	.¾	68.0179	.¾	78.6526	.¾	89.2874

APPLICATION OF THE TABLE.

To Ascertain a Square that shall have the same Area as a Given Circle.

ILLUSTRATION.—What is the side of a square that has the same area as a circle of 73.25 inches?

By table of Areas, page 174, opposite to 73¼, is 4214.11 ; and in this table is 64.9161, the side of a square having the same area as a circle of 73¼ inches in diameter.

Table of the Lengths of Circular Arcs.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

H'ght.	Length.	H'ght	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.02645	.152	1.06051	.204	1.10752	.256	1.16649	.308	1.23636
.101	1.02698	.153	1.0613	.205	1.10855	.257	1.16774	.309	1.2378
.102	1.02752	.154	1.06209	.206	1.10958	.258	1.16899	.31	1.23925
.103	1.02806	.155	1.06288	.207	1.11062	.259	1.17024	.311	1.2407
.104	1.0286	.156	1.06368	.208	1.11165	.26	1.1715	.312	1.24216
.105	1.02914	.157	1.06449	.209	1.11269	.261	1.17275	.313	1.2436
.106	1.0297	.158	1.0653	.21	1.11374	.262	1.17401	.314	1.24506
.107	1.03026	.159	1.06611	.211	1.11479	.263	1.17527	.315	1.24654
.108	1.03082	.16.	1.06693	.212	1.11584	.264	1.17655	.316	1.24801
.109	1.03139	.161	1.06775	.213	1.11692	.265	1.17784	.317	1.24946
.110	1.03196	.162	1.06858	.214	1.11796	.266	1.17912	.318	1.25095
.111	1.03254	.163	1.06941	.215	1.11904	.267	1.1804	.319	1.25243
.112	1.03312	.164	1.07025	.216	1.12011	.268	1.18162	.32	1.25391
.113	1.03371	.165	1.07109	.217	1.12118	.269	1.18294	.321	1.25539
.114	1.0343	.166	1.07194	.218	1.12225	.27	1.18428	.322	1.25686
.115	1.0349	.167	1.07279	.219	1.12334	.271	1.18557	.323	1.25836
.116	1.03551	.168	1.07365	.22	1.12445	.272	1.18688	.324	1.25987
.117	1.03611	.169	1.07451	.221	1.12556	.273	1.18819	.325	1.26137
.118	1.03672	.17	1.07537	.222	1.12663	.274	1.18969	.326	1.26286
.119	1.03734	.171	1.07624	.223	1.12774	.275	1.19082	.327	1.26437
.12	1.03797	.172	1.07711	.224	1.12885	.276	1.19214	.328	1.26588
.121	1.0386	.173	1.07799	.225	1.12997	.277	1.19345	.329	1.2674
.122	1.03923	.174	1.07888	.226	1.13108	.278	1.19477	.33	1.26892
.123	1.03987	.175	1.07977	.227	1.13219	.279	1.1961	.331	1.27044
.124	1.04051	.176	1.08066	.228	1.13331	.28	1.19743	.332	1.27196
.125	1.04116	.177	1.08156	.229	1.13444	.281	1.19887	.333	1.27349
.126	1.04181	.178	1.08246	.23	1.13557	.282	1.20011	.334	1.27502
.127	1.04247	.179	1.08337	.231	1.13671	.283	1.20146	.335	1.27656
.128	1.04313	.18	1.08428	.232	1.13786	.284	1.20282	.336	1.2781
.129	1.0438	.181	1.08519	.233	1.13903	.285	1.20419	.337	1.27964
.13	1.04447	.182	1.08611	.234	1.1402	.286	1.20558	.338	1.28118
.131	1.04515	.183	1.08704	.235	1.14136	.287	1.20696	.339	1.28273
.132	1.04584	.184	1.08797	.236	1.14247	.288	1.20828	.34	1.28428
.133	1.04652	.185	1.0889	.237	1.14363	.289	1.20967	.341	1.28583
.134	1.04722	.186	1.08984	.238	1.1448	.29	1.21202	.342	1.28739
.135	1.04792	.187	1.09079	.239	1.14597	.291	1.21239	.343	1.28895
.136	1.04862	.188	1.09174	.24	1.14714	.292	1.21381	.344	1.29052
.137	1.04932	.189	1.09269	.241	1.14831	.293	1.2152	.345	1.29209
.138	1.05003	.19	1.09365	.242	1.14949	.294	1.21658	.346	1.29366
.139	1.05075	.191	1.09461	.243	1.15067	.295	1.21794	.347	1.29523
.14	1.05147	.192	1.09557	.244	1.15186	.296	1.21926	.348	1.29681
.141	1.0522	.193	1.09654	.245	1.15308	.297	1.22061	.349	1.29839
.142	1.05293	.194	1.09752	.246	1.15429	.298	1.22203	.35	1.29997
.143	1.05367	.195	1.0985	.247	1.15549	.299	1.22347	.351	1.30156
.144	1.05441	.196	1.09949	.248	1.1567	.3	1.22495	.352	1.30315
.145	1.05516	.197	1.10048	.249	1.15791	.301	1.22635	.353	1.30474
.146	1.05591	.198	1.10147	.25	1.15912	.302	1.22776	.354	1.30634
.147	1.05667	.199	1.10247	.251	1.16033	.303	1.22918	.355	1.30794
.148	1.05743	.2	1.10348	.252	1.16157	.304	1.33061	.356	1.30954
.149	1.05819	.201	1.10447	.253	1.16279	.305	1.23205	.357	1.31115
.15	1.05896	.202	1.10548	.254	1.16402	.306	1.23349	.358	1.31276
.151	1.05973	.203	1.1065	.255	1.16526	.307	1.23494	.359	1.31437

Table—(*Continued*).

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.36	1.31599	.389	1.36425	.417	1.41324	.445	1.46441	.473	1.51764
.361	1.31761	.39	1.36596	.418	1.41503	.446	1.46628	.474	1.51958
.362	1.31923	.391	1.36767	.419	1.41682	.447	1.46815	.475	1.52152
.363	1.32086	.392	1.36939	.42	1.41861	.448	1.47002	.476	1.52346
.364	1.32249	.393	1.37111	.421	1.42041	.449	1.47189	.477	1.52541
.365	1.32413	.394	1.37283	.422	1.42222	.45	1.47377	.478	1.52736
.366	1.32577	.395	1.37455	.423	1.42402	.451	1.47565	.479	1.52931
.367	1.32741	.396	1.37628	.424	1.42583	.452	1.47753	.48	1.53126
.368	1.32905	.397	1.37801	.425	1.42764	.453	1.47942	.481	1.53322
.369	1.33069	.398	1.37974	.426	1.42945	.454	1.48131	.482	1.53518
.37	1.33234	.399	1.38148	.427	1.43127	.455	1.4832	.483	1.53714
.371	1.33399	.4	1.38322	.428	1.43309	.456	1.48509	.484	1.5391
.372	1.33564	.401	1.38496	.429	1.43491	.457	1.48699	.485	1.54106
.373	1.3373	.402	1.38671	.43	1.43673	.458	1.48889	.486	1.54302
.374	1.33896	.403	1.38846	.431	1.43856	.459	1.49079	.487	1.54499
.375	1.34063	.404	1.39021	.432	1.44039	.46	1.49269	.488	1.54696
.376	1.34229	.405	1.39196	.433	1.44222	.461	1.4946	.489	1.54893
.377	1.34396	.406	1.39372	.434	1.44405	.462	1.49651	.49	1.5509
.378	1.34563	.407	1.39548	.435	1.44589	.463	1.49842	.491	1.55288
.379	1.34731	.408	1.39724	.436	1.44773	.464	1.50033	.492	1.55486
.38	1.34899	.409	1.399	.437	1.44957	.465	1.50224	.493	1.55685
.381	1.35068	.41	1.40077	.438	1.45142	.466	1.50416	.494	1.55854
.382	1.35237	.411	1.40254	.439	1.45327	.467	1.50608	.495	1.56083
.383	1.35406	.412	1.40432	.44	1.45512	.468	1.508	.496	1.56282
.384	1.35575	.413	1.406	.441	1.45697	.469	1.50992	.497	1.56481
.385	1.35744	.414	1.40788	.442	1.45883	.47	1.51185	.498	1.5668
.386	1.35914	.415	1.40966	.443	1.46069	.471	1.51378	.499	1.56879
.387	1.36084	.416	1.41145	.444	1.46255	.472	1.51571	.5	1.57079
.388	1.36254								

To Ascertain the Length of an Arc of a Circle by the preceding Table.

RULE.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next right-hand column. Multiply the length thus obtained by the base of the arc, and the product will give the length of the arc.

EXAMPLE.—What is the length of an arc of a circle, the base or span of it being 100 feet, and the height 25 feet?

25 ÷ 100 = .25; and .25, per table, = 1.15912, *the length of the base, which, being multiplied by* 100 = 115.912 *feet*.

NOTE.—When, in the division of a height by the base, the quotient has a remainder after the third place of decimals, and great accuracy is required,

Take the length for the first three figures, subtract it from the next following length; multiply the remainder by the said fractional remainder, add the product to the first length, and the sum will be the length for the whole quotient.

EXAMPLE.—What is the length of an arc of a circle, the base of which is 35 feet, and the height or versed sine 8 feet?

8 ÷ 35 = .2285714; the tabular length for .228 = 1.13331, and for .229 = 1.13444, the difference between which is .00113. Then .5714 × .00113 = .000645682.

Hence .228 = 1.13331,
and .0005714 = .000645682

1.133955682, the sum by which the base of the arc is to be multiplied; and 1.133955682 × 35 = 39.68845 *feet*.

Table of the Lengths of Semi-Elliptic Arcs.

The Transverse Diameter of an Ellipse assumed to be Unity, and divided into 1000 equal Parts.

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.04162	.153	1.09669	.206	1.1572	.259	1.22284	.312	1.29285
.101	1.04262	.154	1.0978	.207	1.15838	.26	1.22412	.313	1.29421
.102	1.04362	.155	1.09891	.208	1.15957	.261	1.22541	.314	1.29557
.103	1.04462	.156	1.10002	.209	1.16076	.262	1.2267	.315	1.29603
.104	1.04562	.157	1.10113	.21	1.16196	.263	1.22799	.316	1.29829
.105	1.04662	.158	1.10224	.211	1.16316	.264	1.22928	.317	1.29965
.106	1.04762	.159	1.10335	.212	1.16436	.265	1.23057	.318	1.30102
.107	1.04862	.16	1.10447	.213	1.16557	.266	1.23186	.319	1.30239
.108	1.04962	.161	1.1056	.214	1.16678	.267	1.23315	.32	1.30376
.109	1.05063	.162	1.10672	.215	1.16799	.268	1.23445	.321	1.30513
.11	1.05164	.163	1.10784	.216	1.1692	.269	1.23575	.322	1.3065
.111	1.05265	.164	1.10896	.217	1.17041	.27	1.23705	.323	1.30787
.112	1.05366	.165	1.11008	.218	1.17163	.271	1.23835	.324	1.30924
.113	1.05467	.166	1.1112	.219	1.17285	.272	1.23966	.325	1.31061
.114	1.05568	.167	1.11232	.22	1.17407	.273	1.24097	.326	1.31198
.115	1.05669	.168	1.11344	.221	1.17529	.274	1.24228	.327	1.31335
.116	1.0577	.169	1.11456	.222	1.17651	.275	1.24359	.328	1.31472
.117	1.05872	.17	1.11569	.223	1.17774	.276	1.2448	.329	1.3161
.118	1.05974	.171	1.11682	.224	1.17897	.277	1.24612	.33	1.31748
.119	1.06076	.172	1.11795	.225	1.1802	.278	1.24744	.331	1.31886
.12	1.06178	.173	1.11908	.226	1.18143	.279	1.24876	.332	1.32024
.121	1.0628	.174	1.12021	.227	1.18266	.28	1.2501	.333	1.32162
.122	1.06382	.175	1.12134	.228	1.1839	.281	1.25142	.334	1.323
.123	1.06484	.176	1.12247	.229	1.18514	.282	1.25274	.335	1.32438
.124	1.06586	.177	1.1236	.23	1.18638	.283	1.25406	.336	1.32576
.125	1.06689	.178	1.12473	.231	1.18762	.284	1.25538	.337	1.32715
.126	1.06792	.179	1.12586	.232	1.18886	.285	1.2567	.338	1.32854
.127	1.06895	.18	1.12699	.233	1.1901	.286	1.25803	.339	1.32993
.128	1.06998	.181	1.12813	.234	1.19134	.287	1.25936	.34	1.33132
.129	1.07001	.182	1.12927	.235	1.19258	.288	1.26069	.341	1.33272
.13	1.07204	.183	1.13041	.236	1.19382	.289	1.26202	.342	1.33412
.131	1.07308	.184	1.13155	.237	1.19506	.29	1.26335	.343	1.33552
.132	1.07412	.185	1.13269	.238	1.1963	.291	1.26468	.344	1.33692
.133	1.07516	.186	1.13383	.239	1.19755	.292	1.26601	.345	1.33833
.134	1.07621	.187	1.13497	.24	1.1988	.293	1.26734	.346	1.33974
.135	1.07726	.188	1.13611	.241	1.20005	.294	1.26867	.347	1.34115
.136	1.07831	.189	1.13726	.242	1.2013	.295	1.27	.348	1.34256
.137	1.07937	.19	1.13841	.243	1.20255	.296	1.27133	.349	1.34397
.138	1.08043	.191	1.13956	.244	1.2038	.297	1.27267	.35	1.34539
.139	1.08149	.192	1.14071	.245	1.20506	.298	1.27401	.351	1.34681
.14	1.08255	.193	1.14186	.246	1.20632	.299	1.27535	.352	1.34823
.141	1.08362	.194	1.14301	.247	1.20758	.3	1.27669	.353	1.34965
.142	1.08469	.195	1.14416	.248	1.20884	.301	1.27803	.354	1.35108
.143	1.08576	.196	1.14531	.249	1.2101	.302	1.27937	.355	1.35251
.144	1.08684	.197	1.14646	.25	1.21136	.303	1.28071	.356	1.35394
.145	1.08792	.198	1.14762	.251	1.21263	.304	1.28205	.357	1.35537
.146	1.08901	.199	1.14888	.252	1.2139	.305	1.28339	.358	1.3568
.147	1.0901	.2	1.15014	.253	1.21517	.306	1.28474	.359	1.35823
.148	1.09119	.201	1.15131	.254	1.21644	.307	1.28609	.36	1.35967
.149	1.09228	.202	1.15248	.255	1.21772	.308	1.28744	.361	1.36111
.15	1.0933	.203	1.15366	.256	1.219	.309	1.28879	.362	1.36255
.151	1.09448	.204	1.15484	.257	1.22028	.31	1.29014	.363	1.36399
.152	1.09558	.205	1.15602	.258	1.22156	.311	1.29149	.364	1.36543

Table—(*Continued*).

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.365	1.36588	.421	1.44913	.477	1.53469	.533	1.62216	.589	1.71065
.366	1.36833	.422	1.45064	.478	1.53625	.534	1.62372	.59	1.71225
.367	1.36978	.423	1.45214	.479	1.53781	.535	1.62528	.591	1.71286
.368	1.37123	.424	1.45364	.48	1.53937	.536	1.62684	.592	1.71546
.369	1.37268	.425	1.45515	.481	1.54093	.537	1.6284	.593	1.71707
.37	1.37414	.426	1.45665	.482	1.54249	.538	1.62996	.594	1.71868
.371	1.37662	.427	1.15815	.483	1.54405	.539	1.63152	.595	1.72029
.372	1.37708	.428	1.45966	.484	1.54561	.54	1.63309	.596	1.7219
.373	1.37854	.429	1.46167	.485	1.54718	.541	1.63465	.597	1.7235
.374	1.38	.43	1.46268	.486	1.54875	.542	1.63623	.598	1.72511
.375	1.38146	.431	1.46419	.487	1.55032	.543	1.6378	.599	1.72672
.376	1.38292	.432	1.4657	.488	1.55189	.544	1.63937	.6	1.72833
.377	1.38439	.433	1.46721	.489	1.55346	.545	1.64094	.601	1.72994
.378	1.38585	.434	1.46872	.49	1.55503	.546	1.64251	.602	1.73155
.379	1.38732	.435	1.47023	.491	1.5566	.547	1.64408	.603	1.73316
.38	1.38879	.436	1.47174	.492	1.55817	.548	1.64565	.604	1.73477
.381	1.39024	.437	1.47326	.493	1.55974	.549	1.64722	.605	1.73638
.382	1.39169	.438	1.47478	.494	1.56131	.55	1.64879	.606	1.73799
.383	1.39314	.439	1.4763	.495	1.56289	.551	1.65036	.607	1.7396
.384	1.39459	.44	1.47782	.496	1.56447	.552	1.65193	.608	1.74121
.385	1.39605	.441	1.47934	.497	1.56605	.553	1.6535	.609	1.74283
.386	1.39751	.442	1.48086	.498	1.56763	.554	1.65507	.61	1.74444
.387	1.39897	.443	1.48238	.499	1.56921	.555	1.65665	.611	1.74605
.388	1.40043	.444	1.48391	.5	1.57089	.556	1.65823	.612	1.74767
.389	1.40189	.445	1.48544	.501	1.57234	.557	1.65981	.613	1.74929
.39	1.40335	.446	1.48697	.502	1.57389	.558	1.66139	.614	1.75091
.391	1.40481	.447	1.4885	.503	1.57544	.559	1.66297	.615	1.75252
.392	1.40627	.448	1.49003	.504	1.57699	.56	1.66455	.616	1.75414
.393	1.40773	.449	1.49157	.505	1.57854	.561	1.66613	.617	1.75576
.394	1.40919	.45	1.49311	.506	1.58009	.562	1.66771	.618	1.75738
.395	1.41065	.451	1.49465	.507	1.58164	.563	1.66929	.619	1.759
.396	1.41211	.452	1.49618	.508	1.58319	.564	1.67087	.62	1.76062
.397	1.41357	.453	1.49771	.509	1.58474	.565	1.67245	.621	1.76224
.398	1.41504	.454	1.49924	.51	1.58629	.566	1.67403	.622	1.76386
.399	1.41651	.455	1.50077	.511	1.58784	.567	1.67561	.623	1.76548
.4	1.41798	.456	1.5023	.512	1.5894	.568	1.67719	.624	1.7671
.401	1.41945	.457	1.50383	.513	1.59096	.569	1.67877	.625	1.76872
.402	1.42092	.458	1.50536	.514	1.59252	.570	1.68036	.626	1.77034
.403	1.42239	.459	1.50689	.515	1.59408	.571	1.68195	.627	1.77197
.404	1.42386	.46	1.50842	.516	1.59564	.572	1.68354	.628	1.77359
.405	1.42533	.461	1.50996	.517	1.5972	.573	1.68513	.629	1.77521
.406	1.42681	.462	1.5115	.518	1.59876	.574	1.68672	.630	1.77684
.407	1.42829	.463	1.51304	.519	1.60032	.575	1.68831	.631	1.77847
.408	1.42977	.464	1.51458	.52	1.60188	.576	1.6899	.632	1.78009
.409	1.43125	.465	1.51612	.521	1.60344	.577	1.69149	.633	1.78172
.41	1.43273	.466	1.51766	.522	1.605	.578	1.69308	.634	1.78335
.411	1.43421	.467	1.5192	.523	1.60656	.579	1.69467	.635	1.78498
.412	1.43569	.468	1.52074	.524	1.60812	.580	1.69626	.636	1.7866
.413	1.43718	.469	1.52229	.525	1.60968	.581	1.69785	.637	1.78823
.414	1.43867	.47	1.52384	.526	1.61124	.582	1.69945	.638	1.78986
.415	1.44016	.471	1.52539	.527	1.6128	.583	1.70105	.639	1.79149
.416	1.44165	.472	1.52691	.528	1.61436	.584	1.70264	.64	1.79312
.417	1.44314	.473	1.52849	.529	1.61592	.585	1.70424	.641	1.79475
.418	1.44463	.474	1.53004	.53	1.61748	.586	1.70584	.642	1.70038
.419	1.44613	.475	1.53159	.531	1.61904	.587	1.70745	.643	1.79801
.42	1.44763	.476	1.53314	.532	1.6206	.588	1.70905	.644	1.79964

Table—(*Continued*).

H'ght	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.645	1.80127	.701	1.89352	.757	1.98794	.813	2.0848	.869	2.18475
.646	1.8029	.702	1.89519	.758	1.98964	.814	2.08656	.87	2.18656
.647	1.80454	.703	1.89685	.759	1.99134	.815	2.08832	.871	2.18837
.648	1.80617	.704	1.89851	.76	1.99305	.816	2.09008	.872	2.19018
.649	1.8078	.705	1.90017	.761	1.99476	.817	2.09198	.873	2.192
.65	1.80943	.706	1.90184	.762	1.99647	.818	2.0936	.874	2.19382
.651	1.81107	.707	1.9035	.763	1.99818	.819	2.09536	.875	2.19564
.652	1.81271	.708	1.90517	.764	1.99989	.82	2.09712	.876	2.19746
.653	1.81435	.709	1.90684	.765	2.0016	.821	2.09888	.877	2.19928
.654	1.81599	.71	1.90852	.766	2.00331	.822	2.10065	.878	2.2011
.655	1.81763	.711	1.91019	.767	2.00502	.823	2.10242	.879	2.20292
.656	1.81928	.712	1.91187	.768	2.00673	.824	2.10419	.88	2.20474
.657	1.82091	.713	1.91355	.769	2.00844	.825	2.10596	.881	2.20656
.658	1.82255	.714	1.91523	.77	2.01016	.826	2.10773	.882	2.20839
.659	1.82419	.715	1.91691	.771	2.01187	.827	2.1095	.883	2.21022
.66	1.82583	.716	1.91859	.772	2.01359	.828	2.11127	.884	2.21205
.661	1.82747	.717	1.92027	.773	2.01531	.829	2.11304	.885	2.21388
.662	1.82911	.718	1.92195	.774	2.01702	.83	2.11481	.886	2.21571
.663	1.83075	.719	1.92363	.775	2.01874	.831	2.11659	.887	2.21754
.664	1.8324	.72	1.92531	.776	2.02045	.832	2.11837	.888	2.21937
.665	1.83404	.721	1.927	.777	2.02217	.833	2.12015	.889	2.2212
.666	1.83568	.722	1.92868	.778	2.02389	.834	2.12193	.89	2.22303
.667	1.83733	.723	1.93036	.779	2.02561	.835	2.12371	.891	2.22486
.668	1.83897	.724	1.93204	.78	2.02733	.836	2.12549	.892	2.2267
.669	1.84061	.725	1.93373	.781	2.02907	.837	2.12727	.893	2.22854
.67	1.84226	.726	1.93541	.782	2.0308	.838	2.12905	.894	2.23038
.671	1.84391	.727	1.9371	.783	2.03252	.839	2.13083	.895	2.23222
.672	1.84556	.728	1.93878	.784	2.03425	.84	2.13261	.896	2.23406
.673	1.8472	.729	1.94046	.785	2.03598	.841	2.13439	.897	2.2359
.674	1.84885	.73	1.94215	.786	2.03771	.842	2.13618	.898	2.23774
.675	1.8505	.731	1.94383	.787	2.03944	.843	2.13797	.899	2.23958
.676	1.85215	.732	1.94552	.788	2.04117	.844	2.13976	.9	2.24142
.677	1.85379	.733	1.94721	.789	2.0429	.845	2.14155	.901	2.24325
.678	1.85544	.734	1.9489	.79	2.04462	.846	2.14334	.902	2.24508
.679	1.85709	.735	1.95059	.791	2.04635	.847	2.14513	.903	2.24691
.68	1.85874	.736	1.95228	.792	2.04809	.848	2.14692	.904	2.24874
.681	1.86039	.737	1.95397	.793	2.04983	.849	2.14871	.905	2.25057
.682	1.86205	.738	1.95566	.794	2.05157	.85	2.1505	.906	2.2524
.683	1.8637	.739	1.95735	.795	2.05331	.851	2.15229	.907	2.25423
.684	1.86535	.74	1.95994	.796	2.05505	.852	2.15409	.908	2.25606
.685	1.867	.741	1.96074	.797	2.05679	.853	2.15589	.909	2.25789
.686	1.86866	.742	1.96244	.798	2.05853	.854	2.1577	.91	2.25972
.687	1.87031	.743	1.96414	.799	2.06027	.855	2.1595	.911	2.26155
.688	1.87196	.744	1.96583	.8	2.06202	.856	2.1613	.912	2.26338
.689	1.87362	.745	1.96753	.801	2.06377	.857	2.16309	.913	2.26521
.69	1.87527	.746	1.96923	.802	2.06552	.858	2.16489	.914	2.26704
.691	1.87693	.747	1.97093	.803	2.06727	.859	2.16668	.915	2.26888
.692	1.87859	.748	1.97262	.804	2.06901	.86	2.16848	.916	2.27071
.693	1.88024	.749	1.97432	.805	2.07076	.861	2.17028	.917	2.27254
.694	1.8819	.75	1.97602	.806	2.07251	.862	2.17209	.918	2.27437
.695	1.88356	.751	1.97772	.807	2.07427	.863	2.17389	.919	2.2762
.696	1.88522	.752	1.97943	.808	2.07602	.864	2.1757	.92	2.27803
.697	1.88688	.753	1.98113	.809	2.07777	.865	2.17751	.921	2.27987
.698	1.88854	.754	1.98283	.81	2.07953	.866	2.17932	.922	2.2817
.699	1.8902	.755	1.98453	.811	2.08128	.867	2.18113	.923	2.28354
.7	1.89186	.756	1.98623	.812	2.08304	.868	2.18294	.924	2.28537

Table—(*Continued*).

H'ght.	Length.	H'ght.	Length	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.925	2.2872	.941	2.31666	.956	2.34483	.971	2.37334	.986	2.40208
.926	2.28903	.942	2.31852	.957	2.34673	.972	2.37525	.987	2.404
.927	2.29086	.943	2.32038	.958	2.34862	.973	2.37716	.988	2.40592
.928	2.2927	.944	2.32224	.959	2.35051	.974	2.37908	.989	2.40784
.929	2.29453	.945	2.32411	.96	2.35241	.975	2.381	.99	2.40976
.93	2.29636	.946	2.32598	.961	2.35431	.976	2.38291	.991	2.41169
.931	2.2982	.947	2.32785	.962	2.35621	.977	2.38482	.992	2.41362
.932	2.30004	.948	2.32972	.963	2.3581	.978	2.38673	.993	2.41556
.933	2.30188	.949	2.3316	.964	2.36	.979	2.38864	.994	2.41749
.934	2.30373	.95	2.33348	.965	2.36191	.98	2.39055	.995	2.41943
.935	2.30557	.951	2.33537	.966	2.36381	.981	2.39247	.996	2.42136
.936	2.30741	.952	2.33726	.967	2.36571	.982	2.39439	.997	2.42329
.937	2.30926	.953	2.33915	.968	2.36762	.983	2.39631	.998	2.42522
.938	2.31111	.954	2.34104	.969	2.36952	.984	2.39823	.999	2.42715
.939	2.31295	.955	2.34293	.97	2.37143	.985	2.40016	1.	2.42908
.94	2.31479								

To Ascertain the Length of a Semi-Elliptic Arc (right Semi-Ellipse) by the preceding Table.

Rule.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next right-hand column. Multiply the length thus obtained by the base of the arc, and the product will be the length of the arc.

Example.—What is the length of the arc of a semi-ellipse, the base being 70 feet, and the height 30.10 feet?

$30.10 \div 70 = .43$; and .43, per table, $= 1.46268$.

Then $1.46268 \times 70 = 102.3876$ *feet*.

When the Curve is not that of a Right Semi-Ellipse, the Height being half of the Transverse Diameter.

Rule.—Divide half the base by twice the height, then proceed as in the preceding example; multiply the tabular length by twice the height, and the product will be the length required.

Example.—What is the length of the arc of a semi-ellipse, the height being 35 feet, and the base 60 feet?

$60 \div 2 = 30$, and $30 \div \overline{35 \times 2} = .428$, the tabular length of which is 1.45966.

Then $1.45966 \times \overline{35 \times 2} = 102.1762$ *feet*.

Note.—If in the division of a height by the base there is a remainder, proceed in the manner given for the Lengths of Circular Arcs, page 200.

Table of the Areas of the Segments of a Circle.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area	Versed Sine.	Seg. Area.
.001	.00004	.053	.01601	.105	.04391	.157	.07892	.209	.11908
.002	.00012	.054	.01646	.106	.04452	.158	.07965	.21	.1199
.003	.00022	.055	.01691	.107	.04514	.159	.08038	.211	.12071
.004	.00034	.056	.01737	.108	.04575	.16	.08111	.212	.12153
.005	.00047	.057	.01783	.109	.04638	.161	.08185	.213	.12235
.006	.00062	.058	.0183	.11	.047	.162	.08258	.214	.12317
.007	.00078	.059	.01877	.111	.04763	.163	.08332	.215	.12399
.008	.00095	.06	.01924	.112	.04826	.164	.08406	.216	.12481
.009	.00113	.061	.01972	.113	.04889	.165	.0848	.217	.12563
.01	.00133	.062	.0202	.114	.04953	.166	.08554	.218	.12646
.011	.00153	.063	.02068	.115	.05016	.167	.08629	.219	.12728
.012	.00175	.064	.02117	.116	.0508	.168	.08704	.22	.12811
.013	.00197	.065	.02165	.117	.05145	.169	.08779	.221	.12894
.014	.0022	.066	.02215	.118	.05209	.17	.08853	.222	.12977
.015	.00244	.067	.02265	.119	.05274	.171	.08929	.223	.1306
.016	.00268	.068	.02315	.12	.05338	.172	.09004	.224	.13144
.017	.00294	.069	.02336	.121	.05404	.173	.0908	.225	.13227
.018	.0032	.07	.02417	.122	.05469	.174	.09155	.226	.13311
.019	.00347	.071	.02468	.123	.05534	.175	.09231	.227	.13394
.02	.00375	.072	.02519	.124	.056	.176	.09307	.228	.13478
.021	.00403	.073	.02571	.125	.05666	.177	.09384	.229	.13562
.022	.00432	.074	.02624	.126	.05733	.178	.0946	.23	.13646
.023	.00462	.075	.02676	.127	.05799	.179	.09537	.231	.13731
.024	.00492	.076	.02729	.128	.05866	.18	.09613	.232	.13815
.025	.00523	.077	.02782	.129	.05933	.181	.0969	.233	.139
.026	.00555	.078	.02835	.13	.06	.182	.09767	.234	.13984
.027	.00587	.079	.02889	.131	.06067	.183	.09845	.235	.14069
.028	.00619	.08	.02943	.132	.06135	.184	.09922	.236	.14154
.029	.00653	.081	.02997	.133	.06203	.185	.1	.237	.14239
.03	.00686	.082	.03052	.134	.06271	.186	.10077	.238	.14324
.031	.00721	.083	.03107	.135	.06339	.187	.10155	.239	.14409
.032	.00756	.084	.03162	.136	.06407	.188	.10233	.24	.14494
.033	.00791	.085	.03218	.137	.06476	.189	.10312	.241	.1458
.034	.00827	.086	.03274	.138	.06545	.19	.1039	.242	.14665
.035	.00864	.087	.0333	.139	.06614	.191	.10468	.243	.14751
.036	.00901	.088	.03387	.14	.06683	.192	.10547	.244	.14837
.037	.00938	.089	.03444	.141	.06753	.193	.10626	.245	.14923
.038	.00976	.09	.03501	.142	.06822	.194	.10705	.246	.15009
.039	.01015	.091	.03558	.143	.06892	.195	.10784	.247	.15095
.04	.01054	.092	.03616	.144	.06962	.196	.10864	.248	.15182
.041	.01093	.093	.03674	.145	.07033	.197	.10943	.249	.15268
.042	.01133	.094	.03732	.146	.07103	.198	.11023	.25	.15355
.043	.01173	.095	.0379	.147	.07174	.199	.11102	.251	.15441
.044	.01214	.096	.03849	.148	.07245	.2	.11182	.252	.15528
.045	.01255	.097	.03908	.149	.07316	.201	.11262	.253	.15615
.046	.01297	.098	.03968	.15	.07387	.202	.11343	.254	.15702
.047	.01339	.099	.04027	.151	.07459	.203	.11423	.255	.15789
.048	.01382	.1	.04087	.152	.07531	.204	.11503	.256	.15876
.049	.01425	.101	.04148	.153	.07603	.205	.11584	.257	.15964
.05	.01468	.102	.04208	.154	.07675	.206	.11665	.258	.16051
.051	.01512	.103	.04269	.155	.07747	.207	.11746	.259	.16139
.052	.01556	.104	.0431	.156	.0782	.208	.11827	.26	.16226

Table—(*Continued*).

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
.261	.16314	.309	.20645	.357	.25167	.405	.29827	.453	.34557
.262	.16402	.31	.20738	.358	.25263	.406	.29925	.454	.34676
.263	.1649	.311	.2083	.359	.25359	.407	.30024	.455	.34776
.264	.16578	.312	.20923	.36	.25455	.408	.30122	.456	.34875
.265	.16666	.313	.21015	.361	.25551	.409	.3022	.457	.34975
.266	.16755	.314	.21108	.362	.25647	.41	.30319	.458	.35075
.267	.16844	.315	.21201	.363	.25743	.411	.30417	.459	.35174
.268	.16931	.316	.21294	.364	.25839	.412	.30515	.46	.35274
.269	.1702	.317	.21387	.365	.25936	.413	.30614	.461	.35374
.27	.17109	.318	.2148	.366	.26032	.414	.30712	.462	.35474
.271	.17197	.319	.21573	.367	.26128	.415	.30811	.463	.35573
.272	.17287	.32	.21667	.368	.26225	.416	.30909	.464	.35673
.273	.17376	.321	.2176	.369	.26321	.417	.31008	.465	.35773
.274	.17465	.322	.21853	.37	.26418	.418	.31107	.466	.35872
.275	.17554	.323	.21947	.371	.26514	.419	.31205	.467	.35972
.276	.17643	.324	.2204	.372	.26611	.42	.31304	.468	.36072
.277	.17733	.325	.22134	.373	.26708	.421	.31403	.469	.36172
.278	.17822	.326	.22228	.374	.26804	.422	.31502	.47	.36272
.279	.17912	.327	.22321	.375	.26901	.423	.316	.471	.36371
.28	.18002	.328	.22415	.376	.26998	.424	.31699	.472	.36471
.281	.18092	.329	.22509	.377	.27095	.425	.31798	.473	.36571
.282	.18182	.33	.22603	.378	.27192	.426	.31897	.474	.36671
.283	.18272	.331	.22697	.379	.27289	.427	.31996	.475	.36771
.284	.18361	.332	.22791	.38	.27386	.428	.32095	.476	.36871
.285	.18452	.333	.22886	.381	.27483	.429	.32194	.477	.36971
.286	.18542	.334	.2298	.382	.27580	.43	.32293	.478	.37071
.287	.18633	.335	.23074	.383	.27677	.431	.32391	.479	.3717
.288	.18723	.336	.23169	.384	.27775	.432	.3249	.48	.3727
.289	.18814	.337	.23263	.385	.27872	.433	.3259	.481	.3737
.29	.18905	.338	.23358	.386	.27969	.434	.32689	.482	.3747
.291	.18995	.339	.23453	.387	.28067	.435	.32788	.483	.3757
.292	.19086	.34	.23547	.388	.28164	.436	.32887	.484	.3767
.293	.19177	.341	.23642	.389	.28262	.437	.32987	.485	.3777
.294	.19268	.342	.23737	.39	.28359	.438	.33086	.486	.3787
.295	.1936	.343	.23832	.391	.28457	.439	.33185	.487	.3797
.296	.19451	.344	.23927	.392	.28554	.44	.33284	.488	.3807
.297	.19542	.345	.24022	.393	.28652	.441	.33384	.489	.3817
.298	.19634	.346	.24117	.394	.2875	.442	.33483	.49	.3827
.299	.19725	.347	.24212	.395	.28848	.443	.33582	.491	.3837
.3	.19817	.348	.24307	.396	.28945	.444	.33682	.492	.3847
.301	.19908	.349	.24403	.397	.29043	.445	.33781	.493	.3857
.302	.2	.35	.24498	.398	.29141	.446	.3388	.494	.3867
.303	.20092	.351	.24593	.399	.29239	.447	.3398	.495	.3877
.304	.20184	.352	.24689	.4	.29337	.448	.34079	.496	.3887
.305	.20276	.353	.24784	.401	.29435	.449	.34179	.497	.3897
.306	.20368	.354	.2488	.402	.29533	.45	.34278	.498	.3907
.307	.2046	.355	.24976	.403	.29631	.451	.34378	.499	.3917
.308	.20553	.356	.25071	.404	.29729	.452	.34477	.5	.3927

To Ascertain the Area of a Segment of a Circle by the preceding Table.

RULE.—Divide the height or versed sine by the diameter of the circle; find the quotient in the column of versed sines. Take the area noted in the next column, multiply it by the square of the diameter, and it will give the area.

EXAMPLE.—Required the area of a segment, its height being 10, and the diameter of the circle 50 feet.

$10 \div 50 = .2$, and .2, per table, $= .11182$; then $.11182 \times 50^2 = 279.55$ *feet.*

NOTE.—If, in the division of a height by the base, the quotient has a remainder after the third place of decimals, and great accuracy is required,

Take the area for the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area; the sum will be the area for the whole quotient.

2. What is the area of a segment of a circle, the diameter of which is 10 feet, and the height of it 1.575 feet?

$1.575 \div 10 = .1575$; the tabular area for $.157 = .07892$, and for $.158 = .07965$, the difference between which is .00073.

Then $.5 \times .00073 = .000365$.

Hence $.157 = .07892$
$.0005 = .000365$
$.079285$, the sum by which the square of the diameter of the circle is to be multiplied; and $.079285 \times 10^2 = 7.9285$ *feet.*

Table of the Areas of the Zones of a Circle.

The Diameter of a Circle assumed to be Unity, and divided into 1000 *equal Parts.*

H'ght	Area.	Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.001	.001	.035	.03497	.069	.06878	.103	.10227	.137	.13527
.002	.002	.036	.03597	.07	.06977	.104	.10325	.138	.13623
.003	.003	.037	.03697	.071	.07076	.105	.10422	.139	.13719
.004	.004	.038	.03796	.072	.07175	.106	.1052	.14	.13815
.005	.005	.039	.03896	.073	.07274	.107	.10618	.141	.13911
.006	.006	.04	.03996	.074	.07373	.108	.10715	.142	.14007
.007	.007	.041	.04095	.075	.07472	.109	.10813	.143	.14103
.008	.008	.042	.04195	.076	.0755	.11	.10911	.144	.14198
.009	.009	.043	.04295	.077	.07669	.111	.11008	.145	.14294
.01	.01	.044	.04394	.078	.07768	.112	.11106	.146	.1439
.011	.011	.045	.04494	.079	.07867	.113	.11203	.147	.14485
.012	.012	.046	.04593	.08	.07966	.114	.113	.148	.14581
.013	.013	.047	.04693	.081	.08064	.115	.11398	.149	.14677
.014	.014	.048	.04793	.082	.08163	.116	.11495	.15	.14772
.015	.015	.049	.04892	.083	.08262	.117	.11592	.151	.14867
.016	.016	.05	.04992	.084	.0836	.118	.1169	.152	.14962
.017	.017	.051	.05091	.085	.08459	.119	.11787	.153	.15058
.018	.018	.052	.0519	.086	.08557	.12	.11884	.154	.15153
.019	.019	.053	.0529	.087	.08656	.121	.11981	.155	.15248
.02	.02	.054	.05389	.088	.08754	.122	.12078	.156	.15343
.021	.021	.055	.05489	.089	.08853	.123	.12175	.157	.15438
.022	.022	.056	.05588	.09	.08951	.124	.12272	.158	.15533
.023	.023	.057	.05688	.091	.0905	.125	.12369	.159	.15628
.024	.024	.058	.05787	.092	.09148	.126	.12469	.16	.15723
.025	.025	.059	.05886	.093	.09246	.127	.12562	.161	.15817
.026	.02599	.06	.05986	.094	.09344	.128	.12659	.162	.15912
.027	.02699	.061	.06085	.095	.09443	.129	.12755	.163	.16006
.028	.02799	.062	.06184	.096	.0954	.13	.12852	.164	.16101
.029	.02898	.063	.06283	.097	.09639	.131	.12949	.165	.16195
.03	.02998	.064	.06382	.098	.09737	.132	.13045	.166	.1629
.031	.03098	.065	.06482	.099	.09835	.133	.13141	.167	.16384
.032	.03198	.066	.0658	.1	.09933	.134	.13238	.168	.16478
.033	.03298	.067	.0668	.101	.10031	.135	.13334	.169	.16572
.034	.03397	.068	.0678	.102	.10129	.136	.1343	.17	.16667

Table—(*Continued*).

H'ght.	Area.	Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.171	.16761	.227	.21894	.283	.26706	.339	.31085	.395	.34879
.172	.16855	.228	.21983	.284	.26789	.34	.31159	.396	.3494
.173	.16948	.229	.22072	.285	.26871	.341	.31232	.397	.35001
.174	.17042	.23	.22161	.286	.26953	.342	.31305	.398	.35062
.175	.17136	.231	.2225	.287	.27035	.343	.31378	.399	.35122
.176	.1723	.232	.22335	.288	.27117	.344	.3145	.4	.35182
.177	.17323	.233	.22427	.289	.27199	.345	.31523	.401	.35242
.178	.17417	.234	.22515	.29	.2728	.346	.31595	.402	.35302
.179	.1751	.235	.22604	.291	.27362	.347	.31667	.403	.35361
.18	.17603	.236	.22692	.292	.27443	.348	.31739	.404	.3542
.181	.17697	.237	.2278	.293	.27524	.349	.31811	.405	.35479
.182	.1779	.238	.22868	.294	.27605	.35	.31882	.406	.35538
.183	.17883	.239	.22956	.295	.27686	.351	.31954	.407	.35596
.184	.17976	.24	.23044	.296	.27766	.352	.32025	.408	.35654
.185	.18069	.241	.23131	.297	.27847	.353	.32096	.409	.35711
.186	.18162	.242	.23219	.298	.27927	.354	.32167	.41	.35769
.187	.18254	.243	.23306	.299	.28007	.355	.32237	.411	.35826
.188	.18347	.244	.23394	.3	.28088	.356	.32307	.412	.35883
.189	.1844	.245	.23481	.301	.28167	.357	.32377	.413	.35939
.19	.18532	.246	.23568	.302	.28247	.358	.32447	.414	.35995
.191	.18625	.247	.23655	.303	.28327	.359	.32517	.415	.36051
.192	.18717	.248	.23742	.304	.28406	.36	.32587	.416	.36107
.193	.18809	.249	.23829	.305	.28486	.361	.32656	.417	.36162
.194	.18902	.25	.23915	.306	.28565	.362	.32725	.418	.36217
.195	.18994	.251	.24002	.307	.28644	.363	.32794	.419	.36272
.196	.19086	.252	.24089	.308	.28723	.364	.32862	.42	.36326
.197	.19178	.253	.24175	.309	.28801	.365	.32931	.421	.3638
.198	.1927	.254	.24261	.31	.2888	.366	.32999	.422	.36434
.199	.19361	.255	.24347	.311	.28958	.367	.33067	.423	.36488
.2	.19453	.256	.24433	.312	.29036	.368	.33135	.424	.36541
.201	.19545	.257	.24519	.313	.29115	.369	.33203	.425	.36594
.202	.19636	.258	.24604	.314	.29192	.37	.3327	.426	.36646
.203	.19728	.259	.2469	.315	.2927	.371	.33337	.427	.36698
.204	.19819	.26	.24775	.316	.29348	.372	.33404	.428	.3675
.205	.1991	.261	.24861	.317	.29425	.373	.33471	.429	.36802
.206	.20001	.262	.24946	.318	.29502	.374	.33537	.43	.36853
.207	.20092	.263	.25021	.319	.2958	.375	.33604	.431	.36904
.208	.20183	.264	.25116	.32	.29656	.376	.3367	.432	.36954
.209	.20274	.265	.25201	.321	.29733	.377	.33735	.433	.37005
.21	.20365	.266	.25285	.322	.2981	.378	.33801	.434	.37054
.211	.20156	.267	.2537	.323	.29886	.379	.33866	.435	.37104
.212	.20546	.268	.25455	.324	.29962	.38	.33931	.436	.37153
.213	.20637	.269	.25539	.325	.30039	.381	.33996	.437	.37202
.214	.20727	.27	.25623	.326	.30114	.382	.34061	.438	.3725
.215	.20818	.271	.25707	.327	.3019	.383	.34125	.439	.37298
.216	.20908	.272	.25791	.328	.30266	.384	.3419	.44	.37346
.217	.20998	.273	.25875	.329	.30341	.385	.34253	.441	.37393
.218	.21088	.274	.25959	.33	.30416	.386	.34317	.442	.3744
.219	.21178	.275	.26043	.331	.30491	.387	.3438	.443	.37487
.22	.21268	.276	.26126	.332	.30566	.388	.34444	.444	.37533
.221	.21358	.277	.26209	.333	.30641	.389	.34507	.445	.37579
.222	.21447	.278	.26293	.334	.30715	.39	.34569	.446	.37624
.223	.21537	.279	.26376	.335	.3079	.391	.34632	.447	.37669
.224	.21626	.28	.26459	.336	.30864	.392	.34694	.448	.37714
.225	.21716	.281	.26541	.337	.30938	.393	.34756	.449	.37758
.226	.21805	.282	.26624	.338	.31012	.394	.34818	.45	.37802

Table—(*Continued*).

H'ght.	Area.	Height.	Area.	Height.	Area.	Height.	Area.	Height.	Area.
.451	.37845	.461	.38255	.471	.38617	.481	.38923	.491	.39156
.452	.37888	.462	.38294	.472	.3865	.482	.3895	.492	.39175
.453	.37931	.463	.38332	.473	.38683	.483	.38976	.493	.39192
.454	.37973	.464	.38369	.474	.38715	.484	.39001	.494	.39208
.455	.38014	.465	.38406	.475	.38747	.485	.39026	.495	.39223
.456	.38056	.466	.38443	.476	.38778	.486	.3905	.496	.39236
.457	.38096	.467	.38479	.477	.38808	.487	.39073	.497	.39248
.458	.38137	.468	.38514	.478	.38838	.488	.39095	.498	.39258
.459	.38177	.469	.38549	.479	.38867	.489	.39117	.499	.39266
.46	.38216	.47	.38583	.48	.38895	.49	.39137	.5	.3927

This Table is computed only for Zones, the longest chord of which is diameter.

To Ascertain the Area of a Zone by the preceding Table.

RULE 1.—*When the Zone is Less than a Semicircle*, Divide the height by the diameter, and find the quotient in the column of heights. Take out the area opposite to it in the next column on the right hand, and multiply it by the square of the longest chord; the product will be the area of the zone.

EXAMPLE.—Required the area of a zone the diameter of which is 50, and its height 15.

$15 \div 50 = .3$; and .3, as per table, $= .28088$.

Hence $.28088 \times 50^2 = 702.2$ *area*.

RULE 2.—*When the Zone is Greater than a Semicircle*, Take the height on each side of the diameter of the circle, and ascertain, by Rule 1, their respective areas; add the areas of these two portions together, and the sum will be the area of the zone.

EXAMPLE.—Required the area of a zone, the diameter of the circle being 50, and the heights of the zone on each side of the diameter of the circle 20 and 15 respectively.

$20 \div 50 = .4$; .4, as per table, $= .35182$; and $.35182 \times 50^2 = 879.55$.
$15 \div 50 = .3$; .3, as per table, $= .28088$; and $.28088 \times 50^2 = 702.2$.

Hence $879.55 + 702.2 = 1581.75$ *area*.

NOTE.—When, in the division of a height by the chord, the quotient has a remainder after the third place of decimals, and great accuracy is required,

Take the area for the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area; the sum will be the area for the whole quotient.

EXAMPLE.—What is the area of a zone of a circle, the greater chord being 100 feet, and the breadth of it 14 feet 3 inches?

14 feet 3 inches $= 14.25$, and $14.25 \div 100 = .1425$; the tabular length for $.142 = .14007$, and for $.143 = .14103$, the difference between which is .00096.

Then $.5 \times .00096 = .00048$.

Hence $.142 = .14007$
$.0005 = .00048$

.14055, the sum by which the square of the greater chord is to be multiplied; and $.14055 \times 100^2 = 1405.5$ *feet*.

S*

Table of Squares, Cubes, and Square and Cube Roots, of all Numbers from 1 to 1600.

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1.	1.
2	4	8	1.4142 136	1.2599 21
3	9	27	1.7320 508	1.4422 496
4	16	64	2.	1.5874 011
5	25	125	2.2360 68	1.7099 759
6	36	216	2.4494 897	1.8171 206
7	49	343	2.6457 513	1.9129 312
8	64	512	2.8284 271	2.
9	81	729	3.	2.0800 837
10	1 00	1 000	3.1622 777	2.1544 347
11	1 21	1 331	3.3166 248	2.2239 801
12	1 44	1 728	3.4641 016	2.2894 286
13	1 69	2 197	3.6055 513	2.3513 347
14	1 96	2 744	3.7416 574	2.4101 422
15	2 25	3 375	3.8729 833	2.4662 121
16	2 56	4 096	4.	2.5198 421
17	2 89	4 913	4.1231 056	2.5712 816
18	3 24	5 832	4.2426 407	2.6207 414
19	3 61	6 859	4.3585 989	2.6684 016
20	4 00	8 000	4.4721 36	2.7144 177
21	4 41	9 261	4.5825 757	2.7589 243
22	4 84	10 648	4.6904 158	2.8020 393
23	5 29	12 167	4.7958 315	2.8438 67
24	5 76	13 824	4.8989 795	2.8844 991
25	6 25	15 625	5.	2.9240 177
26	6 76	17 576	5.0990 195	2.9224 96
27	7 29	19 683	5.1961 524	3.
28	7 84	21 952	5.2915 026	3.0365 889
29	8 41	24 389	5.3851 648	3.0723 168
30	9 00	27 000	5.4772 256	3.1072 325
31	9 61	29 791	5.5677 644	3.1413 806
32	10 24	32 768	5.6568 542	3.1748 021
33	10 89	35 937	5.7445 626	3.2075 343
34	11 56	39 304	5.8309 519	3.2396 118
35	12 25	42 875	5.9160 798	3.2710 663
36	12 96	46 656	6.	3.3019 272
37	13 69	50 653	6.0827 625	3.3322 218
38	14 44	54 872	6.1644 14	3.3619 754
39	15 21	59 319	6.2449 98	3.3912 114
40	16 00	64 000	6.3245 553	3.4199 519
41	16 81	68 921	6.4031 242	3.4482 172
42	17 64	74 088	6.4807 407	3.4760 266
43	18 49	79 507	6.5574 385	3.5033 981
44	19 36	85 184	6.6332 496	3.5303 483
45	20 25	91 125	6.7082 039	3.5568 933
46	21 16	97 336	6.7823 3	3.5830 479
47	22 09	103 823	6.8556 546	3.6088 261
48	23 04	110 592	6.9282 032	3.6342 411
49	24 01	117 649	7.	3.6593 057
50	25 00	125 000	7.0710 678	3.6840 314
51	26 01	132 651	7.1414 284	3.7084 298
52	27 04	140 608	7.2111 026	3.7325 111
53	28 09	148 877	7.2801 099	3.7562 858
54	29 16	157 464	7.3484 692	3.7797 631
55	30 25	166 375	7.4161 985	3.8029 525

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
56	31 36	175 616	7.4833 148	3.8258 624
57	32 49	185 193	7.5498 344	3.8485 011
58	33 64	195 112	7.6157 731	3.8708 766
59	34 81	205 379	7.6811 457	3.8929 965
60	36 00	216 000	7.7459 667	3.9148 676
61	37 21	226 981	7.8102 497	3.9364 972
62	38 44	238 328	7.8740 079	3.9578 915
63	39 69	250 047	7.9372 539	3.9790 571
64	40 96	262 144	8.	4.
65	42 25	274 625	8.0622 577	4.0207 256
66	43 56	287 496	8.1240 384	4.0412 401
67	44 89	300 763	8.1853 528	4.0615 48
68	46 24	314 432	8.2462 113	4.0816 551
69	47 61	328 509	8.3066 239	4.1015 661
70	49 00	343 000	8.3666 003	4.1212 853
71	50 41	357 911	8.4261 498	4.1408 178
72	51 84	373 248	8.4852 814	4.1601 676
73	53 29	389 017	8.5440 037	4.1793 39
74	54 76	405 224	8.6023 253	4.1983 364
75	56 25	421 875	8.6602 54	4.2171 633
76	57 76	438 976	8.7177 979	4.2358 236
77	59 29	456 533	8.7749 644	4.2543 21
78	60 84	474 552	8.8317 609	4.2726 586
79	62 41	493 039	8.8881 944	4.2908 404
80	64 00	512 000	8.9442 719	4.3088 695
81	65 61	531 441	9.	4.3267 487
82	67 24	551 368	9.0553 851	4.3444 815
83	68 89	571 787	9.1104 336	4.3620 707
84	70 56	592 704	9.1651 514	4.3795 191
85	72 25	614 125	9.2195 445	4.3968 296
86	73 96	636 056	9.2736 185	4.4140 049
87	75 69	658 503	9.3273 791	4.4310 476
88	77 44	681 472	9.3808 315	4.4479 602
89	79 21	704 969	9.4339 811	4.4647 451
90	81 00	729 000	9.4868 33	4.4814 047
91	82 81	753 571	9.5393 92	4.4979 414
92	84 64	778 688	9.5916 63	4.5143 574
93	86 49	804 357	9.6436 508	4.5306 549
94	88 36	830 584	9.6953 597	4.5468 359
95	90 25	857 375	9.7467 943	4.5629 026
96	92 16	884 736	9.7979 59	4.5788 57
97	94 09	912 673	9.8488 578	4.5947 009
98	96 04	941 192	9.8994 949	4.6104 363
99	98 01	970 299	9.9498 744	4.6260 65
100	1 00 00	1 000 000	10.	4.6415 888
101	1 02 01	1 030 301	10.0498 756	4.6570 095
102	1 04 04	1 061 208	10.0995 049	4.6723 287
103	1 06 09	1 092 727	10.1488 916	4.6875 482
104	1 08 16	1 124 864	10.1980 39	4.7026 694
105	1 10 25	1 157 625	10.2469 508	4.7176 94
106	1 12 36	1 191 016	10.2956 301	4.7326 235
107	1 14 49	1 225 043	10.3440 804	4.7474 594
108	1 16 64	1 259 712	10.3923 048	4.7622 032
109	1 18 81	1 295 029	10.4403 065	4.7768 562
110	1 21 00	1 331 000	10.4880 885	4.7914 199
111	1 23 21	1 367 631	10.5356 538	4.8058 995

Table—(*Continued*).

Number.	Square.	Cube	Square Root.	Cube Root.
112	1 25 44	1 404 928	10.5830 052	4.8202 845
113	1 27 69	1 442 897	10.6301 458	4.8345 881
114	1 29 96	1 481 544	10.6770 783	4.8488 076
115	1 32 25	1 520 875	10.7238 053	4.8629 442
116	1 34 56	1 560 896	10.7703 296	4.8769 99
117	1 36 89	1 601 613	10.8166 538	4.8909 732
118	1 39 24	1 643 032	10.8627 805	4.9048 681
119	1 41 61	1 685 159	10.9087 121	4.9186 847
120	1 44 00	1 728 000	10.9544 512	4.9324 242
121	1 46 41	1 771 561	11.	4.9460 874
122	1 48 34	1 815 848	11.0453 61	4.9596 757
123	1 51 29	1 860 867	11.0905 365	4.9731 898
124	1 53 76	1 906 624	11.1355 287	4.9866 31
125	1 56 25	1 953 125	11.1803 399	5.
126	1 58 76	2 000 376	11.2249 722	5.0132 979
127	1 61 29	2 048 383	11.2694 277	5.0265 257
128	1 63 84	2 097 152	11.3137 085	5.0396 842
129	1 66 41	2 146 689	11.3578 167	5.0527 743
130	1 69 00	2 197 000	11.4017 543	5.0657 97
131	1 71 61	2 248 091	11.4455 231	5.0787 531
132	1 74 24	2 299 968	11.4891 253	5.0916 434
133	1 76 89	2 352 637	11.5325 626	5.1044 687
134	1 79 56	2 406 104	11.5758 369	5.1172 299
135	1 82 25	2 460 375	11.6189 5	5.1299 278
136	1 84 96	2 515 456	11.6619 038	5.1425 632
137	1 87 69	2 571 353	11.7046 999	5.1551 367
138	1 90 44	2 628 072	11.7473 401	5.1676 493
139	1 93 21	2 685 619	11.7898 261	5.1801 015
140	1 96 00	2 744 000	11.8321 596	5.1924 941
141	1 98 81	2 803 221	11.8743 421	5.2048 279
142	2 01 64	2 863 288	11.9163 753	5.2171 034
143	2 04 49	2 924 207	11.9582 607	5.2293 215
144	2 07 36	2 985 984	12.	5.2414 828
145	2 10 25	3 048 625	12.0415 946	5.2535 879
146	2 13 16	3 112 136	12.0830 46	5.2656 374
147	2 16 09	3 176 523	12.1243 557	5.2776 321
148	2 19 04	3 241 792	12.1655 251	5.2895 725
149	2 22 01	3 307 949	12.2065 556	5.3014 592
150	2 25 00	3 375 000	12.2474 487	5.3132 928
151	2 28 01	3 442 951	12.2882 057	5.3250 74
152	2 31 04	3 511 008	12.3288 28	5.3368 033
153	2 34 09	3 581 577	12.3693 169	5.3484 812
154	2 37 16	3 652 264	12.4096 736	5.3601 084
155	2 40 25	3 723 875	12.4498 996	5.3716 854
156	2 43 36	3 796 416	12.4899 96	5.3832 126
157	2 46 49	3 869 893	12.5299 641	5.3946 907
158	2 49 64	3 944 312	12.5698 051	5.4061 202
159	2 52 81	4 019 679	12.6095 202	5.4175 015
160	2 56 00	4 096 000	12.6491 106	5.4288 352
161	2 59 21	4 173 281	12.6885 775	5.4401 218
162	2 62 44	4 251 528	12.7279 221	5.4513 618
163	2 65 69	4 330 747	12.7671 453	5.4625 556
164	2 68 96	4 410 944	12.8062 485	5.4737 037
165	2 72 25	4 492 125	12.8452 326	5.4848 066
166	2 75 56	4 574 296	12.8840 987	5.4958 647
167	2 78 89	4 657 463	12.9228 48	5.5068 784

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
168	2 82 24	4 741 632	12.9614 814	5.5178 484
169	2 85 61	4 826 809	13.	5.5287 748
170	2 89 00	4 913 000	13.0384 048	5.5396 583
171	2 92 41	5 000 211	13.0766 968	5.5504 991
172	2 95 84	5 088 448	13.1148 77	5.5612 978
173	2 99 29	5 177 717	13.1529 464	5.5720 546
174	3 02 76	5 268 024	13.1909 06	5.5827 702
175	3 06 25	5 359 375	13.2287 566	5.5934 447
176	3 09 76	5 451 776	13.2664 992	5.6040 787
177	3 13 29	5 545 233	13.3041 347	5.6146 724
178	3 16 84	5 639 752	13.3416 641	5.6252 263
179	3 20 41	5 735 339	13.3790 882	5.6357 408
180	3 24 00	5 832 000	13.4164 079	5.6462 162
181	3 27 61	5 929 741	13.4536 24	5.6566 528
182	3 31 24	6 028 568	13.4907 376	5.6670 511
183	3 34 89	6 128 487	13.5277 493	5.6774 114
184	3 38 56	6 229 504	13.5646 6	5.6877 34
185	3 42 25	6 331 625	13.6014 705	5.6980 192
186	3 45 96	6 434 856	13.6381 817	5.7082 675
187	3 49 69	6 539 203	13.6747 943	5.7184 791
188	3 53 44	6 644 672	13.7113 092	5.7286 543
189	3 57 21	6 751 269	13.7477 271	5.7387 936
190	3 61 00	6 859 000	13.7840 488	5.7488 971
191	3 64 81	6 967 871	13.8202 75	5.7589 652
192	3 68 64	7 077 888	13.8564 065	5.7689 982
193	3 72 49	7 189 057	13.8924 4	5.7789 966
194	3 76 36	7 301 384	13.9283 883	5.7889 604
195	3 80 25	7 414 875	13.9642 4	5.7988 9
196	3 84 16	7 529 536	14.	5.8087 857
197	3 88 09	7 645 373	14.0356 688	5.8186 479
198	3 92 04	7 762 392	14.0712 473	5.8284 867
199	3 96 01	7 880 599	14.1067 36	5.8382 725
200	4 00 00	8 000 000	14.1421 356	5.8480 355
201	4 04 01	8 120 601	14.1774 469	5.8577 66
202	4 08 04	8 242 408	14.2126 704	5.8674 673
203	4 12 09	8 365 427	14.2478 068	5.8771 307
204	4 16 16	8 489 664	14.2828 569	5.8867 653
205	4 20 25	8 615 125	14.3178 211	5.8963 685
206	4 24 36	8 741 816	14.3527 001	5.9059 406
207	4 28 49	8 869 743	14.3874 946	5.9154 817
208	4 32 64	8 998 912	14.4222 051	5.9249 921
209	4 36 81	9 129 329	14.4568 323	5.9344 721
210	4 41 00	9 261 000	14.4913 767	5.9439 22
211	4 45 21	9 393 931	14.5258 39	5.9533 418
212	4 49 44	9 528 128	14.5602 198	5.9627 32
213	4 53 69	9 663 597	14.5945 195	5.9720 926
214	4 57 96	9 800 344	14.6287 388	5.9814 24
215	4 62 25	9 938 375	14.6628 783	5.9907 264
216	4 66 56	10 077 696	14.6969 385	6.
217	4 70 89	10 218 313	14.7309 199	6.0092 45
218	4 75 24	10 360 232	14.7648 231	6.0184 617
219	4 79 61	10 503 459	14.7986 486	6.0276 502
220	4 84 00	10 648 000	14.8323 97	6.0368 107
221	4 88 41	10 793 861	14.8660 687	6.0459 435
222	4 92 84	10 941 048	14.8996 644	6.0550 489
223	4 97 29	11 089 567	14.9331 845	6.0641 27

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
224	5 01 76	11 239 424	14.9666 295	6.0731 779
225	5 06 25	11 390 625	15.	6.0822 02
226	5 10 76	11 543 176	15.0332 964	6.0911 994
227	5 15 29	11 697 083	15.0665 192	6.1001 702
228	5 19 84	11 852 352	15.0996 689	6.1091 147
229	5 24 41	12 008 989	15.1327 46	6.1180 332
230	5 29 00	12 167 000	15.1657 509	6.1269 257
231	5 33 61	12 326 391	15.1986 842	6.1357 924
232	5 38 24	12 487 168	15.2315 462	6.1446 337
233	5 42 89	12 649 337	15.2643 375	6.1534 495
234	5 47 56	12 812 904	15.2970 585	6.1622 401
235	5 52 25	12 977 875	15.3297 097	6.1710 058
236	5 56 96	13 144 256	15.3622 915	6.1797 466
237	5 61 69	13 312 053	15.3948 043	6.1884 628
238	5 66 44	13 481 272	15.4272 486	6.1971 544
239	5 71 21	13 651 919	15.4596 248	6.2058 218
240	5 76 00	13 824 000	15.4919 334	6.2144 65
241	5 80 81	13 997 521	15.5241 747	6.2230 843
242	5 85 64	14 172 488	15.5563 492	6.2316 797
243	5 90 49	14 348 907	15.5884 573	6.2402 515
244	5 95 36	14 526 784	15.6204 994	6.2487 998
245	6 00 25	14 706 125	15.6524 758	6.2573 248
246	6 05 16	14 886 936	15.6843 871	6.2658 266
247	6 10 09	15 069 223	15.7162 336	6.2743 054
248	6 15 04	15 252 992	15.7480 157	6.2827 613
249	6 20 01	15 438 249	15.7797 338	6.2911 946
250	6 25 00	15 625 000	15.8113 883	6.2996 053
251	6 30 01	15 813 251	15.8429 795	6.3079 935
252	6 35 04	16 003 008	15.8745 079	6.3163 596
253	6 40 09	16 194 277	15.9059 737	6.3247 035
254	6 45 16	16 387 064	15.9373 775	6.3330 256
255	6 50 25	16 581 375	15.9687 194	6.3413 257
256	6 55 36	16 777 216	16.	6.3496 042
257	6 60 49	16 974 593	16.0312 195	6.3578 611
258	6 65 64	17 173 512	16.0623 784	6.3660 968
259	6 70 81	17 373 979	16.0934 769	6.3743 111
260	6 76 00	17 576 000	16.1245 155	6.3825 043
261	6 81 21	17 779 581	16.1554 944	6.3906 765
262	6 86 44	17 984 728	16.1864 141	6.3988 279
263	6 91 69	18 191 447	16.2172 747	6.4069 585
264	6 96 96	18 399 744	16.2480 768	6.4150 687
265	7 02 25	18 609 625	16.2788 206	6.4231 583
266	7 07 56	18 821 096	16.3095 064	6.4312 276
267	7 12 89	19 034 163	16.3401 346	6.4392 767
268	7 18 24	19 248 832	16.3707 055	6.4473 057
269	7 23 61	19 465 109	16.4012 195	6.4553 148
270	7 29 00	19 683 000	16.4316 767	6.4633 041
271	7 34 41	19 902 511	16.4620 776	6.4712 736
272	7 39 84	20 123 648	16.4924 225	6.4792 236
273	7 45 29	20 346 417	16.5227 116	6.4871 541
274	7 50 76	20 570 824	16.5529 454	6.4950 653
275	7 56 25	20 796 875	16.5831 24	6.5029 572
276	7 61 76	21 024 576	16.6132 477	6.5108 3
277	7 67 29	21 253 933	16.6433 17	6.5186 839
278	7 72 84	21 484 952	16.6783 32	6.5265 189
279	7 78 41	21 717 639	16.7032 931	6.5343 351

Table—(*Continued*).

Number	Square.	Cube	Square Root.	Cube Root.
280	7 84 00	21 952 000	16.7332 005	6.5421 326
281	7 89 61	22 188 041	16.7630 546	6.5499 116
282	7 95 24	22 425 768	16.7928 556	6.5576 722
283	8 00 89	22 665 187	16.8226 038	6.5654 144
284	8 06 56	22 906 304	16.8522 995	6.5731 385
285	8 12 25	23 149 125	16.8819 43	6.5808 443
286	8 17 96	23 393 656	16.9115 345	6.5885 323
287	8 23 69	23 639 903	16.9410 743	6.5962 023
288	8 29 44	23 887 872	16.9705 627	6.6038 545
289	8 35 21	24 137 569	17.	6.6114 89
290	8 41 00	24 389 000	17.0293 864	6.6191 06
291	8 46 81	24 642 171	17.0587 221	6.6267 054
292	8 52 64	24 897 088	17.0880 075	6.6342 874
293	8 58 49	25 153 757	17.1172 428	6.6418 522
294	8 64 36	25 412 184	17.1464 282	6.6493 998
295	8 70 25	25 672 375	17.1755 64	6.6569 302
296	8 76 16	25 934 336	17.2046 505	6.6644 437
297	8 82 09	26 198 073	17.2336 879	6.6719 403
298	8 88 04	26 463 592	17.2626 765	6.6794 2
299	8 94 01	26 730 899	17.2916 165	6.6868 831
300	9 00 00	27 000 000	17.3205 081	6.6943 295
301	9 06 01	27 270 901	17.3493 516	6.7017 593
302	9 12 04	27 543 608	17.3781 472	6.7091 729
303	9 18 09	27 818 127	17.4068 952	6.7165 7
304	9 24 16	28 094 464	17.4355 958	6.7239 508
305	9 30 25	28 372 625	17.4642 492	6.7313 155
306	9 36 36	28 652 616	17.4928 557	6.7386 641
307	9 42 49	28 934 443	17.5214 155	6.7459 967
308	9 48 64	29 218 112	17.5499 288	6.7533 134
309	9 54 81	29 503 609	17.5783 958	6.7606 143
310	9 61 00	29 791 000	17.6068 169	6.7678 995
311	9 67 21	30 080 231	17.6151 921	6.7751 69
312	9 73 44	30 371 328	17.6635 217	6.7824 229
313	9 79 69	30 664 297	17.6918 06	6.7896 613
314	9 85 96	30 959 144	17.7200 451	6.7968 844
315	9 92 25	31 255 875	17.7482 393	6.8040 921
316	9 98 56	31 554 496	17.7763 888	6.8112 847
317	10 04 89	31 855 013	17.8044 938	6.8184 62
318	10 11 24	32 157 432	17.8325 545	6.8256 242
319	10 17 61	32 461 759	17.8605 711	6.8327 714
320	10 24 00	32 768 000	17.8885 438	6.8399 037
321	10 30 41	33 076 161	17.9164 729	6.8470 213
322	10 36 84	33 386 248	17.9443 584	6.8541 24
323	10 43 29	33 698 267	17.9722 008	6.8612 12
324	10 49 76	34 012 224	18.	6.8682 855
325	10 56 25	34 328 125	18.0277 564	6.8753 433
326	10 62 76	34 645 976	18.0554 701	6.8823 888
327	10 69 29	34 965 783	18.0831 413	6.8894 188
328	10 75 84	35 287 552	18.1107 703	6.8964 345
329	10 82 41	35 611 289	18.1383 571	6.9034 359
330	10 89 00	35 937 000	18.1659 021	6.9104 232
331	10 95 61	36 264 691	18.1934 054	6.9173 964
332	11 02 24	36 594 368	18.2208 672	6.9243 556
333	11 08 89	36 926 037	18.2482 876	6.9313 088
334	11 15 56	37 259 704	18.2756 669	6.9382 321
335	11 22 25	37 595 375	18.3030 052	6.9451 496

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
336	11 28 96	37 933 056	18.3303 028	6.9520 533
337	11 35 69	38 272 753	18.3575 598	6.9589 434
338	11 42 44	38 614 472	18.3847 763	6.9658 198
339	11 49 21	38 958 219	18.4119 526	6.9726 826
340	11 56 00	39 304 000	18.4390 889	6.9795 321
341	11 62 81	39 651 821	18.4661 853	6.9863 681
342	11 69 64	40 001 688	18.4932 42	6.9931 906
343	11 76 49	40 353 607	18.5202 592	7.
344	11 83 36	40 707 584	18.5472 37	7.0067 962
345	11 90 25	41 063 625	18.5741 756	7.0135 791
346	11 97 16	41 421 736	18.6010 752	7.0203 49
347	12 04 09	41 781 923	18.6279 36	7.0271 058
348	12 11 04	42 144 192	18.6547 581	7.0338 497
349	12 18 01	42 508 549	18.6815 417	7.0405 806
350	12 25 00	42 875 000	18.7082 869	7.0472 987
351	12 32 01	43 243 551	18.7349 94	7.0540 041
352	12 39 04	43 614 208	18.7616 63	7.0606 967
353	12 46 09	43 986 977	18.7882 942	7.0673 767
354	12 53 16	44 361 864	18.8148 877	7.0740 44
355	12 60 25	44 738 875	18.8414 437	7.0806 988
356	12 67 36	45 118 016	18.8679 623	7.0873 411
357	12 74 49	45 499 293	18.8944 436	7.0939 709
358	12 81 64	45 882 712	18.9208 879	7.1005 885
359	12 88 81	46 268 279	18.9472 953	7.1071 937
360	12 96 00	46 656 000	18.9736 66	7.1137 866
361	13 03 21	47 045 831	19.	7.1203 674
362	13 10 44	47 437 928	19.0262 976	7.1269 36
363	13 17 69	47 832 147	19.0525 589	7.1334 925
364	13 24 96	48 228 544	19.0787 84	7.1400 37
365	13 32 25	48 627 125	19.1049 732	7.1465 695
366	13 39 56	49 027 896	19.1311 265	7.1530 901
367	13 46 89	49 430 863	19.1572 441	7.1595 988
368	13 54 24	49 836 032	19.1833 261	7.1660 957
369	13 61 61	50 243 409	19.2093 727	7.1725 809
370	13 69 00	50 653 000	19.2353 841	7.1790 544
371	13 76 41	51 064 811	19.2613 603	7.1855 162
372	13 83 84	51 478 848	19.2873 015	7.1919 663
373	13 91 29	51 895 117	19.3132 079	7.1984 05
374	13 98 76	52 313 624	19.3390 796	7.2048 322
375	14 06 25	52 734 375	19.3649 167	7.2112 479
376	14 13 76	53 157 376	19.3907 194	7.2176 522
377	14 21 29	53 582 633	19.4164 878	7.2240 45
378	14 28 84	54 010 152	19.4422 221	7.2304 268
379	14 36 41	54 439 939	19.4679 223	7.2367 972
380	14 44 00	54 872 000	19.4935 887	7.2431 565
381	14 51 61	55 306 341	19.5192 213	7.2495 045
382	14 59 24	55 742 968	19.5448 203	7.2558 415
383	14 66 89	56 181 887	19.5703 858	7.2621 675
384	14 74 56	56 623 104	19.5959 179	7.2684 824
385	14 82 25	56 066 625	19.6214 169	7.2747 864
386	14 89 96	57 512 456	19.6468 827	7.2810 794
387	14 97 69	57 960 603	19.6723 156	7.2873 617
388	15 05 44	58 411 072	19.6977 156	7.2936 33
389	15 13 21	58 863 869	19.7230 829	7.2998 936
390	15 21 00	59 319 000	19.7484 177	7.3061 436
391	15 28 81	59 776 471	19.7737 199	7.3123 828

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
392	15 36 64	60 236 288	19.7989 899	7.3186 114
393	15 44 49	60 698 457	19.8242 276	7.3248 295
394	15 52 36	61 162 984	19.8494 332	7.3310 369
395	15 60 25	61 629 875	19.8746 069	7.3372 339
396	15 68 16	62 099 136	19.8997 487	7.3434 205
397	15 76 09	62 570 773	19.9248 588	7.3495 966
398	15 84 04	63 044 792	19.9499 373	7.3557 624
399	15 92 01	63 521 199	19.9749 844	7.3619 178
400	16 00 00	64 000 000	20.	7.3680 63
401	16 08 01	64 481 201	20.0249 844	7.3741 979
402	16 16 04	64 964 808	20.0499 377	7.3803 227
403	16 24 09	65 450 827	20.0748 599	7.3864 373
404	16 32 16	65 939 264	20.0997 512	7.3925 418
405	16 40 25	66 430 125	20.1246 118	7.3986 363
406	16 48 36	66 923 416	20.1494 417	7.4047 206
407	16 56 49	67 419 143	20.1742 41	7.4107 95
408	16 64 64	67 917 312	20.1990 099	7.4168 595
409	16 72 81	68 417 929	20.2237 484	7.4229 142
410	16 81 00	68 921 000	20.2484 567	7.4289 589
411	16 89 21	69 426 531	20.2731 349	7.4349 938
412	16 97 44	69 934 528	20.2977 831	7.4410 189
413	17 05 69	70 444 997	20.3224 014	7.4470 342
414	17 13 96	70 957 944	20.3469 899	7.4530 399
415	17 22 25	71 473 375	20.3715 488	7.4590 359
416	17 30 56	71 991 296	20.3960 781	7.4650 223
417	17 38 89	72 511 713	20.4205 779	7.4709 991
418	17 47 24	73 034 632	20.4450 483	7.4769 664
419	17 55 61	73 560 059	20.4694 895	7.4829 242
420	17 64 00	74 088 000	20.4939 015	7.4888 724
421	17 72 41	74 618 461	20.5182 845	7.4948 113
422	17 80 84	75 151 448	20.5426 386	7.5007 406
423	17 89 29	75 686 967	20.5669 638	7.5066 607
424	17 97 76	76 225 024	20.5912 603	7.5125 715
425	18 06 25	76 765 625	20.6155 281	7.5184 73
426	18 14 76	77 308 776	20.6397 674	7.5243 652
427	18 23 29	77 854 483	20.6639 783	7.5302 482
428	18 31 84	78 402 752	20.6881 609	7.5361 221
429	18 40 41	78 953 589	20.7123 152	7.5419 867
430	18 49 00	79 507 000	20.7364 414	7.5478 423
431	18 57 61	80 062 991	20.7605 395	7.5536 888
432	18 66 24	80 621 568	20.7846 097	7.5595 263
433	18 74 89	81 182 737	20.8086 52	7.5653 548
434	18 83 56	81 746 504	20.8326 667	7.5711 743
435	18 92 25	82 312 875	20.8566 536	7.5769 849
436	19 00 96	82 881 856	20.8806 13	7.5827 865
437	19 09 69	83 453 453	20.9045 45	7.5885 793
438	19 18 44	84 027 672	20.9284 495	7.5943 633
439	19 27 21	84 604 519	20.9523 268	7.6001 385
440	19 36 00	85 184 000	20.9761 77	7.6059 049
441	19 44 81	85 766 121	21.	7.6116 626
442	19 53 64	86 350 888	21.0237 96	7.6174 116
443	19 62 49	86 938 307	21.0475 652	7.6231 519
444	19 71 36	87 528 384	21.0713 075	7.6288 837
445	19 80 25	88 121 125	21.0950 231	7.6346 067
446	19 89 16	88 716 536	21.1187 121	7.6403 213
447	19 98 09	89 314 623	21.1423 745	7.6460 272

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
448	20 07 04	89 915 392	21.1660 105	7.6517 247
449	20 16 01	90 518 849	21.1896 201	7.6574 138
450	20 25 00	91 125 000	21.2132 034	7.6630 943
451	20 34 01	91 733 851	21.2367 606	7.6687 665
452	20 43 04	92 345 408	21.2602 916	7.6744 303
453	20 52 09	92 959 677	21.2837 967	7.6800 857
454	20 61 16	93 576 664	21.3072 758	7.6857 328
455	20 70 25	94 196 375	21.3307 29	7.6913 717
456	20 79 36	94 818 816	21.3541 565	7.6970 023
457	20 88 49	95 443 993	21.3775 583	7.7026 246
458	20 97 64	96 071 912	21.4009 346	7.7082 388
459	21 06 81	96 702 579	21.4242 853	7.7138 448
460	21 16 00	97 336 000	21.4476 106	7.7194 426
461	21 25 21	97 972 181	21.4709 106	7.7250 325
462	21 34 44	98 611 128	21.4941 853	7.7306 141
463	21 43 69	99 252 847	21.5174 348	7.7361 877
464	21 52 96	99 897 344	21.5406 592	7.7417 532
465	21 62 25	100 544 625	21.5638 587	7.7473 109
466	21 71 56	101 194 696	21.5870 331	7.7528 606
467	21 80 89	101 847 563	21.6101 828	7.7584 023
468	21 90 24	102 503 232	21.6333 077	7.7639 361
469	21 99 61	103 161 709	21.6564 078	7.7694 62
470	22 09 00	103 823 000	21.6794 834	7.7749 801
471	22 18 41	104 487 111	21.7025 344	7.7804 904
472	22 27 84	105 154 048	21.7255 61	7.7859 928
473	22 37 29	105 823 817	21.7485 632	7.7914 875
474	22 46 76	106 496 424	21.7715 411	7.7969 745
475	22 56 25	107 171 875	21.7944 947	7.8024 538
476	22 65 76	107 850 176	21.8174 242	7.8079 254
477	22 75 29	108 531 333	21.8403 297	7.8133 892
478	22 84 84	109 215 352	21.8632 111	7.8188 456
479	22 94 41	109 902 239	21.8860 686	7.8242 942
480	23 04 00	100 592 000	21.9089 023	7.8297 353
481	23 13 61	111 284 641	21.9317 122	7.8351 688
482	23 23 24	111 980 168	21.9544 984	7.8405 949
483	23 32 89	112 678 587	21.9772 61	7.8460 134
484	23 42 56	113 379 904	22.	7.8514 244
485	23 52 25	114 084 125	22.0227 155	7.8568 281
486	23 61 96	114 791 256	22.0454 077	7.8622 242
487	23 71 69	115 501 303	22.0680 765	7.8676 13
488	23 81 44	116 214 272	22.0907 22	7.8729 944
489	23 91 21	116 930 169	22.1133 444	7.8783 684
490	24 01 00	117 649 000	22.1359 436	7.8837 352
491	24 10 81	118 370 771	22.1585 198	7.8890 946
492	24 20 64	119 095 488	22.1810 73	7.8944 468
493	24 30 49	119 823 157	22.2036 033	7.8997 917
494	24 40 36	120 553 784	22.2261 108	7.9051 294
495	24 50 25	121 287 375	22.2485 955	7.9104 599
496	24 60 16	122 023 936	22.2710 575	7.9157 832
497	24 70 09	122 763 473	22.2934 968	7.9210 994
498	24 80 04	123 505 992	22.3159 136	7.9264 085
499	24 90 01	124 251 499	22.3383 079	7.9317 104
500	25 00 00	125 000 000	22.3606 798	7.9370 053
501	25 10 01	125 751 501	22.3830 293	7.9422 931
502	25 20 04	126 506 008	22.4053 565	7.9475 739
503	25 30 09	127 263 527	22.4276 615	7.9528 477

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
504	25 40 16	128 024 064	22.4499 443	7.9581 144
505	25 50 25	128 787 625	22.4722 051	7.9633 743
506	25 60 36	129 554 246	22.4944 438	7.9686 271
507	25 70 49	130 323 843	22.5166 605	7.9738 731
508	25 80 64	131 096 512	22.5388 553	7.9791 122
509	25 90 81	131 872 229	22.5610 283	7.9843 444
510	26 01 00	132 651 000	22.5831 796	7.9895 697
511	26 11 21	133 432 831	22.6053 091	7.9947 883
512	26 21 44	134 217 728	22.6274 17	8.
513	26 31 69	135 005 697	22.6495 033	8.0052 049
514	26 41 96	135 796 744	22.6715 681	8.0104 032
515	26 52 25	136 590 875	22.6936 114	8.0155 946
516	26 62 56	137 388 096	22.7156 334	8.0207 794
517	26 72 89	138 188 413	22.7376 340	8.0259 574
518	26 83 24	138 991 832	22.7596 134	8.0311 287
519	26 93 61	139 798 359	22.7815 715	8.0362 935
520	27 04 00	140 608 000	22.8035 085	8.0414 515
521	27 14 41	141 420 761	22.8254 244	8.0466 03
522	27 24 84	142 236 648	22.8473 193	8.0517 479
523	27 35 29	143 055 667	22.8691 933	8.0568 862
524	27 45 76	143 877 824	22.8910 463	8.0620 18
525	27 56 25	144 703 125	22.9128 785	8.0671 432
526	27 66 76	145 531 576	22.9346 899	8.0722 62
527	27 77 29	146 363 183	22.9564 806	8.0773 743
528	27 87 84	147 197 952	22.9782 506	8.0824 8
529	27 98 41	148 035 889	23.	8.0875 794
530	28 09 00	148 877 000	23.0217 289	8.0926 723
531	28 19 61	149 721 291	23.0434 372	8.0977 589
532	28 30 24	150 568 768	23.0651 252	8.1028 39
533	28 40 89	151 419 437	23.0867 928	8.1079 128
534	28 51 56	152 273 304	23.1084 4	8.1129 803
535	28 62 25	153 130 375	23.1300 67	8.1180 414
536	28 72 96	153 990 656	23.1516 738	8.1230 962
537	28 83 69	154 854 153	23.1732 605	8.1281 447
538	28 94 44	155 720 872	23.1948 27	8.1331 87
539	29 05 21	156 590 819	23.2163 735	8.1382 23
540	29 16 00	157 464 000	23.2379 001	8.1432 529
541	29 26 81	158 340 421	23.2594 067	8.1482 765
542	29 37 64	159 220 088	23.2808 935	8.1532 939
543	29 48 49	160 103 007	23.3023 604	8.1583 051
544	29 59 36	160 989 184	23.3238 076	8.1633 102
545	29 70 25	161 878 625	23.3452 351	8.1683 092
546	29 81 16	162 771 336	23.3666 429	8.1733 02
547	29 92 09	163 667 323	23.3880 311	8.1782 888
548	30 03 04	164 566 592	23.4093 998	8.1832 695
549	30 14 01	165 469 149	23.4307 49	8.1882 441
550	30 25 00	166 375 000	23.4520 788	8.1932 127
551	30 36 01	167 284 151	23.4733 892	8.1981 753
552	30 47 04	168 196 608	23.4946 802	8.2031 319
553	30 58 09	169 112 377	23.5159 52	8.2080 825
554	30 69 16	170 031 464	23.5372 046	8.2130 271
555	30 80 25	170 953 875	23.5584 38	8.2179 657
556	30 91 36	171 879 616	23.5796 522	8.2228 985
557	31 02 49	172 808 693	23.6008 474	8.2278 254
558	31 13 64	173 741 112	23.6220 236	8.2327 463
559	31 24 81	174 676 879	23.6431 808	8.2376 614

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
560	31 36 00	175 616 000	23.6643 191	8.2425 706
561	31 47 21	176 558 481	23.6854 386	8.2474 74
562	31 58 44	177 504 328	23.7065 392	8.2523 715
563	31 69 69	178 453 547	23.7276 21	8.2572 635
564	31 80 96	179 406 144	23.7486 842	8.2621 492
565	31 92 25	180 362 125	23.7697 286	8.2670 294
566	32 03 56	181 321 496	23.7907 545	8.2719 039
567	32 14 89	182 284 263	23.8117 618	8.2767 726
568	32 26 24	183 250 432	23.8327 506	8.2816 255
569	32 37 61	184 220 009	23.8537 209	8.2864 928
570	32 49 00	185 193 000	23.8746 728	8.2913 444
571	32 60 41	186 169 411	23.8956 063	8.2961 903
572	32 71 84	187 149 248	23.9165 215	8.3010 304
573	32 83 29	188 132 517	23.9374 184	8.3058 651
574	32 94 76	189 119 224	23.9582 971	8.3106 941
575	33 06 25	190 109 375	23.9791 576	8.3155 175
576	33 17 76	191 102 976	24.	8.3203 353
577	33 29 29	192 100 033	24.0208 243	8.3251 475
578	33 40 84	193 100 552	24.0416 306	8.3299 542
579	33 52 41	194 104 539	24.0624 188	8.3347 553
580	33 64 00	195 112 000	24.0831 891	8.3395 509
581	33 75 61	196 122 941	24.1039 416	8.3443 41
582	33 87 24	197 137 368	24.1246 762	8.3491 256
583	33 98 89	198 155 287	24.1453 929	8.3539 047
584	34 10 56	199 176 704	24.1660 919	8.3586 784
585	34 22 25	200 201 625	24.1867 732	8.3634 466
586	34 33 96	201 230 056	24.2074 369	8.3682 095
587	34 45 69	202 262 003	24.2280 829	8.3729 668
588	34 57 44	203 297 472	24.2487 113	8.3777 188
589	34 69 21	204 336 469	24.2693 222	8.3824 653
590	34 81 00	205 379 000	24.2899 156	8.3872 065
591	34 92 81	206 425 071	24.3104 916	8.3919 423
592	35 04 64	207 474 688	24.3310 501	8.3966 729
593	35 16 49	208 527 857	24.3515 913	8.4013 981
594	35 28 36	209 584 584	24.3721 152	8.4061 180
595	35 40 25	210 644 875	24.3926 218	8.4108 326
596	35 52 16	211 708 736	24.4131 112	8.4155 419
597	35 64 09	212 776 173	24.4335 834	8.4202 46
598	35 76 04	213 847 192	24.4540 385	8.4249 448
599	35 88 01	214 921 799	24.4744 765	8.4296 383
600	36 00 00	216 000 000	24.4948 974	8.4343 267
601	36 12 01	217 081 801	24.5153 013	8.4390 098
602	36 24 04	218 167 208	24.5356 883	8.4436 877
603	36 36 09	219 256 227	24.5560 583	8.4483 605
604	36 48 16	220 348 864	24.5764 115	8.4530 281
605	36 60 25	221 445 125	24.5967 478	8.4576 906
606	36 72 36	222 545 016	24.6170 673	8.4623 479
607	36 84 49	223 648 543	24.6373 7	8.467
608	36 96 64	224 755 712	24.6576 56	8.4716 471
609	37 08 81	225 866 529	24.6779 254	8.4762 892
610	37 21 00	226 981 000	24.6981 781	8.4809 261
611	37 33 21	228 099 131	24.7184 142	8.4855 579
612	37 45 44	229 220 928	24.7386 338	8.4901 848
613	37 57 69	230 346 397	24.7588 368	8.4948 065
614	37 69 96	231 475 544	24.7790 234	8.4994 233
615	37 82 25	232 608 375	24.7991 935	8.5040 35

Table—(*Continued*).

Number.	Square	Cube	Square Root.	Cube Root.
616	37 94 56	233 744 896	24.8193 473	8.5086 417
617	38 06 89	234 885 113	24.8394 847	8.5132 435
618	38 19 24	236 029 032	24.8596 058	8.5178 403
619	38 31 61	237 176 659	24.8797 106	8.5224 321
620	38 44 00	238 328 000	24.8997 992	8.5270 189
621	38 56 41	239 483 061	24.9198 716	8.5316 009
622	38 68 84	240 641 848	24.9399 278	8.5361 78
623	38 81 29	241 804 367	24.9599 679	8.5407 501
624	38 93 76	242 970 624	24.9799 92	8.5453 173
625	39 06 25	244 140 625	25.	8.5498 797
626	39 18 76	245 134 376	25.0199 92	8.5544 372
627	39 31 29	246 491 883	25.0399 681	8.5589 899
628	39 43 84	247 673 152	25.0599 282	8.5635 377
629	39 56 41	248 858 189	25.0798 724	8.5680 807
630	39 69 00	250 047 000	25.0998 008	8.5726 189
631	39 81 61	251 239 591	25.1197 134	8.5771 523
632	39 94 24	252 435 968	25.1396 102	8.5816 809
633	40 06 89	253 636 137	25.1594 913	8.5862 047
634	40 19 56	254 840 104	25.1793 566	8.5907 238
635	40 32 25	256 047 875	25.1992 063	8.5952 38
636	40 44 96	257 259 456	25.2190 404	8.5997 476
637	40 57 69	258 474 853	25.2388 589	8.6042 525
638	40 70 44	259 694 072	25.2586 619	8.6087 526
639	40 83 21	260 917 119	25.2784 493	8.6132 48
640	40 96 00	262 144 000	25.2982 213	8.6177 388
641	41 08 81	263 374 721	25.3179 778	8.6222 248
642	41 21 64	264 609 288	25.3377 189	8.6267 063
643	41 34 49	265 847 707	25.3574 447	8.6311 83
644	41 47 36	267 089 984	25.3771 551	8.6356 551
645	41 60 25	268 336 125	25.3968 502	8.6401 226
646	41 73 16	269 585 136	25.4165 301	8.6445 855
647	41 86 09	270 840 023	25.4361 947	8.6490 437
648	41 99 04	272 097 792	25.4558 441	8.6534 974
649	42 12 01	273 359 549	25.4754 784	8.6579 465
650	42 25 00	274 625 000	25.4950 976	8.6623 911
651	42 38 01	275 894 451	25.5147 016	8.6668 31
652	42 51 04	277 167 808	25.5342 907	8.6712 665
653	42 64 09	278 445 077	25.5538 647	8.6756 974
654	42 77 16	279 726 264	25.5734 237	8.6801 237
655	42 90 25	281 011 375	25.5929 678	8.6845 456
656	43 03 36	282 300 416	25.6124 969	8.6889 63
657	43 16 49	283 593 393	25.6320 112	8.6933 759
658	43 29 64	284 890 312	25.6515 107	8.6977 843
659	43 42 81	286 191 179	25.6709 953	8.7021 882
660	43 56 00	287 496 000	25.6904 652	8.7065 877
661	43 69 21	288 804 781	25.7099 203	8.7109 827
662	43 82 44	290 117 528	25.7293 607	8.7153 734
663	43 95 69	291 434 247	25.7487 864	8.7197 596
664	44 08 96	292 754 944	25.7681 975	8.7241 414
665	44 22 25	294 079 625	25.7875 939	8.7285 187
666	44 35 56	295 408 296	25.8069 758	8.7328 918
667	44 48 89	296 740 963	25.8263 431	8.7372 604
668	44 62 24	298 077 632	25.8456 96	8.7416 246
669	44 75 61	299 418 309	25.8650 343	8.7459 846
670	44 89 00	300 763 000	25.8843 582	8.7503 401
671	45 02 41	302 111 711	25.9036 677	8.7546 913

Table—(*Continued*).

Number.	Square.	Cube	Square Root.	Cube Root.
672	45 15 84	303 464 448	25.9229 628	8.7590 383
673	45 29 29	304 821 217	25.9422 435	8.7633 809
674	45 42 76	306 182 024	25.9615 1	8.7677 192
675	45 56 25	307 546 875	25.9807 621	8.7720 532
676	45 69 76	308 915 776	26.	8.7763 83
677	45 83 29	310 288 733	26.0192 237	8.7807 084
678	45 96 84	311 665 752	26.0384 331	8.7850 296
679	46 10 41	313 046 839	26.0576 284	8.7893 466
680	46 24 00	314 432 000	26.0768 096	8.7936 593
681	46 37 61	315 821 241	26.0959 767	8.7979 679
682	46 51 24	317 214 568	26.1151 297	8.8022 721
683	46 64 89	318 611 987	26.1342 687	8.8065 722
684	46 78 56	320 013 504	26.1533 937	8.8108 681
685	46 92 25	321 419 125	26.1725 047	8.8151 598
686	47 05 96	322 828 856	26.1916 017	8.8194 474
687	47 19 69	324 242 703	26.2106 848	8.8237 307
688	47 33 44	325 660 672	26.2297 541	8.8280 099
689	47 47 21	327 082 769	26.2488 095	8.8322 85
690	47 61 00	328 509 000	26.2678 511	8.8365 559
691	47 74 81	329 939 371	26.2868 789	8.8408 227
692	47 88 64	331 373 888	26.3058 929	8.8450 854
693	48 02 49	332 812 557	26.3248 932	8.8493 44
694	48 16 36	334 255 384	26.3438 797	8.8535 985
695	48 30 25	335 702 375	26.3628 527	8.8578 489
696	48 44 16	337 153 536	26.3818 119	8.8620 952
697	48 58 09	338 608 873	26.4007 576	8.8663 375
698	48 72 04	340 068 392	26.4196 896	8.8705 757
699	48 86 01	341 532 099	26.4386 081	8.8748 099
700	49 00 00	343 000 000	26.4575 131	8.8790 4
701	49 14 01	344 472 101	26.4764 046	8.8832 661
702	49 28 04	345 948 408	26.4952 826	8.8874 882
703	49 42 09	347 428 927	26.5141 472	8.8917 063
704	49 56 16	348 913 664	26.5329 983	8.8959 204
705	49 70 25	350 402 625	26.5518 361	8.9001 304
706	49 84 36	351 895 816	26.5706 605	8.9043 366
707	49 98 49	353 393 243	26.5894 716	8.9085 387
708	50 12 64	354 894 912	26.6082 694	8.9127 369
709	50 26 81	356 400 829	26.6270 539	8.9169 311
710	50 41 00	357 911 000	26.6458 252	8.9211 214
711	50 55 21	359 425 431	26.6645 833	8.9253 078
712	50 69 44	360 944 128	26.6833 281	8.9294 902
713	50 83 69	362 467 097	26.7020 598	8.9336 687
714	50 97 96	363 994 344	26.7207 784	8.9378 433
715	51 12 25	365 525 875	26.7394 839	8.9420 14
716	51 26 56	367 061 696	26.7581 763	8.9461 809
717	51 40 89	368 601 813	26.7768 557	8.9503 438
718	51 55 24	370 146 232	26.7955 22	8.9545 029
719	51 69 61	371 694 959	26.8141 754	8.9586 581
720	51 84 00	373 248 000	26.8328 157	8.9628 095
721	51 98 41	374 805 361	26.8514 432	8.9669 57
722	52 12 84	376 367 048	26.8700 577	8.9711 007
723	52 27 29	377 933 067	26.8886 593	8.9752 406
724	52 41 76	379 503 434	26.9072 481	8.9793 766
725	52 56 25	381 078 125	26.9258 24	8.9835 089
726	52 70 76	382 657 176	26.9443 872	8.9876 373
727	52 85 29	384 240 583	26.9629 375	8.9917 62

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
728	52 99 84	385 828 352	26.9814 751	8.9958 899
729	53 14 41	387 420 489	27.	9.
730	53 29 00	389 017 000	27.0185 122	9.0041 134
731	53 43 61	390 617 891	27.0370 117	9.0082 229
732	53 58 24	392 223 168	27.0554 985	9.0123 288
733	53 72 89	393 832 837	27.0739 727	9.0164 309
734	53 87 56	395 446 904	27.0924 344	9.0205 293
735	54 02 25	397 065 375	27.1108 834	9.0246 239
736	54 16 96	398 688 256	27.1293 199	9.0287 149
737	54 31 69	400 315 553	27.1477 439	9.0328 021
738	54 46 44	401 947 272	27.1661 554	9.0368 857
739	54 61 21	403 583 419	27.1845 544	9.0409 655
740	54 76 00	405 224 000	27.2029 41	9.0450 417
741	54 90 81	406 869 021	27.2213 152	9.0491 142
742	55 05 64	408 518 488	27.2396 769	9.0531 831
743	55 20 49	410 172 407	27.2580 263	9.0572 482
744	55 35 36	411 830 784	27.2763 634	9.0613 098
745	55 50 25	413 493 625	27.2946 881	9.0653 677
746	55 65 16	415 160 936	27.3130 006	9.0694 22
747	55 80 09	416 832 723	27.3313 007	9.0734 726
748	55 95 04	418 508 992	27.3495 887	9.0775 197
749	56 10 01	420 189 749	27.3678 644	9.0815 631
750	56 25 00	421 875 000	27.3861 279	9.0856 03
751	56 40 01	423 564 751	27.4043 792	9.0896 352
752	56 55 04	425 259 008	27.4226 184	9.0936 719
753	56 70 09	426 957 777	27.4408 455	9.0977 01
754	56 85 16	428 661 064	27.4590 604	9.1017 265
755	57 00 25	430 368 875	27.4772 633	9.1057 485
756	57 15 36	432 081 216	27.4954 542	9.1097 669
757	57 30 49	433 798 093	27.5136 33	9.1137 818
758	57 45 64	435 519 512	27.5317 998	9.1177 931
759	57 60 81	437 245 479	27.5499 546	9.1218 01
760	57 76 00	438 976 000	27.5680 975	9.1258 053
761	57 91 21	440 711 081	27.5862 284	9.1298 061
762	58 06 44	442 450 728	27.6043 475	9.1338 034
763	58 21 69	444 194 947	27.6224 546	9.1377 971
764	58 36 96	445 943 744	27.6405 499	9.1417 874
765	58 52 25	447 697 125	27.6586 334	9.1457 742
766	58 67 56	449 455 096	27.6767 05	9.1497 576
767	58 82 89	451 217 663	27.6947 648	9.1537 375
768	58 98 24	452 984 832	27.7128 129	9.1577 139
769	59 13 61	454 756 609	27.7308 492	9.1616 869
770	59 29 00	456 533 000	27.7488 739	9.1656 565
771	59 44 41	458 314 011	27.7668 868	9.1696 225
772	59 59 84	460 099 648	27.7848 88	9.1735 852
773	59 75 29	461 889 917	27.8028 775	9.1775 445
774	59 90 76	463 684 824	27.8208 555	9.1815 003
775	60 06 25	465 484 375	27.8388 218	9.1854 527
776	60 21 76	467 288 576	27.8567 766	9.1894 018
777	60 37 29	469 097 433	27.8747 197	9.1933 474
778	60 52 84	470 910 952	27.8926 514	9.1972 897
779	60 68 41	472 729 139	27.9105 715	9.2012 286
780	60 84 00	474 552 000	27.9284 801	9.2051 641
781	60 99 61	476 379 541	27.9463 772	9.2090 962
782	61 15 24	478 211 768	27.9642 629	9.2130 25
783	61 30 89	480 048 687	27.9821 372	9.2169 505

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
784	61 46 56	481 890 304	28.	9.2208 726
785	61 62 25	483 736 625	28.0178 515	9.2247 914
786	61 77 96	485 587 656	28.0356 915	9.2287 068
787	61 93 69	487 443 403	28.0535 203	9.2326 189
788	62 09 44	489 303 872	28.0713 377	9.2365 277
789	62 25 21	491 169 069	28.0891 438	9.2404 333
790	62 41 00	493 039 000	28.1069 386	9.2443 355
791	62 56 81	494 913 671	28.1247 222	9.2482 344
792	62 72 64	496 793 088	28.1424 946	9.2521 3
793	62 88 49	498 677 257	28.1602 557	9.2560 224
794	63 04 36	500 566 184	28.1780 056	9.2599 114
795	63 20 25	502 459 875	28.1957 444	9.2637 973
796	63 36 16	504 358 336	28.2134 72	9.2676 798
797	63 52 09	506 261 573	28.2311 884	9.2715 592
798	63 68 04	508 169 592	28.2488 938	9.2754 352
799	63 84 01	510 082 399	28.2665 881	9.2793 081
800	64 00 00	512 000 000	28.2842 712	9.2831 777
801	64 16 01	513 922 401	28.3019 434	9.2870 44
802	64 32 04	515 849 608	28.3196 045	9.2909 072
803	64 48 09	517 781 627	28.3372 546	9.2947 671
804	64 64 16	519 718 464	28.3548 938	9.2986 239
805	64 80 25	521 660 125	28.3725 219	9.3024 775
806	64 96 36	523 606 616	28.3901 391	9.3063 278
807	65 12 49	525 557 943	28.4077 454	9.3101 75
808	65 28 64	527 514 112	28.4253 408	9.3140 19
809	65 44 81	529 475 129	28.4429 253	9.3178 599
810	65 61 00	531 441 000	28.4604 989	9.3216 975
811	65 77 21	533 411 731	28.4780 617	9.3255 32
812	65 93 44	535 387 328	28.4956 137	9.3293 634
813	66 09 69	537 367 797	28.5131 549	9.3331 916
814	66 25 96	539 353 144	28.5306 852	9.3370 167
815	66 42 25	541 343 375	28.5482 048	9.3408 386
816	66 58 56	543 338 496	28.5657 137	9.3446 575
817	66 74 89	545 338 513	28.5832 119	9.3484 731
818	66 91 24	547 343 432	28.6006 993	9.3522 857
819	67 07 61	549 353 239	28.6181 76	9.3560 952
820	67 24 00	551 368 000	28.6356 421	9.3599 016
821	67 40 41	553 387 661	28.6530 976	9.3637 049
822	67 56 84	555 412 248	28.6705 424	9.3675 051
823	67 73 29	557 441 767	28.6879 766	9.3713 022
824	67 89 76	559 476 224	28.7054 002	9.3750 963
825	68 06 25	561 515 625	28.7228 132	9.3788 873
826	68 22 76	563 559 976	28.7402 157	9.3826 752
827	68 39 29	565 609 283	28.7576 077	9.3864 6
828	68 55 84	567 663 552	28.7749 891	9.3902 419
829	68 72 41	569 722 789	28.7923 601	9.3940 206
830	68 89 00	571 787 000	28.8097 206	9.3977 964
831	69 05 61	573 856 191	28.8270 706	9.4015 691
832	69 22 24	575 930 368	28.8444 102	9.4053 387
833	69 38 89	578 009 537	28.8617 394	9.4091 054
834	69 55 56	580 093 704	28.8790 582	9.4128 69
835	69 72 25	582 182 875	28.8963 666	9.4166 297
836	69 88 96	584 277 056	28.9136 646	9.4203 873
837	70 05 69	586 376 253	28.9309 523	9.4241 42
838	70 22 44	588 480 472	28.9482 297	9.4278 936
839	70 39 21	590 589 719	28.9654 967	9.4316 423

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
840	70 56 00	592 704 000	28.9827 535	9.4353 8
841	70 72 81	594 823 321	29.	9.4391 307
842	70 89 64	596 947 688	29.0172 363	9.4428 704
843	71 06 49	599 077 107	29.0344 623	9.4466 072
844	71 23 36	601 211 584	29.0516 781	9.4503 41
845	71 40 25	603 351 125	29.0688 837	9.4540 719
846	71 57 16	605 495 736	29.0860 791	9.4577 999
847	71 74 09	607 645 423	29.1032 644	9.4615 249
848	71 91 04	609 800 192	29.1204 396	9.4652 47
849	72 08 01	611 960 049	29.1376 046	9.4689 661
850	72 25 00	614 125 000	29.1547 595	9.4726 824
851	72 42 01	616 295 051	29.1719 043	9.4763 957
852	72 59 04	618 470 208	29.1890 39	9.4801 061
853	72 76 09	620 650 477	29.2061 637	9.4838 136
854	72 93 16	622 835 864	29.2232 784	9.4875 182
855	73 10 25	625 026 375	29.2403 83	9.4912 2
856	73 27 36	627 222 016	29.2574 777	9.4949 188
857	73 44 49	629 422 793	29.2745 623	9.4986 147
858	73 61 64	631 628 712	29.2916 37	9.5023 078
859	73 78 81	633 839 779	29.3087 018	9.5059 98
860	73 96 00	636 056 000	29.3257 566	9.5096 854
861	74 13 21	638 277 381	29.3428 015	9.5133 699
862	74 30 44	640 503 928	29.3598 365	9.5170 515
863	74 47 69	642 735 647	29.3768 616	9.5207 303
864	74 64 96	644 972 544	29.3938 769	9.5244 063
865	74 82 25	647 214 625	29.4108 823	9.5280 794
866	74 99 56	649 461 896	29.4278 779	9.5317 497
867	75 16 89	651 714 363	29.4448 637	9.5354 172
868	75 34 24	653 972 032	29.4618 397	9.5390 818
869	75 51 61	656 234 909	29.4788 059	9.5427 437
870	75 69 00	658 503 000	29.4957 624	9.5464 027
871	75 86 41	660 776 311	29.5127 091	9.5500 589
872	76 03 84	663 054 848	29.5296 461	9.5537 123
873	76 21 29	665 338 617	29.5465 734	9.5573 63
874	76 38 76	667 627 624	29.5634 91	9.5610 108
875	76 56 25	669 921 875	29.5803 989	9.5646 559
876	76 73 76	672 221 376	29.5972 972	9.5682 982
877	76 91 29	674 526 133	29.6141 858	9.5719 377
878	77 08 84	676 836 152	29.6310 648	9.5755 745
879	77 26 41	679 151 439	29.6479 342	9.5792 085
880	77 44 00	681 472 000	29.6647 939	9.5828 397
881	77 61 61	683 797 841	29.6816 442	9.5864 682
882	77 79 24	686 128 968	29.6984 848	9.5900 937
883	77 96 89	688 465 387	29.7153 159	9.5937 169
884	78 14 56	690 807 104	29.7321 375	9.5973 373
885	78 32 25	693 154 125	29.7489 496	9.6009 548
886	78 49 96	695 506 456	29.7657 521	9.6045 696
887	78 67 69	697 864 103	29.7825 452	9.6081 817
888	78 85 44	700 227 072	29.7993 289	9.6117 911
889	79 03 21	702 595 369	29.8161 03	9.6153 977
890	79 21 00	704 969 000	29.8328 678	9.6190 017
891	79 38 81	707 347 971	29.8496 231	9.6226 03
892	79 56 64	707 932 288	29.8663 69	9.6262 016
893	79 74 49	712 121 957	29.8831 056	9.6297 975
894	79 92 36	714 516 984	29.8998 328	9.6333 907
895	80 10 25	716 917 375	29.9165 506	9.6369 812

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
896	80 28 16	719 323 136	29.9332 591	9.6405 69
897	80 46 09	721 734 273	29.9499 583	9.6441 542
898	80 64 04	724 150 792	29.9666 481	9.6477 367
899	80 82 01	726 572 699	29.9833 287	9.6513 166
900	81 00 00	729 000 000	30.	9.6548 938
901	81 18 01	731 432 701	30.0166 62	9.6584 684
902	81 36 04	733 870 808	30.0333 148	9.6620 403
903	81 54 09	736 314 327	30.0499 584	9.6656 096
904	81 72 16	738 763 264	30.0665 928	9.6691 762
905	81 90 25	741 217 625	30.0832 179	9.6727 403
906	82 08 36	743 677 416	30.0998 339	9.6763 017
907	82 26 49	746 142 643	30.1164 407	9.6798 604
908	82 44 64	748 613 312	30.1330 383	9.6834 166
909	82 62 81	751 089 429	30.1496 269	9.6869 701
910	82 81 00	753 571 000	30.1662 063	9.6905 211
911	82 99 21	756 058 031	30.1827 765	9.6940 694
912	83 17 44	758 550 825	30.1993 377	9.6976 151
913	83 35 69	761 048 497	30.2158 899	9.7011 583
914	83 53 96	763 551 944	30.2324 329	9.7046 989
915	83 72 25	766 060 875	30.2489 669	9.7082 369
916	83 90 56	768 575 296	30.2654 919	9.7117 723
917	84 08 89	771 095 213	30.2820 079	9.7153 051
918	84 27 24	773 620 632	30.2985 148	9.7188 354
919	84 45 61	776 151 559	30.3150 128	9.7223 631
920	84 64 00	778 688 000	30.3315 018	9.7258 883
921	84 82 41	781 229 961	30.3479 818	9.7294 109
922	85 00 84	783 777 448	30.3644 529	9.7329 309
923	85 19 29	786 330 467	30.3809 151	9.7364 484
924	85 37 76	788 889 024	30.3973 683	9.7399 634
925	85 56 25	791 453 125	30.4138 127	9.7434 758
926	85 74 76	794 022 776	30.4302 481	9.7469 857
927	85 93 29	796 597 983	30.4466 747	9.7504 93
928	86 11 84	799 178 752	30.4630 924	9.7539 979
929	86 30 41	801 765 089	30.4795 013	9.7575 002
930	86 49 00	804 357 000	30.4959 014	9.7610 001
931	86 67 61	806 954 491	30.5122 926	9.7644 974
932	86 86 24	809 557 568	30.5286 75	9.7679 922
933	87 04 89	812 166 237	30.5450 487	9.7714 845
934	87 23 56	814 780 504	30.5614 136	9.7749 743
935	87 42 25	817 400 375	30.5777 697	9.7784 616
936	87 60 96	820 025 856	30.5941 171	9.7829 466
937	87 79 69	822 656 953	30.6104 557	9.7854 288
938	87 98 44	825 293 672	30.6267 857	9.7889 087
939	88 17 21	827 936 019	30.6431 069	9.7923 861
940	88 36 00	830 584 000	30.6594 194	9.7958 611
941	88 54 81	833 237 621	30.6757 233	9.7993 336
942	88 73 64	835 896 888	30.6920 185	9.8028 036
943	88 92 49	838 561 807	30.7083 051	9.8062 711
944	89 11 36	841 232 384	30.7245 83	9.8097 362
945	89 30 25	843 908 625	30.7408 523	9.8131 989
946	89 49 16	846 590 536	30.7571 13	9.8166 591
947	89 68 09	849 278 123	30.7733 651	9.8201 169
948	89 87 04	851 971 392	30.7896 086	9.8235 723
949	90 06 01	854 670 349	30.8058 436	9.8270 252
950	90 25 00	857 375 000	30.8220 7	9.8304 757
951	90 44 01	860 085 351	30.8382 879	9.8339 238

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
952	90 63 04	862 801 408	30.8544 972	9.8373 695
953	90 82 09	865 523 177	30.8706 981	9.8408 127
954	91 01 16	868 250 664	30.8868 904	9.8442 536
955	91 20 25	870 983 875	30.9030 743	9.8476 92
956	91 39 36	873 722 816	30.9192 477	9.8511 28
957	91 58 49	876 467 493	30.9354 166	9.8545 617
958	91 77 64	879 217 912	30.9515 751	9.8579 929
959	91 96 81	881 974 079	30.9677 251	9.8614 218
960	92 16 00	884 736 000	30.9838 668	9.8648 483
961	92 35 21	887 503 681	31.	9.8682 724
962	92 54 44	890 277 128	31.0161 248	9.8716 941
963	92 73 69	893 056 347	31.0322 413	9.8751 135
964	92 92 96	895 841 344	31.0483 494	9.8785 305
965	93 12 25	898 632 125	31.0644 491	9.8819 451
966	93 31 56	901 428 696	31.0805 405	9.8853 574
967	93 50 89	904 231 063	31.0966 236	9.8887 673
968	93 70 24	907 039 232	31.1126 984	9.8921 749
969	93 89 61	909 853 209	31.1287 648	9.8955 801
970	94 09 00	912 673 000	31.1448 23	9.8989 83
971	94 28 41	915 498 611	31.1608 729	9.9023 835
972	94 47 84	918 330 048	31.1769 145	9.9057 817
973	94 67 29	921 167 317	31.1929 479	9.9091 776
974	94 86 76	924 010 424	31.2089 731	9.9125 712
975	95 06 25	926 859 375	31.2249 9	9.9159 624
976	95 25 76	929 714 176	31.2409 987	9.9193 513
977	95 45 29	932 574 833	31.2569 992	9.9227 379
978	95 64 84	935 441 352	31.2729 915	9.9261 222
979	95 84 41	938 313 739	31.2889 757	9.9295 042
980	96 04 00	941 192 000	31.3049 517	9.9328 839
981	96 23 61	944 076 141	31.3209 195	9.9362 613
982	96 43 24	946 966 168	31.3368 792	9.9396 363
983	96 62 89	949 862 087	31.3528 308	9.9430 092
984	96 82 56	952 763 904	31.3687 743	9.9463 797
985	97 02 25	955 671 625	31.3847 097	9.9497 479
986	97 21 96	958 585 256	31.4006 369	9.9531 138
987	97 41 69	961 504 803	31.4165 561	9.9564 775
988	97 61 44	964 430 272	31.4324 673	9.9598 389
989	97 81 21	967 361 669	31.4483 704	9.9631 981
990	98 01 00	970 299 000	31.4642 654	9.9665 549
991	98 20 81	973 242 271	31.4801 525	9.9699 095
992	98 40 64	976 191 488	31.4960 315	9.9732 619
993	98 60 49	979 146 657	31.5119 025	9.9766 12
994	98 80 36	982 107 784	31.5277 655	9.9799 599
995	99 00 25	985 074 875	31.5436 206	9.9833 055
996	99 20 16	988 047 936	31.5594 677	9.9866 488
997	99 40 09	991 026 973	31.5753 068	9.9899 9
998	99 60 04	994 011 992	31.5911 38	9.9933 289
999	99 80 01	997 002 999	31.6069 613	9.9966 656
1000	1 00 00 00	1 000 000 000	31.6227 766	10.
1001	1 00 02 01	1 003 003 001	31.6385 84	10.0033 222
1002	1 00 40 04	1 006 012 008	31.6543 836	10.0066 622
1003	1 00 60 09	1 009 027 027	31.6701 752	10.0099 899
1004	1 00 80 16	1 012 048 064	31.6859 59	10.0133 155
1005	1 01 00 25	1 015 075 125	31.7017 349	10.0166 389
1006	1 01 00 36	1 018 108 216	31.7175 03	10.0199 601
1007	1 01 40 49	1 021 147 343	31.7332 633	10.0232 791

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1008	1 01 60 64	1 024 192 512	31.7490 157	10.0265 958
1009	1 01 80 81	1 027 243 729	31.7647 603	10.0299 104
1010	1 02 01 00	1 030 301 000	31.7804 972	10.0332 228
1011	1 02 01 21	1 033 364 331	31.7962 262	10.0365 33
1012	1 02 41 44	1 036 433 728	31.8119 474	10.0398 41
1013	1 02 61 69	1 039 509 197	31.8276 609	10.0431 469
1014	1 02 81 96	1 042 590 744	31.8433 666	10.0464 506
1015	1 03 02 25	1 045 678 375	31.8590 646	10.0497 521
1016	1 03 22 56	1 048 772 096	31.8747 549	10.0530 514
1017	1 03 42 89	1 051 871 913	31.8904 374	10.0563 485
1018	1 03 63 24	1 054 977 832	31.9061 123	10.0596 435
1019	1 03 83 61	1 058 089 859	31.9217 794	10.0629 364
1020	1 04 04 00	1 061 208 000	31.9374 388	10.0662 271
1021	1 04 24 41	1 064 332 261	31.9530 906	10.0695 156
1022	1 04 44 84	1 067 462 648	31.9687 347	10.0728 02
1023	1 04 65 29	1 070 599 167	31.9843 712	10.0760 863
1024	1 04 85 76	1 073 741 824	32.	10.0793 684
1025	1 05 06 25	1 076 890 625	32.0156 212	10.0826 484
1026	1 05 26 76	1 080 045 576	32.0312 348	10.0859 262
1027	1 05 47 29	1 083 206 683	32.0468 407	10.0892 019
1028	1 05 67 84	1 086 373 952	32.0624 391	10.0924 755
1029	1 05 88 41	1 089 547 389	32.0780 298	10.0957 469
1030	1 06 09 00	1 092 727 000	32.0936 131	10.0990 163
1031	1 06 29 61	1 095 912 791	32.1091 887	10.1022 835
1032	1 06 50 24	1 099 104 768	32.1247 568	10.1055 487
1033	1 06 70 89	1 102 302 937	32.1403 173	10.1088 117
1034	1 06 91 56	1 105 507 304	32.1558 704	10.1120 726
1035	1 07 12 25	1 108 717 875	32.1714 159	10.1155 314
1036	1 07 32 96	1 111 934 656	32.1869 539	10.1185 882
1037	1 07 53 69	1 115 157 653	32.2024 844	40.1218 428
1038	1 07 74 44	1 118 386 872	32.2180 074	10.1260 953
1039	1 07 95 21	1 121 622 319	32.2335 229	10.1283 457
1040	1 08 16 00	1 124 864 000	32.2490 31	10.1315 941
1041	1 08 36 81	1 128 111 921	32.2645 316	10.1348 403
1042	1 08 57 64	1 131 366 088	32.2800 248	10.1380 845
1043	1 08 78 49	1 134 626 507	32.2955 105	10.1413 266
1044	1 08 99 36	1 137 893 184	32.3109 888	10.1445 667
1045	1 09 20 25	1 141 166 125	32.3264 598	10.1478 047
1046	1 09 41 16	1 144 445 336	32.3419 233	10.1510 406
1047	1 09 62 09	1 147 730 823	32.3573 794	10.1542 744
1048	1 09 83 04	1 151 022 592	32.3728 281	10.1575 062
1049	1 10 04 01	1 154 320 649	32.3882 695	10.1607 359
1050	1 10 25 00	1 157 625 000	32.4037 035	10.1639 636
1051	1 10 46 01	1 160 935 651	32.4191 301	10.1671 893
1052	1 10 67 04	1 164 252 608	32.4345 495	10.1704 129
1053	1 10 88 09	1 167 575 877	32.4499 615	10.1736 344
1054	1 11 09 16	1 170 905 464	32.4653 662	10.1768 539
1055	1 11 30 25	1 174 241 375	32.4807 635	10.1800 714
1056	1 11 51 36	1 177 583 616	32.4961 536	10.1832 868
1057	1 11 72 49	1 180 932 193	32.5115 364	10.1865 002
1058	1 11 93 64	1 184 287 112	32.5269 119	10.1897 116
1059	1 12 14 81	1 187 648 379	32.5422 802	10.1929 209
1060	1 12 36 00	1 191 016 000	32.5576 412	10.1961 283
1061	1 12 57 21	1 194 389 981	32.5729 949	10.1993 336
1062	1 12 78 44	1 197 770 328	32.5883 415	10.2025 369
1063	1 12 99 69	1 201 157 047	32.6036 807	10.2057 382

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1064	1 13 20 96	1 204 550 144	32.6190 129	10.2089 375
1065	1 13 42 25	1 207 949 625	32.6343 377	10.2121 347
1066	1 13 63 56	1 211 355 496	32.6496 554	10.2153 3
1067	1 13 84 89	1 214 767 763	32.6649 659	10.2185 233
1068	1 14 06 24	1 218 186 432	32.6802 693	10.2217 146
1069	1 14 27 61	1 221 611 509	32.6955 654	10.2249 039
1070	1 14 49 00	1 225 043 000	32.7108 544	10.2280 912
1071	1 14 70 41	1 228 480 911	32.7261 363	10.2312 766
1072	1 14 91 84	1 231 925 248	32.7414 111	10.2344 599
1073	1 15 13 29	1 235 376 017	32.7566 787	10.2376 413
1074	1 15 34 76	1 238 833 224	32.7719 392	10.2408 207
1075	1 15 56 25	1 242 296 875	32.7871 926	10.2439 981
1076	1 15 77 76	1 245 766 976	32.8024 389	10.2471 735
1077	1 15 99 29	1 249 243 533	32.8176 782	10.2503 47
1078	1 16 20 84	1 252 726 552	32.8329 103	10.2535 186
1079	1 16 42 41	1 256 216 039	32.8481 354	10.2566 881
1080	1 16 64 00	1 259 712 000	32.8633 535	10.2598 557
1081	1 16 85 61	1 263 214 441	32.8785 644	10.2630 213
1082	1 17 07 24	1 266 723 368	32.8937 684	10.2661 85
1083	1 17 28 89	1 270 238 787	32.9089 653	10.2693 467
1084	1 17 50 56	1 273 760 704	32.9241 553	10.2725 065
1085	1 17 72 25	1 277 289 125	32.9393 382	10.2756 644
1086	1 17 93 96	1 280 824 056	32.9545 141	10.2788 203
1087	1 18 15 69	1 284 365 503	32.9696 83	10.2819 743
1088	1 18 37 44	1 287 913 472	32.9848 45	10.2851 264
1089	1 18 59 21	1 291 467 969	33.	10.2882 765
1090	1 18 81 00	1 295 029 000	33.0151 48	10.2914 247
1091	1 19 02 81	1 298 596 571	33.0302 891	10.2945 709
1092	1 19 24 64	1 302 170 688	33.0454 233	10.2977 153
1093	1 19 46 49	1 305 751 357	33.0605 505	10.3008 577
1094	1 19 68 36	1 309 338 584	33.0756 708	10.3039 982
1095	1 19 90 25	1 312 932 375	33.0907 842	10.3071 368
1096	1 20 12 16	1 316 532 736	33.1058 907	10.3102 735
1097	1 20 34 09	1 320 139 673	33.1209 903	10.3134 083
1098	1 20 56 04	1 323 753 192	33.1360 83	10.3165 411
1099	1 20 78 01	1 327 373 299	33.1511 689	10.3196 721
1100	1 21 00 00	1 331 000 000	33.1662 479	10.3228 012
1101	1 21 22 01	1 334 633 301	33.1813 2	10.3259 284
1102	1 21 44 04	1 338 273 208	33.1963 853	10.3290 537
1103	1 21 66 09	1 341 919 727	33.2114 438	10.3321 77
1104	1 21 88 16	1 345 572 864	33.2264 955	10.3352 985
1105	1 22 10 25	1 349 232 625	33.2415 403	10.3384 181
1106	1 22 32 36	1 352 899 016	33.2565 783	10.3415 358
1107	1 22 54 49	1 356 572 043	33.2716 095	10.3446 517
1108	1 22 76 64	1 360 251 712	33.2866 339	10.3477 657
1109	1 22 98 81	1 363 938 029	33.3016 516	10.3508 778
1110	1 23 21 00	1 367 631 000	33.3166 625	10.3539 88
1111	1 23 43 21	1 371 330 631	33.3316 666	10.3570 964
1112	1 23 65 44	1 375 036 928	33.3466 64	10.3602 029
1113	1 23 87 69	1 378 749 897	33.3616 546	10.3633 076
1114	1 24 09 96	1 382 469 544	33.3766 385	10.3664 103
1115	1 24 32 25	1 386 195 875	33.3916 157	10.3695 113
1116	1 24 54 56	1 389 928 896	33.4065 862	10.3726 103
1117	1 24 76 89	1 393 668 613	33.4215 499	10.3757 076
1118	1 24 99 24	1 397 415 032	33.4365 07	10.3788 03
1119	1 25 21 61	1 401 168 159	33.4514 573	10.3818 965

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1120	1 25 44 00	1 404 928 000	33.4664 011	10.3849 882
1121	1 25 66 41	1 408 694 561	33.4813 381	10.3880 781
1122	1 25 88 84	1 412 467 848	33.4962 684	10.3911 661
1123	1 26 11 29	1 416 247 867	33.5111 921	10.3942 523
1124	1 26 33 76	1 420 034 624	33.5261 092	10.3973 366
1125	1 26 56 25	1 423 828 125	33.5410 196	10.4004 192
1126	1 26 78 76	1 427 628 376	33.5559 234	10.4034 999
1127	1 27 01 29	1 431 435 383	33.5708 206	10.4065 787
1128	1 27 23 84	1 435 249 152	33.5857 112	10.4096 557
1129	1 27 46 41	1 439 069 689	33.6005 952	10.4127 31
1130	1 27 69 00	1 442 897 000	33.6154 726	10.4158 044
1131	1 27 91 61	1 446 731 091	33.6303 434	10.4188 76
1132	1 28 14 24	1 450 571 968	33.6452 077	10.4219 458
1133	1 28 36 89	1 454 419 637	33.6600 653	10.4250 138
1134	1 28 59 56	1 458 274 104	33.6749 165	10.4280 8
1135	1 28 82 25	1 462 135 375	33.6897 61	10.4311 443
1136	1 29 04 96	1 466 003 456	33.7045 991	10.4342 069
1137	1 29 27 69	1 469 878 353	33.7194 306	10.4372 677
1138	1 29 50 44	1 473 760 072	33.7340 556	10.4403 267
1139	1 29 73 21	1 477 648 619	33.7490 741	10.4433 839
1140	1 29 96 00	1 481 544 000	33.7638 860	10.4464 393
1141	1 30 18 81	1 485 446 221	33.7786 915	10.4494 929
1142	1 30 41 64	1 489 355 288	33.7934 905	10.4525 448
1143	1 30 64 49	1 493 271 207	33.8082 83	10.4555 948
1144	1 30 87 36	1 497 193 984	33.8230 691	10.4586 431
1145	1 31 10 25	1 501 123 625	33.8378 486	10.4616 896
1146	1 31 33 16	1 505 060 136	33.8526 218	10.4647 343
1147	1 31 56 09	1 509 603 523	33.8673 884	10.4677 773
1148	1 31 79 04	1 512 953 792	33.8821 487	10.4708 185
1149	1 32 02 01	1 516 910 949	33.8969 025	10.4738 579
1150	1 32 25 00	1 520 875 000	33.9116 499	10.4768 955
1151	1 32 48 01	1 524 845 951	33.9263 909	10.4799 314
1152	1 32 71 04	1 528 823 808	33.9411 255	10.4829 656
1153	1 32 94 09	1 532 808 577	33.9558 537	10.4859 98
1154	1 33 17 16	1 536 800 264	33.9705 755	10.4890 286
1155	1 33 40 25	1 540 798 875	33.9852 91	10.4920 575
1156	1 33 63 36	1 544 804 416	34.	10.4950 847
1157	1 33 86 49	1 548 816 893	34.0147 027	10.4981 101
1158	1 34 09 64	1 552 836 312	34.0293 99	10.5011 337
1159	1 34 32 81	1 556 862 679	34.0440 89	10.5041 556
1160	1 34 56 00	1 560 896 000	34.0587 727	10.5071 757
1161	1 34 79 21	1 564 936 281	34.0734 501	10.5101 942
1162	1 35 02 44	1 568 983 528	34.0881 211	10.5132 109
1163	1 35 25 69	1 573 037 747	34.1027 858	10.5162 259
1164	1 35 48 96	1 577 098 944	34.1174 442	10.5192 391
1165	1 35 72 25	1 581 167 125	34.1320 963	10.5222 506
1166	1 35 95 56	1 585 242 296	34.1467 422	10.5252 604
1167	1 36 18 89	1 589 324 463	34.1613 817	10.5282 685
1168	1 36 42 24	1 593 413 632	34.1760 15	10.5312 749
1169	1 36 65 61	1 597 509 809	34.1906 42	10.5342 795
1170	1 36 89 00	1 601 613 000	34.2052 627	10.5372 825
1171	1 37 12 41	1 605 723 211	34.2198 773	10.5402 837
1172	1 37 35 84	1 609 840 448	34.2344 855	10.5432 832
1173	1 37 59 29	1 613 964 717	34.2490 875	10.5462 81
1174	1 37 82 76	1 618 096 024	34.2636 834	10.5492 771
1175	1 38 06 25	1 622 234 375	34.2782 73	10.5522 715

Table—(*Continued*).

Number.	Square	Cube.	Square Root.	Cube Root.
1176	1 38 29 76	1 626 379 776	34.2928 564	10.5552 642
1177	1 38 53 29	1 630 532 233	34.3074 336	10.5582 552
1178	1 38 76 84	1 634 691 752	34.3220 046	10.5612 445
1179	1 39 00 41	1 638 858 339	34.3365 694	10.5642 322
1180	1 39 24 00	1 643 032 000	34.3511 281	10.5672 181
1181	1 39 47 61	1 647 212 741	34.3656 805	10.5702 024
1182	1 39 71 24	1 651 400 568	34.3802 268	10.5731 849
1183	1 39 94 89	1 655 595 487	34.3947 67	10.5761 658
1184	1 40 18 56	1 659 797 504	34.4093 011	10.5791 449
1185	1 40 42 25	1 664 006 625	34.4238 289	10.5821 225
1186	1 40 65 96	1 668 222 856	34.4383 507	10.5850 983
1187	1 40 89 69	1 672 446 203	34.4528 663	10.5880 725
1188	1 41 13 44	1 676 676 672	34.4673 759	10.5910 45
1189	1 41 37 21	1 680 914 629	34.4818 793	10.5940 158
1190	1 41 61 00	1 685 159 000	34.4963 766	10.5969 85
1191	1 41 84 81	1 689 410 871	34.5108 678	10.5999 525
1192	1 42 08 64	1 693 669 888	34.5253 53	10.6029 184
1193	1 42 32 49	1 697 936 057	34.5398 321	10.6058 826
1194	1 42 56 36	1 702 209 384	34.5543 051	10.6088 451
1195	1 42 80 25	1 706 489 875	34.5687 72	10.6118 06
1196	1 43 04 16	1 710 777 536	34.5832 329	10.6147 652
1197	1 43 28 09	1 715 072 373	34.5976 879	10.6177 228
1198	1 43 52 04	1 719 374 392	34.6121 366	10.6206 788
1199	1 43 76 01	1 723 683 599	34.6265 794	10.6236 331
1200	1 44 00 00	1 728 000 000	34.6410 162	10.6265 857
1201	1 44 24 01	1 732 323 601	34.6554 469	10.6295 367
1202	1 44 48 04	1 736 654 408	34.6698 716	10.6324 86
1203	1 44 72 09	1 740 992 427	34.6842 904	10.6354 338
1204	1 44 96 16	1 745 337 664	34.6987 031	10.6383 799
1205	1 45 20 25	1 749 690 125	34.7131 099	10.6413 244
1206	1 45 44 36	1 754 049 816	34.7275 107	10.6442 672
1207	1 45 68 49	1 758 416 743	34.7419 055	10.6472 085
1208	1 45 92 64	1 762 790 912	34.7562 944	10.6501 48
1209	1 46 16 81	1 767 172 329	34.7706 773	10.6530 86
1210	1 46 41 00	1 771 561 000	34.7850 543	10.6560 223
1211	1 46 65 21	1 775 956 931	34.7994 253	10.6589 57
1212	1 46 89 44	1 780 360 128	34.8137 904	10.6618 902
1213	1 47 13 69	1 784 770 597	34.8281 495	10.6648 217
1214	1 47 37 96	1 789 188 344	34.8425 028	10.6677 516
1215	1 47 62 25	1 793 613 375	34.8568 501	10.6706 799
1216	1 47 86 56	1 798 045 696	34.8711 915	10.6736 066
1217	1 48 10 89	1 802 485 313	34.8855 271	10.6765 317
1218	1 48 35 24	1 806 932 232	34.8998 567	10.6794 552
1219	1 48 59 61	1 811 386 459	34.9141 805	10.6823 771
1220	1 48 84 00	1 815 848 000	34.9284 984	10.6852 973
1221	1 49 08 41	1 820 316 861	34.9428 104	10.6882 16
1222	1 49 32 84	1 824 793 048	34.9571 166	10.6911 331
1223	1 49 57 29	1 829 276 567	34.9714 169	10.6940 486
1224	1 49 81 76	1 833 767 244	34.9857 114	10.6969 625
1225	1 50 06 25	1 838 265 625	35.	10.6998 748
1226	1 50 30 76	1 842 771 176	35.0142 828	10.7027 855
1227	1 50 55 29	1 847 284 083	35.0285 598	10.7056 947
1228	1 50 79 84	1 851 804 352	35.0428 309	10.7086 023
1229	1 51 04 41	1 856 331 989	35.0570 963	10.7115 083
1230	1 51 29 00	1 860 867 000	35.0713 558	10.7144 127
1231	1 51 53 61	1 865 409 391	35.0856 096	10.7173 155

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1232	1 51 78 24	1 869 959 168	35.0998 575	10.7202 168
1233	1 52 02 89	1 874 516 337	35.1140 997	10.7231 165
1234	1 52 27 56	1 879 080 904	35.1283 361	10.7260 146
1235	1 52 52 25	1 883 652 875	35.1425 568	10.7289 112
1236	1 52 76 96	1 888 232 256	35.1567 917	10.7318 062
1237	1 53 01 69	1 892 819 053	35.1710 108	10.7346 997
1238	1 53 26 44	1 897 413 272	35.1852 242	10.7375 916
1239	1 53 51 21	1 902 014 919	35.1994 318	10.7404 819
1240	1 53 76 00	1 906 624 000	35.2136 337	10.7433 707
1241	1 54 00 81	1 911 240 521	35.2278 299	10.7462 579
1242	1 54 25 64	1 915 864 488	35.2420 204	10.7491 436
1243	1 54 50 49	1 920 495 907	35.2562 051	10.7520 277
1244	1 54 75 36	1 925 134 784	35.2703 842	10.7549 103
1245	1 55 00 25	1 929 781 125	35.2845 575	10.7577 913
1246	1 55 25 16	1 934 434 936	35.2987 252	10.7606 708
1247	1 55 50 09	1 939 096 223	35.3128 872	10.7635 488
1248	1 55 75 04	1 943 764 992	35.3270 435	10.7664 252
1249	1 56 00 01	1 948 441 249	35.3411 941	10.7693 001
1250	1 56 25 00	1 953 125 000	35.3553 391	10.7721 735
1251	1 56 50 01	1 957 816 251	35.3694 784	10.7750 453
1252	1 56 75 04	1 962 515 008	35.3836 12	10.7779 156
1253	1 57 00 09	1 967 221 277	35.3977 4	10.7807 843
1254	1 57 25 16	1 971 945 064	35.4118 624	10.7836 516
1255	1 57 50 25	1 976 656 375	35.4259 792	10.7865 173
1256	1 57 75 36	1 981 385 216	35.4400 903	10.7893 815
1257	1 58 00 49	1 986 121 593	35.4541 958	10.7922 441
1258	1 58 25 64	1 990 865 512	35.4682 957	10.7951 053
1259	1 58 50 81	1 995 616 979	35.4823 9	10.7979 649
1260	1 58 76 00	2 000 376 000	35.4964 787	10.8008 23
1261	1 59 01 21	2 005 142 581	35.5105 618	10.8036 797
1262	1 59 26 44	2 009 916 728	35.5246 393	10.8065 348
1263	1 59 51 69	2 014 698 447	35.5387 113	10.8093 884
1264	1 59 76 96	2 019 487 744	35.5527 777	10.8122 404
1265	1 60 02 25	2 024 284 625	35.5668 385	10.8150 909
1266	1 60 27 56	2 029 089 096	35.5808 937	10.8179 4
1267	1 60 52 89	2 033 901 163	35.5949 434	10.8207 876
1268	1 60 78 24	2 038 720 832	35.6089 876	10.8236 336
1269	1 61 03 61	2 043 548 109	35.6230 262	10.8264 782
1270	1 61 29 00	2 048 383 000	35.6370 593	10.8293 213
1271	1 61 54 41	2 053 225 511	35.6510 869	10.8321 629
1272	1 61 79 84	2 058 075 648	35.6651 09	10.8350 03
1273	1 62 05 29	2 062 933 417	35.6791 255	10.8378 416
1274	1 62 30 76	2 067 798 824	35.6931 366	10.8406 788
1275	1 62 56 25	2 072 671 875	35.7071 421	10.8435 144
1276	1 62 81 76	2 077 552 576	35.7211 422	10.8463 485
1277	1 63 07 29	2 082 440 933	35.7351 367	10.8491 812
1278	1 63 32 84	2 087 336 952	35.7491 258	10.8520 125
1279	1 63 58 41	2 092 240 639	35.7631 095	10.8548 422
1280	1 63 84 00	2 097 152 000	35.7770 876	10.8576 704
1281	1 64 09 61	2 102 071 841	35.7910 603	10.8604 972
1282	1 64 35 24	2 106 997 768	35.8050 276	10.8633 225
1283	1 64 60 89	2 111 932 187	35.8189 894	10.8661 464
1284	1 64 86 56	2 116 874 304	35.8329 457	10.8689 687
1285	1 65 12 25	2 121 824 125	35.8468 966	10.8717 897
1286	1 65 37 96	2 126 781 656	35.8608 421	10.8746 091
1287	1 65 63 69	2 131 746 903	35.8747 822	10.8774 271

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1288	1 65 89 44	2 136 719 872	35.8887 169	10.8802 436
1289	1 66 15 21	2 141 700 569	35.9026 461	10.8830 587
1290	1 66 41 00	2 146 689 000	35.9165 699	10.8858 723
1291	1 66 66 81	2 151 685 171	35.9304 884	10.8886 845
1292	1 66 92 64	2 156 689 088	35.9444 015	10.8914 952
1293	1 67 18 49	2 161 700 757	35.9583 092	10.8943 044
1294	1 67 44 36	2 166 720 184	35.9722 115	10.8971 123
1295	1 67 70 25	2 171 747 375	35.9861 084	10.8999 186
1296	1 67 96 16	2 176 782 336	36.	10.9027 235
1297	1 68 22 09	2 181 825 073	36.0138 862	10.9055 269
1298	1 68 48 04	2 186 875 592	36.0277 671	10.9083 29
1299	1 68 74 01	2 191 933 899	36.0416 426	10.9111 296
1300	1 69 00 00	2 197 000 000	36.0555 128	10.9139 287
1301	1 69 26 01	2 202 073 901	36.0693 776	10.9167 265
1302	1 69 52 04	2 207 155 608	36.0832 371	10.9195 228
1303	1 69 78 09	2 212 245 127	36.0970 913	10.9223 177
1304	1 70 04 16	2 217 342 464	36.1109 402	10.9251 111
1305	1 70 30 25	2 222 447 625	36.1247 837	10.9279 031
1306	1 70 56 36	2 227 560 616	36.1386 22	10.9306 937
1307	1 70 82 49	2 232 681 443	36.1524 55	10.9334 829
1308	1 71 08 64	2 237 810 112	36.1662 826	10.9362 706
1309	1 71 34 81	2 242 946 629	36.1801 05	10.9390 569
1310	1 71 61 00	2 248 091 000	36.1939 221	10.9418 418
1311	1 71 87 21	2 253 243 231	36.2077 34	10.9446 253
1312	1 72 13 44	2 258 403 328	36.2215 406	10.9475 074
1313	1 72 39 69	2 263 571 297	36.2353 419	10.9501 88
1314	1 72 65 96	2 268 747 144	36.2491 379	10.9529 673
1315	1 72 92 25	2 273 930 875	36.2626 287	10.9557 451
1316	1 73 18 56	2 279 122 496	36.2767 143	10.9585 215
1317	1 73 44 89	2 284 322 013	36.2904 946	10.9612 965
1318	1 73 71 24	2 289 529 432	36.3042 697	10.9640 701
1319	1 73 97 61	2 294 744 759	36.3180 396	10.9668 423
1320	1 74 24 00	2 299 968 000	36.3318 042	10.9696 131
1321	1 74 50 41	2 305 199 161	36.3455 637	10.9723 825
1322	1 74 76 84	2 310 438 248	36.3593 179	10.9751 505
1323	1 75 93 29	2 315 685 267	36.3730 67	10.9779 171
1324	1 75 29 76	2 320 940 224	36.3868 108	10.9806 823
1325	1 75 56 25	2 326 203 125	36.4005 494	10.9834 462
1326	1 75 82 76	2 331 473 976	36.4142 829	10.9862 086
1327	1 76 09 29	2 336 752 783	36.4280 112	10.9889 696
1328	1 76 35 84	2 342 039 552	36.4417 343	10.9917 293
1329	1 76 62 41	2 347 334 289	36.4554 523	10.9944 876
1330	1 76 89 00	2 352 637 000	36.4691 65	10.9972 445
1331	1 77 15 61	2 357 947 691	36.4828 727	11.
1332	1 77 42 24	2 363 266 368	36.4965 752	11.0027 541
1333	1 77 68 89	2 368 593 037	36.5102 725	11.0055 069
1334	1 77 95 56	2 373 927 704	36.5239 647	11.0082 583
1335	1 78 22 25	2 379 270 375	36.5376 518	11.0110 082
1336	1 78 48 96	2 384 621 056	36.5513 388	11.0137 569
1337	1 78 75 69	2 389 979 753	36.5650 106	11.0165 041
1338	1 79 02 44	2 395 346 472	36.5786 823	11.0192 5
1339	1 79 29 21	2 400 721 219	36.5923 489	11.0219 945
1340	1 79 56 00	2 406 104 000	36.6060 104	11.0247 377
1341	1 79 82 81	2 411 494 821	36.6196 668	11.0274 795
1342	1 80 09 64	2 416 893 688	36.6333 181	11.0302 199
1343	1 80 36 49	2 422 300 607	36.6469 644	11.0329 59

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1344	1 80 63 36	2 427 715 584	36.6606 056	11.0356 967
1345	1 80 90 25	2 433 138 625	36.6742 416	11.0384 33
1346	1 81 17 16	2 438 569 736	36.6878 726	11.0411 68
1347	1 81 44 09	2 444 008 923	36.7014 986	11.0439 017
1348	1 81 71 04	2 449 456 192	36.7151 195	11.0466 339
1349	1 81 98 01	2 454 911 549	36.7287 353	11.0493 649
1350	1 82 25 00	2 460 375 000	36.7423 461	11.0520 945
1351	1 82 52 01	2 465 846 551	36.7559 519	11.0548 227
1352	1 82 79 04	2 471 326 208	36.7695 526	11.0575 497
1353	1 83 06 09	2 476 813 977	36.7831 483	11.0602 752
1354	1 83 33 16	2 482 309 864	36.7967 39	11.0629 994
1355	1 83 60 25	2 487 813 875	36.8103 246	11.0657 222
1356	1 83 87 36	2 493 326 016	36.8239 053	11.0684 437
1357	1 84 14 49	2 498 846 293	36.8374 809	11.0711 639
1358	1 84 41 64	2 504 374 712	36.8510 515	11.0738 828
1359	1 84 68 81	2 509 911 279	36.8646 172	11.0766 003
1360	1 84 96 00	2 515 456 000	36.8781 778	11.0793 165
1361	1 85 23 21	2 521 008 881	36.8917 335	11.0820 314
1362	1 85 50 44	2 526 569 928	36.9052 842	11.0847 449
1363	1 85 77 69	2 532 139 147	36.9188 299	11.0874 571
1364	1 86 04 96	2 537 716 544	36.9323 706	11.0901 679
1365	1 86 32 25	2 543 302 125	36.9459 064	11.0928 775
1366	1 86 59 56	2 548 895 896	36.9594 372	11.0955 857
1367	1 86 86 89	2 554 497 863	36.9729 631	11.0982 926
1368	1 87 14 24	2 560 108 032	36.9864 84	11.1009 982
1369	1 87 41 61	2 565 726 409	37.	11.1037 025
1370	1 87 69 00	2 571 353 000	37.0135 11	11.1064 054
1371	1 87 96 41	2 576 987 811	37.0270 172	11.1091 07
1372	1 88 23 84	2 582 630 848	37.0405 184	11.1118 073
1373	1 88 51 29	2 588 282 117	37.0540 146	11.1145 064
1374	1 88 78 76	2 593 941 624	37.0675 06	11.1172 041
1375	1 89 06 25	2 599 609 375	37.0899 924	11.1199 004
1376	1 89 33 76	2 605 285 376	37.0944 74	11.1225 955
1377	1 89 61 29	2 610 969 633	37.1079 506	11.1252 893
1378	1 89 88 84	2 616 662 152	37.1214 224	11.1279 817
1379	1 90 16 41	2 622 362 939	37.1348 893	11.1306 729
1380	1 90 44 00	2 628 072 000	37.1483 512	11.1333 628
1381	1 90 71 61	2 633 789 341	37.1618 084	11.1360 514
1382	1 90 99 24	2 639 514 968	37.1752 606	11.1387 386
1383	1 91 26 89	2 645 248 887	37.1887 079	11.1414 246
1384	1 91 54 56	2 650 991 104	37.2021 505	11.1441 093
1385	1 91 82 25	2 656 741 625	37.2155 881	11.1467 926
1386	1 92 09 96	2 662 500 456	37.2290 209	11.1494 747
1387	1 92 37 69	2 668 267 603	37.2424 489	11.1521 555
1388	1 92 65 44	2 674 043 072	37.2558 72	11.1548 35
1389	1 92 93 21	2 679 826 869	37.2692 903	11.1575 133
1390	1 93 21 00	2 685 619 000	37.2827 037	11.1601 903
1391	1 93 48 81	2 691 419 471	37.2961 124	11.1628 659
1392	1 93 76 64	2 697 228 288	37.3095 162	11.1655 403
1393	1 94 04 49	2 703 045 457	37.3229 152	11.1682 134
1394	1 94 32 36	2 708 870 984	37.3363 094	11.1708 852
1395	1 94 60 25	2 714 704 875	37.3496 988	11.1735 558
1396	1 94 88 16	2 720 547 136	37.3630 834	11.1762 25
1397	1 95 16 09	2 726 397 773	37.3764 632	11.1788 93
1398	1 95 44 04	2 732 256 792	37.3898 382	11.1815 598
1399	1 95 72 01	2 738 124 199	37.4032 084	11.1842 252

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1400	1 96 00 00	2 744 000 000	37.4165 738	11.1868 894
1401	1 96 28 01	2 749 884 201	37.4299 345	11.1895 523
1402	1 96 56 04	2 755 776 808	37.4432 904	11.1922 139
1403	1 96 84 09	2 761 677 827	37.4566 416	11.1948 743
1404	1 97 12 16	2 767 587 264	37.4699 88	11.1975 334
1405	1 97 40 25	2 773 505 125	37.4833 296	11.2001 913
1406	1 97 68 36	2 779 431 416	37.4966 665	11.2028 479
1407	1 97 96 49	2 785 366 143	37.5099 987	11.2055 032
1408	1 98 24 64	2 791 309 312	37.5233 261	11.2081 573
1409	1 98 52 81	2 797 260 929	37.5366 487	11.2108 101
1410	1 98 81 00	2 803 221 000	37.5499 667	11.2134 617
1411	1 99 09 21	2 809 189 531	37.5632 799	11.2161 12
1412	1 99 37 44	2 815 166 528	37.5765 885	11.2187 611
1413	1 99 65 69	2 821 151 997	37.5898 922	11.2214 089
1414	1 99 93 96	2 827 145 944	37.6031 913	11.2240 054
1415	2 00 22 25	2 833 148 375	37.6164 857	11.2267 007
1416	2 00 50 56	2 839 159 296	37.6297 754	11.2293 448
1417	2 00 78 89	2 845 178 713	37.6430 604	11.2319 876
1418	2 01 07 24	2 851 206 632	37.6563 407	11.2346 292
1419	2 01 35 61	2 857 243 059	37.6696 164	11.2372 696
1420	2 01 64 00	2 863 288 000	37.6828 874	11.2399 087
1421	2 01 92 41	2 869 341 461	37.6961 536	11.2425 465
1422	2 02 20 84	2 875 403 448	37.7094 153	11.2451 831
1423	2 02 49 29	2 881 473 967	37.7226 722	11.2478 185
1424	2 02 77 76	2 887 553 024	37.7359 245	11.2504 527
1425	2 03 06 25	2 893 640 625	37.7491 722	11.2530 856
1426	2 03 34 76	2 899 736 776	37.7624 152	11.2557 173
1427	2 03 63 29	2 905 841 483	37.7756 535	11.2583 478
1428	2 03 91 84	2 911 954 752	37.7888 873	11.2609 77
1429	2 04 20 41	2 918 070 589	37.8021 163	11.2636 05
1430	2 04 49 00	2 924 207 000	37.8153 408	11.2662 318
1431	2 04 77 61	2 930 345 991	37.8285 606	11.2688 573
1432	2 05 06 24	2 936 493 568	37.8417 759	11.2714 816
1433	2 05 34 89	2 942 649 737	37.8549 864	11.2741 047
1434	2 05 63 56	2 948 814 504	37.8681 924	11.2767 266
1435	2 05 92 25	2 954 987 875	37.8813 938	11.2793 472
1436	2 06 20 96	2 961 169 856	37.8945 906	11.2819 666
1437	2 06 49 69	2 967 360 453	37.9077 828	11.2845 849
1438	2 06 78 44	2 973 559 672	37.9209 704	11.2872 019
1439	2 07 07 21	2 979 767 519	37.9341 535	11.2898 177
1440	2 07 36 00	2 985 984 000	37.9473 319	11.2924 323
1441	2 07 64 81	2 992 209 121	37.9605 058	11.2950 457
1442	2 07 93 64	2 998 442 888	37.9736 751	11.2976 579
1443	2 08 22 49	3 004 685 307	37.9868 398	11.3002 688
1444	2 08 51 36	3 010 936 384	38.	11.3028 786
1445	2 08 80 25	3 017 196 125	38.0131 556	11.3054 871
1446	2 09 09 16	3 023 464 536	38.0263 067	11.3080 945
1447	2 09 38 09	3 029 741 623	38.0394 532	11.3107 006
1448	2 09 67 04	3 036 027 392	38.0525 952	11.3133 056
1449	2 09 96 01	3 042 321 849	38.0657 326	11.3159 094
1450	2 10 25 00	3 048 625 000	38.0788 655	11.3185 119
1451	2 10 54 01	3 054 936 851	38.0919 939	11.3211 132
1452	2 10 83 04	3 061 257 408	38.1051 178	11.3237 134
1453	2 11 12 09	3 067 586 677	38.1182 371	11.3263 124
1454	2 11 41 16	3 073 924 664	38.1313 519	11.3289 102
1455	2 11 70 25	3 080 271 375	38.1444 622	11.3315 067

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1456	2 11 99 36	3 086 626 816	38.1575 681	11.3341 022
1457	2 12 28 49	3 092 990 993	38.1706 693	11.3366 964
1458	2 12 57 64	3 099 363 912	38.1837 662	11.3392 894
1459	2 12 86 81	3 105 745 579	38.1968 585	11.3418 813
1460	2 13 16 00	3 112 136 000	38.2099 463	11.3444 719
1461	2 13 45 21	3 118 535 181	38.2230 297	11.3470 614
1462	2 13 74 44	3 124 943 128	38.2361 085	11.3496 497
1463	2 14 03 69	3 131 359 847	38.2491 829	11.3522 368
1464	2 14 32 96	3 137 785 344	38.2622 529	11.3548 227
1465	2 14 62 25	3 144 219 625	38.2753 184	11.3574 075
1466	2 14 91 56	3 150 662 696	38.2883 794	11.3599 911
1467	2 15 20 89	3 157 114 563	38.3014 36	11.3625 735
1468	2 15 50 24	3 163 575 232	38.3144 881	11.3651 547
1469	2 15 79 61	3 170 044 709	38.3275 358	11.3677 347
1470	2 16 09 00	3 176 523 000	38.3405 79	11.3703 136
1471	2 16 38 41	3 183 010 111	38.3536 178	11.3728 914
1472	2 16 67 84	3 189 506 048	38.3666 522	11.3754 679
1473	2 16 97 29	3 196 010 817	38.3796 821	11.3780 433
1474	2 17 26 76	3 202 524 424	38.3927 076	11.3806 175
1475	2 17 56 25	3 209 046 875	38.4057 287	11.3831 906
1476	2 17 85 76	3 215 578 176	38.4187 454	11.3857 625
1477	2 18 15 29	3 222 118 333	38.4317 577	11.3883 332
1478	2 18 44 84	3 228 667 352	38.4447 656	11.3909 028
1479	2 18 74 41	3 235 225 239	38.4577 691	11.3934 712
1480	2 19 04 00	3 241 792 000	38.4707 681	11.3960 384
1481	2 19 33 61	3 248 367 641	38.4837 627	11.3986 045
1482	2 19 63 24	3 254 952 168	38.4967 53	11.4011 695
1483	2 19 92 89	3 261 545 587	38.5097 39	11.4037 332
1484	2 20 22 56	3 268 147 904	38.5227 206	11.4062 959
1485	2 20 52 25	3 274 759 125	38.5356 977	11.4088 574
1486	2 20 81 96	3 281 379 256	38.5486 705	11.4114 177
1487	2 21 11 69	3 288 008 303	38.5616 389	11.4139 769
1488	2 21 41 44	3 294 646 272	38.5746 03	11.4165 349
1489	2 21 71 21	3 301 293 169	38.5875 627	11.4190 918
1490	2 22 01 00	3 307 949 000	38.6005 181	11.4206 476
1491	2 22 30 81	3 314 613 771	38.6134 691	11.4242 022
1492	2 22 60 64	3 321 287 488	38.6264 158	11.4267 556
1493	2 22 90 49	3 327 970 157	38.6393 582	11.4293 079
1494	2 23 20 36	3 334 661 784	38.6522 962	11.4318 591
1495	2 23 50 25	3 341 362 375	38.6652 299	11.4344 092
1496	2 23 80 16	3 348 071 936	38.6781 593	11.4369 581
1497	2 24 10 09	3 354 790 473	38.6910 843	11.4395 059
1498	2 24 40 04	3 361 517 992	38.7040 05	11.4420 525
1499	2 24 70 01	3 368 254 499	38.7169 214	11.4445 98
1500	2 25 00 00	3 375 000 000	38.7298 335	11.4471 424
1501	2 25 30 01	3 381 754 501	38.7427 412	11.4496 857
1502	2 25 60 04	3 388 518 008	38.7556 447	11.4522 278
1503	2 25 90 09	3 395 290 527	38.7685 439	11.4547 688
1504	2 26 20 16	3 402 072 064	38.7814 389	11.4573 087
1505	2 26 50 25	3 408 862 625	38.7943 294	11.4598 474
1506	2 26 80 36	3 415 662 216	38.8072 158	11.4623 85
1507	2 27 10 49	3 422 470 843	38.8200 978	11.4649 215
1508	2 27 40 64	3 429 288 512	38.8329 757	11.4674 568
1509	2 27 70 81	3 436 115 229	38.8458 491	11.4699 911
1510	2 28 01 00	3 442 951 000	38.8587 184	11.4725 242
1511	2 28 31 21	3 449 795 831	38.8715 834	11.4750 562

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1512	2 28 61 44	3 456 649 728	38.8844 442	11.4775 871
1513	2 28 91 69	3 463 512 697	38.8973 006	11.4801 169
1514	2 29 91 96	3 470 384 744	38.9101 529	11.4826 455
1515	2 29 52 25	3 477 265 875	38.9230 009	11.4851 731
1516	2 29 82 56	3 484 156 096	38.9358 447	11.4876 995
1517	2 30 12 89	3 491 055 413	38.9486 841	11.4902 249
1518	2 30 43 24	3 497 963 832	38.9615 194	11.4927 491
1519	2 30 73 61	3 504 881 359	38.9743 505	11.4952 722
1520	2 31 04 00	3 511 808 000	38.9871 774	11.4977 942
1521	2 31 34 41	3 518 743 761	39.	11.5003 151
1522	2 31 64 84	3 525 688 648	39.0128 184	11.5028 348
1523	2 31 95 29	3 532 642 667	39.0256 326	11.5053 535
1524	2 32 25 76	3 539 605 824	39.0384 426	11.5078 711
1525	2 32 56 25	3 546 578 125	39.0512 483	11.5103 876
1526	2 32 86 76	3 553 559 576	39.0640 499	11.5129 03
1527	2 33 17 29	3 560 558 183	39.0768 473	11.5154 173
1528	2 33 47 84	3 567 549 552	39.0896 406	11.5179 305
1529	2 33 78 41	3 574 558 889	39.1024 296	11.5204 425
1530	2 34 09 00	3 581 577 000	39.1152 144	11.5229 535
1531	2 34 39 61	3 588 604 291	39.1279 951	11.5254 634
1532	2 34 70 24	3 595 640 768	39.1407 716	11.5279 722
1533	2 35 00 89	3 602 686 437	39.1535 439	11.5304 799
1534	2 35 31 56	3 609 741 304	39.1663 12	11.5329 865
1535	2 35 62 25	3 616 805 375	39.1790 76	11.5354 92
1536	2 35 92 96	3 623 878 656	39.1918 359	11.5379 965
1537	2 36 23 69	3 630 961 153	39.2045 915	11.5404 998
1538	2 36 54 44	3 638 052 872	39.2173 431	11.5430 021
1539	2 36 85 21	3 645 153 819	39.2300 905	11.5455 033
1540	2 37 16 00	3 652 264 000	39.2428 337	11.5480 034
1541	2 37 46 81	3 659 383 421	39.2555 728	11.5505 025
1542	2 37 77 64	3 666 512 088	39.2683 078	11.5530 004
1543	2 38 08 49	3 673 650 007	39.2810 387	11.5554 973
1544	2 38 39 36	3 680 797 184	39.2937 654	11.5579 931
1545	2 38 70 25	3 687 953 625	39.3064 88	11.5604 878
1546	2 39 01 16	3 695 119 336	39.3192 065	11.5629 815
1547	2 39 32 09	3 702 294 323	39.3319 208	11.5654 74
1548	2 39 63 04	3 709 478 592	39.3446 311	11.5679 655
1549	2 39 94 01	3 716 672 149	39.3573 373	11.5704 559
1550	2 40 25 00	3 723 875 000	39.3700 394	11.5729 453
1551	2 40 56 01	3 731 087 151	39.3827 373	11.5754 336
1552	2 40 87 04	3 738 308 608	39.3954 312	11.5779 208
1553	2 41 18 09	3 745 539 377	39.4081 21	11.5804 069
1554	2 41 49 16	3 752 779 464	39.4208 067	11.5828 919
1555	2 41 80 25	3 760 028 875	39.4334 883	11.5853 759
1556	2 42 11 36	3 767 287 616	39.4461 658	11.5878 588
1557	2 42 42 49	3 774 555 693	39.4588 393	11.5903 407
1558	2 42 73 64	3 781 833 112	39.4715 087	11.5928 215
1559	2 43 04 81	3 789 119 879	39.4841 74	11.5953 013
1560	2 43 36 00	3 796 416 000	39.4968 353	11.5977 799
1561	2 43 67 21	3 803 721 481	39.5094 925	11.6002 576
1562	2 43 98 44	3 811 036 328	39.5221 457	11.6027 342
1563	2 44 29 69	3 818 360 547	39.5347 948	11.6052 097
1564	2 44 60 96	3 825 641 144	39.5474 399	11.6076 841
1565	2 44 92 25	3 833 037 125	39.5600 809	11.6101 575
1566	2 45 23 56	3 840 389 496	39.5727 179	11.6126 299
1567	2 45 54 89	3 847 751 263	39.5853 508	11.6151 012

Table—(*Continued*).

Number.	Square.	Cube.	Square Root.	Cube Root.
1568	2 45 86 24	3 855 123 432	39.5979 797	11.6175 715
1569	2 46 17 61	3 862 503 009	39.6106 046	11.6200 407
1570	2 46 49 00	3 869 883 000	39.6232 255	11.6225 088
1571	2 46 80 41	3 877 292 411	39.6358 424	11.6249 759
1572	2 47 11 84	3 884 701 248	39.6484 552	11.6274 42
1573	2 47 43 29	3 892 119 517	39.6610 64	11.6299 07
1574	2 47 74 76	3 899 547 224	39.6736 688	11.6323 71
1575	2 48 06 25	3 906 984 375	39.6862 696	11.6348 339
1576	2 48 37 76	3 914 430 976	39.6988 665	11.6372 957
1577	2 48 69 29	3 921 887 033	39.7114 593	11.6397 566
1578	2 49 00 84	3 929 352 552	39.7240 481	11.6422 164
1579	2 49 32 41	3 936 827 539	39.7366 329	11.6446 751
1580	2 49 64 00	3 944 312 000	39.7492 138	11.6471 329
1581	2 49 95 61	3 951 805 941	39.7617 907	11.6495 895
1582	2 50 27 24	3 959 309 368	39.7743 636	11.6520 452
1583	2 50 58 89	3 966 822 287	39.7869 325	11.6544 998
1584	2 50 90 56	3 974 344 704	39.7994 975	11.6569 534
1585	2 51 22 25	3 981 876 625	39.8120 585	11.6594 059
1586	2 51 53 96	3 989 418 056	39.8241 155	11.6618 574
1587	2 51 85 69	3 996 969 003	39.8376 686	11.6643 079
1588	2 52 17 44	4 004 529 472	39.8497 177	11.6667 574
1589	2 52 49 21	4 012 099 469	39.8622 628	11.6692 058
1590	2 52 81 00	4 019 679 000	39.8748 04	11.6716 532
1591	2 53 12 81	4 027 268 071	39.8873 413	11.6740 996
1592	2 53 44 64	4 034 866 688	39.8998 747	11.6765 449
1593	2 53 76 49	4 042 474 857	39.9124 041	11.6789 892
1594	2 54 08 36	4 050 092 584	39.9249 295	11.6814 325
1595	2 54 40 25	4 057 719 875	39.9374 511	11.6838 748
1596	2 54 72 16	4 065 356 736	39.9499 687	11.6863 161
1597	2 55 04 09	4 073 003 173	39.9624 824	11.6887 563
1598	2 55 36 04	4 080 659 192	39.9749 922	11.6911 955
1599	2 55 68 01	4 088 324 799	39.9874 98	11.6936 337
1600	2 56 00 00	4 096 000 000	40.	11.6960 709

The uses of the preceding table may be greatly extended by aid of the following Rules:

To Ascertain the Square or Cube of a higher Number than is contained in the Table.

When the Number is divisible by a Number without leaving a Remainder.

Rule.—If the number exceed by 2, 3, or any other number of times, any number contained in the table, multiply the square or cube of that number in the table by the square of 2, 3, etc., and the product will give the result.

Example.—Required the square of 1700.

1700 is 10 *times* 170, and the square of 170 is 2 89 00.

Then $28900 \times 10^2 = 2890000$.

Ex. 2.—What is the cube of 2400?

2400 is 2 *times* 1200, and the cube of 1200 is 1 728 000 000.

Then $1\,728\,000\,000 \times 2^3 = 13\,824\,000\,000$.

When the Number is an Odd Number.

Rule.—Take the two numbers nearest to each other, which, added together, make that sum; then from the sum of the squares or cubes of these two numbers, as per table, multiplied by 2, subtract 1, and the remainder will give the result.

EXAMPLE.—What is the square of 1745?

The nearest two numbers are $\left\{\begin{matrix}873\\872\end{matrix}\right\} = 1745$.

Then, per table, $\left\{\begin{matrix}873^2 = 76\ 21\ 29\\872^2 = 76\ 03\ 84\end{matrix}\right.$

$\overline{1\ 52\ 25\ 13} \times 2 = 3045026 - 1 = 3\ 04\ 50\ 25.$

To Compute the Squares or Cubes of Numbers following each other in Arithmetical Progression.

RULE.—Take the squares of the two first numbers in the usual way, and subtract the less from the greater. Add the difference to the greatest square, with the addition of 2 as a constant quantity; the sum will give the square of the next number.

EXAMPLE.—What are the squares of 1001, 1002, and 1003?

$1000^2 =$	1 00 00 00	
$999^2 =$	99 80 01	
	19 99	
Add $1000^2 =$	1 00 00 00	
Add	2	
	1 00 20 01	*Square* of 1001.
Difference, 20 01 + 2 =	20 03	
	1 00 40 04	*Square* of 1002.
Difference, 20 03 + 2 =	20 05	
	1 00 60 09	*Square* of 1003.

RULE 2.—Take the cubes of the two first numbers, and subtract the less from the greater; then multiply the least of the two numbers cubed by 6; add the product, with the addition of 6, to the difference, and continue this the first series of differences.

For the second series of differences, add the cube of the highest of the above numbers to the difference, and the sum will be the cube of the next number.

EXAMPLE.—What are the cubes of 1001, 1002, and 1003?

First Series.

Cube of 1000 =	1000000000	
Cube of 999 =	997002999	
	2997001	*Difference.*
999 × 6 + 6 =	6000	
	3003001	*Diff.* of 1000.
6000 + 6 =	6006	
	3009007	*Diff.* of 1001.
6006 + 6 =	6012	
	3015019	*Diff.* of 1002.

Second Series.

Cube of 1000	= 1000000000	
Diff. for 1000,	3003001	
	1003003001	= *Cube* of 1001.
Diff. for 1001,	3009007	
	1006012008	= *Cube* of 1002.
Diff. for 1002,	3015019	
	1009027027	= *Cube* of 1003.

To Compute the Square or Cube Root of a higher Number than is contained in the Table.

When the Number is divisible by 4 or 8 without leaving a Remainder.

RULE.—Divide the number by 4 or 8 respectively, as the square or cube root is required; take the root of the quotient in the table, multiply it by 2, and the product will give the root required.

EXAMPLE.—What are the square and cube roots of 3200?

$3200 \div 4 = 800$, and $3200 \div 8 = 400$.

Then the square root for 800, per table, is 28.2842712, which, being $\times 2 = 56.5\ 68\ 54\ 24$, *the root.*

The cube root for 400, per table, is 7.368063, which, being $\times 2 = 14.73\ 61\ 26$, *the root.*

When the Root (which is taken as the Number) does not exceed 1600.

The *Numbers* in the table are the roots of the squares or cubes, which are to be taken as the numbers.

ILLUSTRATION.—The square root of 6400 is 80, and the cube root of 512000 is 80.

When a Number has Three or more Ciphers at its right hand.

RULE.—Point off the number into periods of two or three figures each, according as the square or cube root is required, until the remaining figures come within the limits of the table; then take the root for these figures, and remove the decimal point one figure for every period pointed off.

EXAMPLE.—What are the square or cube roots of 1500000?

1500000 = 150, the remaining figure, the square root of which = 12.24745; hence 1224.745, *the square root.*

1500000 = 1500, the remaining figures, the cube root of which = 11.44714; hence 114.4714, *the cube root.*

To Ascertain the Cube Root of any Number over 1600.

RULE.—Find by the table the nearest cube to the number given, and call it the assumed cube; multiply it and the given number respectively by 2; to the product of the assumed cube add the given number, and to the product of the given number add the assumed cube.

Then, as the sum of the assumed cube is to the sum of the given number, so is the root of the assumed cube to the root of the given number.

EXAMPLE.—What is the cube root of 224 809?

By table, the nearest cube is 216 000, and its root is 60.

$$216\,000 \times 2 + 224\,809 = 656\,809,$$
$$\text{And } 224\,809 \times 2 + 216\,000 = 665\,618.$$

Then 656 809 : 665 618 : : 60 : 60.804 +, *the root.*

To Ascertain the Square or Cube Root of a Number consisting of Integers and Decimals.

RULE.—Multiply the difference between the root of the integer part and the root of the next higher integer by the decimal, and add the product to the root of the integer given; the sum will be the root of the number required.

This is correct for the square root to three places of decimals, and for the cube root to seven.

EXAMPLE.—What is the square root of 53.75, and the cube root of 843.75?

$\sqrt{54}$ =	7.3484	$\sqrt[3]{844}$ =	9.4503
$\sqrt{53}$ =	7.2801	$\sqrt[3]{843}$ =	9.4466
	.0683		.0037
	75		.75
	.051225		.002775
$\sqrt{53}$ =	7.2801	$\sqrt[3]{843}$ =	9.4466
$\sqrt{53.75}$ =	7.331325	$\sqrt[3]{843.75}$ =	9.449375

When the Square Root is required for Numbers not exceeding the Roots given in the Table.

The *Numbers* in the table are the squares and cubes of the roots.

ILLUSTRATION.—The square of 27.313 is 746, and the cube of 11.01925 is 1338.

RULE.—Find, by the table, in the column of numbers, that number representing the figures of the integer and decimals for which the root is required, and point it off decimally by places of 2 or 3 figures as the square or cube root is required; and opposite to it, in the column of roots, take the root and point off 1 or 2 additional places of decimals to those in the root, as the square or cube root is required, and the result is the root required.

EXAMPLE.—What are the square roots of .15, 1.50, and 15.00?

In the table 15 has for its root 3.87298; hence .387298 = *the square root for* .15.
150 has for its root 12.24745; hence 1.224745 = *the square root for* 1.50.
1500 has for its root 38.7298; hence 3.87298 = *the square root for* 15.

EX. 2.—What are the cube roots of .15, 1.50, and 15.00?

Add a cipher to each, to give the numbers three places of figures.

In the table 150 has for its root 5.3133; hence .53133 = *the cube root of* .15.
1500 has for its root 11.447; hence 1.1447 = *the cube root of* 1.50.
15 has for its root 2.4662; and 15.000, by the addition of 3 places of figures, has 24.662; hence 2.4662 = *the cube root of* 15.00.

To Ascertain the Square or Cube Roots of Decimals alone.

RULE.—Point off the number from the decimal point into periods of two or three figures each, as the square or cube root is required. Ascertain from the table or by calculation the root of the number corresponding to the decimal given, the same being read off by removing the decimal point one place to the left for every period of 2 figures if the square root is required, and one place for every period of 3 figures if the cube root is required.

EXAMPLE.—What are the square and cube roots of .810, .081, and .0081?

.810,	when pointed off = $.8\dot{1}$,	and $\sqrt{\ } = 9.$	which becomes	.9;
.081,	" " " = $.0\dot{8}1$,	" $\sqrt{\ } = 2.846$	" "	.2846;
.0081,	" " " = $.0\dot{0}8\dot{1}$,	" $\sqrt{\ } = 9.$	" "	.09.

.810,	when pointed off = $.81\dot{0}$,	and $\sqrt[3]{\ } = 9.3217$,	which becomes	.93217;
.081,	" " " = $.08\dot{1}$,	" $\sqrt[3]{\ } = 4.3267$,	" "	.43267;
.0081,	" " " = $.00\dot{8}1$,	" $\sqrt[3]{\ } = 2.0083$,	" "	.20083.

To Compute the 4th Root of a Number.

RULE.—Take the square root of its square root.

EXAMPLE.—What is the $\sqrt[4]{\ }$ root of 1600?

$$\sqrt{1600} = 40, \text{ and } \sqrt{40} = 6.3\,245\,553.$$

To Compute the 6th Root of a Number.

RULE.—Take the cube root of its square root.

EXAMPLE.—What is the $\sqrt[6]{\ }$ of 441?

$$\sqrt{441} = 21, \text{ and } \sqrt[3]{21} = 2.7\,589\,243.$$

To Extract the Root of any Given Number of any Power.

RULE.—Divide the given number by any assumed root, *raised* to the next less power to that given; to the quotient add the assumed root, *multiplied* by the next less power to that given; divide the sum by the given power for a new root, with which repeat the operation if necessary.

EXAMPLE.—Ascertain the cube root of 64.

$64 \div 4^2 = 4$ = *quotient of given number divided by assumed root of* 4, *raised to the next less power (its square).*

$4 + \overline{4 \times 2} = 12$ = *sum of above quotient, and the assumed root multiplied by the next less power.*

$12 \div 3 = 4.$ = *quotient of above sum* ÷ *the given power* = *the root required.*

Ex. 2.—Ascertain the cube root of 216. Assume the root to be 4.

$216 \div 4^2 = 13.5$, and $13.5 + \overline{4 \times 2} = 21.5$, which $\div 3 = 7.1667$, *which is too great.*

Then $216 \div 7.1667^2 = 4.2054$, and $4.2054 + \overline{7.1667 \times 2} = 18.5388$, which $\div 3 =$ 6.796, *which is also too great.*

Again, $216 \div 6.796^2 = 5.6563$, and $5.6563 + \overline{6.1796 \times 2} = 18.0155$, which $\div 3 =$ 6.0052, *which is also too great.*

Finally, $216 \div 6.0052^2 = 5.99$, and $5.99 + \overline{6.0052 \times 2} = 18$, which $\div 3 = 6$, *the root.*

Ex. 3.—Ascertain the fifth root of 6436314.

Assume 20, the fourth power of which = 160000.

Then $6436343 \div 16000 = 40.224$, and $40.227 + \overline{20 \times 4} = 120.227$, which $\div 5 =$ 24.045, *which is too great.*

Assume 24, the fourth power of which = 331776.

Then $6436343 \div 331776 + \overline{24 \times 4} = 115.4$, which $\div 5 = 23.08$, *which is also too great.*

Assume 23, the fourth power of which = 279841.

Then $6436343 \div 279841 = 23$, *the root.*

Table of the 4th and 5th Powers of Numbers.

Number.	4th Power.	5th Power.	Number.	4th Power.	5th Power.
1	1	1	65	17850625	1160290625
2	16	32	66	18974736	1252332576
3	81	243	67	20151121	1350125107
4	256	1024	68	21381376	1453933568
5	625	3125	69	22667121	1564031349
6	1296	7776	70	24010000	1680700000
7	2401	16807	71	25411681	1804229351
8	4096	32768	72	26873856	1934917632
9	6561	59049	73	28398241	2073071593
10	10000	100000	74	29986576	2219006624
11	14641	161051	75	31640625	2373046875
12	20736	248832	76	33362176	2535525376
13	28561	371293	77	35153041	2706784157
14	38416	537824	78	37015056	2887174368
15	50625	759375	79	38950081	3077056399
16	65536	1048576	80	40960000	3276800000
17	83521	1419857	81	43046721	3486784401
18	104976	1889568	82	45212176	3707398432
19	130321	2476099	83	47458321	3939040643
20	160000	3200000	84	49787136	4182119424
21	194481	4084101	85	52200625	4437053125
22	234256	5153632	86	54708016	4704270176
23	279841	6436343	87	57289761	4984209207
24	331776	7962624	88	59969536	5277319168
25	390625	9765625	89	62742241	5584059449
26	456976	11881376	90	65610000	5904900000
27	531441	14348907	91	68574961	6240321451
28	614656	17210368	92	71639296	6590815232
29	707281	20511149	93	74805201	6956883693
30	810000	24300000	94	78074896	7339040224
31	923521	28629151	95	81450625	7737809375
32	1048576	33554432	96	84034656	8153726976
33	1185921	39135393	97	88529281	8587340257
34	1336336	45435424	98	92236816	9039207968
35	1500625	52521875	99	96059601	9509900499
36	1679616	60466176	100	100000000	10000000000
37	1874161	69343957	101	104060401	10510100501
38	2085136	79235168	102	108243216	11040808032
39	2313441	90224199	103	112550881	11592740743
40	2560000	102400000	104	116985856	12166529024
41	2825761	115856201	105	121550625	12762815625
42	3111696	130691232	106	126247696	13382255776
43	3418801	147008443	107	131079601	14025517307
44	3748096	164916224	108	136048896	14693280768
45	4100625	184528125	109	141158161	15386239549
46	4477456	205962976	110	146410000	16105100000
47	4879681	229345007	111	151807041	16850581551
48	5308416	254803968	112	157351936	17623416832
49	5764801	282475249	113	163047361	18424351793
50	6250000	312500000	114	168896016	19254145824
51	6765201	345025251	115	174900625	20113581875
52	7311616	380204032	116	181063936	21003416576
53	7890481	418195493	117	187388721	21924480357
54	8503056	459165024	118	193877776	22877577568
55	9150625	503284375	119	200533921	23863536599
56	9834496	550731776	120	207360000	24883200000
57	10556001	601692057	121	214358881	25937424601
58	11316496	656356768	122	221533456	27027081632
59	12117361	714924299	123	228886641	28153056843
60	12960000	777600000	124	236421376	29316250624
61	13845841	844596301	125	244140625	30517578125
62	14776336	916132832	126	252047376	31757969376
63	15752961	992436543	127	260144641	33038369407
64	16777216	1073741824	128	268435456	34359738368

Table—(*Continued*).

Number.	4th Power.	5th Power.	Number.	4th Power.	5th Power.
129	276 922 881	35 723 051 649	140	384 160 000	53 782 400 000
130	285 610 000	37 129 300 000	141	395 254 161	55 730 836 701
131	294 499 921	38 579 489 651	142	406 586 896	57 735 339 232
132	303 595 776	40 074 642 432	143	418 161 601	59 797 108 943
133	312 900 721	41 615 795 893	144	429 981 696	61 917 364 224
134	322 417 936	43 204 003 424	145	442 050 625	64 097 340 625
135	332 150 625	44 840 334 375	146	454 371 856	66 338 290 976
136	342 102 016	46 525 874 176	147	466 948 881	68 641 485 507
137	352 275 361	48 261 724 457	148	479 785 216	71 008 211 968
138	362 673 936	50 049 003 168	149	492 884 401	73 439 775 749
139	373 301 641	51 888 844 699	150	506 250 000	75 937 500 000

To Ascertain the 4th Power of a Number greater than is contained in the Table.

Rule.—Ascertain the square of the number by the preceding table or by calculation, and square it; the product is the power required.

Example.—What is the 4th power of 15?

$15^2 = 225$, and $225^2 = 50\,625$.

To Ascertain the 5th Power of a Number greater than is contained in the Table.

Rule.—Ascertain the cube of the number by the preceding table or by calculation, and multiply it by its square; the product is the power required.

Example.—What is the 5th power of 15?

$15^3 = 3375$, which $\times\ 15^2(225) = 759\,375$.

To Ascertain the 4th and 5th Powers by another Method.

Rule.—Reduce the number by 2 until it is one contained within the table. Take the power which is required of that number, and multiply it by 16, 16^2, and 16^3 respectively for each division by 2 for the 4th power, and by 32, 32^2, 32^3 respectively for each division by 2 for the 5th power.

Example.—What are the 4th and 5th powers of 600?

$600 \div 2 = 300$, and $300 \div 2 = 150$.

The 4th power of 150, per table, $= 506\,250\,000$, which $\times\ 16^2\ (256)$, the multiplier for a second division $= 129\,600\,000\,000$, *the 4th power.*

Again, the 5th power of 150 $= 75\,937\,500\,000$, which $\times 32^2(1024) = 77\,760\,000\,000\,000$, *the 5th power.*

To Compute the 6th Power of a Number.

Rule.—Square its cube.

Example.—What is the 6th power of 2?

$(2^3)^2 = 8^2$, and $8^2 = 64$.

To Ascertain the 4th or 5th Root of a Number.

Rule.—Find in the column of 4th and 5th powers the number given, and the number from which that power is derived will be the root required.

Example.—What is the 5th root of 3200000?

3200000 in the table is the 5th power of 20; hence 20 is the root required.

RECIPROCALS.

The Reciprocal of a number is the quotient arising from dividing 1 by the number; thus the reciprocal of 2. is $1 \div 2 = .5$.

The product of a number and its reciprocal is always equal to 1.; thus, $2 \times .5 = 1$.

The reciprocal of a vulgar fraction is the denominator divided by the numerator; thus, $\frac{2}{1} = .5$.

MENSURATION OF AREAS, LINES, AND SURFACES.

PARALLELOGRAMS.

DEFINITION.—Quadrilaterals, having their opposite sides parallel.

To Compute the Area of a Square, a Rectangle, a Rhombus, or a Rhomboid—Figs. 1, 2, 3, and 4.

RULE.—Multiply the length by the breadth or height.

Or, $l \times b =$ area, l representing the length, and b the breadth.

Fig. 1.

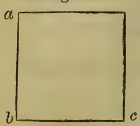


Fig. 2.

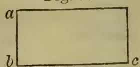


Fig. 3.

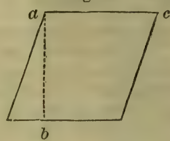
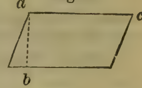


Fig. 4.



EXAMPLE.—The sides *a b*, *b c*, Fig. 1, are 5 feet 6 ins.; what is the area?

$5.5 \times 5.5 = 30.25$ *square feet.*

NOTE.—The side of a square is equal to the square root of its area.

2. The opposite angles of a Rhombus and a Rhomboid are equal.

GNOMON.

DEFINITION.—The space included between the lines forming two similar parallelograms, of which the smaller is inscribed within the larger, so that one angle in each is common to both.

To Compute the Area of a Gnomon.

RULE.—Ascertain the areas of the two parallelograms, and subtract the less from the greater; the difference will give the area.

Or, $a - a' =$ area, a and a' representing the areas.

EXAMPLE.—The sides of a gnomon are 10 by 10 and 6 by 6 ins.; what is its area? $10 \times 10 = 1000$, and $6 \times 6 = 36$. Then $100 - 36 = 64$ *square ins.*

TRIANGLES.

DEFINITION.—Plain superficies having three sides and angles.

To Compute the Area of a Triangle—Figs. 5, 6, and 7.

RULE.—Multiply the base by the height, and divide the product by 2.

Or, $\frac{ab \times cd}{2}$. Or, $\frac{b \times h}{2} =$ area, b representing the base, and h the height.

NOTE.—The *Hypothenuse* of a right angle is the side opposite to the right angle.

2. The *perpendicular height* of a triangle = twice its area divided by its base.

Fig. 5.

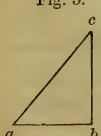


Fig. 6.

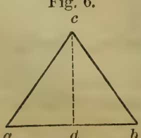
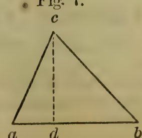


Fig. 7.



EXAMPLE. — The base *a b*, Fig. 5, is 4 feet, and the height *c b*, 6; what is the area?

$4 \times 6 = 24$, and $24 \div 2 = 12$ *square feet.*

To Compute the Area of a Triangle by the Length of its Sides—Figs. 6 and 7.

RULE.—From half the sum of the three sides subtract each side separately; then multiply the half sum and the three remainders continually together, and take the square root of the product.

Or, $\sqrt{s(s-a)\times(s-b)\times(s-c)}=$ area, a, b, c representing the sides, and s half the sum of the three sides.

When all the Sides are Equal.

RULE.—Square the length of a side, and multiply the product by .433.

Or, $S^2\times.433=$ area, S representing length of a side.

EXAMPLE.—The sides of a triangle are 30, 40, and 50 feet; what is the area?

$\frac{30+40+50}{2}=\frac{120}{2}=60$, or half sum of the sides. $\left.\begin{array}{l}60-30=30\\60-40=20\\60-50=10\end{array}\right\}$ *remainders.*

Whence $30\times20\times10\times60=360000$, and $\sqrt{360000}=600$ *square feet.*

To Compute the Length of one Side of a Right-angled Triangle, the Length of the other two Sides being given—Fig. 5.

When the two Legs are given, to Ascertain the Hypothenuse.

RULE.—Add together the squares of the two legs, and take the square root of their sum.

Or, $\sqrt{ab^2+bc^2}=$ hypothenuse. Or, $\sqrt{b^2+h^2}$.

EXAMPLE.—The base ab is 30 ins., and the height bc 40; what is the length of the hypothenuse?

$30^2+40^2=2500$, and $\sqrt{2500}=50$ *ins.*

To Ascertain the other Leg, When the Hypothenuse and one of the Legs are given—Fig. 5.

RULE.—Subtract the square of the given leg from the square of the hypothenuse, and take the square root of the remainder.

Or, $\sqrt{hyp.^2-\left\{\begin{array}{l}b^2=h.\\h^2=b.\end{array}\right.}$ Or, $\sqrt{ac^2-\left\{\begin{array}{l}ab^2=bc.\\bc^2=ab.\end{array}\right.}$

EXAMPLE.—The base of a triangle is 30 feet, and the hypothenuse 50; what is the height of it?

$50^2-30^2=2500-900$, and $2500-900=1600$. Then $\sqrt{1600}=40$ *feet.*

To Compute the Length of a Side, When the Hypothenuse of a Right-angled Triangle of equal Sides alone is given—Figs. 8 and 9.

RULE.—Divide the hypothenuse by 1.414213.

Or, $\frac{hyp.}{1.414213}=$ the length of a side.

EXAMPLE.—The hypothenuse of a right-angled triangle is 300 feet; what is the length of its sides?

$300\div1.414213=212.1321$ *feet.*

To Compute the Perpendicular or Height of a Triangle, When the Base and Area alone are given.

RULE.—Divide twice the area by its base. Or, $2a \div b = h$.

EXAMPLE.—The area of a triangle is 10 feet, and the length of its base 5; what is its perpendicular?

$$10 \times 2 = 20, \text{ and } 20 \div 5 = 4 \textit{ feet}.$$

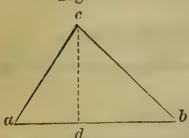
To Compute the Perpendicular or Height of a Triangle, When the two Sides and the Base are given.

RULE.—As the base is to the sum of the sides, so is the difference of the sides to the difference of the divisions of the base. Half this difference being added to or subtracted from half the base will give the two divisions thereof. Hence, as the sides and their opposite division of the base constitute a right-angled triangle, the perpendicular thereof is readily ascertained by preceding rules.

$$\text{Or, } \frac{\overline{bc + ca} \times \overline{bc \backsim ca}}{ba} = bd \backsim da.$$

$$\text{Or, } \frac{ac^2 + ab^2 - bc^2}{2ab} = ad; \text{ whence } \sqrt{ac^2 - ad^2} = dc.$$

Fig. 8.



EXAMPLE.—The three sides of a triangle, $a\,b\,c$, Fig. 8, are 9.928, 8, and 5 feet; what is the length of the perpendicular on the longest side?

As $9.928 : 8 + 5 :: 8 \backsim 5 : 3.928$, *the difference of the divisions of the base.*

Then $3.928 \div 2 = 1.964$, which, added to $\frac{9.928}{2} = 4.964 + 1.964 = 6.928$, *the length of the longest division of the base.*

Hence we have a right-angled triangle with its base 6.928, and its hypothenuse 8; consequently, its remaining side or perpendicular is $\sqrt{(8^2 - 6.928^2)} = 4$ *feet.*

When any two of the Dimensions of a Triangle and one of the corresponding Dimensions of a similar Figure are given, and it is required to ascertain the other corresponding Dimensions of the last Figure.

Fig. 9.

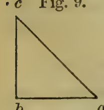
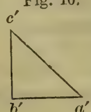


Fig. 10.



Let $abc, a'b'c'$, be two similar triangles, Figs. 9 and 10. Then $ab : bc :: a'b' : b'c'$, or $a'b' : b'c' :: ab : bc$.

NOTE.—The same proportion holds with respect to the similar lineal parts of any other similar figures, whether plane or solid.

EXAMPLE.—The shadow of a vertical cone 4 feet in length was 5 feet; at the same time, the shadow of a tree on level ground was 83 feet; what was the height of the tree?

$$5\,a'b' : 4\,b'c' :: 83\,ab : 66.2\text{-}5\,bc \textit{ feet}.$$

TRAPEZIUM.

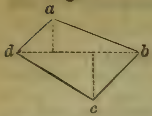
DEFINITION.—Quadrilaterals having unequal sides.

To Compute the Area of a Trapezium—Fig. 11.

RULE.—Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and divide the product by 2.

$$\text{Or, } \frac{db \times \overline{a + c}}{2} = \text{area}.$$

Fig. 11.



EXAMPLE.—The diagonal $d\,b$, Fig. 11, is 125 feet, and the perpendiculars a and c 50 and 37 feet; what is the area?

$125 \times \overline{50+37} = 10875$, and $10875 \div 2 = 5437.5$ *square feet.*

When the two opposite Angles are Supplements to each other, that is, when a Trapezium can be inscribed in a Circle, the Sum of its opposite Angles being equal to two Right Angles, or 180°.

RULE.—From half the sum of the four sides subtract each side severally; then multiply the four remainders continually together, and take the square root of the product.

EXAMPLE.—In a trapezium the sides are 15, 13, 14, and 12 feet; its opposite angles being supplements to each other, required its area.

$$15 + 13 + 14 + 12 = 54, \text{ and } \frac{54}{2} = 27.$$

$$\begin{array}{cccc} 27 & 27 & 27 & 27 \\ 15 & 13 & 14 & 12 \end{array}$$

$\overline{12} \times \overline{14} \times \overline{13} \times \overline{15} = 32760$, and $\sqrt{32760} = 180.997$ *square feet.*

TRAPEZOID.

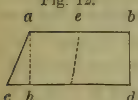
DEFINITION.—A Quadrilateral with only one pair of opposite sides parallel.

To Compute the Area of a Trapezoid—Fig. 12.

RULE.—Multiply the sum of the parallel sides by the perpendicular distance between them, and divide the product.

Or, $\dfrac{a\,b + d\,c \times a\,h}{2}$. Or, $\dfrac{s + s' \times h}{2}$ = area, s and s' representing the sides.

Fig. 12.



EXAMPLE.—The parallel sides $a\,b$, $c\,d$, Fig. 12, are 100 and 132 feet, and the distance between them 62.5 feet; what is the area?

$100 + 132 \times 62.5 = 14500$, and $14500 \div 2 = 7250$ *square feet.*

POLYGONS.

DEFINITION.—Plane figures having three or more sides, and are either regular or irregular, according as their sides or angles are equal or unequal, and they are named from the number of their sides and angles.

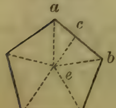
Regular Polygons.

To Compute the Area of a Regular Polygon—Fig. 13.

RULE.—Multiply the length of a side by the perpendicular distance to the centre; multiply the product by the number of sides, and divide it by 2.

Or, $\dfrac{a\,b \times c\,e \times n}{2}$ = area, n representing the number of sides.

Fig. 13.



EXAMPLE.—What is the area of a pentagon, the side $a\,b$, Fig. 13, being 5 feet, and the distance $c\,e$ 4¼ feet?

$5 \times 4\frac{1}{4} \times 5\ (n) = 106\frac{1}{4}$ = product of length of a side, the distance to the centre, and the number of sides.

Then $106\frac{1}{4} \div 2 = 53.125$ *feet.*

To Compute the Area of a Regular Polygon, When the Length of a Side only is given.

RULE.—Multiply the square of the side by the multiplier opposite to the name of the polygon in the following table:

No. of Sides.	Name of Polygon.	Area.	A. Radius of Circumscribed Circle.	B. Length of the Side	C. Radius of Circumscribing Circle.	D. Radius of Inscribed Circle.
3	Trigon	.433013	2.	1.732	.5773	.2887
4	Tetragon	1.	1.414	1.4142	.7071	.5
5	Pentagon	1.720477	1.238	1.1756	.8506	.6882
6	Hexagon	2.598076	1.156	1.	1.	.866
7	Heptagon	3.633912	1.11	.8677	1.1524	1.0383
8	Octagon	4.828427	1.083	.7653	1.3066	1.2071
9	Nonagon	6.181824	1.064	.684	1.4619	1.3737
10	Decagon	7.694209	1.051	.618	1.618	1.5388
11	Undecagon	9.36564	1.042	.5634	1.7747	1.7028
12	Dodecagon	11.196152	1.037	.5176	1.9319	1.866

EXAMPLE.—What is the area of a square when the length of its sides is 7.0710678 inches?

$$7.0710678^2 = 50, \text{ and } 50 \times 1. = 50 \textit{ ins.}$$

To Compute the Radius of a Circle that contains a Given Polygon, When the Length of a Perpendicular from the Centre alone is given.

RULE.—Multiply the distance from the centre to a side of the polygon by the unit in column A.

EXAMPLE.—What is the radius of a circle that contains a hexagon, the distance to the centre being 4.33 inches?

$$4.33 \times 1.156 = 5 \textit{ ins.}$$

To Compute the Length of a Side of a Polygon that is contained in a Given Circle, When the Radius of the Circle is given.

RULE.—Multiply the radius of the circle by the unit in column B.

EXAMPLE.—What is the length of the side of a pentagon contained in a circle 8.5 feet in diameter?

$$8.5 + 2 = 4.25 \text{ radius, and } 4.25 \times 1.1756 = 5 \textit{ feet.}$$

To Compute the Radius of a Circumscribing Circle, When the Length of a Side is given.

RULE.—Multiply the length of a side of the polygon by the unit in column C.

EXAMPLE.—What is the radius of a circle that will contain a hexagon, a side being 5 inches?

$$5 \times 1 = 5 \textit{ ins.}$$

To Compute the Radius of a Circle that can be Inscribed in a Given Polygon, When the Length of a Side is given.

RULE.—Multiply the length of a side of the polygon by the unit in column D.

EXAMPLE.—What is the radius of the circle that is bounded by a hexagon, its sides being 5 inches?

$$5 \times .866 = 4.33 \textit{ ins.}$$

To Compute the Length of a Side and Radius of a Regular Polygon, When the Area alone is given.

RULE.—Multiply the square root of the area of the polygon by the multiplier in column E of the following table for the length of the side; by the multiplier in column G for the radius of the circumscribing circle; and by the multiplier in column H for the radius of the inscribed circle or perpendicular.

No. of Sides	Name of Polygon.	E. Length of the Side.	G. Radius of Circumscribing Circle.	H. Radius of Inscribed Circle.	Angle.	Angle of Polygon.	Tangents.
3	Trigon	1.5197	.8774	.4387	120°	60°	.57735
4	Tetragon	1.	.7071	.5	90	90	1.
5	Pentagon	.7624	.6485	.5247	72	108	1.37638
6	Hexagon	.6204	.6204	.5373	60	120	1.73205
7	Heptagon	.5246	.6045	.5446	51 25′	128 4-7	2.07652
8	Octagon	.4551	.5946	.5493	45	135	2.41421
9	Nonagon	.4022	.588	.5525	40	140	2.74747
10	Decagon	.3605	.5833	.5548	36	144	3.07768
11	Undecagon	.3268	.5799	.5564	32 43′	147 3-11	3.40568
12	Dodecagon	.2989	.5774	.5577	30	150	3.73205

EXAMPLE.—The area of a square (tetragon) is 16 inches; what is the length of its side?

$\sqrt{16} = 4$, and $4 \times 1 = 4$ *ins.*

Additional Uses of the foregoing Table.—The 6th and 7th columns of the table facilitate the construction of these figures with the aid of a sector. Thus, if it is required to describe an octagon, opposite to it, in column 6th, is 45; then, with the chord of 60 on the sector as radius, describe a circle, taking the length 45 on the same line of the sector; mark this distance off on the circumference, which, being repeated around the circle, will give the points of the sides.

The 7th column gives the angle which any two adjoining sides of the respective figures make with each other; and the 8th gives the tangent of the angle in column 6th.

REGULAR BODIES.

To Compute the Surface or Linear Edge of any Regular Solid Body.

RULE.—Multiply the square of the linear edge, or the radius of the circumscribed or inscribed sphere, by the units in the following table, under the head of the dimension used:

No. of Sides.	Names of Figures.	Surface.	Radius of Circumscribed Circle.	Radius of Inscribed Circle.	Linear Edge by Surface.
4	Tetrahedron	1.73205	1.63299	4.89898	.75984
6	Hexahedron	6.	1.1547	2.	.40825
8	Octahedron	3.4641	1.41421	2.44949	.53729
12	Dodecahedron	20.64578	.71364	.89806	.22008
20	Icosahedron	8.66025	1.05146	1.32317	.33981

EXAMPLE.—What is the surface of a hexahedron or cube having sides of 5 inches?

$5^2 \times 6 = 25 \times 6 = 150$ *ins.*

To Compute the Linear Edge when the Surface alone is given.

RULE.—Multiply the square root of the surface by the multiplier under the head of Linear Edge by Surface.

EXAMPLE.—What is the linear edge of a hexahedron, the surface being 6 inches?

$\sqrt{6} \times 40825 = 1$ *linear edge.*

Irregular Polygons.

DEFINITION.—Figures with unequal sides.

To Compute the Area of an Irregular Polygon—Figs. 14 and 15.

Fig. 14.

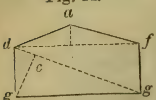
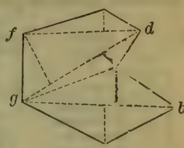


Fig. 15.



RULE.—Draw diagonals, as *d f*, *d g*, *g c*, and *g b*, to divide the figures into triangles and quadrilaterals: ascertain the areas of these separately, and take their sum.

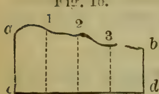
NOTE.—To ascertain the area of mixed or compound figures, or such as are composed of rectilineal and curvilineal figures together, compute the areas of the several figures of which the whole is composed, then add them together, and the sum will be the area of the compound figure. In this manner any irregular surface or field of land may be measured by dividing it into trapeziums and triangles, and computing the area of each separately.

When any Part of a Figure is bounded by a Curve the Area may be ascertained as follows: Erect any number of perpendiculars upon the base at equal distances, and ascertain their lengths.

Add the lengths of the perpendiculars thus found together, and their sum, divided by their number, will give the *mean breadth*; then multiply the mean breadth by the length of the base.

To Compute the Area of a long Irregular Figure—Fig. 16.

Fig. 16.



RULE.—Take the breadth at several places, and at equal distances apart; add them together, divide their sum by the number of breadths for the mean breadth, and multiply this by the length of the figure.

$$\text{Or, } \frac{b + b' + b''}{2} = B \times l = \textit{area}.$$

CIRCLE.

The *Diameter* is a right line drawn through its centre, bounded by its periphery.

The *Radius* is a right line drawn from its centre to its circumference.

The *Circumference* is assumed to be divided into 360 equal parts, termed *degrees*; each degree is divided into 60 parts, termed *minutes*; each minute into 60 parts, termed *seconds*; and each second into 60 parts, termed *thirds*, and so on.

To Compute the Circumference of a Circle.

RULE.—Multiply the diameter by 3.1416.

Or, as 7 is to 22, so is the Diameter to the Circumference.

Or, as 113 is to 355, so is the Diameter to the Circumference.

EXAMPLE.—The diameter of a circle is $1\frac{1}{4}$ inches; what is its circumference?

$$1\tfrac{1}{4} \times 3.1416 = 3.927 \textit{ ins.}$$

To Compute the Diameter of a Circle.

Divide the circumference by 3.1416.

Or, as 22 is to 7, so is the Circumference to the Diameter.

NOTE.—Divide the area by .7854, and the square root of the quotient will give the diameter of the circle.

To Compute the Area of a Circle.

RULE.—Multiply the square of the diameter by .7854.

Or, multiply the square of the circumference by .07958.
Or, multiply half the circumference by half the diameter.
Or, multiply the square of the radius by 3.1416.
Or, $p\,r^2 = area$, *r representing the radius.*

EXAMPLE.—The diameter of a circle is 8 inches; what is the area of it?

8^2 or $8 \times 8 = 64$, and $64 \times .7854 = 50.2656$ *ins.*

USEFUL FACTORS.

In which p represents the Circumference of a Circle, the Diameter of which is 1.

$p = 3.141592653589+$	$\frac{1}{4}\,p = .785398163397+$	1-12 $p = .261799$
$2\,p = 6.283185307179+$	4-3 $p = 4.18879$	1-360 $p = .008726$
$4\,p = 12.566370614359+$	$\frac{1}{6}\,p = .523598$	$\frac{1}{2}\sqrt{p} = .886226$
$\frac{1}{2}\,p = 1.570796326794+$	$\frac{1}{8}\,p = .392699$	$36\,p = 113.097335$

In which the Diameter of a Circle is 10.

1. Chord of the arc of the semicircle	= 10.
2. Chord of half the arc of the semicircle	= 7.071067
3. Versed sine of the arc of the semicircle	= 5.
4. Versed sine of half the arc of the semicircle	= 1.464466
5. Chord of half the arc, of the half of the arc of the semicircle	= 3.82683
6. Half the chord, of the chord of half the arc	= 3.535533
7. Length of arc of semicircle	= 15.707963
8. Length of half the arc of the semicircle	= 7.853981
9. Square of the chord, of half the arc of the semicircle (2)	= 50.
10. Square root of versed sine of half the arc (4)	= 1.210151
11. Square of versed sine of half the arc (4)	= 2.144664
12. Square of the chord of half the arc, of half the arc of the semicircle (5)	= 14.64467
13. Square of half the chord, of the chord of half the arc (6)	= 12.5

NOTE.—In all the calculations p is taken at 3.1416, $\frac{1}{4}\,p$ at .7854, $\frac{1}{6}\,p$ at .5236; and whenever the decimal figure next to the one last taken exceeds 5, one is added. Thus, 3.14159 for four places of decimals is taken as 3.1416.

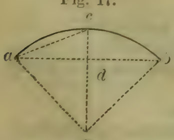
To Compute the Length of an Arc of a Circle, Fig. 17, When the Number of Degrees and the Radius are given.

RULE 1.—Multiply the number of degrees in the arc by 3.1416 times the radius, and divide by 180.

2. Multiply the radius of the circle by .01745329, and the product by the degrees in the arc.

If the length is required for minutes, multiply the radius by .000290889; if for seconds, multiply by .000004848.

Fig. 17.



EXAMPLE.—The number of degrees in an arc, $o\,a\,b$, are 90, and the radius, $o\,b$, 5 inches; what is the length of the arc?

$90 \times (3.1416 \times 5) = 1413.72$, which $\div 180 = 7.854$ *ins.*

Ex. 2.—The radius of an arc is 10, and the measure of its angle 44° 30′ 30″; what is the length of the arc?

$10 \times .01745329 = .1745329$, which $\times 44 = 7.6794476$, *the length for* 44°.

$10 \times .000290889 = .00290889$, which $\times 30 = .0872667$, *the length for* 30′.

$10 \times .000004848 = .00004848$, which $\times 30 = .0014544$, *the length for* 30″.

Then $\left.\begin{array}{r} 7.6794476 \\ .0872667 \\ .0014544 \end{array}\right\} = 7.7681687$, *length.* Or, reduce the minutes and seconds to the decimal of a degree, and multiply by it.

See Rule, page 46. 30′ 30″ = .5083, and .1745329 from above $\times 44.5083 = 7.768163$.

When the Chord of half the Arc and the Chord of the Arc are given.

RULE.—From 8 times the chord of half the arc subtract the chord of the arc, and one third of the remainder will give the length nearly.

Or, $\frac{8\,c'-c}{3}$, *c′ representing the chord of half the arc, and c the chord of the arc.*

EXAMPLE.—The chord of half the arc, *a c*, Fig. 17, is 30 inches, and the chord of the arc, *a b*, 48; what is the length of the arc?

$30 \times 8 = 240 = 8$ *times the chord of half the arc;* $240 - 48 = 192$, and $192 \div 3 = 64$ *ins.*

When the Chord of the Arc and the Versed Sine of the Arc are given.

RULE.—Multiply the square root of the sum of the square of the chord, and four times the square of the versed sine (equal to twice the chord of half the arc), by ten times the square of the versed sine; divide this product by the sum of fifteen times the square of the chord and thirty-three times the square of the versed sine; then add this quotient to twice the chord of half the arc,* and the sum will give the length of the arc very nearly.

Or, $\frac{\sqrt{c^2+4\,v^2}\times 10\,v^2}{15\,c^2+33\,v^2}+2\,c'$, *v representing the versed sine.*

EXAMPLE.—The chord of an arc is 80, and its versed sine 30; what is the length of the arc?

$80^2 = 6400 =$ *square of the chord;* $30^2 = 900 =$ *square of the versed sine.*

$\sqrt{(6400+\overline{900\times 4})} = 100 =$ *square root of the square of the chord and four times the square of the versed sine = twice the chord of half the arc.*

Then $100 \times \overline{30^2 \times 10} = 900000 =$ *product of ten times the square of the versed sine and the root above obtained.*

$80^2 \times 15 = 96000 = 15$ *times the square of the chord.*
$30^2 \times 33 = 29700 = 33$ *times the square of the versed sine.*
$\overline{125700}$

And $\frac{100\times 30^2\times 10}{125700} = \frac{900000}{125700} = 7.1599$, *which, added to* 100, *or twice the chord of half the arc* $= 107.1599$.

When the Diameter and Versed Sine are given.

RULE.—Multiply twice the chord of half the arc by 10 times the versed sine; divide the product by 27 times the versed sine subtracted from 60 times the diameter, and add the quotient to twice the chord of half the arc; the sum will give the length of the arc very nearly.

Or, $\frac{2c'\times 10\,v}{60d-27\,v}+2\,c'=c.$

EXAMPLE.—The diameter of a circle is 100 feet, and the versed sine of the arc 25; what is the length of the arc?

$\sqrt{25\times 100} = 50 =$ *chord of half the arc.*

$50 \times 2 \times \overline{25 \times 10} = 25000 =$ *twice the chord of half the arc by* 10 *times the versed sine.*

$\overline{100\times 60} - \overline{25\times 27} = 5325 = 27$ *times the versed sine subtracted from* 60 *times the diameter.*

Then $\frac{25000}{5325} = 4.6948$, and $4.6948 + \overline{50\times 2} = 104.6948$ *feet.*

* The square root of the sum of the square of the chord and four times the square of the versed sine is equal to twice the chord of half the arc.

To Compute the Chord of an Arc, When the Chord of half the Arc and the Versed Sine are given.

RULE.—From the square of the chord of half the arc subtract the square of the versed sine, and take twice the square root of the remainder.

$$\text{Or, } \sqrt{(c'^2 - v^2)} \times 2 = c.$$

EXAMPLE.—The chord of half the arc is 60, and the versed sine 36; what is the length of the chord of the arc?

$$60^2 - 36^2 = 2304, \text{ and } \sqrt{2304} \times 2 = 96.$$

When the Diameter and Versed Sine are given.

RULE.—Multiply the versed sine by 2, and subtract the product from the diameter; then subtract the square of the remainder from the square of the diameter, and take the square root of that remainder.

$$\text{Or, } \sqrt{(v \times 2 - d)^2 - d^2} = c.$$

EXAMPLE.—The diameter of a circle is 100, and the versed sine of half the arc is 36; what is the length of the chord of the arc?

$$100 - 36 \times 2 = 28; \text{ then } 100^2 - 28^2 = 9216, \text{ and } \sqrt{9216} = 96.$$

To Compute the Chord of half an Arc, When the Chord of the Arc and the Versed Sine are given.

RULE 1.—Divide the square root of the sum of the square of the chord of the arc and four times the square of the versed sine by two.

RULE 2.—Take the square root of the sum of the squares of half the chord of the arc and the versed sine.

$$\text{Or, } \frac{\sqrt{c^2 + 4\,v^2}}{2} = c'. \quad \text{Or, } \sqrt{\left(\frac{c}{2}\right)^2 + v^2} = c'.$$

When the Diameter and Versed Sine are given.

RULE.—Multiply the diameter by the versed sine, and take the square root of their product.

$$\text{Or, } \sqrt{d \times v} = c'.$$

To Compute a Diameter.

RULE 1.—Divide the square of the chord of half the arc by the versed sine.

$$\text{Or, } c'^2 \div v = diameter.$$

RULE 2.—Add the square of half the chord of the arc to the square of the versed sine, and divide this sum by the versed sine.

$$\text{Or, } \frac{(c \div 2)^2 + v^2}{v} = d.$$

To Compute the Versed Sine.

RULE.—Divide the square of the chord of half the arc by the diameter.

$$\text{Or, } \frac{c'^2}{d} = v.$$

When the Chord of the Arc and the Diameter are given.

RULE.—From the square of the diameter subtract the square of the chord, and extract the square root of the remainder; subtract this root from the diameter, and halve the remainder.

$$\text{Or, } \frac{d - \sqrt{d^2 - c^2}}{2} = v.$$

When it is greater than a Semidiameter.

RULE.—Proceed as before, but add the square root of the remainder (of the squares of the diameter and chord) to the diameter, and halve the sum.

$$\text{Or, } \frac{d + \sqrt{d^2 - c^2}}{2} = v.$$

Proportions of the Circle, its Equal, Inscribed, and Circumscribed Squares.

CIRCLE.

1. Diameter ×.8862 } = Side of an Equal Square.
2. Circumference ×.2821 }
3. Diameter ×.7071 }
4. Circumference ×.2251 } = Side of the Inscribed Square.
5. Area ×.9003 ÷ diam. }

SQUARE.

6. A Side ×1.1442 = Diameter of its Circumscribing Circle.
7. " ×4.443 = Circumference of its Circumscribing Circle.
8. " ×1.128 = Diameter }
9. " ×3.545 = Circumference } of an Equal Circle.
10. Square inches ×1.273 = Round inches }

NOTE.—The square described within a circle is one half the area of one described without it.

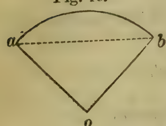
SECTOR OF A CIRCLE.

DEFINITION.—A part of a circle bounded by an arc and two radii.

To Compute the Area of a Sector of a Circle, When the Degrees in the Arc are given—Fig. 18.

RULE.—As 360 is to the number of degrees in a sector, so is the area of the circle of which the sector is a part to the area of the sector.

Fig. 18.



Or, $\frac{d \times a}{360}$ = *area, d representing the degrees in the arc, and a the area of the circle.*

EXAMPLE.—The radius of a circle, *o a*, is 5 inches, and the number of degrees of the sector, *a b o*, is 22° 30′; what is the area?

Area of a circle of 5 inches radius = 78.54 inches.

Then, as 360° : 22° 30′ : : 78.54 : 4.90875 *ins.*

When the Length of the Arc, etc., are given—Fig. 17.

RULE.—Multiply the length of the arc by half the length of the radius, and the product is the area.

Or, $b \times \overline{r \div 2}$ = *area, b representing the arc, and r the radius.*

SEGMENT OF A CIRCLE.

DEFINITION.—A part of a circle bounded by an arc and a chord.

To Compute the Area of a Segment of a Circle, Fig. 17, When the Chord and Versed Sine of the Arc, and Radius or Diameter of the Circle are given.

RULE.—*When the Segment is less than a Semicircle, as a b c, Fig.* 17, Ascertain the area of the sector having the same arc as the segment; then ascertain the area of the triangle formed by the chord of the segment and the radii of the sector, and take the difference of these areas.

NOTE.—Subtract the versed sine from the radius; multiply the remainder by one half of the chord of the arc, and the product will give the area of the triangle.

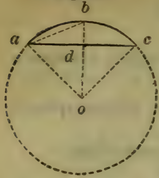
Or, $a - a'$ = *area, a a′ representing areas of the sector and the triangle.*

RULE.—*When the Segment is greater than a Semicircle,* Ascertain, by the preceding rule, the area of the lesser portion of the circle; subtract it from the area of the whole circle, and the remainder will give the area.

Or, $a - a'$ = *area, a a′ representing areas of circle and the lesser portion.*

See Table of Areas, page 205.

Fig. 19.



EXAMPLE.—The chord, *a c*, is 14.142; the diameter, *b e*, is 20; and the versed sine, *b d*, is 2.929 inches; what is the area of the segment?

$14.142 \div 2 = 7.071 =$ *half the chord of the arc.*

$\sqrt{7.071^2 + 2.929^2} = 7.654 =$ *the square root of the sum of the squares of half the chord of the arc and versed sine, which is the chord a b of half the arc a b c.*

By Rule, page 252,

$7.654 \times 2 \times 10 \times 2.929 = 448.371 =$ *twice the chord of half the arc by 10 times the versed sine.*

$\overline{20 \times 60} - \overline{2.929 \times 27} = 1120.917 = 60$ *times the diameter subtracted from 27 times the versed sine.*

Then $448.371 \div 1120.917 = .400$, and .400 added to $\overline{7.654 \times 2}$ (twice the chord of half the arc) $= 15.708$ *inches, the length of the arc.* By Rule, page 254, $15.708 \times \frac{10}{2} = 78.54 =$ *the arc multiplied by half the length of radius,* $=$ *the area of the sector.* $10 - 2.929 = 7.071 =$ *the versed sine subtracted from a radius, which is the height of the triangle a o c,* and $7.071 \times \frac{14.142}{2} = 50 =$ *area of the triangle.*

Consequently, $78.54 - 50 = 28.54$ *ins.*

When the Chords of the Arc, and of the half of the Arc, and the Versed Sine are given.

RULE.—To the chord of the whole arc add the chord of half the arc and one third of it more; multiply this sum by the versed sine, and this product, multiplied by .40426, will give the area nearly.

$$\text{Or, } c + c' + \frac{c'}{3} v \times .40426 = \textit{area nearly.}$$

EXAMPLE.—The chord of a segment, *a c*, is 28 feet; the chord of half the arc, *a b*, is 15; and the versed sine, *b d*, 6; what is the area of the segment?

$28 + 15 + \frac{15}{3} =$ *the chord of the arc added to the chord of half the arc and one third of it more.* $48 \times 6 = 288 =$ *product of above sum and the versed sine.* Then $288 \times .40426 = 116.427$ *feet.*

When the Chord of the Arc (or Segment) and the Versed Sine only are given.

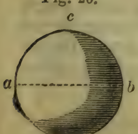
RULE.—Ascertain the chord of half the arc, and proceed as before.

SPHERE.

DEFINITION.—A figure, the surface of which is at a uniform distance from the centre.

To Compute the Convex Surface of a Sphere—Fig. 20.

Fig. 20.



RULE.—Multiply the diameter by the circumference, and the product will give the surface.

Or, $d\,c =$ *surface, d representing diameter, and c the circumference.*

Or, $4 p r^2 =$ *surface.**

Or, $p d^2 =$ *surface.*

EXAMPLE.—What is the convex surface of a sphere of 10 inches diameter?

$$10 \times 31.416 = 314.16 \textit{ ins.}$$

* *p* or π represents in this, and in all cases where it is used, the ratio of the circumference of a circle to its diameter, or 3.1416.

SEGMENT OF A SPHERE.

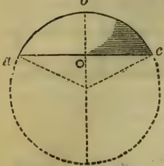
DEFINITION.—A section of a sphere.

To Compute the Surface of a Segment of a Sphere —Fig. 21.

RULE.—Multiply the height by the circumference of the sphere, and add the product to the area of the base.

Or, $h\,c + b =$ *surface, h representing the height, and b area of base.*
Or, $2\,p\,r\,h =$ *convex surface alone.*

Fig. 21.



EXAMPLE.—The height, *b o*, of a segment, *a b c*, is 36 inches, and the diameter, *b e*, of the sphere 100; what is the convex surface, and what the whole surface?

$36 \times 100 \times 3.1416 = 11309.76 =$ *height of segment multiplied by the circumference of the sphere.*

Then, to ascertain the area of the base; the diameter and versed sine being given, the diameter of the base of the segment, being equal to the chord of the arc, is, by Rule, page 253,

$$100 - \overline{36 \times 2} = 28;\ \sqrt{100^2 - 28^2} = 96.$$

$96^2 \times .7854 = 7238.2464 =$ *convex surface*, and $7238.2464 + 11309.76 = 18548.0064 =$ *convex surface added to area of base = the surface.*

NOTE.—When the convex surface of a figure alone is required, the area or areas of the base or ends must be omitted.

When the Diameter of the Base of the Segment and the Height of it are alone given.

RULE.—Add the square of half the diameter of the base to the square of the height; divide this sum by the height, and the result will give the diameter of the sphere.

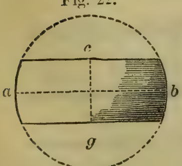
Or, $\overline{d \div 2}^2 + h^2 \div h =$ *diameter.*

SPHERICAL ZONE (OR FRUSTRUM OF A SPHERE).

DEFINITION.—The part of a sphere included between two parallel chords.

To Compute the Surface of a Spherical Zone— Fig. 22.

Fig. 22.



RULE.—Multiply the height by the circumference of the sphere, and add the product to the area of the two ends.

Or, $h\,c + a + a' =$ *surface.*
Or, $2\,p\,r\,h =$ *convex surface alone.*

EXAMPLE.—The diameter of a sphere, *a b*, from which a zone *c g* is cut, is 25 inches, and the height of it, *c g*, is 8; what is the convex surface?

$25 \times 3.1416 \times 8 = 628.32 =$ *height × circumference of sphere = convex surface.*

When the Diameter of the Sphere is not given, Multiply the mean length of the two chords by half their difference; divide this product by the breadth of the zone, and to the quotient add the breadth. To the square of this sum add the square of the lesser chord, and the square root of their sum will give the diameter of the sphere.

SPHEROIDS OR ELLIPSOIDS.

DEFINITION.—Figures generated by the revolution of a semi-ellipse about one of its diameters.

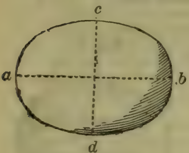
When the revolution is about the transverse diameter they are Prolate, and when it is about the conjugate they are Oblate.

To Compute the Surface of a Spheroid—Fig. 23.

When the Spheroid is Prolate.

RULE.—Square the diameters, and multiply the square root of half their sum by 3.1416, and this product by the conjugate diameter.

Fig. 23.



Or, $\sqrt{\frac{d^2+d'^2}{2}} \times 3.1416 \times d =$ *surface*, *d representing conjugate diameter.*

EXAMPLE.—A prolate spheroid has diameters of 10 and 14 inches; what is its surface?

$10^2 + 14^2 = 296 =$ *sum of squares of diameters.*

$296 \div 2 = 148$, and $\sqrt{148} = 12.1655 =$ *square root of half the sum of the squares of the diameters.*

$12.1655 \times 3.1416 \times 10 = 382.191$ *ins.* = *product of root above obtained* $\times 3.1416$, *and that product by the conjugate diameter.*

When the Spheroid is Oblate.

RULE.—Square the diameters, and multiply the square root of half their sum by 3.1416, and this product by the transverse diameter.

Or, $\sqrt{\frac{d^2+d'^2}{2}} \times 3.1416 \times d' =$ *surface*, *d′ representing transverse diameter.*

EXAMPLE.—An oblate spheroid has diameters of 14 and 10 inches; what is its surface?

$14^2 + 10^2 = 296 =$ *sum of squares of diameters.*

$296 \div 2 = 148$, and $\sqrt{148} = 12.1655 =$ *square root of half the sum of the squares of the diameter.*

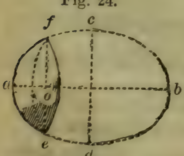
$12.1655 \times 3.1416 \times 14 = 535.0679$ *ins.* = *product of root above obtained* $\times 3.1416$, *and that product by the transverse diameter.*

To Compute the Convex Surface of a Segment of a Spheroid—Figs. 24 and 25.

RULE.—Square the diameters, and take the square root of half their sum; then, as the diameter from which the segment is cut is to this root, so is the height of the segment to the proportionate height required. Multiply the product of the other diameter and 3.1416 by the proportionate height of the segment, and this last product will give the surface.

$$\text{Or, } \frac{\sqrt{\frac{(d^2+d'^2)}{2}} \times h}{d \text{ or } d'} \times d' \text{ or } d \times 3.1416 = \textit{surface.}$$

Fig. 24.

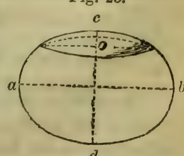


EXAMPLE.—The height, *a o*, of a segment, *e f*, of a prolate spheroid, Fig. 24, is 4 inches, the diameters being 10 and 14 inches; what is the convex surface of it?

Square root of half the sum of the squares of the diameters, as by previous examples, 12.1655.

Then $14 : 12.1655 :: 4 : 3.4758 =$ *height of segment, proportionate to the mean of the diameters.*

Fig. 25.



$10 \times 3.1416 \times 3.4758 = 109.1957$ *ins.* = *remaining diameter* $\times 3.1416$, *and again by proportionate height of segment.*

Ex. 2.—The height, *c o*, of a segment of an oblate spheroid, Fig. 25, is 4 inches, the diameters being 14 and 10 inches; what is the convex surface of it?

214.0272 *ins.*

To Compute the Convex Surface of a Frustrum or Zone of a Spheroid—Figs. 25 and 26.

RULE.—Proceed as by previous rule for the surface of a segment, and obtain the proportionate height of the frustrum; then multiply the product of the diameter parallel to the base of the frustrum and 3.1416 by the proportionate height of the frustrum, and it will give the surface. Or, d or $d' \times 3.1416 \times h =$ *surface.*

Fig. 26.

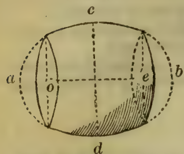
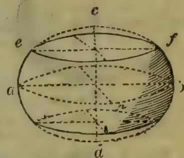


Fig. 27.



EXAMPLE.—The middle frustrum, *o e*, of a prolate spheroid, Fig. 25, is 6 inches, the diameters of the spheroid being 10 and 14 inches; what is its convex surface?

Mean diameter, as per example, page 257, is 12.1655.

Diameter parallel to base of frustrum is 10.

Then 14 : 12.1655 : : 6 : 5.2138 = *proportionate height of frustrum*, and $10 \times 3.1416 \times 5.2138 = 163.7967$ *ins.*

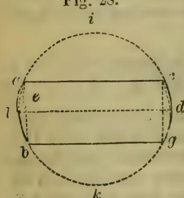
CIRCULAR ZONE.

DEFINITION.—A part of a circle included between two parallel chords.

To Compute the Area of a Circular Zone—Fig. 28.

RULE.—To the area of the trapezoid, *a b c d*, or of the parallelogram, *a h c g*, as the case may be, add the area of the segments, *a b*, *c d*, or *a h*, *c g*, and the sum will give the area.

Fig. 28.



Or, subtract the areas of the segments, *a i c*, *h k g*, from the area of the circle.

Or, $a + a' = s$, *a representing area of trapezoid, or parallelogram, and a' area of segments.*

See Table of Areas of Zones, page 267.

When the Diameter of the Circle is not given, Multiply the mean length of the two chords by half their difference; divide this product by the breadth of the zone, and to the quotient add the breadth.

To the square of this sum add the square of the lesser chord, and the square root of their sum will give the diameter of the circle.

EXAMPLE.—The greater chord, *b d*, is 96 inches; the lesser, *a c*, is 60; and the breadth of the zone, *a e*, is 26; what is its area?

$\frac{96+60}{2} = 78 =$ *mean length of chords;* $\frac{96-60}{2} = 18 =$ *half their difference.*

$\frac{78 \times 18}{26} = 54 =$ *product of chords and their difference* ÷ *by the breadth of the zone.*

$54 + 26 = 80 =$ *sum of above quotient and breadth of zone.*

$80^2 + 60^2 = 10000 =$ *sum of square of above sum and lesser chord.*

Then $\sqrt{10000} = 100 =$ *diameter*, and $78 \times 26 = 2028 =$ *area of trapezoid.*

To Compute the Area of the Segments, It is necessary, *first*, to ascertain the chord of their arcs; *second*, the versed sine of their arcs.

To Ascertain the Chord.—The breadth of the zone is the perpendicular, *a e*, of the triangle, of which either chord, *a b*, *c d*, is the hypothenuse. Further, half the difference of the chords *a c* and *b d* of the zone is the length of the base, *b e*, of this triangle.

Hence, having the base and the perpendicular, the hypothenuse or chord of the arc of the segment is readily computed.

Thus, 26 = *breadth of the zone or perpendicular of triangle;* 96 — 60 ÷ 2 = 18 = *length of base of triangles.*

Then $18^2 + 26^2 = 1000$, and $\sqrt{1000} = 31.6228$ = *chord of arc of segments a b, c d.*

To Ascertain the Versed Sine.—From the square of the radius subtract the square of half the chord, and subtract the square root of the remainder from the radius.

Thus, 100 ÷ 2 = 50, and $50^2 = 2500$ = *square of radius;* 31.6228 ÷ 2 = 15.8114, and $15.8114^2 = 250$ = *square of half the chord,* 2500 — 250 = 2250, and $\sqrt{2250}$ = 47.4342 = *square root of the difference of the squares of the radius and half the chord.* Then 50 — 47.4342 = 2.5658 = *versed sine*

Having obtained the versed sine (2.5658), the diameter of the circle (100), then, by Rule, page 253, $\sqrt{100 \times 2.5658} = 16.0181$ = *chord of half the arc.*

And by Rule, page 252, to compute the length of an arc, 32.1747 = *length of the arc;* 32.1747 × 50 ÷ 2 = 804.3675 = *the product of the length of the arc and half the radius of the circle = area of sector.*

And $804.3675 - \dfrac{31.6228 \times 47.4342}{2} = 54.3664$ = *area of the triangle subtracted from the area of the sector = area of each segment,* 54.3664 × 2 = 108.7328 = *area of segments. Area of trapezoid* = 2028 = 2136.7328 *ins* = *area of zone.*

CYLINDER.

Definition.—A figure formed by the revolution of a right-angled parallelogram around one of its sides.

To Compute the Surface of a Cylinder—Fig. 29.

Rule.—Multiply the length by the circumference, and add the product to the area of the two ends.

Fig. 29.

Or, $l\,c + 2\,a = s$, *a representing area of end.*

Note.—When the internal or convex surface alone is wanted, the areas of the ends are omitted.

Example.—The diameter of a cylinder, *b c*, is 30 inches, and its length, *a b*, 50 inches; what is its surface?

30 × 3.1416 = 94.248 *ins.* = *circumference;* 94.248 × 50 = 4712.4 = *area of body*

And $30^2 \times .7854$ = 706.86 = *area of one end,* 706.86 × 2 = 1413.72 = *area of both ends.*

Then 4712.4 + 1413.72 = 6125.12 *ins.*

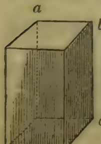
PRISMS.

Definition.—Figures the sides of which are parallelograms, and the ends equal and parallel.

Note.—When the ends are triangles, they are termed *triangular prisms;* when they are square, *square* or *right prisms*, and when they are pentagon, *pentagonal prisms*, etc., etc.

To Compute the Surface of a Right Prism—Figs. 30 and 31.

Fig. 30.



Rule.—Ascertain the areas of the ends and sides, and add them together.

Or, $2\,a + n\,a' = s$, *a representing area of the ends, and a′ the area of the sides.*

Example.—The side, *a b*, Fig. 30, of a square prism is 12 inches, and the length, *b c*, 30: what is the surface?

12 × 12 = 144 = *area of one end;* 144 × 2 = 288 = *area of both ends;* 12 × 30 = 360 = *area of one side;* 360 × 4 = 1440 = *area of four sides.*

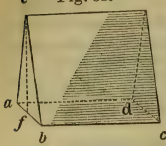
Then 288 + 1440 = 1728 *ins.*

Fig. 31.

WEDGE.

DEFINITION.—A wedge is a prolate triangular prism, and its surface is computed by the rule for that of a right prism.

Fig. 32.



EXAMPLE.—The back of a wedge, *a b c d*, Fig. 32, is 20 by 2 inches, and its end, *e f*, 20 by 2 inches; what is its surface?

$20^2 + \overline{2 \div 1}^2 = 401 =$ *sum of the squares of half the base, a f, and the height, e f, of the triangle, e f a.*

$\sqrt{401} = 20.025 =$ *square root of above sum = length of e a.*
Then $20.025 \times 20 \times 2 = 808 =$ *area of sides.*

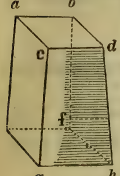
And $20 \times 2 = 40 =$ *area of back;* and $\overline{20 \times 2 \div 2 \times 2} = 40 =$ *area of ends.* Hence $801 + 40 + 40 = 881 =$ *surface.*

PRISMOIDS.

DEFINITION.—Figures are alike to a prism, but having only one pair of their sides parallel.

To Compute the Surface of a Prismoid—Fig. 33.

Fig. 33.



RULE.—Ascertain the area of the ends and sides as by the rules for squares, triangles, etc., and add them together.

EXAMPLE.—The ends of a prismoid, *e f g h* and *a b c d*, Fig. 33, are 10 and 8 inches square, and its slant height 25; what is its surface?

$10 \times 10 = 100 =$ *area of base;* $8 \times 8 = 64 =$ *area of top.*

$\frac{10 + 8}{2} \times 25 = 225$, and $225 \times 4 = 900 =$ *area of sides*

Then $100 + 64 + 900 = 1064 =$ *surface.*

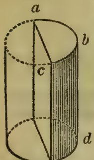
UNGULAS.

DEFINITION.—Cylindrical ungulas are the parts (including all or part of the base) left by a plane cutting a cylinder through any portion and at any angle.

To Compute the Curved Surface of an Ungula—Figs. 34, 35, 36, and 37.

RULE.—1. *When the Section is parallel to the Axis of the Cylinder,* Fig. 34, Multiply the length of the arc of one end by the height.

Fig. 34.

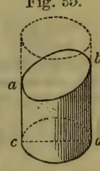


EXAMPLE.—The diameter of a cylinder from which an ungula is cut is 10 inches, its length 50, and the versed sine or depth of the ungula is 5 inches; what is the curved surface?

$10 \div 2 = 5 =$ *radius of cylinder.*

Hence the radius and versed sine are equal; the arc, therefore, of the ungula is one half the circumference of the cylinder, which is $31.416 \div 2 = 15.708$, and $15.708 \times 50 = 785.4$ *ins.*

Fig. 35.



RULE.—2. *When the Section passes obliquely through the opposite Sides of the Cylinder,* Fig. 35, Multiply the circumference of the base of the cylinder by half the sum of the greatest and least heights of the ungula.

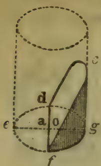
EXAMPLE.—The diameter of a cylindrical ungula is 10 inches, and the greater and less heights, *b d* and *a c*, are 25 and 15 inches; what is its curved surface?

10 *diameter* = 31.416 *circumference;* $25 + 15 = 40$, and $40 \div 2 = 20$. Hence $31.416 \times 20 = 628.32$ *ins.*

RULE.—3. *When the Section passes through the Base of the Cylinder and one of its Sides, and the Versed Sine does not exceed the Sine,* Fig. 36, Multiply the sine, *a d*, of halt the arc, *d g*, of the base, *d g f*, by the diameter, *e g*, of the cylinder, and from this product subtract the product* of the arc and cosine, *a o*. Multiply the

* When the cosine is 0, this product is 0.

Fig. 36.



difference thus found by the quotient of the height, *g c*, divided by the versed sine, *a g*.

EXAMPLE.—The sine, *a d*, of half the arc of the base of an ungula is 5, the diameter of the cylinder is 10, and the height of the ungula 10 inches; what is the curved surface?

$5 \times 10 = 50 =$ *sine of half the arc by the diameter*

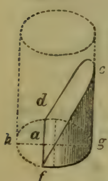
Length of arc, the versed sine and radius being equal, under Rule, page 252 = 15.708.

Again, as the versed sine and the radius are equal, the cosine is 0.

Hence, when the cosine is 0, the product is 0. $50 - 0 = 50 =$ *the difference of the product before obtained and the product of the arc and the cosine.*

$50 \times \overline{10 \div 5} = 50 \times 2 = 100 =$ *the difference multiplied by the height divided by the versed sine, which is the surface.*

Fig. 37.



RULE.—4. *When the Section passes through the Base of the Cylinder, and the Versed Sine, a g, exceeds the Sine.* Fig. 37, Multiply the sine of half the arc of the base by the diameter of the cylinder, and to this product add the product of the arc and the excess of the versed sine over the sine of the base.

Multiply the sum thus found by the quotient of the height divided by the versed sine.

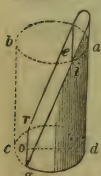
EXAMPLE.—The sine, *a d*, of half the arc of an ungula is 12 inches; the versed sine, *a g*, is 16; the height, *c g*, 16; and the diameter of the cylinder, *h g*, 25 inches; what is the curved surface?

$12 \times 25 = 300 =$ *sine of half the arc by the diameter of the cylinder.*

Length of arc of base, Rule, p. 252 = *arc of d h f — circumference of base* = 46.392.

Then $\overline{46.392 \times 16 - 12.5} = 162.372 =$ *product of arc and the excess of the versed sine over the sine*; $300 + 162.372 = 462.372 =$ *the sum of the above products*; $16 \div 16 = 1 =$ *quotient of height divided by the versed sine*; $462.372 \times 1 = 462.372$ *ins.* = *the sum of the products and the height divided by the versed sine = the curved surface.*

Fig. 38.



NOTE.—When the sine of an arc is 0, the versed sine is equal to the diameter.

RULE.—5. *When the Section passes obliquely through both Ends of the Cylinder*, Fig. 38, Conceive the section to be continued till it meets the side of the cylinder produced; then, as the difference of the versed sines of the arcs of the two ends of the ungula is to the versed sine of the arc of the less end, so is the height of the cylinder to the part of the side produced.

Ascertain the surface of each of the ungulas thus found by Rules 3 and 4, and their difference will give the curved surface.

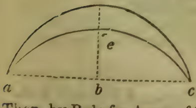
LUNE.

DEFINITION.—The space between the intersecting arcs of two eccentric circles.

To Compute the Area of a Lune--Fig. 39.

RULE.—Ascertain the areas of the two segments from which the lune is formed, and their difference will give the area.

Fig. 39.



EXAMPLE.—The length of the chord *a c* is 20, the height *e d* is 3, and *e b* 2 inches; what is the area of the lune?

By Rule 2, page 253, the diameters of the circles of which the lune is formed are thus ascertained:

For *a d c*, $\frac{10^2 + (3+2)^2}{5} = 25$. For *a e c*, $\frac{10^2 + 2^2}{2} = 52$.

Then, by Rule for Areas of Segments of a Circle, p. 254, the area of *a d c* is 70.5577 *ins*

" *a e c* " 27.1638 "

their difference 43.3939 *ins*

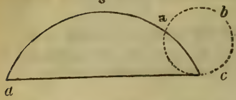
NOTE.—If semicircles be described on the three sides of a right-angled triangle as diameters, two lunes will be formed, and their united areas will be equal to that of the triangle.

CYCLOID.

DEFINITION.—A curve generated by the revolution of a circle on a plane.

To Compute the Area of a Cycloid—Fig. 40.

Fig. 40.



RULE.—Multiply the area of the generating circle, *a b c*, by 3.

EXAMPLE.—The generating circle of a cycloid has an area of 115.45 inches; what is the area of the cycloid?

$$115.45 \times 3 = 346.35 \textit{ ins.}$$

To Compute the Length of a Cycloidal Curve—Fig. 40.

RULE.—Multiply the diameter of the generating circle by 4.

EXAMPLE.—The diameter of the generating circle of a cycloid, Fig. 40, is 8 inches; what is the length of the curve *d s c*?

$$8 \times 4 = 32 = \textit{product of diameter and } 4 = \textit{ins}$$

NOTE.—The curve of a cycloid is the line of swiftest descent; that is, a body will fall through the arc of this curve, from one point to another, in less time than through any other path.

RINGS.

CIRCULAR RINGS.

DEFINITION.—The space between two concentric circles.

To Compute the Sectional Area of a Circular Ring.

RULE.—From the area of the greater circle subtract that of the less.

Or, $a - a' = \textit{area}$.

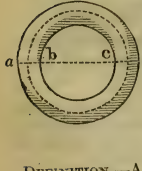
CYLINDRICAL RINGS.

DEFINITION.—A ring formed by the curvature of a cylinder.

To Compute the Surface of a Cylindrical Ring—Fig. 41.

RULE.—To the thickness of the ring add the inner diameter; multiply this sum by the thickness, and the product by 9.8696.

Fig. 41.



Or, $d + d'\, d\ 9.8696 = \textit{surface}$.

EXAMPLE.—The thickness of a cylindrical ring, *a b*, is 2 inches, and the inner diameter, *b c*, is 18; what is the surface of it?

$2 + 18 = 20 =$ *thickness of ring added to the inner diameter.*

$20 \times 2 \times 9.8696 = 394.784$ *ins.* = *the sum above obtained* × *the thickness of the ring, and that product by* 9.8696.

LINK.

DEFINITION.—An elongated ring.

To Compute the Surface of a Link—Figs. 42 and 43.

RULE.—Multiply the circumference of a section of the body of the link by the length of its axis.

Or, $c \times l = \textit{surface}$.

NOTE.—To Compute the Circumference or Length of the Axes.

When the Ring is Elongated.—To the less diameter add its thickness, and multiply the sum by 3.1416; multiply the difference of the diameters by 2, and take the sum of these products.

Fig. 42.

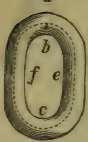
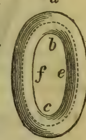


Fig. 43.



When the Ring is Elliptical.—Square the diameters of the axes of the ring, and multiply the square root of half their sum by 3.1416.

EXAMPLE.—The link of a chain, Fig. 42, is 1 inch in diameter of body, *a b*, and its inner diameters, *b c* and *e f*, are 12.5 and 2.5 inches; what is its circumference?

$2.5 + 1 \times 3.1416 = 10.9956 =$ *length of axis of ends.*
$12.5 - 2.5 \times 2 = 20 =$ *length of sides of body.*
Then $10.9956 + 20 = 30.9956 =$ *length of axis of link*, which $\times 3.1416$ (cir. of 1 in.) $= 97.3758$ *ins.*

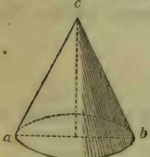
CONES.

DEFINITION. — A figure described by the revolution of a right-angled triangle about one of its legs.

For Sections of a Cone, see Conic Sections, page 289.

To Compute the Surface of a Cone—Fig. 44.

Fig. 44.



RULE.—Multiply the perimeter or circumference of the base by the slant height, or side of the cone; halve the product, and add it to the area of the base.

Or, $c \times h \div 2 + a' =$ *surface, c representing perimeter.*

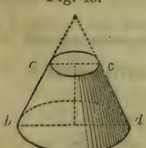
EXAMPLE.—The diameter, *a b*, of the base of a cone is 3 feet, and the slant height, *a c*, 15; what is the surface of the cone?

Circum. of 3 feet $= 9.4248$, and $\frac{9.4248 \times 15}{2} = 70.686 =$ *surface of side, and area* $3 = 7.068 = 70.686 + 7.068 = 77.754$ *sq. feet.*

To Compute the Surface of the Frustrum of a Cone—Fig. 45.

RULE.—Multiply the sum of the perimeters of the two ends by the slant height of the frustrum; halve the product, and add it to the areas of the two ends.

Fig. 45.



Or, $\frac{c + c' \times h}{2} + a + a' =$ *surface.*

EXAMPLE.—The frustrum, *a b c d*, Fig. 45, has a slant height of 26 inches, and the circumferences of its ends are 15.71 and 22 inches respectively; what is its surface?

$\frac{15.71 + 22. \times 26}{2} = 490.23 =$ *surface of sides*; $\left(\frac{15.71}{3.1416}\right)^2 \times .7854 + \left(\frac{22.}{3.1416}\right)^2 \times 7854 = 58.119 =$ *areas of ends.* Then $490.23 + 58.119 = 548.349$ *inches.*

PYRAMIDS.

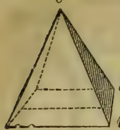
DEFINITION.—A figure the base of which has three or more sides, and the sides of which are plain triangles.

To Compute the Surface of a Pyramid—Figs. 46 and 47.

RULE.—Multiply the perimeter of the base by the slant height; halve the product, and add it to the area of the base.

Or, $\frac{c \times h}{2} + a =$ *surface.*

Fig. 46.



EXAMPLE.—The side of a quadrangular pyramid, $a\ b$, Fig. 46, is 12 inches, and its slant height, $a\ c$, 40; what is its surface?

$12 \times 4 = 48 =$ *perimeter of base.*

$\frac{48 \times 40}{2} = 960 =$ *area of sides.*

Then $12 \times 12 + 960 = 1104.$ *ins.*

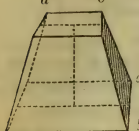
Fig. 47.

To Compute the Surface of the Frustrum of a Pyramid—Fig. 48.

RULE.—Multiply the sum of the perimeters of the two ends by the slant height or side; halve the product, and add it to the areas of the ends.

Or, $\frac{c + c' \times h}{2} + a + a' =$ *surface.*

Fig. 48.



EXAMPLE.—The sides, $a\ b$, $c\ d$, Fig. 48, of the frustrum of a quadrangular pyramid are 10 and 9 inches, and its slant height, $a\ c$, 20; what is its surface?

$10 \times 4 = 40$, and $9 \times 4 = 36 = 76 =$ *sum of perimeters.*

$76 \times 20 = 1520$, and $\frac{1520}{2} = 760 =$ *area of sides;* $10 \times 10 = 100$, and $9 \times 9 = 81$.

Then $100 + 81 + 760 = 941 =$ *ins.*

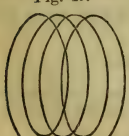
HELIX (SCREW).

DEFINITION.—A line generated by the progressive rotation of a point around an axis and equidistant from its centre.

To Compute the Length of a Helix—Fig. 49.

RULE.—To the square of the circumference described by the generating point add the square of the distance advanced in one revolution, and take the square root of their sum multiplied by the number of revolutions of the generating point.

Fig. 49.



Or, $\sqrt{(c^2 + h^2)}n =$ *length, n representing number of revolutions.*

EXAMPLE.—What is the length of a helical line running 3.5 times around a cylinder of 22 inches in circumference and advancing 16 inches in each revolution?

$22^2 + 16^2 = 740 =$ *sum of squares of circumference and of the distance advanced.** Then $\sqrt{740} \times 3.5 = 95.21$ *inches.*

SPIRALS.

DEFINITION.—Lines generated by the progressive rotation of a point around a fixed axis.

A *Plane Spiral* is when the point rotates around a central point.

A *Conical Spiral* is when the point rotates around an axis at a progressing distance from its centre, or around a cone.

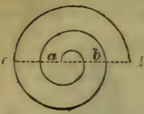
To Compute the Length of a Plane Spiral Line—Fig. 50.

RULE.—Add together the greater and less diameters; divide their sum by two; multiply the quotient by 3.1416, and again by the num-

* When the spiral is other than a line, measure the diameters of it from the middle of the body composing it.

ber of revolutions. Or, when the circumferences are given, take their mean length, and multiply it by the number of revolutions.

Fig. 50.



Or, $\overline{d + d' \div 2} \times 3.1416\, n$ = *length of line*; $p \times n$ = *radius*, and $pr^2 \div l$ = *pitch*.

EXAMPLE.—The less and greater diameters of a plane spiral spring, as *a b*, *c d*, Fig. 50, are 2 and 20 inches, and the number of revolutions 10; what is the length of it?

$\overline{2 + 20} \div 2 = 11$ = *sum of diameters* $\div 2$; $11 \times 3.1416 = 34.5576$ = *above quotient* $\times 3.1416$.

Then $34.5576 \times 10 = 345.576$ *inches.*

NOTE.—The above rule is applicable to winding engines where it is required to ascertain the length of a rope, its thickness, the number of revolutions, diameter of drum, etc., etc.

To Compute the Length of a Conical Spiral Line—Fig. 51.

RULE.—Add together the greater and less diameters; divide their sum by two, and multiply the quotient by 3.1416.

To the square of the product of this circumference and the number of revolutions of the spiral, add the square of the height of its axis, and take the square root of the sum.

Fig. 51.

Or, $\sqrt{(\overline{d + d' \div 2 \times 3.1416\, n}^2 + l^2)}$ = *length of line.*

EXAMPLE.—The greater and less diameters of a conical spiral, Fig. 51, are 20 and 2 inches; its height, *c d*, 10; and the number of revolutions 10; what is the length of it?

$20 + 2 \div 2 = 11 \times 3.1416 = 34.5576$ = *sum of diameters* $\div 2$, and $\times 3.1416$; $34.5576 \times 10 = 345.576$, and $345.576^2 = 119422.77$ = *square of the product of the circumference and number of revolutions.*

Then $\sqrt{119422.77 + 10^2} = 345.72$ *inches.*

SPINDLES.

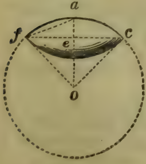
DEFINITION.—Figures generated by the revolution of a plane area, when the curve is revolved about a chord perpendicular to its axis, or about its double ordinate, and they are designated by the name of the arc or curve from which they are generated, as Circular, Elliptic, Parabolic, etc., etc.

CIRCULAR SPINDLE.

T Compute the Convex Surface of a Circular Spindle, Zone, or Segment of it—Fig. 52, 53, and 54.

RULE.—Multiply the length by the radius of the revolving arc; multiply this arc by the central distance, or distance between the centre of the spindle and centre of the revolving arc; subtract this product from the former, double the remainder, and multiply it by 3.1416.

Fig. 52.



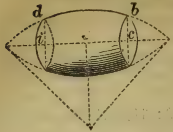
Or, $l r - \left(a \sqrt{r^2 - \left(\frac{c}{2}\right)^2}\right) 2 p$ = *surface, a representing length of arc, and c the chord.*

EXAMPLE.—What is the surface of a circular spindle, Fig. 52, the length of it, *f c*, being 14.142 inches, the radius of its arc, *o c*, 10, and the central distance, *o e*, 7.071?

$14.142 \times 10 = 141.42$ = *length* $\times$ *radius.* *Length of arc, f a c,* by Rules, page 252 = 15.708.

$15.708 \times 7.071 = 111.0713$ = *length of arc* $\times$ *central distance*; $141.42 - 111.0713 = 30.3487$ = *difference of products.* Then $30.3487 \times 2 \times 3.1416 = 190.687$ *inches.*

Fig. 53.



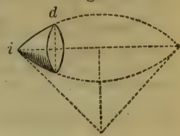
EXAMPLE.—What is the convex surface of the zone of a circular spindle, Fig. 53, the length of it, *i c*, being 7.653 inches, the radius of its arc 10, the central distance 7.071, and the length of its side or arc, *d b*, 7.854 inches?

$7.653 \times 10 = 76.53 =$ *length* $\times$ *radius*; $7.854 \times 7.071 = 55.5356$ $=$ *length of arc* $\times$ *central distance*; $76.53 - 55.5356 = 20.9944 =$ *difference of products*.

Then $20.9944 \times 2 \times 3.1416 = 131.912$ *inches*.

EXAMPLE.—What is the convex surface of a segment of a circular spindle, Fig. 54, the length of it being 3.2495 inches, the radius of its arc 10, the central distance 7.071, and the length of its side, *i d*, 3.927 inches?

Fig. 54.



$3.2495 \times 10 = 32.495 =$ *length* $\times$ *radius*; $3.927 \times 7.071 = 27.7678 =$ *length of arc* $\times$ *central distance*; $32.495 - 27.7678 = 4.7272 =$ *difference of products*.

Then $4.7272 \times 2 \times 3.1416 = 29.702$ *inches*.

GENERAL FORMULA.—$S = 2(l\,r - a\,c)\,p =$ *surface*, *l representing length of spindle, segment, or zone, a the length of its revolving arc, r the radius of the generating circle, and c the central distance.*

ILLUSTRATION.—The length of a circular spindle is 14.142 inches, the length of its revolving arc is 15.708, the radius of its generating circle is 10, and the distance of its centre from the centre of the circle from which it is generated is 7.071; what is its surface?

$$2 \times (14.142 \times 10 - \overline{15.708 \times 7.071}) \times 3.1416 = 190.687 \text{ inches}.$$

NOTE.—The surface of a frustrum of a spindle may be obtained by the division of the surface of a zone.

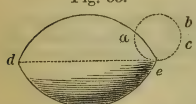
CYCLOIDAL SPINDLE.

To Compute the Convex Surface of a Cycloidal Spindle--Fig. 55.

RULE.—Multiply the area of the generating circle by 64, and divide it by 3.

Or, $\frac{a \times 64}{3} =$ *surface*.

Fig. 55.



EXAMPLE.—The area of the generating circle, *a b c*, of a cycloidal spindle, *d e*, is 32 inches; what is the surface of the spindle?

$32 \times 64 = 2048 =$ *area of circle* $\times 64$; and $2048 \div 3 = 682.667$ *inches*.

NOTE.—The area of the greatest section of a cycloidal spindle is twice the area of the cycloid.

ELLIPSOID, PARABOLOID, OR HYPERBOLOID OF REVOLUTION.

DEFINITION.—Figures alike to a cone generated by the revolution of a conic section around its axis.

NOTE.—These figures are usually known as Conoids.

When they are generated by the revolution of an ellipse, they are called Ellipsoids, and when by a parabola, Paraboloids, etc., etc.

The revolution of an arc of a conic section around the axis of the curve will give a segment of a conoid.

ELLIPSOID.

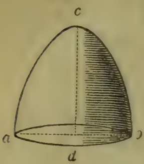
To Compute the Convex Surface of an Ellipsoid—Fig. 56.

RULE.—Add together the square of the base and four times the square of the height; multiply the square root of half their sum by 3.1416, and this product by the radius of the base.

Or, $\sqrt{\frac{b^2 + 4h^2}{2}}\,3.1416\,r =$ *surface, h representing height of the ellipsoid.*

To Compute the Convex Surface of a Segment, Frustrum, or Zone of an Ellipsoid.

Fig. 56.



See Rules for the Convex Surface of a Segment, Frustrum, or Zone of a Spheroid or Ellipsoid, pages 257, 258.

EXAMPLE.—The base, *a b*, of an ellipsoid, Fig. 56, is 10 inches, and the vertical height, *c d*, 7; what is its surface?

$10^2 + \overline{7^2 \times 4} = 296 =$ *sum of the square of the base and 4 times the square of the height;* $296 \div 2 = 148$, and $\sqrt{148} = 12.1655$ $=$ *square root of half the above sum.*

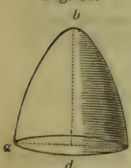
Then $12.1655 \times 3.1416 \times \frac{10}{2} = 191.0957$ *inches.*

PARABOLOID.

To Compute the Convex Surface of a Paraboloid—Fig. 57.

RULE.—From the cube of the square root of the sum of four times the square of the height, and the square of the radius of the base, subtract the cube of the radius of the base; multiply the remainder by the quotient of 3.1416 times the radius of the base divided by six times the square of the height.

Fig. 57.



Or, $(\sqrt{4h^2 + r^2})^3 - r^3 \times \frac{r \times p}{6 \times h^2} =$ *surface.*

EXAMPLE.—The axis, *b d*, of a paraboloid, Fig. 57, is 40 inches; the radius, *a d*, of its base is 18 inches; what is its convex surface?

$40^2 \times 4 = 6400 = 4$ *times the square of the height;* $6400 + 18^2 = 6724 =$ *sum of the above product and the square of the radius of the base;* $(\sqrt{6724})^3 - 18^3 = 545536 =$ *the remainder, of the cube of the radius of the base subtracted from the cube of the square root of the preceding sum;* $3.1416 \times 18 \div (6 \times 40^2) = .0058905 =$ *the quotient of* 3.1416 *times the radius of the base* $\div$ 6 *times the square of the height.*

Then $515536 \times .0058905 = 3213.48$ *inches.*

ANY FIGURE OF REVOLUTION.

To Ascertain the Convex Surface of any Figure of Revolution—Figs. 58, 59, and 60.

RULE.—Multiply the length of the generating line by the circumference described by its centre of gravity.

Or, $l\ 2\ r\ p =$ *surface, r representing radius of centre of gravity.*

Fig. 58.

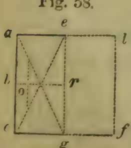


ILLUSTRATION.—If the generating line, *a c*, of the cylinder, *a c d f*, 10 inches in diameter, Fig. 58, is 10, then the centre of gravity of it will be in *b*, the radius of which is $b\ r = 5$.

Hence $10 \times \overline{5 \times 2} \times 3.1416 = 314.16$ *inches = the convex surface of the cylinder.*

Again, If the generating line is *e a c g*, and it is ($e\ a = 5$, $a\ c = 10$, and $c\ g = 5$) $= 20$, then the centre of gravity, *o*, will be in the middle of the line joining the centres of gravity of the triangles *e a c* and $a\ c\ g = 3.75$ from *r*.

Hence $20 \times \overline{3.75 \times 2} \times 3.1416 = 471.24$ *inches = the entire surface of the cylinder.*

PROOF.	Convex surface as above...	314.16
	Area of each end, $10^2 \times .7854 = 78.54$, and $78.54 \times 2 =$.........	157.08
		471.24

Fig. 59.

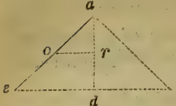


ILLUSTRATION 2.—If the generating elements of a cone, Fig. 59, are $a\ d = 10, d\ c = 10$, and $a\ c$ the generating line $= 14.142$, the centre of gravity of which is in o, and $o\ r = 5$.

Then $14.142 \times \overline{5 \times 2} \times 3.1416 = 444.285 =$ *the convex surface of the cone*, and $\overline{10 \times 2}^2 \times .7854 = 314.16 =$ *area of base.*

Hence $444.285 + 314.16 = 758.445 =$ *the entire surface of the cone.*

Fig. 60.

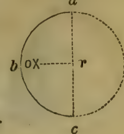


ILLUSTRATION 3.—If the generating elements of a sphere, Fig. 60, are $a\ c = 10$, $a\ b\ c$ will be 15.708, the centre of gravity of which is in o, and by Rule, page 339, $o\ r = 3.183$.

Hence $15.708 \times \overline{3.183 \times 2} \times 3.1416 = 314.16$ *inches = the surface of the sphere.*

To Ascertain the Area of an Irregular Figure.

RULE.—Take a uniform piece of board or pasteboard, weigh it, cut out the figure of which the area is required, and weigh it; then, as the weight of the board or pasteboard is to the entire surface, so is the weight of the figure to its surface.

CAPILLARY TUBE.

To Compute the Diameter of a Capillary Tube.

RULE.—Weigh the tube when empty, and again when filled with mercury; subtract the one weight from the other; reduce the difference to Troy grains, and divide it by the length of the tube in inches. Extract the square root of this quotient, multiply it by .0192245, and the product will give the diameter of the tube in inches.

Or, $\sqrt{\frac{w}{l}} \times .0192245 =$ *diameter, w representing difference in weights in Troy grains and l the length of the tube.*

EXAMPLE. — The difference in the weights of a capillary tube when empty and when filled with mercury is 90 grains, and the length of the tube is 10 inches; what is the diameter of it?

$90 \div 10 = 9 =$ *weight of mercury ÷ length of tube;* $\sqrt{9} = 3$, and $3 \times .0192245 = .0576735 =$ *the square root of the above quotient* $\times .0192245$ *inches = diameter of tube.*

PROOF.—The weight of a cubic inch of mercury is 3442.75 Troy grains, and the diameter of a circular inch of equal area to a square inch is 1.128 (page 254).

If, then, 3442.75 grains occupy 1 cubic inch, 90 grains will require .0261419 cubic inch, which, ÷ 10 for the height of the tube = .00261419 inch for the area of the section of the tube.

Then $\sqrt{.00261419} = .051129 =$ *side of the square of a column of mercury of this area.*

Hence $.051129 \times 1.128$, which is the ratio between the side of a square and the diameter of a circle of equal area $= .0576735$.

ADDITIONAL RULES FOR ELEMENTS OF REGULAR POLYGONS.

To Compute the Radius of the Inscribed or Circumscribed Circles.

RULE.— *When the Radius of the Circumscribing Circle is given,* Multiply the radius given by the unit in column E, in the following Table, opposite to the figure for which the radius is required.

RULE.— *When the Radius of the Inscribed Circle is given,* Multiply the radius given by the unit in column F, in the following Table, opposite to the figure for which the radius is required.

To Compute the Area, When the Radii of the Inscribed or Circumscribing Circles are given.

Rule.—Square the radius given, and multiply it by the unit in columns G or H, in the following Table, and opposite to the figure for which the area is required.

To Compute the Length of a Side, When the Radius of the Inscribed or Circumscribing Circle is given.

Rule.—Multiply the radius given by the unit in column K, in the following Table, and opposite to the figure for which the length is required.

No. of Sides.	Name of Polygon.	E. Radius of Inscribed Circle. By Circumscrib. Circle.	F. Radius of Circumscr. Circle. By Inscribing Circle.	G. Area. By Radius of Inscrib'd Circle.	H. Area. By Radius of Circumscribing Circle.	I. Area. By Length of Side.	K. Length of Side. By Radius of Inscrib'd Circle.
3	Trigon	.5	2.	5.19615	1.29904	.43301	3.4641
4	Tetragon	.70711	1.41421	4.	2.	1.	2.
5	Pentagon....	.80902	1.23607	3.63272	2.37764	1.72048	1.45308
6	Hexagon	.86602	1.1547	3.4641	2.59808	2.59808	1.1547
7	Heptagon ...	.90097	1.10992	3.37102	2.73641	3.63391	.96315
8	Octagon.....	.92388	1.08239	3.31371	2.82842	4.81843	.82843
9	Nonagon	.93969	1.06418	3.27573	2.89254	6.18182	.72794
10	Decagon	.95106	1.05146	3.2492	2.93893	7.69421	.64984
11	Undecagon ..	.95949	1.04222	3.22989	2.97353	9.36564	.58725
12	Dodecagon ..	.96593	1.03528	3.21539	3.	11.19615	.5359

MENSURATION OF SOLIDS.

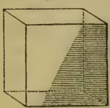
CUBES AND PARALLELOPIPEDONS.

CUBE.

DEFINITION.—A solid contained by six equal square sides.

To Compute the Volume of a Cube—Fig. 61.

Fig. 61.



RULE.—Multiply a side of the cube by itself, and that product again by a side.

Or, $s^3 = V$, *s representing the length of a side, and* V *the volume.*

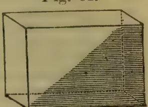
EXAMPLE.—The side *a b*, Fig. 61, is 12 inches; what is the volume of it?

$12 \times 12 \times 12 = 1728$ *cubic inches.*

PARALLELOPIPEDON.

DEFINITION.—A solid contained by six quadrilateral sides, every opposite two of which are equal and parallel.

Fig. 62.



To Compute the Volume of a Parallelopipedon—Fig. 62.

RULE.—Multiply the length by the breadth, and that product again by the depth.

Or, $l b d = V$.

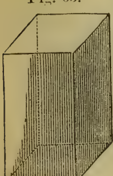
PRISMS, PRISMOIDS, AND WEDGES.

PRISMS.

DEFINITION.—Solids, the ends of which are equal, similar, and parallel planes, and the sides of which are parallelograms.

NOTE.—When the ends of a prism or prismoid are triangles, it is called a *triangular prism* or *prismoid;* when rhomboids, a *rhomboidal prism*, etc.; when squares, a *square prism*, etc.; when rectangles, a *rectangular prism*, etc.

Fig. 63.



To Compute the Volume of a Prism—Figs. 63 and 64.

Fig. 64.

RULE.—Multiply the area of the base by the height.

Or, $a \times h = V$.

EXAMPLE.—A triangular prism, *a b c d e*, Fig. 64, has sides of 2.5 feet, and a length, *c d*, of 10 feet; what is its volume?

By Rule, page 245, $2.5^2 \times .433 = 2.70625$ = *area of end a b c*, and $2.70625 \times 10 = 27.0625$ *cubic feet*

PRISMOIDS.*

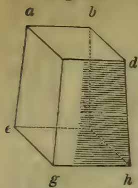
To Compute the Volume of a Prismoid—Fig. 65.

RULE.—To the sum of the areas of the two ends add four times the area of the middle section, parallel to them, and multiply this sum by ⅙ of the perpendicular height.†

* An excavation or embankment of a road, when terminated by parallel cross sections, is a rectangular prismoid.

† This is the general rule, and applies equally to figures of proportionate or dissimilar ends.

Fig. 65.



Or, $a + a' + 4m = \overline{h \div 6} = V$, *a and a′ representing areas of ends, and m area of middle section.*

EXAMPLE.—What is the volume of a rectangular prismoid, Fig. 65, the lengths and breadths, *e g* and *g h*, and *a b* and *b d*, of the two ends being 7×6 and 3×2 inches, and the height 15 feet?

$7\times6+\overline{3\times2}=42+6=48=$ *sum of the areas of the two ends*; $7+3\div2=5=$ *length of the middle section*; $6+2\div2=4=$ *breadth of the middle section*; $5\times4\times4=80=$ *four times the area of the middle section.*

Then $48+80\times\frac{15\times12}{6}=128\times30=3840$ *cubic inches.*

NOTE.—The length and breadth of the middle section are respectively equal to half the sum of the lengths and breadths of the two ends.

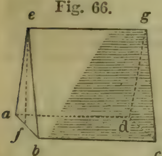
2.—Prismoids, alike to prisms, derive their designation from the figure of their ends, as triangular, square, rectangular, pentagonal, etc.

WEDGE.

To Compute the Volume of a Wedge—Fig. 66.

RULE.—To the length of the edge add twice the length of the back; multiply this sum by the perpendicular height, and then by the breadth of the back, and take ⅙ of the product.

Fig. 66.



Or, $(l+\overline{l'\times2}\times h\,b)\div6=V$.

EXAMPLE.—The back of a wedge, *a b c d*, is 20 by 2 inches, and its height, *e f*, 20 inches; what is its volume?

$20+\overline{20\times2}=60=$ *length of the edge added to twice the length of the back*; $60\times20\times2=2400=$ *above sum multiplied by the height, and that product by the breadth of the back.*

Then $2400\div6=400$ *cubic inches.*

NOTE.—When a wedge is a true prism, as represented by Fig. 66, the volume of it is equal to the area of an end multiplied by its length.

REGULAR BODIES (POLYHEDRONS).

DEFINITION.—A regular body is a solid contained under a certain number of similar and equal plane faces,* all of which are equal regular polygons.

NOTE.—The whole number of regular bodies which can possibly be formed is five.

2.—A sphere may always be inscribed within, and may always be circumscribed about a regular body or polyhedron, which will have a common centre.

Fig. 67.

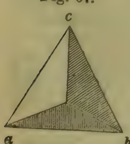


Fig. 68.

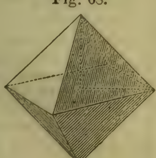


Fig. 69.

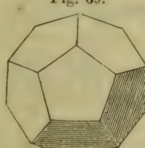
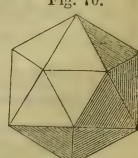


Fig. 70.



1. The Tetrahedron, or Pyramid, Fig. 67, which has four triangular faces.
2. The Hexahedron, or Cube, Fig. 61, which has six square faces.
3. The Octahedron, Fig. 68, which has eight triangular faces.
4. The Dodecahedron, Fig. 69, which has twelve pentagonal faces.
5. The Icosahedron, Fig. 70, which has twenty triangular faces.

* The angle of the adjacent faces of a polygon is called the diedral angle.

Table of Units for Elements of any Regular Body.

	Figure.	A. Radius of Circumscribing Sphere. By Linear Edge.	B. Radius of Inscribed Sphere. By Linear Edge.	C. Radius of Circumscribing Sphere. By Surface.	D. Radius of Inscribed Sphere. By Surface.	E. Radius of Circumscribing Sphere. By Volume.	F. Radius of Inscribed Sphere. By Volume.	G. Radius of Inscribed Sphere. By Circumscribing Sphere.	H. Radius of Circumscribing Sphere. By Inscribed Sphere.	I. Linear Edge. By Radius of Circumscribing Sphere.	K. Linear Edge. By Radius of Inscribed Sphere.
4	Tetrahedron ..	.61237	.20412	.4653	.1551	1.24896	.41634	.33333	3.	1.63299	4.89898
6	Hexahedron ..	.86602	.5	.35355	.20412	.86602	.5	.57735	1.73205	1.1547	2.
8	Octahedron ...	.70711	.40825	.37992	.21935	.90806	.52456	.57735	1.73205	1.41421	2.44949
12	Dodecahedron.	1.40126	1.11352	.30839	.24507	.71075	.5648	.79465	1.25841	.71364	.89806
20	Icosahedron ..	.95106	.75576	.32318	.25681	.73329	.58271	.79465	1.25841	1.05146	1.32317

	Figure.	L. Linear Edge. By Surface.	M. Linear Edge. By Volume.	N. Surface. By Radius of Circumscribing Sphere.	O. Surface. By Radius of Inscribed Sphere.	P. Surface. By Linear Edge.	Q. Surface. By Volume.	R. Volume. By Linear Edge.	S. Volume. By Radius of Circumscribing Sphere.	T. Volume. By Radius of Inscribed Sphere.	U. Volume. By Surface.
4	Tetrahedron ..	.75984	2.03955	4.6188	41.56922	1.73205	7.20562	.11785	.5132	13.85641	.0517
6	Hexahedron ..	.40825	1.	8.	24.	6.	6.	1.	1.5396	8.	.06804
8	Octahedron ...	.53729	1.2849	6.9282	20.78461	3.4641	5.7191	.4714	1.33333	6.9282	.07311
12	Dodecahedron.	.22008	.50722	10.51462	16.65087	20.64573	5.31161	7.66312	2.78517	5.55029	.08169
20	Icosahedron ..	.33981	.77102	9.57454	15.16217	8.66025	5.14835	2.1817	2.53615	5.05406	.0856

To Compute the Elements of any Regular Body—Figs. 67, 68, 61, 69, and 70.

To Compute the Radius of a Sphere that will Circumscribe a given Regular Body, and the Radius of one also that may be Inscribed within it.

RULE.—*When the Linear Edge is given*, Multiply it by the multiplier opposite to the body in the columns A and B in the preceding Table, under the head of the element required.

EXAMPLE.—The linear edge of a hexahedron or cube, Fig. 61, is 2 inches; what are the radii of the circumscribing and inscribed spheres?

$2 \times .86602 = 1.73204$ *inches = radius of circumscribing sphere;* $2 \times .5 = 1$ *inch = radius of inscribed sphere.*

RULE.—*When the Surface is given*, Multiply the square root of it by the multiplier opposite to the body in columns C and D in the preceding Table, under the head of the element required.

RULE.—*When the Volume is given*, Multiply the cube root of it by the multiplier opposite to the body in columns E and F in the preceding Table, under the head of the element required.

RULE.—*When one of the Radii of the Circumscribing or Inscribed Sphere alone is required, the other being given*, Multiply the given radius by the multiplier opposite to the body in columns G and H in the preceding Table, under the head of the other radius.

To Compute the Linear Edge of a Polyhedron.

RULE.—*When the Radius of the Circumscribing or Inscribed Sphere is given*, Multiply the radius given by the multiplier opposite to the body in columns I and K in the preceding Table.

RULE.—*When the Surface is given*, Multiply the square root of it by the multiplier opposite to the body in column L in the preceding Table.

RULE.—*When the Volume is given*, Multiply the cube root of it by the multiplier opposite to the body in column M in the preceding Table.

To Compute the Surface of a Polyhedron.

RULE.—*When the Radius of the Circumscribing Sphere is given*, Multiply the square of the radius by the multiplier opposite to the body in column N in the preceding Table.

RULE.—*When the Radius of the Inscribed Sphere is given*, Multiply the square of the radius by the multiplier opposite to the body in column O in the preceding Table.

RULE.—*When the Linear Edge is given*, Multiply the square of the edge by the multiplier opposite to the body in column P in the preceding Table.

RULE.—*When the Volume is given*, Extract the cube root of the volume, and multiply the square of the root by the multiplier opposite to the body in column Q in the preceding Table.

To Compute the Volume of a Polyhedron.

RULE.—*When the Linear Edge is given*, Cube the linear edge, and multiply it by the multiplier opposite to the body in column R in the preceding Table.

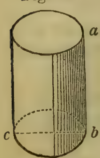
RULE.—*When the Radius of the Circumscribing Sphere is given*, Multiply the cube of the radius given by the multiplier opposite to the body in column S in the preceding Table.

RULE.—*When the Radius of the Inscribed Sphere is given*, Multiply the cube of the radius given by the multiplier opposite to the body in column T in the preceding Table.

RULE.—*When the Surface is given*, Cube the surface given, extract the square root, and multiply the root by the multiplier opposite to the body in column U in the preceding Table.

CYLINDER.

Fig. 71.



To Compute the Volume of a Cylinder—Fig. 71.

RULE.—Multiply the area of the base by the height.

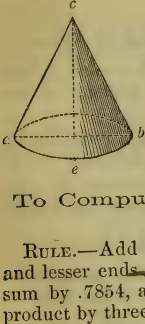
Or, $a \times h = V$.

EXAMPLE.—The diameter of a cylinder, *b c*, is 3 feet, and its length, *a b*, 7 feet; what is its volume?

Area of 3 feet = 7.068. Then $7.068 \times 7 = 49.476$ *cubic feet.*

CONE.

Fig. 72.



To Compute the Volume of a Cone—Fig. 72.

RULE.—Multiply the area of the base by the perpendicular height, and take one third of the product.

Or, $a\,h \div 3 = V$.

EXAMPLE.—The diameter, *a b*, of the base of a cone is 15 inches, and the height, *c e*, 32.5 inches; what is its volume?

Area of 15 inches = 176.7146. Then $\frac{176.715 \times 32.5}{3}$ 1914.4125 *cubic inches.*

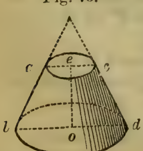
To Compute the Volume of the Frustrum of a Cone—Fig. 73.

RULE.—Add together the squares of the diameters of the greater and lesser ends, and the product of the two diameters; multiply their sum by .7854, and this product by the height; then divide this last product by three. Or, add together the squares of the circumferences of the greater and lesser ends and the product of the two circumferences; multiply their sum by .07958, and this product by the height; then divide this last product by three.

Or, $d^2 + d'^2 + \overline{d \times d'} \times .7854\,h \div 3 = V$.

Or, $c^2 + c'^2 + \overline{c \times c'} \times .07958 \div 3 = V$.

Fig. 73.



EXAMPLE.—What is the volume of the frustrum of a cone, the diameters of the greater and lesser ends, *b d*, *a c*, Fig. 73, being respectively 5 and 3 feet, and the height, *e o*, 9 feet?

$5^2 + 3^2 + \overline{5 \times 3} = 49$ = *the sum of the squares and the product of the diameters*; $49 \times .7854 = 38.4846$ = *the above sum by* .7854.

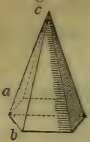
$\frac{38.4846 \times 9}{3} = 115.4538$ *cubic feet.*

PYRAMID.

NOTE.—The volume of a pyramid is equal to one third of that of a prism having equal bases and altitude.

To Compute the Volume of a Pyramid—Fig. 74.

Fig. 74.



RULE.—Multiply the area of the base by the perpendicular height, and take one third of the product.

Or, $ah \div 3 = N$.

EXAMPLE.—What are the contents of a hexagonal pyramid, Fig. 74, a side, *a b*, being 40 feet, and its height, *c e*, 60 feet?

$40^2 \times 2.5981$ (tabular multiplier, page 248) $= 4156.96 =$ *area of base.*

$$\frac{4156.96 \times 69}{3} = 83139.2 \text{ cubic feet.}$$

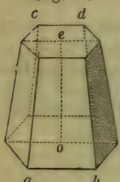
To Compute the Volume of the Frustrum of a Pyramid—Fig. 75.

RULE.—Add together the squares of the sides of the greater and lesser ends and the product of these two sides; multiply the sum by the tabular multiplier for areas in Table, page 248, and this product by the height; then divide the last product by three.

Or, $s^2 + s'^2 + \overline{s \times s'} \times$ *tab. mult.* $\times h \div 3 = V$, *s and s′ representing the lengths of the sides.*

When the areas of the ends are known, or can be obtained without reference to a tabular multiplier, use the following.

Fig. 75.



Or, $a + a' + \sqrt{a \times a'} \times h \div 3 = V$, *a and a′ representing areas of the ends.*

EXAMPLE.—What are the contents of the frustrum of a hexagonal pyramid, Fig. 75, the lengths of the sides of the greater and lesser ends, *c d*, *a b*, being respectively 3.75 and 2.5 feet, and its perpendicular height, *e o*, 7.5 feet?

$3.75^2 + 2.5^2 = 20.3125 =$ *sum of the squares of sides of greater and lesser ends;* $20.3125 + \overline{3.75 \times 2.5} = 29.6875 =$ *above sum added to the product of the two sides;* $29.6875 \times 2.5981 \times 7.5 = 578.48 =$ *the last sum × tab. mult., and again by the height*, which, $\div 3 =$ 192.83 *cubic feet.*

When the Ends of a Pyramid are not those of a Regular Polygon, or when the Areas of the Ends are given.

RULE.—Add together the areas of the two ends and the square root of their product; multiply the sum by the height, and take one third of the product.

Or, $a + a' + \sqrt{a \times a'} \times h \div 3 = V$.

EXAMPLE.—What are the contents of an irregular-sided frustrum of a pyramid, the areas of the two ends being 22 and 88 inches, and the length 20 inches.

$22 + 88 = 110 =$ *sum of areas of ends;* $22 \times 88 = 1936$, and $\sqrt{1936} = 44 =$ *square root of product of areas.* Then $\frac{110 + 44 \times 20}{3} = 1026.66$ *cubic inches.*

SPHERICAL PYRAMID.

A Spherical Pyramid is that part of a sphere included within three or more adjoining plane surfaces meeting at the centre of the sphere. The spherical polygon defined by these plane surfaces of the pyramid is called the base, and the lateral faces are sectors of circles.

To compute the Elements of Spherical Pyramids, see Docharty and Hackley's Geometry.

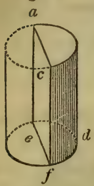
CYLINDRICAL UNGULAS.

DEFINITION.—Cylindrical Ungulas are frustra of cylinders. Conical Ungulas are frustra of cones.

To Compute the Volume of a Cylindrical Ungula—Fig. 76.

1. *When the Section is parallel to the Axis of the Cylinder.*

Fig. 76.



RULE.—Multiply the area of the base by the height of the cylinder.

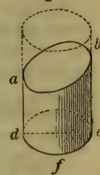
Or, $a \times h = \text{V}$.

EXAMPLE.—The area of the base, *d e f*, Fig. 76, of a cylindrical ungula is 15.5 inches, and its height, *e a*, 20; what is its volume?

$15.5 \times 20 = 310$ *cubic inches.*

2. *When the Section passes Obliquely through the opposite sides of the Cylinder*—Fig. 77.

Fig. 77.



RULE.—Multiply the area of the base of the cylinder by half the sum of the greatest and least lengths of the ungula.

Or, $a \times \overline{l + l'} \div 2 = \text{V}$.

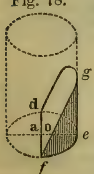
EXAMPLE.—The area of the base, *d e f*, of a cylindrical ungula, Fig. 77, is 25 inches, and the greater and less heights of it, *a d*, *b e*, are 15 and 17 inches; what is its volume?

$$25 \times \frac{15+17}{2} = 400 \text{ cubic inches.}$$

3. *When the Section passes through the Base of the Cylinder and one of its Sides, and the Versed Sine does not exceed the Sine*—Fig. 78.

RULE.—From ⅔ of the cube of the sine of half the arc of the base subtract the product of the area of the base and the cosine* of the half arc. Multiply the difference thus found by the quotient arising from the height, divided by the versed sine.

Fig. 78.



Or, $\frac{2\,s^3}{3} - \overline{a\,c} \times \frac{h}{vs} = \text{V}$, *vs representing the versed sine.*

EXAMPLE.—The sine, *a d*, of half the arc, *d e f*, of the base of an ungula, Fig. 78, is 5 inches, the diameter of the cylinder is 20, and the height, *e g*, of the ungula 10; what are the contents of it?

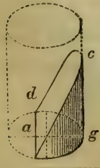
⅔ of $5^3 = 83.333 =$ *two thirds of the cube of the sine.* As the versed sine and radius of the base are equal, the cosine is 0. Hence area of base $\times$ cosine $= 0$.

$83.333 - 0 \times 10 \div 5 = 166.666$ *cubic inches.*

4. *When the Section passes through the Base of the Cylinder, and the Versed Sine exceeds the Sine*—Fig. 79.

RULE.—To ⅔ of the cube of the sine of half the arc of the base add the product of the area of the base and the cosine. Multiply the sum thus found by the quotient arising from the height, divided by the versed sine.

Fig. 79.



Or, $\frac{2\,s^3}{3} + \overline{a\,c} \times \frac{h}{vs} = \text{V}$.

EXAMPLE.—The sine *a d* of half the arc of an ungula, Fig. 79, is 12 inches, the versed sine *a g* is 16, the height *g c* 10, and the diameter of the cylinder 25 inches; what is its volume?

⅔ of $12^3 = 1152 =$ *two thirds of cube of sine of half the arc of the base.* Area of base $= 331.78$; $1152 + \overline{331.78 \times 16 - 12.5} = 2313.23$ *= sum of ⅔ of the cube of the sine of half the arc of the base, and product of area of base and cosine.*

Then $2313.23 \times 20 \div 16 = 2891.5375$ *cubic inches.*

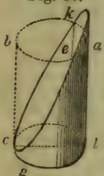
* When the cosine is 0, the product is 0.

5. *When the Section passes Obliquely through both Ends of the Cylinder* —Fig. 80.

Rule.—Conceive the section to be continued till it meets the side of the cylinder produced; then, as the difference of the versed sines of the arcs of the two ends of the ungula is to the versed sine of the arc of the less end, so is the height of the cylinder to the part of the side produced.

Ascertain the volume of each of the ungulas by Rules 3 and 4, and take their difference.

Fig. 80.



Or, $\frac{v'\,h}{v - v'} = h'$, *v and v' representing the versed sines of the arcs of the two ends, h the height of the cylinder, and h' the height of the part produced.*

Example.—The versed sines, *a e*, *d c*, and sines, *e k* and *c*, of the arcs of the two ends of an ungula, Fig. 80, are assumed to be respectively 8.5 and 25, and 11.5 and 0 inches, the length of the ungula, *b c*, within the cylinder, cut from one having 25 inches diameter, *d c*, is 20 inches; what is the height of the ungula produced beyond the cylinder, and what is the volume of it?

$25 \backsim 8.5 : 8.5 :: 20 : 10.303 =$ *height of ungula produced beyond the cylinder.*

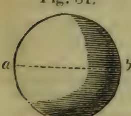
Greater ungula, the sine *c* being 0, the versed sine = *the diameter.* Base of ungula being a circle of 25 inches diameter, area = 490.875. The versed sine and diameter of the base being equal (25), the sine = 0. $490.875 \times 25 \backsim \frac{25}{2} = 6135.9375 =$ *product of area of base and cosine, or excess of versed sine over the sine of the base.* $30.303 \div 25 = 1.21212 =$ *quotient of height ÷ versed sine.*

Then $6135.9375 \times 1.21212 = 7437.4926$ *cubic inches;* and by Rules 3 and 4, volumes of less and greater ungulas = 515.444, and 6922.0486 = 7437.4926 *cubic inches.*

SPHERE.

Definition.—A solid, the surface of which is at a uniform distance from the centre.

Fig. 81.



To Compute the Volume of a Sphere—Fig. 81.

Rule.—Multiply the cube of the diameter by .5236.

Or, $d^3 \times .5236 = V$, *d representing the diameter.*

Example.—What is the volume of a sphere, Fig. 81, its diameter, *a b*, being 10 inches?

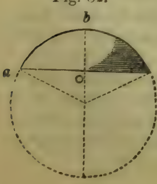
$10^3 = 1000$, and $1000 \times .5236 = 523.6$ *cubic inches.*

SEGMENT OF A SPHERE.

Definition.—A section of a sphere.

To Compute the Volume of a Segment of a Sphere —Fig. 82.

Fig. 82.



Rule 1.—To three times the square of the radius of its base add the square of its height; multiply this sum by the height, and the product by .5236.

Or, $\overline{3\,r^2 + h^2}\,h \times .5236 = V$.

2.—From three times the diameter of the sphere subtract twice the height of the segment; multiply this remainder by the square of the height, and the product by .5236.

Or, $\overline{3\,d - 2\,h}\,h^2 \times .5236 = V$.

EXAMPLE.—The segment of a sphere, Fig. 82, has a radius, *a o*, of 7 inches for its base, and a height, *b o*, of 4 inches; what is its volume?

$7^2 \times 3 + 4^2 = 163$ = *the sum of three times the square of the radius and the square of the height;* $163 \times 4 \times .5236 = 331.3872$ *cubic inches.*

SPHERICAL ZONE (OR FRUSTRUM OF A SPHERE).

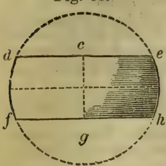
DEFINITION.—The part of a sphere included between two parallel chords.

To Compute the Volume of a Spherical Zone—Fig. 83.

DEFINITION.—The part of a sphere included between two parallel planes.

RULE.—To the sum of the squares of the radii of the two ends add ⅓ of the square of the height of the zone; multiply this sum by the height, and again by 1.5708.

Fig. 83.



Or, $r^2 + r'^2 + \overline{h^2 \div 3} \times h \times 1.5708 = V.$

EXAMPLE.—What are the contents of a spherical zone, Fig. 83, the greater and less diameters, *f h* and *d e*, being 20 and 15 inches, and the distance between them, or height of the zone, *c g*, being 10 inches.

$10^2 + 7.5^2 = 156.25$ = *sum of the squares of the radii of the two ends;* $156.25 + \overline{10^2 \div 3} = 189.58$ = *the above sum added to one third of the square of the height.*

Then $189.58 \times 10 \times 1.5708 = 2977.9226$ *cubic inches.*

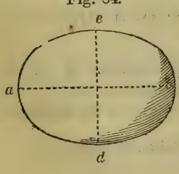
SPHEROIDS (ELLIPSOIDS).

DEFINITION.—Solids generated by the revolution of a semi-ellipse about one of its diameters. When the revolution is about the transverse diameter they are Prolate, and when about the conjugate they are Oblate.

To Compute the Volume of a Spheroid—Fig. 84.

RULE.—Multiply the square of the revolving axis by the fixed axis, and this product by .5236.

Fig. 84.



Or, $a^2 a' \times .5236 = V$, *a and a′ representing the revolving and fixed axes.*

Or, $4 \div 3 \times 3.1416\, r^2 r' = V$, *r and r′ repres'g the semi-axes.*

EXAMPLE.—In a prolate spheroid, Fig. 84, the fixed axis, *a b*, is 14 inches, and the revolving axis, *c d*, 10; what is its volume?

$10^2 \times 14 = 1400$ = *product of square of revolving axis and fixed axis.* Then $1400 \times .5236 = 733.04$ *cubic inches.*

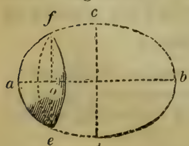
NOTE.—The volume of a spheroid is equal to ⅔ of a cylinder that will circumscribe it.

SEGMENTS OF SPHEROIDS.

To Compute the Volume of the Segment of a Spheroid.

When the Base, e f, is Circular, or parallel to the revolving Axis, as c d, Fig. 85, *or as e f to the Axis a b,* Fig. 86.

Fig. 85.



RULE.—Multiply the fixed axis by 3, the height of the segment by 2, and subtract the one product from the other; multiply the remainder by the square of the height of the segment, and the product by .5236. Then, as the square of the fixed axis is to the square of the revolving axis, so is the last product to the volume of the segment.

$$\text{Or, } \frac{\overline{3a - 2h} \times h^2 \times .5236 \times a'^2}{a^2} = V.$$

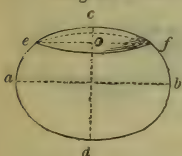
EXAMPLE.—In a prolate spheroid, Fig. 85, the fixed or transverse axis, *a b*, is 100 inches, the revolving or conjugate, *c d*, 60, and the height of the segment, *a o*, 10 inches; what is its volume?

$100 \times 3 - \overline{10 \times 2} = 280$ = *twice the height of the segment subtracted from three times the fixed axis*; $280 \times 10^2 \times .5236 = 14660.8$ *inches* = *product of above remainder, the square of the height, and* .5236. Then $100^2 : 60^2 :: 14660.8 : 5277.888$ *cubic inches.*

When the Base, e f, is Elliptical, or perpendicular to the revolving Axis a b, Fig. 85, *or e f,* Fig. 86, *to the Axis c d.*

RULE.—Multiply the revolving axis by 3, and the height of the segment by 2, and subtract the one from the other; multiply the remainder by the square of the height of the segment, and the product by .5236. Then, as the revolving axis is to the fixed axis, so is the last product to the volume of the segment.

Fig. 86.



Or, $\frac{\overline{3\,a' - 2\,h} \times h^2 \times .5236 \times a}{a'} = V.$

EXAMPLE.—The diameters of an oblate spheroid, Fig. 86, are 100 and 60 inches, and the height of a segment thereof is 12 inches; what is its volume?

$100 \times 3 - \overline{12 \times 2} = 276$ = *twice the height of the segment subtracted from three times the revolving axis*; $276 \times 12^2 \times .5236 = 20809.9584$ = *product of above remainder, the square of the height, and* 5236.
Then $100 : 60 :: 20809.9584 : 12485.975$ *cubic inches.*

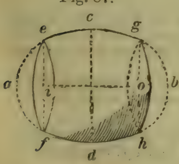
FRUSTRA OF SPHEROIDS.

To Compute the Volume of the Middle Frustrum of a Spheroid.

When the Ends, e f and g h, are Circular, or parallel to the revolving Axis, as c d, Fig. 87, *or a b,* Fig. 88.

RULE.—To twice the square of the revolving axis add the square of the diameter of either end; multiply this sum by the length of the frustrum, and the product by .2618.

Fig. 87.



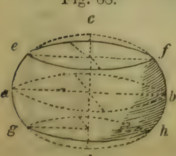
Or, $\overline{2\,a'^2 + d^2} \times l\,.2618 = V.$

EXAMPLE.—The middle frustrum of a prolate spheroid, *i o*, Fig. 87, is 36 inches in length, the diameter of it being, in the middle, *c d*, 50 inches, and at its ends, *e f and g h*, 40; what is its volume?

$50^2 \times 2 + 40^2 = 6600$ = *sum of twice the square of the middle diameter added to the square of the diameter of the ends.* Then $6600 \times 36 \times .2618 = 62203.68$ *cubic inches.*

When the Ends, e f and g h, are Elliptical, or perpendicular to the revolving Axis a b, Fig. 87, *or e f and g h to the Axis c d,* Fig. 88.

Fig. 88.



RULE.—To twice the product of the transverse and conjugate diameters of the middle section add the product of the transverse and conjugate of either end; multiply this sum by the length of the frustrum, and the product by .2618.

Or, $\overline{d\,d' \times 2 + \overline{d\,d'}} \times l \times .2618 = V.$

EXAMPLE.—In the middle frustrum of a prolate spheroid, Fig. 88, the diameters of its middle section are 50 and 30 inches, its ends 40 and 24 inches, and its length, *o i*, 18 inches; what is its volume?

$50\times30\times2=3000=$ *twice the product of the transverse and conjugate diameters;*
$3000+\overline{40\times24}=3960=$ *sum of the above product and the product of the transverse and conjugate diameters of the ends.*
Then $3960\times18\times.2618=18661.104$ *cubic inches.*

CYLINDRICAL RING.

DEFINITION.—A ring formed by the curvature of a cylinder.

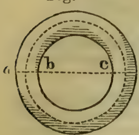
To Compute the Volume of a Cylindrical Ring—Fig. 89.

RULE.—To the diameter of the body of the ring add the inner diameter of the ring; multiply the sum by the square of the diameter of the body, and the product by 2.4674.

Or, $\overline{d+d'}\times d^2\ 2.4674=V$, *d and d' representing the diameter of the body and inner diameter.*

Or, $a\times l=V$, *a representing area of section of body, and l the length of the axis of the body.*

Fig. 89.



EXAMPLE.—What is the volume of an anchor ring, Fig. 89, the diameter of the metal, *a b*, being 3 inches, and the inner diameter of the ring, *b c*, 8 inches?

$\overline{3+8}\times3^2=99=$ *product of sum of diameters and the square of diameter of body of ring.*
Then $99\times2.4674=244.2726$ *cubic inches.*

LINKS.

DEFINITION.—Elongated or Elliptical rings.

ELONGATED OR ELLIPTICAL LINKS.

To Compute the Volume of an Elongated or Elliptical Link—Figs. 90 and 91.

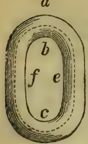
RULE.—Multiply the area of a section of the body of the link by its length, or the circumference of its axis.

Or, $a\times l=V$.

NOTE.—By Rule, page 263, the circumference or length of the axis of an Elongated link = the sum of 3.1416 times the sum of the less diameter added to the thickness of the ring, and the product of twice the remainder of the less diameter subtracted from the greater.

Also, the circumference or length of the axis of an Elliptical ring = the square root of half the sum of the diameters added to the thickness of the ring or the axes squared $\times3.1416$.

Fig. 90.

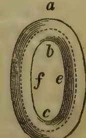


EXAMPLE.—The elongated link of a chain, Fig. 90, is 1 inch in diameter of body, *a b*, and its inner diameters, *e c* and *e f*, are 10 and 2.5 inches; what is its volume?

Area of 1 inch $=.7854$; $\overline{2.5+1}\times3.1416=10.9956=3.1416$ *times the sum of the less diameter and thickness of the ring = length of axis of ends;* $\overline{10-2.5}\times2=15=$ *twice the remainder, of the less diameter subtracted from the greater = length of sides of body.*

Then $10.9956+15=25.9956=$ *length of axis of link.*
Hence $.7854\times25.9956=20.417$ *cubic inches.*

Fig. 91.



Ex. 2.—The elliptical link of a chain, Fig. 91, is of the same dimensions as the preceding; what is its volume?

$\overline{2.5+1}^2+\overline{10+1}^2=133.25=$ *diameter of axes squared;* $\sqrt{\frac{133.25}{2}}\times3.1416$ $=25.643=$ *square root of half sum of diameters squared* $\times3.1416$ *= circumference of axis of ring.* Area of 1 inch $=.7854$.
Then $25.643\times.7854=20.14$ *cubic inches.*

SPHERICAL SECTOR.

DEFINITION.—A figure generated by the revolution of a sector of a circle about a straight line through the vertex of the sector as an axis.

NOTE.—The arc of the sector generates the surface of a zone, termed the base of the sector of a sphere, and the radii generate the surfaces of two cones, having a vertex in common with the sector at the centre of the sphere.

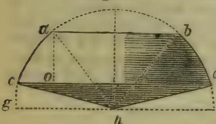
To Compute the Volume of a Spherical Sector—Fig. 92.

RULE.—Multiply the surface of the zone, which is the base of the sector, by ⅓ of the radius of the sphere.

Or, $a \times r \div 3 = V$, *a representing the area of the base.*

EXAMPLE.—What is the volume of a spherical sector, Fig. 92, generated by the sector, *c a h*, the height of the zone, *a b c d*, being, *a o*, 12 inches, and the radius, *g h*, of the sphere 15 inches?

Fig. 92.



$12 \times 94.248 = 1130.976$ = *height of zone × circumference of sphere = surface of zone* (see page 256).

$1130.976 \times \overline{30} \div 6$ (= ⅓ *of radius*) $= 5654.88$ *cub. ins.*

NOTE.—The surface of a spherical sector = the sum of the surface of the zone and the surfaces of the two cones.

SPINDLES.

DEFINITION.—Figures generated by the revolution of a plane area bounded by a curve, when the curve is revolved about a chord perpendicular to its axis or about its double ordinate, and they are designated by the name of the arc from which they are generated, as Circular, Elliptic, Parabolic, etc.

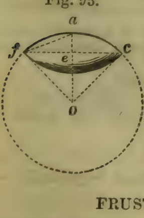
CIRCULAR SPINDLE.

To Compute the Volume of a Circular Spindle—Fig. 93.

RULE.—Multiply the central distance by half the area of the revolving segment; subtract the product from ⅓ of the cube of half the length, and multiply the remainder by 12.5664.

Or, $\frac{(l \div 2)^3}{3} - \left(c \times \frac{a}{2}\right) \times 12.5664 = V$, *a repres'ing the area of the revolving segment.*

Fig. 93.



EXAMPLE.—What is the volume of a circular spindle, Fig. 93, when the central distance, *o e*, is 7.071067 inches, the length, *f c*, 14.14213, and the radius, *o c*, 10 inches?

$7.071067 \times 14.27 = 100.9041$ = *central distance × half area of revolving segment;* $\frac{7.07167^3}{3} - 100.9041 = 16.947$ = *remainder of above product and ⅓ of cube of half the length.* Then $16.497 \times 12.5664 = 212.9628$ *cubic inches.*

NOTE.—The area of the revolving segment, *f e*, being = the side of the square that can be inscribed in a circle of 20, is $\overline{20^2 \times .7854 - 14.14213^2} \div 4 = 28.54$.

FRUSTRUM OR ZONE OF A CIRCULAR SPINDLE.*

To Compute the Volume of a Frustrum or Zone of a Circular Spindle—Fig. 94.

RULE.—From the square of half the length of the whole spindle take ⅓ of the square of half the length of the frustrum, and multiply

* The middle frustrum of a Circular Spindle is one of the various forms of casks.

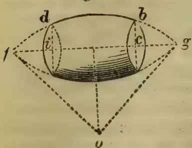
the remainder by the said half length of the frustrum; mnltiply the central distance by the revolving area which generates the frustrum; subtract this product from the former, and multiply the remainder by 6.2832.

Or, $\overline{l \div 2}^2 - \frac{\overline{l' \div 2}^2}{2} \times \frac{l'}{2} - (c \times a) \times 6.2832 = V$, *l and l′ representing the lengths of the spindle and of the frustrum, and a area of the revolving section of frustrum.*

NOTE.—The revolving area of the frustrum can be obtained by dividing its plane into a segment of a circle and a parallelogram.

EXAMPLE.—The length of the middle frustrum of a circular spindle, *i c*, Fig. 94, is 6 inches; the length of the spindle, *f g*, is 8 inches; the central distance, *o e*, is 3 inches; and the area of the revolving or generating segment is 10 inches; what is the volume of the frustrum?

Fig. 94.



$(8 \div 2)^2 - \frac{(6 \div 2)^2}{3} = 13$ and $13 \times 3 = 39 =$ *product of half the length of the frustrum, and the remainder of ⅓ the square of half the length of the frustrum subtracted from the square of half the length of the spindle;* $39 - \overline{3 \times 10} = 9 =$ *product of the central distance and the area of the segment subtracted from preceding product.*

Then $9 \times 6.2832 = 56.5488$ *cubic inches.*

SEGMENT OF A CIRCULAR SPINDLE.

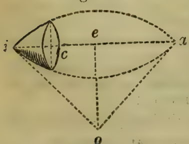
To Compute the Volume of a Segment of a Circular Spindle—Fig. 95.

RULE.—Subtract the length of the segment from the half length of the spindle; double the remainder, and ascertain the volume of a middle frustrum of this length. Subtract the result from the volume of the whole spindle, and halve the remainder.*

Or, $C - c \div 2 = V$, *C and c representing the volume of spindle and middle frustrum.*

EXAMPLE.—The length of a circular spindle, *i a*, Fig. 95, is 14.14213; the central distance, *o e*, is 7.07107; the radius of the arc, *o a*, is 10; and the length of the segment, *i c*, is 3.53553 inches; what is its volume?

Fig. 95.



$\frac{14.14213}{2} - 3.53553 \times 2 = 7.07107 =$ *double the remainder, of the length of the segment subtracted from half the length of the spindle = length of the middle frustrum.*

NOTE.—The area of the revolving or generating segment of the whole spindle is 28.54 inches, and that of the middle frustrum is 19.25.

The volume of the whole spindle is .... 212.9628 *cubic inches.*
" " middle frustrum is .... 162.8982 " "

Hence .... $50.0646 \div 2 = 25.0323$ *cubic inches.*

CYCLOIDAL SPINDLE.†

To Compute the Volume of a Cycloidal Spindle—Fig. 96.

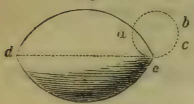
RULE.—Multiply the product of the square of twice the diameter of the generating circle and 3.927 by its circumference, and divide this product by 8.

* This rule is applicable to the segment of any Spindle or any Conoid, the volume of the figure and frustrum being first obtained.

† The volume of a Cycloidal Spindle is equal to ⅝ of its circumscribing cylinder.

Or, $\frac{\overline{2d}^3 \times 3.927 \times d \times 3.1416}{8} = V$, *d representing the diameter of the circle, or half width of the spindle.*

Fig. 96.



EXAMPLE.—The diameter of the generating circle, *a b c*, of a cycloid, Fig. 96, is 10 inches; what is the volume of the spindle, *d e?*

$\overline{10\times2}^2 \times 3.927 = 1570.8$ = *product of twice the diameter squared and* 3.927.

Then $1570.8 \times 10 \times 3.1416 \div 8 = 6168.5316$ *cubic inches.*

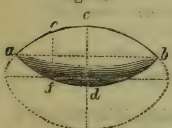
ELLIPTIC SPINDLE.

To Compute the Contents of an Elliptic Spindle—Fig. 97.

RULE.—To the square of its diameter add the square of twice the diameter at ¼ of its length; multiply the sum by the length, and the product by .1309.†

Or, $d^2 + \overline{2d'}^2 \times l.1309 = V$, *d and d′ representing the diameter as above.*

Fig. 97.



EXAMPLE.—The length of an elliptic spindle, *a b*, Fig. 97, is 75 inches, its diameter, *c d*, 35, and the diameter, *e f*, at ¼ of its length, 25; what is its volume?

$35^2 + \overline{25\times2}^2 = 3725$ = *sum of squares of diameter of spindle and of twice its diameter at ¼ of its length;* $3725 \times 75 = 279375$ = *above sum × length of the spindle.*

Then $279375 \times .1309 = 36570.1875$ *cubic inches.*

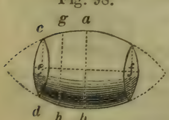
NOTE.—For all such solid bodies this rule is exact when the body is formed by a conic section, or a part of it, revolving about the axis of the section, and will always be very near when the figure revolves about another line.

To Compute the Volume of the Middle Frustrum or Zone of an Elliptic Spindle—Fig. 98.

RULE.—Add together the squares of the greatest and least diameters, and the square of double the diameter in the middle between the two; multiply the sum by the length, and the product by .1309.*

Or, $d^2 + d'^2 + \overline{2d''}^2 \times l.1309 = V$, *d, d′, and d″ representing the different diameters.*

Fig. 98.



EXAMPLE.—The greatest and least diameters, *a b* and *c d*, of the frustrum of an elliptic spindle, Fig. 98, are 68 and 50 inches, its middle diameter, *g h*, 60, and its length, *e f*, 75; what is its volume?

$68^2 + 50^2 + \overline{60\times2}^2 = 21524$ = *sum of squares of greatest and least diameters and of double the middle diameter.*

Then $21524 \times 75 \times .1309 = 211311.87$ *cubic inches.*

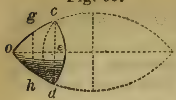
To Compute the Volume of a Segment of an Elliptic Spindle—Fig. 99.

RULE.—Add together the square of the diameter of the base of the segment and the square of double the diameter in the middle between the base and vertex; multiply the sum by the length of the segment, and the product by .1309.*

Or, $d^2 + \overline{2d''}^2 \times l \times .1309 = V$, *d and d″ representing the diameters.*

* See Note above.

Fig. 99.



EXAMPLE.—The diameters, *c d* and *g h*, of the segment of an elliptic spindle, Fig 99, are 20 and 12 inches, and the length, *o e*, is 16 inches; what is its volume?

$20^2 + \overline{12\times2}^2 = 976$ = *sum of squares of diameter at base and in the middle.*

Then $976\times116\times.1309 = 2044.134$ *cubic inches.*

PARABOLIC SPINDLE.

To Compute the Volume of a Parabolic Spindle—Fig. 100.

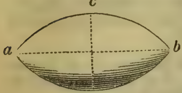
RULE 1.—Multiply the square of the diameter by the length, and the product by .41888.*

Or, $d^2\,l\times.41888 = V.$

RULE 2.—To the square of its diameter add the square of twice the diameter at ¼ of its length; multiply the sum by the length, and the product by .1309.†

Or, $d^2 + \overline{2\,d'}^2 \times l \times .1309 = V.$

Fig. 100.



EXAMPLE.—The diameter of a parabolic spindle, *a b*, Fig. 100, is 40 inches, and its length, *c d*, 10; what is its volume?

$40^2\times10 = 16000$ = *square of diameter* × *the length.*

Then $16000\times.41888 = 6702.08$ *cubic inches.*

Again, If the middle diameter at ¼ of its length is 30, then, by Rule 2, $40^2 + \overline{30\times2}^2 \times 40 \times .1309 = 6806.8$ *cubic ins.*

To Compute the Volume of the Middle Frustrum of a Parabolic Spindle—Fig. 101.

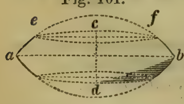
RULE 1.—Add together 8 times the square of the greatest diameter, 3 times the square of the least diameter, and 4 times the product of these two diameters; multiply the sum by the length, and the product by .05236.

Or, $d^2\,8 + \overline{d'^2 3} + d\,\overline{d'\times4}\,l.\times05236 = V.$

RULE 2.—Add together the squares of the greatest and least diameters and the square of double the diameter in the middle between the two; multiply the sum by the length, and the product by .1309.

Or, $d^2 + d'^2 + \overline{2\,d''}^2 \times l \times .1309 = V$, d'' *representing the diameter between the two.*

Fig. 101.



EXAMPLE.—The middle frustrum of a parabolic spindle, Fig. 101, has diameters, *a b* and *e f*, of 40 and 30 inch s, and its length, *c d*, is 10 inches; what is its volume?

$40^2\times8 + 30^2\times3 + \overline{40\times30}\times4 = 20300$ = *the sum of* 8 *times the square of the greatest diameter*, 3 *times the square of the least diameter, and* 4 *times the product of these.*

Then $20300\times10\times.05236 = 10629.08$ *cubic inches.*

To Compute the Volume of a Segment of a Parabolic Spindle—Fig. 102.

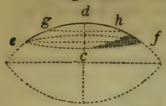
RULE.—Add together the square of the diameter of the base of the segment and the square of double the diameter in the middle between the base and vertex; multiply the sum by the length of the segment, and the product by .1309.

Or, $d^2 + d''^2 \times l. \times .1309 = V.$

* 8-15 of .7854.

† See Note, page 283.

Fig. 102.



EXAMPLE.—The segment of a parabolic spindle, Fig. 102, has diameters, *e f* and *g h*, of 15 and 8.75 inches, and the length, *c d*, is 2.5 inches; what is its volume?

$15^2+\overline{8.75\times2}^2=531.25=$ *sum of square of base and of double the diameter in the middle of the segment.*

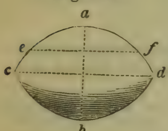
Then $531.25\times2.5\times.1309=173.852$ *cubic inches.*

HYPERBOLIC SPINDLE.

To Compute the Volume of a Hyperbolic Spindle—Fig. 103.

RULE.—To the square of the diameter add the square of double the diameter at ¼ of its length; multiply the sum by the length, and the product by .1309.*

Fig. 103.



Or, $d^2+\overline{2d'}^2\times l\times.1309=V.$

EXAMPLE.—The length, *a b*, Fig. 103, of a hyperbolic spindle is 100 inches, and its diameters, *c d* and *e f*, are 150 and 110 inches; what is its volume?

$150^2+\overline{110\times2}^2\times109=7090000=$ *product of the sum of the squares of the greatest diameter and of twice the diameter at ¼ of the length of the spindle and the length.*

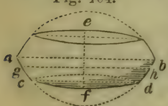
Then $7090000\times.1309=928081$ *cubic inches.*

To Compute the Volume of the Middle Frustrum of a Hyperbolic Spindle—Fig. 104.

RULE.—Add together the squares of the greatest and least diameters and the square of double the diameter in the middle between the two; multiply this sum by the length, and the product by .1309.†

Or, $d^2+d'^2+(2\,d'')^2\times l\times.1309=V.$

Fig. 104.



EXAMPLE.—The diameters, *a b* and *c d*, of the middle frustrum of a hyperbolic spindle, Fig. 104, are 150 and 110 inches; the diameter, *g h*, 140 inches; and the length, *e f*, 50; what is its volume?

$150^2+110^2+\overline{140\times2}^2=113000=$ *sum of squares of greatest and least diameters and of double the middle diameter.*

Then $113000\times50\times.1309=739585$ *cubic inches.*

To Compute the Volume of a Segment of a Hyperbolic Spindle—Fig. 105.

RULE.—Add together the square of the diameter of the base of the segment and the square of double the diameter in the middle between the base and vertex; multiply the sum by the length of the segment, and the product by .1309.

Or, $d^2+d''^2\,l\times.1309=V.$

ELLIPSOID, PARABOLOID, AND HYPERBOLOID OF REVOLUTION‡ (CONOIDS).

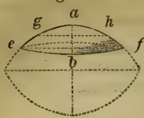
DEFINITION.—Figures like to a cone, described by the revolution of a conic section around and at a right angle to the plane of their fixed axes.

* See Note, page 283.

† Ibid.

‡ These figures have been known as Conoids. For the definition of a Conoid, see *Haswell's Mensuration*, page 233.

Fig. 105.



EXAMPLE.—The segment of a hyperbolic spindle, Fig. 105, has diameters, *e f* and *g h*, of 110 and 65 inches, and its length, *a b*, 25; what is its volume?

$110^2 + \overline{65 \times 2}^2 = 29000 =$ *sum of squares of diameter of base and of double the middle diameter.*

Then $29000 \times 25 \times .1309 = 94902.5$ *cubic inches.*

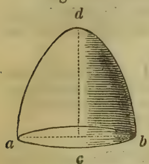
ELLIPSOID OF REVOLUTION (SPHEROID).

DEFINITION.—An ellipsoid of revolution is a semi-spheroid. (See page 257.)

PARABOLOID OF REVOLUTION.*

To Compute the Volume of a Paraboloid of Revolution—Fig. 106.

Fig. 106.



RULE.—Multiply the area of the base by half the altitude. Or, $a \times h \div 2 = V$.

NOTE.—This rule will hold for any segment of the paraboloid, whether the base be perpendicular or oblique to the axis of the solid.

EXAMPLE.—The diameter, *e b*, of the base of a paraboloid of revolution, Fig. 106, is 20 inches, and its height, *d c*, 20 inches; what is its volume?

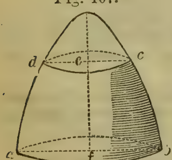
Area of 20 inches diameter of base $= 314.16$.

Then $314.16 \times 20 \div 2 = 3141.6$ *cubic inches.*

FRUSTRUM OF A PARABOLOID OF REVOLUTION.

To Compute the Volume of a Frustrum of a Paraboloid of Revolution—Fig. 107.

Fig. 107.



RULE.—Multiply the sum of the squares of the diameters by the height of the frustrum, and this product by .3927.

Or, $\overline{d^2 + d'^2} \times h \times .3927 = V$.

EXAMPLE.—The diameters, *a b* and *d c*, of the base and vertex of the frustrum of a paraboloid of revolution, Fig. 107, are 90 and 11.5 inches, and its height, *e f*, 12.6; what is its volume?

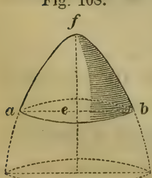
$20^2 + 11.5^2 = 532.25 =$ *sum of squares of the diameters.*

Then $532.25 \times 12.6 \times .3927 = 2633.5837$ *cubic inches.*

SEGMENT OF A PARABOLOID OF REVOLUTION.

To Compute the Volume of the Segment of a Paraboloid of Revolution—Fig. 108.

Fig. 108.



RULE.—Multiply the area of the base by half the altitude.

Or, $a \times h \div 2 = V$.

NOTE.—This rule will hold for any segment of the paraboloid, whether the base be perpendicular or oblique to the axis of the solid.

EXAMPLE.—The diameter, *a b*, of the base of a segment of a paraboloid of revolution, Fig. 108, is 11.5 inches, and its height, *e f*, is 7.4; what is its volume?

Area of 11.5 inches diameter of base $= 103.869$.

Then $103.869 \times 7.4 \div 2 = 384.315$ *cubic inches.*

---

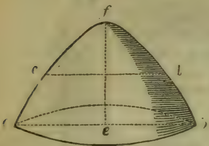
* The volume of a Paraboloid of Revolution is $= \frac{1}{2}$ of its circumference.

HYPERBOLOID OF REVOLUTION.

To Compute the Volume of a Hyperboloid of Revolution—Fig. 109.

RULE.—To the square of the radius of the base add the square of the middle diameter; multiply this sum by the height, and the product by .5236.

Fig. 109.



Or, $r^2 + d^2 \times h \times .5236 = V$, *d representing middle diameter.*

EXAMPLE.—The base, *a b*, of a hyperboloid of revolution, Fig. 109, is 80 inches; the middle diameter, *c d*, 66; and the height, *e f*, 60; what is its volume?

$\overline{80 \div 2}^2 + 66^2 = 5956 =$ *sum of square of radius of base and middle diameter.*

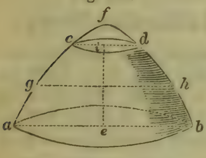
Then $5956 \times 60 \times .5236 = 87113.7$ *cubic inches.*

FRUSTRUM OF A HYPERBOLOID OF REVOLUTION.

To Compute the Volume of the Frustrum of a Hyperboloid of Revolution—Fig. 110.

RULE.—Add together the squares of the greatest and least semi-diameters and the square of the diameter in the middle of the two; multiply this sum by the height, and the product by .5236.

Fig. 110.



Or, $\left(\frac{d}{2}\right)^2 + \left(\frac{d'}{2}\right)^2 + d''^2 \times h \times .5236 = V$, *d*, *d'*, *and d'' representing the several diameters.*

EXAMPLE.—The frustrum of a hyperboloid of revolution, Fig. 110, is in height, *e i*, 50 inches; the diameters of the greater and lesser ends, *a b* and *c d*, are 110 and 42; and that of the middle diameter, *g h*, is 80; what is the volume?

$110 \div 2 = 55$, and $42 \div 9 = 21$. Hence $110^2 + 21^2 + 80^2 = 9866 =$ *sum of the squares of the semi-diameters of the ends and of the middle diameter.*

Then $9866 \times 50 \times .5236 = 258291.88$ *cubic inches.*

SEGMENT OF A HYPERBOLOID OF REVOLUTION.

To Compute the Volume of the Segment of a Hyperboloid of Revolution, as Fig. 109.

RULE.—To the square of the radius of the base add the square of the middle diameter; multiply this sum by the height, and the product by .5236.

Or, $r^2 + d^2 \times h \times .5236 = V$, *r representing radius of base.*

EXAMPLE.—The radius, *a e*, of the base of a segment of a hyperboloid of revolution, as Fig. 109, is 21 inches; its middle diameter, *c d*, is 30; and its height, *e f*, 15; what is its volume?

$21^2 + 30^2 \times 15 = 20115 =$ *the product of the sum of the squares of the radius of the base and the middle diameter multiplied by the height.*

Then $20115 \times .5236 = 10532.214$ *cubic inches.*

ANY FIGURE OF REVOLUTION.

To Compute the Volume of any Figure of Revolution—Fig. 111.

RULE.—Multiply the area of the generating surface by the circumference described by its centre of gravity.

Or, $a\,2\,r\,p = V$, *r representing radius of centre of gravity.*

Fig. 111.

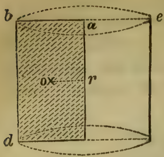


Illustration.—If the generating surface, $a\ b\ c\ d$, of the cylinder, $b\ e\ d\ f$, Fig. 111, is 5 inches in width and 10 in height, then will $a\ b = 5$ and $b\ d = 10$, and the centre of gravity will be in o, the radius of which is $r\ o = 5 \div 2 = 2.5$.

Hence $10 \times 5 = 50$ = *area of generating surface.*

Then $50 \times \overline{2.5 \times 2} \times 3.1416 = 785.4$ = *area of generating surface × circumference of its centre of gravity = the volume of the cylinder.*

Proof.—Volume of a cylinder 10 inches in diameter and 10 inches in height. $10^2 \times .7854 = 78.54$, and $78.54 \times 10 = 785.4$.

Fig. 112.

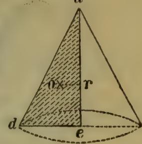


Illustration 2.—If the generating surface of a cone, Fig. 112, is $a\ e = 10$, $d\ e = 5$, then will $a\ d = 11.18$, and the area of the triangle $= 10 \times 5 \div 2 = 25$, the centre of gravity of which is in o, and $o\ r$, by Rule, page 339 = 1.666.

Hence $25 \times \overline{1.666 \times 2} \times 3.1416 = 261.8$ = *area of generating surface × circumference of its centre of gravity = the volume of the cone.*

Fig. 113.

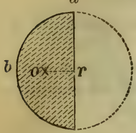


Illustration 3.—If the generating surface of a sphere, Fig. 113, is $a\ b\ c$, and $a\ c = 10$, $a\ b\ c$ will be $\left(\frac{10^2 \times .7854}{2}\right) = 39.27$, the centre of gravity of which is in o, and by Rule, page 339, $o\ r = 2.122$.

Hence $39.27 \times \overline{2.122 \times 2} \times 3.1416 = 523.6$ = *area of generating surface × circumference of its centre of gravity = the volume of the sphere.*

To Compute the Volume of an Irregular Body.

Rule.—Weigh it both in and out of fresh water, and note the difference in pounds; then, as 62.5* is to this difference, so is 1728† to the number of cubic inches in the body. Or, divide the difference in pounds by 62.5, and the quotient will give the volume in cubic feet.

Note.—If salt-water is to be used, the ascertained weight of a cubic foot of it, or 64, is to be used for 62.5.

Example.—An irregular-shaped body weighs 15 pounds in water, and 30 out; what is its volume in cubic inches?

$30 - 15 = 15$ = *difference of weights in and out of water.*
$62.5 : 15 :: 1728 : 414.72$ = *volume in cubic inches.*
Or, $15 \div 62.5 = .24$, and $.24 \times 1728 = 414.72$ = *volume in cubic inches.*

* The weight of a cubic foot of fresh water.
† The number of inches in a cubic foot.

CONIC SECTIONS.

A *Cone* is a figure described by the revolution of a right-angled triangle about one of its legs, or it is a solid having a circle for its base, and terminated in a vertex.

Conic Sections are the figures made by a plane cutting a cone.

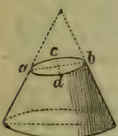
The *Axis* is the line about which the triangle revolves.

The *Base* is the circle which is described by the revolving base of the triangle.

NOTES.—If a cone is cut by a plane through the vertex and base, the section will be a triangle.

If a cone is cut by a plane parallel to its base, the section will be a circle.

Fig. 1.

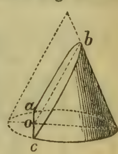


An *Ellipse* is a figure generated by an oblique plane cutting a cone above its base.

The *transverse axis* or diameter is the longest right line that can be drawn in it, as *a b*, Fig. 1.

The *conjugate axis* or diameter is a line drawn through the centre of the ellipse perpendicular to the transverse axis, as *c d*.

Fig. 2.

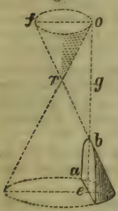


A *Parabola* is a figure generated by a plane cutting a cone parallel to its side, as *a b c*, Fig. 2.

The *axis* is a right line drawn from the vertex to the middle of the base, as *b o*.

NOTE.—A parabola has no conjugate diameter.

Fig. 3.



A *Hyperbola* is a figure generated by a plane cutting a cone at any angle with the base greater than that of the side of the cone, as *a b c*, Fig. 3.

The *transverse axis* or diameter, *o b*, is that part of the axis, *c b*, which, if continued, as at *o*, would join an opposite cone, *o f r*.

The *conjugate axis* or diameter is a right line drawn through the centre, *g*, of the transverse axis, and perpendicular to it.

The straight line through the *foci* is the indefinite transverse axis; that part of it between the vertices of the curves, as *o b*, is the definite transverse axis. Its middle point, *g*, is the centre of the curve.

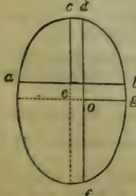
The *eccentricity* of a hyperbola is the ratio obtained by dividing the distance from the centre to either *focus* by the semi-transverse axis.

The *asymptotes* of a hyperbola are two right lines to which the curve continually approaches, touches at an infinite distance, but does not pass; they are prolongations of the diagonals of the rectangle constructed on the extremes of the axes.

Two hyperbolas are *conjugate* when the transverse axis of the one is the conjugate of the other, and contrariwise.

GENERAL DEFINITIONS.—An *Ordinate* is a right line from any point of a curve to either of the diameters, *a e* and *d o*, Fig. 4; *a b* and *d f* are double ordinates.

Fig. 4.



An *abscissa* is that part of the diameter which is contained between the vertex and an ordinate, as *c e*, *g o*.

The *parameter* of any diameter is equal to four times the distance from the *focus* to the vertex of the curve; the parameter of the axis is the least possible, and is termed the parameter of the curve.

The *parameter* of the curve of a conic section is equal to the chord of the curve drawn through the *focus* perpendicular to the axis.

The *parameter* of the transverse axis is the least, and is termed the parameter of the curve.

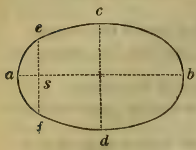
The *parameter* of a conic section and the *foci* are sufficient elements for the construction of the curve.

NOTES.—In the *Parabola* the parameter of any diameter is a third proportional to the abscissa and ordinate of any point of the curve, the abscissa and ordinate being referred to that diameter and the tangent at its vertex.

In the *Ellipse* and *Hyperbola* the parameter of any diameter is a third proportional to the diameter and its conjugate.

To Determine the Parameter of an Ellipse or Hyperbola.

Fig. 5.

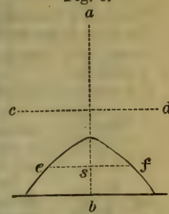


RULE.—Divide the product of the conjugate diameter, multiplied by itself, by the transverse, and the quotient is equal to the parameter.

In the annexed Figs. 5 and 6, of an *Ellipse* and *Hyperbola*, the transverse and conjugate diameters, *a b*, *c d*, are each 30 and 20.

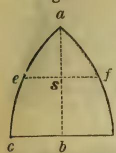
Then 30 : 20 : : 20 : 13.333 = *parameter*.

Fig. 6.



The parameter of the curve = *e f*, a double ordinate passing through the focus, *s*.

Fig. 7.



In a *Parabola*, Fig. 7. The abscissa, *a b*, and ordinate, *c b*, are also equal to 30 and 20.

A *Focus* is a point on the principal axis where the double ordinate to the axis, through the point, is equal to the parameter, as *e f* in the preceding figures.

It may be determined arithmetically thus: Divide the square of the ordinate by four times the abscissa, and the quotient will give the focal distances, *a s* and *s*, in the preceding figures.

A *Conoid* is a warped surface generated by a right line being moved in such a manner that it will touch a straight line and curve, and continue parallel to a given plane. The straight line and curve are called *directrices*, the plane a plane *directrix*, and the moving line the *generatrix*.

The *Directrix* of a conic section is a straight line, such that the ratio obtained by dividing the distance from any point of the curve to it by the distance from the same point to the focus shall be constant. It is always perpendicular to the principal axis; and if the curve is given, it is readily constructed. (See *Haswell's Mensuration*, page 232.)

Ellipsoid, *Paraboloid*, and *Hyperboloid of Revolution*—figures generated by the revolution of an ellipse, parabola, etc., around their axes. (See *Mensuration of Surfaces and Solids*.)

NOTE.—All the figures which can possibly be formed by the cutting of a cone are mentioned in these definitions, and are the five following: viz., a *Triangle*, a *Circle*, an *Ellipse*, a *Parabola*, and a *Hyperbola*; but the last three only are termed the *Conic Sections*.

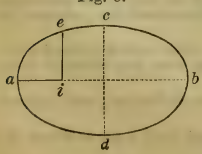
ELLIPSE.

To Describe Ellipses.

When any three of the four following Terms of an Ellipse are given, viz., the Transverse and Conjugate Diameters, an Ordinate, and its Abscissa, to Ascertain the remaining Terms.

To Compute the Ordinate, the Transverse and Conjugate Diameters and the Abscissa being given—Fig. 8.

Fig. 8.



RULE.—As the transverse diameter is to the conjugate, so is the square root of the product of the abscissæ to the ordinate which divides them.

Or, $\frac{c}{t}\times\sqrt{a\times(t-a')}=o$, *t representing the transverse diameter, c the conjugate, a′ the less abscissa, and o the ordinate.*

EXAMPLE. — The transverse diameter, *a b*, of an ellipse, Fig. 8, is 25; the conjugate, *c d*, is 16; and the abscissa, *a i*, 7; what is the length of the ordinate, *i e*?

$25-7=18=$ *second abscissa;* $\sqrt{7\times 18}=11.225=$ *square root of the abscissæ.*
Hence 25 : 16 : : 11.225 : 7.184 *inches = length of the ordinate.*

To Compute the Abscissæ, the Transverse and Conjugate Diameters and the Ordinate being given—Fig. 8.

RULE.—As the conjugate diameter is to the transverse, so is the square root of the difference of the squares of the ordinate and semi-conjugate to the distance between the ordinate and centre; and this distance being added to, or subtracted from the semi-transverse, will give the abscissæ required.

Or, $\frac{t}{c}\times\sqrt{\left(\frac{c}{2}\right)^2-o^2}=x \begin{cases} t\div 2+x=a, \\ t\div 2-x=a', \end{cases}$ *x representing the distance obtained, and a a′ the greater and lesser abscissæ.*

EXAMPLE.—The transverse diameter, *a b*, of an ellipse, Fig. 8, is 25; the conjugate, *c d*, 16; and the ordinate, *i e*, 7.184; what is the abscissa, *i b?*

$\sqrt{8^2-7.1842^2}=3.519943=$ *square root of difference of squares of semi-conjugate and ordinate.*
Hence, as 16 : 25 : : 3.52 : 5.5 = *distance between ordinate and centre.*
Then $25\div 2=12.5$, and $12.5+5.5=18=b\ i,$ } *abscissæ.*
$25\div 2=13.5$, and $12.5-5.5=\ \ 7=a\ i,$ }

To Compute the Transverse Diameter, the Conjugate, Ordinate, and Abscissa being given—Fig. 8.

RULE.—To or from the semi-conjugate, according as the greater or lesser abscissa is used, add or subtract the square root of the difference of the squares of the ordinate and semi-conjugate. Then, as this sum or difference is to the abscissa, so is the conjugate to the transverse.

$$\text{Or, } \frac{a\times c}{\left.\begin{matrix} c\div 2+ \\ c\div 2- \end{matrix}\right\}\sqrt{o^2-\left(\frac{2}{2}\right)^2}}=t.$$

EXAMPLE.—The conjugate diameter, *e d*, of an ellipse, Fig. 6, is 16; the ordinate, *i e*, is 7.184; and the abscissæ, *b i e, a*, are 18 and 7; what is the length of the transverse diameter?

$\sqrt{(16\div 2)^2-7.184^2}=3.52=$ *square root of difference of squares of ordinate and semi-conjugate.*
$16\div 2+3.52$: 18 : : 16 : 25; $16\div 2-3.52$: 7 : : 16 : 25 = *transverse diameter.*

To Compute the Conjugate Diameter, the Transverse, Ordinate, and Abscissa being given—Fig. 8.

RULE.—As the square root of the product of the abscissæ is to the ordinate, so is the transverse diameter to the conjugate.

$$\text{Or, } o\times t\div\sqrt{a\times a'}=c.$$

EXAMPLE.—The transverse diameter, *a b*, of an ellipse, Fig. 6, is 25; the ordinate, *i e*, is 7.184; and the abscissæ, *b i* and *i a*, 18 and 7; what is the length of the conjugate diameter?

$\sqrt{18\times 7}=11.225=11.225=$ *square root of product of abscissæ.*
11.225 : 7.184 : : 25 : 16 = *conjugate diameter.*

To Compute the Circumference of an Ellipse—Fig. 8.

RULE.—Multiply the square root of half the sum of the squares of the two diameters by 3.1416.

$$\text{Or, } \sqrt{\frac{d^2+d'^2}{2}}\times 3.1416=\textit{circumference.}$$

EXAMPLE.—The transverse and conjugate diameters, *a b* and *c d*, of an ellipse, Fig. 6, are 24 and 20; what is its circumference?

$\frac{24^2+20^2}{2}=488$, and $\sqrt{488}=22.09=$ *square root of half the sum of the squares of the diameters.*
Hence $22.09\times 3.1416=69.398=$ *the above root* $\times 3.1416=$ *area.*

To Compute the Area of an Ellipse--Fig. 8.

RULE.—Multiply the diameters together, and the product by .7854. Or, multiply one diameter by .7854, and the product by the other.

Or, $d \times d' \times .7854 = area$.

EXAMPLE.—The transverse diameter of an ellipse, *a b*, Fig. 6, is 12, and its conjugate, *c d*, 9; what is its area?

$12 \times 9 \times .7854 = 84.8232 =$ *product of diameters and* $.7854 =$ *area.*

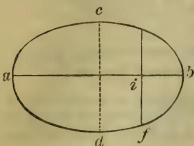
SEGMENT OF AN ELLIPSE.

To Compute the Area of a Segment of an Ellipse when its Base is parallel to either Axis, as e i f, Fig. 9.

RULE.—Divide the height of the segment, *b i*, by the diameter or axis, *a b*, of which it is a part, and find in the Table of Areas of Segments of a Circle, page 205, a segment having the same versed sine as this quotient; then multiply the area of the segment thus found and the two axes of the ellipse together.

Or, $h \div d \times tab.\ area \times d.d' = area$.

Fig. 9.



EXAMPLE.—The height, *b i*, Fig. 9, is 5, and the axes of the ellipse are 30 and 20; what is the area of the segment?

$5 \div 30 = .1666 =$ *tabular versed sine, the area of which* (page 205) *is* .08554.

Hence $.08554 \times 30 \times 20 = 51.324 =$ *area.*

NOTE.—The area of an elliptic segment may also be found by the following rule:

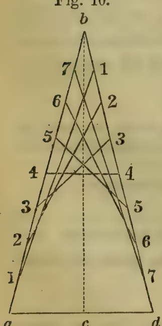
Ascertain the segment of the circle described upon the same axis to which the base of the segment is perpendicular. Then, as this axis is to the other axis, so is the circular segment to the elliptical segment.

ILLUSTRATION.—In the above example, the axis to which the base of the segment is perpendicular is the conjugate, 50, and the height of the segment 25. *Also, the area of the segment is one half of that of a circle of* 50 *diameter* $= \frac{1963.4954}{2} =$ 981.7477.

Hence 50 : 70 : : 981.75 : 1374.45 = *area of elliptic segment.*

PARABOLA.

Fig. 10.



To Describe a Parabola, the Base and Height being given--Fig. 10.

OPERATION.—Draw an isosceles triangle, as *a b d*, Fig. 10, the base of which shall be equal to, and its height, *c b*, twice that of the proposed parabola.

Divide each side, *a b*, *d b*, into any number of equal parts; then draw lines, 1 1, 2 2, 3 3, etc., and their intersection will define the curve of a parabola.

To Compute either Ordinate or Abscissa of a Parabola, the other Ordinate and the Abscissæ, or the other Abscissa and the Ordinates being given--Fig. 11.

RULE.—As either abscissa is to the square of its ordinate, so is the other abscissa to the square of its ordinate.

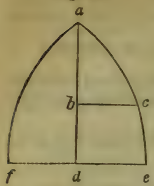
Or, 1. $\frac{o^2 \times a'}{a} = o'^2$. 2. $\frac{o'^2 \times a}{a'} = o^2$.

3. $\frac{o^2 \times a'}{o'^2} = a$. 4. $\frac{o'^2 \times a}{o^2} = a'$.

Or, as the square root of any abscissa is to its ordinate, so is the square root of any other abscissa to its ordinate.

Hence $\frac{o \times \sqrt{a'}}{\sqrt{a}} = o'$.

Fig. 11.



EXAMPLE.—The abscissa, *a b*, of the parabola, Fig. 11, is 9; is ordinate, *b c*, 6; what is the ordinate, *d e*, the abscissa of which, *a d*, is 16?

Hence $9 : 6^2 :: 16 : 64$, and $\sqrt{64} = 8 =$ *length of ordinate.*
Or, $\sqrt{9} : 6 :: \sqrt{16} : 8 =$ *ordinate.*

EX. 2.—The abscissæ of a parabola are 9 and 16, and their corresponding ordinates 6 and 8; any three of these being taken, it is required to find the fourth.

1. $\sqrt{\frac{6^2 \times 16}{9}} = 8 =$ *ordinate.* 2. $\sqrt{\frac{8^2 \times 9}{16}} = 6 =$ *ordinate.*

3. $\frac{6^2 \times 16}{8^2} = 9 =$ *less abscissa.* 4. $\frac{8^2 \times 9}{6^2} = 16 =$ *abscissa.*

PARABOLIC CURVE.

To Compute the Length of the Curve of a Parabola cut off by a Double Ordinate—Fig. 11.

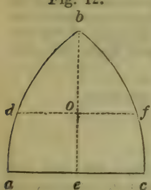
RULE.—To the square of the ordinate add 4-3 of the square of the abscissa, and the square root of this sum, multiplied by two, will give the length of the curve nearly.

$$\text{Or, } \sqrt{\left(o^2 + \frac{4\,a^2}{3}\right)}\, 2 = \textit{length of curve.}$$

EXAMPLE.—The ordinate, *d e*, Fig. 11, is 8, and its abscissa, *a d*, 16; what is the length of the curve, *f a e*?

$8^2 + \frac{4 \times 16^2}{3} = 405.333 =$ *sum of square of the ordinate and* 4-3 *of the square of the abscissa, and* $\sqrt{405.333} = 20.133$, which $\times 2 = 40.267 =$ *length.*

Fig. 12.



To Compute the Area of a Parabola—Fig. 12.

RULE.—Multiply the base by the height, and take ⅔ of the product.*

$$\text{Or, } \frac{2}{3}\, b \times h = \textit{area.}$$

EXAMPLE.—What is the area of the parabola, *a b c*, Fig. 12, the height, *b e*, being 16, and the base, or double ordinate, *a c*, 16?

$16 \times 16 = 256 =$ *product of base and height, and* ⅔ *of* $266 = 170.667 =$ *area.*

To Compute the Area of a Segment of a Parabola—Fig. 12.

RULE.—Multiply the difference of the cubes of the two ends of the segment, *a c*, *d f*, by twice its altitude, *e o*, and divide the product by three times the difference of the squares of the ends.

Or, $\frac{d^3 \infty d'^3 \times 2\,h}{d^2 \infty d'^2 \times 3} =$ *area, d and d′ representing the lengths of the base and lesser end.*

EXAMPLE.—The ends of a segment of a parabola, *a c* and *d f*, Fig. 12, are 10 and 6, and the height, *e o*, is 10; what is its area?

$10^3 \infty 6^2 \times \overline{10 \times 2} = 15680 =$ *difference of cubes of the ends × twice the height.*

$15680 \div \overline{10^2 \infty 6^2 \times 3} = 81.667 =$ *preceding product ÷ 3 times the difference of the squares of the ends = area.*

NOTE.—Any parabolic segment is equal to a parabola of the same altitude, the base of which is equal to the base of the segment, increased by a third proportional to the sum of the two ends and the lesser end.

ILLUSTRATION.—In Example 1 the base and end are 10 and 6.

Then $10 + 6 : 6 :: 6 : 2.25 =$ *third proportional to the sum of the two ends and the lesser end.*

Hence $10 + 2.25 = 12.25 =$ *sum of length of base of parabola and third proportional, and the area then, the height being* $10 = 81.667$.

* *Corollary.*—A parabola is ⅔ of its circumscribing parallelogram.

HYPERBOLA.

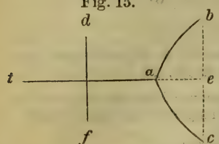
To Describe a Hyperbola.

(See *Haswell's Mensuration*, page 246.)

To Compute the Ordinate of a Hyperbola, the Transverse and Conjugate Diameters and the Abscissæ being given—Fig. 13.

RULE.—As the transverse diameter is to the conjugate, so is the square root of the product of the abscissæ to the ordinate required.

Fig. 13.



Or, $\frac{c \times \sqrt{a \times a'}}{t} = ordinate.$

EXAMPLE. — The hyperbola, *a b c*, Fig. 13, has a transverse diameter, *a t*, of 120; a conjugate, *d f*, of 72; and the abscissa, *a e*, is 40; what is the length of the ordinate, *e c?*

$40 + 120 = 160 =$ *sum of lesser abscissa and axis = greater abscissa.*

$120 : 72 :: \sqrt{(40 \times 160)} : 48 =$ *ordinate.*

NOTE.—In hyperbolas the lesser abscissa, added to the axis (the transverse diameter), gives the greater.

2.—The difference of two lines drawn from the *foci* of any hyperbola to any point in the curve is equal to its transverse diameter.

To Compute the Abscissæ, the Transverse and Conjugate Diameters and the Ordinate being given—Fig. 13.

RULE.—As the conjugate diameter is to the transverse, so is the square root of the sum of the squares of the ordinate and semi-conjugate to the distance between the ordinate and the centre, or half the sum of the abscissæ. Then the sum of this distance and the semi-transverse will give the greater abscissa, and their difference the lesser abscissa.

$$\text{Or, } \frac{t\sqrt{o^2 + (c \div 2)^2}}{c} = \frac{a + a'}{2} = \textit{half the sum of the abscissæ.}$$

$$\frac{a + a'}{2} + \frac{t}{2} = a, \text{ and } \frac{a + a'}{2} - \frac{t}{2} = a'.$$

EXAMPLE.—The transverse diameter, *a t*, of a hyperbola, Fig. 13, is 120; the conjugate, *d f*, 72; and the ordinate, *e c*, 48; what are the lengths of the abscissæ, *t e* and *a e?*

$72 : 120 :: \sqrt{48^2 + (72 \div 2)^2} = 60 : 100 =$ *half the sum of the abscissæ.*

$100 + (120 \div 2) = 160 =$ *above sum added to the semi-transverse = the greater abscissa;* and

$100 - (120 \div 2) = 40 =$ *above sum subtracted from the semi-transverse = the lesser abscissa.*

To Compute the Conjugate Diameter, the Transverse Diameter, the Abscissæ, and Ordinate being given—Fig. 13.

RULE.—As the square root of the product of the abscissæ is to the ordinate, so is the transverse diameter to the conjugate.

$$\text{Or, } \frac{o \times t}{\sqrt{(a \times a')}} = \textit{conjugate diameter.}$$

EXAMPLE.—The transverse diameter, *a b*, of a hyperbola, Fig. 13, is 120; the ordinate, *e c*, 48; and the abscissæ, *t e* and *a e*, 160 and 40; what is the length of the conjugate, *d f?*

$$\sqrt{40 \times 160} = 80 : 48 :: 120 : 72 = \textit{conjugate.}$$

To Compute the Transverse Diameter, the Conjugate, the Ordinate, and an Abscissa being given—Fig. 13.

RULE.—Add the square of the ordinate to the square of the semi-conjugate, and extract the square root of their sum.

Take the sum or difference of the semi-conjugate and this root, according as the greater or lesser abscissa is used.* Then, as the square of the ordinate is to the product of the abscissa and conjugate, so is the sum or difference above ascertained to the transverse diameter required.

Or, a or $a' \times c \times (\sqrt{o^2 + (c \div 2)^2} \pm \overline{c \div 2}) \div o^2 =$ *transverse diameter.*

EXAMPLE.—The conjugate diameter, *d f*, of a hyperbola, Fig. 13, is 72; the ordinate, *e c*, 48; and the lesser abscissa, *a e*, 40; what is the length of the transverse diameter, *a t?*

$\sqrt{48^2 + (72 \div 2)^2} = 60 =$ *square root of the squares of the ordinate and semi-conjugate.*

$60 + \overline{72 \div 2} = 96 =$ *sum of above root and the semi-conjugate* (the lesser abscissa being used).

$40 \times 72 = 2880 =$ *product of abscissa and conjugate.*

$48^2 : 2880 :: 96 : 120 =$ *transverse diameter.*

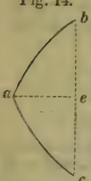
To Compute the Length of any Arc of a Hyperbola, commencing at the Vertex—Fig. 14.

RULE.—To 19 times the transverse diameter add 21 times the parameter of the axis, and multiply the sum by the quotient of the lesser abscissa divided by the transverse diameter.

To 9 times the transverse diameter add 21 times the parameter, and multiply the sum by the quotient of the lesser abscissa divided by the transverse diameter.

To each of the products thus ascertained add 15 times the parameter, and divide the former by the latter; then this quotient, being multiplied by the ordinate, will give the leng.h of the arc nearly.

Fig. 14.



Or, $$\frac{t \times 19 + \overline{21 \times p} \times \frac{a}{t} + \overline{15 \times p}}{t \times 9 + \overline{21 \times p} \times \frac{a}{t} + \overline{15 \times p}} \times o = \textit{arc nearly.}$$

EXAMPLE.—In the hyperbola, *a b c*, Fig. 14, the transverse diameter is 120, the conjugate 80, the ordinate, *e c*, 48, and the lesser abscissa, *a e*, 40: what is the length of the arc, *a b?*

$120 : 80 :: 80 : 53.3333 =$ *parameter.*

$\overline{120 \times 19} + \overline{53.3333 \times 21} \times \frac{40}{120} = 1133.3333 =$ *product of the sum of* 19 *times the transverse and* 21 *times the parameter, by the quotient of the lesser abscissa and the transverse.*

$\overline{120 \times 9} + \overline{53.3333 \times 21} \times \frac{40}{120} = 733.3333 =$ *product of the sum of* 9 *times the transverse and* 21 *times the parameter, by the quotient of the lesser abscissa and transverse.*

$1133.3333 + \overline{53.3333 \times 15} \div (33.3333 + \overline{53.3333 \times 15}) = 1.2609 =$ *quotient of former product and* 15 *times the parameter* ÷ *latter product and* 15 *times the parameter.*

$1.2609 \times 48 = 60.5232 =$ *above quotient* × *the ordinate* = *length.*

NOTE.—As the transverse diameter is to the conjugate, so is the conjugate to the parameter. (See Rule, page 290.)

To Compute the Area of a Hyperbola, the Transverse, Conjugate, and lesser Abscissa being given—Fig. 14.

RULE.—To the product of the transverse diameter and lesser abscissa add 5-7 of the square of this abscissa, and multiply the square root of the sum by 21.

Add 4 times the square root of the product of the transverse diameter and lesser abscissa to the product last ascertained, and divide the sum by 75.

Divide 4 times the product of the conjugate diameter and lesser abscissa by the transverse diameter, and this last quotient, multiplied by the former, will give the area nearly.

Or, $$\frac{\sqrt{t \times a + \frac{5}{7} a'^2} \times 21 + (\sqrt{t \times a'} \times 4)}{75} \times \frac{c \times a' \times 4}{t} = \textit{area.}$$

* When the greater abscissa is used, the difference is taken, and contrariwise.

EXAMPLE.—The transverse diameter of a hyperbola, Fig. 14, is 60, the conjugate 36, and the lesser abscissa or height, *a e*, 20; what is the area of the figure?

$60 \times 20 + \frac{5}{7}$ of $20^2 = 1485.7143 =$ *sum of the product of the transverse and abscissa and $\frac{5}{7}$ of the square of the abscissa.*

$\sqrt{1485.7143 \times 21} = 809.424 = 21$ *times the square root of the above sum.*

$\sqrt{60 \times 20 \times 4} + 809.424 = 947.988 =$ *sum of above result and the root of 4 times the product of the transverse and abscissa.*

$947.988 \div 75 = 12.6398 =$ *quotient of above result* $\div 75$.

$\frac{36 \times 20 \times 4}{60} \times 12.6398 = 606.7104 =$ *product of 4 times the product of the conjugate and abscissa* $\div$ *the transverse and the above quotient = area.*

PLANE TRIGONOMETRY.

By *Plane Trigonometry* is ascertained how to compute or determine four of the seven elements of a plane or rectilinear triangle from the other three when one of the given quantities is a side, or the area.

The determination of the mutual relation of the Sines, Tangents, Secants, etc., of the sums, differences, multiples, etc., of arcs or angles is also classed under this head.

For Explanation of Terms, see Geometry, page 163.

When any three elements of a Plane Triangle are given, one of which being a side, or its area, the remaining elements may be determined; and this operation is termed *solving the triangle.*

RIGHT-ANGLED TRIANGLES.

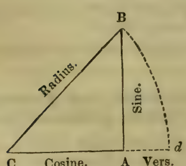
For Solution by Lines and Areas, see also Mensuration, page 245.

In the following figures, A = 90°, B = 45°, C = 45°, Radius = 1, Secant = 1-4142, Cosine = .7071, Sin 45° = .7071, Tangent = 1, Area = .25, Figs. 1 and 2, and = .5, Figs. 3 and 4.

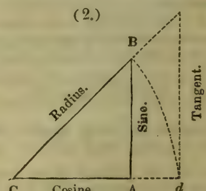
By Sin., Tan., Sec., etc., etc., A, B, etc., is expressed the Sine, Tangent, Secant, etc., of the angle, A, B, etc.

To Compute Sides A C and B A—Figs. 1 and 2.

(1.)



(2.)

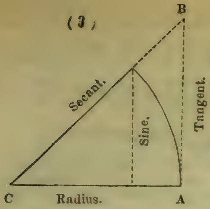


Given	Hyp. B C,	R. 1 : B C 1 : : sin. B .7071 : A C : .7071, Fig. 1.
	Leg. B A,	R. 1 : B C 1 : : sin. C .7071 : B A : 7071, Fig. 1.
	and Angles	Sin. C .7071 : : sin. B. .7071 : : B A .7071 : A C .7071, Figs. 1 & 2.

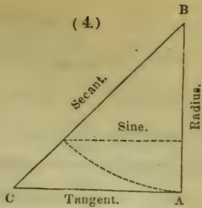
To Compute Sides B A and B C—Figs. 3 and 4.

Given	A C	R. 1 : A C 1 : : tan. C 1 : B A 1, Fig. 3.
	and	R. 1 : A C 1 : : sec. C 1.4142 : B C 1.4142, Fig. 3.
	Angles	Sin. B .7071 : A C 1 : : R. 1 : B C 1 4142, Fig. 4.

(3.)



(4.)



To Compute the Angles and Side, A C--Figs. 1 and 2.

Given { Hyp. B C and Leg B A { B C 1 : R. 1 : : B A .7071 : sin. C .7071—Fig. 1.
R. 1 : B C 1 : : sin. B .7071 : A C. 7071—Fig. 2.

To Compute Angles and Side, B C--Fig. 2.

Given both Legs { A C .7071 : R. 1 : : B A .7071 : tan. C 1—Fig. 2.
Sin. C .7071 : B A .7071 : : R. 1 : B C 1—Fig. 2.
R 1 : A C .7071 : : sec. C 1.4142 : B C 1—Fig. 2.

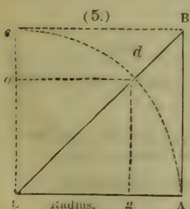
To Compute Area--Fig. 1.

Given B A and A C. $BA \times AC \div 2 = .7071 \times .7071 \div 2 =$ area .25.

Given { Hyp. B C and Angle C {
4 : B C² 1 : : sin. 2 C 1 : area .25—Fig. 1.
$\frac{AC^2}{2} \times \tan. C = \frac{.5}{2} \times 1 =$ area .25—Fig. 1.
$\frac{BA^2}{2} \times \cot. C = \frac{.5}{2} \times 1 =$ area. 25—Fig. 1.

Given { $\frac{BA}{2}\sqrt{(BC + BA) \times (BC - BA)} = \frac{.7071}{2} \times \sqrt{(1 + .7071) \times 1 - .7071} = .3535$
$\times .7071 =$ area .25—Fig. 1.

(5.)



Let B A C be a right-angled triangle, in which C A is assumed to be radius; B A is the tangent of C, and B C its secant to that radius; or, dividing each of these by the base, there is obtained the tangent and secant of C respectively to radius 1.

Radius	C A = 1.	Sine	d g = .7071.
Secant	C B = 1.4142.	Cosine	C g or o d = .7071.
Tangent	A B = 1.	Versed sine	g A = .2929.
Co-secant	C B = 1.4142.	Co-versed sine	o e = .2929.
Cotangent	e B = 1.	Angle	C A B = 90°.

$$\frac{\text{perp.}}{\text{base}} = \text{tan. angle C} = \frac{.7071}{.7071} = 1.$$

$$\frac{\text{hyp.}}{\text{base}} = \text{sec. angle C} = \frac{1.4142}{1} = 1.4142.$$

$$\frac{\text{perp.}}{\text{hyp.}} = \text{sin. angle C} = \frac{1}{1.4142} = .7071.$$

$$\frac{\text{base}}{\text{perp.}} = \text{tan. angle B} = \frac{1}{1} = 1.$$

$$\frac{\text{hyp.}}{\text{perp.}} = \text{sec. angle B} = \frac{1.4142}{1} = 1.4142.$$

$$\frac{\text{base}}{\text{hyp.}} = \text{sin. angle B} = \frac{1}{1.4142} = .7071.$$

Formulæ—Fig. 5.

1. $\frac{B\,A}{B\,C} = \sin.\ C = \frac{1}{1.4142} = .7071.$ 2. $\frac{A\,C}{B\,C} = \cos.\ C = \frac{1}{1.4142} = .7071.$

3. $\frac{B\,A}{A\,C} = \tan.\ C = \frac{1}{1} = 1.$ 4. $\frac{2\ Area}{A\,C^2} = \tan.\ C = \frac{.5 \times 2}{1^2} = 1.$

5. $\frac{Sin.\ C}{Cos.\ C} = \tan.\ C = \frac{.7071}{.7071} = 1.$ 6. $\frac{1}{Cot.\ C} = \tan.\ C = \frac{1}{1} = 1.$

7. $1 - \cos.\ C = versin\ C = 1 - .7071 = .2929.$

8. $\frac{1}{Cos.\ C} = \sec.\ C = \frac{.7071}{1} = 1.4142.$ 9. $\frac{B\,C}{B\,A} = \sec.\ C = \frac{1.4142}{1} = 1.4142.$

10. $\sqrt{A\,C^2 + B\,A^2} = hyp.\ B\,C = \sqrt{1+1} = 1.4142.$

11. $\frac{B\,A}{Sin.\ C} = hyp.\ B\,C = \frac{1}{.7071} = 1.4142.$ 12. $\frac{A\,C}{Cos.\ C} = hyp.\ B\,C = \frac{1}{.7071} = 1.4142.$

13. $2\sqrt{\frac{Area}{Sin.\ 2\,C}} = hyp.\ B\,C = 2\sqrt{\frac{.5}{1.}} = 1.4142.$

14. $B\,C \times \cos.\ C = radius = 1.4142 \times .7071 = 1.$

15. $B\,A \times \cot.\ C = radius = 1 \times 1 = 1.$

16. $B\,C \times \sin.\ C = radius = 1.4142 \times .7071 = 1.$

17. $B\,A \times \tan.\ B = radius = 1 \times 1 = 1.$ 18. $\sqrt{\frac{2\ Area}{Tan.\ C}} = radius = \sqrt{\frac{5 \times 2}{1.}} = 1.$

19. $Sin.\ C^2 + Cos.\ ^2 = radius\ ^2 = .7071^2 + .7071^2 = 1.$

20. $B\,C \times \sin.\ C = perp.\ B\,A = 1.4142 \times .7071 = 1.$

21. $A\,C \times \tan.\ C = perp.\ B\,A = 1 \times 1 = 1.$

22. $\frac{1}{Sin.\ C} = cosec.\ C = \frac{1}{.7071} = 1.4142.$ 23. $\frac{Cos.\ C}{Sin.\ C} = \cot.\ C = \frac{.7071}{.7071} = 1.$

24. $\frac{1}{Tan.\ C} = \cot.\ C = \frac{1}{1} = 1.$ 25. $1 - \sin.\ C = co\text{-}versin. = 1 - .7071 = .2929.$

26. $2 \sin.\ C.\ \cos.\ C = \sin.\ 2\,C = 2 \times .7071 \times .7071 = 1.$

27. $\frac{1}{2}\sqrt{\sin.\ C^2 + versin.\ C^2} = \sin.\ \frac{1}{2}\,C = \sqrt{\frac{.7071^2 + .2929^2}{2}} = .38268.$

28. $Sin.\ C.\ \cos.\ B \pm \sin.\ B \cos.\ C = \sin.\ (C \pm B) = .7071 \times .7071 + .7071 \times .7071$
$= 1$, or $(45° - 45°) = 0$; $.7071 \times .7071 - .7071 \times .7071 = 0.$

ILLUSTRATION.—The side, B A, of a right-angled triangle is 100 feet; the angle C = 30°; and B = 60°; what are the lengths of the sides, B C and A C?

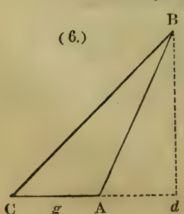
$\frac{100}{Sin.\ 30°} = \frac{100}{.5} = 200$ *feet*, B C; $200 \times \cos.\ 30° = 200 \times .86603 = 173.206$ *feet*, A C.

2.—Side B C = 200 feet, and angle C = 30°; what is the length of side B A?

$200 \times \sin.\ 30° = 200 \times .5 = 100$ *feet*.

OBLIQUE-ANGLED TRIANGLES.

(6.)



(7.)

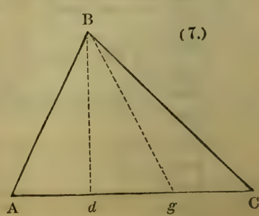


Fig. 6.	Fig. 7.
A = 116°.30′; B = 18°.30′; C = 45°.	A = 63°.30′; B = 71°.30′; C = 45°.

B A = 1.1174; A C = .5014; B C = 1.4142.
Area = .2507; S = ½ sum of sides = 1.5165.
B A = 1.1174; A C = 1; B C = 1.4142.
Area = .7493; S = ½ sum of sides = 2.0151.

To Compute B A and A C—Fig. 6.

Given { Angles and Side B C { Sin. A .89493 : B C 1.4142 : : sin. C .7071 : B A 1.1174.
Sin. A .89493 : B C 1.4142 : : sin. B .3173 : A C .5014.

To Compute Angles B and C and Side A C—Figs. 6 and 7.

Given { A B, B C, and Angle B { B C 1.4142 : sin. A .89493 : : B A 1.1174 : sin. C .7071, which angle added to A and the sum subtracted from 180° = angle of B—Fig. 6 = 18°.30′, and which added to A and the sum subtracted from 180° = angle of B—Fig. 7 = 71°.30′.
Sin. A .89493 : B C 1.4142 : : sin. B .3173 : A C .5014—Fig. 6.
Sin. A .89493 : B C 1.4142 : : sin. B .94832 : A C 1.4986—Fig. 7.

To Compute B C and B A—Figs. 6 and 7.

Given { A C and Angles { Sin. B .3173 : A C .5014 : : sin. A .89493 : B C 1.4142—Fig. 6.
Sin. B .94832 : A C 1.4986 : : sin. A .89493 : B C 1.4142—Fig. 7.
Sin. B .3173 : A C .5014 : : sin. C. .7071 : B A 1.1174—Fig. 6.
Sin. B .94832 : A C 1.4986 : : sin. C .7071 : B A 1.1174—Fig. 7.

To Compute Angles B and C and Side B C—Figs. 6 and 7.

Given { A C, A B, and Angle A { Subtract half of the given angle, A, from 90°; the remainder is half the sum of the other angles.
Then, as the sum of the sides, A C, A B, is to their difference, so is the tangent of the half sum of the other angles to the tangent of half their difference, which, added to and subtracted from the half sum, will give the two angles B and C, the greatest angle being opposite to the greatest side.

OPERATION. $90° - \angle A \div 2 = 90'' - \overline{116°.30' \div 2} = 31°.45' = \frac{45° + 18°.30'}{2} =$ *half sum of the other angles.*

Then A C + A B = .5014 + 1.1174 = 1.6188 : A C ∾ A B = 1.1174 − .5014 = .6160 : : tan. ∠ B + C ÷ 2 = $\overline{18°.30' + 45°} \div 2$ = 31°.45′ = .61882 : tan. ∠ B ∾ C ÷ 2 = $\overline{45° - 18°.30'} \div 2$ = 13°.15′ = .23547; which, being added to 31°.45′ (the half sum) = 13°.15′ + 31°.45′ = 45° = ∠ C, opposite to side A B, and 31°.45′ − 13°.15′ = 18°.30′ ∠ B, opposite side A C.

To Compute all the Angles—Figs. 6 and 7.

Given all three sides. { Let fall a perpendicular, B *d*, opposite to the required angle.
Then, as A C : sum of A B, B C : : their difference : twice *d g*, the distance of the perpendicular, B *d*, from the middle of the base.
Hence A *d*, C *g* are known, and the triangle, A B C, is divided into two right-angled triangles, B C *d*, B A *d*; then, by the rules in right-angled triangles, ascertain the angle A or C.

OPERATION.—A C, Fig. 6, .5014 : A B + B C 1.1174 + 1.4142 = 2.5316 : : A B ∾ B C 1.4142 − 1.1174 = .2968 : 2 × *d g* = 1.4986.

Hence A $d = \overline{d\,g - A\,C \div 2} = \frac{1.4986}{2} - \frac{.5014}{2} = .4986$, and C *d* = A *d* + A *c* = 1.

Consequently, the triangle B *d* C is divided into two triangles, B A C and B *d* A.
Again, A C (Fig. 7) 1.4986 : A B + B C 1.1174 + 1.4142 = 2.5316 : : A B ∾ B C 1.4142 − 1.1174 = .2968 : 2 × *d g* = 5014.

Hence A $d = \overline{d\,g - A\,C \div 2}$ = .5014 ÷ 2 − $\overline{1.4986 \div 2}$ = .4986, and *d* C = A C − A *d* = 1.4986 − .4986 = 1.

Consequently, the triangle A B C is divided into two triangles, B *d* C and B *d* A. Then, by the preceding rules for right-angled triangles, ascertain the angle A or C.

Formulæ—Figs. 6 and 7.

1. $\frac{BC \, . \sin. B}{AC} = \sin. A = \frac{1.4142 \times .3173}{.5014} = .89493.$

2. $\frac{BC \, . \sin. C}{BA} = \sin. A = \frac{1.4142 \times .7071}{1.1174} = .89493.$

3. $\frac{AC \, . \sin. A}{BC} = \sin. B = \frac{.5014 \times .89493}{1.4142} = .3173.$

4. $\frac{AC \, . \sin. C}{BA} = \sin. B = \frac{.5014 \times .7071}{1.1174} = .3173.$

5. $\frac{2 \text{ Area}}{AC \, . BC} = \sin. C = \frac{.2507 \times 2}{.5014 \times 1.4142} = .7071.$

6. $\frac{BA \, . \sin. A}{BC} = \sin. C = \frac{1.1174 \times .89493}{1.4142} = .7071.$

7. $\frac{BA \, . \sin. A}{\sin. C} = \text{hyp. } BC = \frac{1.1174 \times .89493}{.7071} = 1.4142.$

8. $\frac{AC \, . \sin. A}{\sin. B} = \text{hyp. } BC = \frac{.5014 \times .89493}{.3173} = 1.4142.$

9. $\sqrt{\frac{2 \text{ Area} \, . \sin. A}{\sin. C \, . \sin. (A + C)}} = \text{hyp. } BC = \sqrt{\frac{.2507 \times 2 \times .89493}{.7071 \times .3173}} = \sqrt{2} = 1.4142.$

10. $\frac{BC \, . \sin. C}{\sin. A} = BA = \frac{1.4142 \times .7071}{.89493} = 1.1174.$

11. $\frac{BC \, . \sin. C}{\sin. (C + B)} = BA = \frac{1.4142 \times .7071}{.89493} = 1.1174.$

12. $\frac{2 \text{ Area}}{AC \, . \sin. A} = BA = \frac{.2507 \times 2}{.5014 \times .89493} = 1.1174.$

13. $\sqrt{\frac{2 \text{ Area} \, . \sin. C}{\sin. B \, . \sin. (C + B)}} = BA = \sqrt{\frac{.2507 \times 2 \times .7071}{.3173 \times .89493}} = 1.1174.$

14. $\frac{AC \, . \sin. C}{\sin. B} = BA = \frac{.5014 \times .7071}{.3173} = 1.1174.$

15. $\frac{BC \, . \sin. B}{\sin. a} = AC = \frac{1.4142 \times .3173}{.89493} = .5014.$

16. $\frac{2 \text{ Area}}{BC \, . \sin. C} = AC = \frac{.2507 \times .2}{1.4142 \times .7071} = .5014.$

17. $\sqrt{\frac{2 \text{ Area} \, . \sin. (C + A)}{\sin. C \, . \sin. A}} = AC = \sqrt{\frac{.2507 \times 2 \times .3173}{.7071 \times .89493}} = .5014.$

18. $\frac{AC \, . BC \, . \sin. C}{2} = \text{area} = \frac{.5014 \times 1.4142 \times .7071}{2} = .2507.$

19. $\frac{BA \, . AC \, . \sin. a}{2} = \text{area} = \frac{1.1174 \times .5014 \times .89493}{2} = .2507.$

20. $\frac{BC^2 \, . \sin. C. \sin. B}{2 \sin. (B + C)} = \text{area} = \frac{1.4142^2 \times .7071 \times .3173}{2 \times .89493} = .2507.$

21. $\sqrt{S \, . (S - BC)(S - BA)(S - AC)} = \text{area} = \sqrt{1.5165 \times (1.5165 - 1.4142) \times (1.5165 - 1.1174) \times (1.5165 - .5014)} = \sqrt{.06285048} = .2507.$

Table of Natural Sines and Cosines.

Prop. parts		0°		1°		2°		3°		4°			Prop parts
29	'	N.sine.	N. cos.	N. sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.		2
0	0	00000	1.	01745	99985	0349	99939	05234	99863	06976	99756	60	2
0	1	00029	1.	01774	99984	03519	99938	05263	99861	07005	99754	59	2
1	2	00058	1.	01803	99984	03548	99937	05292	9986	07034	99752	58	2
1	3	00087	1.	01832	99983	03577	99936	05321	99858	07063	9975	57	2
2	4	00116	1.	01862	99983	03606	99935	0535	99857	07092	99748	56	2
2	5	00145	1.	01891	99982	03635	99934	05379	99855	07121	99746	55	2
3	6	00175	1.	0192	99982	03664	99933	05408	99854	0715	99744	54	2
3	7	00204	1.	01949	99981	03693	99932	05437	99852	07179	99742	53	2
4	8	00233	1.	01978	9998	03723	99931	05466	99851	07208	9974	52	2
4	9	00262	1.	02007	9998	03752	9993	05495	99849	07237	99738	51	2
5	10	00291	1.	02036	99979	03781	99929	05524	99847	07266	99736	50	2
5	11	0032	.99999	02065	99979	0381	99927	05553	99846	07295	99734	49	2
6	12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48	2
6	13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47	2
7	14	00407	99999	02152	99977	03897	99924	0564	99841	07382	99727	46	2
7	15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45	2
8	16	00465	99999	02211	99976	03955	99922	05698	99838	0744	99723	44	1
8	17	00495	99999	0224	99975	03984	99921	05727	99836	07469	99721	43	1
9	18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42	1
9	19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41	1
10	20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40	1
10	21	00611	99998	02356	99972	041	99916	05844	99829	07585	99712	39	1
11	22	0064	99998	02385	99972	04129	99915	05873	99827	07614	9971	38	1
11	23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37	1
12	24	00698	99998	02443	9997	04188	99912	05931	99824	07672	99705	36	1
12	25	00727	99997	02472	99969	04217	99911	0596	99822	07701	99703	35	1
13	26	00756	99997	02501	99969	04246	9991	05989	99821	0773	99701	34	1
13	27	00785	99997	0253	99968	04275	99909	06018	99819	07759	99699	33	1
14	28	00814	99997	0256	99967	04304	99907	06047	99817	07788	99696	32	1
14	29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31	1
15	30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30	1
15	31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29	1
15	32	00931	99996	02676	99964	0442	99902	06163	9981	07904	99687	28	1
16	33	0096	99995	02705	99963	04449	99901	06192	99808	07933	99685	27	1
16	34	00989	99995	02734	99963	04478	999	06221	99806	07962	99683	26	1
17	35	01018	99995	02763	99962	04507	99898	0625	99804	07991	9968	25	1
17	36	01047	99995	02792	99961	04536	99897	06279	99803	0802	99678	24	1
18	37	01076	99994	02821	9996	04565	99896	06308	99801	08049	99676	23	1
18	38	01105	99994	0285	99959	04594	99894	06337	99799	08078	99673	22	1
19	39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21	1
19	40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20	1
20	41	01193	99993	02938	99957	04682	9989	06424	99793	08165	99666	19	1
20	42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18	1
21	43	01251	99992	02996	99955	0474	99888	06482	9979	08223	99661	17	1
21	44	0128	99992	03025	99854	04769	99886	06511	99788	08252	99659	16	1
22	45	01309	99991	03054	99953	04798	99885	0654	99786	08281	99657	15	1
22	46	01338	99991	03083	99952	04827	99883	06569	99784	0831	99654	14	0
23	47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13	0
23	48	01396	9999	03141	99951	04885	99881	06627	9978	08368	99649	12	0
24	49	01425	9999	0317	9995	04914	99879	06656	99778	08397	99647	11	0
24	50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10	0
25	51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9	0
25	52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8	0
26	53	01542	99988	03286	99946	0503	99873	06773	9977	08513	99637	7	0
26	54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6	0
27	55	016	99987	03345	99944	05088	9987	06831	99766	08571	99632	5	0
27	56	01629	99987	03374	99943	05117	99869	0686	99764	086	9963	4	0
28	57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3	0
28	58	01687	99986	03432	99941	05175	99866	06918	9976	08658	99625	2	0
29	59	01716	99985	03461	9994	05205	99864	06947	99758	08687	99622	1	0
29	60	01745	99985	0349	99939	05234	99863	06976	99756	08716	99619	0	0
		N. cos.	N. sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	N. cos.	N.sine.	'	
		89°		88°		87°		86°		85°			

Table—(*Continued*).

Prop. parts	′	5° N. sine.	5° N. cos.	6° N. sine.	6° N. cos.	7° N.sine.	7° N. cos.	8° N sine.	8° N. cos.	9° N. sine.	9° N cos.		Prop. parts
29													4
0	0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60	4
0	1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59	4
1	2	08774	99614	10511	99446	12245	99248	13975	99019	15701	9876	58	4
1	3	08803	99612	1054	99443	12274	99244	14004	99015	1573	98755	57	4
2	4	08831	99609	10569	9944	12302	9924	14033	99011	15758	98751	56	4
2	5	0886	99607	10597	99437	12331	99237	14061	99006	15787	98746	55	4
3	6	08889	99604	10626	99434	1236	99233	1409	99002	15816	98741	54	4
3	7	08918	99602	10655	99431	12389	9923	14119	98998	15845	98737	53	4
4	8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52	3
4	9	08976	99596	10713	99424	12447	99222	14177	9899	15902	98728	51	3
5	10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50	3
5	11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49	3
6	12	09063	99588	108	99415	12533	99211	14263	98978	15988	98714	48	3
6	13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47	3
7	14	09121	99583	10858	99409	12591	99204	1432	98969	16046	98704	46	3
7	15	0915	9958	10887	99406	1262	992	14349	98965	16074	987	45	3
8	16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44	3
8	17	09208	99575	10945	99399	12678	99193	14407	98957	16132	9869	43	3
9	18	09237	99572	10973	99396	12706	99189	14436	98953	1616	98686	42	3
9	19	09266	9957	11002	99393	12735	99186	14464	98948	16189	98681	41	3
10	20	09295	99567	11031	9939	12764	99182	14493	98944	16218	98676	40	3
10	21	09324	99564	1106	99386	12793	99178	14522	9894	16246	98671	39	3
11	22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38	3
11	23	09382	99559	11118	9938	12851	99171	1458	98931	16304	98662	37	2
12	24	09411	99556	11147	99377	1288	99167	14608	98927	16333	98657	36	2
12	25	0944	99553	11176	99374	12908	99163	14637	98923	16361	98652	35	2
13	26	09469	99551	11205	9937	12937	9916	14666	98919	1639	98648	34	2
13	27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33	2
14	28	09527	99545	11263	99364	12995	99152	14723	9891	16447	98638	32	2
14	29	09556	99542	11291	9936	13024	99148	14752	98906	16476	98633	31	2
15	30	09585	9954	1132	99357	13053	99144	14781	98902	16505	98629	30	2
15	31	09614	99537	11349	99354	13081	99141	1481	98897	16533	98624	29	2
15	32	09642	99534	11378	99351	1311	99137	14838	98893	16562	98619	28	2
16	33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27	2
16	34	097	99528	11436	99344	13168	99129	14896	98884	1662	98609	26	2
17	35	09729	99526	11465	99341	13197	99125	14925	9888	16648	98604	25	2
17	36	09758	99523	11494	99337	13226	99122	14954	98876	16677	986	24	2
18	37	09787	9952	11523	99334	13254	99118	14982	98871	16706	98595	23	2
18	38	09816	99517	11552	99331	13283	99114	15011	98867	16734	9859	22	1
19	39	09845	99514	1158	99327	13312	9911	1504	98863	16763	98585	21	1
19	40	09874	99511	11609	99324	13341	99106	15069	98858	16792	9858	20	1
20	41	09903	99508	11638	9932	1337	99102	15097	98854	1682	98575	19	1
20	42	09932	99506	11667	99317	13399	99098	15126	98649	16849	9857	18	1
21	43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17	1
21	44	0999	995	11725	9931	13456	99091	15184	98841	16906	98561	16	1
22	45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15	1
22	46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14	1
23	47	10077	99491	11812	993	13543	99079	1527	98827	16992	98546	13	1
23	48	10106	99488	1184	99297	13572	99075	15299	98823	17021	98541	12	1
24	49	10135	99485	11869	99293	136	99171	15327	98818	1705	98536	11	1
24	50	10164	99482	11898	9929	13629	99067	15356	98814	17078	98531	10	1
25	51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9	1
25	52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98531	8	1
26	53	1025	99473	11985	99279	13716	99055	15442	988	17164	98516	7	0
26	54	10279	9947	12014	99276	13744	99051	15471	98796	17193	98511	6	0
27	55	10308	99467	12043	99272	13773	99047	155	98791	17222	98506	5	0
27	56	10337	99464	12071	99269	13802	99043	15529	98787	1725	98501	4	0
28	57	10366	99461	121	99265	13831	99039	15557	98782	17279	98496	3	0
28	58	10395	99458	12129	99262	1386	99035	15586	98778	17308	98491	2	0
29	59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1	0
29	60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0	0
		N. cos.	N. sine.	N. cos.	N.sine.	N. cos.	N.sine.	N.cos.	N. sine.	N. cos.	N. sine.	′	
		84°		83°		82°		81°		80°			

Table—(*Continued*).

Prop. parts.		10°		11°		12°		13°		14°			Prop. parts.
28	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		6
0	0	17365	98481	19081	98163	20791	97815	22495	97437	24192	9703	60	6
0	1	17393	98476	19109	98157	2082	97809	22523	9743	2422	97023	59	6
1	2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58	6
1	3	17451	98466	19167	98146	20877	97797	2258	97417	24277	97008	57	6
2	4	17479	98461	19195	9814	20905	97791	22608	97411	24305	97001	56	6
2	5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55	6
3	6	17537	9845	19252	98129	20962	97778	22665	97398	24362	96987	54	5
3	7	17565	98445	19281	98124	2099	97772	22693	97391	2439	9698	53	5
4	8	17594	9844	19309	98118	21019	97766	22722	97384	24418	96973	52	5
4	9	17623	98435	19338	98112	21047	9776	2275	97378	24446	96966	51	5
5	10	17651	9843	19366	98107	21076	97754	22778	97371	24474	96959	50	5
5	11	1768	98425	19395	98101	21104	97748	22807	97365	24503	96952	49	5
6	12	17708	9842	19423	98096	21132	97742	22835	97358	24531	96945	48	5
6	13	17737	98414	19452	9809	21161	97735	22863	97351	24559	96937	47	5
7	14	17766	98409	19481	98084	21189	97729	22892	97345	24587	9693	46	5
7	15	17794	98404	19509	98079	21218	97723	2292	97338	24615	96923	45	5
7	16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44	4
8	17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43	4
8	18	1788	98389	19595	98061	21303	97705	23005	97318	247	96902	42	4
9	19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41	4
9	20	17937	98378	19652	9805	2136	97692	23062	97304	24756	96887	40	4
10	21	17966	98373	1968	98044	21388	97686	2309	97298	24784	9688	39	4
10	22	17995	98368	19709	98039	21417	9768	23118	97291	24813	96873	38	4
11	23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37	4
11	24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36	4
12	25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35	4
12	26	18109	98347	19823	98016	2153	97655	23231	97264	24925	96844	34	3
13	27	18138	98341	19851	9801	21559	97648	2326	97257	24954	96837	33	3
13	28	18166	98336	1988	98004	21587	97642	23288	97251	24982	96829	32	3
14	29	18195	98331	19908	97998	21616	97636	23316	97244	2501	96822	31	3
14	30	18224	98325	19937	97992	21644	9763	23345	97237	25038	96815	30	3
14	31	18252	9832	19965	97987	21672	97623	23373	9723	25066	96807	29	3
15	32	18281	98315	19994	97981	21701	97617	23401	97223	25094	968	28	3
15	33	18309	9831	20022	97975	21729	97611	23429	97217	25122	96793	27	3
16	34	18338	98304	20051	97969	21758	97604	23458	9721	25151	96786	26	3
16	35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25	3
17	36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24	2
17	37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23	2
18	38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22	2
18	39	18481	98277	20193	9794	21899	97573	23599	97176	25291	96749	21	2
19	40	18509	98272	20222	97934	21928	97566	23627	97169	2532	96742	20	2
19	41	18538	98267	2025	97928	21956	9756	23656	97162	25348	96734	19	2
20	42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18	2
20	43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17	2
21	44	18624	9825	20336	9791	22041	97541	2374	97141	25432	96712	16	2
21	45	18652	98245	20364	97905	2207	97534	23769	97134	2546	96705	15	2
21	46	18681	9824	20393	97899	22098	97528	23797	97127	25488	96697	14	1
22	47	1871	98234	20421	97893	22126	97521	23825	9712	25516	9669	13	1
22	48	18738	98229	2045	97887	22155	97515	23853	97113	25545	96682	12	1
23	49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11	1
23	50	18795	98218	20507	97875	22212	97502	2391	971	25601	96667	10	1
24	51	18824	98212	20535	97869	2224	97496	23938	97093	25629	9666	9	1
24	52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8	1
25	53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7	1
25	54	1891	98196	2062	97851	22325	97476	24023	97072	25713	96638	6	1
26	55	18938	9819	20649	97845	22353	9747	24051	97065	25741	9663	5	1
26	56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4	0
27	57	18995	98179	20706	97833	2241	97457	24108	97051	25798	96615	3	0
27	58	19024	98174	20734	97827	22438	9745	24136	97044	25826	96608	2	0
28	59	19052	98168	20763	97821	22467	97444	24164	97037	25854	966	1	0
28	60	19081	98163	20791	97815	22495	97437	24192	9703	25882	96593	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		79°		78°		77°		76°		75°			

Table—(*Continued*).

Prop. parts.		15°		16°		17°		18°		19°			Prop. parts
27	′	N. sine.	N. cos.	N sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		9
0	0	25882	96593	27564	96126	29237	9563	30902	95106	32557	94552	60	9
0	1	2591	96585	27592	96118	29265	95622	30929	95097	32584	94542	59	9
1	2	25938	96578	2762	9611	29293	95613	30957	95088	32612	94533	58	9
1	3	25966	9657	27648	96102	29321	95605	30985	95079	32639	94523	57	9
2	4	25994	96562	27676	96094	29348	95596	31012	9507	32667	94514	56	8
2	5	26022	96555	27704	96086	29376	95588	3104	95061	32694	94504	55	8
3	6	2605	96547	27731	96078	29404	95579	31068	95052	32722	94495	54	8
3	7	26079	9654	27759	9607	29432	95571	31095	95043	32749	94485	53	8
4	8	26107	96532	27787	96062	2946	95562	31123	95033	32777	94476	52	8
4	9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51	8
5	10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50	8
5	11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49	7
5	12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48	7
6	13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47	7
6	14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46	7
7	15	26303	96479	27983	96005	29654	95502	31316	9497	32969	94409	45	7
7	16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44	7
8	17	26359	96463	28039	95989	2971	95485	31372	94952	33024	9439	43	6
8	18	26387	96456	28067	95981	29737	95476	31399	94943	33051	9438	42	6
9	19	26415	96448	28095	95972	29765	95467	31427	94933	33079	9437	41	6
9	20	26443	9644	28123	95964	29793	95459	31454	94924	33106	94361	40	6
9	21	26471	96433	2815	95956	29821	9545	31482	94915	33134	94351	39	6
10	22	265	96425	28178	95948	29849	95441	3151	94906	33161	94342	38	6
10	23	26528	96417	28206	9594	29876	95433	31537	94897	33189	94332	37	6
11	24	26556	9641	28234	95931	29904	95422	31565	94888	33216	94322	36	5
11	25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35	5
12	26	26612	96394	2829	95915	2996	95407	3162	94869	33271	94303	34	5
12	27	2664	96386	28318	95907	29987	95398	31648	9486	33298	94293	33	5
13	28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32	5
13	29	26696	96371	28374	9589	30043	9538	31703	94842	33353	94274	31	5
14	30	26724	96363	28402	95882	30071	95372	3173	94832	33381	94264	30	5
14	31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29	4
14	32	2678	96347	28457	95865	30126	95354	31786	94814	33436	94245	28	4
15	33	26808	9634	28485	95857	30154	95345	31813	94805	33463	94235	27	4
15	34	26836	96332	28513	95849	30182	95337	31841	94795	3349	94225	26	4
16	35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25	4
16	36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24	4
17	37	2692	96308	28597	95824	30265	9631	31923	94768	33573	94196	23	3
17	38	26948	96301	28625	95816	30292	95301	31951	94758	336	94186	22	3
18	39	26976	96293	28652	95807	3032	95293	31979	94749	33627	94176	21	3
18	40	27004	96285	2868	95799	30348	95284	32006	9474	33655	94167	20	3
18	41	27032	96277	28708	95791	30376	95275	32034	9473	33682	94157	19	3
19	42	2706	96269	28736	95782	30403	95266	32061	94721	3371	94147	18	3
19	43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17	3
20	44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16	2
20	45	27144	96246	2882	95757	30486	9524	32144	94693	33792	94118	15	2
21	46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14	2
21	47	272	9623	28875	9574	30542	95222	32199	94674	33846	94098	13	2
22	48	27228	96222	28903	95732	3057	95213	32227	94665	33874	94088	12	2
22	49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11	2
23	50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10	2
23	51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9	1
23	52	2734	9619	29015	95698	3068	95177	32337	94627	33983	94049	8	1
24	53	28368	96182	29042	9569	30708	95168	32364	94618	34011	94039	7	1
24	54	27396	96174	2907	95681	30736	95159	32392	94609	34038	94029	6	1
25	55	27424	96166	29098	95673	30763	9515	32419	94599	34065	94019	5	1
25	56	27452	96158	29126	95664	30791	95142	32447	9459	34093	94009	4	1
26	57	2248	9615	29154	95656	30819	95133	32474	9458	3412	93999	3	0
26	58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2	0
27	59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1	0
27	60	27564	96126	29237	9563	30902	95106	32557	94552	34202	93969	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		74°		73°		72°		71°		70°			

Table—(*Continued*).

Prop. parts. 27	′	20° N. sine.	20° N. cos.	21° N. sine.	21° N. cos.	22° N. sine.	22° N. cos.	23° N. sine	23° N. cos	24° N. sine	24° N. cos		Prop. parts. 11
0	0	34202	93969	35837	93358	37461	92718	39073	9205	40674	91355	60	11
0	1	34229	93959	35864	93348	37488	92707	391	92039	407	91343	59	11
1	2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58	11
1	3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57	10
2	4	34311	93929	35945	93316	37569	92675	3918	92005	4078	91307	56	10
2	5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55	10
3	6	34366	93909	36	93295	37622	92653	39234	91982	40833	91283	54	10
3	7	34393	93899	36027	93285	37649	92642	3926	91971	4086	91272	53	10
4	8	34421	93889	36054	93274	37676	92631	39287	91959	40886	9126	52	10
4	9	34448	93879	36081	93264	37703	9262	39314	91948	40913	91248	51	9
5	10	34475	93869	36108	93253	3773	92609	39341	91936	40939	91236	50	9
5	11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49	9
5	12	3453	93849	36162	93232	37784	92587	39394	91914	40992	91212	48	9
6	13	34557	93839	3619	93222	37811	92576	39421	91902	41019	912	47	9
6	14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46	8
7	15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45	8
7	16	34639	93809	36271	9319	37892	92543	39501	91868	41098	91164	44	8
8	17	34666	93799	36298	9318	37919	92532	39528	91856	41125	91152	43	8
8	18	34694	93789	36325	93169	37946	92521	39555	91845	41151	9114	42	8
9	19	34721	93779	36352	93159	37973	9251	39581	91833	41178	91128	41	8
9	20	34748	9376	36379	93148	37999	92499	39608	91822	41204	91116	40	7
9	21	34775	93759	36406	93137	38026	92488	39635	9181	41231	91104	39	7
10	22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38	7
10	23	3483	93738	36461	93116	3808	92466	39688	91787	41284	9108	37	7
11	24	34857	93728	36488	93106	38107	92455	39715	91775	4131	91068	36	7
11	25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35	6
12	26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34	6
12	27	34939	93698	36569	93074	38188	92421	39795	91741	4139	91032	33	6
13	28	34966	93688	36596	93063	38215	9241	39822	91729	41416	9102	32	6
13	29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31	6
14	30	35021	93667	3665	93042	38268	92388	39875	91706	41469	90996	30	6
14	31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29	5
14	32	35075	93647	36704	9302	38322	92366	39928	91683	41522	90972	28	5
15	33	35102	93637	36731	9301	38349	92355	39955	91671	41549	9096	27	5
15	34	3513	93626	36758	92999	38376	92343	39982	9166	41575	90948	26	5
16	35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25	5
16	36	35184	93606	36812	92978	3843	92321	40035	91636	41628	90924	24	4
17	37	35211	93596	36839	92967	38456	9231	40062	91625	41655	90911	23	4
17	38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22	4
18	39	35266	93575	36894	92945	3851	92287	40115	91601	41707	90887	21	4
18	40	35293	93565	36921	92935	38537	92276	40141	9159	41734	90875	20	4
18	41	3532	93555	36948	92924	38564	92265	40168	91578	4176	90863	19	3
19	42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18	3
19	43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17	3
20	44	35402	93524	37029	92892	38644	92231	40248	91543	4184	90826	16	3
20	45	35429	93514	37056	92881	38671	9222	40275	91531	41866	90814	15	3
21	46	35456	93503	37083	9287	38698	92209	40301	91519	41892	90802	14	3
21	47	35484	93493	3711	92859	38725	92198	40328	91508	41919	9079	13	2
22	48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12	2
22	49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11	2
23	50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10	2
23	51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9	2
23	52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8	1
24	53	35647	93431	37272	92794	38886	9213	40488	91437	42077	90717	7	1
24	54	35674	9342	37299	92784	38912	92119	40514	91425	42104	90704	6	1
25	55	35701	9341	37326	92773	38939	92107	40541	91414	4213	90692	5	1
25	56	35728	934	37353	92762	38966	92096	40567	91402	42156	9068	4	1
26	57	35755	93389	3738	92751	38993	92085	40594	9139	42183	90668	3	1
26	58	35782	93379	37407	9274	3902	92073	40621	91378	42209	90655	2	0
27	59	3581	93368	37434	92729	39046	92062	40647	91366	42235	90643	1	0
27	60	35837	93358	37461	92718	39073	9205	40674	91355	42262	90631	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine	N. cos.	N. sine.	′	
		69°		68°		67°		66°		65°			

Table—(*Continued*).

Prop. parts.	′	25°		26°		27°		28°		29°			Prop. parts.
26	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		11
0	0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60	14
0	1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59	14
1	2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58	14
1	3	42341	90594	43916	89841	45477	89061	47024	88254	48557	8742	57	13
2	4	42367	90582	43942	89828	45503	89048	4705	8824	48583	87406	56	13
2	5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55	13
3	6	4242	90557	43994	89803	45554	89021	47101	88213	48634	87377	54	13
3	7	42446	90545	4402	8979	4558	89008	47127	88199	48659	87363	53	12
3	8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52	12
4	9	42499	9052	44072	89764	45632	88981	47178	88172	4871	87335	51	12
4	10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50	12
5	11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49	11
5	12	42578	90483	44151	89726	4571	88942	47255	8813	48786	87292	48	11
6	13	42604	9047	44177	89713	45736	88928	47281	88117	48811	87278	47	11
6	14	42631	90458	44203	897	45762	88915	47306	88103	48837	87264	46	11
7	15	42657	90446	44229	89687	45787	88902	47332	88089	48862	8725	45	11
7	16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44	10
7	17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43	10
8	18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42	10
8	19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41	10
9	20	42788	90383	44359	89623	45917	88835	4746	8802	48989	87178	40	9
9	21	42815	90371	44385	8961	45942	88822	47486	88006	49014	87164	39	9
10	22	42841	90358	44411	89597	45968	88808	47511	87993	4904	8715	38	9
10	23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37	9
10	24	42894	90334	44464	89571	4602	88782	47562	87965	4909	87121	36	8
11	25	4292	90321	4449	89558	46046	88768	47588	87951	49116	87107	35	8
11	26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34	8
12	27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33	8
12	28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32	7
13	29	43025	90271	44594	89506	46149	88715	4769	87896	49217	8705	31	7
13	30	43051	90259	4462	89493	46175	88701	47716	87882	49242	87036	30	7
13	31	43077	90246	44646	8948	46201	88688	47741	87868	49268	87021	29	7
14	32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28	7
14	33	4313	90221	44698	89454	46252	88661	47793	8784	49318	86993	27	6
15	34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26	6
15	35	43182	90196	4475	89428	46304	88634	47844	87812	49369	86964	25	6
16	36	43209	90183	44776	89415	4633	8862	47869	87798	49394	86949	24	6
16	37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23	5
16	38	43261	90158	44828	89389	46381	88593	4792	8777	49445	86921	22	5
17	39	43287	90146	44854	89376	46407	8858	47946	87756	4947	86906	21	5
17	40	43313	90133	4488	89363	46433	88566	47971	87743	49495	86892	20	5
18	41	4334	9012	44906	8935	46458	88553	47997	87729	49521	86878	19	4
18	42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18	4
19	43	43392	90095	44958	89324	4651	88526	48048	87701	49571	86849	17	4
19	44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16	4
20	45	43445	9007	4501	89298	46561	88499	48099	87673	49622	8682	15	4
20	46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14	3
20	47	43497	90045	45062	89272	46613	88472	4815	87645	49672	86791	13	3
21	48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12	3
21	49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11	3
22	50	43575	90007	4514	89232	4669	88431	48226	87603	49748	86748	10	2
22	51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9	2
23	52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8	2
23	53	43654	89968	45218	89193	46767	8839	48303	87561	49824	86704	7	2
23	54	4368	89956	45243	8918	46793	88377	48328	87546	49849	8669	6	1
24	55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5	1
24	56	43733	8993	45295	89153	46844	88349	48379	87518	49899	86661	4	1
25	57	43759	89918	45321	8914	4687	88336	48405	87504	49924	86646	3	1
25	58	43785	89905	45347	89127	46896	88322	4843	8749	4995	86632	2	0
26	59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1	0
26	60	43837	89879	45399	89101	46947	88295	48481	87462	5	86603	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		64°		63°		62°		61°		60°			

Table—(*Continued*).

Prop. parts	′	30°		31°		32°		33°		34°			Prop. parts
25		N. sine.	N. cos.	N sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		11
0	0	5	86603	51504	85717	52992	84805	54464	83867	55919	82904	60	16
0	1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59	16
1	2	5005	86573	51554	85687	53041	84774	54513	83835	55968	82871	58	15
1	3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57	15
2	4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56	15
2	5	50126	8653	51628	85642	53115	84728	54586	83788	5604	82822	55	15
3	6	50151	86515	51653	85627	5314	84712	5461	83772	56064	82806	54	14
3	7	50176	86501	51678	85612	53164	84697	54635	83756	56088	8279	53	14
3	8	50201	86486	51703	85597	53189	84681	54659	8374	56112	82773	52	14
4	9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51	14
4	10	50252	86457	51753	85567	53238	8465	54708	83708	5616	82741	50	13
5	11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49	13
5	12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48	13
5	13	50327	86413	51828	85521	53312	84604	54781	8366	56232	82692	47	13
6	14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46	12
6	15	50377	86384	51877	85491	53361	84573	54829	83629	5628	82659	45	12
7	16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44	12
7	17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43	11
8	18	50453	8634	51952	85446	53435	84526	54902	83581	56353	8261	42	11
8	19	50478	86325	51977	85431	5346	84511	54927	83565	56377	82593	41	11
8	20	50503	8631	52002	85416	53484	84495	54951	83549	56401	82577	40	11
9	21	50528	86295	52026	85401	53509	8448	54975	83533	56425	82561	39	10
9	22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38	10
10	23	50578	86266	52076	8537	53558	84448	55024	83501	56473	82528	37	10
10	24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36	10
10	25	50628	86237	52126	8534	53607	84417	55072	83469	56521	82495	35	9
11	26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34	9
11	27	50679	86207	52175	8531	53656	84386	55121	83437	56569	82462	33	9
12	28	50704	86192	522	85294	53681	8437	55145	83421	56593	82446	32	9
12	29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31	8
13	30	50754	86163	5225	85264	5373	84339	55194	83389	56641	82413	30	8
13	31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29	8
13	32	50804	86133	52299	85234	53779	84308	55242	83356	56689	8238	28	7
14	33	50829	86119	52324	85218	53804	84292	55266	8334	56713	82363	27	7
14	34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26	7
15	35	50879	86089	52374	85188	53853	84261	55315	83308	5676	8233	25	7
15	36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24	6
15	37	50929	86059	52423	85157	53902	8423	55363	83276	56808	82297	23	6
16	38	50954	86045	52448	85142	53926	84214	55388	8326	56832	82281	22	6
16	39	50979	8603	52473	85127	53951	84198	55412	83244	56856	82264	21	6
17	40	51004	86015	52498	85112	53975	84182	55436	83228	5688	82248	20	5
17	41	51029	86	52522	85096	54	84167	5546	83212	56904	82231	19	5
18	42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18	5
18	43	51079	8597	52572	85066	54049	84135	55509	83179	56952	82198	17	5
18	44	51104	85956	52597	85051	54073	8412	55533	83163	56976	82181	16	4
19	45	51129	85941	52621	85035	54097	84104	55557	83147	57	82165	15	4
19	46	51154	85926	52646	8502	54122	84088	55581	83131	57024	82148	14	4
20	47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13	3
20	48	51204	85896	52696	84989	54171	84057	5563	83098	57071	82115	12	3
20	49	51229	85881	5272	84974	54195	84041	55654	83082	57095	82098	11	3
21	50	51254	85866	52745	84959	5422	84025	55678	83066	57119	82082	10	3
21	51	51279	85851	5277	84943	54244	84009	55702	8305	57143	82065	9	2
22	52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8	2
22	53	51329	85821	52819	84913	54293	83978	5575	83017	57191	82032	7	2
23	54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6	2
23	55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5	1
23	56	51404	85777	52893	84866	54366	8393	55823	82969	57262	81982	4	1
24	57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3	1
24	58	51454	85747	52943	84836	54415	83899	55871	82936	5731	81949	2	1
25	59	51479	85732	52967	8482	5444	83883	55895	8292	57334	81932	1	0
25	60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine	N. cos.	N. sine.	′	
		59°		58°		57°		56°		55°			

Table—(*Continued*).

Prop. parts.	′	35°		36°		37°		38°		39°			Prop. parts.
25	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		18
0	0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60	18
0	1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59	18
1	2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58	17
1	3	57429	81865	58849	8085	60251	79811	61635	78747	63	7766	57	17
2	4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56	17
2	5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55	17
2	6	57501	81815	5892	80799	60321	79758	61704	78694	63068	77605	54	16
3	7	57524	81798	58943	80782	60344	79741	61726	78676	6309	77586	53	16
3	8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52	16
3	9	57572	81765	5899	80748	6039	79706	61772	7864	63135	7755	51	15
4	10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50	15
4	11	57619	81731	59037	80713	60437	79671	61818	78604	6318	77513	49	15
5	12	57643	81714	59061	80696	6046	79653	61841	78586	63203	77494	48	14
5	13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47	14
5	14	57691	81681	59108	80662	60506	79618	61887	7855	63248	77458	46	14
6	15	57715	81664	59131	80644	60529	796	61909	78532	63271	77439	45	14
6	16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44	13
7	17	57762	81631	59178	8061	60576	79565	61955	78496	63316	77402	43	13
7	18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42	13
7	19	5781	81597	59225	80576	60622	7953	62001	7846	63361	77366	41	12
8	20	57833	8158	59248	80558	60645	79512	62024	78442	63383	77347	40	12
8	21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39	12
8	22	57881	81546	59295	80524	60691	79477	62069	78405	63428	7731	38	11
9	23	57904	8153	59318	80507	60714	79459	62092	78387	63451	77292	37	11
9	24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36	11
10	25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35	11
10	26	57976	81479	59389	80455	60784	79406	6216	78333	63518	77236	34	10
10	27	57999	81462	59412	80438	60807	79388	62183	78315	6354	77218	33	10
11	28	58023	81445	59436	8042	6083	79371	62206	78297	63563	77199	32	10
11	29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31	9
12	30	5807	81412	59482	80386	60876	79335	62251	78261	63608	77162	30	9
12	31	58094	81395	59506	80368	60899	79318	62274	78243	6363	77144	29	9
12	32	58118	81378	59529	80351	60922	793	62297	78225	63653	77125	28	8
13	33	58141	81361	59552	80334	60945	79282	6232	78206	63675	77107	27	8
13	34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26	8
13	35	58189	81327	59599	80299	60991	79247	62365	7817	6372	7707	25	8
14	36	58212	8131	59622	80282	61015	79229	62388	78152	63742	77051	24	7
14	37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23	7
15	38	5826	81276	59669	80247	61061	79193	62433	78116	63787	77014	22	7
15	39	58283	81259	59693	8023	61084	79176	62456	78098	6381	76996	21	6
15	40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20	6
16	41	5833	81225	59739	80195	6113	7914	62502	78061	63854	76959	19	6
16	42	58354	81208	59763	80178	61153	79122	62524	78043	63877	7694	18	5
16	43	58378	81191	59786	8016	61176	79105	62547	78025	63899	76921	17	5
17	44	58401	81174	59809	80143	61199	79087	6257	78007	63922	76903	16	5
17	45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15	5
18	46	58449	8114	59856	80108	61245	79051	62615	7797	63966	76866	14	4
18	47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13	4
18	48	58496	81106	59902	80073	61291	79016	6266	77934	64011	76828	12	4
19	49	58519	81089	59926	80056	61314	78998	62683	77916	64033	7681	11	3
19	50	58543	81072	59949	80038	61337	7898	62706	77897	64056	76791	10	3
20	51	58567	81055	59972	80021	6136	78962	62728	77879	64078	76772	9	3
20	52	5859	81038	59995	80003	61383	78944	62751	77861	641	76754	8	2
20	53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7	2
21	54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6	2
21	55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5	2
21	56	58684	8097	60089	79934	61474	78873	62842	77788	6419	76679	4	1
22	57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3	1
22	58	58731	80936	60135	79899	6152	78837	62887	77751	64234	76642	2	1
23	59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1	0
23	60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		54°		53°		52°		51°		50°			

Table—(*Continued*).

Prop. parts.	′	40°		41°		42°		43°		44°			Prop. parts.
22		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		19
0	0	64279	76604	65606	75471	66913	74314	682	73135	69466	71934	60	19
0	1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59	19
1	2	64323	76567	6565	75433	66956	74276	68242	73096	69508	71894	58	18
1	3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57	18
1	4	64368	7653	65694	75395	66999	74237	68285	73056	69549	71853	56	18
2	5	6439	76511	65716	75375	67021	74217	68306	73036	6957	71833	55	17
2	6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54	17
3	7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53	17
3	8	64457	76455	65781	75318	67086	74159	6837	72976	69633	71772	52	16
3	9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51	16
4	10	64501	76417	65825	7528	67129	7412	68412	72937	69675	71732	50	16
4	11	64524	76398	65847	75261	67151	741	68434	72917	69696	71711	49	16
4	12	64546	7638	65869	75241	67172	7408	68455	72897	69717	71691	48	15
5	13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47	15
5	14	6459	76342	65913	75203	67215	74041	68497	72857	69758	7165	46	15
6	15	64612	76323	65935	75184	67237	74022	68518	72837	69779	7163	45	14
6	16	64635	76304	65956	75165	67258	74002	68539	72817	698	7161	44	14
6	17	64657	76286	65978	75146	6728	73983	68561	72797	69821	7159	43	14
7	18	64679	76267	66	75126	67301	73963	68582	72777	69842	71569	42	13
7	19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41	13
7	20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40	13
8	21	64746	7621	66066	75069	67366	73904	68645	72717	69904	71508	39	12
8	22	64768	76192	66088	7505	67387	73885	68666	72697	69925	71488	38	12
8	23	6479	76173	66109	7503	67409	73865	68688	72677	69946	71468	37	12
9	24	64812	76154	66131	75011	6743	73846	68709	72657	69966	71447	36	11
9	25	64834	76135	66153	74992	67452	73826	6873	72637	69987	71427	35	11
10	26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34	11
10	27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33	10
10	28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32	10
11	29	64923	76059	6624	74915	67538	73747	68814	72557	7007	71345	31	10
11	30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30	10
11	31	64967	76022	66284	74876	6758	73708	68857	72517	70112	71305	29	9
12	32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28	9
12	33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27	9
12	34	65033	75965	66349	74818	67645	73649	6892	72457	70174	71243	26	8
13	35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25	8
13	36	65077	75927	66393	7478	67688	7361	68962	72417	70215	71203	24	8
14	37	651	75908	66414	7476	67709	7359	68983	72397	70236	71182	23	7
14	38	65122	75889	66436	74741	6773	7357	69004	72377	70257	71162	22	7
14	39	65144	7587	66458	74722	67752	73551	69025	72357	70277	71141	21	7
15	40	65166	75851	6648	74703	67773	73531	69046	72337	70298	71121	20	6
15	41	65188	75832	66501	74683	67795	73511	69067	72317	70319	711	19	6
15	42	6521	75813	66523	74664	67816	73491	69088	72297	70339	7108	18	6
16	43	65232	75794	66545	74644	67837	73472	69109	72277	7036	71059	17	5
16	44	65254	75775	66566	74625	67859	73452	6913	72257	70381	71039	16	5
17	45	65276	75756	66588	74606	6788	73432	69151	72236	79401	71019	15	5
17	46	65298	75738	6661	74586	67901	73413	69172	72216	70422	70998	14	4
17	47	6532	75719	66632	74567	67923	73393	69193	72196	70443	70978	13	4
18	48	65342	757	66653	74548	67944	73373	69214	72176	70463	70957	12	4
18	49	65364	7568	66675	74528	67965	73353	69235	72156	70484	70937	11	3
18	50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10	3
19	51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9	3
19	52	6543	75623	6674	7447	68029	73294	69298	72095	70546	70875	8	3
19	53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7	2
20	54	65474	75585	66783	74431	68072	73254	6934	72055	70587	70834	6	2
20	55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5	2
21	56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4	1
21	57	6554	75528	66848	74373	68136	73195	69403	71995	70649	70772	3	1
21	58	65562	75509	6687	74353	68157	73175	69424	71974	7067	70752	2	1
22	59	65584	7549	66891	74334	68179	73155	69445	71954	7069	70731	1	0
22	60	65606	75471	66913	74314	682	73135	69466	71934	70711	70711	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. sine.	N. cos.	′	
		49°		48°		47°		46°		45°			

The preceding Table contains the Natural Sine and Cosine for every minute of the quadrant to the radius of 1, and although the decimal point is not put in the Table, it is always to be prefixed.

If the Degrees are taken at the head of the columns, the minutes, sine, and cosine must be taken from the head also; and if they are taken at the foot of the column, they must be taken from the foot also.

ILLUSTRATION.—.3173 is the sine of 18° 30′, and the cosine of 71° 30′.

To Compute the Sine or Cosine for Seconds.

When the Angle is less than 45°.

Ascertain the sine or cosine of the angle for degrees and minutes from the Table; then take the difference between it and the sine or cosine of the angle next below it. Look for this difference or remainder,* if the *Sine* is required, at the head of the column of *Proportional Parts*, on the left side; and if the *Cosine* is required, at the head of the column on the right side; and in these respective columns, opposite to the number of seconds of the angle in the column, is the number or correction in seconds to be added to the Sine, or subtracted from the Cosine of the angle.

EXAMPLE.—What is the sine of 8° 9′ 10″?

Sine of 8° 9′, per Table = .14177; }
Sine of 8° 10′, " = .14205; } *difference*, .00028.

In left-side column of proportional parts, under 29, and opposite to 10′, is 5, the correction for 10′, which, being added to .14177 = .14182, *the sine.*

EX. 2.—What is the cosine of 8° 9′ 10″?

Cosine of 8° 9′, per Table = .98990; }
Cosine of 8° 10′ " = .99986; } *difference*, .00004.

In right-side column of proportional parts, under 4, and opposite to 10′, is 1, which, being subtracted from .98990 = .98989, *the cosine.*

When the Angle exceeds 45°.

Ascertain the sine or cosine for the angle in degrees and minutes from the Table, taking the degrees at the foot of it; then take the difference between it and the sine or cosine of the angle next above it.

Look for the remainder, if the Sine is required, at the head of the column of *Proportional Parts*, on the right side; and if the Cosine is required, at the head of the column on the left side; and in these respective columns, opposite to the seconds in the angle, is the number or correction in seconds to be added to the Sine, or subtracted from the Cosine of the angle.

EXAMPLE.—What is the sine of 81° 50′ 50″?

Sine of 81° 50′, per Table = .98986; }
Sine of 81° 51′, " = .98990; } *difference*, .00004.

In right-side column of proportional parts, and opposite to 50′, is 3, which, being added to .98986 = .98989, *the sine.*

EX. 2.—What is the cosine of 81° 50′ 50″?

Cosine of 81° 50′, per Table = .14205; }
Cosine of 81° 51, " = .14177; } *difference*, .00025.

In left-side column of proportional parts, and opposite to 50′, is 24, which, being subtracted from .14205 = .14181, *the cosine.*

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* The Tables in some instances, as in the example given, will give a unit too much, but this, in general, is of little importance.

To Compute the Number of Degrees, Minutes, and Seconds of a given Sine or Cosine.

When the Sine is given.

RULE.—If the given *Sine* is found in the Table, the degrees of it will be found at the top or bottom of the page, and the minutes in the marginal column, at the left or right side, according as the sine corresponds to an angle less or greater than 45°.

If the given sine is not found in the Table, take the sine in the Table which is the next *less* than the one for which the degrees, etc., are required, and note the degrees, etc., for it. Subtract this sine from the tabular sine next greater than it, and also from the given sine.

Then, as the tabular difference is to the difference between the given sine and the tabular sine, so is 60 seconds to the seconds for the sine given.

EXAMPLE.—What are the degrees, minutes, and seconds for the sine of .75000?

The next less sine is .74992, *the arc for which is* 48° 35′. The next greater sine is .75011, *the difference between which and the next less is* .75011 — .74992 = 19. *The difference between the less tabular sine and the one given is* .75000 — .74992 = 8. Then 19 : 8 : : 60 : 25 +, which, added to 48° 35′ = 48° 35′ 25″.

When the Cosine is given.

RULE.—If the given *Cosine* is found in the Table, the degrees of it will be found as in the manner specified when the *Sine* is given.

If the given cosine is not found in the Table, take the cosine in the Table which is the next *greater* than the one for which the degrees, etc., are required, and note the degrees, etc., for it. Subtract this cosine from the tabular cosine next *less* than it, and also from the given cosine.

Then, as the tabular difference is to the difference between the given cosine and the tabular cosine, so is 60 seconds to the seconds for the cosine given.

EXAMPLE.—What are the degrees, minutes, and seconds for the cosine of .75000?

The next greater cosine is .75011, *the arc for which is* 41° 24′. The next less cosine is .74992, *the difference between which and the next greater is* .75011 — 74992 = 19. *The difference between the greater tabular cosine and the one given is* .75011 — .75000 = 11. Then 19 : 11 : : 60 : 35 —, which, added to 41° 24′ = 41° 24′ 35″.

To Compute the Versed Sine of an Angle.

Subtract the cosine of the angle from 1.

EXAMPLE.—What is the versed sine of 21° 30′?

The cosine of 21° 30′ is .93042, which. — 1 = .06958, *the versed sine.*

To Compute the Co-versed Sine of an Angle.

Subtract the sine of the angle from 1.

EXAMPLE.—What is the co-versed sine of 21° 30′?

The sine of 21° 30′ is .3665, which, — 1 = .6335, *the co-versed sine.*

To Compute the Chord of an Angle.

Double the sine of half the angle.

EXAMPLE.—What is the chord of 21° 30′?

Sine of $\frac{21° 30'}{2}$ = sine of 10° 45′ = .18652, which, × 2 = .37304, *the chord.*

Table of Natural Secants and Co-secants.

SECANTS.

Prop. parts to 1″.	Deg.	0′	10′	20′	30′	40′	50′	Deg.	Prop. parts to 1′.
.004	0	1.	1.	1.00002	1.00004	1.00007	1.00011	89	.25
.013	1	1.00015	1.00021	1.00027	1.00034	1.00042	1.00051	88	.76
.021	2	1.00061	1.00072	1.00083	1.00095	1.00108	1.00122	87	1.27
.03	3	1.00137	1.00153	1.00169	1.00187	1.00205	1.00224	86	1.78
.038	4	1.00244	1.00265	1.00287	1.00309	1.00333	1.00357	85	2.3
.047	5	1.00382	1.00408	1.00435	1.00462	1.00491	1.0052	84	2.81
.056	6	1.00551	1.00582	1.00614	1.00647	1.00681	1.00715	83	3.34
.064	7	1.00751	1.00787	1.00825	1.00863	1.00902	1.00942	82	3.86
.073	8	1.00983	1.01024	1.01067	1.01111	1.01155	1.012	81	4.39
.082	9	1.01246	1.01294	1.01342	1.0139	1.0144	1.01491	80	4.94
.091	10	1.01543	1.01595	1.01649	1.01703	1.01758	1.01814	79	5.48
.101	11	1.01872	1.0193	1.01989	1.02049	1.02109	1.02171	78	6.04
.11	12	1.02234	1.02298	1.02362	1.02428	1.02494	1.02562	77	6.6
.12	13	1.02631	1.027	1.0277	1.02841	1.02914	1.02987	76	7.18
.13	14	1.03061	1.03137	1.03213	1.0329	1.03368	1.03447	75	7.77
.139	15	1.03528	1.03609	1.03691	1.03774	1.03858	1.03944	74	8.37
.15	16	1.0403	1.04117	1.04205	1.04295	1.04385	1.04477	73	8.99
.16	17	1.04569	1.04663	1.04757	1.04853	1.0495	1.05047	72	9.62
.171	18	1.05146	1.05246	1.05347	1.05449	1.05552	1.05657	71	10.26
.182	19	1.05762	1.05869	1.05976	1.06085	1.06195	1.06306	70	10.93
.194	20	1.06418	1.06531	1.06645	1.06761	1.06878	1.06995	69	11.61
.205	21	1.07114	1.07235	1.07356	1.07479	1.07602	1.07727	68	12.32
.217	22	1.07853	1.07981	1.08109	1.08239	1.0837	1.08502	67	13.04
.23	23	1.08636	1.08771	1.08907	1.09044	1.09183	1.09322	66	13.79
.243	24	1.09464	1.09606	1.09749	1.09895	1.10041	1.10189	65	14.57
.256	25	1.10338	1.10488	1.1064	1.10793	1.10947	1.11103	64	15.37
.27	26	1.1126	1.11419	1.11579	1.1174	1.11903	1.12067	63	16.21
.285	27	1.12233	1.124	1.12568	1.12738	1.1291	1.13083	62	17.07
.3	28	1.13257	1.13433	1.1361	1.13789	1.1397	1.14152	61	17.97
.315	29	1.14335	1.14521	1.14707	1.14896	1.15085	1.15277	60	18.91
.332	30	1.1547	1.15665	1.15861	1.16059	1.16259	1.16461	59	19.89
.349	31	1.16663	1.16868	1.17075	1.17283	1.17493	1.17704	58	20.91
.366	32	1.17918	1.18133	1.1835	1.18569	1.18789	1.19012	57	21.98
.385	33	1.19236	1.19462	1.19691	1.1992	1.20152	1.20386	56	23.09
.404	34	1.20622	1.20859	1.21099	1.21341	1.21584	1.2183	55	24.26
.425	35	1.22077	1.22327	1.22579	1.22833	1.23089	1.23347	54	25.49
.446	36	1.23607	1.23869	1.24134	1.244	1.24669	1.2494	53	26.78
.469	37	1.25214	1.25489	1.25767	1.26047	1.2633	1.26615	52	28.14
.493	38	1.26902	1.27191	1.27483	1.27778	1.28075	1.28374	51	29.57
.518	39	1.28676	1.2898	1.29287	1.29597	1.29909	1.30223	50	31.08
.545	40	1.30541	1.30861	1.31183	1.31509	1.31837	1.32168	49	32.68
.573	41	1.32501	1.32838	1.33177	1.33529	1.33864	1.34212	48	34.37
.603	42	1.34563	1.34917	1.35274	1.35634	1.35997	1.36363	47	36.16
.634	43	1.36733	1.37105	1.37481	1.3786	1.38242	1.38627	46	38.06
.668	44	1.39016	1.39409	1.39804	1.40203	1.40606	1.41012	45	40.08
.704	45	1.41421	1.41834	1.42251	1.42672	1.43096	1.43524	44	42.24
.742	46	1.43956	1.44391	1.44831	1.45274	1.45721	1.46173	43	44.53
.783	47	1.46628	1.47087	1.47551	1.48014	1.48491	1.48967	42	47.
.827	48	1.49448	1.49933	1.50422	1.50916	1.51414	1.51918	41	49.63
.874	49	1.52425	1.52938	1.53455	1.53977	1.54504	1.55036	40	52.45
.925	50	1.55572	1.56114	1.56661	1.57213	1.57771	1.58333	39	55.49
		60′	50′	40′	30′	20′	10′	Deg.	

CO-SECANTS.

Table—(*Continued*).

SECANTS.

Prop. parts to 1''.	Deg.	0'	10'	20'	30'	40'	50'	Deg.	Prop. parts to 1'.
.979	51	1.58902	1.59475	1.60054	1.60639	1.61229	1.61825	38	58.75
1.038	52	1.62427	1.63035	1.63648	1.64268	1.64894	1.65526	37	62.29
1.102	53	1.66164	1.66809	1.6746	1.68117	1.68782	1.69452	36	66.1
1.171	54	1.70132	1.70815	1.71506	1.72205	1.72911	1.73624	35	70.24
1.246	55	1.73445	1.75073	1.75888	1.76552	1.77303	1.78062	34	74.74
1.327	56	1.78829	1.79604	1.80388	1.8118	1.81981	1.8279	33	79.64
1.416	57	1.83608	1.84435	1.85271	1.86116	1.8697	1.87834	32	85.
1.515	58	1.88708	1.89591	1.90485	1.91388	1.92302	1.93226	31	90.87
1.622	59	1.9416	1.95106	1.96062	1.97029	1.98008	1.98998	30	97.33
1.74	60	2.	2.01014	2.02039	2.03077	2.04128	2.05191	29	104.44
1.872	61	2.0626	2.07356	2.08458	2.09574	2.10704	2.11847	28	112.33
2.017	62	2.13005	2.14178	2.15365	2.16568	2.17786	2.19019	27	121.06
2.18	63	2.20269	2.21535	2.22817	2.24116	2.25432	2.26766	26	130.8
2.362	64	2.28117	2.29487	2.30875	2.32282	2.33708	2.35154	25	141.72
2.564	65	2.3662	2.38106	2.39614	2.41142	2.42692	2.44264	24	153.84
2.797	66	2.45859	2.47477	2.49119	2.50784	2.52474	2.5419	23	167.85
2.06	67	2.5593	2.57697	2.59491	2.61313	2.63162	2.6504	22	183.6
2.36	68	2.66947	2.68839	2.70851	2.7285	2.74881	2.76945	21	201.6
3.705	69	2.79043	2.81175	2.83342	2.85545	2.87785	2.90063	20	222.29
4.104	70	2.9238	2.94372	2.97135	2.99574	3.02057	3.04583	19	246.25
4.57	71	3.07155	3.09774	3.1244	3.15154	3.1792	3.20737	18	274.19
5.117	72	3.23607	3.26531	3.29512	3.32551	3.35649	3.38808	17	307.06
5.768	73	3.4203	3.45317	3.48671	3.52094	3.55587	3.59154	16	346.09
6.549	74	3.62795	3.66515	3.70311	3.74198	3.78166	3.82222	15	392.91
7.496	75	3.8637	3.90612	3.94952	3.99393	4.03938	4.08591	14	449.77
8.662	76	4.13357	4.18238	4.23239	4.28366	4.33621	4.39012	13	519.74
10.12	77	4.44541	4.50216	4.56041	4.62023	4.68167	4.74482	12	607.21
11.975	78	4.80973	4.87649	4.94517	5.01585	5.08863	5.16359	11	718.52
14.553	79	5.24084	5.32049	5.40263	5.4874	5.57493	5.66533	10	863.21
17.602	80	5.75877	5.85539	5.95536	6.05886	6.16607	6.27719	9	1056.1
	81	6.39245	6.51208	6.63633	6.76547	6.8998	7.03962	8	
	82	7.1853	7.33719	7.49571	7.6613	7.83443	8.01564	7	
	83	8.20551	8.40466	8.61379	8.83367	9.06515	9.30917	6	
	84	9.56677	9.83912	10.1275	10.4334	10.7585	11.1045	5	
	85	11.4737	11.8684	12.2912	12.7455	13.2347	13.7631	4	
	86	14.3556	14.9579	15.6368	16.3804	17.1984	18.1026	3	
	87	19.1073	20.2303	21.4937	22.9256	24.5621	26.4504	2	
	88	28.6537	31.2576	34.3823	38.2015	42.9757	49.1141	1	
	89	57.2987	68.7574	85.9456	114.593	171.888	343.775	0	
		60'	50'	40'	30'	20'	10'	Deg.	

CO-SECANTS.

The preceding Table contains the Natural Secants and Co-secants for every ten minutes of the quadrant, to the radius of 1.

The degrees in the column on the left side and the minutes at the head of the page are for Secants, and contrariwise for Co-secants.

If the degrees are taken in the column on the left side, the minutes and seconds must be taken from the head of the page; and if they are taken from the column on the right side, the minutes and co-secant must be taken from the foot.

ILLUSTRATION.—1.16059 is the secant of 30° 30′, and the co-secant of 59° 30′.

To Compute the Secant or Co-secant for Minutes not given at the Head or Foot of the Columns.

Ascertain from the Table the Secant or Co-secant of the angle for degrees, and the next less number of minutes given in the line opposite to the degrees. Take the correction or number for one minute from the right-hand column of Proportional Parts, and opposite to the degrees given; multiply it by the number of minutes, and add the product to the result for degrees and minutes before obtained, if the Secant is required, and subtract it if the Co-secant is required.

EXAMPLE.—What is the secant of 25° 25′?

Secant of 25° 20′, per Table = 1.10640.

The correction for 1′ over 25° is 15.37, which, multiplied by 5″ = 77; and 1.10640 + 77 = 1.10717, *the secant.*

Ex. 2.—What is the co-secant for 64° 35′?

Co-secant of 64° 30′, per Table = 1.10793.

The correction for 1′ over 64° is 15.37, which, multiplied by 5″ = 77; and 1.10753 — 77 = 1.10716, *the co-secant.*

To Compute the Secant or Co-secant of Seconds.

Ascertain from the Table the Secant or Co-secant of the angle for degrees and minutes. Take the correction or number for one second from the left-hand column of Proportional Parts, and multiply it by the number of seconds; add the product to the result for degrees and minutes before obtained, if the Secant is required, and subtract it if the Co-secant is required.

EXAMPLE.—What is the secant of 22° 40′ 22″?

Secant of 22° 40′, per Table = 1.08370.

The correction for 1″ over 22° is .217, which, multiplied by .22″ = 4.77; and 1.08370 + 4.77 = 1.08375, *the secant.*

Ex. 2.—What is the co-secant of 67° 19′ 38″?

Co-secant of 67° 19′, per previous Rules = 1.08383.

The correction for 1″ over 67° is .217, which, multiplied by 38″ = 8.25; and 1.08383 — 8.25 = 1 08375, *the co-secant.*

To Compute the Secant or Co-secant of any Angle in Degrees, Minutes, and Seconds.

Divide 1 by the cosine of the angle for the Secant, and by the sine for the Co-secant.

EXAMPLE.—What is the secant of 25° 25′?

Cosine of this angle = .90321. Then 1 ÷ .90321 = 1.10716, *secant.*

Ex. 2.—What is the co-secant of 64° 35′?

Sine of this angle = .90321. Then 1′ ÷ .90321 = 1.10716, *co-secant.*

To Compute the Number of Degrees, Minutes, and Seconds of a given Secant or Co-secant.

When the Secant is given,

Proceed as by Rule, page 311, for Sines, substituting Secants for Sines.

EXAMPLE.—What is the secant for 1.56685?

The next less secant is 1.55036, *the arc for which is* 49° 50′.
The next greater secant is 1.58333, *the difference between which and the next less is* 1.58333 — 1.55036 = 3297.
Difference between the less tab. sine and the one given is 1.56685 — 1.55036 = 1649.
Then 3297 : 1649 : : 60 : 30, which, added to 49° 50′ = 49° 50′ 30″, *degrees.*

When the Co-secant is given,

Proceed as by Rule, p. 311, for Cosines, substituting Co-secants for Cosines.

Table of Natural Tangents and Cotangents.

TANGENTS.

Prop. parts to 1′.	Deg.	0′	10′	20′	30′	40′	50′	Deg.	Prop. parts to 1′.
.485	0	.	.00291	.00582	.00873	.01164	.01454	89	29.
.485	1	.01745	.02036	.02327	.02619	.0291	.03201	88	29.
.486	2	.03492	.03783	.04075	.04366	.04658	.04949	87	29.
.487	3	.05241	.05532	.05824	.06116	.06408	.067	86	29.
.488	4	.06993	.07285	.07577	.0787	.08163	.08456	85	29.
.489	5	.08749	.09042	.09335	.09629	.09923	.10216	84	29.
.491	6	.1051	.10805	.11099	.11394	.11688	.11983	83	29.
.493	7	.12278	.12574	.12869	.13165	.13461	.13758	82	30.
.496	8	.14054	.14351	.14648	.14945	.15243	.1554	81	30.
.498	9	.15838	.16137	.16435	.16734	.17033	.17333	80	30.
.501	10	.17633	.17933	.18233	.18534	.18835	.19136	79	30.
.505	11	.19438	.1974	.20042	.20345	.20648	.20952	78	30.
.509	12	.21256	.2156	.21864	.2217	.22475	.22781	77	31.
.513	13	.23087	.23393	.237	.24008	.24316	.24624	76	31.
.517	14	.24933	.25242	.25552	.25862	.26172	.26483	75	31.
.522	15	.26795	.27107	.27419	.27732	.28046	.2836	74	31.
.527	16	.28675	.2899	.29305	.29621	.29938	.30255	73	32.
.533	17	.30573	.30891	.3121	.3153	.3185	.32171	72	32.
.539	18	.32492	.32814	.33136	.33459	.33783	.34108	71	32.
.546	19	.34433	.34758	.35085	.35412	.3574	.36068	70	33.
.553	20	.36397	.36727	.37057	.37388	.3772	.38053	69	33.
.560	21	.38386	.3872	.39055	.39391	.39727	.40065	68	34.
.568	22	.40403	.40741	.41081	.41421	.41763	.42105	67	34.
.576	23	.42448	.42791	.43136	.43481	.43828	.44175	66	34.
.585	24	.44523	.44872	.45222	.45573	.45924	.46277	65	35.
.595	25	.46631	.46985	.47341	.47698	.48055	.48414	64	36.
.605	26	.48773	.49134	.49495	.49858	.50222	.50587	63	36.
.616	27	.50953	.5132	.51687	.52057	.52427	.52798	62	37.
.628	28	.53171	.53545	.53914	.54296	.54673	.55051	61	38.
.64	29	.55431	.55812	.56194	.56577	.56962	.57348	60	38.
.653	30	.57735	.58123	.58513	.58904	.59297	.59691	59	39.
.667	31	.60086	.60483	.60881	.6128	.61681	.62083	58	40.
.682	32	.62487	.62892	.63299	.63707	.64117	.64528	57	41.
.697	33	.64941	.65355	.65771	.66189	.66608	.67028	56	42.
.714	34	.67451	.67875	.68301	.68728	.69157	.69588	55	43.
.731	35	.70021	.70455	.70891	.71329	.71769	.72211	54	44.
.75	36	.72654	.731	.73547	.73996	.74447	.749	53	45.
.77	37	.75355	.75812	.76272	.76733	.77196	.77661	52	46.
.792	38	.78129	.78598	.7907	.79544	.8002	.80498	51	47.
.814	39	.80978	.81461	.81946	.82434	.82923	.83415	50	49.
.838	40	.8391	.84407	.84906	.85408	.85912	.86419	49	50.
.864	41	.86929	.87441	.87955	.88472	.88992	.89515	48	52.
.892	42	.9004	.90568	.91099	.91633	.9217	.92709	47	53.
.921	43	.92552	.93797	.94345	.94896	.95451	.96008	46	55.
.953	44	.96569	.97133	.977	.9827	.98843	.9942	45	57.
.99	45	1.	1.00583	1.0117	1.01761	1.02355	1.02952	44	59.
1.02	46	1.03553	1.04158	1.04766	1.05378	1.05994	1.06613	43	61.
1.06	47	1.07237	1.07864	1.08496	1.09131	1.0977	1.10414	42	63.
1.1	48	1.11061	1.11713	1.12369	1.13029	1.13694	1.14363	41	66.
1.15	49	1.15037	1.15715	1.16398	1.17085	1.17777	1.18474	40	69.
1.2	50	1.19175	1.19882	1.20593	1.2131	1.22031	1.22758	39	72.
		60′	50′	40′	30′	20′	10′	Deg.	

COTANGENTS.

Table—(*Continued*).

TANGENTS.

Prop. parts to 1''.	Deg.	0'	10'	20'	30'	40'	50'	Deg.	Prop. parts to 1'.
1.25	51	1.2349	1.24227	1.24969	1.25717	1.26471	1.2723	38	75.
1.31	52	1.27994	1.28764	1.29541	1.30323	1.3111	1.31904	37	78.
1.37	53	1.32704	1.33511	1.34323	1.35142	1.35968	1.368	36	82.
1.44	54	1.37638	1.38484	1.39336	1.40195	1.41061	1.41934	35	86.
1.51	55	1.42815	1.43703	1.44598	1.45501	1.46411	1.4733	34	90.
1.59	56	1.48256	1.4919	1.50133	1.51084	1.52043	1.5301	33	95.
1.68	57	1.53986	1.54972	1.55966	1.56969	1.57981	1.59002	32	100.
1.78	58	1.60033	1.61074	1.62125	1.63185	1.64256	1.65337	31	107.
1.88	59	1.66428	1.6753	1.68643	1.69766	1.70901	1.72047	30	113.
2.	60	1.73205	1.74375	1.75556	1.76749	1.77955	1.79174	29	120.
2.13	61	1.80405	1.81649	1.82906	1.84177	1.85462	1.8676	28	128.
2.27	62	1.88073	1.894	1.90741	1.92098	1.9347	1.94858	27	136.
2.44	63	1.96261	1.97681	1.99116	2.00569	2.02039	2.03526	26	146.
2.62	64	2.0503	2.06553	2.08094	2.09654	2.11233	2.12832	25	157.
2.82	65	2.14451	2.1609	2.17749	2.1943	2.21132	2.22857	24	169.
3.05	66	2.24604	2.26374	2.28167	2.29984	2.31826	2.33693	23	183.
3.31	67	2.35585	2.37504	2.39449	2.41421	2.43422	2.45451	22	199.
3.61	68	2.47509	2.49597	2.51715	2.53865	2.56046	2.58261	21	217.
3.95	69	2.60509	2.62791	2.65109	2.67462	2.69853	2.72281	20	235.
4.35	70	2.74748	2.77254	2.79802	2.82391	2.85023	2.877	19	261.
4.82	71	2.90421	2.93189	2.96004	2.98868	3.01783	3.04749	18	289.
5.36	72	3.07768	3.10842	3.13972	3.17159	3.20406	3.23714	17	322.
6.01	73	3.27085	3.30521	3.34023	3.37594	3.41236	3.44951	16	360.
6.79	74	3.48741	3.52609	3.56557	3.60588	3.64705	3.68909	15	407.
7.73	75	3.73205	3.77595	3.82083	3.86671	3.91364	3.96165	14	464.
8.9	76	4.01078	4.06107	4.11256	4.1653	4.21933	4.27471	13	534.
10.35	77	4.33148	4.38969	4.44942	4.51071	4.57363	4.63825	12	621.
12.2	78	4.70463	4.77286	4.843	4.91516	4.9894	5.06584	11	732.
14.6	79	5.14455	5.22566	5.30928	5.39552	5.48451	5.57638	10	876.
17.8	80	5.67128	5.76937	5.8708	5.97576	6.08444	6.19703	9	1068.
22.19	81	6.31375	6.43484	6.56055	6.69116	6.82694	6.96823	8	1331.
28.46	82	7.11537	7.26873	7.42871	7.59575	7.77035	7.95302	7	1708.
37.83	83	8.14435	8.34496	8.55555	8.77689	9.00983	9.2553	6	2270.
52.8	84	9.5144	9.7882	10.078	10.3854	10.7119	11.0594	5	3168.
78.8	85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	4	4728.
120.1	86	14.3007	14.9244	15.6048	16.3499	17.1693	18.075	3	7806.
	87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	2	
	88	28.6363	31.2416	34.3628	38.1885	42.9641	49.1039	1	
	89	57.29	68.7501	85.9398	114.589	171.885	343.774	0	
		60'	50'	40'	30'	20'	10'	Deg.	

COTANGENTS.

The preceding Table contains the Natural Tangents and Co-tangents for every ten minutes of the quadrant, to the radius of 1.

The degrees in the column on the left side and the minutes at the head of the page are for Tangents, and contrariwise for Cotangents.

If the degrees are taken in the column on the left side, the minutes and tangents must be taken from the head of the page; and if they are taken from the column on the right side, the minutes and cotangents must be taken from the foot.

ILLUSTRATION.—.1974 is the tangent for 11° 10′, and the cotangent for 78° 50′.

To Compute the Tangent or Cotangent for Minutes not given at the Head or Foot of the Columns.

Ascertain from the Table the Tangent or Cotangent of the angle for degrees, and the next less number of minutes given in the line opposite to the degrees. Take the correction or number for one minute from the right-hand column of Proportional Parts, and opposite to the degrees given; multiply it by the number of minutes, and add the product to the result for degrees and minutes before obtained, if the Tangent is required, and subtract it if the Cotangent is required.

EXAMPLE.—What is the tangent of 10° 15′?

Tangent of 10° 10′, per Table = .17933.

The correction for 1′ over 10° is 30, which, multiplied by 5 (15 — 10) = 150, and .17933 + 150 = .18083, *the tangent.*

Ex. 2.—What is the cotangent of 79° 45′?

Cotangent of 79° 40′ per Table = .18233.

The correction for 1′ over 40′ is 30, which, multiplied by 5 (15 — 10) = 150, and .18233 — 150 = .18083, *the cotangent.*

To Compute the Tangents or Cotangents for Seconds.

Ascertain from the Table the Tangent or Cotangent of the angle for degrees and minutes. Take the correction or number for one second from the left-hand column of Proportional Parts, and multiply it by the number of seconds; add the product to the result for degrees and minutes before obtained, if the Tangent is required, and subtract it if the Cotangent is required.

EXAMPLE.—What is the tangent of 54° 40′ 40″?

Tangent of 54° 40′, per Table = 1.41061.

The correction for 1″ over 54° is 1.44, which, multiplied by 40″ = 58, and 1.41061 + 58 = 1.41119, *the tangent.*

To Compute the Tangent or Cotangent of any Angle in Degrees, Minutes, and Seconds.

Divide the Sine of the angle by the Cosine for the Tangent, and the Cosine by the Sine for the Cotangent.

EXAMPLE.—What is the tangent of 25° 18′?

The sine of this angle = .42736; the cosine of this angle = .90408.

Then $\frac{.42736}{.90408}$ = .4727, *the tangent.*

To Compute the Number of Degrees, Minutes, and Seconds of a given Tangent or Cotangent.

When the Tangent is given,

Proceed as by Rule, page 311, for Sines, substituting Tangents for Sines.

EXAMPLE.—What is the tangent for 1.41119?

The next less tangent is 1.41061, *the arc for which is* 54° 40′.

The next greatest tangent is 1.41934, *the difference between which and the next less is* 1.41934 — 1.41061 = 873.

The difference between the less tabular tangent and the one given is 1.11061 — 1.41119 = 58.

Then 873 : 580 (58×10 for tangent of 10′) : : 60 : 40, which, added to 54° 40′ = 54° 40′ 40″.

When the Cotangent is given,

Proceed as by Rule, page 311, for Cosines, substituting Cotangents for Cosines.

MECHANICS.

MECHANICS is the science which treats of and investigates the effects of forces, the motion and resistance of material bodies, and of equilibrium: it is divided into two parts—STATICS and DYNAMICS.

STATICS treats of the equilibrium of forces or bodies at rest. DYNAMICS of the forces that produce motion, or bodies in motion.

These bodies are further divided into the *Mechanics of solid, fluid,* and *aeriform bodies;* hence the following combinations:

1. *Statics of Solid Bodies, or Geostatics.*
2. *Dynamics of Solid Bodies, or Geodynamics.*
3. *Statics of Fluids, or Hydrostatics.*
4. *Dynamics of Fluids, or Hydrodynamics.*
5. *Statics of Aeriform Bodies, or Aerostatics.*
6. *Dynamics of Aeriform Bodies, Pneumatics or Aerodynamics.*

Forces are various, and are divided into moving forces or resistances; as,

Gravity,	*Heat or Caloric,*	*Inertia,*
Muscular,	*Magnetism,*	*Cohesion,*
Elasticity and Contractility,	*Percussion,*	*Adhesion.*

STATICS.

Composition and Resolution of Forces.

When two forces act upon a body in the same or in an opposite direction, the effect is the same as if only one force acted upon it, being the sum or difference of the forces.

Hence, when a body is drawn or projected in directions immediately opposite by two or more unequal forces, it is affected as if it were drawn or projected by a single force equal to the difference between the two or more forces, and acting in the direction of the greater force.

This single force, derived from the combined action of two or more forces, is their *Resultant.*

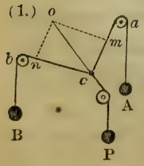
The process by which the *resultant* of two or more forces, or a single force equidistant in its effect to two or more forces, is determined, is termed the *Composition of Forces*, and the inverse operation; or, when the combined effects of two or more forces are equivalent to that of a single given force, the process by which they are determined is termed the *Decomposition or Resolution of Forces.*

Two or more forces which are equivalent to a single force are termed *Components.*

When two forces act on the same point their intensities are represented by the sides of a parallelogram, and their combined effect will be equivalent to that of a single force acting on the point in the direction of the diagonal of the parallelogram, the intensity of which is proportional to the diagonal.

ILLUSTRATION.—Attach three cords to a fixed point, c, Fig. 1; let $c\,a$ and $c\,b$ pass over fixed rollers, and suspend the weights A and B therefrom.

(1.)



The point c will be drawn by the forces A and B in the directions $a\,c$ and $b\,c$. Now, in order to ascertain which single force, P, would produce the same effect upon it, set off the distances $c\,m$ and $c\,n$ on the cords in the same proportion of length as the weights of A and B; that is, so that $c\,m : c\,n :: A : B$; then draw the parallelogram $c\,m\,o\,n$ and the diagonal $o\,c$, and it will represent a single force, P, acting in its direction, and having the same ratio to the weights A or B as it has to the sides $c\,m$ or $c\,n$ of the parallelogram. Consequently, it will produce the same effect on the point c as the combined actions of A and B.

The parallelogram, which is constructed from the lateral forces, and the diagonal of which is the mean force, is termed the *Parallelogram of Forces.*

(2.)

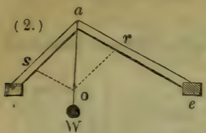


ILLUSTRATION.—Assume a weight, W, Fig. 2, to be suspended from a; then, if any distance, $a\ o$, is set off in numerical value upon the vertical line, a W, and the parallelogram, $o\ r\ a\ s$, is completed, $a\ s$ and $a\ r$, measured upon the scale $a\ o$, will represent the strain upon $a\ c$ and $a\ e$ in the same proportion that $a\ o$ bears to the weight W.

In like manner, when three or more forces are combined upon one point, it follows:

If several forces act upon the same point, and their intensities taken in order are represented by the sides of a polygon, except one, a single force proportioned to and acting in the direction of that one side will be their resultant.

Equilibrium of Forces.

Two bodies which act directly against each other in the same line are in equilibrium when their quantities of motion are equal; that is, when the product of the mass of one, into the velocity with which it moves or tends to move, is equal to the product of the mass of the other, into its actual or virtual* velocity.

When the velocities with which bodies are moved are the same, their forces are proportional to their masses or quantities of matter. Hence, when equal masses are in motion, their forces are proportional to their velocities.

The relative magnitudes and directions of any two forces may be represented by two right lines, which shall bear to each other the relations of the forces, and which shall be inclined to each other in an angle equal to that made by the direction of the forces.

(3.)

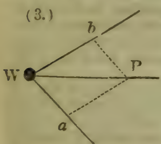


ILLUSTRATION.—Assume a body, W, to weigh 150 lbs., and resting upon a smooth surface, to be drawn by two forces, a and b, Fig. 3 = 24 and 30 lbs., which make with each other an angle; a W b = 105°, in which direction and with what acceleration will motion occur?

Cos. a W b = 105°, and cos. 180° — 105° = cos. 75°, *the mean force.*

$$P = \sqrt{30^2 + 24^2 - 2 \times 30 \times 24 \text{ cos. } 75^\circ} = \sqrt{900 + 576 - 1440 \text{ cos. } 75^\circ} = \sqrt{1476 - (1440 \times .25882)} = \sqrt{1103.3} = 33.21 \text{ } lbs.$$

The acceleration is $\frac{P g}{W} = \frac{33.21 \times 32.166}{150} = 7.1215$ *feet.*

The *Angle of Repose* is the greatest inclination of a plane to the horizon at which a body will remain in equilibrio upon it.

Hence the greatest angle of obliquity of pressure between two planes, consistent with stability, is the angle the tangent of which is equal to the co-efficient of the friction of the two planes.

Angles of Equilibrium at which various Substances will Repose, as determined by a Clinometer.

Angle measured from a Horizontal Plane.

	Degrees.		Degrees.
Lime-dust falling from a spout	45	Malt corn fall'g from a spout	37
Wheat flour " " "	44	Common mold " "	37
Malt flour " " "	40	Peas " "	35
Saw-dust " " "	44	Coarse gravel heaps	35 to 38
Dry sand " " "	40	Common gravel	35 to 36
Sand less dry " " "	39.6	Large flints	40 to 45
Wheat corn " " "	37	Flints, half size	35

* Virtual velocity is the velocity which a body in equilibrium would acquire were the equilibrium to be disturbed.

Table of Coefficients of Friction and Angles of Repose.

The Coefficient of Friction is the Tangent of the Angle of Repose measured from a Horizontal Plane.

Material.	Coefficient.	Angle.	Cotangent of Angle of Friction, or Exponent of Stability of Material.
Dry masonry and brick-work....	.6 to .7	31° to 35°	1.67 to 1.43
Timber on stone................	.4	22°	2.5
Iron on stone	.3 to .7	16° 60′ to 35°	3.35 to 1.43
Wood on wood	.2 to .5	11° 20′ to 26° 30′	5 to 2
Wood on stone	.2 to .6	11° 20′ to 31°	5 to 1.67
Metals on metals..............	.15 to .25	8° 30′ to 14°	6.67 to 4
Masonry on dry clay	.51	27°	1.96
Masonry on moist clay	.33	18° 15′	3.
Earth on earth	.25 to 1.	14° to 45°	4 to 1
Earth on dry sand, clay, and earth	.38 to .75	21° to 37°	2.63 to 1.33
Earth on earth, damp clay	1.	45°	1.
Earth on earth, wet clay	.31	17°	3.23
Earth on earth, shingle, or gravel	.81 to 1.11	39° to 48°	1.23 to .9
Dry earth........................	.81	39°	1.24
Fine sand.......................	.6	31°	1.67

Revetment Walls.

When a wall sustains the pressure of earth, sand, or any loose material, it is called a revetment wall.

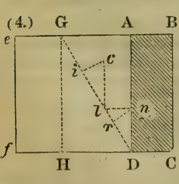
The thrust of earth, etc., upon a wall is caused by a certain portion, in the shape of a wedge, tending to break away from the general mass. The pressure thus caused is similar to that of water, but the weight of the material must be reduced by a particular ratio dependent upon the angle of natural slope, which varies from 45° to 60° (measured from the vertical) in earth of mean quality.

The angle which the line of rupture makes with the vertical is one half of the angle which the line of natural slope, or *angle of repose*, makes with the same vertical line. When the earth is level at the top, the pressure of the earth may be ascertained by considering it as a fluid, the weight of a cubic foot of which is equal to the weight of a cubic foot of the earth, multiplied by the square of the tangent of half the angle included between the natural slope and the vertical.

Therefore the squares of the tangents of .5 of 45° and .5 of 60° = 22° 30′, and .3333 = .1716, which are the multipliers to be used in ordinary cases to reduce a cubic foot of the material to a cubic foot of equivalent fluid, will have the same effect as the earth by its pressure upon the wall.

Pressure of Earth against Revetment Walls.

(4.)



Let A B C D, Fig. 4, be the vertical section of a revetment wall, behind which is a bank of earth, A D *f e*; let D G represent the *angle of repose*, the *line of rupture*, or *natural slope* which the earth would assume but for the resistance of the wall.

In sandy or loose earth the angle G D H is generally 30°; in firmer earth it is 36°; and in some instances it is 45°.

If the upper surface of the earth and the wall which supports it are both in one horizontal plane, then the resultant, *l n*, of the pressure of the bank, behind a vertical wall, is at a distance D *n* of ⅓ A D.

Equilibrium of Piers.

For a Diagram, Formula, and Illustration, see *Gregory's Mathematics*, p. 220, 221.

Thickness of Walls, both Faces Vertical.

Wall of Bricks.—Weight of a cubic foot, 109 lbs. avoirdupois, bank of vegetable earth behind it, A B = .16 A D.

Wall of uncut Stone.—135 lbs. per cubic foot, bank as before, A B = .15 A D.

Wall of Bricks.—Bank of clay, well rammed, A B = .17 A D.

Wall of cut Freestone.—170 lbs. per cubic foot, bank of vegetable earth, A B = .13 A D; if the bank is of clay, A B = .14 A D.

Wall of Bricks.—Bank of sand, A B = .33 A D.

Wall of uncut Stone.—Bank of sand, A B = .3 A D.

Wall of cut Freestone.—Bank of sand, A B = .26 A D.

Wall of Bricks.—Bank of earth and gravel = .19 A D.

Wall of uncut Freestone.—Bank of earth and gravel = .17 A D.

Wall of cut Freestone.—Bank of earth and gravel = .16 A D.

Wall of cut Stone.—Bank of common earth = .13 A D.

Wall of cut Stone.—Bank of sand = .26 A D.

The Friction in vegetable earths is .5; the pressure : in sand .4.

When vegetable earths are cut in turfs and well laid in courses, the thrust is reduced .66.

NOTE.—When the bank is liable to be saturated with water, the thickness of the wall should be doubled.

The *Line of Rupture* behind a wall supporting a bank of vegetable earth is at a distance, A G, from the interior face, A D = .618 the height of it.

When the bank is of sand, A G = .677 h; when of earth and small gravel = .646 h; and when of earth and large gravel = .618 h.

The prism, the vertical section of which is A D G, has a tendency to descend along the inclined plane, G D, by its gravity; but it is retained in its place by the resistance of the wall, and by its cohesion to and friction upon the face, G D. Each of these forces may be resolved into one which will be perpendicular to G D, and into another which will be parallel to G D. The lines $c\ i$, $i\ l$ represent the components of the force of gravity, which is represented by the vertical line $c\ l$, drawn from the centre of gravity, c, of the prism. The lines $n\ r$, $l\ r$ represent the components of the forces of cohesion and friction, which is represented by the horizontal line $n\ l$. The force that gives the prism a tendency to descend is $i\ l$, and the force opposed to this is $r\ l$, together with the effects of cohesion and friction.

Thus $i\ l = r\ l +$ cohesion + friction. Consequently the solution of problems of this nature must be in a great measure experimental.

It has been found, however, and confirmed experimentally, that the angle formed with the vertical, by the prism of earth that exerts the greatest horizontal stress against a wall, is *half the angle* which the *angle of repose* or *natural slope* of the earth makes with the vertical.

The condition of equilibrium, therefore, of a vertical *Revetment* or *Retaining Wall* exposed to the thrust of a bank of earth is, $.5\,A\,D \times D\,C^2 \times S = .166\,A\,D^3 \times S \times \tan.^2 .5\,A\,D\,G$,

S and s *representing the specific gravities or weights of the wall and earth.*

Then the above equation becomes $.166 \times A\,D^3 \times s \times \tan.^2 \frac{A\,D\,G}{2} = \frac{A\,D}{2} \times D\,C^2 \times S$.

Or, $A\,D \sqrt{\frac{\tan.^2 .5\,A\,D\,G \times s}{3\,S}} \times$ *breadth*, or D C.

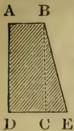
ILLUSTRATION.—A revetment wall, Fig. 4, having a specific gravity of 2000, is 40 feet in height, and it sustains earth of a specific gravity of 1428, having a natural slope of 52° 24′; what should be the thickness of the wall to have equilibrium?

$$\text{Tangent}^2\ .52^\circ\ 24' \div 2 = .242.$$

$$40\sqrt{\frac{.242 \times 1428}{3 \times 2000}} = 40\sqrt{\frac{345.576}{6000}} = 40\sqrt{.0576} = 40 \times .24 = 9.6\ \textit{feet}.$$

When the Wall has the Section of a Prismoid, or an Exterior Slope or Batter, as B E—Fig. 5.

(5.)



$$\sqrt{\frac{3\,(D\,C + C\,E)^2 s - A\,D^2 \times S \times \tan.^2 \frac{A\,D\,G}{2}}{s}} = \textit{batter}, \text{ or } C\,E.$$

ILLUSTRATION.—A trapezoidal embankment, Fig. 5, has a depth of 12 feet, and a bottom width of 4 feet: what should be the width of the crown, the weight of the material being 100 lbs. per cubic foot, and the angle of repose of the bank 45°?

Tangent^2 45° $\div$ 2 = .1714.

$$\sqrt{\left(\frac{3 \times 4^2 \times 100 - 12^2 \times 62.5 \times .1714}{100}\right)} = \sqrt{\frac{719.88}{100}} = \sqrt{7.2} = 2.68 \textit{ feet}.$$ Consequently, $4 - 2.68 = 1.32$ *feet, the top width.*

When the Wall has the Section of a Prismoid, or an Interior Slope.

$$A\,D \sqrt{\frac{1}{3\,n^2} + \tan.^2 \frac{A\,D\,G}{2} \times \frac{s}{S}} - \frac{A\,D}{n} = \textit{breadth}, \text{ or } D\,C,$$ *n representing the base of the slope, or the* $\frac{1}{n}$ *of the height.*

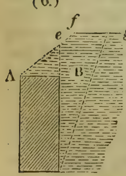
NOTE.—When the co-efficient of friction is known, use it for $\tan.^2 \frac{A\,D\,G}{2}$.

ILLUSTRATION.—The height of a wall is 20 feet, the slope of the base is $\frac{1}{20}$ of the height, the specific gravities of the bank and wall are 14 and 26, and the coefficient of the material is .166; what must be the thickness of the wall at the crown?

$$20\sqrt{\frac{1}{3 \times 20^2} + .166 \times \frac{14}{26}} - \frac{20}{20} = 20\sqrt{.000833 + .089385} - 1 = 20 \times .3004 - 1 = 5.008$$ *feet; and the thickness at bottom will be* $5.008 + \frac{1}{20}$ *of* $20 = 5.008 + 1 = 6.008$ *feet.*

Surcharged Revetments.

(6.)



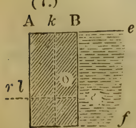
When the earth stands above the wall, A B C, Fig. 6, with its natural slope A f, A B C is termed a *Surcharged Revetment*, C g being the line of rupture, and therefore A $f g$ C is the part of the earth that presses upon the wall, which part must be taken into the calculation, with the exception of the portion A B e, which rests upon the wall; that is, the calculation must be for the part C $e f g$, which must be reduced to its equivalent quantity of fluid by multiplying the weight of a cubic foot of it by the square of the tangent of the angle e C g = the angle of the line of rupture, or half the angle which the natural slope makes with the vertical, and then proceed as in the previous cases.

Embankments and Walls.

To Compute the Conditions of Equilibrium of Embankments or Walls sustaining Water.

When both Faces are Vertical—Fig. 7.

(7.)



Assume the Perpendicular embankment or wall, A B C D, to sustain the pressure of the water, B C e f.

Let $k\,i$ be a vertical line passing through o, the centre of gravity of the wall, c the centre of pressure of the water, the distance C c being = ⅓ B C. Draw $c\,l$ perpendicular to A D; then, since the section A C of the wall is rectangular, the centre of gravity, o, is in the centre of the wall, and therefore D i = ½ D C. Now l D i is to be considered as a bent lever, the fulcrum of which is D, the weight of the wall acting in the direction of the centre of gravity, o, on the arm D i, and the pressure of the water on the arm D i, or a force equal to that pressure thrusting in the direction $l\,r$.

Put P = *pressure of the water, and* W *the weight of the wall.*

$$\text{Then } P \times D\,h = P \times \frac{B\,C}{3} = W \times \frac{D\,C}{2}, \text{ or } P = \frac{3\,D\,C\,.\,W}{2\,B\,C}.$$

NOTE.—When this equation holds, the wall or embankment will just be on the point of overturning; but in order that the wall may have complete stability, this equation should give a larger value to P than its actual amount.

The following formulæ are for embankments of one foot in length; for, if they have stability for that length, they will be stable for any other length.

Let *a represent depth of water and embankment, which are here supposed to be equal, b breadth of the embankment,* S *weight of water, and s that of the wall per cubic foot.*

Then $P = \frac{a^2}{2} \times 1 \times S$, also $W = a \times b \times 1 \times s$, each value being for 1 foot in length, which being substituted in the above equation, there will result

$$\frac{a^2}{2}S = \frac{3\,b \times a\,b\,s}{2\,a}, \text{ or } a^2\,S = 3\,b^2\,s;\ \ b\sqrt{\frac{3\,s}{S}} = a, \text{ and } a\sqrt{\frac{S}{3\,s}} = b;$$

which gives the breadth of an embankment or retaining wall that will just sustain the pressure of the water; the wall must therefore be made broader than shown by this equation, to give it due stability.

ILLUSTRATION.—The height of a wall, B C, equal to the depth of the water, is 12 feet, and the respective weights of the water and the wall are 62.5 lbs. and 120 lbs. per cubic foot; required the breadth of the wall, so that it may have complete stability to sustain the pressure of fresh water.

$$b = a\sqrt{\frac{S}{3\,s}} = 12\sqrt{\frac{62.5}{3 \times 120}} = 12 \times .4166 = 5\ \textit{feet},$$

the breadth that will just sustain the pressure of the water; therefore 1 foot should be added to this to give the wall complete stability; hence 5 + 1 = 6, *the required width of the wall.*

ILLUS. 2.—The width of a wall is 3 feet, and the weight of a cubic foot of it is 150 lbs.; required the height of the wall to resist the pressure of fresh water at the top.

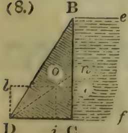
$$a = b\sqrt{\frac{3s}{S}} = 3\sqrt{\frac{3 \times 150}{62.5}} = 3 \times 2.683 = 8.069\ \textit{feet}.$$

ILLUS. 3.—Required the thickness of a rectangular embankment or retaining wall, the height being 12 feet, and the weight of a cubic foot of its material 133.33 lbs., so that it may have just sufficient stability to retain its equilibrium against sea-water.

$$12\sqrt{\frac{64}{133.33 \times 3}} = 12\sqrt{\frac{64}{400}} = 12\sqrt{.16} = 12 \times .4 = 4.8\ \textit{feet}.$$

When the Section is a Triangle—Fig. 8.

(8.)



Assume the Triangular embankment or wall, B C D, to sustain the pressure of the water, B C $e\,f$.

Draw D n bisecting B C in n; from the centre of pressure, c, draw $c\,l$, perpendicular to B C, cutting D n in o, which is the centre of gravity of the triangular section of the wall; also, draw $o\,i$ D l respectively perpendicular to D C, $c\,l$. Now l D i is to be considered as a bent lever, the fulcrum of which is D, the pressure of the water acting at l, and the weight of the wall in the direction of the centre of gravity, o, on D i.

Put B C = a, D C = b, and the weights per cubic foot of the water and wall, S and s, as in the preceding cases.

Then $c\,C = o\,i = l\,D = \frac{1}{3}\,a$, and consequently $D\,i = \frac{2}{3}\,D\,C = \frac{2}{3}\,b$; the weight of 1 foot in length of the wall $= \frac{1}{2}\,a\,b\,s$, and the pressure at c of the same length of water $= \frac{1}{2}\,a^2\,S$; hence $.666\,b \times .5\,a\,b\,s = .333\,a \times .5\,a^2\,S$; whence $a\sqrt{\frac{S}{2\,s}} = b$, and $b\sqrt{\frac{2\,s}{S}} = a$.

NOTE.—The embankment, B C D, has equal resistance at any portion of its height for the corresponding depth of water.

2.—An embankment or retaining wall with a triangular section as above has greater resistance than one with a rectangular section for equal heights, and like volumes and qualities of materials, in the proportion of 8 to 3.

ILLUSTRATION.—There is a triangular embankment of brick-work, weighing 117 lbs. per cubic foot; its depth is 14 feet; required its width at the base to resist the pressure of fresh water standing at the surface.

$$D\,C = b = a\sqrt{\frac{S}{2\,s}} = 14\sqrt{\frac{62.5}{2 \times 117}} = 14 \times .517 = 7.238\,\textit{feet}.$$

Hence the breadth of the base of the embankment must be at least 8 feet to insure perfect stability.

When the Surface of the Water is below the Top of the Wall.

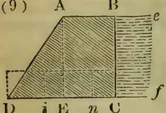
Put d = depth of the water; then $c = .5\,d^2\,S$, and $W = .5\,a\,b\,s$, as before; therefore $.666\,b \times .5\,a\,b\,s = .333\,d \times .5\,d^2\,S$; whence $d\sqrt{\frac{d\,S}{2\,a\,s}} = b$.

ILLUSTRATION.—A triangular embankment is 12 feet in depth; the weight of the material is 130 lbs. per cubic foot; required its width at the base to resist the pressure of fresh water 10.5 feet deep.

$$10.5\sqrt{\frac{10.5 \times 62.5}{2 \times 12 \times 130}} = 10.5\sqrt{\frac{656.25}{3120}} = 10.5 \times .458 = 4.809\,\textit{feet}.$$

When the Wall has the Section of a Prismoid, or an Exterior Slope or Batter, as A D—Fig. 9.

(9)



Assume the Prismoidal embankment, A B C D, to sustain the pressure of the water, B C $e\,f$.

Draw A E perpendicular to D C; put B C, as before $= a$, the top breadth $A\,B = E\,C = b$, and the bottom width, D E, of the sloping part, $A\,E\,D = c$.

Then the weights of the portions A C and A E D respectively for one foot in length are $a\,b\,s$ and $.5\,a\,c\,s$, these weights acting at the points n and i respectively.

Now $D\,n = D\,i + .5\,E\,C = c + .5\,b$, and $D\,i = .666\,D\,E = .666\,c$; hence the sum of the moments of the embankment is $a\,b\,s\,(c + .5\,b) + .5\,a\,c\,s \times .666\,c = .5\,(b^2 + 2\,b\,c + .666\,c^2)\,a\,s$, which must be equal to the pressure of the water. $\therefore .5\,(b^2 + 2\,b\,c + .666\,c^2)\,a\,s = .333\,a \times .5\,a^2\,S$, or $(b^2 + 2\,b\,c + .666\,c^2)\,s = .333\,a^2\,S$.

Hence, *when the depth, a, and the bottom width, b + c, are given,*

$$\sqrt{\left(\frac{3\,(+c)^2\,s - a^2\,S}{s}\right)} = c.$$

ILLUSTRATION.—A trapezoidal embankment has a depth of 12 feet, and a bottom width of 6 feet; required the top width, to resist the pressure of an equal depth of fresh water, the weight of the material being 100 lbs. per cubic foot.

$$\sqrt{\left(\frac{3 \times 6^2 \times 100 - 12^2 \times 62.5}{100}\right)} = \sqrt{\frac{1800}{100}} = \sqrt{18} = 4.24\,\textit{feet}.$$

Consequently, $6 - 4.24 = 1.76$ *feet, the top width.*

NOTE.—It frequently occurs that the face of an embankment has also a slope or batter; in this case the section of the embankment is to be divided into two triangles and a parallelogram, and the moments of the several parts added together, as in the last problem.

MECHANICAL POWERS.

POWER is a compound of *weight*, or *force* and *velocity:* it can not be increased by mechanical means.

The Powers are three in number—viz., LEVER, INCLINED PLANE, and PULLEY.

NOTE.—The Wheel and Axle is a *continuous* or *revolving lever*, the Wedge a *double* inclined plane, and the Screw a *revolving* inclined plane.

LEVER.

LEVERS are straight, bent, curved, single, or compound.

To Compute the Length of a Lever, the Weight and Power being given.

RULE.—Divide the weight by the power, and the quotient is the difference of leverage, or the distance from the fulcrum at which the power supports the weight.

Or, $\frac{W}{P}=p$, W *representing the weight*, P *the power, and p the distance of the power from the fulcrum.*

EXAMPLE.—A weight of 1600 lbs. is to be raised by a power or force of 80 lbs.; required the length of the longest arm of the lever, the shortest being 1 foot.

$$\frac{1600}{80}=20 \textit{ feet.}$$

To Compute the Weight that can be raised by a Lever, its Length, the Power, and the Position of its Fulcrum being given.

RULE.—Multiply the power by its distance from the fulcrum, and divide the product by the distance of the weight from the fulcrum.

$$\text{Or, } \frac{P \times p}{w}=W.$$

EXAMPLE.—What weight can be raised by a power of 375 lbs. suspended from the end of a lever 8 feet from the fulcrum, the distance of the weight from the fulcrum being 2 feet?

$$\frac{375 \times 8}{2}=1500 \textit{ lbs.}$$

To Compute the Position of the Fulcrum, the Weight and Power and the Length of the Lever being given.

When the Fulcrum is between the Weight and the Power.

RULE.—Divide the weight by the power, add 1 to the quotient, and divide the length by the sum thus obtained.

Or, $L \div \left(\frac{W}{P}+1\right)=l$, *l representing length of lever between the weight and fulcrum.*

EXAMPLE.—A weight of 2460 lbs. is to be raised with a lever 7 feet long and a power of 300 lbs.; at what part of the lever must the fulcrum be placed?

$\frac{2460}{300}=8.2$, and $8.2+1=9.2$. Then $84\ (7 \times 12) \div 9.2=9.13$ *inches.*

When the Weight is between the Fulcrum and the Power.

RULE.—Divide the length by the quotient of the weight, divided by the power.

$$\text{Or, } L \div \frac{W}{P}=l.$$

To Compute the Length of Arm of the Lever to which the Weight is attached, the Weight, Power, and Length of Arm of the Lever to which the Power is applied being given.

RULE.—Multiply the power by the length of the arm to which it is applied, and divide the product by the weight.

$$\text{Or, } \frac{P \times d}{W} = d'.$$

EXAMPLE.—A weight of 1600 lbs. suspended from the fulcrum of a lever is supported by a power of 80 lbs. applied at the other end of the arm, 20 feet in length; what is the length of the arm?

$$\frac{80 \times 20}{1600} = 1.\ foot.$$

NOTE.—These rules apply equally *when the fulcrum (or support) of the lever is between the weight and the power;* when the fulcrum is at one extremity of the lever, and the power, or the weight, at the other;† and when the arms of the lever are equally or unequally bent or curved.*

To Compute the Power required to Raise a given Weight, the Length of the Lever and the Position of the Fulcrum being given.

RULE.—Multiply the weight to be raised by its distance from the fulcrum, and divide the product by the distance of the power from the fulcrum.

$$\text{Or, } \frac{W \times w}{p} = P.$$

EXAMPLE.—The length of a lever is 10 feet, the weight to be raised is 3000 lbs., and its distance from the fulcrum is 2 feet; what is the power required?

$$\frac{3000 \times 2}{10 - 2} = \frac{6000}{8} = 750\ lbs$$

To Compute the Length of Arm of the Lever to which the Power is applied, the Weight, Power, and Distance of the Fulcrum being given.

RULE.—Multiply the weight by its distance from the fulcrum, and divide the product by the power.

$$\text{Or, } \frac{W \times w}{P} = p.$$

EXAMPLE.—A weight of 400 lbs. suspended 15 inches from the fulcrum is supported by a power of 50 lbs. applied at the other; what is the length of the arm?

$$\frac{400 \times 15}{50} = 120\ inches.$$

NOTE.—When the arms of a lever are bent or curved, the distances taken from perpendiculars, drawn from the lines of direction of the weight and power, must be measured on a line running horizontally through the fulcrum.

The GENERAL RULE, therefore, for ascertaining the relation of POWER TO WEIGHT in a lever, whether it be straight or curved, is,

The power multiplied by its distance from the fulcrum is equal to *the weight multiplied by its distance from the fulcrum.*

Or, $P : W :: w : p$, or $P \times p = W \times w$; and

1. $\frac{W \times w}{p} = P.$ 2. $\frac{P \times p}{w} = W.$ 3. $\frac{W \times w}{P} = p.$ 4. $\frac{P \times p}{W} = w.$

* The pressure upon fulcrum is equal to the sum of the weight and the power.

† The pressure upon fulcrum is equal to the difference of the weight and the power.

If several weights or powers act upon one or both ends of a lever, the condition of equilibrium is

$$P \times p + P' \times p' + P'' \times p'', \text{ etc.}, = W \times w + W' \times w', \text{ etc.}$$

In a system of levers, either of similar, compound, or mixed kinds, the condition is

$$\frac{P \times p \times p' \times p''}{w \times w' \times w''} = W.$$

ILLUSTRATION.—Let $P = 1$ lb., p and p' each 10 feet, p'' 1 foot; and if w and w' be each 1 foot, and w'' 1 inch, then

$\frac{1 \times 120 \times 120 \times 12}{12 \times 12 \times 1} = \frac{172800}{144} = 1200$; *that is,* 1 *lb. will support* 1200 *lbs.* with levers of the lengths above given.

NOTE.—The weights of the levers in the above formulæ are not considered, the centre of gravity being assumed to be over the fulcrums.

WHEEL AND AXLE.

A WHEEL AND AXLE is a revolving lever.

The power, multiplied by the radius of the wheel, is equal to the weight, multiplied by the radius of the axle.

Or, $P \times R = W \times r$. Or, $P \times V = W \times v$, R *and* r *representing the radii, and* V *and* v *the velocities of wheel and axle.*

As the radius of the wheel is to the radius of the axle, so is the effect to the power.

$$\text{Or, } R : r :: W : P.$$

When a series of wheels and axles act upon each other, either by belts or teeth, the weight or velocity will be to the power or unity as the product of the radii, or circumferences of the wheels, to the product of the radii, or circumferences of the axles.

EXAMPLE.—If the radii of a series of wheels are 9, 6, 9, 10, and 12, and their pinions have each a radius of 6 inches, and the power applied is 10 lbs., what weight will it raise?

$$\frac{10 \times 9 \times 6 \times 9 \times 10 \times 12}{6 \times 6 \times 6 \times 6 \times 6} = 75 \textit{ lbs.}$$

Or, if the 1st wheel make 10 revolutions, the last will make 75 in the same time.

To Compute the Power of a Combination of Wheels and an Axle or Axles, as in Cranes, etc., etc.

RULE.—Divide the product of the driven teeth by the product of the drivers, and the quotient is their relative velocity, which, multiplied by the length of the winch and the power applied to it in lbs., and divided by the radius of the barrel, will give the weight that can be raised.

Or, $\frac{v\,w\,P}{r} = W$, *w representing length of winch, r radius of barrel.*

Or, $W r = v\,w\,P$, P " *power.*

Or, $\frac{W r}{v\,w} = P$, v " *velocity, and* W *weight.*

EXAMPLE.—A power of 18 lbs. is applied to the winch of a crane, the length of it being 8 inches; the pinion having 6, the driving-wheel 72 teeth, and the barrel 6 inches diameter.

$\frac{72}{6} = 12$, and $12 \times 8 \times 18 = 1728$, which, $\div 3$, *the radius of the barrel* $= 576$ *lbs.*

EX. 2.—A weight of 94 tons is to be raised 360 feet in 15 minutes, by a power the velocity of which is 220 feet per minute; what is the power required?

$360 \div 15 = 24$ *feet per minute.* Hence $\frac{24 \times 94}{220} = 10.2542$ *tons.*

COMPOUND AXLE, OR CHINESE WINDLASS.

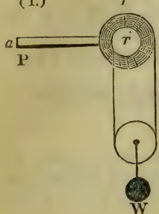
The axle or drum of the windlass consists of two parts, the diameter of one being less than that of the other.

The operation is thus: At a revolution of the axle or drum, a portion of the sustaining rope or chain equal to the circumference of the larger axle, is wound up, and at the same time a portion equal to the circumference of the lesser axle is unwound. The effect, therefore, is to wind up or shorten the rope or chain, by which a weight or stress is borne, by a length equal to the difference between the circumferences of the two axles. Consequently, half that portion of the rope or chain will be shortened by half the difference between the circumferences.

To Compute the Elements of a Wheel and Compound Axle, or Chinese Windlass—Fig. 1.

RULE.—Multiply the power by radius of the wheel, arm, or bar to which it is applied, and divide the product by half the difference of the radii of the axle, and the quotient is the weight that can be sustained.

(1.)



$$\text{Or, } \frac{P \times R}{\frac{1}{2}(r - r')} = W.$$

$P \times R = W \times \frac{1}{2}(r - r')$, R *representing radius of wheel, etc., and* r *and* r' *radii of axle at its greatest and least diameter.*

EXAMPLE.—What weight can be raised by a capstan, the radius of its bar, a, being 5 feet, the power applied 50 lbs., and the radii, $r\ r'$, of the axle or drum 6 and 5 inches?

$\frac{50 \times \overline{5 \times 12}}{\frac{1}{2}(6-5)} = \frac{3000}{.5} = 6000$ *lbs.* = *product of power and length of bar in inches* ÷ ½ *difference of radii of axle.*

WHEEL AND PINION COMBINATIONS, OR COMPLEX WHEEL-WORK.

The power, multiplied by the product of the radii or circumferences, or number of teeth of the wheels, is equal to the weight, multiplied by the product of the radii or circumferences, or number of leaves of the pinions.

$$\text{Or, } P \times R \times R' \times R'', \text{ etc.} = W \times r \times r' \times r'', \text{ etc.}$$

NOTE.—The cogs on the face of the wheel are termed *teeth*, and those on the surface of the axle are termed *leaves*; the axle itself in this case is termed a *pinion.*

RACK AND PINION.

To Compute the Power of a Rack and Pinion.

RULE.—Multiply the weight to be sustained by the quotient of the radius of the pinion, divided by the radius of the crank, and the product is the power required.

$$\text{Or, } W \times \frac{r}{R} = P.$$

When the Pinion on the Crank Axle communicates with a Wheel and Pinion.

RULE.—Multiply the weight to be sustained by the quotient of the product of radii of the pinions, divided by the radii of the crank and the wheel, and the product is the power required.

$$\text{Or, } W \times \frac{r \times r'}{R \times R'} = P.$$

EXAMPLE.—The radii of the pinions of a jack-screw are each 1 inch; of the crank and wheel 10 and 5 inches; what power will sustain a weight of 750 lbs.?

$$750 \times \frac{1 \times 1}{10 \times 5} = \frac{750}{50} = 15 \textit{ lbs}$$

INCLINED PLANE.

To Compute the Length of the Base, Height, or Length, when any Two of them are given.

When the Line of Direction of the Power or Traction is Parallel to the Face of the Plane.

Proceed as in Mensuration or Trigonometry to determine the side of a right-angled triangle, any two of the three being given.

To Compute the Power necessary to Support a Weight on an Inclined Plane, the Height and Length being given.

RULE.—Multiply the weight by the height of the plane, and divide the product by the length.

Or, $\frac{W \times h}{l} = P$, *h and l representing the height and length of the plane*

EXAMPLE.—What is the power necessary to support 1000 lbs. on an inclined plane 4 feet high and 6 feet in length?

$$\frac{1000 \times 4}{6} = 666.67 \textit{ lbs.}$$

To Compute the Weight that may be Sustained by a given Power on an Inclined Plane, the Height and Length of the Plane being given.

RULE.—Multiply the power by the length of the plane, and divide the product by the height.

$$\text{Or, } \frac{P \times l}{h} = W.$$

EXAMPLE.—What is the weight that can be sustained on an inclined plane 5 feet high and 7 feet in length by a power of 700 lbs.?

$$\frac{700 \times 7}{5} = 980 \textit{ lbs.}$$

NOTE.—In estimating the power required to overcome the resistance of a body, being drawn up or supported upon an inclined plane, and contrariwise, if the body is descending; the weight of the body, in the proportion of the power of the plane (*i. e.*, as its length to its height), must be added to the *resistance* if being drawn up or supported, or to the *moment* if descending.

To Compute the Height or Length of an Inclined Plane, the Weight and Power and one of the required Elements being given.

When the Height is required.

RULE.—Multiply the power by the length, and divide the product by the weight.

When the Length is required.

RULE.—Multiply the weight by the height, and divide the product by the power.

$$\text{Or, } \frac{P \times l}{W} = h, \text{ and } \frac{W \times h}{P} = l.$$

To Compute the Pressure on an Inclined Plane.

Rule.—Multiply the weight by the length of the base of the plane, and divide the product by the length of the face.

Or, $\frac{W \times b}{l}$ = *pressure, b representing length of base of plane.*

Example.—The weight on an inclined plane is 100 lbs., the base of the plane is 4 feet, and the length of it 5; required the pressure on the plane.

$$\frac{100 \times 4}{5} = 80 \text{ lbs.}$$

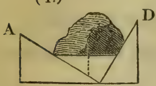
When two Bodies on two Inclined Planes sustain each other, as by the Connection of a Cord over a Pulley, their Weights are directly as the Lengths of the Planes.

Illustration.—If a weight of 50 lbs. upon an inclined plane, of 10 feet rise in 100, be sustained by a weight on another plane of 10 feet rise in 90 of an inclination, what is the weight of the latter?

Therefore 100 : 90 : : 50 : 45, *the weight that on the shortest plane would sustain that on the largest*

When a Body is Supported by two Planes, as Fig. 1.

(1.)



The pressure upon them will be reciprocally as the sines of the inclinations of the planes.

Thus the weight is as sin. A B D.
The pressure on A B as sin. D B I.
The pressure on B D as sin. A B H.

Assume the angle A B D to be 90°, and D B I, 60; then the angle A B H will be 30°; and as the sines of 90°, 60°, and 30° are respectively .1, .866, and .5. if the weight = 100 lbs., then the pressures on A B and D B will be 86.6 and 50 lbs., *the centre of gravity of the weight being in its centre.*

When the Line of Direction of the Power is parallel to the Base of the Plane.

The power is to the weight as the height of the plane to the length of its base.

Or, P : W : : *h* : *b*, *b representing the length of its base.*

Hence $P = \frac{W \times h}{b}$; $W = \frac{P \times b}{h}$; $h = \frac{P \times b}{W}$; $b = \frac{W \times h}{P}$.

When the Line of Direction of the Power is neither parallel to the Face of the Plane nor to its Base, but in some other Direction, as P′, Fig. 2.

The power is to the weight as the sine of the angle of the plane's elevation, to the cosine of the angle which the line of the power or traction describes with the face of the plane.

Thus, P′ : W : : sin A : cos. P′ *c o*.
Sin. A : cos. P′ *c o* : : P′ : W.
Cos. P′ *c o* : sin. A : : W : P′.

(2.)

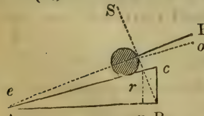


Illustration.—A weight of 500 lbs. is required to be sustained on a plane, the angle of elevation of which, *c* A B, is 10°; the line of direction of the power or traction, P′ *e c*, is 50°; what is the sustaining power required?

Cos. P′ *e c* (5°) = .9962 : sin. A (10°) = .1737 : : 500 : 87.18 *lbs.*

Or, draw a line, B *s*, perpendicular to the direction of the power's action from the end of the base line (at the back of the plane), and the intersection of this line on the length, A *c*, will determine the length and height, *n r*, of the plane.

ILLUSTRATION.—Of the last Example.

By Trigonometry (page 296), A B, assumed to be 1, A r and n r are $= .985$ and .171.

Hence $\frac{500 \times .171}{.985} = 86.8$ *lbs = the product of the weight = the height of the plane ÷ the length of it*

NOTE.—When the line of direction of the power is parallel to the plane, the power is least.

WEDGE.

A WEDGE is a double inclined plane.

To Compute the Power.

1. *When two Bodies or two Parts of a Body are Forced or Sustained in a Direction parallel to the Back of the Wedge.*

RULE.—Multiply the weight or resistance to be sustained by half the depth of the back of the wedge, and divide the product by the length of the wedge.

Or, $\frac{W \times \overline{d \div 2}}{l} = P$, *d representing the depth of the back, and l the length.*

EXAMPLE.—The depth of the back of a double-faced wedge is 6 inches, and the length of it through the middle 10 inches; what is the power necessary to sustain or overcome a resistance of 150 lbs.?

$$\frac{150 \times \overline{6 \div 2}}{10} = \frac{450}{10} = 45 \text{ lbs.}$$

2. *When one Body only is to be Forced or Sustained.*

RULE.—Multiply the weight or resistance to be sustained by the depth of the back of the wedge, and divide the product by the length of its base.

EXAMPLE.—What power, applied to the back of a wedge 6 inches deep, will raise a weight of 15,000 lbs., the wedge being 100 inches long on its base?

$$\frac{15000 \times 6}{100} = \frac{90000}{100} = 900 \text{ lbs.}$$

To Compute the Elements of a Wedge.

1. $\frac{P \times l}{d \div 2} = W.$ 2. $\frac{P \times l}{d} = W.$

1. $\frac{W \times \overline{d \div 2}}{P} = l.$ 2. $\frac{W \times d}{P} = l.$ 1. $\frac{W \times d}{l} \div 2 = P.$

1. $\frac{P \times l}{W} = d \div 2.$ 2. $\frac{P \times l}{W} = d.$ 2. $\frac{W \times d}{l} = P.$

NOTE.—As the power of the wedge in practice depends upon the split or rift in the wood to be cleft, or in the rise of the body to be raised, the above rules as regards the length of the wedge are only theoretical when a rift or rise exists.

SCREW.

A SCREW is a revolving inclined plane.

To Compute the Length and Height of the Plane of a Screw.

As the screw is an inclined plane wound around a cylinder, the length of the plane is ascertained by adding the square of the circumference of the screw to the square of the distance between the threads, and taking the square root of the sum; and the height or *pitch* of the screw is the distance between its consecutive threads.

To Compute the Power.

RULE.—Multiply the weight or resistance, to be sustained by the pitch of the threads, and divide the product by the length of the plane.

Or, $\frac{W \times p}{l} = P$, *p representing the pitch.*

EXAMPLE.—What is the power requisite to raise a weight of 8000 lbs. by a screw of 12 inches circumference and 1 inch pitch?

$12^2 + 1^2 = 145$, and $\sqrt{145} = 12.0416$. Then $\frac{8000 \times 1}{12.0416} = 664.36$ *lbs.*

To Compute the Weight.

RULE.—Proceed as above, substituting the power for the weight, and transposing the length of the plane and the pitch of the threads.

Or, $\frac{P \times l}{p} = W$.

When the Diameters or Circumferences of the Screw or of the Point at which the Power is applied are given.

RULE.—Ascertain the length of the plane from the diameter or circumference given, and proceed as before.

NOTE.—When a lever is used to transmit the power, the circumference described by the power is not that due to the radius of the lever alone, but it is the path described in one revolution, *i. e.*, the hypothenuse of the triangle (length of the plane), of which the circumference of the screw or lever is the base and the pitch of the threads the height of it. Hence the diameter of a screw is not a necessary element in determining the weight it will support, when the point at which the power is applied is given.

The preceding formulæ then become $\frac{W \times p}{d} = P$. $\frac{P \times d}{p} = W$, *d representing the distance described by the power.*

EXAMPLE.—If a lever of 30 inches in length was added to the circumference of the screw in the preceding example.

Then $12 \div 3.1416 = 3.819$ = *diameter of screw*; $\frac{3.819}{2} + 30 = 31.9095$ = *sum of radius of screw and length of lever*; and $31.9095 \times 2 \times 3.1416 = 200.4938$ = *the circumference described by the end of the lever = the base of the triangle.*

Hence $\sqrt{200.4938^2 + 1^2} = 200.5$ = *length of plane described by the power.*

Consequently, $\frac{8000 \times 1}{200.5} = 39.96$ *lbs.*

When a Screw and Lever is combined with a Wheel and Axle, etc.

RULE.—Multiply the power by the product of the circumference described by it and the radius of the wheel, and divide this product by the product of the pitch of the screw and the radius of the axle of the wheel.

Or, $\frac{P \times c \times R}{p \times r} = W$, *c representing the circumference described by the power,* R *and* r *the radii of the wheel and its axle.*

NOTE.—As the screw applied to the wheel is an endless one, *i. e.*, it revolves without advancing, the circumference due to the radius of the lever or crank is the distance described by the power.

EXAMPLE.—What is the power of a screw having a pitch of ⅞ths of an inch, driven by a lever 30 inches in length, with a force of 50 lbs. applied at its extremity?

$30 \times 2 \times 3.1416 = 188.5$ = *circumference described by lever = base of triangle*; ⅞ths $= .875$ = *height of triangle.*

Then $\sqrt{188.5^2 + .875^2} = 188.5$ = *length of plane described by power.*

$\frac{50 \times 188.5}{.875} = 107714$ *lbs.*

NOTE.—*If there is more than one thread to a screw the pitch must be increased as many times as there are threads.*

Ex. 2.—What weight can be raised with a power of 10 lbs. applied to a crank 32 inches long, turning an endless screw of 3½ inches diameter and 1 inch pitch, applied to a wheel of 20 inches diameter, upon an axle of 5 inches?

$32 \times 2 \times 3.1416 = 201$ *inches = circumference of* 64 *inches.*

$\frac{10 \times 201 \times (20 \div 2)}{1 \times (5 \div 2)} = \frac{10 \times 2012}{2.5} = 8040$ = *product of power and product of circumference described by it and the radius of the wheel ÷ product of pitch of screw and radius of axle.*

When a Series of Wheels and Axles are in Connection with each other.

The weight is to the power as the continued product of the radii of the wheels is to the continued product of the radii of the axles.

$W : P :: Rn : rn.$

Or, $r\ n : Rn :: P : W$, *n representing the number of wheels or axles.*

EXAMPLE.—If a power of 150 lbs. is applied to a crank of 20 inches radius, turning an endless screw with a pitch of half an inch, geared to a wheel, the pinion of which is geared to another wheel, and the pinion of the second wheel is geared to a third wheel, to the axle or barrel of which is suspended a weight; it is required to know what weight can be sustained in that position, the diameter of the wheels being 18, and the pinions and the axle 2 inches.

$\frac{150 \times 20 \times 2 \times 3.1416}{.5} = 37680$ = *power applied to face of first wheel.*

The diameters of wheels and pinions being 28 and 2, *their radii are* 9 *and* 1. Hence $1 \times 1 \times 1 : 9 \times 9 \times 9 : 37680 :: 27468720$ *lbs.*

Differential Screw.

When a hollow screw revolves upon one of less diameter and pitch (as designed by Mr. Hunter), the effect is the same as that of a single screw, in which the distance between the threads is equal to the difference of the distances between the threads of the two screws.

Therefore the power, to the effect or weight sustained, is as the difference between the distances of the threads of the two screws : to the circumference described by the power.

ILLUSTRATION.—If the external screw has 20 threads, and the internal one 21 threads in an inch pitch, and the power applied describes a line of 35 inches, the result is as $\frac{1}{21} \backsim \frac{1}{20} = \frac{1}{420}$, or .00238. Hence $\frac{35}{.00238} = 14706$.

PULLEY.

PULLEYS are designated as *Fixed* and *Movable*, according as the cord is passed over a fixed or a movable pulley. A *movable* pulley is when the cord passes through a second pulley or block in suspension; a single movable pulley is termed a *runner;* and a combination of pulleys is termed a *system of pulleys.*

To Compute the Power required to Raise a given Weight, the Number of Parts of the Cord supporting the Lower Block being given.

When only one Cord or Rope is used.

RULE.—Divide the weight to be raised by the number of parts of the cord supporting the lower or movable block.

Or, $\frac{W}{n} = P$. Or, $n \times P = W$, *n representing the number of parts of the cord sustaining the lower block.*

EXAMPLE.—What power is required to raise 600 lbs. when the lower block contains six sheaves and the end of the cord is fastened to the upper block, and what power when fastened to the lower block?

1. $\frac{600}{6\times2}=50$ *lbs.* = *weight ÷ number of parts of rope sustaining lower block*

2. $\frac{600}{6\times2+1}=46.15$ *lbs.* = *weight ÷ number of parts of rope sustaining lower block.*

To Compute the Weight a given Power will Raise, the Number of Parts of the Cord supporting the Lower Block being given.

RULE.—Multiply the power by the number of parts of the cord supporting the lower block.

Or, $P\times n=W$.

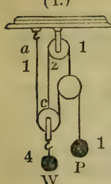
To Compute the Number of Cords necessary to Sustain the Lower Block, the Weight and Power being given.

RULE.—Divide the weight by the power, and the quotient is the number of parts of cord required.

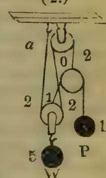
Or, $\frac{W}{P}=n$.

When more than one Cord or Rope is used.

(1.)



(2.)

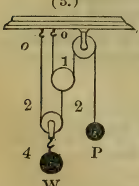


In a Spanish Burton, Fig. 1, where the ends of one cord, *a* P, are fastened to the support and the power, and the ends of the other, *c o*, to the lower and upper blocks, the weight is to the power as 4 to 1.

In another, Fig. 2, where there are two cords, *a* and *o*, two movable pulleys, and one fixed pulley, with the ends of one rope fastened to the support and upper movable pulley, and the ends of the other fastened to the lower block and the power, the weight is to the power as 5 to 1.

In a System of Pulleys, Figs. 3 and 4, *with any Number of Cords, o o, the Ends being fastened to the Support.*

(3.)

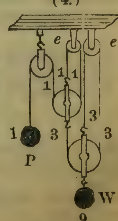


$\frac{W}{2n}=P$; $2n\times P=W$; $\frac{W}{P}=2n$, *n representing the number of distinct cords.*

EXAMPLE.—What weight will a power of 1 lb. sustain in a system of four movable pulleys and four cords?

$1\times2\times2\times2\times2=16$ *lbs.*

(4.)



When fixed Pulleys, e e, are used in the place of Hooks, to Attach the Ends of the Rope to the Support—Fig. 4.

$\frac{W}{3n}=P$; $3n\times P=W$; $\frac{W}{P}=3n$.

EXAMPLE.—What weight will a power of 5 lbs. sustain with four movable and four fixed pulleys, and four cords?

$5\times3\times3\times3\times3=405$ *lbs.*

When the Ends of the Cord or the fixed Pulleys are fastened to the Weight, as by an Inversion of the last Figures, putting the Supports for the Weights, and contrariwise—Figs. 3 and 4.

Fig. 3. $\frac{W}{(2n-1)} = P$; $(2n-1) \times P = W$; $\frac{W}{P} = (2n-1)$.

Fig. 4. $\frac{W}{(3n-1)} = P$; $(3n-1) \times P = W$; $\frac{W}{P} = (2n-1)$.

Example.—What weight will a power of 1 lb. sustain in a system of two movable pulleys and two cords?

$1 \times 2 \times 2 - 1 = 3$ *lbs.*

Ex. 2.—What weight will a power of 1 lb. sustain with a system of two movable and two fixed pulleys and two cords.

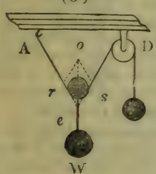
$1 \times 3 \times 3 - 1 = 8$ *lbs.*

And in the two examples preceding the last,

$1 \times 2 \times 2 \times 2 \times 2 = 16 - 1 = 15$ *lbs.*; $5 \times 3 \times 3 \times 3 \times 3 = 405 - 1 = 404$ *lbs.*

When the Cords by which the Pulleys are sustained are not in a Vertical Direction—Fig. 5.

(5)



e o, Fig. 5, is the vertical line through which the weight bears, and from *o* draw *o r*, *o s* parallel to D *e* and A *e*.

The forces acting at *e* are represented by the lines *e s*, *e r*, and *e o*; and as the tension of every part of the cord is the same, and equal to the power P, the sides *o s* and *o r* of the parallelogram must be equal, and therefore the diagonal *e o* divides the angle *r o s* into two equal portions. Hence the weight will always fall into the position in which the two parts of the cord A *e* and *e* D will be equally inclined to the vertical line, and it will bear to the power the same ratio as *e o* to *e s*.

Therefore W : P : : 2 cos. ½ *e* : 1, *e representing the angle* A *e* D.

Or, $2\,P \times \cos. \frac{1}{2} e = W$. That is, twice the power, multiplied by the cosine of half the angle of the cord, at the point of suspension of the weight, is equal to the weight.

Example.—What weight will be sustained by a power of 5 lbs., with an oblique movable pulley, Fig. 5, having an angle A *e* D of 30°?

$5 \times 2 \times .96593 = 9.6593$ *lbs. = twice the power × cos.* 15°.

When the Direction of the Cord is Irregular, the Weight not resting in the Centre of it.

$W \times P = \sin.(a+b) \times \sin. a$; $\frac{P \times \sin.(a+b)}{\sin. a} = W$; $\frac{W \times \sin. a}{\sin.(a+b)} = P$, *a* and *b representing the greater and lesser angles of the cord at e.*

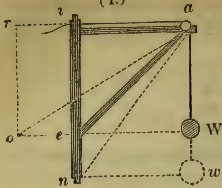
CRANES.

When the Post is Supported at both the Top and Foot.

The usual form of a Crane is that of a right-angled triangle, the three sides being the post or upright, the jib or arm, and the stay or strut, which is the hypothenuse of the triangle.

When the jib and the post are equal in length, and the stay is the diagonal of a square, this form is theoretically the strongest, as the whole stress or weight is borne by the stay, tending to compress it in the direction of its length; the stress upon it, compared to the weight supported, being as the diagonal to the side of the square, or as 1.4142 to 1. Consequently, if the weight borne by the crane is 1000 lbs., the thrust or compression upon the stay will be 1414.2 lbs., or as *a e* to *e* W, Fig. 1.

(1.)



The weight W is sustained by the rope or chain, and the tension is equal upon both parts of it; that is, on the two sides of the square, *i a* and *e* W. Consequently the jib, *i a*, has no stress upon it, and serves merely to retain the stay, *a e*.

If the foot of the stay be set at *n*, the thrust upon it, as compared with the weight, will be as *a n* to *a w;* and if the chain or rope from *i* to *a* is removed, and the weight is suspended from *a*, the tension on the jib will be as *i a* to *a* W.

If the foot of the stay is raised to *o*, the thrust, as compared with the weight, will be as the line *a o* is to *a* W, and the tension on the jib will be as the line *a r*.

By dividing the line representing the weight into equal parts, to represent pounds or tons, and using it as a scale, the stress upon any other part may be measured upon the parallelogram.

When the Post is Supported at the Foot only.

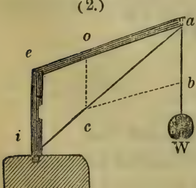
If the post is wholly unsupported at top, and its foot is secured up to the line *o* W, then the weight W, acting with the leverage, *e* W, will tend to rupture the post at *e*, with the same intensity or effect as if twice that weight was laid upon the middle of a beam equal to twice the length of *e* W, the point *e* being at the middle of the beam, which is assumed to be supported at both ends, the dimensions of the beam being alike to those of the post, and the depth being that of the line of rupture of the post.

Or, the force exerted to rupture the post will be represented by the weight or stress, W, multiplied by 4 times the length of the lever, *e* W, divided by the depth or thickness of the post in the line of the stress, squared, and multiplied by the breadth of it and the *Value* of the material of which it is composed.

The post of a crane is in the condition of half a beam supported at one end, the weight suspended from the other; consequently, it must be estimated as a beam of twice the length supported at both ends, the stress applied in the middle.

To Compute the Stress on the Jib or Tension-rods, and on the Stay or Strut—Fig. 2.

(2.)



On the diagram of the crane, Fig. 2, mark off on the line of the chain, as *a* W, a distance, *a b*, representing the weight on the chain; from the point *b* draw a line, *b c*, parallel to the tension-rod or jib, *a e*, as the case may be, and where this intersects the stay or strut, draw a vertical line, *c o*, extending to the jib or tension-rod, and the distances from *a* to the points *b c* and *o c*, measured upon a scale of equal parts, will represent the proportional strain.

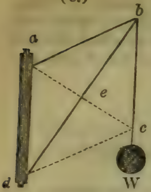
Thus, in the figure, the weight being 10 tons, the stress on the stay or strut compressing, *a c*, will be 31 tons, and on the jib or tension-rods, *a o*, 26 tons.

By dividing the line representing the weight, as *a* W or *a w*, into equal parts, to represent tons or pounds, and using it as a scale, the stress upon any other part may be measured upon the described parallelogram.

Thus, as the length of *a* W, compared to *a e*, is as 1 to 1.4142 : if *a* W is divided into 10 parts representing tons, *a e* would measure 14.142 parts or tons.

In Fig. 3, the angle *a b e* and *e b c* being equal, the chain or rope is represented by *a b c*, and the weight by W; the stress upon the stay *b d*, as compared with the weight, is as *b d* to *a b* or *b c*.

(3.)



In practice, however, it is not prudent to consider the chain as supporting the stay; but it is proper to disregard the chain or rope as forming part of the system, and the crane should be designed to support the load independent of it. It is also proper that the angles on each side of the diagonal stay, in this case, should not be equal. If the side *a b* is formed of tension-rods of wrought iron, the point *a* should be depressed, so as to lengthen that side, and decrease the angle *a b e*; but if it be of timber, the point *a* should be raised, and the angle *a b e* increased.

(4)

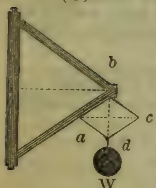
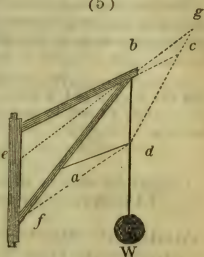


Fig. 4 shows the parts composing a crane, arranged in the form of an equilateral triangle, in which the weight *b d*, the tension *b c*, and the thrust *a b* are all equal to each other, the weight W being suspended from the point *b*.

Fig. 5 shows a form of crane very generally used; the angles are the same as in Fig. 3, and the weight suspended from it, being attached to the point *d*, is represented by the line *b d*. The tension, which is equal to the weight, is shown by the length of the line *b c*, and the thrust by the length of the line *b a b*, measured by a scale of equal parts, into which the line *b d*, representing the weight, is supposed to be divided.

(5)



But if *b e* be the direction of the jib, then *b g* will show the tension, and *b f* the thrust (*d f* being taken parallel to *b e*), both of them being now greater than before; the line *b d* representing the weight, and being the same in both cases.

To Compute the Stress upon the Stay of a Crane.

RULE.—Multiply the length of the stay in feet by the weight to be borne in pounds; divide the product by the height of the jib from the point of bearing of the stay in feet, and the quotient will give the stress or thrust in pounds.

EXAMPLE.—The length of the stay of a crane is 28.284 feet, the height of the post is 26.457 feet, and the weight to be borne is 22400 lbs.; what is the stress?

$$\frac{28.284 \times 22400}{26.457} = \frac{633561.6}{26.457} = 23947 \textit{ lbs.}$$

To Compute the Dimensions of the Post of a Crane.

When the Post is Supported at the Feet only.

RULE.—Multiply the weight or stress to be borne in pounds by the length of the jib in feet measured upon a horizontal plane; divide the product by the *Value* of the material to be used, and the product, divided by the breadth in inches, will give the square of the depth, also in inches.

EXAMPLE.—The stress upon a crane is to be 22400 lbs., and the distance of it from the centre of the post is 20 feet; what should be the dimension of the post if of American white oak?

Value of American white oak 50. Assumed breadth 12 inches.

$$\frac{22400 \times 20}{50} = 8960, \text{ and } \frac{8960}{12} = 746.67. \text{ Then } \sqrt{746.67} = 27.32 \textit{ inches.}$$

When the Post is Supported at both Ends.

RULE.—Multiply the weight or stress to be borne in pounds by twice the length of the jib in feet measured upon a horizontal plane; divide the product by the *Value* of the material to be used, and the product, divided by four times the breadth in inches, will give the square of the depth, also in inches.

EXAMPLE.—Take the same elements as in the preceding case.

Assumed breadth 10 inches.

Then $\frac{22400\times20\times2}{50}=17920$, and $\frac{17920}{4\times10}=448$, and $\sqrt[2]{448}=21.166$ *inches.*

To Compute the Stress on the Jib or Tension-rods, on the Stay of a Crane.

On the diagram of the crane mark off on the line of the chain or rope a distance that represents the weight into a scale of equal parts, and by applying this scale to the sides of the parallelogram representing the thrusts, the measure of each is obtained by inspection.

ILLUSTRATION.—The distance $b\ d$, Fig. 5, is divided into 10 parts representing tons, and the length of the sides, $b\ c$ and $a\ b$, representing the thrust and tension, have respectively 10 and 14 parts; consequently the stress on them is in their proportion in tons.

CHAINS AND ROPES.

Chains for cranes should be made of short oval links, and should not exceed 1 inch in diameter.

Table of Short-linked Crane Chains and Ropes, showing the Dimensions and Weight of each, and the Proof of the Chain in Tons.

Diam. of Chains.	Weight per Fathom.	Proof Strain.	Circumf. of Rope.	Weight of Rope pr. Fath.	Diam. of Chains.	Weight per Fathom.	Proof Strain.	Circumf. of Rope.	Weight of Rope pr. Fath.
Ins.	Lbs.	Tons.	Ins.	Lbs.	Ins.	Lbs.	Tons.	Ins.	Lbs.
5/16	6	.75	2.½	1.5	11/16	28	6.5	7.	10.5
3/8	8.5	1.5	3.¼	2.5	3/4	32	7.75	7.½	12.
7/16	11.	2.5	4.	3.75	13/16	36	9.25	8.¼	15.
1/2	14.	3.5	4.¾	5.	7/8	44	10.75	9.	17.5
9/16	18.	4.5	5.½	7.	15/16	50	12.5	9.½	19.5
5/8	24.	5.25	6.¼	8.7	1.	56	14.	10.	22.

The ropes of the sizes given are considered to be of equal strength with the chains, which, being short-linked, are made without studs.

A crane chain will stretch under a proof of 15 tons, half an inch to a fathom.

CENTRES OF GRAVITY.

The CENTRE OF GRAVITY of a body, or any system of bodies connected together, is the point about which, if suspended, all the parts will be in equilibrium.

A body or system of bodies, suspended at a point *out* of the centre of gravity, will rest with its centre of gravity vertical under the point of suspension.

A body or system of bodies, suspended at a point *out* of the centre of gravity, and successively suspended at two or more such points, the vertical line through these points of suspension will intersect each other at the centre of gravity of the body or bodies.

The centre of gravity of a body is not always within the body itself.

If the centres of gravity of two bodies, as B C, be connected by a line, the distances of B and C from the common centre of gravity, *a*, *will be as the weights of the bodies.*

Thus, B : C : : C *a* : *a* B.

LINE.

Circular Arc.—$\frac{r\,c}{l}$ = *distance from the centre, r representing radius, c the chord, and l the length of the arc.*

SURFACES.

Square, Rectangle, Rhombus, Rhomboid, Gnomon, Cube, Regular Polygons, Circle, Sphere, Spheroids or Ellipsoids, Spheroidal Zones, Cylinder, Circular Ring, Cylindrical Ring, Links, Helix, Plain Spiral, Spindles, all Regular Figures, and Middle Frustra of all Spheroids, Spindles, etc. The centre of gravity of the surfaces of these figures is *in their geometrical centre.*

Triangles.—On a line drawn from any angle to the middle of the opposite side, *at ⅔ of the distance from the angle.*

Trapezium.—Draw the two diagonals, and ascertain the centres of gravity of each of the four triangles thus formed; join each opposite pair of these centres, *and it is at the intersection of the two lines.*

Trapezoid.—$\left(\frac{B + 2\,b}{B + b}\right) \times \frac{a}{3}$ = *distance from* B *on a line joining the middle of the two parallel sides* B *b*, *a representing the middle line.*

Sector of a Circle.—$\frac{2\,c\,r}{3\,l}$ = *distance from the centre of the circle.*

Semicircle.—$\frac{4\,r}{3 \times 3.1416}$ = *distance from the centre.*

Semi-semicircle.—.424 *r* = *distance from both base and height and at their intersection.*

Segment of a Circle.—$\frac{c^3}{12\,a}$ = *distance from the centre, a representing area of segment.*

Sector of a Circular Ring.—$\frac{4}{3} \times \frac{\sin.\ \frac{1}{2} \angle}{\angle} \times \frac{r^3 - r_1^3}{r^2 - r_1^2}$ = *distance from centre of arcs, r and r′ representing the radii.*

Hemisphere, Spherical Segment, and Spherical Zone at the centre of their heights.

Circular Zone.—Ascertain the centres of gravity of the trapezoid and the segments comprising the zone; draw a line (equally dividing the zone) perpendicular to the chords; connect the two centres of the segments by a line cutting the perpendicular to the chords; then will the centre of gravity of the figure be *on the perpendicular, toward the lesser chord, at such proportionate distance of the difference between the centres of gravity of the trapezoid and line connecting the centres of the segments* as the area of the two segments bears to the area of the trapezoid.

Prisms and Wedge.—When the end is a Parallelogram, *in their geometrical centres;* when the end is a Triangle, Trapezium, etc., it is *in the middle of its length*, at the same distance from the base as that of the triangle or trapezoid, of which it is a section.

Prismoid.—*At the same distance from its base* as that of the trapezoid or trapezium, which is a section of it.

Lune.—*On a line connecting the centres of gravity of the two arcs* at a point proportionate to the respective areas of the arcs.

Cycloid.—⅚ of radius of generating circle = *distance from the centre of the chord of the curve.*

Cone, Frustrum of a Cone, Pyramid, Frustrum of a Pyramid, and Ungula.—At the same distance from the base as in that of the triangle, parallelogram, or semicircle, which is a right section of them.

Spirals.—Plane, in its geometrical centre. Conical, at a distance from the base, ¼ of the line joining the vertex and centre of gravity of the base.

Frustrum of a Circular Spindle.—$\frac{r^2 - r'^2}{2(h - D.z)}$ = *distance from the centre of the spindle, h representing the distance between the two bases,* D *the distance of the centre of the spindle from the centre of the circle, and z the generating arc, expressed in units of the radius.*

Paraboloid of Revolution.—$\frac{1}{10p}\ \frac{(e^2 + b^2)^{\frac{3}{2}}(3b^2 - 2e^2) + 2e^5}{(e^2 + b^2)^{\frac{3}{2}} - e^3}$ = *distance from vertex, a representing altitude, b radius of base, and* $e = \frac{b^2}{2a}$.

Any Plane Figure.—Divide it into triangles, and ascertain the centre of gravity of each; connect two centres together, and ascertain their common centre; then connect this common centre and the centre of a third, and ascertain the common centre, and so on, connecting the last-ascertained common centre to another centre till the whole are included, and *the last* common centre will give *the centre required.*

Parabola.—2-5 of the height = *distance from the base.*

Semi-spheroid or Ellipsoid and its Segment, and Segment of a Circular Spindle.—See *Haswell's Mensuration*, pages 281–283.

SOLIDS.

Cube, Parallelopipedon, Hexahedron, Octahedron, Dodecahedron, Icosahedron, Cylinder, Sphere, Spherical Zone, Spheroids or Ellipsoids, Cylindrical Ring, Links, Spindles, all Regular Bodies, and Middle Frustra of all Spheroids and Spindles, etc. The centre of gravity of these figures *is in their geometrical centre.*

Tetrahedron.—Is the common centre of the centres of gravity of the triangles made by a section through the centre of each side of the figures.

Cone and Pyramid.—¼ of the line joining the vertex and centre of gravity of the base = *distance from the base.*

Frustrum of a Cone or Pyramid.—$\frac{(r + r_{\prime})^2 + 2r^2}{(r + r_{\prime})^2 - r r_{\prime}} \times \frac{1}{4} h$ = *distance from centre of lesser end, r and $r_{\prime}$ in a pyramid representing the sides.*

Spherical Segment.— $3.1416\, vs^2 \left(r - \frac{vs}{2}\right)^2 \div v$ = *distance from the centre, vs representing the versed sine, and v the volume of the segment.*

$\left(\frac{8r - 3h}{12r - 4h}\right) \times h$ = *distance from the vertex, h representing the height.*

Hemisphere.—⅜ r = *distance from the centre.*

Spherical Sector.—¾ $(r - \frac{1}{2} h)$ = *distance from the centre.*

$\frac{2r + 3h}{8}$ = *distance from the vertex.*

Frustrum of a Sphere.—$\frac{8r - 3h}{13r - 4h}$ = *distance from the vertex of the frustrum.*

Frustra of Spheroids.—Prolate. ¾ $\frac{h(2a^2 - h^2)}{3a^2 - h^2}$ = *distance from centre of spheroid, a representing semi-transverse diameter in a prolate frustrum, and the semi-conjugate in an oblate frustrum*

Any Frustrum. $\frac{3}{4}\ \frac{(d + d') \times (2\,a^2 - d'^2 + d^2)}{3\,a^2 - d'^2 + d'\,d + d^2}$ = *distance from centre of spheroid, d and d′ representing the distances of the base and end of the segments from the centre of the spheroid.*

Semi-spheroids.—Prolate. $\frac{3}{8}\,a$.—*Oblate.* $\frac{3}{8}\,a$ = *distance from the centre.*

Segments of Spheroids.—Prolate. $\frac{3}{4}\ \frac{(a + d)^2}{2\,a + d}$.—*Oblate.* $\frac{3}{4}\ \frac{(a + d')^2}{2\,a + d'}$ = *distance from the centre of the spheroid, d and d′ representing the distances of the base of the segments from the centre of the spheroid.*

Segment of an Elliptic Spindle at $\frac{2}{3}$ *of height from the vertex.*

Segment of a Circular Spindle and of a Parabolic Spindle.—See *Haswell's Mensuration*, pages 192 and 199.

Segment of a Hyperbolic Spindle, at $\frac{3}{4}$ *of the height from the vertex.*

Paraboloid of Revolution, at $\frac{2}{3}$ *of the height from the vertex.*

Hyperboloid of Revolution.—$\frac{4\,b + 3\,h}{6\,b + 4\,h} \times h$ = *distance from the vertex, b representing the diameter of the base.*

Frustrum of Paraboloid of Revolution.—$\frac{2\quad{}^2 + r_1}{r^2 + r_1}\ \frac{1}{3}\,h$ = *distance from the vertex, r and r_1 representing radii of base and diameter.*

Frustrum of Hyperboloid of Revolution.—$\frac{3}{4}\ \frac{(d + d')\,(2\,a^2 - d'^2 + d^2)}{3\,a^2 - d'^2 + d'd + d^2}$ = *distance from centre of the base, a representing the semi-transverse axis, or distance from centre of the curve to vertex of figure; d and d′ the distances from the centre of the curve to the centre of the lesser and greater diameter of the frustrum.*

Segment of Paraboloid of Revolution, at $\frac{2}{3}$ *of the height from the vertex.*

Segment of Hyperboloid of Revolution.—$\frac{4\,b + 3\,h}{6\,b + 4\,h} \times h$ = *distance from the vertex.*

Of an Irregular Body of Rotation.

Divide the figure into four or six equidistant divisions; ascertain the volume of each, their moments with reference to the first horizontal plane or base, and then connect them thus:

$(A + 4\,A_1 + 2\,A_2 + 4\,A_3 + A_4)\ \frac{h}{12} = V$, $A\ A_1$, etc., *representing the volume of the divisions and h the height of the body from the base;*

and $\frac{(0\,A + 1 \times 4\,A_1 + 2 \times 2\,A_2 + 3 \times 4\,A_3 + 4\,A_4)}{A + 4\,A_1 + 2\,A_2 + 4\,A_3 + A_4} \times \frac{h}{4}$ = *distance of centre of gravity from base.*

Illustration.—A vessel generated by the rotation of a curve is divided into 4 sections; viz., $A = 1$, $A_1 = 2.4$, $A_2 = 1.6$, $A_3 = 1$, and $A_4 = .15$.

Then $\frac{(0 \times 1 + 1 \times 4 \times 2.4 + 2 \times 2 \times 1.6 + 3 \times 4 \times 1 + 4 \times .15)}{1 + 4 \times 2.4 + 2 \times 1.6 + 4 \times 1 + .15} \times \frac{2.5}{4} = \frac{71.5}{71.8} = .9958$ *ins.;*

and $(1 + 4 \times 2.4 + 2 \times 1.6 + 4 \times 1 + .15) \times \frac{2.5}{12} =$ 5.[illegible]83 *cubic inches, the volume.*

To Compute the Common Centre of Gravity of the Engine, Boilers, etc., etc., or of any Number of Weights in a Vessel.

Rule.—Multiply the several weights on each side of ⊠* by their respective distances from it; divide the sum of the products by the sum of the weights, and the quotients will give the distances at which the sum of the weights on each side will produce the same effects as the several weights at their respective distances.

* This is the symbol for *Dead flat*, and it is the point in the length of a vessel where her frame has the greatest dimensions.

Reduce the weights to units, and add them together; divide the sum of the distances by the sum of the units, and the product of the quotient and the respective units will give the distances of the contrary weights from the common centre of gravity.

EXAMPLE.—The weights and the mean distances from ⊠ of the engine, boilers, water in boilers, water-wheels, coal, coal-bunkers, spars and rigging, extra pieces, and engine stores in a steam vessel are as follows; where is their common centre of gravity?

Engine,	105 tons at 20 feet.	Coal,	140 tons at 4 feet.
Boilers,	60 " " 26 "	Bunkers,	20 " " 3 "
Water,	40 " " 26 "	Spars and rigging,	30 " " 4.33 "
Water-wheels,	20 " " 3.5 "	Extra pieces and stores,	5 " " 22. "

Forward.		Aft.	
60 tons at 26 feet	= 1560	105 tons at 20 feet	= 2100
40 " " 26 "	= 1040	140 " " 4 "	= 560
30 " " 4.33 "	= 130	20 " " 3 "	= 60
		20 " " 3.5 "	= 70
		5 " " 22 "	= 110
130	2730	290	2900

$$\frac{2730}{130} = 21 \textit{ feet, and } \frac{2900}{290} = 10 \textit{ feet.}$$

130 and 290, when reduced, become respectively 1.3 and 2.9. Then $1.3 + 2.9 = 4.2$, and $\frac{21+10}{4.2} = 7.3809$.

Hence $7.3809 \times 1.3 = 9.5952$ *feet = the distance of* 2.9 or 290 *from the common centre*, and $7.3809 \times 2.9 = 21.4046$ *feet* = the distance of 1.3 or 130 *from the common centre.*

Consequently $9.5952 \times 290 = 21.4046 \times 130$, and $21.4046 - 21 = .4046$ *feet = the distance aft of* ⊠ *of the common centre of gravity of the weights of the entire mass.*

FRICTION.

Friction is termed *sliding* when surfaces move parallel with one another, as on a slide or over a pin; and *rolling* when a body rotates upon the surface of some other, so that new parts of both surfaces are continually being brought in contact with each other.

The force necessary to abrade the fibres of a body is termed the *measure of the friction;* this is determined by ascertaining what portion of the weight of a moving body must be exerted to overcome the resistance arising from this cause: the fraction expressing the ratio is termed the *Coefficient of the Friction.*

To Compute the Coefficient of the Friction of Bodies.

Place them upon a horizontal plane, attach a cord to them, and lead it in a direction parallel to the plane over a pulley, and suspend from it a scale in which weights are to be placed until the body moves.

The weight that moves a body is the numerator, and the weight of the body moved is the denominator of a fraction, which represents the coefficient required.

ILLUSTRATION.—If by a pressure of 320 lbs. the friction amounts to 80 lbs., the coefficient of friction in this case would be $\frac{80}{320} = .25$.

Hence, if the coefficient of friction of a wagon over a gravel road was .25, and the load 8400 lbs., the power required to draw it would be $8400 \times .25 = 2100$ *lbs.*

Experiments and *Investigations* have adduced the following observations and results:

1. The amount of friction in surfaces of like material is very nearly proportioned to the pressure perpendicularly exerted on such surfaces.

2. With equal pressure and similar surfaces, friction increases as the dimensions of the surfaces are increased.

3. A regular velocity has no considerable influence on friction; if the velocity is increased the friction is greater, but this depends on the secondary or incidental causes, as the generation of heat and the resistance of the air.

4. Similar substances excite a greater degree of friction than dissimilar. If the pressures are light, the hardest bodies excite the least friction.

5. Friction is diminished by unguents. In the choice of unguents, those of a viscous nature are best adapted for rough or porous surfaces, as tar and tallow are suitable for the surfaces of woods, and oils best adapted for the surfaces of metals.

6. A rolling motion produces much less friction than a sliding one.

7. The friction of metals and woods varies with their hardness.

8. Hard metals and woods have less friction than soft.

9. That without unguents or lubrication, and within the limits of 33 lbs. pressure per square inch, the friction of hard metals upon each other may be generally estimated at about one sixth the pressure.

10. That within the limits of abrasion the friction of metals is nearly alike.

11. That with greatly increased pressures friction increases in a very sensible ratio, being greatest with steel or cast iron, and least with brass or wrought iron.

12. With woods and metals, without lubrication, velocity has very little influence in augmenting the friction, except under peculiar circumstances.

13. When no unguent is interposed, the amount of the friction is, in every case, wholly independent of the extent of the surfaces of contact; so that, the force with which two surfaces are pressed together being the same, their friction is the same, whatever may be the extent of their surfaces of contact.

14. The friction of a body sliding upon another will be the same, whether the body moves upon its face or upon its edge.

15. When the fibres of materials cross each other, friction is less than when they run in the same direction.

16. Friction is greater between surfaces of the same character than between those of different characters.

17. That with hard substances, and within the limits of abrasion, friction is as the pressure, without regard to surfaces, time, or velocity.

18. The influence of the duration of contact (friction of rest) varies with the nature of the substances; thus, with hard bodies resting upon each other, the effect reaches a maximum very quickly; with soft bodies, very slowly; with wood upon wood, the limit is attained in a few minutes; and with metal on wood, the greatest effect is not attained for some days.

Experiments with Unguents interposed.—[Morin.]

Materials and Surfaces in Contact.	Coef. of Friction. During Motion.	Coef. of Friction. After Quiesc.	Unguents
Beech upon oak, fibres parallel ...	.055	...	Tallow.
Brass upon brass...............	.053	...	Olive-oil.
Brass upon cast iron	.086	.106	Tallow.
Brass upon wrought iron	.081	...	Tallow.
Cast iron upon brass	.103	...	Tallow.
Cast iron upon brass	.078	...	Olive-oil.
Cast iron upon cast iron	.314	...	Water.
Cast iron upon cast iron	.197	...	Soap.
Cast iron upon oak, fibres parallel	.189	...	Dry soap.
Cast iron upon oak, " "	.218	.646	Greased and saturated with water.
Cast iron upon oak, " "	.078	.1	Tallow.
Cast iron upon wrought iron	.103	.1	Tallow.
Copper upon cast iron	.072	.103	Tallow or olive-oil.
Copper upon oak, fibres parallel..	.069	.1	Tallow.
Elm upon cast iron...............	.066	...	Tallow.
Elm upon oak, fibres parallel	.07	.142	Tallow.
Oak upon cast iron	.08	...	Tallow.
Oak upon elm, fibres parallel.....	.136	...	Dry soap.
Oak upon elm, " "	.073	.178	Tallow.
Oak upon oak, " "	.164	.44	Dry soap.
Oak upon oak, " "	.075	.164	Tallow.
Oak upon oak, fibres perpendicular	.083	.254	Tallow.
Oak upon oak, " "	.25	...	Water.
Oak upon wrought iron...........	098	...	Tallow.
Steel upon cast iron	.105	.108	Tallow.
Steel upon cast iron	.079	...	Olive-oil.
Steel upon wrought iron	.093	...	Tallow.
Steel upon brass	.056	...	Tallow or olive-oil.
Tanned ox-hide upon cast iron...	.365	...	Greased and saturated with water.
Wrought iron upon brass	.103	...	Tallow.
Wrought iron upon cast iron	.103	...	Tallow.
Wrought iron upon cast iron	.066	.1	Olive-oil.
Wrought iron upon elm, fibres par.	.078	...	Tallow.
Wrought iron upon oak, " "	.256	.649	Greased and saturated with water.
Wrought iron upon oak, " "	.214	...	Dry soap.
Wrought iron upon oak, " "	.085	.108	Tallow.
Wrought iron upon wrought iron .	.082	...	Tallow.

Note.—The extent of the surfaces bore such a relation to the pressure as to separate them from one another by an interposed stratum of the unguent.

Deductions from the above Table, showing the Relative Values of some of the Substances and Unguents to reduce Friction.

Substances.	Relat. Value.	Unguents used.
WOOD UPON WOOD.		
Beech upon oak, fibres parallel	.96	Tallow.
Oak upon elm, fibr. par.	.75	Tallow.
Oak upon elm, " "	.73	Tallow.
Oak upon elm, " "	.74	Tallow.
Oak upon oak, " "	.21	Water.
METALS UPON WOOD.		
Cast iron upon elm, fibres parallel	.87	Olive-oil.
Cast iron upon oak, fibres parallel	.76	Tallow.
Cast iron upon oak, fibres parallel.......	.69	Lard and tallow.
Cast iron upon oak, fibres parallel.......	.66	" "
Wrought iron upon oak, fibres parallel	.21	Greased and wet.
METALS UPON METALS.		
Brass upon brass	.91	Olive-oil.
Cast iron upon brass...	.69	Olive-oil.
Cast iron upon cast iron	.83	Olive-oil.
Cast iron upon cast iron	.64	Water.
Cast iron upon cast iron	.27	Soap.
Cast iron upon copper..	.8	Olive-oil.
Steel upon brass.......	1.	Olive-oil.
Steel upon cast iron....	.66	Olive-oil.
Steel upon wrought iron	.57	Olive-oil.
Wrought iron upon cast iron	.8	Olive-oil.
Wrought iron upon wrought iron........	.75	Olive-oil.

Relative Value of Unguents to reduce Friction.

Unguents.	Wood upon Wood.	Wood upon Metals.	Metals upon Metals.
Dry soap	.4	.32	.27
Lard	.82	.85	.7
Lard and plumbago	..	.67	.96
Olive-oil	..	1.	1.
Tallow	1.	.93	.8
Water	.22	.24	.18

Table of the Coefficients of Friction under Pressures increased gradually up to the Limit of Abrasion.

[G. Rennie.]

Pressure per Square Inch.	Coefficient.			
	Wrought Iron upon Wrought Iron.	Wrought Iron upon Cast Iron.	Steel upon Cast Iron.	Brass upon Cast Iron.
Lbs.				
32.5	.14	.174	.166	.157
186	.25	.275	.3	.225
224	.271	.292	.333	.219
298	.297	.329	.344	.211
336	.312	.333	.347	.215
390	.376	.363	.353	.205
485	.403	.366	.356	.221
523	.409	.366	.357	.223
600	...	.367	.359	.234
672	...	.376	.403	.233
710	...	.434	...	.234
820	...	...	...	.273

Results of Experiments upon Oils to determine their Relative Permanent Fluidity as exposed in Lubrication.

[Naysmith.]

Description of Oil.	Duration of Fluidity.	Relative Fluidity.
Gallipoli	9	31.6
Lard	5	17.3
Linseed	7	27.5
Sperm, common	9	100.
Sperm, best	7	80.1
Rape-seed	8	29.

Coefficients of Friction.—Leather belts over wooden drums .47 of the pressure, and over turned cast-iron pulleys .28 of the pressure.

Comparative Friction of Steam-engines, the Side Lever taken as the Standard of Comparison.

Direct-action Engine, with rollers to slides, has a gain of .8 per cent., and with parallel motion a gain of 1.3 per cent.

Vibrating Engine has a gain of 1.1 per cent.

Direct-action Engine, with slides, has a loss of 1.8 per cent.

Experiments upon different steam-engines have determined that the friction, when the pressure on the piston is about 12 lbs. per square inch, does not exceed 1.5 lbs., or about one tenth of the power exerted.

The friction of a double cylinder (50-inch diameter) direct-acting condensing propeller engine is 1.25 lbs. per square inch of piston = 10.3 per cent. of the total power developed; the friction of the load is .9 lbs. per square inch of piston = 7.5 per cent. of the total pressure; and the friction of the screw is 1.3 lbs. per square inch of piston = 10.8 per cent. of the total power = 28.6 per cent.

The friction of a double cylinder (70-inch diameter) inclined condensing water-wheel engine with its load is 15 per cent. of the total power developed.

The power required to work the air-pumps is 5 per cent., and to work the feed-pumps 1 per cent.

Results of Experiments upon the Friction of Machinery.

[DAVISON.]

Steam-engine, vertical beam, one tenth its power; 190 feet horizontal, and 180 feet vertical shafting, with 34 bearings, having an area of 3300 square inches, with 11 pair of spur and bevel wheels; 7.65 horses power.

A set of three-throw pumps, 6 inches in diameter, delivering 5000 gallons per hour at an elevation of 165 feet; 4.7 horses power, or about 13 per cent.

Two pair iron rollers and an elevator, grinding and raising 320 bushels malt per hour; 8.5 horses power.

An ale-mashing machine 800 bushels malt at a time; 5.68 horses power.

Ninety-five feet of Archimedes screw 15 inches in diameter, and an elevator conveying 320 bushels malt per hour to a height of 65 feet; 3.13 horses power.

Friction Clutch.—Driven by a leather belt 14 inches in width; face of clutch 5 inches deep; broke a cast-iron shaft 6.5 inches in diameter.

Wood Bearings for Propeller Shaft.

[*Results of Experiments by* MR. JOHN PENN.]

Bearings.	Pressure per Square Inch.	Time of Operation.	Result.
	Lbs.		
Babbit's metal on iron	1600	8 minutes.	Rolled out sidewise.
Box on brass................	4480	5 minutes.	Not cut.
Box on iron	448	30 minutes.	No wear.
Brass on brass..............	448	30 minutes.	Little or no cutting.
Brass on iron...............	448	30 minutes.	Little or no cutting.
Brass on iron...............	675	1 hour.	Abraded.
Brass on iron...............	4480	—	Set fast immediately.
Cam-wood on brass..........	8007	5 minutes.	No wear.
Lignum-vitæ on brass.......	4000	5 minutes.	No wear.
Lignum-vitæ on iron........	1250	36 hours.	No wear.
Snake-wood on brass........	4000	5 minutes.	No wear.

Marine Railway.—To draw 3000 tons upon greased slides a power of 250 tons was necessary to move it, but when started 150 tons would draw it.

Friction and Resistances (Screw Steamer).

[By Vice-admiral C. R. MOORSOM, R.N.]

Moving friction of hull	.07
Moving friction of load..	.063
Moving friction of rotation of blades of screw	.09
Slip of screw	.171
Resistance of hull	.606
	1.

Side Lever Steam-engine.—[J. V. MERRICK.]

Friction to work the air-pump	.585	to	.7
Friction of weight of parts	.5	"	.5
Friction of cylinder packing	.15	"	.3
Friction of air-pump packing..........................	.046	"	.092
Friction of valves, parallel motion, resistance to air, etc.	.169	"	.178
	1.45		1.85

Hence $\frac{1.45 + 1.85}{2} = 1.65$ lbs. per square inch. If the journals are kept constantly lubricated, as with automaton lubrications, the friction of weight will be reduced to .33, and the pressure will be reduced from $1.65 - .33$ to 1.32 pounds per square inch of piston to work the engine without load. The friction of the load, according as the journals are lubricated, the ends keyed up, etc., will range from 2 to 5 per cent.

Friction of Steam-engines in Pounds per Square Inch of Piston.—(*Condensing.*)

Diameter of Cylinder.	Oscillating and Trunk	Beam and Geared.	Direct-acting and Vertical.	Diameter of Cylinder.	Oscillating and Trunk	Beam and Geared.	Direct-acting and Vertical.
10	5	6	7	50	2.5	2.7	3.3
15	4	5	6	60	2.4	2.6	3
20	3.5	4	5	70	2.3	2.5	2.7
25	3	3.6	4.5	80	2	2.3	2.6
30	3	3.5	4	100	1.6	2.2	2.5
35	2.6	3	3.5	110	1.5	2	2.1

Useful Effect of several Machines.—The useful effect or modulus of a machine is the fraction which expresses the value of the work compared with the power applied, which is expressed by unity.

Bucket wheel	.60	Inclined chain pump	.40
Crab	.80	Screw press	.33
Endless screw	.50	Vertical chain pump	.50

Application of the preceding Results.

ILLUSTRATION.—A vessel, including the cradle, weighing 1000 tons, is to be drawn upon an inclined plane having a rise of 10 feet in 100 of its length; what will be the resistance to be overcome, the cradle being supported on wrought-iron axles in cast-iron rollers, running on cast-iron rails?

$\frac{1000\times10}{100} = 100$ *tons = the power required to draw the vessel independent of friction*

Ratio of friction to pressure of wrought iron on cast, in an axle and its bearing, .075. Ratio of ditto of cast iron upon cast, say .005.

Hence $.075 + .005 = .08$ of 1000 tons $= 80$ *tons*, which, added to the 100 tons before deducted, gives 180 *tons, or the resistance to be overcome*

The power or effect lost by friction in axles and their bearing may be expressed by the formulæ

$\frac{Wfdr}{230} = P$, *f representing the coefficient of the friction, d the diameter of the axle in inches, and r the number of revolutions per minute.*

ILLUSTRATION.—The pressure on the piston of a steam-engine is 20.000 lbs., the number of revolutions 20, and the diameter of the driving shaft of wrought iron in a brass journal is 8 inches; what is the effect of the friction?

$$\frac{20.000\times.07\times8\times20}{230} = 973.93 \; lbs.$$

Hence $\frac{Pv}{33\,000}$ = *horses power, v representing circumference of shaft in feet × by revolutions per minute.*

The power or effect lost by friction in guides or slides may be expressed by the following formulæ:

$\frac{Wfsr}{60\times\sqrt{(5\,l^2-s^2)}} = P$, *s representing the stroke of the cross-head, and l the length of the connecting rod in feet.*

On the Traction of Carriages, and the Destructive Effects they produce on Roads.—[*Experiments by* M MORIN]

The effects produced when a carriage is moved over a road are divided into two parts—the traction of the carriage, and its action upon the road.

For woods, plasters, and hard bodies in general, the resistance to rolling is nearly,

1. Proportional to the pressure. 2. Inversely proportional to the diameter of the body on rollers. 3. Greater as the breadth of the part in contact is diminished.

The relation or ratio between the load and the traction, upon a level road, is approximately given by the following formulæ:

For Carriages with Two Wheels: $\frac{Ufs}{r} = \frac{F}{W}$.

For Carriages with Four Wheels. $\frac{2(Ufs)}{t \times t'} = \frac{F}{W}$, U *representing the constant multiplier in the following table, f the coefficient of friction, s the mean radius of the journals, r the radius of the wheels, t and t' the radii of the fore and hind wheels,* F *the horizontal component of the traction, and* W *the total weight or pressure on the road*

Table of the Results of some Experiments which were made at a Walking Pace.

Carriages	Roads.	Weight in Pounds.	Traction in Pounds.	Ratio of Traction to Load.
Artillery wagon	Good sand and dry	15417 10099	398 251	1 to 38.6 1 to 40.2
Cart without springs	Hard gravel and dry	15713 9812 7563	306 206 151	1 to 51.3 1 to 47.7 1 to 50.2
Cart with springs	Ordinary sand and muddy.	3528 11017	87 300	1 to 40.8 1 to 36.8
Carriages with six wheels Two connected carriages, with six wheels each	Sand, rutted and muddy	6615 13230	306 632	1 to 21.6 1 to 21

An examination of the table furnishes the following results among others:

On hard roads the traction is sensibly proportionate to the weights of the carriages, other elements being equal, and within certain limits the traction is independent of the number of wheels.

Influence of the Diameter of Wheels.—The results of experiments show that on solid roads it may be admitted as a law that the traction is inversely proportionate to the diameters of the wheels.

Influence of the Width of Wheels.—Experiments made upon wheels of different breadths, having the same diameter, give the following results:

1. *On soft grounds*, the resistance to rolling increases as the width of the felloe. 2. *On hard ground and roads of like firmness*, the resistance is very nearly independent of the width of the felloe.

Influence of Velocity.

Vehicles.	Roads.	Loads.	Speed in Miles per Hour.	Traction.	Ratio of Traction to Load.
		Lbs.		Lbs.	
Carriage on a brass shaft	Wet gravel and soft	2300	3.13 6.26	364 370	1 to 6.3 1 to 6.2
16-lb. carriage and piece	Hard gravel, even and dry	8270	2.82 3.4 8.45	203 203 267	1 to 40.8 1 to 40.8 1 to 31
Wagon on six springs	Good sand	7250	2.77 5.28	318 355	1 to 22.8 1 to 20.8
Wagon on six springs	Good sand	7400	8.05	406	1 to 18.3

The results show that on soft grounds traction has no sensible augmentation with an increase of velocity, but that on solid and uneven-surfaced roads it increases with an increase of velocity and in a greater degree, as the surface is uneven and the vehicle has less spring.

Experiments made with the carriage of a siege train on a solid gravel road and on a good sand road gave the following deductions: 1. That at a walk the traction on a good sand road is less than that on a good firm gravel road. 2. That at high speeds the traction on a good sand road increases very rapidly with the velocity. Thus a vehicle without springs, on a good sand road, gave a traction 2.64 times greater than with a similar vehicle on the same road with springs.

Friction of Roads.—According to Babbage and others, the friction of roads is as follows: A wagon with its load weighing 1000 lbs. requires a traction,

In loose sand	of 250 lbs.,	or	.25 *of the load.*
Fresh earth	" 140 "	or	.125 "
Common by-roads	" 106 "	or	.1 "
Hard dry meadow	" 40 "	or	.04 "
Dry high road	" 25 "	or	.025 "
Macadamized road	" 33 "	or	.033 "
Rail-roads	6 lbs. to 3.5 "	or	.0035 " / .0059 "

Upon well-paved roads the friction is	$\frac{1}{71}$st *part of the loaa.*
Upon graveled roads the friction is	$\frac{1}{35}$ "
Upon fresh earth the friction is	$\frac{1}{16}$ "

Friction and Rigidity of Cordage.

Experiments by Amonton and Coulomb, with an apparatus of Amonton's, furnish the following deductions:

1. That the resistance caused by the stiffness of cords about the same or like pulleys varies directly as the suspended weight.

2. That the resistance caused by the stiffness of cords increases not only in the direct proportion of the suspended weights, but also in the direct proportion of the diameter of the cords.

Consequently, that the resistance to motion over the same or like pulleys, arising from the stiffness of cords, is in the direct compound proportion of the suspended weight and the diameter of the cords.

3. That the resistance to bending could be represented by an expression consisting of two terms, the one constant for each rope, and each sheave or drum; the other proportional to the tension of the end of the rope which is being bent.

4. That the resistance to bending varied inversely as the diameter of the sheave, or drum.

5. That the complete resistance is represented by the expression $\frac{S + CT}{d}$, S *representing constant for each rope and sheave, and which expresses the natural stiffness of the rope;* T *the tension of the rope which is being bent, which is expressed by* C T; C *being a constant for each rope and sheave; and d the diameter of the sheave, including the diameter of the rope.*

6. That the stiffness of tarred ropes is sensibly greater than that of white ropes.

Extending the results obtained by Coulomb, Morin furnishes the following *formulæ*

For White Ropes: $\frac{12\,n}{d}(.00215 + .00177\,n + .0012\,W) = R.$

For Tarred Ropes: $\frac{12\,n}{d}(.01054 + .0025\,n + .0014\,W) = R$, R *representing the rigidity in pounds, n the number of yarns, d the diameter of the sheave in inches and the rope combined, and* W *the weight in pounds.*

ILLUSTRATION.—What is the value of the stiffness or resistance of a dry white rope having a diameter of 60 yarns, which runs over a sheave 6 inches in diameter in the groove, with an attached weight of 1000 lbs.?

Assume the diameter for 60 yarns to be 1.2 *ins.*

Then $\frac{12 \times 60}{7.2}$ $(.00215 + .00177 \times 60 + .0012 \times 1000) = 100 \times (.00215 + .10620 + 1.2)$ $= 100 \times 1.30835 = 130.835$ *lbs*

To Compute the Diameter of a Rope from the Number of Yarns, and contrariwise.

For White Ropes: $\sqrt{(.020739\,n)}$ = *diam. in ins.* *For Tarred Ropes:* $\sqrt{(.02883\,n)}$ = *diam in ins.* *For White Ropes:* $d^2 \div .020739$ = *number of yarns.* *For Tarred Ropes:* $d^2 \div .02883$ = *number of yarns.*

EXAMPLE.—The number of yarns in a white rope is 60; what is its diameter?

$$\sqrt{.020739 \times 60} = 1.116 \textit{ inches} \quad \text{Or,} \quad \frac{1.116^2}{.020739} = 60 \textit{ yarns.}$$

The value of the natural stiffness of ropes increases as the square of the number of threads nearly, and the value of the stiffness proportional to the tension is directly as the number of threads, being a constant number. Hence, having the rigidity for any number of threads, the rigidity for a greater or lesser number is readily ascertained.

Wire Ropes.—Weisbach deduced from his experiments on wire ropes that their rigidity for diameters capable of supporting equal strains with hemp ropes is considerably less.

Wire ropes, newly tarred or greased, have about 40 per cent. less rigidity than untarred ropes.

Friction of Axles.—With axles the friction of motion has alone been experimented upon. When the weight upon the axle and the radius of its journal is given, the *mechanical effect* of the friction may be readily determined.

The mechanical effect absorbed by, or of friction, increases with the pressure or weight upon the journal of the axle and the number of revolutions.

The friction of an axle is greater the deeper it lies in its bearing.

Coefficients of Axle Friction.—[M. MORIN.]

Substances.	Condition of the Surfaces and Unguents.				
	Dry and a little Greasy	Greasy and wetted with Water.	Oil, Tallow, or Lard. In the usual way.	Oil, Tallow, or Lard. Continuously	Very soft and purified Carriage Grease.
Bell metal upon bell metal	...	...	.097	...	...
Cast iron upon bell metal	...	...	...	.049	...
Cast iron upon bell metal	.194	.161	.075	.054	.065
Cast iron upon cast iron	...	.079	.075	.054	...
Cast iron upon lignum-vitæ	.185	...	.1	.032	.10
Lignum-vitæ upon cast iron	...	...	.116	...	...
Lignum-vitæ upon lignum-vitæ	...	...	...	.07	...
Wrought iron upon bell metal	.251	.189	.075	.054	.09
Wrought iron upon cast iron	...	...	.075	.054	...
Wrought iron upon lignum-vitæ	.188	...	.125	...	...

The friction of the journal of an axle which presses on one side only, as in a worn bearing, is less than when it presses at all points, the difference being about .005.

If the journal of an axle revolves in a nave or eye, the radius of the friction path is the radius of the nave or eye.

If the journal of an axle lies in a prismatic bearing, as in a triangle, etc., the friction is greater, as there is more pressure on, and consequently greater friction in contact. in a triangular bearing it is about double that of a cylindrical bearing.

To Compute the Mechanical Effect of Friction on the Journal of an Axle.

$\frac{p\,t\,f\,W\,r}{30} = F$, *p representing the ratio of the circumference of a circle to its diameter, t the number of revolutions, and r the radius of the journal in feet.*

EXAMPLE.—The weight of a wheel, with its axle or shaft resting on its journals, is 360 pounds; the diameter of the journals 2 inches; and the number of revolutions 30; what is the mechanical effect of the friction, the coefficient of it being .16?

$$\frac{3.1416 \times 30 \times .16 \times 360 \times \overline{1 \div 12}}{30} = \frac{452.4}{30} = 15.08\ lbs.$$

By the application of friction-wheels (rollers) the friction is much reduced, and the mechanical effect then becomes, when the weights of the friction-wheels are disregarded,

$\frac{p\,t\,f\,W\,r}{30} \times \frac{r'}{a'\ cos\ \overline{a \div 2}} = F$, *r′ representing the radii of the axles of the friction-wheels, a′ the radii of the friction-wheels, and a the angle of the lines of direction between the axis of the roller and the axis of the friction-wheels.*

When a single Friction-wheel is used, $\frac{2\,p\,r\,t}{60} \times f\,W = F$, and $\frac{F}{r' \div a'} = F'$, *F′ representing the mechanical effect.*

EXAMPLE. — A wheel and its shaft, making 5 revolutions per minute, weighs 30000 lbs.; its diameter and that of its journals are 32 feet and 10 inches. The journals rest upon a friction-wheel, the radius of which is 5 times greater than its axle.

1. What is the power at the circumference of the wheel necessary to overcome the friction? 2. What is the mechanical effect of the friction? 3. What is the reduction of friction by the use of the friction-wheel?

The coefficient of friction is assumed to be .075.

1st. $\frac{\overline{32 - 2} \times 12}{10 \div 2} = 38.4$ *times, i. e., the circumference of the wheel = 38.4 times that of the axle.* Hence $\frac{30000 \times .075}{38.4} = 58.59$ *lbs. = power required at circumference to overcome friction at axle.*

2d. $\frac{10 \times 3.1416}{12} = 2.618$ *feet = distance passed by friction.*

Consequently, $\frac{2.618 \times 5}{60} = .2181$ *feet = distance passed by friction at 5 revolutions in one second.* Hence $.2181 \times 2250\ (30000 \times .075) = 490.71$.

3d. $1 \div 5 = .2 =$ *radius of friction-axle ÷ by radius of friction-wheel.* Hence $38.4 \times .2 = 7.68 =$ *friction referred to circumference of wheel, and* $\frac{490.71}{5} = 98.142$ *= mechanical effect by application of friction-wheel = a reduction of four fifths.*

Friction of Pivots.—Friction on Pivots is independent of their velocity, and increases in a greater degree than their pressures.

Friction on Conical Bearings is greater than with like elements on plane surfaces.

The figure of the point of a pivot, as to its acuteness, affects the friction: with great pressure the most advantageous angle for the figure ranges from 30° to 45°; with less pressure it may be reduced to 10° and 12°.

Relative Value of Angles of Pivots.

6°	.1.	15°	.66	45°	.39

Relative Values of different Materials for use as Pivots.

Agate	.83	Granite	1.	Tempered steel	.44
Glass	.55	Rock crystal	.76		

The friction of a pivot approximates so near to that of sliding and axle friction, that the following coefficients are used:

Coefficients of the Friction of Motion.

Substances.		Condition of the Surfaces and Unguents.						
		Dry.	Water.	Olive-oil.	Lard.	Tallow.	Dry Soap.	Greasy and wet.
Hemp cords, etc.	On wood	.45	.33	..	..	..	..	..
	On iron.	..	..	.15	..	.19	..	..
Metal upon wool	Mean...	.18	.31	.07	.09	.09	.20	.13
Sole-leather, smooth, upon wood	Raw ...	.54	.36	.16	..	.20	..	..
or meta'	Dry....	.34	.31	.14	..	.14	..	..
Wood upon metal	Mean...	.42	.24	.06	.07	.08	.20	.14
Wood upon wood		.36	.25	..	.07	.07	.15	.12

To Compute the Mechanical Effect absorbed by the Friction of Pivots.

It is necessary to have the coefficients of friction, and to know the mean space which the base of the point describes in a revolution.

When the Section is that of a Parallelogram, $\frac{4}{3} p f \mathrm{W} r = \mathrm{F}$, *r representing the radius of the base in feet.*

EXAMPLE.—A vertical shaft, having a pivot or journal at its base, 1 inch in diameter, with its superincumbent wheel, weighs 360 lbs., and makes 100 revolutions per minute; what is the friction, and what the loss of mechanical effect?

The coefficient of friction is assumed to be .1; then $.1 \times 360 = 36$ lbs. The space per revolution $= \frac{4}{3} \times 3.14166 \times .5$ of $\frac{1}{12} = 4.188 \times .0416 = .1746$ *feet.*

Hence the mechanical effect per revolution $= 36 \times .1746 = 6.2856$ *lbs.*; and, as the shaft makes 100 revolutions per minute, $\frac{100}{60} \times 6.2856 = 10.476$ *lbs.*

When the Section is that of a Ring having Sections of a Parallelogram,

$$\frac{4}{3}\left(\frac{r^3 - r'^3}{r^2 - r'^2}\right) = \mathrm{F}.$$

EXAMPLE.—The elements the same as in the preceding example, with the exception that the radii of the ring are taken instead of the radius of the journals. Thus, the external and internal diameters of the ring are 1 inch and 5 inches.

$$\frac{4}{3} \times 3.1416 \times \left(\frac{.5^3 - .25^3}{.5^2 - .25^2}\right) \text{ of } \frac{1}{12} = 4.88 \times \frac{.1094}{.1875} \text{ of } \frac{1}{12} = 4.88 \times \frac{.1094}{2.25} = .2036.$$

Hence the mechanical effect per revolution $= 36 \times .2036 = 7.3296$ *lbs.*, and $\frac{100}{60} \times 8.3296 = 12.216$ *lbs.*

When the Section is that of a Triangle, $\frac{4}{3} p f \frac{\mathrm{W} r}{\sin. a} = \mathrm{F}$. *a = the half of the angle of the vertex of the triangle.*

EXAMPLE.—The elements the same as in the first example, with the addition of the angle $a = 30°$.

$$\frac{4}{3} \times 3.1416 \times \frac{.5}{\sin. a} \times \frac{1}{24} = 4.188 \times \frac{.5}{.5} \times .04166 = .1746.$$

Hence the mechanical effect per revolution $= 36 \times .1746 = 6.286$ *lbs.*, and $\frac{100}{60} \times 6.286 = 10.476$ *lbs.*

NOTE.—When the ends of pivots on vertical shafts are rounded, the friction is not diminished thereby.

Rolling Friction.—*Rolling Friction* increases with the pressure, and is inversely as the diameter of the rolling body.

For rolling upon compressed wood, $f = .019$ to $.031$.

When a Body is moved upon Rollers and the Power applied at the Base of the Body, $(f+f')\frac{W}{r} = F$, *f and f' representing the coefficients of friction of the two surfaces upon which the rollers act.*

When the Power is applied at the Circumference of the Roller, $f\frac{W}{r} = F$.

When the Power is applied at the Axis of the Roller, $f\frac{W}{r \div 2} = F$.

FRICTION OF MACHINERY AND TRAINS ON RAILWAYS.

To Compute the Resistance to a Train on a Railway.

Let P *represent the power requisite to draw a weight* W, *including its friction, on a plane with a rise of h feet in* 100; R *the power requisite to draw* W *on the plane exclusive of friction; and let* F, *the friction, be* $\frac{W}{n}$ = *an n part of* W.

By the formulæ for inclined planes, $R = \frac{h\,W}{100}$, and $P = R + F = \left(\frac{h\,W}{100} + \frac{W}{n}\right)$.

Hence $\frac{100\,n\,P}{h\,n + 100} = W$, and $\frac{h\,n + 100 \times W}{100\,n} = P$.

Illustration.—If a train of 30 tons' weight is drawn on a level rail-road, what power is necessary to overcome the resistance or friction, it being 8 lbs. per ton?

$2240 \div 8 = 280$ = hence the resistance = $\frac{1}{280}$ *of the weight.*

$$P = F = \frac{W}{n} = \frac{30 \times 2240}{280} = 240 \text{ lbs.}$$

2.—The grade of a railway is 2 feet in 100 feet; what power is required to draw a load of 50 tons up the grade, the coefficient of friction being $\frac{1}{280}$, or $n = 280$?

$$\frac{2 \times 280 + 100 \times 50 \times 2240}{100 \times 280} = \frac{73920000}{28000} = 2640 \text{ lbs.}$$

To Compute the Power, Speed, or Time of Running a Locomotive or Train upon a Railway.

On a Horizontal Plane.—Let H *represent the horses' power,* S *the space in miles passed over in the time t in minutes, and* W *the weight in tons.*

$$\frac{1.28^{*}\,W \times S}{t} = H.\ \frac{H\,t}{1.28\,W} = S;\ \frac{1.28\,W \times S}{H} = t;\ \frac{H\,t}{1.28\,S} = W.$$

Illustration.—What is the required power of a locomotive to draw a train of 45 tons at the rate of 50 miles per hour?

$$\frac{1.28 \times 45 \times 50}{60} = \frac{2880}{60} = 48 \text{ horses.}$$

2.—In what time will a locomotive of 50 horses' power, drawing a train of 135 tons, run a distance of 80 miles?

$$\frac{1.28 \times 135 \times 80}{50} = \frac{13824}{50} = 276.24 \text{ minutes.}$$

On an Inclined Plane.—Let *h represent the rise or fall of the plane in every* 100 *feet.*

$$\frac{256\,(5 \pm 14\,h)\,W \times S}{1000\,t} = H.\ \frac{1000\,t \times H}{256\,(5 \pm 14\,h)\,W} = S;\ \frac{256\,(5 \pm 14\,h)\,W \times S}{1000\,H} = t;$$

$$\frac{1000\,t \times H}{256\,(5 \pm 14\,h)\,S} = W;\ \frac{1000\,t \times H - 1280\,W \times S}{3584\,W \times S} = h.$$

* This constant represents a coefficient of friction of 1-280 of the weight of the train.

ILLUSTRATION.—A train of 40 tons in weight ascends a road having a rise of 2 feet in 100 at a speed of 15 miles per hour; what is the power of the engine?

$$\frac{256\times(5+\overline{14\times 2})\times 40\times 15}{1000\times 60}=\frac{5068800}{6000}=84.48\ horses.$$

NOTE.—When the train or load descends a plane, h must be taken negatively, or —, as gravity in this case assists the moving power; it also appears that, when $-h$ $=5\text{-}14$ of a foot, no moving power is required to draw the train.

2.—A train of 60 tons descends a road falling 3 inches in 100 feet with a speed of 50 miles per hour; what is the horses' power exerted by the engine?

Here h is negative, and 3 inches $=.25$ feet.

$$\frac{256\times(5-\overline{14.25})\times 50\times 60}{1000\times 60}=\frac{1152000}{6000}=19.2\ horses.$$

Friction of Locomotives and Railway Trains.

Locomotive moving friction	15 lbs. per ton.
Trains " "	6 " " "

Friction developed in the Launching of Vessels.

Experiments made by a committee of the Franklin Institute on the friction of launching vessels gave, when the pressure or weight was from 2280 to 3560 per square foot, a coefficient of $\frac{1}{29.8}=.0335$.

STABILITY.

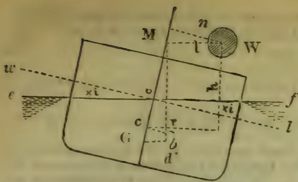
STABILITY, *Strength*, and *Stiffness* are necessary to the permanence of a structure, under all the variations or distributions of the load or stress to which it may be subjected.

Stability of a Fixed Body is the power of remaining *in equilibrio* without sensible deviation of position, notwithstanding the load or stress to which it may be submitted may have certain directions.

Stability of a Floating Body.—A body floating in a fluid is balanced, or at rest, when it displaces a volume of the fluid, the weight of which is equal to the weight of the floating body, and when the centre of gravity of the floating body and that of the volume, from which the fluid is displaced, are in the same vertical plane.

When a body *in equilibrio* is free to move, and is caused to deviate in a small degree from its position of equilibrium, if it does not tend to deviate further, or to recover its original position, its equilibrium is termed *Indifferent;* when it tends to deviate further from its original position, its equilibrium is *Unstable;* and when it tends to return to its original position, its equilibrium is termed *Stable.*

A body *in equilibrio* may be stable for one direction of stress, and unstable for another.



Assume figure to represent the cross-section of the hull of a vessel, G the centre of gravity of the hull, *w l* the water-line, and *c* the centre of buoyancy of the immersed section in the position of equilibrium. Conceive the vessel to be heeled or inclined over, so that *e f* becomes the water-line, and *b* the centre of buoyancy of the immersed section; produce *b* M, and the point M is the *meta-centre** of the hull of the vessel.

The *Comparative Stability* of different hulls or vessels is proportionate to the distance of G M for the same angles of heeling, or of the distance G *d*. The oscillations of the hull of a vessel may be revolved into a rolling about its longitudinal axis, pitching about its transverse axis and vertical pitching, consisting in rising and sinking below and above the position of equilibrium.

If the transverse section of the hull of a vessel is such that, when the vessel heels, the level of the centre of gravity is not altered, then her rolling will be about a permanent longitudinal axis traversing her centre of gravity, and it will not be accompanied by any vertical oscillations or pitchings, and the moment of her *inertia* will be constant while she rolls. But if, when the vessel heels, the level of her centre of gravity is altered, then the axis about which she rolls becomes an instantaneous one, and the moment of her *inertia* will vary as she rolls; her rolling must, then, necessarily be accompanied by vertical oscillations.

Such oscillations tend to strain a vessel and her spars, and it is desirable, therefore, that the transverse section of her hull should be such that the centre of its gravity should not alter as she rolls, a condition which is always secured if all the water-lines, as *w l* and *e f*, are tangents to a common sphere described about G; or, in other words, if the point of their intersections, *o*, with the vertical plane of the keel, is always equidistant from the centre of gravity of the hull.

The *Momentum of Stability* of a floating body is equal to the product of the weight of the fluid displaced, and the horizontal distances between the two centres of gravity of the body and of the displacement.

To Determine the Measure of the Stability of the Hull of a Vessel or of a Floating Body.

The measure of the stability of a floating body depends essentially upon the horizontal distance, G *d*, of the *meta-centre* of the body from the centre of gravity of the body; and it is the product of the force of the water, or resistance to displacement of it, acting upward, and the distance of G *d*, or P × G *d*. If the distance, *c* M, is represented by *r*, and the angle of rolling, *c* M *b*, by M°, the measure of stability on S is determined by P *r*, sin. M° = S; and this is therefore the greater, the greater the weight of the body, the greater the distance of the *meta-centre* from the centre of gravity of the body, and the greater the angle of inclination of this or of *c* M *b*.

* The *meta-centre* depends upon the position of the centre of buoyancy, for it is that point where a vertical line drawn from the centre intersects a line passing through the centre of gravity, or the hull of the vessel perpendicular to the plane of the keel.

The point of the *meta-centre* may be the same, or it may differ slightly for different angles of heeling. The angle of direction adopted to ascertain the position of the meta-centre should be the greatest which, under ordinary circumstances, is of probable occurrence; in different vessels this angle ranges from 60° to 20°.

If the *meta-centre* is above the centre of gravity, the equilibrium is Stable; if it coincides with it, the equilibrium is Indifferent; and if it is below it, the equilibrium is Unstable.

ILLUSTRATION.—The weight of a floating body is 5515 lbs., the distance between its centre of gravity and the *meta-centre* is 11-32 feet, and the angle M is 20°.

Hence $S = 5515 \times 11.32 \times .34202 = 21352.24$ *lbs*.

To Compute the Elements of Stability of a Floating Body.

$A\,s = A_{,}a$; $\frac{A_{,}}{A}a = s$; $\frac{s}{\sin. M} = g$; $\frac{A_{,}a}{A \sin. M} = g$, $\frac{A_{,}a}{A} \pm \overline{e \sin. M} = c$; $\frac{c}{\sin. M} = r$;

and $\sin. M\, r = c$, A *representing area of immersed section;* $A_{,}$ *the section immersed by the careening of the body, as* w o e; s *the horizontal distance,* c r, *between the centres of buoyancy;* a *the horizontal distance between the centre of gravity,* u, *of the areas immersed and emerged by careening;* g *the distance,* c M, *below the centre of buoyancy of the body, or of the water displaced and the meta-centre;* r *the distance between the centre of gravity of the body and the meta-centre;* c *the horizontal distance,* G d, *between the centre of gravity of the body and of the line of displacement of it when careened, all in feet; and* M *the angle of careening of the body.*

NOTE.—When the centre of gravity, G, is below that of the displacement, *c*, then *e* is +; when it is above *c*, it is —; and when it coincides with *c*, it is 0; or *e* is — when $\frac{S}{P} < s$; and a body will roll over when $e \sin. M =$ or $> s$.

The other elements of the body above given, deduced by the formulæ, etc., are, $A = 86$, $A_{,} = 21.5$, $a = 14.8$, $s = 3.7$, $r = 11.32$, $g = 10.82$, $c = 3.87$, $e = +.5$, $b = 21.5$, and $P = 5515$, *b representing the breadth in feet, and* P *the weight or displacement of the body in lbs.*

Of the Hull of a Vessel.—$\left(\frac{b^3}{10.7 \text{ to } 13\,A^*} \pm e\right)P, \sin. M = S$; $d \cos. \tfrac{1}{2} M = d'$, *d representing depth of centre of gravity of displacement under water in equilibrium, and* d' *the depth when out of equilibrium, all in feet.*

$\frac{b^3}{10.7 \text{ to } 13\,A} = g$; $\frac{1}{\sin. M}\left(\frac{S}{P} - s\right) = \pm e$; $P\left(\frac{b\,a}{A} + \overline{e \sin. M}\right) = S$; and $P(s \pm \overline{e \sin. M}) = S$.

ILLUSTRATION.—The displacement of a vessel is 10 000 000 lbs.; area of immersed section 800 square feet; vertical distance from centre of gravity of hull up to the centre of buoyancy or displacement, 1.9 feet; and horizontal distance between the centres of gravity of the areas immersed and emerged, when careened to an angle of 9° 10′ = 33.4 feet, the immersed area being 50 square feet.

Hence $s = \frac{50}{800} \times 33.4 = 2.0875$ *feet*; $g = \frac{2.0875}{.1593} = 13.1$ *feet*; $r = \frac{2.39}{.1593} = 15$ *feet*; $c = \frac{50 \times 33.4}{800} + \overline{1.9 \times .1593} = 2.39$ *feet*; $S = 10\,000\,000\left(\frac{50 \times 33.4}{800} + \overline{1.9 \times .1593}\right) = 23\,901\,700$ *lbs.*; or $S = 10\,000\,000\,(2.0875 + 1.9 \times .1593) = 23\,901\,700$ *lbs*.

2.—The length of a hull is 300, and breadth of beam 50 feet.

Hence $S = \left(\frac{50^3}{11.93 \times 800}\right) + 1.910\,000\,000 \times .1593 \times 23\,905\,396$ *lbs.*; $g = \frac{50^3}{11.93 \times 800} = 13.1$ *feet*; and $e = \frac{1}{.1593}\left(\frac{23\,901\,700}{10\,000\,000} - 2.0875\right) = 1.9$ *feet*.

To Compute the Elements of the Power, etc., required to Careen a Body or Vessel.

$W\,l\,r = P\,c$, and $W\,l = S$, W *representing the weight or power exerted, and* l *the distance at which the weight or effort acts to careen the body taken from the centre of gravity of the displacement, perpendicular to the careening force in feet.*

$\text{Sin. } M\,(h - \overline{n \sin. M}) + \overline{n \sec. M} - s = l$, and $\frac{b^3}{10.7 \text{ to } 13\,A}\sqrt[3]{\frac{P}{64.125\,L\,A}} = m$,

h representing the vertical height from centre of gravity of displacement to centre of the weight or effort to careen the body when it is in equilibrium, n *the horizontal distance from the centre of the vessel to the centre of the weight or effort,* L *the length of the vessel, and* m *the meta-centre, all in feet.*

* The unit for the section of a parallelogram is 10.7; of a semicircle, 12; and of a triangle, 12.8.

ILLUSTRATION.—A weight is placed upon the deck of a vessel at a mean distance of 3.87 feet from the centre line of the hull; the height at which it is placed is 10.82 feet, and the other elements as in the first case given.

Then $h = 10.82$, $n = 3.87$, and $.34202\ (10.82 - \overline{3.87 \times .34202}) + 3.87 \times 1.06418 - 3.7 = .34202 \times 9.5 + 4.12 - 3.7 = 3.67$ *feet.*

To Compute the Measure of the careening Power of a Sailing Vessel.

F f sin. w, cos. $s =$ P, F *representing area of sails, i square feet, f force of wind in lbs. per square foot, w angle of the wind to the sails, and s angle of sails to the plane of the vessel.*

To Compute the Measure of the Sailing Power of a Vessel.

F f sin. w, sin. $s =$ P.

To Compute the Alteration of the Trim of a Vessel.

$\frac{w\ a}{\text{M}} =$ *distance in feet, w representing the weight or weights in tons moved through a mean distance, a; and* $\text{M} = \text{D} \times \frac{l\,m}{\text{W}\,l}$, D *representing displacement in tons, l m distance of longitudinal meta-centre above the centre of gravity, and* W l *the length of the vessel at her load water-line.*

ILLUSTRATION.—If a steamer has a displacement of 8625 tons, a length of water-line of 380 feet, and the height of her longitudinal *meta-centre* is 475 feet; what would be the alteration of her trim if 6 guns of 6 tons' weight were moved aft a distance of 248 feet?

$$8625 \times \frac{475}{380} = 10781. \quad \text{Then } \frac{6 \times 6 \times 248}{10781} = .83 \textit{ feet} = 9.96 \textit{ inches.}$$

Results of Experiments upon the Stability of Rectangular Blocks of Wood of uniform Length and Depth, but of different Breadths.—[W. BLAND.]

(Length 15, Depth 2, and Depression 1 inch.)

Width.	Weight.	Ratios of Stability.			
		As Observed.	With like Weights.	By Squares of the Widths.	By Cubes of the Width.
Ins.	Oz.				
3.	24	1.	1.	1.	1.
4.5	35	3.5	2.4	2.25	3.375
6.	45	7.	3.7	4.	8.
7.5	55	11.	4.8	6.25	15.625

Hence it appears that rectangular and homogeneous bodies of a uniform length, depth, weight, and immersion in a fluid, but of different widths, have stability for uniform depressions at their sides (heeling) nearly *as the squares of their width;* and that, when the weights are directly as their widths, their stability under like circumstances is nearly *as the cubes of their width.*

Further experiments deduced the following results:

1. That rectangular and homogeneous bodies of a uniform width, depth, and immersion in a fluid, but of different lengths, have stability for uniform depressions at their sides nearly *as their weight*, and without reference to their lengths, and that, when the weights are directly as their lengths, their stability under like circumstances is nearly directly as their lengths.

2. That like bodies of a uniform width, length, an immersion of half their depth, but of different depths, have stability for uniform depressions at their sides nearly *inversely as their depths*, and that, when the weights are directly as the depths, their stability is *inversely as their depths.*

Results of Experiments upon the Stability and Speed of Models having Amidship Sections of different Forms, but of uniform Length, Width, and Weights.—(W. Bland.)

(Immersion different, depending upon form of Section.)

Form of Immersed Section.	Stability.	Speed.
Half-depth triangle; the other half rectangle	12.	4.
Rectangle	14.	3.
Right-angled triangle*	7.	3.
Semicircle	9.	2.

. See page 611 for further results.

MODELS.

The classes of forces to which Models are subjected are,

1. To draw them asunder by *tensile* strain. 2. To break them by *transverse* strain. 3. To crush them by *compression.*

The stress upon the side of a model is to the corresponding side of a structure as the cube of its corresponding magnitude. Thus, if the structure is six times greater than the model, the stress upon it is as 6^3 to $1 = 216$ to 1. But the resistance of rupture increases only as the squares of the corresponding magnitudes, or as 6^2 to $1 = 36$ to 1. A structure, therefore, will bear as much less resistance than its model as its side is greater.

To Compute the Dimensions of the Beam, etc., which a Structure can bear.

Rule.—Divide the greatest weight which the beam, etc., in the model can bear by the greatest weight which it actually sustains, and the quotient, multiplied by the length of the beam, etc., in the model, will give the length of the beam, etc., in the structure.

Example.—A beam in a model is 7 inches in length, and sustains a weight of 4 pounds, but it is capable of bearing a weight of 26 lbs.; what is the greatest length that the corresponding beam can be made in the structure?

$$26 \div 4 = 6.5, \text{ and } 6.5 \times 7 = 45.5 \textit{ inches.}$$

The resistance in a model to crushing increases directly as its dimensions; but as strain increases as the cubes of the dimensions, a model is stronger than the structure, inversely as the squares of their comparative magnitude.

Hence the greatest magnitude of a structure is ascertained by taking the square root of the quotient, as obtained by the preceding rule, instead of the quotient itself.

Example.—If the greatest weight which a column in a model can sustain is 26 lbs., and it actually bears 4 lbs.; then, if the height of the column is 18 inches, what will be the height of it in the structure?

$$\sqrt{\left(\frac{26}{4}\right)} = \sqrt{6.5} = 2.55, \text{ and } 2.55 \times 18 = 45.9 \textit{ ins., height of the column in the structure.}$$

If, when the length or height and the breadth are retained, and it is required to give to the beam, etc., such a thickness or depth that it will not break in consequence of its increased dimensions,

Then $\sqrt{\left(\frac{26}{4}\right)} = \sqrt{6.5} = 2.55$, which, $\times$ the square of the relative size of the model $=$ *the thickness required.*

If it were required to compute the resistance of a bridge from an experiment made with a model,

$$n^2 W - \frac{n^2}{2}(n-1)\,w = \textit{the load the bridge will bear in its centre.}$$

* Draught of water or immersion double that of the rectangle.

Suppose l the length of the platform of the model between the centres of its repose upon the piers to be 12 feet, its weight w 30 lbs., and the weight W it will just sustain at its centre 350 lbs.

Let the comparative magnitudes, n, of the model and the bridge be as 20, and the actual length of the bridge 240 feet; what is the weight the bridge will sustain?

$$20^2 \times 350 - \frac{400}{2} \times (20 - 1) \times 30 = 140000 - 3800 \times 30 = 26000 \textit{ lbs.}$$

HYDRAULICS.

Descending Fluids are actuated by the same laws as *Falling Bodies.*

A Fluid will fall through 1 foot in ¼ of a second, 4 feet in ½ of a second, and through 9 feet in ¾ of a second, and so on.

The Velocity of a fluid, flowing through an aperture in the side of a vessel, reservoir, or bulkhead, is the same that a heavy body would acquire by falling freely from a height equal to that between the surface of the fluid and the middle of the aperture.

The Velocity of a fluid flowing out of an aperture is as the square root of the height of the head of the fluid. The *Theoretical* velocity, therefore, in feet per second is as the square root of the product of the space fallen through in feet and 64.333; consequently, for 1 foot it is $\sqrt{64.333} = 8.02$ *feet.* The *Mean* velocity, however, of a number of experiments gives 5.4 feet, or .673.

In short ajutages accurately rounded, and having the form of the contracted vein, or *vena contracta*, the coefficient of discharge = .974 that of the theoretical.

Fluids subside to a natural level, or curve similar to the earth's convexity; the apparent level, or level taken by any instrument for that purpose, is only a tangent to the earth's circumference; hence, in leveling for canals, etc., the difference caused by the earth's curvature must be deducted from the apparent level to obtain the true level.

Deductions from Experiments on the Discharge of Fluids from Reservoirs.

1. That the quantities of a fluid discharged in equal times by the same apertures from the same head are nearly as the areas of the apertures.

2. That the quantities of a fluid discharged in equal times by the same apertures, under different heads, are nearly as the square roots of the corresponding heights of the fluid above the surface of the apertures.

3. That the quantities of a fluid discharged during the same time by different apertures, under different heights of the fluid, are to one another in the compound ratio of the areas of the apertures and of the square roots of the heights of the fluid above the centre of the apertures.

4. That, on account of the friction, small-lipped or thin orifices discharge proportionally more fluid than those which are larger and of similar figure, under the same height of fluid.

5. That in consequence of a slight augmentation which the contraction of the fluid vein undergoes, in proportion as the height of a fluid increases, the flow is a little diminished.

6. That circular apertures are the most effective, as they have less rubbing or frictional surface for the same area.

7. That the discharge of a fluid through a cylindrical horizontal tube, the diameter and length of which are equal to one another, is the same as through a simple aperture.

8. That if a cylindrical horizontal tube is of greater length than the diameter, the discharge of a fluid is much increased, and may be increased with advantage, up to a length of tube of four times the diameter of the aperture.

9. That the discharge of a fluid by a vertical pipe is augmented, on the principle of the gravitation of falling bodies; consequently, the greater the length of a pipe, the greater the discharge of the fluid.

10. That the discharge of a fluid by an inclined pipe is greater than by a horizontal pipe of the same dimensions.

11. That the discharge of a fluid is inversely as the square root of its density.

12. That the velocity of a fluid line passing from a reservoir at any point is equal to the ordinate of a parabola, of which twice the action of gravity ($2g$) is the parameter, the distance of this point below the surface of the reservoir being the abscissa.*

13. The volume of water discharged through an aperture from a prismatic vessel which empties itself is only half of what it would have been during the time of emptying if the flow had taken place constantly under the same head and corresponding velocity as at the commencement of the discharge; consequently, the time in which such a vessel empties itself is double the time in which all its fluid would have run out if the head had remained uniform.

14. The mean velocity of a fluid flowing from a rectangular slit in the side of a reservoir is two thirds of that due to the velocity at the sill or lowest point, or it is that due to a point four ninths of the whole height from the surface of the reservoir.

15. The velocity of efflux increases as the square root of the pressure on the surface of a fluid.

16. In efflux under water, the difference of levels between the surfaces must be taken as the head of the flowing water.

17. To attain the greatest mechanical effect, or *vis viva*, of water flowing through an opening, it should flow through a circular aperture in a thin plate in preference to a prismatic tube.

18. The discharge of fluids through apertures slightly under water is nearly equal to the discharge in air.

It is ascertained in practice that water issuing from a circular aperture in a thin plate contracts its section at a distance of half its diameter from the aperture to very nearly .8, the diameter of the aperture, so as to reduce its area from 1 to about .617.† The velocity at this point is also ascertained to be about .974 times the theoretical velocity due to a body falling from a height equal to the head of water. The mean velocity *in the aperture* is therefore .974, which, $\times$ .617 = .6, the theoretical discharge; and in this case .6 becomes the *coefficient of discharge*, which, if expressed generally by C, will give for the discharge itself

$a\sqrt{2\,g\,h} \times C = D$, D *representing the discharge per second, and a the area of the aperture.*

For *Square Apertures* it is .615, and for *Rectangular* .621.

To Compute the Difference between True and Apparent Level.

When the distance is in	Feet,	multiply the square of the distance by	.000000287	and the product is the difference in inches when refraction is not taken into account.
	Yards,		.000002583	
	Chains,		.00125	

* See D'Aubuisson, p. 66

† Bayer, .61; Bossut, .666; Venturi, .637; Eytelwein, .64; Michelotti, .64 The observed discharges of water coincide nearer to the unit of Bayer than that of others.

NOTE.—If the distance is considerable, and the refraction must be considered, diminish the distance by .0833.

EXAMPLE.—What is the difference between the true and apparent level at a distance of 18 chains, when the refraction is taken into account?

$18 \times .0833 = 1.5$, $18 - 1.5 = 16.5$, and $16.5^2 \times .00125 = .3403$ *inch.*

Deduction from Experiments on the Discharge of Water by Horizontal Conduit or Conducting Pipes.—[M. BOSSUT.]

1. The less the diameter of the pipe, the less is the proportional discharge of the fluid.

2. The greater the length of conduit pipe, the greater the diminution of the discharge.

3. The discharges made in equal times by horizontal pipes of different lengths, but of the same diameter, and under the same altitude of fluid, are to one another in the inverse ratio of the square roots of their lengths.

4. In order to have a perceptible and continuous discharge of fluid, the altitude of it in a reservoir, above the plane of the conduit pipe, must not be less than .082 inches for every 100 feet of the length of the pipe.

5. In the construction of hydraulic machines, it is not enough that elbows and contractions be avoided, but also any intermediate enlargements, the injurious effects of which are proportionate, as in the following Table:

Relative Velocity of the Discharge of like Quantities of Fluid under like Heads in Pipes having a different Number of Enlarged Parts.

Number of Parts.	Relative Velocity.	Number of Parts.	Relative Velocity.	Number of Parts.	Relative Velocity.	Number of Parts.	Relative Velocity.
0	1.	1	.741	3	.569	5	.451

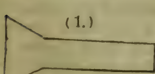
Friction.

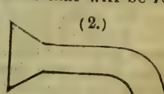
The friction of a fluid gliding over a solid surface exerts a resistance to its motion which is proportional to the surfaces of contact and to the density of the fluid, and approximately to the square of the velocity of its motion; that is, the resistance is approximately proportional to the weight of a prism of the fluid, the base of which is the surface of contact, and its height that due to its velocity.

$C\,W\,s\,\frac{v^2}{2g} = F$, C *representing the coefficient of friction,* W *the weight of a unit of volume of the fluid in pounds, and s the surface of contact in square feet.*

The flowing of liquids through pipes or in natural channels is liable to be materially affected by friction. Liquids flow smoothly and with least retardation when the course is perfectly smooth and straight.

Thus, if equal quantities of water were to be discharged through pipes of equal diameters and lengths, but of the following forms:


(1.)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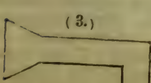

(3.)

and the time that the quantity discharged through the first is represented by 1; the time that will be required to discharge an equal quantity through the second will be 1.11; and the time for the same quantity through the third, 1.55; and the Velocities of motion that would result under like heads would be as 1 for the first, .72 for the second, and .64 for the third.

When a fluid issues out of a circular aperture in a thin plate in the bottom or side of a reservoir, the issuing stream tends to converge to a point at the distance of about half its diameter outside the aperture, and this contraction of the stream reduces the area of its section from 1 to .644.

When a fluid issues through a short tube, the vein is less contracted than in the preceding case, in the proportion of 16 to 13; and if it issues through an aperture which is alike to the frustrum of a cone, the base of which is the aperture, the height of the frustrum half the diameter of the aperture, and the area of the small end to the area of the large end as 10 to 16, there will be no contraction of the vein. Hence this form of aperture will give the greatest attainable discharge of a fluid.

The volume of a fluid that will flow out of a Vertical Rectangular Opening or Slit, that reaches as high as the surface of the fluid, is .666 of that which would flow out of the same aperture if it were horizontal at the depth of the base.

Discharges from Compound or Divided Reservoirs.

The velocity in each may be considered as generated by the difference of the heights in the two contiguous reservoirs; consequently, the square root of the difference will represent the velocities, which, if there are several apertures, must be inversely as their respective areas.

NOTE.—When water flows into a vacuum, 32.166 feet must be added to the height of it; and when into a rarefied space only, the height due to the difference of the external and internal pressure only must be added.

VELOCITY OF WATER OR OF FLUIDS.

Coefficients of Discharge.

The *Coefficient of Discharge* or *Efflux* is the product of the coefficients of *Contraction* and *Velocity*.

The quantity of water or a fluid discharged in a given time from an aperture of a given area depends on the head, form of the aperture, and nature of the approaches. Two cases present themselves in practice: first, apertures in thin plates in the sides or bottom of a vessel or reservoir; and, second, apertures at the ends of short or long tubes. If v represent the velocity per second acquired by falling from a height h, and g the velocity acquired at the end of a second, then

$$\sqrt{2gh} = v.$$

The value of g varies slightly with the latitude and altitude of the place of observation, but may be assumed for most practical purposes equal to 32.166 feet; then, for measures in feet,

$v = \sqrt{64.333\,h}$, and $h = \frac{v^2}{64.333}$, *h representing the height measured to the centre of the opening.*

The *head* or *height*, h, may be measured from the surface of the water to the centre of the aperture without practical error, for it has been proved by Mr. John Neville that for circular apertures, having their centre at the depth of their radius below the surface, and therefore the circumference touching the surface, the error can not exceed four per cent. in excess of the true theoretical discharge, and that for depths exceeding three times the diameter the error is practically immaterial. For rectangular apertures it is also shown that, when their upper side is at the surface of the water, as in notches, the extreme error can not exceed 4.17 per cent. in excess; and when the upper is three times the depth of the aperture below the surface, the excess is inappreciable. For notches, weirs, slits, etc., however, it is usual to take the full depth for the head, when the above equation must be multiplied by two thirds to ascertain the discharge.

Experiments show, 1. That the coefficient for similar apertures in thin plates, for small apertures and low velocities, is greater than for large apertures and high velocities, and that for elongated and small apertures it is greater than for apertures which have a regular form, and which approximate to the circle.

2. That it increases with the ratio of the aperture to the approaching channel when formed at the end of a short tube.

3. That it increases when the aperture is at the side or bottom of a vessel, and when the contraction is partial.

4. That it increases when the dimensions of the aperture and the head of water decrease.

When the head, measured from the surface to the centre of an aperture, is the same, the discharge from them, whether horizontal or lateral, if of equal area, is practically the same.

When the Discharge of a Fluid is under the Surface of another body of a like Fluid, the difference of the levels between the two surfaces must be taken as the head of the water or fluid.

$$\text{Or, } \sqrt{2\,g\,(h-h')}=v.$$

When the Outer Side of the opening of a discharging Vessel is pressed by a Force, the difference of the height of the head of the fluid and the quotient of the pressures on the two sides of the vessel, divided by the density of the fluid, must be taken as the heads of the fluid.

$$\text{Or, } \sqrt{2\,g\left(h-\frac{p-p'\times 144}{S}\right)}=v,$$ S *representing the density of the fluid.*

ILLUSTRATION.—Assume the head of water in the open reservoir of a steam boiler is 12 feet above the water-line in the boiler, and the pressures of the atmosphere and the steam are respectively 14.7 and 19.7 lbs.

$$\text{Then } \sqrt{2\,g\left(12-\frac{19.7-14.7\times 144}{62.5}\right)}=\sqrt{64.333\times\left(12-\frac{5\times 144}{62.5}\right)}=\sqrt{64.333\times .48}$$
$$=5.557\,\textit{feet}$$

When Water flows into a rarefied Space, as into the Condenser of a Steam-engine, and is either pressed upon or open to the Atmosphere, the height due to the mean pressure of the atmosphere within the condenser, added to the height of the water above the internal surface of it, must be taken as the head of the water.

$$\text{Or, } \sqrt{2\,g\,(h+h)}=v.$$

ILLUSTRATION.—Assume the head of water in the condenser of a steam-engine to be 3 feet, the vacuum gauge to indicate a column of mercury of 26.467 ins. (=13 lbs.), and a column of water of 13 lbs. = 29.9 *feet*.

$$\text{Then } \sqrt{2\,g\,(3+29.9)}=\sqrt{64.333\times 32.9}=\sqrt{2116.566}=46\,\textit{feet}.$$

Relative Velocity of the Discharge of Water through different Apertures and under like Heads.

The velocity of motion that would result from the direct, unretarded action of the column of water which produces it, being a *constant*, or	1.
The velocity through a cylindrical aperture in a thin plate	.625
Through a tube from two to three diameters in length, projecting outward	.8125
Through a tube of the same length, projecting inward	.6812
Through a conical tube of the form of the contracted vein	.974

COEFFICIENTS FOR THE DISCHARGE OR EFFLUX OF WATER FOR VARIOUS OPENINGS AND APERTURES.

Rectangular Weir.

Height measured from the Surface of the Water to the Sill.

Coefficients for the Discharge over Weirs.

[*From the Experiments of* JAMES B FRANCIS, *Lowell, Mass*, 1852.]

Mean Head.	Length of Opening	Mean Discharge per Second.	Mean Coefficient.
.62 to 1.55 feet.	10 feet.	32.9 cubic feet.	.623

Mean, .603; Small oblong openings as they approach the surface, .705 (Poncelet and Lesbros); Large square apertures as they approach the surface, .572; Opening, 10 feet in length and 1 inch in depth, .591; 9 inches deep, .781; mean, .723 (Blackwell).

[*Deduced from the Experiments of* Mr. BLACKWELL.]

Heads in inches, measured from still water in the reservoir, 1 to 14. Thin plates, 3 feet long, .617; 10 feet long, .667. Planks 2 inches thick, square, 3 feet long, .571; 6 feet long, .563; 10 feet long, .547; 10 feet long, wing-boards, making an angle of 60°, .714.

The principal causes for the variation in the coefficients derived from most experiments giving the discharge of water over weirs arises from,

1. The depth being taken from only one part of the surface, for it has been proved that the heads *on*, *at*, *and above a weir* should be taken in order to determine the true discharge.

2. The unequal widths of the crest, which, increasing the friction, reduce the coefficients, particularly for smaller depths, very considerably.

3. The nature of the approaches, including the ratio of the water-way in the channel above, to the water-way on the weir.

When a weir extends from side to side of a channel, the contraction is less than when it forms a notch, or Poncelet weir, and the coefficient sometimes rises as high as .667. When the weir or notch extends only one fourth, or a less portion of the width, the coefficient has been found to vary from .584 to .6.

When the overfall is a thin plate, it discharges a greater proportionate quantity, when the stream is only 1 inch deep, than with greater depths, the vein contracting with the increased head.

When the length of the weir is 10 feet, the coefficient is greatest with a depth of 5 inches; and when wing-boards are added at an angle of about 64°, the coefficient is greater than even when the head is less.

When the heads remain the same, the coefficients decrease, at first more, and then less rapidly than the breadths of the weirs.

Rectangular Notches, or Vertical Apertures or Slits.

Height measured from the Surface of the Head of the Water to the Sill.

Opening, 8 inches by 5 inches, mean .606 (Poncelet and Lesbros).

The coefficient increases as the depth decreases, or as the ratio of the length of the notch to its depth increases.

Opening, 18.4 inches by 1.8 and 6.75 inches, .648 to .63 (Du Buat).

When the sides and under edge of a notch increase in thickness, so as to be converted into a short or small channel, open at the top, the coefficients reduce very considerably, and to an extent beyond what the increased resistance from friction, particularly for small depths, indicates. Poncelet and Lesbros found, for apertures 8×8 inches, that the addition of a horizontal shoot 21 inches long reduced the coefficient from .604 to .601, with a head of about 4 feet; but for a head of 4½ inches the coefficient fell from .572 to .483. For notches 8 inches wide, with the addition of a horizontal shoot 9 feet 10 inches long, the coefficient fell from .582 to .479 for a head of 8 inches, and from .622 to .34 for a head of 1 inch. Castel also found for a notch 8 inches wide, with the addition of a shoot 8 inches long, inclined 4° 18′, the coefficient for heads from 2 to 45 inches, to be .527 nearly.

Triangular Notch.

Professor Thomson deduced that for discharges from 2 to 10 cubic feet per minute in a thin plate the coefficient for a right-angled triangular notch was .617.

Rectangular Openings or Sluices, or Horizontal Slits.

Height measured from the Surface of the Water to the Centre of Pressure of the Opening.

Opening, 1 inch by 1 inch.	Head, 7 to 23 feet,	.621	Michelotti.
" 3 " " 3 "	" 7 " 23 "	.614	Michelotti.
" 1 " " 1 "	" 1 " 5 "	.637	Smeaton.
" 2 " " 2 "	" 12 feet	.617	Bossut.
Equilateral triangle, 1 square inch, base down,		.593	Rennie.
" " 1 " " base up,		.585	Rennie.

Poncelet and Lesbros deduced that the coefficient of discharge increases with small and very oblong apertures as they approach the surface, and decreases with large and square apertures under like circumstances.

The coefficients ranged, in square apertures of 8 by 8 inches, under a head of 6 inches to rectangular apertures, 8 inches by .4 inches; under a head of 10 feet, from 572 to .745.

In applying an open channel or canal to the exit of an opening in a reservoir, etc., the bottom and sides corresponding with the dimensions of the opening, Poncelet and Lesbros obtained the following coefficients:

Without channel. mean, .623; with channel, mean, .628. With a channel, at a declination of .01, or 34′, the coefficients were sensibly the same as when it was horizontal; and when the declination was increased to .1, or 5° 44′, the coefficients were increased.

In a Thin Plate = .616 (Bossut); .61 (Michelotti).

Sluice-boards or Convergent Sides.—When one side is inclined to the horizon, the coefficients for angles of 45° and 63° 30′ are as .467 and .447.

Circular Openings or Sluices.

Height measured from the Surface of the Head of the Water to the Centre of the Opening.

Simple Aperture.	Head of 6 inches; diameter of opening,	1. inch	= .649
	Head of 1 foot; diameter of opening,	6. inches	= .642
	Head of 10 feet; diameter of opening,	.5 "	= .609
	Head of 10 feet; diameter of opening,	6. "	= .601

Mean for 5 feet, and diameter of opening of 3 inches = .615 (Poncelet and Lesbros).

In a Thin Plate = 666 (Bossut); .631 (Venturi); .64 (Eytelwein).

Contraction of section from 1 to .633, and reduction of velocity to .974; hence .633×.974 = .617 (Neville).

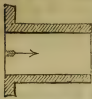
1 to 3 inches in diameter, and heads from 6 to 23 feet, .614; and 3 to 6 inches in diameter, with like heads, .62 (Michelotti).

Cylindrical Ajutages, or Additional Tubes, give a greater discharge than apertures in a thin side, the head and area of the opening being the same; but it is necessary that the flowing water should entirely fill the mouth of the ajutage.

The mean coefficient, as deduced by Castel, Bossut, and Eytelwein, is .82.

Short Tubes and Mouth-pieces.

If an aperture be placed in the side of a vessel of from 1½ to 2½ diameters in thickness, it is converted thereby into a short tube, and the coefficient, instead of being reduced by the increased friction, is increased from the mean value up to about .815, when the bore is cylindrical, as in Fig. 1; and when the junction is rounded, as in Fig. 2, to the form of the contracted vein, the coefficient increases to .974.

(1.)


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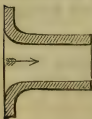

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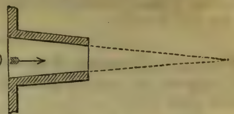
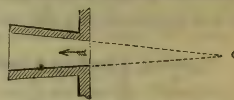


In the conically divergent tube, Fig. 4, the coefficient of discharge is greater than for the same tube placed convergent, the fluid filling in both cases, and the smaller diameters, or those at the same distance from the centres, o o, being used in the calculations. A tube, the angle of convergence, o, of which is 5° nearly, with a head of from 1 to 10 feet, the axial length of which is 3½ inches, small diameter 1 inch, and large diameter 1.3 inch, gives, when placed as at Fig. 3, .921 for the coefficient; but when placed as at Fig. 4, the coefficient increases up to .948. The coefficient of velocity is, however, larger for Fig. 3 than for Fig. 4, and the discharging jet has greater amplitude in falling. If a prismatic tube project beyond the sides into a vessel, the coefficient will be reduced to .715 nearly.

(4.)


Cylindrical Prolongations or Ajutages.

Length of prolongation in Diameters of Aperture	Coefficient of Discharge	Length of prolongation in Diameters of Aperture.	Coefficiennt of Discharge.	Length of prolongation in Diameters of Aperture	Coefficient of Discharge.
1 and under	.62	4 to 12	.77	37 to 48	.63
2 to 3	.82	25 " 36	.68	49 " 60	.6

The coefficients for prismatic tubes increase as the depths decrease, the same as for simple apertures. Bossut's experiments gave a mean of .807.

Conically Convergent and Divergent Tubes.

The form of tube which gives the greatest discharge is that of a truncated cone, the lesser base being fitted to the reservoir (Fig. 4). Venturi concluded from his experiments that the tube of the greatest discharge has a length 9 times the diameter of the lesser opening (base), and a diverging angle of 5° 6′—the discharge being 2.5 greater than that through a thin plate, 1.9 times greater than through a short cylindrical tube, and 1.46 greater than the theoretic discharge.

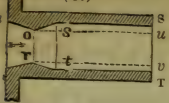
D'Aubuisson and Castel's experiments give, for conically convergent tubes, Fig. 3, the following coefficients (Mr. Neville):

Converging Angle at 0.	Coefficient of Velocity	Coefficient of Discharge.	Converging Angle at 0.	Coefficient of Velocity.	Coefficient of Discharge.
1°	.858	.858	16°	.970	.937
5°	.916	.920	30°	.976	.895
8°	.933	.931	50°	.985	.844

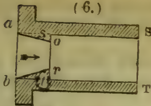
Compound Mouth-pieces and Ajutages.

The following Table gives the coefficients of discharge for the different figures here given, and it will be found of great value, as the coefficients are calculated for the large as well as small diameters.

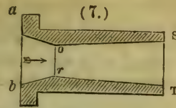
(5.)



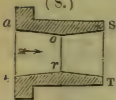
(6.)



(7.)



(8.)



The necessity for taking into consideration the form of the junction of a pipe with a reservoir will be understood from the following results:

(9.)

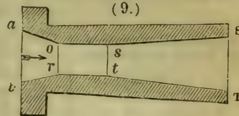


Table of Coefficients for Mouth-pieces and Short Tubes.

[*Calculated and reduced by* Mr. NEVILLE, *from* VENTURI'S *Experiments.*]

Description of Aperture, Mouth-piece, or Short Tube.	Coefficients for the Diameter *a b*.	Coefficients for the Diameter *o r*.
1. An aperture 1½ ins. diameter in a thin plate..........	.622	.974
2. Cylindrical tube 1½ ins. diameter, and 4½ ins. long, Fig. 1	.823	.823
3. Cylindrical tube, Fig. 2, having the junction rounded to the form of the contracted vein......................	.611	.956
4. Short conical convergent mouth-piece, Fig. 3	.607	.934
5. The like tube divergent, with the smaller diameter at the junction with the reservoir; length 3½ ins., lesser diameter 1 in., and greater diameter 1.3 ins.	.561	.948
6. The tube, Fig. 5, when *a b* = 1½ ins., *o r* = 1.21 ins., *u v* = 1.21 in., and *o u* = 2 ins., the cylindrical portion being shown by dotted lines	.6	.923
7. The same tube when *o u* = 11 ins.	.567	.873
The same tube when *o u* = 23 ins.....................	.531	.817
8. The tube *a b*, *o r*, *s t*, *u v*, Fig. 5, in which *a b* = *s t* = S T = 1½ ins. from *a* to *s* 1¾ ins., and *s* S = 3 ins..........	.823	1.266
9. The tube, Fig. 6, *a b* = 1½ ins.........	.804	1.237
10. The same tube, having the spaces *a s* O and *r t b* between the mouth-piece and *s t*, and the cylindrical tube *a s t b* open to the influx of the fluid........................	.785	1.209
11. The double conical tube, *a o* S T *r b*, Fig. 7, when *a b* = S T = 1½ ins., *o r* = 1.21 ins., *a o* = .92 in., and *o* S = 4.1 ins.	.928	1.428
12. The like tube when, as in Fig. 8, *a o r b* = *o* S T *r*, and *a o* S = 1.84 ins..	.823	1.266
13. The like tube when S T = 1.46 ins., and *o* S = 2.17 ins....	.823	1.266
14. The like tube when S T = 3 ins., and *o* S = 9½ ins.	.911	1.4
15. The like tube when *o* S = 6½ ins., and S T enlarged to 1.92 ins.	1.02	1.569
16. The like tube when S T = 2¼ ins., and *o* S = 12.½ ins. ...	1.215	1.855
17. A tube, Fig. 9, when *o s* = *r t* = 3 ins., *o r* = *s t* = 1.21 ins., and the tube *o* S T *r* the same as described in No. 11, viz., S T = 1½ ins., and *s* S = 4.1 ins..................	.895	1.377
18. The tube, Fig. 9, when S T is enlarged to 1.97 ins., and *s* S to 7 ins., the other dimensions remaining as in No. 8 ...	.945	1.454
19. When the junction of *o s r t* with *s* S T *t*, Fig. 5, is improved, the other parts remaining as described in No. 8.	.85	1.309

Cylindrical Tubes or Pipes.

The mean of various experiments with tubes of .5 to 3 inches in diameter, and with a head of fluid of from 3 to 20 feet, gives a coefficient of discharge of .813; and as the mean for circular apertures in a thin plate is .63, it follows that, under otherwise similar circumstances and relations, $\frac{.813}{.63} = 1.29$ times as much fluid flows through a cylindrical tube as through a like aperture in a thin plate.

These coefficients increase as the diameter of the aperture or tube is decreased, and but slightly with an increase of the velocity of efflux or height of the head of fluid.

Thus, with tubes .65 of an inch in diameter, and with a head of from 9 inches to 10 feet, the coefficient is from .946 to .957.

Coefficients of Discharge from Short Cylindrical Tubes with Square Junctions.

Relation of the Length of the Tube to the Diameter.	Coefficient of Discharge.	Relation of the Length of the Tube to the Diameter.	Coefficient of Discharge.
0 to 1½ diameters	.617	30 diameters	.65
2 "	.814	50 "	.59
10 "	.75	100 "	.48

DISCHARGE FROM RECTANGULAR WEIRS AND NOTCHES, AND VERTICAL APERTURES OR SLITS.

A *Notch* is an opening, either vertical or oblique, in the side of a vessel, reservoir, etc., alike to a narrow and deep weir.

Vertical Apertures or *Slits* are narrow notches, running to or near to the bottom of the vessel or reservoir.

NOTE.—The mean velocity of a fluid issuing through a rectangular opening in the side of a vessel is ⅔ of that due to the velocity at the sill or lower edge of the opening, or it is that due to a point $\frac{4}{9}$ of the whole height from the surface of the fluid.

To Compute the Volume of Fluid which will Flow out of any of the above Openings.

Height measured from the Surface of the Head of the Water to the Sill of the Opening.

RULE.—Multiply the square root of the product of 64.333 and the height or whole depth of the fluid in feet by the area in feet, and by the coefficient for the opening, and ⅔ of this product will give the discharge in cubic feet per second.

Or, $\frac{2}{3} b h \sqrt{2 g h}\, C = V$, and $\frac{V}{\frac{2}{3} b h \sqrt{2 g h}\, C} = t$, *t representing the time in seconds.*

EXAMPLE.—The sill of a weir is 1 foot below the surface of the water, and its length is 10 feet; what volume of water will it discharge in one second?

$C = .623$, and $\sqrt{64.33 \times 1} \times \overline{10 \times 1} = 80.2$, and $\frac{2}{3}\ \overline{80.2 \times .623} = 33.318$ *cubic feet.*

NOTE.—The mean* coefficient of discharge of weirs, the breadth of which is no more than the third part of the breadth of the stream, is $\frac{2}{3}$ of $6 = .4$; and for weirs which extend the whole width of the stream it is $\frac{2}{3}\,.666 = .444$.

Rectangular Sluices and Horizontal Slits.

Height measured from the Surface of the Head of the Water to the upper Side and to the Sill of the Opening.

RULE.—Multiply the square root of 64.333 and the breadth of the opening in feet by the coefficient for the opening, and by the difference of the products of the heights of the water and their square roots, and ⅔ of the whole product will give the discharge in cubic feet per second.

Or, $\frac{2}{3} b \sqrt{2 g}\, (h' \sqrt{h'} - h \sqrt{h})\, C = V$, *h and h′ representing the heights of the aperture and sill in feet.*

EXAMPLE.—The sill of a rectangular sluice, 6 feet in width by 5 feet in height, is 9 feet below the surface of the water; what is the discharge in cubic feet per second?

$C = .625$, $\frac{2}{3} \sqrt{2 g} = 5.348$, and $h' = 9 - 5 = 4$; $5.348 \times 6\, (9\sqrt{9} - 4\sqrt{4}) \times .625 = 32.088 \times 19 \times .625 = 381.04$ *cubic feet.*

* This includes the element of ⅔ in the formula.

DISCHARGE FROM CIRCULAR SLUICES, ETC.

Height measured from the Surface of the Head of the Water to the Centre of the Opening.

RULE.—Multiply the square root of the product of 64.333 and the depth of the centre of the opening from the surface of the water by the area of the opening in square feet, and this product by the coefficient for the opening, and the whole product will give the discharge in cubic feet per second.

Or, $\sqrt{2gh}, a\,C = V$, *a representing the area in square feet, and h the height of the surface of the fluid from the centre of the opening in feet.*

EXAMPLE.—The diameter of a circular sluice is 1 foot, and its centre is 1.5 feet below the surface of the water; what is the discharge in cubic feet per second?

Area of 1 foot $= \frac{113.09}{144} = .7854$; C $= .64$; $\sqrt{64.333 \times 1.5} \times .7854 \times .64 = 9.823 \times .7854 \times .64 = 4.938$ *cubic feet.*

When the Circumference reaches the Surface of the Water.

$\sqrt{2gr}, .9604\,a\,C = V$, *r representing the radius of the circle.*

Semicircular Sluices.

When the Diameter is either Upward or Downward.

$\sqrt{2gh}\,a\,C = V$, *h representing the depth of the centre of gravity of the figure from the surface.*

Semicircular Weirs or Notches.

When the Diameter is Uppermost and Horizontal.

$$\sqrt{2gr}\ .6103\,a\,C = V.$$

When the Diameter as above is at the Depth z, below the Surface.

$$\sqrt{2gz}\ 1.188\,a\,C = V.$$

When the Circumference is Uppermost and Horizontal.

$\sqrt{2gr}\ .7324\,a\,C = V$, *r representing radius of semicircle.*

EXAMPLE.—In what time will 800 cubic feet of water be discharged through a circular opening of .025 square feet, the centre of which is 8 feet below the surface of the water?

$$C = .63.$$

$$\frac{800}{\sqrt{2gh} \times .025 \times .63} = \frac{800}{22.68 \times .025 \times .63} = \frac{800}{.35721} = 2239.58 = 37\ min., 19.6\ sec.$$

DISCHARGE FROM TRAPEZOIDAL WEIRS OR NOTCHES.

Height measured from the Surface of the Head of the Water to the Sill of the Opening.

When either the Greater or Lesser Breadth is uppermost.

$\frac{2}{15} h \sqrt{2gh}\,(2b + 3b')\,C = V$, *b and b′ representing the upper and lower breadths.*

Trapezoidal Sluices or Slits.*

When the Greater Breadth is uppermost.

$\frac{2}{3}\sqrt{2g}\left(b'h'^{\frac{3}{2}} - b\,h^{\frac{3}{2}} + \frac{2}{5}(b - b')\,\frac{h'^{\frac{5}{2}} - h^{\frac{5}{2}}}{h' - h}\right)C = V$, *h and h′ representing the depth from the surface to the upper and lower edges of the opening.*

* For Triangular Sluices, etc., see Weisbach, vol. i, p. 359, 360.

When the Lesser Breadth is uppermost.

$$\frac{2}{3}\sqrt{2g}\left(b'h'^{\frac{3}{2}} - b\,h^{\frac{3}{2}} - \frac{2}{5}(b'-b)\,\frac{h'^{\frac{5}{2}} - h^{\frac{5}{2}}}{h'-h}\right) C = V.$$

ILLUSTRATION.—The sill of a trapezoidal sluice 5 feet in height, the lesser breadth being uppermost, is 9 feet below the surface of the water, and the breadths of it are 5.5 and 6.5 feet; what is the discharge in cubic feet per second?

$C = .62$, $\frac{2}{3}\sqrt{2g} = 5.348$, $h' = 9 - 5 = 4$; $5.348\left(6.5 \times \sqrt{9^3} - 5.5 \times \sqrt{4^3} - \frac{2}{5}(6.5 - 5.5)\frac{\sqrt{9^5} - \sqrt{4^5}}{9-5}\right.\left(\times .62 = 5.348 \times \left(175.5 - 44 - \frac{2}{5}\,1 \times \frac{243-32}{4}\right) \times .62 = 5.348 \times (131.5 - 21.1) \times .62 = 366.06\right.$ *cubic feet.*

Triangular Weirs and Notches.

Height measured from the Surface of the Head of the Water to the Vertex and Base of the Opening.

When the Vertex is uppermost, $\frac{6}{15}\sqrt{2g}\,b'\sqrt{h^3}\,C = V.$

When the Base is uppermost, $\frac{4}{15}\sqrt{2g}\,b\sqrt{h^3}\,C = V.$

When the Notch is a Parallelogram, the longest Diagonal being Vertical,

$\sqrt{2g}\,\frac{4b}{15h}\left(h'^{\frac{5}{2}} - 2d^{\frac{5}{2}}\right) C = V$, *h and d representing the depth from the surface to the lower and centre points of the opening. When the Figure is a Square,* $b = 2d$.

When the Notch is a Right Angle.

1. *When the Base is at the Surface of the Water,* $\frac{4}{15}\,b\,h\sqrt{2gh}\,C = V.$
2. *When the Vertex is at the Surface of the Water,* $\frac{2}{5}\,b\,h\sqrt{2gh}\,C = V.$

The foregoing formulæ* furnish an expression for the discharge from any rectilineal aperture whatever, as it can be divided into triangles, the discharge from each of which can be determined; as any triangular aperture can be divided into two others by a line through one of the angles.

NOTE.—For the greater number of apertures at any depth below the surface of the water, the product of the area, and the velocity due to the depth of the centre, or centre of gravity when it is practicable to obtain it, will give the discharge with sufficient accuracy.

DISCHARGE FROM WEDGE- AND PYRAMIDAL-SHAPED VESSELS.

Horizontal Triangular Prism—A Side uppermost. Paraboloid of Revolution—Base uppermost. $\frac{4}{3} \times \frac{V}{C\,a\sqrt{2gh}} = t.$

NOTE.—In this efflux the time is ⅓ greater than if the initial velocity remained uniform.

Quadrangular Pyramid—Base uppermost. $\frac{6}{5} \times \frac{V}{C\,a\sqrt{2gh}} = t$

NOTE.—As in this efflux the initial velocity of it decreases gradually to 0, the time of efflux is $\frac{1}{5}$ greater than if the initial velocity remained uniform.

EXAMPLE.—In what time will a pond of water, the surface of which has an area of 765000 square feet, be discharged through a conduit 15 feet below its surface, 15 inches in diameter, and 50 feet in length?

$$C = .6. \quad V = \frac{765000 \times 15}{2} = 5737500 = \textit{volume of pond},\ a = 1.2272.$$

$$\frac{4}{3} \times \frac{5737500}{.6 \times 1.2272 \times 31.0644} = \frac{22950000}{68.62} = 334450\ \textit{sec} = 92\ \textit{h.},\ 54\ \textit{min.},\ 10\ \textit{sec.}$$

* For other formulæ, see Neville's Treatise, p. 51–58.

SPHERICAL AND PRISMOIDAL-SHAPED VESSELS.

Sphere: $\frac{16 p r^2 \sqrt{2 r}}{15 C a \sqrt{2 g}} = t$, *2 r being equal h. Hemisphere — Base uppermost:* $\frac{14 p r^2 \sqrt{r}}{15 C a \sqrt{2 g}} = t$, *r being equal h. Spherical Segment — Base uppermost:* $\frac{2}{5}$ $\frac{A h}{C a \sqrt{2 g h}} = t$. *Prismoid—Trilateral or Multilateral Pyramid—Bases uppermost:* $(3 G + 8 G_1 + 4 \sqrt{G G_1}) \frac{2 \sqrt{h}}{15 C a \sqrt{2 g}} = t$, G and G′ *repr'g the base and lower surface.*

EXAMPLE.—A prismoidal reservoir filled with water is at top 5 feet in length by 3 feet in breadth, and at a depth of 4 feet, at the point of insertion of a short horizontal discharge, 1 inch in diameter by 3 inches in length; it is 4 feet in length and 2 feet in breadth; what is the time required for the water in it to subside 2.5 feet?

$C = .815$, $a = .00545$, $G = 15$, $G_1 = 8$.

$(3 \times 15 + 8 \times 8 + 4 \sqrt{15 \times 8}) \frac{2 \times \sqrt{4}}{15 \times .815 \times .00545 \times 8.02} = 152.818 \times \frac{4}{.534} = 1144.7$ *seconds, the time of emptying the whole vessel.*

Then $4 - 2.5 = 1.5$; hence $G = 4.375 \times 2.375 = 10.39$.

$(3 \times 10.39 + 8 \times 8 + 4 \sqrt{10.39 \times 8}) \frac{2 \times \sqrt{1.5}}{.534} = 131.638 \times \frac{2.45}{.534} = 603.9$ *seconds, and the difference of these times gives the time in which the level of the water subsides 2.5 feet;* viz., $1144.7 - 603.9 = 540.8$ *seconds.*

DISCHARGE FROM IRREGULAR-SHAPED VESSELS, AS A POND, LAKE, ETC.

To Compute the Time and Volume discharged.

Divide the whole mass of water into four or six strata of equal depths.

Then, *for 4 Strata,* $\frac{h - h_4}{12 C a \sqrt{2 g}} \times \left(\frac{A}{\sqrt{h}} + \frac{4 A_1}{\sqrt{h_1}} + \frac{2 A_2}{\sqrt{h_2}} + \frac{4 A_3}{\sqrt{h_3}} + \frac{A_4}{\sqrt{h_4}}\right) = t$, h, h', etc., *representing the depths of the strata at* A, A_1, etc., *commencing at the surface;* A_1, A_2, etc., *being the areas of the first, second, etc., transverse sections of the pond,* etc.; and $\frac{h - h_4}{12} \times A + 4 A_1 \times 2 A_2 + 4 A_3 + A_4 = V$.

EXAMPLE.—In what time will the depth of water in a lake subside 6 feet, the surfaces of its strata having the following areas, the outline of the sluice being a semicircle, 18 inches wide, 9 inches deep, and 60 feet in length?

$A = 20$	feet	(h)	depth of water	$=$	area of	600000	square feet.
$A_1 = 18.5$	"	(h_1)	" "	$=$	"	495000	" "
$A_2 = 17$	"	(h_2)	" "	$=$	"	410000	" "
$A_3 = 15.5$	"	(h_3)	" "	$=$	"	325000	" "
$A_4 = 14$	"	(h_4)	" "	$=$	"	265000	" "

$a =$ area of $18 \div 2 = .8836$ *square feet;* $C = .537$.

Then $\frac{20 - 14}{12 \times .537 \times .8836 \times 8.02} \times \left(\frac{600000}{4.472} + \frac{4 \times 495000}{4.301} + \frac{2 \times 410000}{4.123} + \frac{4 \times 325000}{3.937} + \frac{265000}{3.742}\right) = \frac{6}{45.6648} \times 1194429 = 156938$ *sec.* $= 43$ *h.*, 35 *min.*, 38 *sec.*

And the discharge $= 6 \div \times (600000 + 4 \times 495000 + 2 \times 410000 + 4 \times 325000 + 265000) = 6 \div 12 \times 4965000 = 2482500$ *cubic feet.*

For 6 Strata, put $2 A_4$ instead of A_4, and $4 A_5$ and A_6 additional, and divide by 18 instead of 12.

Discharge through Pipes or Canals, when the Form and Dimensions of the Vessel of Efflux are not known.

The volume discharged may be estimated by observing the heads of the water at equal intervals of time.

Then $C a t \sqrt{2g}\left(\frac{\sqrt{h}+\sqrt{h_1}}{2}\right)=V$, *for* 1 *depth;* $C a t \sqrt{2g}\left(\frac{\sqrt{h}+4\sqrt{h_1}+\sqrt{h_2}}{3}\right)$ $=V$ *for* 2 *depths;* and $C a t \sqrt{2g}\left(\frac{\sqrt{h}+4\sqrt{h_1}+2\sqrt{h_2}+4\sqrt{h_3}+\sqrt{h_4}}{3}\right)=V$ *for* 4 *depths.*

NOTE.—At the end of half the time of discharge, the head of water will be ¼ of the whole height from the surface to the delivery.

When discharged through Weirs or Notches.

$\frac{2}{9} C b t \sqrt{2g}\,(\sqrt{h^3}+4\sqrt{h^3_1}+\sqrt{h^3_2})=V$, *b representing the breadth in feet.*

EXAMPLE.—A prismatic reservoir 9 feet in depth is discharged through a notch 2.222 feet wide, the surface subsiding 6.75 feet in 935 seconds; what is the volume discharged?

$$C=.6;\ h_1=9-6.75=2.25\ feet.$$

$\frac{2}{9}.6\times 2.222\times 935\times 8.02\,(\sqrt{9^3}+4\sqrt{2.25^3}+\sqrt{0^3})=2221.6\times(27+13.5+0)=2221.6$ $\times 40.5=89974.8$ *cubic feet.*

When there is an Influx and Efflux.

If a reservoir or vessel during an efflux from it has an influx into it, the determination of the time in which the surface of the water rises or falls a certain height becomes so complicated that an approximate determination is here alone essayed.

A state of permanency or constant height occurs whenever the head of the water is increased or decreased by $\frac{1}{2g}\left(\frac{I}{C a}\right)^2=k$, I *representing the influx in cubic feet per second.*

The time in which the variable head of water (x) increases by the volume V is given by the following formula: $\frac{A_1 V}{I-C a\sqrt{2gx}}$; and the time in which it sinks the height, k, by $\frac{A_1 V}{C a\sqrt{2gx}-I}$

The time of efflux, in which the subsiding surface falls from A to A_1, etc., and the head of water from h to h_1, when k is represented by $\frac{I}{C a\sqrt{2g}}=\sqrt{k}$, is $\frac{h-h_4}{12 C a\sqrt{2g}}$ $\left(\frac{A}{\sqrt{h}-\sqrt{k}}+\frac{4A_1}{\sqrt{h_1}-\sqrt{k}}+\frac{2A_2}{\sqrt{h_2}-\sqrt{k}}+\frac{4A_3}{\sqrt{h_3}-\sqrt{k}}+\frac{A_4}{\sqrt{h_4}-\sqrt{k}}\right)=t.$

EXAMPLE.—In what time will the surface of the water in a pond, as in a previous example, fall 6 feet if there is an influx into it of 3.0444 cubic feet per second?

$$\sqrt{k}=.8,\ C=.537.$$

$$\frac{20-14}{12\times .537\times .8836\times 8.02}\times\left(\frac{600000}{4.472-.8}+\frac{4\times 495000}{4.301-.8}+\frac{2\times 410000}{4.123-.8}+\frac{4\times 325000}{3.937-.8}+\frac{265000}{3.742-.8}\right)=\frac{6}{45.6648}\times 1480201=194498\ sec.=54\ h,\ 1\ min.,\ 8\ sec.$$

Prismatic Vessels.—If the vessel has a uniform transverse section, A.

Then $\frac{2A}{C a\sqrt{2g}}\left(\sqrt{h}-\sqrt{h_1}+\sqrt{k}\times \text{hyp. log.*}\left(\frac{\sqrt{h}-\sqrt{k}}{\sqrt{h_1}-\sqrt{k}}\right)\right)=t=$ *the time in which the head of the water flows from h to* h_1.

EXAMPLE.—A reservoir has a surface of 500000 square feet, a depth of 20 feet; it is fed by a stream affording a supply of 3.0444 cubic feet per second, and the outlet has an area of .8836 square feet; in what time will it subside 6 feet?

$$\sqrt{k},\ \text{as before}=.8,\ C=.537.$$

* For hyperbolic logarithm substitute the ordinary by multiplying by 2.303.

$$\frac{2\times500000}{C\,a\sqrt{2g}}\times(\sqrt{20}-\sqrt{14}+.8\times\log.\left(\frac{\sqrt{20}-.8}{\sqrt{14}-.8}\right)\times2.303)=\frac{1000000}{3.8054}\times4.472-3.742$$

$$+.8\times\log.\left(\frac{4.472-.8}{3.742-.8}\right)\times2.303=262784.5\times(.73+.8\times.09621\times2.303)=262784.5\times$$

$(.73+.17726)=238414$ *sec.* $=66$ *h.*, 13 *min*, 34 *sec.*

To Compute the Fall in a given Time.

This is determining the head h_1 at the end of that time, and it should be subtracted from the head h at the commencement of the discharge. Put into the preceding equation several values of h_1, until one is found to meet the condition.

ILLUSTRATION.—Take a prismatic pond having a surface of 38750 square feet, a depth to the centre of the opening of the sluice of 10.5 feet, a supply of 33.6 cubic feet, and a discharge of 40 cubic feet per second.

$$\sqrt{k}=.84.$$

Putting these numerical values into the equation, and assuming different values for h_1, a value which nearly satisfies the equation is 4. Consequently, $10.5-4=6.5$ *feet, the fall.*

For a Weir or Notch.

$$t=\frac{A\,k}{3\,I}\left[\text{hyp. log.}\,\frac{h_1+\sqrt{h_1k}+k}{(\sqrt{h_1}-\sqrt{k})^2}+\sqrt{12}\ \text{arc}\left(\text{tang.}=\frac{-\sqrt{3h_1}}{2\sqrt{k}+\sqrt{h_1}}\right)\right];$$

$$k=\left(\frac{I}{\frac{2}{3}C\,b\sqrt{2g}}\right)^{\frac{2}{3}};\ \text{arc (tang.}=y),\ \textit{the arc the tangent of which}=y.$$

According as k is $\lessgtr h$, and the influx of water, $I \gtrless \frac{2}{3}C\,l\sqrt{2gh^3}$, there is a rise or fall of the fluid surface, the condition of permanency occurring when $h_1=k$.

EXAMPLE.—In what time will the water in a rectangular tank, 12 feet in length by 6 feet in breadth, rise from the sill of a notch, 6 inches broad, to 2 feet above it, when 5 cubic feet of water flow into the tank per second?

$$h_1=2,\ h=0,\ A=12\times6=72,\ I=5,\ b=.5,\ C=.6.$$

$$k=\left(\frac{5}{\frac{2}{3}.6\times.5\times8.02}\right)^{\frac{2}{3}}=\sqrt[3]{3.117^2}=2.1338.$$

Then $\frac{72\times2.1338}{3\times5}\left(\text{hyp. log.}\,\frac{2+\sqrt{2\times2.1338}+2.1338}{(\sqrt{2}-\sqrt{2.1338})^2}+\sqrt{12}\ \text{arc}\left(\text{tang.}=\frac{-\sqrt{3\times2}}{2\sqrt{2.1338}+\sqrt{2}}\right)\right)=10.2423\times\text{hyp. log.}\,\frac{6.1996}{.002162}-3.4641\times\text{arc}\left(\text{tang.}\,\frac{\sqrt{6}}{4.3356}\right)$ $=10.2423\times[7.961-(3.461\times$ arc, the tangent of which $=.56497$, or $29°\,28'=29.466$, the length of which $=.5143)=1.781]=10.2423-7.961-1.781=10.2423\times6.18=$ 63.297 *sec.*

DISCHARGE OF WATER UNDER VARIABLE PRESSURES.

To Compute the Time, the Rise and Fall, and the Volume of Water discharged under Variable Pressures.

$\frac{a}{A}\sqrt{2gx}=v$, *x representing the variable head,* A *area of transverse horizontal section of vessel, and v theoretical velocity of efflux.*

Discharge from Reservoirs or Vessels not receiving any supply of Water.

For prismatic vessels the general law applies, that twice as much would be discharged from like apertures if the vessels were kept full during the time which is required for emptying them.

To Compute the Time. $\frac{2A\sqrt{h}}{C\,a\sqrt{2g}}=\frac{2A\,h}{D}=t.$

EXAMPLE.—A rectangular cistern has a transverse horizontal section of 14 feet, a depth of water of 4 feet, and a circular opening in its bottom of 2 inches in diameter; in what time will it discharge its volume of water, the depth being maintained at 4 feet, and in what time when the supply to it is cut off and the cistern allowed to be emptied of its contents?

$$h = 4\,feet,\ a = 2^2 \times .7854 \div 144 = .0218,\ C = .6.$$

$$\sqrt{2gh} \times a \times C = 16.04 \times .0218 \times .6 = .2098\ cubic\ feet\ per\ second.$$

Hence $\frac{2 \times 14 \times 4}{.2098} \div 2 = 266.9$ *seconds, the depth being maintained.*

Under a diminishing head of water the coefficient of efflux is increased; hence, in the following case, it is taken at a mean of .613, *and the volume discharged becomes* .2143.

Then $\frac{2 \times 14 \times 4}{.2143} = 522.6$ *seconds, the vessel being emptied.*

To Compute the Time and Fall.

The depression or subsiding of the surface of the water in a vessel, corresponding to a given time of efflux, is $h - h' = s$, h' *representing the lesser depth.*

$$\frac{2\,A}{C\,a\sqrt{2g}}(\sqrt{h} - \sqrt{h'}) = t. \quad \text{Inversely, } \left(\sqrt{h} - \frac{C\,a\sqrt{2g}}{2\,A}t\right)^2 = h'.$$

Example.—In what time will the water in the cistern, as given in the preceding example, subside 1.6 feet?

$$A = 14,\ C = .6,\ a = .0218,\ \sqrt{2g} = 8.02,\ h = 4,\ h' = 4 - 1.6 = 2.4.$$

$$\frac{2 \times 14}{.6 \times .0218 \times 8.02} \times (\sqrt{4} - \sqrt{2.4}) = \frac{28}{.1049} \times (2 - 1.55) = 120.1\ sec.$$

To Compute the Volume.

$A\,y = V$, y *representing the extent of the fall, and* V *the volume of water discharged*, as $h - h'$.

Discharge from Vessels when the Reservoir of Supply is maintained at a uniform Height.

To Compute the Time. $\frac{2\,A\sqrt{h}}{C\,a\sqrt{2g}} = t.$

Example.—In what time will the level of the water in a receiving vessel having a section of 14 square feet attain the height of that in the supply, through a pipe 2 inches in diameter, placed 4 feet below the level of the supply?

$$C = .613. \quad \frac{2 \times 14 \times \sqrt{4}}{.613 \times .0218 \times 8.02} = \frac{56}{.1072} = 522.3\ sec.$$

When the Vessel of Supply has no Influx, and is not indefinitely great compared with the Receiving Vessel.

$\frac{2\,A\,A'\sqrt{h}}{C\,a\,(A + A')\sqrt{2g}} = t$, A′ *representing section of receiving vessel, t the time in which the two surfaces of water attain the same level; and the time within which the level falls from h to h′ is* $\frac{2\,A\,A'(\sqrt{h} - \sqrt{h'})}{C\,a\,(A + A')\sqrt{2g}} = t.$

Discharge from a Notch in the Side of a Vessel when it has no Influx.*

$\frac{3\,A}{C\,b \times \sqrt{2g}} \times \left(\frac{1}{\sqrt{h'}} - \frac{1}{\sqrt{h}}\right) = t$, *b representing the breadth of the notch in feet.*

* When the notch extends to the bottom of the reservoir, etc., the time for the water to run out is indefinite, as $h' = 0$.

DISCHARGE FROM VESSELS IN MOTION.

When the Vessel moves uniformly up or down. $\sqrt{2gh} = V$, *g representing the accelerating force.*

When it Ascends with a retarded Motion. $\sqrt{2(g-p)h} = V$, *p representing the power applied.*

When it Descends with the same Retardation. $\sqrt{2(g+p)h} = V$.

EXAMPLE.—A vessel containing water weighing 350 lbs. is drawn by a running weight of 450 lbs. over a roller; what is its accelerating force, and what the velocity of its discharge?

$$p = \frac{450 - 350}{450 + 350},\ g = \frac{100}{800} = .125;\ \text{hence } \sqrt{2 \times (\tfrac{1}{8} + 8)\, g\, h} = \textit{velocity of discharge}.$$

If the head of water is 4 feet, then $\sqrt{2 \times \frac{9}{8} \times 32.16 \times 4} = 17.01$ *feet.*

When the Vessel revolves. $\sqrt{2gh + o^2} = V$, *o representing the velocity of rotation of the aperture in feet per second.*

FLOW OF WATER IN RIVERS AND CANALS.

Running Water.—Water flows either in a natural or artificial bed. In the first case it forms Streams, Brooks, and Rivers; in the second, Drains, Cuts, and Canals.

The Bed of a flowing water-course is formed of a *bottom* and the two *banks* or *shores*.

The *Transverse Section* is a vertical plane at right angles to the course of the flowing water; the *Perimeter* is the length of this section in its bed.

The *Longitudinal Section* or *Profile* is a vertical plane in the course of the flowing water.

The *Slope* or *Declivity* is the mean angle of inclination of the surface of the water to the horizon.

The *Fall* is the vertical distance of the two extreme points of a defined length of the flowing course, measured upon a horizontal plane, and this fall serves to assign the angle for the defined length of the course.

The *Line of Current* is the point when the flowing water attains its maximum velocity.

The *Mid-channel* is the deepest point of the bed. The *Velocity* is greatest at the surface and in the middle of the current; and the surface of flowing water is highest in the current, and lowest at the banks or shore.

A river, canal, etc., is in a state of *permanency* when an equal quantity of water flows through each of its transverse sections in an equal time, or when V, *the product of the area of the section, and the mean velocity through the whole extent of the stream, is a constant number.* Hence, in the permanent motion of water, the mean velocities in two transverse sections are to each other inversely as the areas of these sections.

To Compute the Mean Depth of Flowing Water.

RULE.—Set off the breadth of the stream, etc., into any convenient number of divisions; ascertain the mean depths of these divisions; then divide their sum by the number of divisions, and the quotient is the mean depth.

To Compute the Mean Area of Flowing Water.

RULE 1.—Multiply the breadth or breadths of the stream, etc., by the mean depth or depths, and the product is the area.

2.—Divide the volume flowing in cubic feet per second by the mean velocity in feet per second, and the qnotient is the area in square feet.

To Compute the Volume of Flowing Water.

RULE.—Multiply the area of the stream, etc., by the mean velocity of its flow in feet, and the product is the volume in cubic feet.

To Compute the Mean Velocity of Flowing Water.

RULE.—Divide the velocity of the flow in feet per second by the area of the stream, etc., and the quotient will give the velocity in feet.

From the experiments of Ximenes, Du Buat, and others, it is deduced that the coefficient of the mean velocity of a flowing stream is from .81 to .83 of the maximum velocity or of that of the line of the current, and, contrariwise, the maximum velocity is 1.24 to 1.19 times that of the mean.

The mean velocity at half depth of a stream has been ascertained to be as .915 to 1, and at the bottom of it as .83 to 1, compared with the velocity at the surface.

Thus, let v_1, v_2, v_3, etc., be the surface velocities of a whole transverse profile of not a very variable depth: the corresponding velocities at a mean depth are .915 v, .915 v_1, .915 v_2, etc.; hence the mean velocity in the whole profile,

$.915 \frac{v_1 + v_2 + v_3}{n} = v$, *n representing the number of velocities put in the formula.*

Again, the velocity diminishes from the line of current toward the banks, and, to obtain the mean superficial velocity,

$$\frac{v_1 + v_2 + v_3}{n} = .915\, v; \text{ hence,}$$

To Compute the Mean Velocity in the whole Profile of a River, etc.,

$.915 \times .915 \times v = .837\ C = .83$ to $.84$ *per cent. of the maximum velocity, or of that of the line of current.*

ILLUSTRATION.—In the line of current of a brook, the velocity of the flow of the water is 4 feet per second, and the depth 6 feet; what is the mean velocity of the flow, and what is its velocity at the bottom of the line?

Assume $C = .32$. Then $4 \times .82 = 3.28$ *feet*; $4 \times .83 = 3.32$ *feet.*

The upper surface of flowing water is not exactly horizontal, as the water at its surface flows with different velocities with respect to each other, and consequently exert on each other different pressures.

If v and v_1 are the velocities at the line of current and bank of a stream, the difference of the two levels is $\frac{v^2 - v_1^2}{2g} = h$.

ILLUSTRATION.—If $v = 5$ *feet*, and v_1 $.9v$; then $\frac{5^2 - \overline{.9 \times 5}^2}{2g} = \frac{4.75}{64.333} = .0738$ *feet.*

A velocity of 7 to 8 ins. per second is necessary to prevent the deposit of slime and the growth of grass, and 15 ins. is necessary to prevent the deposit of sand.

The *maximum velocity* of water in a canal should depend on the character of the bed of the channel.

Thus, the *Mean Velocity* should not exceed,

	Second.		Second.		Second.
Over a slimy bed...	8 ins.	Over small gravel....	1 ft.	Over stones........	6 ft.
Over common clay..	6 "	Over large shingle...	3 "	Over rocks........	10 "
Over river sand	1 ft.	Over broken stones ..	4 "		

To Compute the Velocity of the Flow or Discharge of Water in Canals, Streams, Pipes, etc.

1. *When the Volume discharged per Minute is given in Cubic Feet, and the Area of the Canal, etc., in Square Feet.*

RULE.—Divide the volume by the area, and the quotient, divided by 60, will give the velocity in feet per second.

2. *When the Volume is given in Cubic Feet, and the Area in Square Inches.*

RULE.—Divide the volume by the area; multiply the quotient by 144, and divide the product by 60.

3. *When the Volume is given in Cubic Inches, and the Area in Sq. Inches.*

RULE.—Divide the volume by the area, and again by 12 and by 60.

EXAMPLE.—The flow of water through a drain of 20 ins. area is 25 cubic feet per minute; what is the velocity of the flow?

$$\frac{25}{20}\times 144\div 60=3\ feet.$$

To Compute the Flow or Volume of the Discharge.

1. *When the Area is given in Square Feet.*

RULE.—Multiply the area of the flow by its velocity in feet per second, and the product, multiplied by 60, will give the volume in cubic feet.

2. *When the Area is given in Square Inches.*

RULE.—Multiply the area by its velocity, and again by 60, and divide the product by 144.

To Compute the Height of the Head of Flowing Water.

1. *When the Volume and the Area of the Flow are given in Feet.*

RULE.—Divide the volume in feet per second by the area, and the square of the quotient, divided by 64.333, will give the height in feet.

2. *When the Area is given in Square Inches.*

RULE.—Divide the volume by the area; multiply the quotient by 144, and the square of the product, divided by 64.333, will give the height required.

NOTE 1.—The velocities and discharges here deduced are theoretical, the actual results depending upon the coefficient of efflux used. The mean velocity, however, as before given, page 359, may be taken at $\sqrt{2}\,g\,.673=5.4$ feet instead of 8.02 feet.

2.—As a rule, with large bodies, as ships, etc., their floating velocity is somewhat greater than that of the flow of the water, not only because in floating they descend an inclined plane, formed by the surface of the water, but because they are but slightly affected by the irregular intimate motion of the water: the variation for small bodies is so slight that it may be neglected.

Illustrations of the preceding Rules.

The breadth of a stream is set off into three divisions—viz., 3.1, 5.4, and 4.3 feet; the mean depths of the sections of these divisions are 2.5, 4.5, and 3 feet, and their mean velocities are 2.9, 3.7, and 3.2 feet per second; what is the volume of the water flowing, and what the mean velocity?

Hence $3.1\times2.5\times2.9+5.4\times4.5\times3.7+4.3\times3\times3.2=153.665$ *cubic feet*; and $3.1\times2.5+5.4\times4.5+4.3\times3=44.95$ *sq. feet, area of the sections*; and $\frac{153.665}{44.95}=3.419$ *ft.*

CANAL LOCKS.

When a fluid passes from one level or reservoir to another, through an aperture covered by the fluid in the latter, the effective head on each point of the aperture, and consequently the head due to the velocity of the efflux at each instant, is the difference of the levels of the two reservoirs at that instant.

Hence $C\,a\sqrt{2gh''}=V$ per second, h'' *representing the difference of the levels.*

To Compute the Time of Filling and Discharging a Single Lock.

When the Aperture or Sluice in the Upper Gate is entirely under Water, and above the Lower Level.

$\frac{A\,h'}{C\,a\sqrt{2\,g\,h}}=$ *time of filling up to centre of sluice, h representing the height of the centre of the sluice in the upper gate from the surface of the canal or reservoir, and h′ the height of the centre of the sluice in the upper gate from the lower surface, or* *the water in the lock or river, all in feet;* and $\frac{2\,A\,h}{C\,a\sqrt{2\,g\,h}}$ = *time of filling the remaining space, where a gradual diminution of the head of water occurs.*

Consequently, $\frac{(h'+2\,h)\,A}{C\,a\sqrt{2\,g\,h}}$ = *time of filling a single lock.*

When the Aperture or Sluice in the Lower Gate is entirely under Water, and above the Lower Level.

$\frac{2\,A\sqrt{h+h'}}{C\,a'\sqrt{2\,g}}$ = *time of emptying or discharging it, a′ representing area of sluice, h height of upper surface of canal from centre of sluice, and h′ height of centre of sluice from lower surface.*

Example.—The mean dimensions of a lock are 200 feet in length by 24 in breadth; the height of the centre of the aperture of the sluice from the upper and lower surfaces is 5 feet; the breadth of both upper and lower sluices is 2.5 feet; the height of the upper is 4 feet, and of the lower—entirely under water—5 feet; required the times of filling and discharging.

$h=5$, $h'=5$, $A=200\times24=4800$, $C=.615$. $a=4\times2.5=10$, $a'=5\times2.5=12.5$.

$\frac{4800\times5}{.615\times10\times\sqrt{2\,g\,h}}=\frac{24000}{110.286}=217.62$ *seconds = time of filling the lock up to the centre of the sluice;* and $\frac{2\times4800\times5}{.615\times10\times\sqrt{2\,g\,h}}=\frac{48000}{110.286}=435.23$ *seconds = time of filling the remaining space, or the lock above the centre of sluice,* and $217.62+435.23=652.82$ *seconds, the whole time.*

Or, $\frac{(5+2\times5)\times4800}{.615\times10\times\sqrt{2\,g\,h}}=\frac{72000}{110.286}=652.85$ *seconds = time of filling.*

$\frac{2\times4800\sqrt{5+5}}{.615\times12.5\times\sqrt{2\,g}}=\frac{30358.08}{61.654}=492.39$ *seconds = time of discharging.*

When the Aperture or Sluice in the Upper Gate is entirely under Water, and below the Lower Level.

$$\frac{2\,A\sqrt{h-h'}}{C\,a\sqrt{2\,g}} = \textit{time of filling the lock.}$$

When the Aperture or Sluice in the Lower Gate is in part above the Surface of the Lower Level and in part below it.

$\frac{2\,A\,(h+h')}{C\,b\sqrt{2\,g}\left(d\sqrt{h+h'-\frac{d}{2}}+d'\sqrt{h+h'}\right)}$ = *time of discharging, d and d′ representing the distances of the part of the aperture above and below the surface of the lower water, b the breadth of the aperture, and h and h′ as before.*

Example.—Assume the sluice in the preceding example to be 1 foot above the lower level of the water, or that of the lower canal; what is the time of the discharge of the lock? $d=1$, $d_1=4$.

$\frac{2\times4800\,(5+5)}{.615\times2.5\times8.02\,[1\times\sqrt{5+5-(1\div2)}+4\times\sqrt{5+5}]}=\frac{96000}{12.33\times(3.082+12.65)}=\frac{96000}{193.976}$ $=494.9$ *seconds = time of discharging.*

Double Lock.

A double lock is not a duplication of a single lock in its operation, for in the lower chamber the supply of water is from the upper one, having no influx, instead of a uniform supply flowing directly from the surface level of the canal or feeder.

The operation, therefore, of a double lock is complex, the addition to the formula for a single lock being that of the discharging of the water in the upper lock to fill the lower, the head of water gradually decreasing in the chamber, which is closed from the upper reach during the discharge into the lower.

To Compute the Time of Filling the Lower Lock.

When the Sluice in the Middle Gate is wholly below the Level of the Lower Lock.

$\frac{2\,A\,A'\sqrt{h}}{C\,a\sqrt{2g}\,(A + A')} = t$, *A' representing area of the lower or receiving lock, and h the initial height or difference of the levels of the surfaces of the water.*

EXAMPLE. — A double lock has the following dimensions — viz., area of upper chamber 2300 square feet, and of lower 2200 square feet; height of surface of water in upper chamber from surface of water in lower chamber 13 feet, and area of sluice 13.5 square feet; what is the time of filling the lower chamber, or that in which the level of the water is the same in both chambers?

$$C = .55. \qquad \frac{2 \times 2300 \times 2200 \times \sqrt{13}}{.55 \times 13.5 \times 8.02\,(2300 + 2200)} = 135.9 \textit{ sec.}$$

NOTE.—This is also the time of emptying of the upper lock or chamber.

When the Sluice in the Middle Gate is wholly above the Level of the Lower Lock.

$\frac{2\,A\,A'\sqrt{h'}}{C\,a\sqrt{2g}\,(A + A')}$ = *time of filling up to centre of sluice, h' representing the distance from the lower level to the centre of the sluice;* and $\frac{2\,A\,A'\sqrt{h}}{C\,a\sqrt{2g}\,(A + A')}$ = *time of filling the remaining space.*

OVERFALL WEIRS.

Weirs are designated as *Perfect* when their sill is above the surface of the natural or down stream, and as *Imperfect* or *Submerged* when their sill is below that surface.

To Compute the Volume of Water discharged over a Weir.

$\frac{2}{3} C b \sqrt{2g}\left[(h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] = V$, *h representing head or depth of water over sill, b breadth of the weir, and k height due to the velocity of the water as it flows to the weir* $= \frac{v^2}{2g}$.

This formula, however, is not directly applicable to the determination of the discharge, because k, or the height due to the velocity, is dependent upon V, or the volume. When, therefore, k can not be determined by observation, it will answer to put $\frac{2}{3} C b \sqrt{2gh} = V$.

To Compute the Depth of the Flow over a Sill or Saddle that will Discharge a given Volume of Water.

$$\left(\frac{3\,V}{2\,C\,b\sqrt{2g}} + k^{\frac{3}{2}}\right)^{\frac{2}{3}} - k = h.$$

When the back-water is raised considerably, say 2 feet, the velocity of the water approaching the weir (k) may be neglected.

Then $a + h - \left(\frac{3\,V}{2\,C\,b\sqrt{2g}}\right)^{\frac{2}{3}} = x$, *a representing the original depth of the stream or of the back-water below the weir, h the depth of water over the sill, h' the height to which the surface of the natural stream has been raised by the dam, and x the height of the dam.*

Hence $h' + a = h + x$, and $a + h' - = x$.

ILLUSTRATION.—A stream 30 feet wide and 3 feet deep discharges 310 cubic feet of water per second. It is required to raise it at this point 4.5 feet by the aid of a dam; what should be the height of it?

NOTE.—As the height of water to be raised is considerable, the simple formula may be used.

$$a=3,\ h'=4.5,\ V=310,\ b=30,\ C=.75,\ \sqrt{2g}=8.02.$$

$$3+4.5-\left(\frac{3\times310}{2\times.75\times30\times8.02}\right)^{\frac{2}{3}}=7.5-\left(\frac{930}{360.9}\right)^{\frac{2}{3}}=7.5-\sqrt[3]{2.577^2}=5.62\ \textit{feet.}$$

If it were required to raise the water 1.5 feet, the dam would not be required to be raised above the level of the natural stream, and hence the weir would be *submerged.*

Applying then the following formula in this case, which is that of a submerged weir,

$$h-h'+\frac{V}{Cb\sqrt{2g(h'+k)}}-\frac{2}{3}\frac{(h'+k)^{\frac{3}{2}}-k^{\frac{3}{2}}}{(h'+)k^{\frac{1}{2}}}=h.$$

Putting $k=\frac{v^2}{2g}=\left(\frac{V}{(a+h')\,b}\right)^2=.0155\left(\frac{310}{4.5\times10}\right)^2=.0155\times5.272=.0817$ *feet.*

Assuming C in this case $=.8$.

Then $\frac{310}{.8\times30\times\sqrt{64.333\,(1.5+.0817}}-\frac{2}{3}\frac{(1.582)^{\frac{3}{2}}-(.0817)^{\frac{3}{2}}}{\sqrt{1.582}}=1.283-\frac{2}{3}\,\frac{1.986-.0233}{1.257}$
$=1.283-1.041=.242$ *feet.*

Hence, as $a+h'=h+x$, $h=h'+\overline{a-x}$, and $h=1.5+.242=1.742$, $x=3+1.5-1.742=2.758$ *feet.*

SLUICE WEIRS OR SLUICES.

The discharge of water by Sluices occurs under three forms—viz., *Unimpeded*, *Impeded*, or *Partly Impeded.*

To Compute the Discharge when Unimpeded.

$Cdb\sqrt{2gh}=V$, *d representing the depth of the opening.*

To Compute the Discharge when Impeded.

$Cdb\sqrt{2gh}=V$, *h representing the difference of level between the supply and the back-water.*

To Compute the Discharge when it is partly Impeded.

$Cb\sqrt{2g}\left(d\sqrt{h-\frac{d}{2}}+d'\sqrt{h}\right)=V$, *d' representing the depth or height of the back-water above the upper edge of the sluice.*

ILLUSTRATION.—The dimensions of a sluice are 18 feet in breadth by .5 feet in depth; the height of the back-water is .7 feet, and the difference between the levels of the supply and back-water is 2 feet; what is the discharge per second?

$$.6\times18\times8.02\left(.7\sqrt{2-\frac{.7}{2}}+.5\sqrt{2}\right)=86.62\times1.606=139.11\ \textit{cubic feet.}$$

FLOW OF WATER IN BEDS.

The flow of water in beds is either *Uniform* or *Variable.* It is uniform when the mean velocity at all transverse sections is the same, and consequently when the areas of the sections are equal; it is variable when the mean velocities, and therefore the areas of the sections, vary.

To Compute the Fall of the Flow.

$F\frac{lp}{A}\times\frac{v^2}{2g}=h$, *F representing the coefficient of friction, l the length of the flow, p the perimeter of the sides and bottom of the river, and h the fall in feet.*

To Compute the Velocity of the Flow.

$$\sqrt{\frac{A}{F \times l p} 2 g h} = v, \text{ or } 92.35 \sqrt{\frac{A h}{p l}} = v,$$ *A representing area of vertical section.*

Table of the Coefficients of Friction of the Flow of Water in Beds, as in Rivers, Canals, Streams, etc.

IN FEET PER SECOND.

Velocity	Coefficient.	Velocity.	Coefficient.	Velocity	Coefficient.	Velocity.	Coefficient.
.3	.00815	.7	.00773	1.5	.00759	5	.00745
.4	.00797	.8	.00769	2	.00752	8	.00744
.5	.00785	.9	.00766	2.5	.00751	10	.00743
.6	.00778	1.	.00763	3	.00749	12	.00742

By experiments of Du Buat and others, reduced by Eytelwein, $F = .007565$ for a velocity of 1.5 feet, and $v = 92.35$ for measures in feet.

ILLUSTRATION 1.—A canal 2600 feet in length has breadths of 3 and 7 feet, a depth of 3 feet, with a flow of 40 cubic feet per second; what is its fall?

$$p = 3 + 2\sqrt{\left(\frac{7-3}{2}\right)^2 + 3^2} = 10.2; \quad A = \frac{\overline{7+3} \times 3}{2} = 15; \quad v = \frac{40}{15} = 2.66.$$

Hence $.007565^* \times \frac{2600 \times 10.2}{15} \times \frac{2.66^2}{2g} = 1.467$ *feet*

2.—A canal 5800 feet in length has breadths of 4 and 12 feet, a depth of 5 feet, and a fall of 3 feet; what is the velocity and volume of the flow?

$$p = 4 + 2\sqrt{\left(\frac{12-4}{2}\right)^2 + 5^2} = 16.8; \quad A = \frac{\overline{12+4} \times 5}{2} = 40.$$

Hence $92.35\sqrt{\frac{40 \times 3}{16.8 \times 5800}} = 3.24$ *feet*, and $40 \times 3.24 = 129.6$ *cubic feet.*

Forms of Transverse Sections.

The resistance or friction which the bed of a stream, etc., opposes to the flow of water, in consequence of its adhesion or viscosity, increases with the surface of contact between the bed and the water, and therefore with the perimeter of the water profile, or of that portion of the transverse section which comprises the bed.

The friction of the flow of water in a bed is inversely as the area of it.

That the friction of a flowing stream may be the least practicable of attainment, its transverse section, omitting any part of its surface in contact with the air, must have that form in which the perimeter for a given area is a minimum, or the area for a given perimeter a maximum.

Of all regular figures, that which has the greatest number of sides has for the same area the least perimeter; hence, for inclosed conduits, the nearer its transverse profile approaches to a regular figure, the less the coefficient of its friction; consequently, the circle has the profile which presents the minimum of friction.

Th trapezoidal and rectangular sections are those generally given to canals, cuts, etc.

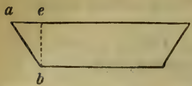
The order of the regular figures applicable to Canals, Cuts, etc., in their least resistance to the flow of water, is as follows:

1. Semicircles. 2. Half a Decagon. 3. Half a Hexagon, or a Trapezium. 4. Half a Square.

When a canal is cut in earth or sand and not walled up, the slope of its sides should not exceed 45°.

* See Note on Table of Coefficients, above.

To Compute the Form of Profile corresponding to a given Slope, as *a b e*.



$\sqrt{\frac{A \sin. \angle a}{2 - \cos. \angle a}} = d$, and $\frac{A}{d} - d$, cotang. $\angle a = b$, *d representing the depth, and b the breadth at the bottom.*

EXAMPLE. — What dimensions must be given to the transverse section of a canal when its banks are to have a slope of 40°, and which is to convey a volume of 75 cubic feet of water with a mean velocity of 3 feet?

$$A = \frac{75}{3} = 25 \textit{ square feet.}$$

$$\sqrt{\frac{25 \sin. 40^\circ}{2 - \cos. 40^\circ}} = \sqrt{\frac{25 \times .64279}{2 - .76604}} = \sqrt{\frac{16.0698}{1.234}} = 3.609 \textit{ feet depth.}$$

$\frac{25}{3.609} - 3.609 \times$ cotang. $\angle 40^\circ = 6.927 - \overline{3.609 \times 1.1917} = 2.626$ *feet breadth at bottom.*

The slope *a e*, or cut of the bank $= 3.609 \times$ cotang. $40^\circ = 4.301$. Hence $4.301 \times 2 + 2.626 = 11.228$ *feet, the breadth at the top*, and $b + \frac{2d}{\sin. \angle a} =$ *perimeter of the water profile;* then $2.626 + \frac{2 \times 3.609}{.64279} = 13.855$ *feet*, and $\frac{p}{A} = \frac{13.855}{25} = .5542$, *the ratio determining the friction.*

Table of the Profiles which correspond to different Angles of Slope.

Angle of Slope.	Relative Slope.	Dimensions of Transume Profile. Depth. d	Lower Breadth. b	Slope. n	Upper Breadth. $b + 2n$	Quotient of $\frac{p}{A}$
90°	0.	$.707\sqrt{A}$	$1.414\sqrt{A}$	0.	$1.414\sqrt{A}$	$\frac{2.828}{\sqrt{A}}$
60°	.577	$.76\sqrt{A}$	$.877\sqrt{A}$	$.439\sqrt{A}$	$1.755\sqrt{A}$	$\frac{2.632}{\sqrt{A}}$
45°	1.	$.74\sqrt{A}$	$.613\sqrt{A}$	$.74\sqrt{A}$	$2.092\sqrt{A}$	$\frac{2.704}{\sqrt{A}}$
40°	1.192	$.722\sqrt{A}$	$.525\sqrt{A}$	$.86\sqrt{A}$	$2.246\sqrt{A}$	$\frac{2.771}{\sqrt{A}}$
36° 52′	1.333	$.707\sqrt{A}$	$.471\sqrt{A}$	$.943\sqrt{A}$	$2.537\sqrt{A}$	$\frac{2.818}{\sqrt{A}}$
35°	1.402	$.697\sqrt{A}$	$.439\sqrt{A}$	$.995\sqrt{A}$	$2.43\sqrt{A}$	$\frac{2.87}{\sqrt{A}}$
30°	1.732	$.664\sqrt{A}$	$.356\sqrt{A}$	$1.15\sqrt{A}$	$2.656\sqrt{A}$	$\frac{3.012}{\sqrt{A}}$
26° 34′	2.	$.636\sqrt{A}$	$.3\sqrt{A}$	$1.272\sqrt{A}$	$2.844\sqrt{A}$	$\frac{3.144}{\sqrt{A}}$
Semicircle		$.798\sqrt{A}$			$1.596\sqrt{A}$	$\frac{2.507}{\sqrt{A}}$

The Angle of Slope = angle a b e. Relative Slope = length of a e to *b e.*
n = length of slope cut off the banks.

From this Table it appears that the quotient $\frac{p}{A}$ is least for the semicircle, and greatest for the trapezium of 26° 34′.

ILLUSTRATION.—What dimensions must be given to a profile having an area of 49 square feet, and a slope of its banks of 35°?

By the preceding Table, $.697\sqrt{40} = 4.408$ *feet depth*; $.439\sqrt{40} = 2.776$ *feet, lower breadth*; $.995\sqrt{40} = 6.292$ *feet, slope*; $2.43\sqrt{40} = 15.367$ *feet, upper breadth*; and $\frac{2.87}{\sqrt{40}} = .4538$, *the ratio determining the friction.*

To Compute the Transverse Section when the Volume and Fall are given.

$.0268\left(\frac{m\,l\,V^2}{h}\right)^{\frac{2}{5}} = A$, and $\left(F\,\frac{m\,l\,V^2}{2\,g\,h}\right)^{\frac{2}{5}} = A$, *m representing the unit in the preceding Table.*

Example.—A trench for a length of 3650 feet, with a fall of 1 foot, is to discharge 12 cubic feet of water per second; what dimensions are to be given to the transverse profile, it being of a semi-hexagonal figure?

$$.0268\left(\frac{2.632\times3650\times12^2}{1}\right)^{\frac{2}{5}} = 7.665 \textit{ square feet}; \; v = \frac{12}{7.665} = 1.530 \textit{ feet.}$$

Hence F, per Table $= .00758$, and $\left(.00758\,\frac{2.632\times3650\times12^2}{2\times32.166\times1}\right)^{\frac{2}{5}} = 7.67$ *square feet.*

Therefore the depth $= .76\sqrt{A} = 2.104$ *feet*, the lower breadth $= .877\sqrt{A} = 2.428$ *feet*, and the upper breadth $= 2\times2.428 = 4.856$ *feet.*

Variable Motion.

The variable motion of water in beds of rivers or streams may be reduced to the rules of uniform motion when the resistance of friction for an observed length of the river can be taken as constant.

To Compute the Volume of Water flowing through a River.

$$\frac{\sqrt{2gh}}{\sqrt{\frac{1}{A^2_1} - \frac{1}{A^2} + F\,\frac{l\,p}{A_1 + A}\left(\frac{1}{A^2_1} + \frac{1}{A^2}\right)}} = V,$$

A and A_1 *representing the areas of the upper and lower transverse sections of the flow.*

Example.—A stream having a mean perimeter of water profile of 40 feet for a length of 300 feet has a fall of 9.6 inches; the area of its upper section is 70 feet, and of its lower 60 square feet; what is the volume of its discharge?

To obtain P for the velocity due to this case, $92.35\sqrt{\frac{70 + 60\times\frac{9.6}{12}}{40\times300}} = 8.59$ *feet*, the coefficient for which, see following Table $= .00744$.

$$\frac{\sqrt{64.333\times\frac{9.6}{12}}}{\sqrt{\frac{1}{70^2} - \frac{1}{60^2} + .00744\,\frac{300\times40}{70+60}\left(\frac{1}{70^2} + \frac{1}{60^2}\right)}} = \frac{7.174}{\sqrt{.00040457}} = 358.7 \textit{ cubic feet};$$

the mean velocity of which $= \frac{358.7}{\frac{70+60}{2}} = 5.52$ *feet*, the exact coefficient for which is .00745.

Friction in Pipes and Sewers.

The *Resistance of Friction* in the flow of water through pipes, etc., of a uniform diameter is independent of the pressure, and increases directly as the length, very nearly as the square of the velocity of the flow, and inversely as the diameter of the pipe.

With wooden pipes the friction is 1.75 times greater than in metallic.

The time occupied in the flowing of an equal quantity of water through Pipes or Sewers of equal lengths, and with equal heads, is proportionally as follows: In a Right Line as 90, in a True Curve as 100, and in a Right Angle as 140.

When Pipes branch off from Mains, or when they are deflected at right angles, the radius of the curvature should be proportionate to their diameter. Thus,

	Ins.	Ins.	Ins.	Ins.	Ins.
Diameter..........	2 to 3	3 to 4	6	8	10
Radius	18	20	30	42	60

Discharge of Water in Pipes or Sewers for any Length and Head, and for Diameters from 1 Inch to 10 Feet.

[BEARDMORE.]

IN CUBIC FEET PER MINUTE.

Diameter. Ft. Ins.	Tabular No.	Diameter. Ft. Ins.	Tabular No.	Diameter. Ft. Ins.	Tabular No.
1	4.71	1.7	7433.	3.7	57265.
1¼	8.48	1.8	8449.	3.8	60648.
1½	13.02	1.9	9544.	3.9	64156.
1¾	19.15	1.1	10722.	3.1	67782.
2	26.69	1.11	11983.	3.11	71526.
2½	46.67	2.	13328.	4.	75392.
3	73.5	2.1	14758.	4.1	79380.
3½	108.14	2.2	16278.	4.2	83492.
4	151.02	2.3	17889.	4.3	87730.
4½	194.84	2.4	19592.	4.6	101207.
5	263.87	2.5	21390.	4.9	115854.
6	416.54	2.6	23282.	5.	131703.
7	612.32	2.7	25270.	5.3	148791.
8	854.99	2.8	27358.	5.6	167139.
9	1147.6	2 9	29547.	5.9	186786.
10	1493.5	2.10	31834.	6.	207754.
11	1894.9	2.11	34228.	6.6	253781.
1.	2356.	3.	36725.	7.	305437.
1.1	2876.7	3.1	39329.	7.6	362935.
1.2	3463.3	3.2	42040.	8.	426481.
1.3	4115.9	3.3	44863.	8.6	496275.
1.4	4836.9	3.4	47794.	9.	572508.
1.5	5628.5	3.5	50835.	9.6	655369.
1.6	6493.1	3.6	53995.	10.	745038.

NOTE.—This Table is applicable to Sewers and Drains by taking the same proportion of the tabular numbers that the cross-section of the water in the sewer or drain bears to the area of the whole area of the sewer or drain.

The formula upon which this Table is constructed is,

$2356 \times \frac{\sqrt{d^5}}{\sqrt{\frac{l}{h}}} = V$, *d representing diameter, and h height of fall of the water in feet.*

APPLICATION OF THE TABLE.

To Compute the Volume of Fluid discharged, the Length of the Pipe or Sewer, the Height or Fall, and the Diameter being given.

RULE.—Divide the tabular number, opposite to the diameter of the tube, by the square root of the rate of inclination, and the quotient will give the volume required.

EXAMPLE.—A pipe has a diameter of 9 inches, and a length of 4750 feet; what is its discharge per second under a head of 17.5 feet?

$$\sqrt{\frac{4750}{17.5}} = \sqrt{271.4} = 16.47, \text{ and tabular number for 9 ins.} = 1147.61.$$

Then $\frac{1144.61}{16.47} = 69.67$ *cubic feet per minute.*

To Compute the Diameter, the Length, Fall, and Discharge being given.

Rule.—Multiply the discharge by the square root of the ratio of inclination; take the nearest corresponding number in the Table, and opposite to it is the diameter required.

Example.—Take the elements of the preceding case.

$$69.67 \times \sqrt{\frac{4750}{17.5}} = 1147.61, \text{ and opposite to this is } 9 \textit{ inches.}$$

To Compute the Head, the Length, the Discharge, and the Diameter being given.

Rule.—Divide the tabular number for the diameter by the discharge, square the quotient, and divide the length of the pipe by it; the quotient will give the head necessary to force the given volume of water through the pipe in one minute.

Example.—Take the elements of the preceding cases.

$$\frac{1147.61}{69.67} = 16.47\text{; } 16.47^2 = 271.4\text{; } 4750 \div 271.4 = 17.5 \textit{ feet.}$$

To Compute the whole Head necessary to furnish the requisite Discharge.

See Formula and Illustration, page 386.

To Compute the Velocity, the Volume and the Diameter alone being given.

Rule.—Divide the volume when in feet by the area in feet, and the quotient, divided by 60, will give the velocity in feet per second.

Example.—Take the elements of the preceding case.

$$\frac{69.67}{.75^2 \times .7854} \div 60 = 2.63 \textit{ feet.}$$

*When the Volume is not given.**

Rule.—Multiply the square root of the product of the height of the pipe by the diameter in feet, divided by the length in feet, by 50, and the product will give the velocity in feet per second.

Example.—Take the elements of the preceding case.

$$\sqrt{\frac{17.5 \times .75}{4750}} \times 50 = 2.63 \textit{ feet.}$$

To Compute the Volume of Water discharged from a Pipe.†

$$39.27\sqrt{\frac{h\,d^5}{l}} = \text{V } \textit{in cubic feet per second.}$$

Illustration.—The diameter of a pipe is 1 foot, the head of the flow 9, and the length of the pipe 9000 feet; what is the volume of the discharge?

$$39.27 \times \sqrt{\frac{9 \times 1}{9000}} = 39.27 \times \sqrt{.001} = 1.242 \textit{ cubic feet.}$$

To Compute the Diameter of the Pipe, the Volume of the Flow, the Head, and the Length of the Pipe being given.

$$\sqrt[5]{\left(\frac{\text{V}}{39.27}\right)^2 \times \frac{l}{h}} = d \textit{ in feet.}$$

Illustration.—Take the elements of the preceding case.

$$\sqrt[5]{\left(\frac{1.242}{39.27}\right)^2 \times \frac{9000}{9}} = \sqrt[5]{.001 \times 1000} = \sqrt{1} = 1 \textit{ foot.}$$

* Beardmore.

† Ibid.

To Compute the Inclination of a Pipe, the Volume of the Flow, the Diameter and Length of the Pipe being given.

$$\left(\frac{V}{39.27}\right)^2 \frac{1}{d^5} = \frac{h}{l}.$$

ILLUSTRATION.—Take the elements of the preceding case.

$$\left(\frac{1.242}{39.27}\right)^2 \times \frac{1}{1} = .001 \times 1 = .001 = \textit{ratio of height to length.}$$

FRICTION OF WATER IN PIPES.—[WEISBACH.]

To Compute the Head necessary to overcome the Friction of the Pipe.

$\left(.0144 + \frac{.01746}{\sqrt{v}}\right) \times \frac{l}{d} \times \frac{v^2}{5.4} = h'$, *h' representing the head to overcome the friction of the flow in the pipe in feet, l the length of pipe in feet, d internal diameter of pipe in inches, and v velocity of the water in feet per second.*

ILLUSTRATION.—The length of a conduit-pipe is 1000 feet, its diameter 3 inches, and the required velocity of its discharge 4 feet per second; what is the required head of water to overcome the friction of the flow in the pipe?

$$\left(.0144 + \frac{.01746}{\sqrt{4}}\right) \times \frac{1000}{3} \times \frac{16}{5.4} = .02313 \times 333.333 \times 2.963 = 22\ 845 \textit{ feet.}$$

The head here deduced is the height necessary to overcome the friction of the water in the pipe alone.

The whole or entire head or fall includes, in addition to the above, the height between the surface of the supply and the centre of the opening of the pipe at its upper end. Consequently, it is the whole height or vertical distance between the supply and the centre of the outlet.

To Compute the whole Head, or the Height from the Surface of the Supply to the Centre of the Discharge.

$$(C \times \frac{l}{d} + 1.5) \times \frac{v^2}{2g} = h.$$

1.5 is taken as a mean, and is the coefficient of friction for the interior orifice, or that of the upper portion of the pipe.

ILLUSTRATION.—Take the elements of the preceding case.

$$(.0231 \times \frac{1000 \times 12}{3} + 1\ 5) \times \frac{4^2}{64.333} = 93.9 \times \frac{16}{64.333} = 23.35 \textit{ feet}$$

NOTE.—In the preceding formula l was taken in feet, as the multiplier of 12 for inches was canceled by taking 5.4 for $2g$, but in the above formula it is necessary to restore this multiplier.

For facilitating the calculation, the following Table of the coefficient of resistance is introduced:

Coefficients of Friction of Water in Pipes at different Velocities.

(A Reduction of the following Formula.)

Ft. Ins.	C.	Ft. Ins.	C.	Ft. Ins.	C.	Ft. Ins.	C.
4	.0443	3.4	.0239	6.4	.0213	11.	.0196
8	.0356	3.8	.0234	6.8	.0211	11.6	.0195
1.	.0317	4.	.0231	7.	.0209	12.	.0194
1.4	.0294	4.4	.0227	7.4	.0208	12.6	.0193
1.8	.0278	4.8	.0224	7.8	.0206	13.	.0191
2.	.0266	5.	.0221	8.	.0205	13.6	.019
2.4	.0257	5.4	.0219	8.6	.0204	14.	.0189
2.8	.025	5.8	.0217	9.	.0202	15.	.0188
3.	.0244	6.	.0215	10.	.0199	16.	.0187

ILLUSTRATION.—The coefficient due to a velocity of 4 feet per second is .0231.

Thus, by the Formula $\left(.0144+\frac{.01746}{\sqrt{4}}\right)=.0231$; and by the preceding Table a velocity of 4 feet per second $=.0231$ for its coefficient.

Hence for $\left(.0144+\frac{.01746}{\sqrt{v}}\right)$ read, when practicable to do so, C, a coefficient.

Table showing the Velocity of Water flowing from Pipes and Sewers, as computed by the Formulæ of Beardmore and Weisbach.

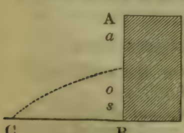
Diameter of Pipe.	Head.	Length.	Discharge per Minute.	(Beardmore.) $50\sqrt{\frac{h}{l}\times d}=v.$	(Weisbach.) $\frac{\sqrt{2gh}}{\sqrt{C\times\frac{l}{d}+1.5}}=v.$	Actual Velocity.
Ins.	Feet.	Feet.	Cubic Feet.	Feet per Second	Feet per Second.	Feet.
1	1	100	.471	1.44	C = .0294 1.32	1.43
1	9	100	1.41	4.33	C = .0227 4.49	4.31
1	9	1800	.333	1.02	C = .0317 .92	1.02
1	25	225	1.57	4.81	C = .0224 5.09	4.79
2	25	225	8.89	6.81	C = .0211 7.32	6.79
2	25	5250	1.84	1.41	C = .0294 1.32	1.4
3	25.6	1000	11.78	4.	C = .0231 4.19	4.
4	4	144	25.17	4.8	C = .0224 4.8	4.8
4	36	300	52.25	10.	C = .0199 10.92	9.99
6	4	144	69.42	5.9	C = .0217 5.76	5.89
6	8	144	98.24	8.33	C = .0204 8.35	8.33
6	50	1000	93.2	7.91	C = .0205 8.65	7.91
12	1	1000	74.5	1.58	C = .0294 1.44	1.59
12	9	441	336.5	7.15	C = .0209 7.35	7.14
12	9	9000	74.5	1.58	C = .0209 1.75	1.59
12	64	1600	471.2	10.	C = .0199 11.11	9.99
24	3	300	1332.8	7.07	C = .0209 6.44	7.07
24	9	900	1332.8	7.07	C = .0209 7.28	7.97
24	9	9000	421.5	2.24	C = .026 2.21	2.24
24	70	13720	95.2	5.05	C = .0221 5.43	5.05
36	6	600	3672.5	8.66	C = .0204 8.32	8.66
36	6	15000	734.5	1.73	C = .0356 1.47	1.73
36	9	2025	2448.3	5.76	C = .0217 5.99	5.77
36	70	43750	1469.	3.46	C = .0239 3.57	3.46

(Beardmore), $l=$ *representing length of pipe, and d its diameter in feet.*
(Weisbach), $l=$ *representing length of pipe, and d its diameter in inches.*

NOTE.—For values of C and the constant of 1.5, see page 386.

The preceding cases are selected with a view to give the range of ordinary operations, and to show the application of the Formulæ to high and low heads, large and small diameters, and extremes of length.

To Compute the Distance a Jet of Water will be projected from a Vessel through an Opening in its Side.



B C is equal to twice the square root of A $o\times o$ B.

If s is 4 times as deep below A as a is, s will discharge twice the quantity of water that will flow from a in the same time, as 2 is the $\sqrt{}$ of A s and 1 is the $\sqrt{}$ of A a.

NOTE. — The water will spout the farthest when o is equidistant from A and B; and if the vessel is raised above a plane, B must be taken upon the plane.

The quantities of water passing through equal apertures in the same time are as the square roots of their depths from the surface.

RULE.—Multiply the square root of the product of the distance of the opening from the surface of the water, and its height from the plane upon which the water flows, in feet by 2, and the product will give the distance in feet.

Example.—A vessel 20 feet deep is raised 5 feet above a plane; how far will a jet reach that is 5 feet from the bottom of the vessel?

$$20-5\times\overline{5+5}=150, \text{ and } \sqrt{150\times2}=24.495 \textit{ feet.}$$

The velocity of a jet of water flowing from a cylindrical tube is determined to be .82 of that due to the height of the reservoir. Hence the volume of the discharge through a cylindrical opening $= .82\,a\sqrt{2gh}$.

Jets d'Eau.

That a jet may ascend to the greatest practicable height, the communication with the supply should be perfectly free.

Short tubes shaped alike to the contracted fluid vein, and conically convergent pipes, are those which give the greatest velocities of efflux. Hence, to attain the greatest effect, as in fire-engines, long and slightly conically convergent tubes or pipes should be applied.

In order to diminish the resistance of the descending water, a jet must be directed with a slight inclination from the vertical.

The effect of the combined causes which diminish the height of a jet from that due to the elevation of its supply can only be determined by experiments. Great jets rise higher than small ones.

With cylindrical tubes, the velocity being reduced in the ratio of 1 to .82, and as the heights of the jets are as the squares of these coefficients or ratios, or as 1 to .67, the height of a jet through a cylindrical tube is ⅔ that of the head of the water from which it flows.

With conical tubes, the velocity being from .55 to .95, the heights of the jets are as the squares of the coefficients 1 and .9 (a mean), or as 1 to .81, which is equal to $\frac{8}{10}$ that of the head of the water from which it flows. Hence the relative values of Cylindrical and Conical tubes are as .67 to .81.

To Compute the Vertical Height of a Stream projected from the Pipe of a Fire-engine or Pump.

Note.—In Fire-engines, the difference between the actual discharge and that as computed by the capacity and stroke of the cylinder, as ascertained by Mr. Larned, 1850, is 18 per cent.

Rule.—Ascertain the velocity of the stream by computing the quantity of water running or forced through the opening in a second; then, by Rule in Gravitation, page 397, ascertain the height to which the stream would be elevated if wholly unobstructed, which multiply by a coefficient for the particular case.

Example.—If a fire-engine actually discharges 14 cubic feet of water through a pipe ¾ inch in diameter in one minute, how high will the water be projected, the pipe being directed vertically?

$14\times1728\div.4417$ area of pipe, $\div$ 12 inches in a foot, $\div$ 60 seconds $= 76.07$ *feet* velocity; and as the velocity of a stream of water from a vessel is but ⅔ that due to its head, then $76.07\times\frac{3}{2}=114.1$ *feet.*

Then, by Rule, page 397, $114.1\div8.02=14.22$, and $14.22^2=202.21$ *feet.*

According to the elements furnished by observation, the mean coefficient in this case would be .5; hence $202.21\times\ 5=101.15$ *feet.*

In great heights and with small apertures, the coefficient should be reduced. In consequence of the varying elements and conditions of operation of fire-engines, it is difficult to assign a coefficient for them.

A steam fire-engine of the Portland Company, discharging a stream 1⅛ in. in diameter, through 100 feet 2½ in. hose, gave a theoretical head, computed from the actual discharge, of 225 feet, and the stream vertically projected was 200 feet; hence the coefficient in this case was .88.

Table of the Proportional Rise of Water in Rivers, occasioned by the Erection of Piers, etc.

Velocity of Current in Feet per Second.	Amount of Obstruction compared with Area of Section of the River.								
	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1	.0157	.0377	.0698	.1192	.2012	.3521	.678	1.609	6.639
2	.0277	.0665	.1231	.2102	.3548	.6208	1.196	2.838	11.71
3	.0477	.1144	.2118	.3618	.6107	1.069	2.058	4.885	20.15
4	.076	.1822	.3372	.5759	.9719	1.701	3.276	7.775	32.07
5	.1165	.2793	.5168	.8782	1.49	2.607	5.02	11.92	49.15
6	.1558	.3736	.6912	1.181	1.993	3.487	6.715	15.94	65.75
7	.2078	.4983	.9221	1.575	2.658	4.651	8.958	21.26	87.71
8	.2678	.6423	1.188	2.03	3.426	5.995	11.54	27.4	113.
9	.3359	.8054	1.49	2.557	4.296	7.517	14.48	34.36	141.7
10	.4119	.9877	1.827	3.122	5.268	9.219	17.75	42.14	173.8

Table of the Velocities of Flow necessary to clear Circular Drains or Sewers.

Diam. Ins.	Velocity per Min.	Gradial.	Diam. Ins.	Velocity per Min.	Gradial.	Diam. Ins.	Velocity per Min.	Gradial.
4	240	1 in 36	12	190	1 in 175	30	180	1 in 490
6	220	1 in 65	15	180	1 in 244	36	180	1 in 588
7	220	1 in 76	18	180	1 in 294	42	180	1 in 686
8	220	1 in 87	21	180	1 in 343	48	180	1 in 784
10	210	1 in 119	24	180	1 in 392	54	180	1 in 882

Friction of Water upon a Plane Surface.

By the experiments of Beaufoy, it was ascertained that the friction increased *very nearly* as the square of the velocity, and that a surface of 50 square feet, at a velocity of 6 feet per second, presented a resistance of 6 lbs. Hence $\frac{50}{6} = 8.33$ square feet $= 1$ lb. resistance at a velocity of 6 feet; and, consequently, $\frac{1}{8.33} = .12$ lbs. resistance per sq. foot at the same velocity.

For a velocity of 10 feet, $6^2 : 10^2 :: .12 : .333$ *lbs.*

To Compute the Horses' Power necessary to Raise Water to any given Elevation.

Rule.—Multiply the weight of the column of the water by its velocity in feet per minute, and divide the product by 33000.

Example.—It is required to raise 1000 gallons of fresh water per minute to an elevation of 140 feet, through a cast-iron pipe 560 feet in length; what is the required diameter of the pipe, and what the power?

1000 galls. fresh water $= 1000 \times 231$ (page 23) $= 231000$ *cubic inches*, and $\frac{231000}{1728} =$ 133.68 *cubic feet per minute.*

Then, by Formula, page 385, $133.68 \times \sqrt{\frac{560}{140}} = 267.36$; and opposite to the nearest tab. number, page 384, is $5 +$ *inch* $=$ *diam. of pipe.*

Hence $133.68 \times 62.5 \times 140 \div 33.000 = 35.41$ *horses' power.*

Volume of Water per Acre of Area.

A depth of 1 inch = 3630 *cubic feet.*

MISCELLANEOUS ILLUSTRATIONS.

1. The head required to discharge 252 cubic inches of water per second through an opening of 6 square inches is determined by

$$h = \frac{1}{2g}\left(\frac{I}{c\,a}\right)^2, \text{ or } .015545\left(\frac{252 \div 12}{.6 \times 6}\right)^2 = .015545 \times .5833^2 = .5289 \textit{ feet.}$$

.6 is here taken as the coefficient of efflux.

2. If the external height of fresh water, at 60° above the injection opening in the condenser of a steam-engine, is 3 feet, and the indicated vacuum at 23 inches, the velocity of the water flowing into the condenser is determined thus:

$v = \sqrt{2g(h + h')}$, *h' representing the height of a column of water equivalent to the pressure of the atmosphere within the condenser.*

A column of 2.03593 inches of mercury = 1 *lb. pressure per square inch;* 1 inch of mercury = .490774 *lbs.*

Assuming the mean pressure of the atmosphere = 14.723 *lbs. per square inch*, the height of a column of fresh water equivalent thereto = 33.95 *feet.*

Then, if 1 inch = .490774 *lbs.*, 23 inches = 11.2878 *lbs.*; and if 14.723 *lbs.* = 33.95 *feet;* 11.2878 *lbs.* = 26.013 *feet.*

Hence $v = \sqrt{2g(3 + 26.013)} = 43.203$ *feet*, less the retardation due to the coefficient of efflux.

3. If a stream of water has a mean velocity of 2.25 feet per second at a breadth of 560 feet, and a mean depth of 9 feet, what will be its mean velocity when it has a breadth of 320 feet, and a mean depth of 7.5 feet?

$$\frac{560 \times 9 \times 2.25}{320 \times 7.5} = \frac{11340}{2400} = 4.725 \textit{ feet.}$$

AEROSTATICS.

One cubic foot of Atmospheric Air at the surface of the earth, when the barometer is at 30 inches, and at a temperature of 34°, weighs 527.04 grains = .07529144 lbs. avoirdupois, being 829.43 times lighter than water.

Specific gravity compared with water, at 62.4491 = .00120567.

The mean weight of a column of air a foot square, and of an altitude equal to the height of the atmosphere, is equal to 2120.14 lbs. avoirdupois, being equal to the support of 33.95 feet of water.

13.817 cubic feet of air weigh a pound avoirdupois.

It consists, by volume, of oxygen 21, and nitrogen 79 parts; and in 10000 parts there are 4.9 parts of carbonic acid gas. By weight, it consists of 77 parts of oxygen, and 23 of nitrogen.

The rate of expansion of Air, *and all other Elastic Fluids* for all temperatures, is uniform. From 32° to 212° they expand from 1000 to 1376, equal to $\frac{1}{479}$ = .002088 for each degree of their bulk for every degree of heat. From 212° to 680° they expand from 1376 to 2322 = .002 for each degree of heat.

The Standard pound is computed with a mercurial barometer at 30 inches; hence, as a cubic inch of mercury at 60° weighs .490774 lbs., the pressure of the atmosphere at a temperature of 60° = 14.72322 lbs. per square inch.

The *Elasticity* of air is inversely as the space it occupies, and directly as its density.

When the altitude of the air is taken in arithmetical proportion, its *Rarity* will be in geometric proportion. Thus, at 7 miles above the surface of the earth, the air is 4 times rarer or lighter than at the earth's surface; at 14 miles, 16 times; at 21 miles, 64 times, and so on.

At the temperature of 32°, the mean velocity of sound is 1092.5 feet per second. It is increased or diminished half a foot for each degree of temperature above or below 32°.

The velocity of sound in water is estimated at 4900 feet per second.

The motions of air and all gases, by the force of gravity, are precisely alike to those of fluids.

The head or altitude of the atmosphere at the ordinary density is equal to a column of mercury 30 inches in height, divided by the specific gravity of air compared with mercury.

Hence 30 ins. = 2.5 feet, which, divided by .00008878, the specific gravity of air compared with mercury = 28160 *feet* = 53.3 *miles*.

The theoretical velocity, therefore, with which air will flow into a vacuum, if wholly unobstructed, is $\sqrt{2gh} = 1347.4$ *feet per second*. In operation, however, it is $1347.4 \times .707 = 952.61$ *feet*.

The coefficients for the efflux of air through openings are as follows:

Circular aperture in a thin plate	.65 to .7
Cylindrical ajutage	.92
Conical ajutage	.93

To Compute the Velocity of Sound through Air.

$1092.5\sqrt{1 + .002088\,(t - 32)} = v$ *in feet per second, t representing temperature of air in degrees.*

EXAMPLE.—The flash of a cannon from a vessel was observed 13 seconds before the report was heard; the temperature of the air was 60°; what was the distance to the vessel?

$1092.5 \times 13\sqrt{1 + .002088\,(60° - 32)} = 14623$ *feet*, or 2.77 *miles*.

Height of the Barometer at different Levels above the Surface of the Earth.

Feet.	Inches	Feet.	Inches.	Miles.	Inches.	Miles.	Inches
1000	28.91	4000	25.87	2	20.29	5	11.28
2000	27.86	5000	24.93	3	16.68	10	4.24
3000	26.85	1 mile	24.67	4	13.72	15	1.6

Measurement of Heights by a Barometer.

APPROXIMATE RULE.—For a mean temperature of 55°, *x required difference in height in feet, h the height of the mercury at the lower station, and h′ the height of the mercury at the upper station.*

$55000 \times \frac{h - h'}{h + h'} = x$. Add $\frac{1}{440}$ of this result for each degree which the mean temperature of the air at the two stations exceeds 55°, and deduct as much for each degree below 55°.

To Compute what Degree of Rarefaction may be effected in a Vessel.

Let the quantity of air in the vessel, tube, and pump be represented by 1, and the proportion of the capacity of the pump to the vessel and tube by .33; consequently, it contains $\frac{1}{4}$ of the air in the united apparatus.

Upon the first stroke of the piston this fourth will be expelled, and $\frac{3}{4}$ of the original quantity will remain; $\frac{1}{4}$ of this will be expelled upon the second stroke, which is equal to $\frac{3}{16}$ of the original quantity; and, consequently, there remains in the apparatus $\frac{9}{16}$ of the original quantity. Proceeding in this manner, the following Table is deduced:

No. of Strokes.	Air expelled at each Stroke.	Air remaining in the Vessel.
1	$\frac{1}{4} = \frac{1}{4}$	$\frac{3}{4} = \frac{3}{4}$
2	$\frac{3}{16} = \frac{3}{4 \times 4}$	$\frac{9}{16} = \frac{3 \times 3}{4 \times 4}$
3	$\frac{9}{64} = \frac{3 \times 3}{4 \times 4 \times 4}$	$\frac{27}{64} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4}$

And so on, continually multiplying the air expelled at the preceding stroke by 3, and dividing it by 4; and the air remaining after each stroke is ascertained by multiplying the air remaining after the preceding stroke by 3, and dividing it by 4.

Distances at which different Sounds are Audible.

	Feet.	Miles.
A full human voice speaking in the open air, calm	460	.087
In an observable breeze, a powerful human voice with the wind can be heard	15840	3
Report of a musket	16000	3.02
Drum	10560	2
Music, strong brass band	15840	3
Cannonading, very heavy	575000	90

In the Arctic, conversation has been maintained over water a distance of 6696 feet.

Velocity and Force of Wind.

Miles per Hour.	Feet per Minute.	Pressure on a Sq. Ft in Lbs.	Description of the Wind.	Miles per Hour.	Feet per Minute.	Pressure on a Square Foot in Lbs.	Description of the Wind.
1	88	.005	Barely observable.	25	2200	3.125	Very brisk.
2	176	.02	Just perceptible.	30	2640	4.5	High wind.
3	264	.045		35	3080	6.125	
4	352	.08	Light breeze.	40	3520	8.	Very high wind.
5	440	.125	Gentle, pleasant wind.	45	3960	10.125	Gale.
6	528	.18		50	4400	12.5	Storm.
8	704	.32		60	5280	18.	Great storm.
10	880	.5	Fresh breeze.	80	7040	32.	Hurricane.
15	1320	1.125	Brisk blow.	100	8800	50.	Tornado.
20	1760	2.	Stiff breeze.				

To Compute the Capacity and Diameter of a Balloon, see Page 162.

For Expansion of Air, see *Practical Mechanics' Journal*, vol. viii., 1st series, pages 231–252.

For Treatise on Aerometry, of D'Aubuisson de Voissons, see *Journal of Franklin Institute*, pages 124, 186, 234, 313, vol. 39.

Table for foretelling the Weather through the Lunations of the Moon.—[Dr. Herschell and Adam Clarke]

This Table and the accompanying remarks are the result of many years' actual observation, and will, by inspection, show the observer what kind of weather will most probably follow the entrance of the moon into any of its Quarters.

If the New Moon, the First Quarter, the Full Moon, or the Last Quarter enters	In Summer.	In Winter.
Between midnight and 2 A.M.	Fair.	Hard frost, unless the wind is S. or E.
Between 2 and 4 A.M.	Cold, with frequ't showers	Snowy and stormy.
Between 4 and 6 A.M.	Rain.	Rain.
Between 6 and 8 A.M.	Wind and rain.	Stormy.
Between 8 and 10 A.M.	Changeable.	Cold rain, if the wind be W.; snow if E.
Between 10 and 12 A.M.	Frequent showers.	Cold, and high wind.
At 12 o'clock M. and 2 P.M.	Very rainy.	Snow or rain.
Between 2 and 4 P.M.	Changeable.	Fair and mild.
Between 4 and 6 P.M.	Fair.	Fair.
Between 6 and 8 P.M.	Fair, if wind N.W.; rainy, if S. or S.E.	Fair and frosty, if the wind is N. or W. Rain or snow, if S. or S.E.
Between 8 and 10 P.M.	Ditto.	Ditto.
Between 10 and midnight.	Fair.	Fair and frosty.

Observations.—1. The nearer the time of the moon's change, first quarter, full, and last quarter are to *midnight*, the fairer will be the weather during the seven following days. The range for this is from 10 at night till 2 next morning.

2. The nearer to *mid-day*, or *noon*, the phases of the moon happen, the more foul or wet weather may be expected during the following seven days. The range for this calculation is from 10 in the forenoon till 2 in the afternoon.

These observations refer principally to the summer, though they affect spring and autumn nearly in the same ratio.

3. The moon's change, first quarter, full, and last quarter, entering during six of the afternoon hours, *i. e.*, from 4 to 10, may be followed by fair weather; but this is mostly dependent on the *wind*, as noted in the Table.

4. Though the weather, from a variety of irregular causes, is more uncertain in the latter part of autumn, the whole of winter, and the beginning of spring, yet, as a general rule, the above observations will apply to those periods also.

Classification of Clouds.

1. *Cirrus* — Like to a feather. 2. *Cirro-cumulus* — Small round clouds. 3. *Cirro-stratus*—The concave or undulated stratus. 4. *Cumulus* — When in conical, round clusters. 5. *Cumulo-stratus* — The two latter mixed. 6. *Nimbus*—A cumulus spreading out in arms, and precipitating rain beneath it. 7. *Stratus*—A level sheet.

Note.—The *Cirrus* is the most elevated.

Classification of Lightning.

1. *Striped* or *Zigzag*—Developed with great rapidity. 2. *Sheet*—Covering a large surface. 3. *Globular*—When the electric fluid appears condensed, and it is developed at a comparatively lower velocity. 4. *Phosphoric*—When the flash appears to rest upon the edges of the clouds.

DYNAMICS.

DYNAMICS is the investigation of the laws of *Motion of Solid Bodies*, or *of Matter*, *Force*, *Velocity*, *Space*, and *Time*.

The *Mass* of a body is the quantity of matter of which it is composed.

Force is divided into Motive, Accelerative, or Retardative.

Motive Force, or the *Momentum* of a body, is the product of its mass and its velocity, and is its quantity of motion. This force can, therefore, be ascertained and compared in any number of bodies when these two quantities are known.*

Accelerative or *Retardative Force* is that which respects the velocity of the motion only, accelerating or retarding it; and it is denoted by the quotient of the motive force, divided by the mass or weight of the body. Thus, if a body of 5 lbs. is impelled by a force of 40 lbs., the accelerating force is 8 lbs.; but if a force of 40 lbs. act upon a body of 10 lbs., the accelerating force is then only 4 lbs., or half the former, and will produce only half the velocity.

With equal masses, the velocities are proportional to their forces.

With equal forces, the velocities are inversely as the masses.

With equal velocities, the forces are proportional to the masses.

Motion.—The succession of positions which a body in its motion progressively occupies forms a line which is termed the trajectory, or path of the moving body.

A motion is *Uniform* when equal spaces are described by it in equal times, and *Variable* when this equality does not occur. When the spaces described in equal times increase continuously with the time, a variable motion is termed accelerated, and when the spaces decrease, retarded; and when equal spaces are described within certain intervals only, the motion is termed *Periodic*, and the intervals periods. Uniform motion is illustrated in the progressive motion of the hands of a watch; Variable motion in the progressive velocity of falling and upwardly projected bodies; and Periodic motion by the oscillation of a pendulum or the strokes of a piston of a steam-engine.

Let *body, force, velocity, space, and time be represented by b f v s t, gravity by g, and momentum or quantity of motion by m; this being the effect produced by a body in motion.*

If two or more bodies, etc., *are compared, two or more corresponding letters, as* B, *b*, *b′*, V, *v*, *v′*, etc., *are employed.*

Uniform Motion.—The space described by a body moving uniformly is represented by the product of the velocity into the time.

With *momenta*, *m* varies as *b v*.

ILLUSTRATION.—Two bodies, one of 20, the other of 10 lbs., are impelled by the same momentum, say 60. They move uniformly, the first for 8 seconds, the second for 6; what are the spaces described by both?

$$\frac{60}{20}=3=\mathrm{V}, \text{ and } \frac{60}{10}=6=v.$$

Then $\mathrm{T\,V}=3\times 8=24=\mathrm{S}$, and $t\,v=6\times 6=36=s$, *the spaces respectively.*

* It is compared, because it is not referable to any standard, as a ton, pound, etc. Thus, suppose a cannon-ball weighing 15 lbs., projected with a velocity of 1500 feet per second, strike a resisting body, its momentum, according to the above rule, would be $15\times 1500=22500$; not pounds, for weight is a pressure with which it can not be compared.

Uniform Variable Motion.—The space described by a body having uniform variable motion is represented by the sum or difference of the velocity, and the product of the acceleration and the time, according as the motion is accelerated or retarded.

ILLUSTRATION.—A sphere rolling down an inclined plane with an initial velocity of 25 feet, acquires in its course an additional velocity at each second of time of 5 feet; what will be its velocity after 3 seconds?

$$25+\overline{5\times 3}=40 \textit{ feet.}$$

2.—A locomotive having an initial velocity of 30 feet per second is so retarded that in each second it loses 4 feet; what is its velocity after 6 seconds?

$$30-\overline{4\times 6}=6 \textit{ feet.}$$

Motion Uniformly Accelerated.

In this motion, the velocity acquired at the end of any time whatever is equal to the product of the accelerating force into the time, and the space described is equal to the product of half the accelerating force into the square of the time, or half the product of the velocity and the time of acquiring the velocity.

The spaces described in successive seconds of time are as the odd numbers, 1, 3, 5 7, 9, etc.

Gravity is a constant force, and its effect upon a body falling freely in a vertical line is represented by g. and the motion of such body is uniformly accelerated.

The following theorems are applicable to all cases of motion uniformly accelerated by any constant force, F:

$$s=\tfrac{1}{2}\,t\,v=\tfrac{1}{2}\,g\,F\,t^2=\frac{v^2}{2\,g\,F}. \qquad t=\frac{t\,s}{v}=\frac{v}{g\,F}=\sqrt{\frac{s}{\frac{1}{2}\,g\,F}}.$$

$$v=\frac{2s}{t}=g\,F\,t=\sqrt{2\,g\,f\,s}. \qquad F=\frac{v}{g\,t}=\frac{2\,s}{g\,t^2}=\frac{v^2}{2\,g\,s}.$$

When gravity acts alone, as when a body falls in a vertical line, F is omitted. Thus,

$$s=\tfrac{1}{2}\,g\,t^2=\frac{v^2}{2\,g}=\tfrac{1}{2}\,t\,v \qquad t=\frac{v}{g}=\frac{2\,s}{v}=\sqrt{\frac{2\,s}{g}}.$$

$$v=g\,t=\frac{2\,s}{t}=\sqrt{2\,g\,s} \qquad g=\frac{v}{t}=\frac{2\,s}{t^2}=\frac{v^2}{2\,s}.$$

If, instead of a heavy body falling freely, it be projected vertically upward or downward with a given velocity, v, then $s=t\,v\mp\tfrac{1}{2}\,g\,t^2$; an expression in which $-$ must be taken when the projection is upward, and $+$ when it is downward.

ILLUSTRATION.—If a body in 10 seconds has acquired a velocity by uniformly accelerated motion of 26 feet, what is the accelerating force, and what the space described, in that time?

$$26\div 10=2.6=\textit{accelerating force};\ \frac{2.6}{2}\times 10^2=130 \textit{ feet} = \textit{the space described}$$

2. A body moving with an acceleration of 15.625 feet describes in 1.5 seconds a space $=\dfrac{15.625\times(1.5)^2}{2}=17.578$ *feet.*

3. A body propelled with an initial velocity $=3$ feet, and with an acceleration $=5$ feet, describes in 7 seconds a space $=3\times 7+5\times\dfrac{7^2}{2}=143.5$ *feet*

4. A body which in 180 seconds changes its velocity from 2.5 to 7.5 feet, traverses in this time a distance of $\dfrac{2.5+7.5}{2}\times 180=900$ *feet.*

5. A body which rolls up an inclined plane with an initial velocity of 40 feet, by which it suffers a retardation of 8 feet per second, ascends only $\dfrac{40}{8}=5$ *seconds*, and $\dfrac{40^2}{2\times 8}=100$ *feet* in height, then rolls back, and returns, after 10 seconds, with a velocity of 40 feet, to its initial point; and after 12 seconds arrives at a distance of $40\times 12-\overline{4\times 12^2}=96$ *feet below the point, assuming the plane to be extended backward.*

GRAVITATION.

Gravity is an attraction common to all material substances, and they are effected by it in exact proportion to their mass.

This attraction is termed *terrestrial gravity*, and the force with which any body is drawn toward the centre of the earth is termed the *weight* of that body.

The force of gravity differs a little at different latitudes: the law of the variation, however, is not accurately ascertained; but the following theorems represent it very nearly;

$$g' = \begin{cases} g\,(1 - .002837 \text{ cos. 2 lat.}), \\ g\,(1 + .002837), \text{ at the poles}, \\ g\,(1 - .002837), \text{ at the equator}, \end{cases}$$ *g representing the force of gravity at latitude* 45°, *and* g' *the force at the other places*

In bodies descending freely by their own weight, their *velocities* are as the *times* of their descent, and the *spaces* passed through as the square of the times.

The *Times*, then, being 1, 2, 3. 4, etc., the *Velocities* will be 1, 2, 3, 4, etc. The *Spaces* passed through will be as the square of the velocities acquired at the end of those times, as 1, 4, 9, 16, etc.; and the *spaces* for each time as 1, 3, 5, 7, 9, etc.

A body falling freely will descend through 16.0833 feet in the first second of time, and will then have acquired a velocity which will carry it through 32.166 feet in the next second.

The velocity acquired at any period is equal to twice the mean velocity during that period.

The motion of a falling body being uniformly accelerated by gravity, the motion of a body projected vertically upward is uniformly retarded in the same manner.

A body projected perpendicularly upward with a velocity equal to that which it would have acquired by falling from any height, will ascend to the same height before it loses its velocity.

Table exhibiting the Relation of Time, Space, and Velocities.

Seconds from the beginning of the Descent.	Velocity acquired at the End of that Time.	Squares of the Time.	Space fallen through in that Time.	Spaces for this Time	Space fallen through in the last Second of the Fall.
1	32.166	1	16.083	1	16.08
2	64.333	4	64.333	3	48.25
3	96.5	9	144.75	5	80.41
4	128.665	16	257.33	7	112.58
5	160.832	25	402.08	9	144.75
6	193.	36	579.	11	176.91
7	225.166	49	788.08	13	209.08
8	257.333	64	1029.33	15	241.25
9	289.5	81	1302.75	17	273.42
10	321.666	100	1608.33	19	305.58

and in the same manner the Table may be continued to any extent.

Note.—In considering the action of gravitation on bodies not far distant from the surface of the earth, it is assumed, without sensible error, that the directions in which it acts are parallel, or perpendicular to the horizontal plane.

A distance of one mile only produces a deviation from parallelism less than one minute, or the 60th part of a degree.

To Compute the Time which a Body will be in falling through a given Space.

RULE.—Divide the space in feet by 16.083, and the square root of the quotient will give the required time in seconds.

EXAMPLE.—How long will a body be in falling through 402.08 feet of space?

$$\sqrt{402.08 \div 16.083} = 5 \textit{ seconds}$$

To Compute the Time which a Body will be in falling, the Velocity per Second being given.

RULE.—Divide the given velocity by 32.166, and the quotient is the time.

EXAMPLE.—How long must a body be in falling to acquire a velocity of 800 feet per second? $800 \div 32.166 = 24.87$ *seconds*.

Ex. 2.—Compute the time of generating a velocity of 193 feet per second, and the whole space descended.

$$193 \div 32.166 = 6 \textit{ seconds}; \; 6^2 \times 16.083 = 579 \textit{ feet}$$

To Compute the Velocity a Body will acquire by falling from any given Height.

RULE.—Multiply the space in feet by 64.333, and the square root of the product will give the velocity acquired in feet per second.

EXAMPLE.—Required the velocity a body acquires in descending through 579 feet.

$$\sqrt{579 \times 64.333} = 193 \textit{ feet}.$$

As the velocity acquired at any period is equal to twice the mean velocity during that period.

Ex. 2.—If a ball fall through 2316 feet in 12 seconds, with what velocity will it strike?

$$2316 \div 12 = 193, \textit{ mean velocity}, \text{ which } \times 2 = 386 \textit{ feet} = \textit{velocity}.$$

To Compute the Velocity a Falling Body will acquire in any given Time.

RULE.—Multiply the time in seconds by 32.166, and the product will give the velocity in feet per second.

EXAMPLE.—What is the velocity acquired by a falling body in 6 seconds?

$$32.166 \times 6 = 192.996 \textit{ feet}.$$

To Compute the Space fallen through, the Velocity being given.

RULE.—Divide the velocity by 8.02, and the square of the quotient will give the distance fallen through to acquire that velocity.

EXAMPLE.—If the velocity of a cannon-ball is 579 feet per second, from what height must a body fall to acquire the same velocity?

$$579 \div 8.02 = 72.2 \text{ and } 72.2^2 = 5212.84 \textit{ feet}.$$

To Compute the Space through which a Body will fall in any given Time.

RULE.—Multiply the square of the time in seconds by 16.083, and it will give the space in feet.

EXAMPLE.—Required the space fallen through in 5 seconds.

$$5^2 = 25, \text{ and } 25 \times 16.083 = 402.08 \textit{ feet}.$$

The distance fallen through in feet is very nearly equal to the square of the time in fourths of a second.

Ex. 2.—A bullet being dropped from the spire of a church was 4 seconds in reaching the ground; what was the height of the spire?

$$4 \times 4 = 16, \text{ and } 16^2 = 256 \textit{ feet}.$$

By Rule, $4 \times 4 \times 16.0833 = 257.33$ *feet*.

Ex. 3.—What is the depth of a well, a bullet being 2 seconds in reaching the bottom? $2 \times 4 = 8$, and $8^2 = 64$ *feet.*

By Rule, $2 \times 2 \times 16.0833 = 64.33$ *feet.*

By Inversion.—In what time will a bullet fall through 256 feet?

$$\sqrt{256} = 16, \text{ and } 16 \div 4 = 4 \textit{ sec.}$$

Let s *represent the space described by any falling body,* t *the time,* v *the velocity acquired in feet per second, and* x *the space in feet which the body falls in the* tth *second.*

Then $v = 2\sqrt{16.083\,s}$, or $32.166\,t$, or $\frac{2s}{t}$; $x = 32.166\,(t - \frac{1}{2})$.

$$s = 16.083\,t^2, \text{ or } \frac{t\,v}{2}, \text{ or } \frac{v^2}{64.33};\ t = \sqrt{\frac{s}{16.083}}, \text{ or } \frac{v}{32.166}, \text{ or } \frac{2s}{v}.$$

Ascending bodies are retarded in the same ratio that descending bodies are accelerated. Hence a body projected upward is ascending for one half of the time it is in motion, and descending the other half.

To Compute the Space moved through by a Body projected Upward or Downward with a given Velocity.

If projected Upward.

Rule.—From the product of the given velocity and the time in seconds subtract the product of 32.166, and half the square of the time, and the remainder will give the space in feet.

$$\text{Or, } t^2 \times 16.083 - \overline{v \times t} = s$$

Example.—If a body be projected upward with a velocity of 30 feet per second, through what space will it ascend before it begins to return?

$30 \div 32.166 = .9326 =$ *the time to acquire this velocity.*

Then $30 \times .9326 = 27.98 =$ *product of velocity of projection and the time.*

$32.166 \times \frac{.9326^2}{2} = 13.98 =$ *product of* 32.166, *and half the square of the time*

Hence $27.98 - 13.98 = 14$ *feet.*

Ex. 2.—If a body be projected upward with a velocity of 96.5 feet per second, it is required to ascertain the point of the body at the end of 10 seconds.

$96.5 \div 32.166 = 3$ *seconds, the time to acquire this velocity,* and $3^2 \times 16.083 = 144.75$, *the height the body reached with its initial velocity.*

Then $10 - 3 = 7$ *seconds* left for the body to fall in.

Hence, by Rule (page 397), $7^2 \times 16.083 = 788.07$, and $788.07 - 144.75 = 643.32$ *feet* = *the distance below the point of projection.*

Or, $10^2 \times 16.083 = 1608.3$ *feet, the space fallen through under the effect of gravity,* and $96.5 \times 10 = 965$ *feet, the space if gravity did not act.* Hence $1608.3 - 965 = 643.3$ *feet.*

Ex. 3.—If a shot discharged from a gun return to the earth in 12 seconds, how high did it ascend?

The shot is half the time in ascending.

$6^2 \times 16.083 = 579$ *feet* = *product of the square of the time and* 16.083.

If projected Downward.

Rule.—Proceed as before, and the sum of the products will give the space in feet.

$$\text{Or, } t^2 \times 16.083 + \overline{v \times t} = s.$$

Example.—If a body be projected downward with a velocity of 96.5 feet per second, through what space must it descend to acquire a velocity of 193 feet per second?

$96.5 \div 32.166 = 3$ *seconds, the time to acquire this velocity.*
$193 \div 32.166 = 6$ *seconds, the time to acquire this velocity.*

Hence $6 - 3 = 3$ *seconds, the time of the body falling.*

Then $96.5 \times 3 = 289.5 =$ *product of velocity of projection and the time.*

$32.166 \times \frac{3^2}{2} = 144.75 =$ *product of* 32.166, *and half the square of the time.*

Therefore $289.5 - 144.75 = 144.75$ *feet.*

Promiscuous Examples.

1. A ball is 1 minute in falling, how far will it fall in the last second?

Space fallen through = square of the time, and 1 *minute* = 60 *seconds*.

$60^2 \times 16.083 = 57898$ *feet* for 60 *seconds*,
$59^2 \times 16.083 = 55984$ " " 59 "
1914 — 1 "

2. Compute the time of generating a velocity of 193 feet per second, and the whole space descended.

$193 \div 32.166 = 6$ *seconds*; $6^2 \times 16.083 = 579$ *feet*.

3. If a ball fall through 2316 feet in 12 seconds, with what velocity will it strike?

$2316 \div 12 = 193 \times 2 = 386$ *feet*.

Motion and Gravitation of Bodies on Inclined Planes.

The space which a body describes upon an inclined plane, *when descending the plane by the force of gravity*, is to the space it would freely fall in the same time as the height of the plane is to its length; and the spaces being the same, the times will be inversely in this proportion.

If a body descend in a curve, it suffers no loss of velocity.

If two bodies begin to descend from rest, and from the same point, the one upon an inclined plane, and the other falling freely, their velocities at all equal heights below the surface will be equal.

Example.—What distance will a body roll down an inclined plane 300 feet long and 25 feet high in one second by the force of gravity alone?

As 300 : 25 : : 16.083 : 1.34025 *feet*.

Hence, if the proportion of the height to the length of the above plane is reduced from 25 to 300 to 25 to 600, the time required for the body to fall 1.34025 feet would be determined as follows:

As 25 : 600 : : 1.34025 : 32.166 = 16.083×2 = *twice the time required for one half the proportion of height to length.*

$$\text{Or, as } \frac{300}{25} : \frac{600}{25} : : 1.34025 : 32.166, \textit{ as above.}$$

Notes.—The times of descending different planes of the same height are to one another as the lengths of the planes.

A body acquires the same velocity in descending any inclined plane as by falling freely through a distance equal to the height of the plane.

When bodies move down inclined planes, the accelerating force is expressed by $\frac{h}{l}$; the quotient of the height ÷ the length of the plane; or, what is equivalent thereto, the sine of the inclination of the plane, *i e* sin. *i*.

The Formulæ to determine the several elements are:

1. $s = \frac{1}{2} g t^2 \sin. i = \frac{v^2}{2 g \sin. i} = \frac{1}{2} t v$;
2. $v = g t \sin. i = \sqrt{(2 g s \sin. i)} = \frac{2 s}{t}$;
3. $t = \sqrt{\left(\frac{2 s}{g \sin. i}\right)} = \frac{2 s}{v}$;
4. $v = V \mp g t \sin. i.$
5. $s = V t \mp \frac{1}{2} g t^2 \sin. i = \frac{V^2}{2 g \sin. i}$;

s representing the space fallen through in feet, v the velocity in feet per second, t the time in seconds, g 32.166 *feet, and* V *the velocity in feet per second of the body when projected.*

Illustration.—An inclined plane having a height of one half its length, the space fallen through in any time would be *one half of that which it would fall freely.*

All of the preceding elements are required, the time assumed to be 5 seconds, and the velocity with which a body is projected upward being 96.5 feet, and downward 16.083 feet.

The velocity which a body rolling down such a plane would acquire in 5 seconds is 80.416 feet.

1. $s=\frac{32.166}{2}\times 5^2\times .5=201.04=\frac{80.416^2}{2\times 32.166\times .5}=201.04=\frac{1}{2}\,5\times 80.416=201.04.$

2. $v=32.166\times 5\times .5=80.416=\sqrt{(2\times 32.166\times 201.04\times .5)}=80.416=\frac{2\times 201.04}{5}=80.416.$

3. $t=\sqrt{\left(\frac{2\times 201.04}{32.166\times .5}\right)}=5=\frac{2\times 201.04}{80.416}=5.$

4. $v=16.083+32.166\times 5\times .5=96.499=$ *velocity acquired at the end of* 5 *seconds when projected downward with a velocity of* 16.083 *feet per second.*

5. $s=96.5\times 5-\frac{32.166}{2}\times 5^2\times .5=281.4625$ *feet* = *the space through which the body will be projected upward in* 5 *seconds*

Or, $\frac{96.5^2}{32.166\times 2\times .5}=289.5$ *feet* = *the space through which the body will be projected upward before its motion is lost.*

What time will it take for a ball to roll 38 feet down an inclined plane, the angle $t=12°\ 20'$, and what velocity will it attain at 38 feet from the starting-point?

$t=\sqrt{\frac{2\,s}{g\ \text{sin.}\ i}}=\sqrt{\frac{2\times 38}{32.166\times .2136}}=3.33$ *seconds;* $v=g\,t\ \text{sin.}\ i=32.166\times 3.33\times .2136$ $=22.88$ *feet per second.*

Retarded Motion.—Bodies projected vertically will obtain *inversely* the same velocity as when descending, as the same force acts upon them, and causes *retarded motion* when they ascend, and *accelerated* when they descend.

The Formulæ to determine the several elements are:

1. $s=V\,t-\frac{32.166\,t^2}{2}=t\,v+\frac{32.166\,t^2}{v}.$

2. $v=V-32.166\,t=\frac{s}{t}-\frac{32.166\,t}{2};$ 4. $t=\frac{V}{32.166}=\frac{2\,h}{V}=\sqrt{\frac{2\,h}{32.166}}=\frac{\sqrt{h}}{4.01}.$

3. $t'=\frac{V-v}{32.166}=\frac{V}{32.166}-\sqrt{\frac{V^2}{32.166^2}-\frac{2\,h}{32.166}};$ 5. $h=\frac{32.166\,t'^2}{2}=\frac{V\,t'}{2}=\frac{V^2}{64.083};$

t' representing any time less than t, and h height in feet to which the body will ascend.

Illustration.—An ascending ball starts with a velocity of 135 feet per second; with what velocity will it strike an object 60 feet above?

$t=\frac{135}{32.166}-\sqrt{\frac{135^2}{32.166^2}-\frac{2\times 60}{32.166}}=.41$ *seconds, until it strikes.*

Then $v=135-32.166\times .41=131.89$ *feet per second.*

If a cannon-ball is projected at an angle to the horizon, there are two forces acting on the ball at the same time—viz., the force of gunpowder, which propels the ball uniformly in a right line, and the force of gravity, which causes the ball to gravitate at an accelerated motion; these two motions (uniform and accelerated) cause the ball to move in a curved line (Parabola).

$$V=2800\sqrt{\frac{p}{W}};\ p=\frac{W\,V^2}{7840000};\ b=243781\ \text{sin.}\ i,\ \text{cos.}\ i\ \frac{p}{W};$$

V representing velocity of the ball, W *weight of the ball in pounds, s the greatest height of ball over horizontal line, t the time of flight, p pounds of powder in the charge, b the horizontal range, and x angle with the horizon.*

Illustration.—A cannon loaded to give a ball a velocity of 900 feet per second, the angle $i=45°$; what is the horizontal range, and what the time t?

$$b=\frac{900^2\times \text{sin. } 45°\times \text{cos. } 45°}{32.166}=\frac{900^2\times .5}{32.166}=1259.09\ \textit{feet.}$$

Note.—As the distance b will be greatest when the angle $i=45°$, the product of *sine* and *cosine* is greatest for that angle. Sin. 45° × cos. 45 = .5.

To Compute the Velocity of a Falling Stream of Water per Second at the End of any given Time, the perpendicular Distance being given.

EXAMPLE.—What is the distance a stream of water will descend on an inclined plane 10 feet high, and 100 feet long at the base, in 5 seconds?

$5^2 \times 16.083 = 402.08$ *feet = the space a body will freely fall in this time.*

Then, as 100 : 10 : : 402.08 : 40.21 *feet = the proportionate velocity on a plane of these dimensions to the velocity when falling freely.*

Let *s represent the space described by any falling body, t the time, and v the velocity acquired in feet.*

Then $s = 16.08\,t^2$, or $\frac{tv}{2}$, or $\frac{v^2}{64.3}$; $t = \sqrt{\frac{s}{16.08}}$, or $\frac{v}{32.16}$, or $\frac{2s}{v}$; $v = 2\sqrt{16.08\,s}$, or $32.16\,t$, or $\frac{2s}{t}$.

Or, $v = 32.2$, $t = 8.02\sqrt{s}$, $s = 16.1$, $t^2 = .0155\,v^2$, and $t = .031\,v = .249\sqrt{s}$.

ANIMAL STRENGTH.

MEN.

The mean effect of the power of a man, unaided by a machine, working to the best practicable advantage, is the raising of 70 lbs. 1 foot high in a second, for 10 hours in a day.

Two men, working at a windlass at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

The result of observation upon animal power furnishes the following as the maximum daily effect:

1. When the effect produced varied from ⅓ to .2 of that which could be produced without velocity during a brief interval.

2. When the velocity varied from ¼ to ⅙ for a man, and from .08 to .066 for a horse, of the velocity which they were capable for a brief interval, and not producing any effort.

3. When the duration of the daily work varied from ½ to ⅓ for a brief interval, during which the work could be constantly sustained without prejudice to the health of the man or the animals; the time not extending beyond 18 hours per day, however limited may be the daily task, so long as it represents a constant attendance in the shop.

By Mr. Field's experiments in 1838, the maximum power of a strong man, exerted for 2½ minutes = 18000 lbs. raised one foot in a minute.

A man of ordinary strength exerts a force of 30 lbs. for 10 hours in a day, with a velocity of 2½ feet in a second = 4500 lbs. raised one foot in a minute = .2 of the work of a horse.

A man can travel, without a load, on level ground, during 8½ hours a day, at the rate of 3.7 miles an hour, or 31¼ miles a day. He can carry 111 lbs. 11 miles in a day. Daily allowance of water for a man, 1 gallon for all purposes; and he requires from 220 to 240 cubic feet of air per hour.

A porter going short distances, and returning unloaded, can carry 135 lbs. 7 miles a day. He can transport, in a wheelbarrow, 150 lbs. 10 miles in a day.

The muscles of the human jaw exert a force of 534 lbs.

Mr. Buchanan ascertained that, in *working a pump, turning a winch, in ringing a bell, and in rowing a boat*, the effective power of a man is as the numbers 100, 167, 227, and 248.

A man *drawing a boat in a Canal* can transport 110000 lbs. for a distance of 7 miles, and produce 156 times the effect of a man weighing 154 lbs. and walking 31¼ miles in a day; he can also produce an effect upon a *tread-wheel* of 30 lbs., with a velocity of 2.3 feet in a second, for 8 hours in a day, and can *draw* or *push* on a horizontal plane 30 lbs. with a velocity of 2 feet in a second, for 8 hours in a day. He can raise by a *single pulley* 38 lbs., with a velocity of .8 of a foot per second, for 8 hours in a day, and he can pass over 12½ times the space horizontally that he can vertically.

A foot-soldier travels in 1 minute, in common time, 90 steps = 70 yards.
" " " " in quick-time, 110 " = 86 "
" " " " in double quick-time, 140 " = 110 "

He occupies in the ranks a front of 20 inches, and a depth of 13, without a knapsack: the interval between the ranks is 13 inches.

Average weight of men, 150 lbs. each.

Five men can stand in a space of 1 square yard.

Table of the Effective Power of Men for a Short Period.

Manner of Application.	Force	Manner of Application.	Force.
	Lbs.		Lbs.
Bench-vice or chisel	72	Screw-driver, one-hand	84
Brace-bit	16	Small screw-driver	14
Drawing-knife or auger	100	Thumb and fingers	14
Hand-plane	50	Thumb-vice	45
Hand-saw	36	Windlass or pincers	60

HORSES.

A Horse can travel 400 yards, at a walk, in 4½ minutes; at a trot, in 2 minutes; and at a gallop, in 1 minute. He occupies in the ranks a front of 40 inches, and a depth of 10 feet; in a stall, from 3½ to 4½ feet front; and at a picket, 3 feet by 9; and his average weight = 1000 lbs.

A Horse, carrying a soldier and his equipments (225 lbs.), can travel 25 miles in a day (8 hours).

A Draught-horse can draw 1600 lbs. 23 miles a day, weight of carriage included.

The ordinary work of a horse may be stated at 22500 lbs., raised 1 foot in a minute, for 8 hours a day.

In a horse-mill, a horse moves at the rate of 3 feet in a second. The diameter of the track should not be less than 25 feet.

A horse-power in machinery is estimated at 33000 lbs., raised 1 foot in a minute: but as a horse can exert that force but 6 hours a day, one machinery horse-power is equivalent to that of 4⅖ horses.

The expense of conveying goods at 3 miles per hour per horse teams being 1, the expense at 4⅓ miles will be 1 33, and so on, the expense being doubled when the speed is 5⅛ miles per hour.

The strength of a horse is equivalent to that of 5 men.

The daily allowance of water for a horse should be 4 gallons.

Table of the Amount of Labor a Horse of average Strength is capable of performing, at different Velocities, on Canals, Rail-roads, and Turnpikes.

Force of Traction estimated at 83.3 lbs.

Velocity per Hour.	Duration of Work.	Useful Effect for One Day, drawn 1 Mile. On a Canal.	On a Rail-road.	On a Turnpike.	Velocity per Hour.	Duration of Work.	Useful Effect for One Day, drawn 1 Mile. On a Canal.	On a Rail-road.	On a Turnpike.
Miles.	Hours.	Tons.	Tons.	Tons.	Miles.	Hours.	Tons	Tons.	Tons.
2½	11.5	520	115	14	6	2.	30	48	6
3	8.	243	92	12	7	1.5	19	41	5.1
4	4.5	102	72	9	8	1.125	12.8	36	4.5
5	2.9	52	57	7.2	10	.75	6.6	28.8	3.6

The actual labor performed by horses is greater, but they are injured by it.

A Horse in a mill can produce an effect of 106 lbs., at a velocity of 3 feet in a second, for 8 hours in a day. A Mule can produce, under a like velocity and time, an effect of 71 lbs.; and an Ass, 37 lbs.

An Ox, walking at a velocity of 2 feet in a second (1.34 miles per hour), will draw 154 lbs. for 8 hours in a day.

A Horse requires a space 7 feet by 2½ for transportation in a vessel; and a Beeve requires 6¼ feet by 26 ins., without manger, and 2 feet additional length with one. 3 Beeves or 15 Sheep require the food of 2 Horses.

Table showing the Amount of Labor produced by Animal Power under different Circumstances.

MANNER OF APPLICATION.	Power	Velocity per Second	Weight raised. Foot per Minute.	Horses' Power for the Period given.
10 HOURS PER DAY.	Lbs.	Feet.	Lbs.	No.
Man, throwing earth with a shovel, a height of 5 feet	6	1⅓	480	8.7
Man, wheeling a loaded barrow up an inclined plane, height one-twelfth of length	132	⅝	4950	90.
Man, raising and pitching earth in a shovel to a horizontal distance of 13 feet	6	2¼	810	14.7
Man, pushing and drawing alternately in a vertical direction	13	2½	1950	35.5
Man, transporting weight upon a barrow, and returning unloaded	132	1	7920	144
Man, walking upon a level	143	5	42900	780
Horse, drawing a 4-wheeled carriage at a walk	154	3	27720	504
Horse, with load upon his back, at a walk	264	3¾	59400	1080
Horse, transporting a loaded wagon, and returning unloaded at a walk	1540	2	184800	3360
Horse, drawing a loaded wagon at a walk	1540	3¾	346500	6300
8 HOURS PER DAY				
Man, ascending a slight elevation, unloaded	143	½	4290	62
Man, walking, and pushing or drawing in a horizontal direction	26	2	3120	45.2
Man, turning a crank	18	2½	2790	39
Man, upon a tread-mill	140	½	4200	60.9
Man, rowing	26	5	7800	113
Horse, upon a revolving platform, at a walk	100	3	18000	260.8
Ox, upon a revolving platform, at a walk	132	2	15840	229.5
Mule, upon a revolving platform, at a walk	66	3	11880	172.2
Ass, upon a revolving platform, at a walk	32	2¾	5280	76.5
7 HOURS PER DAY				
Man, walking with a load upon his back	88	2½	13200	167.9
6 HOURS PER DAY.				
Man, transporting a weight upon his back, and returning unloaded	140	1¾	14700	160.5
Man, transporting a weight upon his back up a slight elevation, and returning unloaded	140	.2	1680	19
Man, raising a weight by the hands	44	½	1320	14.4
4½ HOURS PER DAY.				
Horse, upon a revolving platform at a trot	66	6¾	26730	218.7
Horse, drawing an unloaded 4-wheeled carriage at a trot	97	7¼	43195	353.5
Horse, drawing a loaded 4-wheeled carriage at a trot	770	7¼	334950	2741

How many men are required upon a tread-mill, 20 feet in diameter, in order to raise a weight of 900 lbs., the crank being 9 inches in length?

The weight of the wheel and its load is estimated at 5000 lbs., and the friction at .015 = 75 *lbs*. The labor of a man upon such a mill is estimated at 25 lbs. Length of crank = .75 *feet*.

Then $900 \times .75 + 5000 \times .015 = 750$ *lbs., the resistance of the wheel;* and $\frac{750}{20 \div 2} =$ 75 *lbs., the power required at the circumference of the wheel.*

Therefore, $75 \div 25 = 3$ *men.*

The *draught* of Man and Animals by traces is as follows:
Man...... 150 lbs.; Horse...... 600 lbs.; Mule...... 500 lbs.; Ass...... 360 lbs.

A man rowing a boat 1 mile in 7 minutes performs the labor, while rowing, of 6 fully-worked laborers at ordinary occupations of 10 hours.

Table showing the Effects of a Traction of 100 Lbs. at different Velocities on Canals.

Velocity per Hour.	Velocity per Second.	Mass moved.	Useful Effect.	Velocity pr. Hour.	Velocity per Second.	Mass moved.	Useful Effect.
Miles.	Feet.	Lbs.	Lbs.	Miles.	Feet.	Lbs.	Lbs.
2½	3.66	55500	39408	6	8.8	9635	6840
3	4.4	38542	27361	7	10.26	7080	5026
3½	5.13	28316	20100	8	11.73	5420	3848
4	5.86	21680	15390	9	13.2	4282	3040
5	7.33	13875	9850	10	14.66	3468	2462

The load carried, added to the weight of the vessel which contains it, forms the total mass moved, and the useful effect is the load.

The force of traction on a railroad or turnpike is constant, but the mechanical power necessary to move the carriage increases as the velocity; on a canal the force of traction varies as the square of the velocity.

Labor upon Embankments.—[Ellwood Morris.]

Single Horse and Cart.—A horse with a loaded dirt-cart, employed in excavation and embankment, will make 100 lineal feet of trip, or 200 feet in distance per minute, while moving. The time lost in loading, dumping, awaiting, etc. = 4 minutes per load.

A medium laborer will load with a cart in 10 hours, of the following Earths, measured in the bank:

Gravelly Earth, 10; *Loam*, 12; and *Sandy Earth*, 14 cubic yards.

Earth from a natural excavation occupies $\frac{1}{9}$ more space than when transported to an embankment.

Carts are loaded as follows: *Descending Hauling*, $\frac{1}{3}$ of a cubic yard in bank; *Level Hauling*, $\frac{2}{7}$ of a cubic yard in bank; *Ascending Hauling*, $\frac{1}{4}$ of a cubic yard in bank.

Loosening, etc.—In *Loam*, a three-horse plow will loosen from 250 to 800 cubic yards per day of 10 hours.

The cost of loosening earth to be loaded will be from 1 to 8 cents per cubic yard when wages are 105 cents per day.

The cost of Trimming and Bossing is about 2 cents per cubic yard.

Scooping.—A Scoop load will measure $\frac{1}{10}$ of a cubic yard, measured in excavation.

The time lost in loading, unloading, and turning, per load, is 1⅛ minute.

The time lost for every 70 feet of distance, from excavation to bank, and returning, is 1 minute.

In *Double Scooping*, the time lost in loading, turning, etc., will be 1 minute; and in *Single Scooping* it will be 1⅓ minutes.

Volumes of Excavation and Embankment.

The volume of earth in *embankment* is less than in excavation, as the *compression* of earth in an embankment is in excess of the *expansion* of its volume in a natural state, the proportion being as follows:

Sand............ $\frac{1}{7}$; Clay............ $\frac{1}{9}$; Gravel........... $\frac{1}{4}$.

The volume of rock in bank exceeds that in excavation in the proportion of 3 to 2.

Stone.

Hauling Stone.—A cart drawn by horses over an ordinary road will travel 1.1 miles per hour of trip.

A four-horse team will haul from 25 to 36 cubic feet of limestone at each load.

The time expended in loading, unloading, etc., including delays, averages 35 minutes per trip. The cost of loading and unloading a cart, using a horse-crane at the quarry, and unloading by hand, when labor is $1 25 per day, and a horse 75 cents, is 25 cents per perch=24.75 cubic feet.

The work done by an animal is greatest when the velocity with which he moves is ⅙ of the greatest with which he can move when not impeded, and the force then exerted .45 of the utmost force the animal can exert at a dead pull.

PERFORMANCES OF MEN, HORSES, ETC.

The following notes are designed to furnish an authentic summary of the fastest or most successful *recorded* performances in each of the feats, matches, or races, etc., etc., given.

Note.—Parties desirous of maintaining such a record by the contribution of results are requested to address them to the Author.

WALKING.

Man.

1865, — *Hill*, Brooklyn, Long Island, N. Y., ½ mile, backward, in 7 *min.*

1874, *Wm. Perkins*, London, Eng., ½ mile, in 2 *min.* 56 *sec.*, ¾ in 4 *min.* 40 *sec.*, 1 mile, in 6 *min.* 23 *sec.*, 2 in 13 *min.* 20 *sec.*, and 3 in 20 *min.* 47 *sec.*; and 1875, 8 in 59 *min.* 5 *sec.*

1870, *J. Stockwell*, London, Eng., 4 miles in 29 *min.* 6 *sec.*, 5 in 36 *min.* 42 *sec.*, and 6 in 44 *min.* 31 *sec.*

1856, *William Spooner*, London, Eng., 7 miles, in 52 *min.*

1868, *George Topley*, New York, N. Y., 7½ miles, in 56 *min.* 10 *sec.*; London, Eng., 9 in 1 *hour* 12 *min.* 48 *sec.*; and New York, N.Y., 42 in 7 *hours* 3 *min.* 22 *sec.*

1868, *N. Young*, Mansfield, Ohio, 7¾ miles, in 56 *min.* 58 *sec.*

1865, *J. Brighton*, Liverpool, Eng., 11½ miles, in 1 *hour* 31 *min.*

1869, *Geo. Davison*, Hackney Wick, Eng., 7 miles 1 380 yards in 1 *hour*, 10 miles in 1 *hour* 17 *min.* 33 *sec.*, 12 in 1 *hour* 32 *min.* 26 *sec.*, 15 in 1 *hour* 57 *min.* 41 *sec.*, 15 and 580 yards in 2 *hours*, and 21 in 2 *hours* 53 *min.* 34 *sec.*

1849, *C. Westhall*, London, Eng., 25 miles, in 3 *hours* 58 *min.* 45 *sec.*; 1858, turnpike road, Newmarket, Eng., 21 miles in 2 *hours* 59 *min.* 1 *sec.*

1875, *A. Clark*, Fulham, Eng., 50 miles, in 9 *hours* 24 *min.* 16½ *sec.*, including stops.

1868, *M. Wesley*, Troy, N. Y., 50 miles, country road, in 9 *hours* 22 *min.*, including 36 min. in rests.

1801, *Capt.* — *Barclay*, Eng., country road, 110 miles, in training, in 19 *hours*, exclusive of rests, and 90 in 20 *hours* 22 *min.* 4 *sec.*, including rests; 1803, ¼ mile in 56 *sec.*, and Charing Cross to Newmarket, 64 in 10 *hours*, including rests; 1806, Ury to Crathynaird and back, 28 in 4 *hours*, and 100 in 19 *hours*, including 1 *hour* 30 *min.* in rests; 1809, 1 000 in 1 000 consecutive *hours*, walking a mile only at the commencement of each hour.

1830, — *Newsam*, Philadelphia, Penn., 1 000 miles, in 18 *days*.

1818, *Jos. Eaton*, Stowmarket, Eng., 4 032 quarter miles, in 4 032 consecutive *quarter hours*; 1846 (70 years of age), Canada, 1 000 miles, in 1 000 consecutive *hours*; and 1847, Boston, 1 000 quarter miles, in 1 000 consecutive *quarter hours*.

1842, — *Ellsworth*. and 1857, — *Lambeth*, of Boston, Mass.; 1869, *J. De Witt*, Chicago, Ill.; and 1874, — *Richards* [young woman], Stapleton, Eng., each 1 000 miles, in 1 000 consecutive *hours*.

1874, *E. P. Weston*, New York, N. Y., 100 miles, in 20 *hours* 37 *min.* 45 *sec.*, 115 in 23 *hours* 50 *min.* 37 *sec.*, and 430 in 6 *days*.

JUMPING, LEAPING, ETC.

Man.

1774, *Anthony Thorpe*, Artillery Ground, London, Eng., 1 mile, in a sack, in 11 *min.* 30 *sec.*

1829, *Samuel Patch*, over Genesee Falls, N. Y., 125 *feet* perpendicular.

1848, *P. M'Neely*, Petersburg, Ky., 10 jumps, standing, 110 *feet* 4 *ins.*

1854, *J. Howard*, Chester, Eng., 1 jump, board raised 4 inches in front, running start, two 5 lb. weights, 29 *feet* 7 *ins.*

1854, — *Nelson*, New York, N.Y., 10 hops, running start, 112 *feet.*

1856, *Thos. King*, San Francisco, Cal., 1 jump, spring board, running start, over 9 horses, 31 *feet* 7¼ *ins.*

1868, *Geo. M. Kelley*, Corinth, Miss., leaped over 17 horses standing side by side.

1869, *J. P. Naylor*, Manchester, Eng., 6 jumps backward, standing, 54 *feet* 5 *ins.*

1869, *J. Parker*, Leeds, Eng., hopped on one leg 100 yards in 14 *sec.*

1869, *Geo. Harris*, Staten Island, 3 jumps, standing, 38 *feet* 7 *ins.*

1869, *W. E. Evans*, Brooklyn, L. I., standing leap, 4 *feet* 11½ *ins.*

1870, *Edward Searles*, Utica, N. Y., dumb bells, standing jump, 13 *feet* 5¾ *ins.* C. H. Loomis covered 13 feet 7⅝ ins., but it was declared foul.

1870, *P. Frazer*, New York, N.Y., hop, skip, and jump, 40 *feet* 10 *ins.*

1870, *Katie Murphy* [young woman], Dorchester, Mass., 1 jump, standing, 11 *feet* 2½ *ins.*

1870, *D. Dinnie*, New York, N. Y., running leap, 5 *feet* 4 *ins.*

1873, *T. Davis*, Dublin, Ireland, running leap, 5 *feet* 10½ *ins.*

1875, *J. Greaves*, Hazlehurst, Eng., standing jump, 13 *feet* 7 *ins.*

RUNNING.

Man.

18—, *Wm. Bingham*, Toronto, Can., 75 yards, in 7 *sec.*

1869, *P. Perry*, Trenton, N. J., 75 yards, in 7¼ *sec.*

1844, *Geo. Seward*, Manchester, Eng., 100 yards, in 9¼ *sec.*; 150 in 14½ *sec.*, and 200, running start, in 19½ *sec.*

1868, *John Thomas*, Philadelphia, Penn., 100 yards, in 9¾ *sec.*, and 200 in 20 *sec.*; 1868, *W. H. Young*, Paterson, N. J., *H. E. Buermayer* (an amateur), and 1867, *E. D. Davis*, Chicago, Ill., 100 yards, in 9¾ *sec.*

1868, *J. W. Cozad*, Long Island, N. Y., 125 yards, in 12¾ *sec.*

1851, *Charles Westhall*, Manchester, Eng., 150 yards, in 15 *sec.*, and 200 in 19½ *sec.*

1869, *Geo. Forbes*, Providence, R. I., 150 yards, in 15 *sec.*

1863, *Jas. Nuttall*, Manchester, Eng., 300 yards, in 31½ *sec.*; and 1864, 600 in 1 *min.* 13 *sec.*

1873, *R. Buttery*, Newcastle, Eng., ¼ mile, in 48¼ *sec.*

1871, *Frank Hewitt*, Australia, ½ mile, in 1 *min.* 53½ *sec.*

1861, — *White*, Long Island, N.Y., 1 mile, drawing a sulky, in 6 *min.* 24½ *sec.*

1863, *Wm. Lang*, Newmarket, Eng., 1 mile, in 4 *min.* 2 *sec.*, descending ground, Manchester, 2 in 9 *min.* 11½ *sec.*; and 1865, 1 mile, in 4 *min.* 17¼ *sec.*, a dead heat with Richards; 8 in 40 *min.* 57 *sec.*, 9 in 46 *min.* 15 *sec.*, 10 in 51 *min.* 26 *sec.*, 11 in 56 *min.* 52 *sec.*, and 12 in 1 *hour* 2 *min.* 2½ *sec.*

1867, *James Fleet*, Manchester, Eng., 1½ miles, in 6 *min.* 50 *sec.*

1764, A man, Barnet Course, Eng., 2 miles, in a sack, in 56 *min.*

1710, *Levi Whitehead*, Branham Moor, Eng., 4 miles, in 19 *min.*

1863, *J. White*, Hackney Wick, Eng., 3 miles, in 14 *min.* 36 *sec.*, 4 in 19 *min.* 35 *sec.*; Manchester, 5 in 24 *min.* 40 *sec.*; and Hackney Wick, 7 in 34 *min.* 45 *sec.*

1852, *William Howitt*, alias *Jackson*, "American Deer," London, Eng., 5 miles, in 24 *min.* 57 *sec.*; 6 in 29 *min.* 30 *sec.*; 10 in 51 *min.* 34 *sec.*, walking in last 200 yards, computed time, if run, 51 *min.* 20 *sec.*; and 15 in 1 *hour* 22 *min.*

1863, *L. Bennett*, "Deerfoot," Hackney Wick, Eng., 10 miles, in 51 *min.* 29 *sec.*, 11 miles 790 yards, in 59 *min.* 44 *sec.*=11½ miles per hour; and Dublin, Ireland, 12 in 1 *hour* 5 *min.* 6 *sec.*

1845, *T. Maxfield*, Slough, Eng., 20 miles, in 1 *hour* 58 *min.* 30 *sec.*

1860, *H. Howard*, Bridgewater, Eng., 41 miles, public road, in 5 *hours* 36 *min.*, inclusive of 1 *hour* 18 *min.* in rests.

1749, *J. Manser*, Peterborough to Lincoln, Eng., 50 miles, in 7 *hours* 30 *min.*

17—, *A Courier*, East Indies, 102 miles, in 24 *hours.*

LIFTING.

1825, *Thomas Gardner*, of New Brunswick, N. S., a barrel of pork, 320 lbs., under each arm; also transported across a pier an anchor, 1 200 lbs.

1866, *A. O. Butts*, Auburn, Mass., 2 737¾ lbs., in harness, and 900 lbs. by hands alone.

18—, *Wm. B. Curtis*, New York, N. Y., 3 300 lbs., in harness, and 1230 lbs. by hands alone.

1874, *R. A. Pennell*, New York, N. Y., raised dumb-bell, 201 lbs., by one hand.

THROWING WEIGHTS.

1870, *D. Dinnie*, New York, N. Y., *light stone*, 18 lbs., 43 *feet; light hammer*, 16 lbs., 112 *feet; heavy stone*, 24 lbs., 34 *feet* 6 *ins.; heavy hammer*, 24 lbs., 83 *feet* 8 *ins.;* and a 56 lb. weight, 25 *feet;* at Dundee, Scotland, *light hammer*, 16 lbs., 128 *feet* 11 *ins.*

1870, *Jos. Stewart*, Virginia, Nev. Ter., a piece of lead, 3 oz., 100 *yards* 18 *ins.*

FLY ROD CASTING.

1860, *Seth Green*, Rochester, N. Y., rod 12 feet 6 inches in length, standing 1½ feet above the water, wind calm, 100 *feet.*

SWIMMING.

1835, *S. Bruck*, ————, 15 miles, in a rough sea, in 7 *hours* 30 min.

1846, *A Native*, off Sandwich Islands, 7 miles at sea, with a live pig under one arm.

1869, *T. Morris*, Serpentine, London, Eng., 1000 yards short, in 16 *min.* 43⅗ *sec.*

1870, *Chas. Whyte*, London Bridge to Clock at Greenwich Hospital, Eng., favorable current, first 3 miles, in 35 *min.* 28 *sec.*, and 5 miles, in 1 *hour* 4 *min.* 23 *sec.*

1870, *Pauline Rohn* [young woman], Milwaukee, Wis., 650 feet, still water, in 2 *min.* 43 *sec.*

1872, *J. B. Johnson*, English Channel, 7 miles, in 1 *hour* 5 *min.*;* 1874, New York Bay, N.Y., 3 miles (defined by estimate), smooth water. in 1 *hour* 10 *min.* 30 *sec.*

1874, *H. Parker*, London, Eng., 500 yards, in 7 *min.* 27.4 *sec.*

1874, *E. T. Jones*, ——, Eng., 1 mile, still water, in 30 *min.* 3 *sec.*

1875, *Capt. M. Webb*, Dover to Calais, 23 miles, crossing two full and two half tides = 50 miles, in 21 *hours* 45 *min.*

1875, *Agnes Beckwith* [young woman], London Bridge to Greenwich Hospital, Eng., favorable current, 5 + miles, in 1 *hour* 9 *min.*

SKATING.

18—, *Wm. Clarke*, Madison, Wis., 1 mile, in 1 *min.* 56 *sec.*

1867, *Chas. Ochford*, Detroit, Mich., for 60 consecutive hours, stopping 12 minutes in each 12 hours.

1867, *T. Prentiss*, Quincy to Lagrange, Ill., 15 miles, in 50 *min.*

1868, *John Conyers*, Lake Simcoe, Can., 8 miles, in 18 *min.* 40½ *sec.*

1868, *E. St. Clair*, Cincinnati, Ohio, 100 miles, in 11 *hours* 46 *min.*, and for 24 hours without rest.

1868, *Annie C. Jagerisky* [young woman], 30 hours, with 30 minutes rest.

1870, — *Hills*, Chetney Wade, Eng., 1½ miles, in 3 *min.* 6 *sec.*

NOTE.—The *Sporting Magazine*, London, vol. ix., p. 135, reports a man in 1767 to have skated a mile upon the Serpentine, Hyde Park, London, in 57 seconds.

SNOW SHOES.

1871, *J. F. Scholes*, Montreal, Can., ½ mile, in 2 *min.* 39½ *sec.*, and 1 in 5 *min.* 39¼ *sec.*

1871, *J. D. Armstrong*, Montreal, Can., ¼ mile, in 1 *min.* 5 *sec.*

1869, *J. James*, Montreal, Can., ¼ mile, 2d heat, in 1 *min.* 15 *sec.*

1871, *Kerar-onwe* (Indian), Montreal, Can., 2 miles in 11 *min.* 30 *sec.*, 3 in 17 *min.* 52 *sec.*, and 4 in 24 *min.* 4 *sec.*

* In an attempt to cross the British Channel, in which he failed.

RUNNING.

Horse.

ONE MILE.

1850, "Black Doctor," Doncaster, Eng., 2 years, 87 lbs., in 1 *min.* 40 *sec.*

1854, "Lecomte," New Orleans, La., 4 years, 103 lbs., 3d mile of a 2d 4-mile heat, in 1 *min.* 46 *sec.*

1855, "Henry Perritt," New Orleans, La., 4 years, 83 lbs., 1st mile of 2d heat of 2 miles, in 1 *min.* 42½ *sec.*

1868, "Climax," Jerome Park, N.Y., 9 years, 148¼ lbs., in 1 *min.* 48¼ *sec.*

1869, "Lobelia," Fashion Course, L. I., 6 years, 143 lbs., 4 hurdles, in 1 *min.* 51¾ *sec.*

1870, "Judge Curtis," Saratoga, N. Y., 5 years, 114 lbs., in 1 *min.* 43¼ *sec.*

1873, "Thad. Stevens," Sacramento, Cal., 7 years, 115 lbs., 3d heat, in 1 *min.* 43½ *sec.*

1874, "Springbok," Utica, N. Y., 4 years, 108 lbs., 2d heat, in 1 *min.* 42¾ *sec.*

1874, "Gray Planet," Saratoga, N.Y., 5 years, 110 lbs., in 1 *min.* 42½ *sec.;* 1st quarter in 25 *sec.*, 2d in 50 *sec.*, and the third in 1 *min.* 16 *sec.*

1875, "Searcher," now "Leander," Lexington, Ky., 2 years, 90 lbs., in 1 *min.* 41½ *sec.*

1875, "Kadi," Hartford, Conn., 6 years, (catch) 82 lbs., 2d heat, in 1 *min.* 41¼ *sec.*

TWO MILES.

18—, "Child of the Islands" (Arabian), India, in 3 *min.* 48 *sec.*

1834, "Inheritor," Liverpool, Eng., 3 years, 87 lbs., in 3 *min.* 25 *sec.**

1847, "Inheritress," Liverpool, Eng., aged, 115 lbs., in 3 *min.* 27 *sec.**

1850, "Hegira," New Orleans, La., 4 years, feather (71½ lbs.), in 3 *min.* 34¼ *sec.*

1867, "Ruthless," Saratoga, N.Y., 3 years, 112 lbs., in 3 *min.* 37 *sec.*

1867, "Blackbird," Saratoga, N. Y., aged, 161 lbs., 8 hurdles, in 3 *min.* 57¾ *sec.*

1868, "Jonesboro," New Orleans, La., 4 years, 132 lbs., 8 hurdles, in 3 *min.* 51½ *sec.*

1871, "Harry Bassett," Saratoga, N. Y., 3 years, 110 lbs., in 3 *min.* 35¾ *sec.*

1872, "Aureola," Lexington, Ky., 4 years, 101 lbs., 2d heat, in 3 *min.* 35½ *sec.*

1874, "Tom Bowling," Lexington, Ky., 4 years, 104 lbs., in 3 *min.* 27¾ *sec.*

THREE MILES.

1854, "Virago," Warwick, Eng., 3 years, 101 lbs., in 5 *min.* 29 *sec.*

1855, "Brown Dick," New Orleans, La., 3 years, 86 lbs., 2d heat, in 5 *min.* 28 *sec.*

1855, "Rataplan," Warwick, Eng., 5 years, 127 lbs., in 5 *min.* 27 *sec.*

1861, "Mollie Jackson," Louisville, Ky., 4 yrs., 101 lbs., 3d heat, in 5 *min.* 28¾ *sec.*

1865, "Fleetwing," Saratoga, N. Y., 5 years, 114 lbs., in 5 *min.* 31¾ *sec.*

1865, "Norfolk," Sacramento, Cal., 4 years, 100 lbs., in 5 *min.* 27½ *sec.*

FOUR MILES.

1710, "Bay Bolton," York, Eng., 5 years, 168 lbs., in 7 *min.* 43 *sec.*

1752, "Skewball," Kildare, Ireland, 11 years, 119 lbs., in 7 *min.* 51 *sec.*

1760, "Bay Malton," York, Eng., 6 years, 119 lbs., in 7 *min.* 43½ *sec.*

1767, "Selim," Philadelphia, Penn., 8 years, 140 lbs., in 8 *min.* 2 *sec.*

1769, "Eclipse," Winchester, Eng., 5 years, 168 lbs., in 8 *min.*

1823, "Sir Henry," Long Island, N. Y., 4 years, 108 lbs., in 7 *min.* 37½ *sec.*

1828, "Ariel," Newmarket, Va., 6 years, 118 lbs., 4th heat, in 8 *min.* 4 *sec.*

1832, "Black Maria," Long Island, N.Y., 6 years, 118 lbs., 5th heat, in 8 *min.* 47 *sec.*, and the 5 heats, in 41 *min.* 40 *sec.*

1833, "Lady Elizabeth," Doncaster, Eng., 5 years, 135 lbs., in 7 *min.* 46 *sec.*

1842, "Fashion," Long Island, N. Y., 5 years, 111 lbs., in 7 *min.* 32½ *sec.*

1855, "Lexington," New Orleans, La., 5 years, 103¾ lbs., in 7 *min.* 23¾ *sec.;* and with a running start, in 7 *min.* 19¾ *sec.*

1863, "Idlewild," Long Island, N. Y., 6 years, 117 lbs., in 7 *min.* 26¼ *sec.*, and last 3 miles in 5 *min.* 27¼ *sec.;* track heavy.

1871, "Abd-el-Koree," Jerome Park, N.Y., 3 years, 95 lbs., in 7 *min.* 33 *sec.*

1873, "Thad. Stevens," San Francisco, Cal., 7 years, 115 lbs., 2d heat, in 7 *min.* 30 *sec.*

1874, "Fellowcraft," Saratoga, N.Y., 4 years, 108 lbs., in 7 *min.* 19½ *sec.*

* When it is considered that in England no official record of time is taken, the accuracy of the times here given as the performances of both Inheritor and Inheritress is much doubted.

Various Distances and Performances.

English.

1701, *Mr. Sinclair*, on the Swift at Carlisle, a gelding, 1000 miles in 1000 consecutive *hours.*

1721, "Childers" (*Flying*), Newmarket, R. C., 6 years, 128 lbs., 3 miles, 6* furlongs, and 93 yards (3.8029 miles) in 6 min. 43* sec.=3 *miles* in 5 *min.* 18 *sec.*

NOTE.—It is *related* he ran 4 miles, 1 furlong, 138 yards (4.204 miles) in 7 *min.* 30 *sec.*=4 *miles* in 7 *min.* 8.2 *sec.* There is a well-founded doubt about the accuracy of this performance, arising from its unequaled time, the relation being more of a traditionary character than a record, and the circumstance that at that date timing watches were not in use; added to which, Bay Malton's time of 7 *min.* 43½ *sec.*, in 1766, is recorded as 7½ *sec.* less than was ever before accomplished.

1745, *Cooper Thornhill*, between Stilton and London 3 times, 213 miles, by 14 horses, in 12 *hours* 7 *min.*, including 33 *min.* 8 *sec.* in rests.

1751, *Samuel Bendell* (aged 76), 1 horse, 1000 miles in 1000 consecutive *hours.*

1752, *Spedding's Mare*, 100 miles, in 12 *hours* 30 *min.*, for two consecutive days.

1754, A Galloway mare of Daniel Corker's, Newmarket Heath, 300 miles, by one rider, 67 lbs., in 64 *hours* 20 *min.*

1759, *J. Shafto*, Newmarket Heath, 50 miles, by 10 horses, in 1 *hour* 49 *min.* 17 *sec.*

1761, *John Woodcock*, Newmarket Heath, 100 miles per day, by one horse each day, for 29 consecutive days, 14 horses; one day 160 miles, a horse breaking down at the 60th mile.

1786, "Quibbler," 6 years, Newmarket, R. C., 77 lbs., 23 miles in 57 *min.* 10 *sec.*

1791, *Mr. Wilde*, Curragh, Ireland, 127 miles, by 10 horses, in 6 *hours* 21 *min.*

1793, — *Delme, Jr.*, Colnbrook to London, 17 miles, in less than 44 *min.*

1801, *Capt. Newland*, Longdown Hill, 100 miles, by hack horses, in 5 *hours* 5 *min.*, and 140 miles, in 7 *hours* 34 *min.*

1814, *An Officer of 14th Dragoons*, Blackwater, 12 miles, 1 horse, in 25 *min.* 11 *sec.*

1823, "Hampden," Newmarket, R. C., 4 years, 144 lbs., 3 miles, 4 furlongs, 187 yards (3.606 miles), in 7 *min.* 4 *sec.*=4 *miles* in 7 *min.* 50 *sec.*

1831, *Geo. Osbaldeston*, Newmarket, 156 lbs., 60 miles, by 11 horses in 2 *hours* 33 *min.*, 100 by 16 horses in 4 *hours* 19 *min.* 40 *sec.*, and 200 by 28 horses in 8 *hours* 39 *min.*, including 1 *hour* 2 *min.* 56 *sec.* in rests; 1 horse, "Tranby," 16 miles (4 times 4 miles) in 33 *min.* 15 *sec.*

1846, "Sir Tatton Sykes," Doncaster, "St. Leger," 3 years, 122 lbs., 1 mile, 6 furlongs, 132 yards (1.775 miles), in 3 *min.* 16 *sec.*=1¾ *miles* in 3 *min.* 13 *sec.*

1847, "The Widow," Newmarket, "Cambridgeshire," 8 years, 98 lbs., 1 mile, 240 yards (1.136 miles), in 1 *min.* 58 *sec.*=1 *mile* in 1 *min.* 43.8+*sec.*

1854, "Stockwell," Newmarket, B. C., 5 years, 140 lbs., 4 miles, 1 furlong, 138 yards (4.204 miles) in 7 *min.* 52 *sec.*=4 *miles* in 7 *min.* 29 *sec.*

1854, "West Australian," Ascot Heath, 4 years, 119 lbs., and "Kingston," 5 years, 126 lbs., 2½ miles in 4 *min.* 27 *sec.*=2 *miles* in 3 *min.* 33.6 *sec.*

1855, "Mr. Sykes," Newmarket, "Cesarewitch," 5 years, 92 lbs., 2 miles, 2 furlongs, and 28 yards (2.266 miles), in 3 *min.* 55 *sec.*=2 *miles* in 3 *min.* 27+*sec.*

1857, "Saunterer," Newmarket, 3 years, 119 lbs., 1 mile, 2 furlongs, and 73 yards (1.292 miles) in 2 *min.* 10 *sec.*=1 *mile* in 1 *min.* 40.6 *sec.*

1861, "Diophantus," Newmarket, R. M., 3 years, 119 lbs., 1 mile, 17 yards (1.0097 miles) in 1 *min.* 43 *sec.*=1 *mile* in 1 *min.* 42+*sec.*

1864, "Blair Athol," Epsom, "Derby," 3 years, 122 lbs., and 1861, "Kettledrum," 3 years, 119 lbs., 1½ miles, in 2 *min.* 43 *sec.*=1 *mile* in 1 *min.* 48.6+*sec.*

1866, "Sultan," Goodwood, 4 years, 115 lbs., ¾ mile in 1 *min.* 15 *sec.*

American.

1846, *J. F. Tyler*, Alabama, 14 years, 70 lbs., 188 miles (2 miles in a row-boat), by 13 horses, country road, in 12 *hours* 30 *min.*

1858, *J. Powers*, San Francisco, Cal., 150 miles, 25 horses, in 6 *hours* 43 *min.* 34 *sec.*, including rests.

1868, *N. H. Mowry*, San Francisco, Cal., race track, 160 lbs., 300 miles, by 30 horses (Mexican), in 14 *hours* 9 *min.*, including 40 minutes for rests; the first 200 in 8 *hours* 2 *min.* 48 *sec.*, and the fastest mile in 2 *min.* 8 *sec.*

* Whyte gives the distance in furlongs as 4, the *Sporting Magazine* in yards as 103; Johnson the time as 48 seconds, and the *Sporting Magazine* as 40.

1869, *Nell Coher*, San Pedro, Texas, 61 miles, in 2 *hours* 55 *min.* 15 *sec.*, including rests.
1870, *John Faylor*, Carson City, Nevada, 50 miles, 18 horses, in 1 *hour* 58 *min.* 33 *sec.*; and Omaha, Neb., 56 miles, in 2 *hours* 26 *min.*, including rests.
1872, *Chan Reticker*, Greenland Course, Ky., 50 miles, 10 horses, in 2 *hours* 5 *min.* 20 *sec.*, including 12 *min.* 48½ *sec.* in rests.
1874, "Olitipa," Saratoga, N. Y., 2 years, 97 lbs., ½ mile, in 47¾ *sec.*
1870, "Enchantress," Reading, Penn., aged, 100 lbs., ½ mile, 3d heat, in 51 *sec.*
1875, "Madge," Saratoga, N. Y., 4 years, 105 lbs., ¾ mile, in 1 *min.* 15½ *sec.*
1875, "Bob Wooley," Lexington, Ky., 3 years, 90 lbs., 1⅛ miles, in 1 *min.* 54 *sec.*
1875, "Grinstead," Saratoga, N. Y., 4 years, 108 lbs., 1¼ miles, in 2 *min.* 8¾ *sec.*
1874, "Tom Bowling," Lexington, Ky., 4 years, 104 lbs., 1½ miles, in 2 *min.* 34¾ *sec.*
1871, "Harry Bassett," Saratoga, N. Y., 3 years, 110 lbs., 1⅝ miles, in 2 *min.* 56 *sec.*
1875, "Ten Broeck," Lexington, Ky., 3 years, 90 lbs., 1⅝ miles, in 2 *min.* 49¼ *sec.*
1875, "D'Artagnan," Saratoga, N.Y., 3 years, 90 lbs., 1¾ miles, in 3 *min.* 6¼ *sec.*
1874, "Reform," Saratoga, N.Y., 3 years, 83 lbs., 1¾ miles, in 3 *min.* 5¾ *sec.*
1875, "Preakness," aged, and "Springbok," 5 years, Saratoga, N.Y., 114 lbs., 2¼ miles, in 3 *min.* 56¼ *sec.*
1874, "Katie Pease," Buffalo, N. Y., 4 years, 105 lbs., 2½ miles, in 4 *min.* 28½ *sec.*
1866, "Kentucky," Jerome Park, N.Y., 5 years, 124 lbs., 2¾ miles in 5 *min.* 4 *sec.*
1872, "Tom Bowling," Long Branch, N. J., 2 years, 100 lbs., ¾ mile, in 1 *min.* 16¾ *sec.*
1873, "Hubbard," Saratoga, N.Y., 4 years, 108 lbs., 2¾ miles, in 4 *min.* 58¾ *sec.*
1875, "Red Lad," Houston, Texas, 48 miles, without rests, in 5 *hours* 30 *min.*

French.

1865, "Gladiateur," Doncaster, "St. Leger," Eng., 3 years, 122 lbs., 1 mile, 6 furlongs, and 132 yards (1.775 miles), in 3 *min.* 20 *sec.*=1 *mile* in 1 *min.* 52.7 *sec.*

Arabian.

1828, "Chapeau de Paille," India, 1½ miles, 115 lbs., in 2 *min.* 53 *sec.*=1 mile in 1 *min.* 55.33 *sec.*; "Patrician," 2 miles, 6 furlongs, and 160 yards (2.841 miles), 126 lbs., in 5 *min.* 34 *sec.*=3 miles in 5 *min.* 52.7 *sec.*
183–, *Capt. Horne*, Madras to Bungalore, India, 200 miles, in less than 10 *hours.*
1869, A barb, New South Wales, 4 years, 2 miles, 148 lbs., in 3 *min.* 40½ *sec.*, and 3 miles, 139 lbs., in 5 *min.* 53 *sec.*

TROTTING.

One Mile.

1796, A gelding of Mr. Jex, Denham and Norwich road, Eng., in 2 *min.* 49 *sec.**
1818, "Boston Blue," Boston, Mass., sulky, in less than 3 *min.*—the exact time is not now attainable.
1824, "Albany Pony," Long Island, N. Y., harness, turnpike, in 2 *min.* 40 *sec.*
1849, "Lady Suffolk," Cambridge, Mass., saddle, 7 heats, in 17 *min.* 43 *sec.*
1860, "Cora" (3 years), Louisville, Ky., harness, 2d heat, in 2 *min.* 37¾ *sec.*
1867, "Ethan Allen" and running mate, Long Island, N.Y., wagon, in 2 *min.* 15 *sec.*, 2 *min.* 16 *sec.*, and 2 *min.* 19 *sec.*; 3d quarter of 3d heat, in 31 *sec.*
1868, "Lady Thorn," Long Island, N.Y., wagon, in 2 *min.* 24 *sec.*
1869, "Dexter,"† Prospect Park, L. I., road wagon, driver and wagon 319 lbs., in 2 *min.* 21¾ *sec.*, saddle in 2 *min.* 17¾ *sec.*; 1867, wagon, 2d heat, in 2 *min.* 24 *sec.*; 1866, Buffalo,‡ N.Y., saddle, 3d heat, in 2 *min.* 18 *sec.*; Long Island, N.Y., harness, 5th heat, in 2 *min.* 24¼ *sec.*; and 1865, Fashion Course, Long Island, N. Y., saddle, 145 lbs., in 2 *min.* 18.2 *sec.*, one quarter in 34 *sec.*, and one in 32½ *sec.*=½ mile in 1 *min.* 6½ *sec.*

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* *Sporting Magazine*, London, vol. ix., p. 46.
† Public performance, but not recorded.
‡ Track 27 ft. 8 in. over a mile = .72 *sec.*, or 3-5ths of a second less when timed by a watch.

1869, "Blackwood" (3 years), Lexington, Ky., harness, 150 lbs., in 2 *min.* 31 *sec.*
1872, "Jos. Elliott,"* Mystic Park, Mass., harness, in 2 *min.* 15½ *sec.*
1872, "Lady Stout" (1 year), Lexington, Ky., harness, in 3 *min.* 5¾ *sec.*, and 3 *min.* 4¾ *sec.*
1872, "Doble" (2 years), Lexington, Ky., harness, in 2 *min.* 40½ *sec.*
1874, "Goldsmith Maid," Mystic Park, Boston, Mass., harness, in 2 *min.* 14 *sec.*; Rochester, N.Y., in 2 *min.* 14¾ *sec.*
1874, "Smuggler," stallion, Buffalo, N.Y., saddle, in 2 *min.* 20¾ *sec.*
1874, "Mambrino Gift," stallion, Rochester, N.Y., harness, 3 heats, in 2 *min.* 21 *sec.*, 2 *min.* 20 *sec.*, and 2 *min.* 23 *sec.*=total time in 7 *min.* 4 *sec.*
1874, "Lulu," Rochester, N.Y., harness, 2d, 3d, and 4th heats in 2 *min.* 16½ *sec.*, 2 *min.* 15½ *sec.*, and 2 *min.* 17 *sec.*=total time in 6 *min.* 49 *sec.*
1874, "Jay Gould," stallion, Baltimore, Md., harness, in 2 *min.* 19½ *sec.*

Two Miles.

1852, "Tacony," Long Island, N.Y., saddle, in 5 *min.* 2 *sec.*
1859, "Flora Temple," Long Island, N.Y., harness, in 4 *min.* 50½ *sec.*
1863, "General Butler," Long Island, N.Y., wagon, in 4 *min.* 56¼ *sec.*
1865, "Dexter," Long Island, N.Y., wagon, 2d heat, in 4 *min.* 56¼ *sec.*

Three Miles.

1861, "Flora Temple," Long Island, N.Y., wagon, in 7 *min.* 47 *sec.*
1861, "Ethan Allen" and running mate, Long Island, N.Y., wagon, in 7 *min.* 3¾ *sec.*, and 1 mile in 2 *min.* 19¾ *sec.*
1872, "Huntress," Long Island, N.Y., harness, in 7 *min.* 21¼ *sec.*

Four Miles.

1828, "Top Gallant," Philadelphia, Penn., saddle, 4th heat, in 12 *min.* 15 *sec.*
1836, "Dutchman," Long Island, N.Y., saddle, in 10 *min.* 51 *sec.*
1849, "Trustee," Long Island, N.Y., harness, in 11 *min.* 6 *sec.*
1869, "Longfellow," San Francisco, Cal., wagon, 2d heat, in 10 *min.* 34½ *sec.*

Five Miles.

1837, "Dolly," Long Island, N.Y., wagon; driver and man 310 lbs., in 16 *min.* 45 *sec.*
1863, "Little Mac," Long Island, N.Y., wagon, in 13 *min.* 43¾ *sec.*
1864, "Fillmore," San Francisco, Cal., harness, in 13 *min.* 16 *sec.*
1868, "Morrissey," Detroit, Mich., harness, in 13 *min.* 11 *sec.*

Various Distances and Performances.

1865, "Young Pocahontas," Long Island, N.Y., ¼ mile, harness, in 33 *sec.*
1870, "Dexter," Fleetwood, N.Y., ½ mile, road-wagon, driver and wagon 305 lbs., in 1 *min.* 6½ *sec.*;* and 1869, Long Island, N.Y., harness, ½ mile, in 1 *min.* 4 *sec.**
1871, "Bruno," Fleetwood, N.Y., ½ mile, saddle, in 1 *min.* 5¼ *sec.**
1853, "Kemble Jackson," Long Island, N.Y., 1 mile, wagon, 250 lbs., in 8 *min.* 3 *sec.*
1865, "Mountain Maid," Long Island, N.Y., 1 mile, wagon, 2058 lbs., in 3 *min.* 24½ *sec.*
1860, "Lady Palmer," Long Island, N.Y., 2 miles, wagon and driver 335 lbs., 2d heat in 5 *min.* 7 *sec.*
1850, "Sally Green," Long Island, N.Y., 4 miles, wagon, 255 lbs., in 13 *min.* 56 *sec.*
1830, "Whalebone," Long Island, N.Y., 6 miles, saddle, in 18 *min.* 52 *sec.*
1849, "Kentucky Prince," Long Island, N.Y., 10 miles, harness, in 28 *min.* 8½ *sec.*
1868, "John Stewart," Boston, Mass., half-mile track, 10 miles, wagon, in 28 *min.* 2½ *sec.*, and Long Island, N.Y., 20 miles, wagon, in 59 *min.* 23 *sec.* 1867, Boston, Mass., half-mile track, 20 miles, harness, in 58 *min.* 5¾ *sec.*, and 20½ miles in 59 *min.* 31¼ *sec.*

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* Public performances, but not recorded.

1830, "Top Gallant," Philadelphia, Penn., 12 miles, harness, in 38 *min.*

1829, "Tom Thumb," Sunbury Common, Eng., 16½ miles, harness, 248 lbs., in 56 *min.* 45 *sec.*, and 100 miles in 10 *hours* 7 *min.*, including 37 *min.* in rests.

1800, "Phenomenon," Cambridge and Huntingdon road, Eng., 12 years, saddle, feather (70 lbs.), 17 miles, in less than 53 *min.*

1823, "Boston Blue," ———————, Eng., 18 miles, harness, in 1 *hour.*

1869, "Morning Star," Doncaster, Eng., 18 miles, harness (sulky 100 lbs.), in 57 *min.* 27 *sec.*

1833, "Paul Pry," Long Island, N.Y., 18 miles 36 yards, saddle, 138 lbs., in 58 *min.* 52 *sec.*

1848, "Trustee," Long Island, N.Y., 20 miles, harness; sulky and driver 295 lbs., in 59 *min.* 35½ *sec.*

1831, "Chancellor," Philadelphia, Penn., 32 miles, saddle, 90 lbs., in 1 *hour* 58 *min.* 31 *sec.*

1831, "Whalebone," Philadelphia, Penn., 32.3 miles, harness, in 1 *hour* 58 *min.* 5 *sec.*

1832, "Rattler," Newmarket road, Eng., 34 miles, saddle, 154 lbs., in 2 *hours* 18 *min.* 56 *sec.*

1846, "Ariel," Albany, N. Y., 50 miles, harness, driver 60 lbs., in 3 *hours* 55 *min.* 40½ *sec.*

1835, "Black Joke," Providence, R. I., 50 miles, saddle, 175 lbs., in 3 *hours* 57 *min.*

1855, "Spangle," Long Island, N.Y., 50 miles, wagon and driver 400 lbs., in 3 *hours* 59 *min.* 4 *sec.*

1837, "Mischief," Jersey City, N. J., to Philadelphia, Penn., 84¼ miles, harness, very hot day and sandy road, in 8 *hours* 30 *min.*

1853, "Conqueror," Long Island, N. Y., 100 miles, harness, in 8 *hours* 55 *min.* 53 *sec.*, including 15 short rests.

1845, "Fanny Jenks," Albany, N. Y., 101 miles, harness, in 9 *hours* 42 *min.* 57 *sec.*, including 18 *min.* 27 *sec.* in rests. 101st mile in 4 *min.* 23 *sec.*

1783, *S. Halliday*, Leeds to York, Eng., and return, 110 miles, saddle, 196+ lbs., in less than 18 hours.

1873, *M. Delaney's* mare, St. Paul's, Minn., 200 miles, race track, harness, in 44 *hours* 20 *min.*, including 15 *hours* 49 *min.* in rests.

DOUBLE TEAMS.

1867, "Bruno" and "Brunette," Long Island, N. Y., ½ mile, road-wagon, in 1 *min.* 10½ *sec.*, and 1 mile, in 2 *min.* 25¼ *sec.**

1854, "Cinderella" and "Tom Wonder," Long Island, N. Y., 1 mile, wagon, 2d heat in 2 *min.* 32 *sec.*

1862, "Lady Palmer" and "Flatbush Maid," Long Island, N. Y., 1 mile, road wagon, in 2 *min.* 26 *sec.*, 2 miles in 5 *min.* 1¼ *sec.*; 2d quarter of 2d mile, in 33 *sec.**

1870, "Idol" and "Kirkwood," Prospect Park, L. I., 1 mile, wagon, in 2 *min.* 29 *sec.*

1870, "Kirkwood" and "License," Prospect Park, L. I., 1 mile, wagon, 4th and 5th heats, in 2 *min.* 28½ *sec.* each.

1870, "Jesse Wales" and "Darkness," Narragansett Park, R. I., 1 mile, wagon, 3d heat, in 2 *min.* 27¾ *sec.*

1834, "Master Burke" and "Robin," Long Island, N. Y., 100 miles, wagon, in 10 *hours* 17 *min.* 22 *sec.*, including 28 *min.* 34 *sec.* in rests.

PACING.

One Mile.

1839, "Drover," Hoboken, N. J., saddle, in 2 *min.* 30 *sec.*

1852, "Pet," Long Island, N. Y., harness, 2d heat, in 2 *min.* 18½ *sec.*

1855, "Pocahontas," Long Island, N. Y., wagon and driver 265 lbs., in 2 *min.* 17½ *sec.*

1868, "Billy Boyce," Buffalo, N. Y., saddle, 3d heat, in 2 *min.* 14¼ *sec.*, and 1867, St. Louis, Mo., harness, 4th heat, in 2 *min.* 19 *sec.*

Two Miles.

1850, "James K. Polk," Philadelphia, Penn., saddle, 2d heat, in 4 *min.* 57½ *sec.*

1853, "Hero," Long Island, N. Y., harness, in 4 *min.* 56½ *sec.*

1859, "Young America," San Francisco, Cal., wagon, 2d heat, in 4 *min.* 58½ *sec.*

* Public performances, but not recorded.

Three Miles.

1843, "Oneida Chief," Hoboken, N. J., saddle, in 7 *min.* 44 *sec.*
1847, "James K. Polk," Long Island, N. Y., harness, in 7 *min.* 44 *sec.*
1852, "Pet," Long Island, N. Y., wagon, 2d heat, in 7 *min.* 59½ *sec.*

Ten Miles.

1860, "Mary Miller," Maysville, Cal., in less than 30 *min.*

TANDEM.

1867, "Kingston" and mate, Providence, R. I., 1 mile, in 2 *min.* 37 *sec.*
1839, Burke's team, Bromwich Road, Eng., 45 miles, in 2 *hours* 55½ *sec.*

STAGE-COACHING, ETC.

1750, *Four horses, by the Duke of Queensberry, Newmarket, Eng.*, 19 miles, in 53 *min.* 24 *sec.*

1789, *Messrs. Bush and Matthews*, of London, Eng., a post-chaise and pair, from London to Bath, 108 miles, in 8 *hours* 40 *min.*

——, *London to Cambridge, Eng.*, 52 miles, in 3 *hours*, including rests.

——, *Leeds to London, Eng.*, 201 miles, in 13 *hours* 34 *min.*

——, *Dover to London, Eng.*, Express, 72 miles, in 5 *hours* 15 *sec.*

1830, *London to Birmingham, Eng.*, "Tally-ho," 109 miles, in 7 *hours* 50 *min.*, including stop for breakfast of passengers.

SLEIGHING.

1868, "Black Maria," Providence, R. I., to Boston, Mass., 42 miles, in 3 *hours* 25 *min.*

LEAPING.*

Horse.

1752, *Sir C. Turner*, Fell near Richmond, Eng., 10 miles, in 36 *min.*, making 40 leaps of 4 *feet* 4 *ins.* in height.

1821, A horse of Mr. Mane, at Loughborough, Leicestershire, Eng., 173 lbs., over a hedge 6 feet in height, 35 *feet.*†

1821, A horse of Lieut. Green, Third Dragoon Guards, at Inchinnan, Eng., ridden by a heavy dragoon, over a wall 6 *feet* in height and 1 *foot* in width at top.

1839, "Lottery," Liverpool, Eng., over a wall, 33 *feet.*
1847, "Chandler," Warwick, Eng., over water, 39 *feet.*
18—, "Emblem," Birmingham, Eng., 36 *feet* 3 *ins.*
——, "King of the Valley," Leicestershire, Eng., over Wissendine brook, 35 *feet.*

NOTE.—The maximum stride of a horse is estimated to be 28 *feet* 9 *ins.*; "Eclipse" has covered 25 feet. The maximum stride of an elk is 31 *feet*, and of an elephant 14 *feet.*

FLYING.

VELOCITY OF THE FLIGHT OF BIRDS PER HOUR.

Vulture.. 150 miles; *Wild Goose and Swallow*.. 90 miles; *Crow*.... 25 miles.

PIGEON FLYING.

18—, *Carrier Pigeon*, 950 miles, in 14 *hours*=68 *miles* per hour.
1868, *Carrier Pigeons* (6), Eng., 180 miles, in 3 *hours* 32 *min.*
1870, *Carrier Pigeons*, Pesth to Cologne, Germany, 600 miles, in 8 *hours.*
1875, *Carrier Pigeon*, Dundee Lake to Paterson, N. J., 3 miles, in 3 *min.* 24 *sec.*

COURSING AND CHASING.

A Greyhound and Hare have ran 12 miles in 30 *min.*
1794, A Fox, at Brende, Eng., ran 50 miles in 6½ *hours.*
A Greyhound, at Bushy Park, Eng., leaped over a brook 30 *feet* 6 *ins.*

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* A Salmon can leap a dam 14 feet in height.—*Sporting Magazine*, London, vol. xii., p. 79.
† *Sporting Magazine*, London, vol. ix., p. 143.

CENTRAL FORCES.

All bodies moving around a centre or fixed point have a tendency to fly off in a straight line: this is termed *Centrifugal Force;* it is opposed to a *Centripetal Force*, or that power which maintains a body in its curvilineal path.

The *Centrifugal Force* of a body, moving with different velocities in the same circle, is proportional to the square of the velocity. Thus the centrifugal force of a body making 10 revolutions in a minute is 4 times as great as the centrifugal force of the same body making 5 revolutions in a minute. Hence, in equal circles, the forces are inversely as the squares of the times of revolution.

If the times are equal, the velocities and the forces are as the radii of the circle of revolution.

The squares of the times are as the cubes of the distances of the centrifugal force from the axis of revolution.

The centrifugal forces of two unequal bodies, having the same velocity, and at the same distance from the central body, are to one another as the respective quantities of matter in the two bodies.

The centrifugal forces of two bodies, which perform their revolutions in the same time, the quantities of matter of which are inversely as their distances from the centre, are equal to one another.

The centrifugal forces of two equal bodies, moving with equal velocities at different distances from the centre, are inversely as their distances from the centre.

The centrifugal forces of two unequal bodies, moving with equal velocities at different distances from the centre, are to one another as their quantities of matter, multiplied by their respective distances from the centre.

The centrifugal forces of two unequal bodies, having unequal velocities, and at different distances from their axes, are, in the compound ratio of their quantities of matter, the squares of their velocities, and their distances from the centre.

To Compute the Centrifugal Force of any Body.

Rule.—Divide its velocity in feet per second by 4.01, also the square of the quotient by the diameter of the circle; this quotient is the centrifugal force, assuming the weight of the body as 1. Then this quotient, multiplied by the weight of the body, will give the centrifugal force required.

Example.—What is the centrifugal force of the rim of a fly-wheel having a diameter of 10 feet, and running with a velocity of 30 feet per second?

$30 \div 4.01 = 7.48$, and $7.48^2 \div 10 = 5.59$, or the *times the weight of the rim.*

Note.—The diameter of a fly-wheel should be measured from the centres of gravity of the rim.

When great accuracy is required, ascertain the centre of gyration of the body, and take twice the distance of it from the axis for the diameter.

Rule 2.—Multiply the square of the number of revolutions in a minute by the diameter of the circle of the centre of gyration in feet, and divide the product by the constant number 5217; the quotient is the centrifugal force when the weight of the body is 1. Then, as in the previous Rule, this quotient, multiplied by the weight of the body, is the centrifugal force required.

Example.—What is the centrifugal force of a grindstone weighing 1200 lbs., 42 inches in diameter, and turning with a velocity of 400 revolutions in a minute?

Centre of gyration = rad. $(42 \div 2) \times .7071 = 14.85$ *ins.*, which $\div 12$, and $\times 2 = 2.475$ *feet = the diameter of the circle of gyration.* Then $\frac{400^2 \times 2.475}{5217} \times 1200 = 91080$ *lbs.*

Or, let *v represent velocity of body in feet per second, w weight of body, r radius of circle of revolution in feet, and c centrifugal force.*

Then $\frac{v^2 \times w}{r \times 32.166} = c$; $\frac{v^2 \times w}{c \times 32.166} = r$; $\frac{c \times 32.166 \times r}{v} = w$; and $\sqrt{\left(\frac{r \times 32.166 \times c}{w}\right.} = v.$

EXAMPLE.—If the diameter of a grindstone is 45.254 ins., its weight 1200 lbs., and it revolves 1146 times in a minute, what is its centrifugal force?

Centre of gyration $= 22.627 \times .7071$, which $\div 12.$ and $\times 2 = 2.666$ *feet.*

$$\frac{2.666 \times 3.1416 \times 1146}{60} = 160 \textit{ ft. velocity per second.} \text{ Then } \frac{160^2 \times 1200}{1.333 \times 32.166} = 701187.72 \textit{ lbs.}$$

EX. 2.—If a fly-wheel, 12 feet in diameter and 3 tons in weight, revolves in 8 seconds, and another of like weight revolves in 6 seconds, what should be the diameter of the second when their centrifugal forces are equal? and what would be the ratio of the weights of these wheels, their forces being equal?

NOTE.—The centrifugal forces of two bodies are as the radii of the circles of revolution directly, and as the squares of the times inversely.

Then $3 : 3 :: \frac{12}{8^2} : \frac{x}{6^2}$; or $x = \frac{12 \times 6^2}{8^2} = \frac{12 \times 36}{64} = 6.75$ *feet, x representing the unknown element.*

NOTE.—The centrifugal forces of two bodies, when the weights are unequal, are directly as the squares of the times.

Then $3 : x :: 6^2 : 8^2$, or $x = \frac{3 \times 8^2}{6^2} = \frac{3 \times 64}{36} = 5.333$ *tons.*

Fly-wheels.—For Rules for weights of, and Examples, see Steam-engine, page 416.

CENTRE OF GYRATION

The *Centre of Gyration* is that point in any revolving body or system of bodies in which, if the whole quantity of matter were collected, the *angular velocity* would be the same; that is, the *momentum* of the body or system of bodies is centred at this point, and the position of it is a mean proportional between the centres of oscillation and gravity.

If a straight bar of uniform dimensions was struck at this point, the stroke would communicate the same *angular velocity* to the bar as if the whole bar was collected at that point.

The *Angular Velocity* of a body or system of bodies is the motion of a line connecting any point with the axis of motion, and is the same in all parts of the same revolving system.

When a body revolves on an axis, and a force is impressed upon it sufficient to cause it to revolve on another, it will revolve on neither, but on a line in the plane of the axes, dividing the angle which they contain; so that the sine of each part will be in the inverse ratio of the angular velocities with which the bodies would have revolved about these axes separately.

The weight of the revolving body, multiplied into the height due to the velocity with which the centre of gyration moves in its circle, is the energy of the body, or the mechanical power which must be communicated to it to give it that motion.

To Compute the Elements of Gyration.

$$\frac{G \times W \times v}{r \times t \times 32.166} = P. \qquad \frac{P \times r \times t \times 32.166}{W \times v} = G. \qquad \frac{G \times W \times v}{P \times t \times 32.166} = r.$$

$$\frac{P \times r \times t \times 32.166}{G \times v} = W. \qquad \frac{G \times W \times v}{P \times r \times 32.166} = t. \qquad \frac{P \times r \times t \times 32.166}{G \times W} = v;$$

G representing the distance of the centre of gyration from the axis of rotation, W the weight of the body, P the power acting upon the body, t the time the power acts in seconds, v the velocity in feet per second acquired by the revolving body in that time, and r the distance of the point of application of the power from the axis of the body, as the length of the crank, etc.

ILLUSTRATION.—What is the distance of the centre of gyration in a fly-wheel, the power being 224 lbs., the length of the crank 7 feet, the time of rotation 10 seconds, the weight of the wheel 5600 lbs., and the velocity of it 8 feet per second?

$$\frac{224\times7\times10\times32.166}{5600\times8}=\frac{504373}{42800}=11.78\ \textit{feet.}$$

2.—What should be the weight of a fly-wheel making 12 revolutions per minute, its diameter 8 feet, the power applied at 2 feet from its axis 84 lbs., the time of rotation 6 seconds, and the distance of the centre of gyration of the wheel 3.5 feet?

$$\frac{8\times3.1416\times12}{60}=5.0265\ \textit{feet}=\textit{velocity}.\ \text{Then}\ \frac{84\times2\times6\times32.166}{3.5\times5.0625}=1829.9\ \textit{lbs.}$$

To Compute the Centre of Gyration.

When the Body is a Compound one.

RULE.—Multiply the weight of the several particles or bodies by the squares of their distances in feet from the centre of motion or rotation, and divide the sum of their products by the weight of the entire mass; the square root of the quotient will give the distance of the centre of gyration from the centre of motion or rotation.

EXAMPLE.—If two weights, of 3 and 4 lbs. respectively, be laid upon a lever (which is here assumed to be without weight) at the respective distances of 1 and 2 feet, what is the distance of the centre of gyration from the centre of motion (the fulcrum)?

$$3\times1^2=3;\ 4\times2^2=16;\ \frac{3+16}{3+4}=\frac{19}{7}=2.71,\ \text{and}\ \sqrt{2.71}=1.64\ \textit{feet.}$$

That is, a single weight of 7 lbs., placed at 1.64 feet from the centre of motion, and revolving in the same time, would have the same *momentum* as the two weights in their respective places.

When the Centre of Gravity is given.

RULE.—Multiply the distance of the centre of oscillation, from the centre or point of suspension, by the distance of the centre of gravity from the same point, and the square root of the product will give the distance of the centre of gyration.

EXAMPLE.—The centre of oscillation of a body is 9 feet, and that of its gravity 4 feet from the centre of rotation or point of suspension; at what distance from this point is the centre of gyration?

$$9\times4=36,\ \text{and}\ \sqrt{36}=6\ \textit{feet.}$$

To Compute the Centre of Gyration of a Water-wheel.

RULE.—Multiply severally twice the weight of the rim, as composed of buckets, shrouding, etc., and twice that of the arms and that of the water in the buckets (when the wheel is in operation) by the square of the radius of the wheel in feet; divide the sum by twice the sum of these several weights, and the square root of the product will give the distance in feet.

EXAMPLE.—In a wheel 20 feet in diameter, the weight of the rim is 3 tons, the weight of the arms 2 tons, and the weight of the water in the buckets 1 ton; what is the distance of the centre of gyration from the centre of the wheel?

Rim $=3$ tons $\times10^2\times2=600$
Buckets $=2$ tons $\times10^2\times2=400$
Water $=1$ ton $\times10^2\ldots=100$
1100

$3+2+1\times2=12.$

Hence $\sqrt{\frac{1100}{12}}=\sqrt{91.67}=9.57$ *feet.*

The following are the distances of the centres of gyration from the centre of motion in various revolving bodies, as given by Mr. Farey:

In a straight, uniform Rod or *Cylinder*, revolving about one end; length of rod $\times .5773$, and revolving about its centre; length $\times .2886$.

In a Circular Plane, revolving on its centre; radius of the circle $\times .7071$; revolving about one of its diameters as an axis; radius $\times .5$.

In a Wheel of uniform thickness, or *in a Cylinder* revolving about its axis; radius $\times .7071$.

In a Solid Sphere, revolving about one of its diameters as an axis; radius $\times .6325$.

In a thin, hollow Sphere, revolving about one of its diameters as an axis; radius $\times .8164$.

In a Sphere, at a distance from the axis of revolution $= \sqrt{l^2 + \frac{2}{5} r^2}$, *l representing the length of the connection to the centre of the sphere;* and *in a Cylinder* $= \sqrt{l^2 + \frac{1}{2} r^2}$.

In a Cone, revolving about its axis; radius of base $\times .5447$; revolving about its vertex $= \sqrt{12 h^2 + 3 r^2} \div 20$, *h representing the height, and r radius of the base;* revolving about its base $= \sqrt{2 h^2 + 3 r^2} \div 20$.

In a Circular Ring, as the *Rim of a Fly-wheel*, revolving about its diameter $= \sqrt{R^2 + r^2} \div 2$, R *representing radius of periphery of ring.*

In a Fly-wheel $= \sqrt{\dfrac{6W(R^2 + r^2) + w(4r^2 + l^2)}{12(W + w)}}$, W *and w representing the weights of the rim and of the arms and hub, and l the length of the arms from the axis of the wheel.*

In a Straight Lever $= \sqrt{\dfrac{R^3 + r^3}{3(R - r)}}$.

Illustration.—In a solid sphere revolving about its diameter, the diameter being 2 feet, the distance of the centre of gyration is $12 \times .6325 = 7.59$ *inches.*

General Formulæ.—Let P *represent power,* H *horses' power,* F *the force applied to rotate the body in lbs*, M *mass of the revolving body in lbs., r radius upon which* F *acts in feet, d distance from axis of motion to centre of gyration in feet, t time the force is applied in seconds, n number of revolutions in time t, v angular velocity, or number of revolutions per minute at the end of time t, and* $G = \dfrac{32.166\, F r^2}{M d^2}$.

$$\sqrt{\frac{4prn}{G}} = t; \qquad \frac{2prx}{60\,G} = t; \qquad \frac{M x d^2}{153.5\, t r} = F;$$

$$\frac{M n d^2}{2.56\, t^2 F} = r; \qquad \frac{2.56\, t^2 F r}{M d^2} = n; \qquad \frac{153.5\, t F r}{M d^2} = x;$$

$$\frac{244\, t P}{x^2 d^2} = M; \qquad \frac{x^2 M d^2}{244\, t} = P; \qquad \frac{x^2 M d^2}{134100\, t} = H.$$

Illustration.—The rim of a fly-wheel weighing 7000 lbs. has radii of 6.5 and 5.75 feet; what is its centre of gyration, and what force must be applied to it 2 feet from the axis of motion to give it an angular velocity of 130 revolutions per minute in 40 seconds? and how many revolutions will it make in 40 seconds?

Centre of gyration $= \sqrt{\dfrac{6.5^2 + 5.75^2}{2}} = 6.14$ *feet.* Then $F = \dfrac{130 \times 7000 \times 6.14^2}{153.5 \times 40 \times 2} = \dfrac{34406636}{12280} = 2802$ *lbs.*, and $\dfrac{2.56 \times 40^2 \times 2802 \times 2}{7000 \times 6.14^2} = 86.97$ *revolutions.*

CENTRES OF OSCILLATION AND PERCUSSION.

The Centre of Oscillation of a body, or a system of bodies, is that point in the axis of vibration, of a vibrating body, in which, if the whole matter of the body were collected, and it was acted upon by a like force to that acting upon the body, it would, if suspended or supported from the same axis of motion, perform its oscillations or vibrations in the same time and with the same *angular velocity.*

It is in a right line passing through the centre of gravity of the body, and perpendicular to the axis of motion.

The *angular velocity* of a body or system of bodies is the motion of a line connecting any point and the centre or axis of motion: it is the same in all parts of the same revolving body.

In different unconnected bodies, each oscillating about a common centre, their *angular velocity* is as the velocity directly, and as the distance from the centre inversely. Hence, if their velocities are as their radii, or distances from the axis of motion, their angular velocities will be equal.

The Centre of Percussion of a body, or a system of bodies, revolving about a point or axis, is that point at which, if resisted by an immovable obstacle, all the motion of the body, or system of bodies, would be destroyed, and without impulse on the point of suspension.

The Centres of Oscillation and Percussion are in the same point.

As in bodies at rest, the whole weight may be considered as collected in the centre of gravity; so in bodies in vibration, the whole force may be considered as concentrated in the centre of oscillation; and in bodies in motion, the whole force may be considered as concentrated in the centre of percussion.

If the centre of oscillation is made the point of suspension, the point of suspension will become the centre of oscillation.

The *Angle of Oscillation* or *Percussion* is determined by the angle delineated by the vertical plane of the body in vibration, in the plane of motion of the body.

The *Velocity of a Body in Oscillation or Percussion* through its vertical plane is equal to that acquired by a body freely falling through a vertical line equal in height to the versed sine of the arc.

The centre of percussion is also that point of a revolving body which would strike any obstacle with the greatest effect, and from this property it has received the name of percussion.

To Compute the Centre of Oscillation or Percussion of a Body of Uniform Density and Figure.

Rule.—Multiply the weight of the body by the distance of its centre of gravity from the point of suspension; multiply also the weight of the body by the square of its length, and divide the product by 3.

Divide this last quotient by the product of the weight of the body and the distance of its centre of gravity, and the quotient is the distance of the centres from the point of suspension.

$$\text{Or, } \frac{W \times l^2}{3} \div W \times g = \textit{distance from axis.}$$

Example.—Where is the centre of oscillation in a rod 9 feet in length from its point of suspension, and weighing 9 lbs.?

$9 \times \frac{9}{2} = 40.5 =$ *product of the weight and its centre of gravity*; $\frac{9 \times 9^2}{3} = 243 =$ *quotient of product of weight of body and the square of its length* $\div 3$; $\frac{243}{40.5} = 6$ *feet.*

Point of Centres of Oscillation and Percussion in Bodies of Various Figures.

When the Axis of Motion is in the Vertex of the Figure.

When the Oscillation or Motion is Facewise.

1. *In a Right Line, on any figure of uniform shape and density* $= .66\,l$.
2. *In an Isosceles Triangle* $= .75\,h$.
3. *In a Circle* $= 1.25\,r$.
4. *In a Parabola* $= .714\,h$.

When the Oscillation or Motion is Sidewise.

1. *In a Right Line, or any figure of uniform shape and density* $= .66\,l$.
2. *In a Circle* $= .75\,d$.
3. *In a Rectangle*, suspended at one angle $= .66$ *of diagonal.*
4. *In a Parabola*, if suspended by its vertex $= .714$ of axis $+ .33$ parameter; if suspended by the middle of its base $= .57$ of axis $+ .5$ parameter.
5. *In a Sector of a Circle* $= \frac{3 \times \text{arc} \times r}{4 \times c}$, *c representing chord of the arc, and r the radius of the base.*
6. *In a Cone* $= \frac{4}{5}\,\text{axis} + \frac{r^2}{5 \times \text{axis}}$.
7. *In a Sphere* $= \frac{2 \times r^2}{5\,(c + r)} + r + c$, *c representing the length of the cord by which it is suspended.*

To Ascertain the Centre of Oscillation and Percussion experimentally.

Suspend the body very freely by a fixed point, and make it vibrate in small arcs, counting the number of vibrations it makes in a minute, and let the number of vibrations made in a minute be called n; then will the distance of the centre of oscillation from the point of suspension be $= \frac{140850}{n^2}$ *inches.*

For the length of a pendulum vibrating seconds, or 60 times in a minute, being $39\frac{1}{8}$ inches, and the lengths of the pendulums being reciprocally as the squares of the number of vibrations made in the same time, therefore $n^2 : 60^2 :: 39\frac{1}{8} : \frac{60^2 \times 39\frac{1}{8}}{n^2} = \frac{140850}{n^2}$, *being the length of the pendulum which vibrates n times in a minute, or the distance of the centre of oscillation below the axis of motion.*

Illustration.—Where is the centre of percussion of a rod 23 inches in length?

$$.66 \text{ of } 23 = 15.18 \textit{ inches.}$$

2.—In a sphere 10 inches in diameter, suspended by a cord 20 inches in length, where is the centre of percussion or oscillation?

$$\frac{2 \times 5^2}{5\,(20 + 5)} + 5 + 20 = \frac{50}{125} + 25 = 25.4 \textit{ inches.}$$

To Compute the Centres of Oscillation or Percussion of a System of Particles or Bodies.

Rule.—Multiply the weight of each particle or body by the square of its distance from the point of suspension, and divide the sum of their products by the sum of the weights, multiplied by the distance of the centre of gravity from the point of suspension, and the quotient will give the centre required measured from the point of suspension.

$$\text{Or, } \frac{W \times d^2 + W' \times d'^2}{W \times g + W' \times g'} = \textit{distance of centre.}$$

Example.—The length of a suspended rod being 20 feet, and the weight of a foot in length of it equal 100 oz., has a ball attached at the under end weighing 100 oz., at what point of the rod from the point of suspension is the centre of percussion?

$100 \times 20 = 2000 =$ *weight of rod;* $2000 \times \frac{20}{2} = 20000 =$ *momentum of rod, or product of its weight, and distance of its centre of gravity;* $\frac{2000 \times 20^2}{3} = 266666.66 =$ *force of rod;* $1000 \times 20^2 = 400000 =$ *force of ball.*

Then $\frac{266666.66 + 400000}{20000 + 20000} = 16.66$ *feet.*

Or, $\frac{\frac{1}{3}\,l b \times l^2 + c\,l^2}{\frac{1}{2}\,l b \times l + l c} =$ *centre of percussion, etc.; l representing length of rod, b weight of a foot in length of rod, and c weight suspended from end.*

Ex. 2.—Assume a rod 12 feet in length, and weighing 2 lbs. for each foot of its length, with 2 balls of 3 lbs. each—one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$12\times2\times\frac{12}{2}=144=$ *momentum of rod.*
$3\times6 \quad =18=$ " *of 1st ball.*
$3\times12 \quad =36=$ " *of 2d ball.*
198 *sum of moments.*

$\frac{24\times12^2}{3}=\frac{3456}{3}=1152=$ *force of rod.*
$3\times6^2=3\times36=108=$ " *of 1st ball.*
$3\times12^2=3\times144=432=$ " *of 2d ball.*
1692 *sum of forces.*

Then $\frac{1692}{198}=8.545$ *feet.*

FLY-WHEELS.

A Fly-wheel should always have high velocity.

The diameter should be from 3 to 4 times that of the stroke of the driving engine.

The weight of the rim should be about 85 to 95 lbs. per actual horse-power, the momentum of the wheel being $4\frac{1}{2}$ times that of the piston.

When the Engine to which a Fly-wheel is to be attached is single-acting, it is customary to make the weight of the wheel 5 times greater than when it is to be attached to a double-acting engine.

The weight of a fly-wheel in engines that are subjected to irregular motion, as in a cotton-press, rolling-mill, etc., must be greater than in others where so sudden a check is not experienced.

To Compute the Weight of the Rim of a Fly-wheel.

Rule.—Multiply the mean effective pressure upon the piston in lbs. by its stroke in feet, and divide the product by the product of the square of the number of revolutions, the diameter of the wheel and .00023.

Note.—If a light wheel is required, multiply by .0003; and if a heavy one, by .00016.

To Compute the Dimensions of the Rim.

Rule.—Multiply the weight of the wheel in lbs. by .1, and divide the product by the mean diameter of the rim in feet; the quotient will give the sectional area of the rim in square inches of cast iron.

Example.—A non-condensing engine, having a diameter of cylinder of 14 inches, and a stroke of piston of 4 feet, working full stroke, at a pressure of 65 lbs per mercurial gauge, and making 40 revolutions per minute, develops about 65 horses' power; what should be the dimensions of its fly-wheel, adapted to ordinary work?

Area of cylinder, 154 *ins.* stroke, $4\times3\frac{1}{2}=14$ *feet* = *diam. of wheel; mean pressure* = 50 *lbs.*; $50\times154\times4=30800$ = *product of pressure upon piston in lbs.; and the stroke of the piston*, which $\div 40^2\times14\times.00023=5978$ *lbs., weight of the wheel.*

Assume the mean diameter of the wheel $13\frac{1}{4}$ feet. Then $5978\times.1\div13.25=45.12$ *square ins. in the rim.*

Ex. 2.—If a fly-wheel, 16 feet in diameter and 4 tons in weight, is sufficient to regulate an engine when it revolves in 4 seconds, what should be the weight of a second fly-wheel, 12 feet in diameter, revolving in 2 seconds, so that it may have like centrifugal force?

Note.—The centrifugal forces of two bodies are as the radii of the circles of revolution directly, and as the squares of the times inversely.

Then $\frac{4\times16}{4^2}=\frac{x\times12}{2^2}$. Or, $x=\frac{4\times16\times2^2}{12\times4^2}=\frac{4\times16\times4}{12\times16}=1.333$ *tons.*

IMPACT OR COLLISION.

IMPACT is *Direct* or *Oblique*. If the impact of two elastic bodies is direct, their *relative* velocities will be the same, both before and after impact. Bodies are Elastic or Inelastic.*

The product of the mass and velocity of a body is the *momentum* of the body.

The principle upon which the motions of bodies from percussion or collision are determined belongs both to elastic and inelastic bodies; thus there exists in bodies the same momentum or quantity of motion, estimated in any one and the same direction, both before collision and after it.

Action and reaction are always equal and contrary. If a body impinge obliquely upon a plane, the force of the blow is as the sine of the angle of incidence.

The effect of the blow of an elastic body upon a plane is double that of an inelastic one, the velocity and mass being equal in each; for the force of the blow from the inelastic body is as its mass and velocity, which is only destroyed by the resistance of the plane; but in the elastic body that force is not only destroyed, being sustained by the plane, but another, also equal to it, is sustained by the plane, in consequence of the restoring force, and by which the body is repelled with an equal velocity; hence the intensity of the blow is doubled.

If two perfectly elastic bodies impinge on one another, their relative velocities will be the same, both before and after the impulse; that is, they will recede from each other with the same velocity with which they approached and met.

The general laws regarding the Collision of Bodies, assuming them to be inelastic and of equal volumes, are:

1. If two solid bodies are moving in the same direction, the common velocity, after collision, is equal to the sum of the products of the masses and their velocities, divided by the sum of the masses.

2. If two bodies are moving in opposite directions, the common velocity is equal to the difference of the products of the masses and velocities, divided by the sum of the masses.

3. If one body is at rest and the other in motion, the common velocity is equal to the product of the mass and velocity, divided by the sum of the masses.

When the ratio of elasticity of a substance is considered, the effect of collision of two bodies, as A and B, is determined as follows:

When two Bodies move in the same Direction.

$$\frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times \text{E} + (1+e)\,\text{B} \times \text{its vel.}}{\text{A}+\text{B}} = \textit{velocity of A after impact.}$$

$$\frac{\text{B} \times \text{its vel.} - \text{A} \times \text{B}^3\text{ vel.} \times e + (1+e)\,\text{A} \times \text{its vel.}}{\text{B}+\text{A}} = \textit{velocity of B after impact,}$$

e representing ratio of elasticity of the bodies.

When two Bodies move in opposite Directions.

$$\frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times e - (1+e)\,\text{B} + \text{its vel.}}{\text{A}+\text{B}} = \textit{velocity of A.}$$

$$\frac{\text{A} \times \text{B vel.} \times e - \text{B} \times \text{its vel.} + (1+e)\,\text{B} \times \text{its vel.}}{\text{B}+\text{A}} = \textit{velocity of B.}$$

* The division of bodies into *hard* and *elastic* is wholly at variance with these properties; as, for instance, glass, which is among the hardest of bodies, is the most elastic of all.

When one Body moves and the other is at Rest.

$\frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times e}{\text{A} + \text{B}} = \textit{velocity of } \text{A}.$ $\frac{\text{A} \times \text{its vel.} (1 + e)}{\text{A} + \text{B}} = \textit{velocity of } \text{B}.$

Motion over a Fixed Pulley.

$\frac{W - w}{W + w} = F$, in the formula where F is used; so that $s = \frac{W - w}{W + w} \frac{1}{2} g t^2$, *W and w representing the two weights which are connected by the cord that passes over the pulley.*

Or, if the resistance of the friction and inertia of the pulley be represented by r, then $s = \frac{W - w}{W + w + r} \frac{1}{2} g t^2$.

Illustration.—If by experiment it is ascertained that with two weights of 5 and 3 lbs. over a pulley, the heavier weight descended only 50 feet in 4 seconds, what is the measure of *r?*

If r is not considered, the heavier weight would fall $\frac{5-3}{5+3} \times .5 \times 32.166 \times 4^2 =$ 64.333 *feet.* Then $\frac{W - w}{W + w + r} \frac{1}{2} g t^2 = 50$ *feet.*

And, as $5 + 3 + r : 5 + 3 :: 64.333 : 50$;
That is, . . $r : 5 + 3 :: 14.333 : 50$. Whence $r = \frac{8 \times 14.333}{50} = 2.293$ *lbs.*

To Compute the Common Velocity of Two Elastic Bodies after Impact.

When both Bodies moved in the same Direction.

$\frac{B v + b v'}{B + b} = V$, *B and b representing the weight of the two bodies in lbs., v and v' their velocities before impact in feet per second, and V the velocity.*

Illustration.—An elastic body, b, weighing 30 lbs., having a velocity of 3 feet, is struck by another body, B, of 50 lbs., having a velocity of 7 feet; their velocities after impact will be

$$\frac{50 \times 7 + 30 \times 3}{50 + 30} = \frac{440}{80} = 5.5 \textit{ feet}.$$

When the Bodies moved in opposite Directions, $\frac{B v - b v'}{B + b} = V$. *When the lesser Body was at Rest,* $\frac{B v}{B + b} = V$. Consequently, $\frac{V - v}{B + b} \times b =$ *velocity lost by* B.

To Compute the Velocity of each Body.

When both Bodies moved in the same Direction, $\frac{2 b v + \overline{(B - b) V}}{P + b} =$ *velocity of* B, and $\frac{2 B V - \overline{(B - b) v}}{B + b} =$ *velocity of b.* Or, $V - \frac{2 b}{B + b}(V - v) =$ *velocity of* B, and $v + \frac{2 B}{B + b}(V - v) =$ *velocity of b.*

$\frac{2 b}{B + b}(V - v) =$ *velocity lost by* B; and $\frac{2 B}{B + b}(V - v) =$ ***velocity lost by b,*** which two velocities are in the ratio of b to B.

When the Bodies moved in opposite Directions, $\frac{(B - b) V - 2 b v}{B + b} =$ *velocity of* B, and $\frac{(B - b) v + 2 B V}{B + b} =$ *velocity of b.*

When the lesser Body was at Rest, $\frac{B - b}{B + b} V =$ *velocity of* B, and $\frac{2 B}{B + b} =$ *velocity of b.*

To Compute the Velocity of Two Imperfect or Inelastic Bodies after Impact.

When both Bodies moved in the same Direction, $V - \frac{m+n}{m} \times \frac{b}{B+b}(V-v) =$ *velocity of* B; and $v + \frac{m+n}{m} \times \frac{B}{B+b}(V-v) =$ *velocity of b, m and n representing the ratio of perfect to imperfect elasticity.*

When the Bodies moved in opposite Directions, $\frac{V(B-mb)-2bv}{B+b} =$ *velocity of* B, and $\frac{2BV-v(b-nB)}{B+b} =$ *velocity of b.*

When the lesser Body was at Rest, $\frac{V(B-bm)}{B+b} =$ *velocity of* B, and $\frac{2VB}{B+b} =$ *velocity of b.*

If two bodies are imperfectly elastic, the sum of their moments will be the same, both before and after collision, but the velocities after will be less than in the case of perfect elasticity, in the ratio of the imperfection.

PILE-DRIVING.

The effect of the blow of a ram, or monkey, of a pile-driver, is as the square of its velocity; but the impact is not to be estimated directly by this rule, as the degree and extent of the yielding of the pile materially affects it. The rule, therefore, is of value in application only as a means of comparison.

By my experiments in 1852, to determine the *dynamical* effect of a falling body, it appeared that while the effect was directly as the velocity, it was far greater than that estimated by the usual formula $\sqrt{s\,2g}$, which, for a weight of 1 lb. falling 2 feet, would be 11.34, giving a momentum of 11.34 ft. *lbs.;* whereas, by the effect shown by the record of actual observations, it would be v W 4.426 = 50 *lbs.*

Piles are distinguished according to their position and purpose: thus, *Gauge Piles* are driven to define the limit of the ground to be inclosed, or as guides to the permanent piling.

Sheet or *Close Piles* are driven between the gauge piles to form a continuous inclosure of the work.

The weight which is required of each pile to sustain should be computed as if it stood unsupported by any surrounding earth.

When the length of an oak pile does not exceed 16 times its diameter, it may be loaded permanently with a weight of 450 lbs. per square inch of its sectional area.

A heavy ram and a low fall is the most effective condition of operation of a pile-driver, provided the height is such that the force of the blow will not be expended in merely overcoming the inertia of the pile, and at the same time not from such a height as to generate a velocity which will be expended in crushing the fibres of the head of the pile.

The *refusal* of a pile intended to support a weight of 13½ tons can be safely taken at 10 blows of a ram of 1350 lbs., falling 12 feet, and depressing the pile .8 of an inch at each stroke.

Pneumatic Piles.—A hollow pile of cast iron, 2½ feet in diameter, was depressed into the Goodwin Sands 33 feet 7 inches in 5½ hours.

Nasmyth's Steam Pile-hammer has driven a pile 14 inches square, and 18 feet in length, 15 feet into a coarse ground, imbedded in a strong clay, in 17 seconds, with 20 blows of the hammer, or monkey, making 70 strokes per minute.

By the extended observations of Brevet Major John Sanders, U. S. Engineers, he deduced the following rule whereby to estimate the weight that can be safely borne upon a pile: "As many times the weight of the ram as the distance which the pile is sunk the last blow, is contained in the distance which the ram falls in making the blow, divided by 8," which, when reduced to a formula, becomes $\frac{(R \times h \div d)}{8} = W$, R *representing the weight of the ram in lbs., h the height of the fall, and d the distance the pile is depressed by the blow, both in feet.*

Here, then, is obtained a formula whereby to compute the limit of operation of a driver, which is essentially all that is required.

Illustration.—A ram weighing 3500 lbs., falling 3½ feet, depressed a pile 4.2 ins. Then $\frac{3500 \times (42 \div 4.2)}{8} = \frac{35000}{8} = 4375$ *lbs*, the weight which the pile would bear with safety.

By the ordinary formula, $\sqrt{v\,2\,g}\,W$, $15 \times 3500 = 52750$ *lbs.*, the computed force; hence, assuming the rule of Maj. Sanders as a guide, $\frac{4375}{52750} = .0814$, which may be taken as the coefficient whereby to reduce the momentum of a ram to the weight a pile can bear with safety.

Dr. Whewell deduced:

1. A slight increase in the hardness of a pile or in the weight of a ram will considerably increase the distance a pile may be driven.

2. The resistance being great, the lighter a pile the faster it may be driven.

3. The distance driven varies as the cube of the weight of the ram.

The weight of a pile bears so small proportion to the resistance of the earth that it may be neglected, for a pile 25 feet in length and 1 foot square weighs about ½ a ton; and if the fall of a ram weighing 1 ton is 10 feet, and the distance driven by the blow is 2 ins., then the resistance of the earth will be to the weight of the ram as 120 ins. to 2 ins.; that is, it will be 60 tons, of which ½ a ton is the $\frac{1}{120}$ part, and may therefore be neglected.

To Compute the Space through which a Pile is driven.

$\frac{Rh}{C} = s$, C *representing the resistance of the earth.* Hence, by inversion,

To Compute the Coefficient of the Resistance of the Earth.

$$\frac{Rh}{s} = C.$$

Weisbach gives the following formula: The resistance of the bed of earth being constant, the mechanical effect expended in the penetration of the pile will be $\frac{R^2 h}{P + Rs} = W$. Taking the elements of the preceding case, with the addition of the weight of the pile at 1500 lbs., the result would be $\frac{3500^2 \times 3.5}{1500 + 3500 \times (4.2 \div 12)} = \frac{42875000}{1750} = 24500$ *lbs.*

PENDULUMS.

Pendulums are *Simple* or *Compound*, the former being a material point, or single weight suspended from a fixed point, about which it oscillates, or vibrates, by a connection void of weight; and the latter, a like body or number of bodies suspended by a rod or connection. Any such body will have as many centres of oscillation as there are given points of suspension to it, and when any one of these centres are determined the others are readily ascertained. Thus, $s\,o \times s\,g$ = a *constant product*, and $s\,r = \sqrt{s\,o \times s\,g}$, *s g o and r representing the points of suspension, gravity, oscillation, and gyration.*

Or, any body, as a cone, a cylinder, or of any form, regular or irregular, so suspended as to be capable of vibrating, is a compound pendulum, and the distance of its centre of oscillation from any assumed point of suspension is considered as the length of an equivalent simple pendulum.

All vibrations of the same pendulum, whether great or small, are performed very nearly in the same time.

The *Number of Oscillations* of two different pendulums in the same time and at the same place are in the inverse ratio of the square roots of the lengths of these pendulums. The *Length of a Pendulum* vibrating seconds is in a constant ratio to the force of gravity.

The *Times of the Vibration* of pendulums are proportional to the square roots of their lengths. Consequently, the lengths of pendulums for different vibrations are as follows:

Latitude of Washington.

39.0958 ins. for one second.	4.344 for third of a second.
9.774 ins. for half a second.	2.4435 for quarter of a second.

Lengths of Pendulums vibrating Seconds at the Level of the Sea in several Places.

	Ins.		Ins		Ins.
Equator.......	39.0152	New York	39.1017	Paris	39.1284
Washington ...	39.0958	London.......	39.1393	Lat. 45°......	39.127

$\pi\sqrt{l \div g} = t$, *l representing the length of a pendulum vibrating seconds in inches, g the measure of the force of gravity* (32.155 at Washington, and 32.191 at London), *and t the time of one oscillation.*

ILLUSTRATION. — The length of a simple pendulum vibrating seconds, and the measure of the force of gravity at Washington, are 39.0958 *ins.*, and 32.155 *feet.*

$$3.1416\sqrt{\frac{39.0958}{32.155 \times 12}} = 3.1416 \times \sqrt{1.013} = 3.1416 \times .3183 = 1 \textit{ second.}$$

To Compute the Length of a Simple Pendulum for a given Latitude.

$39.127 - .09982 \cos. 2\,L = l$, L *representing the latitude.*

ILLUSTRATION.—Required the length of a simple pendulum vibrating seconds in the latitude of 50° 31′.

$L = 50° 31'$ $\cos. 2\,L = 2 \times 50° 31' = \cos. 180° - \overline{50° 31' \times 2} = \cos. 78° 58' = .19138 -$ $39.127 + \overline{.19138 \times .09982}$ (the two − or negative = an affirmative or +) = 39.1461 *ins.*

To Compute the Length of a Simple Pendulum for a given Number of Vibrations.

$\frac{L' t^2}{n^2} = l$, L′ *representing the length for the latitude, t the time in seconds, and n the number of vibrations.*

To Compute the Length of a Simple Pendulum, the Vibrations of which will be the same in Number as the Inches in its Length.

$$\sqrt[3]{(60\sqrt{L'})^2} = l \text{ in inches.}$$

Example.—What will be the length of a pendulum in New York, the vibrations of which will be the same number as the inches in its length?

$$\sqrt[3]{(\sqrt{39.1013 \times 60})^2} = 7.211^2 = 52 \text{ inches.}$$

To Compute the Number of Vibrations of a Simple Pendulum in a given Time.

$$\frac{\sqrt{L'}t}{\sqrt{l}} = n, \ \frac{t}{n} \text{ representing time of one vibration in seconds.}$$

To Compute the Time of Vibration of a Simple Pendulum, the Length being given.

$$\sqrt{l \div L'} \times 3.1416 = t \text{ in seconds.}$$

Example.—The length of a pendulum is 156.8 inches; what is the time of its vibration in New York?

$$\sqrt{\frac{156.8}{39.1017}} \times 3.1416 = 2 \text{ seconds.}$$

To Compute the Measure of Gravity, the Length of the Pendulum and the Number of its Vibrations being given.

$$\frac{.82246\, l n^2}{t^2} = g, \ g \text{ representing the measure of gravity in feet.}$$

To Compute the Centre of Gravity of a Compound Pendulum of Two Weights connected in a Right Line.

When the Weights are both on one Side of the Point of Suspension, $\frac{l\,W + l'w}{W + w} = o =$ *distance of centre of gravity from the point of suspension. When the Weights are on opposite Sides of the Point of Suspension,* $\frac{l\,W - l'w}{W + w} = o =$ *distance of centre of gravity of the greater weight from the point of suspension.*

Illustration.—A compound pendulum, composed of two weights alike to two cannon-balls, on the opposite sides of the point of suspension, and connected by a rod in a right line between them, has the following elements:

Lengths of pendulum from centre of points of suspension 25 and 28 inches. Weights of balls and connections from points of suspension, for 25 inches 38 lbs., and for 18 inches 12 lbs.

The length of it as a simple pendulum in latitude of New York, and that number of vibrations in one second, would be $\frac{25 \times 38 - 18 \times 12}{38 + 12} = 14.68$ *inches = distance of centre of gravity from point of suspension*; and $\frac{l^2W + l'^2w}{o\,(W + w)} = \frac{25^2 \times 38 + 18^2 \times 12}{14.68\,(38 + 12)} = 47.06$ *inches, the length of it as a simple pendulum; that is, from the point of suspension to a point extending below the greater weight.*

Hence $\sqrt{\frac{39.0958 \times 1}{\sqrt{47.06}}} = \frac{6.2527}{6.86} = .911$ *vibrations.*

2.—If the two weights were both on the same side of the point of suspension. Lengths of pendulum being $25 + 18 = 43$ and 18 *ins.*

$\frac{43 \times 38 + 18 \times 12}{38 + 12} = 46.25$ *ins. = distance of centre of gravity from point of suspension*; and $\frac{43^2 \times 38 + 18^2 \times 12}{46.25 \times (38 + 12)} = 40.08$ *ins., the length of it as a simple pendulum.*

GOVERNORS.

The operation of the *Governor* or *Conical Pendulum* depends upon the principles of Central Forces.

When in a *Ball Governor* the *Balls* diverge, the ring on the vertical shaft raises, and in proportion to the increase of the velocity of the balls squared, or the square roots of the distances of the ring from the fixed point of the arms, corresponding to two velocities, will be as these velocities.

Thus, if a governor makes 6 revolutions in a second when the ring is 16 inches from the fixed point or top, the distance of the ring will be 5.76 inches when the speed is increased to 10 revolutions in the same time.

For 10 : 6 : : $\sqrt{16}$: 2.4, which, squared = 5.76 *ins.*, *the distance of the ring from the top.* Or, $6^2 : 10^2 : 5.76 : 16$ *ins.*

A governor performs in one minute half as many revolutions as a pendulum vibrates, the length of which is the perpendicular distance between the plane in which the balls move and the fixed point or centre of suspension.

To Compute the Number of Revolutions of a Ball Governor per Minute to maintain the Balls at any given Height.

$\frac{188}{\sqrt{H}}$ = *revolutions*, H *representing the vertical height between the plane of the Balls and the points of their suspension in inches.*

To Compute the Vertical Height between the Plane of the Balls and their Points of Suspension.

$\left(\frac{188}{r}\right)^2$ = *vertical height in ins., r representing the number of revolutions in a minute.*

PNEUMATICS.—AERODYNAMICS.

The motion of gases by the operation of gravity is the same as that for liquids. The force or effect of wind increases as the square of its velocity.

If a volume of air, and of the temperature of 32°, is heated t degrees without assuming a different tension, the volume becomes $(1+.002088\,t)$ = V; and if it acquires the temperature t', it will then assume the volume $(1+.002088\,\overline{t'-32°})$. When air passes into a medium of less density, its velocity is determined by the difference of the densities. Under like conditions, a conduit will discharge 30.55 times more air than water.

The force of wind upon a surface perpendicular to its direction has been observed as high as 57¾ lbs. per sq. foot; velocity = 159 feet per second.

To Compute the Volume of Air discharged through an Opening.

$a\,C\sqrt{2\,g\,h}$ = V *in cubic feet, a representing area of opening in square feet, and* C *coefficient of efflux.*

To Compute the Velocity of Air in its Passage from a Greater to a Lesser Density.

$\frac{1347.4}{d}\,a\,C\sqrt{M\,(d+M)\,T}$ = V *in cubic feet, d representing density of the atmosphere,* M *the pressure in the reservoir from which it flows in inches of mercury, and* T $= 1 + .002088\,(t - 32°)$.

ILLUSTRATON.—What is the volume of air at a barometric pressure of 29.7 ins. which a reservoir will furnish, upon which a manometer indicates 1.2 ins. through a cylindrical pipe 3 ins. in diameter, temperature of the air 56°?

$$C = .93,\ T = 1 + .2002088\,(56 - 32) = 1.05,\ a = .05.$$

$$\text{Then } \frac{1347.4}{29.7} \times .05 \times .93 \times \sqrt{1.2 \times 29.7 + 1.2 \times 1.05} = 21.096 \times 6.24 = 13.16 \textit{ cubic feet.}$$

To Compute the Resistance of a Plane Surface to the Air.

$.0022\,a\,v^2 = P$, *a representing area of plane in square feet, v velocity of it in the direction of the wind in feet per second, + when it moves opposite, and — when it moves with the wind.*

Dr. Hutton deduced that the resistance of air varied as the square of the velocity nearly, and to an inclined surface as the 1.84 power of the sine × cosine.

The figure of a plane makes no appreciable difference in the resistance, but the convex surface of a hemisphere, with a surface double the base, has only half the resistance.

At high velocities, experiments upon railways show that the resistance becomes nearly a constant quantity.

The resistance of the air to a train of cars in a dead calm was found to be $\frac{1}{89}$ of their weight.

The velocity of a train of cars which give a resistance of the $\frac{1}{96}$ of the load with a fair wind was 34½ miles per hour, and only 27¾ with an adverse wind.

To Compute the Resistance of a Plane Surface when moving at an Angle to the Air.

$$\frac{v^2\,a\,\sin.^2 x}{450} = P \textit{ in pounds, x representing the angle of incidence.}$$

For other elements, etc., see Treatise on Aerometry of D'Aubuisson de Voissin, pages 124, 186, 234, and 313, vol. xxxix. *Journal of Franklin Institute.*

WIND-MILLS.

The driving shaft of a wind-mill should be set at an elevating angle with the horizon when set upon low ground, and at a depressing angle when set upon elevated ground. The range of these angles is from 3° to 35°. A velocity of wind of 10 feet per second is not generally sufficient to drive a loaded wind-mill, and if the velocity exceeds 35 feet per second the force is generally too great for the ordinary structure.

The angle of the sails should be from 18° to 30° at their least radius, and from 7° to 17° at their greatest radius, the mean angle being from 15° to 17° to the plane of motion of the sails. The length of an arm (whip) is divided into 7 parts, the sails extending over 6 parts.

Deductions from Velocities varying from 4 to 9 Feet per Second.—[MR. SMEATON.]

1. The velocity of wind-mill sails, so as to produce a maximum effect, is nearly as the velocity of the wind, their shape and position being the same.
2. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.
3. The effects of the same sails, at a maximum, are nearly, but somewhat less than, as the cubes of the velocity of the wind.
4. The load of the same sails, at the maximum, is nearly as the squares, and their effect as the cubes of their number of turns in a given time.

5. When sails are loaded so as to produce a maximum effect at a given velocity, and the velocity of the wind increases, the load continuing the same—1st, the increase of effect, when the increase of the velocity of the wind is small, will be nearly as the squares of those velocities; 2dly, when the velocity of the wind is double, the effects will be nearly as 10 to 27½; but, 3dly, when the velocities compared are more than double of that when the given load produces a maximum, the effects increase nearly in the simple ratio of the velocity of the wind.

6. In sails where the figure and position are similar, and the velocity of the wind the same, the number of revolutions in a given time will be reciprocally as the radius or length of the sail.

7. The load, at a maximum, which sails of a similar figure and position will overcome at a given distance from the centre of motion, will be as the cube of the radius.

8. The effects of sails of similar figure and position are as the square of the radius.

9. The velocity of the extremities of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably greater than that of the wind.

Results of Experiments on the Effect of Wind-mill Sails.

When a vertical wind-mill is employed to grind corn, the mill-stone usually makes 5 revolutions to 1 of the sail.

1. When the velocity of the wind is 19 feet per second, the sails make from 11 to 12 revolutions in a minute, and a mill will grind from 880 to 990 lbs. in an hour, or about 22000 in 24 hours.

2. When the velocity of the wind is 30 feet per second, a mill will carry all sail, and make 22 revolutions in a minute, grinding 1984 lbs. of flour in an hour, or 47609 lbs. in 24 hours.

The velocity of the wind in a brisk gale is from 15 to 20 miles per hour, exerting a pressure of 1 to 2 lbs. per square foot. A high wind moves at the rate of 30 to 40 miles per hour, with a pressure of 4½ to 8 lbs. per square foot; while in a storm it may vary from 50 to 60 miles per hour, and exert a pressure of 12½ to 18 lbs. per square foot.

To Compute the Mechanical Effect and Elements of Wind-mills.

$.00048\, n\, v^3\, a\, u = P$, *n representing number of arms, v velocity of wind per second, a area of sails in square feet, and u number of revolution of arms per minute.*

$$.1047\, u = \textit{angular velocity};\quad \frac{1144000}{v^3} = \textit{area of sails}.$$

$$\sqrt{\frac{R^2 + r^2}{2}} = r' = \textit{radius of centre of percussion of arms in feet}.$$

A wind-mill with four arms 70 feet in extreme diameter, and 6 feet wide, will raise 1000 lbs. 218 feet in 1 minute, and if working on an average of 8 hours per day, it is equal to 34 men. It is estimated that 25 square feet of canvas will perform the work of a man.

From 10 to 11 yards of sail will grind and dress 11 bushels of wheat.

$\frac{3.16\, v}{r' \sin. x}$ = *proper number of revolutions, x representing the mean angle of the sails to the plane of motion, and* $\frac{11.5\, v}{r'}$ = *number of revolutions when* $x = 16°$.

Illustration.—The number of arms of a wind-mill is 4, the velocity of the wind 16 feet per second, the area of the sails 250 square feet, and the revolutions of the arms 6 per minute.

Then $.00048 \times 4 \times 250 \times 16^3 \times 6 = 3.58$ *horses.*

2. Taking the preceding elements, the inner radius being 4 feet, the length of the arms 28, and the mean angle $= 16°$; then

$$.1047 \times 6 = .6282, \textit{ angular velocity; } \frac{1144000 \times 3.58}{16^3} = 1000, \textit{ area of sails.}$$

$$\sqrt{\frac{28^2 + 4^2}{2}} = 20 \textit{ feet radius; } \frac{3.16\,v}{r' \sin. x} = \frac{3.16 \times 16}{17.2 \times .27564} = \frac{50.56}{4.74} = 10.66 \textit{ revolutions.}$$

$$\text{Or, } \frac{11.5 \times 16}{20} = 9.2 \textit{ revolutions.}$$

HYDRODYNAMICS.

Hydrodynamics treats of the force of the action of Liquids or Inelastic Fluids, and it embraces *Hydrostatics* and *Hydraulics*: the former of which treats of the pressure, weight, and equilibrium of liquids in a state of rest, and the latter of liquids in motion, as the flow of water in pipes, the raising of liquids by pumps, etc.

Fluids are of two kinds, aeriform and liquid, or elastic and inelastic; Fluids press equally in all directions, and any pressure communicated to a fluid at rest is equally transmitted throughout the whole fluid.

The *Pressure* of a fluid at any depth is as the depth or vertical height, and the pressure upon the bottom of a containing vessel is as the base and perpendicular height, whatever may be the figure of the vessel. The pressure, therefore, of a fluid *upon any surface*, whether *Vertical*, *Oblique*, or *Horizontal*, is equal to the weight of a column of the fluid, the base of which is equal to the surface pressed, and the height equal to the distance of the centre of gravity of the surface pressed, below the surface of the fluid.

The pressure upon a number of surfaces is ascertained by multiplying the sum of the surfaces into the depth of their common centre of gravity, below the surface of the fluid.

The side of any vessel sustains a pressure equal to its area, multiplied by half the depth of the fluid, and the whole pressure upon the bottom and against the sides of a vessel is equal to three times the weight of the fluid.

When a body is partly or wholly immersed in a fluid, the vertical pressure of the fluid tends to raise the body with a force equal to the weight of the fluid displaced; hence the weight of any quantity of a fluid displaced by a buoyant body equals the weight of that body.

The bottom of a Conical, Pyramidal, or Cylindrical vessel, or of one the section of which is that of an inverted frustrum of a Cone or Pyramid, sustains a pressure equal to the area of the bottom and the depth of the fluid.

The *Centre of Pressure* is that point of a surface against which any fluid presses, to which, if a force equal to the whole pressure were applied, it would keep the surface at rest. Hence the distance of the centre of pressure of any given surface from the surface of the fluid is the same as that of the *Centre of Percussion*.

Centres of Pressure.

Of a Parallelogram, When the Side, Base, Tangent, or Vertex of the Figure is at the Surface of the Fluid, is at ⅔ of the line (measuring downward) that joins the centres of the two horizontal sides.

Of a Triangular Plane, When the Base is uppermost, is at the centre of a line, raised vertically from the vertex, and joining it with the centre of the base; and *When the Vertex is uppermost*, it is at ¾ of a line let fall perpendicularly from the vertex, and joining it with the centre of the base.

Of a Right-angled Triangle, When the Base is uppermost, is at the intersection of a line extended from the centre of the base to the extremity of the triangle by a line running horizontally from the centre of the side of the triangle. *When the Vertex or Extremity is uppermost*, it is at the intersection of a line extended from the centre of the base to the vertex, by a line running horizontally from ⅜ of the side of the triangle, measured from the base.

Of a Trapezoid, When either of the parallel Sides are in the Surface, $\frac{b+3b'}{2b+4b'}\times n = d$, *b and b′ representing the breadth of the figure.*

Of a Circle, is at ¾ of its radius, measured from the upper edge.

Of a Semicircle, When the Diameter is in the Surface of the Fluid, $\frac{3pr}{16} = d$, *d representing distance from surface of the fluid, and r radius of circle.*

When the Diameter is downward, $\frac{15pr-32r}{12p-16} = d$.

When the Side, Base, or Tangent of the Figure is below the Surface of the Fluid.

Of a Rectangle or Parallelogram, $\frac{2}{3}\times\frac{h'^3-h^3}{h'^2-h^2} = d$, *h and h′ representing the depths of the upper and under surfaces of the figure from the surface of the fluid.*

Or, $\frac{3o^2+m^2}{3o} = m$, *m representing half the depth of the figure, and o the depth of the centre of gravity of the figure from the surface of the fluid.*

Or, $\frac{3mo+m^2}{3o}$ = *distance from upper side of figure.* Or, $\frac{m^2}{3o}$ = *distance from centre of gravity.*

Of a Trapezoid, When either of the parallel Sides are Horizontal, $\frac{(b'^2+4bb'+b^2)\times n^2+18(b'+b)^2o^2}{18(b'+b)^2o} = d$, *n representing height of figure.*

Of a Triangular Plane, When the Vertex is uppermost, $\frac{n^2+18o^2}{18o} = d$, *distance let fall perpendicularly from the surface of the fluid upon a line joining the vertex and centre of gravity of the figure.*

Or, $\frac{n^2}{18o}$ = *distance from centre of gravity of the figure; and* $h+\frac{2n}{3}$ = *distance of centre of gravity of the figure below surface of fluid, h representing the depth of the vertex below the surface.*

When the Base is uppermost, $\frac{n^2+18o^2}{18o} = d$.

Of a Circle, $\frac{4o^2+r^2}{4o} = d$ Or, $\frac{r^2}{4o}$ = *distance below centre of the circle.*

Of a Semicircle, When the Diameter is Horizontal, and upward or downward, $\frac{n^2}{4o}-\frac{16n^2}{9po}+o = d$; $\frac{4n}{3p}$ = *distance of centre of gravity from the diameter in the first case, and* $\frac{3pn-4n}{3p}$ = *distance from the centre of the circumference in the second case.* And $\frac{n^3}{4o}-\frac{16n^2}{9po}$ = *distance of centre of pressure below centre of gravity.*

PRESSURE.

To Compute the Pressure of a Fluid upon the Bottom of its Containing Vessel.

RULE.—Multiply the area of the base by the height of the fluid in feet, and the product by the weight of a cubic foot of the fluid.

To Compute the Pressure of a Fluid upon a Vertical, Inclined, Curved, or any Surface.

RULE.—Multiply the area of the surface by the height of the centre of gravity of the fluid in feet, and the product by the weight of a cubic foot of the fluid.

EXAMPLE.—What is the pressure upon a sloping side of a pond of fresh water 10 feet square, the depth of the pond being 8 feet?

Centre of gravity, $8 \div 2 = 4$ *feet from the surface.*

Then $10^2 \times 4 = 400$, which $\times 62.5 = 25000$ *lbs.*

Ex. 2.—What is the pressure upon the staves of a cylindrical reservoir when filled with fresh water, the depth being 6 feet, and the diameter of the base 5 feet?

$5 \times 3.1416 = 15.708$ *feet curved surface of reservoir*, which must be considered as a plane. $15.708 \times 6 \times \overline{6 \div 2} = 282.744$, which $\times 62.5 = 17671.5$ *lbs.*

Ex. 3.—What is the pressure of fresh water upon a gate or embankment in the form of a trapezoid, its breadths at top and bottom being 11 and 9 feet, and its depth 10 feet?

$.333 \times \overline{11 - 9} + 9 = 9.666$, which $\div 2 = 4.833 =$ *the centre of gravity of the fluid.*

$\frac{11+9}{2} \times 10 \times 4.833 = 483.33$, which $\times 62.5 = 30208.125$ *lbs.*

Sluice-gates.

The stress upon a Sluice-gate is determined by its area, and the distance of its centre of gravity from the surface of the fluid.

EXAMPLE.—What is the pressure on a sluice-gate 3 feet square, its centre of gravity being 30 feet below the surface of a pond of fresh water?

$3 \times 3 \times 30 = 270$, which $\times 62.5 = 16875$ *lbs.*

Flood-gates.

The stress upon a Flood-gate is identical with that upon a Sluice-gate or any vertical surface.

EXAMPLE.—A rectangular flood-gate in fresh water is 25 feet in length by 12 feet deep; what is the pressure upon it?

$25 \times 12 \times \overline{12 \div 2} = 1800$, which $\times 62.5 = 112500$ *lbs.*

When water presses against both sides of a plane surface, there arises from the resultant forces, corresponding to the two sides, a new resultant, which is obtained by the subtraction of the former, as they are opposed to each other.

ILLUSTRATION.—The depth of water in a canal is 7 feet; in its adjoining lock it is 4 feet, and the breadth of the gates is 15 feet; what mean pressure have they to sustain, and what is the depth of the point of its application below the surface?

$7 \times 15 = 105$, and $4 \times 15 = 60$ *sq. feet.* $7 - 4 = 3$ *feet*, $\frac{1}{3} \times 7^2 = \frac{49}{3}$, and $\frac{1}{3} \times 4^2 = \frac{16}{3}$.

Hence $(105 \times \frac{7}{2} - \overline{60 \times 2}) \times 62.5 = 1546.875$ *lbs., the mean pressure.*

Then $1546.875 \div 62.5 = 247.5 =$ number of cubic feet pressing upon gates upon the high side, and $247.5 \div \overline{15 \times 7} = 2.35$ *feet = depth of centre of gravity of mean pressure.*

To Compute the Pressure of a Column of a Fluid per Square Inch.

RULE.—Multiply the height of the column in feet by the weight of a cubic foot of the fluid, and divide the product by 144; the quotient will give the weight or pressure per square inch in pounds.

NOTE.—When the height is given in inches, omit the division by 144.

EXAMPLE.—The height of a column of fresh water is 23 feet; what is its pressure per square inch?

$23 \times 62.5 = 1437.5$, which $\div 144 = 9.983$ *lbs.*

PIPES.

To Compute the required Thickness of a Pipe.

Rule.—Multiply the pressure in pounds per square inch by the diameter of the pipe in inches, and divide the product by twice the tensile resistance of a square inch of the material of which the pipe is constructed.

By experiment, it has been found that a cast-iron pipe 15 inches in diameter, and ¾ of an inch thick, will support a head of water of 600 feet; and that one of oak, of the same diameter, and 2 inches thick, will support a head of 180 feet.

Example.—The pressure upon a cast-iron pipe 15 inches in diameter is 300 lbs. per square inch; what is the required thickness of the metal?

$$300 \times 15 = 4500, \text{ which } \div \overline{3000 \times 2} = .75 \textit{ inch.}$$

Note.—Here 3000 is taken as the *value* of the tensile strength of cast iron in ordinary small water-pipes. This is in consequence of the liability of such castings to be imperfect from honey-combs, springing of the core, etc.

Ex. 2.—The pressure upon a lead pipe 1 inch in diameter is 150 lbs. per square inch; what is the required thickness of the metal?

$$150 \times 1 = 150, \text{ which } \div \overline{500 \times 2} = .15 \textit{ inch.}$$

HYDROSTATIC PRESS.

To Compute the Elements of a Hydrostatic Press.

$\frac{P\,l\,A}{l'\,a} = W$; $\frac{W\,l'\,a}{P\,l} = A$; $\frac{W\,l'\,a}{A} = P$; $\frac{P\,A\,l}{W\,l'} = a$, P *representing the power or pressure applied*, W *the weight or resistance in pounds*, l *and* l' *the lengths of the lever and fulcrum in inches or feet, and* A *and* a *the areas of the ram and piston in square inches.*

Illustration.—The areas of a ram and piston are 86.6 and 1 square inches, the lengths of the lever and fulcrum 4 feet and 9 inches, and the power applied 20 lbs.; what is the weight that may be borne?

$$\frac{20 \times 4 \times 86.6}{.75 \times 1} = \frac{6928}{.75} = 1924 \textit{ lbs.}$$

To Compute the Thickness of the Metal to resist a given Pressure.

Rule.—Multiply the pressure per square inch in pounds by the diameter of the cylinder in inches, and divide the product by twice the estimated practical tensile resistance or *value* of the metal in pounds per square inch, and the quotient will give the thickness of the metal required.

Example.—The pressure required is 9000 lbs. per square inch, and the diameter of the cylinder is 5.3 inches; what is the required thickness of the metal or cast iron?

$\frac{9000 \times 5.3}{6000 \times 2} = \frac{47.700}{12000} = 3.975$ *ins.* The *value* of the metal is here taken at 6000.

HYDRAULIC RAM.

The useful effect of a Hydraulic Ram, as determined by Eytelwein, varied from .9 to .18 of the power expended. When the height to which the water is raised compared to the fall is low, the effect is greater than with any other machine; but it diminishes as the height increases.

To Compute the Useful Effect of a Hydraulic Ram.

$1.2\,V\,(h - .2\sqrt{h\,h'}) = v\,h'$, V *and* v *representing the volume of water expended and raised, and* h *and* h' *the heights of the fall and of the water raised.*

Illustration.—The heights of a fall and of the elevation are 10 and 26.3 feet, and the volumes expended and raised are 1.71 and .543 cubic feet per minute.

$1.2 \times 1.71\,(10 - .2\sqrt{10 \times 26.3}) = 2.052 \times (10 - 3.243) = 2.052 \times 6.757 = 13.865$ *useful effect = product of volume and height to which it is raised.*

To Compute the Volume to be expended, When the Height and Volume required are given.

$$\frac{v\,h' \div (h - .2\sqrt{h\,h'})}{1.2} = V = \frac{13.865 \div (10 - .2\sqrt{10 \times 26.3})}{1.2} = 1.71\ feet.$$

To Compute the Height of Fall required, When the Volumes and Height of Elevation are given.

$$\left(\frac{\sqrt{h'}}{10} \pm \sqrt{\frac{v\,h'}{1.2\,V} + \frac{h'}{100}}\right)^2 = h = (.513 + \sqrt{6.757 + .263})^2 = (.513 + 2.6435)^2 = 10\ ft.$$

Table of Results of Operations of Hydraulic Rams.

Number of Strokes.	Height of Fall.	Height of Elevation.	Water Expended.	Water Raised.	Useful Effect.
Min.	Feet.	Feet.	Cubic Feet.	Cubic Feet.	
66	10.06	26.3	1.71	.543	.9
50	9.93	38.6	1.93	.421	.85
36	6.05	38.6	1.43	.169	.75
31	5.06	38.6	1.29	.113	.67
15	3.22	38.6	1 98	.058	.35
10	1.97	38.6	1.58	.014	.18
—	22.8	196.8	.38	.029	.67

WATER POWER.

Water acts as a moving power, either by its *weight* or by its *vis viva*, and in the latter case it acts either by *pressure* or by *impact*.

The *Natural Effect* or *Power* of a fall of water is equal to the weight of its volume and the vertical height of its fall.

If water is made to impinge upon a machine, the velocity with which it impinges may be estimated in the effect of the machine. The result or effect, however, is in nowise altered; for in the first case $P = V\,w\,h$, and in the latter $= \frac{v^2}{2\,g}\,V\,w$, V *representing the volume in cubic feet, w the weight in lbs., and v the velocity of the flow in feet per second.*

To Compute the Power of a Fall of Water.

Rule.—Multiply the volume of the flowing water in cubic feet per minute by 62.5, and this product by the vertical height of the fall in feet.

Note.—*When the Flow is over a Weir or Notch*, the height is measured from the surface of the tail-race to a point 4-9 of the height of the weir, or to the centre of velocity or pressure of the opening of the flow.

When the Flow is through a Sluice or Horizontal Slit, the height is measured from the surface of the tail-race to the centre of pressure of the opening.

Example.—What is the power of a stream of water when flowing over a weir 1 foot in depth by 5 feet in width, and having a fall of 20 feet from the centre of pressure of the flow?

By Rule, page 379, $\frac{2}{3}\,5 \times 1\sqrt{2\,g\,1} \times .623 = 16.65$ *cubic feet per second.*

$16.65 \times 60 \times 62.5 \times 20 = 1248750$ *lbs.*, which $\div\ 33000 = 37.84$ *horses' power.*

Water sometimes acts by its weight and *vis viva* simultaneously, by combining the effect of an acquired velocity with the fall through which it flows upon the wheel or instrument.

In this case the mechanical effect $= \left(h + \frac{v^2}{2\,g}\right) V \times 62.5.$

Sluices.

The methods of admitting water to an Overshot or Breast Wheel are various, consisting of the *Overfall*, the *Guide-bucket*, and the *Penstock*.

An *Overfall Sluice* is a saddle-beam with a curved surface, so as to direct the current of water tangentially to the buckets; a *Guide-bucket* is an apron by which the water is guided in a course tangential to the buckets; and a *Penstock* is the sluice-board or gate, placed as close to the wheel as practicable, and of such thickness at its lower edge as to avoid a contraction of the current. The bottom surface of the penstock is formed with a parabolic lip.

WATER-WHEELS.

Water-wheels are divided into two classes, Vertical and Horizontal. The vertical consist of the *Overshot*, *Breast*, and *Undershot*; and the Horizontal of the *Turbine* or *Reaction* wheels.

Vertical wheels are limited by construction to falls of less than 60 feet. Turbines are applicable to falls of any height from 1 foot upward.

Vertical wheels applied to a fall of from 20 to 40 feet give a greater effect than a Turbine, and for very low falls Turbines give a greater effect.

The order of effect of these wheels is as follows:

Ratio of Effect to Power.

Overshot and high breast	from .6 to .8 to 1	Undershot, Poncelet's	from .6 to .4 to 1
Turbine	from .6 to .8 to 1	Undershot do.	from .27 to .45 to 1
Breast	from .45 to .65 to 1	Impact and Reaction	from .3 to .5 to 1

The efficiency of *Turbines* for very high falls is less than for lower falls, on account of the hydraulic resistance involved, and which increases as the square of the velocity.

Turbines, being operated at a higher number of revolutions than Vertical Wheels, are more generally applicable to mechanical purposes; but in operations requiring but low velocities the Vertical Wheel is preferred. For variable resistances, as rolling-mills, etc., the Vertical Wheel is far preferable, as its mass serves to regulate the motion better than a small wheel.

In economy of construction there is no essential difference between a Vertical Wheel and a Turbine. When, however, the fall of water and the quantity of it are great, the Turbine is the least expensive. Variations in the supply of water affect vertical wheels less than Turbines.

The durability of a Turbine is less than that of a Vertical Wheel; and it is indispensable to its operation that the water should be free from sand, branches, leaves, etc.

With Overshot and Breast Wheels, when only a small quantity of water is available, or when it is required or becomes necessary to produce only a portion of the power of the *fall*, their efficiency is relatively increased, from the buckets being but proportionately filled; but with Turbines the effect is contrary, as when the sluice is lowered or the supply decreased the water enters the wheel under circumstances involving greater loss of effect. To produce the maximum effect of a stream of water upon a wheel, it must flow without impact upon it, and leave it without velocity; and the distance between the point at which the water flows upon a wheel and the level of the water in the reservoir should be as small as practicable.

Small wheels give less effect than large, in consequence of their making a greater number of revolutions and having a smaller water arc.

Shrouding.

The *Shrouding* of a wheel consists of the plates at its periphery, which form the sides of the bucket.

The height of the *fall* of a water-wheel is measured between the surfaces of the water in the *penstock* and in the *tail-race*, and ordinarily two thirds of the height between the level of the reservoir and the point at which the water strikes a wheel is lost for all effective operation.

The velocity of a wheel at the centre of percussion of the fluid should be from .5 to .6 that of the flow of the water.

By the deductions of Weisbach it appears that the effect of impact is only half the available effect under the most favorable circumstances; hence the least practicable part of a fall should be used to produce impact.

Under the circumstances of a variable supply of water, the *Breast-wheel* is better calculated for effective duty than the *Overshot*, as it can be made of a greater diameter; whereby it affords an increased facility for the reception of the water into its buckets, also for its discharge at the bottom; and further, its buckets more easily overcome the retardation of back-water, enabling it to be worked for a longer period in the back-water consequent upon a flood.

Friction of the Gudgeons.

A very considerable portion of the mechanical effect of a wheel is lost in the effect absorbed by the friction of the gudgeons.

To Compute the Friction of the Gudgeons of a Water-wheel.

$W\,r\,n\,C\,.0086 = f$, W *representing the weight of the wheel. r the radius of the gudgeon in inches, and n the number of revolutions of the wheel.*

For well-turned surfaces and good bearings, $C = .075$ with oil or tallow; when the best of oil is well supplied $= .054$; and, as in ordinary circumstances, when a black-lead unguent is alone applied $= .11$.

Illustration.—A wheel weighing 25000 lbs. has gudgeons 6 inches in diameter, and makes 6 revolutions per minute; what is the loss of effect?

Assume $C = .08$. Then $25000 \times \frac{6}{2} \times 6 \times .08 \times .0086 = 309.6$ *lbs.*

OVERSHOT-WHEELS.

In an *Overshot-wheel* the flow of water acts in some degree by impact, but chiefly by its weight.

The lower the speed of the wheel at its circumference, the greater will be the mechanical effect of the water. A proper velocity is about 5 feet per second.

The number of buckets should be as great as practicable, and they should retain water so long as practicable. The maximum effect is attained when the buckets are so numerous and close that the water surface in the bucket commencing to empty itself should come in contact with the under side of the bucket next above it.

Curved buckets give the greatest effect, and Radial give but .78 of the effect of Elbow-buckets. A wheel 40 feet in diameter should have 152 buckets.

Small wheels give a less effect than large, in consequence of their greater centrifugal action, and discharging the water from the buckets at an earlier period than with larger wheels, or when their velocity is lower.

When the head of water bears to the fall or the height of the wheel a proportion as great as 1 to 4 or 5, the ratio of effect to power is reduced to .8 and even .75. The general law, therefore, is, that the ratio of effect to power decreases as the proportion of *head* to the *total head and fall* increases.

A wheel with shallow *Shrouding* acts more efficiently than one where it is deep, and the depth is usually made 10 or 12 inches, but in some cases it has been increased to 15 inches.

The breadth of a wheel depends upon the capacity necessary to give the buckets to receive the required volume of the water.

Form of Buckets.—Radial buckets—that is, when the bottom is a right line—involve so great a loss of mechanical effect as to render their use incompatible with economy; and when a bucket is formed of two pieces, the lower or inner piece is termed the *bottom* or *floor*, and the outer piece the *arm* or *wrist*. The former is usually placed in a line with the radius of the wheel.

The line of a circle passing through the *elbow*, made by the junction of the *floor* and *arm*, is termed the division circle, or *bucket pitch*, and it is usual to put this at one half the depth of the shrouding.

When the *arm* of a bucket is included in the division angle of the buckets, that is, $\frac{360°}{n}$, *n representing the number of buckets*, the cells are not sufficiently covered, except for very shallow shrouding; hence it is best to extend the arm of a bucket over five fourths of the division angle, so as to cover or overlap the *elbow* of the bucket next in advance of it.

The least section of a cell should be somewhat greater than the section of the water flowing on to the wheel, and the cells should be in the plane of the flow of the water.

Fairbairn gives the area of the opening of a bucket in a wheel of great diameter, compared to the volume of it, as 5 to 24.

Buckets having a bottom of two planes, that is, with two bottoms, and two division circles or bucket pitches and an arm, give a greater effect than with one bottom.

When an opening is made in the base of the buckets, so as to afford an escape of the air contained within it without a loss of the water admitted, the buckets are termed *ventilated*, and the effective power of the wheel is much greater than with the close buckets.

To Compute the Radius of a Wheel, the Number of Revolutions, and the Height of the Fall of Water upon an Overshot-wheel,

When the whole Fall and the Velocity of the Flow, etc., are given.

$\frac{h - h'}{1 + \cos. a} = r$; $\frac{h\,c}{3.1416\,r} = n$, *h representing the height of the whole fall, h′ the height between the centre of gravity of the discharge and the half depth of the bucket upon which the water flows, a the angle which the point of entrance of the water into a bucket makes with the summit of the wheel, n the number of revolutions, c the velocity of the wheel at its circumference, and r its radius.*

NOTE.—As a proportion of the velocity of the flow is lost, it is proper to assume the height h' as but $= \frac{v^2}{2\,g}\,1.1$.

ILLUSTRATION.—A fall of water is 30 feet, the velocity of its flow is 16, the angle of its impact upon the buckets is 12°, and the required velocity of the wheel is 8 feet per second; what is the required radius, number of revolutions, and the height of the fall upon the wheel?

$h' = \frac{16^2}{2\,g} \times 1.1 = 4.38$ *feet*; $\cos. 12° = .97815$. Then $\frac{30 - 4.38}{1 + .978} = \frac{25.62}{1.978} = 12.95$ *feet radius*; $\frac{30 \times 8}{3.1416 \times 12.95} = \frac{240}{40.684} = 5.9$, *number of revolutions.*

When the Number of Revolutions and the Ratio between the Velocities of the Flow and at the Circumference of the Wheel are given.

$\frac{\sqrt{.000772\,(x\,n)^2\,h + (1 + \cos. a)^2} - (1 + \cos. a)}{.000386\,(x\,n)^2} = r$, *x representing* $\frac{v}{c}$, and $c = \frac{3.1416\,n\,r}{30}$.

ILLUSTRATION.—If the number of revolutions are 5, $x = 2$, and the fall, etc., as in the previous case; what is the radius of the wheel and the velocity of the flow?

$\frac{\sqrt{.000772\,(2 \times 5)^2 \times 30 + (1.978)^2} - 1.978}{.000386\,(2 \times 5)^2} = \frac{\sqrt{2.316 + 3.9125} - 1.978}{.0386} = \frac{.5177}{.0386} =$ 13.41 *feet*; $\frac{3.1416 \times 5 \times 13.41}{30} = 7.03$ *feet velocity at circumference of wheel*, and as $x = 2$. Hence $7.03 \times 2 = 14.06$ *velocity of flow.*

Further, if the velocity of the flow is 14.06 feet, what is the height of it?

$$\frac{14.06^2}{64.333}\times 1.1 = 3.37\ \textit{feet.}$$

To Compute the Width of an Overshot-wheel.

$\frac{o\,V}{s\,c'} = w$, *o representing a coefficient* $= 3$, *when the buckets are filled to an excess, and* 5 *when they are deficiently filled; s depth of shrouding in feet; c' the velocity of the wheel at the centre of the shrouding; and w the width of the buckets in feet.*

Illustration.—A wheel is to be 30 feet in diameter, with a depth of shrouding of 1 foot, and is required to make 5 revolutions per minute under a discharge of 10 cubic feet per second; what should be the width of the buckets?

Assume $o = 4$, and $c' = \frac{30\times 3.1416\times 5}{60} = 7.854$. Then $\frac{4\times 10}{1\times 7.854} = 5.09$ *feet.*

To Compute the Number of Buckets.

$7\left(1+\frac{s}{10}\right)$ = *distance between the buckets, s representing depth of shrouding in inches;* and $\frac{p\,d\,s}{v}$ = *number of buckets.*

Illustration.—Take the elements of the preceding case.

Then $7\left(1+\frac{12}{10}\right) = 7\times 2.2 = 15.4$ *ins.*, and $\frac{30\times 3.1416\times 12}{15.4} = 73.4$, say 72 *buckets;* hence $\frac{360^\circ}{72} = 5^\circ$, *angle of subdivision of buckets.*

To Compute the Effect of an Overshot Water-wheel.

$\frac{V\,h'\,w - \left(\frac{v'^2}{2\,g}V\,w + f\right)}{V\,h\,w} = P$, V *representing the volume of the water flowing per second, w the weight of the water, and v' the velocity of the water discharged at the tail of the wheel.*

Illustration.—A volume of 12 cubic feet per second has a fall of 10 feet, the wheel using but 8.5 feet of it, and the velocity of the water discharged is 9 feet per second; what is the effect of the fall?

The friction of the wheel is assumed to be 750 *lbs.*

$$\frac{12\times 8.5\times 62.5 - \left(\frac{9^2}{64.333}\times 12\times 62.5 + 750\right)}{12\times 10\times 62.5} = \frac{6375-(1.26\times 750+750)}{7500} = \frac{4680}{7500} = .624$$

= *ratio of effect to power;* and 4680×60 *seconds* $\div 33000 = 8.59$ *horses' power.*

To Compute the Power of an Overshot-wheel.

Rule.—Multiply the weight of water in lbs. discharged upon the wheel in one minute by the height or distance in feet from the centre of the opening in the gate to the surface of the tail-race; divide the product by 33000, and multiply the quotient by the assumed or determined ratio of effect to power. Or, for general purposes, divide the product by 50000, and the quotient is the horses' power.

The *Mechanical Effect* of water is the product of its weight into the height from which it falls.

Example.—The volume of water discharged upon an overshot-wheel is 640 cubic feet per minute, and the effective height of the fall is 22 feet; what are the horses' power?

$\frac{640\times 62.5\times 22}{33000} = \frac{880000}{33000} = 26.67$, which, $\times .75$ = the assumed ratio of effect to power in such a case = 20 *horses.*

Useful Effect of an Overshot-wheel.

With a large wheel running in the most advantageous manner, .84 of the power may be taken for the effect.

The velocity of a wheel bears a constant ratio, for maximum effects, to that of the flowing water, and this ratio is at a mean .55.

The ratio of effect to power with radial-buckets is .78 that of elbow-buckets. The ratio of effect decreases as the proportion of head to the total head and fall increases. Thus a wheel of 10 feet in diameter gave with heads of water above the gate, ranging from .25 to 3.75 feet, a ratio of effect decreasing from .82 to .67 of the power.

NOTE.—For the Theoretical Effect of a wheel, whether Overshot, Undershot, or Breast, see Formulæ of Weisbach and Morin. (Weisbach, vol. ii., pages 184–215.)

BREAST-WHEELS.

Breast-wheels are designed for falls of water varying from 5 to 15 feet, and for flows of from 5 to 80 cubic feet per second. They are constructed with either ordinary buckets or with blades confined by a *Curb*.

When buckets are inclosed in a *curb*, they are not required to hold water; hence they may be set *radial*. The buckets should be numerous, as the loss of water escaping between the wheel and the curb is less the greater their number; and that they may not lift or carry up water with them from the tail-race, it is proper to give the bucket such a plane that it may leave the water as nearly vertical as may be practicable.

The distance between two buckets should be equal to the depth of the shrouding, or at from 10 to 15 ins.

It is essential that there should be air-holes in the floor of the buckets, to prevent the air from impeding the flow of water into them, as the water admitted is nearly as deep as the interval between them; and the velocity of the wheel should be such that the buckets should be filled to ½ or ⅝ of their volume.

The inclosure within which the water flows to a breast-wheel as it leaves the sluice is termed a *Curb* or *Mantle*.

When wheels are constructed of iron, and are accurately set in masonry, a clearance of .5 of an inch is sufficient.

High Breast-wheels are used when the level of the water in the *tail-race* and *penstock* or *forebay* are subject to variation of heights, as the wheel revolves in the direction in which the water flows from the buckets, and *back-water* is therefore less disadvantageous, added to which, penstocks can be so constructed as to admit of an adjustable point of opening for the water to flow upon the wheel.

The effect of this wheel is equal to that of the overshot, and in some instances, from the advantageous manner in which the water is admitted to it, it is greater when both wheels have the same general proportions.

With a wheel 30 feet in diameter, having 96 buckets, the water admitted at a point 50° from the summit of it, with a velocity of 8 feet per second, the wheel having a velocity of 5 feet, the ratio of effect was .69. When the water flows at from 10° to 12° above the horizontal centre of the wheel, Fairbairn gives the area of the opening of the bucket, compared with the volume of it, as 8 to 24.

To Compute the Proportion and Effect of a Breast-wheel.

ILLUSTRATION.—The flow of water is 15 cubic feet per second; the height of the fall, measured from the centre of pressure of the opening to the tail-race, is 8.5 feet; velocity of revolution 5 feet per second; and depth of buckets 1 foot, filled to .5 of their volume.

Width of wheel $=\frac{V}{s\,v}$, *s representing depth, and v velocity of buckets*; $\frac{15}{1\times5}=3$, and as the buckets are but .5 filled, $3\div.5=6$ *feet.*

Assume the water is to flow with double the velocity of the rotation of the wheel; hence $v=5\times2=10$ *feet;* and the fall required to generate this velocity $=\frac{v^2}{2g}\times1.1$ $=h'=\frac{100}{64.333}\times1.1=1.71$ *feet.*

Deducting this height from the total fall, there remains for the height of the curb, or for the fall during which the weight of the water alone acts, $h-h'=8.5-1.71=6.79$ *feet.*

Making the radius of the wheel 12 feet, and the radius of the bucket circle 11.5 feet, the whole of the mechanical effect of the flow of the water $=15\times62.5\times8.5=7963.75$ *lbs.*

The theoretical effect, as determined by Weisbach, vol. ii., p. 210 = 7273 *lbs.*, from which are to be deducted the losses, which he computes as follows:

Loss by escape of water between the wheel and the curb	=	916
Loss by escape at sides of wheel and the curb	=	180
Friction and resistance of the water	=	160
		1256 *lbs.*

Assuming the weight of the wheel 16500 lbs., the radius of the gudgeons to be 2.5 inches, C, as before, = .08, and $n=\frac{5\times60}{12\times2\times3.1416}=4.$

Then $16500\times2.5\times4\times.08\times.0086=113.5$ *lbs.*, and $1256+113.5=1369.5$; hence $7273-1369.5=5903.5$; $\frac{5903.4}{7968.75}=.74$, and $5903.5\times60\div33000=10.73$ *horses' power.*

To Compute the Power of a Breast-wheel.

Rule.—Proceed as per rule for an overshot-wheel, using 55000 and .6 with a high breast, and 62500 and .6 for a low breast.

The Committee of the Franklin Institute ascertained that, with a high breast-wheel 20 feet in diameter, the water admitted under a head of 9 inches, and at 17 feet above the bottom of the wheel the elbow-buckets gave a ratio of effect to power of .731 at a maximum, and radial-buckets .653. With the water admitted at a height of 33 feet 8 ins., the elbow-buckets gave .658, and the radial .628.

At 10.96 feet above the bottom of the wheel, with a head of 4.29 feet, the elbow-buckets gave .544, and the radial .329.

At 7 feet above the bottom of the wheel, and a head of 2 feet, this *low breast* gave .62 for elbow-buckets, and .531 for radial.

At 3 feet 8 inches above the bottom of the wheel, and a head of 1 foot, the elbow-buckets gave .555, and the radial .533.

UNDERSHOT-WHEELS.

Undershot-wheels are usually set in a curb, with as little clearance for the escape of water as practicable; hence a curb concentric to this wheel is more effective than one set straight or tangential to it.

The computations for an undershot-wheel and the rules for construction are nearly identical with those for a breast-wheel.

The buckets are usually set radially, but they may be inclined upward, so as to be more effectively relieved of water upon their return side, and they are usually filled from .5 to .6 of their volume. The depth of the shrouding should be from 15 to 18 inches, in order to prevent the overflow of water within the wheel, which would retard it.

The sluice-gate should be set at an inclination to the plane of the curb, or tangential to the wheel, in order that its aperture may be as close to the wheel as practicable; and in order to prevent the partial contraction of the flow of water, the lower edge of the sluice should be rounded.

The effect of undershot-wheels is less than that of breast-wheels, as the fall available as weight is less than with the latter.

To Compute the Power of an Undershot-wheel.

Proceed as per rule for an overshot-wheel, using 93750 for 50000, and .4 for .75.

PONCELET'S WHEEL.

In a *Poncelet Wheel* the buckets are curved, so that the flow of water is along their concave side, pressing upon them without impact; and the effect is greater than when the water impinges at nearly right angles to plane-surfaced buckets.

This wheel is advantageous for application to falls under 6 feet, as their effect is greater than that of other undershot-wheels with a curb, and for falls from 3 to 6 feet their effect is equal to that of a Turbine.

In their arrangement, the aperture of the sluice should be brought close to the face of the wheel. The first part of the course should be inclined from 4° to 6°; the remainder of the course, which should cover or embrace at least 3 buckets, is carried concentric to the wheel, and at the end of it a quick fall of 6 ins. is made, to guard against the effect of back-water. The sluice should not be opened over 1 foot in any case, and 6 ins. is a suitable height for falls of 5 and 6 feet.

The distance between two buckets should not exceed 8 or 10 ins., and the radius of the wheel should not be less than 40 ins., or more than 8 feet.

The plane of the stream or head of water should meet the periphery of the wheel at an angle of from 24° to 30°. The space between the wheel and its curb should not exceed .4 of an inch.

The depth of the shrouding should be such as to prevent the water from flowing through it and over the buckets, and the width of the wheel should be equal to that of the stream of flowing water.

The effect of this wheel increases with the depth of the water flow, and, therefore, other elements being equal, as the filling of the buckets, to obtain the maximum effect, the water should flow to the buckets without impact, and the velocity of rotation of wheel should be only a little less than half the velocity of the water flowing upon the wheel. The effect is a maximum when $c = .5\, v \cos. y$, and then $\frac{v^2 \cos. y^2}{2g} V w = L.$

To Compute the Proportions of a Poncelet Wheel.

NOTE.—As it is impracticable to arrive at the results by a direct formula, they must be obtained by gradual approximation.

EXAMPLE.—The height of the fall is 4.5 feet; the volume of the flowing water 40 cubic feet per second; the radius of the wheel $= 2h$, or 9 feet; the depth of the stream $= \frac{1}{6} h = .75$ *feet;* and C assumed to be .9.

h representing height of fall in feet,
V volume of flowing water in cubic feet,
n number of revolutions $= \frac{30\,c}{p\,n}$,
c velocity of circumference of wheel,
a radius of wheel,
C coefficient of resistance of flow of water,
d depth of shrouding $= \frac{1}{4} \cdot \frac{v^2}{2g} + d'$,
d′ depth of opening of sluice,
e width of sluice,
r radius of curvature of buckets $= \frac{d}{\cos. z}$.
v velocity of flowing water,

x angle between plane of flowing water and that of the circumference of the wheel at the point of contact, sin. *of* $\frac{x}{2} = \sqrt{\cos. z}$.

z angle made by circumference of wheel with end of buckets $= 2$ tang. y,

y angle of direction of flowing water from circumference of wheel $= \frac{pc}{2a}\sqrt{\frac{d}{g+\frac{c^2}{a}}}$.

Then $v = .9\sqrt{2g\left(h - \frac{d'}{2}\right)} = .9 \times 16.29 = 14.66$ *feet = velocity of flowing water* ∴ *the velocity of the wheel,* being less than half the velocity of the flowing water; $c = \frac{14.66 - .66}{2} = 7$ *feet; and the depth of the shrouding* $= \frac{1}{4} \times \frac{14.66^2}{2g} + .75 = \frac{214.92}{257.33} + .75 = 1.58$ *feet;* $y = \frac{3.1416 \times 7}{2 \times 9} \times \sqrt{\frac{1.58}{32.166 + \frac{7^2}{9}}} = 1.22 \times \sqrt{.042} = .25$, the angle corresponding to which $= 14° 30'$; $n = \frac{30 \times 7}{3.1416 \times 9} = 7.43$ *revolutions;* $z = 2$ tang. $y = 2 \times .25862 = .51724$ ∴ $z = 27° 20'$; $e = \frac{40}{.75 \times 14.66} = 3.63$ *feet;* $r = \frac{1.58}{\cos. 27° 20'} = \frac{1.58}{.88835} = 1.78$ *feet;* $x =$ sin. $\frac{x}{2} = \sqrt{\cos. z} = \sqrt{\cos. 27° 20'} = .943 =$ sin. of $70° 34'$ ∴ $x = 141° 8'$.

IMPACT AND REACTION WHEELS.

If the buckets are given increased length, and formed to such a hollow curve that the water leaves the wheel in nearly a horizontal direction, the water then both impinges on the buckets and exerts a pressure upon them; therefore the effect is greater than with an impact wheel alone.

Impact Wheels.—Impact Turbines are the most simple but least efficient form of impact wheel. They consist of a series of rectangular buckets or blades, set upon a wheel at an angle of 50° to 70° to the horizon; the water flows to the blades through a pyramidal trough set at an angle of 20° to 40°, so that the water impinges nearly at right angles to the blades. The effect is estimated at about .5 the entire mechanical effect, which is increased by inclosing the blades in a border or frame.

Reaction Wheels.—The reaction of water issuing from an orifice of less capacity than the section of the vessel of supply is *equal to the weight of a column of water, the basis of which is the area of the orifice or of the stream, and the height of which is twice the height due to the velocity of the water discharged.*

Hence the expression is $2 \cdot \frac{v^2}{2g} a w = L$, *w representing the weight of a cubic foot of water, and a the area of the opening.*

If the water flows out at the side of the vessel, the direction of the reaction is horizontal; and if the water vein is contracted, C, a coefficient of contraction, must be introduced, as $2 \cdot \frac{v^2}{2g} C w a = L$.

In the discharge through a thin plate, Mr. Ewart, of Manchester, determined C = .96; and when the orifice was provided with a mouth-piece in the form of the *vena contracta*, it was .94.

To Compute the Effect of a Reaction-wheel.

$h + \frac{v^2}{2g} = h'$ = *height determining the pressure of water upon the orifice, h representing the height of the centre of the discharge from the centre of pressure of the opening of the supply-vessel.*

Hence $C\sqrt{2gh+v^2}-v'=s$ = *absolute velocity of the water, v' representing velocity of rotation of the wheel in the opposite direction to the efflux of the water, and* $\frac{v(\sqrt{2gh+v^2}-v)}{g}Vw=L$.

This efflux increases as v, the maximum effect depending upon the wheel acquiring an infinite velocity, and as the velocity increases the resistances increase.

When this wheel is loaded, so that the height due to the velocity, corresponding to the velocity of rotation v, is equal to the fall, or $\frac{v^2}{2g}=h$, or $v=\sqrt{2gh}$, there is a loss of 17 per cent. of the available effect; and when $\frac{v^2}{2g}=2h$, there is a loss of but 10 per cent.; and when $\frac{v^2}{2g}=4h$, there is a loss of but 6 per cent. Consequently, for moderate falls, and when a velocity of rotation exceeding the velocity due to the height of the fall may be adopted, this wheel works very effectively.

The efficiency of the wheel is but one half that of an undershot-wheel.

When the sluice is lowered, so that only a portion of the wheel is opened, the efficiency of a Reaction-wheel is less than that of a Pressure Turbine.

TURBINES.

In *High-pressure Turbines* the reservoir (of the wheel) is inclosed at top, and the water is admitted through a pipe at its side. In *Low-pressure*, the water flows into the reservoir, which is open.

In Turbines working under water, the height (h) is measured from the surface of the water in the supply to the surface of the discharged water or race; and when they work in air, the height is measured from the surface in the supply to the centre of the wheel.

In order to obtain the maximum effect from the water, the velocity of it, when leaving a Turbine, should be the least practicable.

The efficiency is greater when the sluice or supply is wide open, and it is less affected by head than by variations in the supply of water. It varies but little with the velocity, as it was ascertained by experiment that when 35 revolutions gave an effect of .64, 55 gave but .66.

When Turbines operate under water, the flow is always full through them; hence they become *reaction-wheels*, which are the most efficient.

The experiments of Morin gave results of the efficiency of Turbines as high as .75 of the power expended.

The angle of the plane of the water entering a Turbine with the inner periphery of it should be greater than 90°, and the angle which the plane of the water leaving the reservoir makes with the inner circumference of the Turbine should be less than 90°.

When Turbines are constructed without a *guide curve*,* the angle of plane of flowing water and inner circumference of wheel = 90°.

Great curvature involves greater resistance to the efflux of the water; and hence it is advisable to make the angle of the plane of the entering water rather obtuse than acute, say 100°; the angle of the plane of the water leaving, then, should be 50°, if the internal pressure is to balance the external; and if the wheel operates free of water, it may be reduced to 25° and 30°.

* Guide curves are plates upon the centre body of a Turbine, which give direction to the flowing water, or to the blades of the wheel which surround them.

The angle made by the plane of the discharged water with the water periphery should never exceed 20°.

Fourneyron's work either in or out of water, are applicable to high and low falls, and are either high or low pressure Turbines. They are best adapted for very low falls, and those of moderate height, say up to 30 feet, with large supplies of water. The pressure upon their step is confined to the weight of the wheel alone.

Fourneyron makes the angle of the plane of the water entering a Turbine = 90°, and the angle of the plane of the water leaving = 30°.

Jonval's.—This wheel is essentially alike in its principal proportions to Fontaine's, and in the principle of operation it is the same. The water in the race must be at a certain depth below the wheel.

The efficiency of this wheel decreases as the volume of water is diminished, or as the sluice is contracted.

Fontaine's.—In the operation of this wheel the water in the race is in immediate contact with the wheel, and its efficiency is greatest when the sluice is fully opened. Its efficiency, also, is less affected by variations of the head of the flow than in the volume of the water supplied; hence they are adapted for *Tide-mills.*

The pressure upon the step, in addition to the weight of the wheel, includes that of the contained water.

Whitelaw's.—This wheel is best adapted for high falls and small volumes of water.

Poncelet's.—This wheel is alike to one of his undershot-wheels set horizontally, and it is the most simple of all the horizontal wheels.

The Ratio of Effect to Power of the several Turbines is as follows:

Poncelet	.65 to .75 to 1	Jonval	.6 to .7 to 1
Fourneyron	.6 to .75 to 1	Fontaine	.6 to .7 to 1
Whitelaw	.6 to .75 to 1		

A *Tremont* Turbine, as observed by Mr. Francis, in his experiments at Lowell, Mass., gave a ratio of effect to power as .79375 to 1.

To Compute the Horses' Power of a Turbine.

$.0425\, D^2 h \sqrt{h} = P$, D *representing the diameter of the wheel at the outer extremities of the buckets.* Or, $.085\, V h = P$, V *representing the volume of water expended in cubic feet per second.*

EXAMPLE.—The diameter of a Turbine is 8.3 feet, and the fall of the flowing water is 16 feet; what is the power of it?

$$.0425 \times 8.3^2 \times 16 \times \sqrt{16} = .0425 \times 68.89 \times 16 \times 4 = 187.38 \textit{ horses.}$$

DIMENSIONS OF WATER-WHEELS AND TURBINES.

To Compute the Diameter of a Shaft, When the Stress is considered as being laid uniformly along its Length.

When of Cast Iron.

$\dfrac{\sqrt[3]{W \times l}}{9.6} = d$, W *representing the weight or load in lbs*, l *the length of the shaft between the journals in feet, and* d *the diameter of the shaft in its body in inches.*

When the Shaft has to resist both Lateral and Torsional Stress, Ascertain the diameter for each stress, and the cube root of the sum of their cubes will give the diameter required.

To Compute the Diameter of the Gudgeons,

The Length being considered 1.25 *times its Diameter.*

$.048 \sqrt{W} = d$, W *representing the weight upon each gudgeon.*

For other Rules, see Strength of Materials, page 447.

To Compute the Dimensions of the Arms.

When of Cast Iron.

$\frac{1.7d}{\sqrt[3]{n}} = w$, *d representing the diameter of the shaft in inches, n the number of arms, and w the width of the arm in inches* ; $\frac{w}{5} = t$, *t representing thickness of the arm*

When the Arm is of Oak, *w* should be 1.4 times that of iron, and the thickness .7 that of the width.

For the Elements of a Turbine, see Experiments of J. B. Francis, pages 44, 54, and *Nystrom's Mechanics*. p. 226, 6th Edition, omitting No. 25.

Barker's Mill.—The effect of this mill is considerably greater than that which the same quantity of water would produce if applied to an undershot-wheel, but less than that which it would produce if properly applied to an overshot-wheel.

For a description of it, see *Grier's Mechanics' Calculator*, page 234; and for its formulæ, see *London Artisan*, 1845, page 229.

The higher an overshot-wheel is, in proportion to the whole descent of the water, the greater will be its effect. The effect is as the product of the volume of water and its perpendicular height.

The weight of the arch of loaded buckets in an overshot-wheel in lbs. is ascertained by multiplying $\frac{4}{9}$ of their number by the number of cubic feet in each, and that product by 40.

MEMORANDA.

A volume of water of 17.5 cubic feet per second, with a fall of 25 feet, applied to an undershot-wheel, will drive a hammer of 1500 lbs. in weight from 100 to 120 blows per minute, with a lift of from 1 to 1.5 feet.*

A volume of water of 21.5 cubic feet per second, with a fall of 12.5 feet, applied to a wheel having a great height of water above its summit, being 7.75 feet in diameter, will drive a hammer of 500 lbs. in weight 100 blows per minute with a lift of 2 feet 10 inches. Estimate of power 31.5 horses.

A Stream and Overshot Wheel at Fishkill Creek, N. Y., of the following dimensions—viz., Height of head to centre of opening, 24⅞ ins.; opening, 1¾ by 80 ins.; wheel, 22 feet diameter by 8 feet face; 52 buckets, each 1 foot in depth, making 3½ revolutions per minute—drives 3 run of 4½ feet stones 130 revolutions in a minute, with all the attendant machinery, and grinds and dresses 25 bushels of wheat per hour.

A Breast-wheel and Stream of the following dimensions—viz., Head, 20 feet; height of water upon wheel, 16 feet; opening, 18 feet by 2 ins.; diameter of wheel, 26 feet 4 inches; face of wheel, 20 feet 9 inches; depth of buckets, 15¾ ins.; number of buckets, 70; revolutions, $4\frac{5}{10}$ per minute—drives at the Rocky Glen Factory, Fishkill, N. Y., 6144 self-acting mule spindles; 160 looms, weaving printing-cloths 27 ins. wide of No. 33 yarn (33 hanks to a lb.), and producing 24000 hanks in a day of 11 hours.

4½ bushels Southern and 5 bushels Northern wheat are required to make 1 barrel of flour.

IMPULSE AND RESISTANCE OF FLUIDS.

Impulse and Resistance of Water.—Water or any other fluid, when flowing against a body, imparts a force to it by which its condition of motion is altered. The resistance which a fluid opposes to the motion of a body does not essentially differ from Impulse.

The Impulse of one and the same mass of fluid under otherwise similar circumstances is proportional to the relative velocities $o \mp v$ of the fluid.

* The volume of water required for a hammer increases in a much greater ratio than the velocity to be given to it, it being nearly as the cube of the velocity.

For an equal transverse section of a stream, the impulse against a surface at rest increases as the square of the velocity of the water.

Impulse against Plane Surfaces.—The impulse of a stream of water depends principally upon the angle under which, after the impulse, it leaves the water; it is nothing if the angle is 0, and a maximum if it is deflected back in a line parallel to that of its flow, or 180°, $P = 2 \frac{c \mp v'}{g} V w$.

When the Surface of Resistance is a Plane, and $= 90°$, then $P = \frac{c \mp v'}{g} V w$.

If the surface is at rest, $P = 2 A h w$, *c and v' representing the velocities of the water and of the surface upon which it impinges, w the weight of the fluid per cubic foot*, A *the transverse section of the stream in sq. ins., and* $c \mp v'$ *the relative motions of the water and surface.*

The normal impulse of water against a plane surface is equivalent to the weight of a column which has for its base the transverse section A of the stream, and for altitude twice the height due to its velocity, $2h = 2\frac{v^2}{2g}$.

The resistance of a fluid to a body in motion is the same as the impulse of a fluid moving with the same velocity against a body at rest.

Maximum Effect of Impulse.—The effect of impulse depends principally on the velocity v of the impinged surface. It is, for example, 0, both when $v = o$ and $v = 0$; hence there is a velocity for which the effect of impulse is a maximum $= (o - v)\, v$; that is, $v = \frac{o}{2}$, and the maximum effect of the impulse of water is obtained when the surface impinged moves from it with half the velocity of the water.

ILLUSTRATION.—A stream of water having a transverse section of 40 square inches discharges 5 cubic feet per second against a plane surface, and flows off with a velocity of 12 feet per second; the effect of its impulse, then, is, $o = \frac{5 \times 144}{40} = 18$;

$$g = 32.166,\ w = 62.5;\ \frac{18 - 12}{32.166} \times 5 \times 62.5 = .1865 \times 5 \times 62.5 = 58.28\ lbs.$$

Hence the mechanical effect upon the surface $= P v = 58.28 \times 12 = 699.36$ *lbs.*

The maximum effect would be $v = \frac{o}{2} = \frac{1}{2} \times \frac{5 \times 144}{40} = 9$ *feet, and* $\frac{1}{2} \times \frac{18^2}{2g} \times 5 \times 62.5 = \frac{1}{2} \times 5.0363 \times 312.5 = 786.92$ *lbs.; and the hydraulic pressure* $= \frac{786.92}{9} = 87.44$ *lbs.*

When the Surface is a Plane and at an Angle, then $P = (1 - \cos. a)\frac{o}{g} V w$.

ILLUSTRATION.—A stream of water having a transverse section of 64 square inches discharges 17.778 cubic feet per second against a fixed cone, having an angle of convergence from the flow of the stream of 50°, the hydraulic pressure in the direction of the stream; then $o = \frac{17.778}{64 \div 144} = 40$; $\cos. 50° = .64279$.

$$(1 - .64279)\ \frac{40}{32.166} \times 17.778 \times 62.5 = .35721 \times 1375.431 = 491.318\ lbs.$$

When the Surface of Resistance is a Plane at 90°, *and has Borders added to its Perimeter*, the effect will be greater, depending upon the height of the border and the ratio of the transverse section between the stream and the part confined.

Oblique Impulse.—In oblique impulse against a plane, the stream may flow in one, two, or in all directions over the plane.

When the Stream is confined at three Sides, $P = (1 \cos. a)\ \frac{o - v}{g} V w$.

When the Stream is confined at two Sides, $P = \frac{o - v}{g} \sin. a^2\ V w$.

The normal impulse of a stream increases as the sine of the angle of incidence; the parallel impulse as the square of the sine of the angle, and the lateral impulse as double the angle.

When an Inclined Surface is not Bordered, then the stream can spread over it in all directions, and the impulse is greater, because of all the angles by which the water is deflected; a is the least; hence each particle that does not move in the normal plane exerts a greater pressure than the particle in that plane, and $P = \frac{2 \sin. a^2}{1 + \sin. a^2} \times \frac{o - v}{g} V w$.

Impulse and Resistance against Surfaces.

The coefficient of resistance, C, or the number with which the height due to the velocity is to be multiplied, to obtain the height of a column of water measuring this hydraulic pressure, varies for bodies of different figures, and only for surfaces which are at right angles to the direction of motion is it nearly a definite quantity.

ILLUSTRATION.—If a wind impinges with a velocity of 20 feet per second against the four arms of a wind-mill, each having an area of 200 square feet, and an angle of inclination of 75° to the direction of the wind, then is the impinging force of the wind in the direction of the axis of the wheel $= 1.85 \times \frac{2 (\sin. 75°)^2}{1 + (\sin. 75°)^2} \times \frac{20^2}{2g} \times 4 \times 200 \times .081$; $.081 =$ *the assumed density of the wind, as given by* Weisbach, § 201.

Hence $1.85 \times \frac{2 \times .96593^2}{1 + .96593^2} \times \frac{400}{.64333} \times 64.8 = 719.5$ *lbs.*

According to the experiments of Du Buat and Thibault, $C = 1.85$ for the impulse of air or water against a plane surface at rest, and for the resistance of air or water against a surface in motion, $C = 1.4$. In each case about .66 of the effect is expended upon the front surface, and .34 upon the rear.

The resistance of air to a surface revolving in a circle has been found by Hutton and others to vary, but it may be expressed by $C = 1.5$. If the surface is not at right angles to the direction of the motion, but makes with it an acute angle a, then for C put $\frac{2 C \sin. a^2}{1 + \sin. a^2}$.

PERCUSSION OF FLUIDS.

When a stream strikes a plane perpendicular to its action, the force with which it strikes is estimated by the product of the area of the plane, the density of the fluid, and the square of its velocity.

Or, $A d V^2 = P$, A *representing the area in square feet, d the weight of the fluid in lbs., and* V *the velocity in feet per second.*

If the plane is itself in motion, then the force becomes $A d (V - v)^2 = P$, *v representing the velocity of the plane.*

If C represent a coefficient to be determined by experiment, and h the height due to the velocity V, then $V^2 = 2 g h$, and the expression for the force becomes $A C 2 g h = P$.

MOTION OF BODIES IN FLUIDS.

From the following Table several practical inferences may be drawn.

1. That the resistance is nearly as the surface, the resistance increasing but a very little above that proportion in the greater surfaces.

2. The resistance to the same surface is nearly as the square of the velocity, but gradually increasing more and more above that proportion as the velocity increases.

3. When the after parts of the bodies are of different forms, the resistances are different, though the fore parts be alike.

4. The resistance on the base of a hemisphere is to that on the convex side nearly as 2.4 to 1, instead of 2 to 1, as theory assigns the proportion.

5. The resistance on the base* of a cone is to that on the vertex nearly as 2.3 to 1. And in the same ratio is radius to the sine of the angle of the inclination of the side of the cone to its path or axis. So that, in this instance, the resistance is directly as the sine of the angle of incidence, the transverse section being the same, instead of the square of the sine.

6. Hence we can ascertain the altitude of a column of air, the pressure of which is equal to the resistance of a body moving through it with any velocity.

Thus, let a = *area of the section of the body, similar to any of those in the Table, perpendicular to the direction of motion;* R = *the resistance to the velocity in the Table; and* x = *the altitude sought, of a column of air the base of which* = a *and its pressure* R. Then $a\,x$ = the volume of the column in feet, and $1.2\,a\,x$ its weight in ounces.

Therefore, $1.2\,a\,x = R$, and $x = \frac{5}{6} \times \frac{R}{a}$ is the altitude sought in feet, viz., $\frac{5}{6}$ of the quotient of the resistance of a body divided by its transverse section, which is a constant quantity for all similar bodies, however different in magnitude, since the resistance R is as the section a.

ILLUSTRATION.—The convex side of the large hemisphere, the resistance of which = .634, or at a velocity of 16 feet per second; then R = .634, and $x = \frac{5}{6} \times \frac{R}{a}$ 2.38 *feet*, the altitude of a column of air, the pressure of which is equal to the resistance upon a spherical surface, at a velocity of 16 feet.

Resistance of different Figures at different Velocities in Air.

Velocity per Second.	Cone. Vertex.	Cone. Base.	Sphere.	Cylinder.	Hemisphere. Flat.	Hemisphere. Round.	Small Hemisphere.
Feet.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.
3	.028	.064	.027	.05	.051	.02	.028
4	.048	.109	.047	.09	.096	.039	.048
5	.071	.162	.068	.143	.148	.063	.072
8	.168	.382	.162	.36	.368	.16	.184
9	.211	.478	.205	.456	.464	.199	.233
10	.26	.587	.255	.565	.573	.242	.287
15	.589	1.316	.581	1.327	1.336	.552	.661
20	1.069	2.54	1.057	2.528	2.542	1.033	1.196

The diameter of all the figures but the small hemisphere was 5⅝ ins., and the altitude of the cone 6⅝ ins. The small hemisphere was 4¾ ins.

The angle of the side of the cone and its axis is, consequently, 25° 42′ nearly.

* This is a refutation of the popular assertion that a taper spar can be towed in water easiest when the base is foremost.

If a body move through a fluid at rest, or the fluid move against the body at rest, the resistance of the fluid against the body is as the square of the velocity and the density of the fluid; that is, $R = dv^2$. For the resistance is as the quantity of matter or particles struck, and the velocity with which they are struck. But the quantity or number of particles struck in any time are as the velocity and the density of the fluid; therefore the resistance of a fluid is as the density and square of the velocity.

The resistance to a plane is as the plane is greater or less, and therefore the resistance to a plane is as its area, the density of the medium, and the square of the velocity; that is, $R = a\,d\,v^2$.

If the motion is not perpendicular, but oblique to the plane or to the face of the body, then the resistance in the direction of the motion will be diminished in the triplicate ratio of radius to the sine of the angle of inclination of the plane to the direction of the motion, or as the cube of radius to the cube of the sine of that angle. So that $R = a\,d\,v^2 s^3$, $1 = radius$, and $s = sine\ of\ the\ angle\ of\ inclination$.

Table of the Resistance to an Area of one Square Foot moving through Water, or contrariwise.

Angle of Surface with Plane of Current.	Pressure per Square Foot for the following Velocities per Foot per Minute:							
	60	120	180	240	300	480	600	900
Degrees.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
6	.022	.09	.202	.359	.561	1.435	2.242	5.046
7	.027	.109	.246	.437	.682	1.747	2.73	6.142
8	.033	.133	.298	.53	.829	2.122	3.315	7.459
9	.039	.156	.351	.624	.975	2.496	3.9	8.775
10	.045	.179	.404	.718	1.121	2.87	4.485	10.091
15	.089	.355	.798	1.42	2.218	5.678	8.872	19.963
20	.152	.608	1.369	2.434	3.802	9.734	15.21	34.222
25	.235	.94	2.115	3.76	5.874	15.038	23.497	52.869
30	.338	1.353	3.045	5.413	8.458	21.653	33.832	76.123
35	.449	1.798	4.045	7.192	11.237	28.766	44.947	101.132
40	.565	2.258	5.081	9.032	14.113	36.13	56.452	127.018
45	.665	2.66	5.985	10.639	16.624	42.557	66.495	149.614
50	.749	2.995	6.739	11.981	18.72	47.923	74.88	168.48
55	.812	3.249	7.31	12.995	20.304	51.979	81.217	182.739
60	.864	3.455	7.775	13.822	21.596	55.286	86.385	194.366
65	.902	3.607	8.117	14.43	22.547	57.72	90.187	202.922
70	.932	3.728	8.389	14.914	23.302	59.654	93.21	209.722
75	.953	3.81	8.573	15.241	23.814	60.965	95.257	214.329
80	.966	3.857	8.678	15.428	24.107	61.714	96.427	216.926
85	.973	3.892	8.757	15.569	24.326	62.275	97.305	218.936
90	.975	3.9	8.775	15.6	24.375	62.4	97.5	219.375

The resistance to a plane, from a fluid acting in a direction perpendicular to its face, is equal to the weight of a column of the fluid, the base of which is the plane and altitude equal to that which is due to the velocity of the motion, or through which a heavy body must fall to acquire that velocity.

The resistance to a plane running through a fluid is the same as the force of the fluid in motion with the same velocity on the plane at rest. But the force of the fluid in motion is equal to the weight or pressure which generates that motion, and this is equal to the weight or pressure of a column of the fluid, the base of which is the area of the plane, and its altitude that which is due to the velocity.

1. If a = *the area of a plane*, v *its velocity*, n *the density or specific gravity of the fluid*, $\frac{1}{2}\,g = 16.0833$ *feet*, and the altitude due to the velocity v, being $\frac{v^2}{2g}$, then $a \times n \times \frac{v^2}{2g} = \frac{a\,n\,v^2}{2g}$ = *the resistance* R.

2. If the direction of motion be not perpendicular to the face of the plane, but oblique to it, then $\frac{a\,n\,v^2 s^3}{2g} = R$.

3. If W represent the weight of a body, a being resisted by the absolute force R, then *the retarding force* f, or $\frac{R}{w} = \frac{a\,n\,v^2 s^3}{2\,g\,w}$.

Illustration.—If a plane 1 foot square be moved through water at the rate of 32.166 feet per second, then $\frac{32.166^2}{64.333} = 16.083$, the space a body would require to fall to acquire a velocity of 32.166 feet per second; therefore 1×62.5 (weight of a cubic foot of water) $\times \frac{32.166^2}{64.333} = 1005$ *lbs.* = *resistance of the plane.*

The resistance to a sphere moving through a fluid is but half the resistance to its great circle, or to the end of a cylinder of the same diameter, moving with an equal velocity, $\frac{p\,n\,v^2 r^2}{4\,g} = R$, being the half of that of a cylinder of the same diameter, r *representing radius.*

Illustration.—An iron ball of 9 lbs., having a diameter of 4 ins., when projected at a velocity of 1600 feet per second, will meet a resistance which is equal to a weight of 132.66 lbs. over the pressure of the atmosphere.

The resistance that a body sustains in moving through a fluid is in proportion to the square of the velocity; and it is the same, whether the plane moves against the fluid or the fluid against the plane.

The progression of a solid floating body, as a boat in a channel of still water, gives rise to a displacement of the water surface, which advances with an undulation in the direction of the body, and this undulation is termed the *Wave of Displacement.*

The resistance of a fluid to the progression of a floating body increases as the velocity of the body attains the velocity of the wave of displacement, and it is greatest when the two velocities are equal.

In the motion of elastic fluids, it appears from experiments that oblique action produces nearly the same effect as in the motion of water, in the passage of curvatures, apertures, etc.

STRENGTH OF MATERIALS.

ELASTICITY AND STRENGTH.

The component parts of a rigid body adhere to each other with a force which is termed *Cohesion.*

Elasticity is the resistance which a body opposes to a change of form.

Strength is the resistance which a body opposes to a permanent separation of its parts.

Elasticity and *Strength*, according to the manner in which a force is exerted upon a body, are distinguished as *Tensile Strength*, or Absolute Resistance; *Transverse Strength*, or Resistance to Flexure; *Crushing Strength*, or Resistance to Compression; *Torsional Strength*, or Resistance to Torsion; and *Detrusive Strength*, or Resistance to Shearing.

The limit of Stiffness is flexure, and the limit of Strength or Resistance is fracture.

Resiliénce, or toughness of bodies, is strength and flexibility combined; hence any material or body which bears the greatest load, and bends the most at the time of fracture, is the toughest.

The Specific Gravity of Iron is ascertained to indicate very correctly the relative degree of its strength.

The *Neutral Axis*, or *Line of Equilibrium*, is the line at which extension terminates and compression begins.

The resistance of cast iron to crushing and tensile strains is, as a mean, as 4.3 to 1.*

English cast iron has a higher resistance to compression, and a less tensile resistance, than American.

The mean tensile strength of American cast iron, as determined by Major Wade for the U. S. Ordnance Corps, is 31829 lbs. per square inch of section; the mean of English, as determined by Mr. E. Hodgkinson for the Railway Commission, etc., in 1849, is 19484 lbs.; and by Col. Wilmot at Woolwich, in 1858, for gun-metal, is 23257 lbs.

The ultimate extension of cast iron is the 500th part of its length.

The mean transverse strength of American cast iron, also determined by Major Wade, is 681 lbs. per square inch, suspended from a bar fixed at one end and loaded at the other; and the mean of English, as determined by Fairbairn, Barlow, and others, is 500 lbs.

The resistance of wrought iron to crushing and tensile strains is, as a mean, as 1.5 to 1 for American; and for English 1.2 to 1.

The mean tensile strength of American wrought iron, as determined by Prof. Johnson, is 55900 lbs.; and the mean of English, as determined by Capt. Brown, Barlow, Brunel, and Fairbairn, is 53900 lbs.†

The ultimate extension of wrought iron is the 600th part of its length.

The resistance to flexure, acting evenly over the surface, is nearly ½ the tensile resistance.

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* The experiments of Mr. Hodgkinson on iron of low tensile strength gives a mean of 6.595 to 1.

† The results, as given by Mr. Telford, included experiments upon Swedish iron; hence they are omitted in this summary.

Modulus of Elasticity.

The *Modulus* or *Coefficient of the Elasticity* of any substance is the measure of its elastic reaction or force, and is the height of a column of the same substance, capable of producing a pressure on its base, which is to the weight causing a certain degree of compression, as the length of the substance is to the diminution of its length.

It is computed by this analogy: As the extension or diminution of the length of any given substance is to its length in inches, so is the force that produced that extension or diminution to the modulus of its elasticity.

Or, $x : P :: l : w = \frac{P}{x}$, *x representing the length a substance* 1 *inch square and* 1 *foot in length would be extended or diminished by the force* P, *and w the weight of the modulus in lbs.*

To Compute the Weight of the Modulus of Elasticity of a Substance.

Rule.—As the extension or compression of the length of any substance is to its length, so is the weight that produced that extension or compression to the modulus of elasticity in pounds avoirdupois.

Example.—If a bar of cast iron, 1 inch square and 10 feet in length, is extended .008 inch, with a weight of 1000 lbs., what is the weight of its modulus of elasticity?

$.008 : 120\ (10 \times 12) :: 1000 : 15\,000\,000$ *lbs.*

Note.—When the weight of the modulus of elasticity of a substance is known, the height of it can be readily computed by dividing the weight by the weight of a bar of the substance 1 inch square and 1 foot in length.

Ex. 2.—If a wrought-iron chain, 60 feet in length and .2 inch in diameter, is subjected to a strain of 150 lbs., what will it be extended?

The modulus of elasticity of iron wire is 26 808 000 *lbs.*, and the area of chain $.2^2 \times .7854 = .31416$.

$$\frac{150}{.31416} = 477.463 \text{ lbs. per square inch, and } 60 \times 12 = 720 \text{ ins.}$$

$$\text{Then } 477.463 \times \frac{120}{26\,808\,000} = \frac{343\,773.36}{26\,808\,000} = .0128 \text{ inch.}$$

To Compute the Weight when the Height is given.

Rule.—Multiply the weight of 1 foot in length of the material by the height of its modulus in feet, and the product will give the weight.

To Compute the Height of the Modulus of Elasticity.

Rule.—Divide the weight of the modulus of elasticity of the material by the weight of 1 foot of it, and the quotient will give the height in feet.

From a series of elaborate experiments by Mr. E. Hodgkinson for the Railway Commission, he deduced the following formulæ for the extension and compression of cast and wrought iron:

Cast-iron Extension: $13\,934\,040\,\frac{e}{l} - 2\,907\,432\,000\,\frac{e^2}{l^2} = W.$

Cast-iron Compression: $12\,931\,560\,\frac{c}{l} - 522\,979\,200\,\frac{c^2}{l^2} = W$, *e and c representing the extension and compression, and l the length in inches.*

Illustration.—What weight will extend a bar of cast iron, 4 inches square and 10 feet in length, to the extent of .2 inch?

$13\,934\,040 \times \frac{.2}{120} - 2\,907\,432\,000\,\frac{.2^2}{120^2} = 23223.4 - 8076.2 = 15147.2$, which $\times$ 4 *ins.* $= 60588.8$ *lbs.*

Modulus of Elasticity and Weight of various Substances.

Substances.	Height in Feet.	Weight in Lbs.	Substances.	Height in Feet.	Weight in Lbs.
Ash	4 970 000	1 656 670	Lignum-vitæ	1 850 000	1 080 400
Brass, yellow	2 460 000	8 464 000	Limestone	2 400 000	3 300 000
" wire	4 112 000	14 632 720	Mahogany	6 570 000	2 071 000
Copper, cast	4 800 000	18 240 000	Marble, white	2 150 000	2 508 000
Elm	5 680 000	1 499 500	Oak	4 750 000	1 710 000
Fir, red	8 330 000	2 016 000	Pine, pitch	8 700 000	2 430 000
Glass	4 440 000	5 550 000	" white	8 970 000	1 830 000
Gun-metal	2 790 000	8 844 300	Steel, cast	8 530 000	26 650 000
Hempen fibres	5 000 000	170 000	" wire	9 000 090	28 689 000
Ice	6 000 000	2 370 000	Stone, Portland	1 672 000	1 718 800
Iron, cast	5 750 900	1 796 850	Tin, cast	1 053 000	3 510 000
" wrought	7 550 000	25 820 000	Willow	6 200 000	1 426 000
" wire	8 377 000	28 230 500	Yellow pine, mean.	10 500 000	2 100 000
Lead, cast	146 000	720 000	Zinc	4 480 000	13 440 000

The elasticity of Ivory, as compared to Glass, is as .95 to 1.

To Compute the Length of a Prism of a Material which would be severed by its own Weight when Suspended.

RULE.—Divide the tensile resistance of the material by the weight of a foot of it in length, and the quotient will give the length.

Modulus of Cohesion, or Length in Feet required to Tear asunder the following Substances:

Rawhide.... 15375 feet; Hemp twine.... 75000 feet; Catgut.... 25000 feet.

TENSILE STRENGTH.

Tensile Strength is the resistance of the fibres or particles of a body to separation. It is therefore proportional to their number, or to the area of its transverse section.

The fibres of wood are strongest near the centre of the trunk or limb of a tree.

Cast Iron.—Experiments on Cast-iron bars give a tensile strength of from 4000 lbs. to 5000 lbs. per square inch of its section, as just sufficient to balance the elasticity of the metal; and as a bar of it is extended the 5500th part of its length for every ton of direct strain per square inch of its section, it is deduced that its elasticity is fully excited when it is extended less than the 3000th part of its length, and the extension of it at its limit of elasticity is estimated at the 1200th part of its length.

The mean tensile strength, then, of cast iron being from 16000 to 20000 lbs., the *Value* of it, when subjected to a tensile strain, may be safely estimated at from ¼ to ⅛ of this, or of its breaking strain.

A bar of cast iron will contract or expand .0000062173, or the 162000th of its length for each degree of heat; and assuming the extreme range of the temperature in this country 140° (— 20° + 120°), it will contract or expand with this change .0008642, or the 1157th part of its length. It shrinks in cooling from .0104 to .0118th of its length.

It follows, then, that as 2240 lbs. will extend a bar the 5500th part of its length, the contraction or extension for the 1157th part will be equivalent to a force of 10648 lbs. (4¾ tons) per square inch of section.

Cast iron (Greenwood) at three successive meltings gave tenacities of 21300, 30100, and 35700 lbs.

Cast iron at 2.5 tons per square inch will extend the same as wrought iron at 5.6 tons.

The mean tensile strength of four kinds of English cast iron, as determined by the Commissioners on the application of iron to Railway Structures, was 15711 lbs. per square inch (7.014 tons); and the mean ultimate extension was, for lengths of 10 feet, .1997 inch, being the 600th part of its length; and this weight would compress a bar the 775th part of its length.

Tensile strength of the strongest piece of cast iron ever tested—45970 lbs. This was a mixture of grades 1, 2, and 3 of Greenwood iron, and at the 3d fusion.

Wrought Iron.—Experiments on Wrought-iron bars give a tensile strength of from 18000 lbs. to 22400 lbs. per square inch of its section, as just sufficient to balance the elasticity of the metal; and as a bar of it is extended the 10000th part of its length for every ton of direct strain per square inch of its section, it is deduced that its elasticity is fully excited when it is extended the 1000th part of its length, and the extension of it at its limit of elasticity is estimated at the 1520th part of its length.

The mean tensile strength of wrought iron being from 55000 to 65000 lbs., the *Value* of it, when subjected to a tensile strain, may be safely estimated at from ¼ to ⅛ of this, or of its breaking strain.

A bar of wrought iron will expand or contract .000006614, or the 151200th part of its length for each degree of heat; and assuming, as before stated for cast iron, that the extreme range of temperature in the air in this country is 140°, it will contract or expand with this change .000926, or the 1080th of its length, which is equivalent to a force of 20740 lbs. (9¼ tons) per square inch of section.

Experiments upon wrought iron, to determine the results from repeated heating and laminating, furnished the following:

From 1 to 6 reheatings and rollings, the tensile stress increased from 43904 lbs. to 61824 lbs., and from 6 to 12 it was reduced to 43904 again.

The tensile force of metals varies with their temperature, generally decreasing as the temperature is increased. In silver the tenacity decreases more rapidly than the temperature; in copper, gold, and platinum it decreases less rapidly than the temperature.

In iron, the tensile strength at different temperatures is as follows: 60°, 1; 114°, 1.14; 212°, 1.2; 250°, 1.32; 270°, 1.35; 325°, 1.41; 435°, 1.4.

Stirling's Mixed or Toughened Iron.—By the mixture of a portion of malleable iron with cast iron, carefully fused in a crucible, a tensile strain of 25764 lbs. has been attained. This mixture, when judiciously managed and duly proportioned, increases the resistance of cast iron about one third; the greatest effect being obtained with a proportion of about 30 per cent. of malleable iron.

Bronze (gun-metal) varies in tenacity from 23000 to 54500 lbs.

Elements connected with the Tensile Resistance of various Substances.

Substances.	Tensile Strain per Sq. Inch for limit of Elasticity.	Ratio of Strain to that causing Rupture.	Substances.	Tensile Strain per Sq. Inch for limit of Elasticity.	Ratio of Strain to that causing Rupture.
	Lbs.			Lbs.	
Beech	3355	.3	Wrought iron, ordinary.	17600	.3
Cast iron, English	4000	.22	" " Swedish.	24400	.34
" American	5000	.2	" " English.	18850	.35
Oak	2856	.23		22400	.35
Steel plates, blue temp'd.	93720	.62	" " American	21000	.26
" wire	35700	.5	" wire, No. 9, unannealed	47532	.46
Yellow pine	5332	.23	" " " annealed	36300	.45

TENSILE STRENGTH OF MATERIALS.

Weight or Power required to Tear asunder one Square Inch.

METALS.

	Lbs.		Lbs.
Copper, wrought	34000	Iron, plates, mean, English	51000
" rolled	36000	" " lengthwise	53800
" cast, American	24250	" " crosswise	48800
" wire	61200	" inferior, bar	30000
" bolt	36800	" wire, American	73600
Iron, cast, Low Moor, No. 2	14076	" " " 16 diam.	80000
" Clyde, No. 1	16125	" scrap	53400
" " No. 3	23468	Lead, cast	1800
" Calder, No. 1	13735	" milled	3320
" Stirling, mean	25764	" wire	2580
" mean of American	31829	Platinum, wire	53000
" mean* of English	19484	Silver, cast	40000
" Greenwood, American	45970	Steel, cast, maximum	142000
" gun-metal, mean	37232	" " mean	88657
" wrought wire	103000	" blistered, soft	133000
" best Swedish bar	72000		104000
" Russian bar	59500	" shear	124000
" English bar	56000	" chrome, mean	170980
" rivets, American	53300	" puddled, extreme	173817
" bolts	52250	" American Tool Co	179980
" hammered	53913	" plates, lengthwise	96300
" mean of English	53900	" " crosswise	93700
" rivets, English	65000	" razor	150000
" crank shaft	44750	Tin, cast, block	5000
" turnings	55800	" Banca	2122
" plates, boiler, American	48000	Zinc	3500
	62000	" sheet	16000

Lake Superior and Iron Mountain charcoal bloom iron has resisted 90000 lbs. per square inch.

MISCELLANEOUS SUBSTANCES.

	Lbs.		Lbs.
Brick, well burned	750	Limestone	670
" fire	65		2800
" inferior	290	Marble, Italian	5200
	100	" white	9000
Cement, blue stone	77	Mortar, 12 years old	60
" hydraulic	234	Plaster of Paris	72
" Harwich	30	Rope, Manila	9000
" Portland, 6 mos.	414	" hemp, tarred	15000
" Sheppy	24	" wire	37000
" Portland 1, sand 3.	380	Sandstone, fine grain	200
Chalk	118	Slate	12000
Glass, crown	2346	Stone, Bath	352
Gutta-percha	3500	" Craigleth	400
Hydraulic lime	140	" Hailes	360
" " mortar	140	" Portland	857
Ivory	16000		1000
Leather belts	330	Whalebone	7600

* By Commissioners, on application of iron to Railway Stations.

COMPOSITIONS.

	Lbs.		Lbs.
Gold 5, Copper 1	50000	Copper 10, Tin 1	32000
Brass	42000	" 8, " 1, gun-met.	30000
" yellow	18000	" 8, " 1, sm'l bars	50000
Bronze, least	17698	Tin 10, Antimony 1	11000
" greatest	56788	Yellow metal	48700

WOODS.

	Lbs.		Lbs.
Ash	14000	Maple	10500
Beech	11500	Oak, American white	11500
Box	20000	" English	10000
Bay	14000	" seasoned	13600
Cedar	11400	" African	14500
Chestnut, sweet	10500	Pear	9800
Cypress	6000	Pine, pitch	12000
Deal, Christiana	12400	" larch	9500
Elm	13400	" American white	11800
Lance	23000	Poplar	7000
Lignum-vitæ	11800	Spruce, white	10290
Locust	20500	Sycamore	13000
Mahogany	21000	Teak	14000
" Spanish	12000	Walnut	7800
" "	8000	Willow	13000

Results of Experiments on the Tensile Strength of Wrought-iron Tie-rods.

Common English Iron, 1 3/16 Inches in Diameter.

Description of Connection.	Breaking Weight.
	Lbs.
Semicircular hook fitted to a circular and welded eye	14000
Two semicircular hooks hooked together	16220
Right-angled hook or goose-neck fitted into a cylindrical eye	29120
Two links or welded eyes connected together	48160
Straight rod without any connection articulation	56000

Iron bars when cold rolled are materially stronger than when only hot rolled, the difference being in some cases as great as 3 to 2.

WIRE ROPES.

Result of Experiments on the Tensile Strength of Iron and Steel Wire Ropes.

Charcoal Iron Wire Rope. Circum.	Weight per Foot.	Breaking Weight.	Steel Wire Rope. Circum.	Stretch in 6 Feet.	Weight per Foot.	Breaking Weight.
Ins.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.
1 7/8	1/2	13440	1 15/16	1 3/8	1/2	33600
3 3/8	1 1/2	44800	2 3/8	5/8	7/8	56000

Tensile Strength of Copper at different Temperatures.

Temp.	Strength in Lbs.	Temp.	Strength in Lbs.	Temp.	Strength in Lbs.
122°	33079	482°	26981	801°	18854
212°	32187	545°	25420	912°	14789
302°	30872	602°	22302	1016°	11054

Extension of Cast-iron Bars, When suspended Vertically.

1 *Inch Square and* 10 *Feet in Length. Weight applied at one End.*

Weight applied.	Extension.	Set.	Weight applied.	Extension.	Set.
Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.
529	.0044	—	4234	.0397	.00265
1058	.0092	.000015	8468	.0871	.00855
2117	.0190	.000059	14820	.1829	.02555

Steel.

The tensile strength of steel increases by reheating and rolling up to the second operation, but decreases after that.

Ratio of the Ductility and Malleability of Metals.

In the order of Wire-drawing Ductility.	In the order of Laminable Ductility	In the order of Wire-drawing Ductility.	In the order of Laminable Ductility	In the order of Wire-drawing Ductility.	In the order of Laminable Ductility.
Gold.	Iron.	Tin.	Gold.	Tin.	Zinc.
Silver.	Copper.	Lead.	Silver.	Platinum.	Iron.
Platinum.	Zinc.	Nickel.	Copper.	Lead.	Nickel.

The relative resistance of Wrought Iron and Copper to tension and compression is as 100 to 54.5.

TRANSVERSE STRENGTH.

The Transverse or Lateral Strength of any Bar, Beam, Rod, etc., is in proportion to the product of its breadth and the square of its depth; in like-sided beams, bars, etc., it is as the cube of the side, and in cylinders as the cube of the diameter of the section.

When one End is fixed and the other projecting, the strength is inversely as the distance of the weight from the section acted upon; and the strain upon any section is directly as the distance of the weight from that section.

When both Ends are supported only, the strength is 4 times greater for an equal length, when the weight is applied in the middle between the supports, than if one end only is fixed.

When both Ends are fixed, the strength is 6 times greater for an equal length, when the weight is applied in the middle, than if one end only is fixed.

The strength of any beam, bar, etc., to support a weight in the centre of it, *when the ends rest merely upon two supports*, compared to one *when the ends are fixed*, is as 2 to 3.

When the Weight or Strain is uniformly distributed, the weight or strain that can be supported, compared with that when the weight or strain is applied at one end or in the middle between the supports, is as 2 to 1.

In Metals, the less the dimension of the side of a beam, etc., or the diameter of a cylinder, the greater its proportionate transverse strength: this is in consequence of their having a greater proportion of chilled or hammered surface compared to their elements of strength, resulting from dimensions alone.

The strength of a *Cylinder*, compared to a *Square* of like diameter or sides, is as 6.25 to 8. The strength of a *Hollow Cylinder* to that of a *Solid Cylinder*, of the same length and volume, is as the greater diameter of the former is to the diameter of the latter.

The strength of an *Equilateral Triangle, fixed at one End and loaded at the other*, having an *edge up*, compared to a *Square* of the same area, is as

22 to 27; and the strength of an equilateral triangle, having an *edge down*, compared to one with an *edge up*, is as 10 to 7.

NOTE.—In these comparisons, the beam, bar, etc., is considered as one end being *fixed*, the weight suspended from the other. In Barlow and other authors the comparison is made when the beam, etc., rested upon *supports*. Hence the stress is contrariwise.

Detrusion is the resistance that the particles or fibres of materials oppose to their sliding upon each other. Punching and shearing are detrusive strains.

Deflection.—When a bar, beam, etc., is deflected by a cross-strain, the side of the beam, etc., which is bounded by the concave surface, is *compressed*, and the opposite side is *extended.*

In Stones and Cast metals, the resistance to compression is greater than the resistance to extension.

In Woods, the resistance to extension is greater than the resistance to compression.

The general law regarding deflection is, that it increases, *cæteris paribus*, directly as the cube of the length of the beam, bar, etc., and inversely as the breadth and cube of the depth.

The resistance of *Flexure* of a body at its cross-section is very nearly $\frac{9}{10}$ of its tensile resistance.

The *stiffest* bar or beam that can be cut out of a cylinder is that of which the depth is to the breadth as the square root of 3 to 1; the *strongest*, as the square root of 2 to 1; and the most *resilient*, that which has the breadth and depth equal.

Relative Stiffness of Materials to Resist a Transverse Strain.

Ash	.089	Elm	.073	Wrought iron	1.3
Beech	.073	Oak	.095	Yellow pine	.087
Cast iron	1.	White pine	.1		

The strength of a Rectangular Beam in an *inclined position*, to resist a vertical stress, is to its strength in a horizontal position as the square of radius to the square of the cosine of elevation; that is, as the square of the length of the beam to the square of the distance between its points of support, measured upon a horizontal plane.

Experiments upon bars of cast iron, 1, 2, and 3 inches square, give a result of transverse strength of 447, 348, and 338 lbs. respectively; being in the ratio of 1, .78, and .756.

The strongest rectangular bar or beam that can be cut out of a cylinder is one of which the squares of the breadth and depth of it, and the diameter of the cylinder, are as 1, 2, and 3 respectively.

The ratio of the *crushing* to the *transverse* strength is nearly the same in glass, stone, and marble, including the hardest and softest kinds.

Green sand iron castings are 6 per cent. stronger than dry, and 30 per cent. stronger than chilled; but when the castings are chilled and annealed, a gain of 115 per cent. is attained over those made in green sand.

Chilling the under side of cast iron very materially increases its strength.

Woods.—Beams of wood, when laid with their annual or annular layers vertical, are stronger than when they are laid horizontal, in the proportion of 8 to 7.

Woods are denser at the roots and at the centre of their trunks. Their strength decreases with the decrease of their density.

Oak loses strength in drying.

Transverse Strength of Materials, deduced from the Experiments of U. S. Ordnance Department, Barlow, Rennie, Stephenson, Hodgkinson, Fairbairn, Pasley, Hatfield, and the Author.

Reduced to the uniform Measure of One Inch Square, and one Foot in Length; Weight suspended from one End.

Materials.	Breaking Weight.	Value for general Use.
METALS.	Lbs.	
Cast iron, American (means of	507	125 to 160
four divi-	632	155 " 210
sions of	733	180 " 240
grades ..)	772	192 " 250
" mean by Maj. Wade	681	170 " 225
" West Pt. Foundry, extreme	980	250 " 325
" English, Low Moor, cold blast	472	110 " 140
" Ponkey, cold......	581	145 " 190
" hot blast, mean ...	500	125 " 165
" cold " " ...	516	130 " 170
" Ystalyfera, cold bl't	770	195 " 255
" mean of 65 kinds..	500	125 " 165
" mean of 15 kinds, direct fr. the pig, cold blast	641	160 " 215
" planed bar	518	130 " 170
" rough bar	534	133 " 175
Steel, greatest.......	1918	350 " 450
Steel, puddled (permanent bend).....	800	170 " 225
WOODS.		
Ash	168	55
Beech	130	32
Birch	160	40
Chestnut	160	53
Deal, Christiana....	137	45
Elm.................	125	30
Hickory	250	55
Locust	295	80
Maple...............	202	65
Norway pine	123	40
Oak, African........	208	50
" American white	230	50
" " live .	245	55
" Canadian......	145	36
" Dantzic	122	30
" English	140	35
" " superior	188	45
Pitch pine	136	45
" American.	160	50
Riga fir.............	94	30
Teak................	206	60
White pine	92	30
" American	130	45
Whitewood	116	38

Materials.	Breaking Weight.	Value for general Use.
WROUGHT IRON.	Lbs.	
American...........	700 / 650 / 600	160 to 200
English	400	100 " 130
"	550	135 " 180
Swedish*	665	165 " 210
MIXTURE OF CAST AND WROUGHT IRON, etc.		
Cast iron, Blaenavon .	—	145
" 10 per ct. of wr't	—	175
" 30 " "	—	230
" 50 " "	—	185
" and 2½ per ct. of nickel, mean	—	180
" Stirling, 2d qu.	—	154
" " 3d "	—	125
Copper	—	55
Brass	—	58
STONES (American).		
Flagging, blue	31.	10
Freestone, Conn.	13.	4
" Dorchester	10.8	3½
" N. Jersey.	20.1	6½
	17.8	6
" N. York ..	24.	8
Granite, blue, coarse.	18.	6
" Quincy, Mass.	26.	8½
STONES (English).		
Adelaide marble	4 5	1½
Arbroath	17.	5½
Bangor slate	90.	30
Bath	5.2	1¾
Caithness, paving, &c.	68.	22
Cornish granite	22.	7
Craigleth sandstone .	10.7	3½
Darley sandst., Vict'a	1.3	4
Kentish rag	35.8	12
Limestone	11	3½
Llangollen slate	43.	14
Park Spring sandst'e	4.3	1.4
Portland oolite......	21.2	7
Valentia, paving, Irel.	68 5	23
Welsh, "	157.	55
Yorkshire, blue	26.	8½
" landing ..	22.5	7½
" paving...	10.4	3½

Increase in Strength of several Woods by Seasoning.

Ash ..... 44.7 per cent.	Elm ..... 12.3 per cent.	White pine. . 9 per cent.
Beech .. 61.9 "	Oak ..... 26.1 "	

* With 840 lbs. the deflection was 1 inch, and the elasticity of the metal destroyed.

Concretes, Cements, etc.

Materials.	Breaking Weight.	Materials.	Breaking Weight.
CONCRETES (English).		BRICKS (English).	
Fire-brick beam, Portl'd cement	3.1	Best stock	11.8
" sand 3 parts, lime 1 part	.7	Fire-brick	14.
CEMENTS (English).		New brick	10.7
Blue clay and chalk	5.4	Old brick	9.1
Portland....................	37.5	Stock-brick, well burned	5.8
	10.2	" inferior, burned ..	2.5
Sheppy	5.		

Transverse Strength of Cast-iron Bars and Oak Beams of Various Figures.

Reduced to the uniform Measure of One Inch Square of Sectional Area, and One Foot in Length. Fixed at one End; Weight suspended from the other.

Form of Bar or Beam.	Breaking Weight.	Form of Bar or Beam.	Breaking Weight.
CAST IRON.	Lbs.		Lbs.
Square..................	673	Equilateral triangle, an edge up	560
Square, diagonal vertical.	568	Equilateral triangle, an edge down	958
Cylinder	573	2 ins. deep × 2 ins. wide × .268 ins. depth......	2068
Hollow cylinder; greater diameter twice that of lesser................	794	2 ins. deep × 2 ins. wide × .268 ins. depth......	555
Rectangular prism, 2 ins. deep × ½ in. depth	1456	OAK. Equilateral triangle, an edge up...............	114
" 3 ins. deep × ⅓ in. depth	2392	Equilateral triangle, an edge down	130
" 4 " × ¼ "	2652		

Transverse Strength of Solid and Hollow Cylinders of various Materials.

One Foot in Length. Fixed at one End; Weight suspended from the other.

Materials.	Solid External Diameter.	Hollow Internal Diameter.	Breaking Weight.	Breaking Weight for 1 Inch external Diam., and proportionate internal Diam.
WOODS.	Ins.	Ins.	Lbs.	Lbs.
Ash	2.	—	685	86
"	2.	1.	604	75
Fir*	2.	—	772	97
White pine	1.	—	75	75
"	2.	—	610	76
METALS.				
Cast iron, cold blast	3.	—	12000	444
STONE-WARE.				
Rolled pipe of fine clay	2.87	1.928	190	8

Brick-work.

A brick arch, having a rise of 2 feet, and a span of 15 feet 9 inches, and 2 feet in width, with a depth at its crown of 4 inches, bore 358400 lbs. laid along its centre.

* An inch-square batten, from the same plank as this specimen, broke at 139 lbs.

To Compute the Transverse Strength of a Rectangular Beam or Bar.

When a Beam or Bar is Fixed at one End, and Loaded at the other. RULE.—Multiply the *Value* of the material in the preceding Tables, or, as may be ascertained, by the breadth and square of the depth in inches, and divide the product by the length in feet.

NOTE.—When the beam is loaded uniformly throughout its length, the result must be doubled.

EXAMPLE.—What are the weights each that a cast and wrought iron bar, 2 inches square and projecting 30 ins. in length, will bear without permanent injury?

The *values* for cast and wrought iron in this and the following calculations are assumed to be 225 and 180.

Hence $225 \times 2 \times 2^2 = 1800$, which. $\div 2.5 = 720$ *lbs.*; and $180 \times 2 \times 2^2 = 1440$, which, $\div 2.5 = 576$ *lbs.*

If the Dimensions of a Beam or Bar are required to Support a given Weight at its End. RULE.—Divide the product of the weight and the length in feet by the *Value* of the material, and the quotient will give the product of the breadth and the square of the depth.

EXAMPLE.—What is the depth of a wrought-iron beam, 2 inches broad, necessary to support 576 lbs. suspended at 30 ins. from the fixed end?

$\frac{576 \times 2.5}{180} = 8$, which, $\div$ 2 ins. for the breadth $= 4$, and $\sqrt{4} = 2$ *ins., the depth.*

When a Beam or Bar is Fixed at both Ends, and Loaded in the Middle. RULE.—Multiply the *Value* of the material by 6 times the breadth and the square of the depth in inches, and divide the product by the length in feet.

NOTE.—When the beam is loaded uniformly throughout its length, the result must be doubled.

EXAMPLE.—What weight will a bar of cast iron, 2 ins. square and 5 feet in length, support in the middle, without permanent injury?

$$225 \times \overline{2 \times 6} \times 2^2 = 10800, \text{ which, } \div 5 = 2160 \textit{ lbs.}$$

Or, *If the Dimensions of a Beam or Bar are required to Support a given Weight in the Middle, between the Fixed Ends.* RULE.—Divide the product of the weight and the length in feet by 6 times the *Value* of the material, and the quotient will give the product of the breadth and the square of the depth.

EXAMPLE.—What dimensions will a cast-iron square bar 5 feet in length require to support without permanent injury a stress of 2160 lbs.?

$\frac{2160 \times 5}{225 \times 6} = \frac{10800}{1350} = 8$, which, $\div$ 2 ins. for the assumed breadth, $= 4$, and $\sqrt{4} = 2$ *ins., the depth.*

When the Breadth or Depth is required. RULE.—Divide the product obtained by the preceding rules by the square of the depth, and the quotient is the breadth; or by the breadth, and the square root of the quotient is the depth.

ILLUSTRATION.—If 128 is the product, and the depth is 8; then $128 \div 8^2 = 2$, *the breadth.* Also, $128 \div 2 = 64$, and $\sqrt{64} = 8$, *the depth.*

When the Weight is not in the Middle between the Ends. RULE.—Multiply the *Value* of the material by 3 times the length in feet, and the breadth and square of the depth in inches, and divide the product by twice the product of the distances of the weight, or stress from either end.

EXAMPLE.—What is the weight a cast-iron bar, fixed at both ends, 2 ins. square and 5 feet in length, will bear without permanent injury, 2 feet from one end?

$$\frac{225 \times 3 \times 5 \times 2 \times 2^2}{2 \times 2 \times 3} = \frac{27000}{12} = 2250 \textit{ lbs.}$$

When a Beam or Bar is Supported at both Ends, and Loaded in the Middle. RULE.—Multiply the *Value* of the material by 4 times the breadth and the square of the depth in inches, and divide the product by the length in feet.

NOTE.—When the beam is loaded uniformly throughout its length, the result must be doubled.

EXAMPLE.—What weight will a cast-iron bar, 5 feet between the supports, and 2 ins. square, bear in the middle, without permanent injury?

$$225 \times \overline{2 \times 4} \times 2^2 = 72000, \text{ which, } \div 5 = 1440 \textit{ lbs.}$$

Or, *If the Dimensions are required to Support a given Weight.* RULE.—Divide the product of the weight and length in feet by 4 times the *Value* of the material, and the quotient will give the product of the breadth, and the square of the depth.

When the Weight is not in the Middle between the Supports. RULE.—Multiply the *Value* of the material by the length in feet, and the breadth and the square of the depth in inches, and divide the product by the product of the distances of the weight, or stress from either support.

EXAMPLE.—What weight will a cast-iron bar, 2 ins. square and 5 feet in length, support without permanent injury, at a distance of 2 feet from one end, or support?

$$\frac{225 \times 5 \times 2 \times 2^2}{2 \times (5-2)} = \frac{9000}{6} = 1500 \textit{ lbs.}$$

To Compute the Pressure upon the Ends or upon the Supports.

RULE.—1. Divide the product of the weight and its distance from the nearest end or support by the whole length, and the quotient will give the pressure upon the end or support farthest from the weight.

2. Divide the product of the weight and its distance from the farthest end, or support, by the whole length, and the quotient will give the pressure upon the end or support nearest the weight.

EXAMPLE.—What is the pressure upon the supports in the case of the preceding example?

$\frac{1500 \times 2}{5} = 600$ *lbs. upon support farthest from the weight;* $\frac{1500 \times 2}{5} = 900$ *lbs. upon support nearest to the weight.*

When a Beam or Bar, Fixed or Supported at both Ends, bears two Weights at unequal Distances from the Ends, Let *m and n represent distances of greatest and least weights from their nearest end, W and w greatest and least weights, L whole length, l distance from least weight to farthest end, and l′ distance of greatest weight from farthest end.*

Then $\frac{m \times W}{L} + \frac{l \times w}{L} =$ *pressure at w end*, and $\frac{n \times w}{L} + \frac{l' \times W}{L} =$ *pressure at* W *end.*

ILLUSTRATION.—A beam 10 feet in length, having both ends fixed in a wall, bears two weights, viz., one of 1000 lbs. at 4 feet from one of its ends, and the other of 2000 lbs. at 4 feet from the other end; what is the pressure upon each end?

$\frac{4 \times 2000}{10} + \frac{6 \times 1000}{10} = 1400$ *lbs., pressure upon w end;* $\frac{4 \times 1000}{10} + \frac{6 \times 2000}{10} = 1600$ *lbs., pressure at* W *end.*

When the Plane of the Beam or Bar projects obliquely Upward or Downward. When Fixed at one End and Loaded at the other. RULE.—Multiply the *Value* of the material by the breadth and square of the depth in inches, and divide the product by the product of the length in feet and the cosine of the angle of elevation or depression.

NOTE.—When the weight is laid uniformly along its length, the result must be doubled.

EXAMPLE.—What is the weight an ash beam, 5 feet in length, 3 inches square, and projecting upward at an angle of 7° 15′, will bear without permanent injury?

$$55 \times 3 \times 3^2 = 1485, \text{ which, } \div \overline{5 \times \cos. 7° 15'} = 1485 \div \overline{5 \times .992} = 299.39 \; lbs.$$

To Compute the Transverse Strength of Cylinders, Ellipses, etc.

When a Cylinder, Rectangle (the diagonal being vertical), Hollow Cylinder, or Beams having sections of an Ellipse, are either Fixed at one End and loaded at the other, or Supported at both Ends, the Load applied in the Middle, or between the Supports. RULE.—Proceed in all cases as if for a rectangular beam, taking for the breadth and depth, and *Value* of the material, as follows:

Cylinder, diameter^3 ×.6; Rectangle,* side^3 ×.7; Hollow Cylinder (diam.^{2} — diam.^{3}) ×.6; Ellipse, transverse diam. vertical conj. × transverse^2, ×.6; and Ellipse, conj. diam. vert. transverse × conj.^{2} ×.6 of *Value.*

When an Equilateral Triangle, or ⊤ *Beam.* RULE.—Proceed in all cases as if for a rectangular beam, taking the following proportions of the *Value* of the material.

Fixed at one or both Ends.	Equilateral triangle,	edge up,	$b \times d^2$,	×.2	of *Value.*
	Equilateral triangle,	edge down,	$b \times d^2$,	×.34	"
	⊤ beam or bar,	edge down,	$b \times d^2$,	×.42	"
Supported at both Ends.	Equilateral triangle,	edge up,	$b \times d^2$,	×.34	"
	Equilateral triangle,	edge down,	$b \times d^2$,	×.2	"
	⊤ beam or bar,	edge up,	$b \times d^2$,	×.42	"

To Compute the Diameter of a Solid Cylinder to Support a given Weight.

When Fixed at one End, and Loaded at the other. RULE.—Multiply the weight to be supported in pounds by the length of the cylinder in feet; divide the product by .6 of the *Value* of the material, and the cube root of the quotient will give the diameter.

NOTE.—When the cylinder is loaded uniformly throughout its length, the cube root of half the quotient will give the diameter.

EXAMPLE.—What should be the diameter of a cast-iron cylindrical beam, 8 ins. in length, to support 15000 lbs. without permanent injury?

$$8 \; ins. = .66 \; feet; \; \frac{15000 \times .66}{.6 \times 225} = 74.07; \text{ and } \sqrt[3]{74.07} = 4.2.$$

When Fixed at both Ends, and Loaded in the Middle. RULE.—Multiply the weight to be supported in pounds by the length of the cylinder between the supports in feet; divide the product by .6 of the *Value* of the material, and the cube root of ⅙ of the quotient will give the diameter.

NOTE.—When the cylinder is loaded uniformly along its length, the cube root of half the quotient will give the diameter.

EXAMPLE.—What should be the diameter of a cast-iron cylinder, 2 feet between the supports, that will support 19305 lbs. without permanent injury?

$$\frac{19305 \times 2}{.6 \times 225} = 286, \text{ and } \sqrt[3]{\frac{286}{6}} = 3.61 \; ins.$$

When Supported at both Ends, and Loaded in the Middle. RULE.—Multiply the weight to be supported in pounds by the length of the cylinder between the supports in feet; divide the product by .6 of the *Value* of the material, and the cube root of ¼ of the quotient will give the diameter.

NOTE.—When the cylinder is loaded uniformly along its length, the cube root of half the quotient will give the diameter.

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* The strength of a Rectangle, the diagonal being vertical, compared to that of its circumscribing rectangle, when the direction of the strain is parallel to the side of it, is as 2.45 to 1.

EXAMPLE.—What should be the diameter of a cast-iron cylinder, 2 feet between the supports, that will support 54000 lbs. without permanent injury?

$$\frac{54000\times 2}{.\times 225}=800, \text{ and } \sqrt[3]{\frac{800}{4}}=5.85 \textit{ ins.}$$

And what its diameter if loaded uniformly along its length?

$$\frac{800\div 2}{4}=100, \text{ and } \sqrt[3]{100}=4.64 \textit{ ins.}$$

To Compute the Relative Value of Materials to resist a Transverse Strain.

Let V *represent this value in a Beam, Bar, or Cylinder, one foot in length, and one inch square, side, or in diameter;* W *the weight;* l *the length in feet;* b *the breadth, and* d *the depth in inches;* m *the distance of the weight from one end; and* n *the distance of it from the other in feet.*

NOTE.—In cylinders, for $b\,d^2$ put d^3.

1. *Fixed at one End*, weight suspended from the other, $\frac{l\,W}{b\,d^2}=V$.

2. *Fixed at both Ends*, weight suspended from the middle, $\frac{l\,W}{6\,b\,d^2}=V$.

3. *Supported at both Ends*, weight suspended from the middle, $\frac{l\,W}{4\,b\,d^2}=V$.

4. *Supported at both Ends*, weight suspended at any other point than the middle, $\frac{m\,n\,W}{l\,b\,d^2}=V$.

5. *Fixed at both Ends*, weight suspended at any other point than the middle, $\frac{2\,m\,n\,W}{3\,l\,b\,d^2}=V$.

From which formulæ, the weight that may be borne, or any of the dimensions, may be computed by the following:

1. $\frac{V\,b\,d^2}{l}=W$; $\frac{V\,b\,d^2}{W}=l$; $\frac{l\,W}{V\,d^2}=b$; $\sqrt{\frac{l\,W}{b\,V}}=d$. In rectangular beams, etc., b and $d=\sqrt[3]{\frac{l\,W}{V}}$.

2. $\frac{6\,b\,d^2\,V}{l}=W$; $\frac{6\,b\,d^3\,V}{W}=l$; $\frac{l\,W}{6\,d^2\,V}=b$; $\sqrt{\frac{l\,W}{6\,b\,V}}=d$. In rectangular beams, etc., b and $d=\sqrt[3]{\frac{l\,W}{6\,V}}$.

3. $\frac{4\,b\,d^2\,V}{l}=W$; $\frac{4\,b\,d^2\,V}{W}=l$; $\frac{l\,W}{4\,d^2\,V}=b$; $\sqrt{\frac{l\,W}{4\,b\,V}}=d$. In rectangular beams, etc., b and $d=\sqrt[3]{\frac{l\,W}{4\,V}}$.

4. $\frac{l\,b\,d^2\,V}{m\,n}=W$; $\frac{m\,n\,W}{b\,d^2\,V}=l$; $\frac{m\,n\,W}{l\,d^2\,V}=b$; $\sqrt{\frac{m\,n\,W}{l\,b\,V}}=d$. In rectangular beams, etc., b and $d=\sqrt[3]{\frac{m\,n\,W}{l\,V}}$.

5. $\frac{3\,l\,b\,d^2\,V}{2\,m\,n}=W$; $\frac{2\,m\,n\,W}{3\,b\,d^2\,V}=l$; $\frac{2\,m\,n\,W}{3\,l\,d^2\,V}=b$; $\sqrt{\frac{2\,m\,n\,W}{3\,l\,b\,V}}=d$. In rectangular beams, etc., b and $d=\sqrt[3]{\frac{2\,m\,n\,W}{3\,l\,V}}$.

When the weight is uniformly distributed, the same formulæ will apply, W representing only half the required or given weight.

Girders, Beams, Lintels, etc.

The Transverse or Lateral Strength of any Girder, Beam, Brest-summer, Lintel, etc., is in proportion to the product of its breadth and the square of its depth, and also to the area of its cross-section.

The best form of section for Cast-iron girders or beams, etc., is deduced from the experiments of Mr. E. Hodgkinson, and such as have this form of section ⊥ are known as Hodgkinson's.

The rule deduced from his experiments directs that the area of the bottom flange should be 6 times that of the top flange—flanges connected by a thin vertical web, sufficiently rigid, however, to give the requisite lateral stiffness, and tapering both upward and downward from the neutral axis; and in order to set aside the risk of an imperfect casting, by any great disproportion between the web and the flanges, it should be tapered so as to connect with them, with a thickness corresponding to that of the flange.

As both Cast and Wrought iron resist crushing or compression with a greater force than extension, it follows that the flange of a girder or beam of either of these metals, which is subjected to a crushing strain, according as the girder or beam is *supported at both ends, or fixed at one end*, should be of less area than the other flange, which is subjected to extension or a tensile strain.

When girders are subjected to impulses, and are used to sustain vibrating loads, as in bridges, etc., the best proportion between the top and bottom flange is as 1 to 4: as a general rule, they should be as narrow and deep as practicable, and should never be deflected to more than one five-hundredth of their length.

In Public Halls, Churches, and Buildings where the weight of people alone are to be provided for, an estimate of 175 pounds per square foot of floor surface is sufficient to provide for the weight of flooring and the load upon it.

In Churches, Buildings, etc., the weight to be provided for should be estimated at that which may at any time be placed thereon, or which at any time may bear upon any portion of their floors; the usual allowance, however, is for a weight of 280 lbs. per square foot of floor surface for stores and factories, and 175 lbs. per square foot when the weight of people alone is to be provided for.

In all uses, such as in buildings and bridges, where the structure is exposed to sudden impulses, the load or stress to be sustained should not exceed from $\frac{1}{5}$ to $\frac{1}{6}$ of the breaking weight of the material employed; but when the load is uniform or the stress quiescent, it may be increased to $\frac{1}{3}$ and $\frac{1}{4}$ of the breaking weight.

An open-web girder or beam, etc., is to be estimated in its resistance on the same principle as if it had a solid web. In cast metals, allowance is to be made for the loss of strength due to the unequal contraction in cooling of the web and flanges.

In cast iron, the mean resistance to Crushing or Extension is as 4.3 to 1, and in wrought iron as 1.35 to 1; hence the mass of metal below the neutral axis will be greatest in these proportions when the stress is intermediate between the ends or supports of the girders, etc.

Wooden Girders or Beams, when sawed in two or more pieces, and have slips set between them, and the whole bolted together, are made stiffer by the operation, and are rendered less liable to decay.

Girders cast with a face up are stronger than when cast on a side, in the proportion of 1 to .96, and they are strongest also when cast with the bottom flange up.

The following results of the resistances of metals will show how the material should be distributed in order to obtain the *maximum* of strength with the *minimum* of material:

	To Tension.	To Crushing.
Cast iron	21000 / 32000	90300 / 140500
Copper	24250	117000
Wrought iron	45000 / 72000	40000 / 83000

The best iron has the greatest tensile strength, and the least compressive or crushing.

The most economical construction of a Girder or Beam, with reference to attaining the greatest strength with the least material, is as follows: The outline of the top, bottom, and sides should be a curve of various forms, according as the breadth or the depth throughout is equal, and as the girder or beam is loaded only at one end, or in the middle, or uniformly throughout.

To Compute the Dimensions and Form of a Girder or Beam.

When a Girder or Beam is Fixed at one End, and Loaded at the other.

1. *When the Depth is uniform throughout the entire Length,* The section at every point must be in proportion to the product of the length, breadth, and square of the depth, and as the square of the depth is in every point the same, the breadth must vary directly as the length; consequently, each side of the beam must be a vertical plane, tapering gradually to the end.

2. *When the Breadth is uniform throughout the entire Length,* The depth must vary as the square root of the length; hence the upper or lower sides, or both, must be determined by a parabolic curve.

3. *When the Section at every point is similar — that is, a Circle, an Ellipse, a Square, or a Rectangle, the sides of which bear a fixed proportion to each other,* The section at every point being a regular figure, for a circle, the diameter at every point must be as the cube root of the length; and for an ellipse, or a rectangle, the breadth and depth must vary as the cube root of the length.

When a Girder or Beam is Fixed at one End, and Loaded uniformly throughout its Length.

1. *When the Depth is uniform throughout its entire Length,* The breadth must increase as the square of the length.

2. *When the Breadth is uniform throughout its entire Length,* The depth will vary directly as the length.

3. *When the Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle,* The section at every point being a regular figure, the cube of the depth must be in the ratio of the square of the length.

When a Girder or Beam is supported at both Ends.

1. *When Loaded in the Middle,* The *constant* of the beam, or the product of the breadth and the square of the depth, must be in proportion to the distance from the nearest support; consequently, whether the lines forming the beam are straight or curved, they meet in the centre, and of course the two halves are alike: the beam, therefore, may be considered as one of half the length, the supported end corresponding with the free end in the case of beams, one end being fixed, and the middle of the beams similarly corresponding with the fixed end.

1. *When the Depth is uniform throughout,* The breadth must be in the ratio of the length.

2. *When the Breadth is uniform throughout,* The depth will vary as the square root of the length.

3. *When the Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle,* The section at every point being a regular figure, the cube of the depth will be as the square of the distance from the supported end.

When a Girder or Beam is Supported at both Ends, and Loaded uniformly throughout its Length.

1. *When the Depth is uniform,* The breadth will be as the product of the length of the beam and the length of it on one side of the given point, less the square of the length on one side of the given point.

2. *When the Breadth is uniform,* The depth will be as the square root of the product of the length of the beam and the length of it on one side of the given point, less the square of the length on one side of the given point.

3. *When the Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle,* The section at every point being a regular figure, the cube of the depth will be as the product of the length of the beam and the length of it on one side of the given point, less the square of the length on one side of the given point.

General Deductions from the Experiments of Stephenson, Fairbairn, Cubitt, Hughes, etc.

Fairbairn shows in his experiments that with a stress of about 12320 lbs. per square inch on cast iron, and 28000 lbs. on wrought iron, the sets and elongations are nearly equal to each other.

A cast-iron beam will be bent to one third of its breaking weight if the load is laid on gradually; and one sixth of it, if laid on at once, will produce the same effect, if the weight of the beam is small compared with the weight laid on. Hence beams of cast iron should be made capable of bearing more than 6 times the greatest weight which will be laid upon them.

In wrought-iron beams, if fixed at both ends, the upper flange should be larger than the lower, in the ratio of 1.35 to 1.

The breaking weights in similar beams are to each other as the squares of their like linear dimensions; that is, the breaking weights of beams are computed by multiplying together the area of their section, their depth, and a *constant*, determined from experiments on beams of the particular form under investigation, and dividing the product by the distance between the supports.

Cast and wrought iron beams, having similar resistances, have weights nearly as 2.44 to 1.

The range of the comparative strength of girders of the same depth, having a top and bottom flange, and those having bottom flange alone, is from having but a little area of bottom flange to a large proportion of it, from ½ to ¼ greater strength.

A box beam or girder, constructed of plates of wrought iron, compared to a single rib and flanged beam I, of equal weights, has a resistance as 100 to 93.

The resistance of beams or girders, where the depth is greater than their breadth, when supported at top, is much increased. In some cases the difference is fully one third.

When a beam is of equal thickness throughout its depth, the curve should be an *ellipse* to enable it to support a uniform load with equal resistance in every part; and if the beam is an open one, the curve of equilibrium, for a uniform load, should be that of a *parabola*. Hence, when the middle portion is not wholly removed, the curve should be a compound of an ellipse and a parabola, approaching nearer to the latter as the middle part is decreased.

Girders of cast iron, up to a span of 40 feet, involve a less cost than of wrought iron.

Cast iron beams and girders should not be loaded to exceed one fifth of their breaking weight; and when the strain is attended with concussion and vibration, this proportion must be increased.

Simple cast-iron girders may be made 50 feet in length, and the best form is that of Hodgkinson: when subjected to a fixed load, the flange should be as 1 to 6, and when to a concussion, etc., as 1 to 4.

The forms of girders for spaces exceeding the limit of those of simple cast iron are various; the principal ones adopted are those of the straight or arched cast-iron girders in separate pieces, and bolted together — the Trussed, the Bow-string, and the wrought-iron Box and Tubular.

A *Straight* or *Arched Girder* is formed of separate castings, and is entirely dependent upon the bolts of connection for its strength.

A *Trussed* or *Bow-string Girder* is made of one or more castings to a single piece, and its strength depends, other than upon the depth or area of it, upon the proper adjustment of the tension, or the initial strain, upon the wrought-iron truss.

A *Box* or *Tubular Girder* is made of wrought iron, and is best constructed with cast-iron tops, in order to resist compression: this form of girder is best adapted to afford lateral stiffness.

Floor Beams, Girders, etc.

The condition of the stress borne by a floor beam is that of a beam supported at both ends and uniformly loaded; but from the irregularity in its loading and unloading, and from the necessity of its possessing great rigidity, it is impracticable to estimate its capacity other than as a beam having the weight borne upon the middle of its length.

To Compute the Depth of a Floor Beam.

When the Length and Breadth are given, and the Distance between the Centres of the Beam is One Foot. RULE.—Divide the product of the square of the length in feet and the weight to be borne in pounds per square foot of floor, by the product of 4 times the breadth and the *value* of the material from the Table (page 455), and the square root of the quotient will give the depth of the beam in inches.

EXAMPLE.—A white pine beam is 2 ins. wide, and 12 feet in length between the supports; what should be the depth of it to support a weight of 175 lbs. per square foot?

$$\frac{12^2 \times 175}{2 \times 4 \times 30} = 105, \text{ and } \sqrt{105} = 10.25 \textit{ ins.}$$

When the Distance between the Centres of the Beam is greater or less than one Foot. RULE.—Divide the product of the square of the depth for a beam, *when the distance between the centres is one foot*, by the distance given in inches by 12, and the square root of the quotient will give the depth of the beam in inches.

EXAMPLE.—Assume the beam in the preceding case to be set 15 ins. from the centres of its adjoining beams; what should be its depth?

$$\frac{10.25^2 \times 15}{12} = 131.25, \text{ and } \sqrt{131.25} = 11.45 \textit{ ins.}$$

Header and Trimmer Beams.

The conditions of the stress borne or to be provided for by them are as follows:

Header or *Trimmer* beams support ½ of the weight of and upon the tail beams inserted into or attached to them.

Trimmer Beams support, in addition to that borne by them directly as a floor beam, each ½ the weight on the headers.

The stress, therefore, upon a header is due directly to its length, or the number of tail beams it supports; and the stress upon the trimmer beams is that of their own stress as a floor beam, and ½ of the weight upon the header supported by them.

NOTE.—The distance between the support of the trimmer beams and the point of connection with the header does not in anywise affect the stress upon the trimmer beams; for in just proportion as this distance is increased, and the stress upon them consequently increased, by the suspension of the header from them nearer to the middle of their length, so is the area of the surface supported by the header reduced, and, consequently, the load to be borne by it.

Girder.

The condition of the stress borne by a Girder* is that of a beam fixed or supported at both ends, as the case may be, supporting the weight borne

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* When a girder has four or more supports, its condition as regards a stress upon its middle is that of a beam fixed at both ends.

by all of the beams resting thereon, at the points at which they rest; and its dimensions must be proportionate to the stress upon it, and the distance between its points of insertion or support.

Illustration.—It is required to determine the dimensions of a pitch-pine girder, 15 feet between its several points of supports, to support the ends of two lengths of beams each 20 feet in length, having a superincumbent weight, including that of the beams, of 200 lbs. per square foot.

The condition of the stress upon such a girder would be that of a number of beams, 40 feet in length (20×2), supported at both ends, and loaded uniformly along their length, with 200 lbs. upon every superficial foot of their area.

Hence the amount of the weight to be borne is determined by $20\times2\times15\times200=$ 120 000 *lbs.* = *the product of twice the length of a beam, the distance between the supports of the girder and the weight borne per square foot of area;* and the resistance to be provided for is that to be borne by a beam, 15 feet in length, fixed at both ends, and supporting 120 000 lbs. uniformly laid along its length, equal to 60 000 lbs. supported at its centre.

Consequently, $\frac{15\times60\,000}{6\times50}=3000=$ *quotient of the product of the length and weight* ÷ *the product of* 6 *times the Value of the material;* and assuming the girder to be 12 inches wide, then $\sqrt{\frac{3000}{12}}=15.8$ *ins.*

Formulæ to Compute the Values and the Dimensions of Beams, Bars, etc., of various Sections.—[Tredgold.]

For a Square, Rectangle, Rectangle the diagonal being vertical, and Cylinder, they are alike to those already given, substituting in the rectangles for $b\,d^2$, S^3.

For a Grooved or Double-flanged, Open, and Single-flanged Beam they are as follows:

	Grooved. 	Open. 
1. *Fixed at one End,* Weight suspended from the other,	$\frac{l\,W}{b\,d^2\,(1-q\,y^3)}=V.$	$\frac{l\,W}{b\,d^2\,(1-y^3)}=V.$
2. *Fixed at both Ends,* Weight suspended from the middle,	$\frac{l\,W}{b\,d^2\,(1-q\,y^3)}=V.$	$\frac{l\,W}{b\,d^2\,(1-y^3)}=V.$
3. *Supported at both Ends,* Weight suspended from the middle,	$\frac{l\,W}{b\,d^2\,(1-q\,y^3)}=V.$	$\frac{l\,W}{b\,d^2\,(1-y^3)}=V.$
4. *Supported at both Ends,* Weight suspended at any other point than the middle,	$\frac{m\,n\,W}{b\,d^2\,\overline{m+n}\,(1-q\,y^3)}=V.$	$\frac{m\,n\,W}{b\,d^2\,\overline{m+n}\,(1-y^3)}=V.$
5. *Fixed at both Ends,* Weight suspended at any other point than the middle,	$\frac{m\,n\,W}{b\,d^2\,\overline{m+n}\,(1-q\,y^3)}=V.$	$\frac{m\,n\,W}{b\,d^2\,\overline{m+n}\,(1-y^3)}=V.$

 *Single-flanged.* 1. $\dfrac{l\,W}{\dfrac{b\,d^2\,(1-q\,y^3)\,(1-q)}{(\sqrt{1-q\,y^3}+\sqrt{1-q})^2}}=V.$

2., 3., 4., 5. For the other conditions of a Beam, Bar, etc., use the same formula as the above, multiplying the *Value* obtained above by 6, 4, 1, and 1.5 respectively, *y and q representing* $\frac{\textit{depth of groove}}{\textit{whole depth of beam}}$ $=y$, *and* $\frac{\textit{whole breadth of beam — width of web}}{\textit{whole breadth of beam}}=q.$

Transverse Resistance from End Pressure applied Horizontally.

 *Wrought Iron.*—7½ feet in length; flanges, 6×3½ ins. × ⅝ depth; area, 5½ square ins.; 50000 lbs. produced no set; 58240 lbs. produced a set of 1¾ ins.

White Oak.—Rectangle 10 feet in length, 11×4½ ins.; 33600 lbs. gave a deflection of .⅜ in.; 50400 lbs. gave a deflection of .5 in.; 67200 lbs. gave a deflection of .⅝, and with 78400 it broke.

Transverse Strength of Cast-iron Girders and Beams, deduced from the Experiments of Barlow, Hodgkinson, Hughes, Tredgold, etc.

Reduced to a uniform Measure of One Inch in Depth, one Foot in Length, supported at both Ends; the Stress or Weight applied in the Middle.

Section of Girder or Beam.	Flanges. Top.	Flanges. Bottom.	Width of Vertical Web.	Depth of Girder.	Breadth of Girder.	Area of Section in Centre.	Breaking Weight at Length of one Foot.	Strength per Sq. Inch of Section.	*Value* for Breaking Weight = $\frac{l\,W}{A\,d}$ = V.‡
	Sq. Ins.	Sq. Ins.	Inch.	Inch.	Inch.	Sq. In.	Lbs.	Lbs.	Lbs.
 Eq. area of flange at top & bottom,	1.75×.42 = .735	1.77×.39 = .69	.29	5.125	1.77	2.82	30150	10768	2100
 do.	2.02×.515 = 1.045	2.02×.515 = 1.045	.51*	2.02	2.02	2.50	10276	3952	1900
 Area of sec. of top & bot. 1 to 6,	2.23×.31 = .72	6.67×.66 = 4.4	.236	5 125	6.67	6.23	117450	18852	3650
	—	5 ×.3 = 1.5	.365	1.56	5.	1.96	7280	3714	2350
	5 ×.3 = 1.5	—	.365	1.56	5	1.96	2366	1213	750
	—	23.9×3.12 = 74.56	3.3	36.1	23.9	183.5	8066240	43958	1200
	.5 ×.5 = .25	1.5×.5 = .75	.5	4.†	1.5	1.	19980	19980	5000
	1.5 ×.5 = .75	.5×.5 = .25	.5	4.†	1.5	1.	7252	7252	1800
	4 × 2 = 8	—	2.	4.	4.	12.	33600	2800	700
	5.1 ×2.33 = 11.88	12.1×2.07 = 25.04	2.08	30.5	11.1	90.8	4793800	52795	1700
 Rectangular Prism,	—	—	.994	2.012	2.994	2.025	9440	4662	2350
 Open Beam.	1.005×.98	1.005×.99	1.005	2.51	1.005	1.98	12340	6232	2450
	.995×1.01	.995× 1	.995	3.01	.995	2.	15420	7710	2550
	1.005×.98	1.005×.99	1.005	.4	1.005	1.98	21765	10992	2700
	.771×1.51	.771×1.5	.771	4.04	.771	2.322	25705	11070	2750
	1.507×.74	1.507×.74	1.507	4.04	1.507	2.23	25735	11540	2850
	1.525×.78	1.525×.78	1.525	4.07	1.525	2.35	30000	12689	3100
 Square Prism, Stress at Side,	—	—	1.02	1.01	1.02	1.032	2635	2552	2500
 Cylinder,	—	—	1.122	1.122	1.122	.989	2370	2396	2150
 Square Prism, angle up	—	—	.4431	1.443	1.443	1.041	2269	2182	1500

* Horizontal web. † Depth of opening, 3 inches.

‡ *A representing area of section, d the depth in inches, l the length in feet, and* W *the breaking weight in pounds.*

Comparative Resistance or Strength of Girders, Beams, etc., of Equal Sectional Areas and Depths.

	Description of Girder or Beam.	Comparative Strength.
	Rectangular beam	1.
	Grooved beam, top and bottom flanges of equal areas, of uniform thickness of metal throughout, and the depth three times the breadth (Tredgold)..	1.16
	Single-flanged beam; width of flange five twelfths of height; width of rib half the depth of flange (Watt and Fairbairn)...........	1.27
	Open beam, the space half the depth	1.5
	Double-flanged beam; area of top flange one sixth of that of bottom: depth of top flange half that of bottom; width of bottom flange 1¼ the depth of the beam (Hodgkinson)................	1.66

To Compute the Transverse Strength, or the Breaking Weight of Cast-iron Girders or Beams, of various Figures and Sections, when Supported at both Ends, the Weight applied in the Middle.

When the Section of the Girder or Beam is that of a Rectangle, a Grooved, Open, Single or Double Flanged Beam, and is alike to any of the Examples given in the preceding Table.

Rule 1.*—Divide the product of the area of the section, the depth, and the *Value* for the girder, etc., from the Table, by the length between the supports in feet, and the quotient will give the breaking weight in pounds.

Example.—The dimensions of a Hodgkinson beam, having top and bottom flanges in the proportion of 1 to 6, give an area of section of 25.6 square inches, a depth of 20.5 inches, and a length between its supports of 18 feet; what is its breaking weight?

Note.—In consequence of the increased area of the metal over the example given in the Table, the unit of *Value* of 3650 is (page 465) reduced to 3500.

Then $\frac{25.6\times20.5\times3500}{18}\times\frac{1836800}{18}=102044.4$ *lbs.*

2. From the product of the breadth and square of the depth in inches of the rectangular solid, the dimensions of which are the depth and the greatest breadth of the beam in its centre, subtract the product of the breadths and the square of the depths of that part of the beam which is wanting to make it a uniform solid, and then proceed to determine its resistance by the rule for the particular case as to its being supported or fixed, etc.

Note 1.—These rules are applicable to all cases where the flange of the beam is set, as shown in the Table, when the beam rests upon two supports, or contrariwise, as to position of flange, when the beam is fixed at one end only.

2.—When the case under consideration is alike in its general characters to one in the Table, but differs in some one or more points, an increase or decrease of the metal is obtained by a reduction or increase of the *value*, according as the differences may affect the resistance of the beam.

Example.—What is the load that will break a Hodgkinson beam of the following dimensions, 10 feet in length between its supports, the load applied in its middle?

* The utility of these rules in preference to those of Hodgkinson, Fairbairn, Tredgold, Hughes, and Barlow, is manifest, as in the one case the *Value* of the metal is considered, and in the other cases the metal is assumed to be of a uniform value or strength; and when the range in this element, both in weight and cost, render it imperative that, in a structure of iron of the highest transverse strength, the weight due to the requirements of dimensions of the lowest transverse strength should not be increased, and contrariwise.

The only variable element not embraced in this rule is that consequent upon any peculiarity of form of section; as, for instance, in that of the Hodgkinson, or like beams, when the area of one flange greatly exceeds the rest of the section, and this flange is other than below when the beam rests upon two supports or is fixed at both ends, or than above when the beam is fixed at one end.

Top flange	7×1 inch.	Whole depth of beam	21 ins.
Bottom flange	21×2 "	Area of whole section	63.4 "
Width of web	.8 "	Dimensions of rectangle	21×21 "

Hence $21 \times 21^2 = 9261$ *ins.*

$7 - .8 = 6.2$ *ins.* = *width of space between both extremities of top flange and rib.*
$21 - \overline{2+1} = 18$ = *depth of space between top and bottom flanges.*

Hence $6.2 \times 18^2 = 2008.8$.

$21 - 7 = 14$ = *width of space between both extremities of top and bottom flanges.*
$21 - 2 = 19$ = *width of space above bottom and outside of top flange.*

Hence $14 \times 19^2 = 5054$. *Sum*, 7062.8.

And $9261 - 7062.8 \times 4 \times 460$* = 4044688 = *difference of products of the breadth and of the depth of the circumscribing solid, and the breadth and square of the depth of the parts wanting to complete the rectangle, multiplied by four times the value of the metal, which* ÷ 10 *for the length* = 404468.8 *lbs.*

In the example given above, the formulæ of various authors give the following results:

Hodgkinson.—$\frac{2}{3\,d\,l} \times (b\,d^3 - \overline{(b - b')\,d'^3}) = W$ *in tons, d representing depth of beam, d′ depth to bottom flange, b breadth of bottom flange, b′ thickness of vertical web, all in inches, l length in feet, and* W *weight in tons.*

$\therefore \frac{2}{3 \times 21 \times 10} \times (21 \times 21^3 - \overline{(21 - .8) \times 19^3}) = 177.55$, which $\times 2240 = 397\,722$ *lbs.*

Fairbairn.—$\frac{2.166\,a\,d}{l} = W$, *a represent'g area of bottom flange* ; $\therefore \frac{2.166 \times 42 \times 21}{10} = 191.1$, which $\times 2240 = 428\,064$ *lbs.*

Hughes.—$\frac{2\,a\,d}{l} = W$; $\therefore \frac{2 \times 42 \times 21}{10} = 176.4$, which $\times 2240 = 395\,136$ *lbs.*

Barlow.—$\frac{1.13\,A\,d}{l} = W$; $\therefore \frac{1.13 \times 63.4 \times 21}{10} = 150.4$, which $\times 2240 = 336\,896$ *lbs.*

Experiments upon the breaking weight of girders of English cast iron have given the following results:

Dimensions of Girders.	1 and 2.	3.
Top flange	3¼×1¼ *ins.*	4⅛×1½ *ins.*
Web	1¼ "	1½ "
Bottom flange	9 ×1¼ "	1½×2¼ "
Whole depth	22 "	24¼ "
Area of bottom flange	11¼ "	33¾ "
Whole area	39.69 *sq.* "	70.69 *sq.* "
Length between supports	19 *ft.*	30¾ *ft.*
Breaking weight	1 — 116 550 *lbs.* 2 — 125 350 "	3 — 145 208 *lbs.*

Breaking Weights computed by various Formulæ.

	1 and 2.	3.		1 and 2.	3.
By Hughes	58 352 *lbs.*	119 240 *lbs.*	By Hodgkinson	94 998 *lbs.*	139 082 *lbs.*
" Fairbairn	63 213 "	129 272 "	" Barlow	116 323 "	141 120 "

Formula of Table, page 466, using 3300 and 2600† .... 105 701 " 144 820 "

Comparative Values of Cast-iron Bars, Hollow Girders, or Tubes of various Figures (English Iron).

Square bar, small	1.	Rectangular tubes, uniform thickness	.85
Square bar, large	.75	Circular tubes, uniform thickness	.9
Round bar, small	.675	Elliptic tubes, uniform thickness	.95
Square tubes, uniform thickness	1.075		

* Assumed breaking weight of the metal, if of American iron. In connection with this, it is to be borne in mind that, the greater the area of the section of the metal, the less its strength, and the longer the beam, the greater the risk of deflection from a flaw in its structure.

† This is an interesting case, as it exhibits the great reduction of the *value* consequent upon an increase of dimensions, as the proportionate *value* for a girder of the proportion of flanges, but of small dimensions, would be 3200, whereas it is but 2600.

Transverse Strength of Wrought-iron Girders and Beams, deduced from the Experiments of Barlow, Fairbairn, Hughes, etc.

Reduced to a uniform Measure of One Inch in Depth, one Foot in Length, supported at both Ends; the Stress or Weight applied in the Middle.

Section of Girder or Beam.		Flanges. Top.	Flanges. Bottom.	Width of Vertical Web.	Depth of Girder.	Breadth of Girder.	Area of Section in Centre.	Breaking Weight at Length of one Foot.	Strength per Sq. Inch of Section.	Value for Breaking Weight $= \frac{l\,W}{A\,d} = V.$
		Sq. Ins.	Sq. Ins.	Inch.	Inch.	Inch.	Sq. In.	Lbs.	Lbs.	Lbs.
	Solid,	2.5 × 1 = 2.5	4 × .38 = 1.52	.325	8.38	4.	6.295	182000	20952	2500
	"	2.85 × .38 = 1.08	—	.31	2.5	2.85	1.73	12560	7260	2900
	"	—	2.85 × .38 = 1.08	.31	2.5	2.85	1.73	12032	6955	2750
	"	—	3.5 × .6 = 2.1	.8	3.5	3.5	6.25	49280	7822	2200
	Riv'd,	2.86 × .33 = .944	2.86 × .33 = .944	.66	3.7	2.86	3.88	50000	14433	3800
	"	5 × .25 = 1.25; 2 of 2.25 × 2.25 × .3 = 2.82	—	.54	2.6	5.	4.07	23485	5770	2250
	"	2 of 3.5 × 3.5 × .5 = 7	2 of 3.5 × 3.5 × .5 = 7	.37	16.	7.37	19.92	768000	38593	2400
		2 of 2.125 × .28 = 1.19	2 of 2.125 × .30 = 1.27	.25	7.	4.5	4.26	170660	40345	5800
				Thick. of Plates.						
	"	—	—	.065	5.8	3.8	1.24	23670	14089	3200
		—	—	.061	3.	1.95	.6	9450	15750	5200
		—	—	.1325	6.	4.	2.62	75600	28855	4700
		—	—	.124	24.	15.	9.6	375000	39063	1600
		—	—	.272	23.75	15.5	21.2	1536000	72452	3000
		—	—	.525	24.	16.	41.45	3864000	93221	3900
		—	—	.75	36.	24.	87.75	4310400	149333	4900
	"	9.6 × .252 = 2.419	9.6 × .075 = .72	.074	Feet. 9.5	Feet. 9.5	4.36	146528	33607	3450
		9.25 × .149 = 1.378	9.25 × .269 = 2.488	.059	18.25	9.25	6.03	119210	19768	1050
		2.25 × .26 = .585	2.25 × .26 = .585	.131	15.	2.25	5.1	452400	88706	5500
		1 × .282 = .282	1 × .116 = .116	.067	8.	1.	1.47	123794	84214	10300
		24.*	128		54.	2.92	45.82	9443400	206096	3800
	Tubes, Ellip.†	—	—	.375 t. .25 b. .125 s.	24.	16.	12.94	188160	14540	6050
			—	.143	15.	9.75	5.56	278250	50045	3300
	Tubes,	—	—	.0408	12.	12.	1.4	44200	31571	2600
		—	—	.095	24.	24.	7.13	298629	41743	1725.

* Thickness of plates, bottom, .156; top, .147; sides, .099. Area of bottom, 8.8 ins.
† The lateral strength of this was ascertained to be 38080, or .613 of its vertical strength. The ultimate deflection was 2¾ ins.

The above and many of the preceding results are deduced from girders of the length of from 20 to 30 feet; hence, when the length is less, the breaking weight may be increased, in consequence of the increased stability of the girder.

These results are very conclusive of the correctness of the formula used, viz., $\frac{A\,d\,V}{l}$, as will be seen in the cases here given, in the 10th and 15th cases, where the relations between breadth, depth, and thickness are nearly identical; and in the 9th and 15th cases, where the relations between breadth are the same, but the thickness and consequent area differ.

To Compute the Transverse Strength, or the Loads that may be borne by Wrought-iron Girders, Beams, or Tubes, of various Figures and Sections, when Supported at both Ends, the Load applied in the Middle.

When the Section of the Girder or Beam is that of any of the Figures in the preceding Table. RULE.—Divide the product of the area of the section, the depth, and the *Value* for the girder, etc., from the Table, by the length between the supports in feet, and the quotient will give the destructive weight in pounds.

NOTE 1.—The Rule given on page 467 for cast-iron girders, etc., will also apply here, when the metal is of such thickness as to give the girder, etc., full resistance to lateral flexure, and when the construction is such as to bring the stress upon the tension and compression of the metals, and not upon the rivets.

For Note 2, see page 467.

3.—The *Values* here given are based altogether upon experiments with English iron.

EXAMPLE.—What is the load that will destroy a wrought-iron solid grooved beam of the following dimensions, and 10 feet in length between the supports?

Top flanges	3×1¼ ins.	Width of web	.4 inch.
Bottom flange	4× .5 "	Depth of beam	9. "

$3 \times 1.25 + 4 \times .5 = 3.75 + 2 = 5.75$ ins., which $+ 9 - \overline{1.25 + .5} \times .4 = 2.90 = 8.65$ ins. $=$ *area of section.* Then $\frac{8.65 \times 9 \times 3000}{10} = 23355$ *lbs.*

Formulæ for Beams and Tubes of Wrought Iron.

FAIRBAIRN.*

Solid beams, $\frac{2800\,A\,d}{l} = L$. Plate beams, $\frac{2912\,A\,d}{l} = L$.

Cylindrical tubes, $\frac{1792 \text{ to } 2800\,A\,d}{l} = L$. Elliptical tubes, $\frac{1680 \text{ to } 5510\,A\,d}{l} = L$.

HODGKINSON.

Rectangular beams, $\frac{60000 \text{ to } 90000\,(b\,d^3 - b'\,d'^3)}{3\,l\,d} = L$.

Cylindrical tubes, $\frac{3.1416 \times 22500 \text{ to } 35500}{A\,l}\,(r^4 - r'^4) = L$.

Elliptical tubes, $\frac{3.1416 \times 29000 \text{ to } 37000\,(c\,t^3 - c'\,t'^3)}{A\,l} = L$;

b, b', and d, d' representing the external and internal breadths and depths, r and r' the external and internal radii, and c c' and t t' semi-conjugate and semi-transverse diameters, and l the length in inches.

Comparative Value of Wrought-iron Bars, Hollow Girders, or Tubes of various Figures (*English Iron*).

Square bar	250	Round bar	195
Rectangular tubes, plates at top and bottom thick, at sides thin	425		

Welded Tubes without Rivets.

Rectangular, uniform thickness	375	Rectangular tubes, riveted	280
Circular, uniform thickness	325	Elliptic tubes, riveted	250
Elliptic, uniform thickness	350	Flanged beams	240
Circular tubes, riveted	190	Plate beams	320

* See Report of Commissioners on Railway Structures, 1849.

CRUSHING STRENGTH.

The *Crushing Strength* of any body is in proportion to the area of its section, and inversely as its height.

In tapered columns, the strength is determined by the least diameter.

Crushing Strength of various Materials, deduced from the Experiments of Maj. Wade, Hodgkinson, and Capt. Meigs, U.S.A.

Reduced to a uniform Measure of One Square Inch.

Figures and Material.	Crushing Weight.
Prisms.	Lbs.
CAST IRON.	
American, gun-metal	174803
" mean	129000
English, Low Moor, No. 1	62450
" " No. 2	92330
" Clyde, No. 3	106039
" Stirling, mean of all	122395
" " extreme	134400
WROUGHT IRON.	
American	127720
" mean	83500
English	65200 40000
VARIOUS METALS.	
Fine brass	164800
Cast copper	117000
Cast steel	295000
Cast tin	15500
Lead	7730
WOODS.	
Ash	6663
Beech	6963
Birch	7969
Box	10513
Cedar, red	5968
Chestnut	5350
Elm	6831
Hickory, white	8925
Locust	9113
Mahogany, Spanish	8198
Maple	8150
Oak, American white	6100
" Canadian white	5982
" " live	6850
" English	6500 6484
Pine, pitch	8947
" white	5775
" yellow	8200
Spruce, white	5050
Sycamore	7082
Teak	12100
Walnut	6645
STONES, CEMENTS, ETC.	
Brick, hard	2000 4368
" common	4000 800

Figures and Material.	Crushing Weight.
	Lbs.
Clay, fine, baked	175
" " rolled and baked	400
Common brick masonry	800 500
Crown glass	31000
Craigleith Limestone, English	7300 2185
Aberdeen granite "	8400 10363
Arbroath "	7884
Caithness "	6493
Limestone "	3065
Portland "	15583 4570
Portland cement "	15000
" mean "	8300
Portland oolite "	3850
Fire-brick, Stourbridge	1717
Freestone, Belleville	3522
" Caen	1088
" Connecticut	3319
" Dorchester	3069
" Little Falls	2991
Gneiss	19600
Granite, Patapsco	5340
" Quincy	15300
Marble, Baltimore, large	8057
" " small	18061
" East Chester‡	13917
" Hastings, N. Y.	18941
" Italian	12624
" Lee, Mass.	22702
" Montgomery co., Pa.	8950
" Stockbridge§	10382
" Symington, large	11156
" " fine crystal	18248
" " strata horizontal	10124
" " strata vertical	9324
Mortar, good	240
" common	120
Normandy Caen	1543
Portland cement 1, sand 1	1280
Roman "	342
Sandstone, Adelaide	2800
" Acquia Creek*	5340
" Seneca†	10762
Stock brick	2177
Sydney "	2228

* Same as that of the Capitol, Treasury Department, and Patent Office, Washington, D. C.
† Same as that of the Smithsonian Institute.
‡ Same as that of the General Post-office, Wash'n.
§ Same as that of the City Hall, New York.
|| Same as that of the Nat. Wash. Monument.

When the height of a prism or column is not 5 times its side or diameter, the crushing strength is at its maximum.

Experiments upon cast-iron bars give a crushing stress of 5000 lbs. per square inch of section as just sufficient to overcome the elasticity of the metal; and when the height exceeds 3 times the diameter, the iron yields by bending.

When it is 10 times, it is reduced as 1 to 1.75; when it is 15 times, it is reduced as 1 to 2; when it is 20 times, it is reduced as 1 to 3; when it is 30 times, it is reduced as 1 to 4; and when it is 40 times, it is reduced as 1 to 6.

The experiments of Mr. Hodgkinson have determined that an increase of strength of about one eighth of the breaking weight is obtained by enlarging the diameter of a column in its middle.

In cast-iron columns of the same thickness, the strength is inversely proportional to the $^{1 \cdot 7}$ power of the length nearly. Thus, in solid columns, the ends being flat, the strength is as $\frac{d^{3 \cdot 6}}{l^{1 \cdot 7}}$, *l representing the length, and d the diameter.*

Hollow columns, having a greater diameter at one end than the other, have not any additional strength over that of uniform cylindrical columns.

Experiments upon wrought iron give a mean crushing stress of 74250 lbs. per square inch. Cast iron is decreased in length nearly double what wrought iron is by the same weight; but wrought iron will sink to any degree with little more than 26680 lbs. per square inch, while cast iron will bear 97500 lbs. to produce the same effect.

A wrought bar will bear a compression of $\frac{1}{863}$ of its length, without its utility being destroyed.

With cast iron, a pressure beyond 26680 lbs. per square inch is of little, if any, use in practice.

For equal decrements of length, wrought iron will sustain double the pressure of cast iron.

Glass and the hardest stones have a crushing strength from 7 to 9 times greater than tensile; hence an approximate value of their crushing strength may be obtained from their tensile, and contrariwise.

Various experiments show that the power of stones, etc., to resist the effects of freezing is a fair exponent of that to resist compression.

Wrought-iron Plates, Cylindrical Tubes.

Length.	Width.	Thickness.	Area.	Crush. Weight.
PLATES.	Ins.	Ins.	Ins.	Lbs.
10 feet	2.98	.497	1.48	815
10 "	3.01	.766	2.3	3379
HOLLOW CYLINDERS.	External.	Internal.		
10 feet	1.495	1.292	.444	14661
10 "	2.49	2.275	.804	29779
10 "	6.366	6.106	2.547	35886
RECTANGULAR TUBES.				
10 } lap-riveted	4.1	4.1	.504	10980
10 } lap-riveted	4.1	4.1	1.02	19261
10 } lap-riveted	4.25	4.25	2.395	21585
10 } lap-riveted	8.4	4.25	6.89	29981
10 } lap-riveted	8.1	8.1	2.07	132760
10 lap-riveted, and two internal diaphragm plates	8.1	8.1	3.551	19800

Comparative Resistance of Cast and Wrought Iron Bars to bear Compression in the Direction of their Length, set Vertical, and inclosed in a Frame to maintain them in that Position.

One Inch Square, and Ten Feet in Length.

DECREASE IN LENGTH.

Weight.	Cast Iron.	Wrought Iron.	Weight.	Cast Iron.	Wrought Iron.
Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.
5054	.054	.028	27498	.3	.143
9578	.102	.052	31978	.357	.174
14058	.151	.073	40938	.503	

Ultimate Practical Resistance.

Cast Iron.—Mean weight, 12800 lbs.; mean decrease, .135 ins.

Hence, the length of the bars being 10 feet = 120 ins., $.\frac{120}{135} = 888$, a cast-iron bar will bear a compression of $\frac{1}{888}$ of its length without its utility being destroyed, although its elasticity will be materially injured.

Wrought Iron.—Mean weight, 26650 lbs.; mean compression, .139 ins.

Hence, the length of the bars being 10 feet = 120 ins., $.\frac{120}{139} = 863$, a wrought-iron bar will bear a compression of $\frac{1}{863}$ of its length without its utility being destroyed.

To Compute the Crushing Strength of a Solid Cylindrical Column of Cast Iron.

$\frac{d^{3.6}}{l^{1.7}} \times 100000 = W$, *d representing the diameter of the column in inches, l its length in feet, and* W *the crushing weight in pounds.*

EXAMPLE.—What is the resistance to crushing of a solid cylinder, 2 inches in diameter and 5 feet in length?

$$\frac{2^{3.6}}{5^{1.7}} = \frac{12.125}{15.426} \times 100000 = 78601 \text{ lbs.}$$

For Rectangular Columns put 160000 for the multiplier.

To Compute the ultimate Crushing Strength of a Hollow Cylindrical Column of Cast Iron.

$\frac{D^{3.6} - d^{3.6}}{l^{1.7}} \times 100000 = W$, D *representing the greater diameter.*

EXAMPLE.—What is the resistance to crushing of a hollow cylindrical column having diameters of 2 and 1.25 inches, and a length of 7 feet?

$$\frac{2^{3.6} - 1.25^{3.6}}{7^{1.7}} = \frac{12.125 - 2.233}{27.332} \times 100000 = 36190 \text{ lbs.}$$

For Wrought Iron and Oak.

Solid cylinder, wrought iron	170000
Solid cylinder, oak	10880
Solid cylinder, pine	8200
Hollow cylinder, wrought iron ...	170000
Hollow cylinder, oak	10880
Hollow cylinder, pine	8200
Rectangular column, wrought iron	270000
Rectangular column, oak	17400
Rectangular column, pine	11280

The above formulæ are those of Hodgkinson for the breaking or crushing weight. The formulæ of Euler, which are for the incipient breaking weight, are applicable for small diameters, and are thus: $\frac{d^4}{l^2} \times 100000 = W$, for solid cylinders, and $\frac{D^4 - d^4}{l^2} \times 100000 = W$, for hollow cylinders.

The safe load that may be borne by a column of cast iron, independent of any considerations, regarding the operation of its ends as to their being flat or rounded, etc., is from 5000 to 8000 lbs. per square inch for short or stable bodies, or about ¼ of the result by the above rule.

NOTE.—The preceding formulæ apply to all columns where the length is not less than about 30 times the external diameter; for columns shorter than this, a modification of the formulæ is necessary, as in shorter columns the breaking weight is a large portion of that necessary to crush the column.

For Columns, the Length of which exceed 5 *Diameters and is less than* 30.

$\frac{w\,c}{w+.75\,c}=W$, *w representing the weight as obtained from the preceding formula, and c the crushing resistance of the material in pounds.*

Weight that can be borne with safety by Cast-iron Columns in 1000 Lbs.—(*Trenton Iron Co.*)

Length. Feet.	2 Ins.	3 Ins.	4 Ins.	5 Ins.	6 Ins.	7 Ins.	8 Ins.	9 Ins.	10 Ins.	11 Ins.	12 Ins.	13 Ins.	14 Ins.	15 Ins.
5	12.4	44	102	184	288	414	560	728	916	1126	1354	—	—	—
6	9.4	36	88	164	264	386	532	698	884	1082	1320	1570	—	—
7	7.2	30	76	146	242	360	502	660	859	1056	1282	1530	1798	2086
8	—	24	66	130	218	332	470	630	812	1016	1240	1486	1754	2040
9	—	20	56	114	198	306	440	596	774	974	1196	1440	1706	1992
10	—	18	48	102	180	282	410	560	739	932	1152	1392	1656	1940
12	—	—	38	80	136	238	354	494	658	846	1056	1292	1550	1828
14	—	—	28	64	122	200	304	432	586	774	966	1192	1440	1712
16	—	—	—	52	100	170	262	378	520	686	878	1094	1332	1596
18	—	—	—	44	84	144	226	332	462	616	796	1000	1228	1482
20	—	—	—	—	72	124	196	292	410	552	720	912	1130	1372

For Tubes or Hollow Columns, subtract the weight that may be borne by a column of the diameter of the internal diameter of the tube. The thickness of metal should not be less than one twelfth their diameter.

Relative Value of various Woods, their Crushing Strength and Stiffness being combined.

Ash	3571	English oak	4074	Sycamore	1833
Beech	3079	Mahogany	2571	Teak	6555
Cedar	700	Quebec oak	2927	Walnut	2378
Elm	3468	Spruce	2522	Yellow pine	2193

Comparative Value of Long Columns of various Materials.

Cast Iron	1000	Oak	108.8	Pine	78.5

Bridges.

Iron bridges with a circular arc should have a rise of .1 of the chord line, and a width of pier of .1 of span.

Girders combined with Suspension Chains.—[P. W. BARLOW.]

In a suspended girder, the stress is resisted by back chains or wire rope.

The economy of metal in a suspension-bridge, under the average circumstances of its attainable depth, is from ¼ to ½ of that in a tubular or simple girder-bridge of equal strength and rigidity.

Comparison between the Two largest Railway Bridges.

Niagara—Wire. Having a roadway, and a single railway of three gauges in a span of 820 feet; weighs 1000 tons.

Britannia—Tubular. Having a double line of railway in a span of 460 feet; weighs 3000 tons.

Trussed Beams or Girders.

Wrought and cast iron possess different powers of resistance to tension and compression; and when a beam is so constructed that these two materials act in unison with each other *at the stress due to the load required to be borne*, their combination will effect an essential saving of material. In consequence of the difficulty of adjusting a tension-rod to the strain required to be resisted, it is held to be impracticable to construct a perfect truss beam.

Fairbairn declares that it is better for the tension of the truss rod to be low than high, which position is fully supported by the following elements of the two metals:

Wrought Iron has great tensile strength, and, having great ductility, it undergoes much elongation when acted upon by a tensile force. On the contrary, *Cast Iron* has great crushing strength, and, having but little ductility, it undergoes but little elongation when acted upon by a tensile force; and, when these metals are released from the action of a high tensile force, the *set* of the one differs widely from that of the other, that of wrought iron being the greatest. Under the same increase of temperature, the expansion of wrought is considerably greater than that of cast iron; 1.81* tons per square inch is required to produce in wrought iron the same extension as in cast iron by 1 ton.

Fairbairn, in his experiments upon English metals, deduced that within the limits of strain of 13440 lbs. per square inch for cast iron, and 30240 lbs. per square inch for wrought iron, the tensile force applied to wrought iron must be 2.25 times the tensile force applied to cast iron, to produce equal elongations.

The relative tensile strengths of cast and wrought iron being as 1 to 1.85, and their resistance to extension as 1 to 2.25, therefore, where no initial tension is applied to a truss rod, the cast iron must be ruptured before the wrought iron is sensibly extended.

The resistance of cast iron in a trussed beam is not wholly that of tensile strength, but it is a combination of both tensile and crushing strengths, or a transverse strength; hence, in estimating the resistance of a girder, the transverse strength of it is to be used in connection with the tensile strength of the truss.

The mean transverse strength of a cast-iron bar, one inch square and one foot in length, supported at both ends, the stress applied in the middle, is about 900 lbs.; and as the mean tensile strength of wrought iron is about 20000 lbs. per square inch, the ratio between the sections of the beams and of the truss should be in the ratio of the transverse strength per square inch of the beam and of the tensile strength of the truss.

The girders under consideration are those alone in which the truss is attached to the beam at its lower flange, in which case it presents the following conditions:

1. *When the truss runs parallel to the lower flange.* 2. *When the truss runs at an inclination to the lower flange, being depressed below its centre.* 3. *When the beam is arched upward, and the truss runs as a chord to the curve.*

Consequently, in all these cases the section of the beam is that of an open one with a cast-iron upper flange and web, and a wrought-iron lower flange, increased in its resistance over a wholly cast-iron beam in proportion to the increased tensile strength of wrought iron over cast iron for equal sections of metals.

As the deductions of Fairbairn as to the initial strain proper to be given to the truss are based upon a cast-iron beam with the truss inserted into the upper flange of the beam, whereby it was submitted almost wholly to a tensile strain, they will not apply to the two constructions of trussed beams under consideration.

As each construction of trussed beam will produce a strain upon the truss in accordance with the position of the neutral axis of the section of the whole beam, and as the extension of the truss will vary according as it is more or less ductile, it is impracticable, in the absence of the necessary elements, to give an amount of initial strain that would be applicable as a rule.

From the various experiments made upon trussed beams, it is shown:

1. That their rigidity far exceeds that of simple beams; in some cases it was from 7 to 8 times greater. 2. That when the truss resists rupture, the upper flange of the beam being broken by compression, there is a great gain in strength. 3. That their strength is greatly increased by the upper flange being made larger than the lower one. 4. That their strength is greater than that of a wrought-iron tubular beam containing the same area of metal.

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* The elongation of cast and wrought iron being 5500 and 10000, hence 10000 ÷ 5500 = 1.81.

Results of Experiments upon the Deflection of Bars, Beams, etc., of various Sections, etc.; by U. S. Ordnance Corps, Barlow, Fairbairn, Hodgkinson, Stephenson, etc.

Bar, Beams, etc., supported at both Ends; Stress or Weight applied in the Middle.

Material and Section.	Length of Bearing.		Breadth.	Depth.	Depth of Opening.	Weight.	Deflection.	Value for Deflection given. $\frac{l^3 W}{16\, b\, d^3 D}$ = V.
	Ft.	Ins.	Inch.	Inch.	Inch.	Lbs.	Inch.	Lbs.
Woods.								
Fir. Rectangle	3		1.5	2.	—	120	.09	187
" Square	6		1.5	2.	—	180	1.	202
Ash. Cylinder	3	10	2.	2.	—	715	2.7*	—
" " hollow	3	10	2.	2.	.5	657	2.5*	—
" Square	7		2.	2.	—	225	1.266†	288
Yellow pine. Square	5		.75	.75	—	16	1.5	250
Oak. Square	3		1.	1.	—	158	2.95	91
Pine. Rectangle	40		7.5	9.25	—	1700	5.25	218
Metals.								
Cast iron, English	2	10	1.	1.	—	300	.16†	2660
" dry sand, square	1	8	2.	2.	—	10800	.11	1756
" green sand, "	1	8	2.	2.	—	5000	.045†	1988
 Flange, 5×.3	6	6	.36	1.55	—	112	.273	5302
	6	6	.36	1.55	—	336	1.03	4203
 Flange, 5×.3	6	6	.36	1.55	—	112	.27	5303
	6	6	.36	1.55	—	336	.895	4837
 Flange, 1.5×.5	3	1	.5	3.	—	2016	.079	450
 Flange, 23.9×3.125	23	1	3.29	36.1	—	60000	.1	2986
" 6.5×1 area 18	15		.91	14.	—	4480	.3	1261
 " 4.5× .875 9 ×1.25	22		1.12	36.	—	22400	.094	3035
 Rectangular, area 1.965	4	6	.975	2.015	—	712	.28	1815
 Open beam, area 2	4	6	1.	2.5	.5	712	.132	1969
Wrought iron. Square	2	9	2.	2.	—	2240	.068	2678
" Rectangle.	2	9	1.5	3.	—	2240	.074	971
 Flange, 4.5×.5 Rib, 3.25 diameter,	10	3	.5	10.	—	3136	.375‡	1126
 Flanges, 2 of 2.25×.28 " 2.25×.3	7		.25	7.	—	16480	.25	16480
 Tubes, thickness .03 in.	3	9	1.9	3.	—	448	.1	289
.525 "	30		15.5	24.	—	5685	.12	373
 Tubes, thickness .037 in.	17		12.	12.	11.925	2755	.65*	—
Corrugated plates	31	6	3.1	8.	—	4480	.62	8893
 Tubes, thickness .0416 in.	17		9.25	14.62	13.535	2262	.62*	—
.143 "	17		9.75	15.	14.714	16800	1.39*	—
Steel, cast, soft	3	2	.23	.52	—	22	.331	4107
Brass, cast	1		.7	.45	—	60	.04†	1488

* Breaking weight. † Elasticity perfect. ‡ Permanent set.

The *Value* in the preceding Table is for the *Deflection given;* hence, when the deflection that a bar, beam, etc., may be permitted to bear is given, the weight it may bear with that deflection will be determined by the formula, substituting for D the deflection it may bear.

Deflection of Bars, Beams, Girders, etc.

The experiments of Barlow upon the deflection of wood battens determined that the deflection of a beam from a transverse strain varied as the breadth directly, and as the cubes of both the depth and length, and that with like beams and within the limits of elasticity it was directly as the weight.

In bars, beams, etc., of an elastic material, and having great length compared to their depth, the deductions of Barlow will apply with sufficient accuracy for all practical purposes; but in consequence of the varied proportions of depth to length of the varied character of materials, of the irregular resistance of beams constructed with scarfs, trusses, or riveted plates, and of the unequal deflection at initial and ultimate strains, it is impracticable to give any positive laws regarding the degrees of deflection of different and dissimilar bars, beams, etc.

In the experiments of Hodgkinson, it was farther shown that the sets from deflections was very nearly as the squares of the deflections.

In a rectangular bar, beam, etc., the position of the neutral axis is in its centre, and it is not sensibly altered by variations in the amount of strain applied. In bars, beams, etc., of cast and wrought iron, the position of the neutral axis varies in the same beam, and is only fixed while the elasticity of the beam is perfect. When a bar, beam, etc., is bent so as to injure its elasticity, the neutral line changes, and continues to change during the loading of the beam, until it breaks.

When bars, beams, etc., are of the same length, the deflection of one, the weight being suspended from one end, compared with that of a beam uniformly loaded, is as 8 to 3; and when a beam is supported at both ends, the deflection in like cases is as 5 to 8. Whence, if a bar, etc., is in the first case supported in the middle, and the ends permitted to deflect; and in the second, the ends supported, and the middle permitted to descend, the deflection in the two cases is as 3 to 5.

Of three equal and similar bars or beams, one inclined upward, one downward at the same angle, and the other horizontal, that which has its angle upward is the weakest, the one which declines is the strongest, and the one horizontal is a mean between the two.

When a bar, beam, etc., is *Uniformly Loaded*, the deflection is as the weight, and approximately as the cube of the length or as the square of the length; and the element of deflection and the strain upon the beam, the weight being the same, will be but half of that when the weight is suspended from one end.

The deflection of a bar, beam, etc., *Fixed at one End, and Loaded at the other*, compared to that of a beam of twice the length, *Supported at both ends, and Loaded in the Middle*, the strain being the same, is as 2 to 1; and when the length and the loads are the same, the deflection will be as 16 to 1, for the strain will be four times greater on the beam fixed at one end than on the one supported at both ends; therefore, all other things being the same, the element of deflection will be four times greater; also, as the deflection is as the element of deflection into the square of the length, then, as the lengths at which the weights are borne in their cases are as 1 to 2, the deflection is as $1 : \overline{2^2 \times 4} = 1$ to 16.

The deflection of a bar, beam, etc., having the section of a triangle, and supported at its ends, is $\frac{1}{3}$ greater when the edge of the angle is up than when it is down.

When the Length is uniform, with the same weight, the deflection is inversely as the breadth and square of the depth into the element of deflection, which is inversely as the depth. Hence, other things being equal, the deflection will vary inversely as the breadth and cube of the depth.

Illustration.—The deflections of two pine battens, of uniform breadth and depth, and equally loaded, but of the lengths of 3 and 6 feet, were as 1 to 7.8.

If a bar or beam is cylindrical, the deflection is 1.7 times that of a square beam, other things being equal.

Bars, Beams, etc.—*When Fixed at one End, and Loaded at the other*, $\frac{l^3 W}{b d^3 D} = V$, *a constant quantity.*

When Fixed at one End, and uniformly Loaded, $\frac{3 l^3 W}{8 b d^3 D} = V$.

When Fixed at both Ends, and Loaded in the Middle, $\frac{l^3 W}{24 b d^3 D} = V$.

When Supported at both Ends, and Loaded in the Middle, $\frac{l^3 W}{16 b d^3 D} = V$.

When Supported at both Ends, and uniformly Loaded, $\frac{5 l^3 W}{8 \times 16 b d^3 D} = V$.

When Supported in the Middle, and the Ends uniformly Loaded, $\frac{3 l^3 W}{5 \times 16 b d^3 D} = V$.

When Supported at both Ends, and the Weight suspended from any other Point than the Middle, $\frac{m^2 n^2 W}{l b d^3 D} = V$, *$l$ representing the length in feet, b its breadth, d its depth, W the weight or stress with which it is loaded, $m\,n$ the distances of the weight from the supports, and D the deflection in inches.*

Hence, in order to preserve the same stiffness in bars, beams, etc., the depth must be increased in the same proportion as the length, the breadth remaining constant.

The deflection of different bars, beams, etc., arising from their own weight, having their several dimensions proportional, will be as the square of either of their like dimensions.

Note.—In the construction of models on a scale intended to be executed in full dimensions, this result should be kept in view.

Results of Experiments upon the Transverse Strength and Deflection of Wrought-iron Rails.—[Barlow.]

Rails.	Weight per Yard.	Length of Bearing.	Depth.	Area.	Weight.	Deflection. For Weight.	Deflection. For each Ton.
	Lbs.	Feet.	Inch.	Inch.	Lbs.	Inch.	Inch.
 Flanges, 2.25..... rib, .65..	60	2.75	4.5	6.166	2240	.027	—
	60	2.75	4.5	6.166	4480	.031	.004
	60	2.75	4.5	6.166	17920	.057	.005
	60	2.75	4.5	6.166	26680	.087	.01
" 2.6×1.25ins. rib, .85.....	75	4.5	5.	7.5	4480	.05	—
	75	4.5	5.	7.5	20160*	.165	.023
 " 3.5×.6 in... rib, 8.......	57	2.75	3.5	5.85	4480	.05	—
	57	2.75	3.5	5.85	17920*	.152	.039
 Head, 2.25×1 in. rib, .75... Flange, 3.5×.8.	60	2.75	4.	6.7	4480	.034†	—
	60	2.75	4.	6.7	17920	.064	.082
 Head, 2.5×.6... rib, .6.... Bottom, 1.25×.88	51	3.	4.5	5.55	6720‡	.024	—

* Destructive weights.
† The deflection between this and a like bar to this, reversed, was, for between 5 and 10 tons weight, as .0074 and .0059.
‡ Destructive weight 7 tons.

As it is impracticable to give any general rule for the deflection of bars, beams, etc., of different lengths and sections, reference must be had to the results of previous experiments upon bars, beams, etc., of a like character to that of those for which the deflection is required.

Thus, in the preceding Tables, page 476 to 478, are given the deflections ascertained in very many cases, added to which is given a *value* or *constant*, obtained by the formula $\frac{l^3 W}{16 d b^3 D}$.

In the first and second examples of the Table are results of two experiments with a like material, but of differing dimensions.

In order, then, to determine the relative values of the *Constants*, the varying elements of the case must be reduced to a uniform measure.

In the examples referred to, the *Values* or *Constants* are as 187 and 202, their sections ($b d^3$) as 12 and 12, the weights applied as 120 and 180, and the lengths as 3^3 and 6^3.

If the deflections were in conformance with the formula, the *Values* here deduced would be equal, instead of 187 and 202; the proportion of which is obtained by $\frac{187}{202} = .92$ *of the deflection given by the formula.* The deflection as furnished by the Table for the second experiment is 1; hence, as $.92 : 1 :: 1 : 1.09 =$ *the calculated deflection of it.*

When it is required to estimate the Deflection for Differing Weights, Lengths, and Sections, and contrariwise, to estimate Weights, Lengths, and Sections for a given Deflection.

RULE.—Divide the deflections by the cubes of the lengths and by the weights; or, multiply the deflections by the sections ($b d^3$).

Thus, if deflections are as .15 and 1.2 inches, weights as 125 and 250 lbs., lengths as 1 and 2 feet, and sections as 1×2^3 and 2×2^3 inches.

Then $\frac{.15}{1.2} \div \frac{1^3}{2^3} = \frac{.15}{.15} =$ *quotient of the deflection ÷ the cubes of the lengths, which, being equal, shows the deflections to be as the cubes of the lengths.*

$\frac{.15}{.15} \div \frac{125}{250} = \frac{.0012}{.0006} = \frac{2}{1} =$ *quotient of the reduced deflection ÷ the weights; hence the deflections are but one half of that due to the weights.*

$\frac{2}{1} \times \frac{1 \times 2^3}{2 \times 2^3} = \frac{16}{16} = \frac{1}{1} =$ *product of the preceding quotient and the sections* ($b d^3$); *hence the reduced deflections are as the sections.*

Relative Elasticity of various Materials.—[TRUMBULL.]

Ash	2.9	Elm	2.9	Pine, yellow	2.6
Beech	2.1	Oak	2.8	" pitch	2.9
Cast iron	1.	Pine, white	2.4	Wrought iron	.86

Comparative Strength and Deflection of Cast-iron Flanged Beams.

Description of Beam.	Comp. Strength.	Description of Beam.	Comp. Strength.
Beam of equal flanges	.58	Breadth with flanges as 1 to 4.5	.78
Beam with only bottom flange	.72	Breadth with flanges as 1 to 5.5	.82
Beam with flanges as 1 to 2	.63	Breadth with flanges as 1 to 6	1.
Beam with flanges as 1 to 4	.73	Breadth with flanges as 1 to 6.73	.92

General Deductions.

In Cast Iron, the permanent deflection is from ⅓ to ¼ of its breaking weight, and the deflection should never exceed ⅓ of the ultimate deflection.

All rectangular bars of Wrought Iron, having the same bearing length, and loaded in their centre to the full extent of their elastic power, will be so deflected that their deflection, being multiplied by their depth, the product will be a constant quantity, whatever may be their breadth or other dimensions, provided their lengths are the same.

The heaviest running weight that a bridge is subjected to is that of a locomotive and tender, which is equal to 1.5 tons per lineal foot.

Girders should not be deflected to exceed the $\frac{1}{40}$ of an inch to a foot in length.

In cast iron, the $\frac{1}{20}$ to $\frac{1}{30}$ of the breaking weight will give a visible set.

When a load on a girder is supported by the bottom flange of it alone, it produces a torsional strain.

A continuous weight, equal to that a beam, etc., is suited to sustain, will not cause the deflection of it to increase unless it is subjected to considerable changes of temperature.

The heaviest load on a railway girder should not exceed ⅙ of that of the breaking weight of the girder when laid on at rest.

Deflection consequent upon Velocity of the Load.—Deflection is very much increased by instantaneous loading; by some authorities it is estimated to be doubled.

The momentum of a railway train in deflecting girders, etc., is greater than the effect from the dead weight of it, and the deflection increases with the velocity.

Experiments made by the Commissioners of Railway Structures of 1849 showed that a passing load produced a greater effect on a beam than a load at rest.

A carriage was moved at a velocity of 10 miles per hour; the deflection was .8 inch, and when at a velocity of 30 miles the deflection was 1½ inches.

In this case, 4150 lbs. would have been the breaking weight of the bars if applied in their middle, but 1778 lbs. would have broken them if passed over them with a velocity of 30 miles per hour.

Cast iron will bend to ⅓ of its ultimate deflection with less than ⅓ of its breaking weight if it is laid on gradually, and but ⅙ if laid on rapidly.

When motion is given to the load on a beam, etc., the point of greatest deflection does not remain in the centre of the beam, etc., as beams broken by a traveling load are always fractured at points beyond their centres, and often into several pieces.

Chilled bars of cast iron deflect more readily than unchilled.

Results of Experiments on the Subjection of Iron Bars to continual Strains.—[*Rep. Comm. on Railway Structures.*]

Cast-iron bars subjected to a regular depression, equal to the deflection due to a load of ⅓ of their statical breaking weight, bore 10 000 successive depressions, and when broken by statical weight gave as great a resistance as like bars subjected to a like deflection by statical weight.

Of two bars subjected to a deflection equal to that carried by half of their statical breaking weight, one broke with 28 602 depressions, and the other bore 30 000, and did not appear weakened to resist statical pressure.

Hence Cast-iron bars will not bear the continual applications of ⅓ of their breaking weight.

A bar of Wrought Iron, 2 inches square and 9 feet in length between its supports, was subjected to 100 000 vibratory depressions, each equal to the deflection due to a load of $\frac{5}{9}$ of that which permanently injured a similar bar, and their depressions only produced a permanent set of .015 inch.

The greatest deflection which did not produce any permanent set was due to rather more than ½ the statical weight, which permanently injured it.

A wrought-iron box girder, 6×6 inches and 9 feet in length, was subjected to vibratory depressions, and a strain corresponding to 3762 lbs., repeated 43 370 times, did not produce any appreciable effect on the rivets.

Mr. Tredgold, in his experiments upon Cast Iron, has shown that a load of 300 lbs., suspended from the middle of a bar 1 inch square and 34 inches between its supports, gave a deflection of .16 of an inch, while the elasticity of the metal remained unimpaired. Hence a bar 1 inch square and 1 foot in length will sustain 3412 lbs., and retain its elasticity.

TORSIONAL STRENGTH.

The *Torsional Strength* of any square bar or beam is as the cube of its side, and of a cylinder as the cube of its diameter. Hollow cylinders or shafts have greater torsional strength than solid ones containing the same volume of material.

The *Torsional Angle* of a bar, etc., under equal pressures will vary as the length of the bar, etc. Hence the torsional strength of bars of like diameters is inversely as their lengths.

The strength of a cylindrical prism compared to a square is as 1 to .85.

When a bar, beam, etc., having a length greater than its diameter, is subjected to a torsional strain, the direction of the greatest strain is in the line of the diagonal of a square, and if a square be drawn on the surface of the bar, etc., in its primitive form, it will become a rhombus by the action of the strain.

Torsional Strength of Cast Iron, deduced from the Experiments of Major Wade, U. S. A.

Reduced to a uniform Measure of One Inch Square or in Diameter; Weight or Stress applied at One Foot from Centre of Axis of the Material, and at the Face of the Axis or Journal.

Area of Cross-section.	Breaking Weight of Figures.					
	Square, $b\,d^2$.	Cylinder, d^3.	Area of Section.	Hollow Cylinder, $d^3 - d'^3$.	Area of Section.	Hollow Cylinder, $d^3 - d'^3$.
Inch.	Lbs.	Lbs.	Inch.	Lbs.	Inch.	Lbs.
1	730	613	.995	$d' = .5$ 563	1.012	$d' = .6$ 585
2	677	698	1.931	$d' = .5$ 597	1.967	$d' = .7$ 579
3	840	636	2.728	$d' = .5$ 540	2.966	$d' = .8$ 476
Mean.....	749	649		567		547

Summary of Results.

All of the bars were from the same mixture of common foundry iron, of a mean torsional strength of 644 lbs. per square inch of section.

From these results it appears that solid square shafts have about $\frac{1}{5}$ less strength than solid cylinders of equal areas.

The stress which will give a bar a permanent set of ½° is about $\frac{7}{10}$ of that which will break it, and this proportion is quite uniform, even when the strength of the material may vary essentially.

The strongest bars give the longest fractures.

Wrought Iron, compared with Cast Iron, has equal strength under a stress which does not produce a permanent set, but this set commences under a less force in wrought iron than cast, and progresses more rapidly thereafter. The strongest bar of wrought iron acquired a permanent set under a less strain than a cast-iron bar of the lowest grade. The mean *Values* of cast and wrought iron and bronze, for bars of small diameters for a permanent set of ½°, are as 1, .6, and .33.

The torsional strength and rigidity of Puddled Steel does not differ essentially from that of cast iron.

The coefficients for the torsional breaking stress of iron and bronze, as determined by Major Wade, are: Wrought Iron, 640; Cast Iron, 560; Bronze, 460.

Torsional Strength of Cast and Wrought Iron and Bronze, with their Values for different Diameters.

Reduced to a uniform Measure of One Inch Square or in Diameter; Weight or Stress applied at One Foot from Centre of Axis of the Material, and at the Face of the Axis or Journal

Length of Journal, or of the Bar or Beam submitted to Stress, for which the Values are given, three times the Diameter or Side of the Shaft.

Figures	Specific Gravity.	Length of Journal or Side.	Breaking Weight.	*Value* for Diameter of 2 Ins.	5 Ins.	10 Ins.	15 Ins.
		Inch	Lbs	Lbs.	Lbs	Lbs.	Lbs
CYLINDER (*Cast Iron*).							
Good common castings	7.18	8.	583	170	115	105	100
" cold blast, mean of 8 trials......	—	8.	705	175	120	110	105
Gun iron, small bars	7.32	8.	750	190	130	120	115
" greatest extreme	7.724	8.	833	200	135	125	120
CYLINDER (*Wrought Iron*).							
Begins to yield, permanent set .	7.855	8.	300	130	128	125	123
Bends without breaking	—	8.	642				
CYLINDER (*Bronze*).							
Begins to yield, permanent set .	8.71	8.	192	55	45	35	33
Bends without breaking	—	8.	458				
SQUARE (*Cast Iron*)	7.2	3.	730	220	150	140	134
" "	—	4.8	840				
" (*Wrought Iron*).......	7.855	3.	—	170	165	160	162

The Torsional Strength of Cast Steel is about double that of Cast Iron.

The experiments above given were made with bars not exceeding 2 inches in diameter; the relations given, therefore, do not hold, as the diameters are increased, in consequence of the shrinking of the cast metals in cooling, which, by cooling at the outer surface first, draws the metal from the centre, and in effect gives to a bar or shaft the properties of a hollow cylinder. In shafts of 10 inches in diameter, the torsional strength of wrought iron is fully equal to that of cast iron; and with larger diameters it would be much greater, but that it suffers deterioration as its diameter increases, from the increased difficulty in effecting welding and the reduction of the metal to a fibrous texture.

The following rules are purposed to apply in all instances to the diameters of the journals of shafts, or to the diameter or side of the bearings of the beams, etc., where the length of the journal or the distance upon which the strain bears does not greatly exceed the diameter of the journal or side of beam, etc.; hence, when the length or distance is greatly increased, the diameter or side must be correspondingly increased.

To Compute the Torsional Strength of Square or Round Shafts, etc.

RULE.—Multiply the *Value* in the preceding Table by the cube of the side or of the diameter of the shaft, etc., and divide the product by the distance from the axis at which the stress is applied in feet; the quotient will give the resistance in pounds.

EXAMPLE.—What torsional stress may be borne by a cast-iron shaft of the best material, 2 inches in diameter, the power being applied at two feet from its axis?

$$200 \times 2^3 = 1600, \text{ and } \frac{1600}{2} = 800 \ lbs.$$

To Compute the Diameter of a Square or Round Shaft, etc., to resist Torsion.

RULE.—Multiply the extreme of pressure upon the crank-pin, or at the pitch-line of the pinion, or at the centre of effect upon the blades of the wheel, etc., that the shaft may at any time be subjected to, by the length of the crank or radius of the wheel, etc., in feet; divide their product by the *Value* in the preceding Table, and the cube root of the quotient will give the diameter of the shaft or its journal in inches.

EXAMPLE.—What should be the diameter for the journal of a wrought-iron water-wheel shaft, the extreme pressure upon the crank-pin being 59 400 lbs., and the crank 5 feet in length?

$$\frac{59400 \times 5}{125} = 2376, \text{ and } \sqrt[3]{2276} = 13.34 \ inches.$$

When two Shafts are used, as in Steam-vessels with one Engine, etc.

RULE.—Divide three times the cube of the diameter for one shaft by four, and the cube root of the quotient will give the diameter of the shaft in inches.

EXAMPLE.—The area of the journal of a shaft is 113 inches; what should be the diameter, two shafts being used?

Diameter for area of 113 = 12.

Then $\frac{3 \times 12^3}{4} = 1296$, and $\sqrt[3]{1296} = 10.9$ *inches.*

NOTE.—The examples here given are deduced from instances of successful practice; where the diameter has been less, fracture has almost universally taken place, the strain being increased beyond the ordinary limit.

EX. 2.—When the work to be performed is of a regular character, and the stress is consequently uniform, the proportion of ¾ may be reduced to ⅝.

Relative Values of Cast and Wrought Iron.

When shafts of less diameter than 12 inches are required, the *Values* here given may be slightly reduced, according to the quality of the iron and the diameter of the shaft to be used; but when they exceed this diameter, the *Values* may not be increased in a like manner, as the strength of a cast or a wrought-iron shaft decreases as their diameters increase.

To Compute the Torsional Strength of Hollow Shafts and Cylinders.

RULE.—From the fourth power of the exterior diameter subtract the fourth power of the interior diameter, and multiply the remainder by the *Value* of the material; divide this product by the product of the exterior diameter and the length or distance from the axis at which the stress is applied in feet; the quotient will give the resistance in pounds.

EXAMPLE.—What torsional stress may be borne by a cast-iron hollow shaft, having diameters of 3 and 2 inches, the power being applied at 1 foot from its axis?

$$3^4 - 2^4 \times 105 = 81 - 16 \times 105 = 6825, \text{ which} \div 3 \times 1 = \frac{6825}{3} = 2275 \ lbs.$$

The order of shafts, with reference to the degree of torsional stress to which they are subjected, is as follows:

1. Fly-wheel.
2. Water-wheel.
3. Secondary.
4. Tertiary, etc.

Hence the diameters of their journals may be reduced in this order.

Relative Value of different Figures to Resist Torsion, having equal Sectional Areas.

Solid Cylinder.	Solid Square.	Hollow Cylinders, the interior and exterior Diameters of which are in the Proportion of				
		4 to 10.	5 to 10.	6 to 10.	7 to 10.	8 to 10.
1	.875	1.2656	1.4433	1.7	2.0864	2.7377

Various Qualities of different Metals,

As determined by the Experiments of Major Wade for the U. S. Ordnance Corps.

Metals.		Specific Gravity.	Tensile Strength.	Transverse Strength.	Torsional Strength for Diam. 1 Inch, and Length of 1 Foot.		Crushing Strength.	Hardness.
					At ½ Degree.	Ultimate.		
			Per Sq. In.	Per Sq In	Lbs.	Lbs.	Per Sq. In.	
Bronze......	Least...	7.978	17 698	—	620	1687	—	4.57
	Greatest	8.953	56 786	—	833	1020	—	5.94
Cast iron....	Least...	6.9	9 000	416	1006	1660	84 529	4.57
	Greatest	7.4	45 970	958	2500	3060	174 120	33.51
"	Mean ..	7.225	31 829	680	1930	2760	144 916	22.34
Cast steel ...	Least...	7.729	—	—	—	5511	198 944	—
	Greatest	8.953	128 000	1916	—	—	391 985	—
Wrought iron	Least...	7.704	38 027	542	970	1296	40 000	10.45
	Greatest	7.858	74 592	—	1320	1836	127 720	12.14

DETRUSIVE STRENGTH.

The *Detrusive Strength* of any body is directly as its strength, or thickness, or area.

Results of Experiments upon the Detrusive Strength of Metals with a Punch.

Metals.	Diameter of Punch.	Thickness of Metal.	Power exerted.	Power required for a Surface of Metal of One Square Inch.	
	Inch	Inch.	Lbs	Lbs.	
Brass....................	.1	.045	5 448	37 000	NOTE.—The free use of oil reduces the power required very materially.
Cast iron................	—	—	—	30 000	
Copper................	.5	.08	3 983	30 000	
	.5	.17	7 823		
	.1	.3	21 250	22 300	
Steel....................	.5	.25	34 720	90 000	
Wrought iron..........	.5	.08	6 025	45 000	
	.5	.17	11 950		
	.5	.24	17 000		
	.1	.615	82 870	43 900	
	.2	1.06	297 400	44 300	

To Compute the Power necessary to Punch Iron, Brass, or Copper Plates.

RULE.—Multiply the product of the diameter of the punch and the thickness of the metal by 150 000 if for wrought iron, by 128 000 if for brass, and by 96 000 if for copper, and the product will give the power required in pounds.

Comparison between Detrusive and Transverse Strengths.

Assuming the compression and abrasion of the metal in the application of a punch of one inch in diameter to extend to ⅛ of an inch beyond the diameter of the punch, the comparative resistance of wrought iron to *detrusive* and *transverse* strain, the latter estimated at 600 lbs. per square inch, for a bar one foot in length, is as 2.5 to 1.

Results of Experiments upon the Detrusive Strength of Metals with Shears.

Made by Parallel Cutters.

Wrought Iron.—Thickness from .5 to 1 inch, 50 000 lbs. per square inch.

Made by Inclined Cutters, angle 1 in 8 = 7°.

Sheet Metals.	Thickness.	Power.	Bolts.	Diameter.	Power.
	Ins.	Lbs.		Ins.	Lbs.
Brass	.05	540	Brass	1.11	29 700
Copper	.297	11 196	Copper	.775	11 310
Steel	.24	14 930	Steel	.775	28 720
Wrought iron	.51	39 150	Wrought iron	1.142	35 410
	1.	44 800		.32	3 093

The resistance of wrought iron to shearing is about 75 per cent. of its resistance to tensile stress.

The resistance to shearing of plates and bolts is not in a direct ratio. It approximates to that of the square of the depth of the former, and to the square of the diameter of the latter.

Character of Strains to which Connecting Rods, Straps, Gibs, and Keys are subjected.

Heads of Rods.—At sides of keyholes, *tensile* and *crushing;* at front of keyholes, *detrusive.*

Straps.—At crown and at the sides of keyhole, *tensile;* at back of key holes, *detrusive.*

Gib.—*Transverse*, uniformly loaded along its length, fixed at both ends.

Key.—With single gib, *transverse*, uniformly loaded along its length, fixed at both ends.

Key.—With double gib, *transverse*, uniformly loaded along its length, fixed at both ends.

Woods.

When a beam or any piece of wood is let in (not mortised) at an inclination to another piece, so that the thrust will bear in the direction of the fibres of the beam that is cut, the depth of the cut *at right angles to the fibres* should not be more than .2 of the length of the piece, the fibres of which, by their cohesion, resist the thrust.

Shafts and Gudgeons.

Shafts are divided into *Shafts* and *Spindles*, according to their magnitude.

A *Gudgeon* is the metal journal or arbor upon which a wooden shaft revolves.

Shafts are subjected to Torsion and Lateral Stress combined, or to Lateral Stress alone.

Lateral Stiffness and Strength.—Shafts of equal length have *lateral stiffness* as their breadth and the cube of their depth, and have *lateral strength* as their breadth and the square of their depths. Hence, in shafts of equal lengths, their stiffness by any increase of depth increases in a greater proportion than their strength.

Shafts of different lengths have *lateral stiffness*, directly as their breadth and the cube of their depth, and inversely as the cube of their length; and have *lateral strength* directly as their breadth and as the square of their depth, and inversely as their length. Hence, in shafts of different lengths, their stiffness by any increase of their length decreases in a greater proportion than their strength.

Hollow shafts having equal lengths and equal quantities of material have *lateral stiffness* as the square of their diameter, and have *lateral strength* as their diameters. Hence, in hollow shafts, one having twice the diameter of another will have four times the stiffness, and but double the strength; and when having equal lengths, by an increase in diameter they increase in stiffness in a greater proportion than in strength.

The stress upon a shaft from a weight upon it is proportional to the product of the parts of the shaft multiplied into each other. Thus, if a shaft is 10 feet in length, and a weight upon the centre of gravity of the stress is at a point 2 feet from one end, the parts 2 and 8, multiplied together, are equal to 16; but if the weight or stress were applied in the middle of the shaft, the parts 5 and 5, multiplied together, would produce 25.

The ends of a shaft having to support the whole weight, the end which is nearest the weight has to support the greatest proportion of it, in the inverse proportion of the distance of the weight from the end. Hence, when a shaft is loaded in the middle, each of the journals or gudgeons has half the weight or stress to support.

When the load upon a shaft is uniformly distributed over any part of it, it is considered as united in the middle of that part; and if the load is not uniformly distributed, it is considered as united at its centre of gravity.

When the transverse section of a shaft is a regular figure, as a square, circle, etc., and the load is applied in one point, in order to give it equal resistance throughout its length, the curve of the sides becomes a cubic parabola; but when the load is uniformly distributed over the shaft, the curve of the sides becomes a semi-cubical parabola.

The deflection of a shaft produced by a load which is uniformly distributed over its length is the same as when ⅝ of the load is applied at the middle of its length.

The resistance of the body of a shaft to lateral stress is as its breadth and the square of its depth; hence the diameter will be *as the product of the length of it and the length of it on one side of a given point, less the square of that length.*

Illustration.—The length of a shaft between the centres of its journals is 10 feet; what should be the relative cubes of its diameters when the load is applied at 1, 2, and 5 feet from one end? and what when the load is uniformly distributed over the length of it?

$l \times l^1 - l^3 = d^3$; and when uniformly distributed, $d^3 \div 2 = d^1$.

$10 \times 1 = 10 - 1^2 = 9 =$ *cube of diameter at* 1 *foot*; $10 \times 2 = 20 - 2^2 = 16 =$ *cube of diameter at* 2 *feet*; $10 \times 5 = 50 - 5^2 = 25 =$ *cube of diameter at* 5 *feet.*

When a load is uniformly distributed, the stress is greatest at the middle of the length, and is equal to half of it; $25 \div 2 = 12.5 =$ *cube of diameter at* 5 *feet.*

CYLINDRICAL OR SOLID SHAFTS.

To Compute the Diameter of a Shaft of Cast Iron to resist Lateral Stress alone.

When the Stress is in or near the Middle. Rule.—Multiply the weight by the length of the shaft in feet; divide the product by 500, and the cube root of the quotient will give the diameter in inches.

Example.—The weight of a water-wheel upon a shaft is 50 000 lbs., its length 30 feet, and the centre of stress of the wheel 7 feet from one end; what should be the diameter of its body?

$$\sqrt[3]{\left(\frac{50\,000 \times l}{500}\right)} = 14.422 \text{ ins.}, \textit{ if the weight was in the middle of its length.}$$

Hence the diameter at 7 feet from one end will be, as by preceding Rule, $30 \times 7 - 7^2 = 161 =$ *relative cube of diameter at* 7 *feet*; $30 \times 15 - 15^2 = 225 =$ *relative cube of diameter at* 15 *feet.*

Then, as $\sqrt[3]{225} : 14.42 :: \sqrt[3]{161} : 12.89$ *ins., the diameter of the shaft at* 7 *feet from one end.*

When the Stress is uniformly laid along the Length of the Shaft. RULE.—Divide the cube root of the product of the weight and the length by 9.3, and the quotient will give the diameter in inches.

EXAMPLE.—Apply the rule to the preceding case.

$$\frac{\sqrt[3]{50\,000\times30}}{9.3}=12.31 \textit{ ins.}$$

Or, *When the Diameter for the Stress applied in the Middle is given.* RULE.—Take the cube root of ⅝ of the cube of the diameter, and this root will give the diameter required.

EXAMPLE.—The diameter of a shaft when the stress is uniformly applied along its length is 14.422 ins.; what should be its diameter, the stress being applied in the middle?

$$\sqrt[3]{\tfrac{5}{8}\times14.422^3}=\sqrt[3]{\tfrac{5}{8}\times3000}=12.33 \textit{ ins.}$$

HOLLOW SHAFTS OF CAST IRON.

When the Stress is in or near the Middle. RULE.—Divide the continued product of .012 times the cube of the length, and the number of times the weight of the shaft in pounds by the square of the internal diameter added to 1, and twice the square root of the quotient added to the internal diameter, will give the whole diameter in inches.

EXAMPLE.—The weight of a water-wheel upon a hollow shaft 30 feet in length is 2.5 times its own weight, and the internal diameter is 9 ins.; what should be the whole diameter of the shaft?

$$\sqrt{\left(\frac{.012\times30^3\times2.5}{1+9^2}\right.}=\sqrt{\frac{810}{82}}=3.14 \textit{ ins.}$$

Then $3.14\times2+9=15.28$ *ins., the diameter.*

To Compute the Diameter of a Solid Shaft of Cast Iron to resist its own Weight alone.

RULE.—Multiply the cube of its length by .007, and the square root of the product will give the diameter in inches.

EXAMPLE.—The length of a shaft is 30 feet; what should be its diameter in the body?

$$\sqrt{(30^3\times.007)}=189, \text{ and } \sqrt{189}=13.75 \textit{ ins.}$$

To Compute the Diameter of a Shaft, the Stress being applied in the Middle, when it has to resist both Torsional and Lateral Stress combined.

RULE.—Ascertain the diameter for each stress, and the cube root of the sum of their cubes will give the diameter required.

EXAMPLE.—The diameter of the journal of a shaft to resist torsional stress is ascertained to be 17 ins., and the diameter of its body in the centre to resist lateral stress has also been ascertained to be 14.422 ins.; what should be the diameter of the body?

$$\sqrt[3]{(17^3+14.422^3)}=7913, \text{ and } \sqrt[3]{7913}=19.927 \textit{ ins.}$$

The strength of a cylindrical shaft compared to a square one, the diameter of the one being equal to the side of the other, is as 1 to 1.2, and of a square shaft to a cylindrical as 1 to .85.

To Compute the Deflection of a Cylindrical Shaft.

RULE.—Divide the square of three times the length in feet by the product of the following *Constants* and the square of the diameter in inches, and the quotient will give the deflection.

Cast iron, cylindrical	1500	Wrought iron, cylindrical	1980
Cast iron, square	2560	Wrought iron, square	3360

EXAMPLE.—The length of a cast-iron cylindrical shaft is 30 feet, and its diameter in the centre 15 ins.; what is its deflection?

$$\frac{\overline{30\times3}^2}{1500\times15^2}=\frac{8100}{337500}=.024 \textit{ ins.}$$

To Compute the Diameter of Shafts of Wrought Iron, Oak, and Pine.

Multiply the diameter ascertained for Cast Iron as follows: Wrought Iron by .935, Oak by 1.83, Yellow Pine by 1.716.

To Compute the Length of a Cylindrical Shaft.

RULE.—Multiply the preceding *Constant* by the deflection, and the square of the diameter and ⅓ of the square root of the product will give the length in feet.

EXAMPLE.—The diameter of a cast-iron cylindrical shaft is 15 ins., and the deflection assigned to it is .024; what should be its length?

$$\frac{\sqrt{1500\times.024\times15^2}}{3}=\frac{90}{3}=30\,feet.$$

GUDGEONS.

To Compute the Diameter of a Single Gudgeon of Cast Iron, to Support a given Weight or Stress.

RULE.—Divide the square root of the weight in pounds by 25 for cast iron, and 26 for wrought iron, and the quotient will give the diameter in inches.

EXAMPLE.—The weight upon a gudgeon of a cast-iron water-wheel shaft is 62 500 lbs.; what should be its diameter?

$$\frac{\sqrt{62\,500}}{25}=\frac{250}{25}=10\ ins.$$

To Compute the Diameter of Two Gudgeons of Cast Iron, to Support a given Stress or Weight.

RULE.—Multiply the square root of the weight of half the wheel by .048, and the product will give the diameter in inches.

SPRINGS.

The flexure of a spring is proportional to its load and to the cube of its length.

Deflection of a Carriage Spring.

A railway-carriage spring, consisting of 10 plates 5/16 thick and 2 of ⅜ inch, length 2 feet 8 ins., width 3 ins., and *camber* or spring 6 ins., deflected as follows, without any permanent set:

½ ton, ½ inch.	1½ ton, 1½ inch.	3 tons, 3 inches.
1 " 1 "	2 " 2 "	4 " 4 "

Compression of an India-rubber Buffer of 3 *ins. Stroke.*

1 ton, 1.3 inch.	2 tons, 2 inches.	5 tons, 2¾ inches.
1½ " 1¾ "	3 " 2⅝ "	10 " 3 "

TUBES AND FLUES.

Resistance of Wrought-iron Tubes to External and Internal Pressure.—[W. FAIRBAIRN.]

It has been considered a rule that a cylindrical tube, such as a boiler-flue, when subjected to a uniform external pressure, was equally strong in every part, and that the length did not affect the strength of a tube so placed. Although this rule may be true when applied to tubes of indefinite lengths, it is very far from true where the lengths are restricted within certain apparently constant limits, and where the ends are securely fastened, as in heads or tube sheets, which prevent their yielding to an external force, or where, as in flues constructed in courses, the laps present a ring which greatly increases their resistance.

In some experimental tests to prove the efficiency of large boilers, it was ascertained that flues 35 feet long were distorted with considerable less force than others of a similar construction 25 feet long.

Results of Experiments upon the Resistance of Wrought-iron Tubes and Flues to External Pressure or Collapse.

Welded Tubes, and Ends secured to Head Plates.

Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.	Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.	Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.
Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.
4	19	.043	170	6	59	.043	32	10	50	.043	19
4	40	.043	65	6	30	.043	65	10	30	.043	33.
4	60	.043	43	8	30	.043	39	12	60	.043	12.5
6	30	.043	48	8	39	.043	32	12	30	.043	22

Tubes and Flues, Lap and Abut Joints.

Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.
Ins.	Ins.	Ins.	Lbs.
Tube 18¾	61	.25	420
Lap 9	37	.14	262
Abut 9	37	.14	378

Riveted Flues, Over-lap Joints, Ends closed.

Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.
Ins.	Ins.	Ins.	Lbs.
14⅛ by 14¾	60	.125	125

Rivets ⅛ in., and 1¼ ins. apart.

Cylindrical and Elliptical Riveted Flues. Abut Joints.

Flues.	Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.	Flues.	Diameter.	Length.	Thickness of Plates.	Pressure per Square Inch.
	Ins.	Ins.	Ins.	Lbs.		Ins.	Ins.	Ins.	Lbs.
Cylindrical	18.¾	61	.25	420	Elliptical	20¾×15⅛	61	.25	127.5
"	12.	61	.043	12.5	"	14. ×10¼	62	.043	6.5

To Compute the Collapsing Pressure upon a Tube or Flue.

The total external pressure upon a tube or flue varies directly as its longitudinal section, that is, as the product of the length and the diameter. $P' l d C = P$; P' *representing the pressure to which the tube is subjected in pounds per square inch, l the length of the tube in feet, d diameter in inches, and C a constant to be determined.*

It has been ascertained by experiment that the resistance of thin metal plates to a force tending to crush or to crumple them, varies directly as a certain power (x) of their thickness.

Hence the *Value* of a tube, etc., to resist collapse is as $\frac{P}{t^x}$, *t representing the thickness of the metal in inches.*

The mean of the product of $P' l d$ in the several experiments here given, where the metal was of a uniform thickness of .043 inches, is 850 for a thickness of ⅛ inch, 9140, etc.; and the mean of the value of x for all thicknesses is $^{2.19}$, which is assumed at ^{2}.

By taking ^{2}, therefore, instead of $^{2.19}$ for the index of t, this formula becomes $V \times \frac{t^2}{l d} = P'$, *the collapsing pressure*, which is the general formula for calculating the strength of wrought-iron tubes and short flues subjected to external pressure—that is, provided their length is not less than 1.5 feet, and not greater than 10 feet.

For thick tubes of considerable diameter and length, this formula is sufficiently exact for practical purposes.

V varies with the thickness of the tubes and flues, and may be safely estimated, as in the following Table:

When a Flue is constructed of *courses*, the above rule will apply by estimating the length of it to be the distance between the centres of two continuous laps, if the whole length of the flue does not exceed three times the length of a *course;* when, however, the length does exceed that proportion, the estimate of its resistance is to be made by taking the units from the following Tables:

In one experiment, the tube was divided into three parts by two rigid rings soldered upon its exterior, and its powers of resistance were thus increased in the ratio of three to one; *virtually*, the length was reduced in this ratio, and the strength was *actually* increased from 43 to 140 lbs. per square inch.

For Lengths from 1½ *to* 10 *Feet.*

From .043 to ⅛ inch,	380 000	to 520 000
From ⅛ to ¼ inch,	520 000	to 650 000
From ¼ to ⅜ inch,	650 000	to 720 000
From ⅜ to ½ inch,	720 000	to 800 000

For Lengths from 10 *to* 18 *Feet.*

From ⅛ to ¼ inch,	650 000	to 720 000
From ¼ to ⅜ inch,	720 000	to 810 000
From ⅜ to ½ inch,	810 000	to 910 000

For Lengths from 18 *to* 25 *Feet.*

From ⅛ to ¼ inch,	720 000	to 810 000
From ¼ to ⅜ inch,	810 000	to 920 000
From ⅜ to ¾ inch,	920 000	to 1 020 000

For Lengths from 25 *to* 35 *Feet.*

From ⅛ to ¼ inch,	812 000	to 920 000
From ¼ to ⅜ inch,	920 000	to 1 020 000
From ⅜ to ½ inch,	1 020 000	to 1 120 000

NOTE.—In selecting the above units, regard should be had to the length of the flue, independent of the ordinary conditions of strength of the materials and character of the riveting; as the nearer the length is to the limit of the length at the head of each table, the higher the unit is to be taken.

ILLUSTRATION.—Let $t = .043$ in., $l = 2.5$ feet, and $d = 6$ ins.; then $\frac{.043^2}{2.5 \times 6} \times 400\,000 = \frac{.001849}{15} \times 400\,000 = 49.3$ *lbs.*

Experiment gave 50 *lbs.* for a length of but 2.5 feet.

2. Let $t = \frac{1}{4}$ in., $l = 5$ feet, and $d = 18\frac{3}{4}$ ins.; then $\frac{.25^2}{5 \times 18.75} \times 585\,000 = 390$ *lbs.*

Experiment gave 420 *lbs.* for a length of but 5 feet 1 inch.

3. Let $t = \frac{3}{8}$ in., $l = 25$ feet, and $d = 42$ ins.; then $\frac{.375^2}{25 \times 42} \times 920\,000 = 123.2$ *lbs.*

Results of Experiments upon the Resistance of Wrought-iron Tubes or Flues to Internal Pressure or Bursting.

Diameter.	Length.	Thickness.	Pressure per Square Inch.	Diameter.	Length.	Thickness.	Pressure per Square Inch.
Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.
6	12	.043	475	6	30	.043	230
6	24	.043	235	12	60	.043	110

Formulæ of Resistance of Cylindrical Tubes or Flues to Internal Pressure.

$\frac{T \times 2t}{d} = P$; $\frac{P \times d}{2T} = P'$; and then $\frac{2t \times P'}{P} = d$, T *representing the tensile resistance of the material per square inch in pounds*, P *the pressure requisite to produce rupture of the tube or flue in pounds per square inch, and* P′ *the total pressure exerted.*

MEAN OF THE RESULTS OF EXPERIMENTS UPON THE RESISTANCE OF WROUGHT-IRON CYLINDRICAL TUBES TO INTERNAL PRESSURE.

To Compute the Thickness of a Wrought-iron riveted Tube or Flue.

When the Diameter of the Tube and the Pressure in Pounds per Square Inch are given. RULE.—Multiply the pressure in pounds per square inch by the diameter of the tube in inches, and divide the product by twice the tensile resistance of the metal in pounds per square inch.

EXAMPLE.—The diameter of a wrought-iron tube is 6 ins., and the pressure to which it is to be submitted is 425 lbs. per square inch; what should be the thickness of the metal?

Assume the tensile strength to be 29 651 *lbs.*

$$\frac{425 \times 6}{29\,651 \times 2} = \frac{2550}{59\,302} = .043 \textit{ ins.}$$

The tenacity or tensile resistance of wrought-iron boiler plates ranges from 42 000 to 62 000 lbs.* per square inch.

Tubes or flues subjected to internal pressure or bursting have much greater resistance than when subjected to external pressure or collapsing; in some cases, where the lengths of the collapsed tubes were 2.5 feet, the difference was about 6.2 times.

The difference, however, between these strains can not be determined as a rule, for the reason that the resistance to internal pressure is inversely as the diameter of the tube or flue alone, without regard to its length; whereas, with the resistance to collapse, the stress is inversely as the product of the diameter and the length.

Application to Construction of the Results of the Experiments.

With *drawn* or brazed tubes, when there are no courses and laps, their length is an essential element in an estimate of their resistance to collapse; but with riveted flues, constructed in courses, the objection to length is removed, as the addition of the laps is a source of great resistance to collapse, rendering the flue alike to a series of lengths, each equal to the distance between the centres of the courses.

In a boiler of the ordinary construction, of 30 feet in length and 3½ feet in diameter, with two flues 16 ins. in diameter, the cylindrical external shell has 2.8 times resistance to the force tending to burst it that the flues have to resist the same force to collapse them.

To Compute the Ultimate Collapsing Resistance of a Flue.

RULE.—Take the square of the thickness of the metal in decimals of an inch, or that due to the number of it, if given by a wire gauge, and multiply it by its proportional unit or multiplier from the Table (page 490), the thickness and length being duly considered, and divide the product by the product of the diameter of the flue in inches and the length of it in feet.

EXAMPLE.—The diameter of a flue is 18 ins., the thickness of the metal No. 3 U. S. wire gauge (.23 in.), and the length of it 30 feet; what is its ultimate resistance to collapse per square inch?

Multipliers for thicknesses from ⅛ to ¼ in., and for a length of 30 feet, are 810 000 to 920 000, the difference of which is 920 000 — 810 800 = 110 000, and the difference in thickness .25 — .125 = .125. Then, as .125 : 110 000 : : .105 (.23 — .125) : 92 400.

Difference in length, 35 — 25 = 10. Then, as 10 : 110 000 : : 5 (35 — 30) : 55 000.

Consequently, $\frac{92\,400 + 55\,000}{2} = 73\,700$, *a mean multiplier of thickness and length*, which, added to 810 000, the multiplier for ⅛ in. in thickness and 25 feet in length, = 883 700. Hence $\frac{.23^2}{30 \times 15} \times 883\,700 = \frac{.0529}{450} \times 883\,700 = 103.88$ *lbs.*

* Including English plates.

The following Table exhibits the collapsing pressure of flues, and bursting pressure of boilers of different diameters and thickness of metal:

Resistance of Wrought-iron Flues to an External or Collapsing Pressure, and of the Shells of Boilers to an Internal or Bursting Pressure.

Tensile Resistance of the Plates without Riveting is taken at a Mean of 55 000 *pounds per Square Inch.*

Flues.				Shells.			
Diameter.	Length.	Thickness.	Collapsing Pressure per Square Inch.	Diameter.	Thickness.	Bursting Pressure per Square Inch.	
						Single Riveted.	Double Riveted.
Ins.	Feet.	Ins.	Lbs.	Feet.	Ins.	Lbs.	Lbs.
6	10	.2	417	2	.$\frac{1}{4}$	573	745
6.5	10	.2	385	2.6	.$\frac{1}{4}$	458	596
7	10	.2	357	3	.$\frac{1}{4}$	382	496
	10	.$\frac{1}{4}$	580	3.4	.$\frac{1}{4}$	318	414
7.5	10	.2	333		.$\frac{5}{16}$	398	518
	10	.$\frac{1}{4}$	542	3.6	.$\frac{1}{4}$	327	426
8	10	.2	312		.$\frac{5}{16}$	409	532
	10	.$\frac{1}{4}$	508	4	.$\frac{1}{4}$	286	372
8.5	10	.2	294		.$\frac{5}{16}$	358	465
	10	.$\frac{1}{4}$	478	4.6	.$\frac{1}{4}$	254	331
9	10	.2	278		.$\frac{5}{16}$	318	413
	10	.$\frac{1}{4}$	451	5	.$\frac{1}{4}$	229	298
9.5	10	.2	263		.$\frac{5}{16}$	286	372
	10	.$\frac{1}{4}$	427	5.6	.$\frac{1}{4}$	208	270
10	12	.2	227		.$\frac{5}{16}$	260	338
	12	.2$\frac{1}{4}$	354		.$\frac{3}{8}$	312	406
	12	.$\frac{5}{16}$	612	6	.$\frac{1}{4}$	191	248
10.5	12	.2	216		.$\frac{5}{16}$	239	311
	12	.$\frac{1}{4}$	337		.$\frac{3}{8}$	286	372
	12	.$\frac{5}{16}$	583	6.6	.$\frac{5}{16}$	220	287
11	12	.2	206		.$\frac{3}{8}$	264	344
	12	.$\frac{1}{4}$	322	7	.$\frac{5}{16}$	204	266
	12	.$\frac{5}{16}$	557		.$\frac{3}{8}$	245	319
11.5	12	.2	197	7.6	.$\frac{5}{16}$	191	248
	12	.$\frac{1}{4}$	308		.$\frac{3}{8}$	229	298
	12	.$\frac{5}{16}$	532	8	.$\frac{5}{16}$	179	233
12	15	.2	153		.$\frac{3}{8}$	215	279
	15	.$\frac{1}{4}$	239	8.6	.$\frac{5}{16}$	168	219
	15	.$\frac{5}{16}$	415		.$\frac{3}{8}$	202	263
12.5	15	.$\frac{1}{4}$	229	9	.$\frac{5}{16}$	159	207
	15	.$\frac{5}{16}$	398		.$\frac{3}{8}$	191	248
13	15	.$\frac{1}{4}$	220	9.6	.$\frac{5}{16}$	150	196
	15	.$\frac{5}{16}$	384		.$\frac{3}{8}$	181	235
13.5	15	.$\frac{1}{4}$	212	10	.$\frac{5}{16}$	143	186
	15	.$\frac{5}{16}$	369		.$\frac{3}{8}$	172	224
14	18	.$\frac{1}{4}$	176		.$\frac{1}{2}$	229	298
	18	.$\frac{5}{16}$	305	10.6	.$\frac{5}{16}$	136	177
14.5	18	.$\frac{1}{4}$	168		.$\frac{3}{8}$	163	212
	18	.$\frac{5}{16}$	294		.$\frac{1}{2}$	218	284
15	20	.$\frac{1}{4}$	157	11	.$\frac{3}{8}$	156	203
	20	.$\frac{5}{16}$	276		.$\frac{1}{2}$	208	271
15.5	20	.$\frac{1}{4}$	152	11.6	.$\frac{3}{8}$	149	194
	20	.$\frac{5}{16}$	267		.$\frac{1}{2}$	199	259
16	20	.$\frac{1}{4}$	148	12	.$\frac{3}{8}$	143	166
	20	.$\frac{5}{16}$	231		.$\frac{1}{2}$	191	248

NOTE.—The single-riveted are estimated at .5 the resistance of the plates, and the staggered riveted at .65; this reduction from .56 and .7, as determined by Fairbairn, is to meet defects of rivets, cracks of plates from the pinning of rivet holes, etc., his deductions being taken from experiments made with rivets and plates in a normal condition.

From the results given in the Table and deduced from the rules, such allowances for the resistance and wear of the plates, oxydation, etc., are to be made, as the character of the metal, the nature of the service, and the circumstance of using fresh or salt water, etc., will render necessary.

In riveted plates, it is customary in practice to estimate the safe tensile resistance of the metal of a boiler or tube, when exposed to salt-water, at one fifth of its ultimate resistance or bursting pressure; and, when exposed to fresh-water alone, at one fourth of it.

To Compute the Ultimate Bursting Resistance of the Shell of a Boiler.

RULE.—Double the thickness given or ascertained by a wire gauge; multiply the sum by the tensile resistance of the material as it may be constructed, and divide the product by the diameter of the boiler or flue in inches.

EXAMPLE.—The diameter of the shell of a wrought-iron boiler, single riveted, is 5 feet, and the thickness of the metal is .28 in.; what is the ultimate resistance to a bursting pressure?

$.28 + .28 \times 55\,000$, which $\times .5$ for reduction of resistance of the plates for single riveting $= 15\,400$, and $\frac{15\,400}{60} = 256.6$ *lbs.*

DEDUCTIONS.—1. The resistance of Tubes or Flues to an External or Internal Pressure varies directly and inversely as their diameters. 2. The resistance of a Tube or Flue to External Pressure, up to the lengths experimented upon, is inversely as its length. Consequently, the resistance of tubes or flues to external pressure, of different diameters but of equal lengths, varies inversely as their diameter, and contrariwise. 3. The Tubes or Flues, with lap-joints, have one third less resistance to external pressure than when their joints are abutted. 4. A Cylindrical Tube or Flue has three times the resistance to external pressure of an Elliptical Tube or Flue, of the proportionate diameter given in the experiments noticed. 5. The length of Tubes or Flues, to resist Internal pressure, has no essential effect. 6. With Tubes or Flues of like thickness, their resistance varies inversely as the product of their lengths by their diameters.

Results of Experiments upon the Resistance of Elliptical Flues to External Pressure or Collapse.

By comparing the results of Experiments upon Elliptical tubes with those upon Cylindrical tubes, it appears the preceding general formula will apply approximately to elliptical tubes, by substituting for d in that formula the diameter of the circle of curvature touching the extremity of the minor axis. Thus:

Diameter of the circle of curvature (page 489, Ex. 4, 19th line from bottom) = $\frac{2\,r^2}{r} = \frac{2 \times 7^2}{5.125} = 19.12$ *ins.*

The pressure upon this tube was 6.5 *lbs.*, which, reduced to unity of length and diameter = 621.4 *lbs.* $(19.12 \times 5 \times 6.5)$.

Comparison between the Resistance to External and Internal Pressure in Wrought Iron Single-riveted Flues of different Diameters and Lengths.

Diameter.	Thickness.	Length.	External Pressure per Square Inch.	Internal Pressure per Square Inch.	Ratio.
Ins.	Ins.	Feet.	Lbs.	Lbs.	
6	.15	10	205	1375	1 to 6.7
13	.2	15	163	917	1 to 5.6
18	.¼	20	135	764	1 to 5.6

Resistance of Lead Tubes to Internal Pressure.

Diameter.	Length.	Thickness.	Pressure of Rupture per Square Inch.
Ins.	Ins.	Ins	Lbs.
3	14½	¼	374
3	31	¼	364

Assume 370 as the mean of the pressure of rupture of lbs. per square inch.

To Compute the Thickness of a Lead Pipe when the Diameter and the Pressure in Pounds per Square Inch is given.

RULE.—Multiply the pressure in pounds per square inch by the diameter of the pipe in inches, and divide the product by twice the tensile resistance of the metal in pounds per square inch.

EXAMPLE.—The diameter of a lead pipe is 3 inches, and the pressure to which it is to be submitted is 370 lbs. per square inch; what should be the thickness of the metal?

$$\frac{370 \times 3}{2220 \times 2} = \frac{1110}{4440} = .25 \text{ } in.$$

Resistance of Glass Globes and Cylinders to Internal Pressure and Collapse.

GLOBES (FLINT GLASS). CYLINDER.

Bursting Pressure.

Diameter.	Thickness.	Per Square Inch.	Diameter.	Length.	Thickness.	Per Square Inch.
Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.
4	.024	84	4	7	.079	282
4	.038	150	Elliptical (Crown Glass).			
5	.022	90	4.1	7	.019	109
6	.059	152				

Collapsing Pressure.

5	.014	292	3	14	.014	85
4	.025	1000*	4	7	.034	202
6	.059	900*	4	14	.064	297

MEMORANDA.

Repetition of Stress.—A piece of cast iron submitted to transverse stress broke at the 1956th strain, with a stress three fourths of that of its original ultimate resistance.

Resistance to Bursting of Thick Cylinders.—The mean resistance to bursting of the chambers of cast-iron guns, from experiments of Major Rodman, is as follows:

Thickness of metal = 1 calibre, length = 3 calibres, 52 217 *lbs.* per sq. in.
Thickness of metal = ½ calibre, length = 3 calibres, 49 100 *lbs.* per sq. in.
The tensile strength of the iron being 18 820 *lbs.*

Diam. of cylinder 2 ins., length 12 ins., metal 2 ins., 80 229 *lbs.* per sq. in.
Diam. of cylinder 3 ins., length 12 ins., metal 3 ins., 93 702 *lbs.* per sq. in.
The tensile strength of the iron being 26 866 *lbs.*

Average Tensile strength of Gun-metal (cast iron), 37 774 *lbs.*

Wire Ropes.

The ultimate strength of iron wire ropes is 4480 *lbs.* for each pound in weight per fathom, and for galvanized steel ropes 6720 lbs.

* Unbroken.

IRON.

The foreign substances which iron contains modify its essential properties. *Carbon* adds to its hardness, but destroys some of its qualities, and produces Cast Iron or Steel according to the proportion it contains. *Sulphur* renders it fusible, difficult to weld, and brittle when heated or "*hot short*." *Phosphorus* renders it "*cold short*," but may be present in the proportion of $\frac{2}{1000}$ to $\frac{3}{1000}$ without affecting injuriously its tenacity. *Antimony*, *Arsenic*, and *Copper* have the same effect as sulphur, the last in a greater degree.

Cast Iron.

The process of making cast iron depends much upon the description of fuel used; whether charcoal, coke, bituminous or anthracite coals. A larger yield from the same furnace, and a great economy in fuel, are effected by the use of a *hot blast*. The greater heat thus produced causes the iron to combine with a larger per-centage of foreign substances.

Cast iron for purposes requiring great strength should be smelted with a *cold blast*. *Pig-iron*, according to the proportion of carbon which it contains, is divided into *Foundry Iron* and *Forge Iron*, the latter adapted only to conversion into malleable iron; while the former, containing the largest proportion of carbon, can be used either for castings or bars.

There are many varieties of cast iron, differing by almost insensible shades; the two principal divisions are *gray* and *white*, so termed from the color of their fracture. Their properties are very different.

Gray Iron is softer and less brittle than white iron; it is in a slight degree malleable and flexible, and is not sonorous; it can be easily drilled or turned in a lathe, and does not resist the file. It has a brilliant fracture, of a gray, or sometimes a bluish-gray, color; the color is lighter as the grain becomes closer, and its hardness increases at the same time. It melts at a lower heat than white iron, and preserves its fluidity longer. The color of the fluid metal is red, and deeper in proportion as the heat is lower; it does not adhere to the ladle; it fills the molds well, contracts less, and contains fewer cavities than white iron; the edges of its castings are sharp, and the surfaces smooth and convex. A medium-sized grain, bright gray color, fracture sharp to the touch, and a close, compact texture, indicate a good quality of iron. A grain either very large or very small, a dull, earthy aspect, loose texture, dissimilar crystals mixed together, indicate an inferior quality.

Gray iron is used for machinery and ordnance purposes where the pieces are to be bored or fitted. Its tenacity and specific gravity are *diminished* by annealing. Its mean specific gravity is 7.2.

White Iron is very brittle and sonorous; it resists the file and the chisel, and is susceptible of high polish; the surface of its castings is concave; the fracture presents a silvery appearance, generally fine-grained and compact, sometimes radiating or lamellar. When melted it is white, and throws off a great number of sparks, and its qualities are the reverse of those of gray iron; it is, therefore, unsuitable for machinery purposes. Its tenacity is *increased*, and its specific gravity *diminished* by annealing. Its mean specific gravity is 7.5.

Mottled Iron is a mixture of white and gray; it has a spotted appearance; it flows well, and with few sparks; its castings have a plane surface, with edges slightly rounded. It is suitable for shot, shells, etc.

A fine mottled iron is the only kind suitable for castings which require great strength, such as beam centres, cylinders, and cannon. The kind of mottle will depend much upon the size of the casting.

Besides these general divisions, the different varieties of pig-iron are more particularly distinguished by numbers, according to their relative hardness.

No. 1 is the softest iron, possessing in the highest degree the qualities belonging to gray iron; it has not much strength, but on account of its fluidity when melted, and of its mixing advantageously with old or scrap iron and with the harder kinds of cast iron, it is of great use to the founder, and commands the highest price.

No. 2 is harder, closer grained, and stronger than No. 1; it has a gray color and considerable lustre. It is the character of iron most suitable for shot and shells.

No. 3 is still harder than No. 2. Its color is gray, but inclining to white; it has considerable strength, but it is principally used for mixing with other kinds of iron.

No. 4 is *bright* iron; No. 5, *mottled;* and No. 6, *white*, which is unfit for general use by itself.

The qualities of these various descriptions depend upon the proportion of carbon, and upon the state in which it exists in the metal; in the darker kinds of iron, where the proportion is sometimes 7 per cent., it exists partly in the state of graphite or plumbago, which makes the iron soft. In white iron, the carbon is thoroughly combined with the metal, as in steel.

Cast iron frequently retains a portion of foreign ingredients from the ore, such as earths or oxides of other metals, and sometimes sulphur and phosphorus, which are all injurious to its quality. Sulphur hardens the iron, and, unless in a very small proportion, destroys its tenacity.

These foreign substances, and also a portion of the carbon, are separated by melting the iron in contact with air, and soft iron is thus rendered harder and stronger. The effect of remelting varies with the nature of the iron and the character of ore from which it has been extracted; that from the hard ores, such as the magnetic oxides, undergoes less alteration than that from the hematites, the latter being sometimes changed from No. 1 to *white* by a single remelting in an air furnace.

The color and texture of cast iron depend greatly upon the volume of the casting and the rapidity of its cooling; a small casting, which cools quickly, is almost always *white*, and the surface of large castings partakes more of the qualities of white metal than the interior.

All cast iron expands at the moment of becoming solid, and contracts in cooling; gray iron expands more and contracts less than other iron.

The contraction is about $\frac{1}{100}$ for gray and strongly-mottled iron, or $\frac{1}{8}$ of an inch per foot.

Remelting iron improves its tenacity; thus, a mean of 14 cases for two fusions gave, for 1st fusion, a tenacity of 29 284 lbs.; for 2d fusion, 33 790 lbs. For 2 cases—for 1st fusion, 15 129 lbs.; for 2d fusion, 35 786 lbs.

Wrought Iron.

Wrought iron is made from the pig-iron in a *Bloomery Fire* or in a *Puddling Furnace*—generally in the latter. The process consists in melting it and keeping it exposed to a great heat, constantly stirring the mass, bringing every part of it under the action of the flame until it loses its remaining carbon, when it becomes malleable iron. When, however, it is desired to obtain iron of the best quality, the pig-iron should be *refined.*

Refining.—This operation deprives the iron of a considerable portion of its carbon; it is effected in a *Blast Furnace*, where the iron is melted by means of charcoal or coke, and exposed for some time to the action of a great heat; the metal is then run into a cast-iron mold, by which it is formed into a large broad plate. As soon as the surface of the plate is chilled, cold water is poured on to render it brittle.

The *Bloomery* resembles a large forge fire, where charcoal and a strong blast are used; and the *refined* metal or the pig-iron, after being broken into pieces of the proper size, is placed before the blast, directly in contact with charcoal; as the metal fuses, it falls into a cavity left for that purpose below the blast, where the bloomer works it into the shape of a *ball*, which he places again before the blast, with fresh charcoal; this operation is generally again repeated, when the ball is ready for the *Shingler*.

The *Puddling Furnace* is a reverberatory furnace, where the flame of bituminous coal is brought to act directly upon the metal. The metal is first melted; the puddler then stirs it, exposing each portion in turn to the action of the flame, and continues this as long as he is able to work it. When it has lost its fluidity, he forms it into balls, weighing from 80 to 100 lbs., which are next passed to the shingler.

Shingling is performed in a strong *squeezer* or under the trip-hammer. Its object is to press out as perfectly as practicable the liquid cinder which the ball still contains; it also forms the ball into shape for the puddle rolls. A heavy hammer, weighing from 6 to 7 tons, effects this object most thoroughly, but not so cheaply as the squeezer. The ball receives from 15 to 20 blows of a hammer, being turned from time to time as required: it is now termed a *Bloom*, and is ready to be rolled or hammered; or the ball is passed once through the squeezer, and is still hot enough to be passed through the puddle rolls.

Puddle Rolls.—By passing through different grooves in these rolls, the bloom is reduced to a *rough bar* from three to four feet in length, its name conveying an idea of its condition, which is rough and imperfect.

Piling.—To prepare rough bars for this operation, they are cut, by a pair of *shears*, into such lengths as are best adapted to the size of the finished bar required; the sheared bars are then piled one over the other, according to the volume required, when the pile is ready for balling.

Balling.—This operation is performed in the balling furnace, which is similar to the puddling furnace, except that its bottom or hearth is made up, from time to time, with sand; it is used to give a welding-heat to the piles to prepare them for rolling.

Finishing Rolls.—The *balls* are passed successively between rollers of various forms and dimensions, according to the shape of the finished bar required.

The *quality* of the iron depends upon the description of pig-iron used, the skill of the puddler, and the absence of deleterious substances in the furnace.

The strongest cast irons do not produce the strongest malleable iron.

For many purposes, such as sheets for tinning, best boiler-plates, and bars for converting into steel, *charcoal iron* is used exclusively; and, generally, this kind of iron is to be relied upon, for strength and toughness, with greater confidence than any other, though iron of superior quality is made from pigs made with other fuel, and with a hot blast. Iron for gun-barrels has been lately made from anthracite hot-blast pigs.

Iron is improved in quality by judicious working, reheating it, and hammering or rolling: other things being equal, the best iron is that which has been wrought the most.

STEEL.

Steel is a compound of Iron and Carbon, in which the proportion of the latter is from 1 to 5 per cent., and even less in some kinds. Steel is distinguished from iron by its fine grain, and by the action of diluted nitric acid, which leaves a black spot upon steel, and upon iron a spot which is lighter colored in proportion to the carbon it contains.

There are many varieties of steel, the principal of which are:

Natural Steel, obtained by reducing rich and pure descriptions of iron ore with charcoal, and refining the cast iron, so as to deprive it of a sufficient portion of carbon to bring it to a malleable state. It is used for files and other tools.

Indian steel, termed *Wootz*, is said to be a natural steel, containing a small portion of other metals.

Blistered Steel, or Steel of Cementation, is prepared by the direct combination of iron and carbon. For this purpose, the iron in bars is put in layers, alternating with powdered charcoal, in a close furnace, and exposed for seven or eight days to a heat of about 9000°, and then put to cool for a like period. The bars, on being taken out, are covered with blisters, have acquired a brittle quality, and exhibit in the fracture a uniform crystalline appearance. The degree of carbonization is varied according to the purposes for which the steel is intended, and the best qualities of iron (Russian and Swedish) are used for the finest kinds of steel.

Tilted Steel is made from blistered steel moderately heated, and subjected to the action of a tilt hammer, by which means its tenacity and density are increased.

Shear Steel is made from blistered or natural steel, refined by piling thin bars into fagots, which are brought to a welding heat in a reverberatory furnace, and hammered or rolled again into bars; this operation is repeated several times to produce the finest kinds of shear steel, which are distinguished by the names of *half shear*, *single shear*, and *double shear*, or steel of 1, 2, or 3 *marks*, etc., according to the number of times it has been piled.

Cast Steel is made by breaking blistered steel into small pieces and melting it in close crucibles, from which it is poured into iron molds; the *ingot* is then reduced to a bar by hammering or rolling. Cast steel is the best kind of steel, and best adapted for most purposes; it is known by a very fine, even, and close grain, and a silvery, homogeneous fracture; it is very brittle, and acquires extreme hardness, but is difficult to weld without the use of a flux. The other kinds of steel have a similar appearance to cast steel, but the grain is coarser and less homogeneous; they are softer and less brittle, and weld more readily. A fibrous or lamellar appearance in the fracture indicates an imperfect steel. A material of great toughness and elasticity, as well as hardness, is made by forging together steel and iron, forming the celebrated *damasked Steel*, which is used for sword-blades, springs, etc.; the damask appearance of which is produced by a diluted acid, which gives a black tint to the steel, while the iron remains white.

Various *fancy steels*, or alloys of steel with *silver*, *platinum*, *rhodium*, and *aluminum*, have been made with a view to imitating the Damascus steel, wootz, etc., and improving the fabrication of some of the finer kinds of surgical and other instruments.

Properties of Steel.—After being tempered it is not easily broken; it welds readily; it does not crack or split; it bears a very high heat, and preserves the capability of hardening after repeated working.

Hardening and Tempering.—Upon these operations the quality of manufactured steel in a great measure depends.

Hardening is effected by heating the steel to a cherry-red, or until the scales of oxide are loosened on the surface, and plunging it into a liquid, or placing it in contact with some cooling substance; the degree of hardness depends upon the heat and the rapidity of cooling. Steel is thus rendered so hard as to resist the hardest files, and it becomes at the same time extremely brittle. The degree of heat, and the temperature and nature of the cooling medium, must be chosen with reference to the quality of the steel and the purpose for which it is intended. Cold water gives a greater hardness than oils or other fatty substances, sand, wet-iron scales, or cinders, but an inferior degree of hardness to that given by acids. Oil, tallow, etc., prevent the cracks which are caused by too rapid cooling. The lower the heat at which the steel becomes hard, the better.

Tempering.—Steel in its hardest state being too brittle for most purposes, the requisite strength and elasticity are obtained by tempering—or *letting down the temper*, as it is termed—which is performed by heating the hardened steel to a certain degree and cooling it quickly. The requisite heat is usually ascertained by the color which the surface of the steel assumes from the film of oxide thus formed. The degrees of heat to which these several colors correspond are as follows:

At 430°, a very faint yellow At 450°, a pale straw color.	Suitable for hard instruments; as hammer-faces, drills, etc.
At 470°, a full yellow...... At 490°, a brown color.....	For instruments requiring hard edges without elasticity; as shears, scissors, turning tools, etc.
At 510°, brown, with purple spots.................. At 538°, purple............	For tools for cutting wood and soft metals; such as plane-irons, knives, etc.
At 550°, dark blue......... At 560°, full blue..........	For tools requiring strong edges without extreme hardness; as cold-chisels, axes, cutlery, etc.
At 600°, grayish-blue, verging on black	For spring-temper, which will bend before breaking; as saws, sword-blades, etc.

If the steel is heated higher than this, the effect of the hardening process is destroyed.

Case-hardening.

This operation consists in converting the surface of wrought iron into steel, by cementation, for the purpose of adapting it to receive a polish or to bear friction, etc.; this is effected by heating iron to a cherry-red, in a close vessel, in contact with carbonaceous materials, and then plunging it into cold water. Bones, leather, hoofs, and horns of animals are generally used for this purpose, after having been burned or roasted so that they can be pulverized. Soot is also frequently used.

LIMES, CEMENTS, MORTARS, AND CONCRETES.

Limestones.

The calcination of marble or any pure limestone produces *lime* (*quicklime*). The pure limestones burn white, and give the richest limes.

The finest calcareous minerals are the rhombohedral prisms of calcareous spar, the transparent double-reflecting Iceland spar, and white or statuary marble.

The property of hardening under water, or when excluded from air, conferred upon a paste of lime, is effected by the presence of foreign substances—as silicum, alumina, iron, etc.—when their aggregate presence amounts to $\frac{1}{10}$ of the whole.

Limes are classed: 1. The common or fat limes. 2. The poor or meagre. 3. The hydraulic. 4. The hydraulic cements. 5. The natural puzzuolanas, including puzzuolana properly so called, trass or terras, the arènes, ochreous earths, basaltic sands, and a variety of similar substances.

Rich Limes are fully dissolved in water frequently renewed, and they remain a long time without hardening; they also increase greatly in volume, from 2 to $3\frac{1}{2}$ times their original bulks, and will not harden without the action of the air. They are rendered *hydraulic* by the admixture of *puzzuolana* or *trass*.

Rich, fat, or *common Limes* usually contain less than 10 per cent. of impurities..

Hydraulic Limestones are those which contain iron and clay, so as to enable them to produce cements which become solid when under water.

The pastes of fat limes shrink, in hardening, to such a degree that they can not be used as mortar without a large dose of sand.

Poor Limes have all the defects of rich limes, and increase but slightly in bulk.

The poorer limes are invariably the basis of the most rapidly-setting and most durable cements and mortars, and they are also the only limes which have the property, when in combination with silica, etc., of indurating under water, and are therefore applicable for the admixture of hydraulic cements or mortars. Alike to rich limes, they will not harden if in a state of paste under water or in wet soil, or if excluded from contact with the atmosphere or carbonic acid gas. They should be employed for mortar only when it is impracticable to procure common or hydraulic lime or cement, in which case it is recommended to reduce them to powder by grinding.

Lime absorbs, in slaking, a mean of $2\frac{1}{2}$ times its volume, and $2\frac{1}{4}$ times its weight of water.

Hydraulic Limes are those which readily harden under water. The most valuable or *eminently hydraulic set* from the 2d to the 4th day after immersion; at the end of a month they become hard and insoluble, and at the end of 6 months they are capable of being worked like the hard, natural limestones. They absorb less water than the pure limes, and only increase in bulk from $1\frac{3}{4}$ to $2\frac{1}{2}$ times their original volume.

The inferior grades, or *moderately hydraulic*, require a longer period, say from 15 to 20 days' immersion, and continue to harden for a period of 6 months.

The resistance of hydraulic limes increase if sand is mixed in the proportion of 50 to 180 per cent. of the part in volume; from thence it decreases.

Slaked Lime is a hydrate of lime.

M. Vicat declares that lime is rendered hydraulic by the admixture with it of from 33 to 40 per cent. of clay and silica, and that a lime is obtained which does not slake, and which quickly sets under water.

Artificial Hydraulic Limes do not attain, even under favorable circumstances, the same degree of hardness and power of resistance to compression as the natural limes of the same class.

The close-grained and densest limestones furnish the best limes.

Hydraulic limes lose or depreciate in value by exposure to the air.

Arènes is a species of ochreous sand. It is found in France. On account of the large proportion of clay it contains, sometimes as great as $\frac{7}{10}$, it can be made into a paste with water without any addition of lime; hence it is sometimes used in that state for walls constructed *en pisé*, as well as for mortar. Mixed with rich lime, it gives excellent mortar, which attains great hardness under water, and possesses great hydraulic energy.

Puzzuolana is of volcanic origin. It comprises trass or terras, the arènes, some of the ochreous earths, and the sand of certain graywackes, granites, schists, and basalts; their principal elements are silica and alumina, the former preponderating. None contain more than 10 per cent. of lime.

When finely pulverized, without previous calcination, and combined with the paste of fat lime in proportions suitable to supply its deficiency in that element, it possesses hydraulic energy to a valuable degree. It is used in combination with rich lime, and may be made by slightly calcining clay and driving off the water of combination at a temperature of 1200°.

Brick or *Tile Dust* combined with rich lime possesses hydraulic energy.

Trass or *Terras* is a blue-black trap, and is also of volcanic origin. It requires to be pulverized and combined with rich lime to render it fit for use, and to develop any of its hydraulic properties.

General Gillmore* designates the varieties of hydraulic limes as follows: If, after being slaked, they harden under water in periods varying from 15 to 20 days after immersion, *slightly hydraulic;* if from six to eight days, *hydraulic;* and if from one to four days, *eminently hydraulic.*

Pulverized silica burned with rich lime produces hydraulic lime of excellent quality. Hydraulic limes are injured by air-slaking in a ratio varying directly with their hydraulicity, and they deteriorate by age.

For foundations in a damp soil or exposure, hydraulic limes must be exclusively employed.

Cements.

Hydraulic Cements contain a larger proportion of silica, alumina, magnesia, etc., than any of the preceding varieties of lime; they do not slake after calcination, and are superior to the very best of hydraulic limes, as some of them set under water at a moderate temperature (65°) in from 3 to 4 minutes; others require as many hours. They do not shrink in hardening, and make an excellent mortar without any admixture of sand.

Roman Cement is made from a lime of a peculiar character, found in England and France, derived from argillo-calcareous kidney-shaped stones termed "Septaria."

Rosendale Cement is from Rosendale, New York.

Portland Cement is made in England and France. It requires less water than the Roman cement, sets slowly, and can be remixed with additional water after an interval of 12 or even 24 hours from its first mixture.

The property of setting slow may be an obstacle to the use of some designations of this cement, as the Boulogne, when required for localities having to contend against immediate causes of destruction, as in sea constructions having to be executed under water and between tides. On the other hand, a quick-setting cement is always difficult of use; it requires special workmen and an active supervision. A slow-setting cement, however, like the natural Portland, possesses the advantage of being managed by ordinary workmen, and it can be remixed with additional water after 12 or even 24 hours.

Artificial Cement is made by a combination of slaked lime with unburned clay in suitable proportions.

Artificial Puzzuolana is made by subjecting clay to a slight calcination.

Salt-water has a tendency to decompose cements of all kinds.

Mortars.

Lime or Cement paste is the cementing substance in mortar, and its proportion should be determined by the rule that *the volume of the cementing substance should be somewhat in excess of the volume of voids or spaces in the sand or course material to be united*, the excess being added to meet imperfect manipulation of the mass.

Hydraulic Mortar, if re-pulverized and formed into a paste after having once *set*, immediately loses a great portion of its hydraulicity, and descends to the level of the moderate hydraulic limes.

All mortars are much improved by being worked or manipulated; and as rich limes gain somewhat by exposure to the air, it is advisable to work mortar in large quantities, and then render it fit for use by a second manipulation.

For an Analysis of Limestones, etc., etc., see Gen. Gilmore's Treatise, p. 22, 125.

White lime will take a larger proportion of sand than brown lime.

The use of salt-water in the composition of mortar injures the adhesion of it.

* See his Treatises on Limes, Hydraulic Cements, and Mortars, of Papers on Practical Engineering, Engineer Department, U. S. A.

Mortar.—When a small quantity of water is mixed with slaked lime, a stiff paste is made, which, upon becoming dry or hard, has but very little tenacity, but, by being mixed with sand or like substances, it acquires the properties of a cement or mortar.

The proportion of sand that can be incorporated with mortar depends partly upon the degree of fineness of the sand itself, and partly upon the character of the lime. For the rich limes, the resistance is increased if the sand is in proportions varying from 50 to 240 per cent. of the paste in volume; beyond this proportion the resistance decreases.

Stone Mortar.—8 parts cement, 3 parts lime, and 31 parts of sand.

Brick Mortar.—8 parts cement, 3 parts lime, and 27 parts of sand.

Brown Mortar.—Lime 1 part, sand 2 parts, and a small quantity of hair.

Lime and sand, and cement and sand, lessen about ⅓ in volume when mixed together.

Calcareous Mortar, being composed of one or more of the varieties of lime or cement, natural or artificial, mixed with sand, will vary in its properties with the quality of the lime or cement used, the nature and quality of sand, and the method of manipulation.

Mortar.—Lime, 1; clean sharp sand, 2½. An excess of water in slaking the lime swells the mortar, which remains light and porous, or shrinks in drying; an excess of sand destroys the cohesive properties of the mass.

It is indispensable that the sand should be sharp and clean.

Turkish Plaster, or Hydraulic Cement.—100 lbs. fresh lime reduced to powder, 10 quarts linseed-oil, and 1 to 2 ounces cotton. Manipulate the lime, gradually mixing the oil and cotton, in a wooden vessel, until the mixture becomes of the consistency of bread-dough.

Dry, and, when required for use, mix with linseed-oil to the consistency of paste, and then lay on in coats. Water-pipes of clay or metal, joined or coated with it, resist the effect of humidity for very long periods.

Exterior Plaster or Stucco.—1 volume of cement powder to 2 volumes of dry sand.

In India, to the water for mixing the plaster is added 1 lb. of sugar, or molasses, to 8 Imperial gallons of water, for the first coat; and for the second or finishing, 1 lb. sugar to 2 gallons water.

Powdered slaked lime and Smith's forge scales, mixed with blood in suitable proportions, make a moderate hydraulic mortar, which adheres well to masonry previously coated with boiled oil.

The plaster should be applied in two coats laid on in one operation, the first coat being thinner than the second. The second coat is applied upon the first while the latter is yet soft.

The two coats should form one of about 1½ inches in thickness, and when finished it should be kept moist for several days.

This process may be modified by substituting for the first coat a wash of thick cream of pure cement, applied with a stiff brush just before the plaster is laid on.

When the cement is of too dark a color for the desired shade, it may be mixed with white sand in whole or in part, or lime paste may be added until its volume equals that of the cement paste.

Khorassar, or Turkish Mortar, used for the construction of buildings requiring great solidity, ⅓ powdered brick and tiles, ⅔ fine sifted lime. Mix with water to the required consistency, and lay on layers of 5 and 6 inches in thickness between the courses of brick or stones.

Interior Plastering.—The mortars used for inside plastering are termed Coarse, Fine, Gauge or hard finish, and Stucco.

Coarse Stuff.—Common lime mortar, as made for brick masonry, with a small quantity of hair; or by volumes, lime paste (30 lbs. lime) 1 part, sand 2 to 2¼ parts, hair ⅙ part.

When full time for hardening can not be allowed, substitute from 15 to 20 *per cent.* of the lime by an equal proportion of hydraulic cement.

For the second or *brown coat* the proportion of hair may be slightly diminished.

Fine Stuff (lime putty).—Lump lime slaked to a paste with a moderate volume of water, and afterward diluted to the consistency of cream, and then to harden by evaporation to the required consistency for working.

In this state it is used for a *slipped coat*, and when mixed with sand or plaster of Paris, it is used for the *finishing coat.*

Gauge Stuff, or Hard finish, is composed of from 3 to 4 volumes fine stuff and 1 volume plaster of Paris, in proportions regulated by the degree of rapidity required in hardening; for cornices, etc., the proportions are equal volumes of each, fine stuff and plaster.

Stucco is composed of from 3 to 4 volumes of white sand, to 1 volume of fine stuff, or lime putty.

Scratch Coat.—The first of three coats when laid upon laths, and is from ¼ to ⅜ of an inch in thickness.

One-coat Work.—Plastering in one coat without finish, either on masonry or laths—that is, *rendered* or *laid.*

Two-coat Work.—Plastering in two coats is done either in a *laying coat and set*, or in a *screed coat and set.*

The *Screed coat* is also termed a *Floated coat. Laying* the first coat in two-coat work is resorted to in common work instead of *screeding*, when the finished surface is not required to be exact to a straight-edge. It is laid in a coat of about ½ an inch in thickness.

The laying coat, except for very common work, should be *hand-floated.*

The firmness and tenacity of plastering is very much increased by hand-floating.

Screeds are strips of mortar 6 to 8 inches in width, and of the required thickness of the first coat, applied to the angles of a room, or edge of a wall and parallelly, at intervals of 3 to 5 feet over the surface to be covered. When these have become sufficiently hard to withstand the pressure of a straight-edge, the inter-spaces between the screeds should be filled out flush with them, so as to produce a continuous and straight, even surface.

Slipped Coat is the smoothing off of a brown coat with a small quantity of lime putty, mixed with 3 per cent. of white sand, so as to make a comparatively even surface.

This finish answers when the surface is to be finished in distemper, or paper.

Hard Finish.—Fine stuff applied with a trowel to the depth of about ⅛ of an inch.

Estimate of Materials and Labor for 100 Square Yards of Lath and Plaster.

Materials and Labor.	Three Coats Hard Finish.	Two Coats Slipped.	Materials and Labor.	Three Coats Hard Finish.	Two Coats Slipped.
Lime..........	4 casks.	3½ casks.	White sand...	2½ bushels.	
Lump lime....	⅔ "		Nails.........	13 lbs.	13 lbs.
Plaster of Paris	½ "		Masons.......	4 days.	3½ days.
Laths.........	2000.	2000.	Laborer......	3 "	2 "
Hair..........	4 bushels.	3 bushels.	Cartage......	1 "	¾ "
Sand..........	7 loads.	6 loads.			

Concrete or Beton

Is a mixture of mortar (generally hydraulic) with coarse materials, as gravel, pebbles, stones, shells, broken bricks, etc. Two or more of these materials, or all of them, may be used together. As lime or cement paste is the cementing substance in mortar, so is mortar the cementing substance in concrete or beton. The original distinction between cement and beton was, that the latter possessed hydraulic energy, while the former did not.

Hydraulic.—1½ parts unslacked hydraulic lime, 1½ parts sand, 1 part gravel, and 2 parts of a hard broken limestone.

This mass contracts one fifth in volume. Fat lime may be mixed with concrete, without serious prejudice to its hydraulic energy.

Various Compositions of Concrete.—Forts Richmond and Tompkins, U. S.

Hydraulic.—308 lbs. cement = 3.65 to 3.7 cubic feet of stiff paste. 12 cubic feet of loose sand = 9.75 cubic feet of dense.

For Superstructure.—11.75 cubic feet of mortar as above, and 16 cubic feet of stone fragments.

In the foundations of Fort Tompkins, about $\frac{1}{12}$ of its volume was composed of stones from ¼ to ¾ of a cubic foot in volume, rammed into the wall as the concrete was laid.

Sea Wall.—Boston Harbor.—Hydraulic.—308 lbs. cement, 8 cubic feet of sand, and 30 cubic feet of gravel. The whole producing 32.3 cubic feet.

Superstructure.—308 lbs. cement, 80 lbs. lime, and 14.6 cubic feet dense sands. The whole producing 12.825 cubic feet.

Cost of labor and materials expended in laying concrete foundation at Fort Tompkins, during the year 1849, per cubic yard as laid, $2.26.

Transverse Strength of Concretes, Cements, Mortars, Puzzuolana, and Trass, deduced from the Experiments of Generals Totten and Gillmore, U. S. A., General Treussart, and M. Voisin.

Reduced to a uniform Measure of One Inch Square and One Foot in Length. Supported at both Ends.

$\frac{2}{3}\,\frac{l\,W}{4\,b\,d^2} = V$ *per square inch of section, representing value for general use, being ⅔ of ultimate breaking strain.*

EXPERIMENTS OF VOISIN, 1857.

Mortar.		Volume produced.	Concrete.				Mortar.		Volume produced.	Concrete.			
One Volume of Sand.			One Volume of Pebbles.		Value.		One Volume of Sand.			One Volume of Pebbles.		Value.	
Cement.	Water.		Mortar.	Volume produced.	10 Days.	60 Days.	Cement.	Water.		Mortar.	Volume produced	10 Days.	60 Days.
					Lbs.	Lbs.						Lbs.	Lbs.
1	.62	1.69	1	1.56	2.3	2.9	⅓	.38	1.12	½	1.03	.58	1.2
			½	1.03	1.7	3.2	¼	.35	1.05	1	1.4	.48	1.
			⅓	1.	1.8	3.1				½	1.01	.35	.85
			¼	1.	1.	1.	⅕	.34	1.	1	1.45	.3	.83
½	.43	1.24	1	1.45	1.6	2.7				½	1.03	.44	.65
			½	1.	1.	1.9	⅙	.32	.96	1	1.45	.41	.81
⅓	.83	1.12	1	1.4	.86	.91				½	1.03	.36	.79

EXPERIMENTS OF GENERAL TOTTEN, 1837.

Concrete.*	Mortar.			Concrete.*	Mortar.		
	Cement 1.	Cement 1. Sand .5.	Cement 1. Sand 1.		Cement 1.	Cement 1. Sand .5.	Cement 1. Sand 1.
	Lbs.	Lbs.	Lbs.		Lbs.	Lbs.	Lbs.
Granite . 1, Mortar .. 1	2.9	2.4	2.3	Stone....., Gravel....	1.9	1.	.6
Gravel .. 1, Mortar .. 2	1.4	2.4	.7	Brick....., Gravel....	.9	1.4	1.6

* The granite, bricks, etc., were broken into fragments or spalls of the required size.

Tensile Strength of various Cements, Mortars, and Masonry, deduced from the Experiments of Vicat and Chatoney at Cherbourg, Gen. Gillmore, U. S. A., Crystal Palace, London, etc.

Weight or Power required to Tear asunder One Square Inch.

Materials and Mixtures.	Ultimate Resistance.	Materials and Mixtures.	Ultimate Resistance
	Lbs.		Lbs.
Boulogne, 100 parts, water 50 ...	112	Portland, English, 320 days, cement 1, sand 2	7[illegible]3
90 days, 100 parts, water 50	52	" 45 days, pure and mixed, stiff	20[illegible]
Boulogne, 1 year, Portland (natural)	675	" English, pure, 1 month..	393
English, 1 year, Portland (artificial)	462	" " " 6 mos. ...	424
Portland, 42 days, cement 1, sand 1	142	Roman, 1 year, from Septaria...	191
" 15 "	134	" 42 days, cement 1, sand 1	284
" 135 "	233	" " " 1, " 2	199
" English, 320 days, pure.	1152	" " " 1, " 3	166
" " " cement 1, sand 1,	948	Stone masonry, Roman cement, 5 mos.	77

Brick and Granite Masonry, 320 Days.		Lbs
Cement, Delafield and Baxter	Pure............	68.56
	Cement .... 4, Sand 1	68.5
	Cement .... 5, Siftings 1	79.87
	Cement .... 1, Siftings 2	74.5
" Lawrence Co	Pure............	87.37
	Pure............	52.68
" James River	Cement .... 4, Sand 1	62.
" Newark Lime and Cement Co...........	Pure	93.25
	Cement .... 1, Sand 2	39.62
" Brighton and Rosendale................	Pure............	80 25
" Newark and Rosendale	Pure............	75.81
" Pure upon bricks		31.
" 1, sand 1 pure upon bricks..................................		16.
" 1, " 3 " "		7.
" Pure upon granite..		27.
" 1, water .5		20.
" 1, " .42		27.
" Pure upon bricks, without mortar, mean		45.
Common lime .... 1, " sand .... 2½ "		6.
Lime paste .... 1, Sand .... 3 upon bricks		6.
Lime paste .... 1, Sand .... 2 "		4.13
Lime paste .... 1, Sand .... 3, Cement paste .... 5 "		11 41

Crushing Strength of Cements, Stone, etc.—(*Crystal Palace, London.*)

Reduced to a uniform Measure of One Square Inch.

Material.	Ultimate Pressure.	Material.	Ultimate Pressure.
	Lbs.		Lbs.
Portland cement, area 1, height 1	1680	Portland cement 1, " sand 4	1244
" cement, " sand	1244	Roman cement, pure..........	342

Experiments of General Gillmore.

Materials.	Cements and Mixtures.	Value.
		Lbs.
Delafield and Baxter	Stiff paste	6.*
High Falls (N. Y.), 270 days	Pure	11.3
James River	Cement, Sand	5.9
James River, 59 days	Cement... 4. Water.... 2.6	1.9
	Cement... 4. Water.... 1.4	3.4*
Portland (Eng.), 320 days	Pure cement	10.6
	Cement .... 1, Sand....... 2	6.6
Portland Pure (Eng.), 100 days	Cement .... 1, Sand 1	12.5
		13.
	Cement .... 1, Sand....... 2	8.5
Roman (Eng.), 100 days	Cement .... 1, Sand....... 1	4.
Rosendale, 95 days	Pure	7.
	Cement.... 1, Lime...... ¼	6.7
	Cement .... 1, Lime 1	3.9
Rosendale (Hoffman), 320 days	Stiff paste	4.4*
	Thin "	4.8*

Cement.	Value. Pure.	Cement 1. Sand 1.	Cement 1. Sand 2.
		Lbs.	Lbs.
Akron, New York	5.2	4.4	4.1
Brighton and Rosendale	4.9	3.8	3.4
Cumberland, Md.	6.5	6.3	3.8
James River, Va.	—	4.2	4.4
Newark and Rosendale.	5.8	3.8	3.4
Portland, English	10.5	8.6	6.5
Remington, Conn.	6.5	4.8	3.4
Round Top, Md.	—	4.1	—
Rosendale, Hoffman	5.8	4.1	—
" Lawrence	5.3	—	—
Sandusky, Ohio	3.8	3.2	—
Shepherdstown, Va.	5.1	4.2	3.1
Utica, Ill.	5.1	1.2	3.8

Note.—When the paste is not subjected to compression during setting, a thin paste produces as strong a mortar as a stiff one.

Experiments of General Treussart.

Puzzuolana and Trass—Mortar.			Value.
			Lbs.
Strasburgh	Puzzuolana 1, Sand ...... 1, Trass 1	5 days	2.8
	Lime...... 1, Sand...... 1, Puzzuolana 1	4 "	3.4
Strasburgh	Lime paste. 1, Puzzuolana 2½	5 days	3.8
	Lime paste. 1, Trass 2	8 "	3.1
White Marble.	Lime ...... 1, Sand ...... 1, Trass...... 1	5 "	2.1

Cement paste, 95 days	13.8
" 1, lime paste ¼	13.6
" 1, " ½	11.3
" 1, " 1	7.9
Cement paste ½, lime paste 1	4.2
Fire-brick beam†	2.1
Portland cement, 4 mos.	21.3
Roman " 4 "	14.8

Deductions.—1. Particles of unground cement exceeding $\frac{1}{80}$ of an inch in diameter may be allowed in cement paste without sand, to the extent of 50 per cent. of the whole, without detriment to its properties, while a corresponding proportion of sand injures the strength of mortar about 40 per cent.

2. When these unground particles exist in cement paste to the extent of 66 per cent. of the whole, the adhesive strength is diminished about 28 per cent. For a corresponding proportion of sand the diminution is 68 per cent.

3. The addition of siftings exercises a less injurious effect upon the cohesive than upon the adhesive property of cement. The converse is true when sand, instead of siftings, is used.

* All except the first were submitted to a pressure of 32 lbs. per square inch.
† Loaded partly along the bricks, and broke through them.

4. In all the mixtures with siftings, even when the latter amounted to 66 per cent. of the whole, the cohesive strength of the mortars exceeded its adhesion to the bricks. The same results appear to exist when the siftings are replaced by sand, until the volume of the latter exceeds 20 per cent. of the whole, after which the adhesion exceeds the cohesion.

5. At the age of 320 days (and perhaps considerably within that period) the cohesive strength of pure cement mortar exceeds that of Croton front bricks. The converse is true when the mortar contains 50 per cent. or more of sand.

6. When cement is to be used without sand, as may be the case when *grouting* is resorted to, or when old walls are to be repaired by injections of thin paste, there is no advantage in having it ground to an impalpable powder.

7. For economy it is customary to add lime to cement mortars, and this may be done to a considerable extent when in positions where hydraulic activity and strength are not required in an eminent degree.

Slaking.—The volume of water required to slake lime will vary with limes from 2.5 to 3 times the volume of the lime (quicklime), and it is important that all the water required to reduce the lime to a proper consistency should be given to it before the temperature of the water first given becomes sensibly elevated.

Immediately upon the lime being provided with the requisite volume of water, it should be covered, in order to confine the heat, and it should not be stirred while slaking. When the paste is required for *grouting* or *whitewashing*, the water required should be given at once, and in larger volume than when the paste is required for mortar, and when slaked the mass should be transferred to tight casks to prevent the loss of water. When the character of the limes, as with those of hydraulic energy, will not readily reduce, their reduction, which is an indispensable condition, must be aided by mechanical means, as a mortar mill.

The process here given is termed *drowning*. When the lime is retained in a barrel, or like instrument, immersed in water, and then withdrawn before reduction occurs, it is termed *immersion*, and when it is reduced by being exposed to the atmosphere, and gradually absorbing moisture therefrom, it is termed *air-slaked*.

Bricks should be well wetted before use. *Sea sand* should not be used in the composition of mortar, as it contains salt and its grains are round, being worn by attrition, and consequently having less tenacity than sharp-edged grains.

Fine Clay.—The fusibility of clay arises from the presence of impurities, such as lime, iron, and manganese. These may be removed by steeping the clay in hot muriatic acid, then washing it with water. Crucibles from common clay may be made in this manner.

Pisé is made of clay or earth rammed in layers of from 3 to 4 inches in depth. In moist climates, it is necessary to protect the external surface of a wall constructed in this manner with a coat of mortar.

Asphalt Composition.—Mineral pitch 1 part, bitumen 11, powdered stone, or wood ashes, 7 parts.

2. Ashes 2 parts, clay 3 parts, and sand 1 part, mixed with a little oil, makes a very fine and durable cement, suitable for external use.

Mastic.—Pulverized burnt clay 93 parts, litharge ground very fine 7 parts, mixed with a sufficient quantity of pure linseed oil.

3. Silicious sand 14, pulverized calcareous stone 14, litharge 2, and linseed oil 4 parts by weight.

The powders to be well dried in an oven, and the surface upon which it is to be applied must be saturated with oil.

4. *For Roads.*—Bitumen 16.875 parts, asphaltum 225 parts, oil of resin 6.25 parts, and sand 135 parts. Thickness, from 1¼ to 1⅝ inches.

Asphaltum 55 lbs. and gravel 28.7 lbs. will cover an area of 10.75 square feet.

Notes by General Gillmore, U. S. A.—All the lime necessary for any required quantity or *batch* of mortar should be slaked at least one day before it is mixed with the sand.

All the water required to slake the lime should be poured on at one time, the lime should be submerged, and the mass should then be covered with a tarpaulin or canvas, and allowed to remain undisturbed for a period of 24 hours.

The ingredients should be thoroughly mixed, and then heaped for use as required.

Recent experiments have developed that most American cements will sustain, without any great loss of strength, a dose of lime paste equal to that of the cement paste, while a dose equal to ½ to ¾ the volume of cement paste may be safely added to any Rosendale cement without producing any essential deterioration of the quality of the mortar. Neither is the hydraulic activity of the mortars so far impaired by this limited addition of lime paste as to render them unsuited for concrete under water, or other submarine masonry. By the use of lime is secured the double advantages of slow setting and economy.

Pointing Mortar is composed of a paste of finely-ground cement and clean sharp silicious sand, in such proportions that the volume of cement paste is slightly in excess of the volume of voids or spaces in the sand. The volume of sand varies from 2½ to 2¾ that of the cement paste, or by weight, 1 of cement powder to 3 to 3⅓ of sand. The mixture should be made under shelter, and in quantities not exceeding from 2 to 3 pints at a time.

Before pointing, the joints should be reamed, and in close masonry they must be open to $\frac{1}{5}$ of an inch, then thoroughly saturated with water, and maintained in a condition that they will neither absorb water from the mortar or impart any to it. Masonry should not be allowed to dry rapidly after pointing, but it should be well driven in by the aid of a caulking iron and hammer.

In the pointing of rubble masonry the same general directions are to be observed.

Notes by General Totten, U. S. A.—240 lbs. lime = 1 cask, will make from 7.8 to 8.15 cubic feet of stiff paste.

308* lbs. of finely-ground cement will make from 3.7 to 3.8 cubic feet of stiff paste; 79 to 83 lbs. of cement powder will make 1 cubic foot of stiff paste.

1 cubic foot of dry cement powder, measured when loose, will measure .78 to .8 cubic foot when packed, as at a manufactory.

100 yards of lath and plaster work, with wages of masons at $1.75 per day, and Rockland lime at $1 per cask, cost, respectively:

3 Coats hard finish work .... $25.50 | 2 Coats slipped work......... $19.95

Mural Efflorescences.—White alkaline efflorescences upon the surface of brick walls laid in mortar, of which natural hydraulic lime or cement is the basis.

The crystallization of these salts within the pores of bricks, into which they have been absorbed from the mortar, causes disintegration.

Asphalte Flooring.—8 lbs. of composition will cover 1 sup. foot, ¾ inch thick.

Plastering.—1 bushel, or 1¼ cubic foot of cement, mortar, etc., will cover 1½ square rods ¾ inch thick. 75 volumes are required upon brick work for 70 upon laths.

Cost of Masonry, of various Kinds, per Cubic Yard, and the Volume of Mortar required for each.—[Gen. Gilmore, U. S. A.]

Mortar.	Volume.	Lime, no Cement used.	Cement, no Lime used.	Difference of Cost with Cement or Lime Mortar.	Cost. Lime Mortar.	Cost. Cement Mortar.
	Cu. Ft.	Bbls.	Bbls.	$ Cts.	$ Cts.	$ Cts.
Rough, in rubble or gravel, from ⅛ to .1 cubic foot in volume	10.8	.5 5	1.22	90	4.10	5.
Blocks, large and small, not in courses; joints hammer-dressed...	8.1	.423	.92	62	7.	7.63
Large masses; headers and stretchers dovetailed; hammer-dressed; beds and joints laid close..............	1.	.05	.11	08	9.	9.08
Ordinary; courses 20 to 32 in rise ...	1.5	.08	.17	12	5.70	
Ordinary; courses 12 to 20 in rise...	2.	1.05	.22	16	2.19	
Brick	8.	.42	.9	66	5.70	6.10
Concrete, good....................	11.	.54	1.75	1.21	2.19	3.20
" medium	9.	.41	1.06	65	1.56	2.21
" inferior..................	8.	.37	.97	60	1.45	2.05
Rubble, without mortar					3. to 3.30	

Cost of materials assumed as follows: Cement, $1.25 per barrel; Lime, $1; Bricks, $4.25 per M; Sand and Gravel, 80 cents per ton; Granite spalls, 55 cents per cubic yard; Labor, $1 per day.

* 300 lbs. net is the standard barrel, but it usually weighs 308 lbs.

WHEEL GEARING.

The *Pitch Line* of a wheel, is the circle upon which the pitch is measured, and it is the circumference by which the diameter, or the velocity of the wheel, is measured.

The *Pitch*, is the arc of the circle of the pitch line, and is determined by the number of the teeth in the wheel.

The *True Pitch* (*Chordial*), or that by which the dimensions of the tooth of a wheel are alone determined, is a straight line drawn from the centres of two contiguous teeth upon the pitch line.

The *Line of Centres*, is the line between the centres of two wheels.

The *Radius* of a wheel, is the semi-diameter running to the periphery of a tooth. The *Pitch Radius*, is the semi-diameter running to the pitch line.

The *Length of a Tooth*, is the distance from its base to its extremity.

The *Breadth of a Tooth*, is the length of the face of wheel.

A *Cog Wheel*, is the general term for a wheel having a number of cogs or teeth set upon or radiating from its circumference.

A *Mortice Wheel*, is a wheel constructed for the reception of teeth or cogs, which are fitted into recesses or sockets upon the face of the wheel.

Plate Wheels, are wheels without arms.

A *Rack*, is a series of teeth set in a plane.

A *Sector*, is a wheel which reciprocates without forming a full revolution.

A *Spur Wheel*, is a wheel having its teeth perpendicular to its axis.

A *Bevel Wheel*, is a wheel having its teeth at an angle with its axis.

A *Crown Wheel*, is a wheel having its teeth at a right angle with its axis.

A *Mitre Wheel*, is a wheel having its teeth at an angle of 45° with its axis.

A *Face Wheel*, is a wheel having its teeth set upon one of its sides.

An *Annular* or *Internal Wheel*, is a wheel having its teeth convergent to its centre.

Spur Gear.—Wheels which act upon each other in the same plane.

Bevel Gear.—Wheels which act upon each other at an angle.

When the tooth of a wheel is made of a material different from that of the wheel, it is termed a *cog*: in a pinion it is termed a *leaf*, and in a trundle a *stave*.

A wheel which impels another is termed the *Spur*, *Driver*, or *Leader*; the one impelled is the *Pinion*, *Driven*, or *Follower*.

A series of wheels in connection with each other is termed a *Train*.

When two wheels act upon one another, the greater is termed the *Wheel* and the lesser the *Pinion*.

A *Trundle*, *Lantern*, or *Wallower* is when the teeth of a pinion are constructed of round brass or solid cylinders set in to two discs.

A *Trundle* with less than eight staves can not be operated uniformly by a wheel with any number of teeth.

The material of which cogs are made is about one fourth the strength of cast iron. The product of their $b\,d^2$ should be four times that of iron teeth.

Buchanan: Rules that to increase or diminish velocity in a given proportion, and with the least quantity of wheel-work, the number of teeth in each pinion should be to the number of teeth in its wheel as 1 : 3.59. Even to save space and expense, the ratio should never exceed 1 : 6.

The least number of teeth that it is practicable to give to a wheel is regulated by the necessity of having at least one pair always in action, in order to provide for the contingency of a tooth breaking.

The teeth of wheels should be as small and numerous as is consistent with strength.

When a *Pinion is driven by a wheel*, the number of teeth in the pinion should not be less than eight.

When a *Wheel is driven by a pinion*, the number of teeth in the pinion should not be less than ten.

The *Number of teeth* in a wheel should always be prime to the number of the pinion; that is, the number of teeth in the wheel should not be divisible by the number of teeth in the pinion without a remainder. This is in order to prevent the same teeth coming together so often as to cause an irregular wear of their faces. An odd tooth introduced into a wheel is termed a *hunting tooth* or *cog*.

To Compute the Pitch of a Wheel.

RULE.—Divide circumference at the pitch-line by the number of teeth.

EXAMPLE.—A wheel 40 ins. in diameter requires 75 teeth; what is its pitch?

$$\frac{3.1416 \times 40}{75} = 1.6755 \textit{ ins.}$$

To Compute the True or Chordial Pitch.

RULE.—Divide 180° by the number of teeth, ascertain the sine of the quotient, and multiply it by the diameter of the wheel.

EXAMPLE.—The number of teeth is 75, and the diameter 40 inches; what is the true pitch?

$$\frac{180}{75} = 2° \ 24' \text{ and } \textit{sin.} \text{ of } 2° \ 24' = .04188, \text{ which } \times 40 = 1.6752 \textit{ ins.}$$

To Compute the Diameter of a Wheel.

RULE.—Multiply the number of teeth by the pitch, and divide the product by 3.1416.

EXAMPLE.—The number of teeth in a wheel is 75, and the pitch 1.675 ins.; what is the diameter of it?

$$\frac{75 \times 1.6755}{3.1416} = 10 \textit{ ins.}$$

To Compute the Number of Teeth in a Wheel.

RULE.—Divide the circumference by the pitch.

To Compute the Diameter when the True Pitch is given.

RULE.—Multiply the number of teeth in the wheel by the true pitch, and again by .3184.

EXAMPLE.—Take the elements of the preceding case.

$$75 \times 1.6752 \times .3184 = 40 \textit{ ins.}$$

To Compute the Number of Teeth in a Pinion or Follower to have a given Velocity.

RULE.—Multiply the velocity of the driver by its number of teeth, and divide the product by the velocity of the driven.

EXAMPLE.—The velocity of a driver is 16 revolutions, the number of its teeth 54, and the velocity of the pinion is 48; what is the number of its teeth?

$$\frac{16 \times 54}{48} = 18 \textit{ teeth.}$$

2. A wheel having 75 teeth is making 16 revolutions per minute; what is the number of teeth required in the pinion to make 24 revolutions in the same time?

$$\frac{16 \times 75}{24} = 50 \textit{ teeth.}$$

To Compute the Proportional Radius of a Wheel or Pinion.

RULE.—Multiply the length of the line of centres by the number of teeth in the wheel for the wheel, and in the pinion for the pinion, and divide by the number of teeth in both the wheel and pinion.

To Compute the Diameter of a Pinion, when the Diameter of the Wheel and Number of Teeth in the Wheel and Pinion are given.

RULE.—Multiply the diameter of the wheel by the number of teeth in the pinion, and divide the product by the number of teeth in the wheel.

EXAMPLE.—The diameter of a wheel is 25 inches, the number of its teeth 210, and the number of teeth in the pinion 30; what is the diameter of the pinion?

$$\frac{25 \times 30}{210} = 3.57 \textit{ ins.}$$

To Compute the Number of Teeth required in a Train of Wheels to produce a given Velocity.

RULE.—Multiply the number of teeth in the driver by its number of revolutions, and divide the product by the number of revolutions of each pinion, for each driver and pinion.

EXAMPLE.—If a driver in a train of three wheels has 90 teeth, and makes 2 revolutions, and the velocities required are 2, 10, and 18, what are the number of teeth in each of the other two?

$$10 : 90 :: 2 : 18 = \textit{teeth in 2d wheel.}$$
$$18 : 90 :: 2 : 10 = \textit{teeth in 3d wheel.}$$

To Compute the Circumference of a Wheel.

RULE.—Multiply the number of teeth by their pitch.

To Compute the Revolutions of a Wheel or Pinion.

RULE.—Multiply the diameter or circumference of the wheel or the number of its teeth, as the case may be, by the number of its revolutions, and divide the product by the diameter, circumference, or number of teeth in the pinion.

EXAMPLE.—A pinion 10 inches in diameter is driven by a wheel 2 feet in diameter, making 46 revolutions per minute; what is the number of revolutions of the pinion?

$$\frac{2 \times 12 \times 46}{10} = 110.4 \textit{ revolutions.}$$

To Compute the Velocity of a Pinion.

RULE.—Divide the diameter, circumference, or number of teeth in the driver, as the case may be, by the diameter, etc., of the pinion.

When there are a Series or Train of Wheels and Pinions.

RULE.—Divide the continued product of the diameter, circumference, or number of teeth in the wheels by the continued product of the diameter, etc., of the pinions.

EXAMPLE.—If a wheel of 32 teeth drive a pinion of 10, upon the axis of which there is one of 30 teeth, driving a pinion of 8, what are the revolutions of the last?

$$\frac{32}{10} \times \frac{30}{8} = \frac{960}{80} = 12 \textit{ revolutions}$$

Ex. 2. The diameters of a train of wheels are 6, 9, 9, 10, and 12 inches; of the pinions, 6, 6, 6, 6, and 6 inches; and the number of revolutions of the driving shaft or prime mover is 10; what are the revolutions of the last pinion?

$$\frac{6 \times 9 \times 9 \times 10 \times 12 \times 10}{6 \times 6 \times 6 \times 6 \times 6} = \frac{583200}{7776} = 75 \textit{ revolutions.}$$

To Compute the Proportion that the Velocities of the Wheels in a Train should bear to one another.

RULE.—Subtract the less velocity from the greater, and divide the remainder by one less than the number of wheels in the train; the quotient is the number, rising in arithmetical progression from the less to the greater velocity.

EXAMPLE.—What should be the velocities of 3 wheels to produce 18 revolutions, the driver making 3?

$18-3=15$, $3-1=2$; $\frac{15}{2}=7.5=$ *number to be added to velocity of the driver* $=7.5+3=10.5$, and $10.5+7.5=18$ *revolutions*. Hence 3, 10.5, and 18 *are the velocities of the three wheels.*

General Illustrations.

1. A wheel 96 inches in diameter, having 42 revolutions per minute, is to drive a shaft 75 revolutions per minute; what should be the diameter of the pinion?

$$\frac{96 \times 42}{75}=53.76 \text{ ins.}$$

2. If a pinion is to make 20 revolutions per minute, required the diameter of another to make 58 revolutions in the same time.

$58 \div 20=2.9=$ *the ratio of their diameters.* Hence, if one to make 20 revolutions is given a diameter of 30 inches, the other will be $30 \div 2.9=10.345$ *ins.*

3. Required the diameter of a pinion to make 12½ revolutions in the same time as one of 32 ins. diameter making 26.

$$\frac{32 \times 26}{12.5}=66.56 \text{ ins.}$$

4. A shaft, having 22 revolutions per minute, is to drive another shaft at the rate of 15, the distance between the two shafts upon the line of centres is 45 ins.; what should be the diameter of the wheels?

Then, 1st. $22+15:22::45:26.75=$ *inches in the radius of the pinion.*

2d. $22+15:15::45:18.24=$ *inches in the radius of the spur.*

5. A driving shaft, having 16 revolutions per minute, is to drive a shaft 81 revolutions per minute, the motion to be communicated by two geared wheels and two pulleys, with an intermediate shaft; the driving wheel is to contain 54 teeth, and the driving pulley upon the driven shaft is to be 25 inches in diameter; required the number of teeth in the driven wheel, and the diameter of the driven pulley.

Let the driven wheel have a velocity of $\sqrt{16 \times 81}=36$, a mean proportional between the extreme velocities 16 and 81.

Then, 1st. $36:16::54:24=$ *teeth in the driven wheel.*

2d. $81:36::25:11.11=$ *ins. diameter of the driven pulley.*

6. If, as in the preceding case, the whole number of revolutions of the driving shaft, the number of teeth in its wheel, and the diameters of the pulleys are given, what are the revolutions of the shafts?

Then, 1st. $18:16::54:48=$ *revolutions of the intermediate shaft.*

2d. $15:48::25:80=$ *revolutions of the driven shaft.*

To Compute the Diameter of a Wheel for a given Pitch and Number of Teeth.

RULE.—Multiply the diameter in the following table for the number of teeth by the pitch, and the product will give the diameter at the pitch circle.

EXAMPLE.—What is the diameter of a wheel to contain 48 teeth of 2.5 ins. pitch?

$$15.29 \times 2.5=38.225 \text{ ins.}$$

To Compute the Pitch of a Wheel for a given Diameter and Number of Teeth.

RULE.—Divide the diameter of the wheel by the diameter in the table for the number of teeth, and the quotient will give the pitch.

EXAMPLE.—What is the pitch of a wheel when the diameter of it is 50.94 inches, and the number of its teeth 80?

$$\frac{50.94}{25.47}=2 \text{ ins.}$$

To Compute the Number of Teeth of a Wheel for a given Diameter and Pitch.

RULE.—Divide the diameter by the pitch, and opposite to the quotient in the table is given the number of teeth.

PITCH OF WHEELS.

A Table whereby to Compute the Diameter of a Wheel for a given Pitch, or the Pitch for a given Diameter.

From 8 *to* 192 *teeth.*

No. of Teeth.	Diameter.	No of Teeth.	Diameter.	No. of Teeth.	Diameter.	No. of Teeth.	Diameter.	No. of Teeth.	Diameter.
8	2.61	45	14.33	82	26.11	119	37.88	156	49.66
9	2.93	46	14.65	83	26.43	120	38.2	157	49.98
10	3.24	47	14.97	84	26.74	121	38.52	158	50.3
11	3.55	48	15.29	85	27.06	122	38.84	159	50.61
12	3.86	49	15.61	86	27.38	123	39.16	160	50.93
13	4.18	50	15.93	87	27.7	124	39.47	161	51.25
14	4.49	51	16.24	88	28.02	125	39.79	162	51.57
15	4.81	52	16.56	89	28.33	126	40.11	163	51.89
16	5.12	53	16.88	90	28.65	127	40.43	164	52.21
17	5.44	54	17.2	91	28.97	128	40.75	165	52.52
18	5.76	55	17.52	92	29.29	129	41.07	166	52.84
19	6.07	56	17.8	93	29.61	130	41.38	167	53.16
20	6.39	57	18.15	94	29.93	131	41.7	168	53.48
21	6.71	58	18.47	95	30.24	132	42.02	169	53.8
22	7.03	59	18.79	96	30.56	133	42.34	170	54.12
23	7.34	60	19.11	97	30.88	134	42.66	171	54.43
24	7.66	61	19.42	98	31.2	135	42.98	172	54.75
25	7.98	62	19.74	99	31.52	136	43.29	173	55.07
26	8.3	63	20.06	100	31.84	137	43.61	174	55.39
27	8.61	64	20.38	101	32.15	138	43.93	175	55.71
28	8.93	65	20.7	102	32.47	139	44.25	176	56.02
29	9.25	66	21.02	103	32.79	140	44.57	177	56.34
30	9.57	67	21.33	104	33.11	141	44.88	178	56.66
31	9.88	68	21.65	105	33.43	142	45.2	179	56.98
32	10.2	69	21.97	106	33.74	143	45.52	180	57.23
33	10.52	70	22.29	107	34.06	144	45.84	181	57.62
34	10.84	71	22.61	108	34.38	145	46.16	182	57.93
35	11.16	72	22.92	109	34.7	146	46.48	183	58.25
36	11.47	73	23.24	110	35.02	147	46.79	184	58.57
37	11.79	74	23.56	111	35.34	148	47.11	185	58.89
38	12.11	75	23.88	112	35.65	149	47.43	186	59.21
39	12.43	76	24.2	113	35.97	150	47.75	187	59.53
40	12.74	77	24.52	114	36.29	151	48.07	188	59.84
41	13.06	78	24.83	115	36.61	152	48.39	189	60.16
42	13.38	79	25.15	116	36.93	153	48.7	190	60.48
43	13.7	80	25.47	117	37.25	154	49.02	191	60.81
44	14.02	81	25.79	118	37.56	155	49.34	192	61.13

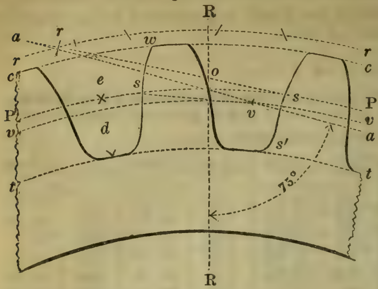
NOTE.—The pitch in this table is the true pitch, as before described.

Change Wheels in Screw-cutting Lathes.

$\frac{T\,S}{t\,t'} I = N$; $\frac{t\,t'}{I\,T} = S$. T *representing number of teeth in traverse screw;* S *number in stud wheel gearing in mandril;* t *number in wheel upon mandril, and* t' *number in gearing upon stud pinion, gearing in* T; I *number of threads per inch upon traverse screw;* N *number to be cut.*

To Construct a Tooth. (J. W. Nystrom.)

Conceive the true pitch to be divided into 10 parts.



The *Length* of a tooth should be .7 of the pitch, .4 of it being below or within the pitch line, and .3 of it above or without it.

The *Depth* of a tooth should be .46 of the pitch.

Draw the radius R R, and strike the pitch line P P.

Through the intersection of these lines, o, draw $a\ a$ at an angle of 75° to R o.

From o set off the depth of the tooth $o\ s = .46$ of the pitch; strike the circles $t\ t$ and $c\ c$, making $d = .4$ and $e = .3$ of the pitch.

Determine the distance $o\ r$ by the formula $\frac{P\,(n+6)}{2\,(n-11)}$, and $o\ v = .11\ P\sqrt[3]{n}$, P *representing the pitch*, and n *the number of teeth in the wheel*, and strike the circles $r\ r$ and $v\ v$; then with $r\ s$ describe $s\ s'$, and with $v\ s$, $s\ w$. Repeat these operations at the uniform distances upon the respective circles determined by the pitch and depth of the tooth, and the teeth of the wheel will be delineated.

When the Teeth of both Wheel and Pinion are required.

$\frac{P\,(n+6)}{2\,(n-11)} = o\ r$, and $.11\ P\sqrt[3]{n} = o\ v$ in the wheel; and $\frac{P\,(n'+6)}{2\,(n'-11)} = o\ r$, and $.11\ P\sqrt[3]{n'} = o\ v$ in the pinion. n' *representing the number of teeth in the pinion.*

When a Wheel has Teeth exceeding those in the Pinion in the Proportion of 4 *to* 1. The depth of a tooth is,

In a wheel, $P\left(.42 + \frac{n'}{700}\right) = o\ s$; in a pinion, $\frac{P}{2}\left(1 - \frac{n'}{350}\right) = o\ s$.

A *Rack* is treated as if it were a wheel of not less than 200 teeth.

Depth of tooth $\times$ 2.174 = *pitch*. Depth of tooth $\times$ 1.522 = *length*.

To Compute the Number of Teeth, Dimensions, etc., of a Single or Pair of Wheels.

$$\frac{N+2}{d} = m;\quad \frac{N}{m} = d';\quad m\,d' = N;\quad \frac{1.445}{m} = t;\quad \frac{N\,V}{v} = n;\quad \frac{b}{2\,m} = a;$$

$$\frac{N+2}{m} = D;\quad 2\,a\,m = b;\quad \frac{n}{m} = d'';\quad \frac{N\,V}{n} = v;\quad \frac{2\,a\,(n+2)}{b} = d.$$

N *and* n *representing number of teeth in large and small wheels;* D *and* d *whole diameters of large and small wheels, and* d' *and* d'' *diameters of pitch circle of large and small wheels;* m *the diametral pitch or number of teeth to one inch of diameter of pitch circle;* t *depth of teeth upon pitch circle;* V *and* v *velocities of large and small wheels;* b *number of teeth in both wheels; and* a *distance between centres of both wheels.*

Assume a single or large wheel of 72 teeth, a small one of 36 teeth, a diameter of large wheel of 123.33 ins., and velocities as 1 and 2.

Then, $\frac{72+2}{123.33} = .6$ *ins. = diametral pitch;* $\frac{72}{.6} = 120$ *ins. = diameter of pitch cir-*

[illegible]; $.6 \times 120 = 72 =$ *number of teeth in large wheel*; $\frac{1.445}{.6} = 2.408$ *ins.* = *depth of teeth*; $\frac{72 \times 1}{2} = 36 =$ *number of teeth in small wheel*; $\frac{72+36}{2 \times .6} = 90$ *ins.* = *distance between centres of both wheels*; $\frac{72+2}{.6} = 123.33$ *ins.* = *diameter of large wheel*; $2 \times 90 \times .6 = 108 =$ *number of teeth in both wheels*; $\frac{2 \times 90\,(36+2)}{72+36} = 63.33$ *ins.* = *diameter of small wheel*; $\frac{72 \times 1}{36} = 2 =$ *velocity of small wheel*; $\frac{36}{.6} = 60 =$ *diameter of pitch circle of small wheel.*

PROPORTIONS OF WHEELS.

Tooth.—In computing the dimensions of a tooth, it is to be considered as a beam fixed at one end, the weight suspended from the other, or face of the beam; and it is essential to consider the element of velocity, as its stress in operation, at high velocity with irregular action, is increased thereby.

The dimensions of a tooth should be much greater than is necessary to resist the direct stress upon it, as but one tooth is proportioned to bear the whole stress upon the wheel, although two or more are actually in contact at all times; but this requirement is in consequence of the great wear to which a tooth is subjected, the shocks it is liable to from lost motion, when so worn as to reduce its depth and uniformity of bearing, and the risk of the breaking of a tooth from a defect.

A tooth running at a low velocity may be materially reduced in its dimensions compared with one running at a high velocity and with a like stress.

The result of operations with toothed wheels, for a long period of time, has determined that a tooth with a pitch of 3 inches and a breadth 7.5 inches will transmit, at a velocity of 6.66 feet per second, the power of 59.16 horses.

To Compute the Dimensions of a Tooth to Resist a given Stress.

Rule.—Multiply the extreme pressure at the pitch-line of the wheel by the length of the tooth in the decimal of a foot, divide the product by the *Value* of the material of the tooth, and the quotient will give the product of the breadth and square of the depth.

Or $\frac{S\,l}{V} = b\,d^2$. S *representing the stress in pounds, and l the length in feet.*

The *Value* of cast iron for this or like purposes may be taken at from 50 to 70.

Note.—It is necessary first to determine the pitch, in order to obtain either the length or depth of a tooth.

Example.—The pressure at the pitch-line of a cast-iron wheel (at a velocity of 6.66 feet per second) is 4886 lbs.; what should be the dimensions of the teeth, the pitch being 3 inches?

$3 \times .7 = 2.1 =$ *length of tooth, which* $\div 12 = .175 =$ *length in decimals of a foot*; $3 \times 2.5 = 7.5 =$ *breadth of tooth.*

The *Value of the material* in this case is taken at 60.

$$\frac{4886 \times .175}{60} = 14.25, \text{ and } \sqrt{\frac{14.25}{7.5}} = 1.38 \textit{ ins. in depth.}$$

When the product b d² is obtained, and it is required to ascertain either dimension, proceed as follows:

As $d = .46$, and as $b = 2.5$ *times the pitch*, b is to d as 5.435 is to 1. Assume the preceding case where $b\,d^2 = 14.25$.

Then $b : d :: 5.435 : 1$; $\therefore b = 5.435\,d$, and $5.435\,d^3 = 14.25$; $\therefore d^3 = \frac{14.25}{5.435} = 2.6219$, and $\sqrt[3]{2.622} = 1.38$ *ins., the depth*; and $\frac{b\,d^2}{d^2} = b$; $\therefore \frac{14.25}{1.38^2} = 7.5$ *ins., the breadth.*

The following Rule* to ascertain the dimensions of a tooth is the result of some consideration of the subject, and is supported by several well-defined cases in operation.

To Compute the Depth of a Cast-iron Tooth

1. *When the Stress is given.*

RULE.—Extract the square root of the stress, and multiply it by .02.

EXAMPLE.—The stress to be borne by a tooth is 4886 lbs.; what should be its depth?

$$\sqrt{4886} \times .02 = 1.4 \textit{ ins.}$$

2. *When the Horses' Power is given.*

RULE.—Extract the square root of the quotient of the horses' power divided by the velocity in feet per second, and multiply it by .466.

EXAMPLE.—The horses' power to be transmitted by a tooth is 60, and the velocity of it at its pitch-line is 6.66 feet per second: what should be the depth of the tooth?

$$\sqrt{\frac{60}{6.66}} \times .466 = 1.398 \textit{ ins.}$$

To Compute the Horses' Power of a Tooth.

RULE.—Multiply the pressure at the pitch-line, by its velocity in feet per minute, and divide the product by 33 000.

EXAMPLE.—What is the horses' power of a tooth of the dimensions and at the velocity given in the preceding example, page 515?

$$\frac{4886 \times 6.66 \times 60''}{33\,000} = 59.16 \textit{ horses.}$$

To Compute the Stress that may be borne by a Tooth.

RULE.—Multiply the *Value* of the material of the tooth to resist a transverse strain, as estimated for this character of stress, by the breadth and square of its depth, and divide the product by the extreme length of it in the decimal of a foot.

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* As an exponent of the necessity of an investigation of the stress of a tooth, the following deductions by the rules of different authors for like elements are submitted:

Pitch.....3 *ins.* Depth.....1.38 *ins.* Breadth.....7.5 *ins.* Length.....2.1 *ins.*

FOR CAST IRON.

Actual power in stress exerted at a velocity of 400 feet per minute, 4886 lbs.	Depth of Tooth.
	Ins.
By above rule $\sqrt{\frac{H}{v}} \times .466 =$	1.398†
" Fairbairn $.025\sqrt{W} =$	1.75
" Imperial Journal $\sqrt{\frac{W}{1576}} =$	1.76
" Rankine $\sqrt{\frac{W}{1500}} =$	1.8
" Tredgold $\sqrt{\frac{3\,P}{4500}} =$	1.8
" " $\frac{3}{4}\sqrt{\frac{H}{v}} =$	2.25
" Buchanan $\sqrt{\frac{.556\,H}{v}} =$	2.24

H *representing horses' power* (60), W *and* P *the stress in pounds, and* v *the velocity in feet per second.*

† This depth, with a breadth of 7.5 ins., is .1 of the ultimate strength of the average strength of American Cast Iron.

EXAMPLE.—The dimensions of a cast-iron tooth in a wheel are 1.38 ins. in depth by 7.5 ins. in breadth; what is the stress it will bear?

$$\text{Pitch} = 2.174 \times 1.38 = 3\ ins. \quad \text{Length} = .7 \text{ of } 3 = 2.1\ ins.$$

$$\text{Breadth} = 2.5 \times 3 = 7.5\ ins. \quad \frac{60 \times 7.5 \times 1.38^2}{2.1 \div 12} = 4886\ lbs.$$

PROPORTIONS OF WHEELS

With six flat arms and Ribs upon one side of them, as ; *or a Web in the centre, as* .

Rim.—Depth, measured from base of the teeth, .45 to .5 of the pitch of the teeth, having a web upon its inner surface .4 of the pitch in depth and .25 to .3 of it in width.

NOTE.—When the face of the wheel is morticed, the depth of the rim should be 1.5 times the pitch, and the breadth of it 1.5 times the breadth of the tooth or cog.

Hub.—When the eye is proportionate to the stress upon the wheel, the hub should be twice the diameter of the eye. In other cases the depth around the eye should be .75 to .8 of the pitch.

Arm.—Depth .4 to .45 of the pitch. Breadth at rim 1.5 times the pitch, increasing .5 inch per foot of length toward the hub.

The Rib upon one edge of the arm, or the Web in its centre, should be from .25 to .3 the pitch in width, and .4 to .45 of it in depth.

When the section of an arm differs from those above given, as with one with a plane section, as , or with a double rib, as , its dimensions should be proportioned to the form of the section.

In a wheel of greater relative diameter, the length of the hub and the breadth of the arms, or of the rib or web, according as the plane of the arm is in that of the wheel or contrariwise, should be made to exceed the breadth of the face of the wheel (at the hub) in order to give it resistance to lateral strain.

The number of arms in wheels should be as follows:

1.5 to 3.25 feet in diam.......	4.	8.5 to 16 feet in diam.......	8.
3.25 " 5 " "	5.	16 " 24 " "	10.
5 " 8.5 " "	6.		

With light wheels, the number of arms should be increased, in order the better to sustain the rigidity of the rim.

Pitches of Equivalent Strength for Iron and Wood.—Iron 1. Hard wood 1.26.

WINDING ENGINES.

In Winding Engines, for drawing coals, etc., out of a Pit, where it is required to give a certain number of revolutions, it is necessary to know the diameter of the *Drum* and the thickness of the rope, and contrariwise.

To Compute the Diameter of a Drum.

Where flat Ropes are used, and are wound one part over the other.

RULE.—Divide the depth of the pit in inches by the product of the number of revolutions and 3.1416, and from the quotient subtract the product of the thickness of the rope and the number of revolutions; the remainder is the diameter in inches.

EXAMPLE.—If an engine makes 20 revolutions, the depth of the pit being 600 feet, and the rope 1 inch, what should be the diameter of the drum?

$$\frac{600 \times 12}{20 \times 3.1416} - 1 \times 20 = \frac{7200}{62.832} - 20 = 94.59\ ins.$$

To Compute the Diameter of the Roll.

RULE.—To the area of the drum add the area or edge surface of the rope; then ascertain by inspection in the table of areas, or by calculation, the diameter that gives this area, and it is the diameter of the Roll.

EXAMPLE.—What is the diameter of the roll in the preceding example?

Area of $94.59 = 7027.2 +$ area of $\overline{7200\times 1} + 7200 = 14227.2$, and $\sqrt{14227.2} \div .7854 = 134.59$ *ins.*

Or, the radius of the drum is increased the number of the revolutions multiplied by the thickness of the rope; as, $\frac{94.59}{2} + \overline{20\times 1} = 67.295$ *ins.*

To Compute the Number of Revolutions.

RULE.—To the area of the drum add the area of the edge surface of the rope; from the diameter of the circle having that area subtract the diameter of the drum, and divide the remainder by twice the thickness of the rope; the quotient will give the number of revolutions.

EXAMPLE.—The length of a rope is 2600 inches, its thickness 1 inch, and the diameter of the drum 20 inches; what is the number of revolutions?

Area of $20 +$ area of rope $= 314.16$, and $314.16 + 2600 = 2914.16$, the diameter of which is 60.91, and $\frac{60.91 - 20}{1\times 2} = 20.45$ *revolutions.*

Or, subtract the diameter of the drum from the diameter of the roll, and divide the remainder by twice the thickness of the rope; as, $134.59 - 94.59 = 40$, and $40 \div \overline{1\times 2} = 20$ *revolutions.*

To Compute the Place of Meeting of the Ascending and Descending Buckets when two or more are used.

NOTE.—Meetings will always be below half the depth of the pit.

To Compute this Depth.

RULE.—Take the circumference of the drum for the length of the first turn; then, to the diameter of the drum add twice the thickness of the rope, multiplied by the number of revolutions, less 1, for a diameter, and the circumference of this diameter is the length of the last turn; add these two lengths together, multiply their sum by half the number of revolutions, and the product will give the depth of the pit.

EXAMPLE.—The diameter of a drum is 9 feet, the thickness of the rope 1 inch, and the revolutions 20; what is the depth of the pit, and at what distance from the top will the buckets meet?

$9\times 3.1416 = 28.27$ *feet, length of first turn.* $9 + \frac{1\times 2\times \overline{20-1}}{12}\times 3.1416 = 38.23$ *feet, length of last turn.* $28{,}27 + 38.23 \times \frac{20}{2} = 66.5\times 10 = 665$ *feet, or depth of pit.*

2. Divide the sum of the length of the turns of the rope by 2, and to the quotient add the length of the last turn; divide the sum by 2, multiply the quotient by half the number of revolutions, and the product will give the distance from the centre of the drum at which the buckets will meet.

NOTE.—At half the number of revolutions the buckets will meet.

$$\frac{28.27 + 38.23}{2} + 38.23 = 71.48, \text{ and } \frac{71.48}{2}\times\frac{20}{2} = \frac{1429.6}{4} = 357.4 \textit{ feet.}$$

DREDGING MACHINE.

In the operation of a Dredging Machine, in 1855, under Lieut. Meade, U. S. A., the following elements were obtained:

Two non-condensing engines, working at a power of 22 horses, excavated 1075 cubic yards, or 170 tons, of soft and hard clay and mud per hour, at a depth of 11 feet from the water line.

The coefficients deduced from the friction of the materials raised were C =.1 for hard clay with gravel; =.07 for pure hard clay; =.05 for common clay or sand; =.04 for soft clay or loose sand; and =.03 for loose materials.

From which Mr. Nystrom furnishes the following formulæ:

$\left(\frac{h}{700} + C\right) W =$ *horses' power, h representing the total height to which the material is raised, and* W *the weight of it in tons.*

$$\frac{700\ H}{h + 700\ C} = W; \quad \frac{H}{W} + \frac{h}{700} = C.$$

WOOD, TIMBER, ETC.

Selection of Standing Trees.—Wood grown in a moist soil is lighter, and decays sooner than that grown in dry, sandy soil.

The best *Timber* is that grown in a dark soil intermixed with gravel. Poplar, cypress, willow, and all others which grow best in a wet soil, are exceptions.

The hardest and densest woods, and the least subject to decay, grow in warm climates; but they are more liable to split and warp in seasoning.

Trees grown upon plains or in the centre of forests are less dense than those from the edge of a forest, from the side of a hill, or from open ground.

Trees (in the U. S.) should be selected in the latter part of July or first part of August; for at this season the leaves of the sound, healthy trees are fresh and green, while those of the unsound are beginning to turn yellow. A sound, healthy tree is recognized by its top branches being well leaved, the bark even and of a uniform color. A rounded top, few leaves, some of them turned yellow, a rougher bark than common, covered with parasitic plants, and with streaks or spots upon it, indicate a tree upon the decline. The decay of branches, and the separation of bark from the wood, are infallible indications that the wood is impaired.

Felling Timber.—The most suitable time for felling timber is in midwinter and in midsummer. Recent experiments indicate the latter season and in the month of July.

A tree should be allowed to attain full maturity before being felled. Oak matures at 75 to 100 years and upward, according to circumstances. The age and rate of growth of a tree are indicated by the number and width of the rings of annual increase which are exhibited in a cross-section.

A tree should be cut as near to the ground as practicable, as the lower part furnishes the best timber.

Dressing Timber.—As soon as a tree is felled, it should be stripped of its bark, raised from the ground, the sap-wood taken off, and the timber reduced to its required dimensions.

Inspection of Timber.—The quality of wood is in some degree indicated by its color, which should be nearly uniform in the heart, a little deeper toward the centre, and free from sudden transitions of color. White spots indicate decay. The sap-wood is known by its white color; it is next to the bark, and very soon rots.

Defects of Timber.—Wind-shakes are circular cracks separating the concentric layers of wood from each other. It is a serious defect.

Splits, checks, and cracks, extending toward the centre, if deep and strongly marked, render the timber unfit for use, unless the purpose for which it is intended will admit of its being split through them.

Brush-wood is generally consequent upon the decline of the tree from age. The wood is porous, of a reddish color, and breaks short, without splinters.

Belted timber is that which has been killed before being felled, or which has died from other causes. It is objectionable.

Knotty timber is that containing many knots, though sound; usually of stunted growth.

Twisted wood is when the grain of it winds spirally; it is unfit for long pieces.

Dry-rot.—This is indicated by yellow stains. Elm and beech are soon affected, if left with the bark on.

Large or decayed knots injuriously affect the strength of timber.

Seasoning and Preserving Timber.

Timber freshly cut contains about 37 to 48 per cent. of liquids. By exposure to the air in seasoning one year, it loses from 17 to 25 per cent., and when seasoned it yet retains from 10 to 15 per cent.

Timber of large dimensions is improved and rendered less liable to warp and crack in being seasoned by immersion in water for some weeks.

For the purpose of seasoning, timber should be piled under shelter and be kept dry; it should have a free circulation of air about it, without being exposed to strong currents. The bottom pieces should be placed upon skids, which should be free from decay, raised not less than 2 feet from the ground; a space of an inch should intervene between the pieces of the same horizontal layers, and slats or piling-strips placed between each layer, one near each end of the pile, and others at short distances, in order to keep the timber from winding. These strips should be one over the other, and in large piles should not be less than 1 inch thick. Light timber may be piled in the upper portion of the shelter, heavy timber upon the ground floor. Each pile should contain but one description of timber. The piles should be at least 2½ feet apart.

Timber should be repiled at intervals, and all pieces indicating decay should be removed, to prevent their affecting those which are still sound.

Timber houses are best provided with blinds, which keep out rain and snow, but which can be turned to admit air in fine weather, and they should be kept entirely free from any pieces of decayed wood.

The gradual mode of seasoning is the most favorable to the strength and durability of timber, but various methods have been proposed for hastening the process. For this purpose, *steaming* timber has been applied with success; and the results of experiments of various processes of saturating timber with a solution of *corrosive sublimate* and *antiseptic* fluids are very satisfactory. This process hardens and seasons wood, at the same time that it secures it from dry-rot and from the attacks of worms. *Kiln-drying* is serviceable only for boards and pieces of small dimensions, and is apt to cause cracks and to impair the strength of wood, unless performed very slowly. *Charring* or *painting* is highly injurious to any but seasoned timber, as it effectually prevents the drying of the inner part of the wood, in consequence of which fermentation and decay soon take place.

Timber piled in badly-ventilated sheds is apt to be attacked with the *common-rot.* The first outward indications are yellow spots upon the ends of the pieces, and a yellowish dust in the checks and cracks, particularly where the pieces rest upon the piling-strips.

Timber requires from 2 to 8 years to be seasoned thoroughly, according to its dimensions. It should be worked as soon as it is thoroughly dry, for it deteriorates after that time.

Oak timber loses *one fifth of its weight* in seasoning, and about *one third of its weight* in becoming perfectly dry. Seasoning is the extraction or dissipation of the vegetable juices and moisture, or the solidification of the albumen. When wood is exposed to currents of air at a high temperature, the moisture evaporates too rapidly and the wood cracks; and when the temperature is high and sap remains, it ferments, and dry-rot ensues.

Timber is subject to *Common-rot* or *Dry-rot*, the former occasioned by alternate exposure to moisture and dryness. The progress of this decay is from the exterior; hence the covering of the surface with paint, tar, etc., is a preservative.

Painting and charring *green* timber hastens its decay.

Dry or Sap-rot is inherent in timber, and it is occasioned by the putrefaction of the vegetable albumen. Sap wood contains a large proportion of fermentable elements. Insects attack wood for the sugar or gum contained in it, and *Fungi* subsist upon the albumen of wood; hence, to arrest dry-rot, the albumen must be either extracted or solidified.

In the seasoning of timber naturally there is required a period of from 2 to 4 years. Immersion in water facilitates seasoning by solving the sap.

The most effective method of preserving timber is that of expelling or exhausting its fluids, solidifying its albumen, and introducing an antiseptic liquid.

The strength of impregnated timber is not reduced, and its *resilience* is improved.

In desiccating timber by expelling its fluids by heat and air, its strength is increased fully 15 per cent.

In coating unseasoned timber with creosote, tar, etc., the fluids are retained, and decay facilitated thereby.

When timber is saturated with creosote, tar, antiseptics, etc., it is also preserved from the attack of worms. Jarrow wood, from Australia, is not subjected to their attack.

The condition of timber, as to its soundness or decay, is readily recognized when struck a quick blow.

Timber that has been for a long time immersed in water, when brought into the air and dried, becomes brashy and useless.

When trees are barked in the spring, they should not be felled until the foliage is dead.

Timber can not be seasoned by either smoking or charring; but when it is to be used in locations where it is exposed to worms or to produce *fungi*, it is proper to smoke or char it.

Timber may be partially seasoned by being boiled or steamed.

Impregnation of Wood.

The several processes are as follows:

Kyan, 1832. Saturated with corrosive sublimate. Solution 1 lb. of chloride of mercury to 4 gallons of water.

Burnett, 1838. Impregnation with chloride of zinc by submitting the wood endwise to a pressure of 150 lbs. per square inch. Solution 1 lb. of the chloride to 10 gallons of water.

Boucheri. Impregnation by submitting the wood endwise to a pressure of about 15 lbs. per square inch. Solution 1 lb. of sulphate of copper to 12½ gallons of water.

Bethel. Impregnation by submitting the wood endwise to a pressure of 150 to 200 lbs. per square inch, with oil of creosote mixed with bituminous matter.

Louis S. Robbins, 1865. Aqueous vapor dissipated by the wood being heated in a chamber, the albumen solidified, then submitted to the vapor of coal tar, resin, or bituminous oils, which, being at a temperature not less than 325°, readily takes the place of the vapor expelled by a temperature of 212°.

Fluids will pass with the grain of wood with great facility, but will not enter it except to a very limited extent when applied externally.

Absorption of Preserving Solution by different Woods for a Period of 7 Days.

Average Pounds per Cubic Foot.

Black Oak	3.6	Hemlock	2.6	Rock Oak	3.9
Chestnut	3.	Red Oak	3.9	White Oak	3.1

Proportion of Water in various Woods.

Alder (*Betula alnus*)	41.6	Pine (*Pinus Sylvestris L.*)	39.7
Ash (*Fraxinus excelsior*)	28.7	Red Beech (*Fagus sylvatica*)	39.7
Birch (*Betula alba*)	30.8	Red Pine (*Pinus picea dur*)	45.2
Elm (*Ulmus campestris*)	44.5	Sycamore (*Acer pseudo-platanus*)	27.
Horse-chestnut (*Æsculus hippocast.*)	38.2	White Oak (*Quercus alba*)	36.2
Larch (*Pinus larix*)	48.6	White Pine (*Pinus abies dur*)	37.1
Mountain Ash (*Sorbus aucuparia*)	28.3	White Poplar (*Populus alba*)	50.6
Oak (*Quercus robur*)	34.7	Willow (*Salix caprea*)	26.

Comparative Resilience of Timber.

Ash	1.	Chestnut	.73	Larch	.84	Spruce	.64
Beech	.86	Elm	.54	Oak	.63	Teak	.59
Cedar	.66	Fir	.4	Pitch Pine	.57	Yellow Pine	.64

Weight and Strength of Oak and Yellow Pine.

Weight of a Cubic Foot.

Age.	White Oak, Va. Round.	White Oak, Va. Square.	Yellow Pine, Va. Round.	Yellow Pine, Va. Square.	Live Oak.
Green	64.7	67.7	47.8	39.2	78.7
1 Year	53.6	53.5	39.8	34.2	—
2 Years	46.	49.9	34.3	33.5	66.7

In England, Timber sawed into boards is classed as follows:

6½ to 7 ins. in width, *Battens;* 8½ to 10 ins., *Deals;* and 11 to 12 ins., *Planks.*

In a perfectly dry atmosphere the durability of woods is almost unlimited. Rafters of roofs are known to have existed 1000 years, and piles submerged in fresh water have been found perfectly sound 800 years from the period of their being driven.

Distillation.—From a single cord of pitch pine distilled by chemical apparatus, the following substances and in the quantities stated have been obtained:

Charcoal	50 bushels.	Pyroligneous Acid	100 gallons.
Illuminating Gas.... about	1000 cu. feet.	Spirits of Turpentine	20 "
Illuminating Oil and Tar	50 gallons.	Tar	1 barrel.
Pitch or Resin	1½ barrels.	Wood Spirit	5 gallons.

Decrease in Dimensions of Timber by Seasoning.

Woods.	Ins.	Ins.	Woods.	Ins.	Ins.
Cedar, Canada	14	to 13¼	Pitch Pine, South	18⅜	to 18¼
Elm	11	to 10¾	Spruce	8½	to 8⅜
Oak, English	12	to 11⅝	White Pine, American	12	to 11⅞
Pitch Pine, North	10×10	to 9¾×9¾	Yellow Pine, North	18	to 17⅞

The weight of a beam of English oak, when wet, was reduced by seasoning from 972.25 to 630.5 pounds.

HEAT.

Heat, alike to gravity, is a universal force, and is referred to both as cause and effect.

Caloric, is usually treated of as a material substance, though its claims to this distinction are not decided; the strongest argument in favor of this position is that of its power of radiation. Upon touching a body having a higher temperature than our own, caloric passes from it, and excites the feeling of warmth; and when we touch a body having a lower temperature than our own, caloric passes from our body to it, and thus arises the sensation of cold.

To avoid any ambiguity that may arise from the use of the same expression, it is usual and proper to employ the word *Caloric* to signify the principle or cause of the sensation of heat.

Heat is termed *Sensible* when it diffuses itself to all surrounding bodies; hence it is free and uncombined, passing from one substance to another, affecting the senses in its passage, determining the height of the thermometer, etc., etc.

The *Temperature* of a body, is the quantity of sensible heat in it, present at any moment.

Latent Heat, is that which is insensible to the touch of our bodies, and is incapable of being detected by a thermometer.

When a body passes from a solid to a liquid state, or from a liquid to a gaseous, a certain portion of its heat becomes insensible, either by feeling or by a thermometer, and the portion of heat thus combined with the body in its new form is termed *latent*. Hence, when a gas is converted into a liquid, or a liquid into a solid, the same quantity of heat is disengaged, as was held in a latent state by the body before its change of condition.

Specific Heat, is that which is absorbed by different bodies of equal weights or volumes when their temperature is equal, based upon the law *that similar quantities of different bodies require unequal quantities of heat at any given temperature.* It is also the *quantity of heat* requisite to change the temperature of a body any stated number of degrees compared with that which would produce the same effect upon water at 60°.

The *quantity of heat*, therefore, is the quantity necessary to change the temperature of a body by any given amount (as 1°), divided by the quantity of heat necessary to change an equal weight or volume of water 60° by the same amount.

NOTE.—Water has greater specific heat than any known body.

Mechanical power may be expended in the production of heat either by friction or compression, and the quantity of heat produced bears the same proportion to the quantity of mechanical power expended, being 1 unit for the power necessary to raise 1 lb. 772 feet in height. This number of 772 is termed the *mechanical equivalent of heat* (Joules).

Capacity for Heat, is the relative power of a body in receiving and retaining heat, in being raised to any given temperature; while *Specific* applies to the actual quantity of heat so received and retained.

Radiation of Heat, is the diffusion of heat by the projection of it in diverging right lines into space, from a body having a higher temperature than the space surrounding it, or the body or bodies enveloping it.

Reflection of Heat, is the passage of heat from the surface of one substance to another or into space, and it is the converse of radiation.

Heat is reflected from the surface upon which its rays fall in the same manner as light, the angle of reflection being opposite and equal to that of incidence. The metals are the strongest reflectors.

Communication of Heat, is the passage of heat through different bodies with different degrees of velocity. This has led to the division of bodies into *Conductors* and *Non-conductors* of caloric; the former includes such as metals, which allow caloric to pass freely through their substance, and the latter comprise those that do not give an easy passage to it, such as stones, glass, wood, charcoal, etc.

The velocity of cooling, other things being equal, increases with the extent of surface compared with the volume of substance; and of two bodies of the same material, temperature, and form, but differing in volume.

Transmission of Heat, is the passage of heat through different bodies with different degrees of intensity. Gaseous bodies and a vacuum are the highest in the order of transmittents.

Evaporation or *Vaporization*, is the conversion of a fluid into vapor. Evaporation produces cold, because heat is absorbed to form vapor.

Distillation, is the depriving of vapor of its latent heat.

Heat is developed by water when it is violently agitated.

Heat is developed by the percussion of a metal, and it is greatest at the first blow.

The quantities of heat evolved are nearly the same for the same substance, without reference to the temperature of its combustion.

LATENT HEAT.—A pound of water, in passing from a liquid at 212° to steam at 212°, receives as much heat as would be sufficient to raise it through 966.6 thermometric degrees, if that heat, instead of becoming *latent*, had been *sensible*.

If 5½ lbs. of water, at the temperature of 32°, be placed in a vessel, communicating with another one (in which water is kept constantly boiling at the temperature of 212°), until the former reaches the temperature of the latter quantity, then let it be weighed, and it will be found to weigh 6½ lbs., showing that 1 lb. of water has been received in the form of steam through the communication, and reconverted into water by the lower temperature in the vessel. Now this pound of water, received in the form of steam, had, when in that form, a temperature of 212°. It is now converted into the liquid form, and still retains the same temperature of 212°; but it has caused 5½ lbs. of water to rise from the temperature of 32° to 212°, and this without losing any temperature of itself. Now this heat was combined with the steam, but as it is not sensible to a thermometer, it is termed *Latent*.

The quantity of heat necessary to enable ice to resume the fluid state is equal to that which would raise the temperature of the same weight of water 140°; and an equal quantity of heat is set free from water when it assumes the solid form.

Sensible and Latent Heat of Steam.—(Regnault).

Temp.	Latent Heat.	Sum of Sensible and Latent.	Temp.	Latent Heat.	Sum of Sensible and Latent.	Temp.	Latent Heat.	Sum of Sensible and Latent.
Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.
32	1092.6	1124.6	212	966.6	1178.6	302	901.8	1203.8
104	1042.2	1146.2	230	952.2	1182.2	338	874.8	1212.5
140	1017	1157	248	936.6	1187.6	374	849.6	1223.6
176	991.8	1167.8	266	927.	1193.	410	822.6	1232.6

If to a pound of newly-fallen snow were added a pound of water at 172°, the snow would be melted, and 32° will be the resulting temperature.

When a body is *fusing*, no rise in its temperature occurs, however great the additional quantity of heat may be imparted to it, as the increased heat is absorbed in the operation of fusion. The quantity of heat thus made latent varies in different bodies.

Latent Heat of various Substances for a Unit of Weight.

Alcohol	364°	Ether	163°	Phosphorus	9°	Tin	500°
Ammonia	860°	Ice	142.6°	Spermaceti	148°	Water	966.6°
Beeswax	175°	Lead	162°	Steam	966.6°	Zinc	493°
Bismuth	22°	Mercury	157°	Sulphur	17°		

Specific Heat.—Every substance has a specific heat peculiar to itself, whence a change of composition will be attended by a change of its capacity for heat.

The specific heat of a body varies with its form. A solid has a less capacity for heat than the same substance when in the state of a liquid; the specific heat of water, for instance, being 9 in the solid state, and 10 in the liquid.

The specific heat of equal weights of the same gas increases as the density decreases; the exact rate of increase is not known, but the ratio is less rapid than the diminution in density.

Change of capacity for heat always occasions a change of temperature. Increase in the former is attended by diminution of the latter, and contrariwise.

The specific heat multiplied by the atomic weight of a substance will give the constant 37.5 as an average, which shows that the atoms of all substances have equal capacity for heat. This is a result for which as yet no reason has been assigned.

Thus: The atomic weights of lead and copper are respectively 1294.5 and 395.7, and their specific heats are .031 and .095. Hence $1294.5 \times .031 = 40.129$, and $395.7 \times .095 = 37.591$.

It is important to know the relative Specific Heat of bodies. The most convenient method of discovering it is by mixing different substances together at different temperatures, and noting the temperature of the mixture; and by experiments it appears that the same quantity of heat imparts twice as high a temperature to mercury as to an equal quantity of water; thus, when water at 100° and mercury at 40° are mixed together, the mixture will be at 80°, the 20° lost by the water causing a rise of 40° in the mercury; and when weights are substituted for measures, the fact is strikingly illustrated; for instance, on mixing a pound of mercury at 40° with a pound of water at 160°, a thermometer placed in it will fall to 155°. Thus it appears that the same quantity of heat imparts twice as high a temperature to mercury as to an equal volume of water, and that the heat which gives 5° to water will raise an equal weight of mercury 115°, being the ratio of 1 to 23. Hence, if equal quantities of heat be added to equal weights of water and mercury, their temperatures will be expressed in relation to each other by the numbers 1 and 23; or, in order to increase the temperature of equal weights of those substances to the same extent, the water will require 23 times as much heat as the mercury.

Specific Heat of various Substances. (Air as Unity.)

	Equal Volumes.	Equal Weights.		Equal Volumes.	Equal Weights.
Air	1.	1.	Oxygen	.976	.885
Hydrogen	.903	12.34	Steam	—	—

Water as Unity.

	Equal Weights.		Equal Weights.		Equal Weights.		Equal Weights.
Water	1.	Cast iron	.13	Lead	.031	Steam*	.475
Air	.267	Carbonic acid	.216	Lime	.217	Steel	.116
Alcohol	.7			Linseed oil	.53	Sulph. acid	.335
Bismuth	.023	Ether	.517	Mercury	.033	Silver	.057
Brick	.2	Gold	.032	Oxygen	.218	Spts. Turp'e	.467
Charcoal	.241	Glass	.198	Olive-oil	.31	Sulphur	.203
Coke and Clay	.202	Hydrogen	3.405	Petroleum	.468	Tin	.056
		Ice	.504	Phosphorus	.189	Woods	.54
Copper	.095	Iron	.115	Platinum	.034	Zinc	.095

* Steam under a constant volume, that is confined, is estimated at 365.

Illustration.—If 1 lb. of coal will heat 1 lb. of water to 100°, $\frac{1}{.033}=\frac{1}{30.3}$ of a lb. will heat 1 lb. of mercury to 100°.

To Compute the Temperature of a Mixture of like Substances.

$\frac{WT+wt}{W+w}=t'$; $\frac{w(t'-t)}{T-t'}=W$; $\frac{w(t'-t)}{W}+t'=T$. *W representing the weight or volume of a substance of the temperature* T, *w the weight or volume of a like substance of the temperature t, and t' the temperature of the mixture* W + *w*.

Illustration.—When 5 cubic feet of water at a temperature of 150° is mixed with 7.5 cubic feet at 50° (t), what is the resultant temperature of the mixture?

$$\frac{5\times150°+7.5\times50°}{5+7.5}=\frac{1125}{12.5}=90°.$$

2. How much water at (T) 100° should be mixed with 30 gallons (w) at 60°, the temperature required being 80°?

$$\frac{30\ (80°-60°)}{100°-80°}=\frac{600}{20}=30 \textit{ gallons.}$$

To Compute the Temperature of a Mixture of Unlike Substances.

$\frac{WST+wst}{WS+ws}=t'$; $\frac{ws(t'-t')}{S(T-t)}=W$; $\frac{t'(WS+ws)-wst}{WS}=T$. W *and w representing the weights, and* S *and s the specific heat of the substances.*

Illustration.—To what temperature should 20 lbs. iron (W) be heated to raise 150 lbs. (w) of water at a temperature (t) of 50° to 60°?

$s=1$, and $S=.114$.

$$\frac{60°\ (20\times.114+150\times1)-150\times1\times80°}{20\times.114}=\frac{1636.8}{2.28}=718°.$$

Capacity for Heat.—When a body has its density increased, its capacity for heat is diminished. The rapid reduction of air to one fifth of its volume evolves heat sufficient to inflame tinder, which requires 550°.

Relative Capacity for Heat of various Bodies. (Water as Unity.)

	Equal Weights.	Equal Volumes.		Equal Weights.	Equal Volumes.		Equal Weights.	Equal Volumes.
Water..	1.	1.	Gold...	.05	.966	Mercury	.036	—
Brass ..	.116	.971	Ice	.9	—	Silver .	.082	.833
Copper.	.114	1.027	Iron ...	.126	.993	Tin ...	.06	—
Glass ..	.187	.448	Lead ..	.043	.487	Zinc...	.102	—

The rule for ascertaining by calculation, combined with experiment, the relative capacities of different bodies, is as follows:

Multiply the weight of each body by the number of degrees of temperature lost or gained by the mixture, and the capacities of the bodies will be inversely as the products.

Or, if the bodies be mingled in unequal quantities, the capacities of the bodies will be reciprocally as the quantities of matter, multiplied into their respective changes of temperature.

If 1 lb. of water at 156° is mixed with 1 lb. of mercury at 40°, the resultant temperature is 152°.

Thus, $1\times156°-152°=4°$, and $1\times40°\backsim152°=112°$. Hence the capacity of water for heat is to the capacity of mercury as 112° to 4°, or as 28 to 1.

Radiation of Caloric.—Radiation is affected by the nature of the surface of the body; thus, black and rough surfaces radiate and absorb more heat than light and polished surfaces. Bodies which radiate heat best absorb it best.

Radiating Power of various Bodies.

Blackened tin	1.	Glass	.9	Lamp-black	1.	Silver	.12
Bright lead	.19	Gold	.12	Lead	.45	Tin	.12
Clean tin	.12	Ice	.85	Mercury	.2	Water	1.
Copper	.12	India ink	.88	Polished iron	.15	Writing-paper	1.

Loss by Radiation.

To Compute the Loss of Heat per Square Foot.

$\frac{1.7\,l\,(T-t)}{d\,v} = R.$ T *representing temperature of pipe, which is assumed to be* $\frac{1}{20}$ *less than that of the steam ; t temperature of the air ; l length of the pipe in feet ; d diameter in inches ; v velocity of the heat in feet per second ; and* R *radiation in degrees per second.*

Reflection of Heat is the converse of *Radiation ;* the one increases as the other diminishes.

Reflecting Power of various Substances.

Brass	1.	Glass	.1	Silver	.9	Tin	.8
Glass, waxed or oiled	.05	Lamp-black	.0	Steel	.7	Tinfoil	.85
		Lead	.6			" with Mer'y	.14

Conduction or Convection of Heat.—Air and gases are very imperfect conductors. Heat appears to be transmitted through them almost entirely by conveyance, the heated portions of air becoming lighter, and diffusing the heat through the mass in their ascent. Hence, in heating a room with air, the hot air should be introduced at the lowest part. The advantage of double windows for the retention of heat depends, in a great measure, upon the sheet of air confined between them, through which heat is very slowly transmitted.

The *Convection* of heat refers to the transfer and diffusion of heat in a fluid mass, by means of the motion of the particles of the mass.

Relative Conducting Power of various Bodies.

Bismuth	.061	Fire-clay	.011	Marble	.024	Silver	1.
Cast iron	.359	Gold	.981	Mercury	.677	Steel	.397
Copper, rolled	.845	Iron, wrought	.436	Platinum	.38	Tin	.303
Fire-brick	.011	Lead	.18	Porcelain	.012	Zinc	.641

Woods (with Water as Unity).

Water	1.	Apple	.28	Elm	.32	Oak	.33
Ash	.31	Ebony	.22	Lime	.39	Pine	.39

Of Fluids.

Alcohol	.0232	Mercury	1.	Water	.0357

Various Substances compared with each other.

Air	.576	Charcoal	.937	Hare's fur	1.315	Silk, raw	1.284
Ashes (wood)	.927	Cotton	1.046	Lamp-black	1.117	Silk, sewing	.917
Beaver's fur	1.296	Eider-down	1.305	Lint	1.032	Wool	1.118

Various Substances compared with Slate.—(J. Hutchinson.)

Substance.	Conducting Power.	Cooling Power.	Substance.	Conducting Power.	Cooling Power.
Slate	1.	1.	Hair and lime	1.09	.38
Lath and plaster	.26	.75	Pine wood	.28	.69
Asphaltum	.45	1.06	Plaster	.19	.63
Brick	.6	.97	Stone	.75	.95

Relative Power of various Substances to Transmit Heat.

All bodies capable of transmitting heat are more or less transparent, though their powers of transmitting heat and light are not in the same relative proportions.

Air	1.	Flint-glass	.67	Rock-crystal	.62	Sulphuric ether	.21
Alcohol	.15	Gypsum	.2	Rape-seed oil	.3	Turpentine	.31
Crown-glass	.49	Nitric acid	.15	Sulphuric acid	.17	Water	.11

Weight of Steam Condensed by various Substances per Square Foot per Hour.

In Air.	Temperature.	Steam in Lbs.	In Air.	Temperature.	Steam in Lbs.
Cast iron	59°	.36	Tin-plate	59°	.21
Copper	59°	.28			
Glass	59°	.35	In Water.		
Plate iron	59°	.36	Copper	72°	21.5

Practical Deductions from above and preceding Results.

Asphaltum is the best composition for resisting moisture, and, being a slow conductor of heat, it is best adapted where economy of heat and dryness are required.

Slate is a very dry material, but, from its quick conducting power, it is not adapted for the retention of heat.

Cements. Plaster of Paris and Woods are well adapted for the lining of rooms, having low conductive powers, while *Hair and lime*, being a quick conductor, is one of the coldest compositions.

Fire-brick absorbs much heat, and is therefore well adapted for the lining of fire-places, furnaces, etc.; while contrariwise, *Iron*, being a high conductor of heat, is one of the worst of substances for this purpose.

Common brick is not a very slow conductor of heat; it is 1.8 times higher in the scale than oak wood, and about .027 lower than fire-brick.

Relative Transmitting Powers of Various Substances.

Air 1. Glass .67. Alcohol .15. Water .11. Ice .06.

The heat which passes through one plate of glass is less subject to absorption in passing through a second and a third plate. Of 1000 rays, 451 were intercepted by 4 plates as follows:

1st. 381. 2d. 43. 3d. 18. 4th. 9.

EVAPORATION proceeds only from the surface of fluids, and therefore, *other things equal*, must depend upon the extent of surface exposed.

When a liquid is covered by a stratum of dry air, evaporation is rapid, even when the temperature is low.

As a large quantity of heat passes from a sensible to a latent state during the formation of vapor, it follows that cold is generated by evaporation.

Fluids evaporate in a vacuum at from 120° to 125° below their boiling point.

DISTILLATION is the depriving vapor of its latent heat, and, though it may be effected in a vacuum with very little heat, no advantage in regard to a saving of fuel is gained, as the latent heat of vapor is increased proportionately to the diminution of sensible heat.

A temperature of 70° is sufficient for the distillation of water in a vessel exhausted of air.

CONGELATION AND LIQUEFACTION.—Freezing water gives out 140° of heat. All solids absorb heat when becoming fluid.

The particular quantity of heat which renders a substance fluid is termed its caloric of fluidity, or latent heat.

Fluids boil in a vacuum with less of heat than when under the pressure of the atmosphere. On Mont Blanc water boils at 187°; and in a vacuum water boils at 98° to 100°, according as it is more or less perfect.

Water may be reduced to 5° if confined in tubes of from .003 to .005 inch in diameter: this is in consequence of the adhesion of the water to the surface of the tube, interfering with a change in its state. It may also be reduced in its temperature below 32° if it is kept perfectly quiescent.

Effect upon Various Bodies by Heat.

Wedgewood's zero is 1077° of Fahrenheit, and each degree = 130°.

In the designation of degrees of temperature, the symbol + is omitted when the temperature is above 0; but when it is below it, the symbol — must be prefixed.

	Degrees.
Acetification ends	88
Acetous fermentation begins	78
Air Furnace	3300
Ambergris melts	145
Ammonia boils	140
Ammonia (liquid) freezes	—46
Antimony melts	951
Arsenic melts	365
Beeswax melts	151
Bismuth melts	476
Blood (human), heat of	98
" freezes	25
Brandy freezes	—7
Brass melts	1900
Cadmium melts	600
Charcoal burns	800
Coal Tar boils	325
Cold, greatest artificial	—166
" greatest natural	—56
Common fire	790
Copper melts	2548
Glass melts	2377
Gold, fine, melts	2590
Gutta-percha softens	145
Heat, cherry red	1500
" " (Daniell)	1141
" bright red	1860
" red, visible by day	1077
" white	2900
Highest natural temperature, Egypt	117
Ice melts	32
India-rubber and Gutta-percha vulcanize	293
Iron (cast) melts	3479
" (wrought) melts	3980
" bright red in the dark	752
" red hot in twilight	884
Lard melts	95
Lead melts	594
Mercury boils	662
" volatilizes	680
" melts	—39
Milk freezes	30
Naphtha boils	186
Nitric Acid (sp. grav. 1.424) freezes	—45
Nitrous Oxide freezes	—150
Olive-oil freezes	36
Petroleum boils	306
Phosphorus melts	108
" boils	560
Pitch melts	91
Platinum melts	3080
Potassium melts	135
Proof Spirit freezes	—7
Saltpetre melts	600
Sea-water freezes	28
Silver, fine, melts	1250
Snow and Salt, equal parts	0
Spermaceti melts	112
Spirits Turpentine freezes	14
Steel melts	2500
" polished, blue	580
" " straw color	460
Strong Wines freeze	20
Sulphur melts	226
Sulph. Acid (sp. grav. 1.641) freezes	—45
Sulphuric Ether freezes	—46
" " boils	98
Tallow melts	97
Tin melts	421
Vinegar freezes	28
Vinous fermentation	60 to 77
Water in *vacuo* boils	98
Zinc melts	740

Boiling Points of Various Fluids.

	Degrees.
Ether	96 to 104
Alcohol, sp. grav. 813	173.5
Nitric Acid, " 1.5	210
" " 1.42	248
Sea Salt	224.3
Common Salt	226
Sulphuric Acid, sp. grav. 1.848	600
" " 1.3	240
Rectified Petroleum	316
Oil of Turpentine	304
Phosphorus	554
Sulphur	570
Linseed Oil	640
Sweet Oil	412
Sea-water	213.2
Water, distilled	212

Volume of several Liquids at their Boiling Point.

	Steam.
1 Water	1700
1 Alcohol	528
1 Ether	298
1 Turpentine	193

For boiling point of salt water, see Water, p. 541. Water may be heated in a Digester to 400° without boiling.

Boiling Points corresponding to Altitudes of the Barometer between 26 and 31 Inches.

Barom.	Boiling Point.	Barom.	Boiling Point	Barom.	Boiling Point.	Barom.	Boiling Point.
26.	204.91°	27.5	207.55°	29.	210.19°	30.5	212.88°
26.5	205.79°	28.	208.43°	29.5	211.07°	31.	213.76°
27.	206.67°	28.5	209.31°	30.	212.°		

Melting Point of Alloys.

Lead 2, Tin 3, Bismuth 5......... 212°	Tin 8, Bismuth 1 .... 392°
" 1, " 3, " 5......... 210°	Lead 2, Tin 1 (solder) .... 475°
" 1, " 4, " 5......... 240°	" 1, " 2 (soft solder) .... 360°
" 1, " 1......... 286°	Zinc 1, " 1.................... 399°
" 2, " 1......... 336°	Lead 1, " 1.................... 368°
Lead 2, " 3.................... 334°	" 1, " 1, Bism'h 4, Cadm'm 1 155°

For Alloys and Fusible Compounds, see pp. 627 628.

Frigorific Mixtures.

Mixtures.		Degree of Cold produced.	Temperature falls
Nitrate of Ammonia...... / Water....................	1 part / 1 "	46°	From 50° to 4°
Phosphate of Soda......... / Nitrate of Ammonia...... / Dilute Nitric Acid........	9 parts / 6 " / 4 "	71°	From 50° to −21°
Sulphate of Soda .... / Muriatic Acid............	8 parts / 5 "	50°	From 50″ to 0°
Snow.................... / Muriate of Lime..........	2 parts / 3 "	53°	From −15° to −68°
Snow.................... / Dilute Sulphuric Acid....	8 parts / 10 "	22°	From −68° to −90°
Snow.................... / Potash fused	3 parts / 4 "	83°	From 32° to −51°
Sulphate of Soda .... / Diluted Nitrous Acid	3 parts / 2 "	53°	From 50° to −3°
Phosphate of Soda........ / Diluted Nitrous Acid.....	9 parts / 4 "	62°	From 50° to −12°

To Compute the Expansion of a Substance in Parts of its Length.

Divide 1. by the decimal given in the following Table (omitting the unit of 1), and the quotient will give the proportion.

To Compute the Expansion for any Number of Degrees under 180°.

Divide the decimals given in the following Table by the number of degrees, and the quotient will give the expansion for the degrees required.

EXPANSION OR DILATATION OF SOLIDS.—(*Faraday.*)

Lineal.

At 212°, the length of the bar at 32° = 1.

Bismuth....	1.0013908	Gold.......	1.001495	Silver	1.00201
Brass	1.0019062	Granite	1.0007894	Slate	1.0011436
Cast iron ...	1.0011112	Lead	1.0028426	Stock brick .	1.0005502
Cement	1.001435	Marble	1.0011041	Steel	1.0011899
Copper	1.001745	Pavements.	1.0008985	Tin	1.002
Fire-brick ..	1.0004928	Platinum ..	1.0009542	Wroug't iron	1.0012575
Glass	1.0008545	Sandstone..	1.001743	Zinc	1.002942

To Compute the Volume of a Gas, at any Temperature, its Volume at 32° being known, and the Pressure being constant.

RULE.—Divide the difference between the number of degrees in temperature and 32° by 490. Add the quotient to 1 if it is above 32°, and subtract 1 if it is below 32°. Multiply the volume of the gas at 32° by the resulting number, and the product will give the number required.

EXAMPLE.—What volume will 1000 cubic feet of air at 32° acquire by being heated to 1000°?

1000° − 32° = 968°, which ÷ 490 = 1.9755, to which add 1 = 2.9755.

Then, 1000 × 2.9755 = 2975.5 *cubic feet.*

Expansion of Air.—(DALTON.)

Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.
32°	1.	40°	1.021	60°	1.066	80°	1.110	100°	1.152	392°	1.739
33°	1.002	45°	1.032	65°	1.077	85°	1.121	200°	1.354	482°	1.912
34°	1.004	50°	1.043	70°	1.089	90°	1.132	212°	1.376	680°	2.028
35°	1.007	55°	1.055	75°	1.099	95°	1.142	302°	1.558	572°	2.312

Expansion of Water.—(DALTON.)

Temperature.	Expansion.	Temperature.	Expansion.	Temperature.	Expansion.
12°	1.00236	82°	1.00312	152°	1.01934
22	1.00090	92	1.00477	162	1.02245
32	1.00022	102	1.00672	172	1.02575
*40	1.	112	1.00880	182	1.02916
52	1.00021	122	1.01116	192	1.03265
62	1.00083	132	1.01367	202	1.03634
72	1.00180	142	1.01638	212	1.04012

Hence, at 72°, water expands $\frac{1.}{.0018} = 555.55$th part of its original bulk.

Expansion of Fluids.—(URE.)

At 212°, the volume at 32° = 1.

Air	1.376	Ether	1.07	Mercury	1.02
Alcohol	1.11	Oil	1.08	Water	1.04012

The ratio of Expansion for Solids and Liquids increases with the temperature; that of the Gases is uniform for all temperatures.

To Compute the Temperature to which a Substance of a given Length or Dimension must be Submitted or Reduced, to give it a Greater or Less Length or Volume by Expansion or Contraction.

LINEAL.—*When the Length is to be increased.*

$\frac{L-l}{C\,l} + t = T$. L *representing the whole length of the substance when increased, and* l *the primitive length of it in like denominations.* T *and* t *the temperatures of* L *and* l, *and* C *the expansion of the substance for each degree of heat.*

ILLUSTRATION.—A copper rod at 32° is 100 feet in length; to what temperature must it be subjected to increase its length 1.1633 ins.?

The expansion for a unit of length of copper for 180° is .001745. Hence .001745 ÷ 180 = .000009694 for each degree.

$$\frac{100 \times 12 + 1.1633 - \overline{100 \times 12}}{.000009694 \times \overline{100 \times 12}} + 32 = \frac{1.1633}{.011633} + 32 = 132°.$$

* Water is held to be at its greatest density when at 39.83.°

When the Length is to be reduced. $\frac{L-l}{C\,l} - T = t.$

$$l\left(1+\overline{C\,(T-t)}\right) = L;\ \text{and}\ \frac{L}{1+(C\,(T-t))} = l.$$

ILLUSTRATION.—Take the elements of the preceding case. Then, to ascertain L, $1200 \times (1 \times \overline{.000009694 \times (132-32)}) = 1200 \times \overline{1+.0009694} = 1200 \times 1.0009694 =$ 1201.1633 *ins.*

To Compute the Expansion of Fluids in Volume.

RULE.—Proceed by the preceding formulæ for computing the length of a substance. Substitute V and v for the volume, instead of L and l, the length.

ILLUSTRATION.—A closed vessel contains 6 cubic feet of water at a temperature of 40°; to what height will a column of it rise in a pipe 1.152 ins. in area, when it is exposed to a temperature of 130°?

1.152 ins. = .008 square feet. C for water = .00023325.

$6\left(1+\overline{.00023325\,(130-40)}\right) = 6.12595$, and $6 - \frac{6.12595-6}{.008} = 15.744$ *lineal feet.*

TEMPERATURE BY AGITATION.

Results of Experiments with Water inclosed in a Vessel and violently Agitated.—

Temperature of Air, 60.5°; of Water, 59.5°.

Duration of Agitation.	Increase of Temperature.	Duration of Agitation.	Increase of Temperature.	Duration of Agitation	Increase of Temperature.
Hours.		Hours.		Hours.	
.5	10. °	2	19.5°	5	39.5°
1.	14.5°	3	29.5°	6	42.5°

Mean Temperatures of various Localities.

London	51°	Rome	60°	Poles	−13°	Polar Regions	36°
Edinburgh	41°	Equator	82°	Torrid Zone	75°	Globe	50°

Line of Perpetual Congelation, or Snow Line.

Latitude.	Height.	Latitude.	Height.	Latitude.	Height.	Latitude.	Height.
	Feet.		Feet.		Feet.		Feet.
10°	14764	30°	11484	50°	6384	70°	1278
20°	13478	40°	9000	60°	3818	80°	451

At the Equator it is 15260 feet; at the Alps, 8120 feet; and in Iceland, 3084 feet. At the Polar Regions ice is constant at the surface of the earth.

To Reduce the Degrees of a Fahrenheit Thermometer to those of Reaumur and the Centigrade, and contrariwise.

Fahrenheit to Reaumur.—*If above the freezing point.*—Subtract 32 from the number of degrees; multiply the remainder by 4, and divide the product by 9.

Thus, $212° - 32° = 180°$, and $180° \times 4 \div 9 = 80°$.

If below the freezing point.—Add 32 to the number of degrees; multiply the remainder by 4, and divide the product by 9.

Thus, $-40° + 32° = 72°$, and $72° \times 4 \div 9 = -32°$.

Reaumur to Fahrenheit.—Multiply the number of degrees by 9, and divide the product by 4. Then, when they are above the freezing point, add 32 to the quotient, and when they are below, subtract 32.

Thus, $80° \times 9 \div 4 = 180$, and $180 + 32 = 212°$.

" $-32° \times 9 \div 4 = 72$, and $72 - 32 = 40°$.

Fahrenheit to Centigrade.—If above the freezing point.—Subtract 32 from the number of degrees; multiply the remainder by 5, and divide the product by 9.

Thus, $212^\circ - 32^\circ \times 5 \div 9 = 180 \times 5 \div 9 = 100^\circ$.

If below the freezing point.—Add 32 to the number of degrees; multiply the remainder by 5, and divide the product by 9.

Thus, $-40^\circ + 32^\circ \times 5 \div 9 = 72 \times 5 \div 9 = -40^\circ$.

Centigrade to Fahrenheit.—Multiply the number of degrees by 9, and divide the product by 5. Then, when they are above the freezing point, add 32 to the quotient, and when they are below, subtract 32.

Thus, $100^\circ \times 9 \div 5 = 180$, and $180 + 32 = 212^\circ$.
" $-10^\circ \times 9 \div 5 = 18$, and $18 \backsim 32 = 14^\circ$.

Reaumur to Centigrade.—Multiply by .25, and add the product; or divide by 4, and add that product.

Thus, $80^\circ \times .25 = 20$, and $20 + 80 = 100^\circ$.
Or, $80^\circ \div 4 = 20$, and $20 + 80 = 100^\circ$.

Centigrade to Reaumur.—Divide by 5, and subtract the product.

Thus, $100^\circ \div 5 = 20$, and $20 \backsim 100 = 80^\circ$.

Corresponding Degrees upon the Three Scales.

Fah.	Cent.	Reaum.	Fah	Cent.	Reaum.	Fah	Cent.	Reaum.
212	100	80	32	0	0	−40	−40	−32

Temperature of the Earth.—The ratio of increase in its temperature is directly as the depth from the surface, being about 1° for every 65 feet.

WARMING BUILDINGS AND APARTMENTS.

By Low Pressure Steam (1½ to 2 lbs.) *or Hot Water.*

One square foot of plate or pipe surface will heat from 40 to 100 cubic feet of inclosed space to 75° in a latitude where the temperature ranges from −10°, or 10° below zero.

The range from 40 to 100 is to meet the conditions of exposed or corner buildings, of buildings less exposed, as the intermediate ones of a block, and of rooms intermediate between the front and rear.

As a general rule, 1 square foot will heat 75 cubic feet of air in outer or front rooms and 100 in inner rooms.

By High Pressure Steam.

When steam at a pressure exceeding 2 lbs. per square inch is used, the space heated by it will be in proportion to its increase of temperature above that pressure, less the increased radiation of heat in its course to the place of application.

One cubic foot of water evaporated is required for every 2000 cubic feet of inclosed space.

By Hot Water of Low or High Temperatures.

$\frac{(P - t)\,(T - t)}{(P - T)} \times .005\,V =$ *square feet of surface of plate or pipe.* P *representing temperature of plate or pipe,* T *and* t *the required temperature and that of the external air, and* V *the volume or cubic feet of inclosed space.*

Ventilation.

Each person requires from 3 to 4 cubic feet of air per minute. Windows, as ordinarily constructed, will admit about 8 cubic feet per minute.

LIGHT.

LIGHT is similar to Heat in many of its qualities, being emitted in the form of rays, and subject to the same laws of reflection.

It is of two kinds, *Natural* and *Artificial;* the one proceeding from the Sun and Stars, the other from heated bodies.

Solids shine in the dark only at a temperature from 600° to 700°, and in daylight at 1000°.

The *Intensity of Light* is inversely as the square of the distance from the luminous body.

The *Velocity of the Light of the Sun* is 192 500 miles per second.

The Standard of Intensity or of comparison of light between different methods of Illumination is a Sperm Candle "short 6," burning 120 grains per hour.

Loss of Light by Use of Shades.—(F. H. Storer.)

Glass, etc.	Thickness.	Loss.	Glass, etc.	Thickness.	Loss.
	Ins.	Per Cent.		Ins.	Per Cent.
American enameled	1/16	51.23	Window, double, Eng. ..	1/8	9.39
Crown	1/8	13.08	" " German.	1/8	13.
Crystal plate	1/8	8.61	" single, " ..	1/16	4.27
English "	1/3	6.15	" " ground..	1/16	65.75
Porcelain transparency ..	1/16	97.68	" green	1/16	81.95

ILLUMINATION.—GAS, LAMPS, AND CANDLES.

Comparison of several Varieties of Lamps, Fluids, and Candles with Coal Gas, deduced from Reports of Com. of Franklin Institute, and of A. Frye, M.D., etc., etc.*

Lamp and Fluid.	Intensity of Light.	Ratio of Cost per Hour.	Light at Equal Cost.	Time of Burning 1 Pint of Oil.	Relative Costs for Equal Lights.
				Hours.	$ Cts.
Camphene	1.75	.57	3.08	9.31	.32
Carcel, Sperm oil, *maximum*	2.15	1.22	1.8	6.32	.56
" " *mean*	1.22	.86	1.35	9.87	.74
" " *minimum*	.69	.67	1.2	14.6	.83
" Lard oil	.77	.76	.97	11.3	1.03
Gas	1.	1.	1.	—	1.
Semi-solar, Sperm oil	1.15	1.25	.93	6.75	1.07
Solar "	1.76	1.09	1.55	8.42	64

Candle.	Burns.	Intensity of Light.†	Light at Equal Costs.	Cost with Equal Light.	Cost compared with Gas for Equal Light.
	Hours.				
Diaphane	6.6	.7	.5	2.08	15.1
Palm oil	6.6	.7	.77	1.32	10.5
Spermaceti, short 6's	8.	.8	.54	2.16	16.2
Tallow, short 6's, single wick	6.	.58	.85	1.	7.5
" " double "	5.5	1.	1.	1.46	7.1
Wax, short 6's	9.	.8	.61	1.96	14.4
" long 4's	13.	—	—	—	—

* City of Philadelphia.

† Compared with a fish-tail jet of Edinburgh gas, containing 12 per cent. of condensable matter and consuming 1 cubic foot per hour.

Dimensions, Consumption, and Comparative Intensity of Light of Candles.

Candle.	No. in a Pound.	Diameter.	Length.	Consumption per Hour.	Light compared with Carcel.
		Inch.	Inches.	Grains.	
Wax	3	1.	12	135	.09
"	3	⅞	15		
"	6	.8	9		
Spermaceti	3	.9	15	156	.09
"	4	.8	13½		
"	6	.84	8½		
Tallow	3	1.	12½	204	.06 to .08
"	3	.9	15		
"	4	.8	13¾		

The illuminating power of coal gas varies from 4.4 to 1.6 times that of a tallow candle 6 to a pound; the consumption being from 2.3 to 1.5 cubic feet per hour, and the specific gravity from .58 to .42.

The higher the flame from a burner the greater the intensity of the light, the most effective height being 5 inches.

English Cannel coal produces the greatest quantity and the best quality of coal gas. Scotch Parrot coal is next in order.

Water absorbs its own volume of carbonic acid gas.

The greater the proportion of hydrogen, and the less oxygen and sulphur, the better the coal is adapted for generating gas.

Pine-wood gas will give when burning 4.6 cubic feet per hour—a light equal to 18.3 sperm candles per hour; and Oak-wood gas, under like conditions, will give a light equal to 19.17 candles.

Philadelphia City gas is equal to 17.5 sperm candles. 2472 lbs. pine wood produced 12350 cubic feet of gas, 46.8 bushels charcoal, and 4.5 gallons coal tar.

A mean of Coal and Mineral Oils gave an expenditure of 1.6 galls. oil for 1000 cubic feet of gas at .6 the intensity of the light, and 2.6 galls. gave an equal light of 1000 cubic feet of gas per hour. 1.8 galls. Burning Fluid gave .15 the intensity of a gas-light for 1000 cubic feet of gas, and for equal light there was required 11.7 galls. for 1000 cubic feet of gas.

In the combustion of oil in an ordinary lamp, a straight or horizontally cut wick gives great economy over an irregular cut wick.

Relative Intensity of Light from different Candles.

Candles.	Gas = 1.	Burned during Flow of 1000 cub. feet of Gas.	Giving equal Light to 1000 cub. feet of Gas.	Amount burned for equal Light.
		Lbs.	Lbs.	
Paraffine	.098	3.5	35.5	103
Sperm	.095	3.9	41.1	120
Adamantine	.108	5.1	47.2	137
Tallow	.074	5.1	53.8	155

Intensity of Light with Equal Volumes of Gas from different Burners.

Burners.	Expenditure in Cubic Feet per Hour. 1.	2.	3.	4.	At most Effective Height of Flame.
Single jet, 1 foot = candles*	2.6	—	—	—	100.
Fish-tail No. 3, 1 foot = candles.	3.5	4.	4.2	—	138.
Bats'-wing ..... 1 " = "	3.	4.1	4.3	4.5	135.
Argand, 16 holes, 1 " = "	.32	1.9	3.3	3.8	—
Argand, 24 holes, 1 " = "	—	—	—	—	183.5
Argand, 28 holes, 1 " = "	.34	2.3	3.5	5.8	183.
Argand, 42 holes, 1 " = "	—	—	—	—	182.3

* Spermaceti candle burning 120 grains per hour.

Volume of Gas in Cubic Feet required to Produce the Light of One Spermaceti Candle.

Burners.	Expenditure in Cubic Feet per Hour.				
	.4.	.6.	.9.	1.5.	2.6.
Argand, 26 holes	–	–	–	.8	.3
Fish-tail No. 1*............	.37	.28	.2	–	–
" No. 2†	.37	.28	.2	.17	.16

Relative Intensity, Consumption, and Cost of various Modes of Illumination.

Oil at 11 cents per lb. Tallow at 14 cents per lb. Wax at 52 cents per lb. Stearine at 32 cents per lb. 100 cubic feet coal gas at 14 cents. 100 cubic feet of oil gas at 52 cents.

Illuminator.	Intensity.	Consumption of Material per Hour.	Illumination. Carcel Lamp =100.	Actual Cost per Hour.	Cost per Hour for equal Intensity.
				Cents.	
Carcel Lamp........	100.	42.	100.	.87	.87
Lamp with inverted reservoir..........	90.	43.	57.8	.89	.99
Astral Lamp	31.	26.7	48.7	.56	1.78
Petticoat Lamp	6.65	8.	33.6	.16	2.49
Wax Candle 6 to lb.	14.6	9.6	61.6	.92	6.31
Stearine " 5 "	14.4	9.3	66.6	.59	4.13
Tallow " 6 "	10.7	8.5	54.	.25	2.34
Sperm " 6 "	16.	8.8	67.5	.89	5.7
		Cubic feet.			
Coal Gas............	127.	8.7	–	1.16	.91
Oil Gas.............	127.	2.4	–	1.26	.73

1000 cubic feet of 13-candle coal gas is equal to 7.5 gallons sperm oil, 52.9 lbs. mold candles, and 44.6 lbs. sperm candles.

GAS.

A retort produces about 600 cubic feet of gas in 5 hours with a charge of about 1½ cwt. of coal, or 2800 cubic feet in 24 hours.

In estimating the number of retorts required, ¼th should be added for being under repairs, &c.

Purifiers.—Wet purifiers require 1 bushel of lime mixed with 48 bushels of water for 10000 cubic feet of gas.

Dry purifiers require 1 bushel of lime to 10000 cubic feet of gas, and 1 superficial foot for every 400 cubic feet of gas.

A cubic foot of good gas, from a jet $\frac{1}{33}$ of an inch in diameter and height of flame of 4 inches, will burn for 65 minutes.

Internal lights require 4 cubic feet, and external lights about 5 cubic feet per hour. When large or Argand burners are used, from 6 to 10 cubic feet will be required.

The pressure with which gas is forced through pipes should seldom exceed 2½ inches of water at the Works, or the leakage will exceed the advantages to be obtained from increased pressure.

When pipes are laid at an inclination either above or below the horizon, a correction will have to be made in estimating the supply, by adding or deducting $\frac{1}{100}$ of an inch from the initial pressure for every foot of rise or fall in the length of the pipe.

* Fully spread at .85 inch pressure at 1.4 cubic feet per hour.
† " .9 " 2.4 " "

In Winter the average of duration of internal lights per day is 5.08 hours; in Summer it is 2.83; in Spring it is 3.41; and in the Fall, 4.16.

Street-lamps in the city of New York consume 3 cubic feet of gas per hour. In some cities 4 and 5 cubic feet are consumed. Fish-tail burners for ordinary coal gas consume from 4 to 5 cubic feet of gas per hour.

The standard of gas burning is a 15-hole Argand lamp, internal diameter .44 inch, chimney 7 inches in height, and consumption 5 cubic feet per hour, giving a light from ordinary coal gas of from 10 to 12 candles, with Cannel coal from 20 to 24 candles, and with the rich coals of Virginia and Pennsylvania of from 14 to 16 candles.

In Philadelphia, with a fish-tail burner, consuming 4.26 cubic feet per hour, the illuminating power was equal to 17.9 candles, and with an Argand burner, consuming 5.28 cubic feet per hour, the illuminating power was 20.4 candles.

Gas, which at the level of the sea would have a *Value* of 100, would have but 60 in the city of Mexico.

Loss of Light by Glass Globes.

Clear glass, 12 per cent. | Half ground, 35 per cent. | Full ground, 40 per cent.

Resin Gas.—Jet $\frac{1}{33}$, flame 5 inches, 1¼ cubic feet per hour.

1 Chaldron Newcastle coal, 3136 lbs., will furnish 8600 cubic feet of gas at a specific gravity of .4, 1454 lbs. coke, 14.1 gallons tar, and 15 gallons ammoniacal liquor.

Volumes of Gas obtained from a Ton of Coal, Resin, etc.

	Cubic Feet.	Specific Gravity.		Cubic Feet.	Specific Gravity.
Boghead Cannel....	13 334	.42	Oil and Grease......	23 000	.67
Wigan Cannel......	15 426	.73	Pictou and Sidney...	8 000	–
Cannel............	8 960	.42	Pine wood..........	11 800	.66
	15 000	.58	Pittsburg...........	9 520	–
Cape Breton, "Cow Bay," etc.........	9 500	–	Resin...............	15 600	.66
Cumberland.........	–	–	Scotch............	10 300	.55
				15 000	.64
English, mean.......	11 000	.24	Virginia............	8 960	–
Newcastle..........	9 500	.4	" Western....	9 500	–
	10 000	.5	Walls-end..........	12 000	.42

Australian Coal is superior to Welsh in the furnishing of gas.

1 lb. Peat will supply gas for 1 hour's light. 1 ton Wigan Cannel has produced coke, 1326 lbs.; gas, 338 lbs.; tar, 250 lbs.; loss, 326 lbs.

To Compute the Volumes of Gas discharged through Pipes.—(Clegg.)

$1350\, d^2\sqrt{\frac{h\,d}{l\,g}} = V.$ *d representing diameter of the pipe and h the height of the water in inches, denoting the pressure upon the gas, l length of pipe in yards, g specific gravity of the gas, and V the volume in cubic feet per hour.*

g may be assumed for ordinary computation at .42.

Volumes of Gas discharged per Hour under a Pressure of Half an Inch of Water, Specific Gravity of Gas .42.

Diameter of Opening.	Volume.	Diameter of Opening.	Volume.	Diameter of Opening.	Volume.	Diameter of Opening.	Volume.
Ins.	Cubic Feet.	Ins.	Cubic Feet.	Ins.	Cubic Feet.	Ins.	Cubic Feet.
.¼	80	.¾	723	1.⅛	1625	1.½	2885
.½	321	1.	1287	1.¼	2010	5.	46150

GAS PIPES.

Flow of Gas in Pipes.

The flow of Gas is determined by the same rules as govern that of the flow of Water. The pressure applied is indicated and estimated in inches of water.

Diameter and Length of Gas-pipes to transmit given Volumes of Gas to Branch Pipes.—(Dr. Ure.)

Volume per Hour.	Diameter.	Length.	Volume per Hour.	Diameter.	Length.	Volume per Hour.	Diameter.	Length.
Cub. Feet.	Ins.	Feet.	Cub. Feet.	Ins.	Feet.	Cub. Feet.	Ins.	Feet.
50	.4	100	1000	3.16	1000	2000	7.	6000
250	1.	200	1500	3.87	1000	6000	7.75	1000
500	1.97	600	2000	5.32	2000	6000	9.21	2000
700	2.65	1000	2000	6.33	4000	8000	8.95	1000

The volumes of gases of like specific gravities discharged in equal times by a horizontal pipe, under the same pressure and for different lengths, are inversely as the square roots of the lengths.

The velocity of gases of different specific gravities, under like pressure, are inversely as the square roots of their gravities.

By experiment, 30 000 cubic feet of gas, specific gravity of .42, were discharged in an hour through a main 6 ins. in diameter and 22.5 feet in length; and 852 cub. feet, specific gravity .398, were discharged, under a head of 3 ins. of water, through a main 4 ins. in diameter and 6 miles in length.

The loss of volume of discharge by friction, in a pipe 6 ins. in diameter and 1 mile in length, is estimated at 95 per cent.

In distilling 56 lbs. of coal, the volume of gas produced in cubic feet when the distillation was effected in 3 hours was 41.3, in 7 hours 37.5, in 20 hours 33.5, and in 25 hours 31.7.

For Rules and Results of Velocities, etc., see Appleton's Dictionary of Mechanics and Engineering, and Hughes's Treatise on Gas Works. London.

GAS ENGINES.

In the Lenoir engine, the best proportions of air and gas are, for common gas, 8 volumes of air to 1 of gas, and for cannel gas, 11 of air to 1 of gas.

The time of explosion is about the 27th part of a second, and the resultant temperature 2474°.

An engine, having a cylinder 4⅝ ins. in diameter and 8¾ ins. stroke of piston, making 185 revolutions per minute, develops a power of half a horse.

Services for Lamps.

Lamps.	Length from Main.	Diameter of Pipe.	Lamps.	Length from Main.	Diameter of Pipe.	Lamps.	Length from Main.	Diameter of Pipe.
No.	Feet.	Ins.	No.	Feet.	Ins.	No.	Feet.	Ins.
2	40	⅜	10	100	¾	25	180	1¾
4	40	½	15	130	1	30	200	1½
6	50	⅝	20	150	1¼			

Average Composition of London Gas by Volume.

	Common Gas.	Cannel Gas.		Common Gas.	Cannel Gas.
Aqueous vapor......	2.	2.	Light carb'd hyd...	39.5	50.
Carbonic acid	.7	.1	Nitrogen	.5	.4
Carbonic oxide......	7.5	6.8	Olefiant, etc...... .	3.8	13.
Hydrogen...........	46.	27.7			

Combustion, Temperature, and Power of Gases.

	Per Pound of Gas.	Water Heated 1 Degree.		Temp. of Combustion.	Air Heated 1 Degree.
	Oxygen used.	Per lb. of Material.	Per Cub. Ft. of Gas.	Open Flame.	
	Cub. Feet.	Lbs.	Lbs.	Deg.	Cub. Feet.
Alcohol	24.6	12929	1597	4831	—
Camphene	38.9	18573	7134	5026	—
Cannel gas	31.	20140	760	5121	36585
Carbon	31.	14544	—	3026	—
Carbonic oxide	6.7	4825	320	5358	15403
Common coal gas	87.5	21060	650	5228	31299
Ether	30.9	13219	3217	5150	—
Hydrogen	93.4	62080	329.	5744	15837
Marsh gas	47.2	23543	996	4762	47946
Olefiant gas	40.5	21344	1585	5217	76290
Paraffine	40.5	21327	—	5239	—
Rape oil	38.7	17752	—	5087	—
Sperm oil	38.7	17230	—	4937	—
Spermaceti	37.	17589	—	4413	—
Stearine	34.4	18001	—	5095	—
Sulph. hydrogen	16.7	7414	671	4388	—
Wax	37.7	15809	—	4122	—
Wood spirit	25.3	9547	819	4641	—

Regulation of the Diameter and Extreme Length of Tubing and Number of Burners permitted.

Diameter of Tubing.	Length.	Number of Burners.	Capacity of Meters.	Number of Burners.	Diameter of Tubing.	Length.	Number of Burners.	Capacity of Meters.	Number of Burners.
Ins.	Feet.		Light.		Ins.	Feet.		Light.	
¼	6	1	3	6	1	70	35	45	90
⅜	20	3	5	10	1¼	100	60	60	120
½	30	6	10	20	1½	150	100	100	200
⅝	40	12	20	40	2	200	200		
¾	50	20	30	60					

Temperature of Gases.—The combustion of a cubic foot of common gas will heat 65 gallons of water 1°.

WATER.

Fresh Water. The constitution of it by weight and measure is

	By Weight.	By Measure.		By Weight.	By Measure.
Oxygen	88.9	1	Hydrogen	11.1	2

One cubic inch of distilled water at its maximum density of 39°.83, the barometer at 30 inches, weighs 252.6937 grains, and it is 828.5 times heavier than atmospheric air.

A cubic foot weighs 998.068 ounces, or 62.37925 lbs. avoirdupois.

Note.—For facility of computation, the weight of a cubic foot of water is taken at 1000 ounces and 62.5 lbs.

2. By the British Imperial Standard, the weight of a cubic foot of water at 62°, the barometer at 30 ins. = 998.224 ounces.

At a temperature of 212° its weight is 59.675 lbs. Below 39°.83 its density decreases, at first very slow, but progressing rapidly to the point of congelation, the weight of a cubic foot of ice being but 57.25 lbs.

It expands. 089 $= \frac{1}{11.24}$ of its bulk in freezing. From 40° to 12° it expands .00236 of its bulk; and from 40° to 212° it expands .04012, = .00023325 for every degree, giving an increase in volume (from 40° to 212°) of $\frac{1.}{.04012} = 1$ cubic feet in 24.92 feet.

Height of a column of water at 60° (62.4491 lbs.), equivalent to the pressure of	1 lb. per square inch, is 2.306 *feet.*
	the atmosphere is ..... 33.949 "

35.84 cubic feet of water weigh a ton.
39.13 " ice " "

When water is pure it will not become turbid, or produce a precipitate with any of the following *Re-agents.*

Baryta Water, If a precipitate or opaqueness appear, Carbonic Acid is present.

Chloride of Barium, Indicates Sulphates.

Nitrate of Silver, Indicates Chlorides.

Oxalate of Ammonia, Indicates Lime salts.

Sulphide of Hydrogen, slightly acid, Indicates Antimony, Arsenic, Tin, Copper, Gold, Platinum, Mercury, Silver, Lead, Bismuth, and Cadmium.

Sulphide of Ammonium, solution alkaloid by ammonia, Indicates Nickel, Cobalt, Manganese, Iron, Zinc, Alumina, and Chromium.

Chloride of Mercury or Gold and *Sulphate of Zinc*, Indicate organic matter.

Mineral Waters are divided into 5 groups, viz.:

1. Carbonated, containing pure carbonic acid—as, Seltzer, Germany; Spa, Belgium; Pyrmont, Westphalia; Seidlitz, Bohemia; and Sweet Springs, Virginia.

2. Sulphurous, containing sulphuretted hydrogen—as, Harrowgate and Cheltenham, England; Aix-la-Chapelle, Prussia; Blue Lick, Ky.; Sulphur Springs, Va., etc.

3. Chalybeate, containing carbonate of iron—as, Hampstead, Tunbridge, Cheltenham, and Brighton, England; Spa, Belgium; Ballston and Saratoga N. Y.; and Bedford, Penn.

4. Alkaline, containing carbonate of soda—these are rare, as, Vichy, Ems.

5. Saline, containing salts—as, Epsom, Cheltenham, and Bath, England; Baden-Baden and Seltzer, Germany; Kissingen, Plombieres, France; Seidlitz, Bohemia; Lucca, Italy; Yellow Springs, Ohio; Warm Springs, N. C.; Congress Springs, N. Y.; and Grenville, Ky.

Brief Rules for the Qualitative Analysis of Mineral Waters.

The first point to be determined, in the examination of a mineral water, is to which of the above classes does the water in question belong.

1. If the water reddens blue litmus paper before boiling, but not afterward, and the blue color of the reddened paper is restored upon warming, it is carbonated.

2. If it possesses a nauseous odor, and gives a black precipitate, with acetate of lead, it is sulphurous.

3. If, after the addition of a few drops of hydrochloric acid, it gives a blue precipitate, with yellow or red prussiate of potash, the water is a chalybeate.

4. If it restores the blue color to litmus paper after boiling, it is alkaline.

5. If it possesses neither of the above properties in a marked degree, and leaves a large residue upon evaporation, it is a saline water.

River or canal water contains	$\frac{1}{20}$	of its volume of gaseous matter.
Spring or well water "	$\frac{1}{14}$	

Sea-Water. A cubic foot of it weighs 64.3125 lbs.

Height of a column of water at 60° (specific gravity, 1029.), equivalent to the pressure of....	1 lb. per square inch, is 2.239 *feet.*
	the atmosphere is........ 32.966 "

34.83 cubic feet weigh a ton.

Sea-water contains from 4 to 5⅓ ounces of salt in a gallon of water.

Saline Contents of Sea-Water from several Localities.

Baltic	6.6	British Channel	35.5	South Atlantic	41.2
Black Sea	21.6	Mediterranean	39.4	North Atlantic	42.6
Arctic	28.3	Equator	39.42	Dead Sea	385.

There are 62 volumes of carbonic acid in 1000 of sea-water.

Destructive Effect of Sea-Water upon Metals and Alloys per Square Foot.

	Grains.		Grains.		Grains
Steel	40	Copper	9	Galvanized Iron	1.5
Iron	38	Zinc	8	Tin	2.

Sea-water, according to the analysis of Dr. Murray, at the specific gravity of 1.029, contains

Muriate of soda	220.01	Muriate of magnesia	42.08
Sulphate of soda	33.16	Muriate of lime	7.84
			303.09

Or, 1 part sea-water contains .030309 parts of salt $= \frac{1}{33}$ part of its weight.

Boiling Points at different Degrees of Saturation.

Salt, by Weight, in 100 Parts of Sea-water.	Boiling Point	Salt, by Weight, in 100 Parts of Sea-water.	Boiling Point.	Salt, by Weight, in 100 Parts of Sea-water.	Boiling Point.
$3.03 = \frac{1}{33}$	213.2°	$15.15 = \frac{5}{33}$	217.9°	$27.28 = \frac{9}{33}$	222.5°
$6.06 = \frac{2}{33}$	214.4°	$18.18 = \frac{6}{33}$	219. °	$30.31 = \frac{10}{33}$	223.7°
$9.09 = \frac{3}{33}$	215.5°	$21.22 = \frac{7}{33}$	220.2°	$33.34 = \frac{11}{33}$	224.9°
$12.12 = \frac{4}{33}$	216.7°	$24.25 = \frac{8}{33}$	221.4°	$*36.37 = \frac{12}{33}$	226. °

Deposits at different Degrees of Saturation and Temperature.

When 1000 *Parts are reduced by Evaporation.*

Volume of Sea-water.	Boiling Point.	Salt in 100 Parts.	Nature of Deposit.
1000	214°	3.	None.
299	217°	10.	Sulphate of lime.
102	228°	29.5	Common salt.

WAVES OF THE SEA.

Arnott estimated the extreme height of the waves of an ocean, at a distance from land sufficiently great to be freed from any influence of it upon their culmination, to be 20 feet.

The French Exploring Expedition computed waves of the Pacific to be 22 feet in height.

The average force of the waves of the Atlantic Ocean during the summer months, as determined by Thomas Stevenson, was 611 lbs. per square foot; and for the winter months 2086 lbs. During a heavy gale a force of 6983 lbs. was observed.

By the observations of Mr. Douglass in 1853, he deduced that when waves had heights of

8 feet, there were	35	in number in one mile, and	8 per minute.
15 " "	5 and 6	" "	5 "
20 " "	3	" "	4 "

Tidal Waves.—Professor Airey declares that when the length of a wave is not greater than the depth of the water, the velocity depends only upon its length, and is proportionate to the square root of its length.

* Saturated.

When the length of a wave is not less than 1000 times the depth of the water, the velocity of it depends only upon the depth, and is proportionate to the square root of it; the velocity being the same that a body falling free would acquire by falling through a height equal to half the depth of the water. The diurnal and other tidal waves, so far as they are free, may be all considered as running with the same velocity, but the column of the length of the wave must be doubled for the diurnal wave.

Depth of Water in Feet.	Length of Wave in Feet.					
	1	10	100	1000	10000	100 000
	Velocity per Second in Feet.					
1	2.26	5.34	5.67	–	–	–
10	2.26	7.15	16.88	17.92	17.93	–
100	–	7.15	22.62	53.19	56.67	56.71
1000	–	–	22.62	71.54	168.83	179.21
10000	–	–	–	71.54	226.24	533.9

The wave produced by the action of the sun and moon is termed the *Free Tide Wave.* The semi-diurnal tide wave is this, and has a period of 12 hours 24+ minutes.

Semi-Diurnal Free-Tide Wave.

Depth of Water.	Velocity per Second.	Length.	Space described per Hour.	Depth of Water.	Velocity per Second.	Length.	Space described per Hour.
Feet.	Feet.	Miles.	Miles.	Feet.	Feet.	Miles.	Miles.
1	5.7	47.9	3.9	100	56.7	429.5	38.7
4	11.3	95.9	7.7	400	113.4	959.	77.3
10	17.9	151.6	12.3	800	160.4	1356.	109.4
20	25.4	214.4	17.3	1000	179.3	1516.	122.3
40	35.9	303.2	24.5	2000	253.6	2144.	172.9
60	43.9	371.4	29.9	4000	358.7	3032.	244.5

GUNNERY.

A heavy body impelled by a force of projection describes a parabola, the *parameter* of which is four times the height due to the velocity of the projection.

It has been ascertained by experiment that the velocity of a shot projected from a gun varies as the square root of the charge directly, and as the square root of the weight of the shot reciprocally.

To Compute the Velocity of a Shot or Shell.

RULE.—Multiply the square root of treble the weight of the powder in pounds by 1600; divide the product by the square root of the weight of the shot; and the quotient will give the velocity in feet per second.

EXAMPLE.—What is the velocity of a shot of 196 lbs., projected with a charge of 9 lbs. of powder?

$\sqrt{9 \times 3} = 5.2$, and $\sqrt{196} = 14$. Then, $5.2 \times 1600 \div 14 = 594$ *feet.*

To Compute the Range for a Charge, or the Charge for a Range.

When the Range for a Charge is given.—The ranges have the same proportion as the charges of powder; that is, as one range is to its charge, so is any other range to its charge, the elevation of the gun being the same in both cases. *Consequently,*

RULE 1. *To Compute the Range.*—Multiply the range determined by the charge in pounds for the range required, and divide the product by the given charge; the quotient will give the range required.

RULE 2. *To Compute the Charge.*—Multiply the given range by the charge in pounds for the range determined, divide the product by the range determined, and the quotient will give the charge required.

EXAMPLE.—If, with a charge of 9 lbs. of powder, a shot ranges 4000 feet, how far will a charge of 6.75 lbs. project the same shot at the same elevation?

$$4000 \times 6.75 \div 9 = 3000 \textit{ feet.}$$

Ex. 2.—If the required range of a shot is 3000 feet, and the charge for a range of 4000 feet has been determined to be 9 lbs. of powder, what is the charge required to project the same shot at the same elevation?

$$3000 \times 9 \div 4000 = 6.75 \textit{ lbs.}$$

To Compute the Range at one Elevation, When the Range for another is given.

RULE.—As the sine of double the first elevation in degrees is to its range, so is the sine of double another elevation to its range.

EXAMPLE.—If a shot range 1000 yards when projected at an elevation of 45°, how far will it range when the elevation is 30° 16′, the charge of powder being the same?

Sine of $45° \div 2 = 100\,000$; sine of $30°\ 16' \times 2 = 87\,064$.

Then, as 100 000 : 1000 : : 87 064 : 870.64 *feet.*

To Compute the Elevation at one Range, When the Elevation for another is given.

RULE.—As the range for the first elevation is to the sine of double its elevation, so is the range for the elevation required to the sine for double its elevation.

EXAMPLE.—If the range of a shell at 45° elevation is 3750 feet, at what elevation must a gun be set for a shell to range 2810 feet with a like charge of powder?

Sine of $45° \times 2 = 100\,000$.

Then, as 3750 : 100 000 : : 2810 : 74 933 = *sine for double the elevation* = 24° 16′.

INITIAL VELOCITY AND RANGES OF SHOT AND SHELLS.

The *Range* of a shot or shell is the distance of its first graze upon a horizontal plane, the piece mounted upon its proper carriage.

Arms and Ordnance.	Projectile.		Powder	Initial Velocity.	Powder.	Time of Flight.	Elevation	Range.
	Description.	Weight.						
		Grains.	Grains.	Feet.	Lbs.	Sec.	° ′	Yards.
Rifle Musket........	Elongated.	510	60	963	–	–	–	–
Musket, 1841	Round.	412	110	1500	–	–	–	–
		Lbs.	Lbs.					
6-Pounder.........	"	6.15	2.	1741	1.25	3.	2	800
6 "	"	6.15	–	–	1.25	–	5	1523
12 "	"	12.3	4.	1826	2.5	1.75	1	575
24 "	"	24.25	8.	1870	6.	–	2	1147
32 "	"	32.3	8.	1640	8.	–	1	713
42 "	"	42.5	–	–	10.5	–	1	775
42 "	"	42.5	–	–	10.5	–	5	1955
8-inch Columbiad..	"	65.	–	–	10.	14.19	15	3224
10 " " ..	"	127.5	–	–	15.	14.32	15	3281
10 " " ..	"	127.5	–	–	20.	–	39 15	5654
10 " Mortar.....	Shell.	98.	–	–	10.	36.	45	4250
13 " "	"	200.	–	–	20.	–	45	4325
15 " Columbiad..	"	302.	–	–	40.	–	7	1948
15 " " ..	"	315.	–	–	50.	23.29	25	4680
RIFLED.								
10-pounder Parrott.	"	9.75	1.	–	–	21.	20	5000
20 " "	"	19.	2.	–	–	17.25	15	4400
30 " "	"	29.	3.25	–	–	27.	25	6700
60 " "	Elongated.	60.	6.	–	–	–	–	
100 " "	"	100.	10.	–	–	29.	25	6910
100 " "	Shell.	101.	10.	1250	–	28.	25	6820
200 " "	"	150.	16.	–	–	–	4	2200
12-inch Rodman	"		50.	1154	–	5½	40	
Hall's Rockets......	3 inch.	16.		–	–	–	47	1720

Approximate Rule for Time of Flight.

Under 4000 yards, velocity of projectile 900 feet in one second; under 6000 yards, velocity 800 feet; and over 6000 yards, velocity 700 feet.

PENETRATION OF SHOT AND SHELL.

Experiments at Fort Monroe, 1839, and at West Point, 1853.

Ordnance.	Charge.	Projectile.	Distance.	Mean Penetration. Sand Butt.	Compact Earth.	White Oak.	Hard Brick.	Freestone.	Granite.	Concrete.	Iron Plates.
	Lbs.		Yds.	Feet.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins
32-Pounder	8.	Shot.	880	–	–	–	15.25	12.	3.5	–	–
32 "	11.	"	100	–	–	60.	–	–	–	–	–
42 "	10.5	"	100	–	–	54.75	18.	–	4.	–	–
42 "	7.	Shell.	100	–	–	40.75	–	–	–	–	–
42 "	–	–	–	–	–	–	–	–	–	–	–
42 "	–	–	–	–	–	–	–	–	–	–	–
8-inch Howitzer...	6.	Shell.	880	–	8	–	8.5	4.5	1.	–	–
8 " Columbiad..	12.	Shot.	200	–	–	–	–	–	–	24	–
10 " " ..	18.	"	114	–	–	63.5	44.	–	7.75	–	–
10 " " ..	18.	Shell.	100	–	33	56.75	–	–	–	–	–
15 " " ..	–	–	–	–	–	–	–	–	–	–	–
" " ..	–	–	–	–	–	–	–	–	–	–	–
RIFLED.											
10-pounder Parrott.	–	–	1	–	–	–	–	–	–	–	–
20 " "	–	–	–	–	–	–	–	–	–	–	–
30 " "	–	–	–	–	–	–	–	–	–	–	–
60 " "	–	–	–	–	–	–	–	–	–	–	–
100 " "	–	–	–	–	–	–	–	–	–	–	–
200 " "	–	–	–	–	–	–	–	–	–	–	–
12-inch Rodman	55.	Shot.	400	17⅓	–	–	–	–	–	–	4.*

The solid shot broke against the granite, but not against the freestone or brick, and the general effect is less upon brick than upon granite.

The shells broke into small fragments against each of the three materials.

The penetrations in other kinds of earth are found by multiplying the above by .63 for sand mixed with gravel; by .87 for earth mixed with sand and gravel, weighing 125 lbs. per cubic foot; by 1.09 for compact mold and fresh earth mixed with sand, or half clay; by 1.44 for wet potter's clay; by 1.5 for light earth, settled; and by 1.9 for light earth, fresh.

The penetration in other kinds of earth and stone may be obtained by using the coefficients given for the other tables. For woods, use for beech and ash 1, for elm 1.3, for white pine and birch 1.8, and for poplar 2.

Penetration in Ball Cartridge Paper, No. 1.

Musket, with 134 grains, at 13.3 yards.......... 653 sheets.
Common rifle, 92 grains, at 13.3 yards.......... 500 sheets.

Experiments—England.—(HOLLEY.)

Ordnance.	Charge.	Projectile.	Weight	Velocity.	Range	Target and Effects
	Lbs.		Lbs.	Feet.	Yards	
11-inch U. S. Navy..	30	Shot.	169	1400	50	Iron plates, 14 ins.—loosened.
15-inch Rodman....	60	"	400	1480	50	Iron plates 6 ins.—destroyed.
RIFLED.						
7-inch Whitworth ..	25	Shot.	150	1241	200	Inglis's†—destroyed
10.5-inch Armstrong	45	"	307	1228	200	" "
13-inch "	90	‡ "	344.5	1760	200	Solid plates, 11 ins. thick—destroyed.

* Passed through. † 8-in. vertical and 5-in. horizontal slabs, and 7-in vertical and 5-in. horizontal slabs, 9×5 in. ribs and 3-in. ribs. ‡ Steel.

Penetration of Lead Balls in Small Arms.

Experiments at Washington Arsenal in 1839, and at West Point in 1837.

Arm.	Diam. of Ball.	Charge. Powder.	Distance.	Ball.	Penetration. White Oak.	Penetration. White Pine.
	Inch.	Grains.	Yards.	Grains.	Inches.	Inches.
Musket	.64	134	9	397.5	1.6	–
	.64	144	5	397.5	3.	–
Common rifle	–	100	5	219.	2.05	–
	–	92	9	–	1.8	–
Hall's rifle	–	100	5	219.	2.	–
	–	70	9	219.	.6	–
Hall's carbine, musket calibre	.5775	70	5	–	1.7	–
		80	5	219.	.8	–
		90*	5		1.1	–
		100*	5		1.2	–
Pistol	–	51	5	219.	.725	–
Rifle musket	.5775	–	200	500.	–	11.
Altered musket	.685	60	200	730.	–	10.5
Rifle, Harper's Ferry	.5775	70	200	500.	–	9.83
Pistol carbine	.5775	40	200	450.	–	5.75
Sharpe's carbine	.55	60	30	463.	–	7.17
Burnside's "	.55	55	30	350.	–	6.15

The musket discharged at 9 yards distance, with a charge of 134 grains, 1 ball and 3 buckshot, gave for the ball a penetration of 1.15 in., buckshot, .41 in.

Weight and Dimensions of Leaden Balls.

Number of Balls in a Pound, from 1 5/16ths to .237 of an Inch Diam.

Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.
Inch		Inch.		Inch.		Inch.		Inch.		Inch.	
1.67	1	.75	11	.57	25	.388	80	.301	170	.259	270
1.326	2	.73	12	.537	30	.375	88	.295	180	.256	280
1.157	3	.71	13	.51	35	.372	90	.29	190	.252	290
1.051	4	.693	14	.505	36	.359	100	.285	200	.249	300
.977	5	.677	15	.488	40	.348	110	.281	210	.247	310
.919	6	.662	16	.469	45	.338	120	.276	220	.244	320
.873	7	.65	17	.453	50	.329	130	.272	230	.242	330
.835	8	.637	18	.426	60	.321	140	.268	240	.239	340
.802	9	.625	19	.405	70	.314	150	.265	250	.237	350
.775	10	.615	20	.395	75	.307	160	.262	260		

The decimals for the dimensions in the division of an inch are

1 5/16 .... 1.3125	7/8 .... .875	11/16 .... .6875	7/16 .... .4375	1/4 .... .25
15/16 .... .9375	13/16 .... .8125	1/2 5	5/16 .... .3125	3/16 .... .187

Heated shot do not return to their original dimensions upon cooling, but retain a permanent enlargement of about .02 per cent. in volume.

Loss of Force by Windage (24-Pounder Gun).

Powder.	Ball.	Initial Velocity of Ball in Feet per Second. Without Windage.	Windage, .135 Inch.	Windage, .245 Inch.	Windage, .355 Inch.
Lbs.	Lbs.	Feet.	Feet.	Feet.	Feet.
4	24.25	1631	1450	1332	1197
6	24.25	1963	1702	1596	1465

A comparison of these results shows that 4 lbs. of powder give to a ball without windage nearly as great a velocity as is given by 6 lbs. to a ball having .14 inch windage, which is the true windage of a 24-pound ball; or, in other words, this windage causes a loss of nearly *one third* of the force of the charge.

* Charges too great for service.

Vents.—Experiments show that the loss of force by the escape of gas from the vent of a gun is altogether inconsiderable when compared with the whole force of the charge.

The diameter of the *Vent* in U. S. Ordnance is in all cases .2 inch.

Guns and Howitzers take their denomination from the weights of their solid shot in round numbers, up to the 42-pounder; larger pieces, rifled guns, and mortars, from the diameter of their bore.

Effect of different Descriptions of Wadding with a Charge of 77 Grains of Powder.

Wad.	Velocity of Ball per Second.
	Feet
Ball wrapped in cartridge paper, crumpled into a wad	1308 1377
1 felt wad upon powder and 1 upon ball	1346
2 felt wads upon powder and 1 upon ball	1482
1 elastic wad upon powder and 1 upon ball	1132
2 pasteboard wads upon powder	1200
2 elastic wads upon powder	1100

The *felt wads* were cut from the body of a hat, weight 3 grains.

The *pasteboard wads* were .1 of an inch thick, weight 8 grains.

The *cartridge paper* was 3×4.5 inches, weight 12.82 grains.

The *elastic wads* were "Baldwin's indented," a little more than .1 of an inch thick, weight 5.127 grains.

The most advantageous wads are those made of thick pasteboard, or of the ordinary cartridge paper.

Number of Pellets in an Ounce of Lead Shot of the different Sizes.

A A	40	No. 2	112	No. 7	341
A	50	3	135	8	600
B B	58	4	177	9	984
B	75	5	218	10	1726
No. 1	82	6	280	12	2140

No. 14 3150

Proportion of Powder to Shot for the following Numbers of Shot, as determined by Experiment.

No.	Shot.	Powder.	No.	Shot.	Powder.	No.	Shot.	Powder.
	Oz.	Drams.		Oz.	Drams.		Oz.	Drams.
2	2.	1.5	4	1.5	$1\frac{7}{8}$	6	1.25	$2\frac{3}{8}$
3	1.75	1.625	5	1.375	$2\frac{1}{8}$	7	1.125	$2\frac{5}{8}$

NOTE.—2 oz. of No. 2 shot, with 1.5 drams of powder, produced the greatest effect.

The increase of powder for the greater number of pellets is in consequence of the increased friction of their projection.

Numbers of Percussion Caps corresponding with the Birmingham Numbers

Eley's	5	6	7	8	9	24	10	11	18	12	13	14
Birmingham	43	44	46	48	49	50	51 and 52	53 and 54	55 and 56	57	58	

Where there are two numbers of the Birmingham sizes corresponding with only one of Eley's, it is in consequence of two numbers being of the *same size*, varying only in the length of the caps.

GUNPOWDER.

Gunpowder is distinguished as *Musket*, *Mortar*, *Cannon*, *Mammoth*, and *Sporting* powder; it is all made in the same manner, of the same proportions of materials, and differs only in the size of its grain.

Bursting or Explosive Energy.—By the experiments of Captain Rodman, U. S. Ordnance Corps, a pressure of 45 000 lbs. per square inch was obtained with 10 lbs. of powder, and a ball of 43 lbs.

Also, a pressure of 185 000 lbs. per square inch was obtained *when the powder was burned in its own volume*, in a cast-iron shell having diameters of 3.85 and 12 ins.

Properties and Results of Gunpowder, determined by Experiments of Captain A. MORDECAI, *U. S. A.*

24-POUNDER GUN.		MUSKET PENDULUM.	
Weight of ball and wad	24.25 lbs.	Weight of ball	397.5 grains.
" " powder	6. "	" " powder	120. "
Windage of ball	.135 inch.	Windage of ball	.09 inch.

Grain.	Composition.			Manufacture.	Number of Grains in 10 Troy Grains.	Relative Quickness of Burning.	Water absorbed by exposure to Air.	Relative Force.
	Saltpetre.	Charcoal	Sulphur.	Where from.			Per cent.	
Cannon, large ..	76	14.	12.	* Dupont's Mills, Del.	77	275	2.77	.677
" small ..					569	314	3.15	.72
Musket					1134	214	–	.808
Rifle					6174	142	–	.907
Rifle					5344	282	3.55	.728
Musket					1642	–	–	.834
Rifle					13152	–	–	.943
Cannon, uneven.	75	12.5	12.5	† Dupont's Mills, Del.	166	183	2.09	.788
" large ..					103	182	1.91	.756
Sporting	77	13.	10.	* Dupont's Mills, Del.	72808	100	4.42	1.
Blasting, uneven	70	15.	15.		295	212	–	–
Rifle	76	15.	9.	Loomis, Hazard, & Co., Conn.*	2378	204	–	.82
Sporting					–	–	–	.888
Rifle	75	15.	10.	Waltham Abbey, England.*	11600	–	–	.865

Comparison of the Force of a Charge in various Arms.

Arm.	Lock.	Powder, A 5.	Windage.	Weight of Ball.	Velocity.
		Grains.	Inch.	Grains.	Feet.
Ordinary rifle	Percussion.	100	.015	219	2018
" "	"	70	.015	219	1755
‡ Hall's rifle	Flint.	70	.0	219	1490
‡ Hall's carbine	Percussion.	70	.0	219	1240
‡ Jenks's carbine	"	70	.0	219	1687
Cadet's musket	Flint.	70	.045	219	1690
Pistol	Percussion.	35	.015	218.5	947

Deductions of Captain Mordecai.

Proof of Powder.—The common eprouvettes are of no value as instruments for determining the relative force of different kinds of gunpowder.

In the proof of gunpowder a cannon pendulum should be used.

In a 24 pounder gun, new cannon powder should give, with a charge of 6 lbs., an initial velocity of not less than 1600 feet to a ball of medium weight and windage.

For the proof of powder for small arms, a small ballistic pendulum is best adapted.

The initial velocity of a musket ball (18 to the pound), of .05 inch windage, with a charge of 120 grains, should be,

With new musket powder, not less than 1500 feet.
With new rifle powder, not less than 1600 feet.
With fine sporting powder, not less than 1800 feet.

Manufacture of Powder.—The powder of greatest force, whether for cannon or small arms, is produced by incorporation in the "cylinder mills."

* Glazed. † Rough. ‡ Loaded at the breach.

Effect of Wads.—In the service of *cannon*, heavy wads over the ball are in all respects injurious.

For the purpose of retaining the ball in its place, light *grommets* should be used.

On the other hand, it is of great importance, and especially so in the use of small arms, that there should be a good wad over the powder for developing the full force of the charge, unless, as in the rifle, the ball has but very little windage.

Effect of the Size of the Grain.—Within the limits of the difference in the size of grain, which occurs in ordinary cannon powder, the granulation appears to exercise but little influence upon the force of it, unless the grain be exceedingly dense and hard.

Effect of Glazing.—Glazing is favorable to the production of the greatest force, and to the quick combustion of the grains, by affording a rapid transmission of the flame through the mass of the powder.

Effect of using Percussion Primers.—The increase of force by the use of primers, *which nearly closes the vent*, is constant and appreciable in amount, yet not of sufficient value to authorize a reduction of the charge.

Bore, Weight of Charges, and Ranges for U. S. Small Arms.

Arm.	Bore.	Windage of Ball.	Powder.	Ball.	
	Inch.	Inch.	Grains.		
Musket....	.69	* .05	120	397.5 grains.	Wad, 10.2 grains.
"	.69	† .04	110	470.2 "	" 10.2 "
Rifle......	.54	.015	75	218.5 "	" 8.4 "
Pistol.....	.54	.015	35	218.5 "	" 5.5 "
"	.54	.015	30	218.5 "	" 5.5 "

Ranges for Small Arms.—Musket. With a ball of 17 to the pound, and a charge of 110 grains of powder, etc., an elevation of 36′ is required for a range of 200 yards; and for a range of 500 yards, an elevation of 3° 30′ is necessary, and at this distance a ball will pass through a pine board 1 inch in thickness.

Rifle. With a charge of 70 grains, an effective range of from 300 to 350 yards is obtained; but as 75 grains can be used without *stripping* the ball, it is deemed better to use it, to allow for accidental loss, deterioration of powder, etc.

Pistol. With a charge of 30 grains, the ball is projected through a pine board 1 inch in thickness at a distance of 80 yards.

Proof of Powder.

Ordinary Proof of Powder.—One oz. with a 24-lb. ball. The mean range of new, proved at any one time, must not be less than 250 yards; but none ranging below 225 yards is received.

Powder in magazines that does not range over 180 yards is held to be unserviceable.

Good powder averages from 280 to 300 yards; *small grain*, from 300 to 320 yards.

Restoring Unserviceable Powder.—When powder has been damaged by being stored in damp places, it loses its strength, and requires to be worked over. If the quantity of moisture absorbed does not exceed 7 per cent., it is sufficient to dry it to restore it for service. This is done by exposing it to the sun.

When powder has absorbed more than 7 per cent. of water it should be sent to a powder mill to be worked over.

Dimensions of Powder Barrels for 100 *lbs. of Powder*

Whole length	20.5 ins.	Interior diameter at the bilge ..	16. ins.
Length, interior in the clear...	18. "	Thickness of staves and heads..	.5 "
Interior diameter at the head..	14. "	Weight of barrels about	25 lbs.

Diameter of Holes in Sieve to determine the Class of Powder.

Musket powder, No. 1, .03 in.; No. 2, .06 in.	Cannon powder, No. 4, .25 in.; No. 5, .35 in.
Mortar " No. 2, .06 in.; No. 3, .10 in.	Mammoth " No. 6, .6 in.; No. 7, .9 in.

Musket Powder.—None should pass through sieve No. 1; all through No. 2.

Mortar Powder.—None should pass through sieve No. 2; all through No. 3.

Cannon Powder.—None should pass through sieve No. 4; all through No. 5.

* .05679 lbs., or 18 to the pound. † .05861 lbs., or 17 to the pound.

RAILWAYS AND ROADS.

Table of Gradients, Rise per Mile, and Resistance to Gravity.

Gradient of 1. in...	20	25	30	35	40	45	50	60	70	80	90	100
Rise in ft. per mile	264	211	176	151	132	117	106	88	75	66	59	53
Resistance in lbs. per ton of train..	112	89.6	74.7	64	56	50	44.8	37.3	32	28	24.8	22.4

Resistance due to gravity upon any inclination $\frac{2240}{\text{rate of grad.}}$ = *lbs. per ton of train.*

Resistance of Trains upon a Level at different Speeds.

$\frac{V^2}{171} + 8 = R$. V *representing velocity in miles per hour, and* R *resistance in lbs. per ton of train.*

The resistance of curves may be taken at 1 per cent. for each degree of the curve covered by a train.

Irregularities of roads vary from 5 to 40 per cent. Strong side winds resist 20 per cent.

Velocity of train per hour	Miles. 10	15	20	30	40	50	60	70
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Resistance upon straight line per ton..........	8½	9¼	10¼	13¼	17¼	22½	29	36½
Ditto, with sharp curves and strong wind*.....	13	14	15½	20	26	34	43½	55

To Compute the Weight of Rails.

$L \times 12 = W$. L *representing greatest load upon one driving wheel in tons, and* W *weight of rail in lbs. per yard.*

Sectional area of rail in inches × 10.08 = weight of rail in lbs. per yard.

Weight of rail in lbs. per yard × 1.571 = weight of rails per mile of single line in tons.

Points and Crossings, Ordinary Crossing, Narrow Gauge.

Length from point to crossing = 75 feet.
Total length from point to point = 165 "
Radius........................ = 600 "
Angle of crossing............ = 1 in 10.
Length of inner switch....... = 10 feet.
Length of outer switch........ = 15 feet.
Throw of outer switch at point = 4 ins.
Clearance " " = 3½ ins.
Length of guard rail .... = 8 feet.
Clearance of ditto .... = 1½ ins.

Railway Sidings, etc.

$2\sqrt{d\,r - (\frac{1}{2}\,d)^2} = L$. *d representing distance between centres of lines of siding in feet, r radius of curves, and* L *length over the points.*

Coefficients of Adhesion of Locomotives per Ton upon the Driving Wheels.

	Lbs.
When the rails are very dry...	670
When the rails are very wet...	600
In misty weather	350
In frost or snow	200

In coupled engines the adhesion is due to the load upon all the wheels coupled to the drivers.

The adhesion must exceed the traction of an engine upon the rails, otherwise the wheels will slip.

* Equal to 50 per cent. added to resistance upon a straight line.

To Compute the Load which a Locomotive will draw up an Inclination.

$\frac{T}{r + r'} - W = L$. T *representing tractive power of locomotive in lbs.*, r *resistance due to gravity, and* r′ *resistance due to assumed velocity of train in lbs. per ton*, W *weight of locomotive and tender, and* L *load the locomotive can draw in tons, exclusive of its own weight and tender.*

Coefficient of Traction of Locomotives.

Railroads in good order, etc. .... {4 lbs. / 6 "}
Railroads in ordinary condition .... 8 "

Tredgold estimates the resistance to a train from concussions, at a velocity of 10 miles per hour and above this, at ⅙ the velocity.

To Compute the Traction, Retraction, and Adhesive Power of a Locomotive or Train.

When upon a Level. $\frac{a\,s\,P}{D} = T$. *a representing area of one cylinder in sq. ins.*, s *stroke of piston in feet*, P *mean pressure of steam in lbs. per sq. in.*, D *diameter of driving wheels, and* T *traction in lbs.*

$C\,w = A$. C *representing coefficient in lbs. per ton*, w *weight of locomotive upon driving wheels in tons, and* A *adhesion in lbs.*

When upon an Inclination. $\frac{a\,s\,P}{D} - r\,w\,h = T$. r *representing resistance per ton of locomotive, and* h *height of rise in feet per* 100 *feet of road.*

$r\,w\,h = R$, *representing retraction in lbs.*

$\frac{C\,w\,b}{100} = A$. b *representing base of inclination in feet per* 100 *feet of road.*

When the Velocity of a Train is considered.

When upon a Level, $W(c + \sqrt{V}) = R$; *When upon an Inclination*, $W(r\,h + c + \sqrt{V}) = R$. V *representing velocity of train in miles per hour.*

ILLUSTRATION.—A train weighing 300 tons is to be driven up a grade of 52.8 feet per mile, with a velocity of 16 miles per hour; required the retractive power?

52.8 per mile = 1 in 100 feet = r = 22.4 lbs. C = 5.

$200\,(22.4 \times 1 + 8 + \sqrt{16}) = 200 \times \overline{22.4 + 9} = 6280$ *lbs.*

The resistance to traction upon a level is doubled by a radius of curve of 400 feet, and 13 lbs. per ton is the additional friction upon a curve of 300 feet.

By the experiments of Mr. Gooch with a Dynamometer, the resistances of a train were determined to be as follows:

Engine and Tender 50 *tons. Train* 100 *tons.*

Velocity per Hour.	Cars.	Resistance in lbs. per Ton. Engine and Cars.	Engine and Tender.	Atmosphere for Cars.	Oscillation of Cars.
Miles.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
13.1	7.56	9.04	11.97	.62	.87
20 2	8.19	12.28	20.43	1.47	1.35
45 3	14.36	21.84	37.36	7.37	3.02
56.6	21.8	31.16	46.11	11.53	3.77
61.3	19.8	32.86	50.68	13.52	4.00

The atmospheric resistance per bulk of cars alone is estimated as equal to the product of .00002 the bulk of the cars in cubic feet, and their velocity in miles per hour squared.

The oscillating resistance to cars alone is estimated at the quotient of the product of the weight of the cars and their velocity in miles per hour, divided by 1.5.

The resistance of the engine and tender alone, is estimated by the sum of .5 the velocity in miles per hour, added to 5 for the friction of the axles and parts. To this sum add the product of .00004 times the square of the velocity and the weight of the cars, and multiply their sum by the weight of the engine and tender in tons.

ILLUSTRATION.—Assuming a train of 150 tons (engine and tender 50 tons, cars 100), at a velocity of 55 miles per hour and a bulk of cars of 18000 cubic feet.

Then, $.00002 \times 18000 \times 55^2 = 1089.$ *lbs. atmospheric resistance,*

$\frac{100 \times 55}{15} = 366.6$ " *oscillating resistance.*

$\overline{55 \times .5 + 5} + \overline{55^2 \times .00004 \times 100} \times 50 = 2230.$ " *engine and tender,*

$100 \times 6 = 600.$ " *friction of cars.*

4285.6 "

Hence $\frac{4285.6}{150} = 28.57$ *lbs. per ton of the train.*

Experiment gave a resistance of 29. lbs. per ton.

Grades of 200, and even 250 feet, can be advantageously overcome.

To Compute the Maximum Load that can be drawn by an Engine, up the Maximum Grade that it can attain, the Weight and Grade being given.—(MAJOR McCLELLAN, U. S. A.)

$\frac{.2\ A}{.4242\ G + 8} = L$ and $\frac{.2\ A - 8\ L}{.4242\ L} = G$. A *representing the adhesive weight of the engine in lbs.*, G *the grade in feet per mile, and* L *the Load in tons.*

NOTE.—When the rails are out of order, and slippery, etc., for .2 A, put .143 A.

2. With an engine of 4 drivers, put .6 as the weight resting upon the drivers; with 6 drivers the entire weight rests upon them.

ILLUSTRATION.—An engine weighing 30 tons has 6 drivers; what are the *maximum* loads it can draw upon a level, and upon a grade of 250 feet, and what is its *maximum* grade for that load?

$\frac{.2 \times 2240 \times 30}{.4242 + 8} = \frac{13440}{8.4242} = 1595.4$ *tons upon a level.*

$\frac{.2 \times 2240 \times 30}{.4242 \times 250 + 8} = \frac{13440}{114.05} = 117.8$ *tons up a grade of* 250 *feet.*

$\frac{.2 \times 2240 \times 30 - 8 \times 117.8}{.4242 \times 117.8} = \frac{12497}{49.97} = 250.1$ *tons.*

The adhesion of a 4-wheeled locomotive, compared with one of 6 wheels, is as 5 to 8.

Regulations for Railways—(English Board of Trade).

Cast-iron girders to have a breaking weight = 3 times the permanent load, added to 6 times the moving load.

Wrought-iron bridges not to be strained to more than 5 tons per square inch.

Minimum distance of standing work from the outer edge of rail at level of carriage steps, 3.5 feet in England and 4 feet in Ireland.

Minimum distance between lines of railway, 6 feet.

Stations.—Minimum width of platform, 6 feet. Minimum distance of columns from edge of platform, 6 feet. Steepest gradient for stations, 1 in 300. Ends of platforms to be ramped (not *stepped*). Signals and distant signals in both directions.

Carriages.—Minimum space per passenger 20 cubic feet. Minimum area of glass per passenger, 60 superficial ins. Minimum width of seats, 15 ins. Minimum breadth of seat per passenger, 16 ins. Minimum number of lamps per carriage, 2.

Requirements.—Joints of rails to be fished. Chairs to be secured by *iron* spikes. Fang bolts to be used at the joints of flat-bottomed rails.

Friction of Railway Carriages.

The least resistance in parts of the weight, as determined by experiments, is the $\frac{1}{288}$ part, and the greatest resistance the $\frac{1}{115}$ part, equal to a resistance of 7.48 lbs. and of 20.2 lbs. per ton.

The average resistance of a great number of experiments being the $\frac{1}{246}$ part, equal to 8.83 lbs. per ton.

By the experiments of Mr. Geo. Rennie, he determined that the resistance of an axle was directly as its diameter, and but one half in the terms of its length; also that the length of the bearing should be twice its diameter, and that the area of the bearing surface should not be subject to a greater insistent weight than 90 lbs. per square inch.

The resistance to the leading car of a train is about 12 lbs. per ton, and of the intermediate cars, 8 lbs.

The friction of locomotive engines is about 9 per cent., or 2 lbs. per ton of weight.

Case-hardening of wheel-tyres reduces their friction from .14 to .08 part of the load.

The resistance of the atmosphere to a train is as the square of its velocity, being ¼ lb. per square foot for a velocity of 10 miles per hour, 1 lb. for 20 miles, etc.

Roads.

Relative Capacities of different Roads.

Load upon horse's back	1.	Plank road	25.
Inferior gravel or earth roads	3.	Stone track	33.
Macadamized road	9.	Railway	54.

Coefficients of Friction in proportion to Load upon Road Surfaces.

	Per 100.	Per Ton		Per 100.	Per Ton.
Gravel road, new	.083	186	Pavement, street	.015	34
Sand road	.063	141	" "	.01	22
Broken stone, rutted	.052	117	" " very smooth.	.006	13
" fair order.	.028	63	Plank	.01	22
" perfect order	.015	34	Stone track	.05	112
Macadamized road	.033	74	Railway	.0036	8
Earth, good order	.025	56	Common road, bad order	.07	157

For other elements, see Friction, pp. 347–349.

Resistance of Gravity at different Inclinations.

Rise.	Load.	Rise.	Load.	Rise.	Load.
1 in a 100	.9	1 in 40	.72	1 in 24	.5
1 in 50	.81	1 in 30	.64	1 in 20	.4
1 in 44	.75	1 in 26	.54	1 in 10	.25

Inclination of Roads.—The limit of practicable inclination varies with the character of the road and the friction of the vehicle. For the best carriages on the best roads, the limit is 1 in 35.

To secure effective drainage of a road, it should incline 1 in 125 in the direction of its length. The transverse section of a Macadamized road should have an inclination of 1 in 50.

In the construction of Roads the advantage of a level road over that of an inclined one, in the reduction of labor, is superior to the cost of an increased length of road in the avoiding of a hill.

In the construction of a Macadamized road none but cubes of stone should be used, and none, the longest diameter of which exceeds 2½ ins., and when the stone is very hard this may be reduced to 1¼ and 1½ ins.

The dimensions of a hammer for breaking the stone should be, head 6 ins. in length, weighing 1 lb., handle 18 ins. in length; and an average laborer can break from 1½ to 2 cubic yards per day.

The thickness or depth of the stones, *i. e.* the metaling, should be 6 ins., in 2 layers of 3 inches, laid at an interval, enabling the first layer to be fully consolidated before the second is laid on.

A horse can draw upon a plank road three times the load that he can upon an ordinary broken stone or Macadamized road.

To Compute the Tractive Power of a Horse Team.

When upon a Level. $L\,(c+\sqrt{V})=T$; L *representing Load in tons, and c coefficient as before.* $\frac{375}{V\,\sqrt{d}}=T$; *d representing duration of travel in hours.*

When upon an Inclination. $L\,(r\,h+c+\sqrt{V})=T$, and $\frac{375}{V\,\sqrt{d}}-\frac{w'\,h}{100}=T$; *r representing resistance in lbs. per ton, h vertical rise in 100 feet, v velocity in miles per hour, and w' weight of horses in lbs.*

Horses upon Turnpike Roads.

At a speed of 10 miles per hour, a horse will perform 13 miles per day for 3 years. In ordinary staging, a horse will perform 15 miles per day.

Comparative Effect of Horses upon Roads and Canals.

Duty.	Rate per Hour.	Force.	Distance per Day.	Duration per Day	Effect.
	Miles.	Lbs.	Miles.	Hours.	
Railroad	2½	125	20	8.	2500
Turnpike....	10	42	13	1.3	546
Canal.......	9	133	10	1.11	1330

CANALS.

Resistance of Boats at Low and High Velocities.

Low Velocities.

Speed per Hour.	Weight which 1 lb. will draw.	Resistance per Ton.	Speed per Hour.	Weight which 1 lb will draw.	Resistance per Ton.
Miles.	Lbs.	Lbs.	Miles.	Lbs.	Lbs.
4	200	11.2	3	474	4.73
3¾	243	9.22	2½	819	2.73
3½	299	7.5	2	1600	1.4

High Velocities.

Speed per Hour.	Resistance per Ton.		
	Maximum Load.	Minimum Load.	Average Load.
Miles.	Lbs.	Lbs.	Lbs.
4	7.1	13.1	9.2
8¾	49.8	74.9	58.53
10¾	56.8	92.8	72.45

SEWERS.

Sewers are classed as Drains, Sewers, and Culverts.

Drains are the small courses, as from one or more locations leading to a sewer.

Sewers are the courses from a series of locations.

Culverts are the courses that receive the discharge of sewers.

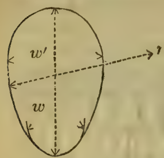
The greatest fall of rain is 2 inches per hour = 54308.6 galls. per acre.

Drainage of Lands by Pipes.

Soils.	Depth of Pipes	Distance apart.	Soils.	Depth of Pipes	Distance apart.
	Ft. Ins.	Feet.		Ft. Ins.	Feet.
Coarse gravel sand ...	4 6	60	Loam with gravel ...	3 3	27
Light sand with gravel	4	50	Sandy loam.........	3 9	40
Light loam	3 6	33	Soft clay............	2 9	21
Loam with clay......	3 2	21	Stiff clay	2 6	15

SEWERS.

Circular. $55\sqrt{x \times 2f} = v$, and $v \times a = V$; *x representing area of sewer ÷ the wetted perimeter, f inclination of do. per mile, and v velocity of flow, in feet per minute; a area of flow in square feet, and V volume of discharge in cubic feet per minute.*



Egg. $\frac{D}{3} = w$, $\frac{2D}{3} = w'$, and $D = r$. D *representing height of sewer, w and w′ width at bottom and top, and r radius of sides.*

In culverts less than 6 feet in depth,* the brick-work should be 9 ins. thick. When they are above 6 feet and less than 9 feet, it should be 14 ins. thick.

If the diameter of top arch = 1, the diameter of inverted arch = .5, and the total depth = the sum of the two diameters, or 1.5; then the radius of the arcs which are tangential to the top, and inverted, will be 1.5.

From this any two of the elements can be deduced, one being known.

Oval. Top and bottom* should be of equal diameters. The diameter .76 depth of culvert; the intersections of the top and bottom circles, as *n*, Fig. 14, p. 168, form the centres for striking the courses connecting the top and bottom circles.

The inclination of sewers should not be less than 1 foot in 240.

Dimensions, Areas, and Volume of Work per Lineal Foot of Egg-shaped Sewers of different Dimensions.

Internal Dimensions.				Volume of Brick-work.		
Depth.	Diameter of Top Arch.	Diameter of Invert.	Area.	4½ Inch Thick.	9 Inch Thick.	13½ Inch Thick.
Feet.	Feet.	Feet.	Sq. Feet.	Cub. Feet.	Cub. Feet.	Cub. Feet.
2.¼	1.5	.75	2.53	2.81	—	—
3.	2.	1.	4.5	3.56	—	—
3.¾	2.5	1.25	7.03	4.31	9.56	—
4.½	3.	1.5	10.12	5.06	10.87	—
5.½	3.5	1.75	13.78	5.81	12.75	—
6.	4.	2.	18.	6.56	14.25	—
6.¾	4.5	2.25	22.78	7.31	15.75	24.75
7.½	5.	2.5	28.12	—	17.06	27.
8.¼	5.5	2.75	34.03	—	18.	28.41
9.	6.	3.	40.5	—	19.69	30.94

In laying large sewers through quicksands, cast-iron inverts are sometimes employed, and with success, to connect the foundation of the whole work together.

Area of Surface from which Circular Sewers will discharge Water equal in Volume to One Inch in Depth upon surface per Hour, including ordinary City Drainage.

Inclination in Feet.	Diameter of Sewers in Feet.					
	2	2½	3	4	5	6
	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.
None	38¾	67¼	120	277	570	1020
1 in 480	48	75	135	308	630	1117
1 in 240	50	87	155	355	735	1318
1 in 160	63	113	203	460	950	1692
1 in 120	78	143	257	590	1200	2180
1 in 80	90	165	295	570	1388	2486
1 in 60	125	182	318	730	15[illegible]0	2675

* Internal dimensions.

ARCHES AND ABUTMENTS.

Approximate Rules and Tables for the Depth of Arches and Thickness of Abutments.

$C\sqrt{r} = D$. C *representing coefficient*, r *radius of arch at crown*, t *thickness of abutment*, h *height of abutment to spring, and* D *depth of crown in feet.*

In single arches, Stone C = .3, Brick .4, and Rubble .45.

Depths required for the Crowns of Arches.

Radius of Curve.	Stone.	Brick.	Radius of Curve.	Stone.	Brick.	Radius of Curve.	Stone.	Brick.	Radius of Curve.	Stone.	Brick.
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
2	.42	.56	10	.95	1.26	24	1.47	1.96	80	2.68	3.58
2½	.47	.63	11	1.	1.33	25	1.5	2.	85	2.77	3.69
3	.52	.69	12	1.04	1.38	30	1.64	2.19	90	2.85	3.8
3½	.56	.75	13	1.08	1.44	35	1.78	2.37	95	2.92	3.9
4	.6	.8	14	1.12	1.5	40	1.9	2.53	100	3.	4.
4½	.64	.85	15	1.16	1.55	45	2.01	2.68	110	3.15	4.2
5	.67	.9	16	1.2	1.6	50	2.12	2.83	120	3.29	4.38
5½	.71	.94	17	1.23	1.65	55	2.22	2.97	130	3.42	4.56
6	.74	.98	18	1.27	1.7	60	2.33	3.1	140	3.55	4.73
7	.8	1.06	19	1.31	1.74	65	2.42	3.22	150	3.67	4.9
8	.85	1.13	20	1.34	1.79	70	2.51	3.35	160	3.8	5.06
9	.9	1.2	22	1.41	1.88	75	2.6	3.46	170	4.13	5.22

Minimum Thickness of Abutments for Arches of 120°, *where their Depth does not exceed* 3 *Feet. Computed from the Formula—*

$$\sqrt{6\,r + \left(\frac{3\,r}{2\,h}\right)^2} - \frac{3\,r}{2\,h} = t.$$

Radius of Arch.	Height of Abutment to Spring in Feet.					Radius of Arch.	Height of Abutment to Spring in Feet.				
	5	7.5	10	20	30		5	7.5	10	20	30
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
4.	3.7	4.2	4.3	4.6	4.7	12.	5.6	6.4	6.9	7.6	7.9
4.5	3.9	4.4	4.6	4.9	5.	15.	6.	7.	7.5	8.4	8.8
5.	4.2	4.9	4.8	5.1	5.2	20.	6.5	7.7	8.4	9.6	10.
6.	4.5	4.7	5.2	5.6	5.7	25.	6.9	8.2	9.1	10.5	11.1
7.	4.7	5.2	5.5	6.	6.1	30.	7.2	9.7	0.7	11.1	12.
8.	4.9	5.5	5.8	6.4	6.5	35.	7.4	9.1	10.2	11.8	12.9
9.	5.1	5.8	6.1	6.7	6.9	40.	7.6	9.4	10.6	12.8	13.6
10.	5.3	6.	6.4	7.1	7.3	45.	7.8	9.7	11.	13.4	14.3
11.	5.5	6.2	6.6	7.3	7.6	50.	7.9	10.	11.4	14.	15.

NOTE.—The abutments are assumed to be without counterforts or wing walls.

KEYSTONES.

To Compute the Depth of Keystones for Segmental Arches of Stone.—(TRAUTWINE.)

First Class of Arch. .36 √ of the radius at the crown.

Second Class of Arch. .4 √ of the radius at the crown.

Brick or Rubble. .45 √ of the radius at the crown.

In *Viaducts* of several Arches. Increase the above units to .42, .46, and .51.

RAILWAY BRIDGES.

For Spans between 25 and 70 feet.

Rise, $\frac{1}{5}$ of the span. *Depth of Arch*, .055 of the span.

Thickness of Abutments, from ¼ to $\frac{1}{5}$ of the Span. *Batter*, 1 in. per foot.

COST OF TUNNELS PRIOR TO 1855.—(*Major McClellan, U. S. A.*)

Location.	Per Cubic Yard.	Location.	Per Cubic Yard.
	$ Cts.		$ Cts.
Black Rock, U. S., greywacke slate	6.60	England, freestone, marble, clay, etc., lined	3.46
Blaisley, France, lined	3.18	Lehigh, U. S., hard granite	4.36
Blisworth, Eng., blue clay, lined	1.55	Schuylkill, U. S., slate	2.
Blue Ridge, U. S.	4.	Union, U. S., slate	2.08

Railway Tunnels.

In soft sandstone, U. S., without lining, per lineal yard ....$ 88.
In loose ground, thick lining, per lineal yard.......................... 710.
Ordinary brick lining, including centering, per cubic yard............. 8.50

Shafts.

Blaisley Tunnel, clay, chalk, and loose earth, per yard in depth $139.11. Deepest 646 feet.

Black Rock, 7 feet in diameter and 139 in depth, hard slate, per yard in depth $79.50, or per cubic yard $18.72.

The time required to drive the heading of the Black Rock Tunnel for 1782.5 feet was 2387 turns of 12 hours each.

IRON WORKS (ENGLAND).

Temperature of hot blast .... 600°

Density of blast and of refining furnace ... $2\frac{1}{2}$ to 3 lbs. per sq. inch.

Revolutions of puddling rolls per minute, 60; rail rolls, 100; rail saw 800.

Horse-power (Indicated) required for different Processes.

Blast furnace	60	Rail rolling train	250
Refining "	26	Small bar train	60
Puddling rolls with squeezers and shears	80	Double rail saw	12
		Straightening	7

ROLLING-MILLS.

10 tons bar iron per day	80	Plates, for each sq. foot rolled	5

FLOUR MILLS, SAW MILLS, WOOD-WORKING MACHINERY.

Flour Mills.

For each pair of 4-feet stones, with all the necessary dressing machinery, etc., there is required 15 horse's power.

One pair of 4-feet stones will grind about 5 bushels of wheat per hour. Each bushel of wheat so ground per hour requires .87 actual or 1.11 indicated horses' power, exclusive of dressing and other machinery.

Stones, 4 feet diam., 120 to 140 revolutions per minute.

Dressing Machines, 21 ins. diam., 450 to 500 revolutions per minute.

Creepers, $3\frac{1}{2}$ ins. pitch, 75 revolutions per minute.

Elevator, 18 ins. diam., 40 revolutions per minute.

Screen, 16 ins. diam., 300 to 350 revolutions per minute.

788 cubic feet of water, discharged at a velocity of 1 foot per second, are necessary to grind and dress 1 bushel of wheat per hour = 1.49 horses' power per bushel.

2000 feet per minute, for the velocity of a stone 4 feet in diameter, may be considered a maximum speed.

Saw-mill.

Gang saw, 30 sq. feet of dry oak, or 45 sq. feet of dry pine, per hour. . 1 horse-power.
Circular saw, 2.5 feet in diam., 270 revolutions per minute, 40 sq. feet of oak, or 70 of dry spruce .... 1 "

300 revolutions per minute. 1.33 square feet of dry pine per minute, kerf $\frac{7}{32}$ inch and 6 ins. deep, requires the power of 1 horse for the saw alone; and 1 square foot, kerf ¼ inch and 1 foot in depth, requires a like power.

4.5 feet in diameter, kerf ¼, and 1 foot in depth, requires 1 horse's power for 1.33 feet per minute.

Oak requires nearly one half more power than pine.

With a kerf of ⅛ inch, 1 horse's power will saw 2.66 square feet per minute.

The speed of the periphery should be about 50 feet per minute.

Velocities of Wood-working Machinery in Feet or Revolutions per Minute.

Circular saws, at periphery, 6000 to 7000 feet.

Band saw, 2500 feet.

Gang saws, 20 in. stroke, 120 strokes per minute.

Scroll saws, 300 strokes per minute.

Planing-machine cutters at periphery, 4000 to 6000 feet.

Work under planing machine, $\frac{1}{20}$th of an inch for each cut.

Molding-machine cutters, 3500 to 4000 feet.

Squaring-up-machine cutters, 7000 to 8000 feet.

Wood-carving drills, 5000 revolutions.

Machine augers, 1½ diam., 900 revolutions.

Machine augers, ¾ diam., 1200 revolutions.

Gang saws require for 45 super. feet of pine per hour, 1 horse power.

Circular saws require for 75 super. feet of pine per hour, 1 horse power.

In oak or hard wood, ¾ths of the above quantity require 1 horse power.

Sharpening Angles of Machine Cutters.

Adzing soft wood across the grain...	30°	Gouges and ploughing machines ...	40°
Planing machines, ordinary soft wood	35°	Hard-wood tool cutters......	50° to 55°

MINING AND BLASTING.

MINING.

In ordinary Soil, $\frac{l^3}{10}$ = *charge of powder in pounds, l representing half the depth of the line of least resistance.*

In Masonry, $l^3 \times C$ = *charge in pounds; C representing a coefficient depending upon the structure.*

In a plane Wall, C = .15, in one with counterforts = 2, and under a foundation when it is supported upon two sides = .4 to .6.

BLASTING.

In small blasts 1 lb. of powder will loosen about 4½ tons.

In large blasts 1 lb. of powder will loosen about 2¾ tons.

50 or 60 lbs. of powder, inclosed in a resisting bag, hung or propped up against a gate or barrier, will demolish any ordinary construction.

One man can bore, with a bit 1 inch in diameter, from 50 to 100 ins. per day of 10 hours in granite, or 300 to 400 ins. per day in limestone.

Two strikers and a holder can bore with a bit 2 inches in diameter 10 feet in a day in rock of medium hardness.

PROJECTION OF WATER.

Heights to which Water may be Projected through Engine Pipes under Pressure.

Pressure per Square Inch.	Equivalent Head of Water.	Height of Jet.	Ratio of Compression of Air in Air-chamber.	Pressure per Square Inch.	Equivalent Head of Water.	Height of Jet.	Ratio of Compression of Air in Air-chamber.
Lbs.	Feet.	Feet.		Lbs.	Feet.	Feet.	
30	68	33	.5	90	204	165	.17
45	102	66	.33	105	238	198	.14
60	136	99	.25	120	272	231	.125
75	170	132	.2	150	340	297	.1

Power required to raise Water from Wells by a Double-acting Lifting-pump.

Diameter of Pump.	Volume per Hour.	Depth from which this Volume can be raised by each Unit of Power.			
		Man turning a Crank.	Donkey working a Gin.	Horse working a Gin.	One Horse-power Engine.
Ins.	Gallons.	Feet.	Feet.	Feet.	Feet.
2	265	80	160	560	880
2½	420	50	100	350	550
3	620	35	70	245	385
3½	830	25	50	175	275
4	1060	20	40	140	220

WATER POWER.

To Compute Water-power.

$.00189\ V h =$ *horse's power, and* $\frac{528\ HP}{V} = V$; V *representing volume of water in cubic feet, per minute, and h head of water from race in feet.*

Effective Horse-power for different Motors.

Theoretical power .... 1.

Undershot wheels	= .4	Reaction wheel	= .2
Poncelet's undershot wheel	= .6	Impact wheel	= .5
Breast wheel (high)	= .55	Turbines	= .6 / .75
" (low)	= .6		
Overshot wheel	= .84 / .64	Tremont turbine	= .79
		Hydraulic ram	= .6

HYDRAULIC RAM.

$\frac{882\ HP}{h} = V$, $.00113\ V h = HP$; V *representing volume of water in cubic feet per minute, h head of water in feet, and* HP *actual horse-power.*

JET PUMP.

The greatest effect of a Jet Pump is when the depth from which the water is drawn through the supply or suction pipe is .9 of the height from which the water fell to give the jet.

The flow up the suction-pipe being .2 of that of the volume of the jet; hence, the effect $= .9 \times 2 = .18$.

Imperial Gallons.

6.2355 Gallons in a Cubic Foot.

WAVES.

The undulations of waves are performed in the same time as the oscillations of a pendulum, the length of which is equal to the breadth of a wave, or to the distance between two neighboring cavities or eminences.

DAMS AND TUNNELS.

DAMS (Earthwork).

Width at top in high dams from 7 to 20 ft. | Breast slopes .... = 3 to 1
Width at top in low dams .. = height. | Back slopes.................. = 2 to 1
Height above surface of water not less than 3.5 feet.

Proportion of Laborers in Bank, Fillers, and Wheelers, in different Soils, Wheelers being Estimated for a Distance of 50 Yards.

	Getters.	Fillers.	Wheelers.		Getters.	Fillers.	Wheelers.
In loose earth, sand, etc.	1	1	1	In hard clay	1	1¼	1¼
In compact earth......	1	2	2	In compact gravel	1	1	1
In marl	1	2	2	In rock	3	1	1

Masonry.

Width at bottom = .7 height; at middle = .5 height; and at top = .3 height.

TUNNELS.—(*From actual practice in Brick-work*).

Purpose.	Formation of Strata.	Extreme Height.		Extreme Width.		Depth at Crown.	
		Feet.	Ins.	Feet.	Ins.	Feet	Ins.
Canal	Various	16	2	17		1	3
Canal	Clay	21	6	20		1	6
Thames Tunnel......	Clay	22	3	37	6	2	6
Railway	Chalk	26	6	27		1	6
"	Various	27	6	27		1	10½
"	Shale	30		30		1	10½
"	Green sand	30	6	30		2	3
"	Freestone	36		36		2	3
Canal	Chalk and earth....	39		35	6	1	2

WIND-MILLS.—(*Molesworth.*)

To Compute the Angles of the Sails.

$23° - \frac{18\,d^2}{r^2}$ = *angle of the sail with the plane of motion at any part of the sail; r representing radius of sail in feet, and d distance of any part of the sail from the axis.*

Axis of Shaft of Wind-mill with Horizon.

8° upon level ground.

Breadth of whip at axis, $\frac{1}{30}$ length of whip.
Depth " " $\frac{1}{40}$ "
Breadth of whip at end, $\frac{1}{60}$ "
Depth " " $\frac{1}{80}$ "
Width of sail " ⅛ "

Divided by the whip in the proportion of 5 to 3, the narrow portion being nearest to the wind.

Width of sail at axis, $\frac{1}{5}$ length of whip; distance of sail from axis, $\frac{1}{7}$th length of whip.

Cross-bars from 16 to 18 inches apart.

STRENGTH OF ICE.

Thickness, 2 ins. will bear infantry.
" 4 " cavalry or light guns.
" 6 " heavy field-guns.
" 8 " upon sledges, a weight not exceeding 1000 lbs per sq. ft

STIFFNESS OF BEAMS.

Stiffness of Beams.—(Tredgold.)

$\sqrt[3]{\frac{l^2 W C}{b}} = d$; $\frac{l^2 W C}{d^3} = b$; *b representing breadth, and d depth in inches, l length in feet, and* W *load in lbs. upon the middle.*

C = Pine .01, Ash .01, Beech .013, Elm .015, Oak .13, Teak .008.

When the beam is uniformly loaded, put .625 W instead of W.

Resistance to Detrusion.

When one beam is let in, at an inclination to the depth of another, so as to bear in the direction of the fibres of the beam that is cut, the depth of the cut *at right angles to the fibres* should not be more than $\frac{1}{5}$ of the length of the piece, the fibres of which, by their cohesion, resist the pressure.

To Compute the Length necessary to resist a given Horizontal Thrust, as in the Case of a Rafter let into a Tie-Beam.

$\frac{4 T}{b c} = l$; *b representing the breadth of the beam in inches,* T *the horizontal thrust in lbs., c the cohesive resistance of the material in lbs. per sq. inch, and l the length in inches.*

REVOLVING DISC.

To Compute the Power.

Rule.—Multiply one half the weight of the disc by the height due to the velocity of its circumference in feet per second.

Example.—A grind-stone 3⅜ feet in diameter, weighing 2000 lbs., is required to make 362¼ revolutions per minute; what power must be communicated to it?

Circum. of 3⅜ = 10.6 *feet*, which × 362.25 and ÷ 60 = 64 *feet per second.* Then 2000 ÷ 2 × 64 = 64000 *lbs. raised* 1 *foot.*

Note.—If the revolving disc is not an entire or solid wheel, being a ring or *annulus*, it must first be computed as if an entire disc, and then the portion wanting must be computed and deducted.

Power Concentrated in Moving Bodies.

Simple power is *force multiplied by its velocity.* Power concentrated in a moving body is the *weight of the body multiplied by the square of its velocity; and the product divided by the accelleratrix*, or the power concentrated in a moving body, is equal to the power expended in generating the motion.

SHRINKAGE OF CASTINGS.

Iron, small cylinders	= 1/16 in. per ft.	Ditto, in length	= ⅛ in 16 ins.
" Pipes	= ⅛ "	Brass, thin	= ⅛ in 9 ins.
" Girders, beams, etc.	= ⅛ in 15 ins.	Brass, thick	= ⅛ in 10 ins.
" Large cylinders, the contraction of diam. at top.	= 1/16 per foot.	Zinc	= 5/16 in a foot.
		Lead	= 5/16 in a foot.
		Copper	= 3/16 in a foot.
" Ditto at bottom	= 1/12 per foot.	Bismuth	= 5/32 in a foot.

VERNIER SCALE.

The *Vernier* Scale is $\frac{11}{10}$, divided into 10 equal parts; so that it divides a scale of 10ths into 100ths when the lines meet in the two scales.

Measurement and Computation of the Tonnage of Vessels under the Act of Congress of 6th May, 1864.

Measurements are expressed in feet and decimals of a foot, and tonnage in tons and hundredths of a ton.

The "Tonnage Length" is the length along the middle line of the vessel upon the *under side* of the tonnage-deck plank, but for convenience is measured upon the *top* of the deck, and is the length between these extremities, which is divided into a number of parts, according to the classification under the law.

The depths are perpendicular and the breadths horizontal; the upper breadth, which in every case passes through the top of the tonnage depth, being at a distance below the deck, at its middle line, equal to one third of the spring of the beam at that point, and thus passing *through* the deck upon each side; and the lower breadth, which is at the bottom of the tonnage depth, being at a distance above the upper side of the floor timber at the inside of the limber-strake, equal to the average thickness of the ceiling, and thus passing *through* the keelson.

The "spring of the beam" is the perpendicular distance from the crown of the tonnage deck at the centre to a line stretched from end to end of the beam, and must be ascertained at each point where it is to be used in the measurement.

The Register of every vessel expresses her length and breadth, together with her depth, and the height under the third or spar deck is ascertained in the following manner: The tonnage deck, in vessels having three or more decks to the hull, is the second deck from below; in all other cases the upper deck of the hull is the tonnage deck. The length from the fore part of the outer planking, upon the side of the stem, to the after part of the main stern-post of screw steamers, and to the after part of the rudder-post of all other vessels, measured upon the top of the tonnage deck, is accounted the vessel's length. The breadth of the broadest part upon the outside of the vessel is accounted the vessel's breadth of beam. A measure from the under side of tonnage-deck plank, amidships, to the ceiling of the hold (average thickness), is accounted the depth of hold. If the vessel has a third deck, then the height from the top of the tonnage-deck plank to the under side of the upper-deck plank is accounted as the height under the spar deck.

The register tonnage of a vessel is her internal cubical capacity in tons of 100 cubic feet each, to be ascertained as follows: From the inside of the inner plank (average thickness) at the side of the stem to the inside of the plank upon the stern timbers (average thickness), deducting from this length what is due to the rake of the bow in the thickness of the deck, and what is due to the rake of the stern timber in the thickness of the deck, and also what is due to the rake of the stern timber in one third of the spring of the beam.

Classes.

CLASS 1. Vessels of which the tonnage length is 50 feet or under.
2. Over 50 feet, and not exceeding 100 feet in length.
3. Over 100 feet, and not exceeding 150 feet in length.
4. Over 150 feet, and not exceeding 200 feet in length.
5. Over 200 feet, and not exceeding 250 feet in length.
6. Over 250 feet in length.

If there is a break, a poop, or any other permanent closed-in space upon the upper decks, or upon the spar deck, available for cargo, or stores, or for the berthing or accommodation of passengers or crew, the tonnage of such space is computed.

If a vessel has a third deck, or spar deck, the tonnage of the space between it and the tonnage deck is computed.

In computing the tonnage of open vessels, the upper edge of the upper strake is to form the boundary-line of measurement, and the depth shall be taken from an athwart-ship line, extending from the upper edge of said strake at each division of the length.

The register of a vessel expresses the number of decks, the tonnage under the tonnage deck, that of the between decks, above the tonnage deck; also that of the poop or other inclose dspaces above the deck, each separately. In every registered U. S. vessel the number denoting the total registered tonnage must be deeply carved or otherwise permanently marked upon her main beam, and shall be so continued; and if it at any time cease to be so continued, such vessel shall no longer be recognized as a registered U. S. vessel.

Recapitulation of Measurements.

Register Length.—Length at the middle of the 2d deck from below, in vessels of two or more decks, and in all other vessels of the upper deck, measured from the fore part of the outer planking upon the side of the stem, to the after part of the main stern-post of single screw propeller steamers, and to the after part of the rudder-post of other vessels, measured upon the top of the tonnage deck.

Tonnage Length.—Length at upper side of tonnage-deck beams, from the inside of the inboard plank, at its average thickness at the side of the stem to the inside of the plank upon the stern timbers at its average thickness, deducting from this length that which is due to the rake of the bow in the thickness of the deck, and of the stern timber in the thickness of the deck, and one third the spring of the beam.

Breadth of Beam.—At the broadest part of the outside of the vessel.

Depth of Hold.—Height measured from the under side of tonnage-deck plank amidships from a point at a distance of one third the spring of the beam to the ceiling of the hold at its average thickness.

Height under Spar Deck.—The mean height from top of tonnage-deck plank to the under side of the upper-deck plank.

Open Vessels.—The upper edge of the upper strake is to be the boundary-line of measurement of length, and the depth is to be measured from a line running athwartships from the upper edge of the upper strake at each division of the length.

By an Act of Congress of 28th February, 1865, the preceding rule of admeasurement was amended as follows: No part of any ship or vessel shall be admeasured or registered for tonnage that is used for cabins or state-rooms, and constructed entirely above the first deck, which is not a deck to the hull.

CARPENTERS' MEASUREMENT.

For a Single-deck Vessel.

RULE.—Multiply the length of keel, the breadth of beam, and the depth of the hold together, and divide by 95.

For a Double-deck Vessel.

RULE.—Multiply as above, taking half the breadth of beam for the depth of the hold, and divide by 95.

BRITISH MEASUREMENT.

Divide the length of the upper deck between the after part of the stem and the fore part of the stern-post into 6 *equal parts*, and note the foremost, middle, and aftermost points of division. Measure the *depths* at these three points in feet and tenths of a foot, also the *depths* from the under side of the upper deck to the ceiling at the limber-strake; or, in case of a break in the upper deck, from a line stretched in continuation of the deck. For the *breadths*, divide each depth into 5 equal parts, and measure the inside breadths at the following points, viz.: at .2 and 8 from the upper deck of the foremost and aftermost depths, and at .4 and .8 from the upper deck of the amidship depth. Take the *length*, at half the amidship depth, from the after part of the stem to the fore part of the stern-post.

Then, to twice the amidship depth, add the foremost and aftermost depths for the *sum of the depths*; and add together the foremost upper and lower breadths, 3 times the upper breadth with the lower breadth at the midship, and the upper and twice the lower breadth at the after division for the *sum of the breadths*.

Multiply together the sum of the depths, the sum of the breadths, and the length, and divide the product by 3500, which will give the number of tons, or register.

If the vessel has a poop or half deck, or a break in the upper deck, measure the inside mean length, breadth, and height of such part thereof as may be included within the bulkhead; multiply these three measurements together, and divide the product by 92.4. The quotient will be the number of tons to be added to the result, as above ascertained.

For Open Vessels.—The depths are to be taken from the upper edge of the upper strake.

For Steam Vessels.—The tonnage due to the engine-room is deducted from the total tonnage computed by the above rule.

To determine this, measure the inside length of the engine-room from the foremost to the aftermost bulkhead; then multiply this length by the amidship depth of the vessel, and the product by the inside amidship breadth at .4 of the depth from the deck, and divide the final product by 92.4.

General Rule to Compute the Work done by any Machine.

Ascertain the distance through which the power, P, applied to the machine has operated in one minute, and represent it by *a*.

Ascertain the distance through which the weight, W, producing useful work, has operated in one minute, and represent it by *b*.

Then, $a\,P - b\,W =$ *work done by friction per minute.*

$a\,P$ $=$ *work applied per minute.*

$b\,W$ $=$ *useful work done per minute.*

Mechanical Laws of Elastic Fluids.

Boyle's or Mariotte's Law.—The elastic force of a gas or air at a given temperature is inversely proportional to the space which it occupies.

Let p and P *represent elastic forces of a gas when they occupy the spaces s and* S.

$$\text{Then } \frac{p\,s}{S} = P.$$

The elastic force of any gas at a given temperature is proportional to its density.

Wrought Iron Beams.

(*Trenton Iron Works*, Cooper, Hewitt, & Co., N. Y.)

Depth.	Thickness of Web.	Width of Flanges.	Weight per Lineal Foot.	Load borne with Safety. $\frac{C}{l} = W.$	Depth.	Thickness of Web.	Width of Flanges	Weight per Lineal Foot	Load borne with Safety. $\frac{C}{l} = W.$
Ins.	Ins.	Ins.	Lbs.	C in Lbs.	Ins.	Ins.	Ins.	Lbs.	C in Lbs.
6	¼	3	13.3	76 000	9	½	4	30	246 000
6	5/16	3¼	16.6	92 000	9	⅝	5⅜	50	448 000
7	⅜	3½	20	124 000	12¼	7/16	4½	40	390 000
9	⅜	3½	23.3	192 000	15	9/16	4 1/16	51.6	640 000
9	7/16	4	28	240 000	15	⅝	5⅜	66.6	908 000

Load uniformly distributed, Beam resting upon two supports, *l representing length in feet*, and W *weight in pounds* $=$ .3 *of breaking or ultimate strain.*

ILLUSTRATION.—What is the weight, uniformly distributed, that may be borne with safety by floor beams of the above description resting upon two supports, 20 feet in length, 9 ins in depth, 7/16 in. width of web, and 4 ins. width of flange?

$$C = 240\,000. \qquad \frac{240\,000}{20} = 12\,000 \textit{ lbs.}$$

See page 470 for other Formulæ and Illustrations.

FUEL.

With equal weights, that which contains most hydrogen ought, in its combustion, to produce the greatest volume of flame where each kind is exposed under like advantageous circumstances. Thus, pine wood is preferable to hard wood, and bituminous to anthracite coal.

When wood is employed as a fuel, it should be as dry as practicable. To produce the greatest quantity of heat, it should be dried by the direct application of heat; as usually employed, it has about 25 per cent. of water mechanically combined with it, the heat necessary for the evaporation of which is lost.

Different fuels require different volumes of oxygen; for the different kinds of coal it varies from 1.87 to 3 lbs. for each lb. of coal. 60 cubic feet of air is necessary to furnish 1 lb. of oxygen; and, making a due allowance for loss, nearly 90 cubic feet of air are required in the furnace of a boiler for each lb. of oxygen applied to the combustion.

Bituminous Coal.

Lignite. Brown Coal or Bituminous Wood.—Presents a distinct woody structure; is devoid of taste, brittle, and burns readily, leaving a white ash. This coal contains and absorbs moisture in some cases fully 40 per cent.

Caking Coal.—Fractures uneven; color varying from a resinous to a gray-black, and when heated breaks into small pieces, which afterward agglomerate and form a compact body. When the proportion of bitumen is great, it fuses into a pasty mass. This coal is unsuited where great heat is required, as the draught of a furnace is impeded by its caking. It is applicable for the production of gas and coke.

Splint or Hard Coal.—Color black or brown-black, lustre resinous and glistening. When broken, the principal fracture appears irregular and slaty, the transverse being fine grained, uneven, and splintery. It kindles less readily than caking coal, but when ignited produces a clear and hot fire.

Cherry or Soft Coal.—Alike to splint coal in its fracture and appearance, but its lustre is more splendent. It does not fuse when heated, is very brittle, ignites readily, and produces a bright fire with a clear yellow flame, but consumes rapidly.

Cannel or Parrot Coal.—Color jet, or gray or brown black, compact and even texture, a shining, resinous lustre. Fractures smooth or flat, conchoidal in every direction, and polishes readily. From its decrepitation when exposed to heat it is termed *parrot* coal.

Experiments upon the practical burning of this description of coal in the furnace of a steam-boiler give an evaporation of from 6 to 10 lbs of fresh water, under a pressure of 30 lbs. per square inch for 1 lb. of coal; Cumberland (Md., U. S.) coal being the most effective, and Scotch the least.

Coals that contain sulphur, and are in progress of decay, are liable to spontaneous combustion.

The limit of evaporation from 212° for 1 lb. of the best, assuming all of the heat evolved from it to be absorbed, would be 14.9 lbs.

Anthracite Coal.

Anthracite or Glance Coal, or Culm.—Is hard, compact, lustrous, and sometimes iridescent, the most perfect being entirely free from bitumen; it ignites with difficulty, and breaks into fragments when heated.

The evaporative power of this coal, in the furnace of a steam-boiler and under pressure, is from 7½ to 9½ lbs. of fresh water per lb. of coal.

Coals from one pit will vary 6 per cent. in evaporative value.

Coke.

Coke.—Coking in a close oven will give an increase of yield of 40 per cent. over coking in heaps, the gain in bulk being 22 per cent. Coals when coked in heaps will lose in bulk.

Cannel and Welsh (Cardiff) coals when coked in retorts will gain 30 per cent. in bulk and lose 36.5 per cent. in weight.

The relative costs of coal and coke for like results, as developed by an experiment in a locomotive boiler, are as 1 to 2.4.

Its evaporative power, in the furnace of a steam-boiler and under pressure, is from 7½ to 8½ lbs. of fresh water per lb. of coke.

Charcoal.

Charcoal.—The best quality is made from Oak, Maple, Beech, and Chestnut.

Wood will furnish, when properly burned, about 23 per cent. of coal.

Charcoal absorbs, upon an average of the various kinds, about 5.5 per cent. of water, Oak absorbing about 4.28, and Pine 8.9.

Its evaporative power, in the furnace of a boiler and under pressure, is 5½ lbs. of fresh water per lb. of coal.

The volume of air chemically required for the combustion of 1 lb. of charcoal is 293.5 cubic feet.

138 bushels charcoal and 432 lbs. limestone, with 2612 lbs. of ore, will produce 1 ton of pig iron.

Produce of Charcoal from various Woods.

Apple	23.8	Birch	24.1	Oak	22.85	Red Pine	23.
Ash	26.7	Elm	25.1	" young	33.3	White Pine	23.5
Beech	21.1	Maple	22.9	Poplar	20.5	Willow	18.6

The produce of charcoal by a slow process of charring is very nearly 50 per cent. greater than by a quick process.

Wood.

Weights and Comparative Values of different Woods.

Woods.	Cord.	Value.	Woods.	Cord.	Value
	Lbs.			Lbs.	
Shell-bark Hickory	4469	1.	New Jersey Pine	2137	.54
Red-heart Hickory	3705	.81	Yellow Pine	1904	.43
White Oak	3821	.81	White Pine	1868	.42
Red Oak	3254	.69	Beech	—	.7
Virginia Pine	2689	—	Spruce	—	.52
Southern Pine	2375	—	Hemlock	—	.44
Hard Maple	2878	.6	Cottonwood	—	—

The evaporative power of 1 cubic foot of pine wood is equal to that of 1 cubic foot of fresh water; or, in the furnace of a steam-boiler and under pressure, it is 4¾ lbs. fresh water for 1 lb. of wood.

Northern Wood.—One cord of *hard* wood and one cord of *soft* wood, such as is used upon Lakes Ontario and Erie, is equal in evaporative effects to 2000 lbs. of anthracite coal.

Western Wood.—One cord of the description used by the river steamboats is equal in evaporative qualities to 12 bushels (960 lbs.) of Pittsburg coal.

9 cords cotton, ash, and cypress wood are equal to 7 cords of yellow pine.

The solid portion (*lignin*) of all woods, wherever and under whatever circumstances of growth, are nearly similar, the specific gravity being as 1.46 to 1.53.

The densest woods give the greatest heat, as charcoal produces greater heat than flame.

For every 14 parts of an ordinary pile of wood there are 11 parts of space; or a cord of wood in pile has 71.68 feet of solid wood and 56.32 feet of space.

Trees in the early part of April contain 20 per cent. more water than they do in the end of January.

Ash.

Proportion of Ash in 100 lbs. of several Woods.

Woods.	Wood.	Leaves.	Woods.	Wood.	Leaves.
	Per Cent.	Per Cent.		Per Cent.	Per Cent
Ash	.5	—	Elm	1.88	11.8
Beech	.35	5.4	Oak	.21	4.
Birch	.34	5.	Pitch Pine	.25	3.15

Peat.

Peat.—The proportion of ash in peat varies very much, ranging from 1.25 per cent. in grass peat to 18.47 per cent. in other varieties, the mean of Irish peat being about 3.5 per cent.

The distillation of peat produces, upon an average, Water 31 parts, Tar 3, Charcoal 29, and Gas 37.

In the distillation of peat, the following products have been obtained:

Charcoal, 41.1 per cent.; Watery Liquor, 19.3; Tar, .6; and Gaseous matter, 39.

Its evaporative power, in the furnace of a steam-boiler and under pressure, is from 3½ to 5 lbs. of fresh water per lb. of fuel.

Average Composition of Fuels.

	Specific Gravity.	Carbon.	Hydrogen.	Nitrogen.	Sulphur.	Oxygen.	Ash.	Percentage of Coke.
Bituminous Coals.								
Welsh	1.32	83.78	4.79	.98	1.43	4.15	4.91	72.6
Duffryn	1.33	88.26	4.66	1.45	1.77	.6	3.26	84.3
Newcastle	1.26	82.24	5.42	1.61	1.35	6.44	2.94	60.67
Scotch	1.26	78.53	5.61	1.	1.11	9.69	4.03	54.22
Derbyshire	1.29	79.85	4.84	1.23	.72	10.96	2.4	59.32
Lancashire	1.28	78.	5.23	1.32	1.	8.75	5.69	60.22
Sydney, S. W.	—	82.39	5.32	1.27	.07	8.32	2.04	58.
Borneo	1.28	64.52	4.74	.8	1.45	20.75	7.74	—
Formosa Island	1.24	78.26	5.7	.64	.49	10.95	3.96	—
Vancouver's Island	—	66.93	5.32	1.02	2.2	8.7	15.83	—
Chili, Conception Bay	1.29	70.55	5.76	.95	1.98	13.24	7.52	—
" Chiriqui	—	38.98	4.01	.58	6.14	13.38	36.91	—
Patagonia	—	62.25	5.05	.63	1.13	17.54	13.4	—
V. Diemen's Land, S. Cape	—	63.4	2.89	1.27	.98	1.01	30.45	—
V. Diemen's Land, Adv'te B.	—	80.22	3.05	1.36	1.9	4.8	8.67	—
Cannel, Wigan	1.23	79.23	6.08	1.18	1.43	7.24	4.84	60.33
Cumberland	—	93.81	1.82	—	—	2.77	1.6	—
Anthracite	1.5	88.54	—	—	.52	—	8.67	—
Oak	—	48.13	5.25	—	—	44.5	1.3	—
White Pine	—	49.95	6.41	—	—	43.65	.31	—
Birch	—	48.12	6.37	—	—	45.	.48	—
Charcoal, Oak	—	87.68	2.83	—	—	6.43	3.06	—
" Pine	—	71.36	5.95	—	—	22.19*	.3	—
" Maple	—	70.07	4.61	—	—	24.89*	.43	—
Peat, dense	—	61.02	5.77	.81	—	32.4	—	—
Patent, Warlich's	1.15	90.02	5.56	—	1.62	—	2.91†	85.1
" Wylam's	1.1	79.91	5.69	1.68	1.25	6.63	4.84	65.8

* Including Nitrogen. † Including Oxygen.

Weights, Evaporative Powers per Weight and Bulk, etc., of different Fuels.—(W. R. Johnson and others.)

Fuel.	Spec. Grav.	Weight per Cubic Foot.	Steam from Water at 212° by 1 lb. of Fuel.	Clinker from 100 lbs.	Cubic Feet required to Stow a Ton.
Bituminous.		Lbs.	Lbs.	Lbs.	No.
Cumberland, *maximum*	1.313	52.92	10.7	2.13	42.3
" *minimum*	1.337	54.29	9.44	4.53	41.2
Duffryn	1.326	53.22	10.14	—	42.09
Cannel, Wigan	1.23	48.3	7.7	—	46.37
Blossburgh	1.324	53.05	9.72	3.4	42.2
Midlothian, *screened*	1.283	45.72	8.94	3.33	49.
" *average*	1.294	54.04	8.29	8.82	41.4
Newcastle, Hartley	1.257	50.82	8.66	3.14	44.
Pictou	1.318	49.25	8.41	6.13	45.
Pittsburg	1.252	46.81	8.2	.94	47.8
Sydney	1.338	47.44	7.99	2.25	47.2
Carr's Hartley	1.262	47.88	7.84	1.86	46.7
Clover Hill, Va.	1.285	45.49	7.67	3.86	49.2
Cannelton, Ind.	1.273	47.65	7.34	1.64	47.
Scotch, Dalkeith	1.519	51.09	7.08	5.63	43.8
Chili	—	—	5.72	—	—
Japanese, Takasmia	1.231	48.3	—	—	—
Anthracite.					
Peach Mountain	1.464	53.79	10.11	3.03	41.6
Forest Improvement	1.477	53.66	10.06	.81	41.7
Beaver Meadow, No. 5	1.554	56.19	9.88	.6	39.8
Lackawanna	1.421	48.89	9.79	1.24	45.8
Welsh, Jones & Co.	1.375	58.25	9.46	—	38.45
Beaver Meadow, No. 3	1.61	54.93	9.21	1.01	40.7
Lehigh	1.59	55.32	8.93	1.08	40.5
Patent, Warlich's	1.15	69.05	10.36	—	32.44
Coke.					
Natural Virginia	1.323	46.64	8.47	5.31	48.3
Midlothian	—	32.7	8.63	10.51	68.5
Cumberland	—	31.57	8.99	3.55	70.9
Charcoal	—	24.	5.5	—	104.
Peat	—	30.	5.	—	75.
Wood.					
Pine wood, dry	—	21.01	4.69	—	106.6

(Sir H. de la Beche and Dr. Lyon Playfair, 1851.)

(*Averages of all Experiments.*)

Fuels.	Rate of Evaporation or lbs. Evaporated per Hour.	Weight per Cubic Foot.	Steam from Water at 212° by 1 lb. of Fuel.
	Lbs.	Lbs.	Lbs.
Coal. Welsh	448	53.1	9.05
Newcastle	411	49.8	8.37
Lancashire	448	49.7	7.94
Scotch	431	50.	7.7
Derbyshire	433	47.2	7.58
Patent Fuel. Warlich's	458	69.	10.36
Livingstone's	484	65.6	10.03
Lyon's	409	61.1	9.58
Wylam's	419	65.	8.9
Bell's	549	65.3	8.53

10 lbs. fresh water have been evaporated in a tubular boiler by 1 lb. of anthracite coal.

Mean Relative Evaporating Power of different Fuels and Total Heat of Combustion.

Fuel.	Water Evaporated from 212°.	Evaporate Power.	Total Heat in Thermal Units.
	Lbs.		
Anthracite coal	9.5	1.	15 225
Bituminous coal	8.75	.92	14 700
" caking	—	—	15 837
" cannel	—	—	15 080
Coke, natural	9.	.95	13 620
" artificial	8.5	.89	12 760
Pine wood	4.35	.45	7 215
Peat	5.5	.58	9 660
Patent fuel, Warlich's, *maximum*	10.36	1.09	—
" Bell's, *minimum*	8.53	.89	—

Relative Values of different Fuels.

Description of Coal, etc.	Pounds of Steam raised from Water at 212° Fahr. by 1 lb. of Fuel.	Relative Evaporative Power for equal Weights of Coal.	Relative Evaporative Power for equal Bulks of Coal.	Relative Rapidities of Ignition.	Relative Freedom from Waste.	Relative Completeness of Combustion.	Relative Weights.
Anthracites.							
Peach Mountain, Pa.	10.7	1.	1.	.505	.633	.725	.945
Beaver Meadow, No. 5	9.88	.923	.982	.207	.748	.6	1.
Bituminous.							
Newcastle	8.66	.809	.776	.595	.887	.346	.904
Pictou (Cunard's)	8.48	.792	.738	.588	.418	1.	.876
Liverpool	7.84	.733	.663	.581	1.	.333	.852
Cannelton, Ind.	7.34	.686	.616	1.	.984	.578	.848
Scotch	6.95	.649	.625	.521	.499	.649	.909
Pine wood, dry	4.69	.436	.175	—	16.417	—	—

Destructive Distillation of various Coals.

Coal.	Coke.	Tar.	Water.	Ammonia.	Carbon. Acid.	Sulph. Hydrogen.	Olefiant Gas and Hydrocarbon.	Other Gases Inflammable.
Anthracite	92.9	—	2.87	.2	.06	.04	—	3.93
Oldcastle Fiery Vein	79.8	5.86	3.39	.35	.44	.12	.27	9.77
Binea Coal	88.1	2.08	3.58	.08	1.68	.09	.31	4.08
Llangennach	83.69	1.22	4.07	.08	3.21	.02	.43	7.28

Miscellaneous.

One pound of anthracite coal in a cupola furnace will melt from 5 to 10 lbs. of cast iron; 8 bushels bituminous coal in an air furnace will melt 1 ton of cast iron.

Small coal produces about ¾ the effect of large coal of the same description.

Experiments by Messrs. Stevens at Bordentown, N. J., gave the following results: *Under a pressure of* 30 *lbs.*, 1 lb. pine wood evaporated 3.5 to 4.75 lbs. water. 1 lb. Lehigh coal, 7.25 to 8.75 lbs.

Bituminous coal is 13 per cent. more effective than coke for equal weights; and in England the effects are alike for equal costs.

Radiation from Fuel.—The proportion which the heat radiated from incandescent fuel bears to the total heat of combustion is,

From Wood .29 | From Charcoal and Peat .5

The least consumption of coal yet attained is 1½ lbs. per indicated horse-power. It usually varies in different engines from 2 to 8 lbs.

The bulk of pine wood is about 5⅓ times as great as its equivalent bulk of bituminous coal.

Experiments undertaken by the Baltimore and Ohio R. R. Co. determined the evaporating effect of 1 ton of Cumberland coal (2240 lbs.) equal to 1.25 tons of anthracite, and 1 ton of anthracite to be equal to 1.75 cords pine wood; also that 2000 lbs. Lackawanna coal were equal to 4500 lbs. best pine wood.

Relative Evaporation of several Combustibles in Pounds of Water, Heated 1° by 1 lb. of the Material.

Combustible.	Composition	Water	Combustible.	Composition.	Water.
		Lbs.			Lbs.
Alcohol....... .812	Hyd. .12, Carb. .45	8120	Oak wood, green...	—	5662
Bituminous Coal..	Hyd. .04, Carb. .75	9830	Olive Oil	Hyd. .13, Carb. .77	14560
Carbon...........	—	14220	Peat, charred......	Carb. .4	5620
Coke.............	Carb. .84	9028	" dry	—	3900
Hydrogen (mean).	—	50854	Pine wood, dry....	—	3618
Oak wood, dry....	Hyd. .06, Carb. .53	6018	Sulphuric Ether. .7	Hyd. 13, Carb. .6	8680
			Tallow............	—	14560

1 lb. Hydrogen will evaporate 62.6 lbs. water from 212°=60.509 lbs. heated 1°.

1 lb. Carbon " 14.6 lbs. " 212°, or raise 12 lbs. water at 60° to steam at 120 lbs. pressure.

A pound of Oxygen will generate the same quantity of heat whether in combustion with hydrogen, carbon, alcohol, or other combustible.

Areas and Productions of Coal Fields.

State.	Sq. Miles.	State	Sq. Miles.	State.	Sq. Miles.
Illinois..........	44000	Ohio............	11900	Tennessee.......	4300
Virginia	21000	Indiana.........	7700	Alabama	3400
Pennsylvania*...	15437	Missouri†.......	6000	Maryland........	550
Kentucky	13500	Michigan†......	5000	Georgia	150

Countries.	Area	Coal raised in 1845	Countries.	Area.	Coal raised in 1845.
	Sq. Miles.	Tons.		Sq. Miles.	Tons.
Great Britain	11 859	31 500 000	France..........	1729	4 141 617
Belgium	529	4 960 977	Prussia	—	3 500 000
United States.....	133 132	4 400 000	Austria	—	659 340

COMBUSTION.

Combustion is one of the many sources of heat, and denotes the combination of a body with any of the substances termed Supporters of Combustion; with reference to the generation of steam, we are restricted to but one of these combinations, and that is Oxygen.

All bodies, when intensely heated, become luminous. When this heat is produced by combination with oxygen, they are said to be ignited; and when the body heated is in a gaseous state, it forms what is termed Flame.

Carbon exists in nearly a pure state in charcoal and in soot. It combines with no more than 2⅔ of its weight of oxygen. In its combustion, 1 lb. of it produces sufficient heat to increase the temperature of 14 500 lbs. of water 1°.

Hydrogen exists in a gaseous state, and combines with 8 times its weight of oxygen, and 1 lb. of it, in burning, raises the heat of 50000 lbs. of water 1°‡

An increase in the rapidity of combustion is accompanied by a diminution in the evaporative efficiency of the combustible.

* Bituminous and Anthracite. † Anthracite. ‡ Mean effect.

COMBUSTION OF FUEL.

The constituents of coal are *Carbon*, *Hydrogen*, *Azote*, and *Oxygen*.

The volatile products of the combustion of coal are hydrogen and carbon, the unions of which (relating to combustion in a furnace) are *Carburetted hydrogen* and *Bi-carburetted hydroyen* or *Olefiant gas*, which, upon combining with atmospheric air, becomes *Carbonic acid* or *Carbonic Oxide*, *Steam*, and uncombined *Nitrogen.*

Carbonic oxide is the result of imperfect combustion, and Carbonic acid that of perfect combustion.

The perfect combustion of carbon evolves heat as 15 to 4.55 compared with the imperfect combustion of it, as when carbonic oxide is produced.

1 lb. carbon combines with 2.66 lbs. of oxygen, and produces 3.66 lbs. of carbonic acid.

Smoke is the combustible and incombustible products evolved in the combustion of fuel, which pass off by the flues of a furnace, and it is composed of such portions of the Hydrogen and Carbon of the fuel gas as have not been supplied or combined with oxygen, and consequently have not been converted either into Steam or Carbonic acid; the Hydrogen so passing away is invisible, but the Carbon, upon being separated from the Hydrogen, loses its gaseous character, and returns to its elementary state of a black pulverulent body, and as such it becomes visible.

The bituminous portion of coal is converted into the gaseous state alone, the carbonaceous portion only into the solid state. It is partly combustible and partly incombustible.

To effect the combustion of 1 cubic foot of coal gas, 2 cubic feet of oxygen are required; and, as 10 cubic feet of atmospheric air are necessary to supply this volume of oxygen, 1 cubic foot of gas requires the oxygen of 10 cubic feet of air.

In furnaces with a natural draught, the volume of air required exceeds that when the draught is produced artificially.

An insufficient supply of air causes imperfect combustion; an excessive supply, a waste of heat.

The quantity of atmospheric air that is chemically required for the combustion of 1 lb. of bituminous coal is 150.35 cubic feet. Of this, 44.64* cubic feet combine with the gases evolved from the coal, and the remaining 105.71 cubic feet combine with the carbon of the coal.

The combination of the gases evolved by combustion gives a resulting volume proportionate to the volume of atmospheric air required to furnish the oxygen, as 11 to 10. Hence the 44.64 cubic feet must be increased in this proportion, and it becomes $44.64 + 4.46 = 49.1$.

The gases resulting from the combustion of the carbon of the coal and the oxygen of the atmosphere, are of the same bulk as that of the atmospheric air required to furnish the oxygen, viz., 105.71 cubic feet. The total volume, then, of the atmospheric air and gases at the bridge wall, flues, or tubes, becomes $105.71 + 49.1 = 154.81$ cubic feet, assuming the temperature to be that of the external air. Consequently, the augmentation of volume due to the increase of the temperature of a furnace is to be considered and added to this volume in the consideration of the capacity of the flue or the *calorimeter* of a furnace.

There is required, then, to be admitted through the grate of a furnace for the combustion of 1 pound of bituminous coal as follows:

Coal containing 80 *per cent. of carbon, or* .7047 *per cent. of coke.*

1 lb. coal × 44.64 cubic feet of gas .... =	44.64
.7047 lb. carbon × 150 cubic feet of air =	105.71
	150.35 *cubic feet.*

* By experiment, 4.464 cubic feet of gas are evolved from 1 lb. of bituminous coal, requiring 44.64 cubic feet of air.

For anthracite, by the observations of W. R. Johnston, an increase of 30 per cent. over that for bituminous coal is required = 195.45 *cubic feet.*

Coke does not require as much air as coal, usually not to exceed 108 cubic feet, depending upon its purity.

The heat of an ordinary furnace may be safely considered at 1000°; hence the air entering the ash-pit and the gases evolved in the furnace under the general law of the expansion of permanently elastic fluids of $\frac{1}{479}$ths of its volume (or .002087) for each degree of heat imparted to it, the 154.81 is increased in volume from 100° (the assumed ordinary temperature of the air at the ash-pit) to 1000° = 900°; then 900 × .002087 = 1.8783 times, or $154.81 + \overline{154.81 \times 1.8783} = 445.59$ *cubic feet.*

If the combustion of the gases evolved from the coal and the air was complete, there would be required to give passage to the volume of but 445.59 cubic feet over the bridge wall or through the flues of a furnace; but by experiments it appears that about one half of the oxygen admitted beneath the grates of a furnace passes off uncombined, the area of the bridge wall, or the flues or tubes, must consequently be increased in this proportion, hence the 445.59 becomes 891.18.

The velocity of the gases passing from the furnace of a proper-proportioned boiler may be estimated at from 30 to 36 feet per second. Then $\frac{891.18}{60' \times 60'' \times 36} = .00687$ square feet, or .99 square inches, of area at the bridge wall for each pound of coal consumed per hour.

A limit, then, is here obtained for the area at the bridge wall, or of the flues or tubes immediately behind it, below which it must not be decreased, or the combustion will be imperfect. In ordinary practice it will be found advantageous to make this area .014 square feet, or 2 square inches for every pound of bituminous coal consumed per square foot of grate per hour, and so on in proportion for any other quantity.

The quantities of heat evolved are very nearly the same for the same substance, whatever the temperature of the combustible.

Relative Volumes of Air required for Combustion of Fuels.

	Lbs.		Lbs.		Lbs.
Charcoal	11.16	Anthracite Coal	12.13	Peat, dry	7.08
Coke	11.28	Bituminous "	10.98	Wood, dry	6.

The volume of air chemically required for the combustion of different woods in cubic feet is as follows:

Pine	158	Birch	153	Beech	152.9	Oak	154.4

Relative Volumes of Gases or Products of Combustion per Pound of Fuel.

Temp. Air.	Supply of Air per lb. of Fuel.			Temp. Air.	Supply of Air per lb. of Fuel.		
	12 lbs. Volume per lb.	18 lbs. Volume per lb.	24 lbs. Volume per lb.		12 lbs. Volume per lb.	18 lbs. Volume per lb.	24 lbs. Volume per lb.
	Cub. Feet.	Cub. Feet.	Cub. Feet.		Cub. Feet.	Cub. Feet.	Cub. Feet.
32°	150	225	300	212°	205	307	409
68°	161	241	322	392°	259	389	519
104°	172	258	344	572°	314	471	628

The perfect combustion of 1 lb. of carbon requires 12 lbs. air; hence the weight = 12 × 1. The total heat of combustion of 1 lb. carbon or charcoal is 14 500 thermal units; the mean specific heat of the products of combustion is .238, which, multiplied by 13 as above = 3094, and 14500*

* Mean of all experiments 13 964.

÷ 3094 = 4689° temperature of a furnace, assuming every atom of oxygen that was ignited in the furnace entered into combination.

If, however, as in the case in ordinary furnaces, twice the volume of air enters, then the products of combustion of 1 lb. of coal will be 25 lbs., which, multiplied by its specific heat as before, and if divided into 14 500, the quotient will be 2437°, which is the temperature of an ordinary furnace.

If 18 lbs. air per lb. of coal are furnished, as per blast or artificial draught, then 3207° is the resultant temperature.

Volumes of Products of Combustion at different Temperatures of Combustion.

Temperature.	Supply of Air per lb. of Carbon 12 lbs	18 lbs.	24 lbs.	Temperature.	Supply of Air per lb of Carbon 12 lbs.	18 lbs.	24 lbs.
	Volume.	Volume.	Volume		Volume.	Volume.	Volume.
	Cub feet.	Cub feet.	Cub feet		Cub feet.	Cub feet.	Cub feet.
32°	150	225	300	1112°	479	718	957
68°	161	241	322	1472°	588	882	1176
104°	172	258	344	1832°	697	1046	1395
212°	205	307	409	2500°	906	1359	1812
392°	259	389	519	3275°	1136	1704	—
752°	369	553	738	4640°	1551	—	—

Ratio of Combustion.—The quantity of fuel burned per hour per square feet of grate varies very much in different classes of boilers. In Cornish boilers it is 3½ lbs. per square foot; in Land boilers, 10 to 20 lbs.; (English) 13 to 14 lbs.; in Marine boilers (natural draught), 10 to 18 lbs.; (blast) 30 to 60 lbs.; and in Locomotive boilers, 80 to 120 lbs.

The volumes of air and smoke for each cubic foot of water converted into steam, is for coal and coke 2000 cubic feet, and for wood 4000 cubic feet; and for each lb. of fuel as follows:

Coal.... 207. Cannel coal .. 315. Coke.... 216. Wood.... 173.

To Compute the Consumption of Fuel in a Steam-Engine.

RULE.—Compute the volume of the cylinder to the point of cutting off the steam. Multiply the result by the number of cylinders, by twice the number of revolutions of the engine and by 60 (minutes), and divide the product by the density of the steam at its pressure in the cylinder, and the quotient will give the number of cubic feet of water expended in steam.

Multiply the number of cubic feet by 64.3125, divide the product by the evaporation of the boiler per lb. of fuel consumed, and the quotient will give the consumption in pounds per hour.

NOTE.—In computing the evaporation of water by the boiler of an engine, or the volume of steam used, the space displacement of the piston in its course is alone considered.

EXAMPLE.—The cylinder of a marine engine is 95 ins. in diameter by 10 feet stroke of piston; the pressure of the steam in the steam chest is 15.3 lbs. per square inch, cut off at ½ stroke; the number of revolutions 14½, and the evaporation estimated at 8 lbs. of salt water per lb. of coal; what is the consumption of coal per hour?

Volume of steam at above pressure, compared with water (15.3 + 14.7) = 883.
Area of 95 ins.=7088.2, which ÷ 144 = 49.22 *cubic feet.*

Point of cutting off 5 feet, and $49.22 \times 5 \times \overline{14.5 \times 2}$ (*strokes of piston*) × 60 (*minutes*) = 428214 *cubic feet = volume of steam per hour.*

Hence, 428214 ÷ 883 = 484.95 *cubic feet of water evaporated per hour*, and 484.95 × 64.3125 lbs. = 31188, which ÷ 8 = 3898.5 *lbs. coal per hour.*

NOTE.—The elements given are those of one engine of the Steamer Arctic, and the consumption of fuel for a run of 12 days (one engine) was 3850 lbs. per hour.

STEAM.

Steam, arising from water at its boiling point, is equal to the pressure of the atmosphere, which is 14.72322 lbs. at 60° upon a square inch.

In all calculations concerning steam, it is necessary to have some or all of the following elements, viz.:

Its *Pressure*, which is termed its tension or elastic force, and is expressed in pounds per square inch.

Its *Temperature*, which is the number of degrees of heat indicated by a thermometer immersed in it.

Its *Density*, which is the weight of a unit of its volume compared with that of water.

Its *Relative volume*, which is the space occupied by a given weight or volume of steam, compared with the weight or volume of the water that produced it.

Steam in contact with water is at its *maximum density.*

Each augmentation of 1 degree of Fahrenheit in the temperature of steam will produce an increase of .00202 of the volume occupied by the fluid at the temperature of 32°.

Under the pressure of the atmosphere alone, the temperature of water can not be raised above its boiling point.

The *Expansive* force of the steam of all fluids is the same at their boiling point.

A cubic inch of water, evaporated under the ordinary atmospheric pressure, is converted into 1700* cubic inches of steam, or, in a unit of measure, very nearly 1 cubic foot, and it exerts a mechanical force equal to the raising of 2120.14 lbs. 1 foot high.

27.2222 cubic feet of steam at the pressure of the atmosphere weigh 1 lb. avoirdupois.

A pressure of 1 lb. upon a square inch will support a column of mercury at a temperature of 60° 2.0376 inches in height; hence it will raise a mercurial siphon gauge one half of this, or 1.0188 inches.

A column of mercury 1 inch in height will counterbalance a pressure of .490774 lbs. upon a square inch.

The *Velocity* of steam, when flowing into a vacuum, is about 1550 feet per second when at an expansive power equal to the atmosphere, when at 10 atmospheres the velocity is increased to but 1780 feet; and when flowing into the air under a similar pressure it is about 650 feet per second, increasing to 1600 feet for a pressure of 20 atmospheres.

The *Boiling Points* of Water, corresponding to different heights of the barometer, is given under Heat, page 530.

The elasticity of the vapor of alcohol, at all temperatures, is about 2.125 times that of steam.

Thus, the volume of a cubic foot of water evaporated into steam is 1700 cubic feet; hence $1 \div 1700 = .00058823$, which represents the density or specific gravity of steam at the pressure of the atmosphere.

* Pole's Formula makes it 1712.

The *Specific Gravity* of steam, compared with air, is as the weight of a cubic foot of it compared with an equal volume of air. Thus the weight of a cubic foot of steam at the pressure of the atmosphere is 257.353 grains, and the weight of a like quantity of air at 34° is 527.04 grains. Hence $257.353 \div 527.04 = .4883$, *the specific gravity of steam* compared with air, and with water it is .00058823.

The volumes here given are from the results of experiments, and accord very nearly with the results deduced by the formulæ of Pambour, and also with that of Pole and Tate, which is,

$$m + n\ P^{a} = V. \quad m = 12.5,\ n = 20570, \text{ and } a = -.9301.$$

In the following table and calculations, the unit of measure is 1700 cubic inches.

Elastic Force, Temperature, Volume, and Density of Steam.

From a Temperature of 32° *to* 387.3°, *and from a Pressure of* .2 *to* 408 *Inches of Mercury.*

Temperature.	Elastic Force per Square Inch.		Volume.	Density.	Temperature.	Elastic Force per Square Inch.		Volume.	Density.
	In Mercury.	In Pounds.				In Mercury.	In Pounds.		
Deg.	Ins.	Lbs.	Cub. Ft.		Deg.	Ins.	Lbs.	Cub. Ft.	
32	.2	.098	187407	.0000053	214.5	31.62	15.5	1618	.000617
35	.221	.108	170267	.0000058	216.3	32.64	16.	1573	.000635
40	.263	.129	144529	.0000069	218.	33.66	16.5	1530	.000653
45	.316	.155	121483	.0000082	219.6	34.68	17.	1488	.000672
50	.375	.184	103350	.0000096	221.2	35.7	17.5	1440	.000694
55	.443	.217	88388	.0000113	222.7	36.72	18.	1411	.000708
60	.524	.257	75421	.0000132	224.2	37.74	18.5	1377	.000726
65	.616	.302	64762	.0000154	225.6	38.76	19.	1343	.000744
70	.721	.353	55862	.0000179	227.1	39.78	19.5	1312	.000762
75	.851	.417	47771	.0000209	228.5	40.8	20.	1281	.00078
80	1.	.49	41031	.0000244	229.9	41.82	20.5	1253	.000798
85	1.17	.573	35393	.0000282	231.2	42.84	21.	1225	.00081
90	1.36	.666	30425	.0000329	232.5	43.86	21.5	1199	.000834
95	1.58	.774	26686	.0000375	233.8	44.88	22.	1174	.000851
100	1.86	.911	22873	.0000437	235.1	45.9	22.5	1150	.000869
103	2.04	1.	20958	.0000477	236.3	46.92	23.	1127	.000886
105	2.18	1.068	19693	.00005	237.5	46.94	23.5	1105	.000904
110	2.53	1.24	16667	.000059	238.7	48.96	24.	1184	.000922
115	2.93	1.431	14942	.000066	239.9	49.98	24.5	1064	.000939
120	3.33	1.632	13215	.000075	241.	51.	25.	1044	.000957
125	3.79	1.857	11723	.000085	243.3	53.04	26.	1007	.000993
130	4.34	2.129	10328	.000096	245.5	55.08	27.	973	.001027
135	5.	2.45	9036	.00011	247.6	57.12	28.	941	.001062
140	5.74	2.813	7938	.000125	249.6	59.16	29.	911	.001097
145	6.53	3.1	7040	.000142	251.6	61.2	30.	883	.001132
150	7.42	3.636	6243	.00016	253.6	63.24	31.	857	.001166
155	8.4	4.116	5559	.000179	255.5	65.28	32.	833	.0012
160	9.46	4.635	4976	.0002	257.3	67.32	33.	810	.001234
165	10.68	5.23	4443	.000225	259.1	69.36	34.	788	.001269
170	12.13	5.94	3943	.000253	260.9	71.4	35.	767	.001304
175	13.62	6.67	3538	.000282	262.6	73.44	36.	748	.001337
180	15.15	7.42	3208	.000311	264.3	75.48	37.	729	.001371
185	17.	8.33	2879	.000347	265.9	77.52	38.	712	.001404
190	19.	9.31	2595	.000385	267.5	79.56	39.	695	.001438
195	21.22	10.4	2342	.000426	269.1	81.6	40.	679	.001472
200	23.64	11.58	2118	.000472	270.6	83.64	41.	664	.001506
205	26.13	12.8	1932	.000517	272.1	85.68	42.	649	.00154
210	28.84	14.13	1763	.000567	273.6	87.72	43.	635	.001574
211	29.41	14.41	1730	.000578	275.	89.76	44.	622	.001607
212	30.	14.7	1700	.000588	276.4	91.8	45.	610	.001639
212.8	30.6	15.	1669	.0005[illegible]9	277.8	93.84	46.	598	.001672

Temperature.	Elastic Force per Square Inch.		Volume.	Density.	Temperature.	Elastic Force per Square Inch.		Volume.	Density.
	In Mercury.	In Pounds.				In Mercury.	In Pounds.		
Deg.	Ins.	Lbs.	Cub. Ft.		Deg.	Ins.	Lbs.	Cub Ft.	
279.2	95.88	47.	586	.001706	314.	159.14	78.	370	.002703
280.5	97.92	48.	575	.001739	314.9	161.18	79.	366	.002732
281.9	99.96	49.	564	.001773	315.8	163.22	80.	362	.002762
283.2	102.	50.	554	.001805	316.7	165.26	81.	358	.002793
284.4	104.04	51.	544	.001838	317.6	167.3	82.	354	.002824
285.7	106.08	52.	534	.001872	318.4	169.34	83.	350	.002828
286.9	108.12	53.	525	.001904	319.3	171.38	84.	346	.00289
288.1	110.16	54.	516	.001937	320.1	173.42	85.	342	.002923
289.3	112.2	55.	508	.001968	324.3	183.62	90.	325	.003076
290.5	114.24	56.	500	.002	328.2	193.82	95.	310	.003225
291.7	116.28	57.	492	.002032	332.	203.99	100.	292	.003389
292.9	118.32	58.	484	.002066	335.8	214.19	105.	282	.003546
294.2	120.36	59.	477	.002096	339.2	224 39	110.	271	.00369
295.6	122.4	60.	470	.002127	342.7	234.59	115.	259	.003861
296.9	124.44	61.	463	.002159	345.8	244.79	120.	251	.003984
298.1	126.48	62.	456	.002192	349.1	254.99	125.	240	.004166
299.2	128.52	63.	449	.002227	352.1	265.19	130.	233	.004291
300.3	130.56	64.	443	.002257	355.	275.39	135.	224	.004464
301.3	132.6	65.	437	.002288	357.9	285.59	140.	218	.004587
302.4	134.64	66.	431	.00232	360.6	295.79	145.	210	.004761
303.4	136.68	67.	425	.002352	363.4	306.	150.	205	.004879
304.4	138.72	68.	419	.002386	366.	316.19	155.	198	.00505
305.4	140.76	69.	414	.002415	368.7	326.39	160.	193	.005181
306.4	142.8	70.	408	.002451	371.1	336.59	165.	187	.005347
307.4	144.84	71.	403	.002481	373.6	346.79	170.	183	.005464
308.4	146.88	72.	398	.002512	376.	357.	175.	178	.005617
309.3	148.92	73.	393	.002544	378.4	367.2	180.	174	.005747
310.3	150.96	74.	388	.002577	380.6	377.1	185.	169	.005919
311.2	153.02	75.	383	.00261	382.9	387.6	190.	166	.006024
312.2	155.06	76.	379	.002638	384.1	397.8	195.	161	.006211
313.1	157.1	77.	374	.002673	387.3	408.	200.	158	.006329

The Temperatures of steam, as deduced from the experiments of the Comm. of the Franklin Institute, range somewhat different from those given in the above table.

Thus, at the pressure of the atmosphere, the temperatures are alike, viz., 212°; but at 6.4 atmospheres = 192 inches of mercury, they are 320° and 327.5°.

To Compute the Pressure of Steam.

When the Height of the Column of Mercury it will Support is given. Rule.—Divide the height of the column of mercury in inches by 2.0376, and the quotient will give the pressure per square inch in pounds.

Example.—The height of a column of mercury is 203.76 inches; what pressure per square inch will it contain?

$$203.76 \div 2.0376 = 100 \; lbs.$$

To Compute the Temperature of Steam.

Rule.—Multiply the 6th root of its force in inches of mercury by 177, and subtract 100 from the product, the remainder will give the temperature in degrees.

Example.—When the elastic force of steam is equal to a pressure of 49 inches of mercury, what is its temperature?

Note.—To extract the 6th root of a number, ascertain the cube root of its square root.

$$\sqrt{\ } \text{ of } 49 = 7, \text{ and } \sqrt[3]{\ } \text{ of } 7 = 1.9129. \quad \text{Hence } 1.9129 \times 177 - 100 = 238°.58.$$

To Compute the Pressure of Steam in Inches of Mercury.

When the Temperature is given. Rule.—Add 100 to the temperature, divide the sum by 177, and the 6th power of the quotient will give the pressure in inches of mercury.

EXAMPLE.—The temperature of steam is 132°; what is its pressure?

$$\frac{100+312}{177}=2.3277, \text{ and } 2.3277^6=150 \text{ ins.}$$

NOTE.—To involve the 6th power of a number, square its cube.

To Compute the Specific Gravity of Steam compared with Air.

RULE.—Divide the constant number 830.11 (1700 × .4883) by the volume of the steam at the temperature of pressure at which the gravity is required.

EXAMPLE.—The pressure of steam is 60 lbs., and the volume of it is 470; what is its specific gravity?

$$830.11 \div 470 = 1.766.$$

NOTE.—The specific gravity of steam compared with water = .00058823.

To Compute the Volume a Cubic Foot of Water occupies in Steam.

When the Elastic Force and Temperature of the Steam are given. RULE.—To 459 add the temperature in degrees, and multiply the sum by 76.5; divide the product by the elastic force of the steam in inches of mercury, and the quotient will give the volume required.

NOTE.—When the force in inches of mercury is not given, multiply the pressure in pounds per square inch by 2.0376.

Or, $\frac{1+.00202\times(t-32)}{p}\,18329 = volume$; *p representing the pressure of the steam per square inch, and t the temperature.*

This formula is based upon the discovery of Gay-Lussac, viz., that if the temperature of a given weight of any elastic fluid be made to vary, its tension being the same, it will receive augmentation of volume exactly proportional to the augmentation of temperature, and for every increase of 1°, will be produced an increase of .00202 of the volume occupied by the fluid at a temperature of 32°.

EXAMPLE.—The temperature of a cubic foot of water evaporated into steam is 376°, and the elastic force is 357 inches; what is its volume?

$$\frac{459+376\times76.5}{357}=\frac{63877.5}{357}=178.93 \text{ cubic feet.}$$

To Compute the Density or Specific Gravity of Steam under different Pressures.

When the Volume is given. RULE.—Divide 1 by the volume in cubic feet, and the quotient will give the density required.

EXAMPLE.—The volume is 210; what is the density?

$$1 \div 210 = .004761.$$

When the Pressure is given.—Take the temperature due to the pressure, and proceed as by the foregoing rule to compute the volume, which, when obtained, proceeds as above.

To Compute the Volume of Water contained in a given Volume of Steam.

When its Density is given. RULE.—Multiply the volume of the steam by its density, and the product will give the volume of the water in cubic feet.

Hence, *To Compute the weight of the water in pounds,* Multiply the product by 62.5.

EXAMPLE.—The density of a volume of 17000 cubic feet of steam is .0011765; what is the weight of it in pounds?

$17000 \times .0011765 = 20 = $ *volume of water*, and $20 \times 62.5 = 1250$ *pounds.*

To Compute the Volume of Steam required to raise a Given Volume of Water to any Given Temperature.

RULE.—Multiply the water to be heated by the difference of temperatures between it and that to which it is to be raised, for a dividend; then to the temperature of the steam add 966°.6, and from that sum take the required temperature of the water for a divisor; the quotient will give the volume of steam in the same terms as the water.

EXAMPLE.—What volume of steam at 212° will raise 100 cubic feet of water at 80° to 212°?

$\frac{100 \times \overline{212^\circ - 80^\circ}}{212^\circ + 966^\circ.6 - 212^\circ} = \frac{13200}{966.6} = 13.66$ *cubic feet of water formed into steam, occupying* (13.66×1700) 232 22 *cubic feet.*

To Compute the Velocity with which Steam Flows into a Vacuum.

RULE.—To the temperature of the steam add the constant 459, and multiply the square root of the sum by 60.2; the quotient will give the velocity in feet per second.

To Compute the Number of Cubic Inches of Water, at any Given Temperature, that must be mixed with a Cubic Foot* of Steam to raise or reduce the Mixture to any Required Temperature.

RULE.—From the required temperature subtract the temperature of the water; then ascertain how often the remainder is contained in the required temperature subtracted from 1178°.6, and the quotient will give the quantity required.

The sum of the Sensible and Latent Heats for several temperatures will be found under Heat, p. 524.

EXAMPLE.—The temperature of the condensing water of an engine is 80°, and the required temperature 100°; what is the proportion of condensing water to that evaporated?

$100 - 80 = 20$. Then, $\frac{1178.6 - 100}{20} = 53.93$ *cubic inches to* 1.

Or, let *w represent temperature of condensing water, t the required temperature, and* S *the sum of sensible and latent heats.*

Then $\frac{S - t}{t - w} = volume.$

When the Temperature of the Steam is given.

$\frac{l + \overline{T - t}}{t - w} = volume$; *l representing the latent heat, and* T *the temperature of the steam in the cylinder.*

EXAMPLE.—The temperature of the steam in a cylinder is 230°, and the other elements the same as in the preceding example; required the volumes of injection water. Sum of sensible and latent heat of steam at 230° = 1182.2.

$$\frac{\overline{1182.2 - 230} + \overline{230 - 100}}{100 - 80} = \frac{1082.2}{20} = 54.11 \textit{ volumes.}$$

To Compute the Temperature of Water in the Condenser or Reservoir of a Steam-engine.

$\frac{l + T + \overline{V \times w}}{V + 1} = t$; V *representing volume of injection water.*

EXAMPLE.—The elements the same as in the preceding example.

$\frac{952^\circ.2 + 230 + \overline{54.11 \times 80}}{54.11 + 1} = \frac{5511}{55.11} = 100^\circ.9$

* The exact volume is assumed to be 1700 cubic ins. Hence, when accuracy is required, the result, as determined by the rule, must be ×1728, and ÷1700.

Elastic Force and Temperatures of the Vapors of Alcohol, Sulphuric Ether, Sulphuret of Carbon, Petroleum, and Turpentine.

Temp.	Force in Inches of Mercury.	Temp.	Force in Inches of Mercury.	Temp.	Force in Inches of Mercury.	Temp.	Force in Inches of Mercury.
ALCOHOL.		175°	30.	94°	24.70	279.5°	300.
32°	.4	180	34.73	96 }	30.	347.	606.
50	.86	200	53.	104 }			
60	1 23	212	67 5	120	39.47	PETROLEUM.	
70	1.76	220	78.5	150	67.6	316°	30.
80	2.45	240	111.24	212	178.	345	44.1
90	3.4	264	166.1	SULPHURET OF CARBON.		375	64.
100	4.5						
120	8.1	ETHER.		53.5°	7.4	OIL OF TURPENTINE.	
130	10.6	34°	6.2	72.5	12.55	304°	30.
140	13.9	54	15.3	110.	80.	340	47.78
160	22.6	74	16.2	212.	126.	362	62.4

Loss of Pressure of Steam in an Engine by Imperfect Condensation.

Temp.	Square Inch.	Temp.	Square Inch.	Temp.	Square Inch.
	Lbs.		Lbs.		Lbs.
60°	.26	100°	.93	140°	2.88
80°	.5	120°	1.65		

Velocity of Flow of Steam into the Atmosphere per Second.

Pressure above Atmosphere.	Velocity.	Pressure above Atmosphere.	Velocity.	Pressure above Atmosphere.	Velocity.
Lbs.	Feet.	Lbs.	Feet.	Lbs.	Feet.
1	482	10	1241	50	1791
3	791	20	1504	70	1877
5	973	30	1643	100	1957

Mechanical Equivalent of Heat contained in Steam.

1 lb. water heated from 32° to 212°, requires as much heat as would raise 180 lbs. 1°. Hence 180
1 lb. water at 212°, converted into steam at 212° (14.7 lbs.), absorbs as much heat for its conversion as would raise 966.6 lbs. water 1°. Hence .... 966°.6

1146°.6

This number, 1146.6, is a *constant*, and expresses the units of heat in 1 lb. of steam from 32° up to the temperature at which the conversion takes place.

Thus, 1 lb. water heated from 32° to 332°, requires as much heat as would raise 300 lbs. 1°. Hence .... 300°
And 1 lb. water at 332°, converted into steam at 332° (100 lbs.), absorbs as much heat for its conversion as would raise 846.6 lbs. water 1°. Hence .... 846°.6

1146°.6

The *Mechanical Equivalent*, or maximum theoretical duty of this quantity of heat, as contained in 1 lb. of steam, is 772 lbs. × 1146.6 *units of heat* = 885175.2 lbs. *raised* 1 *foot high*.

The amount of duty realized in the production and use of 1 lb. of steam falls far short of this theoretical computation.

MIXTURE OF AIR AND STEAM.

Water contains a portion of air or other uncondensable gaseous matter, and when it is converted into steam, this air is mixed with it, and when the steam is condensed it is left in a gaseous state. If means were not taken to remove this air or gaseous matter from the condenser of a steam-engine, it would fill it and the cylinder, and obstruct their operation; but, notwithstanding the ordinary means of removing it (by the air-pump), a certain quantity of it always remains in the condenser.

20 volumes of water absorb 1 volume of air.

STEAM ACTING EXPANSIVELY.

To Compute the mean Pressure of Steam upon a Piston by Hyperbolic Logarithms.

Rule.—Divide the length of the stroke of a piston, added to the clearance in the cylinder at one end, by the length of the stroke at which the steam is cut off, added to the clearance at that end, and the quotient will express the relative expansion of the steam or *number*.

Find in the table the logarithm of the *number* nearest to that of the quotient, to which add 1. The sum is the ratio of the gain.

Multiply the ratio thus obtained by the pressure of the steam (including the atmosphere) *as it enters the cylinder*, divide the product by the relative expansion, and the quotient will give the mean pressure required.

Table of Hyperbolic Logarithms.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1.05	.049	2.65	.975	4.25	1.447	5.8	1.758	7.4	2.001
1.1	.095	2.66	.978	4.3	1.459	5.85	1.766	7.45	2.008
1.15	.14	2.7	.993	4.33	1.465	5.9	1.775	7.5	2.015
1.2	.182	2.75	1.012	4.35	1.47	5.95	1.783	7.55	2.022
1.25	.223	2.8	1.03	4.4	1.482	6.	1.792	7.6	2.028
1.3	.262	2.85	1.047	4.45	1.493	6.05	1.8	7.65	2.035
1.33	.285	2.9	1.065	4.5	1.504	6.1	1.808	7.66	2.036
1.35	.3	2.95	1.082	4.55	1.515	6.15	1.816	7.7	2.041
1.4	.336	3.	1.099	4.6	1.526	6.2	1.824	7.75	2.048
1.45	.372	3.05	1.115	4.65	1.537	6.25	1.833	7.8	2.054
1.5	.405	3.1	1.131	4.66	1.54	6.3	1.841	7.85	2.061
1.55	.438	3.15	1.147	4.7	1.548	6.33	1.845	7.9	2.067
1.6	.47	3.2	1.163	4.75	1.558	6.35	1.848	7.95	2.073
1.65	.5	3.25	1.179	4.8	1.569	6.4	1.856	8.	2.079
1.66	.506	3.3	1.194	4.85	1.579	6.45	1.864	8.05	2.086
1.7	.531	3.33	1.202	4.9	1.589	6.5	1.872	8.1	2.092
1.75	.56	3.35	1.209	4.95	1.599	6.55	1.879	8.15	2.098
1.8	.588	3.4	1.224	5.	1.609	6.6	1.887	8.2	2.104
1.85	.612	3.45	1.238	5.05	1.619	6.65	1.895	8.25	2.11
1.9	.642	3.5	1.253	5.1	1.629	6.66	1.896	8.3	2.116
1.95	.668	3.55	1.267	5.15	1.639	6.7	1.902	8.33	2.119
2.	.693	3.6	1.281	5.2	1.649	6.75	1.91	8.35	2.122
2.05	.718	3.65	1.295	5.25	1.658	6.8	1.917	8.4	2.128
2.1	.742	3.66	1.297	5.3	1.668	6.85	1.924	8.45	2.134
2.15	.765	3.7	1.308	5.33	1.673	6.9	1.931	8.5	2.14
2.2	.788	3.75	1.322	5.35	1.677	6.95	1.939	8.55	2.146
2.25	.811	3.8	1.335	5.4	1.686	7.	1.946	8.6	2.152
2.3	.833	3.85	1.348	5.45	1.696	7.05	1.953	8.65	2.158
2.33	.845	3.9	1.361	5.5	1.705	7.1	1.96	8.66	2.159
2.35	.854	3.95	1.374	5.55	1.714	7.15	1.967	8.7	2.163
2.4	.875	4.	1.386	5.6	1.723	7.2	1.974	8.75	2.169
2.45	.896	4.05	1.399	5.65	1.732	7.25	1.981	8.8	2.175
2.5	.916	4.1	1.411	5.66	1.733	7.3	1.988	8.85	2.18
2.55	.936	4.15	1.423	5.7	1.74	7.33	1.991	8.9	2.186
2.6	.956	4.2	1.435	5.75	1.749	7.35	1.995	8.95	2.192

NOTE.—The Hyp. Log. of any number not in the table may be found by multiplying a common log. by 2.302585053, usually by 2.3.

EXAMPLE.—Assume steam to enter a cylinder at a pressure of 34.7 lbs. per square inch, and to be cut off at ¼ the length of the stroke of the piston, the stroke being 10 feet; what will be the mean pressure?

10 feet + .5 for clearance = 120.5 *ins.*, stroke 10 ÷ 4 + .5 for clearance = 30.5 *ins.* Then 120.5 ÷ 30.5 = 3.95, *the relative expansion.*
Log. of number 3.95 = 1.374, which + 1 = 2.374.

$$\frac{2.374 \times 34.7}{3.95} = \frac{82.3778}{3.95} = 20.855 \textit{ lbs.}$$

When the Relative Expansion or Number falls between two Numbers in the Table, Proceed as follows. Take the difference between the logs. of the two numbers. Then, as the difference between the numbers is to the difference between these logs., so is the excess of the expansion over the least number, which, added to the least log., will give the log. required.

ILLUSTRATION.—The expansion is 4.84, the logs. for 4.8 and 4.85 are 1.569 and 1.579, and their difference .01. Hence, as 4.85 ∞ 4.8 = .05 : 1.579 ∞ 1.569 = .01 : : 4.84 — 4.8 = .04 : .008, and 1.569 + .008=1.577 = *the log. required.*

Effect of Expansion with Equal Volumes of Steam.

The theoretical economy of using steam expansively is as follows. A like volume of steam being expended in each case, and expanded to fill the increased spaces.

Point of Cutting Off.	Expansion Number.	Mean Pressure of Steam.	Gain per Cent. in Power.	Point of Cutting Off.	Expansion Number.	Mean Pressure of Steam.	Gain per Cent. in Power.
.1	10.	3.302	230.	.5	2.	1.693	69.3
.125	8.	3.079	208.	.6	1.66	1.507	50.7
.166	6.	2.791	179.	.625	1.6	1.47	47.
.2	5.	2.609	161.	.666	1.5	1.405	40.5
.25	4.	2.386	139.	.7	1.42	1.351	35.1
.3	3.33	2.203	120.	.75	1.33	1.285	22.3
.333	3.	2.099	110.	.8	1.25	1.223	20.5
.375	2.66	1.978	97.8	.875	1.143	1.131	13.1
.4	2.5	1.916	91.6	.9	1.11	1.104	10.4

In this illustration, no deductions are made for a reduction of the temperature of the steam while expanding or for loss by back pressure.

The same relative advantage follows in expansion as above given, whatever may be the initial pressure of the steam.

Gain in Fuel, and Initial Pressure of Steam required, when Acting Expansively, compared with Non-Expansion or Full Stroke.

Point of Cutting Off.	Gain in Fuel.	Initial Pressure required.		Point of Cutting Off.	Gain in Fuel.	Initial Pressure required.	
		Cutting Off.	Full Stroke.			Cutting Off.	Full Stroke.
Stroke.	Per Cent.	Lbs.	Lbs.	Stroke.	Per Cent.	Lbs.	Lbs.
⅞	11.7	1.01	1.	⅜	49.6	1.32	1.
¾	22.4	1.03	1.	¼	58.2	1.67	1.
⅝	32.	1.09	1.	⅛	67.6	2.6	1.
½	41.	1.18	1.				

The *Relative Effect* of Steam during Expansion is obtained from the preceding rule.

The *Mechanical Effect* of Steam in a cylinder is the product of the mean pressure in lbs., and the distance through which it has passed in feet.

The *Pressure at the End of a Stroke*, or *at any Given Point of the Stroke*, is obtained by dividing the initial pressure by the portion of the stroke performed when the steam is cut off.

The *Per Cent. of Gain by Expansion* is obtained by multiplying the relative expansion by 100.

The *Back Pressure* is the force of the uncondensed steam in a cylinder, consequent upon the impracticability of obtaining a perfect vacuum, and is opposed to the course of a piston. It varies from 3 to 5 lbs. per square inch.

ILLUSTRATIONS.—The initial pressure of steam admitted to a cylinder having a stroke of 9 feet, is 50 lbs. per square inch, cut off at ⅓ the stroke, the back pressure being 2 lbs.; what is the relative effect of the steam, its mechanical effect, its mean pressure upon the piston at the end of the stroke, at ⅔ of the stroke, and the gain per cent.?

The back pressure is assumed at 2 lbs.

(Hyp. log. 9 ÷ 3) + 1 = 2.099 *relative effect.*

$2.099 \times \overline{50 \div 3 - 2} = 30.78$ *lbs.*, and 9 × 30.78 = 277.02 *lbs. mechanical effect.*

50 ÷ 3 − 2 = 14.66 *lbs. at the end of the stroke*, 50 ÷ 2 — 2 = 23 *lbs. at ⅔ the stroke.*

1.099 × 100 = 109.9 *per cent.*

There is an economy in the use of super-heated steam when commixed with steam of about 10 per cent.

When the pressure of steam flowing full stroke is given. Its initial pressure can be ascertained by multiplying the unit in the last column of the preceding table by the pressure of the steam.

The Results there given are theoretical. In practice, from the resistance to expansion of the back pressure in a cylinder, from the loss of temperature by cooling and from the friction of the passages, these results are materially reduced.

Gain in Fuel.

When steam is cut off before the termination of the stroke of a piston, the pressure effected by it, for the portion at which it flowed for full stroke, is represented by 1, and the pressure exerted afterward by the result due to the relative expansion, and the total pressure or work effected is represented by the sum of these units.

Upon the other hand, if the steam had flowed for the full stroke of the piston, the pressure or work effected would have been 1 added to the proportionate distance for which the steam was expended in the case of expansion.

ILLUSTRATION.—What are the relative pressures or work effected in steam cut off at ½ and ¾ stroke, and what at full stroke?

Cut off at ½, by rule and table = 1.693. Then 1 + 1 = 2, and 2 and 1.693 the relative effects.

Cut off at ¾, by rule and table = 1.285. Then 1 + .33 = 1.33, and 1.33 and 1.285 the relative effects.

At full stroke 1 + .33 (the proportionate distance of ¼ in ¾) = 1.33, the relative effect.

To Compute the Gain in Fuel.

RULE.—Divide the relative effect of the steam by the number of times the steam is expanded, and divide 1 by the quotient; the result is the initial pressure of steam required to be expanded to produce a like effect to steam at full stroke.

Divide this pressure by the number of times the steam is expanded, and subtract the quotient from 1, the remainder will give the gain per cent. in fuel.

EXAMPLE.—When steam is cut off at ½ the stroke of the piston, what is the gain in fuel?

Relative effect = 1.693, number of times of expansion = 2. 1.693 ÷ 2 = .8465, and 1 ÷ .8465 = 1.18 *initial pressure.*

1.18 ÷ 2 = .59, and 1 — .59 = 41 *per cent.*

STEAM-ENGINE.

The extremes of proportions here given are for the particular requirements of variations in speed, pressures, and differences in draughts of water, etc., in the varied purposes of Marine, River, and Land practice.

CONDENSING ENGINE.

For a Range of Pressures of from 10 to 60 lbs. (Mercurial Gauge) per Square Inch, Cut Off at One Half the Stroke, and for from 15 to 60 Revolutions per Minute.

Piston Rod. Its diameter .1 that of the cylinder or air-pump for which it is designed. If of steel, .8 the diameter of wrought iron; and if an air-pump rod is of copper or brass, .11 and .125 the diameter of pump.

Condenser (*Jet*). The capacity of it should be from .35 to .45 that of the steam cylinder. (*Surface.*) The area of its condensing surfaces should be .55 to .65 of that of the evaporating surface of the boiler, when a natural draught is employed; but with a blower, or forced draught, this proportion should be increased to .6 and .75.

When a *Circulating pump* is used, the area may be reduced to .45 and .5.

Air-pump (*Single acting and direct connection*). The capacity of it should be from .1 to .2 that of the steam cylinder.

Foot Valve. The dimensions of it should give an area of from .25 to .5 of the area of the air-pump in inches, the mean being .375 for 37½ revolutions.

Delivery Valve. When a solid piston is used in the air-pump, its area should correspond with that of the foot valve; but when an open piston alone is used, this proportion may not be obtained.

Out-board Delivery Valve. The area of it should be from .5 to .8 that of the foot valve.

Steam and Exhaust Valves (*Puppet*), $\frac{a \times s \times r}{24\,000} =$ *area for steam*, and $\frac{a \times s \times r}{20\,000} =$ *area for exhaust*; (*Slide*), $\frac{a \times s \times r}{30\,000} =$ *area for steam*, and $\frac{a \times s \times r}{22\,750} =$ *area for exhaust. a representing area of steam cylinder in sq. ins., s stroke of pistons in ins., and r number of revolutions per minute.*

Injection Cocks. There should be two to each condenser, the area of each sufficient to supply 70 times the quantity of water evaporated when the engine is working at its maximum; and in all *marine* and *river* engines, there should be three, viz., a Bottom, Side, and Bilge.

The Bilge injection is properly a branch of the bottom injection pipe, and may be of less capacity.

The proportions here given will admit of a sufficient volume of water when the engine is in operation in the Gulf Stream, where the water is at times at the temperature of 84°, and the volume required to give the water of condensation a temperature of 100° is 70 times that of the quantity evaporated.

Feed Pump (*Single acting*, *Marine*). Its volume should be .007 to .01 that of the steam cylinder. (*River* and *Land*), or when fresh water alone is used, .0035 to .004.

NON-CONDENSING ENGINE.

For a Range of Pressures of from 50 to 150 lbs. (Mercurial Gauge) per Square Inch, Cut Off at One Half the Stroke, and for 30 to 100 Revolutions per Minute.

Piston Rod. Its diameter should be from .125 to .2 that of the steam cylinder. If of steel, .8 the diameter of wrought iron.

Steam and Exhaust Valves (*Puppet*). Their area is determined by the rule given for them in a condensing engine, using for divisors 30 000 and 22 750.

A decrease in the capacity of the cylinder is not attended with a proportionate decrease of their area. A 12-inch cylinder by 4 feet stroke has 9 ins. area of valve, which is 1 inch in every 600 ins. capacity; and a 6-inch cylinder by 1 foot stroke has 2 ins. area, which is 1 inch in every 125 ins. capacity.

Feed Pump (*Single acting, Marine*). Its volume should be from .025 to .033 that of the steam cylinder. (*River* or *Land*), or where fresh water alone is used .0133 to .014.

GENERAL RULES.

Engines.

Steam Cylinder, Thickness.—(*Vertical.*) Multiply its diameter by the extreme pressure of steam in pounds per square inch that it may be subjected to, and divide the product by 2400; the result will give the thickness in inches. (*Horizontal*), divide by 2000. (*Inclined*), divide as above, in a ratio inversely as the sine of the angle of inclination.

Shafts, Gudgeons, etc.—To resist torsion. See Rules for Torsional strength, pp. 485, 487.

Journals of Shafts.—Their length should be from 1.15 to 1.25 times their diameter, and in *main centres* they should be 1.5 times.

The *bearings* of driving and propeller shafts are not here considered as journals.

Cross Heads, Wrought iron (*Cylinder*). Its section in the centre should be determined by the following formulæ:

$\frac{a \times p \times l}{700} = s$, and $\sqrt{\frac{s}{b}} = d$, or $\frac{s}{d^2} = b$; *a representing area of cylinder in ins., p the extreme pressure in pounds per square inch that it may be subjected to, l the length of the cross head between the centres of its journals in feet, and s the product of the square of the depth d, and the breadth, b, of the section.* (*Air-pump*). $\frac{a \times l}{18} = s$, and as above for d and b.

If the section of either of these cross heads is cylindrical, for s put $\sqrt[3]{s \times 1.7}$.

Diameter of boss twice, and diameter of end journals the same as that of the piston rod. Section at ends .5 that of their centre.

Steam Pipe.—Its area should exceed that of the steam valve.

Connecting Rod.—Its length should be 2.25 times the stroke of the piston when it is at all practicable to afford the space. When, however, it is imperative to reduce this proportion, it may be a little less than twice the stroke.

The comparative friction of long and short connecting rods is, for once the length of stroke of piston, 12 per cent. additional; twice, 3 per cent.; and thrice, 1.33.

Neck.—Its diameter should be from 1 to 1.1 that of the piston rod.

Centre of the body (*Horizontal*), its diameter is ascertained in the following manner:

Multiply the length of the body (between the necks) in feet by the area of a neck in inches, divide the product by ¾ the stroke of the piston or the throw of the rod in feet, and the quotient is the area of the centre in square inches, from which the diameter may be determined. (*Vertical*). The area deduced by this rule may be somewhat less.

Connecting Links.—Their length should be .5 of that of the stroke or of the throw of their attachments.

Where a pair of connecting rods is used, as in some descriptions of engines, and with a pair of connecting links, their necks should have an area of .7 to .75 of that of the attached rod.

When a second set of connecting rods or links are used, as with some air-pump connections, etc., their necks should have an area, in a ratio, inversely as their throw to that of the first set.

Straps of Connecting Rods, Links, etc.—The area of the *Strap* at its least section should be .65 that of the neck of the attached rod. The *Key* should be in thickness .3 the diameter of the neck, the width of *Gib* and *Key* combined should be 1.25 times, and the *Slot* should be 1.35 times that of the same diameter. The *Draft* of keys should be from .5 to .6 of an inch per foot. Distance of *Slot* from end of rod .5 diameter of pin.

Pins. For *Cranks*, *Beams*, etc., their area at their *journals* should be 1.6 to 1.75 times; and for *Air-pump* connections their area should be 1.5 times that of the attached rod; their length 1.3 to 1.5 times their diameter.

Cranks (*Wrought Iron*).—The *Hub*, compared with the neck of the shaft, should be 1.75 the diameter and 1 the depth. The *Eye*, compared with the pin, should be twice its diameter, and 1.5 the depth. The *Web*, at the periphery of the hub, should be, in width, .7 the width, and in depth, .5 the depth of the hub; and at the periphery of the eye it should be, in width, .8 the width, and in depth, .6 the depth of the eye.

(*Cast Iron.*) The diameters of the *Hub* and *Eye* should be, respectively, twice the diameter of the neck of the shaft, and 2.25 times that of the crank pin.

The *Radii* for the fillets of the sides of the web should be one half of the width of the web at the end for which the fillet is designed; for the fillets at the back of the web, they should be one half of that at the sides of their respective ends.

Beams, *Open* or *Trussed*. Their length from centres should be 1.8 to 2 the stroke of the piston, and their depth .5 their length. If strapped, the *Strap* at its smallest dimensions should have at least .9 the area of the piston rod, its depth being equal to .5 its breadth. The *end centre journals* should have each 1, and the *main centre journals* 2.5 times the area of the piston or driving rod.

This proportion for the strap is when the depth of the beam is half of its length, as above; consequently, when its depth is less, the area of the strap must be increased; and when the depth of the strap is greater or less than half its width, its area is determined by the product of its $b\,d^2$, being the same as if its depth was half its width.

(*Cast Iron*). *Area of Section at Centre.* Multiply the extreme pressure upon the piston in pounds by half the length of the beam, and divide the product by 500 times the depth of the centre in inches.

Their depth at their centre should be .5 to .75 the diameter of the cylinder, and, when of uniform thickness, should have a thickness of not less than .1 of their depth.

Vibration of End Centres. $l \div 2 - \sqrt{(l \div 1)^2 - (s \div 2)^2} =$ *vibration at each end; l representing the length of beam, and s the stroke of the piston in feet.*

Plumber Blocks (*Shaft*). Bottom .4, and Binder .45, the diameter of the shaft journal. *Holding-down Bolts.* If two are used .3 to .33, and if four, .22 to .25, the diameter of the shaft.

Cocks. The angles of the sides of their plug should be from 7° to 8° from the plane of it.

Pumps. The velocity of water in pump openings should not exceed 500 feet per minute.

Fly Wheels and Governors. See Rules, p. 423 and 598.

Smoke Pipes and Chimneys. Their area at their base should exceed that of the extremity of the flue or flues with which they are connected.

The intensity of their draught is as the square root of their height.

The relative volumes of their draught is determined by the formula:

$\sqrt{h}$ .1 a = *volume in square feet; h representing the height of the pipe or chimney in feet, and a its area in square feet.*

When wood is consumed their area should be 1.6 times that for coal.

The less the height of a chimney the higher the temperature of its air is required.

Chimneys. The diameter at their base should not be less than from .1 to .11 of their height.

The batter or inclination of their external surface should be .35 of an inch to the foot, which is about equal to 1 brick (½ brick each side) in 25 feet.

The diameter of the base should be determined by the internal diameter at the top, and the necessary batter due to the height.

The thickness of the walls should be determined by the internal diameter at the top; thus, for a diameter of 4 feet and less, the thickness may be 1 brick, but for a diameter in excess of that it should be 1.5 bricks.

Velocities of the Current of heated Air in a Chimney 100 *Feet in Height in Feet per Second.*

External Air.	Air at Base of Chimney.				External Air.	Air at Base of Chimney.			
	150°	250°	350°	450°		150°	250°	350°	450°
	Feet.	Feet.	Feet.	Feet.		Feet.	Feet.	Feet.	Feet.
10°	24	30	33	35	60°	19	26	29	33
32°	22	28	31	34	70°	18	25	29	32
50°	20	27	30	33	80°	17	24	28	32

When the Height of the Chimney is less than 100 *feet.*—Multiply the velocity as obtained for the temperature by one tenth the square root of the height of the chimney in feet.

The draught consequent upon a steam jet in a chimney or smoke-pipe is nearly equal to that of a moderate blast.

Water Wheels (Arms).—Their number should be from .75 to .8 the diameter of the wheel in feet. (*Blades*) *Wood.*—For a distance of from 5 to 5.5 feet between the arms, their thickness should be from .09 to .1 of an inch for each foot of diameter of wheel.

A wrought iron blade .625 inch thick bent at a stress withstood by an oak blade 3.5 inches thick.

Blades (Area).—The area of the blades, compared with the area of immersed amidship section of a vessel, depend upon the dip of the wheels, their distance apart, the model and rig of the vessel.

In *River service*, the area of a single line of blade surface varies from .3 to .4 that of the immersed section; in *Bay* or *Sound service*, the area varies from .15 to .2; and in *Sea service*, the area varies from .07 to .1.

Propellers (Screw).—The *Pitch* of it should vary with the area of the circle described by the screw to the area of the midship section of the vessel.

Area, Two-Bladed.

Area of disc of propeller to midship section being 1 to...............	6	5	4½	4	3½	3	2½	2
Ratio of pitch to the diameter of propeller is 1 to.................	.8	1.02	1.11	1.2	1.27	1.31	1.4	1.47

For *Four-bladed* screws, multiply the ratio of the pitch to the diameter as given above by 1.35. *Length*, .166 diameter.

Slip.—The slip of a screw propeller is directly as its pitch.

The economical effect of a screw is inversely as its pitch, the greater the pitch the less effect.

An expanding pitch has less slip than a uniform pitch, and, consequently, is more effective.

Safety Valve.—Area in square inches .7 to .8 area of grate surface in square feet.

Act of Congress (*U. S*).—For boilers having flat or stayed surfaces, 30 ins. for every 500 square feet of effective* heating surface; for cylindrical boilers, or cylindrical flued, 24 square inches.

Locked Safety Valves.—Effective heating surface, less than 700 square feet, valve 2 ins. in diameter; less than 1500, 3 ins. in diameter; less than 2000, 4 ins. in diameter; less than 2500, 5 ins. in diameter; and above 2500, 6 ins. in diameter.

Memoranda.

The saving of fuel by a fresh-water condenser may be safely estimated at from 15 to 20 per cent., added to an increased speed of engine of from 3 to 4 per cent.

The loss of power in condensing engines by the cooling of the cylinder from exposure to the air, is .01; and in non-condensing engines, about .015.

Friction Clutch, 5 ins. face, driven by a belt 14 ins. wide, broke a cast-iron shaft 6½ ins. in diameter.

A condensing steam-engine in river service, 43¾ inches in diameter of cylinder, with a stroke of piston of 10 feet, pressure of steam 25 lbs., revolutions 25, broke one of a pair of cast-iron water-wheel shafts 11 ins. in diameter.—*Steam-boat Utica.*

A condensing steam-engine in river service, 55 ins. in diameter of cylinder, with a stroke of piston of 10 feet, pressure of steam 25 lbs., revolutions 26, broke one of a pair of cast-iron water-wheel shafts 11 38 ins. in diameter.—*Steam-boat New Philadelphia.*

A non condensing engine, driving a rolling mill, cylinder 13 ins. in diameter, with a stroke of piston of 6 feet, pressure of steam 75 lbs., revolutions 30, broke a single cast-iron shaft of 11 ins. in diameter. *Soft Iron.*

A condensing steam-engine in river service, 65 ins. in diameter of cylinder, with a stroke of piston of 10 feet, pressure of steam 12 lbs., revolutions 25, broke one of a pair of cast-iron water-wheel shafts, 12½ ins. in diameter.—*Steam-boat De Witt Clinton.*

Two steam cylinders, 50 ins. in diameter, by 10 feet stroke of piston, having one air pump disabled, both engines worked by the remaining one, 30 ins. in diameter by 4.75 feet stroke, pressure of steam 25 lbs., cut off at half the stroke; vacuum 25½ ins.—*Steamer Sonora.*

Radial and Feathering Water Wheels.

Radial.—The loss of effect is the sum of the loss by the oblique action of the wheel blades upon the water, their slip, and the thrust and drag of the arms and blades as they enter and leave the water.

The loss by oblique action is computed by taking the mean of the square of the sines of the angles of the blades when fully immersed in the water.

The loss by the oblique action of the blades of the water-wheels of the steamer *Arctic* (for details of which see p. 638), when her wheels were immersed 7 feet 9 ins. and 5 feet 9 ins., was 25½ and 18½ *per cent.*, which was the loss of useful effect of the portion of the total power developed by the engines, which was applied to the wheels.

Feathering.—The loss of effect is confined to the thrust and drag of the arms and blades as they enter and leave the water.

Comparative Effects.

In two wheels of a like diameter (26 feet, and 6 feet immersion), like number and depth of blades, etc., the losses are as follows:

Radial.................. 25.6 *per cent.* | Feathering.............. 15.4 *per cent.*

The loss of effect by thrust and drag in a feathering wheel, having these elements and included in the above given loss, is computed at 2 *per cent.*

The relative loss of effect of the two wheels is, approximately, for ordinary immersions, 20 and 15 *per cent.* from the circumference of the wheel.

The *Centre of Pressure* of an immersed wheel blade, when the upper or inner edge is level with the surface of the water, is ⅓ from the bottom or lower edge.

* By a *rule* of the Inspectors, the fire surface from the grate bars to the water line (deducting half the flue or tube surface only) is alone computed as effective. This construction of the law is altogether arbitrary upon the part of the Inspectors, as all surfaces are more or less effective.

To Compute the Centre of Pressure of Water-wheel Blade when Immersed.

RULE.—Divide the difference of the cubes of the depths of the blade below the surface of the water by the difference of their squares, and ⅔ the quotient will give the distance of the centre of pressure below the surface, from which subtract the depth of the upper edge of the blade, and the remainder will give the position of the centre of pressure required.

In the cases here given, the centres of pressure are as follows:

Radial wheel .... 6.4 ins. from the bottom edge.
Feathering wheel .... 8.5 " "

SLIDE VALVES.

All Dimensions in Inches.

To Compute how much Lap must be given on the Steam Side of a Slide Valve, to cut off the Steam at any given Part of the Stroke of the Piston.

RULE.—From the length of stroke of piston subtract the length of the stroke that is to be made before the steam is cut off; divide the remainder by the stroke of the piston, and extract the square root of the quotient. Multiply this root by half the throw of the valve, from the product subtract half the lead, and the remainder will give the lap required.

EXAMPLE.—Having stroke of piston 60 ins.; stroke of valve 16 ins., lap upon exhaust side ½ in. = $\frac{1}{32}$ of valve stroke, lap upon steam side 3¼ ins., lead 2 ins., steam to be cut off at ⅚ the stroke; what is the lap?

$60 - \frac{5}{6}$ of $60 = 10$. $\frac{10}{60} = .166$. $\sqrt{.166} = .408$. $.408 \times \frac{16}{2} = 3.264$, and $3.264 - \frac{2}{2} =$ 2.264 *ins. or the lap — half the lead.*

To Compute the Lap required on the Steam Side of a Valve, to Cut the Steam off at various Portions of the Stroke of the Piston.

Valve without Lead.

	Distance of the piston from the end of its stroke when the steam is cut off, in parts of the length of its stroke.									
	½	$\frac{5}{12}$	⅓	$\frac{7}{24}$	¼	$\frac{5}{24}$	⅙	⅛	$\frac{1}{12}$	$\frac{1}{24}$
Lap in parts of the stroke ..	.354	.323	.286	.27	.25	.228	.204	.177	.144	.102

ILLUSTRATION.—Take the elements of the preceding case.

Under ⅙ is .204, and $.204 \times 16 = 3.264$ *ins. lap.*

When the Valve is to have Lead.—Subtract half the proposed lead from the lap ascertained by the table, and the remainder will be the proper lap to give to the valve.

If, therefore, as in the last case, the valve was to have 2 ins. lead, then $2 \div 2 - 3.264 = 2.264$ *ins.*

To Compute at what Part of the Stroke of the Piston any given Lap on the Steam Side will cut off the Steam.

RULE.—To the lap on the steam side add the lead; divide the sum by half the length of throw of the valve. From a table of natural sines (p. 301) find the arc, the sine of which is equal to the quotient; to this arc add 90°, and from their sum subtract the arc, the cosine of which is equal to the lap on the steam side, divided by half the throw of the valve. Find the cosine of the remaining arc, add 1 to it, and multiply the sum by half the stroke of the piston, and the product will give the length of that part of the stroke that will be made by the piston before the steam is cut off.

EXAMPLE.—Take the elements of the preceding case.

$\frac{2.25+2}{16 \div 2} = .53125$; sin. $.53125 = 32° 5'$; $32° 5' + 90° = 122° 5'$; $2.25 \div 8 = .28125$ = cos. of $73° 40'$; then $122° 5' - 73° 40' = 48° 25'$; cos. $+1 = 1.66371$, which $\times \frac{60}{2}$ = 50 *ins.*, or ⅚*th stroke.*

Portion of the Stroke of a Piston at which the Exhausting Port is closed and opened.

Lap on the Exhaust Side of the Valve in Parts of its throw.

Lap.	Portion of Stroke at which the Steam is cut off.							
	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
A								
⅛	.178	.161	.143	.126	.109	.093	.074	.053
$\frac{1}{16}$	.13	.118	.1	.085	.071	.058	.043	.027
$\frac{1}{32}$	.113	.101	.085	.069	.053	.043	.033	.024
0	.092	.082	.067	.055	.041	.033	.022	.011
B								
⅛	.033	.026	.019	.012	.008	.004	.001	.001
$\frac{1}{16}$	.06	.052	.04	.03	.022	.015	.008	.002
$\frac{1}{32}$	.073	.066	.051	.042	.033	.023	.013	.004
0	.092	.082	.067	.055	.044	.033	.022	.011

The units in the columns of the table marked A express the distance of the piston, in parts of its stroke, from the end of the stroke when the exhaust port in advance of it is closed; and those in the columns of the table marked B express the distance of the piston, in parts of its stroke, from the end of its stroke when the exhaust port behind it is opened.

ILLUSTRATION.—A slide valve is to cut off at ⅙ from the end of the stroke of the piston, the lap on the exhaust side is $\frac{1}{32}$ of the stroke of the valve (16 ins.), and the stroke of the piston is 60 inches. At what point of the stroke of the piston will the exhaust port in advance of it be closed and the one behind it opened?

Under ⅙ in table A, opposite to $\frac{1}{32}$, is .053, which × 60, the length of the stroke = 3.18 *ins.;* and under ⅙ in table B, opposite to $\frac{1}{32}$, is .033, which × 60 = 1.98 *ins.*

If the lap on the exhaust side of this valve was increased, the effect would be to cause the port in advance of the valve to be closed sooner and the port behind it opened later. And if the lap on the exhaust side was removed entirely, the port in advance of the piston would be shut, and the one behind it open, at the same time.

The lap on the steam side should always be greater than that on the exhaust side, and the difference greater the higher the velocity of the piston.

In fast-running engines alike to locomotives, it is necessary to open the exhaust valve before the end of the stroke of the piston, in order to give more time for the escape of the steam.

To Ascertain the Breadth of the Ports.

Half the throw of the valve should be at least equal to the lap on the steam side, added to the breadth of the port. If this breadth does not give the required area of port, the throw of the valve must be increased until the required area is attained.

To Compute the Stroke of a Slide Valve.

RULE.—To twice the lap add twice the width of a steam port in inches, and the sum will give the stroke required.

Expansion by lap, with a slide valve operated by an eccentric alone, can not be extended beyond ⅓ of the stroke of a piston without interfering with the efficient operation of the valve; with a link motion, however, this distortion of the valve is somewhat compensated. When the lap is increased, the throw of the eccentric should also be increased.

When low expansion is required, a cut-off valve should be resorted to in addition to the main valve.

To Compute the Distance of a Piston from the End of its Stroke, when the Lead produces its Effect.

RULE.—Divide the lead by the width of the steam port, both in inches, and term the quotient sine; multiply its corresponding versed sine by half the stroke, and the product will give the distance of the piston from the end of its stroke, when steam is admitted for the return stroke and exhaustion ceases.

EXAMPLE.—The stroke of a piston is 48 ins., width of port 2½ ins., and the lead ½ in.; what will be the distance of the piston from the end of its stroke when exhaustion commences?

$.5 \div 2.5 = .2 =$ *sine*, and versed sine of $.2 = .0202$. Then $.0202 \times \frac{48}{2} = .4848$ *ins.*

To Compute the Lead, When the Distance of a Piston from the End of its Stroke is given.

RULE.—Divide the distance in inches by half the stroke in inches, and term the quotient versed sine; multiply the corresponding sine by the width of the steam port, and the product will give the lead.

EXAMPLE.—Take the elements of the preceding case.
$.4848 \div 24 = .0202 =$ *versed sine*, and sine of versed sine $.0202 = .2$.
Then $.2 \times 2.5 = .5$ *ins.*

To Compute the Distance of a Piston from the End of its Stroke, when Steam is admitted for its return Stroke.

RULE.—Divide the width of the steam port, and also that width less the lead, by half the stroke of the slide, and term the quotient versed sines *first* and *second*. Ascertain their corresponding arcs, and multiply the versed sine of the difference between the *first* and *second* by half the stroke, and the product will give the distance required.

To Compute the Lap and Lead of Locomotive Valves.

$.22\,t =$ *lap in ins.*, and $.07\,t =$ *lead in ins.*; *t representing the stroke of the valve.*

Giffard's Injector.*

Maximum Temperature of the Feed-water Admissible at different Pressures of Steam.

	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Pressure per square inch	10	20	30	40	50	100
Temperature of feed................	148°	138°	130°	124°	120°	110°

The capacities of Injectors are denoted by the diameters of their *throats* in millimeters; thus, No. 4 has a diameter of 4 millimeters $= 4 \times .0394 = .1576$ *ins.*

The expenditure of steam increases with the proportionate pressure in the boiler.

To Compute the Diameter of Throat and Mean Volume of Discharge.

$.077\sqrt{\frac{V}{\sqrt{P}}} = d.$ $\overline{12.9\,d^2}\sqrt{P} = V$; *d representing diameter of throat in ins.*, V *volume of water required per hour in cubic feet, and* P *pressure of steam in pounds per square inch.*

ILLUSTRATION.—The pressure of the steam is 60 lbs., and the volume of water required to be delivered per hour is 50 cubic feet.

Then $.077\sqrt{\frac{50}{\sqrt{60}}} = .077 \times 2.54 = .196$ *ins. diam. of throat* $= \frac{.196}{.0394} = 5$, or No. 5; and $\overline{12.9 \times .196^2}\sqrt{60} = 6.45 \times 7.75 = 50$ *cubic feet.*

* Made by Wm. Sellers & Co., Philadelphia.

INDEPENDENT STEAM FIRE AND BILGE PUMPS.

H. R. Worthington's.

No.	*Single Pumps.* Diameter of Steam Cylinder.	Diameter of Water Plunger.	Length of Stroke.	Displacement per single Stroke.	*Duplex Pumps.* Diameter of Steam Cylinder.	Diameter of Water Plungers.	Length of Stroke.	Displacement per single Stroke.
	Inches.	Inches.	Inches.	Gallons.	Inches.	Inches.	Inches.	Gallons.
1	3¾	2¼	3	.054	4½	2¾	4	.22
2	5	3	6	.19	6	4	6	.68
3	6½	4½	9	.64	7½	4½	10	1.44
4	12	6	9	1.1	10	6	10	2.55
5	16	7	9	1.56	14	7	10	3.48
6	18½	10¼	9	3.36	14	10¼	10	7.46
7	16	14	9	6.27	16	10¼	10	7.46
3d 7	18½	14	9	6.27	14	14	10	14.

Woodward's.

No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Length of Stroke.	Capacity per Minute.	No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Length of Stroke.	Capacity per Minute.
	Ins.	Ins.	Ins.	Galls.		Ins.	Ins.	Ins.	Galls.
1	4	2	3	8 to 12	7	16	9	8	414 to 518
2	5	2½	6	26 to 39	8	18	12	8	736 to 900
3	7	3½	6	52 to 78	9	20	14	9	900 to 1200
4	9	5	6	85 to 120	10	22	16	9	1500 to 1800
5	12	7	6	167 to 240	11	24	18	10	1700 to 2000
6	12	9	6	276 to 331	12	26	20	10	2000 to 2500

No. 0, Steam Cylinder 2½ ins., discharge 3 to 5 gallons per minute.

A. S. Cameron & Co.'s.

No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Stroke of Piston.	Discharge per Revolution.	No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Stroke of Piston.	Discharge per Revolution.
	Ins.	Ins.	Ins.	Galls.		Ins.	Ins.	Ins.	Galls.
0	3½	1¾	3	.04	4	10	5	10	2
1	4½	2¼	5	.16	5	12	7	10	4
2	6	3	6	.33	6	14	9	12	8
3	7	3½	8	.75	7	16	10½	14	11
3½	8	4	8	1.	8	18	12	16	16

Wm. Sewell's.

No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Stroke of Piston.	Discharge per Revolution.	No.	Diameter of Steam Cylinder.	Diameter of Water Cylinder.	Stroke of Piston.	Discharge per Revolution.
	Ins.	Ins.	Ins.	Galls.		Ins.	Ins.	Ins.	Galls.
1	4½	2¼	6	.2	5	12	7	7	2.33
2	6	3	6	.33	6	12	9	7	5.
3	7	3½	6	.5	7	16	11	9	7.5
3½	8	4	7	.75	8	18	12	12	12.
4	10	4	7	1.2					

Steam Siphon. *An Independent Lifting Pump.*

Capacity for a Discharge Pipe 2 ins. in Diameter.

Height of Water raised.	Pressure of Steam.	Discharge per Minute.	Height of Water raised.	Pressure of Steam.	Discharge per Minute.
Feet. Ins.	Lbs.	Gallons.	Feet. Ins.	Lbs.	Gallons.
14 6	30	63.54	13 2	60	119.68
13 2	40	85.71	13 2	70	138.44
13 2	50	100.	13 2	80	157.47

Land and Portable Steam Engines and Boilers.

LAND. (VARIABLE CUT-OFF.)

Power. Nominal.	Power. Actual.*	Cylinder. Diam. × Stroke.	Driving, or Fly Wheels. Diam. × Fan.	Driving, or Fly Wheels. Revolution.	Weight complete. E. & B.
		Ins.	Ins.	No.	Lbs.
15	25.6	8×24	6×10	100	13850
25	40.	10×24	8×13	100	15025
30	57.6	12×24	9×15	100	16650
40	78.4	14×24	9×17	100	19330
50	94.5	15×36	12×18	70	28530
60	109.	16×32	10×20	80	26000
80	136.	18×36	14×25	70	40560
100	168.	20×36	15×25	70	40600
125	220.2	22×42	16×33	65	48860
150	276.4	24×48	20×49	60	66070

PORTABLE.

Power. Nominal.	Power. Actual.	Cylinder. Diam. × Stroke.	Driving, or Fly Wheels. Diam. × Fan.	Driving, or Fly Wheels. Revolution.	Weight complete. E. & B.
		Ins.	Ins.	No.	Lbs.
4	4.7	4×10	2½ × 6	175	2800
5	7.3	5×10	3 × 7	175	3200
7	10.5	6×10	3½ × 7	175	4200
8	14.3	7×10	3⅔ × 8	175	4900
12	19.2	8×12	4 × 8	150	6100
15	24.3	9×12	5 × 9	150	6900
20	30.9	10×16	6 ×10	116	11200
25	36.3	11×18	6 ×10	100	12800
30	43.2	12×18	6 ×12	100	13800
40	58.8	14×18	7 ×14	100	16700

Face of the fly wheels turned for a belt. All the portable engines have two fly whee's, or driving pulleys.

* Computed at 60 lbs. pressure.

BOILERS.

Natural Draught.

Boilers (*Land*) should be set at an inclination of .5 inch in 10 feet.

Grates (*Coal*). They should have a superficial area of 1 square foot for every 15 lbs. of coal required to be consumed per hour, at a rapid rate of combustion, and they should be set at an inclination toward the bridge wall of 1 inch in every foot of length. When, however, the rate of combustion is not high, in consequence of the low velocity of the draught of the furnace, or the fuel being insufficient, this proportion must be increased to 1 square foot for every 12 lbs. of fuel.

With *Wood* as the fuel, their area should be 1.25 to 1.4 that for coal.

The width of the bars should be the least practicable, and the spaces between them from .5 to .75 of an inch, according to the fuel used.

Ash-pit.—The transverse area of it, for a like combustion of 15 lbs. of coal per hour, should be .25 the area of the grate surface for bituminous coal, and .33 for anthracite.

The velocity of the current of air entering an ash-pit may be estimated at 12 feet per second.

Furnace or Chamber (*Coal*).—The volume of it should be from 2.75 to 3 cubic feet for every square foot of its grate surface. (*Wood.*) The volume should be 4.6 to 5 cubic feet.

Combustion is the most complete with firings or charges at intervals of from 15 to 20 minutes.

The volume of air and smoke for each cubic foot of water converted into steam is from coal 1780 to 1950 cubic feet, and for wood 3900.

Bridge-wall (*Flue boilers*). The cross section of the flues or tubes should have an area of 1.7 to 2 square inches for each lb. of coal consumed per hour, or from 22.5 to 26 square inches for each square foot of grate, for a combustion of 13 lbs. of coal per hour; the difference in the area depending upon the character of the conformation of the section of, and the length of the passage of the gases; the area being inversely with the diameter, and directly as the length of the flues, tubes, or spaces between them. Thus, in *Horizontal tubular* boilers, the area should be increased to 27.5 and 31 square inches; in *Vertical tubular*, to 32.5 and 36 square inches; and when a *Blast* is used, the area may be decreased to 15.5 and 20.5 square inches.

The temperature of a furnace is about 1000°, and the volume of air required for the combustion of 1 lb. of bituminous coal, together with the products of combustion, is 154.81 cubic feet, which, when exposed to the above temperature, makes the volume of heated air at the bridge wall from 450 to 470 cubic feet for each lb. of coal consumed upon the grates.

Hence, at a velocity of the draught of about 36 feet per second, the area over a bridge wall, required to admit of this volume being passed off in an hour, would be .5 of a square inch, but in practice it should be 2 square inches.

When 13 lbs. of coal per hour are consumed upon a square foot of grate, 13×2=26 square inches are required, and in this proportion for other quantities.

The temperature of the heated air at the end of the flues should be about 500°, and their area, and that of the base of the chimney, should be .75 of that over the bridge wall, or 1.5 square inches for each pound of coal consumed per hour.

When the area of the flues is determined upon, and the area over the bridge wall is required, it should be taken at from .7 to .8 the area of the lower flues for a natural draught, and from .5 to .6 for a blast.

Flues.—Their area should decrease with their length, but not in proportion with the reduction of the temperature of the heated air, their area at their termination being from .7 to .8 that of their *calorimeter* or area immediately at the bridge wall.

Large flues absorb more heat than small, as both the volume and intensity of the heat is greater with equal surfaces.

The temperature of the base of the chimney, or the termination of the flues or tubes, is estimated at 500°; and the base of the chimney, or the *calorimeter*, should have an area of 1.33 square inches for every pound of coal consumed per hour. With tubes of small diameter, compared to their length, this proportion may be reduced to 1 inch.

The admission of air behind a bridge wall increases the temperature of the gases, but it must be at a point where their temperature is not below 800°.

Evaporation.—1 square foot of grate surface, at a combustion of 13 lbs. coal per hour, will evaporate 2 cubic feet of salt water per hour.

A square foot of heating surface, at the above combustiou of fuel, will evaporate from 4.33 to 5.33 lbs. of salt water per hour; and at a combustion of 40 lbs. coal per hour (as upon the Western rivers of the U. S.), from 10 to 11 lbs. fresh water, exclusive of that lost by blowing out from the boilers.

12 to 15 square feet of surface will evaporate 1 cubic foot of salt water per hour at a combustion of 13 lbs. coal per hour per square foot of grate.

NOTE.—The boilers of the Steamer Arctic, of N. Y., vertical tubular, having a surface of 33¼ to 1 of grate, consuming 13 lbs. of coal per square foot of grate per hour, evaporated 8.56 lbs. of salt water per lb. of coal, including that lost by blowing out of saturated water.

The relative evaporating powers of iron, brass, and copper are as 1, 1.25, and 1.56.

Water Surface.—At low evaporations, 3 square feet are required for each square foot of grate surface, and at high evaporation 4 to 5 square feet.

Heating Surfaces.

Heating Surfaces (Sea Water).—The grate and heating surfaces should be increased about .07 over that for fresh water.

Relative Value of Heating Surfaces.

Horizontal surface above the flame	=1.	Horizontal beneath the flame	=.1
Vertical	= .5	Tubes and flues	=.56

A scale one sixteenth of an inch in thickness will affect a loss of 14.7 per cent. of fuel.

One square foot of *fire* surface is computed to be as effective as three of *heating* surface.

When the combustion in a furnace is complete, the tubes may be shorter than when it is incomplete.

Tubes should always be set in vertical rows, and the spaces between them should be increased with their number.

Boilers with Internal Furnaces.

For Coal, 13 *lbs. per Hour per Square Foot of Grate.* (*Natural Draught.*)

Pressure of Steam 20 *lbs.* (*Mercurial Gauge*), *and* 20 *Revolutions of the Engine per Minute.*

*Fire and Flue Surface.** (*Arches or Flues and Return Flues.*)—For every cubic foot of steam to be expended in the steam cylinder, for a single stroke of the piston (computed only to the point of cutting off), the length of the flues and steam chimney not exceeding 45 or 50 feet, there should be from 48 to 54 square feet.

(*Arches or Flues, and Tubes, or Return Tubes.*) *Horizontal Return.*—The length of the tubes and steam chimney not exceeding 30 or 35 feet, there should be from 58 to 64 square feet.

Vertical Water Tubes.—From 64 to 70 square feet.

Grates.—For every cubic foot of steam as above, there should be from 1.75 to 2.1 square feet.

For Coal, 30 *lbs. per Hour per Square Foot of Grate.* (*Blast or Exhaust.*)

Pressure of Steam 30 *lbs.* (*Mercurial Gauge*), *and* 20 *Revolutions of the Engine per Minute.*

*Fire and Flue Surface.** (*Arches or Flues and Return Flues.*)—For every cubic foot of steam to be expended in the steam cylinder, for a single stroke of the piston (computed only to the point of cutting off), the length of the flues and steam chimney not exceeding 55 or 60 feet, there should be from 24 to 28 square feet.

(*Arches or Flues and Tubes.*) *Horizontal Return.*—The length of the tubes and steam chimney not exceeding 30 or 35 feet, there should be from 29 to 32 square feet.

Vertical Water Tubes.—From 32 to 35 square feet.

Grates.—For every cubic foot of steam as above, there should be from 1.15 to 1.35 square feet.

Boilers with External Furnace and Internal Flues. (Cylindrical Flue.)

For Coal, 20 *lbs. per Hour per Square Foot of Grate, or for Wood at* 40 *lbs.* (*Natural Draught.*)

Pressure of Steam 100 *lbs.* (*Mercurial Gauge*), *and* 20 *Revolutions of the Engine per Minute.*

Fire and Flue Surface.†—For every cubic foot of steam to be expended in the steam cylinder, for a single stroke of the piston (computed only to the point of cutting off), the length of the flues and steam chimney not exceeding 55 to 60 feet, there should be from 100 to 108 square feet.

Grates.—For every cubic foot of steam as above, there should be from 3.8 to 4. square feet.

Western Boilers.—In the boilers upon the Western lakes and rivers of the United States, where the coal consumed is of the very best quality, and the smoke pipes are carried to a great height, the combustion of coal per square foot of grate per hour readily reaches 40 lbs.

1⅛ cords of Western wood have been burned per hour upon 48 square feet of grate.

In this case, the units above given may be reduced to 50 and 54 for heating surface, and the grate surface decreased to 1.85 and 2.

* Estimated from above the grate bars, including steam chimney, and for sea water.

† Estimated from above the grate bars, including steam chimney, where one exists, and for fresh water.

Boilers with External Furnace and Flue. (Plain Cylindrical.)

For Coal, 20 *lbs. per Hour per Square Foot of Grate, or for Wood at* 40 *lbs.* (*Natural Draught.*)

Pressure of Steam 100 *lbs.* (*Mercurial Gauge*), *and* 20 *Revolutions of the Engine per Minute.*

*Fire and Flue Surface.**—For every cubic foot of steam to be expended in the steam cylinder, for a single stroke of the piston (computed only to the point of cutting off), the length of the flues and steam chimney not exceeding 30 feet, there should be from 85 to 92 square feet.

Grates.—For every cubic foot of steam as above, there should be from 3.8 to 4. square feet.

All of these units are based upon the volume of furnace, area of bridge-wall, or cross-section of flues or tubes, etc., as given in the preceding rules.

The ranges given, of from 48 to 54, 24 to 48, etc., are for the purpose of meeting the ordinary differences of construction, thickness of metal, etc.

When a heater is used, and the temperature of the feed-water is raised above that obtained in a condensing engine, the proportions of surfaces may be correspondingly reduced.

Steam Room.—There should be from 2.5 to 3.5 times the volume of steam room that there are cubic feet of steam expended in the cylinder for each single stroke of the piston for 25 revolutions; or the volume of it should be from 5 to 7 times the volume of the cylinder, increasing in proportion with the number of revolutions.

When there are two engines, or an increased number of revolutions, these proportions of steam room must be increased.

Felt covering to a boiler and steam pipes effects a very material saving in fuel.

NOTES.—Four copper boilers, with a natural draught and bituminous coal, flues 40 feet in length, including steam chimney, with 14 square feet of fire and flue surface, and .6 of a square foot of grate surface for every cubic foot in the cylinders, furnished steam at 20 lbs pressure, cut off at ½ of the stroke of the piston, for 18.5 revolutions.

The mean of four cases, with iron boilers and anthracite coal, with a blast, flues 50 feet in length, gave, with 12.5 square feet of fire and flue surface, and .5 of a square foot of grate surface for every cubic foot in the cylinders, steam at 35 lbs. pressure, cut off at ½ of the stroke of the piston, for 22 revolutions.

The space in the steam room of the boilers and chimney was about 5 times that of the cylinders in the preceding cases.

To Compute the Heating and Grate Surface required for a given Evaporation, or Volume of Cylinder and Revolutions.

OPERATION.—Reduce the evaporation to the required volume of cylinder, number of revolutions of engine, pressure of steam, and point of cutting off; then reduce these results to the range of consumption of fuel per square foot of grate, pressure of steam, and number of revolutions given for the several cases at pp. 593 and 594, and multiply them by the units given for the surfaces required.

ILLUSTRATION.—There is required an evaporation of 492.24 cubic feet of salt water per hour, under a pressure of steam of 17.3 lbs. per square inch, stroke of engine 10 feet, cutting off at ½ stroke, revolutions 15 per minute, and consumption of fuel (coal) 13 lbs. per square foot of grate per hour, in a marine boiler having internal furnaces and vertical tubes.

Volume of steam at this pressure compared with water, 833.

$492.24 \times 833 \div 60 = 6833.93$ *cubic feet of cylinder per minute.*

$6833.93 \div \overline{15 \times 2} = 227.79$ *cubic feet of cylinder at half stroke.*

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* These proportions are for the evaporation of fresh water; if sea water is used, the surface must be increased .066.

Then $\frac{227.79 \times 17.3}{20} = 197.04$ *cubic feet at 17.3 lbs. pressure, and* $\frac{197.04 \times 15}{20} =$ 147.78, which × 66, the unit for heating surface for a vertical tubular boiler at 20 lbs. pressure and 20 revolutions = 9753.48 *square feet.*

And 147.78 × 2 = the unit for grate under like condition = 295.56 *square feet.*

NOTE.—The steamer Baltic has developed all the elements here given, and the surfaces of her boilers and grates (for one engine) were 9742 and 293.9 *square feet.*

To Compute the Consumption of Fuel in the Furnace of a Boiler.

The Dimensions of the Cylinder, the Pressure of the Steam, the Point of Cutting Off, the Revolutions, and the Evaporation of the Boilers per Pound of Fuel per Minute being given.

RULE.—Ascertain the volume of water expended in steam, and multiply it by the weight of a cubic foot of the water used; divide the product by the evaporating power of the fuel in the boiler under computation in pounds of water, and add thereto the loss per cent. by blowing off.

BOILER PLATES, BOLTS, AND JOINTS.

BOILER PLATES AND BOLTS.—The tensile strength of Iron plates and bolts ranges from 42 500 to 62 500 lbs., being increased when subjected to a moderate temperature.

The mean tensile strength of Copper plates and bolts is 33 000 lbs., being reduced when subjected to a temperature exceeding 120°; at 212° being 32 000 lbs., and at 550° but 25 000 lbs.

Bursting and Collapsing Pressure.

For use in salt water, computation for iron plates or bolts, without reference to the riveting, should be based upon a strength of two fifths that of the ultimate strength of the metal, and for use in fresh water upon one half that of its ultimate strength. With copper one half is a safe reduction for all purposes.

The resistance to collapse of a flue or tube is much less than the resistance to bursting; the ratio can not be determined, as the resistance of a flue decreases with its length, or that of its courses.

With an ordinary cylindrical boiler, 4 feet in diameter, single riveted, 20 feet in length, with flues 15½ inches in diameter, shell 5/16ths thick, flues ¼ in., the relative strengths are: Bursting, 350 lbs.; Collapsing, 152 lbs.

The following units are based upon a tensile strength of iron of 52 500 lbs., and for copper of 32 000 lbs.

To Compute the Thicknesses, Maximum Working Pressure, and Diameter of an Iron Boiler or Flue.

For Service in Salt Water.

Thickness. RULE.—Multiply the diameter in feet by the working pressure in lbs.; divide the product by 1260 for *square* riveting, 1170 for *staggered*, and 900 for *single*, and the quotient will give the thickness in decimals of an inch.

Working Pressure. RULE.—Multiply the thickness by 1260, 1170, or 900, as before given; divide the product by the diameter in feet, and the quotient will give the pressure in pounds.

Diameter. RULE.—Multiply the thickness by 1260, 1170, or 900, as before given; divide the product by the working pressure, and the quotient will give the diameter in feet.

EXAMPLE.—The diameter of a single-riveted iron boiler is 4 feet, and the thickness of the plates $\frac{5}{16}$ths; what will be its *maximum* working pressure?

$$\frac{5}{16} = .3125. \quad \frac{.3125 \times 900}{4} = 70.3 \ lbs.$$

For Service in Fresh Water.—The preceding units are increased one fourth, viz., 1575, 1460, and 1125.

To Compute the Diameter of Stay Bolts.

RULE.—Multiply the distance between their centre in ins. by the square root of the quotient of the maximum working pressure, divided by 5530 for salt water and 6900 for fresh, for iron bolts, and by 5000 for copper bolts, and the quotient will give the diameter in inches.

Salt Water.—The strength of iron stay bolts should be computed at one seventh of the ultimate strength of the metal.

Fresh Water.—The strength of iron stay bolts should be computed at one sixth of the ultimate strength of the metal.

The strength of copper bolts may be taken at one fifth the strength of the metal for either salt or fresh water.

EXAMPLE.—The maximum working pressure of an iron boiler, for use in salt water, is 70 lbs., and the distance apart of the bolts is 8 ins.; what should be their diameter?

$$8 \times \sqrt{\frac{70}{5530}} = 8 \times \sqrt{.01266} = 8 \times .1125 = .9 \ in.$$

To Compute the Distance Apart of Stay Bolts.

RULE.—Multiply the square root of the quotient of 5530 for salt water and 6900 for fresh water, for iron bolts, and by 5000 for copper bolts; divided by the maximum working pressure, by the diameter of the bolts, and the product will give the distance in inches.

EXAMPLE.—The maximum working pressure of an iron boiler for use in salt water is 70 lbs., and the diameter of the stay bolts is .9 in.; what should be their distance apart?

$$\sqrt{\frac{5530}{70}} \times .9 = \sqrt{79} \times .9 = 8 \ ins.$$

NOTE.—Where stays are secured by keys, their ends should be $1\frac{1}{4}$ times the diameter of the stay, the depth of the slot 1.6 diam. of stay, and the width .3.

To Compute the Thickness of Flat Surfaces in a Boiler.

Salt Water. RULE.—Multiply the maximum working pressure by the square of the distance, or the area of the surface, between the centres of the stays in inches; divide the product by 45 700, and the square root of the quotient will give the thickness in inches.

EXAMPLE.—Take the elements of the preceding case.

$$\text{Then } \sqrt{\frac{70 \times 8^2}{45\,700}} = \sqrt{.098} = .313 \ in.$$

For fresh water take 57 000, and for copper 41 300.

Stay Bolts.—Iron stay bolts, $\frac{3}{4}$ ins. in diameter, screwed into a Copper plate $\frac{3}{8}$ths thick, drew at a strain of 18 260 lbs.

A like stay bolt, screwed and riveted into an Iron plate, drew at a strain of 28 760 lbs.

A like stay bolt of Copper, screwed and riveted into a Copper plate, drew at a strain of 16 265 lbs.

Hence, Stay bolts when screwed and riveted are $\frac{1}{8}$ stronger than when screwed alone.

Flat Surfaces.

The resistance of a flat surface decreases in a higher ratio than the space between the stays.

Iron plates ⅜ in. thick, with stay bolts 5 ins. apart (from centres), gave way with a strain of 9000 lbs., and with stay bolts 4 ins. apart at 16 000 lbs.

Thickness of Boiler Iron required and Pressures allowed by the Laws of the U. S.

Pressure equivalent to the Standard for a Boiler 42-in. in Diameter and ¼ inch thick.

Wire Gauge.	Thickness in 16ths.	Diameter.						
		34 Inches.	36 Inches.	38 Inches	40 Inches.	42 Inches.	44 Inches.	46 Inches.
No.		Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1	5	169.9	160.4	152.	144.4	137.5	131.2	125.5
2	4½	158.5	149.7	141.8	134.7	128.3	122.5	117.2
3	4¼	147.2	139.1	131.8	125.1	119.2	113.7	108.8
4	4	135.9	128.3	121.6	115.5	110.	105.	100.4
5	3⅔	124.5	117.6	111.4	105.9	100.8	96.2	92.1
6	3⅓	113.2	106.9	101.3	96.2	91.7	87.5	83.7
7	3	101.9	96.2	91.2	86.6	82.5	78.7	75.3

Riveted Joints.

Forms and Proportions of Riveted Joints.—(W. Fairbairn.)

Thickness of Plate	Diameter of Rivets.	Multiplier.	Length of Rivets.	Multiplier.	Centre to Centre of Rivets.	Multiplier.	Lap in Single Joints.	Multipliers.	
								Single Joints.	Double Joints.
Ins.	Ins.		Ins.		Ins.		Ins.		
3/16	⅜	2.	.⅞	4.5	1¼	6.5	1¼	6.8	11.1
¼	½	2.	1.⅛	4.5	1½	6.	1½	6.	10.
5/16	⅝	2.	1.⅜	4.5	1⅝	5.2	1⅞	6.	10.
⅜	¾	2.	1.⅝	4.5	1¾	4.7	2	5.3	8.8
½	13/16	1.5	2.¼	4.5	2	4.	2¼	4.5	7.5
⅝	15/16	1.5	2.¾	4.5	2½	4.	2¾	4.4	7.3
¾	1.⅛	1.5	3.⅜	4.5	3	4.	3¼	4.3	7.2

The Length of a rivet, alike to a bolt, is measured from inside of its head.

The Multipliers are for computing the Diameter, Length, and Distance between centres of the rivets; also for the Laps for Single and Double Joints, by multiplying the thickness of the plate by the Multiplier for the element required.

In Riveted Joints exposed to a tensile strain, the area of the rivets should be equal to the areas of the section of the plates through the line of the rivets, running a little in excess up to 9/16 in., and somewhat less beyond that diameter of rivet.

Relative Strength of Riveted Joints per Square Inch of Single Plate.

Single Lapped.—Machine riveted. Rivets 3 diameters from centres, 25 000 lbs.

Hand riveted. Rivets 3 diameters from centres, 24 000 lbs.

Rivets set "staggered," and equidistant from centres, 30 500 lbs.

Abut Joints.—Hand riveted. Rivets not "staggered," and equidistant from centres, single cover or strip, 30 000 lbs.

Rivets set "square," single cover or strip, 42 000 lbs.; double covers or strips, 55 000 lbs.

Relative Mean Strength of Riveted Joints compared to that of the Plates, Allowances being made for Imperfections of Rivets, etc.

Plates, 100; Double or "square" rivets, .7; "Staggered" rivets, .65; Single rivets, .5.

LOCOMOTIVE ENGINE.

Proportion of Parts based upon Diameter of Cylinder.

Crank Shaft	diameter .4		Depth of Piston	.28
Connecting Rod, end	"	.16	Blast Pipe, diameter	.3
" middle	"	.21	Area Steam Port	$d^2 \times .08$
Piston Rod	"	.15	" Exhaust Port	$d^2 \times .28$

$.0022 \times \frac{H}{g} = w$. $21.2\sqrt{wg} = H$. $.0022\frac{H^2}{w} = g$; H and *g representing areas of heating and grate surfaces in square feet, and w volume of water evaporated in cubic feet per hour.*

Heating Surface.—70 to 85 times that of the grate surface.

Grate Surface.—Area .011 of heating surface when coke is used, and .016 when bituminous coal is used.

Area of Tube surface 10.5 times that of the furnace surface.

Steam Room.—6 to 7 cubic feet per square foot of grate surface.

The Evaporation of water in a locomotive boiler is from 8.5 to 9.5 lbs. water from 212° per lb. of coke consumed.

Memoranda.

1192 square feet of heating surface and 74 feet of grate in a locomotive boiler (tubular direct), gave identical results with a plain cylindrical boiler having 1184 feet of heating surface and 96 feet of grate.

The mean results of an experiment with a passenger train was 18 lbs. coke expended per mile run, and 26.3 lbs. per mile at a speed of 24.5 miles.

There is an increase of effect with large *drums* over small.

.76 cubic feet of water evaporated per hour will produce 1 horse power.

In the Sœmmering locomotive, a power of 4.2 horses has been attained per ton of engine and tender; and 1 lb. of wood has raised 13 600 lbs. 1 foot in height per minute = 14.6 lbs. wood per horse power; and in a Freight locomotive, Northwestern Railway, No. 227 (Eng.), a power of 3.55 horses has been attained per ton of engine and tender.

With two locomotives of R. Stephenson & Co., cylinders 14 ins. in diam. and 22 ins. stroke, driving wheels 3.5 feet, locomotives secured together and operated as one, weight 50 tons, 150 tons were elevated a grade of 1 in 36 at 15 miles per hour.

Speed of Trains.

60. ÷ minutes in running 1 mile = *miles per hour.*
3600 ÷ seconds " 1 mile = " "

Railway Train.—At a speed of 33 miles per hour, a distance of 57 yards is required within which a train can be arrested; and at a speed of 63 miles per hour, a distance of 273 yards is required.

Brakes.—The resistance of brakes, as determined by experiments, is about 129 lbs. per ton of train.

GOVERNOR.

$\frac{187.5}{\sqrt{l}} = n$. $\left(\frac{187.5}{n}\right)^2 = l$. *l representing vertical height from plane of revolution to point of suspension in inches*, and *n number of revolutions per minute.*

Duty of Steam Engines.

The conventional duty of an engine is the number of pounds raised by it 1 foot in height by a bushel of bituminous coal (112 lbs.).

Cornish Engine.—Average duty 70 000 000 lbs.; the highest duty ranging from 47 000 000 to 101 900 000 lbs.

An actual horse-power per hour, in a condensing marine engine, working with steam at 15 lbs. (mercurial gauge), cut off at ½ stroke, will require 2.07 lbs. bituminous coal.

Evaporation 10.5 lbs. water per lb. of Welsh coal consumed.

Portable Engine.—Cylinder 6¼ ins. in diam. by 4 foot stroke of piston, revolutions 115 per minute, consumption of anthracite coal 17.28 lbs. per hour, steam cut off at ½ stroke.

Pumping Engine (Condensing).—The engine of the Brooklyn Waterworks, N. Y., elevated 611 114 lbs. water 1 foot in height per minute, with a consumption of 1 lb. anthracite coal. Friction of engine between cylinder and pumps 7.4 per cent.; loss of action in pumps 1.69 per cent.

This operation is in excess of all previous essays.

The Leeghwater engine, in the Harlaem Meer, elevated 11 926 642 lbs. 1 foot in height per minute.

Relative Cost of various Engines for Equal Effects.

In Pounds of Coal per Horse-power per Hour.

	Lbs.
A theoretically perfect steam-engine	.66
A Cornish condensing steam-engine	2.38
Ericsson's air engine	3.86
A marine condensing steam-engine	2 to 6.

WEIGHTS OF STEAM-ENGINES AND WHEELS OR PROPELLERS.

Side Wheels.—*American Marine* (*Condensing*).

Engine.	Frame.	Water Wheels.	No. of Cylinders.	Volume of Cylinders.	Weight per Cubic Feet.	Service.
				Cub. feet.	Lbs.	
Vertical beam	Wood.*	Wood.	1	63.	1100	River.
do.	Wood.*	Wood.	2	216.	1049†	Coast.
do.	Wood.*	Iron.	1	530.	1500	Sea.
do.	Wood.*	Iron.	1	725.	1080†	Sea.
Steeple	Iron.	Iron.	1	12.8	3800	River and Coast.
Oscillating	Iron.	Iron.	2	—	—	Sea.
do.	Iron.	Iron.	2	1090.	760‡	Sea.
Overhead direct	Iron.	Iron.	2	261.3	1400	Sea (Gorgon).
Inclined direct	Iron.	Iron.	2	534.5	1100	U. S. Navy.
do.	Iron.	Iron.	2	353.	1316	U. S. Navy.
Side Lever	Iron.	Iron.	2	534.5	1390	U. S. Navy.
Horizontal	Wood.	Wood.	2	—	—	River.

English, per Nominal Horse-power.

Side Lever.

	Lbs.
Engines and Wheels	1546.
Boilers (tubular)	798.
Coal Bunkers	77.3
Water in boilers	515.

Oscillating.

	Lbs.
Engines	560
Wheels	246
Boilers	481
Coal Bunkers	67
Water in Boilers	224

* Without frame. † With frame 1109. ‡ Single frame.

Screw Propellers.—*Marine.* (*Condensing.*)

Engine.	No. of Cylinders.	Volume of Cylinder.	Weight per Cubic Feet.	Service.
		Cubic feet.	Lbs.	
Vertical direct............	2	12.5	5600	Sea.
do.	1	69.	4660	Sea.
do.	1	33.	3650	Coast.
Oscillating...............	—	—	—	Sea.
Trunk....................	2	193.	2080	Sea.
Inclined direct............	—	—	—	—
Horizontal back-action....	2	68.	4260	Sea.

Non-condensing.

Engines.	No. of Cylinders.	Volume of Cylinder.	Weight per Cubic Feet.	Service.
		Cubic feet.	Lbs.	
Horizontal direct.........	2	3.9	4990	River.
Oscillating...............	2	7.6	5500	Sea.
Vertical direct............	1	9.3	7800	Coast.
Inclined direct...........	–	—	—	Coast.

Land Engines.—(*Non-condensing.*)

	Engine and Spur Wheel.	Boilers, Grates, etc.	Sugar Mill Complete.	Flour Mill.	Engine per Cub. Ft. of Cylinder.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Vertical beam, 18 ins. × 4 feet.	103 040	26 880	89 600	—	9600
"	—	—	—	—	—
Horizontal, 14 ins. × 2 feet.....	1050	—	—	—	4000
"	—	—	—	—	—

Relative Space occupied and Weight of different Forms of Marine Engines, omitting Water Wheels, Shafts, and Cranks, being common to all.—(Horatio Allen.)

Engine.—Equal Volume of Cylinder of 315 Cubic Feet.	Space.		Weight.	
	Total.	Per Cubic Foot of Cylinder.	Total.	Per Cubic Foot of Cylinder.
	Cub. Feet.	Cub. Feet.	Lbs.	Cub. Feet
Vertical beam (overhead), 1 cylinder....	14 000	44.4	280 000	885
Side lever.............. 1 "	12 000	38.	295 000	935
Oscillating............. 1 "	8 000	25.4	220 000	697
Double cylinder, inclined at right angles, volume of high pressure or full stroke cylinders 150 cub. ft.—(A. G. Stimers.)	10 000	17.	418 000	760

The spaces include all passages about the engine; but with the vertical beam, that portion of the frame and engine which is above the spar deck is not included.

Proportionate Weights, Space occupied, and Cost of English Side-wheel Engines, Boilers, Wheels, or Screw Propeller, etc., per Nominal Horse-power.—(Admiralty.) (Mean of 18 cases.)

WEIGHTS.

	Lbs.		Lbs.
Engines........................	588	Extra pieces.....................	84
Boilers and Appendages...........	336	Coal Bunker for 1680 lbs. per horse-power..........................	84
Water in do.	224		
Wheels or Propeller...............	112	Total......................	1428

SPACE.

Space occupied by Engines alone, exclusive of boilers, coal bunkers, and passage-ways, may be taken at ¾ of a square foot per horse-power, and for Boilers alone at 1 square foot.

COST (1856).

Engines	$120	Wheels	$12.50
Boilers	60	Extra pieces	12.50
Coal Bunkers	10	Total	$215.

WEIGHTS OF BOILERS.

Weights of Iron Boilers (including Doors and Plates, and exclusive of Smoke Pipes and Grates) per Square Foot of Heating Surface.

Measured from Grates to Top of Steam Chimney or Base of Smoke Pipe.

Description of Boiler.		Sea or Land.	Weight. in Lbs.
Double return, Flue*	water bottom..	S.	31.5 to 33.3
Single do. do.†	do.	S.	25.6 to 32.9
Do. do. do.	—	S.	24. to 30.
Drop do. do.†	water bottom..	S.	30.4 to 40.8
Do. do. do., and over furnace....	do.	S.	29. to 38.
Do. do. do. do.	—	S.	27. to 36.
Single do. do., Multiflue†	water bottom..	S.	27. to 45.
Do. do. do. do.	—	S.	25. to 43.
Horizontal return, Tubular‡	water bottom..	S.	22.5 to 35.
Do. do. do.‡	—	S.	21. to 33.
Do. do. do.†	—	S.	17.7 to 26.7
Vertical do. do.‡	water bottom..	S.	18.5 to 26.5
Do. do.,‡ back of furnace	do.	S.	25. to 31.2
Horizontal direct, Tubular†	water bottom..	S.	19.8 to 23.8
Do. do. do.†	—	S.	17. to 21.
Cylindrical, external furnace,§ 36 ins. in diam., ¼ in. thick		L.	23.5 to 24.
Do. Flue do. § 36 to 42 do. ¼ do.		L.	18.1 to 18.6
Horizontal direct, Tubular	Locomotive ...	L.	
Vertical Cylinder direct, Tubular	—	S.	24. to 26.

NOTE.—The range in the units of weight here given arises from peculiarities of construction, consequent upon the proportionate number of furnaces, thicknesses of metal, volume of boilers compared with heating surface, character of staying, etc.

2. The boiler of the *British Admiralty* is a Horizontal return Tubular, with water bottom, and its weight varies from 28 to 33 lbs. per square foot as above.

Consumption of Fuel per Square Foot of Grate per Hour for several Marine Boilers.

Natural Draught.	Lbs.	Blast.	Lbs.
"Costa Rica," anthracite	11.	"San Jacinto," anthracite	20.9
"Wyoming," do.	8.	"Metropolis," do.	
"Adriatic," do.		"Princeton," do.	17.
"Wabash," do.	12.56	"Daniel Drew," do.	
"Arctic," bituminous coal	13.	"Mary Powell," do.	
Western steamboats, do.	40.	Locomotive, bituminous	

SATURATION IN MARINE BOILERS.

Sea water contains 3.03 parts of its weight in saline matter, and is saturated when it contains 36.37 parts.

Blowing Off.

To Compute the Loss of Heat by the Blowing Off of Saturated Water from a Steam-boiler.

$\frac{S-T\times E+t}{t}$ = *proportion of heat lost*, $S-T\times E$ = *degrees of heat re-*

* Section of furnace square. Shell, top arched, bottom square.
† Section of furnace square. Shell cylindrical. ‡ Section of furnace and shell square.
§ Wrought iron heads, $\frac{3}{8}$ths thick, flues ¼ in., and surface computed to half diameter of shell.

quired from the fuel for the water evaporated, and $\frac{t}{S-T \times E+t}$ = *loss of heat per cent.* S *representing sum of sensible and latent heats of water evaporated,* T *temperature of feed water,* t *difference in temperature of water blowed off and that supplied to the boiler,* E *volume of water evaporated, proportionate to that blowed off, the latter being a constant quantity, and represented by* 1.

Values of E at the following degrees of saturation, viz.:

$\frac{1.25}{32}=.25$; $\frac{1.5}{32}=.5$; $\frac{1.75}{32}=.75$; $\frac{2}{32}=1$; $\frac{2.25}{32}=1.25$: $\frac{2.5}{32}=1.5$; $\frac{2.75}{32}=1.75$ $\frac{3}{32}=2$; $\frac{3.25}{32}=2.25$; $\frac{3.5}{32}=2.5$, etc.

Thus, when the water in a boiler is maintained at a density of $\frac{2}{32}$, 1 volume of it is evaporated, and an equal volume, or 1, is blowed off. Hence $1+1-1=1=$ *ratio of volume evaporated to the volume blowed off;* and when it is maintained at $\frac{1.25}{32}$, .25 volumes of it are evaporated, and 1 blowed off. Hence $1 \div .25 = 4$ *volumes blowed off.*

Illustrations.—The point of blowing off is $\frac{2}{32}$, the pressure of the steam 15.3 lbs. mercurial gauge, and the density of the feed water $\frac{1}{32}$.

$S=1202°$, $T=100°$, $t=$ temp. of $15.3 \times 14.7 = 251.6° - 100 = 151.6°$, $E=1$.

Then $\frac{1202-100 \times 1+151.6}{151.6} = 8.27 =$ *proportion of heat lost*

$1202-100 \times 1+151.6° = 1253.6° =$ *total heat required from the fuel;* and $\frac{151.6°}{1253.6°}$ $=12.093$ *per cent loss by blowing off at* $\frac{2}{32}$ *at the temperatures given.*

Note.—If the temperature of the feed water in this case had been 150°, the loss would have been but 8.81 *per cent.*

To Compute the Degree of Saturation to Contain x Parts of Saline Matter.

The quantity of saline matter entering and the quantity blowed off in the same time, will be equal when $3.03\,(s+b)=x\,b$. hence $\frac{3.03\,s}{x-3.03}=b$, and $\frac{3.03\,s}{b}=x-3.03$.

Illustration.—The volume of water used for steam in an engine is 1, and the volume blowed out 4; what is the degree of saturation?

Note.—As Saline Hydrometers are graduated to 3 parts of saline matter in 100, or 1 in 33, it is preferable to use 3 instead of 3.03 as above.

Here x degrees of saturation $=1.25$.

Then $\frac{3 \times 1}{1.25 \times 3 - 3} = \frac{3}{.75} = 4$; and $\frac{3 \times 1}{4} = .75$, or $(1.25 \times 3 - 3) = x$.

To Compute the Volume of Water Blowed Off to that Evaporated. The Degree of Saturation being Given.

Rule.—Divide 1 by the proportionate volume of water evaporated to that blowed off, or the value of E as above, for the degree of saturation given, and the quotient will give the number of volumes blowed off to that evaporated.

ILLUSTRATION.—The degree of saturation in a marine boiler is $\frac{2.25}{32}$; what is the volume of water blowed off?

Value of E 1.25. Then $\frac{1}{1.25} = .8$ *blowed off.*

To Compute the Economy attained by the Use of Fresh Water in a Marine Boiler compared with the Use of Sea Water

$966.6 \div T - t = r$, *or ratio of increase of temperature to which feed water must be subjected.*

$F - E = V$, or *volume to be blowed off.*

$V \div r = v$, *or volume of water which could be converted into steam by the temperature lost by blowing off.*

Hence $v \times$ the pounds of fuel necessary to evaporate a cubic foot of water will give the amount of fuel economized.

T *and* t *representing temperature of fresh water due to the pressure of the steam, and the mean temperature of feed water, and* F *and* E *the volumes of feed-water and that evaporated.*

NOTE.—This economy is exclusive of the loss of heat consequent upon the incrustation upon the heating surfaces of a boiler when sea water is used.

HORSE POWER.

As this is the universal term used to express the capability of first movers, of magnitude, it is essential that the estimate of it should be uniform.

Its estimate is the elevation of 33 000 pounds avoirdupois one foot in height in one minute, and it is designated as being Nominal, Indicated, or Actual.

The first designation being adopted and referred to by Manufacturers of steam-engines in order to express the capacity of an engine, the elements thereof being confined to the dimensions of the steam cylinder, and a conventional pressure of steam and speed of piston; the second to designate the full capacity of an engine, as developed in operation, without any deduction for friction; and the last referring to its actual power as developed by its operation, involving the elements of the mean pressure upon the piston, its velocity, and a just deduction for the friction of the operation of the machine.

In reviewing the various modes for the computation as submitted by Engineers and Manufacturers, there is no proper formula that presents the essential element of being in conformity with any other, and as conformity in a rule for this purpose, if based upon an assimilation to the capacity of an engine, is all that is requisite, it would have been preferable to have adopted an existing formula to the introduction of a new one, had it been practicable to have done so. It occurs, further, that there is not only a want of conformity in the various rules essayed by authors, but they have neither reached the cases of both condensing and non-condensing engines, nor have they properly approached to the actual power of an engine; and as the practice of operating engines since the adoption of existing formulæ has materially altered both in an increase of pressure and velocity of piston, the following rules are submitted.

Nominal Horse's Power. (*Condensing Engine.*)

$\frac{d^2 v}{3000}$ = *horse's power: d representing diam. of cylinder in inches, and v the velocity of the piston in feet per minute*

This is alike to the rule of the British Admiralty, substituting 3000 for

6000, and it is based upon a uniform steam pressure of 10 lbs. per square inch (steam gauge, or *above* the pressure of the atmosphere), cut off at one half the stroke, deducting one fifth* for friction and losses, with a mean velocity of piston of 250 feet per minute for an engine of long stroke, and of 200 feet for one of short stroke.

The rule of the British Admiralty is based upon a uniform and effective pressure of 7·lbs. per square inch at full stroke, and a mean velocity of piston of 205 feet per minute; viz., 170 feet for a stroke of 2.5 feet, and 240 feet for a stroke of 8 feet.

Non-condensing Engine.

$$\frac{d^2\,v}{1000} = \textit{horses' power}.$$

This is based upon a uniform steam pressure of 60 lbs. per square inch (steam gauge), cut off at one half the stroke, deducting one sixth for friction and losses, with a mean velocity of piston of 250 feet per minute.

Nominal Horse Power of several Non-condensing Engines.

Computed from Formula $\frac{d^2\,v}{1000} = \text{H P.}$

Horses' Power.	Diameter and Stroke of Cylinder		Revolutions	Horses' Power	Diameter and Stroke of Cylinder		Revolutions.	Horses' Power	Diameter and Stroke of Cylinder		Revolutions.
No.	Ins	Feet	Min	No	Ins.	Feet.	Min.	No.	Ins	Feet	Min
9.	6×	1.	125	46.1	12×	4.5	32	159.7	22×	5.5	30
9.2	6	1.5	85	55.3	14	3.	47	160.7	22	6.	28
12.2	7	1.	125	56.3	14	3.5	41	163.6	22	6.5	26
12.5	7	1.5	85	58.	14	4.	37	169.4	22	7.	25
16.3	8	1.5	85	60.	14	4 5	34	183.7	24	5.5	29
16.9	8	1.75	75	60.8	14	5.	30	193.5	24	6.	28
21.1	9	1.5	87	64.8	15	3.	48	194.7	24	6.5	26
21.3	9	1.75	75	66.1	15	3.5	42	193.5	24	7.	24
21.4	9	2.	66	66.6	15	4.	37	198.7	24	7.5	23
21.5	9	2.5	53	66.8	15	4.5	33	227.1	26	6.	28
26.1	10	1.5	87	67.5	15	5.	30	228.5	26	6.5	26
26.6	10	1.75	76	77.1	16	3.5	43	227.1	26	7.	24
27.2	10	2.	68	77.8	16	4.	38	233.2	26	7.5	23
27.5	10	2.5	55	78.3	16	4.5	34	237.9	26	8.	22
28.2	10	3.	47	79.4	16	5.	31	266.	28	6.5	26
28 7	10	3.5	41	81.7	16	5.5	29	274.4	28	7.	25
28.8	10	4.	36	82.9	16	6.	27	270.5	28	7.5	23
33.9	11	2.	70	99.1	18	4.5	34	275.8	28	8.	22
33.3	11	2 5	55	103.7	18	5.	32	279.9	28	8.5	21
33.4	11	3.	46	103.4	18	5.5	29	304.2	30	6 5	26
33.9	11	3.5	40	105.	18	6.	27	315.	30	7.	25
34.9	11	4.	36	128.	20	5.	32	324.	30	7.5	24
39.2	12	2.	68	127.6	20	5.5	29	331 2	30	8.	23
39.6	12	2.5	55	129.6	20	6.	27	336.6	30	8.5	22
40.6	12	3.	47	130.	20	6.5	25	340.2	30	9.	21
41.3	12	3,5	41	134.4	20	7.	24	359.1	30	9 5	21
41.5	12	4.	36	154.9	22	5.	32	360.	30	10.	20

Indicated Horse Power.

This is the gross power exerted by an engine, without any deduction for friction, the mean pressure upon the piston being determined by an Indicator, or by a computation based upon the actual initial pressure in the cylinder.

* The friction and losses in a marine engine may be taken at 1.5 to 2 lbs. per square inch for working the engine, and 5 to 7½ per cent. upon the remainder for the friction of the load.

Actual or Effective Power.—*Condensing Engine.*

$\frac{A \times \overline{P^{*} - f^{\dagger}} \times 2\,s\,r}{33\,000}$ = *horse's power.* A *representing area of cylinder in square inches*, P *mean effective pressure upon cylinder piston in lbs. per square inch, inclusive of the atmosphere, f the friction of the engine in all its parts, added to the friction of the load in lbs. per square inch, s stroke of piston in feet, and r number of revolutions per minute.*

The Power required to work the air-pump of an engine varies from .7 to .9 lbs. per square inch upon the cylinder piston.

ILLUSTRATION.—The diameter of cylinder of a marine steam-engine is 60 ins., the stroke of its piston 10 feet, its revolutions 15 per minute, and the pressure of the steam per gauge, cut off at one fourth the stroke, is 20 lbs. per square inch.

A = 2827.4 *sq. ins.* P (per Ex., p. 580) 20.855‡ *lbs.* $f = 1.5 + \overline{20.855 - 1.5} \times .05 = 2.467$ *lbs.* Then $\frac{2827.4 \times \overline{20.855 - 2.467} \times 2 \times 10 \times 15}{33\,000} = 533.24$ *horses' power.*

From which is to be deducted in Marine Engines the power necessary to discharge the water of condensation at the level of the load-line, which is determined by the pressure due to the elevation of the water, the area of the air-pump piston, and the velocity of its discharge in feet per second.

Non-condensing Engine.

$$\frac{A \times \overline{P - (f^{\S} + 14.7)} \times 2\,s\,r}{33\,000} = \textit{horse's power.}$$

The sum of these resistances is from 12½ to 20 per cent., according to the pressure of the steam, being least with the highest pressure.

ILLUSTRATION.—The diameter of cylinder of a non-condensing engine is 10 ins., the stroke of the piston 4 feet, its revolutions 45 per minute, and the mean pressure of the steam in the cylinder (per steam gauge) is 60 lbs. per square inch.

A = 78.54 *sq. ins.* P 60 + 14.7 = 74.7 *lbs.* $f = 2.5 + \overline{(60 + 14.7 - 2.5)} \times .075 = 7.92$ *lbs.* Then $\frac{78.54 \times \overline{(60 + 14.7 - 7.92 + 14.7)} \times 2 \times 4 \times 45}{33\,000} = 44.6$ *horses.*

NOTE.—The power of a non-condensing engine is sensibly affected by the character of its exhaust, as to whether it is into a heater, or through a contracted pipe, to afford a blast to combustion.

NOTE 2. If an Indicator is not used to determine the pressure of the steam in a cylinder, a safe estimate of it, when acting expansively, is .9 of the full pressure, and when at full stroke from .75 to .8.

To Compute the Horses' Power of an Engine necessary to raise Water to any Given Height.

RULE.—Multiply the weight of the column of water by its velocity in feet per minute, and divide the product by 33 000.

EXAMPLE.—It is required to raise a column of fresh water, 16 inches in diameter by 36 feet in height, with a velocity of 128 feet per minute; what power is necessary?

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* This value is best obtained by an *Indicator*: when one is not used, refer to rule and table, page 579. In estimating the value of P, add 14.7 lbs., for the atmospheric pressure, to that indicated by the steam gauge or safety valve. When, however, an Indicator is not used, a safe estimate is .85 that of the boiler pressure.

† This value may be safely estimated in engines of magnitude at 1.5 to 2 lbs per square inch, for the friction of the engine in all its parts, and the friction of the load may be taken at 5 to 7½ per cent. of the remaining pressure.

The sum of these resistances in ordinary marine engines is from 10 to 20 per cent., according to the pressure of the steam, exclusive of the power required to deliver the water of condensation at the level of the load-line. For the pressure representing the friction for different designs and capacities of engines as estimated by English authority, see page 347.

‡ Clearance of piston at each end of cylinder is included in this estimate.

§ This value may be safely estimated at 2.5 lbs. per square inch for the friction of the engine in all its parts, and the friction of the load may be taken at 7½ per cent. of the remaining pressure.

The height of a column of fresh water equal to a pressure of 1 lb. per sq. in. = 2.31. Then 86 ÷ 2.31 = 37.23 *lbs.*

Area of 16 ins. = 201.06 ins., which × 37.23 = 7485.46, and 7485.46 × 128 ÷ 33 000 = 29.03 *horses' power.*

To which should be added an allowance of fully .2 for friction, leakages, waste, etc.

To Compute the Velocity necessary to Discharge a Given Volume of Water in any Given Time.

Rule.—Multiply the number of cubic feet by 144, and divide the product by the area of the pipe or opening in inches.

Example.—The diameter of a pipe is 16 inches, and the volume of water 179 cubic feet per minute; what is its velocity?

Area of pipe 201.06 *ins.* $\frac{179 \times 144}{201.06} = 128.2$ *feet*

To Compute the Area of a required Pipe, the Velocity and Volume of the Water being given.

Rule.—Proceed as above, and divide the product by the velocity.

To Compute the Volume of Water required to be Evaporated in a Steam-engine.

Rule.—Multiply the volume of steam expended in the cylinder and steam chests by twice the number of revolutions, and multiply the product by the density of the steam at the pressure given.

Example.—What quantity of water will an engine require to be evaporated per revolution, the diameter of the cylinder being 70 ins., the stroke of the piston 10 feet, and the pressure of steam 32 lbs. per square inch, including the atmosphere, cut off at one half of the stroke?

Area of cylinder = 3848.4 *ins.* 10 × 12 ÷ 2 = 60 *ins.*, then 60 × 3848.4 = 230904 *cub. ins.*

Add, for clearance at one end and volume of nozzle, steam chest, etc., 17318 *cub. ins.*

Then 230904 + 17318 ÷ 1728 × 2 = 287.29 *cub feet*, which × .0012, the density of steam at 32 lbs. pressure (p. 574) = .3447 *cubic feet.*

Note.—This refers to the expenditure of steam alone; in practice, however, a large quantity of water (differing in different cases) is carried into the cylinder in mechanical combination with the steam.

To Compute the Area of an Injection Pipe.

Ruel.—Ascertain the volume of water required by the rule, p. 576, in cubic inches per second, multiply it by the number of volumes of water required for condensation, by rule, p. 577, and divide it by the velocity due to the flow in feet per second, and again, by 12, and the quotient will give the area in square inches.

Example.—An engine has the following elements at its maximum operation; what should be the area of its injection pipe?

Cylinder, 70 ins. diameter and 10 feet stroke of piston; revolutions, 15 per minute; steam, 17.3 lbs., mercurial gauge, cut off one half.

Volume of cylinder 267.25 cubic feet, cut off at ½ = 133.625.

Density of steam at 32 lbs. (17.3 + 14.7) = .0012. Velocity of flow of injected water (computed from vacuum and elevation of condensing water) 33 feet per second.

Then 133.625 × 15 × 2 × 1728 ÷ 60 = 115 452 *cubic inches steam per second*, and 115 452 × .0012 = 138.54 *cubic inches water per second.*

The *maximum* volume of water required to condense steam is about 70 times the volume of that evaporated, which only occurs in the Gulf of Mexico; the ordinary requirement is not one half of it.

Hence 138.54 + 10.39 ins. (= 7½ per cent. for leakage of valves, etc.) = 148.93, which × 70, as above, = 10 425.1 *cubic ins.*, which ÷ 33 = 315.91, and again by 12 (ins. in a foot) = 26.32 *square ins. area*, which ÷ .7, the coefficient for velocity of flow of water in a pipe under like conditions, = 37.6 *square ins.*

Note.—This is the required capacity for one pipe for navigation in the Gulf of Mexico. It is proper and customary that there should be two pipes, to meet the contingency of the operation of one being arrested.

To Compute the Volume of Discharge through an Injection Pipe.

Rule.—Multiply the square root of the product of 64.333 and the depth of the centre of the opening into the condenser, from the surface of the external water in feet, added to the height in feet of a column of water due to the vacuum in the condenser, by the area of the opening in square inches; and .7 the product divided by 2.4 (144 ÷ 60) will give the volume in cubic feet per minute.

Example.—The diameter of an injection pipe is 5⅜ ins., the height of the external water above the condenser 6.13 feet, and the vacuum 24.45 ins., mercurial gauge; what is the volume of the flow per minute?

Area of 5⅜ ins. = 22.69 ins. Vacuum $\frac{24.45 \text{ ins.}}{2.0376} = 12$ lbs., and 12 lbs. × 2.239 feet (sea water) = 26.87 feet, and 26.87 + 6.13 = 33 *feet.*

Then $\frac{\sqrt{64.333 \times 33 \times 22.69 \times .7}}{2.4} = \frac{731.82}{2.4} = 304.93$ *cubic feet.*

To Compute the Area of a Feed Pump. For Sea Water.

Rule.—Ascertain the volume of water required in cubic inches per minute; divide it by the number of single strokes of the piston of the pump per minute, and divide the quotient by the stroke of the piston in inches; multiply this quotient by 6 (for waste, leaks, "running up," etc.), and the product will give the area of the pump in square inches.

Example.—Take the elements of the preceding cases.

138.54 cubic inches per second = 8312.4 per minute, stroke of pump 3½ feet.

Then $\frac{8312.4}{15} = 554.16$, which ÷ 3.5 × 12 = 13.19, and 13.19 × 6 = 79.14 *square ins.*

Note.—In fresh water, this proportion may be reduced two thirds.

BLOWING ENGINES.

For Smelting.

The volume of oxygen in air is different at different temperatures. Thus, dry air at 85° contains 10 per cent. less oxygen than when it is at the temperature of 32°; and when it is saturated with vapor, it contains 12 per cent. less. If an average supply of 1500 cubic feet per minute is required in winter, 1650 feet will be required in summer.

Manufacture of Pig Iron.

Coke or Anthracite Coal.—18 to 20 tons of air are required for each ton.
Charcoal.—17 to 18 tons of air are required for each ton.
(1 ton of air at 34° = 29 751, and at 60° = 31 366 *cubic feet.*)

Pressure.—The pressure ordinarily required for smelting purposes is equal to a column of mercury from 3 to 7 inches.

Reservoir.—The capacity of it, if dry, should be 15 times that of the cylinder if single acting, and 10 times if double acting.

Pipes.—Their area, leading to the reservoir, should be .2 that of the blast cylinder, and the velocity of the air should not exceed 35 feet per second.

A smith's forge requires 150 cubic feet of air per minute. Pressure of blast ¼ to 2 lbs. per square inch. A ton of iron melted per hour in a cupola requires 3500 cubic feet of air per minute. A finery forge requires 100 000 cubic feet of air for each ton of iron refined. A blast furnace requires 20 cubic feet per minute for each cubic yard capacity of furnace.

To Compute the Power of a Blowing Engine.

$P\,a\,v\,f$ = *power in lbs. to be raised* 1 *foot per minute*, and $\frac{P\,a\,v\,f}{33\,000}$ = *horses' power required.* P *representing pressure of blast in lbs. per square inch; a area of cylinder in square inches; v velocity of piston in feet per minute; f friction of piston and from curvatures, etc., estimated at* 1.25 *per square inch of piston.*

NOTE.—If the cylinder is single acting, divide the result by 2.

To Compute the Dimensions of a Driving Engine.

RULE.—Divide the power in pounds by the product of the mean effective pressure upon the piston of the steam cylinder in pounds per square inch, and the velocity of the piston in feet per minute, and the quotient will give the area of the cylinder in square inches.

2.—Divide the velocity of the piston by twice the number of revolutions, and the quotient will give the stroke of the piston in feet.

The quantity of air at atmospheric density delivered into the reservoir, in consequence of escape through the valves, and the partial vacuum necessary to produce a current, will be about .2 less than the capacity of the cylinder.

See p. 620 for dimensions of Furnace, Engines, etc.

To Compute the Volume of Air transmitted by an Engine, When the Pressure, Temperature, etc., are given.

$34.5\sqrt{h\left(\frac{1+.004\,t}{h+H}\right)}\,C = v.$ *h representing pressure of blast in inches of mercury; t temperature of blast;* H *height of barometer in inches;* and *v velocity in feet per second.*

Then $a\,v \times 60 = V$ *in cubic feet per minute.*

ILLUSTRATION.—A furnace having 2 tuyeres of 5 inches diameter, the pressure and temperature of the blast 3 inches and 350°, and barometer 30 inches; what is the volume of air transmitted per minute?

Coefficient for a conical opening .94.

$34.5\sqrt{3\left(\frac{1+.004\times350°}{3+30}\right)} \times .94 = 34.5\sqrt{3\left(\frac{2.4}{33}\right)} = 34.5\times.467\times.94 = 15.14$ *feet velocity per second.*

Then, area 5 ins. = 19.635, which × 2 = 39.27 ins., and 39.27 × 15.14 × 60 ÷ 144 = 247.73 *cubic feet.*

FAN BLOWERS.

Proportions of Parts.—Blades. Their width and length should be at least equal to ¼ or ⅙ the radius of the fan.

Openings. The inlet should be equal to the radius of the fan; and the outlet, or discharge, should be in depth not less than ⅙ the diameter, its width being equal to the width of the fan.

An increase in the number of blades renders the operation of the fan smoother, but does not increase its capacity.

When the pressure of a blast exceeds .7 inch of mercury per square inch, .2 will be a better proportion for the width and length of the fan than that above given.

The pressure or density of a blast is usually measured in inches of mercury, a pressure of 1 lb. per square inch at 60° = 2.0376 inches.

When water is used as the element of measure, a pressure of 1 lb. = 27.671 inches.

The eccentricity of a fan should be .1 of its diameter.

By the experiments of Mr. Buckle, he deduced

1. That the velocity of the periphery of the blades should be .9 that of their *theoretical* velocity; that is, the velocity a body would acquire in

falling the height of a homogeneous column of air equivalent to the required density.

2. That a diminution of the inlet from the proportions here given involved a greater expenditure of power to produce the same density.

3. That the greater the depth of the blade, the greater the density of air produced with the same number of revolutions.

The operation of a blower requires about 2.5 per cent. of the power of the attached boiler.

To Compute the Density of a Blast.

$\left(\frac{v}{8.02}\right)^2 \div 939.454 = d$ *in inches of mercury. v representing velocity of periphery of fan in feet per second.*

ILLUSTRATION.—The velocity of a blast is 123 feet per second.

$$\overline{123 \div 8.02}^2 \div 939.454 = .25 \textit{ inch.}$$

To Compute the Velocity of a Blast.

$$\sqrt{939.454 \times 64.333} = v \textit{ in feet per second.}$$

ILLUSTRATION.—The required density of the air is 1 inch.

$\sqrt{939.454 \times 64.333} = 245.8$ *feet.*

Hence the velocity of the periphery of the fans should be $.9 \times 245.8 = 221.2$ *feet.*

To Compute the Volume of Air discharged per Minute.

$\frac{a \times v \times 60}{160} = V$ *in cubic feet. a representing area of discharge in square ins.*

ILLUSTRATION.—The area of the discharge is 40 inches, and the velocity 123 feet per second.

$$\frac{40 \times 123 \times 60}{160} = 1845 \textit{ cubic feet.}$$

To Compute the Horses' Power.

$\frac{d \times a \times v \times 60}{2400} = a$ P=*horses' power*, or $.000014\, v^2$. P *representing pressure of blast in lbs. per square inch.*

ILLUSTRATION.—The velocity of air discharged is 123 feet per second, the area of the opening 40 square inches, and the density .25 inch of mercury.

$\frac{.25 \times 40 \times 123 \times 60}{24\,000} = 3.07$ *horses, independent of the friction of the blast in the passages and tuyeres.*

A Ton of pig iron requires for its reduction from the ore 310 000 cubic feet of air, or 5.3 cubic feet of air for each pound of carbon consumed. Pressure, .7 lb. per square inch.

An ordinary Eccentric Fan, 4 feet in diameter, with 5 blades 10 inches wide and 14 inches in length, set 1 9/16 ins. eccentric, with an inlet opening of 17.5 inches in diameter, and an outlet of 12 inches square, making 870 revolutions per minute, will supply air to 40 tuyeres, each of 1 5/8 inches in diameter, and at a pressure per square inch of .5 inch of mercury.

An ordinary eccentric fan blower, 50 inches in diameter, running at 1000 revolutions per minute, will give a pressure of 15 inches of water, and require for its operation a power of 12 horses. Area of tuyere discharge 500 square inches.

A non-condensing engine, diameter of cylinder 8 ins., stroke of piston 1 foot, pressure of steam 18 lbs. (mercurial gauge), and making 100 revolutions per minute, will drive a fan, 4 feet by 2, opening 2 feet by 2, 500 revolutions per minute.

Such a blower was applied as an exhausting draught to the smoke-pipe of the Steamer Keystone State, cylinder 80 ins. by 8 feet, and the evaporation was doubled over that of when the wind was calm.

BELTS.

The resistance of belts to slipping is independent of their breadth, consequently there is no advantage derived in increasing this dimension beyond that which is necessary to enable the belt to resist the strain it is subjected to.

The ratio of friction to pressure for belts over wood drums, is for leather belts, when worn, .47; when new, .5; and when over turned cast-iron pulleys, .24 and .27.

A leather belt will safely and continuously resist a strain of 350 lbs. per square inch of section, and a section of .2 of a square inch will transmit the equivalent of a horse's power at a velocity of 1000 feet per minute over a wooden drum, and .4 of a square inch over a turned cast-iron pulley.

A vulcanized India-rubber belt will sustain a greater stress than leather, added to which its resistance to slipping is from 50 to 85 per cent. greater.

In high speed belting, the tension, or the breadth of the belt should be increased, in order to prevent the belt from slipping. Long belts are more effective than short ones.

To Compute the Stress a Belt or Cord is capable of transmitting.—*Aide Memoire.*

Rule.—Multiply the *value* of C from the following table by the stress in pounds.

Proportion of Arc embraced to the Circumference of the Driving Pulley.	Value of Coefficient C.			
	Leather Belts.		Cords on Wooden Sheaves.	
	On Wood Drums.	On Iron Pulleys.	Rough.	Polished.
.2	1.8	1.4	1.9	1.5
.3	2.4	1.7	2.6	1.9
.4	3.3	2.	3.5	2.3
.5	4.4	2.4	4.8	2.8
.6	5.9	2.9	6.6	3.5
.7	7.9	3.4	9.	4.2

C = *the ratio of the resistance of a drum or pulley to slipping a belt or cord when the resistance of the belt or cord upon the under or slack side is known.*

Example.—What is the stress a belt is capable of transmitting when the arc embraced upon the surface of the driving and wooden drum is .4 of its circumference, and the power or tension of the belt is 200 lbs.?

$$3.3 \times 200 = 660 \; lbs.$$

To Compute the Stress which is transmitted to a Belt or Cord.

Rule.—Divide the power in pounds transmitted to the periphery of the pulley by the velocity of the surface of the drum.

Example.—A cast-iron pulley, 4 feet in diameter, driven by a power of 4 horses, makes 160 revolutions per minute; what is the stress upon the belt?

$33\,000 \times 4 = 132\,000$ *lbs.* 1 *foot per minute.*
$4 \times 3.1416 \times 100 = 1256.64$ *feet velocity.*

Then $\frac{132\,000}{1256.64} = 105$ *lbs.* = *difference of the stress upon the belt and the resistance of the under side of it,* $\frac{S}{C - 1} = S$, and $S + s = P$. P *representing the stress transmitted by a belt*, s *the resistance of its under side*, and P *the sum of* $S + s$, *or the stress and resistance.*

Illustration.—What should be the resistance of the under side of a leather belt running over the semi-circumference of a cast-iron pulley, 1 foot in diameter, driven by a power of 200 lbs.?

$$\frac{200}{2.4 - 1} = 142.85 \; lbs.$$

To Compute the Width of a Leather Belt.

ILLUSTRATION.—An engine of 4 horses' power is to be transmitted through a leather belt over a cast-iron pulley, embracing .4 its circumference, 4 feet in diameter, and making 100 revolutions per minute; what should be the width of the belt?

Power, as per preceding example, 132 000 lbs.
Velocity " " 1256.64 "
S " " 105 " and C = 2.

Then $\frac{S}{C-1} = \frac{105}{2-1} = 105$, and $S + s = P = 105 + 105 = 210$ *lbs.*

The resistance or tensile strength of a leather belt is from 270 to 330 lbs. per square inch; and, assuming the thickness of it to be .15 of an inch, then $300 \times .15 = 45$ lbs. Hence $210 \div 45 = 4.67$ *inches.*

ILLUSTRATION.—A belt, 11 inches in width and .22 inch thick, over a drum 4 feet in diameter, C = .5, making 60 revolutions per minute, is sufficient to transmit the power from an engine working at 990 000 lbs. per minute.

Then $\frac{990\,000}{4 \times 3.1416 \times 60} = \frac{990\,000}{753.98} = 1313.3$ *lbs.*, and $\frac{1313.3}{4.4-1} = 386.17$, which $\times 2 =$ 772.35 *lbs.* Hence, $300 \times .22 = 66$, and $\frac{772.35}{66} = 11.7$ *ins.*

India Rubber Belting.—(*Vulcanized.*)

Results of Experiments upon the Adhesion of India Rubber and Leather Belting.
—(J. H. CHEEVER.)

*Rubber.**			Lbs.	*Leather.*			Lbs.
Rubber belt slipped on iron pulley at			90	Leather belt slipped on iron pulley at			48
"	"	leather "	128	"	"	leather "	64
"	"	rubber "	183	"	"	rubber "	128

Hence it appears that a Rubber Belt for equal resistances with a Leather Belt may be reduced, under the circumstances here given, respectively 46, 50, and 30 per cent. from the results to be obtained by the foregoing Rule.

Memoranda.

Two leather belts, 15 ins. in width, over a driver 6 feet in diameter, running with a velocity of 2128 feet per minute, transmit the power from the water-wheel at Rocky Glen Factory, the dimensions of which are given on page 441.

A belt, 11 ins. in width, over a driver 4 feet in diameter, running from 1200 to 2100 feet per minute, will transmit the power from two steam cylinders, 6 ins. in diameter and 11 ins. stroke, averaging 125 revolutions per minute, with a pressure of 60 lbs. per square inch.

The computations here given are based upon the actual horses' power.

CONSTRUCTION OF VESSELS.

Results of Experiments upon the Form of Vessels.
(WM. BLAND.)

Cubical Models.

Head resistance.—Increases directly with the area of its surface.

Resistance to Weight.—Increases directly as the weight.

Vessel's Models.

Lateral Resistance.—About one twelfth of the length of the body immersed, varying with the speed.

The centre of lateral resistance moved forward as the model progressed.

The centre of gravity had no influence upon the centre of lateral resistance.

* Manufacture of the New York Belting and Packing Co, Park Row, N. Y.

Length.—Increased length gave increased speed or less resistance.

Amidship Section.—Curved sections gave higher speed than angled.

Sides.—Curved sides with one fourth more beam gave equal speeds with straight sides of less beam. *Keels.*—Length of keel has greater effect than depth. *Stern.*—Parallel-sided after body gave greater speed than a tapered sided.

Relative Speeds.

Form of Bow.	Order of Speed.
Isosceles triangle, sides slightly convexed	1
" " " right lines	2
" " " slightly concave at entrance and running out convex	3
With each other.	
Equilateral triangle	1
" " spherical sides	2

Relative Resistance to Lee Way.

Form of Amidship Section.	Order of Resistance.	Form of Amidship Section.	Order of Resistance.
2. Rectangular section	1	4. Elliptic section	3
1. Semicircular "	2	3. Triangular section	4

No. 1 resisted best with a depth of keel of ½ an inch; No. 2, best with varying depths of keel and without any; No. 3 resisted best without any keel, and No. 4 had the least resistance without any keel.

Bodies Inclined Upward from Admidship Section.

1. Model, with bow inclined from ⊠, had less resistance than model without any inclination.

2. Model with a stern inclined from ⊠, had less resistance than model without any inclination.

Model 1 had less resistance than model 2. Model with both bow and stern inclined from ⊠, had less resistance than either 1 or 2.

IMMERSED SURFACE OF VESSELS.

To Compute the External, or Bottom and Side Surface of the Hull of a Vessel.

Bottom and Side. Rule.—Multiply the length of the curve of the midship section, taken from the top of the tonnage or main deck beams upon one side to the same point upon the other (omitting the width of the keel) by the mean of the lengths of the keel and between the perpendiculars in feet, and multiply the product by .85 or .9 (according to the capacity of the vessel), and the product will give the surface required in square feet.

Example.—The lengths of a steamer are as follows: length of keel 201 feet, and between the perpendiculars 210 feet, length of the curved surface of the midship section 76 feet; what is the surface?

Coefficient = .87.

$\overline{210+201} \div 2 = 205.5$, and $76 \times 205.5 \times .87 = 13\,587$ *square feet.*

Note.—The exact surface is 13 650 square feet.

Bottom Surface. Rule.—Multiply the length of hull at the load line by its breadth, and this product by the depth of the immersion (omitting the depth of the keel) in feet; and this product multiplied by from .07 to .08 (according to the capacity of the vessel) will give the surface required in square feet.

EXAMPLE.—The length upon the load line of a vessel is 310 feet, the beam 40 feet, the depth of the keel 1 foot, and the draught of water 20 feet; what is the bottom or wet surface?

Coefficient = .073.

$310 \times 40 \times \overline{20 - 1} \times .073 = 17\,199$ *square feet.*

DISPLACEMENT OF VESSELS.

To Compute the Displacement of a Vessel.

RULE.—Multiplply the length of the vessel at the load line by the breadth, and the product by the depth (from the load line to the underside of the garboard strake) in feet, and this product by the unit of volume or coefficient of the mass, and divide by 35 for salt water and 36 for fresh water; the quotient will give the displacement in tons.

NOTE.—This rule is deduced by Mr. S. M. Pook, Naval Constructor, U. S. N., and he gives the ranges of the units of volume as follows:

Amidship sections range from .7 to .9 of their circumscribing square and the mean of the horizontal lines from .55 to .75 of their respective parallelograms. Hence the ranges for vessels of the least capacity to the greatest are $.7 \times .55 = .385$, and $.9 \times .75 = .675$.

Coefficients or Decimals of Proportionate Volume of Vessels.

Merchant Ship, very full	.6 to .7	Merchant Steamer, medium	.52 to .54
" " medium	.58 to .62	Clipper	.5 to .54
River Steamer, stern wheel	.6 to .65	Schooner, medium	.48 to .52
Ship of the Line	.5 to .6	River Steamer, tug boat, sharp	.45 to .5
Naval Steamer, 1st class	.5 to .6	River Steamer, medium	.45 to .5
" "	.52 to .58	" " sharp	.42 to .45
Merchant Steamer, sharp	.54 to .58	Schooner, sharp	.46 to .5
Half Clipper	.52 to .56	Yachts, sharp	.4 to .45
Brigs, Barks, etc.	.52 to .56	" very sharp	.36 to .4
River Steamer, tug boat, m'd'm	.52 to .56	River Steamers, very sharp	.36 to .42

To Compute the Elements of Power required in a Steam Vessel, the Capacity of another Vessel being given.

*In vessels of like Models.** $\frac{v\,A}{a} = V$. *v and v′ representing the products of the volume of the given and computed cylinders and revolutions in cubic feet*, and *a and* A, *areas of their immersed amidship sections in square feet.*

$\frac{S^3\,v'}{s^3} = V$. *s and* S *representing the speeds of the given and required vessels in miles per hour*, and V *the product of the volume of the cylinder and revolutions of the required vessel.*

$\frac{r'\,v}{r} = V$. *r′* and *r representing the revolutions of the given and required vessel.*

$V \div 2\,r = C$. C *representing the cylinder volume in cubic feet.*

ILLUSTRATION.—A steam vessel having an area of amidship section of 675 square feet, has 2 cylinders of 533.33 cubic feet, and runs 10½ knots per hour, with 15 revolutions of her engines. Required the volume of steam cylinders, with a stroke of 10 feet, for a section of 700 feet, and a speed of 13 knots at 14½ revolutions.

$v = 533.33 \times 15 = 8000$ *cubic feet.* $\frac{8000 \times 700}{675} = 8296.3$ *cubic feet.* $\frac{13^3 \times 8296З}{10.5^3}$ = *cubic feet*, and $\frac{15 \times 15\,745.2}{14.5} = 16\,288.1$ *cubic feet.*

* As a means of comparison, if required, and of models, the elements here given are based upon the performances of the Steamers Arctic and Powhatan, for the capacities of which, see pp. 638 and 639, and the area of cylinders deduced are such as the Arctic would have required at the immersion given and speed required.

Then $\frac{16\,288.1}{14.5} = 1123.3$, which $\div$ 2 for the number of cylinders $= 561.65$, which again divided by 10 for the stroke $= 56.165$, which $\div$ 12 for the inches in a foot, and $\times 1728$ for the inches in a cubic foot $= 8087.76$ *square inches = area of the cylinder*, or a diameter of cylinder of 108.5 — *inches.*

Results of Experiments upon the Resistance of Screw Propellers, at high Velocities and immersed at varying Depths of Water.

Immersion of Screw.	Resistance.	Immersion of Screw.	Resistance.	Immersion of Screw.	Resistance.
Surface.	1.	2 feet.	7.	4 feet.	7.8
1 foot.	5.	3 "	7.5	5 "	8.

Thrust of a Screw Propeller.

Indicated horse power 1000. Resistance of hull or thrust of propeller, 20 000 lbs.

Approximate Rules to Compute Speed and Indicated Horses' Power of Steam-boats and Steamers.

$\sqrt[3]{\frac{C\ IHP}{A}} = V$. $\frac{V^3\ A}{C} = IHP$. C *representing coefficient of vessel*, A *area of immersed amidship section*, and V *velocity of vessel in knots per hour.*

Note.—When there exists rig, or any element that affects the unit or coefficient for the class of vessel given, a corresponding addition to, or decrease of the following units is to be made.

The Range of Coefficients, as deduced from Observation, are as follows:

Side Wheel.

River Steam-boats, *small*,	full,	250	Coast Steam-boats, *large*,	full,	320
	medium,	275		medium,	375
	sharp,	300		sharp,	425
River Steam-boats, *medium*,	full,	260	Sea Steamers, *small*,	full,	350
	medium,	290		medium,	425
	sharp,	320		sharp,	500
River Steam-boats, *large*,	full,	270	Sea Steamers, *medium*,	full,	400
	medium,	310		medium,	575
	sharp,	350		sharp,	550
Coast Steam-boats, *medium*,	full,	290	Sea Steamers, *large*,	full,	450
	medium,	335		medium,	530
	sharp,	385		sharp,	600

Propeller.

River Tug-boats, *small*,	full,	250	Sea Steamers, *medium*,	full,	445
	medium,	270		medium,	525
	sharp,	290		sharp,	615
River Tug-boats, *large*,	full,	260	Sea Steamers, *large*,	full,	530
	medium,	290		medium,	630
	sharp,	320		sharp,	730
River Freight-boats, *large*,	full,	300	Sea Steamers, *auxiliary, medium*,	full,	410
	medium,	360		medium,	490
	sharp,	420		sharp,	570
Sea Steamers, *small*,	full,	375	Sea Steamers, *auxiliary, large*,	full,	500
	medium,	450		medium,	580
	sharp,	525		sharp,	660

Relative Capacities of Screw Propellers and Side Wheels.

H. B. M. Screw-propeller Frigate *Arrogant* and the U. S. Mail Side-wheel Steamer *Pacific* had very similar dimensions, like draughts of water, and area of immersed amidship sections.

The Pacific was 75 feet longer, and, consequently, had easier lines.

Results of Performances.

	Arrogant.	*Pacific.*
Indicated Horses' power..................	729	1964
Mean speed in knots per hour.............	8.35	12

$8.35^3 = 582$, and $12^3 = 1728$. Therefore, $1964 : 1728 :: 729 : 663$, and $\frac{582}{663} = .877$.

Hence, the application of power in the Pacific exceeded that in the Arrogant as 1 to .877.

Peninsular and Oriental Screw-propeller Steamer *Himalaya* and Side-wheel Steamer *Atrato* had similar dimensions of immersed section, displacement, and draught of water, and ran at like speeds.

The Himalaya was 22 feet longer, and, consequently, had easier lines.

Results of Performances.

	Himalaya.	*Atrato.*
Indicated Horses' Power............	2050	3070
Mean speed in knots per hour.......	13.75	13.97

$13.75^3 = 2616$, and $13.97^3 = 2726$.

Therefore, $3070 : 2726 :: 2050 : 1820$, and $\frac{2616}{1820} = 1.437$. Hence, the application of power in the Himalaya exceeded that in the Atrato as 1.437 to 1.

Again: $\frac{\text{speed in knots}^3 \times \text{I M S}}{\text{I H P}} =$ *coefficient of immersed amidship section* $= 716, 494$, and $\frac{716}{494} = 1.44$, as before obtained.

Slip of Propeller, 15 per cent.; of Side-wheel feathering blades, and taking axes of blades as the centre of pressure, 23 per cent.

Resistance of Wind to Steam Vessels.—The resistance of air to a square foot of surface at right angles to the course of a vessel is about .33 lbs., and when the surface is inclined to the direction of the wind, the pressure varies as the *sine*² of the angle of incidence.

The mean of the angles of the surface of a steamer exposed to the wind may be taken at 45°; hence their resistance is about .25 lbs. per square foot when the wind has a velocity of 10 knots per hour.

Assuming the sectional area of a steamer's hull above water to be 750 square feet, the resistance to the air at a speed of 10 knots in a dead calm would be $750 \times .25 = 187.5$ lbs., and the resistance to the smoke-pipe, spars, and rigging (brig rigged) would be 201 lbs.

Location of Masts. — Foremast....... .2; Mainmast...... .6; Mizenmast..... .9 — of the length of the load water line from the stem.

Balance of Sails.—The effect of the jib is equal to that of all the sails upon the mainmast, and the sails upon the mizenmast balance those of the foremast.

Lee-way.—A full modeled vessel, with an immersed section of 1 to 6 of her longitudinal section, and with an area of 36 square feet of sails to 1 of immersed section, will drift to leeward 1 mile in 6. A medium modeled vessel, with an immersed section of 1 to 8, and with like areas of sail and section, will drift 1 in 9.

Motive Power.—A sailing vessel having a length 6 times that of her breadth, requires for a speed of 10 knots per hour, an impelling force of 48 lbs. per square foot of immersed section.

Proportion of Power Utilized in a Steam Vessel.

Side Wheel. $\frac{P - z}{.00000259 \times d^3 \times r^2} = C$. P *representing gross Indicated Horses' Power, z loss of effect by slip and oblique action of wheel (or propeller), d diameter of wheels at centre of effect, r revolutions per minute, and* C *coefficient for the vessel.*

ILLUSTRATION.—The indicated horses' power of the engines of a side-wheel steamer is 1120; the slip of the wheels and loss by oblique action, 33.37 per cent.; the diameter of the centre of effect of the wheels is 29.5 feet, and the number of revolutions 13.5 per minute; what is the coefficient, and what the power applied to propel the vessel?

NOTE.—The slip of the wheels from their centre of effect in this case is 15.37 per cent., and the loss by oblique action 18 per cent. Hence, representing the total power by 100, $100 - (18 + 15.37) = 66.63$ per cent. of the power applied to the wheels.

As the assumed power that operates upon the wheels in this case is taken at 86.12 per cent. of the power exerted by the engines, $86.12 \times 33.37 = 28.74$ per cent. for the sum of loss by the wheels.

$$\frac{1120 - (1120 \times 28.74 \div 100)}{.00000259 \times 29.5^3 \times 13.5^2} = \frac{798.11}{12.16} = 65.63 \text{ coefficient.}$$

The speed of the vessel being 10 knots per hour=17.05 feet per second, the power applied to propel the vessel at this speed $= 65.63 \times 17.05^2 = 19\,076.13$, and the horses' power exerted $= \frac{19\,076.13 \times 17.05 \times 60}{33\,000} = 591.36$ *horses' power.*

	Horses' Power.	Per Cent. of Power.
Friction of engines 1.5 lbs. upon 3848 sq. ins. $\times 13.5$ revolutions $\times 10 \times 2 \div 33\,000 \times 2 =$	94.45	13.88 (together)
Friction of load 6 per cent. upon pressure of steam, less the 2 lbs. for friction of engine, as above =	60.45	
Oblique action of wheels =	201.6	18.
Slip of wheels =	172.14	15.37
Absorbed by propulsion of vessel	591.36	52.8
	1120.	100.

Screw Propeller.	Horses' Power.	Per Cent. of Power.	*Auxiliary Propeller.* Made Per Cent. of Power.
Friction of engines =	96.06	18.86 (together)	17.9
" of load =	81.48		
" of screw surface and resistance of edges of blades =	53.44	6.83	12.
Slip of screw =	205.55	26.27	13.7
Absorbed by propulsion of vessel	375.92	48.04	56.4
	782.45	100.	100.

Area of Sails.

The Area taken at Six times that of the Immersed Section of the Vessel.

Sails.	3 Yards upon each Mast.	4 Yards upon each Mast.	Sails.	3 Yards upon each Mast.	4 Yards upon each Mast.
Jib...........	.08	.08	Mizenmast....	.127	.14
Foremast......	.295	.295	Spanker......	.081	.068
Mainmast......	.417	.417			

Proportional Area of Sails upon each Mast under above Divisions.

Sail.	Fore.		Main.		Mizen.		Proportion to 1.	
Course...............	.115	.097	.162	.138	—	—	.389	.33
Topsail..............	.105	.09	.149	.127	.075	.063	.358	.303
Topgallant sail.......	.075	.063	.106	.089	.052	.045	.253	.215
Royal................	—	.045	—	.063	—	.032	—	.152
Spanker (Driver).....	—	—	—	—	.081	.068	—	—
Jib..................	—	—	—	—	.08	.08	—	—
	.295	.295	.417	.417	.288	.288	1.	1.

The areas of the sails upon the masts of a ship should be in the following proportion:

Fore........ 1.414. Main........ 2. Mizen........ 1.

When, therefore, the main yard has a breadth of sail of 100 feet, the fore yard should have 70.71 feet, and the mizen 50 feet; the topgallant and royal yards and sails being in the same proportion.

Assuming the centre of lateral resistance to be in the middle of the length of a vessel, which is the case when she is upon an even keel, the fore and after moments of sails, the masts and sails being set and proportioned in accordance with the preceding rules, will be alike.

Resistance of Bottoms of Hulls at a Speed of One Knot per Hour.

Smooth wood or paint.....	.01 lb.	Copper..................	.007 lb.
Iron bottom, painted......	.014 "	Grass and small barnacles.	.06 "

To Compute the Resistance to the Wet Surface of the Hull of a Vessel.

$C\,a\,v^2 = R$. C *representing a coefficient of resistance, a area of wet surface in square feet*, and *v velocity of hull in feet per second.*

Values of C,	.007, clean copper.	.014, iron plate.
	.01, smooth paint.	.019, iron plate, moderately foul.

The power required to propel 1 square foot of immersed amidship section at ⊠ is .073 that of smooth wet surface.

Estimate of Volumes and Weights for a Cargo and Passenger Steamer for a Route of 1500 *Miles.*

	Volume. Cubic feet.	Weight. Tons.
Cargo..................................	25 000	500
Passengers, 1st Cabin, 25...............	6 250	2.5
" 2d " 20...............	3 000	2
Engines, boilers, and water.............	7 500	150
Fuel and engine-stores..................	10 000	200
Hull and fittings.......................	17 500	350
Provisions and water....................	2 500	50
Officers, engineers, and crew...........	7 500	10
Equipment and sea-stores................	7 500	150
Spare space and weight..................	3 250	85.5
	90 000	1500

Costs of Vessels per Ton.—(*English*, 1865.)

Hull, Joiner Work, Equipment, and Fittings.

IRON.

Merchantman, 500 tons...........	$88.
Passenger ship or steamer, 800 tons.	115.
Passenger steamer, 1500 tons......	147.
" " 1800 "......	122.

Hull (materials)...........	$29.50
Labor....................	14 50
Plant* and labor...........	14.50
Wood work...............	12.25
Fittings and launching.....	14.25
Equipment................	17.
Cabins and passenger fittings..................	20.
	$122.

WOOD.

Merchantman, 650 tons.

Wood in hull, masts, and spars..	$41.
Yellow metal, iron bolts, and labor	10.30
Joiner work and labor..........	5.15
Labor on hull..................	20.
Boats, etc., outfit.............	12.30
Rope and sails.................	8.
Anchors, chains, and tanks......	4.25
Yellow metal sheathing.........	4.
	$105.

Steamer.

Cabins and fittings, 1st class passn'r $25.
" " 2d " 12.5

Engines, boilers, and machinery complete, $225 to $275 per nom'l horse-power.

* Rent, Machinery Tools, etc.

Ship-building.

Weights.—A man requires in a vessel a displacement of 488 lbs. per month for baggage, stores, water, fuel, etc., in addition to his own weight, which is estimated at 175 lbs.

A man and his baggage alone averages 225 lbs.

A sailing ship, 150 feet in length, 32 feet beam, and 22 feet 10 ins. in depth, or 664 tons, C. H. (old measurement), has stowed 2540 square and 484 round bales of cotton. Total weight of cargo 1 254 448 lbs., equal to 4.57 bales, weighing 1889 lbs., per ton of vessel. And a full built ship of 1625 tons, N. M., can carry 1800 tons' weight of cargo, or stow 4500 bales of cotton, New Orleans pressed.

Hull of the iron steam-boat John Stevens—length 245 feet, beam 31 feet, and hold 11 feet: weight of iron 239 440 lbs. And of one—length 175 feet, beam 24 feet, and 8 feet deep: weight of iron 159 190 lbs.

The average weight of a cubic foot of bottom of the ship Great Republic at ⊠, independent of decks and keelsons, was 180 lbs.

The weight of a square foot of bottom of the iron steamer Great Britain at ⊠, independent of deck and keelsons, was 42 lbs. = 170 cubic inches of metal.

The weight of hull of a vessel with an iron frame and oak planking, compared with a hull entirely of wood, is as 8 to 15. An iron hull weighs about 45 per cent. less than a wooden hull.

The weight of hull of an 80-gun ship-of-the-line, 198.5 feet in length by 55 feet beam, is 1995 tons; ballast, 70 tons; crew, equipment, and stores for four months, including armament (447 tons), 1612 tons. Total displacement at load line, 3677 tons.

Sails.

That a vessel's sail may have the greatest effect to propel her forward, it should be set between the plane of the wind and that of the ship's course, that the tangent of the angle it makes with the wind may be twice the tangent of the angle it makes with the ship's course.

Thus, with the wind abeam, a vessel's sails should be braced at an angle of 54° 45′ from the wind, as the tangent of this angle is twice that of 35° 15′, or 90°—54° 45′.

PASSAGES OF STEAMERS AND SAILING VESSELS.

Distances in Geographical Miles or Knots.

STEAMERS—SIDE-WHEELS.

1807, *Phœnix*, of Hoboken, N. J. (John Stevens), New York to Philadelphia. First passage of a steam vessel at sea.

1814, *Morning Star*, of Eng., River Clyde to London. First passage of an English steamer at sea.

1817, *Caledonia*, of Eng., Margate to Cassel, 180 miles, in 24 *hours*.

1819, *Savannah*, of N. Y., about 340 tons O. M., Tybee Light, Savannah River, to Rock Light, Liverpool, 3640 miles, in 25 *days* 14 *hours*; 6 *days* 21 *hours* of which were under steam.

1825, *Enterprise*, of Eng., 500 tons, Falmouth to Table Bay, in 57 *days*; and to Calcutta, India, in 113 *days*. First passage of a steamer to India.

1830, *Hugh Lindsay*, 411 tons, 80 horse-power, Bombay to Suez, 3 103 miles, in 31 *days* running time.

1839, *Great Western*, of Eng., Liverpool to New York, 3017 miles, in 12 *days* 18 *hours*.

1852, *Arctic*, of N. Y., New York to Liverpool, 3017 miles, in 9 *days* 17 *hours* 15 *min*.

1856, *Persia*, of Eng., New York to Liverpool, 3017 miles, in 9 *days* 1 *hour* 45 *min*.

1856, *Ocean Bird*, of N.Y., New York to Havana, 1167 miles, in 4 *days* 4 *hours*.

1858, *Pacific*, of Eng., from St. John's, N. F., to Galway, 1665 miles, in 6 *days* 1 *hour*.

1859, *Baltic*, of N. Y., Aspinwall to New York, 2000 miles, in 6 *days* 21 *hours*. 1853, Liverpool to New York, 3017 miles, in 9 *days* 16 *hours* 33 *min*.

1859, *Vanderbilt*, of N. Y., from the Needles to New York, 3053 miles, in 9 *days* 9 *hours* 26 *min*. 1857, New York to the Needles, in 9 *days* 8 *hours*; equal to 9 *days* 11 *hours* to Liverpool.

1861, *Adriatic*, of N. Y., ran the measured mile at Stokes's Bay, Eng., at an average speed of 15.9 *knots* per hour. 1860, New York to Liverpool, 3017 miles, in 9 *days* 13 *hours* 30 *min.*; Galway to Quarantine, N. Y., touching at St. John's, N. B., 2865 miles, in 8 *days* 12 *hours* 20 *min*. For 1 *day* ran 365 miles.

1862, *Fire-Cracker*, of N. Y., New York to Singapore, 12 309 miles, in 52 *days* 12 *hours*.

1862, *Columbia*, of N. Y., Washington Navy Yard to the Battery, New York, 425 miles, in 30 *hours*.

1865, *Henry Chauncey*, of N. Y., Aspinwall to Canal Street, N. Y., 2000 miles, in 6 *days* 3 *hours* 40 *min*.

1865, *Colorado*, of N.Y., New York to San Francisco, Cal., 14 549 miles, in 61 *days* 21 *hours* 4 *min*.

1866, *Santiago de Cuba*, of N. Y., Greytown to New York, 2090 miles, in 6 *days* 19 *hours*.

1866, *Guiding Star*, of N. Y., New Orleans to New York, 1699 miles, fully laden, in 5 *days* 11 *hours* 30 *min*.

1866, *Morro Castle*, of N. Y., Havana to New York, 1167 miles, in 3 *days* 15 *hours*.

1866, *Mahroussa*, of Egypt, Southampton to Malta, 2130 miles, in 6 *days* 13 *hours* =13.6 *knots* per hour.

1867, *Nevada*, of N. Y., New York to Panama, 11 370 miles, in 43 *days* 5 *hours*, running time.

1870, *Scotia*, of Eng., Queenstown to Sandy Hook, N.Y., 2780 miles, in 8 *days* 7 *hours* 31 *min*. 1866, New York to Queenstown, 2798 miles, in 8 *days* 2 *hours* 48 *min.*; thence to Liverpool, 270 miles, in 14 *hours* 59 *min.*; total, 8 *days* 17 *hours* 47 *min*.

STEAMERS—SCREW.

1866, *City of Baltimore*, of Eng., New York to Liverpool, Eng., ran 385 knots in 24 hours.

1866, *City of Paris*, of Eng., Queenstown (Roche's Point), Ireland, to Liverpool,* Eng., 270 miles, in 14 *hours* 50 *min.*

1867, *Hammonia*, of Bremen, Southampton, Eng., to Sandy Hook, N. Y., 3 103 miles, in 9 *days* 9 *hours* 25 *min.*

1867, *Weser*, of Bremen, Needles, Eng., to Sandy Hook, N. Y., 3 053 miles, in 9 *days* 8 *hours* 20 *min.*

1868, *E. B. Souder*, of N.Y., Sandy Hook, N.Y., to Charleston Bar, S. C., 615 miles, in 49 *hours.*

1869, *City of Boston*, of Eng., Halifax, N. S., to Queenstown, Ireland, 2 226 miles, in 6 *days* 22 *hours.*

1869, *City of Brussels*, of Eng., from Sandy Hook, N. Y., to Rock Light, Liverpool, Eng., 3 000 miles, in 8 *days* 13 *hours* 5 *min.*

1869, *Wm. Lawrence*, of Boston, Norfolk, Va., to Boston, Mass., 585 miles, in 2 *days* 2 *min.*

1870, *Pereire*, of Havre, Brest, France, to New York, N.Y., 2 933 miles, in 8 *days* 15 *hours* 38 *min.;* 1868, New York to Brest, in 8 *days* 10 *hours* 30 *min.*

1870, *Missouri*, of N.Y., New York to Havana, Cuba, 1 167 miles, in 4 *days* 1 *hour.*

1872, *City of Merida*, of N. Y., Havana, Cuba, to New York, N. Y., 1 167 miles, in 3 *days* 11 *hours* 30 *min.*

1872, *City of Houston*, New York to Key West, Fla., 1 145 miles, in 4 *days* 7 *hours* 25 *min.;* thence to Galveston, Texas, 855 miles, in 3 *days* 1 *hour* 10 *min.*

18—, *R. R. Cuyler*, of N. Y., Savannah, Ga., to New York, 700 miles, in 57 *hours.*

1874, *India Government Boat*, Steel, length 87 feet, beam 12 feet, draught of water 3.75 feet, mean speed for one mile 20.77 miles per hour, and maintained a speed of 18.92 miles in 1 *hour.*

1874, *Colima*, of N.Y., San Francisco, Cal., to Yokohama, Japan, 4 750 miles, in 17 *days* 13 *hours.*

1874, *Schiller* and *Goethe*, of Hamburg, Germany, from Hamburg to New York, N.Y., 3 577 miles, each in 10 *days* 20 *hours.*

1874, *City of Waco*, Galveston, Texas, to New York, anchorage to dock, *via* Key West, 2 000 miles, in 6 *days* 18 *hours* 40 *min.*

1875, *Hudson*, of N.Y., S.W. Pass, La., to Sandy Hook, N.Y., 1 648 miles, in 4 *days* 20 *hours* 56 *min.*

1875, *City of Berlin*, of Eng., Liverpool, Eng., to New York *via* Queenstown, 3 050 miles, in 8 *days* 10 *hours* 33 *min.;* and Queenstown, Ireland, to Sandy Hook, N.Y., 2 780 miles, in 7 *days* 18 *hours* 2 *min.*

1875, *Belgic*, of Eng., Yokohama, Japan, to San Francisco, Cal., 4 750 miles, in 17 *days* 12 *hours.*

SAILING VESSELS.

1851, *Chrysolite* (clipper ship), of Eng., Liverpool, Eng., to Anjer, Java, 13 000 miles, in 88 *days.* The *Oriental*, of N. Y., ran the same course in 89 *days.*

1851, *Flying Cloud* (clipper ship), of Boston, Mass., from New York to San Francisco, Cal., 13 610 miles, in 89 *days* and 18 *hours*, and sailed 374 miles in 1 *day.*

1852–3, *Flying Dutchman* (clipper ship), of N. Y., New York to San Francisco, Cal., and return, discharged and loaded, wharf to wharf, 27 220 miles, in 6 *months* 21 *days;* 1853, San Francisco to the Equator, 2 380 miles, 11 *days* 9 *hours*, and rounded Cape Horn, 6 380 miles, in 35 *days.*

1853, *Trade Wind* (clipper ship), of N. Y., San Francisco, Cal., to New York, 13 610 miles, in 75 *days.*

1854, *Lightning* (clipper ship), of Boston, Boston, Mass., to Liverpool, Eng., 2 827 miles, in 13 *days* — *hours;* and Melbourne, Australia, to Liverpool, Eng., 12 190 miles, in 64 *days.*

* Rock Light.

1854, *Comet* (clipper ship), of N. Y., Liverpool to Hong Kong, 13 040 miles, in 84 *days*.

1854, *Sierra Nevada* (schooner), of N. H., Hong Kong to San Francisco, 6 000 miles, in 34 *days*.

1854, *Red Jacket* (clipper ship), of N. Y., Sandy Hook, N. Y., to Liverpool bar, 3 000 miles, in 13 *days* 11 *hours* 25 *min.*; and New York to Melbourne, 12,720 miles, in 69 *days* 11 *hours* 1 *min.*

1855, *Euterpe* (half-clipper ship), of Rockland, Me., New York to Calcutta, 12 500 miles, in 78 *days*.

1855, *Mary Whitridge* (clipper ship), of Baltimore, Baltimore to Liverpool, from Cape Henry, Va., 3 400 miles, in 13 *days* 7 *hours*.

——, *Richard Busteed* (half-clipper ship), of Boston, Sidney, N. S. W., to Calcutta, 5 800 miles, in 42 *days*.

1860, *Dawn* (clipper bark), of N. Y., Buenos Ayres to New York, 6 010 miles, in 36 *days*.

1860, *Andrew Jackson* (clipper ship), of Boston, New York to San Francisco, 13 610 miles, in 80 *days* 4 *hours*.

1865, *Dreadnought* (clipper ship), of Boston, Honolulu to New Bedford, 13 470 miles, in 82 *days*; 1860, Sandy Hook to off Queenstown, 2 760 miles, in 9 *days* 17 *hours*; and 1859, Sandy Hook, N. Y., to Rock Light, Liverpool, 3 000 miles, in 13 *days* 8 *hours*.

——, *Ocean Telegraph* (half-clipper ship), of Boston, Callao to Boston, 9 970 miles, in 58 *days*.

——, *Sovereign of the Seas* (medium ship), of Boston, in 22 days sailed 5 391 miles =245 miles per day. For 4 days sailed 341.78 miles per day, and for 1 *day*, 375 miles.

——, *Northern Light* (half-clipper ship), of Boston, San Francisco to Boston, 13 550 miles, in 76 *days* 8 *hours*.

——, *North Wind* (medium clipper ship), of N.Y., the Downs, Eng., to Port Philip Head, Australia, 12 500 miles, in 67 *days*.

1866, *Henrietta* (schooner yacht), of N. Y., Sandy Hook, N. Y., to the Needles, Eng., 3 053 miles, in 13 *days* 21 *hours* 55 *min.* 16 *sec.*

1866, *Taeping*, *Ariel*, and *Serica* (clipper ships), of England, Foo-chou-foo Bar, China, to the Downs, Eng., 13 500 miles, in 98 *days*. The *Taeping* reached Blackwall in 99 *days*.

1867, *Thornton* (full ship), of N. Y., Sandy Hook, N. Y., to Rock Light, Liverpool, 3 000 miles, in 13 *days* 9 *hours*.

1867, *John J. Ward* (schooner), of N. Y., Alexandria, Va., to Jersey City, N.J., 425 miles, in 48 *hours*.

1867, *Frank Peren* (schooner), of Chicago, Chicago to Buffalo, N.Y., 945 miles (1 100 statute miles), in 3 *days* 5 *hours* 30 *min.*

1868, *Mercury* (ship), of N.Y., New York to Havre, 3 068 miles, in 12 *days* — *hours*.

1869, *Sappho* (schooner yacht), of N. Y., Light-ship off Sandy Hook to Kinsale Head, 2 754 miles, in 12 *days* 7 *hours* 51 *min.*, and to Queenstown, 2 857 miles run, in 12 *days* 9 *hours* 34 *min.*

1869, *Minnie A. Smith* (half brig), of Millbridge, Me., from New York to Salerno, Italy, 4 210 miles, in 27 *days*.

1869, *Dauntless* (schooner yacht), of N. Y., Light-ship off Sandy Hook, N. Y., to Queenstown, 2 770 miles run, in 12 *days* 17 *hours* 6 *min.* 12 *sec.*

18—, *James Barnes*, of Boston, Boston to Liverpool, 2 827 miles, in 12 *days* 6 *hours*.

1869, *Ocean Spray* (bark), Galveston to River Mersey, Eng., 4 850 miles, in 26 *days*.

1873, *Young America* (clipper ship), of N.Y., Liverpool, Eng., to San Francisco, 13,800 miles, in 96 *days*.

1870, *Telegraph* (bark), of Quebec, New York to Cronstadt, 4 588 miles, in 25 *days*.

1870, *Nunquam Dormio* (medium ship), of N. Y., Mobile to Havre, 4 700 miles, in 25 *days*.

NORTHERN AND EASTERN.

Distances in Statute Miles.

1807, *Clermont*, of N.Y., New York to Albany, 145 miles, in 32 *hours* = 4.53 miles per hour, neglecting effect of the tide.

1849, *Alida*, of N.Y., Caldwell's, N.Y., to Pier 1, North River, 43¼ miles, in 1 *hour* 42 *min.*, ebb tide = 2.75 miles per hour. Speed = 22.19 *miles* per hour.

1852, *Reindeer*, of N.Y., New York to Hudson, 116½ miles, in 4 *hours* 57 *min.*, making 5 landings. Flood tide.

1860, *Alida*, of N.Y., 30th Street, N.Y., to Cozzens's Pier, West Point, 50½ miles, in 2 *hours* 4 *min.*, and to Poughkeepsie, 74¼ miles, in 3 *hours* 27 *min.*, making 5 landings. Flood tide. And 1853, Robinson Street to Kingston Light, 90⅝ miles, in 4 *hours*, making 6 landings. Flood tide.

1864, *Daniel Drew*, of N.Y., Jay Street, N.Y., to Albany, 148 miles, in 6 *hours* 51 *min.*, making 9 landings. Flood tide. Speed of boat = 22.6 *miles* per hour.

1864, *Chauncey Vibbard*, of N.Y., Desbrosses Street, N.Y., to Rhinebeck, 91 miles, in 4 *hours* 42 *min.*, making 4 landings. From Rhinebeck to Catskill, 22 miles, in 1 *hour* 6 *min.* Left New York at time of high-water.

1867, *Mary Powell*, of N.Y., Desbrosses Street, N.Y., to Newburg, 60½ miles, in 2 *hours* 50 *min.*, making 3 landings: to Poughkeepsie, 76 miles, in 3 *hours* 40 *min.*, making 6 landings; and to Rondout, 91⅝ miles, in 4 *hours* 23 *min.*, making 7 landings (from Poughkeepsie to Rondout Light, 15⅝ miles, in 39 *min.*). Flood tide. 1873, Milton to Poughkeepsie, light draft and flood tide, 4 miles, in 9 *min.;* and 1874, Desbrosses Street to Piermont, 24 miles, in 1 *hour;* to Caldwell's, 43¼ miles, in 1 *hour* 50 *min.;* to Cozzens's Pier, 50½ miles, in 2 *hours* 9 *min.;* and to Poughkeepsie, making 6 landings, in 3 *hours* 39 *min.* Speed = 22.77 to 23 *miles* per hour.

Runs from New York to Albany, 146 miles, by different Boats.

1826. *Sun,*	12 *hours* 16 *min.*	1851. *New World,*‖	7 *hours* 43 *min.*
1826. *North America,**	10 " 20 "	1852. *Fr. Skiddy,*‖	7 " 24 "
1840. *Albany,*	8 " 27 "	1852. *Reindeer,*‖	7 " 27 "
1841. *Troy,*†	8 " 10 "	1860. *Armenia,*¶	7 " 42 "
1841. *South America,*‡	7 " 28 "	1864, *Daniel Drew,*‡	6 " 51 "
1849. *Alida,*§	7 " 45 "	1864. *Chn'cy Vibbard,*‡	6 " 42 "

1874, *Sylvan Dell*, 7 *hours* 43 *min.*

* 7 landings. † 4 landings. ‡ 9 landings. § 12 landings. ‖ 6 landings. ¶ 11 landings.

Timing Distance.—From 14th St., Hudson River, N. Y., to College at Mount St. Vincent, 13 miles.

NOTE.—Where landings have been made, and the river crossed, the distance between the points given is correspondingly increased.

SOUTHERN AND WESTERN.

Distances in Statute Miles.

1811, *New Orleans*, of Pittsburg, Penn. (non-condensing and stern-wheel), Pittsburg to Louisville, Ky., 650 miles, in 2 *days* 22 *hours*.

1844, *J. M. White*, of St. Louis, Mo. (non-condensing), St. Louis to New Orleans, La., 1200 miles (at that time), 600 tons' freight, 4.5 to 5.5 miles per hour adverse current, in 3 *days* 16 *hours;* and returned to St. Louis, making all regular landings, and losing 2 *hours* 30 *min.* by wooding, etc., in 3 *days* 23 *hours* 9 *min.*

1850, *Buck Eye State*, of Pittsburg, Penn. (non-condensing), Cincinnati to Pittsburg, 500 miles (200 passengers), making 53 landings, in 1 *day* 19 *hours;* 4 miles per hour adverse current. Speed = 15.63 *miles* and 1.23 landings per hour. Average depth of water in channel 7 feet.

1853, *Shotwell*, of Louisville, Ky. (non-condensing), New Orleans to Louisville, 1450 miles, making 8 landings, in 4 *days* 9 *hours;* 4.5 to 5.5 miles per hour adverse current. Speed = 18.81 *miles* per hour.

NOTE.—In 1817–18 the average duration of a passage from New Orleans to Louisville was 27 *days* 12 *hours;* the shortest, 25 *days*.

1855, *Princess*, of New Orleans (non-condensing), New Orleans, La., to Natchez, Miss., 310 miles, in 17 *hours* 30 *min.;* 3.5 to 4 miles per hour adverse current. Speed = 20.98 *miles* per hour.

1861, *Atlanta*, of St. Louis, Mo. (non-condensing), New Orleans to Hard Times Landing, La., making 1 landing, 365 miles, in 24 *hours;* 4.5 to 5.5 miles per hour adverse current. Speed = 20.29 *miles* per hour.

——, *Telegraph*, No. 3 (non-condensing), of Cincinnati, Ohio, Louisville to Cincinnati, 150 miles, in 9 *hours* 55 *min.;* 5.5 miles per hour adverse current. Speed = 20.63 *miles* per hour.

1870, *Natchez*, of Cincinnati (non-condensing), New Orleans, La., to St. Louis, Mo., 1180 miles, making 14 landings, 4 to 5 miles per hour adverse current, and lost 1 *hour* 35 *min.*, in 3 *days* 21 *hours* 58 *min.;* and from New Orleans to Natchez, Miss., 295 miles, in 16 *hours* 51 *min.* 30 *sec.*

1870, *R. E. Lee*, of St. Louis (non-condensing), New Orleans to St. Louis, Mo., 1180 miles (without passengers or freight), 4 to 5 miles per hour adverse current: to Natchez, in 17 *hours* 11 *min.;* Vicksburg, 1 *day* 38 *min.;* Memphis, 2 *days* 6 *hours* 9 *min.;* Cairo, 3 *days* 1 *hour;* and to St. Louis, 3 *days* 18 *hours* 14 *min.*, inclusive of all stoppages. (*Natchez*, of Cincinnati, Ohio, starting in company with the *R. E. Lee*, with passengers, making 2 landings for passengers alone, in 4 *days* 1 *hour* 8 *min.*, including all stoppages for fuel, repairs, and passengers, which were computed at 7 *hours* 28 *min.*) And from New Orleans to Baton Rouge, 120 miles, in 7 *hours* 40 *min.* 42 *sec.;* to Natchez, 295 miles, in 16 *hours* 36 *min.* 47 *sec.*

18—, *Chrysopolis*, Sacramento to San Francisco, Cal., 125 miles, in 5 *hours* 18 *min.*

Runs from New Orleans to Natchez, 295 *miles, by different Boats.*

1814, *Orleans*, 6 *days* 6 *hours* 40 *min.*
1828, *Tecumseh*, 3 *days* 1 *hour* 20 *min.*
1840, *Edward Shippen*, 1 *day* 8 *hours.*
1844, *Old Sultana*, 19 *hours* 45 *min.*
1856, *New Princess*, 17 *hours* 30 *min.*
1870, *R. E. Lee*, 16 *hours* 36 *min.* 47 *sec.*

ICE-BOATS.

Distances in Statute Miles.

1866, *Una*, of Poughkeepsie, N. Y., Newburg to New Hamburg, 6½ miles, in 7 *min.*

1870, *Ella*, of Poughkeepsie, N. Y., Hudson River, 8 miles South and back, estimated by courses at 24 miles, in 23 *min.* 10 *sec.*

1872, *Haze*, of Poughkeepsie, N. Y., Poughkeepsie to buoy off Milton, 4 miles, in 4 *min.*

1872, *Whiz*, of Poughkeepsie, N. Y., Poughkeepsie to New Hamburg, 8⅜ miles, in 8 *min.*

RIFLE-SHOOTING.

1864, *Col. H. Berdan*, Utica, N. Y., 1200 yards, muzzle rest, 4 consecutive bull's eyes; and 1848, at Chicago, Ill., 40 rods, 10 shots, muzzle rest, 7⅜ *inches*.

1869, *Hamilton*. Montreal *vs.* Hamilton, Can., 6 men on each side, 500, 600, 700, 800, and 1000 yards; 7 shots each range; scored 486 *ins*.

PIGEON-SHOOTING.

1865, *A. B. Holabird*, Cincinnati, Ohio, double birds, 21 yards' rise, 100 yards' bounds, 1½ oz. shot, killed 90 out of 100, and 2 fell out of bounds.

1865, *Wm. Seeds*, Newark Meadows, N. J., single birds, 21 yards' rise, 80 yards' bounds, 1¼ oz. shot, killed 84 out of 91=92¼+in 100.

1867, *Paul Mead*, *R. Robinson*, and *F. H. Palmer*, Long Island, N.Y., single birds, 21 yards, 80 yards' bounds, 1¼ oz. shot, guns 9 lbs., high wind, killed 63 out of 75.

1868, — *Root* and — *Brieth*, Newport, Ky., single birds, 21 yards, 80 yards' bounds, 1½ oz. shot, killed 23 out of 24.

1868, *Major Whittingstall*, London, Eng., single birds, 30 yards' rise, 1¼ oz. shot, killed 14 out of 17.

1869, *Ira Payne*, Secaucus, N. J., single birds, 21 yards' rise, 100 yards' bounds, 1½ oz. shot, killed 44 out of 50 from ground trap, 39 out of 40 and 92 out of 100 from spring trap.

1871, *A. H. Bogardus*, Union Course, L. I., single birds, 21 yards' rise, 80 yards' bounds, 1½ oz. shot, ground trap, killed 46 out of 50, and loaded and killed, from spring and plunge traps, 73 birds in 6 *min.* 37½ *sec.*; 1869, Chicago, Ill., single tame birds, 21 yards' rise, 80 yards' bounds, single barrel, heavy gun, shot unlimited, plunge trap, killed 100 in succession.

1871, *John Taylor*, Greenville, N. J., single birds, 21 yards' rise, 80 yards' bounds, ground trap. 1¼ oz. shot, killed 45 out of 50; and 1865, double birds, 18 yards' rise, 100 yards' bounds, 1½ oz. shot, killed 94 out of 100.

1871, *M. Johnson*, Union Course, L. I., double birds, 18 yards' rise, 100 yards' bounds, 1¼ oz. shot, ground traps, killed 28 out of 30.

BIRD-SHOOTING.

18—, *M. Campbell*, Scotland, 220 brace of grouse in 1 *day*.

1856, *Seth Green*, Charlotte, N.Y., 2 double guns, No. 9, No. 8 shot, 990 pigeons, in flocks, in 3 *hours*.

RAT-KILLING.

1862, *Jacko*, London, Eng., 13 lbs., 60 rats in 2 *min.* 43 *sec.*, 100 in 5 *min.* 28 *sec.*, 200 in 14 *min.* 37 *sec.*, and 1000 in less than 100 *min.*; 1861, 25 in 1 *min.* 28 *sec.*

1861, A dog in Philadelphia, Penn., 25 rats in 1 *min.* 28 *sec.*

CEMENTS.

Rust Joint.—(*Quick Setting*)

1 lb. Sal-ammoniac in powder, 2 lbs. Flower of sulphur, 80 lbs. Iron borings. Made to a paste with water.

(*Slow Setting.*)—2 lbs. Sal-ammoniac, 1 lb. of Sulphur, 200 lbs. Iron borings.

The latter cement is best if the joint is not required for immediate use.

For Steam-boilers, Steam-pipes, etc.

Soft.—Red or white lead in oil, 4 parts; Iron borings, 2 to 3 parts.

Hard.—Iron borings and salt water, and a small quantity of Sal-ammoniac with fresh water.

Maltha, or Greek Mastic.

Lime and Sand mixed in the manner of mortar, and made into a proper consistency with milk or size without water.

For China.

Curd of milk, dried and powdered, 10 oz.; Quick-lime, 1 oz.; Camphor, 2 drachms.

Mix, and keep in closely-stopped bottles. When used, a portion is to be mixed with a little water into a paste.

For Earthen and Glass Ware.

Heat the article to be mended a little above 212°, then apply a thin coating of gum Shellac upon both surfaces of the broken vessel.

Or, dissolve gum Shellac in alcohol, apply the solution, and bind the parts firmly together until the cement is dry.

For Holes in Castings.

Sulphur in powder, 1 part; Sal-ammoniac, 2 parts; powdered Iron turnings, 80 parts. Make into a thick paste.

The ingredients composing this cement should be kept separate, and not mixed until required for use.

For Marble.

Plaster of Paris, in a saturated solution of alum, baked in an oven, and reduced to powder. Mixed with water. It may be mixed with various colors.

For Marble Workers and Coppersmiths.

White of egg, or mixed with finely-sifted Quick-lime, will unite objects which are not submitted to moisture.

Transparent—for Glass.

India-rubber, 1 part in 64 of chloroform; add gum Mastic in powder, 16 to 24 parts. Digest for two days with frequent shaking.

To Mend Iron Ware.

Sulphur 2 parts, fine Black Lead 1 part. Put the sulphur in an iron pan, over a fire, until it melts, then add the lead; stir well; then pour out. When cool, break into small pieces. A sufficient quantity of this compound being placed upon the crack of the ware to be mended, can be soldered by an iron.

For Cisterns and Water-casks.

Melted glue, 8 parts; Linseed-oil, 4 parts; boiled into a varnish with litharge.

This cement hardens in about 48 hours, and renders the joints of wooden cisterns and casks air and water tight.

Hydraulic Cement Paint.

Hydraulic cement mixed with oil forms an incombustible and water-proof paint for roofs of buildings, out-houses, walls, etc.

Entomologists' Cement.

Thick Mastic Varnish and Isinglass size, equal parts.

BROWNING, OR BRONZING LIQUID.

Sulphate of copper, 1 oz.; Sweet spirit of nitre, 1 oz.; Water, 1 pint. Mix. In a few days it will be fit for use.

Browning for Gun Barrels.

Tinct. of Mur. of Iron, 1 oz.; Nitric Ether, 1 oz.; Sulph. of Copper, 4 scruples; rain water, 1 pint. If the process is to be hurried, add 2 or 3 grains of Oxymuriate of Mercury.

When the barrel is finished, let it remain a short time in lime-water, to neutralize any acid which may have penetrated; then rub it well with an iron wire scratch-brush.

Bronzing Fluid for Guns, etc.

Nitric acid, sp. gr. 1.2; Nitric ether, Alcohol, and Muriate of iron, each 1 part. Mix, then add Sulphate of copper 2 parts, dissolved in water 10 parts.

LACKERS.

For Small Arms, or Water-proof Paper.

Beeswax, 13 lbs.; Spirits turpentine, 13 gallons; Boiled linseed-oil, 1 gallon.

All the ingredients should be pure and of the best quality. Heat them together in a copper or earthen vessel over a gentle fire, in a water-bath, until they are well mixed.

For Bright Iron Work.

Linseed-oil, boiled	80.5	White lead, in oil	11.25
Litharge	5.5	Resin, pulverized	2.75

Add the litharge to the oil; let it simmer over a slow fire 3 hours; strain it, and add the resin and white lead; keep it gently warmed, and stir it until the resin is dissolved.

INKS.

Indelible, for Marking Linen, etc.

1. Juice of Sloes, 1 pint; Gum, ½ an ounce.

This requires no "preparation" or mordant, and is very durable.

2. Nitrate of silver, 1 part; Water, 6 parts; Gum, 1 part. Dissolve.

Marking.—Lunar caustic, 2 parts; Sap green and Gum arabic, each 1 part; dissolve with distilled water.

The "Preparation."—Soda, 1 ounce; Water, 1 pint; Sap green, ½ drachm. Dissolve, and wet the article to be marked, then dry and apply the ink.

Perpetual, for Tomb-stones, Marble, etc.—Pitch, 11 parts; Lamp-black, 1 part; Turpentine sufficient. Warm and mix.

Copying Ink.—Add 1 oz. of Sugar to a pint of Ordinary Ink.

GLUES.

For Parchment.

Parchment shavings, 1 lb.; Water, 6 quarts.

Boil until dissolved, then strain and evaporate slowly to the proper consistence.

Rice Glue, or Japanese Cement.

Rice flour; Water, sufficient quantity.

Mix together cold, then boil, stirring it all the time.

Liquid.

Glue, Water, and Vinegar, each 2 parts. Dissolve in a water-bath, then add Alcohol, 1 part.

Or, Cologne or strong glue, 2.2 lbs.; Water, 1 quart; dissolved over a gentle heat; add Nitric acid 36°, 7 oz., in small quantities.

Remove from the fire and cool.

Or, White glue, 16 oz.; White lead, dry, 4 oz.; Rain water, 2 pints. Add Alcohol, 4 oz., and continue the heat for a few minutes.

Marine.

Dissolve India-rubber, 4 parts, in 34 parts of Coal-tar naphtha; add powdered Shellac, 64 parts.

While the mixture is hot it is poured upon metal plates in sheets. When required for use, it is heated, and then applied with a brush.

Or, 1 part India-rubber, 12 parts of Coal tar; heat gently, mix, and add 20 parts of powdered Shellac. Pour out to cool. When used, heat to about 250°.

Or, Glue, 12 parts; Water, sufficient to dissolve; add yellow Resin, 3 parts; and, when melted, add Turpentine, 4 parts.

Mix thoroughly together.

Strong Glue.—Add Powdered chalk to common glue.

Gum Mucilage.—A little oil of cloves poured into a bottle containing gum mucilage prevents it from becoming sour.

Glue to resist Moisture.

5 parts Glue, 4 parts Resin, 2 parts Red ochre, mixed with the least practicable quantity of water.

Or, 4 parts of Glue, 1 part of Boiled oil by weight, 1 part Oxide of iron.

Or, 1 lb. of glue melted in 2 quarts of skimmed milk.

VARNISHES.

Waterproof.

Flower of sulphur, 1 lb.; Linseed-oil, 1 gall.; boil them until they are thoroughly combined.

This forms a good varnish for waterproof textile fabrics.

Another is made of Oxyde of lead, 4 lbs.; Lamp-black, 2 lbs.; Sulphur, 5 oz.; and India-rubber dissolved in turpentine, 10 lbs.

Boil together until they are thoroughly combined.

To Adhere Engravings or Lithographs upon Wood.

Sandarach, 250 parts; Mastic in tears, 64; Resin, 125; Venice turpentine, 250; and Alcohol, 1000 parts by measure.

For Harness.

India-rubber, ½ lb.; Spirits of turpentine, 1 gall.; dissolve into a jelly; then take hot Linseed-oil, equal parts with the mass, and incorporate them well over a slow fire.

For Fastening Leather on Top Rollers.

Gum Arabic, 2¾ oz., dissolved in water, and a like volume of Isinglass dissolved in water.

To Preserve Glass from the Rays of the Sun.

Reduce a quantity of Gum Tragacanth to fine powder, and let it dissolve for 24 hours in white of eggs well beat up.

For Water-color Drawings.

Canada balsam, 1 part; Oil of turpentine, 2 parts, mixed.
Size the drawing before applying the varnish.

For Objects of Natural History, for Shells, Fish, etc.

Mucilage of Gum Tragacanth and Mucilage of Gum Arabic, each 1 oz.
Mix, and add spirit with Corrosive sublimate, so as to precipitate the more stringy part of the gum.

For Articles of Iron and Steel.

Clear grains of Mastic, 10 parts; Camphor, 5 parts; Sandarach, 15 parts; and Elemi, 5 parts. Dissolve in a sufficient quantity of alcohol, and apply without heat.

This varnish will retain its transparency, and the metallic brilliancy of the articles will not be obscured.

For Gun Barrels, after Browning.

Shellac, 1 oz.; Dragon's-blood, ¼ oz.; rectified Spirit, 1 quart. Dissolve and filter.

Black.

Heat to boiling, 10 parts of Linseed-oil varnish with burnt Umber, 2 parts, and powdered Asphaltum, 1 part.

When cooled, dilute with Spirits of turpentine as required.

Balloon.

Melt India-rubber in small pieces with its weight of boiled Linseed-oil. Thin with Oil of turpentine.

Transfer.

Alcohol, 5 oz.; pure Venice turpentine, 4 oz.; Mastic, 1 oz.

To Clean Varnish.

Mix a ley of Potash, or Soda, with a little powdered Chalk.

Composition for rendering Canvas Waterproof and Pliable.

Yellow soap, 1 lb., boiled in 6 pints of water, add, while hot, to 112 lbs. of Paint.

STAINING.

Staining Wood and Ivory.

Yellow. Dilute Nitric acid will produce it on wood.

Red. An infusion of Brazil wood in stale urine, in the proportion of 1 lb. to a gallon for wood, to be laid on when boiling hot, and should be laid over with alum water before it dries. Or, a solution of Dragon's-blood in Spirits of wine.

Black. Strong solution of Nitric acid.

Mahogany. Brazil, Madder, and Logwood, dissolved in water and put on hot.

Blue. For Ivory: soak it in a solution of Verdigris in Nitric acid, which will turn it *green;* then dip it into a solution of Pearlash boiling hot.

Purple. Soak ivory in a solution of Sal-ammoniac into four times its weight of Nitrous acid.

MISCELLANEOUS.

To Clean Marble.

Chalk, powdered, and Pumice-stone, each 1 part; Soda, 2 parts. Mix with water. Wash the spots, then clean and wash off with soap and water.

To Extract Grease from Stone or Marble.

Soft soap, 1 part; Fuller's earth, 2 parts; Potash, 1 part. Mix with boiling water. Lay it upon the spots, and let it remain for a few hours.

Paint for Window Glass.

Chrome green, ¼ oz.; Sugar of lead, 1 lb.; ground fine, in sufficient Linseed-oil to moisten it. Mix to the consistency of cream, and apply with a soft brush.

The glass should be well cleansed before the paint is applied. The above quantity is sufficient for about 200 feet of glass.

Durable Paste.

Make common flour paste rather thick (by mixing some flour with a little *cold* water until it is of uniform consistency, and then stir it well while *boiling* water is being added to it); add a little brown Sugar and Corrosive sublimate, which will prevent fermentation, and a few drops of Oil of lavender, which will prevent it becoming moldy. When this paste dries, it may be used again by dissolving it in water.

It will keep for two or three years in a covered vessel.

Dubbing.

Resin, 2 lbs.; Tallow, 1 lb.; Train-oil, 1 gallon.

Blacking for Harness.

Bees'-wax, ½ lb.; Ivory black, 2 oz.; Spirits of turpentine, 1 oz.; Prussian blue ground in oil, 1 oz.; Copal Varnish, ¼ oz.

Melt the wax and stir it into the other ingredients before the mixture is quite cold; make it into balls. Rub a little upon a brush, and apply it upon the harness, then polish lightly with silk.

To Prevent Iron from Rusting.

Warm it; then rub with White wax; put it again to the fire until the wax has pervaded the entire surface.

Or, immerse tools or bright work in boiled Linseed-oil and allow it to dry upon them.

Paper for Draughtsmen, etc.

Powdered Tragacanth, 1 part; Water, 10 parts.

Dissolve, and strain through clean gauze, then lay it smoothly upon the paper, previously stretched upon a board.

This paper will take either oil or water-colors.

To Remove old Ironmold.

Remoisten the part stained with ink, remove this by the use of Muriatic acid diluted by 5 or 6 times its weight of water, when the old and new stain will be removed.

Pastiles for Fumigating.

Gum Arabic, 2 oz.; Charcoal powder, 5 oz.; Cascarilla bark, powdered, ¾ oz.; Saltpetre, ¼ dram. Mix together with water, and make into shape.

For Writing upon Zinc Labels—Horticultural.

Dissolve 100 gr. of Chloride of Platinum in a pint of water; add a little Mucilage and Lamp-black.

Or, Sal-ammoniac, 1 dr.; Verdigris, 1 dr.; Lamp-black, ½ dr.; water, 10 dr. Mix.

Booth's Grease for Railway Axles.

Water	1 gall.	Palm-oil	6 lbs.	Or, Tallow	8 lbs.
Clean tallow	3 lbs.	Common soda	½ lb.	Palm-oil	10 "

To be heated to about 212°, and to be well stirred until it cools to 70°.

Anti-friction Grease.

100 lbs. Tallow, 70 lbs. Palm-oil. Boiled together, and when cooled to 80°, strain through a sieve, and mix with 28 lbs. of Soda and 1½ gallons of water.

For Winter, take 25 lbs. more oil in place of the tallow.

Or, Black Lead, 1 part; Lard, 4 parts.

Liard.

50 parts of finest Rape-oil and 1 part of Caoutchouc, cut small. Apply heat until it is nearly all dissolved.

Stains.

To Remove.—Stains of *Iodine* are removed by rectified spirit. *Ink* stains by oxalic or superoxalate of potash. *Ironmolds* by the same; but if obstinate, moisten them with ink, then remove them in the usual way.

Red spots upon black cloth from acids are removed by Spirits of Hartshorn, or other solutions of Ammonia.

Stains of Marking-ink, or Nitrate of Silver.—Wet the stain with fresh solution of Chloride of lime, and after 10 or 15 minutes, if the marks have become white, dip the part in solution of Ammonia or of Hyposulphite of soda. In a few minutes wash with clean water.

Or, stretch the stained linen over a basin of hot water, and wet the mark with tincture of Iodine.

Preservative Paste for Objects of Natural History.

White Arsenic, 1 lb.; Powdered Hellebore, 2 lbs.

Paste for Cleaning Metals.

Oxalic acid, 1 part; Rottenstone, 6 parts. Mix with equal parts of Train-oil and Spirits of turpentine.

Watchmaker's Oil, which never Corrodes or Thickens.

Place coils of thin sheet lead in a bottle with Olive-oil. Expose it to the sun for a few weeks, and pour off the clear oil.

Blacking, without Polishing.

Molasses, 4 oz.; Lamp-black, ½ oz.; Yeast, a table-spoonful; Eggs, 2; Olive-oil, a teaspoonful; Turpentine, a teaspoonful. Mix well.

To be applied with a sponge, without brushing.

To Preserve Sails.

Slacked lime, 2 bushels. Draw off the lime-water, and mix it with 120 gallons water, and with Blue Vitriol, ¼ lb.

Whitewash

For outside exposure, slack Lime, ½ a bushel, in a barrel; add Common Salt, 1 lb.; Sulphate of Zinc, ½ lb.; and Sweet Milk, 1 gallon.

To Preserve Woodwork.

Boiled oil and finely-powdered Charcoal, each 1 part; mix to the consistence of paint. Lay on two or three coats with it.

This composition is well adapted for casks, water-spouts, etc.

To Polish Wood.

Rub surface with Pumice-stone and water until the rising of the grain is removed. Then, with powdered Tripoli and boiled Linseed-oil, polish to a bright surface.

Files.

Lay dull files in diluted Sulphuric acid until they are bitten deep enough.

To Clean Brass Ornaments.

Brass ornaments that have not been gilt or lackered may be cleaned, and a very brilliant color given to them, by washing them in alum boiled in strong ley, in the proportion of an ounce to a pint, and afterward rubbing them with strong Tripoli.

Adhesive Cement for Fractures of all Kinds.

White Lead ground with Linseed-oil Varnish, and kept out of contact with the air.

It requires a few weeks to harden.

When stone or iron are to be cemented together, use a compound of equal parts of Sulphur and Pitch.

ALLOYS AND COMPOSITIONS.

Alloy is the proportion of a baser metal mixed with a finer or purer, as when copper is mixed with gold, etc.

Amalgam is a compound of Mercury and a metal—a soft alloy.

All Compositions of copper contract in the admixture, and all Amalgams expand.

In the manufacture of alloys and compositions, the more infusible metals should be melted first.

In compositions of Brass, as the proportion of Zinc is increased, so is the malleability decreased.

The tenacity of Brass is impaired by the addition of Lead or Tin.

Steel alloyed with $\frac{1}{500}$th part of platinum, or silver, is rendered harder, more malleable, and better adapted for cutting instruments.

ALLOYS AND COMPOSITIONS.

	Copper.	Zinc.	Tin.	Nickel.	Lead.	Antimony.	Bismuth.	Silver.	Cobalt of Iron.	Iron.	Arsenic.
Argentan	55.	24.	—	21.	—	—	—	—	—	—	—
Argentiferous	50.	2.5	2.5	40.	2.5	—	—	—	—	2.5	—
Babbitt's metal*	3.7	—	89.	—	—	7.3	—	—	—	—	—
Brass, common	84 3	5.2	10.5	—	—	—	—	—	—	—	—
" "	75.	25.	—	—	—	—	—	—	—	—	—
" " hard	79.3	6.4	14.3	—	—	—	—	—	—	—	—
" Mathematical instruments	92.2	—	7.8	—	—	—	—	—	—	—	—
" pinchbeck	80.	20.	—	—	—	—	—	—	—	—	—
" red tombac	88.8	11.2	—	—	—	—	—	—	—	—	—
" rolled	74.3	22.3	3.4	—	—	—	—	—	—	—	—
" tutenag	50.	31.	—	19.	—	—	—	—	—	—	—
" very tenacious	88.9	2.8	8.3	—	—	—	—	—	—	—	—
" wheels, valves	90.	—	10.	—	—	—	—	—	—	—	—
" white	10.	80.	10.	—	—	—	—	—	—	—	—
" wire	67.	33.	—	—	—	—	—	—	—	—	—
" yellow, fine	66.	34.	—	—	—	—	—	—	—	—	—
Britannia metal	—	—	25.	—	—	25.	—	—	—	—	—
When fused, add	—	—	—	—	—	25.	25.	—	—	—	—
Bronze, red	87.	13.	—	—	—	—	—	—	—	—	—
" red	86.	11.1	2.9	—	—	—	—	—	—	—	—
" yellow	67.2	31.2	1.6	—	—	—	—	—	—	—	—
" Cymbals	80.	—	20.	—	—	—	—	—	—	—	—
" gun metal, large	90.	—	10.	—	—	—	—	—	—	—	—
" " small	93.	—	7.	—	—	—	—	—	—	—	—
" Medals	93.	—	7.	—	—	—	—	—	—	—	—
" Statuary	91.4	5.5	1.4	—	1.7	—	—	—	—	—	—
Chinese silver	65.1	19.3	—	13.	—	—	—	2.48	12.	—	—
Chinese white copper	40.4	25.4	2.6	31.6	—	—	—	—	—	—	—
Church bells	80.	5.6	10.1	—	4.3	—	—	—	—	—	—
"	69.	—	31.	—	—	—	—	—	—	—	—
Clock bells	72.	—	26.5	—	—	—	—	—	—	1.5	—
Cocks, Musical bells	87.5	—	12.5	—	—	—	—	—	—	—	—
German silver	33.3	33.4	—	33.3	—	—	—	—	—	—	—
" "	40.4	25.4	—	31.6	—	—	—	—	—	2.6	—
" " fine	49.5	24.	—	24.	—	—	—	—	—	2.5	—
Gongs	81.6	—	18.4	—	—	—	—	—	—	—	—
House bells	77.	—	23.	—	—	—	—	—	—	—	—
Lathe bushes	80.	—	20.	—	—	—	—	—	—	—	—
Machinery bearings	87.5	—	12.5	—	—	—	—	—	—	—	—
" " hard	77.4	7.	15.6	—	—	—	—	—	—	—	—
Metal that expands in cooling	—	—	—	—	75.	16.7	8.3	—	—	—	—
Muntz metal	60.	40.	—	—	—	—	—	—	—	—	—
Pewter, best	—	—	86.	—	—	14.	—	—	—	—	—
"	—	—	80.	—	20.	—	—	—	—	—	—
Printing characters	—	—	—	—	80.	20.	—	—	—	—	—
Sheathing metal	56.	45.	—	—	—	—	—	—	—	—	—
Speculum "	66.	—	22.	—	—	—	—	—	—	—	—
" "	50.	21.	29.	—	—	—	—	—	—	—	12.
Telescopic mirrors	66.6	—	33.4	—	—	—	—	—	—	—	—
Temper†	33.4	—	66.6	—	—	—	—	—	—	—	—
Type and stereotype plates	—	—	—	—	69.	15.5	15.5	—	—	—	—
White metal	7.4	7.4	28.4	—	—	56.8	—	—	—	—	—
" " hard	69.8	25.8	4.4	—	—	—	—	—	—	—	—
Oreide	73.	12.3	Magnesia .... 4.4, Sal-ammoniac 2.5; Cream of tartar 6.5, Quick-lime 1.3								

* See page 628 for directions.

† For adding small quantities of copper.

Solders.

	Copper.	Tin.	Lead.	Zinc.	Silver.	Bismuth.	Gold.	Calcimine.	Antimony.
Tin	—	25	75	—	—	—	—	—	—
"	—	58	16	—	—	16	—	—	10
" coarse, melts at 500°	—	33	67	—	—	—	—	—	—
Tin, ordinary, melts at 360°	—	67	33	—	—	—	—	—	—
Spelter, soft	50	—	—	50	—	—	—	—	—
" hard	67	—	—	33	—	—	—	—	—
Lead	—	33	67	—	—	—	—	—	—
Steel	13	—	—	5	82	—	—	—	—
Brass or Copper	50	—	—	50	—	—	—	—	—
Fine brass	47	—	—	47	6	—	—	—	—
Pewterers' or Soft	—	33	45	—	—	22	—	—	—
" "	—	50	25	—	—	25	—	—	—
Gold	4	—	—	—	7	—	89	—	—
" hard	66	—	—	34	—	—	—	—	—
" soft	—	66	34	—	—	—	—	—	—
Silver, hard	20	—	—	—	80	—	—	—	—
" soft	12	—	—	—	67	—	—	21	—
Pewter	—	40	20	—	—	40	—	—	—
Iron	66	—	—	33	—	—	—	—	1
Copper	53	47	—	—	—	—	—	—	—

A Plastic Metallic Alloy.—See Journal of Franklin Institute, vol. xxxix., page 55, for its composition and manufacture.

Composition for Welding Cast Steel.

Borax, 10 parts; Sal-ammoniac, 1 part. Grind or pound them roughly together; fuse them in a metal pot over a clear fire, continuing the heat until all spume has disappeared from the surface. When the liquid is clear, pour the composition out to cool and concrete, and grind to a fine powder; then it is ready for use.

To use this composition, the steel to be welded should be raised to a bright yellow heat; then dip it in the welding powder, and again raise it to a like heat as before; it is then ready to be submitted to the hammer.

Fusible Compounds.

Compounds.	Zinc.	Tin.	Lead.	Bismuth.	Cadmium.
Rose's, fusing at 200°	—	25	25	50	—
Fusing at less than 200°	33.3	—	33.3	33.4	—
Newton's, fusing at less than 212°	—	19	31	50	—
Fusing at 150° to 160°	—	12	25	50	13

Soldering Fluid for use with soft Solder.

To 2 fluid oz. of Muriatic acid add small pieces of Zinc until bubbles cease to rise. Add ½ a teaspoonful of Sal-ammoniac and 2 fluid oz. of Water.

By the application of this to Iron or Steel, they may be soldered without their surfaces being previously tinned.

Fluxes for Soldering or Welding.

Iron	Borax.	Zinc	Chloride of zinc.
Tinned iron	Resin.	Lead	Tallow or resin.
Copper and Brass	Sal-ammoniac.	Lead and tin pipes.	Resin and sweet oil.

Steel—Sal-ammoniac, 1 part; Borax, 10 parts. Pound together, and fuse until clear, and, when cool, reduce to powder.

Babbitt's Anti-attrition Metal.

Melt 4 lbs. Copper; add, by degrees, 12 lbs. best Banca tin, 8 lbs. Regulus of antimony, and 12 lbs. more of Tin. After 4 or 5 lbs. Tin have been added, reduce the heat to a dull red, then add the remainder of the metal as above.

This composition is termed *hardening*: for lining, take 1 lb. of this *hardening*, melt with it 2 lbs. Banca tin, which produces the lining metal for use. Hence, the proportions for lining metal are 4 lbs. of copper, 8 of regulus of antimony, and 96 of tin.

MISCELLANEOUS NOTES.

Dimensions of Drawings for Patents.—United States 8.5×12 inches.

Painting of Brick-work.—A square yard of new brick wall requires for the first coat of paint in oil ¾ lb., and for the second, .3, and for the third, .4.

Service Train of a Quartermaster.—The Quartermaster's train of an army averages 1 wagon to every 24 men; and a well-equipped army in the field, with artillery, cavalry, and trains, requires 1 horse or mule, upon the average, to every 2 men.

A *Luminous point*, to produce a *visual* circle, must have a velocity of 10 feet in a second, the diameter not exceeding 15 inches.

All solid bodies become *luminous* at 800 degrees of heat.

Tides.—The difference in time between high water averages about 49 minutes each day.

In sandy soil, the greatest force of a pile-driver will not drive a pile over 15 feet.

A *fall* of .1 of an inch in a mile will produce a *current* in rivers.

Melted snow produces from ¼ to ⅛ of its bulk in water.

At the depth of 45 feet, the *temperature of the earth* is uniform throughout the year.

A *Spermaceti candle* .85 of an inch in diameter consumes an inch in length in 1 hour.

Silica is the base of the mineral world, and *Carbon* of the organized.

Sound passes in water at a velocity of 4708 feet per second.

Metals have five degrees of lustre—*splendent*, *shining*, *glistening*, *glimmering*, and *dull.*

A Marble-saw requires half a horse's power.

Avenues of City of New York run 28° 50′ 30″ east of north.

Wire and Hemp Ropes.—A wire rope 3½ ins. in circuumference, and a hemp shroud 8 ins. in circumference, parted in the rope at 10½ tons = 4600 lbs. per square inch.

Endless Ropes.—The friction or adhesion of Ropes is from .1 to .07 of their weight.

Gold Leaf is the 280 000th part of an inch in thickness.

Gun Barrels.—An ointment of Corrosive sublimate and Lard will preserve them from the corrosive effects of sea air.

Brief Rules for the Computation of the Weights of Cast Iron Pipes and Cast and Wrought Iron Bolts.—(Horatio Allen.)

Cast Iron Pipes.—To the inner diameter of the pipe add the thickness of the pipe in inches, and multiply the sum by 10 times the thickness, and the product will give the weight in pounds per foot.

Wrought Iron Bolts.—Square the radius of the bolt and multiply it by 10, and the product will give the weight in pounds per foot.

For Cast iron, subtract $\frac{2}{27}$, or .074 of the result.

Malleable or Aluminum Bronze.—By weight: Copper, 90; Aluminum, 10. This composition may be forged either when heated or cooled, and becomes extremely dense. Its tensile strength is 100 000 lbs., and when drawn into wire 128 000 lbs., and its elasticity one half that of wrought iron. Specific gravity, 7700.

EQUIVALENTS OF OLD AND NEW U. S. MEASURES.

Length.

	Meters.
1 Inch =	.02540005
1 Foot =	.3048006
1 Yard =	.9144018
1 Chain =	20.1168396
1 Furlong =	201.168396
1 Mile =	1609.347168

Surface.

	Square Meters.
1 Inch =	.000645161
1 Foot =	.092903184
1 Yard =	.836128656
1 Rod =	25.292891844
1 Rood =	1011.71567376
1 Acre =	4046.86269504

Volume.

	Liters.*
1 Fluid Drachm =	.0036967
1 Fluid Ounce =	.0295739
1 Fluid Pound =	.35488656
1 Gill =	.1182955
1 Wine Pint =	.4731821
1 Dry Quart =	1.1012344
1 Wine Quart =	.9463642
1 Wine Gallon =	3.7854579

Weight.

	Grams.
1 Grain =	.0648004
1 Scruple =	1.296008
1 Dwt. =	1.5552096
1 Drachm =	3.888024
1 Ounce (Troy) =	31.104192
1 Ounce† =	28.350175
1 Pound =	453.6028
1 Ton =	1016070.272

NOTE.—A square Meter is 1549.9969 square inches, but by Act of Congress it is declared to be 1550 square inches; hence the Liter (cubic decimeter) = 61.023 377 953 cubic inches. In the Act of Congress, a Liter is declared to be 61.022 cubic inches, which is erroneous, as here shown, by the .001 + of an inch.

MISCELLANEOUS ILLUSTRATIONS.

1. The turret of an Iron-clad steamer has an internal diameter of 21 feet, and is composed of 10 courses of plates, 1 inch in thickness, to which an allowance of half a sixteenth ($\frac{1}{32}$d part) of an inch must be made for space between each course. What is the diameter of the inner surface of the outer course in feet and the decimal of a foot?

21 feet + $(10-1\times\overline{1+1})$ ins. + $(10-1\times\overline{\tfrac{1}{2}\times2})$ sixteenths = 22 *feet* 6 *ins.* 9 *sixteenths.* Hence, to reduce this to feet and the decimal of a foot,

Feet.	Inches.	Sixteenths.
22	6	9
12		
270		
16		
1629		
270		
4329		

and 16)4329
12)270.5625
22.546875 *feet.*

2. How many changes may be rung with 4 bells out of 8?
Operation.—$8\times7\times6\times5=1680$ *changes.*

3. How many changes are there in the throws of 5 dice?
Operation.—$6\times6\times6\times6\times6=7776$ *changes.*

4. It is required to lay out a tract of land in form of a square, to be inclosed with a post and rail fence, 5 rails high, and each rod of fence to contain 10 rails. What must be the side of this square to contain just as many acres as there are rails in the fence?

Operation.—1 mile = 320 *rods.* Then $320\times320\div160$, *the square rods in an acre* = 640 *acres;* and 320×4 *sides* $\times10$ *rails* = 12 800 *rails per mile.*

Then, as 640 *acres* : 12 800 *rails* : : 12 800 *acres* : 256 000 *rails*, which will inclose 256 000 *acres*, and $\sqrt{256\,000}\times69.5701$, *the number of yards in the side of a square acre* and $\div1760$, *yards in a mile* = 20 *miles.*

* 61.028 cubic inches.

† Avoirdupois.

5. A invested in a Company \$150 for 14 months, B a certain sum for one year, and C \$100 for a certain time; some time afterward they ascertained that their stock and profits were equal to \$475, of which sum A was credited \$195, B \$153, and C \$127; how much did B invest, and for how long a time was C's stock invested?

Operation.—A was credited \$195, and he invested \$150; hence $195 - 150 = 45$, *his profit in* 14 *months.*

Then, as 14 months : 45 : : 12 months : 38.57, *profit on* \$150 *in* 12 *months.*
And as 188.57 (150 + 38.57) : 150 : : 153 (B's stock) : 121.7, B's *investment.*
C was credited \$127. Then $(127 - 100) = 27$, C's *profit.*
And as 150 : 45 : : 100 : 30, C's *ratio of profit* for 14 *months.*
Therefore, as 30 : 14 : : 27 : 12.6 *months, the time* C's *stock was invested.*

6. How many fifteens can be counted with four fives?

Operation. $$\frac{4\times3\times2\times1}{1\times2\times3}=\frac{24}{6}=4.$$

7. Assume there are 4 companies, in each of which there are 9 men; it is required to ascertain how many ways 4 men may be chosen, one out of each company.

Operation. $$9\times9\times9\times9=6561.$$

8. What are the chances in favor of throwing one point with three dice?

Operation.—Assume a bet to be upon the ace. Then there will be $6\times6\times6=216$ *different ways which the dice may present themselves, that is, with and without an ace.*

Then, if the ace side of the die is excluded, *there will be* 5 *sides left*, and $5\times5\times5 = 125$ *ways without the ace.*

Therefore there will remain only $216 - 125 = 91$ *ways in which there could be an ace.* The chance, then, in favor of the ace is as 91 to 125; that is, out of 216 throws, the probability is that it will come up 91 *times*, and lose 125 *times.*

9. If a body should move through the length of 1 barley-corn in a second of time, one inch in the second second, three inches in the third, and so continue increasing its motion in triple geometrical proportion, how many yards would it advance in half a minute?

Operation.	0	1	2	3	4	5	6	7	8	9
	1	3	9	27	81	243	729	2187	6561	19683

Then $9 + 9 + 9 + 3 = 30$, *the number of seconds or terms.*

And $19\,683\times19\,683\times19\,683\times27 = 205\,891\,132\,094\,649 = 30$th *power*, which $\div\ \overline{3-1}$, the ratio less $1 = 102\,945\,566\,047\,324$ *barley-corns*, and again by 3, 12, and 3, to reduce it to yards $= 953\,199\,685\,623$ *yards* 1 *foot* 1 *inch* and 1 *barley-corn.*

10. A person expended \$100 for 100 head of live-stock, consisting of cows, sheep, and pigs; for the cows he paid \$10 per head, for the sheep \$1, and for the pigs 50 cents. How many did he purchase of each kind?

Operation.—The average cost of the stock was \$1. Hence, *the cost of a pig was* 50 *cents below the average, and that of a cow* \$9 *above.*

Therefore, if he had purchased cows and pigs only; in the inverse ratio of their cost, he would have had 5 cows and 90 pigs = 95 *animals costing* \$95. Then, by adding 5 sheep, at \$1 per head, he had

5	*cows,* at	\$10.	=	\$50
90	*pigs* at	.50	=	45
5	*sheep,* at	1.	=	5
100				\$100

11. The hour and minute hand of a clock are exactly together at 12; when are they next together?

Operation.—As the minute hand runs 11 times faster than the hour hand, then, 11 : 60 : : 1 : 5 *min.* $27\frac{3}{11}$ *sec.* The time, then, is 5 *min.* $27\frac{3}{11}$ *sec. past* 1 *o'clock.*

12. The time of the day is between 4 and 5, and the hour and minute hands are exactly together; what is the time?

Operation.—The difference of the speed of the hands is as 1 to $12 = 11$.
4 hours $\times 60 = 240$, which $\div 11 = 21$ *min.* 49.09 *sec., which is to be added to* 4 *hours.*

13. Assume a cubic inch of glass to weigh 1.49 ounces troy, the same of sea-water .59, and of brandy .53. A gallon of this liquor in a glass bottle, which weighs 3.84 lbs., is thrown into sea-water. It is proposed to determine if it will sink, and, if so, how much force will just buoy it up?

Operation.—$3.84 \times 12 \div 1.49 = 30.92$ *cubic inches of glass in the bottle.*
231 cubic inches in a gallon $\times .53 = 122.43$ *ounces of brandy.*

Then, bottle and brandy weigh $3.84 \times 12 + 122.43$ *ounces* $= 168.51$ *ounces*, and contain 261.92 *cubic inches*, which $\times .59 = 154.53$ *ounces, the weight of an equal bulk of sea-water.*

And, $168.51 - 154.53 = 13.98$ *ounces, the weight necessary to support it in the water.*

14. Three men, viz., A, B, and C, drink a quantity of wine; A can drink it by himself in 12 days, B in 10, and C in 15, when the days are 12 hours long. In what time can they drink out the whole, drinking together, when the days are 10 hours long, and what will be each one's share?

Operation.—If A can drink it in 12 days, he can drink $\frac{1}{12}$ of it in 1 day; and, for like reason, B can drink $\frac{1}{10}$ of it in 1 day, and C $\frac{1}{15}$ of it in 1 day. Therefore, $\frac{1}{12} + \frac{1}{10} + \frac{1}{15} = \frac{15}{60} = \frac{1}{4}$, *the quantity they unitedly will drink in 1 day.* Consequently, if they drink $\frac{1}{4}$ of it in 1 day, *they will drink the whole of it in* 4 *days of* 12 *hours each*, and $\overline{4 \times 12} \div 10 = 4.8$ *days of* 10 *hours.*

Again: their ability to drink being represented by $\frac{1}{12}$, $\frac{1}{10}$, and $\frac{1}{15}$, *their share of drinking will be* $\frac{10}{12} \cdot \frac{10}{10} \cdot \frac{10}{15} = .833$ for A, 1. for B, and .666 for C.

15. A fountain has 4 supply cocks, A, B, C, and D, and under it is a cistern, which can be filled by the cock A in 6 hours, by B in 8 hours, by C in 10, and by D in 12 hours; now, the cistern has 4 holes, designated E, F, G, and H, and it can be emptied through E in 6 hours, F in 5 hours, G in 4 hours, and H in 3 hours. Suppose the cistern to be full of water, and that all the cocks and holes were opened together, In what time would the cistern be emptied?

Operation.—Assume the cistern to hold 120 gallons.

ho.	gall.	ho.	gall.		ho.	gall.	ho.	gall.
If 6 :	120 ::	1 :	20 *at* A.		6 :	120 ::	1 :	20 *at* E.
8 :	120 ::	1 :	15 *at* B.		5 :	120 ::	1 :	24 *at* F.
10 :	120 ::	1 :	12 *at* C.		4 :	120 ::	1 :	30 *at* G.
12 :	120 ::	1 :	10 *at* D.		3 :	120 ::	1 :	40 *at* H.
Run in in 1 *hour*,			57 *gallons.*		*Run out in* 1 *hour*,			114 *gallons.*
								57

Run out in one hour more than run in, 57 *gallons.*

Then, as 57 *gallons* : 1 *hour* : : 120 *gallons* : 2.158 + *hours.*

16. A cistern, containing 60 gallons of water, has 3 unequal cocks for discharging it; one will empty it in 1 hour, a second in 2 hours, and a third in 3 hours; in what time will it be emptied if they are all opened together?

Operation.—First, $\frac{1}{2}$ would run out in 1 hour by the second cock, and $\frac{1}{3}$ by the third; consequently, by the 3 was the reservoir supplied one hour. $\frac{1}{2} + \frac{1}{3} + 1 = \frac{3}{6} + \frac{2}{6} + \frac{6}{6}$, *being reduced to a common denominator, the sum of these* $3 = \frac{11}{6}$; whence the proportion, 11 : 60 : : 6 : $32\frac{8}{11}$ *minutes.*

17. A reservoir has two cocks, through which it is supplied; by one of them it will fill in 40 minutes, and by the other in 50 minutes; it has also a discharging cock, by which, when full, it may be emptied in 25 minutes. If the three cocks are left open, in what time would the cistern be filled, assuming the velocity of the water to be uniform?

Operation.—The least common multiple of 40, 50, and 25, is 200.

Then, the 1st cock will fill it 5 times in 200 *minutes*, and the 2d, 4 times in 200 *minutes*, or both, 9 *times in* 200 *minutes;* and, as the discharge cock will empty it 8 *times in* 200 *minutes*, hence $9 - 8 = 1$, or once in 200 *minutes* $= 3.2$ *hours.*

18. Out of a pipe of wine containing 84 gallons, 10 were drawn off, and the vessel refilled with water, after which 10 gallons of the mixture were drawn off, and then 10 more of water were poured in, and so on for a third and fourth time. It is required to compute how much pure wine remained in the vessel, supposing the two fluids to have been thoroughly mixed.

Operation.—84 — 10 = 74, *the quantity after the 1st draught.*

Then, 84 : 10 : : 74 : 8.8095, and 74 — 8.8095 = 65.1905, *the quantity after the 2d draught.*

84 : 10 : : 65.1905 : 7.7608, and 65.1905 — 7.7608 = 57.4297, *the quantity after the 3d draught.*

84 : 10 : : 57.4297 : 6.8367, and 57.4297 — 6.8367 = 50.593, *the quantity after the 4th draught, which is the result required.*

19. A reservoir having a capacity of 10 000 cubic feet, has an influx of 750 and a discharge of 1000 cubic feet per day. In what time will it be emptied?

Operation. $\frac{10\,000}{1000 - 750} = 40$ *days.*

Contrariwise: The discharge being 1000 and the influx 1250 cubic feet per hour. In what time will it be filled?

Operation. $\frac{10\,000}{1250 - 1000} = 40$ *hours* = 1 *day* 16 *hours.*

20. A son asked his father how old he was. His father answered him thus: If you take away 5 from my years, and divide the remainder by 8, the quotient will be ⅓ of your age; but if you add 2 to your age, and multiply the whole by 3, and then subtract 7 from the product, you will have the number of years of my age. What were the ages of father and son?

Operation.—Assume the father's age 37.

Then 37 — 5 = 32, and 32 ÷ 8 = 4, and 4×3 = 12, *son's age.* Again: 12 + 2 = 14, and 14×3 = 42, and 42 — 7 = 35. Therefore 37 — 35 = 2, *error too little.*

Again: assume the father's age 45; then 45 — 5 = 40, and 40 ÷ 8 = 5. Therefore 5×3 = 15, *son's age.* Again: 15 + 2 = 17, and 17×3 = 51, and 51 — 7 = 44. Therefore 45 — 44 = 1, *error too little.*

Hence (45 sup. ×2 error)—(37 sup. ×1 error) = 90 — 37 = 53, and 2 — 1 = 1.

Consequently, 53 *is the father's age.* Then 53 — 5 = 48, and 48 ÷ 8 = 6 = ⅓ *of the son's age*, and 6×3 = 18 *years, the son's age.*

21. Two companions have a parcel of guineas. Said A to B, if you will give me one of your guineas I shall have as many as you have left. B replied, if you will give me one of your guineas I shall have twice as many as you will have left. How many guineas had each of them?

Operation.—Assume B had 6.

Then A would have had 4, for 6 — 1 = 4 + 1 = 5. Again: 4 (A's parcel) — 1 = 3, and 6 + 1 = 7, and 3×2 = 6. Therefore 7 — 6 = 1, *error too little.*

Again: assume B had 8.

Then A would have 6, for 8 — 1 = 6 + 1 = 7. Again: 6 (A's parcel) — 1 = 5, and 8 + 1 = 9, and 5×2 = 10. Therefore 10 — 9 = 1, *error too great.*

Hence 8×1 = 8, and 6×1 = 6. Then 8 + 6 = 14, and 1 + 1 = 2. Whence, dividing the products by sum of the errors, 14 ÷ 2 = 7 = B's *parcel*, and 7 — 1 = 5 + 1 = 6 for A *when he had received* 1 *of* B; also $\overline{5 - 1} \times 2 = 7 + 1 = 8$ = B's *parcel when he had received* 1 *of* A.

22. If a traveller leaves New York at 8 o'clock in the morning, and walks toward New London at the rate of 3 miles per hour, without intermission; and another traveller starts from New London at 4 o'clock the same evening, and walks toward New York at the rate of 4 miles per hour continuously; assuming the distance between the two cities to be 130 miles, whereabouts upon the road will they meet?

Operation.—From 8 to 4 o'clock in the morning is 8 hours; therefore, 8×3 = 24 *miles, performed by* A *before* B *set out from New London;* and, consequently, 130 — 24 = 106 *are the miles to be travelled between them after that.*

Hence, as (3 + 4) 7 : 3 : : 106 : $\frac{318}{7}$ = 45$\frac{3}{7}$ *more miles travelled by* A *at the meeting;* consequently, 24+35$\frac{3}{7}$ = 69$\frac{3}{7}$ *miles from New York is the place of their meeting.*

23. There are two casks of equal capacity, the one ⅗ full of wine, the other ⅗ full of water; now, assume the cask containing the wine to be first filled from the water-cask, and then the water-cask to be filled from the wine-cask, and so on, alternately filling the one from the other. Assuming that the fluids mix uniformly at each time, how much wine and how much water will the wine-cask contain, and how much water and how much wine will the water-cask contain, after each have been filled 5 times?

Operation.—First, the wine-cask being filled from the water, it will contain $\frac{2}{3}$ of wine and $\frac{1}{3}$ of water.

Second, the water-cask is filled from the wine-cask by drawing from it $\frac{2}{3}$; that is, $\frac{2}{3}$ *of* $\frac{2}{3}$ *of wine* $=\frac{4}{9}$ *of wine*, and $\frac{2}{3}$ *of* $\frac{1}{3}$ *of water* $=\frac{2}{9}$ *of water;* the water-cask contains, therefore, $\frac{5}{9}$ *of water* and $\frac{4}{9}$ *of wine*, and there remains in the wine-cask $\frac{2}{9}$ *of wine* and $\frac{1}{9}$ *of water.*

Again: the wine-cask is filled, and contains $\frac{14}{27}$ *of wine* and $\frac{13}{27}$ *of water*, and there will remain in the water-cask $\frac{5}{27}$ *of water* and $\frac{4}{27}$ *of wine.*

The Denominators of the fractions expressing the proportion of wine and water are the successive powers of 3, and the Numerators are ascertained by dividing the denominators into two parts, differing from each other by a unit.

Thus, after the 1st filling, *the wine-cask has been filled once*, and *the quantity of wine is* $\frac{\frac{1}{2}(3^1+1)}{3^1}=\frac{2}{3}$, and *the water is* $\frac{\frac{1}{2}(3^1-1)}{3^1}=\frac{1}{3}$.

After the 2d filling, *the water-cask has been filled once*, and *the quantity of wine in it is* $\frac{\frac{1}{2}(3^2-1)}{3^2}=\frac{4}{9}$, and *the water is* $\frac{\frac{1}{2}(3^2+1)}{3^2}=\frac{5}{9}$.

After the 3d filling, *the wine-cask has been filled twice*, and *the quantity of wine is* $\frac{\frac{1}{2}(3^3+1)}{3^3}=\frac{14}{27}$, and *the water is* $\frac{\frac{1}{2}(3^3-1)}{3^3}=\frac{13}{27}$; etc., etc.

Thus,	Water	Wine.	Water.	Wine	
	$\frac{1}{3}$		$\frac{1}{3}$	$\frac{2}{3}$	Wine-cask filled 1st time.
Water-cask filled 1st time.	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	
	$\frac{5}{27}$	$\frac{4}{27}$	$\frac{13}{27}$	$\frac{14}{27}$	2d time.
2d time.	$\frac{41}{81}$	$\frac{40}{81}$	$\frac{13}{81}$	$\frac{14}{81}$	
	$\frac{41}{243}$	$\frac{40}{243}$	$\frac{121}{243}$	$\frac{122}{243}$	3d time.
3d time.	$\frac{365}{729}$	$\frac{364}{729}$	$\frac{121}{729}$	$\frac{122}{729}$	
	$\frac{365}{2187}$	$\frac{364}{2187}$	$\frac{1093}{2187}$	$\frac{1094}{2187}$	4th time.
4th time.	$\frac{3281}{6561}$	$\frac{3280}{6561}$	$\frac{1093}{6561}$	$\frac{1094}{6561}$	
	$\frac{3281}{19683}$	$\frac{3280}{19684}$	$\frac{9841}{19683}$	$\frac{9342}{19683}$	5th time.
5th time.	$\frac{29525}{59049}$	$\frac{29524}{59049}$	$\frac{9841}{59094}$	$\frac{9842}{59049}$	

24. If 9 men or 15 women eat 17 apples in 5 hours, and 15 men and 9 women can eat 47 apples of like size in 12 hours, *the apples growing uniformly*, how many boys can eat 360 apples in 60 hours, assuming that 120 boys can eat as many as 18 men and 26 women?

Operation.—If 9 men $=15$ women, 1 man $=\frac{15}{9}$ or $\frac{5}{3}$ *women*, and 15 men $=15\times\frac{5}{3}=25$ *women*. Therefore 15 men and 9 women $=25+9=34$ *women*.

If 15 women eat 17 apples in 5 hours, 34 women will eat $\frac{34}{17}$ of $17=\frac{578}{15}=38\frac{8}{15}$ *apples in the same time;* and if they eat them in 5 hours, they will eat in 12 hours $\frac{12}{5}$ of $38\frac{8}{15}=92\frac{12}{25}$ *apples, provided* the quantity is uniform. But as they are growing whilst being eaten, $92\frac{12}{25}-47=45\frac{12}{25}$ *apples*, the growth of them in $12-5=7$ *hours*.

Now, if the growth of 47 apples in 7 hours $=45\frac{12}{25}$, the growth of 360 in the same time $=\frac{360}{47}$ of $45\frac{12}{25}=\frac{81864}{235}=348\frac{84}{235}$ *apples*, and in 55 hours $(60-5)$, the growth of 360 will be $\frac{55}{7}$ of $348\frac{84}{235}=\frac{900504}{329}=2737\frac{36}{329}$ *apples*.

Hence, $360+2737\frac{36}{329}=3097\frac{36}{329}$ *apples to be eaten in 60 hours.*

Again: if 15 women eat 17 apples in 5 hours, 1 woman will eat $\frac{1}{15}$ of 17 *apples in the same time*, and in 60 hours she will eat 12 times as many $=12\times\frac{17}{15}=\frac{68}{5}=13\frac{3}{5}$ *apples;* and if 1 woman eat $13\frac{3}{5}$ apples in 60 hours, $3097\frac{36}{329}\div\frac{68}{5}=3097\frac{36}{329}\times\frac{5}{68}=\frac{5094745}{22372}=227\frac{16301}{22372}$ *women to eat the apples.*

If 9 men $=15$ women, 18 men and 26 women $=\frac{18}{9}\times15=30$ women, and $30+26=56$ *women*.

Finally, if 56 women are equivalent to 120 boys, $227\frac{16301}{22372}$ women $=227\frac{16301}{22372}\times\frac{120}{56}=487\frac{155027}{156604}$ *boys*.

25. There is a fish, the head of which is 9 inches long, the tail as long as the head and half the body, and the body as long as both the head and tail. Required the length of the fish.

Operation.—Assume the body to be 24 inches in length. Then $24\div2+9=21$, *the length of the tail.*

Hence $21+9=30$, *the length of the body*, which is 6 inches too great.

Again: assume the body to be 26 inches in length. Then $26\div2+9=22$, *the length of the tail.* Hence $22+9=31$, *the length of the body*, which is 5 inches too great.

Therefore, by *Double Position*, divide *difference* of products (see rule) by *difference* of errors (the errors being alike), $\overline{26\times6}-\overline{24\times5}=36=$ *difference of products*, and $6-5=1=$ *difference of errors.*

Consequently, $36\div1=36$, *the length of the body*, and $36\div2+9=27$, *the length of the tail*, and $36+27+9=72$ *inches*, *the length required*

26. A hare, 50 leaps before a greyhound, takes 4 leaps to the greyhound's 3, but 2 leaps of the hound are equal to 3 of the hare's. How many leaps must the greyhound take before he can catch the hare?

Operation.—As 2 leaps of the greyhound equal 3 of the hare, it follows that 6 of the greyhound equals 9 of the hare.

Whilst the greyhound takes 6 leaps, the hare takes 8; therefore, while the hare takes 8, the greyhound gains upon her 1.

Hence, to gain 50 leaps, she must take $50\times8=400$ *leaps; but, whilst the hare takes 400 leaps, the greyhound would take* 300, *since the number of leaps taken by them are as 4 to 3.*

27. If from a cask of wine a tenth part is drawn out and then it is filled with water; after which a tenth part of the mixture is drawn out; again is filled, and again a tenth part of the mixture is drawn out: now, assume the fluids to mix uniformly at each time the cask is replenished, what fractional part of wine will remain after the process of drawing out and replenishing has been repeated ten times?

Operation.—Since .1 of the wine is drawn out at the first drawing, *there must remain* .9. After the cask is filled with water, .1 of the whole being drawn out, *there will remain .9 of the mixture ; but .9 of the mixture is wine;* therefore, after the second drawing, *there will remain .9 of .9 of wine*, or $\frac{9^2}{10^2}$; and after the third drawing, *there will remain .9 of .9 of .9 of wine*, or $\frac{9^3}{10^3}$.

Hence, the part of wine remaining is expressed by the ratio .9, *raised to a power the exponent of which is the number of times the cask has been drawn from.*

Therefore, *the fractional part of wine is* $\frac{9^{10}}{10^{10}}=.3486784401$

28. If a basket and 1000 eggs were laid in a right line 6 feet apart, and 10 men (designated alphabetically from A to J) were to start from the basket and to run alternately, collect the eggs singly, and place them in the basket as collected, and each man to collect but 10 eggs in his turn, how many yards would each man have to run over, and what would be the entire distance run over?

Operation.—A's course would be 6×2 *feet* (*first term*) $+\overline{10\times6\times2}$ *feet* (*last term*) $=132=$ *sum of first and last terms of the progression.*

Then $\overline{132\div2}\times10=660$ *feet = number of times × half the sum of the extremes = the sum of all the terms, or the distance run by* A *in his first turn.*

B's course would be $\overline{11\times6\times2}=132$ *feet* (*first term*) $+\overline{20\times6\times2}=240$ *feet* (*last term*) $=372=$ *sum of first and last terms.*

Then $\overline{372\div2}\times10=1860=$ *sum of all the times, or* B's *first turn.*

A's last course would be $\overline{901\times6\times2}=10\,812$ *feet for the first term*, and $\overline{910\times6\times2}=10\,920$ *feet for the last term of his last turn.*

Then $\overline{10\,812+10\,920\div2}\times10=108\,660=$ *sum of the terms, or distance run.*

B's last course would be $\overline{911\times6\times2}=10\,932$ *feet for the first term*, and $\overline{920\times6\times2}=11\,040$ *feet for the last term of his last turn.*

Then $\overline{10\,932+11\,040\div2}\times10=109\,860=$ *sum of the terms or distance run.*

Therefore, if A's first and last runs $=660$ and $108\,660$ *feet*, and the number of terms 10, then, by *Progression*, the sum of all the terms $=546\,600$ *feet.*

And if B's first and last runs $=1860$ and $109\,860$ *feet*, and the number of terms 10, then the sum of all the terms $=558\,600$ *feet.*

Consequently, $558\,600-546\,600=12\,000=$ *the common difference of the runs*, which, being added to each man's run = the sum of all the runs, or the entire distance run over.

A's *run*,	546 600 = 182 200 *yds.*		F's *run*,	606 600 = 202 200 *yds.*	
B's "	558 600 = 186 200	"	G's "	618 600 = 206 200	"
C's "	570 600 = 190 200	"	H's "	630 600 = 210 200	"
D's "	582 600 = 194 200	"	I's "	642 600 = 214 200	"
E's "	594 600 = 198 200	"	J's "	654 600 = 218 200	"

$\overline{6\,006\,000}$ *feet*, which $\div5280=1137.5$ *miles.*

29. If, in a pair of scales, a body weighs 90 lbs. in one scale, and but 40 lbs. in the other, what is the true weight?

$$\sqrt{(40\times90)}=60 \text{ lbs.}$$

30. If a steam-boat, running uniformly at the rate of 15 miles per hour through the water, were to run for 1 hour with a current of 5 miles per hour, then to return against that current, what length of time would she require to reach the place from whence she started?

Operation.—$15+5=20$ *miles, the distance run during the hour.*

Then $15-5=10$ *miles is her effective velocity per hour when returning, and* $20\div10=2$ *hours, the time of returning*, and $2+1=3$ *hours, or the whole time occupied.*

Or, let d represent the distance in one direction, t and t' the greater and less times of running (in hours), and c the current or tide.

Then, $\dfrac{d\dfrac{t+t'}{2}}{t\times t'}=$ *velocity of boat through the water*, and $\dfrac{d-\overline{v\times t'}}{t'}=c.$

31. The flood-tide wave in a given river runs 20 miles per hour, the current of it is 3 miles per hour. Assume the air to be quiescent, and a floating body set free at the commencement of the flow of the tide; how long will it drift in one direction, the tide flowing for 6 hours from each point of the river?

Operation.—Let x be the time required; $20x=$ distance the tide has run up, together with the distance which the floating body has moved; $3x=$ whole distance which the body has floated.

Then $20x-3x=6\times20$, *or the length in miles of a tide.*

$x=\dfrac{20}{20-3}\times6=7$ *hours, 3 minutes, 31.7647 seconds.*

32. A steam-boat, running at the rate of 10 miles per hour through the water, descends a river, the velocity of which is 4 miles per hour, and returns in 10 hours; how far did she proceed?

Operation.—Let $x =$ distance required, $\frac{x}{10+4} =$ *time of going*, $\frac{x}{10-4} =$ *time of returning.*

Then, $\frac{x}{14} + \frac{x}{6} = 10$; $6x + 14x = 840$; $20x = 840$; $840 \div 20 = 42$ *miles.*

33. From Caldwell's to Newburg is 18 miles; the current of the river is such as to accelerate a boat descending, or retard one ascending $1\frac{1}{2}$ miles per hour. Suppose two boats, running uniformly at the rate of 15 miles per hour through the water, were to start one from each place at the same time, where will they meet?

Operation.—Let $x =$ the distance from N to the place of meeting; *its distance from* C, *then, will be* $18 - x$.

Speed of descending boat, $15 + 1.5 = 16.5$ *miles per hour.*
Speed of ascending boat, $15 - 1.5 = 13.5$ *miles per hour.*

$\frac{x}{16.5} =$ *time of boat descending to point of meeting.*

$\frac{18 - x}{13.5} =$ *time of boat ascending to point of meeting.*

These times are of course equal; therefore, $\frac{x}{16.5} = \frac{18 - x}{13.5}$.

Then, $13.5x = 297 - 16.5x$, and $13.5x + 16.5x = 297$, or $30x = 297$.

Hence $x = \frac{297}{30} = 9.9$ *miles the distance from Newburg.*

34. There is an island 73 miles in circumference; 3 men start together to walk around it and in the same direction: A walks 5 miles per day, B 8, and C 10; when will they all come aside of each other again?

Operation.—It is evident that A and C will be together every round gone by A; hence it remains to ascertain when A and B will be in conjunction at an even round, as 3 miles are gained every day by B. Therefore, as 3 : 1 : : 73 : 24.33 +; but, as the conjunction is a fractional number, it is necessary to ascertain what number of a multiplier will make the division a whole number.

$73 \div 24.33 + = 3$, *the number of days required in which* A *will go round* 5 *times,* B 8, *and* C 10 *times.*

35. Assume a cow, at the age of two years, to bring forth a cow-calf, and then to continue yearly to do the same, and every one of her produce to bring forth a cow-calf at the age of two years, and yearly afterward in like manner; how many would spring from the cow and her produce in 40 years?

Operation.—The *increase* in the first year would be 0, in the second year 1, in the third 1, in the fourth 2, in the fifth 3, in the sixth 5, and so on to 40 years or terms, *each term being = the sum of the two preceding ones.* The last term, then, will be 165 580 141, from which is to be subtracted 1 for the parent cow, and the remainder, 165 580 140, *will represent the increase required.*

Dimensions and Details of several American and English Naval and Merchant Steamers.

All Dimensions, except Cylinders, in Feet and Square Feet.

	Naval Wood Propellers.					Iron Propellers.		Iron Side Wheels.		Wood Side Wheels.				
	Niagara.	Wabash.	Marlborough.	Fairy.	Princeton.	Himalaya.	Yacht.	Atrato.	——**	Arctic.	Metropolis.	Ethan Allen.†	Commonwealth.	Banshee.
Length between perpendiculars	330.	262.5	245.5	144.8	156.5	340.5	122.	318.	—	280.	340.	142.	—	189.
" upon load line	—	262.33	—	—	—	—	—	—	250.	—	—	—	300.	—
Breadth of beam	55.	51.33	61.2	21.12	30.5	46.	20.25	42.	30.	46.	45.	33.	41.66	27.16
Load draught	23.	21.83	26.3	5.1	17.	16.75	9.3	16.5	2.	19.	10.5	6.	8.25	9.
Immersed amidship section	911.	808.	1191.	82.	338.	560.	123.5	544.	58.	685.	460.	170.	288.	190.
Displacement in tons	5440.	4400.	6050.	196.	1046.	3220.	288.5	3070.	234.	3724.	2200.	400.	1400.	770.
Cylinders { number	3	2	2	2	2	2	2	2	2	2	1	1	1	2
Cylinders { diameter	72	72	82	42	††	77¾	28	96	26	95	105¼	38	72	72
Stroke of piston	3.	3.	4.	3.	—	3.6	1.3	9.	6.	10.	12.	9.	12.	5.5
Revolutions per minute	40.	49.3	57.5	44.	30.	59.	110.	18.5	35.	15.8	16.	28.	18.	30.
Steam pressure in lbs., per steam gauge	26	10	20	—	20	14	20	17	100	17	24	25	30	14
Indicated Horses' Power	1955.	1293.	3023.	321.	530.	2050.	254.	3070.	650.	2290.	1900.	500.	1560.	1660.
Propeller in diameter	18.3	17.33	19	6.2	14.	18.	7.9	—	—	—	—	—	—	—
Water wheels in diameter	—	—	—	—	—	—	—	26.5	20◊	36.	41.	18.5	38.	25.
Pitch of Propeller	29.5	23.	25.3	8.	35.	28.	11.	—	—	—	—	—	—	—
Revolutions of Propeller	45.	49.3	57.5	220.	30.	59.	110.	—	—	—	—	—	—	—
Speed of vessel in knots per hour	10.9	9.11	11.23	11.8	9.5	13.8	10.	14.	14.	12.2	16.5	11.	15.75	18.62
Slip of Propeller or of Wheels, from periphery, per cent.	18.	18.5	21.83	31.5	8.	15.	15.2	23.*	30.	30.	20.8	33.1	25.1	19.48
$\frac{\text{Speed}^3 \times \text{Immersed Section}}{\text{I H P}} = C$‡	603.4	472.4	557.8	419.7	546.5	716.	486.6	494.	244.9	543.4	1087.5	452.5	721.3	739.9
$\frac{\text{Speed}^3 \times \text{Displacement}^{\frac{2}{3}}}{\text{I H P}} = C$	205.	157.	155.	176.	167.	279.	171.	189.	160.	194.	400.	144.	313.	327.
Area of Immersed Section to Disc of Propeller, or cross area of W. W. blades	3.46	3.42	4.2	2.74	‡‡2.2	2.2	2.52	—	—	16.7	7.07	18.8	5.48	5.63

* Side Wheel. † Ferry Boat. ‡ Coefficient. ◊ Feathering. ** Light draught river. †† Volume 108 cubic feet. ‡‡ Ericsson's.

Naval Steamer (Wood).

"POWHATAN," U. S. NAVY—INCLINED ENGINES.—*Length upon deck*, 251.5 *feet; between perpendiculars*, 250 *feet; keel*, 246 *feet; beam*, 45 *feet; hold*, 26.5 *feet.*

Immersed Section at load-line, 675 *square feet.*

Displacement 3600 *tons, at load-draught of* 18.5 *feet.*

Cylinders.—Two, of 70 ins. in diam. by 10 feet stroke of piston; volume of piston space, 534.5 cubic feet. *Condensers.*—Two, volume 190 cubic feet. *Air-pumps.*—Two, of 52⅛ ins. in diam. by 42 ins. stroke of piston; volume 103 cubic feet. *Feed-pumps.*—Four, of 8 ins. in diam.; volume, 4.8 cubic feet.

Water-wheel Shafts.—Journals, 18.3 and 13 ins. in diam., and 20 and 15 ins. in length. *Water-wheels.*—Diam. 31 feet. *Arms*, 23. *Blades* (divided), 23; breadth of do., 10 feet; depth of do., 26 ins. *Dip* at load-line, 5.5 feet.

Boilers.—Four (vertical tubular). *Heating surface*, 12 000 square feet. *Grates*, 338 square feet. *Steam-room*, 2980 cubic feet. *Cross area of tubes*, 53.5 square feet.

Smoke-pipe.—Area 63.6 square feet, and 65 feet in height above the grate level.

Pressure of Steam.—10 lbs. per square inch, cut off at ⅓ the stroke of the piston, throttle ⅓ open. *Revolutions*, 13.5 per minute. *Indicated Horses' Power*, 1100.

Fuel.—Bituminous coal, with a natural draught. *Consumption*, at load-line, moderate sea, and at pressure of 12 lbs. and 12 revolutions, 3950 lbs. per hour = 42.3 tons per day.

Speed, 10 knots per hour. *Coal-bunkers*, 800 tons capacity.

Slip of Wheels from Centre of Pressure, 18.75 per cent.

NOTE.—An average pressure of 17 lbs. and 16 revolutions have been obtained for 70 hours, vessel drawing 17.75 feet, giving a speed of 12.6 knots per hour.

Hull.—Launching draught, 10.62 feet; displacement, 1585 tons. Angles of entrance at 17.5 feet, 48°; at 18 feet, 51° 20′; at 19.5 feet, 54° 40′.

Centre of Displacement.—In the vertical plane of the centre of the water-line, and at 18.5 feet draught, 8.86 feet below the water-line. *Average per Inch.*—From 18 to 19 feet draught, 22.09 tons. *Meta Centre.*—Above centre of gravity of displacement 10.87 feet.

Rig.—Full bark. *Armament.*—3 10-inch chambered guns upon pivots, and 6 8-inch chambered guns upon carriages.

Weight. Wheels.

Cast iron	21 634 lbs.	Fitters and patterns	289 days.
Wrought iron	75 865 "	Planing	22 550 sq. ins.

Weight. Coal-Bunkers.

Iron, brass, and copper	117 367 lbs.	Fitters and laborers	150 days.

Day's work (boilers, water-wheels, and coal-bunkers not included, as they were made by the pound, and omitting extra pieces and hoisting engine), 54 498. *Turning, boring, and planing*, 742 348 square inches.

Weight. Boilers.

Brass tubes	33 896 lbs.	Smoke-pipe	23 978 lbs.
Cast-iron	12 000 "	Grate-bars, etc., of cast-iron	27 711 "
Iron	209 410 "	Valves, cocks, etc., brass	8 531 "
Total			315 526 "

WEIGHTS OF ENGINES, BOILERS, ETC. (*finished*).

Engines, frames, flooring, etc.	491 282 lbs.	Coal-bunkers, deck-plates	117 367 lbs.
Extra pieces	56 135 "	Hoisting engine and boiler	4 569 "
Boilers (iron and brass)	255 306 "	Water-wheels	97 499 "
Appurtenances	60 220 "	Water in boilers	189 050 "
Total (567 tons)			1 271 378 "

Cost. Engines.

Cast iron	262 011 lbs.	Fitters and patterns	39 824 days.
Copper	8 084 "	Laborers	11 268 "
Steel	996 "	Planing	133 757 sq. ins.
Wrought iron	171 761 "	Turning and boring	545 515 "

The cost of the engines, boilers, etc., etc., complete (1849–51) was $135 per ton, C. H. (O. M.), and the cost of the vessel, including engines, etc., $300 per ton; the cost of the boilers proper in metal and labor was, 1862, $

Passenger and Cargo Steamers (Wood).

"ADRIATIC," NEW YORK AND LIVERPOOL—OSCILLATING ENGINES.—*Length upon deck*, 351 *feet; length at load-line*, 343.83 *feet; beam*, 50 *feet; hold*, 33.16 *feet.*

Immersed Section at load-line 880 *square feet.*

Displacement 5233 *tons, at load-draught of* 20 *feet.*

Cylinders.—Two, of 101 ins. in diam. by 12 feet stroke of piston; volume of piston space, 1335 cubic feet. *Condensers.* - Two, surface 24 000 square feet; tubes, ¾ in. in diam.; thickness, No. 17 wire gauge. *Air-pumps.*—Two, double acting; volume 96 cubic feet.

Water-wheels.—Diam. 40 feet. *Arms*, 32. *Blades*, 32; breadth of do., 12 feet; depth of do., 3 feet. *Dip* at load-line, 8.25 feet.

Boilers.—Eight (vertical tubular). *Heating surface*, 31 300 square feet. *Grates*, 966 square feet. *Steam-room*, 9200 cubic feet. *Cross area between tubes*, 186 square feet. *Tubes*, 13 064, 2 ins. in diameter and 3.16 feet in length.

Smoke-pipes.—Two, area 38.5 sq. feet, and 48 feet in height above the grate level.

Length of Engine and Boiler Space, including side coal-bunkers, 130 feet.

Pressure of Steam.—26 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions*, 14 per minute; at 18 feet draught, 17.5 per minute. *Indicated Horses' Power*, 4800.

Speed.—At 18 feet draught of water, 15.9 knots per hour.

Fuel.—Anthracite or Bituminous. *Consumption*, 96 tons per day. *Coal-bunkers*, 1200 tons capacity.

Rig.—Brig.

WEIGHTS OF ENGINES, BOILERS, ETC.

Engines	825 000 lbs.	Hull, launching draught 9.96 feet	4 173 120 lbs.
Coal	2 658 000 "	Spars, Sails, Anchors, etc.	[illegible] "
Boilers	674 000 "	Cargo	1 772 000 "
Water in boilers	1 075 200 "		

Displacement.—Average per inch, from 10.16 to 17.125 feet, (light load-line) 26-43 tons; from 17.125 to 20 feet, (load-line) 28.75 tons; from 20 to 21.5 feet, 31.5 tons.

Accommodation.—*Passengers*, cabin, 350; 2d cabin, 200; steerage, 100.

Freight.—800 tons measurement.

"COSTA RICA," NEW YORK AND ASPINWALL—VERTICAL BEAM ENGINE.—*Length at load-line*, 274 *feet; beam*, 39 *feet; hold*, 19.3 *feet; do. to spar deck*, 27 *feet.*

Immersed Section at load-line, 550 *square feet.*

Displacement, 2300 *tons at load-draught of* 15 *feet.*

Cylinder.—One, of 81 ins. in diam. by 12 feet stroke of piston; volume of piston space, 419 cubic feet. *Condenser.*—Volume, 209 cubic feet. *Air-pump.*—Volume, 85.5 cubic feet.

Water-wheels.—Diam. 33 feet. *Blades*, 28 of 12 and 20 ins. in depth; breadth of do., 8.25 feet. *Dip*, at load-line, 5.25 feet.

Boilers.—Two (horizontal tubular). *Heating Surface*, 9090 square feet. *Grates*, 273 square feet. *Area of tubes*, 3360 square inches. *Tubes*, 3.25 inches by 7-75 feet.

Smoke-pipe.—Area 23.75 square feet, and 56 feet in height above the grate level.

Pressure of Steam.—20 to 25 lbs. per square inch, cut off at .45 of the stroke of the piston. *Revolutions*, 18 per minute. *Indicated Horses' Power*, 1950.

Fuel.—Anthracite or Bituminous. *Consumption*, 3000 lbs. per hour.

Coal-bunkers.—Capacity, 550 tons. *Rig.*—Fore-topsail schooner.

WEIGHTS OF ENGINE, BOILERS, ETC.

Engine	526 764 lbs.	Grates, floors, etc.	168 000 lbs.
Boilers	225 060 "	Water	200 000 "
Smoke-pipe	12 000 "	Total	980 624 lbs.

Passenger and Deck Cargo Steamers (Wood).

"City of Boston," New York and Norwich—Vertical Beam Engine.—*Length upon load-line*, 320 *feet; beam*, 39 *feet; hold*, 12.6 *feet.*

Immersed Section at load-line, 288 *square feet.*

Displacement 1459 *tons, at load-draught of* 8.25 *feet.*

Cylinder.—One, of 80 ins. in diam. by 12 feet stroke of piston; volume of piston space, 419 cubic feet.

Water-wheels.—Diam. 37 feet 8 ins. *Arms*, 36. *Blades*, 37; breadth of do. 10 feet; depth of do., 30 5 ins. *Dip* at load-line, 4.25 feet.

Boilers.—Two (return flue). *Heating surface*, 9200 square feet. *Grates*, 192.7 square feet.

Pressure of Steam.—35 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions* (maximum), 19.75 per minute. *Indicated Horses' Power*, 2500.

Fuel.—Anthracite, with a blast. *Consumption*, at ordinary speed, 5200 lbs. per hour.

WEIGHTS OF ENGINE, BOILERS, ETC.

Engines.		*Boilers and Appurtenances.*	
Cast iron	203 500 lbs.	Boilers	157 808 lbs.
Wrought iron	171 700 "	Cast iron	27 060 "
Brass	12 440 "	Wrought iron	17 050 "
Steel and Lead	575 "	Brass and Copper	620 "
Total			590 753 lbs.

Hull.—800 tons. Light draught of hull and machinery without fuel, water, or furniture, 7 feet.

"Wm. H. Webb," Towing, N. Y. Harbor and Coast—Vertical Beam Engines. —*Length upon deck*, 185.5 *feet; beam*, 30.25 *feet; hold*, 10.8 *feet.*

Immersed Section at load-line, 194 *square feet.*

Displacement 498.25 *tons, at load-draught of* 7.25 *feet.*

Cylinders.—Two, of 44 ins. in diam. by 10 feet stroke of piston; volume of piston space, 211 cubic feet. *Condensers.*—Two, volume 105 cubic feet. *Air-pumps.*—Two, volume 45 cubic feet.

Water-wheels.—Diam., 30 feet. *Blades* (divided), 21; breadth of do., 4.6 feet; depth of do., 2.33 feet. *Dip* at load-line, 3.75 feet.

Boilers.—Two (return flue). *Heating surface*, 3280 square feet. *Grates*, 147.5 square feet.

Smoke-pipe.—Area 11.6 square feet, and 35 feet in height above the grate level.

Pressure of Steam.—35 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions*, 22 per minute. *Indicated Horses' Power*, 1500.

Fuel.—Anthracite or Bituminous. *Consumption*, 1680 lbs. per hour.

WEIGHTS OF ENGINES, BOILERS, ETC.

Engines, Wheels, Frame, and Boilers............ 310 579 lbs.

"Banshee", Holyhead to Dublin (Eng.)—*Length between perpendiculars*, 189 *feet; beam*, 27.16 *feet; hold*, 14.75 *feet.*

Immersed Section at load-line, 190 *square feet.*

Displacement 770 *tons, at light draught of water of* 9 *feet.*

Cylinder.—72 ins. in diam. by 5.5 feet stroke of piston; volume of piston space, 155.5 cubic feet.

Water-wheels.—Diam. 25 feet by 9 feet in width. *Blades*, area 33.75 feet.

Pressure of Steam.—14 lbs. per square inch. *Revolutions*, 30 per minute.

Speed.—18.62 knots per hour at trial, and 13.84 average.

Indicated Horses' Power, at trial, 1660. *Rig.*—Schooner.

Passenger and Cargo Steamers (Iron).

"CLEOPATRA" (ENGLISH)—OSCILLATING ENGINES.—*Length upon load-line*, 202 *feet; beam*, 21 *feet; depth at sides*, 10.5 *feet; Hull*, 138.28 *tons; Engine room* 123.53 *tons; Builder's measurement*, 453.4 *tons.*

Immersed Section at load-line, 122 *square feet.*

Displacement 453 *tons, at load-draught of* 6.25 *feet.*

Cylinders.—Two, of 40 ins. in diam. by 4 feet stroke of piston; volume of piston space, 80 cubic feet.

Water-wheels (Feathering).—Diam. 16 feet. *Blades* 10; breadth of do., 8 feet; depth of do., 2.5 feet.

Boilers.—Two (tubular), length 15 feet, breadth 9 feet, height 8.5 feet. *Heating surface*, 3320 square feet. *Grates*, 150 square feet.

Pressure of Steam.—Average 25 lbs. per square inch, cut off at ⅜ the stroke of the piston. *Revolutions*, average 42 per minute.

Speed.—Average 14.78 knots per hour. *Indicated Horses' power*, 882.

Wet Surface of Hull, 4573 square feet.

Rig.—Schooner. *Sails*, 490 square yards.

Bulkheads, 5. *Coal-bunkers*, 35 tons' capacity.

Saloons.—Three, of 17.12 and 10 feet by 14.17 and 19 feet by 6.25 and 7.25 feet.

WEIGHTS OF ENGINES, BOILERS, ETC.

Hull (iron)	212 800 lbs.	Engine	230 720 lbs.
Boilers	38 080 "	Water in boilers	44 800 "
Cargo	80 640 "	Fuel	78 400 "

"LY-EE-MOON" (ENGLISH)—OSCILLATING ENGINES.—*Length upon load-line*, 270.5 *feet; beam*, 27.8 *feet; hold*, 15.25 *feet.*

Immersed Section at load-line, 282.6 *square feet.*

Displacement at load-draught of 12.5 *feet*, 1317.7 *tons; displacement per inch between light and load lines*, 12.74 *tons.*

Cylinders.—Two, of 70 ins. in diam. by 5.5 feet stroke of piston; volume of piston space, 290 cubic feet.

Water-wheels.—Diam. 22 feet. *Blades*, breadth, 10 feet; depth of do., 4.16 feet.

Boilers.—Four (tubular).

Pressure of Steam.—25 lbs. per square inch.

Speed, 16 knots per hour.

Naval Steamer (Iron Clad).

"WARRIOR," R. N.—TRUNK ENGINES.—*Length between perpendiculars*, 380.1 *feet; beam*, 58 *feet; hold*, 37.33 *feet.*

Immersed Section at load-line, 1193 *square feet.*

Displacement 8997 *tons, at load-draught of* 26 *feet.*

Tonnage (Eng.), 6109. Height out of water, 20 feet.

Cylinders.—Two, of 112 ins. in diam. by 4 feet, effective diam. = 104 ins.; volume of piston space, 560 cubic feet.

Boilers.—Ten (horizontal tubular). *Heating surface*, 23 197 square feet. *Grates*, 868 square feet.

Pressure of Steam.—21 lbs. per square inch. *Revolutions*, 43 per minute. *Indicated Horses' Power*, 5469.

Speed, 13.49 knots per hour.

Armor Plates, 4.5 ins. thick by 1.5 ins. in width.

WEIGHTS OF ARMOR, ENGINES, AND PROPELLER.

Armor plates	1 792 000 lbs.	Engines and Propeller	593 600 lbs.
Fuel	2 128 000 lbs.		

Naval Steamer (Wood).

"GENERAL ADMIRAL," R. I. N.—HORIZONTAL BACK-ACTION ENGINES.—*Length upon deck*, 313.7 *feet; length upon load-line*, 302.83 *feet; beam*, 54.5 *feet; hold*, 33.6 *feet.*

Immersed Section at load-line, 1090 *square feet.*

Displacement 5200 *tons, at load-draught of* 23 *feet.*

Cylinders.—Two, of 84 ins. in diam. by 3.75 feet stroke of piston; volume of piston space, 282.8 cubic feet. *Air-pumps.*—Two, double acting; volume, 27.6 cubic feet.

Propeller.—Diam. 19 feet. *Blades* 2. *Pitch*, 31.5 feet. *Area of disc*, 71.39 feet.

Boilers.—Six (horizontal tubular). *Heating surface*, 19 500 square feet. *Grates*, 700 square feet.

Smoke-pipe.—Area 95 square feet, and 65 feet in height above the grate level.

Pressure of Steam.—18 lbs. per square inch, cut off at ⅓ the stroke of the piston.

Revolutions, 42 per minute; maximum, 52. *Indicated Horses' Power*, 2000.

Fuel.—Anthracite coal, with a natural draught. *Consumption*, 7960 lbs. per hour.

Speed, 11 knots per hour. *Coal-bunkers*, 650 tons' capacity.

Rig.—Ship.

WEIGHTS OF ENGINES, BOILERS, ETC.

Engine	535 600 lbs.	Mountings and fixtures for do.	26 680 lbs
Propeller	27 900 "	Boilers	452 964 "
Total	1 043 144 lbs.		

Naval Steamers (Wood).

"BROOKLYN," U. S. N.—HORIZONTAL DIRECT ENGINES.—*Length upon load-line to forward stern-post*, 233 *feet; to stern-post*, 243 *feet; beam*, 43 *feet; hold*, 22.66 *feet. Tonnage*, O. M., 2060.

Immersed Section at load-line, 551 *square feet.*

Displacement 2532 *tons, at load-draught of* 15.5 *feet.*

Cylinders.—Two, of 61 ins. in diam. by 33 ins. stroke of piston; volume of piston space, 112 cubic feet. *Condensers.*—Two, volume 65.2 cubic feet. *Air-pumps.*—Two, of 19 ins. in diam. by 33 ins. stroke of piston, double acting; volume 11 cubic feet. *Steam Valves*, 162 square ins. *Exhaust*, 234 square ins. *Feed-pumps.*—Four, of 5.5 ins. in diam.; volume, 1.8 cubic feet.

Propeller Shaft.—Journals, 12 and 18 ins. in length.

Boilers.—Two (vertical water tubular, brass tubes). *Heating surface*, 7800 square feet. *Grates*, 252 square feet. *Steam-room*, 1150 cubic feet.

Smoke-pipe.—Area 38.5 square feet, and 50 feet in height above the grate level.

Pressure of Steam.—18 lbs. per square inch, cut off at ⅓ the stroke of the piston, full throttle. *Revolutions*, 51 per minute. *Indicated Horses' Power*, 706.

Fuel.—Anthracite coal, with a natural draught. *Consumption*, at load-line, and at a pressure of 20 lbs., and 50 revolutions, 22.4 tons per day.

Speed, 9 knots per hour. *Coal-bunkers*, 360 tons' capacity.

Slip of Propeller, 26 per cent.

Hull.—Weight, 1350 tons. Angle of entrance at load-line, 56°.

Centre of Displacement.—(Gravity of) at 16 feet 2 ins. draught, 6.5 feet below the water-line. *Average per Inch*, at load-line, 19.3 tons. *Meta Centre*, above centre of displacement (gravity of), 10.44 feet.

Rig.—Full bark. *Sails*, 22 480 square feet.

Armament.—2 10-inch chambered guns upon pivots, and 16 9-inch do. upon carriages.

WEIGHTS OF ENGINES, BOILERS, ETC.

Engines and extras	356 035 lbs.	Water in boilers	135 370 lbs.
Boilers	157 670 "	Grates	15 120 "
Smoke-pipe	8 443 "	Armament	406 113 "

Total (Engines, Boilers, etc.)........ 537 265 lbs.

"WYOMING," U. S. N.—HORIZONTAL DIRECT ENGINES.—*Length upon deck*, 209.75 *feet; between perpendiculars*, 198.5 *feet; keel*, 188 *feet; beam*, 33 *feet; hold*, 15.83 *feet.*

Immersed Section at load-line, 391 *square feet.*

Displacement 1475 *tons, at load-draught of* 12.83 *feet.*

Cylinders.—Two, of 50 ins. in diam. by 2.5 feet stroke of piston; volume of piston space, 67 cubic feet. *Condenser.*—Surface 3000 square feet; 3000 tubes, ⅝ in. in diam. by 6 feet in length. *Vacuum*, 22.5 ins. *Air-pumps.*—(Fresh water.) Two, of 11 ins. in diam. by 2.5 feet stroke of piston; (Salt water.) Two, of 10 ins. in diam. by 2.5 feet stroke of piston.

Propeller Shaft.—Journals, 11.57 and 10 ins. in diam., and 16, 18, 18, and 24 ins. in length.

Propeller (true screw).—Diam. 12.25 feet. *Blades*, 4. *Pitch*, 19 feet; length, 2.5 feet. *Surface*, 84 square feet.

Boilers.—Three (vertical tubular, brass tubes), length, including fire-room, 29 feet; breadth (lengthwise of the vessel), 24.75 feet; height, 10.16 feet. *Heating surface*, 7820 square feet; tubes, 4280 of 2 ins. external diam. by 31½ ins. in length. *Grates*, 242 square feet.

Smoke-pipe.—Area 36.7 square feet, and 52 feet in height above the grate level.

Pressure of Steam.—18.25 lbs. per square inch, cut off at .38 the stroke of the piston, throttle wide open. *Revolutions*, 74.5 per minute; attainable, 85. *Indicated Horses' Power*, 793.

Fuel.—Anthracite coal, with a natural draught. *Consumption*, 1710 lbs. per hour, or about 2.16 lbs. per indicated horses' power = 18.3 tons per day.

Speed, 9.87 knots per hour. *Coal-bunkers*, 235 tons' capacity.

Slip of Propeller, 20 per cent.

NOTE.—A pressure of 27 lbs. per square inch, cut off at ½ the stroke of the piston, throttle valve wide open, and 80.5 revolutions per minute have been attained; draught of water, 13.25 feet; speed, 11.25 knots per hour.

Evaporation, 9.32 lbs. water per lb. of fuel.

Rig.—Full bark. *Sails.*—Area 9705 square feet.

Armament.—2 11-inch pivot guns, and 4 32-pounder chambered guns.

Stores, 6 months. *Provisions*, 3 months.

WEIGHTS OF ENGINES, BOILERS, ETC.

Engines and dependencies..	132 907 lbs.	Boilers, Pipe, etc...........	177 995 lbs.
Propeller, shafting, etc.	51 231 "	Water in boilers...........	89 040 "
Coal-bunkers and bulkheads	31 274 "	Tools, Extra pieces, etc.	14 158 "
Total..			496 605 "

Yacht (Wood).

"CAMPANERA" (ENGLISH)—HORIZONTAL DIRECT ENGINE.—*Length upon load-line*, 108.5 *feet; beam*, 21 *feet; depth at side*, 11.5 *feet.*

Immersed Section at load-line, 125 *square feet.*

Displacement 151 *tons, at load-draught of* 7 *feet.*

Cylinder.—One, of 12 ins. in diam. by 1.5 feet stroke of piston; volume of piston space, 1.18 cubic feet.

Propeller.—Two-bladed; diam. 9 feet. *Revolutions*, 108 per minute. *Indicated Horses' Power*, 50; *Nominal*, 35.

Fuel.—Bituminous coal. *Consumption*, 420 lbs. per hour.

Speed, 9 knots per hour. *Coal-bunkers*, 32 tons' capacity.

Stowage, 36 tons. *Sails*, 506 square yards.

WEIGHTS.—Engine, Boiler, and Water, 53 760 lbs.

Passenger and Cargo Steamer (Iron).

"AUSTRALASIAN" (BRITISH)—VERTICAL DIRECT ENGINES.—*Length upon load-line*, 314.5 *feet; beam*, 42.15 *feet; depth at sides*, 31.25 *feet.*

Immersed Section at load-line, 764 *square feet.*

Displacement 4447 *tons, at load-draught of* 22 *feet.*

Cylinders.—Two, of 90 ins. in diam. by 3.5 feet stroke of piston; volume of piston space, 331.3 cubic feet.

Propeller.—Three-bladed, diam. 19 feet. *Pitch*, 34 feet.

Boilers.—Six (tubular).

Pressure of Steam.—20 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions*, 46 per minute. *Indicated Horses' Power*, 2500.

Speed, 12 knots per hour.

Fuel.—Bituminous coal. *Consumption*, 80 tons per day.

Wet Surface of Hull, 18 370 square feet. *Slip of Propeller*, 28.5 per cent.

Rig.—Bark. *Sails*, 3210 square yards.

Passengers, 142.

WEIGHTS.—*Hull*, 1650 tons. *Cargo*, 1100 tons. *Fuel*, 1250 tons.

Passenger and Deck Cargo Steam-boat (Wood).

"CLIFTON, No. 2," NEW YORK TO STATEN ISLAND—VERTICAL BEAM ENGINE.— *Length upon deck*, 180 *feet; beam*, 32 *feet; hold*, 12.5 *feet.*

Immersed Section at load-line, 147 *square feet.*

Displacement 465 *tons, at load-draught of* 6 *feet.*

Cylinder.—One, 43 ins. in diam. by 10 feet stroke of piston; volume of piston space, 101 cubic feet.

Water-wheel Shafts.—Journals, 11.25 and 8 ins. in diam., and 13 and 10 ins. in length.

Water-wheels.—Diam. 22 feet. *Blades* (divided), 20; breadth of do., 8 feet; depth of do., 2 feet by 2.25 ins. thick. *Dip* at load-line, 34 ins. *Centres*, 3, of 5 feet in diam. *Rims* (iron), 3×½, 3½×¾, and 4×⅞ ins. *Braces*, 10, 1¼ ins. in diam.

Beam, length 19 feet; depth 9 feet. *End centres*, 4.5 ins. in diam. *Main centre*, 6.5 ins. in diam. *Strap*, least section, 5 by 3 ins.

Engine Frame, Yellow-pine, 12 by 16 ins. at foot, and 12 by 12 ins. at head.

Boiler.—One (return flue), 12 feet front by 24 feet in length. Shell, diam. 10 feet; height of front 10½ feet. *Furnaces*, 3, 6.16 feet in length. *Steam Chimney*, 12 feet in height. *Heating surface*, 1724 square feet. *Grates*, 65.2 feet.

Pressure of Steam.—28 lbs. per square inch, cut off at ½ stroke. *Revolutions*, 26 per minute. *Speed*, 13.5 knots per hour. *Indicated Horses' Power*, 570.

Hull.—Floors, molded, 16 ins.; sided, 12 ins. Launching draught, 4 feet.

Passenger Steam-boats (Wood).

"DANIEL DREW," NEW YORK TO ALBANY—VERTICAL BEAM ENGINE.—*Length upon deck*, 251.66 *feet; do. at load-line*, 244 *feet; beam*, 31 *feet; hold*, 9.25 *feet.*

Immersed Section at load-line, 136 *square feet.*

Displacement 380 *tons, at load-draught of* 4.83 *feet.*

Cylinder.—One, 60 ins. in diam. by 10 feet stroke of piston; volume of piston space, 196 cubic feet. *Condenser*, volume 68 cubic feet. *Air-pump*, volume 26 cubic feet.

Water-wheels.—Diam. 29 feet. *Arms*, 24. *Blades*, 24; breadth of do., 9 feet; depth of do., 26 ins. *Dip* at load-line, 2.33 feet.

Boilers.—Two (return flue), 29 feet in length by 9 feet in width at furnace. Shell, diam. 8 feet. *Heating surface*, 3350 square feet. *Grates*, 105 square feet. *Cross area of lower flues*, 15 5 square feet; *of upper*, 13 square feet.

Smoke-pipes.—Two, area 25.13 sq. feet, and 32 feet in height above the grate level.

Pressure of Steam.—35 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions* (*maximum*), 26 per minute. *Indicated Horses' Power*, 1720.

Fuel.—Anthracite coal, with a blast. *Consumption*, 3800 lbs. per hour.

Speed, 22.3 miles per hour. *Slip of Wheels from Centre of Pressure*, 12.5 per cent.

Frames.—Molded, 15¾ ins.; sided, 4 ins., and 20 ins. apart at centres.

Weight of boilers, 80 650 lbs.

"SETH GROSVENOR," AFRICAN COAST AND RIVER—STEEPLE ENGINE.—*Length upon deck*, 95 *feet; beam*, 17.2 *feet; hold*, 5 *feet.*

Immersed Section at load-line, 43 *square feet.*

Displacement 73 *tons, at load-draught of* 3.25 *feet.*

Cylinder.—28 ins. in diam. by 3 feet stroke of piston; volume, 12.8 cubic feet.

Water-wheels.—Diam. 13.5 feet. *Blades*, 14; breadth of do., 3 feet; depth of do., 1.25 feet.

Boiler (return tubular). *Heating surface*, 540 square feet. *Grates*, 22.5 square feet. *Area of tubes*, 367 square inches. *Indicated Horses' Power*, 90.

WEIGHTS.—Boiler, Engine, Wheels, and Frame, 61 556 lbs. = 27.4 tons.

The operation of this vessel was in every way successful, being very fast, economical in fuel, etc., and she would have been improved if the hull had had 15 feet additional length, all other dimensions and capacities remaining the same.

Passenger and Cargo Steam-boats (Wood).

"BUCKEYE STATE," OHIO RIVER—HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck*, 260 *feet; beam*, 30.3 *feet; depth of hold*, 6.5 *feet.*

Immersed Section at load-line, 143 *square feet.*

Displacement 530 *tons, at load-draught of* 5 *feet.*

Cylinders.—Two, of 29.5 ins. in diam. by 8 feet stroke of piston; volume of piston space, 76 cubic feet.

Water-wheel Shafts.—Journals, 17 ins. in diam.

Water-wheels.—Diam. 31 feet 2 ins. *Blades*, 20; breadth of do., 12 feet; depth of do., 2.5 feet.

Boilers.—Five (cylindrical flued), 42 ins. in diam. by 30 feet in length, with 2 return flues in each, 17⅜ ins. in diam. *Heating surface*, 2394 square feet. *Grates*, 86.1 square feet.

Smoke-pipes.—Two, area 47.5 sq. feet, and 76 feet in height above the grate level.

Pressure of Steam.—140 lbs. per square inch, cut off at 9/16 the stroke of the piston. *Revolutions* (maximum), 19 per minute. *Indicated Horses' Power*, 2000.

Fuel.—Bituminous coal or Wood. *Consumption*, 4280 lbs. per hour.

Freight.—180 tons at load-draught. Light draught of water, 3.5 feet.

Hull.—Keelson, 11×17 ins.; bilge do., 3×10.5 ins.; false do., 3×8 ins.; bottom plank, 4 ins.; deck beams, 3.5×6.25 ins.; plankshear, 2.5×25 ins., and deck plank, 2 ins. *Frames.*—Throats, 7 ins.; sides, 3.5 ins.; distance apart from centres, 17 ins.

NOTES.—Areas of immersed section of hull at light draught (142.7 square feet), and of cross blade surface, are as 1 to 2.38. Areas of grate and heating surface, as 1 to 27.8. Fuel consumed upon each square feet of grate, 50 lbs. per hour. Areas of smoke-pipes, 47 feet; of flues, 13.2 feet; and of bridge wall, 27 feet.

"MAGNOLIA," MISSISSIPPI RIVER—HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck*, 295 *feet; beam*, 33 *feet; over wheel guards*, 72 *feet; hold*, 9 *feet.*

Immersed Section at light-draught of 4 *feet*, 132 *square feet.*

Displacement 1000 *tons, at load-draught of* 7 *feet.*

Cylinders.—Two, of 30 ins. in diam. by 10 feet stroke of piston; volume of piston space, 98 cubic feet.

Water-wheel Shafts.—Journals, 18 ins. in diam.

Water-wheels.—Diam. 40 feet. *Blades*, 26; breadth of do., 12.5 feet; depth of do., 2.33 feet.

Boilers.—Six (cylindrical flued), 3.5 feet in diam. by 30 feet in length, with two return flues in each, 16 ins. in diam. *Heating surface*, 2716 square feet. *Area at bridge*, 77 square feet. *Area of Flues*, 16.7 square feet. *Grates*, 98.4 square feet. *Furnace*, 2 feet, and *Bridge wall*, 10 ins. in depth below boiler.

Smoke-pipes.—Two, area 39.25 sq. feet, and 81 feet in height above the grate level.

Pressure of Steam.—130 lbs. per square inch, cut off at .6 the stroke of the piston, throttle wide open. *Safety-valves*, two, of 9½ ins. in diam. *Revolutions*, 16 per minute. *Indicated Horses' Power*, 1380.

Evaporation.—10.9 lbs. fresh water per square foot of heating surface per hour, and 7.05 lbs. per pound of coal.

Speed, 15.13 miles per hour.

Consumption of Fuel.—2.9 cords Pine wood per hour, weighing 2700 lbs. per cord, or 3936 lbs. Bituminous coal.

Freight.—4500 bales of cotton, or 800 tons in weight.

WEIGHTS.—Engines, Wheels, Boilers, and Water, 576 000 lbs.

Passenger Steam-boat (Iron).

"LEE" (ENGLISH)—OSCILLATING ENGINES.—*Length upon deck*, 160 *feet; beam*, 17.5 *feet; hold*, 7.5 *feet; draught of water at load-line*, 3.5 *feet. Quarter-deck raised* 2.5 *feet above main deck.*

Immersed Section at load-line, 60 *square feet.*

Displacement 130 *tons, at load-draught of* 3.5 *feet.*

Cylinders.—Two, of 36 ins. in diam. by 3 feet stroke of piston; volume of piston space, 42 cubic feet.

Water-wheels (Feathering).—Diam. 14.5 feet. *Blades*, 10; breadth of do., 10 feet; depth of do., 2 feet.

Boilers.—Two (tubular). Tubes 376, of brass, 2.75 ins. in external diam. by 6 feet in length. *Grates*, 90 square feet. *Speed*, 13.7 knots per hour.

HULL.—*Keel*, 5×4×.5 ins. *Stem*, 4×1.25 ins. *Sternpost*, 3.5×1.5 ins. *Frames*, L 2.5×2.5×.3125 ins., 2 feet apart from centres at body, and 2.5 feet at ends. *Cross floors*, one to each frame, .25 in engine and boiler space, and .1875 forward and aft; reversed frames L, one on every cross floor, 2.5 × 2.5 × .25 in engine and boiler space; and 2×2×.25, forward and aft, all running up each side of hull, 1 foot above junction of cross floor and frame.

Engine plates, .3125 in. thick, with angle iron L, 2.5×2.5×.3125 ins. thick.

Deck beams, L 3×2.5×.25 ins., one upon every frame, with triangular plate knees at ends, 1 foot each in length and depth.

Engine bearers, of .3125 in. plate, and L 2.5×2.5×.3125 ins. *Water-wheel bearers*, I 8×.5 ins.

Covering Plates, for 75 feet amidships, 12×.25 ins.; forward and aft, 12×.1875 ins. Gunwale, L 2.5×2.5×.25 ins.

Bulkheads.—Three, .1875 in. thick, with L 2.5×2.5×.25 ins., set 2.5 feet apart.

Plating.—Garboard strake, .3125 in.; second strake throughout, and bottom to turn of bilge for 60 feet amidship, .25 in.; remainder of plating .1875 in., except gunwale strake of .25 in.

Wheel-houses.—Inboard siding .125 in., with L 2.25×2.25×.1875 ins., set 2.5 feet apart. Brackets for supporting outboard plumber-blocks, at side of hull (wheels overhung), of .375 in. plates and L 3×3×.375 in.

Rivets.—Keel, Stem, Sternpost, and Garboard strake single riveted with .625 in. rivets, 2.5 ins. apart from centre to centre. Plating rivets, .5 in. and 2 ins. apart; frame rivets, 4.5 ins. apart.

Cost of Hull and Engines, 1861, $26 250.

"ALABAMA," MOBILE BAY—VERTICAL BEAM ENGINE.—*Length between perpendiculars*, 225 *feet; beam*, 32 *feet; hold*, 10 *feet; launching draught*, 2.33 *feet.*

Immersed Section at load-line, 130 *square feet.*

Displacement 440 *tons, at load-draught of* 4.5 *feet.*

Cylinder.—One, 50 ins. in diam. by 10 feet stroke of piston; volume of piston space, 136.4 cubic feet.

Water-wheels.—Diam. 30 feet; breadth of do., 10 feet.

Boiler.—One (return flue).

HULL.—*Keel* U, .625 in. thick. *Keelsons*, two box, and five single I. *Frame* L, 3.5×3.5×.375 ins. thick, 1.5 feet apart from centres. *Cross floors* T, 12 ins. deep by .3125ths thick, with angle iron on top, 3×3×.3125 ins. thick. Garboard strake, .5 in. thick; to bilge, .3125; bilge, .375 in.; wales, .375 in.; clamps, .375 in. by 16 ins. deep.

Bulkheads.—Three, water-tight. *Deck beams*, white pine, 5.5×3.5 ins., 2 feet apart. *Deck*, white pine, 2.5 ins. *Suspension frames over gunwale.*

Hull.—Weight, 336 000 lbs.

Passenger Steam-boat (Iron).

(British) Horizontal Engines (*Non-condensing*).—*Length upon deck*, 250 *feet*; *beam*, 30 *feet*.

Immersed Section at load-line, 58 *square feet*.

Displacement 260 *tons*, *at load-draught of* 2 *feet*.

Cylinders.—Two, of 26 ins. in diam. by 6 feet stroke of piston; volume of piston space, 44 cubic feet.

Pressure of Steam.—100 lbs. per square inch. *Revolutions*, 35 per minute.

Indicated Horses' Power, 1020.

Water-wheels.—Diam. 20 feet. *Blades*, 16; breadth of do., 10 feet; depth of do., 1.5 feet.

Speed, 14 miles per hour.

Launch (Wood).

Launch of a 1st *Class Ship-of-the-Line.*

"Experiment"—Direct Acting Engines (*Non-condensing*).—*Length*, 24 *feet*.

Light draught of water, 2 *feet*.

Cylinders.—Two, of 4 ins. in diam. by 6 ins. stroke of piston; volume of piston space, .087 cubic feet.

Propellers.—Two, diam. 2 feet. *Pitch*, 3.375 feet.

Pressure of Steam.—60 lbs. per square inch. *Revolutions*, 290 per minute.

Indicated Horses' Power, 1.3.

Speed, 6.74 knots per hour.

Engines and Boiler occupy a space of 6 feet 11 ins. by 4 feet 4 ins. With one propeller, 340 revolutions were attained, and a speed of 4.61 knots.

Cutter (Iron).

"La Bonita"—Inclined Engine (*Non-condensing*).—*Length upon deck*, 42 *feet*; *beam*, 9 *feet*; *hold*, 3 *feet*.

Immersed Section at load-line, 8.75 *square feet*.

Displacement 8386 *lbs.*, *at load-draught of* 1.3 *feet*. *Tons*, 9.65, O. M.

Cylinder.—8 ins. in diam. by 1 foot stroke of piston; volume of piston space, .35 cubic foot.

Water-wheels.—Diam. 5.66 feet. *Blades*, 7; breadth of do., 2.3 feet; depth of do., 7 ins.

Boiler.—One (horizontal tubular). *Heating surface*, 95 square feet. *Grates*, 6 square feet.

Fuel.—Coal or Wood. *Exhaust draught.*

Pressure of Steam.—65 lbs. per square inch, full stroke. *Revolutions*, 54 per minute. *Indicated Horses' Power*, 9.

Hull.—Corrugated and galvanized plates, .0625 in. thick.

WEIGHTS OF HULL, ENGINE, BOILER, ETC.

Hull	2876 lbs.	Boiler	2260 lbs.
Engine and wheels	2400 "	Pipes, grates, etc.	750 "

STERN WHEELS.

Passenger and Cargo Steam-boats (Wood).

"VENCEDOR," MAGDALENA RIVER — HORIZONTAL ENGINES (*Non-condensing*).—*Length between perpendiculars*, 150 *feet; beam*, 24 *feet; hold*, 5 *feet.*

Immersed Section at load-line, 90 *square feet.*

Displacement 230 *tons, at load-draught of* 4 *feet.*

Cylinders.—Two, 16 ins. in diam. by 6 feet stroke of piston; volume of piston space, 16.8 cubic feet.

Water-wheel Shaft.—Journal, 8.75 ins. in diam.

Wheel.—One, diam. 16 feet. *Blades*, 15; breadth of do., 17 feet; depth of do., 1.25 feet.

Boiler.—One (horizontal tubular) (locomotive). *Tubes*, 138 of 3 ins. diam. by 12 feet in length. *Heating surface*, 1500 square feet. *Grates*, 42 square feet.

Hull.—Frame, yellow pine, molded 6 ins., sided 4 ins., and 2 feet apart at centres; bottom plank 2½ ins. thick.

HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck*, 90 *feet; beam*, 16 *feet; hold*, 3.5 *feet.*

Immersed Section at load-line, 60 *square feet.*

Displacement 100 *tons, at load-draught of* 4 *feet.*

Cylinders.—Two, of 12 ins. in diam. by 3 feet stroke of piston; volume of piston space, 2.36 cubic feet.

Boiler.—One (horizontal tubular).

Launching draught, 9 ins.

HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck.* 56 *feet; beam*, 12 *feet; hold*, 3.5 *feet.*

Immersed Section at load-line, 24 *square feet.*

Displacement 23 *tons, at load-draught of* 2.16 *feet.*

Cylinders.—Two, of 10 ins. in diam. by 2.5 feet stroke of piston; volume of piston space, 2.73 cubic feet.

Boiler.—One (horizontal tubular).

Iron.

HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck*, 110 *feet; beam*, 14 *feet (deck projecting over*, 4 *feet); hold*, 3.5 *feet.*

Immersed Section at load-line, 10.25 *square feet.*

Displacement 33 *tons, at load-draught of* 1.1 *feet.*

Cylinders.—Two, of 10 ins. in diam. by 3 feet stroke of piston; volume of piston space, 1.6 cubic feet.

Wheel.—Diam. 13 feet. *Blades*, 13; breadth of do., 8.5 feet; depth of do., 8 ins.

Revolutions, 36 per minute.

Boiler.—One (horizontal tubular). *Tubes*, 100 of 2 ins. in diam.

Fuel.—Bituminous coal. *Consumption*, 4480 lbs. in 24 hours.

HULL.—*Plates*, keel, No. 3; bilges, No. 4; bottom, No. 5; sides, Nos. 6 and 7. *Frames*, 2.5×.5 ins., and 20 ins. apart from centres.

Passenger and Cargo Steamers (Wood).

"ASIA," CUNARD LINE—SIDE LEVER ENGINES.—*Length of keel and fore rake*, 267 *feet; beam*, 46.5 *feet; over* W. W. *guards*, 63.5 *feet; depth of hold*, 27.5 *feet. Load draught of water*, 19.5 *feet*.

Hull.—2128 tons, O. M. *Engine space*, 92.5 feet in length; *volume*, 812 tons, O. M.

"GORGON," R. N.—*Length between perpendiculars*, 178 *feet; beam*, 37 *feet; depth of hold*, 23 *feet.*

WEIGHTS.

Hull	630 tons.	Engines and water	270 tons.
Masts and rigging	25 "	Coal	300 "
Anchors and chains	50 "	Stores and provisions	78 "
Crew, etc., etc.	133 "	Total	1486 "

Comparison of Tonnage of Engine and Boiler Space to total Tonnage (English).

Side lever Engine......... 44 per cent. | Direct acting Engine...... 34 per cent.

(Iron).

ENGLISH.—*Length upon deck*, 178 *feet; do. at mean load-line* of 19.16 *feet*, 177 *feet; keel*, 171 *feet; beam*, 32.88 *feet; depth of keel* (mean), 2.75 *feet; hold*, 21.75 *feet.*

Immersed Section at load-line, 387 *square feet.*

Displacement 1385 *tons, at load-draught of* 19.16 *feet;* and, in proportion to its circumscribing parallelopipedon, .524, and 1495 tons, at deep load-draught of 20 feet.

Load-line.—Area at load-draught, 4557 square feet. Angle of entrance, 57°; of clearance, 64°. *Area* in proportion to its circumscribing parallelogram, .784.

Immersed Section.—Area in proportion to its circumscribing parallelogram, .737.

Centre of Gravity, 6.416 feet below mean load-line.

Centre of Displacement (gravity of), 6.25 feet below the load-line; and 4.33 feet before the middle of the length of the load-line.

Immersed Surface.—Bottom, 7370 square feet. *Keel*, 1130 square feet.

Rig.—Ship. *Sails*, 13 282 square feet.

Meta Centre, 6.66 feet above centre of gravity of displacement.

Centre of Effort before centre of displacement, 3.5 feet; height of do. above mean load-line, 55.5 feet.

Surface in proportion to immersed section, 34.32 to 1; do. in proportion to displacement in tons, 9.59 to 1.

SURF BOAT (WOOD), NEW YORK.—*Length over all*, 30 *feet; beam*, 8 *feet; hold*, 2.75 *feet.*

Breadth and Height at Sections of 5 *feet.*

	1.	2.	3.	4.	5.
Breadth of Beam..	5 feet 10 ins.	7 feet 2 ins.	8 feet 0 ins.	6 feet 10 ins.	3 feet 9 ins.
Breadth of Floor..	1 " 3 "	2 " 9 "	3 " 6 "	2 " 4 "	0 " 3 "
Height of Gunwale	2 " 7 "	2 " 4 "	2 " 4 "	2 " 5 "	2 " 6 "

Rake of stem, 2 feet; of stern, 3 feet. *Oars.*—6 single, or 10 double banked.

LIFE BOAT (WOOD)—ENGLISH.—*Length over all*, 36 *feet; at load-line*, 31 *feet; beam*, 9.5 *feet; hold*, 3.5 *feet; shear*, 3 *feet; keel*, 8 *inches.*

Capacity.—300 cubic feet of air-vessel, equal to 8.5 tons or 70 persons.

Thwarts.—7 of 2.25 feet apart and 7 inches below gunwale.

Oars.—12 double banked, with pins and grommets.

Weight, 7500 lbs.

RACE BOAT.—*Length*, 46 *feet. Breadth*, 20 *ins.; at water line*, 18 *ins. Depth*, 8 *ins.*

Oars, 4. *Weight* (of Cedar), 150 lbs.

Sailing Vessels.

"America," Yacht (Wood).—*Length over all*, 98 *feet; upon deck*, 94 *feet; at load-line*, 90.5 *feet; beam* 22.5 *feet, at load-line* 22 *feet; depth of hold*, 9.25 *feet.*

Height at side from under side of garboard strake, 11 *feet. Sheer, forward*, 3 *feet; aft*, 1.5 *feet.*

Immersed Section at load-line, 121.8 *square feet.*

Displacement at load-draught of 8.5 *feet, from under side of garboard strake and of* 11 *feet aft*, 191 *tons; and, in proportion to Volume of circumscribing parallelopipedon*, .375.

Displacement at 4 *feet (from garboard strake)*, 43 *tons; at* 5 *feet*, 66 *tons; at* 6 *feet*, 93 *tons; at* 7 *feet*, 127 *tons; and at* 8 *feet*, 167 *tons.*

Area of Load-line, 1280 square feet. *Mean girths of immersed section to load-line*, 25 feet.

Load-draught.—Forward, 4.91 feet; aft, 11.5 feet. *Rake of Stem*, 17 feet from under side of keel.

Spars.—Mainmast, 81 feet in length by 22 ins. in diam. *Foremast*, 79.5 feet in length by 24 ins. in diam. *Main boom*, 58 feet in length. *Gaff*, 28 feet. *Fore Gaff*, 24 feet. *Rake*, 2.7 ins. per foot. *Drag of Keel*, 3 feet.

Tons.—O. M., U. S., 170.56; O. M., English, 210.

Centre of Gravity.—Longitudinally, 1.75 feet aft of centre of length upon load-line. Sectional, 2.58 feet below load-line. Of Fore body, 14.25 feet forward; and of After body, 19 feet aft. *Meta Centre*, 6.72 feet above centre of gravity.

Centre of Effort. 31.17 feet from load-line. *Centre of Lateral Resistance*, 6.33 feet abaft of centre of gravity.

"Wm. H. Aspinwall," Pilot Boat (Wood).—*Length of keel*, 74 *feet; upon deck*, 80 *feet; beam*, 19 *feet; hold*, 7.6 *feet. Draught of water*, 6 *feet forward; aft*, 9.5 *feet.*

Keel, 22 ins. in depth. *False keel*, 12 ins. in depth at centre.

Spars.—Mainmast, 77 feet in length. *Foremast*, 76 feet. *Main boom*, 46 feet. *Main gaff*, 21 feet. *Fore gaff*, 20 feet.

Tons.—N. M., 46.32.

"Lady Octavia," Clipper Ship (Iron).—*Length of keel and fore rake*, 199 *feet; upon deck*, 205.5 *feet; beam*, 36.75 *feet; hold*, 22.3 *feet. Length of poop*, 47 *feet; breadth of do.*, 29.6 *feet; depth of do.*, 6.6 *feet.*

Keelson.—Box, of .375 in. plates, 2.5 feet deep by 1.75 feet in width. *Sister and bilge keelsons*, two. *Bulkheads*, four.

Frames.—L, 5×3×.625 ins. thick. *Bottom Plates*, clincher laid.

Floors, .4375 thick by 22 ins. in depth, and 15 ins. apart at centres.

Beams.—Lower deck, 9 ins. in depth; Main deck, 8 ins.; and Poop, 6 ins.

Masts, iron. *Decks.*—Two, with top-gallant forecastle.

"Twilight," Clipper Ship (Iron).—*Length of keel and fore rake*, 160 *feet; upon deck*, 167.25 *feet; beam*, 30 *feet; hold*, 20 *feet.*

Load-draught, 17.5 *feet aft;* 17 *feet forward.*

Bulkheads.—Three. *Capacity*, 700 tons cargo.

Decks.—Two, with top-gallant forecastle.

BLOWING ENGINES.

Furnace.—One, diam. 14 *feet. Lonaconing* (Md.).

Engine (*Non-condensing*).—*Cylinder*, 18 ins. in diam. by 8 feet stroke of piston.

Boilers (plain cylindrical).—Five, 3 feet in diam. and 24 feet in length. *Grates*, 50 square feet.

Pressure of Steam.—50 lbs. per square inch, cut off at ⅜ the stroke of the piston. *Revolutions*, 12 per minute.

Blowing Cylinders.—Two, 5 feet in diam. and 8 feet stroke of piston. *Pressure*, 2 to 2½ lbs. per square inch. *Volume of Air*, 3770 cubic feet per minute.

Furnaces.—Four, diam. 14 *feet. Mount Savage* (Md.).
100 *Tons Pig Iron per Week.*

Engine (*Condensing*).—*Cylinder*, 56 ins. in diam. by 10 feet stroke of piston.

Boilers.—Six (cylindrical flue), 60 ins. in diam. and 24 feet in length; 1 flue and return, 22 ins. in diam. *Grates*, 198 square feet.

Revolutions, 15 per minute.

Blowing Cylinder, 126 ins. in diam. by 10 feet stroke of piston. *Pressure*, 4 to 5 lbs. per square inch. *Area of Pipes*, 2300 square inches = .2 of cylinder.

Furnaces.—Two. Fineries, two. (England.)
240 *Tons Forge Pig Iron per Week.*

Engine (*Non-condensing*).—*Cylinder*, 20 ins. in diam. by 8 feet stroke of piston.

Boilers.—Six (plain cylindrical), 36 ins. in diam. and 28 feet* in length. *Grates*, 100 square feet.

Blowing Cylinders.—Two, 62 ins. in diam. by 8 feet stroke of piston. *Pressure*, 2.17 lbs. per square inch. *Revolutions*, 22 per minute.

Pipes, 3 feet in diam. = .166 of the cylinder.

Tuyeres.—One Furnace, 2 of 3 ins. in diam. and 1 of 3¼ ins.; and one, 3 of 3 ins. One Finery, 6 of 1⅛ ins.; and one, 4 of 1⅛ ins.

Temperature of Blast, 600°. *Ore*, 40 to 45 per cent. of iron.

Furnace.—One. Esther, Peru (*Clinton Co.*, N. Y.).
37 *Tons Pig Iron per Week.*

Breast Wheel.—20 feet in diam. by 39 ins. face. *Fall*, 14 feet. *Volume of Water*, 350 cubic feet per minute. *Revolutions*, 9 per minute.

Blowing Cylinders.—Two (single acting), 7 feet in diam. and 18 ins. stroke of piston.

Volume of Air, 1039 cubic feet per minute.

Furnaces.—Eight, diam. 16 *to* 18 *feet. Dowlais Iron Works* (England).
1300 *Tons Forge Iron per Week; discharging* 44 000 *Cubic Feet of Air per minute.*

Engine (*Non-condensing*).—*Cylinder*, 55 ins. in diam. by 13 feet stroke of piston.

Pressure of Steam.—60 lbs. per square inch, cut off at ⅓ the stroke of the piston. *Valves*, 120 ins. in area.

Boilers.—Eight (cylindrical flue, internal furnace), 7 feet in diam. and 42 feet in length; one flue 4 feet in diam. *Grates*, 288 square feet.

Fly Wheel.—Diam. 22 feet; weight, 25 tons.

Blowing Cylinder, 144 ins. in diam. by 12 feet stroke of piston.

Revolutions, 20 per minute. *Blast*, 3¼ lbs. per square inch. *Discharge pipe*, diam. 5 feet, and 420 feet in length. *Valves.*—Exhaust, 56 square feet; Delivery, 16 square feet.

* 40 feet would have afforded economy in fuel.

STEAM FIRE ENGINE.

1st Class. (Amoskeag, N. H.)

Steam Cylinder.—Two of 7⅝ ins. in diam. by 8 ins. stroke of piston.

Water Cylinder.—Two of 4½ ins. in diam.

Boiler (vertical tubular).—*Heating surface*, 175 square feet. *Grates*, 4.75 square feet.

Pressure of Steam.—100 lbs. per square inch. *Revolutions*, 200 per minute.

Discharges.—Two gates of 2½ ins., through hose, one of 1¼ in. and two of 1 in.

Projection.—Horizontal, 1¼ in. stream, 311 feet; two 1 in. streams, 256 feet. Vertical, 1¼ in. stream, 200 feet.

Water Pressure.—With 1⅛ in. nozzle, 200 lbs.

Time of Raising Steam.—From cold water, 25 lbs., 4 min. 45 sec.

Weights.—Engine complete, 6000 lbs.; water, 300 lbs.

2d Class. Portland Co. (Portland, Me.)

Steam Cylinder.—9⅝ ins. in diam. by 10 ins. stroke of piston; volume of piston space, .421 cubic feet.

Water Cylinder.—4¾ ins. in diam. by 10 ins. stroke of piston; volume of piston space, .01025 cubic foot.

Boiler (vertical tubular).—*Heating surface*, 134.3 square feet. *Grates*, 5.5 sq. feet.

Pressure of Steam.—80 lbs. per square inch. *Revolutions*, 200 per minute.

Pressure of water in cylinder double that of steam when discharging through 500 feet of hose.

Discharges.—Two of ⅞ in.; one of 1¼ ins.

Projection.—Horizontal, two ⅞ in. streams, 200 feet; one 1¼ ins., 260 feet. Vertical, one 1⅛ ins., 100 feet hose, 210 feet.

Weights.—Complete, 5000 lbs.; water, 1000 lbs.

Time of Raising Steam.—From cold water, 6 to 8 minutes; from water at 130°, 4 minutes.

SUGAR MILLS.

Expressing 20 000 *lbs. Cane-juice per Day.*

Engine (*Non-condensing*).—*Cylinder*, 15 ins. in diam. by 4 feet stroke of piston.

Boiler (cylindrical flue).—62 ins. in diam. by 30 feet in length; two return flues 18 ins. in diam. *Grates*, 36 square feet. *Weight*, 15 500 lbs.

Pressure of Steam.—50 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions*, 30 per minute.

Rollers.—Two sets of 3 each, 24 ins. in diam. by 5 feet in length; geared 2.5 to 36 of engine. Speed of peripheries, 15.5 feet per minute.

Fly Wheel, 18 feet in diam.; weight, 11 200 lbs.

NOTE.—The arrangement of a second set of rolls is for the purpose of distributing the cane over an increased surface of rollers, reducing their speed, and affording more time for the juice to run off; an increase of 20 per cent. is effected by it.

SUGAR MILLS.—*Continued.*

Expressing 40 000 *lbs. Cane-juice per day, or for a Crop of* 5000 *Boxes of* 450 *lbs. each in four Months' Grinding.*

Engine (Non-condensing).—Cylinder, 18 ins. in diam. by 4 feet stroke of piston.

Boiler (cylindrical flue).—64 ins. in diam. and 36 feet in length; two return flues, 20 ins. in diam. *Heating surface*, 660 square feet. *Grates*, 30 square feet.

Pressure of Steam.—60 lbs. per square inch, cut off at ½ the stroke of the piston. *Revolutions*, 40 per minute.

Rollers.—One set of 3, 28 ins. in diam. by 6 feet in length; geared 1 to 14. *Shafts*, 11 and 12 ins. in diam.

Spur Wheel, 20 feet in diam. by 1 foot in width.

Fly Wheel, 18 feet in diam.; weight, 17 400 lbs.

WEIGHTS OF ENGINE, BOILER, ETC.

Engine	61 460 lbs.	Boiler	18 520 lbs.
Sugar Mill	65 730 "	Appendages	6 730 "
Spur Wheel and Connecting Machinery to Mill	28 680 "	Total	181 120 "

FLOUR MILLS.

30 *Barrels of Flour per Hour.*

Water-wheels—Overshot.—Five, diam. 18 feet by 14.5 feet face. *Buckets*, 15 ins. in depth.

Water.—Head, 2.5 feet. *Opening*, 2.5 ins. by 14 feet in length over each wheel.

5 *Barrels of Flour per Hour, and Elevating* 400 *Bushels of Grain* 36 *feet.*

Water-wheel—Overshot.—Diam. 22 feet by 8 feet face. *Buckets*, 52 of 1 foot in depth.

Water.—Head, from centre of opening, 25 ins. *Opening*, 1¾ ins. by 80 ins. in length.

Revolutions, 3½ per minute. *Stones*, 3 of 4½ feet; revolutions, 130.

Three Run of Stones, Diameter 4 *feet.*

Water-wheel—Overshot.—Diam. 19 feet by 8 feet face. *Buckets*, 14 ins. in depth.

Or,

Steam-engine (Non-condensing).—Cylinder, 13 ins. in diam. by 4 feet stroke of piston.

Boiler (cylindrical flue).—Diam. 5 feet by 30 feet in length; two flues 20 ins. in diameter.

HYDROSTATIC PRESS.

30 Bales of Cotton per Hour.

Engine (Non-condensing).—Cylinder, 10 ins. in diam. by 3 feet stroke of piston.

Pressure of Steam.—50 lbs. per square inch, full stroke. *Revolutions*, 45 to 60 per minute.

Presses.—Two, with 12-inch rams; stroke, 4.5 feet.

Pumps.—Two, diam. 2 ins.; stroke, 6 ins.

COTTON PRESS.

For 1000 Bales in 12 Hours.

Engine (Non-condensing).—Cylinder, 14 ins. in diam. by 4 feet stroke of piston.

Boilers.—Three (plain cylindrical), 30 ins. in diam. by 26 feet in length. *Grates*, 32 square feet.

Pressure of Steam.—40 lbs. per square inch. *Revolutions*, 60 per minute.

Presses.—Four, geared 6 to 1, with two screws, each of 7.5 ins. in diam. by 1.625 in pitch.

Shaft (wrought iron).—Journal, 8½ ins.

Fly Wheel, 16 feet in diam.; weight, 8960 lbs.

PILE-DRIVING.

Driving Two Piles.

Engine (Non-condensing).—Cylinders, two, 6 ins. in diam. by 18 ins. stroke of piston.

Boiler (horizontal tubular).—*Shell*, diam. 3 feet by 6 feet in length. Furnace end 3.75 feet in width, 3.5 feet in length, and 6 feet in height.

Pressure of Steam.—60 lbs. per square inch, full stroke. *Revolutions*, 60 to 80 per minute.

Frame, 8.5 feet in width by 26 feet in length. *Leaders*. 3 feet in width by 24 feet in height.

Rams.—Two, 1000 lbs. each, lifted 5 times in a minute.

Driving One Pile.

Engine (Non-condensing).—Cylinder, 6 ins. in diam. by 1 foot stroke of piston.

Boiler (vertical tubular).—32 ins. in diam. by 6.166 feet in height. *Grates*, 3.7 square feet. *Furnace*, 20 ins. in height. *Tubes*, 35, 2 ins. in diam., 4.5 feet in length.

Revolutions, 150 per minute. *Drum*, 12 ins. in diam., geared 4 to 1. *Leader*, 40 feet in height.

Ram.—One, 2000 lbs., 2 blows per minute.

Fuel, 30 lbs. coal per hour. *Water*, 15 gallons per hour.

HOISTING ENGINE.

Engine, Boiler, etc., as given for Pile Driving upon preceding page.

Performance.—250 to 300 tons of coal in 10 hours.

Fuel, 40 lbs. coal per hour. *Water*, 200 gallons per hour.

Weight of Engine and Boiler, 4500 lbs.

COTTON FACTORIES (*English*).

For driving 13 000 *Spindles (Mules and Throstles), with* 256 *Looms for Cloth three quarters wide, No.* 30.

Engine (Condensing).—Cylinders, two; 22 ins. in diam. by 3 feet stroke of piston; volume of piston space, 15.8 cubic feet.

Pressure of Steam.—15 to 45 lbs. per square inch, cut off at ⅓ the stroke of the piston. *Revolutions*, 50 per minute.

For driving 22 060 *Hand-mule Spindles, with preparation, and* 260 *Looms, with common Sizing.*

Engine (Condensing)—Cylinder, 37 ins. in diam. by 7 feet stroke of piston; volume of piston space, 53.6 cubic feet.

Pressure of Steam.—(Indicated average) 16.73 lbs. per square inch. *Revolutions*, 17 per minute.

Friction of Engine and Shafting.—(Indicated) 4.75 lbs. per square inch of piston.

Indicated Horses' Power, 125.

Total power = 1. Available, deducting friction = .717.

NOTES.—Each Indicated Horse's power will drive
305 hand-mule spindles, *with preparation*,
or 230 self-acting " "
or 104 throstle " "
or 10.5 looms, with common sizing.

Including preparation:
1 throstle spindle = 3 hand-mule, or 2.25 self-acting spindles.
1 self-acting spindle = 1.2 hand-mule spindles.

Exclusive of preparation, taking only the spindle:
1 throstle spindle = 3.5 hand-mule, or 2.56 self-acting spindles.
1 self-acting spindle = 1.375 hand-mule spindles.

The *throstles* are the common, spinning 34 twist for power-loom weaving; the *spindles* revolve 4000 per minute. The *self-acting mules* are, one half spinning 36's weft, spindles revolving 4800; the other half spinning 36's twist, spindles revolving 5200. The *hand-mules* spinning about equal quantities of 36's weft and twist. Weft spindles 4700, and twist spindles 5000 revolutions per minute.

Average breadth of looms 37 ins. (weaving 37 ins. cloth), making 123 picks per minute. All common calicoes about 60 reed, Stockport count, and 68 picks to the inch.

No power consumed by the sizing. When the yarn is dressed instead of sized, one horse's power can not drive so many looms, as the dressing machine will absorb from .17 to .14 of the power.

STEAM FIRE-ENGINES.

1855, "Citizens' Gift," Cincinnati, Ohio, projected a stream through 100 feet of 3½ in. hose and 1¾ in. nozzle, 327 *feet.*

1868, "Mississippi No. 2," 1st class Amoskeag, New Orleans, La., projected a stream through 100 feet of hose, nozzle 1¼ inch, 311 *feet* 8 *in.* solid, and 336 *feet* with spray.

1869, "General Morgan," Galesburg, Ill., steam in 4 *min.*, water in 9 *min.*, and projected a stream 214 *feet* 2 *ins.*

1869, "Cataract No. 10," 2d class Amoskeag, Boston, Mass., steam 20 lbs. in 4 *min.* 30 *sec.*; 85 lbs. steam, 175 lbs. water, projected a stream through 300 feet of hose, 1¼ in. nozzle, 199 *feet* 8 *ins.*

1870, "Eddy," Troy, N. Y., 3 tons, harnessed up and ran 1 mile in 5 *min.* 15 *sec.*

A hand engine, 273 *feet* horizontal, and 196 *feet* vertical.

1873, "Eagle No. 7," New Orleans, La., projected a solid stream through 100 feet of hose, 1½ in. nozzle, 314 *feet* 7 *ins.*

MOWING MACHINE.

1860, — *Kirby's*, Auburn, N.Y., 670 lbs., two horses, 1 acre heavy clover in 46 *min.*

STONE SAWING.

1874, *J. E. Emerson's* 20 horse-power machine, 21¼ *square feet* of Berea sandstone, in 10 *min.*

STEAM WAGON.

1865, Poughkeepsie, N. Y., 1 mile in 2 *min.* 20 *sec.*

RAILROADS.

1830, *Schenectady to Albany, N.Y.*, Aug. 12, Hudson and Mohawk Road, first locomotive engine.

185–, *Albany to New York, N. Y.*, Hudson River R. R., 144 miles, in 2 *hours* 40 *min.*

1855, *Chicago and N. W. R. R.*, Galena division, Ill., engine "Nebraska," 1 mile 8 rods, in 56 *sec.* = 1 mile in 54.6 *sec.*

185–, *Paddington to Slough, Eng.*, 18 miles, in 15 *min.*

186–, *Hamburg to Buffalo, N.Y.*, N.Y. and Erie R.R., 10 miles, in 8 *min.*

1862, *Boston to New York, N.Y., via* Providence and New London, 230 miles, in 5 *hours* 27 *min.* running time.

1868, *Indianapolis, Ind., to Pittsburg, Penn.*, 381 miles, in 8 *hours* running time.

1868, *Janesville, Wis., to Chicago, Ill.*, 91 miles, in 1 *hour* 30 *min.*

1872, *Rochester to Syracuse, N.Y.*, N.Y. Central Railroad, engine "341," a distance of 81 miles, in 82 *min.*

1874, *West Albany to Poughkeepsie, N.Y.*, N.Y. Central and Hudson River R. R., 70 miles, in 1 *hour* 38 *min.*, including 18 *min.* in delays. *Peekskill to Sing Sing, N.Y.*, 10 miles, in 9 *min.* 1855, locomotive "Hamilton Davis," with 6 cars, 14 miles, in 11 *min.*

1874, *Clinton, Iowa, to Chicago, Ill.*, Chicago and N.W. R. R., engine "Wabasha," 138 miles, in 2 *hours* 22 *min.* Revolutions, 316 per minute.

BRICK-LAYING.

1870, *W. D. Cozzens*, Philadelphia, Penn., 702 bricks in 12 *min.*

VELOCIPEDES.

1869, *J. C. Hogel*, Philadelphia, Penn., ¼ mile in 47 *sec.*

1869, *V. Price*, New York, N.Y., floor, ½ mile in 1 *min.* 46½ *sec.*

1869, *J. H. Boyle*, Jersey City, N. J., curriculum, ⅓ mile in 1 *min.*, 1 mile in 3 *min.* 17 *sec.*, and 5 miles in 22 *min.* 3 *sec.*

1870, *W. H. Russell*, Velocipedrome, Jersey City, N. J., 1 mile in 2 *min.* Wheel 36 ins. in diameter, geared.

1874, *J. Moore*, of Paris, Wolverhampton, Eng., 1 mile in 3 *min.* 2¼ *sec.*

186–, *C. W. Littlefield*, Boston, Mass., 6 miles in 26 *min.* 15 *sec.*

1869, *F. Hardy*, Cleveland, Ohio, rink, 50 miles in 3 *hours* 14 *min.* 39 *sec.*, including rests.

1869, *A. P. Meisinger*, Central Hall, N. Y., 100 miles in 7 *hours* 20 *min.*, 100 in 8 *hours* 42 *min.*, 100 in 9 *hours* 45 *min.*, 100 in 9 *hours* 38 *min.*, and 100 in 8 *hours* 42 *min.*; total, 500 miles in 49 *hours* 50 *min.*, including 4 *hours* 43 *min.* in rests.

187–, *J. D. Johnson*, London to Worthing, Eng., 132 miles and back, in 17 *hours* 15 *min.*, including rests.

1874, *D. Stanton*, London, Eng., 2 miles in 7 *min.* 7 *sec.*, 25 in 1 *hour* 33 *min.* 7 *sec.*, 100 in 7 *hours* 35 *min.* 43 *sec.*, 106 in 7 *hours* 58 *min.* 54½ *sec.*, and last mile in 3 *min.* 41½ *sec.*

BASE-BALL AND CRICKET.

1789, 24 *Cricket-players*, England, batted a ball, with a letter inclosed, 50 miles in 1 *hour;* wagered and won by the Duke of Queensberry.

1867, *Geo. Wright*, Indianapolis, Ind., batted a ball nearly 200 *yards.*

187–, *H. Berthrong*, Washington, D. C., ran the bases (120 yards) in 14½ *sec.*

1872, *John Hatfield*, Brooklyn, L. I., base ball, thrown 133 *yards* 1 *foot* 7½ *ins.*

1873, *W. H. Game*, Marston, Eng., threw a cricket ball 127 *yards* 1 *foot* 3 *ins.*, strong wind.

PHYSICAL ENDURANCE.

1754, A man, Pinwire, Eng., outwalked a horse in 12 *hours.*

1817, *George Gyngell*, London, Eng., rowed 1 000 miles, in a Thames wherry, in 20 consecutive *days.*

1826, *Capt. Polhill*, Haigh Park Course, Eng., walked, drove, and rode 50 miles, each in 19 *hours* 5 *min.*, including 1 *hour* 38 *min.* in rests.

1859, *J. Kennovan*, San Francisco, Cal., walked 106 *hours* 30 *min.* without resting; and 400 miles, in 98 *hours* 30 *min.* 1874 (60 years), walked and danced 30 hours with but 5 minutes' rest.

1843, —— *Aymar*, Circus Royal, London, a triple somersault. *Jas. Wheel* also performed the same feat.

1869, *Robert Harriott*, "*Mickey Free*," Flushing, N. Y., walked 110 *consecutive hours* without resting.

1870, *W. Dutcher*, Matteawan, N. Y., walked 115 *consecutive hours*, with allowance of 20 minutes in each 24 hours.

1870, *J. Davidson*, Quincy, Ill., walked 105 hours without sleep.

1871, *Geo. W. Chambers*, Staten Island to Newburg, N. Y., rowed 75 miles' course, in 16 *hours* 45 *min.*, in a boat 17 feet in length.

1872, *J. D. Armstrong*, Lachine, Can., hopped, walked, ran, rode, and rowed ¼ mile in each manner in 12 *min.* 38 *sec.*

STONE-GATHERING.

1869, *Geo. Griffen*, Rock Island, Ill., 100 stones, 5 lbs. each, 1 yard apart, and the first stone 1 yard from the point of starting and delivery = 5 miles 1 300 yards in 1 *hour* 13 *min.* Average weight of the stones, 5 lbs.

ROWING.

NOTE.—No performances but such as have been made over properly measured courses are here given; hence New York Harbor, Hudson and Harlem rivers, and like courses, are omitted, except when the effect of a tidal or fluvial current has been compensated by a "turn," *i. e.*, an equal course with and against the current.

Sculls.

ONE AND ONE HALF MILES.

1870, *W. B. Curtis and W. Snyder*, Detroit, Mich., Shell, double sculls, one turn, in 12 *min.* 40 *sec.*

TWO MILES.

1859, *R. F. Clark*, Boston, Mass., Shell, single sculls, one turn, in 13 *min.* 52 *sec.*

1861, *J. D. Parker and Carpenter*, Boston, Mass., Shell, double sculls, one turn, in 12 *min.* 54½ *sec.*

1871, *Miss Amelia Shean*, Harlem, N. Y., Working boat, single sculls, 3 turns, in 18 *min.* 32½ *sec.*

THREE MILES.

1872, *Edward Smith*, Staten Island, N. Y., Shell, single sculls, one turn, in 21 *min.* 57½ *sec.*; and *Geo. Engelhart*, in 22 *min.* 21½ *sec.*

1874, *James O'Neill*, Saratoga, N. Y., Shell, single sculls, one turn, in 21 *min.* 19½ *sec.*

1874, *E. Smith* and *F. C. Eldred*, Saratoga, N. Y., Shell, double sculls, one turn, in 21 *min.* 52½ *sec.*

FOUR MILES.

1871, *Jos. H. Sadler*, of England, Saratoga, N.Y., Shell, single sculls, one turn, in 30 *min.* 18½ *sec.*

FIVE MILES.

1862, *James Hamill*, Philadelphia, Penn., Shell, single sculls, one turn, in 37 *min.* 39 *sec.*

1874, *George Brown*, near St. John's, N. B., Shell, single sculls, one turn, in 37 *min.*

Oars.

TWO MILES.

1871, *Ward Brothers Crew*, Saratoga, N.Y., Shell, 4 oars, straight course, in 11 *min.* 20 *sec.*

THREE MILES.

1860, *Union Boat Club*, of Boston, Worcester, Mass., Shell, 4 oars, one turn, in 19 *min.* 41 *sec.*

1867, *Vesper Boat Club*, Hoboken, N. J., Barge, 8 oars, one turn, in 21 *min.* 20¼ *sec.*

1868, *Ward Brothers Crew*, Worcester, Mass., Shell, 6 oars, one turn, in 17 *min.* 40½ *sec.*

1874, *Beaverwyck Club*, of Albany, N. Y., Saratoga, N. Y., 4 oars, one turn, in 18 *min.* 34 *sec.*

FOUR MILES.

1871, *Ward Brothers Crew*, Saratoga, N. Y., Shell, 4 oars, one turn, in 24 *min.* 40 *sec.*

FIVE MILES.

1867, "*J. F. Tapley*," Springfield, Mass., Shell, 6 oars, one turn, in 33 *min.* 7½ *sec.*

SIX MILES.

1867, *Ward Brothers Crew*, of New York, Springfield, Mass., Shell, 4 oars, one turn, in 39 *min.* 28 *sec.*

English College Races.

1870, *Oxford University Crew*, Putney to Mortlake, Eng., 4⅜ miles, 8 oars, favorable current, in 20 *min.* 6½ *sec.*; and 1851, Henley, 1.312 miles, straight course, favorable current, in 7 *min.* 45 *sec.*

1873, *Cambridge University Crew*, Mortlake to Putney, Eng., 4⅜ miles, 8 oars, favorable current, in 19 *min.* 35 *sec.*

American College Races.

1865, *Yale University Crew*, Worcester, Mass., Shell, 3 miles, 6 oars, one turn, in 17 *min.* 42½ *sec.*

1868, *Harvard University Crew*, Worcester, Mass., Shell, 3 miles, 6 oars, one turn, in 17 *min.* 48½ *sec.*

1872, *Amherst College Crew*, Springfield, Mass., Shell, 3 miles, 6 oars, straight course, in 16 *min.* 32.8 *sec.*

1874, *Columbia College Crew*, Saratoga, N. Y., Shell, 3 miles, 6 oars, straight course, in 16 *min.* 42½ *sec.*

Various Distances and Performances.

NOTE.—In the following cases the effects of the direct and varying currents have been impracticable of attainment.

1830, *F. Creswell* and *William Lewis*, of England, Thames wherry, Billingsport to Gravesend, up to Richmond Bridge and down to Old Swan, 96 miles, in 11 *hours* 50 *min.*

1817, *George Gyngell*, of Millwall, London, Eng., Thames wherry, Thames River, 1 000 miles, in 20 *days.*

1866, *Kingston Club*, Henley, Eng., Barge, 8 oars, straight course, favorable current, 1.3125 miles, in 7 *min.* 21 *sec.*=1 mile in 5 *min.* 36 *sec.*

1869, *G. W. Chambers*, around Staten Island, N.Y., Working boat, 33½ miles, in 6 *hours* 48 *min.*

1874, *William B. Curtis*, Calumet River, Ill., Paper Shell, 50 miles, rough water, in 10 *hours* 11 *min.* 55 *sec.*

18—, *T. White*, of England, Putney to Mortlake, Eng., Shell, 4¼ miles, straight course, favorable current, in 23 *min.* 13 *sec.*

1864, *Tyne Crew*, of England, Putney to Mortlake, Eng., Shell, double sculls, 4¼ miles, straight course, favorable current, in 21 *min.* 54 *sec.*

International Races.

1866, *Henry Kelley*, of England, Newcastle, Eng., Shell, 4.429 miles, straight course, favorable current, in 33 *min.* 29 *sec.*

1869, *Walter Brown*, of Maine, U. S., Newcastle, Eng., Shell, 3.405 miles, straight course, favorable current, in 21 *min.* 50 *sec.*

1871, *Paris Crew*, of St. John's, N. B., Kennebecassis River, N. B., Shell, 6 miles, 4 oars, one turn, in 38 *min.* 50 *sec.**

1871, *Taylor-Winship Crew*, of England, Halifax, N. S., 6.977 miles, 4 oars, one turn, in 44 *min.* 28 *sec.*

1871, *Ward Brothers Crew*, of New York, Saratoga, N.Y., Shell, 4 miles, 4 oars, one turn, in 24 *min.* 40 *sec.*; and to turn, 2 miles in 11 *min.* 20 *sec.*

* Engelhardt gives 39 *min.* 20¾ *sec.*

DREDGING MACHINES.

Dredging 20 Feet from Water-line, or 180 Tons of Clay and Mud per Hou.
11 Feet from Water-line.

Length upon deck, 123 feet; beam, 26 feet. Breadth over all, 41 feet.

Immersed Section at load-line, 60 square feet.

Displacement 141 tons, at load-draught of 2.83 feet.

Engine (Non-condensing).—Cylinders, two, 12⅛ ins. in diam. by 4 feet stroke of piston.

Boilers.—Two (cylindrical flue), diam. 40½ ins., length 20 feet 3 ins.; two flues 14⅝ in diam. *Heating surface*, 617 square feet. *Grates*, 37 square feet.

Pressure of Steam.—25 lbs. per square inch, throttle ¼ open, cut off at ½ the stroke of the piston. *Revolutions*, 42 per minute.

Buckets.—Two sets of 12, 2½ feet in length by 15 ins. at top and 2 feet deep; volume, 6¼ cubic feet.

Chain Links, 8 ins. in length by ½ in. diam.

Scows.—Four, of 40 tons' capacity each.

Dredging 30 Feet from Water-line, 6 full Buckets per Minute.

Engine (Non-condensing).—Cylinder, 12 ins. in diam. by 5 feet stroke of piston.

Boilers.—Two (cylindrical flue), 20 feet in length by 36 ins. in diam., with one return flue 15 ins. in diam. in each.

Pressure of Steam.—60 to 70 lbs. per square inch. *Revolutions*, 20 per minute.

Ways.—55 feet in length by 6 feet in width.

Buckets.—Ten, of 28 ins. in width by 58 in length, and 14 in depth.

Speed of Buckets, 1 to 30 of engine. At a depth of 18 feet, 10 buckets full of mud are discharged per minute, the engine making 30 revolutions.

Hulls.—Two of 50 feet in length, 12 feet in width, and 9 feet in depth; connected upon deck; space between the ways, 7.5 feet.

Note.—This engine is geared too slow.

Dredging 240 Cubic Yards (396 Tons) of Mud per Hour at a depth of 15 feet of water, and operating in water from 5 to 40 feet in depth.

Immersed Section at Load Line 84 square feet. Displacement, 120 tons at load draught of 3 feet.

Engines.—(Non-condensing) Cylinders, two, 12 ins. in diam. by 15 ins. stroke of piston.

Boiler.—One (Locomotive Tubular), diam. 4 feet 8½ ins., length 14 feet, 62 tubes 3 ins. in diameter, and 2 2½ ins. *Heating Surface*, 432 square feet. *Grates*, 1[illegible] square feet.

Pressure of Steam.—60 lbs. per square inch, cut off at four fifths the stroke of the piston.

Revolutions.—100 per minute. *Fuel.*—300 lbs. anthracite coal per hour.

Scoop or Dipper.—4 cubic yards in volume.

Strokes of Scoop in 15 feet of water, 60 per hour.

Steam Hopper Scow,

Transporting 300 *tons of Material at a Load Draught of Water,* 7 *feet* 6 *in.*

Length of Keel and Fore Rake, 135 feet; *beam*, 23 feet; *hold*, 9 feet 9 ins.

Hopper.—50 feet by 19 feet at top, and 8 feet 3 ins. at bottom.

Immersed Section at Load-line.—160 square feet.

Engines (*Condensing*).—*Cylinders*, two, 22 ins. in diam. by 22 ins. stroke of piston.

Boiler.—Horizontal tubular, 144 tubes, 3¾ ins. in diam.

Pressure of Steam.—25 lbs. per square inch. *Revolutions*, 80 per minute.

Propeller.—8 feet in diam. *Pitch*, 12 feet 6 ins.

Hull.—*Frames*, 3½ × 3½ × 5/16, 2 feet apart. *Floors*, ⅜ × 10 ins. *Plates*, ⅜ and 5/16 in.

Bulkheads.—Three.

Fuel.—Consumption, 672 lbs. bituminous coal per hour.

Speed.—6.09 miles per hour.

[See *Practical Mechanic's Journal*, Glasgow, vol. i., 3d series, p. 73 and plate.]

SAW MILLS.

Two Vertical Saws, 34 *ins. Stroke, Lathes, etc.*

Engine (*Non-condensing*).—*Cylinder*, 10 ins. in diam. by 4 feet stroke of piston.

Boilers.—Three (plain cylindrical), 30 ins. in diam. by 20 feet in length.

Pressure of Steam.—90 lbs. per square inch. *Revolutions*, 35 per minute.

NOTE.—This engine has cut, of yellow-pine timber, 30 feet by 18 ins. in one minute.

Two Circular Saws, cutting 7 812 *feet of* 1-*inch Poplar Boards in One Hour.*

Engine (*Non-condensing*).—*Cylinder*, 12 ins. in diam. by 2 feet stroke of piston.

Boilers (*Cylindrical flue*).—Two of 38 ins. in diam. and 26 feet in length, two flues 14 ins. in diam. *Heating surface*, 765 square feet. *Grate surface*, 425 square feet.

Pressure of Steam.—125 lbs. per square inch; cut-off at .7 the stroke of the piston. *Revolutions*, 250 to 350 per minute.

Saws.—Circular, one each, 66 and 60 ins. in diam.

Fuel.—Saw-dust.

Furnace.—25 ins. in depth, and without a bridge-wall. Hot-air chamber back of grates, 4 feet in depth.

Tender Engine.—Cylinder, 11 ins. in diam. by 3.5 feet stroke of piston. *Duty*, Drawing logs from river, driving saws for cutting logs in lengths, edging boards, and feeding boilers.

ORTHOGRAPHY OF TECHNICAL WORDS AND TERMS.

The orthography in ordinary use of the following words and terms is so varied, that they are here given for the purpose of establishing a more general uniformity of expression.

Abut. To meet, to adjoin to at the end, to border upon. The *abut end* of a log, etc., is that having the greatest diameter or side.

But and *Butt end,* when applied in this manner, are corruptions.

Amidships. The middle or centre of a vessel, either fore and aft or athwartships. The amidship frame of a vessel is at ⊠.

Arabesque. Applied to painted and carved or sculptured ornaments of imaginary foliage and animals, in which there are no perfect figures of either. Synonymous with *Moresque.*

Arbor. The principal axis or spindle of a machine of revolution.

Bagasse. Sugar-cane in its crushed state, as delivered from the rollers of a mill.

Baluster. A small column or pilaster; a collection of them, joined by a rail, forms a *balustrade.*

Banister is a corruption of balustrade.

Bark. A ship without a mizzen-topsail, and formerly a small ship.

Bateau. A light boat, with great length proportionate to its beam, and wider at its centre than at its ends.

Bevel. A term for a plane having any other angle than 45° or 90°.

Binacle. The case in which the compass or compasses is set on board of a vessel.

Bit. The part of a bridle which is put into an animal's mouth. In *Carpentry,* a boring instrument.

Bitter End. The inboard end of a vessel's cable.

Bitts. A vertical frame upon the deck of a vessel, around or upon which is secured cables, hawsers, sheets, etc.

Boomkin. A short spar projecting from the bow or quarter of a vessel, to extend the tack of a sail to windward.

Bowlder. A stone rounded by natural attrition; a rounded mass of rock transported from its original bed.

Breast-summer. A lintel beam in the exterior wall of a building.

Buhr-stone. Mill-stone which is nearly pure silex, full of pores and cavities.

Burden. A load. The quantity that a ship will carry. Hence *burdensome.*

Cag. A small cask, differing from a barrel only in size. Commonly written *Keg.*

Calibers. A compass with arched legs, to measure the diameters of spheres, or the exterior and interior diameters of cylinders, bores, etc.

Callipers is a corruption.

Calk. To stop seams and pay them with pitch, etc. To point an iron shoe so as to prevent its slipping.

Cam. An irregular curved instrument, having its axis eccentric to the shaft upon which it is fixed.

Camboose. The stove or range in which the cooking in a vessel is effected. The cooking-room of a vessel; this term is usually confined to merchant vessels; in vessels of war it is termed *Galley.*

Cantle. A fragment; a piece; the raised portion of the hind part of a saddle.

Capstan. A vertical windlass.

Caravel. A small vessel (of 25 or 30 tons burden) used upon the coast of France in herring fisheries.

Carlings. Pieces of timber set fore and aft from the deck beams of a vessel, to receive the ends of the ledges in framing a deck.

Carvel built. A term applied to the manner of construction of small boats, to signify that the edges of their bottom planks are laid to each other like to the manner of planking vessels. Opposed to the term *Clincher.*

Caster. A small phial or bottle for the table; one of a set of *Casters.*

Castors. Small wheels placed upon the legs of tables, etc., to allow them to be moved with facility.

Catamaran. A small raft of logs, usually consisting of three, and designed for use in an open roadstead and upon the sea-coast.

Chamfer. A slope, groove, or small gutter cut in wood, metal, or stone. *To Chamfer* is to slope, to channel, or to groove.

Chimney. The flue of a fire-place or furnace, constructed of masonry in houses and furnaces, and of metal, as in a steam boiler. See *Pipe.*

Chinse. To chinse, is to calk slightly with a knife or chisel.

Chock. Small pieces of wood used to make good any deficiency in a piece of timber, frame, etc. See *Furrings.*

Choke. To stop, to obstruct, to block up, to hinder, etc.

Cleats. Pieces of wood or metal of various shapes, according to their uses, either to resist or support weights or strains, as *shoar* cleats, *beam* cleats, etc.

Clincher built. A term applied to the construction of vessels, when the lower edge of the bottom planks overlays the next under it.

Coak. In *Mechanics,* a cylinder, cube, or triangle of hard wood let into the ends or faces of two pieces of plank or timber to be secured together. The metallic eyes in a sheave through which the pin runs. In *Naval Architecture,* the oblong ridges banded on the masts of ships.

Coamings. Raised borders around the edges of hatches.

Coble. A small fishing-boat.

Coccoon. The case which certain insects make for a covering during the period of their metamorphosis to the *pupa* state.

Cog. In *Mechanics,* a short piece of wood or other material let into the faces of a body to impart motion to another. A term applied to a tooth in a wheel when it is made of a different material than that of the wheel.

Colter. The fore iron of a plow that cuts earth or sod.

Compass. In *Geometry,* an instrument for describing circles, measuring figures, etc. To say, A pair of compasses, is superfluous and improper.

Contrariwise. Conversely, opposite.
Crossways is a corruption.

Corridor. A *gallery* or passage in or around a building, connected with various departments, sometimes running within a quadrangle: it may be opened or inclosed. In *Fortifications,* a covert way.

Damasquinerie. Inlaying in metal.

Davit. A short boom fitted to hoist an anchor or boat.

Dowel. To fasten two boards or pieces together by pins inserted in their edges. This is very similar to coaking, but is used in a diminutive sense. An illustration of it is had in the manner a Cooper secures two or more pieces in the head of a cask.

Draught. A representation by delineation. The depth which a vessel or any floating body sinks into water. The act of drawing. A detachment of men from the main body, etc., etc.

Edgewise. An edge being put into a particular direction. Hence *endwise* and *sidewise* have similar significations with reference to an edge and a side.
Edgeways is a corruption.

Felloe, Felloes. The pieces of wood which form the rim of a wheel.

Flange. A projection from an end or from the body of an instrument, or any part composing it, for the purpose of receiving, confining, or of securing it to a support or to a second piece.

Frap. To bind together with a rope, as to *frap* a fall, etc.

Frustrum. The part of a solid next the base, left by the removal of the top or segment.

Furrings. Strips of timber or boards fastened to frames, joists, etc., in order to bring their faces to the required shape or level.

Galets. Pieces of stone chipped off by the stroke of a chisel. See *Spall.*

Galeting. Putting galets into pointing-mortar or cement.

Galiot. A small galley built for speed, having one mast, and from sixteen to twenty thwarts for rowers. A Dutch-constructed brigantine.

Gate. In *Mechanics*, the hole through which molten metal is poured into a mold for casting.

Geat and *Gett* are corruptions.

Gearing. A series of teethed or cogged wheels for transmitting motion. To *gear* a machine is to prepare to connect its parts as by an articulation.

Gingle. To shake so as to produce a sharp, clattering noise.

Girt. The circumference of a tree or piece of timber. *Girth.* The band or strap by which a saddle or burden is secured upon the back of an animal, by passing around his belly. In *Printing*, the bands of a press.

Graving. Burning off grass, shells, etc., from a ship's bottom. Synonymous with *Breaming.*

Grommet. A wreath or ring of rope.

Gymbal Ring. A circular *rynd* for the connection of the upper mill-stone to the spindle by which the stone is suspended, so that it may vibrate upon all sides.

Hogging. A term applied to the hull of a vessel when her ends drop below her centre. See *Sagging.*

Horsing. In *Naval Architecture*, calking with a large maul or beetle.

Jam. To press, to crowd, to wedge in. In *Nautical Language*, to squeeze tight.

Jamb. A pier; the sides of an opening in a wall.

Jib. The projecting beam of a crane from which the pulleys and weight are suspended. A sail in a vessel.

Jibe. To shift a boom-sail from one tack to another; hence *Jibing*, the shifting of a boom.

Keelson. The timber within a vessel laid upon the middle of the floor timbers, and exactly over the keel.

Kevel. Large wooden cleats to belay hawsers and ropes to, commonly written *Cavil.*

Lacquer. A spirituous solution of *lac.*

Laitance. A pulpy, gelatinous fluid washed from the cement of concrete deposited in water.

Lapsided. A term expressive of the condition of a vessel or any body when it will not float or sit upright.

Leat. A trench to conduct water to or from a mill-wheel.

Leech. In *Nautical language*, the perpendicular or slanting edge of a sail when not secured to a spar or stay.

Luf. The fullest part of the bow of a vessel.

Mall. A large double-headed wooden hammer.

Mantle. To expand, to spread. *Mantle-piece*, the shelf over a fire-place in front of a chimney.

Marquetry. Checkered or inlaid work in wood.

Matrass. A chemical vessel with a body alike to an egg and a tapering neck.

Mattress. A quilted bed; a bed stuffed with hair, moss, etc., and quilted.

Mitered. In *Mechanics*, cut to an angle of 45°, or two pieces joined so as to make a right angle.

Mizzen-mast. The aftermost mast in a three-masted vessel.

Mold. In *Mechanics*, a matrix in which a casting is formed. A number of pieces of vellum or like substance, between which gold and silver are laid for the purpose of being beaten. Thin pieces of materials cut to curves or any required figure. In *Naval Architecture*, pieces of thin board cut to the lines of a vessel's timbers, etc. Fine earth, such as constitutes soil. A substance which forms upon bodies in warm and confined damp air.

This orthography is by analogy, as *gold, sold, old, bold, cold, fold*, etc.

Molding. In *Architecture*, a projection beyond a wall, from a column, wainscot, etc.

Moresque. See *Arabesque.*

Mortise. A hole cut in any material to receive the end or tenon of another piece.

Net. Clear of deductions, as *net* weight.

Newel. An upright post, around which winding stairs turn.

Nigged. Stone hewed with a pick or pointed hammer instead of a chisel.

Ogee. A molding with a concave and convex outline, like to an S.

Pargeting. In *Architecture*, rough plastering, alike to that upon chimneys.

Paillasse. Masonry raised upon a floor. A bed.

Parquetry. Inlaying of wood in figures. See *Marquetry.*

Pawl. The catch which stops, or holds, or falls on to a ratchet wheel.

Peek. The upper or pointed corner of a sail extended by a gaff, or a yard set obliquely to a mast. To *peek* a yard is to point it perpendicularly to a mast.

Pendant. A short rope over the head of a mast for the attachment of tackles thereto; a tackle, etc.

Pennant. A small pointed flag.

Pile. In *Engineering*, spars pointed at one end and driven into soil to support a superstructure or holdfast.

Spile is a corruption.

Pipe. In *Mechanics*, a metallic tube. The flue of a fire-place or furnace when constructed of metal; usually of a cylindrical form.

The term or application of *Stack* (which refers solely to masonry) to a metallic pipe is a misapplication of terms.

Piragua. A small vessel with two masts and boom sails.

Commonly termed *Perry Augur.*

Plastering. In *Architecture*, covering with plaster cement or mortar upon walls or laths. In England, termed *laying*, if in one or two coat work; and *pricking up*, if in three-coat work.

Plumber block. A bearing to receive and support the journal of a shaft.

Polacre. Masts of one piece, without tops.

Poppets. In *Naval Architecture*, pieces of timber set perpendicular to a vessel's bilge-ways, and extending to her bottom, to support her in launching.

Porch. An arched *vestibule* at the entrance of a building. A vestibule supported by columns. A *portico.*

Portico. A gallery near to the ground, the sides being open. A *piazza* encompassed with arches supported by columns, where persons may walk; the roof may be flat or vaulted.

Prize. In *Mechanics*, to raise with a lever.

To *pry* and a *pry* are corruptions.

Puzzuolana. A loose, porous, volcanic substance, composed of silicious, argillaceous, and calcareous earths and iron.

Rebate. In *Mechanics*, to pare down an edge of a board or a plate for the purpose of receiving another board or plate by lapping. To lap and unite edges of boards and plates. In *Naval Architecture*, the grooves in the side of the keel for receiving the garboard strake of plank.

Commonly written *Rabbet.*

Rarefaction. The act or process of distending bodies, by separating their parts and rendering them more rare or porous. It is opposed to *Condensation.*

Rendering. In *Architecture*, laying plaster or mortar upon mortar or walls. *Rendered* and *Set* refers to two coats or layers, and *Rendered, Floated*, and *Set*, to three coats or layers.

Resin. The residuum of the distillation of turpentine.

Rosin is a corruption.

Riband. In *Naval Architecture*, a long, narrow, flexible piece of timber.

Rimer. A bit or boring tool for making a tapering hole. In *Mechanics*, To *Rime* is to bevel out a hole. *Riming.* The opening of the seams between the planks of a vessel for the purpose of calking them.

Rotary. Turning upon an axis, as a wheel.

Rynd. The metallic collar in the upper mill-stone by which it is connected to the spindle.

Sagging. A term applied to the hull of a vessel when her centre drops below her ends. The converse of *Hogging.*

Scallop. To mark or cut an edge into segments of circles.

Scarf. To join, to piece; to unite two pieces of timber at their ends by running the end of one over and upon the other, and bolting or securing them together.

Sennit. Flat-braided cordage.

Sewage. The system of sewers.

Shaky. Cracked, or split, or as timber loosely put together.

Shammy. Leather prepared from the skin of a chamois goat.

Sheer. In *Naval Architecture*, the curve or bend of a ship's deck or sides. *To sheer*, to slip or move aside.

Sheers. Elevated spars connected at the upper ends, and used to elevate heavy bodies, masts, etc.

Shoal. A great multitude; a crowd; a multitude of fish.

School is a corruption.

Shoar. An oblique brace, the upper end resting against the substance to be supported.

Sholes. Pieces of plank under the heels of shoares, etc.

Shoot. A passage-way on the side of a steep hill, down which wood, coal, etc., are thrown or slid. The artificial or natural contraction of a river. A young pig.

Sidewise. See *Edgewise.*

Signaled. Communicated by signals.

Signalized, when applied to signals, is a misapplication of words.

Sill. A piece of timber upon which a building rests; the horizontal piece of timber or stone at the bottom of a framed case.

Siphon. A curved tube or pipe designed to draw fluids out of vessels.

Skeg. The afterpart of a keel; the part upon which the stern-post is set.

Slantwise. Oblique; not perpendicular.

Sleek. To make smooth. Refuse; small coal.

Sleeker. A spherical-shaped, curved, or plane-surfaced instrument with which to smooth surfaces.

Slue. The turning of a substance upon an axis within its figure.

Snying. A term applied to planks when their edges at their ends are curved or rounded upward, as a strake to the ends of a full-modeled vessel.

Spall. A piece of stone, etc., etc., chipped off by the stroke of a hammer or the force of a blow. *Spalling*, Breaking up of ore into small pieces.

Sponson. An addition to the outer side of the hull of a steam vessel, commencing near the light water-line and running up to the wheel guards; applied for the purpose of shielding them from the shock of a sea.

Sponson-sided. The hull of a vessel is so termed when her frames have the outline of a sponson, and the space afforded by the curvature is included in the hold.

Sponding and *Sponsing*, etc., etc., are corruptions

Stack. In *Masonry*, a number of chimneys or pipes standing together. The chimney of a blast furnace.

The application of this word to the smoke-pipe of a steam boiler is erroneous.

Stiving. The elevation of a vessel's bowsprit, cathead, etc., etc.

Strake. A breadth of plank.

Strut. An oblique brace to support a rafter.

Surcingle. A belt, band, or girth, which passes over a saddle or blanket upon a horse's back.

Swage. To bear or force down. An instrument having a groove on its under side for the purpose of giving shape to any piece subjected to it when receiving a blow from a hammer.

Scend. The settling of a vessel below the level of her keel.

Syphered. Overlapping the chamfered edge of one plank upon the chamfered edge of another in such a manner that the joint shall be a plane surface.

Template. In *Architecture*, a wooden bearing to receive the end of a girder to distribute its weight.

Templet. A mold cut to an exact section of any piece or structure.

Tenon. The end of a piece of wood, cut into the form of a rectangular prism, designed to be set into a cavity of a like form in another piece, which is called the *mortise.*

Terring. The earth overlying a quarry.

Tester. The top covering of a bedstead.

Tholes. The pins in the gunwale of a boat which are used as row-locks.

Thwarts. The athwartship seats in a boat.

Tire. The metal hoop that binds the felloes of a wheel.

Tompion. The stopper of a piece of ordnance. The iron bottom to which grape-shot are secured.

Treenails. Wooden pins employed to secure the planking of a vessel to the frames.

Trestle. The frame of a table; a movable form of support. In *Mast making*, two pieces of timber set horizontally upon opposite sides of a mast head.

Trise. To haul or tie up by means of a rope or trising line.

Tue-iron or *Tuyere.* The nozzle of a bellows or blast pipe in a forge or smelting-furnace.

Vice. In *Mechanics*, a press to hold fast any thing to be worked upon.

Voyal. A purchase applied to the weighing of an anchor, leading to a capstan.

Wagon. An open or partially-inclosed four-wheeled vehicle, adapted for the transportation of persons, goods, etc.

Whipple-tree. The bar to which the traces of harness are fastened.

Windrow. A row or line of hay, etc., raked together.

Woold. To wind; particularly to bind a rope around a spar, etc.

THE END.