William B Snyder
Lancaster
Pa

William B Snyder

Sept. 18th 1886

F. G. Snyder
June 1st 1892

MECHANICS' AND ENGINEERS'

POCKET-BOOK

OF

TABLES, RULES, AND FORMULAS

PERTAINING TO

MECHANICS, MATHEMATICS, AND PHYSICS:

INCLUDING

AREAS, SQUARES, CUBES, AND ROOTS, ETC.;

LOGARITHMS, HYDRAULICS, HYDRODYNAMICS, STEAM AND
THE STEAM-ENGINE, NAVAL ARCHITECTURE,
MASONRY, STEAM VESSELS,
MILLS, ETC.;

LIMES, MORTARS, CEMENTS, ETC.;

ORTHOGRAPHY OF TECHNICAL WORDS AND TERMS, ETC., ETC.

Forty-ninth Edition.—Ninety-seventh Thousand.

BY CHAS. H. HASWELL,

CIVIL, MARINE, AND MECHANICAL ENGINEER, MEMBER OF THE AM. SOC. OF CIVIL ENGINEERS, OF THE INSTITUTION OF CIVIL ENGINEERS, AND OF THE INSTITUTION OF NAVAL ARCHITECTS, ENGLAND, OF THE ENGINEERS' CLUB, PHILADELPHIA, OF THE N. Y. ACADEMY OF SCIENCES, CORRESPONDING MEMBER OF THE AMERICAN INSTITUTE OF ARCHITECTS, OF THE BOSTON SOCIETY OF CIVIL ENGINEERS, AND ASSOCIATE MEMBER OF THE N Y. MICROSCOPICAL SOCIETY, ETC.

An examination of facts is the foundation of science.

NEW YORK:
HARPER & BROTHERS, PUBLISHERS,
FRANKLIN SQUARE.
1885

By CHAS. H. HASWELL,

Civil, Marine, and Mechanical Engineer, M. Am. Soc. C. E.

MENSURATION.

For Tuition and Reference, containing Tables of Weights and Measures; Mensuration of Surfaces, Lines, and Solids, and Conic Sections, Centres of Gravity, &c. To which is added, Tables of the Areas of Circular Segments, Sines of a Circle, Circular and Semi-elliptical Arcs, &c. By Chas. H. Haswell, Civil and Marine Engineer, &c. Fifth Edition. 12mo, Sheep, 90 cents.

MECHANICS' TABLES.

Containing Areas and Circumferences of Circles, Sides of Equal Square; Circumferences of Angled Hoops, angled Outside and Inside; Cutting of Boiler-Plates, Covering of Solids, &c., and Weights of various Metals, &c. Miscellaneous Notes, comprising Dimensions of Materials, Alloys, Paints, Lacquers, &c.; U. S. Tonnage Act, with Diagrams, &c. By Chas. H. Haswell, Civil and Marine Engineer, &c. Second Edition. 12mo, Cloth, 75 cents.

Published by HARPER & BROTHERS, New York.

☞ Either of the above Works, or the Pocket-Book, will be sent by Mail, postage prepaid, on receipt of the price.

INSCRIBED

TO

CAPTAIN JOHN ERICSSON, LL.D.,

AS A SLIGHT TRIBUTE TO HIS GENIUS AND ATTAINMENTS,
AND IN TESTIMONY OF THE SINCERE REGARD
AND ESTEEM OF HIS FRIEND,

THE AUTHOR.

PREFACE

To the Forty-fifth Edition.

The First Edition of this work, consisting of 284 pages, was submitted to the Mechanics and Engineers of the United States by one of their number in 1843, who designed it for a convenient reference to Rules, Results, and Tables connected with the discharge of their various duties.

The Twenty-first Edition was published in 1867, consisted of 664 pages, and, in addition to the original design of the work, it was essayed to embrace some general information upon Mechanical and Physical subjects.

The Tables of Areas and Circumferences of Circles have been extended, and together with those of Weights of Metals, Balls, Tubes, Pipes, etc., of this and some preceding editions were computed and verified by the author.

This edition is a revision and an entire reconstruction of all preceding, embracing amended and much new matter, as Masonry, Strength of Girders, Floor Beams, Logarithms, etc., etc.

To the young Mechanic and Engineer it is recommended to cultivate a knowledge of Physical Laws and to note results of observations and of practice, without which eminence in his profession can never be attained; and if this work shall assist him in the attainment of these objects, one great purpose of the author will be well accomplished.

INDEX.

Q.

R.

Y.

Z.

Addenda.

EXPLANATIONS OF CHARACTERS AND SYMBOLS

Used in Formulas, Computations, etc., etc.

$=$ *Equal to*, signifies equality; as 12 inches $=$ 1 foot, or $8 \times 8 = 16 \times 4$.

$+$ *Plus*, or *More*, signifies addition; as $4 + 6 + 5 = 15$.

$-$ *Minus*, or *Less*, signifies subtraction; as $15 - 5 = 10$.

$\times$ *Multiplied by*, or *Into*, signifies multiplication; as $8 \times 9 = 72$. $a \times d$, $a \, . \, d$, or ad, also signify that a is to be multiplied by d.

$\div$ *Divided by*, signifies division; as $72 \div 9 = 8$.

: *Is to*, :: *So is*, : *To*, signifies *Proportion*, as 2 : 4 :: 8 : 16; that is, as 2 *is to* 4, *so is* 8 *to* 16.

$\therefore$ signifies *Therefore* or *Hence*, and $\because$ *Because*.

$\overline{\quad}$ *Vinculum*, or *Bar*, signifies that numbers, etc., over which it is placed, are to be taken together; as $\overline{8 - 2} + 6 = 12$, or $3 \times \overline{5 + 3} = 24$.

. *Decimal point*, signifies, when prefixed to a number, that that number has some power of 10 for its denominator; as .1 is $\frac{1}{10}$, .15 is $\frac{15}{100}$, etc.

$\backsim$ *Difference*, signifies, when placed between two quantities, that their difference is to be taken, it being unknown which is greater.

$\surd$ *Radical sign*, which, prefixed to any number or symbol, signifies that *square root* of that number, etc., is required; as $\sqrt{9}$, or $\sqrt{a+b}$. The degree of the root is indicated by number placed over the sign, which is termed *index* of the *root* or *radical;* as $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, etc.

$>$ ⅂, $<$ ⅃ signify *Inequality*, or *greater*, or *less than*, and are put between two quantities; as a ⅂ b reads a greater than b, and a ⅃ b reads a less than b.

( ) [ ] *Parentheses* and *Brackets* signify that all figures, etc., within them are to be operated upon as if they were only one; thus, $(3 + 2) \times 5 = 25$; $[8 - 2] \times 5 = 30$.

$\pm$ $\mp$ signify that the formula is to be adapted to two distinct cases, as $c \mp v = c$, either diminished or increased by v. Here there are expressed two values: first, the difference between c and v; second, the sum of c and v. In this and like expressions, the upper symbol takes preference of the lower.

p or π is used to express ratio of circumference of a circle to its diameter $= 3.1416$; $\frac{1}{4} p = .7854$, and $\frac{1}{6} p = .5236$.

° ′ ″ ‴ signify *Degrees*, *Minutes*, *Seconds*, and *Thirds*.

′ ″ set *superior* to a figure or figures, signify, in denoting dimensions, *Feet* and *Inches*.

a' a'' a''' signify a *prime*, a *second*, a *third*, etc.

$_1$, $_2$, added to or set *inferior* to a symbol, reads *sub* 1 or *sub* 2, and is used to designate corresponding values of the same element, as h, h_1, h_2, etc.

^{2}, ^{3}, ^{4}, added or set *superior* to a number or symbol, signify that that number, etc., is to be *squared*, *cubed*, etc.; thus, 4^2 means that 4 is to be multiplied by 4; 4^3, that it is to be *cubed*, as $4^3 = 4 \times 4 \times 4 = 64$. The *power*, or number of times a number is to be multiplied by itself, is shown by the number added, as ^{2}, ^{3}, ^{4}, ^{5}, etc.

$\frac{1}{2}$, $\frac{1}{3}$, etc., set *superior* to a number, signify square or cube root, etc., of the number; as $2^{\frac{1}{2}}$ signifies square root of 2; also $\frac{2}{3}$, $\frac{4}{2}$, $\frac{4}{3}$, $\frac{6}{3}$, etc., set *superior* to a number, signify two thirds power, etc., or cube root of square, or square or cube root of 4th power, or cube root of sixth power; as $8^{\frac{2}{3}} = \sqrt[3]{8^2}$ or $= (\sqrt[3]{8})^2$.

1·7, 3·6, etc., set *superior* to a number, signify tenth root of 17th power, etc.

·02, ·059, set *superior* to a number, signify hundredth root of 2d power, or thousandth root of 59th power, the numerator indicating power to which quantity is to be raised, and denominator indicating root which is to be extracted.

∞ signifies *Infinite*, as $\frac{1}{0}$ or a quantity greater than any assignable quantity. Thus, $\frac{a}{o} = \infty$ signifies that o is contained in any finite quantity an infinite number of times: $\frac{a}{1} = a$, $\frac{a}{.1} = 10a$, etc.

$\propto$ signifies *Varies as.* Thus, $M \propto D \times V$ signifies that mass of a body increases or diminishes in same ratio as product of its density and volume, or $S \propto t^2$, signifies S varies as t^2.

$\angle$ signifies *Angle.* $\perp$ *Perpendicular.* $\triangle$ *Triangle.* $\square$ *Square*, as $\square$ inches; and $\boxtimes$ cube, as cube inches.

NOTES.—Degrees of temperature used are those of Fahrenheit.
g is common expression for gravity $= 32.166$, $2g = 64.33$, $\sqrt{2g} = 8.02$ *feet*.

$\boxtimes$ signifies *Dead Flat*, denoting dimensions or greatest amidship section of hull of a vessel.

ALGEBRAIC SYMBOLS AND FORMULAS.

l	*representing*	*length*,	h'	*representing*	*h prime*,	v	*representing*	*versed sine*,
b	"	*breadth*,	c	"	*chord*,	$h_{,}$	"	*h sub*,
d	"	*depth*,	a	"	*area*,	*sin.*	"	*sine*,
h	"	*height*,	r	"	*radius*,	g	"	*gravity*.

$\frac{l+b}{d}$ = sum of length and breadth divided by depth.

$\frac{l\,b}{d}$ = product of length and breadth divided by depth.

$\frac{l-b}{d}$ = difference of length and breadth divided by depth.

$l^2\,b^3$ = product of square of length and cube of breadth.

$\frac{\sqrt{l}}{\sqrt[3]{b}}$ = square root of length divided by cube root of breadth.

$\frac{\sqrt{l+b}}{d}$ = square root of sum of length and breadth divided by depth.

$\sqrt[3]{\frac{h' \backsim h_{,}}{\sqrt{2g}}}$ = cube root of difference of *h prime* and *h sub*, divided by square root of $2g$.

$\sqrt{a+(c-r)^2} = x$. Add square of difference between the chord and radius to the area, and extract the square root; the result will be equal to x.

NOTE.—It is frequently advantageous to begin interpretation of a formula at its right hand, as in the above case.

$l\sqrt{\frac{(x+y)^2}{y^2}-1}=z$. Divide square of sum of x and y by square of y; subtract *unity* from quotient; extract square root of result; multiply it by length, and product will be equal to z.

$\frac{2(\sin. 75^\circ)^2}{1+(\sin. 75^\circ)^2}$. Divide twice square of sine of the angle of 75° by square of sine of the angle of 75° added to *unity*.

$\frac{2a}{(S\sqrt{2g})^2}\left\{S\sqrt{2g}\,(\sqrt{h}-\sqrt{h'})+2.303\ c.\ log.\ \frac{S\sqrt{2gh}-b}{S\sqrt{2gh'}-b}\right\}=t$. Multiply S by the $\sqrt{}$ of $\overline{2g}$, and this product by difference between square roots of h and *h prime;* add this to 2.303 times common logarithm of quotient arising from dividing product of S into $\sqrt{2gh}$ diminished by b by product of S into $\sqrt{2gh}$ *prime* diminished by b, and multiply this sum by the quotient of $2a$ divided by square of product of S into $\sqrt{2g}$, which will be equal to t.

$2a+3\cos. 98^\circ=2a-3\cos. 82^\circ=$ twice a diminished by three times cosine of 82°.

Cosine of any angle greater than 90° and less than 270° is always — or negative, but is *numerically equal* to cosine of its supplement, *i. e.*, remainder after subtracting angle from 180°.

$39.127-.099\,82\cos. 2\,L=l$. Assuming L less than 45°, as 42°, this equation becomes $39.127-.099\,82\cos. (2\times 42^\circ=84^\circ)=l$; and also, L greater than 45°, as 50°, it becomes $39.127+.099\,82\cos. (180^\circ-2\times 50^\circ=80^\circ)=l$.

$L-10^\circ\ N=L+10^\circ\ S$, as a negative result furnished by a formula indicates a positive result in an opposite direction.

$-\frac{(B-b)\ v+2\,BV}{B+b}=y$. Minus, the fraction B minus b, times v, plus 2 times BV, divided by B plus b, is equal to y.

$\text{Sin.}^{-1}x$, $\tan.^{-1}x$, $\cos.^{-1}x$, signifies the arc, the sine, tangent or cosine of which is x. Thus, if $x=.5$, this is 30°, as 30° is the arc, the sine of which is .5.

$(\text{Sin.}\ x)^{-1}=\frac{1}{\sin.\ x}\quad \frac{1}{a^2}=a^{-2}\quad \frac{1}{b^{-2}}=b^2\quad c^{-3}=\frac{1}{c^3}$, and $\frac{l\,r^n-l}{r^n-r^{n-1}}=S$. Raise r to nth power, *i. e.*, multiply r by itself and this result by r, and so on, until r appears in result as a factor, as many times as there are units in n. Multiply this result by l, diminish this by l; divide remainder by r raised to the nth power, diminished by r raised to a power whose exponent is n diminished by 1, and quotient = or is value of S.

$\sqrt[n-1]{\frac{l}{a}}=r$. Divide l by a and extract that root of the quotient, index of which is n diminished by 1, and this root is = or value of r.

Logarithm of a Number is exponent of the power to which a particular constant quantity must be raised in order to produce that number.

Constant Quantity is termed the base of the system.

Common (or *Brigg's*) *Log.* is the logarithm the base of which is 10.

Hyperbolic Log. is the logarithm the base of which is 2.718 28.

Com. Log. = Hyp. log. × .434 294.

Hyp. Log. = Com. log. × 2.302 585 052 994, ordinarily 2.303 or 2.3026.

Illustration.—When a number, hyp. log. of which = a given figure or number, is required.

Multiply figure or number (hyp. log.) by 434 294 (modulus of com. log.) = com. log. of figure.

Thus, Required the number. hyp. log. of which = .02. $.02 \times .434\,294 = .00\,868\,588$, com. log., and $1.0202 =$ *number.*

Log. $100^{.059} = 059 \times$ log. of $100 = .059 \times 2 = .118$; the number corresponding to log. .118, is 1.3122; hence, $100^{.059} = 1.3122$. That is, if 100 is raised to 59th power, and the 1000th root is extracted, the result will be 1.3122.

In Equation, $u = 3x^2 - 2x$, u is termed a function of x. If it is desired to indicate the fact that u thus depends for its value upon the value of x, without expressing exact value of u in terms of x, following notation is used: $u = f(x)$, $u = F(x)$, or $u = \phi(x)$.

Each of these notations is read, u is a function of x. If in such function of x the value of x is assumed to commence with 0 and to increase uniformly, the notation indicating *rate* of increase is dx, and is read "the differential of x."

Differentiation. d is its symbol, and it is the process of ascertaining the ratio existing between the rate of increase or decrease of a function of a variable and the rate of increase or decrease of the variable itself. If $y = 3x^2$, y or its equal $3x^2$ is the function of x, and x is the independent variable, while the exponent of the variable or the primitive exponent is 2.

By the operation of Calculus, such expressions are differentiated by diminishing the exponent of the variable by unity, multiplying by the primitive exponent, and attaching the dx.

Hence, $dy = 2 \times 3x\,dx = 6x\,dx$. This indicates the relation between the differential of y, the function of x, and the differential of x itself.

Assume that x increasing at rate of 3 per second becomes 4; that is, $x = 4$, and $dx = 3$; hence $dy = 6 \times 4 \times 3 = 72$. That is, if x is increasing at rate of 3 per second, at the time that $x = 4$, the function itself is increasing at rate of 72 per second.

To differentiate an expression of two or more terms, it is necessary to differentiate them separately and connect the results with the signs with which the terms are connected.

Thus, differentiating $u = 3x^2 - 2x$, we have $du = d(3x^2 - 2x) = 6x\,dx - 2\,dx = (6x - 2)\,dx$.

Assuming $x = 4$ and $dx = 3$, we have $du = (6 \times 4 - 2) \times 3 = 66$. This indicates that when $x = 4$, and is increasing at rate of 3 per second, the function u, or $3x^2 - 2x$, is at same instant increasing at rate of 66 per second.

Integration. Its symbol $\int$ was originally letter S, initial of *sum*, the symbol of an operation the reverse of differentiation; and when the operation of integration is to be performed twice, thrice, or more times, it is written $\int\int$, $\int\int\int$, etc.

By the operation of Calculus, expressions are integrated by increasing the exponent of the variable by unity, dividing by the new exponent, and detaching the dx.

Hence, integrating the differential $6x\,dx$, we have $\int 6x\,dx = 3x^2$. This result is the function, the differential of which is $6x\,dx$.

To integrate an expression of two or more terms, it is necessary to integrate the terms separately and connect the results with the signs with which the terms are connected.

Thus, integrating $(6x-2)\,dx$, we have $\int (6x-2)\,dx = \int (6x\,dx - 2\,dx) = 3x^2 - 2x$. This result is the function the differential of which is $(6x - 2)\,dx$ or $(6x - 2x^0)\,dx$.

Note.—A quantity with the exponent ^{0}, as x^0 or 3^0, is equal to unity.

The operation of summation may also be illustrated in use of the symbol $\int$. Assuming $x = 4$, the former of the preceding results becomes $\int 6x\,dx = 3x^2 = 48$, the latter $\int (6x - 2)\,dx = 3x^2 - 2x = 40$.

Here x is assumed to commence at 0 and to continue to increase by infinitely small increments of dx until it becomes 4. The summation is the addition of all these values of x from 0 to 4.

Arithmetically.—The first formula may be written

$6(x' + x'' + x''' + \text{etc.})\,dx$. If then x is to advance from 0 to 4 by increments of 1, we have $6(0 + 1 + 2 + 3 + 4) \times 1 = 60$, which exceeds 48. If dx is assumed to be .5, the result is 54. The correct result is obtained only when dx is taken infinitely small. By Arithmetic this is approximated, but it is reached by the operations of Calculus alone.

The second formula may be written

$(6[x' + x'' + x''' + \text{etc.}] - 2[x^{0'} + x^{0''} + x^{0'''}\ \text{etc.}])\,dx$. Assuming $x = 4$, and $dx = 1$, we have $(6[1 + 2 + 3 + 4] - 2[1 + 1 + 1 + 1]) \times 1 = 52$, which exceeds 40. If $dx = .25$, the result would be 43, and if .125 it would be 41.5, ever approaching but never reaching 40, so long as a finite value is assigned to dx.

Δ, *Delta*, when put before a quantity, signifies an absolute and finite increment of that quantity, and not simply the rate of increase.

Σ, *Sigma*, signifies the summation of finite differences or quantities. Thus, $\Sigma y^2\,\Delta x = (y'^2 + y''^2 + y'''^2 + \text{etc.})\,\Delta x$. Assume $y' = 6$, $y'' = 8$, $y''' = 4$, and Δx the common increment of $x = 5$, then $\Sigma y^2\,\Delta x = (36 + 64 + 16) \times 5 = 580$.

NOTATION.

1 = I.	20 = XX.	1 000 = M, or CIↃ.
2 = II.	30 = XXX.	2 000 = MM.
3 = III.	40 = XL.	5 000 = $\overline{\text{V}}$, or IↃↃ.
4 = IV.	50 = L.	6 000 = $\overline{\text{VI}}$.
5 = V.	60 = LX.	10 000 = $\overline{\text{X}}$, or CCIↃↃ.
6 = VI.	70 = LXX.	50 000 = $\overline{\text{L}}$, or IↃↃↃ.
7 = VII.	80 = LXXX.	60 000 = $\overline{\text{LX}}$.
8 = VIII.	90 = XC.	100 000 = $\overline{\text{C}}$, or CCCIↃↃↃ.
9 = IX.	100 = C.	1 000 000 = $\overline{\text{M}}$, or CCCCIↃↃↃↃ.
10 = X.	500 = D, or IↃ.	2 000 000 = $\overline{\text{MM}}$.

As often as a character is repeated, so many times is its value repeated, as CC = 200.

A less character before a greater diminishes its value, as IV = V − I.

A less character after a greater increases its value, as XI = X + I.

For every Ↄ annexed to IↃ the sum as 500 is increased 10 times.

If C is placed on left side of I as many times as Ↄ is on the right, the number is doubled.

A bar, thus ¯, over any number, increases it 1000 times.

Illustration 1.—1880, MDCCCLXXX. 18 560, $\overline{\text{XVIII}}$DLX.

2.—IↃ = 500. CIↃ = 500 × 2 = 1000. IↃↃ = 500 × 10 = 5000. CCIↃↃ = 5000 × 2 = 10 000. IↃↃↃ = 500 × 10 × 10 = 50 000. CCCIↃↃↃ = 50 000 × 2 = 100 000.

CHRONOLOGICAL ERAS AND CYCLES FOR 1884.

The year 1884, *or the* 109*th year of the Independence of the United States of America, corresponds to*

The year 7392–93 of the Byzantine Era;
" 6597 of the Julian Period;
" 5644–45 of the Jewish Era;
" 2660 of the Olympiads, or the last year of the 665th Olympiad, commencing in July (1884), the era of the Olympiads being placed at 775.5 years before Christ, or near the beginning of July of the 3938th year of the Julian Period;
" 2637 since the foundation of Rome, according to Varro;
" 2196 of the Grecian Era, or the Era of the Seleucidæ;
" 1600 of the Era of Diocletian.

The year 1301 of the Mohammedan Era, or the Era of the Hegira, begins on the 7th of February, 1884.

The first day of January of the year 1884 is the 2,409,178th day since the commencement of the Julian Period.

Dominical Letters	F, E	Lunar Cycle or Golden Number	4
Epact	3	Solar Cycle	17

Roman Indiction was a period of 15 years, in use by the Romans. The precise time of its adoption is not known beyond the fact that the year 313 A.D. was a first year of a Cycle of Indiction.

Julian Period is a cycle of 7980 years, product of the Lunar and Solar Cycles and the Indiction (19 × 28 × 15), and it commences at 4714 years B C.

6513 + (given year — 1800) = year of Julian Period, extending to 3267.

NOTE.—If year of Julian Period is divided by 19, 28, 15, or 32, the remainders will respectively give the *Lunar* and *Solar Cycles*, the *Indiction*, and the *Year of the Dionysian.*

MEASURES OF LENGTH.

Standard of measure is a brass scale 82 inches in length, and the yard is measured between the 27th and 63d inches of it, which, at temperature of 62°, is standard yard.

Lineal.

12	inches	=	1 foot.
3	feet	=	1 yard.
5.5	yards	=	1 rod.
40	rods	=	1 furlong.
8	furlongs	=	1 mile.

Inches.	Feet.	Yards.	Rods.	Furl.
36 =	3.			
198 =	16.5 =	5.5.		
7 920 =	660 =	220 =	40.	
63 360 =	5 280 =	1 760 =	320 =	8.

Inch is sometimes divided into 3 *barleycorns*, or 12 *lines.*
A hair's breadth is .020 83 (48th part) of an inch.

1 yard = .000 568, and 1 inch = .000 015 8 of a mile.

Gunter's Chain.

7.92 inches = 1 link. | 100 links = 1 chain, 4 rods, or 22 yards.
80 chains = 1 mile.

Ropes and Cables.

1 fathom = 6 feet. | 1 cable's length = 120 fathoms.

Geographical and Nautical.

1 degree, assuming the Equatorial radius at 6 974 532.34 yards, as given by Bessel, = 69.043 statute miles = 364 556 feet.
1 mile = 2028.81 yards or 6086.44 feet.
1 league = 3 nautical miles.

Log Lines.

Estimating a mile at 6086.43 feet, and using a 30″ glass,

1 knot = 50 feet 8.64 inches. | 1 fathom = 5 feet .864 inches.

If a 28″ glass is used, and 8 divisions, then

1 knot = 47 feet 4 inches. | 1 fathom = 5 feet 11 inches.

The line should be about 150 fathoms long, having 10 fathoms between chip and first knot for stray line.

NOTE.—This estimate of a mile or knot is that of U. S. Coast Survey, assuming equatorial radius of Earth to be 6 974 532.34 yards and a meter to be 39.368 505 35 inches of the Troughton scale at 62°.

Cloth.

1 nail = 2.25 inches. | 1 quarter = 4 nails. | 5 quarters = 1 ell.

Pendulum.

6 points = 1 line. | 12 lines = 1 inch.

Shoemakers'.

No. 1 is 4.125 inches, and every succeeding number is .333 of an inch.

There are 28 numbers or divisions, in two series or numbers—viz., from 1 to 13, and 1 to 15.

Miscellaneous.

12 lines or 72 points = 1 inch. | 1 hand = 4 inches.
1 palm = 3 inches. | 1 span = 9 inches.
1 cubit = 18 inches.

Vernier Scale.

Vernier Scale is $\frac{11}{10}$, divided into 10 equal parts; so that it divides a scale of 10ths into 100ths when two lines of the two scales meet.

Metric, by Act of Congress of July 28, 1866.

Unit of Measurement is the METER, *which by this Act is declared to be* 39.37 *ins.*

Denominations.	Meters.	Inches.	Feet.	Yards.	Miles.
Millimeter	.100	.0394	—	—	—
Centimeter	.10	.3937	—	—	—
Decimeter	.1	3.937	.328 083	—	—
Meter	1.	39.37	3.280 83	1.093 61	—
Dekameter	10.	393.7	32.808 33	10.936 11	—
Hektameter	100.	—	328.083 33	109.361 11	—
Kilometer	1 000.	—	3280.833 33	1093.611 11	.621 37
Myriameter	10 000.	—	—	—	6.2137

In METRIC system, values of the base of each measure—viz., Meter, Liter, Stere, Are, and Gramme—are decreased or increased by following prefix. Thus,

Milli, 1000th part or .001. | Deci, 10th part or .1. | Hekto, 100 times value.
Centi, 100th " .01. | Deka, 10 times value. | Kilo, 1000 "
Myria, 10 000 times value.

NOTE.—The Meter, as adopted by England, France, Belgium, Prussia, and Russia, is that determined by Capt. A. R. Clarke, R.E., F.R.S., 1866, which at 32° in terms of Imperial standard at 62° F. is 39.370 432 *inches* or 1.093 623 11 *yards*, its legal equivalent by Metric Act of 1864 being 39.3708 *inches*, the same as adopted in France.

Captain Kater's comparison, and the one formerly adopted by the U. S. Ordnance Corps, was = 39.370 797 1 *inches*, or 3.280 899 76 *feet*, and the one adopted by the U. S. Coast Survey, as above noted, is = 39.368 505 35 *inches*.

Equivalent Values in Metric Denominations of U. S.

Denominations.	Value in Meters.	Denominations.	Values in Meters.
Inch.	.0254	Rod.	5.029 209 9
Foot.	.304 800 6	Furlong	201.168 396
Yard.	.914 401 8	Mile.	1609.347 168

Approximate Equivalents of Old and Metric U. S. Measures of Length.

1 Kilometer . . . . = .625 *mile.*
1 Mile . . . . = 1.6 *kilometers.*
1 Pole or Perch . = 5 *meters.*

1 Chain . . . . . = 20 *meters.*
1 Furlong . . . = 200 "
5 Furlongs . . . = 1 *kilometer.*

1 Foot . . . . = 3 *decimeters* or 30 *centimeters.*
1 Metre . . . . . . = 3.280 833 *feet* = 3 *feet* 3 *ins.* and 3 *eighths.*
11 Meters . . . . . = 12 *yards.* | 1 Decimeter . . . = 4 *inches.*
1 Millimeter . . = 1 *thirty-second of an inch.*

To Convert Meters into Inches.—Multiply by 40: and *to Convert Inches into Meters.*—Divide by 40.

Approximate rule for Converting Meters or parts, into Yards.—Add one eleventh or .0909.

Inches Decimally = Millimeters.

Inches.	Milli-meters.	Inches.	Milli-meters.	Inches.	Milli-meters.	Inches.	Milli-meters.	Inches.	Milli-meters.
.01	.25	.2	5.08	.48	12.2	.76	19.3	2	50.8
.02	.51	.22	5.59	.5	12.7	.78	19.8	3	76.2
.03	.76	.24	6.1	.52	13.2	.8	20.3	4	101.6
.04	1.02	.26	6.6	.54	13.7	.82	20.8	5	127
.05	1.27	.28	7.11	.56	14.2	.84	21.3	6	152.4
.06	1.52	.3	7.62	.58	14.7	.86	21.8	7	177.8
.07	1.78	.32	8.13	.6	15.2	.88	22.4	8	203.2
.08	2.03	.34	8.64	.62	15.7	.9	22.9	9	228.6
.09	2.29	.36	9.14	.64	16.3	.92	23.4	10	254
.1	2.54	.38	9.65	.66	16.8	.94	23.9	11	279.4
.12	3.05	.4	10.2	.68	17.3	.96	24.4	12	304.8
.14	3.56	.42	10.7	.7	17.8	.98	24.9	= 1 foot.	
.16	4.06	.44	11.2	.72	18.3	1.	25.4		
.18	4.57	.46	11.7	.74	18.8				

Inches in Fractions = Millimeters.

Eighths.	Six-teenths.	Thirty-seconds.	Milli-meters.	Eighths.	Six-teenths.	Thirty-seconds.	Milli-meters.	Eighths.	Six-teenths.	Thirty-seconds.	Milli-meters.	Eighths.	Six-teenths.	Thirty-seconds.	Milli-meters.
		1	.79			9	7.14			17	13.5			25	19.8
	1	—	1.59		5	—	7.94		9	—	14.3		13	—	20.6
		3	2.38			11	8.73			19	15.1			27	21.4
1	—	—	3.17	3	—	—	9.52	5	—	—	15.9	7	—	—	22.2
		5	3.97			13	10.32			21	16.7			29	23
	3	—	4.76		7	—	11.11		11	—	17.5		15	—	23.8
		7	5.56			15	11.91			23	18.3			31	24.6
2	—	—	6.35	4	—	—	12.7	6	—	—	19	8	—	—	25.4

By means of preceding tables equivalent values of inches and millimeters, equivalent values of inches in centimeters, decimeters, and meters, may be ascertained by altering position of decimal point.

Illustration.—Take 1 millimeter, and remove decimal point successively by one figure to the right; the values of a centimeter, decimeter, and meter become

	In.		Ins.		
1 millimeter. . . .	.0394	1 decimeter	3.94	.32 inch	= 8.13 millimeters.
1 centimeter. . . .	.394	1 meter.	39.4	3.2 inches	= 81.3 "

MEASURES OF SURFACE.

144 square inches = 1 square foot. | 9 square feet = 1 square yard.
Architect's Measure, 100 square feet = 1 square.

Land.

			Yards.	Rods.	Roods.
30.25	square yards	= 1 square rod.	1210.		
40	square rods	= 1 square rood.			
4	square roods	= 1 acre.	4840 =	160.	
10	square chains	= 1 acre.			
640	acres	= 1 square mile.	3 097 600 =	102 400 =	2560.

208.710 326 feet, 69.570 109 yards square, or 220 by 198 feet square = 1 Acre.

Paper.

24 sheets = 1 quire. | 20 quires = 1 ream. | 21.5 quires = 1 printer's ream.
2 reams = 1 bundle. | 5 bundles = 1 bale.

Drawing.

Cap	13 × 16 inches.	Columbier	23 × 34 inches.
Demy	15 × 20 "	Atlas	26 × 34 "
Medium	17 × 22 "	Theorem	28 × 34 "
Royal	19 × 23 "	Doub. Elephant	27 × 40 "
Super-royal	19 × 27 "	Antiquarian	31 × 53 "
Imperial	22 × 30 "	Emperor	40 × 60 "
Elephant	23 × 28 "	Uncle Sam	48 × 120 "

Peerless . . . . 18 × 52 inches.

Tracing.

Double Crown	20 × 30 inches.	Grand Royal	18 × 24 inches.
Double D. Crown	30 × 40 "	Grand Aigle	27 × 40 "
Double D. D. Crown	40 × 60 "	Vellum Writing	18 to 28 ins. in width.

Mounted on cloth, 38 ins. in width.

Miscellaneous.

1 sheet	= 4 pages.	1 duodecimo	= 24 pages.
1 quarto	= 8 "	1 eighteenmo	= 36 "
1 octavo	= 16 "	1 bundle	= 2 reams.

1 piece wall-paper, 20 ins. by 12 yards.
1 " " " French, 4.5 sq. yards.
Roll of Parchment = 60 sheets.

Copying.

100 Words = 1 Folio.

Metric, by Act of Congress of July 28, 1866.

Unit of Surface is Are or Square Dekameter.

A square meter $(39.37^2) = 1549.9969$ sq. ins., but by this Act is declared to be 1550 *sq. ins.*

Denominations.	Sq. Meters.	Sq. Inches.	Sq. Feet.	Sq. Yards.	Acres.
Centimeter	.0001	.155	—	—	—
Decimeter	.01	15.50	.107 638	—	—
Centare or Square Meter	1.	1550.	10.763 888	1.196	—
Are	100.	—	1076.388 88	119.6	.024 71
Hectare	10 000.	—	—	11 960.	2.471

Equivalent Values in Metric Denominations of U. S.

Denominations.	Sq. Meters.	Denominations.	Sq. Meters.	Sq. Hectares.	Sq. Ares.
Sq. Inch.....	.000 645 16	Sq. Chain ...	404.686 47	—	4.046 865
" Foot.....	.092 903 23	" Rood....	1011.716 175	—	10.117 162
" Yard.....	.836 129 07	" Acre	4046.864 699	.404 686	40.468 647
" Rod......	25.292 904	" Mile.....	—	258.999 34	25 899.934 074

Approximate Equivalents of Old and Metric U. S. Square Measures.

6.5 square centimeters = 1 *sq. inch.*	1 acre = 1.16 per cent. over 4000 *sq. meters.*
1 " meter = 10.75 *sq. feet.*	1 square mile = 259 *hectares.*

MEASURES OF VOLUME.

Standard gallon measures 231 cube ins., and contains 8.338 882 2 avoirdupois pounds, or 58 373 Troy grains of distilled water, at temperature of its maximum density (39.1°), barometer at 30 ins.

Standard bushel is the *Winchester*, which contains 2150.42 cube ins., or 77.627 413 lbs. avoirdupois of distilled water at its maximum density.

Its dimensions are 18.5 ins. diameter inside, 19.5 ins. outside, and 8 ins. deep; and when heaped, the cone must not be less than 6 ins. high, equal 2747.715 cube ins. for a true cone.

A struck bushel contains 1.244 45 cube feet.

Liquid.

	Cube Ins.	Gills.	Pints.
4 gills = 1 pint.	28.875	8.	
2 pints = 1 quart.	57.75		
4 quarts = 1 gallon.	231.	32 =	8.

Dry.

	Cube Ins.	Pints.	Quarts.	Galls.
2 pints = 1 quart.	67.2006			
4 quarts = 1 gallon.	268.8025	8.		
2 gallons = 1 peck.	537.605	16 =	8.	
4 pecks = 1 bushel.	2150.42	64 =	32 =	8.

Cube.

	Inches.
1728 cube inches = 1 foot.	
27 cube feet = 1 yard.	46 656

Note.—A cube foot contains 2200 cylindrical inches, or 3300 spherical inches.

Fluid.

	Minims.	Drams.	Ounces.
60 minims = 1 dram.			
8 drams = 1 ounce.	480.		
16 ounces = 1 pint.	7 680 =	128.	
8 pints = 1 gallon.	61 240 =	1024 =	128.

Nautical.

1 ton displacement in salt water .... = 35 cube feet.
1 " registered internal capacity .... = 40 " "

Dimensions of a Barrel.

Diameter of head, 17 ins.; bung, 19 ins.; length, 28 ins.; volume, 7689 cube ins. = 3.5756 bushels.

Miscellaneous.

1 cube foot	7.480 5 gallons.
1 bushel	9.309 18 gallons.
1 chaldron = 36 bushels, or	57.244 cube feet.
1 cord of wood	128 cube feet.
1 perch of stone	24.75 cube feet.

1 quarter = 8 bushels. | 1 load hay or straw = 36 trusses.

		Galls.
1 Barrel		32
1 Tierce		42
Butt of Sherry	35×50	108
Pipe of Port	34×58	115
Pipe of Teneriffe		100
Butt of Malaga	33×53	105
Puncheon of Scotch Whisky		110 to 130
Puncheon of Brandy	34×52	110 to 120
Puncheon of Rum		100 to 110
Hogshead of Brandy	28×40	55 to 60
Pipe of Madeira		92
Hogshead of Claret		46

A Hogshead is one half, a Quarter cask is one fourth, and an Octave is one eighth of a Pipe, Butt, or Puncheon.

Metric, by Act of Congress of July 28, 1866.

Unit or Base of Measurement is a cube Decimeter or Liter, which is declared to be 61.022 *cube ins.*

Cube Measures.

Denominations.	Values.	Cube Inches.	Cube Feet.	Cube Yards.
Cube Centimeter	.001 cube milliliter	.061 022	—	—
" Decimeter	1 cube liter	61.022	.035 313 657	—
" Meter	Kiloliter or stere	—	35.313 657	1.308

Dry Measures.

Denominations.	Values.	Cube Ins.	Quarts.	Pecks.	Bushels.	Cube Yards.
Milliliter	1 cube centimeter.	.061	—	—	—	—
Centiliter	10 " "	.6102	—	—	—	—
Deciliter	.1 " decimeter	6.1022	—	—	—	—
Liter	1 " "	61.022	.908*	.1135	—	—
Dekaliter	10 " "	—	9.08	1.135	.283 75	—
Hektoliter	.1 " meter	—	—	11.35	2.837 5†	.1308
Kiloliter or Stere	1 " "	—	—	—	28.375	1.308

* Or .227 *gallon.* † 3.531 365 7 *cube feet.*

Note.—In practice, term cube Centimeter, abbreviated to cc, is used instead of Milliliter, and cube Meter instead of Kilometer.

Equivalent Values in Metric Denominations of U. S.

Dry Measures.

Denominations.	Centiliters.	Deciliters.	Liters.	Dekaliters.
Inch	—	—	—	—
Pint	—	—	—	—
Quart	—	.110 125	1.10125	11.0125
Gallon	—	.440 5	4.405	44.05
Peck	.0881	.881	8.81	88.1
Bushel	.3524	3.524	35.24	352.4

Liquid Measures.

Denominations.	Liters.	Drams.	Ounces.	Pints.	Quarts.	Gallons.
Milliliter	.001	.27	—	—	—	—
Centiliter	.01	2.7	.338	—	—	—
Deciliter	.1	27	3.38	.211 34	—	—
Liter	1	—	33.8	2.113 4	1.0567	.264 17
Dekaliter	10	—	—	21.134	10.567	2.641 7
Hektoliter	100	—	—	—	—	26.417
Kiloliter or Stere	1000	—	—	—	—	264.17

Approximate Equivalents of Old and Metric U. S. Measures of Volume.

1 Gallon .... = 4.5 *liters.*	1 cube meter .... = 1.33 *cube yards*
1 Liter .... = .26 *gallon.*	1 " yard .... = .75 " *meter.*
1 cube foot ...... = 28.3 *liters.*	1 " kiloliter = 2240 *lbs. nearly of water.*

MEASURES OF WEIGHT.

Standard avoirdupois pound is weight of 27.7015 cube inches of distilled water weighed in air, at (39.83°) barometer at 30 inches.

A cube inch of such water weighs 252.6937 grains.

Avoirdupois.

16 drams = 1 ounce.
16 ounces = 1 pound.
112 pounds = 1 cwt.
20 cwt. = 1 ton.

Drams.	Ounces.	Pounds.
256.		
28 672 =	1 792.	
573 440 =	35 840 =	2240.

1 pound = 14 *oz.* 11 *dwts.* 16 *grs.* Troy, or 7000 *grains.*
1 ounce = 18 *dwts.* 5.5 *grains* Troy, or 437.5 *grains.*
1 dram = 1 *dwt.* 3.343 75 *grains* Troy, or 53.5 *grains.*
1 stone = 14 *pounds.*

Troy.

24 grains = 1 dwt.
20 dwt. = 1 ounce.
12 ounces = 1 pound.

Grains.	Dwt.
480.	
5760 =	240.

7000 Troy grains = 1 lb. avoirdupois.
437.5 " " = 1 oz. "
27.343 75 Troy grains = 1 dram "
175 Troy pounds = 144 lbs. "
175 " ounces = 192 oz. "
1 " ounce = 480 grs. "
1 " pound = .822 857 lb.
1 avoirdupois pound = 1.215 278 *lbs. Troy.*

Apothecaries.

20 grains = 1 scruple.
3 scruples = 1 dram.
8 drams = 1 ounce.
12 ounces = 1 pound.

Grains.	Scruples.	Drams.
60.		
480 =	24.	
5760 =	288 =	96.

45 drops = 1 teaspoonful or a fluid dram.
2 tablespoonfuls = 1 ounce.

The pound, ounce, and grain are the same as in Troy weight.

Diamond.

1 grain = 16 parts.
16 parts = .8 grain.

4 grains = 3.2 grains.
1 carat = 4 grains.

15.5 carats = 1 Troy ounce.

Lead.

A Fodder of lead = 8 pigs.
Sheet lead rolls = 6.5 to 7.5 feet in width and from 30 to 35 feet in length.

Grain.

Standard Weights per Bushel.

	Lbs.
Wheat	60
Corn	56 and 58
Rye	56
Oats	32
Barley	48

Miscellaneous.

COAL.

Anthracite	....	1 cube foot = 1.75 broken.
"	....	50 to 55 lbs. per cube foot.
"	....	41 to 45 cube feet = 1 ton broken.
Bituminous	....	70 to 78 lbs. per heaped bushel.
"	....	40 to 50 lbs. per cube foot.
"	Cumberland	53 " " " "
"	Cannel	50.3 lbs. per cube foot.
"	Welsh	43 cube feet = 1 ton.
"	Lancashire	44 " " = 1 "
"	Newcastle	45 " " = 1 "
"	Scotch	43 " " = 1 "
"	R. N. allowance	48 " " = 1 "
Charcoal, hardwood	....	18.5 lbs. per cube foot.
" pine wood	....	18 " " " "

WOOD.

Virginia pine .. 2700 lbs. = 1 cord. | Southern pine . 3300 lbs. = 1 cord.

EARTH.

River sand .. 21 cube feet = 1 ton.
Coarse gravel, 23 " " = 1 "
Marl or Clay, 28 cube feet = 1 ton.
Mold .... 33 " " = 1 "

Metric, by Act of Congress of July 28, 1866.

Unit of Weight is the GRAM, which is weight of one cube centimeter of pure water weighed in vacuo at temperature of 4° C., or 39.2° F., which is about its temperature of maximum density = 15.432 *grains*.

Denominations.	Values.	Grains.	Ounces.	Lbs.	Ton.
Milligram	1 cube millimeter	—	—	—	—
Centigram	10 " "	.154 32	—	—	—
Decigram	.1 " centimeter	1.543 2	—	—	—
Gram	1 " "	15.432	.035 27	—	—
Dekagram	10 " "	—	.352 7	—	—
Hektogram	1 deciliter	—	3.527	.220 46	—
Kilogram or Kilo ..	1 liter	—	35.27	2.204 6	—
Myriagram	10 "	—	—	22.046	—
Quintal	1 hektoliter	—	—	220.46	.098 419
Millier or Tonneau.	1 cube meter	—	—	2204.6	.984 196

Kilogram = 2.679 17 *lbs. Troy*, or 2 *lbs.* 8 *oz.* 3 *dwts.* .3072 *grain.*

Equivalent Values in Metric Denominations of U. S.

Denominations.	Grams.	Dekagrams.	Denominations.	Grams.	Kilograms.
Grain	.064 8	—	Ounce	28.3502	.028 35
Scruple	1.296	—	" Troy	31.1042	.031 1
Pennyweight	1.555 2	—	Pound	453.6028	.453 6
Drachm	1.771 87	17.7187	" Troy	373.2504	.373 25
" (Apoth.)	3.888	38.88	Ton	—	1016.057 28

Approximate Equivalents of Old and New U. S. Measures of Weight.

The ton and the gram are at nearly equal distances above and below the kilogram. Thus,

1 ton .... = 1 016 057.28 grams. | 1 kilogram ..... = 1000 grams.

1 gram is nearly 15.5 grains (about .5 per cent. less).
1 kilogram about 2.2 pounds avoirdupois (about .25 per cent. more).
1000 kilograms, or a metric ton, nearly 1 Engl. ton (about 1.5 per cent. less).

Electrical. (*British Association.*)

Resistance.—Unit of resistance is termed an *Ohm*, which represents resistance of a column of mercury of 1 sq. millimeter in section and 1.0486 meters in length, at temp. 0° C. Equivalent to resistance of a wire 4 millimeters in diameter and 100 meters in length.

1 000 000 Microhms........ = 1 Ohm.
1 " .... = 10 absolute electro-magnetic units.
1 Ohm............. = 10 000 000 " " " "
1 000 000 Ohms............ = 1 Megohm or 10^{13} " " " "

Electro-motive Force.—Unit of tension or difference of potentials is termed a *Volt*.

1 000 000 Microvolts..... = 1 Volt or .1 of an absolute electro-magnetic unit.
1 Volt........... = 100 000 " " " " "
1 Megavolt...... = 1 000 000 Volts.

Current.—Unit of current is equal to 1 *Weber* per second, or the current in a circuit has an electro-motive force of one *Volt* and a resistance of an *Ohm*.

Volume.—Unit of volume is termed an *Ampere*, and represents that volume of electricity which flows through a circuit having an electro motive force of one *Volt* and a resistance of one *Ohm* in a second, or it represents a Volt diminished by an Ohm.

1 000 000 Microvolts or 100 absolute units of volume...... = 1 Ampere.
1 000 000 Amperes.................................. = 1 Megaweber

Capacity.—Unit of capacity is termed a *Farad*.

1 000 000 Microfarads or 10 000 000 absolute units of capacity...... = 1 Farad.
1 000 000 Farads.. = 1 Megafarad.

Heat.—Unit of heat is quantity required to raise one gram of water to 1° C. of temperature.

Weights of Grain and Roots.

Following weights have been fixed by statute in many of the States; and these weights govern in buying and selling, unless a specific agreement to the contrary has been made.

Pounds in a Bushel.

ARTICLES.	California.	Connecticut.	Delaware.	Illinois.	Indiana.	Iowa.	Kentucky.	Louisiana.	Maine.	Massachusetts.	Michigan.	Minnesota.	Missouri.	N. Hampshire.	New Jersey.	New York.	Ohio.	Oregon.	Pennsylvania.	Rhode Island.	Vermont.	Washington T.	Wisconsin.
Barley	50	—	—	48	48	48	48	32	—	46	48	48	48	—	48	48	48	46	47	—	46	45	48
Beans	—	—	—	60	60	60	60	—	—	—	—	—	60	—	—	62	—	—	—	—	—	—	—
Blue Grass Seed	—	—	—	14	14	14	14	—	—	—	—	—	14	—	—	—	—	—	—	—	—	—	—
Buckwheat	40	45	—	40	50	52	52	—	—	46	42	42	52	—	50	48	—	42	48	—	46	42	42
Castor Beans	—	—	—	46	46	46	—	—	—	—	—	—	46	—	—	—	—	—	—	—	—	—	—
Clover Seed	—	—	—	60	60	60	60	—	—	—	60	60	60	—	64	60	60	60	—	—	—	60	60
Dried Apples	—	—	—	24	25	24	—	—	—	—	28	28	24	—	—	—	—	28	—	—	—	28	28
Dried Peaches	—	—	—	33	33	33	—	—	—	—	28	28	33	—	—	—	—	28	—	—	—	28	28
Flaxseed	—	—	—	56	56	56	56	—	—	—	—	—	56	—	55	55	56	—	—	—	—	—	56
Hemp Seed	—	—	—	44	44	44	44	—	—	—	—	—	44	—	—	—	—	—	—	—	—	—	—
Corn	52	56	56	52	56	56	56	56	—	56	56	56	52	—	56	58	56	56	56	—	56	56	56
Corn in ear	—	—	—	70	68	68	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Corn Meal	—	—	—	48	50	—	50	—	50	50	—	—	—	—	—	—	—	—	—	50	—	—	—
Coal	—	—	—	80	70	—	—	—	—	—	—	—	80	—	—	—	—	—	—	—	—	—	—
Oats	32	28	—	32	32	35	33⅓	32	30	30	32	32	35	30	30	32	32	34	32	—	32	36	32
Onions	—	—	—	57	48	57	57	—	—	52	—	—	57	—	—	—	—	—	—	50	—	50	—
Pease	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	60	—	—	—	—	—	—	—
Potatoes	—	60	—	60	60	60	60	—	60	—	—	—	60	60	60	60	—	60	—	60	60	60	60
Rye	54	56	—	54	56	56	56	60	—	56	56	56	56	—	56	56	56	56	56	—	56	56	56
Rye Meal	—	—	—	—	—	—	—	—	50	50	—	—	—	—	—	—	—	—	—	50	—	—	—
Salt	—	—	—	—	50	50	50	—	—	—	—	—	50	—	—	56	—	—	—	—	—	—	—
Timothy Seed	—	—	—	45	45	45	45	—	—	—	—	—	45	—	—	44	—	—	—	—	—	—	46
Wheat	60	56	60	60	60	60	60	60	—	60	60	60	60	—	60	60	60	60	60	—	60	60	60
Wheat Bran	—	—	—	20	—	20	20	—	—	—	—	—	20	—	—	—	—	—	—	—	—	—	—

Weight of Men and Women.

Average weight of 20000 men and women, weighed in Boston, 1864, was —men, 141.5 lbs.; women, 124.5 lbs. Average of men, women, and children, 105.5 lbs.

Weight of Horses.—(U. S.)

Weight of horses ranges from 800 to 1200 lbs.

WEIGHT OF CATTLE.

To Compute Dressed Weight of Cattle.

RULE.—*Measure as follows in feet:*

1. Girth close behind shoulders, that is, over crop and under plate, immediately behind elbow.

2. Length from point between neck and body, or vertically above junction of cervical and dorsal processes of spine, along back to bone at tail, and in a vertical line with rump.

Then multiply square of girth in feet by length, and multiply product by factors in following table, and quotient will give dressed weight of quarters.

Condition.	Heifer, Steer, or Bullock.	Bull.	Condition.	Heifer, Steer, or Bullock.	Bull.
Half fat	3.15	3.36	Very prime fat	3.64	3.85
Moderate fat	3.36	3.5	Extra fat	3.78	4.06
Prime fat	3.5	3.64			

ILLUSTRATION.—Girth of a prime fat bullock is 7 feet 2 ins., and length measured as above 4 feet 5 ins.

$7'\ 2'' = 7.17$, and $7.17^2 = 51.4$, which $\times\ 4'\ 5''$ and by $3.5 = 794.5$ *lbs.* Exact weight was 799 *lbs.*

NOTE.—1. Quarters of a beef exceed by a little, half weight of living animal.
2. Hide weighs about eighteenth part, and tallow twelfth part of animal.

Comparative Weights of Live Beeves and of Beef.

	Lbs.	Per cent.		Lbs.	Per cent.
Bullocks	2800	72 to 78	Bullocks	1550	61 to 64
Heifers	2600	70 to 76	Heifers	1550	
Bullocks	2600		Bullocks	1260	58 to 61
Heifers	2400	66 to 70	Heifers	1200	
Bullocks	2400		Bullocks	1050	57 to 58
Heifers	2100	64 to 68	Heifers	1050	
Bullocks	2100		Bullocks	980	50 to 56
Heifers	1800	63 to 66	Heifers	950	

Weight of Offal in a Beef and Sheep.

	BEEF. Lbs.	SHEEP. Lbs.		BEEF. Lbs.	SHEEP. Lbs.
Hide and Hair	56 to 98	8 to 16*	Kidneys, Heart, Liver, etc.	31 to 62	6 to 10
Tallow	42 " 140	5 " 14	Stomach, Entrails, etc.,	126 " 196	9 " 18
Head and Tongue	28 " 49	6 " 11†	Blood	42 " 56	4 " 6
Feet	21 " 35	2 " 3			

* Including 2 to 6 lbs. for fleece.

† Including 2 to 5 lbs. for horns.

To Compute Equivalents of Old and New U. S. and of Metric Denominations.

By Act of Congress, July 28, 1866.

RULE.—Divide fourth term by second, multiply quotient by first term, and divide product by third term.

Or, Ascertain relative ratio of first and second terms, and multiply result by ratio of third and fourth terms.

NOTE.—When result is required in French or other Metric denominations than those of U. S., use exact denominations, as, 61.025 387 for 61.022, 39.370 432 for 39.37, etc.

EXAMPLE 1.—If one gallon (1st), per sq. foot, yard, acre, etc. (2d); how many liters, (3d), per sq. foot, yard, acre, etc. (4th)?

$\frac{1}{1} \times 231 \div 61.022 \ldots\ldots = 3.7851$ *liters* or 3.7848 *litres.*

Or, $\frac{231}{144} = 1.604$, and $\frac{144}{61.022} = 2.3598$; hence, $1.604 \times 2.3598 = 3.7851$ *liters.*

NOTE.—In computing ratios, first term is to be divided by second, and fourth by third.

EXAMPLE 2.—If one ton per cube foot, how many kilograms per cube decimeter?

$\frac{61.022}{1728} \times 2240 \div 2.2046 = 35.881$ *liters*, or 35.882 *litres.*

MEASURES.

	By Act of Congress of U. S.		By Metric Computation.
1 Liter per sq. foot, etc. =	.2642 *Gallon per sq. foot,*	or	.264 2 *gallon.*
1 Liter per sq. meter . =	.0245 *Gallon per sq. foot.*	or	.024 5 *gallon.*
1 Gallon per sq. foot . =	40.746 *Liters per sq. meter,*	or	40.745 4 *litres.*
1 Sq. foot per acre . . . =	.2296 *Sq. meters per hectare,*	or	2.296 09 *metres.*

WEIGHTS AND PRESSURES.

Per sq. inch.	By Act of Congress of U. S. Per sq. inch.		By Metric Computation.
1 Centimeter . . . . . =	.1929 *Lb.*	or	.192 92 *lb.*
1 Atmosphere . . . . =	6.6679 *Kilograms,*	or	6.667 8 *kilogrammes.*
1 Inch mercury . . =	2.54 *Centimeters,*	or	2.54 *centimetres.*
1 Pound . . . . =	453.6029 *Grams,*	or	453.592 6 *grammes.*
1 Kilogram . . . . . . =	317.4624 *Lbs. per sq. foot,*	or	317.465 *lbs.*

NOTE.—30 ins. of mercury at 62° = 14.7 *lbs. per sq. inch*; hence, 1 *lb.* = 2.0408 *ins.*, and a centimeter of mercury = 30 ÷ .3937 for U. S. computation, and 30 ÷ .393 704 32 for French or Metric.

POWER AND WORK.

1 Horse-power = *Cheval* or *Cheval-vapeur* = $\overline{4500\ k \times m} = 33\,000 \div (4500 \times 2.2046 \times 39.37 \div 12) = 1.013\,88$ *chevaux.*

1 Cheval or Cheval-vapeur ($75\ k \times m$ per second) = *horse-power.*

$(4500 \times 2.2046 \times 39.37 \div 12) \div 33\,000 = .9863$ *horse-power.*

By Act of Congress of U. S.	By Metric Computation.
1 Foot-pound = Kilogrammeter $k \times m$ = 7.233 *foot-lbs.*; hence,	
1 ÷ (2.2046 × 3.280 833) = .138 26 *Kilogrammeter*,	or .138 25 *kilogrammetre.*
1 Cube foot per IP . . . . = .0279 *Cube meter per cheval,*	or .0279 *cheval.*
1 Pound " " . . = .447 38 *Kilogram per cheval.*	or .447 38 *kilogramme.*
1 Cube meter per cheval = 35.8038 *Cube feet per* IP,	or 35.8058 IP.

TEMPERATURES.

1 Caloric or French unit = 3.968 *Heat-units*, and 1 heat-unit = 1 ÷ 3.968 = .252 *caloric.*

1 U. S. Mechanical equivalent (772 foot-lbs.) = 772 ÷ 7.233 = 106.733 *Kilogrammeters* and 106.733 *kilogrammetres.*

1 French Mechanical equivalent (423.55 $k \times m$) = 3.280 833 × 2.2046 × 423.55 = 3063.505 *foot-lbs.*, or 3063.566 *foot-lbs. Metric.*

1 Heat-unit per pound = .5556 *Kilogram*, or .5556 *kilogramme.*

1 Heat-unit per sq. foot = .2715 *Caloric per sq. meter*, or .2713 *per sq. metre.*

VELOCITIES.

1 Foot per second, minute, etc. = .3047 *Meter per second*, or .3047 *metres.*

1 Mile per hour .... = .447 " " " or .447 "

MEASURES OF TIME.

60 thirds = 1 second.	60 minutes = 1 degree.
60 seconds = 1 minute.	30 degrees = 1 sign.

360 degrees = 1 circle.

True or *apparent time* is that deduced from observations of the Sun, and is same as that shown by a properly adjusted sun-dial.

Mean Solar time is deduced from time in which the Earth revolves on its axis, as compared with the Sun; assumed to move at a mean rate in its orbit, and to make 365.242 218 revolutions in a mean Solar or Gregorian year.

Sidereal time is period which elapses between time of a fixed star being in meridian of a place and time of its return to that place.

Standard unit of time is the *sidereal day.*

Sidereal day = 23 *h.* 56 *m.* 4.092 *sec.* in *solar* or *mean time.*

Sidereal year, or revolution of the earth, 365 *d.* 5 *h.* 48 *m.* 47.6 *sec.* in *solar* or *mean time* = 365.242 24 *solar days.*

Solar day, mean = 24 *h.* 3 *m.* 56.555 *sec.* in *sidereal time.*

Solar year (Equinoctial, Calendar, Civil or Tropical) = 365.242 218 *solar days*, or 365 *d.* 5 *h.* 48 *m.* 47.6 *sec.*

Civil day commences at midnight. *Astronomical day* commences at noon of the civil day, having same designation, that is, 12 hours later than the civil day.

Marine or *sea day* commences 12 hours before civil time or 1 day before astronomical time.

New Style was introduced in England in 1752.

NOTE.—In Russia days are reckoned by Old Style, and are consequently 12 days behind Gregorian record.

MEASURES OF VALUE.

10 mills = 1 cent.	10 dimes = 1 dollar.
10 cents = 1 dime.	10 dollars = 1 eagle.

Standard of gold and silver is 900 parts of pure metal and 100 of alloy in 1000 parts of coin.

Fineness expresses quantity of pure metal in 1000 parts.

Remedy of the Mint is allowance for deviation from exact standard fineness and weight of coins.

Nickel cent (old) contained 88 parts of copper and 12 of nickel.
Bronze cent contains 95 parts of copper and 5 of tin and zinc.
Pure Gold 23.22 grains = $1 00. Hence value of an ounce is $20.67.183+.
Standard Gold, $18.60.465+ per ounce.

WEIGHT, FINENESS, ETC., OF U. S. COINS.

Gold.

Denomination.	Weight of Coin.		Weight of Pure Metal.	Denomination.	Weight of Coin.		Weight of Pure Metal.
	Oz.	Grs.	Grs.		Oz.	Grs.	Grs.
Dollar	.053 75	25.8	23.22	Half Eagle	.268 75	129	116.1
Quarter Eagle	.134 375	64.5	58.05	Eagle	.537 5	258	232.2
Three Dollar	.161 25	77.4	69.66	Double Eagle	1.075	516	464.4

Silver.

Denomination.	Oz.	Grs.	Grs.	Denomination.	Oz.	Grs.	Grs.
Dime	.080 375	38.58	34.722	Half Dollar	.401 875	192.9	173.61
20 Cent	.160 75	77.16	69.444	Trade Dollar	.875	420	378
Quarter Dollar	.200 937 5	96.45	86.805	Silver Dollar	.859 375	412.5	371.25

Copper and Nickel.

	Weight.	Copper.	Tin and Zinc.		Weight.	Copper.	Tin and Zinc.
	Grains.	Per cent.	Per cent.		Grains.	Per cent.	Per cent.
One Cent	48	95	5	Three Cents	30	75	25
Two Cents	96	95	5	Five Cents	77.16	75	25

Tolerance.—*Gold*, Dollar to Half Eagle, .25 grains. Eagles, .5 grains. —*Silver*, 1.5 grains for all denominations. — *Copper*, 1 to 3 cents, 2 grains; 5 cents, 3 grains.

Legal Tenders.—*Gold*, unlimited. — *Silver*. Dollars of 412.5 grains unlimited; for subdivisions of dollar, $10. (Trade dollars [420 grains] are not legal tender.)—*Copper* or *cents*, 25 cents.

Note.—Weight of dollar up to 1837 was 416 grains, thence to 1873, 412.5. Weight of $1000, @ 412.5 gr. = 859.375 *oz.*

British standards are: *Gold*, $\frac{22}{24}$ of a pound,* equal to 11 parts pure gold and 1 of alloy; *Silver*, $\frac{222}{240}$ of a pound, or 37 parts pure silver and 3 of alloy = .925 fine.

A Troy ounce of standard gold is coined into £3 17*s.* 10*d.* 2*f.*, and an ounce of standard silver into 5*s.* 6*d.* 1 lb. silver is coined into 66 shillings.

Copper is coined in proportion of 2 shillings to pound avoirdupois.

£ Sterling (1880) $4 86.65; hence $\frac{1}{240}$ of this = value of 1 penny = 2.027 708 33 cents.

---

* A pound is assumed to be divided into 24 equal parts or carats, hence the proportion is equal to 22 carats.

To Compute Value of Coins.

RULE.—Divide product of weight in grains and fineness, by 480 (grains in an ounce), and multiply result by value of pure metal per ounce.

Or, Multiply weight in ounces by fineness and by value of pure metal per ounce.

EXAMPLE 1.—When fine gold is \$20.67.183+ per oz., what is value of a British sovereign?

By following tables, p. 40, Sovereign weighs .2567 oz., and $.2567 \times 480 = 123.216$ grains, and has a fineness of .9165.

$$\text{Hence, } \frac{123.216 \times .916\,5}{480} \times 20.67.183+ = \$4.86.34.$$

EXAMPLE 2.—When fine silver is \$1.15.5 per oz., what is value of U. S. Trade dollar?

By table, p. 40, Dollar weighs .875 oz. and has a fineness of .900.

$$\text{Hence, } .875 \times .900 \times 1.15.5 = 90.956\,25 \textit{ cents.}$$

EXAMPLE 3.—A 4-Florin (Austrian) weighs 49.92 grains and has a fineness of .900. What is its value?

$$\frac{49.92 \times .900}{480} \times 20.67.183+ = \$1.93.49.$$

To Convert U. S. to British Currency and Contrariwise.

RULE 1.—Divide Cents by 2.027 71 − (2.027 708 33), or, Multiply by .493 12 − (.493 118 26), and result is Pence.

2\. Multiply Pence by 2.027 71 −, or divide by .49312 −, and result is Cents.

EXAMPLE.—What are 100 cents in pence?

$$100 \times .493\,12- = 49.312- \textit{ pence} = 4s.\ 1.312d.$$

2\. What is a Pound sterling in cents?

$$20 \times 12 = 240 \textit{ pence}, \text{ which} \times 2.027\,71- = \$4\ 86.65.$$

FOREIGN MEASURES OF VALUE.

Weight, Fineness, and Mint Values of Foreign Silver and Gold Coins.

By Laws of Congress, Regulations of the Mint, and Reports of its Directors.

Current Value of silver coins is necessarily omitted, as the value of silver is a variable element. Hence, in order to compute current value of a silver coin, the price of fine or a given standard of silver being known,

Proceed as per above rule to compute value of coins.

The price of silver should be taken as that of the London market for British standard (925 fine), it being recognized as the standard value, and governing rates in all countries.

EXAMPLE.—If it is required to determine value of a Mexican dollar in cents.

Weight 867.5 *oz.* .903 *fine.* Value of Silver in London 52.75 *pence* per ounce = 106.9616+ *cents.*

$$\text{Then } \frac{867.5 \times .903}{925} = .846\,867- \text{ and } 106.9616 \times .846\,867 = 90.5822 \textit{ cents.}$$

Weight and Mint Values of Foreign Coins.

Countries given in Italics have not a National Coinage.

Country and Denomination.	Weight.	Fineness.	Pure Silver or Gold.	Current or Nominal.	VALUE. Gold. U. S.	British.
	Oz.	Thous's.	Grains.	Cents.	$ c.	£ s. d.
Arabia.						
Piastre or Mocha Dollar	—	—	—	83.14	—	—
Argentine Republic.						
Dollar = 100 Centisimos	—	—	—	50.69	—	—
(Employs South American and Foreign Coins.)						
Australasia.						
Same as British.						
Australia.						
Sovereign, 1855	.256.5	916	—	—	4.85.7	19 11.5
Pound, 1852	.281	916.5	—	—	5.32.37	1 1 10.5
Austria.						
Kreutzer (copper)	—	—	—	.41	—	.2
Florin, new	.397	900	171.47	—	—	—
Dollar, "	.596	900	257.47	—	—	—
4 Florins	.104	900	—	—	1.93.49	7 11
Ducat	.112	986	—	—	2.28.3	9 4.6
Souverain	.363	900	—	—	6.75.4	1 7 9.1
Belgium.						
Same as France.						
Bolivia.						
Centena	—	—	—	.75	—	.37
Dollar, new	.801	900	346.03	—	—	—
Doubloon, 1827–36	.867	870	362.06	—	15.59.3	3 4 1
Brazil.						
Rei	—	—	—	.547	—	.27
Milreis	.028.8	916.66	12.67	—	.54.59	26.92
Double Milreis	.82	918.5	393.6	—	—	—
20 Milreis, 1854–56	.575	917.5	—	—	10.90.6	2 4 9.84
Moidore, 4000 Reis	.261	914	—	—	4.92	1 0 2.63
Canada.						
Mil, sterling	—	—	—	.1	—	.05
Cent "	—	—	—	1.01	—	.5
20 Cent, currency	.15	925	66.6	—	—	—
25 " "	.187.5	925	83.25	—	—	—
Penny "	—	—	—	1.52	—	.75
Shilling "	—	—	—	—	—	—
Dollar, sterling	—	—	—	—	1	4 2
4 " =20 shillings, currency	—	—	—	—	3.97.43	16 4
Pound " "	—	—	—	—	3.99.97	16 5.25
Cape of Good Hope.						
Same as British.						
Central America.						
4 Reals	.027	875	11.34	—	—	—
Dollar	.866	850	353.33	—	—	—
2 Escudos	.209	853.1	—	—	3.68.8	15 1.88
Doubloon ante 1834	.869	833	—	—	14.96.39	3 1 5.97
Chili.						
Centaro	—	—	—	.9	—	.45
Dollar, new	.801	900.5	346.22	—	—	—
10 Pesos	.492	900	—	—	9.15.4	1 17 7.45
Doubloon	.867	870	—	—	15.59.3	3 4 1
China.						
Cash, Le	—	—	—	.14	—	.07
10 Cents, Leang	.087	901	37.98	—	—	—
Dollar	.866	901	374.63	—	—	—
Cochin China.						
Mas, 60 Sapeks	—	—	—	6.75	—	3.33
10 Mas, 1 Quan	—	—	—	67.52	—	2 9.33

Weight and Mint Values.

Country and Denomination.	Weight.	Fineness.	Pure Silver or Gold.	Current or Nominal.	Value. Gold. U. S.	Value. Gold. British.
	Oz.	Thous's.	Grains.	Cents.	$ c.	£ s. d.
Cuba.						
Same as Spain.						
Colombia.						
Centaro	—	—	—	1.01	—	.5
Peso, new	.801	900	346.03	—	—	—
4 Escudos	.433	844	—	—	7.55.5	1 11 0.58
Doubloon, old	.867	870	—	—	15.59.3	3 4 1
Costa Rica.						
Same as Mexico.						
Denmark.						
Mark, 16 Skilling	—	—	—	8.94	—	4.39
Crown	.025	900	—	—	26.8	13.22
2 Rigsdaler	.927	877	390.23	—	—	—
10 Thaler	.427	895	—	—	7.90	1 12 5.6
East Indies.						
See Hindostan and Japan.						
Ecuador.						
Centaro	—	—	—	1.01	—	.5
Peso	.801	900	346.03	—	—	—
England.						
Penny	.304	—	—	2.02+*	—	1
Groat	.060.4	925	26.82	—	—	—
Shilling, new	.182.5	924.5	80.99	—	—	—
" average	.178	925	79.03	—	—	—
Half Crown	.454.5	925	201.8	—	—	—
Florin	.363.6	925	161.44	—	—	—
Sovereign or Pound, new	.256.7	916.5	—	—	4.86.65	1 0 0
" " average	.256.2	916.5	—	—	4.85.1	1 0 0
Egypt.						
Piastre, 40 Paras	.04	755	14.5	—	04.9	—
Guinea, Bedidlik	.275	875	—	—	5. 0.52	1 0 6.84
Pound	.275	875	—	—	4.97.4	1 0 5.3
Purse, 5 Guineas	1.375	875	—	—	25. 2.6	5 2 10.2
France.						
Centime	.032	—	—	.2	—	.1
Sou, 5 Centimes	.161	—	—	1.01	—	.5
Franc, 100 Centimes	.161	900	69.55	—	—	—
5 Francs	.804	900	347.76	—	—	—
20 Francs, Napoleon, new	.207.5	899	—	—	3.85.8	15 10.26
25 Francs 20 centimes=£1 Stg.						
Germany.						
Groschen, 10 Pfenning	—	—	—	2.38	—	1.175
Mark, 10 Groschen	.012.8	900	—	—	23.8	11.74
10 Marks	.128	900	—	—	2.38.24	9 9.5
Thaler	.595	900	257.04	—	—	—
Ducat	.112	986	—	—	2.28.38	9 4.63
Greece and Ionian Islands.						
Same as France.						
Drachma, 100 Lepta	.010.4	900	—	—	19.3	9.5
5 Drachmas	.719	900	310.61	—	—	—
20 Drachmas	.185	900	—	—	44.2	14 1.75
Pound	—	—	—	—	5. 6.11	1 0 9.6
Guatemala.						
Same as Mexico.						
Guiana, British, French, and Dutch.						
Same as that of their Countries.						
Hanse Towns.						
Mark	.012.8	900	—	—	23.8	11.74
Holland.						
Cent	—	—	—	.4	—	.2

* 2.027 71 cents.

Weight and Mint Values.

Country and Denomination.	Weight.	Fineness.	Pure Silver or Gold.	Current or Nominal.	Value. Gold. U. S.	Value. Gold. British.
	Oz.	Thous'ths.	Grains.	Cents.	$ c.	£ s. d.
Holland.						
Florin or Guilder, 100 cents.	.021.6	900	—	—	40.49	1 8
10 Guilders	.215	899	—	—	3.99.7	16 5.11
Hindostan.						
Rupee	.374	916.5	164.53	—	—	1 10.5
Honduras. Same as Mexico.						
Italy. Same as France.						
Lira, 100 Centimes	.16	835	65.12	—	—	—
Scudo	.864	900	373.24	—	—	—
Indian Empire.						
Pic, nominal	—	—	—	.25	—	.125
Anna "	—	—	—	3.03	—	1.5
Rupee,* 16 Annas	.375	916.5	165	—	—	—
10 Rupees, and 4 Annas	—	—	—	—	4.86.65	1 0 0
Mohur, 15 Rupees	.375	916.5	—	—	6.84.36	1 8 1.5
Japan.						
Sen	—	—	—	1	—	.5
Itzebu, new	.279	890	119.19	—	—	—
Yen, 100 Sen	.866.7	900	374.4	—	—	—
" "	.053.6	900	—	—	99.72	4 1.18
Cobang, old	.289	572	—	—	3.57.6	14 8.35
" new	.362	568	—	—	4.44	18 2.96
20 Yen	1.072	900	—	—	19.94.4	4 1 11.6
Java. Same as Holland.						
Liberia. U. S. Currency.						
Malta.						
12 Scudi = 1 Sovereign	—	—	—	—	4.86.65	1 0 0
Mexico.						
Peso, new	.867.5	903	377.17	—	—	4 2
" Maximilian	.861	.902.5	372.98	—	—	—
Doubloon, new	.867.5	870.5	—	—	15. 6.1	3 4 1.88
20 Pesos, Republic	1.081	873	—	—	19.51.5	4 0 2.4
Morocco.						
Ounce, 4 Blankeels	—	—	—	—	—	—
10 Ounces, Mitkeel	—	—	—	—	—	—
Naples.						
Scudo	.844	830	336.25	—	—	—
6 Ducati	.245	996	—	—	5. 4.4	1 0 8.75
Netherlands. Same as Holland.						
New Brunswick. Same as Canada.						
Newfoundland. Same as Canada.						
New Granada.						
Dollar, 1857	.803	896	—	—	—	—
Doubloon, Popayan	.867	858	—	—	15.37.8	3 3 3.39
Norway. Alike to Denmark.						
Mark, 24 Skillingen	—	—	—	21.63	—	10.66
Nova Scotia. Same as Canada.						
Persia.						
Keran, 20 Shahis	—	—	—	22.81	—	11.25
10 Keran, Toman	—	—	—	—	—	—
Paraguay. Foreign coins.						

* .092 76 of a £ Stg., nominal value = 2 shillings sterling.

Weight and Mint Values.

Country and Denomination.	Weight.	Fineness.	Pure Silver or Gold.	Current or Nominal.	Value. Gold. U. S.	Value. Gold. British.
	Oz.	Thous's.	Grains.	Cents.	$ c.	£ s. d.
Peru.						
Dollar, 1858	.766	900	341.01	—	—	—
Sol	.802	900	346.46	—	—	—
Doubloon, old	.867	868	—	—	15.55.7	3 3 11.22
Portugal.						
Coroa, 1838, 10 000 Reis	.308	912	—	—	5.80.66	2 4 5.5
100 Reis	.095	912	—	—	10.8	—
Roumania.						
2 Lei	.322	835	129.06	—	—	—
Russia.						
Copek	—	—	—	.77	—	.38
100 Copek, Rouble	.667	875	277.73	—	—	—
5 Roubles	.21	916.6	—	—	3.97.6	16 4.8
Sandwich Islands. U. S. Currency.						
Sardinia.						
Lira	.16	835	65.12	—	—	—
Spain.						
Centimo	—	—	—	.19	—	.095
100 Centimo, Peseta	.16	835	64.13	—	—	—
Dollar, 5 Peseta	.8	900	345.6	—	—	—
100 Reals	.268	896	—	—	4.96.4	1 0 4.8
10 Escudos	.270.8	896	—	—	5. 1.5	1 0 7.32
20 Reals vellons=1 U.S. Dollar.						
Sweden.						
Riksdaler, 100 Ore	.273	750	98.28	—	—	—
Rixdollar	1.092	750	393.12	—	—	—
Carolin, 10 Francs	.104	900	—	—	1.93.5	7 11.42
Switzerland. Same as France.						
St. Domingo.						
Gomdes, 100 Cents	—	—	—	6.33	—	3.125
Tunis.						
Piastre, 16 Karubs	—	—	—	11.83	—	5.83
5 Piastre	.511	898.5	220.38	—	—	—
25 Piastre	.161	900	—	—	2.99.5	12 3.7
Turkey.						
Piastre, 40 Paras	—	—	—	4.39	—	2.16
20 Piastre	.77	830	306.77	—	—	—
100 Piastre, Medjidie	.231	915	—	—	4.36.9	18 0
Tuscany.						
Zecchino, Sequin	.112	900	—	—	2.31.3	9 6.1
Tripoli.						
20 Piastres, Mahbub	—	—	—	74.3	—	3 0.89
Uruguay.						
Dollar, 100 Centimes	—	—	—	—	—	—
West Indies, British. Same as England.						
Venezuela.						
Centaro	—	—	—	1	—	.5
Bolivar, 1 Franc	—	—	—	—	—	—

Memoranda.

FRANCE.—Bronze coins 9.5 copper, 4 tin, and 1 zinc.
HANSE TOWNS.—Monetary system same as that of German Empire.
SWITZERLAND.—The Centime is termed a Rappe.
SPAIN.—25 Peseta piece is 19*s.* 9.5*d.* Stg.; Real vellon was 2.5*d.* Stg.
ITALY.—All coins same weight and fineness as those of France.
MALTA.—7 Tari and 4 Grani=1 Shilling Sterling.
EGYPT.—A Para=.061 5*d.* Sterling, and 97.22 Piastres=1 Sovereign.
INDIAN EMPIRE.—1 Lac Rupees=£10 000 Sterling. In CEYLON, Rupee=100 Cents.

ENGLISH AND FRENCH MEASURES AND WEIGHTS.

MEASURES OF LENGTH.

ENGLISH.—Imperial standard yard is referred to a natural standard, which is a pendulum 39.1393 ins. in length vibrating seconds in vacuo in London, at level of sea; measured between two marks on a brass rod, at temperature of 62°.

NOTE.—In consequence of destruction of standard by fire in 1834, and difficulty of replacing it by measurement of a pendulum, the present standard is held to be about 1 part in 17 230 less than that of U. S., equal to 3.67 ins. in a mile.

Miscellaneous.

Land.—Woodland pole or perch or Fen .... = 18 *feet.*
Forest pole .... = 21 "

Irish mile .... = 2240 *yards.* | Scotch mile ..... = 1984 *yards.*

Sea.—10 cables, or 1000 fathoms, or 6086.44 feet, or 1.152 8 statute miles = 1 *Admiralty or Nautical mile or knot.*

3 miles = 1 league. 60 Nautical or 69.168 Statute miles or 20 Leagues = 1 *degree.*

Mean length of a minute of latitude at mean level of the sea = 1.1508 *statute miles.*

Nautical mile is taken as length of a minute at the Equator.

Nautical fathom is 1000th part of a nautical mile, and averages about .0125 longer than the common fathom.

FRENCH.—Standard Metre or unit of measurement is defined as the ten millionth part of the terrestrial meridian, or the distance from the Equator to the Pole, passing through Paris. Actual standard is a platinum metre, deposited in the Palais des Archives, Paris.

Metric Length in Inches, Feet, etc.

Denomination.	Metres.	Inches.	Feet.	Yards.	Miles.
1 Millimetre	.001	.039 37	—	—	—
1 Centimetre	.01	.393 7	—	—	—
1 Decimetre	.1	3.937 04	—	—	—
1 METRE	1	39.370 43	3.280 87	1.093 62	—
1 Dekametre	10	—	32.808 69	10.936 23	—
1 Hektometre	100	—	328.086 9	109.362 31	—
1 KILOMETRE	1 000	—	3280.869	1 093.623 1	.621 38
1 Myriametre	10 000	—	—	10 936.231	6.213 77

NOTE.—For length of metre see p. 27.

Old Measure.

1 Toise ...... = 1.949 *metres.*
1 Mille ...... = 1.949 *kilometres.*
1 Nœud (knot). = 1.855 "

1 Terrestrial league = 4.444 *kilometres.*
1 Nautical league . = 5.555 "
1 Arpent .... = 900 *sq. toises.*

MEASURES OF SURFACE.

ENGLISH.—Same as that of United States of America.

Miscellaneous.

Builders. 1 superficial part .... = 1 *square inch.*
12 parts .... = 1 *inch.*
12 inches.................... = *square foot.*

Boards.—Boards 7 inches in width are termed battens, 9 inches deals, and 12 inches planks.

French.

Metric Surfaces in Square Inches, Feet, etc.

Denomination.	Sq. Inches.	Sq. Feet.	Sq. Yards.	Sq. Acres.
1 Square millimetre...........	.001 55	—	—	—
1 " centimetre...........	.155 003	—	—	—
1 " decimetre...........	15.500 309	.107 641	—	—
1 " *Metre* or *Centiare*	1550.030 916	10.764 104	1.196 01	—
1 " dekametre or *are*.....	—	1076.410 358	119.601 15	.024 711
1 " hektometre or *hectare*	—	—	11 960.115 09	2.471 098
1 " kilometre............	—	—	—	247.109 816
1 " myriametre*.........	—	—	—	24 710.981 6

* Equal 38.610 908 *sq. miles.*

Old System.

1 square inch = 1.135 87 *inches.*
1 toise = 6.394 6 *feet.*
1 arpent (Paris) = 900 square toises = 4089 *square yards.*
1 arpent (woodland) = 100 square royal perches = 6108.24 *square yards.*

MEASURES OF VOLUME.

Imperial gallon measures 277.123 cube ins., but by Act of Parliament 1825 its volume is 277.274 cube ins., equal to 10 lbs. avoirdupois of distilled water, weighed in air, at temperature of 62°, barometer at 30 inches. 6.2355 gallons in a cube foot.

Imperial bushel, 18.5 ins. internal diameter, 19.5 external, and 8.25 in depth, contains 2218.192 *cube ins.*, and when heaped in form of a right cone, at least .75 depth of the measure, must contain 2815.4872 *cube ins.* or 1.6293 *cube feet.*

Grain.—1 quarter = 8 *bushels* or 10.2694 *cube feet.*

Vessels. — 1 ton displacement = 35 *cube feet;* 1 ton freight by measurement = 40 *cube feet.*

1 ton internal capacity = 100 *cube feet*, and 1 ton ship-builders = 94 *cube feet.*

English standard No. 5 is .008 grain heavier than the pound, and U. S. pound is .001 grain lighter than English.

Wine and Spirit Measures.

4 quarts (231 cube ins.)		....	= .8333 *Imperial gallon.*
10 gallons		....	= 1 *anchor.*
18 "	(15 imperial)	....	= 1 *runlet.*
31.5 "	26.25 "	....	= 1 *barrel.*
42 "	35 "	....	= 1 *tierce.*
63 "	52.5 "	....	= 1 *hogshead.*
84 "	70 "	....	= 1 *puncheon.*
126 "	105 "	....	= 1 *pipe* or *butt.*
2 pipes or 3 puncheons		....	= 1 *tun.*

Ale and Beer Measures.

	Imp'l gall's.		Imp'l gall's.
4 quarts (282 cube ins.) .. =	1.017	2 kilderkins = 1 barrel =	36.612
9 gallons = 1 firkin ..... =	9.153	54 gallons = 1 hogshead =	54.918
2 firkins = 1 kilderkin ... =	18.306	108 " = 1 butt .... =	109.836

Apothecaries' or Fluid Measures.

1 drop .... = 1 grain.
60 drops .... = 1 drachm.
4 drachms .... = 1 tablespoon.
2 ounces (875 grains) = 1 wineglass.

Coal Measures.

50 pounds .... = 1 *cube foot.*
88 " .... = 1 *bushel.*
9 bushels .... = 1 *vat.*
80 or 84 pounds = { 1 *London* or *Newcastle bushel.*
90 or 94 " = 1 *Cornish* "
93 pounds .... = 1 *Welsh bushel.*
3 heaped bush. = 1 *sack.*
10 sacks ..... = 1 *ton.*
12 sacks .... = 1 *chaldron.*
1 chaldron ..... = 58.6548 *cube ft.*
5.25 chaldrons .. = 1 *room.*
1 London chaldron = 26.5 *cwts.*
1 Newcastle " = 53 "
1 ton .... = 44.5 *cube feet.*
1 room .... = 7 *tons.*
21 chaldrons ..... = 1 *score.*
1 barge or keel .. = 21.2 *tons.*

Miscellaneous.

1 last corn .... = 80 *bushels.*
1 ton water ...... = 35.9 *cube feet.*
1 dicker hides .... = 10 *skins.*
1 last hides ...... = 20 *dickers.*
1 barrel tar ...... = 26.5 *gallons.*
6 bushels wheat .. = 1 *sack flour.*
1 clove .... = 7 *pounds.*
1 score .... = 20 "
1 sack flour ...... = 28.2 "
1 truss straw ..... = 36 "
1 truss old hay ..... = 50 *pounds.*
1 " new " ..... = 60 "
1 bushel oats ...... = 40 "
1 " barley .... = 47 "
1 " wheat ..... = 60 "
1 cube yard new hay = 84 "
1 " " old " = 126 "
1 quintal .... = 100 "
1 boll .... = 140 "
1 sack wool .... = 364 "

35.9 *cube feet* = 1 ton water.

Liquid.

1 wine gallon = 231 *cube ins.*
1 beer " = 282 " "
1 litre ...... = .220 09 *gallon.*
1 gallon .... = 4.544 *litres.*
1 cube foot .. = 6.2321 *gallons.*
1 anker .... = 8.333 "
1 hogshead wine .. = 52.5 *gallons.*
1 " beer ... = 54.918 "
1 puncheon wine .. = 70 "
1 pipe or butt wine = 105 "
1 " " " beer = 109.836 "
1 tun .... = 210 "

1 ton water 62° = 224 *gallons.*

Builders.

1 solid part .... = 12 *cube ins.*
12 " parts ...... = 1 "*inch.*"
12 "inches" .... = 1 *cube foot.*
1 load timber, rough = 40 " *feet.*
1 " " hewn = 50 " "
1 " lime .... = 32 *bushels.*
1 " sand .... = 36 "
1 square .... = 100 *sq. feet.*
1 bundle laths ..... = 120 *laths.*
1 rod brickwork ... = 306 *cube feet.*
1 rood masonry ... = 648 " "
Batten, in section .. = 7 × 2.5 *ins.*
Deal, " " .. = 9 × 3 "
Plank, " " .. = 11 × 3 "

Metric Volumes in Cube Inches, Feet, etc.

Denominations.	Litres.	Gills.	Pints.	Quarts.	Gallons.	Bushels.	Quarters.
Centilitre.........	.01	.0704	.0176	—	—	—	—
Decilitre..........	.1	.7043	.1761	—	—	—	—
*Litre**............	1	7.0429	1.7607	.8804	.2201	—	—
Dekalitre.........	10	—	—	8.8036	2.2009	.275 11	—
Hectolitre.........	100	—	—	—	22.0091	2.751 13	.3439
Kilolitre..........	1000	—	—	—	220.0908	27.511 35	3.4389

* Equal 61.025 24 *cube ins.*

Wood Measure.

1 Stere or cube metre = 35.3150 *cube feet* or 1.308 *cube yards*.

1 Voie de bois (Paris) = 70.6312 *cube feet;* 1 voie de charbon (charcoal) = 7.063 *cube feet;* 1 corde = 4 cube metres = 141.26 *cube feet*.

MEASURES OF WEIGHT.

BRITISH.—1 Troy grain = .003 961 cube inches of distilled water.
1 Troy pound = 22.815 689 cube inches of water.
1 Avoir. drachm = 27.343 75 Troy grains.

Avoirdupois.

16 drachms, or 437.5 grains	...... = 1 ounce.	8 pounds	.. = 1 stone (for meat).
		14 "	.. = 1 stone.
16 ounces, or 7000 grains	...... = 1 pound.	28 "	.. = 1 quarter.
		112 "	.. = 1 cwt.

20 hundredweights ...... = 1 ton.

The *grain*, of which there are 7000 to the pound avoirdupois, is same as Troy grain, of which there are by the revised table 7000 to the Troy pound.
Hence Troy pound is equal with the Avoirdupois pound.
In Wales, the iron ton is 20 cwt. of 120 lbs. each.

Troy.

24 grains	.... = 1 dwt.	16 ounces	.... = 1 pound.
20 pennyweights, or 437.5 grains	.. = 1 ounce.	25 pounds	.... = 1 quarter.
		4 quarters, or 100 pounds	= 1 cwt.

By this are weighed gold, silver, jewels, and such liquors as are sold by weight.

The old Troy ounce to the Avoirdupois ounce was as 480 grains, the weight of the former, to 437.5 grains, weight of the latter; or, as 1 to .9115.

Apothecaries.*

437.5 grains = 1 ounce. | 16 ounces = 1 pound.

FRENCH.

Metric Weights in Avoirdupois.

Denominations.	Grammes.	Grains.	Ounces.	Pounds.	Ton.
Milligramme	.001	.015 43	—	—	—
Centigramme.........	.01	.154 32	—	—	—
Decigramme	.1	1 543 23	—	—	—
Gramme...............	1	15.432 35	—	—	—
Dekagramme.........	10	154.323 49	.3527	—	—
Hektogramme........	100	1 543.234 87	3.5274	.220 46	—
Kilogramme†.........	1 000	15 432.348 74	35.2739	2.204 62	—
Myriagramme	10 000	—	—	22.046 21	—
Quintal................	100 000	—	—	220.462 12	—
Millier or Ton.........	1 000 000	—	—	2204.621 25	.9842

† Kilogramme = 2 *lbs.* 3 *oz.* 4 *drachms*, 10.4734 *grains*.

NOTE.—For the values of the prefixes, as Milli, Centi, etc., see p. 27.

Old System.

1 grain ..	= 0.8188 *grains Troy*.	1 ounce	= 1.0780 *oz. Avoirdupois*.
1 gross ..	= 58.9548 "	1 livre	= 1.0780 *lbs*.

---

* As by revised Pharmacopœia.

FOREIGN MEASURES AND WEIGHTS.

It being wholly impracticable to give all the denominations of measures and weights of all countries, the following cases are selected as essential and as exponents.

With parent countries, as England, France, etc., their denominations extend to their colonies and dependencies. Thus, the denominations of England extend to Canada, a large portion of the East and West Indies, and parts of South America, and those of France to a part of the West Indies, Algiers, etc.

Abyssinia.

Pic, Stambouili	26.8 ins.
" geometrical	30.37 "
Madega	3.466 bush.
Ardeb	34.66 "
" Musuah	83.184 "
Wakea	400 grains.
Mocha	1 Troy oz.
Rottolo	10 " "

Also, same as in Egypt and Cairo.

Africa, Alexandria, Cairo, and Egypt.

Cubit	20.65 ins.
Derah	25.49 "
Pic, cloth	26.8 "
" geometrical	29.53 "
Kassaba, 4.73 Pics	11.65 ft.
Mile	2146 yds.
Feddan al-risach	.552 48 acre.
Roobak	1.684 galls.
Ardeb	4.9 bush.
Rottol	.9821 lb.

Distances are measured by time.

A Maragha = 15 Déréghé or 1 hour.

Aleppo and Syria.

Dra Mesrour	21.845 ins.
Pic	26.63 "

Road Measures are computed by time.

Algeria.

Rob, Turkish	3.11 ins.
Pic, "	24.92 "
" Arabic	18.89 "

Also Decimal System.

Alicante.

Palmo	8.908 ins.
Vara	35.632 "

Amsterdam.

Voet	11.144 ins.
El	21.979 "
Faden	5.57 ft.
Lieue	6.383 yds.
Maat	1.6728 acres.
Morgen	2.0095 "
Vat	40 cub. ft.

Also Decimal System.

Antwerp.

Fuss	11.275 ins.
Elle, cloth	26.94 "
Corde	24.494 cub. ft.
Bonnier	3.2507 acres.

Also Decimal System.

Arabia, Bassora, and Mocha.

Foot, Arabic	1.0502 ft.
Covid, Mocha	19 ins.
Guz, "	25 "
Kassaba	12.3 ft.
Mile, 6000 feet	2146 yds.
Baryd, 4 farsakh	21 120 "
Feddan	57 600 sq. ft.
Noosfia, Arabic	138 cub. ins.
Gudda	2 galls.
Maund	3 lbs.
Tomand	168 "

Other Measures like those of Egypt.

Argentine Confederation, Paraguay, and Uruguay.

Fanega	1.5 bush.
Arroba	25.35 lbs.
Quintal	101.4 "

Also Decimal System in Argentine Confederation and Paraguay.

Australasia.

Land Section	80 acres.

Other Measures same as English.

Austria.

Zoll	1.0371 ins.
Fuss	1.0371 ft.
Meile	24 000 ft.
Klafter, quadrat	35.854 sq. yds.
Jochart	6.884 "
Cube Fuss	1.1155 cub. ft.
Achtel	1.692 galls.
Eimer	12.774 "
Viertel	3.1143 "
Metze	1.6918 bush.
Unze	.8642 grains.
Pfund (1853, 500 grammes)	1.2347 lbs.
Centner	123.47 "

Also Decimal System.

Babylon.

Pachys Metrios	18.205 ins.

Baden.

Fuss	11.81 ins.
Klafter	5.9055 ft.
Ruthe	9.8427 "
Stunden	4860 yds.
Morgen	.8896 acre.
Stutze	3.3014 galls.
Malter	4.1268 bush.
Pfund	1.1023 lbs.

Also Decimal System.

Bagdad.

Guz	31.665 ins.

Barbary States.

Pic, Tunis linen	18.62 ins.
" " cloth	26.49 "
" Tripoli	21.75 "

Batavia.

Foot	12.357 ins.
Covid	27 "
El	27.75 "

Bavaria.

Fuss	11.49 ins.
Klafter	5.745 36 ft.
Ruthe	3.1918 yds.
Meile	8060 "
Ruthe, quadrat	10.1876 sq. yds.
Morgen or Tagwerk	.8416 acre.
Klafter, cube	4.097 cub. yds.
Eimer	15.058 56 galls.
Scheffel	6.119 "
Metze	1.0196 bush.
Pfund	8642 grains.

Also Decimal System.

Belgium.

Meile	2.132 yds.

Also Decimal System.

Benares.

Yard, Tailor's	33 ins.

Bengal, Bombay, and Calcutta.

Moot	3 ins.
Span	9 "
Ady, Malabar	10.46 ins.
Hath	18 "
Guz, Bombay	27 "
" Bengal	36 "
Corah, minimum	3.417 ft.
Coss, Bengal	1.136 miles.
" Calcutta	1.2273 "
Kutty	9.8175 sq. yds.
Biggah, Bengal	.3306 acre.
" Bombay	.8114 "
Seer, Factory	.68 cub. ins.
Covit, Bombay	12.704 cub. ft.
Seer, Bombay	1.234 pints.
Parah	4.4802 galls.
Mooda	112.0045 "

Liquids and Grain measured by weight.

Bohemia.

Foot, Prague	11.88 ins.
" Imperial	12.45 "

Also same as Austria.

Bolivia, Chili, and Peru.

Vara	33.333 ins.
Fanegada	1.5888 acres.
Gallon	.74 gall.
Fanega	1.572 "
Libra	1.014 lbs.
Arroba	25.36 "

Originally as in Spain; now Decimal System in Chili and Peru.

Brazil.

Palmo, Bahia	8.5592 ins.
Vara	3.566 ft.
Braca	7.132 "
Geora	1.448 acres.

Also same as Portugal, and sometimes as in England.

Buenos Ayres.

Vara	2.84 ft.
Legua	3.226 miles.
Suertes de Estancia	27 000 sq. varas.

Also same as Spain.

Burmah.

Paulgat	1 inch.
Dain	4.277 yds.
Viss	3.6 lbs.
Taim	5.5 "
Saading	22 "

Also same as England.

Canary Isles.

Onza	.927 inch.
Pic, Castilian	11.128 ins.
Almude	.0416 acre.
Fanegada	.5 "
Libra	1.0148 lbs.

Also same as Spain.

Cape of Good Hope.

Foot	11.616 ins.
Morgen	2.116 54 acres.

Also same as in England.

Ceylon.

Seer	1 quart.
Parrah	5.62 galls.

Also same as in England.

China.

Li	.486 inch.
Chih, Engineer's	12.71 ins.
" or Covid	13.125 "
" " legal	14.1 "
Chang	131.25 "
" legal	141 "
Pu	4.05 ft.
Chang, fathom	10.9375 ft.
Lí	486 yds.
Pú or Kung	3.32 sq. yds.
King, 100 Mau	16.485 acres.
Tau	1.13 galls.
Tael	1.333 oz.
Catty	1.333 lbs.

Cochin China.

Thuoc or Cubit	19.2 ins.
Sao	64 sq. yds.
Mao	1.32 acres.
Hao	6.222 galls.
Shita	12.444 "
Nen	.8594 lb.

Colombia and Venezuela.

Libra	1.102 lbs.
Oncha	25 "

Also Decimal System.

Denmark,* Greenland, Iceland, and Norway.

Tomme	1.0297 ins.
Fod	1.0297 ft.
Favn, 3 Alen	6.1783 "
Mil	4.680 55 miles.
" nautical	4.61072 "
Anker	8.0709 galls.
Skeppe	.478 bush.
Fjerdingkar	.9558 "
Pund	1.1023 lbs.
Lispund	17.367 "
Centner	110.23 "

* *Also Decimal System.*

Ecuador.

Decimal System.

Genoa, Sardinia, and Turin.

Palmo	9.8076 ins.
Piede, Manual, 8 oncie	13.488 "
" Liprando, 12 "	20.23 "
Trabuco or Tesa	10.113 ft.
Miglio	1.3835 miles.
Starello	.9804 acre.
Giomaba	.9394 "

Germany.

The old measures of the different States differ very materially; generally, however,

Foot, Rhineland	12.357 ins.
Meile	4.603 miles.

Decimal System made compulsory in 1872.

Greece.

Stadium	.6155 mile.

Also Decimal System.

Guinea.

Jachtan	12 ft.

Hamburg.

Fuss	11.2788 ins.
Klafter	5.6413 ft.
Morgen	2.386 acres.
Cube Fuss	.831 1 cub. ft.
Tehr	99.73 "
Viertel	1.594 7 galls.
Pfund (500 grammes)	1.102 32 lbs.
Ton	2135.8 lbs.

Also Decimal System.

Hanover.

Fuss	11.5 ins.
Morgen	.6476 acre.

Hindostan.

Borrel	1.211 ins.
Gerah	2.387 "
Haut	19.08 "
Kobe	29.065 "
Coss	3.65 miles.
Tuda	1.184 cub. ft.
Candy	14.209 "

Hungary.

Fuss	12.445 ins.
Elle	30.67 "
Meile	9.139 yds.

Also as in Vienna.

Indian Empire.

Guz	27.125 ins.
Cowrie	1 sq. yd.
Sen	61.025 39 cub. ins.
"	2.204 737 lbs.

Uniform standard of multiples of the Sen adopted in 1871.

Italy.

Milan and Venice.

Decimal System.

The Metre is termed Metra; the Are, Ara; the Stere, Stero, the Litre, Litro; the Gramme, Gramma, and the Tonneau, Tonnelata de Mare.

Naples and Two Sicilies.

Palmo	10.381 ins.
Canna	6.921 ft.
Miglio	1.1506 miles.
Migliago	.7467 acre.
Moggia	.86 "
Pezza, Roman	.6529 "

Roman States.

Old Measure.

Foot	11.592 ins.
" Architect's	11.73 "
Braccio	30.73 "
Palmo	8.347 "
Miglio	1628 yds.
Quarta	1.1414 acres.

Lucca and Tuscany.

Pie	11.94 ins.
Palmo	11.49 "
Braccio	22.98 "
Passetto	3.829 ft.
Passo	5.74 "
Miglio	1.0277 miles.
Quadrato	.8413 acre.
Saccato	1.324 "

Japan.

Sun,	.303 03 Metre	1.193* ins.
Shaku,	3.030 3 Metres	11.9305* ins.
Jo,	30.303 "	9.9421* ft.
Ken,	5.5 "	5.9653* "
Ri,	11 880 "	2.4403 miles.
Kai-ri		6080 feet.†
Hiro		4.971* feet.
Momme		3.756 521 7 grammes Fr.
Hiyaku-me		.828 17 lbs.
Kwam-me		8.281 71 "
Hiyak-kin		132.507 32 "
Man's load		57.972 "
Koku		331.268 31 "
Hiyak-koku		33 126.830 8 "

* These are as equivalent as they are practicable of reduction.

† Admiralty knot.

Java.

Duim	1.3 ins.
Ell	27.08 "
Djong	7.015 acres.
Kan	.328 galls.
Tael	593.6 grains.
Sach	61.034 lbs.
Pecul	122.068 "
Catty	1.356 "

Madras.

Ady	10.46 ins.
Covid	18.6 "
Guz	33 "
Culy	20.92 ft.
League	3472 yds.
Puddy	.338 galls.
Marcal	2.704 "
Tola	180 grains.
Seer	.625 lbs.
Viss	3.086 "
Maund	24.686 "

Malabar.

Ady	10.46 ins.

Malacca.

Hasta or Covid	18.125 ins.
Depa	6 ft.
Orlong	80 yds.

Malta.

Palmo	10.3125 ins.
Pié	11.167 "
Canna	82.5 "
Salma	4.44 acres.

Also as in Sicily.

Moldavia.

Foot	8 ins.
Kot, silk	24.86 ins.
Fathom	8 ft.

Molucca Islands.

Covid	18.333 ins.

Morocco.

Tomin	2.81025 ins.
Cadée	20.34 ins.
Cubit	21 "
Muhd	3.08135 galls.
Kula, oil	3.356 "
Rotal or Artal	1.12 lbs.

Liquids other than oil are sold by weight.

Mysore.

Angle	2.12 ins.
Haut	19.1 "
Guz	38.2 "
Candy	500 lbs.

Netherlands.

Elle	39.370432 ins.

Decimal System since 1817.

Persia.

Gereh	2.375 ins.
Gueza, common	25 "
" Monkelrer	37.5 "
Archin, Schah	31.55 ins.
" Arish	38.27 "
Parasang	6076 yds.
Chenica	80.26 cub. ins.
Artaba	1.809 bush.
Miscal	71 grains.
Ratel	2.1136 lbs.
Batman Maund	6.49 "

Liquids are measured by weight.

Poland.

Trewice	14.03 ins.
Precikow	17 ins.
Pretow	4.7245 yds.
Mile, short	6075 yds.
Morgen	1.3843 acres.

Portugal and Mozambique.

Foot	13 ins.
Milha	1.2788 miles.
Almude	3.7 galls.
Fanga	1.488 bush.
Alguieri	3.6 "
Libra	1.012 lbs.

Also Decimal System.

Prussia.

Fuss	12.358 ins.
Ruthe	4.1192 yds.
Meile	24.000 feet.
Quadrat Fuss	1.0603 sq. ft.
Morgen	.63103 acre.
Cube Fuss	1.092 cub. ft.
Scheffel	1.5121 bush.
Anker	7.559 galls.
Pound	7217 grains.
Zollpfund	1.1023 lbs.
Centner	113.43 lbs.

Russia.

Vershok	1.75 ins.
Foot	12 ins.
Arschine	28 "
Rhein Fuss	1.03 ft.
Sajene	7 ft.
Verst	3500 "
Mila	5.5574 miles.
Dessatina	2.4954 acres.
Vedro	2.7049 galls.
Tschel-werha	1.4424 "
Pajak	1.4426 bush.
Tschetwert	5.7704 "
Pound	6317 grains.
Funt	.90285 lbs.

Decimal System adopted in 1872.

Siam.

K'up	9.75 ins.
Covid	18 ins.
Ken	39 "
Jod	.09848 mile.
Roëneng	2.462 miles.

Silesia.

Fuss	11.19 ins.
Ruthe	4.7238 yds.
Meile	7086 yds.
Morgen	1.3825 acres.

Singapore.

Hasta or Cubit .... 18 ins.
Dessa .... 6 ft.
Orlong .... 80 yds.

Smyrna.

Pic .... 26.48 ins.
Indise .... 24.648 "
Berri .... 1828 yds.

Spain, Cuba, Malaga, Manilla, Guatemala, Honduras, and Mexico.

Pie .... 11.128 ins.
Vara .... 33.384 "
Milla .... .865 mile.
Legua, 8000 varas .... 4.2151 miles.
Fanegada .... 1.6374 acres.
Vara, cubo .... 21.531 cub. ft.
Cuartilla .... .888 gall.
Arroba, Castile .... 3.554 galls.
Fanega .... 1.5077 bush.
Libra .... 1.0144 lbs.
Tonelada .... 2028.2 lbs.

Also Decimal System.

Stettin.

Fuss .... 11.12 ins.
Foot, Rhineland .... 12.357 "
Elle .... 25.6 ins.
Morgen .... 1.5729 acres.

Sumatra.

Jankal or Span .... 9 ins.
Elle .... 18 "
Hailoh .... 36 "
Fathom .... 6 ft.
Tung .... 4 yds.

Surat.

Tussoo, cloth .... 1.161 ins.
Guz, " .... 27.864 "
Hath .... 20.9 "
Covid .... 18.5 "
Biggah .... .51 acre.

Sweden.

Fot .... 11.6928 ins.
Ref .... 32.4703 yds.
Faden .... 5.845 ft.
League .... 3.3564 miles.
Meile .... 6.6417 "
Tunnland .... 1.2198 acres.
Anker .... 8.641 galls.
Spann .... 1.962 bush.
Centner .... 112.05 lbs.

Also Decimal System.

Switzerland.

Fuss, Berne .... 11.52 ins.
" .... 11.54 "
Vaud .... 11.81 "
Klafter .... 5.77 ft.
Meile .... 4.8568 miles.
Juchart, Berne .... .85 acre.
Maas .... 2.6412 pints.
Eimer .... 8.918 galls.
Malter .... 4.1268 bush.
Pfund .... 1.1023 lbs.

Also Decimal System.

Tripoli.

Pik, 3 palmi .... 26.42 ins.
Almud .... 319.4 cub. ins.
Killow .... 2023 " "
Barile .... 14.267 galls.
Temer .... .7383 bush.
Rottol .... 7680 grains.
Oke .... 2.8286 lbs.

Turkey.

Pic, great .... 27.9 ins.
" small .... 27.06 "
Berri .... 1.828 yds.
Alma .... 1.154 galls.

Also Decimal System.

Würtemberg.

Fuss .... 11.29 ins.
Elle .... 2.015 ft.
Meile .... 8146.25 yds.
Morgen .... .7793 acre.
Cube Fuss .... .83045 cub. ft.
Eimer .... 64.721 galls.
Scheffel .... 4.878 bush.
Pound .... 7217 grains.

Zurich.

Fuss .... 11.812 ins.
Elle .... 23.625 "
Klafter .... 5.9062 ft.
Meile .... 4.8568 miles.
Jachart .... .808 acre.
Cube Klafter .... 144 cub. ft.

LENGTHS OF ENGLISH RACE-COURSES.

Course.	Miles.	Course.	Miles.	Course.	Miles.
NEWMARKET.		DONCASTER.		GOODWOOD.	
Across the Flat	1.292	Circular	1.915	Cup Course	2.5
Beacon	4.206	Fitzwilliam	1	LIVERPOOL.	
Cambridgeshire	1.136	Red House	.711	New Course	1.5
Cesarewitch	2.266	St. Leger	1.825	NEW CASTLE	1.796
Round	3.579	Cup Course	2.634	OXFORD	2
Rowley Mile	1.009	EPSOM.		YORK.	
Summer Course	2	Craven	1.25	Stakes Course	1.75
Two-year old, new	.702	Derby and Oaks	1.5	Two-mile	1.923
Yearling	.277	Metropolitan	2.25		

SCRIPTURE AND ANCIENT LINEAR MEASURES.

Scripture.

Digit	.912 inch.	Span, 3 palms	10.944 ins.
Palm, 40 digits	3.648 ins.	Cubit. 2 spans	21.888 "

Fathom, 4 cubits .... 7 feet 3.552 ins.

Hebrew and Egyptian.

Nahud cubit	1.475 feet.	Babylonian foot	1.140 feet.
Royal "	1.7216 "	Hebrew "	1.212 "
Egyptian finger	.06145 "	" cubit	1.817 "

Hebrew sacred cubit .... 2.002 feet.

Grecian.

Digit	.7554 inch.	Ancient Greek foot (16 Egyptian fingers)	.9841 foot.
Pous (foot)	1.0073 feet.	Arabian foot	1.095 feet.
Cubit	1.1332 "	Stadium	604.0375 "
Pythic or natural foot	.814 foot.	Olympic stadium	606.29 "
Attic or Olympic "	1.009 feet.		

Mile, 8 stadium .... 4835 feet.

Alexandrian or Phileterian stadium (600 Phil. feet) = 708.65 feet.

Volume.—Keramion or Metretes .... 8.488 gallons.

Jewish.

Cubit	1.824 feet.	Mile, 4000 cubits	7296 feet.
Sabbath day's journey	3648 "	Day's journey	33.164 miles.

Roman Long Measures.

Digit	.72575 ins.	Cubit	1.4505 feet.
Uncia (inch)	.967 "	Passus	4.835 "
Pes (foot)	11.604 "	Mile, milliarium	4842 "

ANCIENT WEIGHTS.

Hebrew and Egyptian.

	Troy grains.		Troy grains.
Attic obolus	8.2* 9.1†	Denarius, Roman	51.9* 62.5†
" drachma	51.9* 54.6† 69‡	" Nero	54‡
		Shekel	92.62
Lesser mina	3.892	Ounce	415.1* 437.2† 431.2‡
Greater mina	5.46		
Egyptian mina	8.326*	Drachm	146.5
Ptolemaic "	8.985*	Libra	4086.1
Alexandrian "	9.992*	Pound	12 Roman ounces.
Obolus	4.63	Talub	581.71 ounces.

Talent (60 minæ) .... 56 lbs. avoirdupois.

Grecian.

	Troy grains.		Troy ounces.
Obolus, ancient	8.33	Mina	10.41
"	11.57	" great	14.472
Gramme	23.15	Talent	625.19
Drachma	50.01	" Attic	863.32
" great	69.47		

Roman.

Ounce	416.82 grains.	Pound	10.41 ounces.

---

* Christiani. † Arbuthnot. ‡ Paucton.

GEOGRAPHIC MEASURES AND DISTANCES.

To Reduce Longitude into Time.

RULE.—Multiply degrees, minutes, and seconds by 4, and product is the time.

EXAMPLE.—Required time corresponding to 50° 31′. 50° 31′ × 4 = 3*h.* 22*m.* 4*s.*

To Reduce Time into Longitude.

RULE.—Reduce hours to minutes and seconds, divide by 4, and quotient is the longitude. *Or*, Multiply them by 15.

EXAMPLE.—Required longitude corresponding to 5*h.* 8*m.* 11.2*s.*

5*h.* 8*m.* 11.2*s.* = 308*m.* 11.2*s.*, which ÷ 4 = 77° 2′ 45.5″.

Or, multiplying by 15: 5*h.* 8*m.* 11.2*s.* × 15 = 77° 2′ 45.5″.

Table of Departures for a Distance run of 1 Mile.

Course.	Departure.	Course.	Departure.	Course.	Departure.
3.5 points.	.773	4.5 points.	.634	5.5 points.	.471
4 "	.707	5 "	.556	6 "	.383

Thus, if a vessel holds a course of 4 points, that is without leeway, for distance of 1 mile, she will make .707 of a mile to windward.

Or, a vessel sailing E.N.E. upon a course of 6 points for 100 miles will make 38.3 (100 × .383) miles of longitude.

Degrees, Minutes, and Seconds of each Point of the Compass with Meridian.

NORTH.	SOUTH.	Points.	° ′ ″	Sin. A.*	Cos. A.*	Tan. A.*
		.25	2 48 45	.0489	.9988	.0491
N.	S.	.5	5 37 30	.098	.9952	.0985
		.75	8 26 15	.1467	.9891	.1484
		1	11 15	.195	.9808	.1989
N. by E.	S. by E.	1.25	14 3 45	.2429	.97	.2504
N. by W.	S. by W.	1.5	16 52 30	.2903	.9569	.3034
		1.75	19 41 15	.3368	.9415	.3578
		2	22 30	.3827	.9239	.4142
N.N.E.	S.S.E.	2.25	25 18 45	.4275	.904	.4729
N.N.W.	S.S.W.	2.5	27 7 30	.4714	.8819	.5345
		2.75	30 56 15	.5141	.8577	.5994
		3	33 45	.5556	.8315	.6682
N.E. by N.	S.E. by S.	3.25	36 33 45	.5957	.8032	.7416
N.W. by N.	S.W. by S.	3.5	39 22 30	.6344	.773	.8207
		3.75	42 11 15	.6715	.7409	.9063
		4	45	.7071	.7071	1
N.E.	S.E.	4.25	47 48 45	.7404	.6715	1.103
N.W.	S.W.	4.5	50 37 30	.773	.6344	1.218
		4.75	53 26 15	.8032	.5957	1.348
		5	56 15	.8315	.5556	1.497
N.E. by E.	S.E. by E.	5.25	59 3 45	.8577	.5141	1.668
N.W. by W.	S.W. by W.	5.5	61 52 30	.8819	.4714	1.871
		5.75	64 41 15	.904	.4275	2.114
		6	67 30	.9239	.3827	2.414
E.N.E.	E.S.E.	6.25	70 18 45	.9415	.3368	2.795
W.N.W.	W.S.W.	6.5	73 7 30	.9569	.2903	3.296
		6.75	75 56 15	.97	.2429	3.941
		7	78 45	.9808	.195	5.027
E. by N.	E. by S.	7.25	81 33 45	.9891	.1467	6.741
W. by N.	W. by S.	7.5	84 22 30	.9952	.098	10.153
		7.75	87 11 15	.9988	.0489	20.555
East or West.	East or West.	8	90	1	.0000	∞

* A, representing course or points from the meridian.

GEOGRAPHIC LEVELLING.

Curvature and Refraction.

Correction for Curvature of Earth, to be subtracted from reading of a levelling-staff, is determined as follows:

Divide square of distance in feet from level to staff, by Earth's Equatorial diameter—viz., 41 852 124 feet.

Or, Two thirds of square of distance in statute miles equal the curvature in feet.

Correction for Refraction is to be added to reading, and as a mean may be taken at about one sixth of that for curvature.

Correction for Curvature and Refraction combined, is to be subtracted from reading on staff.

Formulas of Capt. T. J. Lee, U. S. Engineers.

$\frac{D^2}{2\,R}$ = *correction for curvature*, $\frac{D^2}{R}\,m$ = *correction for refraction*, and $(1-2\,m)\frac{D^2}{2\,R}$ = *correction for curvature and refraction.* D *representing distance*, R *radius of earth, and m a coefficient of refraction* = .075, *all in feet.*

ILLUSTRATION.—A distance is 3 statute miles, what is correction for curvature and refraction?

$$(1-2\times .075)\frac{\overline{5280\times 3}^{\,2}}{41\,852\,124} = .85\times 5.996 = 5.097 \textit{ feet.}$$

Approximately, $\frac{2}{3}D^2$ = *curvature in feet.*

Levelling by Boiling Point of Water.

To Compute Height Above or Below Level of Sea.

$$517\,(212^\circ - T) + (212^\circ - T)^2 = \textit{Height.}$$

ILLUSTRATION.—What is height of an elevation, when boiling point of water is 182°?

$$517\times\overline{212^\circ - 182^\circ} + \overline{212^\circ - 182^\circ}^{\,2} = 517\times 30 + 30^2 = 16\,410 \textit{ feet.}$$

Corrections for Temperature to be made in Connection with Formula.

Temp.	Correction.	Temp.	Correction.	Temp.	Correction.	Temp.	Correction.	Temp.	Correction.	Temp.	Correction.
°		°		°		°		°		°	
0	.936	18	.972	36	1.008	54	1.046	72	1.083	90	1.12
2	.94	20	.976	38	1.012	56	1.05	74	1.087	92	1.124
4	.944	22	.98	40	1.016	58	1.054	76	1.091	94	1.128
6	.948	24	.984	42	1.02	60	1.058	78	1.096	96	1.132
8	.952	26	.988	44	1.024	62	1.062	80	1.1	98	1.136
10	.956	28	.992	46	1.028	64	1.066	82	1.104	100	1.14
12	.96	30	.996	48	1.032	66	1.071	84	1.108	102	1.144
14	.964	32	1	50	1.036	68	1.075	86	1.112	104	1.148
16	.968	34	1.004	52	1.041	70	1.079	88	1.116	106	1.152

ILLUSTRATION.—Assume temperature in preceding illustration to have been 80°.
Then 16 410 × 1.1 = 18 051 *feet.*

Apparent Level of Objects at or upon Surface of Land or Sea, and Differences between True and Apparent Levels, Curvatures, etc.

Distances in Geographic or Nautical Miles.

Distance.	HEIGHT of Line of Sight.	HEIGHT of Curvature above Land.	HEIGHT of Curvature and Refraction. Land.	HEIGHT of Curvature and Refraction. Sea.	Apparent Level including Curvature and Refraction. Land.	Apparent Level including Curvature and Refraction. Sea.	Distance.	HEIGHT of Line of Sight.	HEIGHT of Curvature above Land.	HEIGHT of Curvature and Refraction. Land.	HEIGHT of Curvature and Refraction. Sea.	Apparent Level including Curvature and Refraction. Land.	Apparent Level including Curvature and Refraction. Sea.
Miles.	Feet.	Feet.	Feet.	Feet.	Miles.	Miles.	Miles.	Feet.	Feet.	Feet.	Feet.	Miles.	Miles.
1	.667	.662	.563	.568	1.09	1.08	7.24	35	34.7	29.51	29.77	7.89	7.85
1.22	1	.99	.835	.842	1.33	1.32	7.74	40	39.66	33.73	34.02	8.43	8.39
1.73	2	1.98	1.69	1.7	1.89	1.88	8.21	45	44.62	37.95	38.28	8.94	8.9
2.12	3	2.98	2.53	2.55	2.31	2.29	8.66	50	49.65	42.21	42.59	9.43	9.38
2.45	4	3.97	3.38	3.41	2.67	2.65	9.08	55	54.58	46.42	46.83	9.89	9.83
2.74	5	4.97	4.23	4.26	2.98	2.97	9.48	60	59.48	50.59	51.04	10.33	10.24
3	6	5.96	5.07	5.1	3.27	3.25	10.24	70	69.42	59.03	59.56	11.16	11.1
3.24	7	6.95	5.91	5.96	3.53	3.51	10.95	80	79.37	67.5	68.1	11.93	11.86
3.46	8	7.93	6.74	6.8	3.77	3.75	11.61	90	89.23	75.89	76.56	12.65	12.59
3.67	9	8.92	7.58	7.65	4	3.98	12.24	100	99.18	84.35	85.1	13.33	13.27
3.87	10	9.91	8.43	8.51	4.22	4.19	14.99	150	148.7	126.51	127.63	16.33	16.25
4.06	11	10.91	9.28	9.36	4.42	4.4	17.31	200	198.4	168.7	170.2	18.86	18.77
4.24	12	11.9	10.12	10.21	4.62	4.59	21.2	300	297.6	253.03	255.28	23.09	22.98
4.41	13	12.87	10.95	11.05	4.81	4.78	24.49	400	397	337.66	340.66	26.67	26.54
4.58	14	13.89	11.81	11.91	4.99	4.96	27.37	500	495	421.75	425.5	29.81	29.67
4.74	15	14.87	12.65	12.76	5.16	5.14	29.99	600	595.4	506.36	510.86	32.65	32.51
4.9	16	15.89	13.52	13.64	5.33	5.3	32.39	700	694.5	590.65	595.89	35.28	35.11
5.04	17	16.82	14.3	14.43	5.5	5.46	34.63	800	793.9	675.17	681.17	37.71	37.53
5.19	18	17.83	15.16	15.3	5.66	5.62	36.73	900	893.1	756.16	768.87	40	39.8
5.33	19	18.81	15.99	16.14	5.81	5.78	38.72	1000	992.5	844.07	851.57	42.23	41.96
5.48	20	19.88	16.91	17.06	5.96	5.93	54.76	2000	1 985	1 688.3	1 703.2	59.63	59.34
5.61	21	20.83	17.72	17.87	6.11	6.08	67.06	3000	2 977	2 531.8	2 544.3	73.03	72.68
5.74	22	21.81	18.55	18.72	6.25	6.22	77.44	4000	3 970	3 376.3	3 406.3	84.32	83.92
5.87	23	22.81	19.4	19.57	6.39	6.36	86.58	5000	4 962	4 220.3	4 257.8	94.28	93.83
5.98	24	23.67	20.13	20.31	6.53	6.5	95.52	1 Mile.	6 040	5 136.9	5 182.5	104.03	103.54
6.12	25	24.79	21.08	21.27	6.67	6.63	135.09	2 Miles.	12 081	10 274	10 366	147.1	146.39
6.7	30	29.72	25.27	25.5	7.3	7.26	165.44	3 "	18 119	15 430	15 546	180.16	179.3

NOTE 1.—Height or elevation in second column of table is also curvature of Earth at Ocean.
2.—Refraction is greater and more variable at sunrise and sunset, and comparatively stationary between hours of 10 A. M. and 4 P. M.

ILLUSTRATION. — Curvature of Earth independent of refraction is computed at .667 foot = 8.004 ins. for 1 geographical mile, and as refraction on land is taken as .104 foot or 1.248 ins., and on ocean at .099 foot or 1.188 ins., relative visible distances of an object, including curvature and refraction, for an elevation of

.667	foot	is	1.09	miles	on	land,	and	1.08	miles	at	sea.
1	"	"	1.33	"	"	"	"	1.32	"	"	"
9	feet	"	4	"	"	"	"	3.98	"	"	"
1	mile	"	104.03	"	"	"	"	103.54	"	"	"

Difference between two levels in feet is as square of their distance in miles.

ILLUSTRATION.—At what elevation can an object be seen, at surface of ocean, when it is 2 miles distant?

$$1^2 : 2^2 :: .568 : 2.272\ \textit{feet} = 2\ \textit{feet}\ 3.25+\ \textit{ins.}$$

Difference between two distances in miles is as square root of their heights in feet.

ILLUSTRATION 1. — At an elevation of 9 feet above level of sea, at what distance can an object be seen upon its surface?

$$\sqrt{.568} = .754 : 1 :: \sqrt{9} : 3.98\ \textit{miles.}$$

2.—If a man at the fore-topgallant mast-head of a vessel, 100 feet from water, sees another and a large vessel "hull to," how far are the vessels apart?

A large vessel's bulwarks are at least 20 feet from water.

Then, by table, 100 feet.................= 13.27
20 " ....= 5.93
Distance........ 19.20 *miles.*

When an observation for distance is taken from an elevation, as from a light-house, a vessel's mast, etc., of an object that intervenes between observer and horizon, or contrariwise, observer being at a horizon to elevated object, distance of observer from intervening object can be determined by ascertaining or estimating its elevation from horizon, and subtracting its distance from whole distance between observer and point from which observation is taken, and remainder will give distance of object from observer.

ILLUSTRATION.—Top of smoke-pipe of a steamer, assumed to be 50 feet above surface of water, is in range with horizon from an elevation of 100 feet; what is distance to steamer from elevation?

100 feet....................= 13.27
50 " ....= 9.38
Distance........ 3.89 *miles.*

Approximately.—Curvature less Refraction = .566 D^2 for land and .563 D^2 for sea D *representing distance in miles.*

MAGNETIC VARIATION OF NEEDLE.

America. — Needle reached a Westerly maximum in 1660, and then varied to East until 1800, when it reversed to West.

London (Eng.). — From 1576 to 1815 variation ranged from 11° 15′ East to 24° 27′ West, when it receded gradually to 21° in 1865.

Jamaica (W. I.).—No variation from year 1660.

Diurnal Variation.—There is a small diurnal variation, being greatest in summer (15′), and least in winter (7′ 30″), added to which a change of temperature affects a needle.

Variation in U. S.—Professor Loomis concludes that the Westerly variation is increasing and Easterly diminishing in every part of United States; that this change occurred between 1793 and 1819, and that present annual change is about 2′ in Southern and Western States, from 3′ to 4′ in Middle States, and 5′ to 7′ in Eastern States.

Rules for computation of variation are empirical, except in each particular locality, as the annual and diurnal variations of the needle, added to local attraction, render it altogether unreliable.

Decennial Variation of Needle.

Mr. Schott, U. S. Coast and Geodetic Survey.

From January 1, 1790, *to January* 1, 1880.

LOCATION.	1790.	1800.	1810.	1820.	1830.	1840.	1850.	1860.	1870.	1880.
	W. °	W. °	W. °	W. °	W. °	W. °	W. °	W. °	W. °	W. °
Halifax, N. S.	15.1	15.9	16.7	17.4	18.1	18.7	19.3	19.8	20.1	20.3
Quebec, Can......	—	—	11.2	12.3	13.4	14.4	15.3	16	16.4	—
Portland, Me.	8.5	8.9	9.4	10	10.6	11.23	11.82	12.35	12.8	13.15
Burlington, Vt....	7.7	7.52	7.39	7.58	8.17	8.94	9.62	10.21	10.97	11.97
Newburyport, M's.	7.2	7.4	7.8	8.4	9	9.6	10.23	10.83	11.4	11.8
Portsmouth, N. H.	7.8	8	8.4	8.8	9.35	9.94	10.55	11.15	11.7	12.2
Rutland, Vt.	6.5	6.2	6.14	6.39	6.9	7.64	8.53	9.53	10.54	11.49
Salem, Mass......	6.2	6.2	6.5	7	7.8	8.7	9.8	10.9	11.9	12.8
Boston, Mass.	6.7	7	7.4	7.9	8.43	9.05	9.69	10.32	10.9	11.41
Cambridge, Mass. .	6.9	7.1	7.5	8	8.64	9.33	10.03	10.67	11.21	11.63
Providence, R. I. .	6.24	6.37	6.45	6.73	7.43	8.31	9.09	9.65	10.21	10.94
Hartford, Conn. ..	5.2	5.16	5.24	5.46	5.8	6.24	6.77	7.36	7.99	8.62
New Haven, Conn.	4.8	4.7	4.8	5	5.43	5.99	6.67	7.41	8.18	8.9
New York, N. Y. .	4.29	4.28	4.3	4.47	4.91	5.59	6.34	6.96	7.43	7.84
Philadelphia, Pa. .	2.4	2.1	2.1	2.28	2.71	3.33	4.11	4.99	5.89	6.76
Baltimore, Md....	—	—	.6	.8	1.2	1.7	2.4	2.9	—	—
Albany, N. Y.	—	—	5.4	5.79	6.32	6.97	7.7	8.47	9.2	9.9
		E.								
Buffalo, N. Y.	.14	.01	.05	.3	.74	1.33	2.05	2.85	3.68	4.49
	E.		E.	E.	E.					
Erie, Pa..........	.03	.35	.49	.43	.17	.25	.83	1.5	2.23	2.96
						E.	E.			
Cleveland, O......	2.2	2	1.8	1.5	1.05	.6	.14	.31	.72	1.07
								E.	E.	
Detroit, Mich.....	—	3.18	3.11	2.9	2.55	2.09	1.56	.99	.41	.13
		W.	W.	W.	W.	W.	W.	W.	W.	
Washington, D. C.	.1	—	.3	.6	1	1.49	1.99	2.47	2.9	3.26
		E.	E.	E.	E.	E.	E.	E.	E.	E.
Acapulco, Mex....	7.2	7.8	8.3	8.68	8.88	8.91	8.79	8.5	8.06	7.5
Charleston, S. C...	5.1	4.9	4.5	4.04	3.44	2.78	2.12	1.52	1	.62
Havana, Cuba....	—	6.2	6.26	6.22	6.12	5.94	5.71	5.44	5.1	—
Kingston, W. I....	6.3	6	5.7	5.4	5	4.6	4.2	3.8	3.4	—
San Diego, Cal....	11	11.1	11.3	11.6	11.9	12.2	12.54	12.88	13.2	13.5
Savannah, Ga.....	—	—	4.9	4.8	4.5	4.14	3.65	3.08	2.48	1.89
Mobile, Ala.......	—	7.1	7.2	7.3	7.2	7.1	7	8.8	—	6.3
Key West, Fla....	—	—	—	6.9	6.52	6.03	5.47	4.86	4.24	3.65
Monterey, Cal. ...	11.4	12	12.6	13.3	13.9	14.44	14.95	15.42	15.79	16.08
Mexico, Mex......	7.1	7.7	8.3	8.6	8.8	8.9	8.76	8.48	8.04	7.46
New Orleans, La. .	7	7.5	7.9	8.1	8.2	8.14	7.94	7.61	7.15	6.62
San Blas, Mex. ...	7.41	7.88	8.28	8.61	8.84	8.97	9.9	8.91	—	—
San Francisco, Cal.	12.8	13.4	13.9	14.42	14.92	15.38	15.78	16.11	16.36	16.52
Sitka, Alaska.....	—	26.12	27.11	27.89	28.48	28.88	29.08	29.08	28.88	28.5
Vera Cruz, Mex...	8.37	8.95	9.32	9.48	9.42	9.14	8.66	7.98	7.15	—

For variation in other locations in United States and North America, see Treatises of J. B. Stone, C.E., New York, and Heller and Brightly, Philadelphia, 1878.

Table for Reducing Observed Daily Variation of Needle to Mean Variation of the Day.

U. S. Coast and Geodetic Survey, 1878.

Season.	Needle East of Mean Magnetic Meridian.					Needle West of Mean Magnetic Meridian.							
	A. M.	A. M.	A. M.	A. M.	A. M.	A. M.	NOON.	P. M.	P. M.	P. M.	P. M.	P. M.	P. M.
	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11	Noon.	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6
	′	′	′	′	′	′	′	′	′	′	′	′	′
Spring	3	4	4	3	1	1	4	5	5	4	3	2	1
Summer	4	5	5	4	1	2	4	6	5	4	3	2	1
Autumn	2	3	3	2	—	2	3	4	3	2	1	1	—
Winter	1	1	2	2	1	—	2	3	5	2	1	1	—

Variation of Needle at Locations in United States and Canada, 1875.

U. S. Coast and Geodetic Survey.

EAST.

Location.	Variation. °	′	Location.	Variation. °	′
Astoria, W. T.	21	30	Montgomery, Ala.	5	2
Augusta, Ga.	2	28	Natchez, Miss.	7	26
Austin, Tex.	9	15	Nebraska, Neb.	11	20
Bismarck, Dak.	16	6	New Orleans, La.	6	50
Chicago, Ill.	5		Olympia, W. T.	22	8
Cincinnati, O.	2	55	Omaha, Neb.	11	
Colorado Springs, Col.	14	18	Oregon City, Or.	20	55
Columbia, S. C.	1	45	Paducah, Kan.	6	2
Columbus, O.	1	8	Portland, Or.	21	4
Deadwood, Dak.	16	20	Port Townsend, W. T.	23	
Denver, Col.	14	45	Sacramento, Cal.	17	4
Detroit, Mich.		3	Salt Lake City, Utah	17	
Duluth, Min.	10	12	San Antonio, Tex.	9	17
Galveston, Tex.	8	13	Santa Barbara, "	14	58
Green Bay, Wis.	6		Santa Fé, N. Mex.	13	18
Houston, Tex.	27		Springfield, Ill.	6	3
Indianapolis, Ind.	3	38	St. Augustine, Fla.	2	55
Jackson, Miss.	7		St. Louis, Mo.	6	30
Jacksonville, Fla.	3		St. Paul, Minn.	10	30
Kansas, Kan.	9	20	Tallahassee, Fla.	4	14
Keokuk, Ia.	7	55	Toledo, O.	1	2
Little Rock, Ark.	8	5	Topeka, Kan.	10	12
Louisville, Ky.	4		Vincennes, Ind.	5	
Milwaukee, Wis.	5	48	Yazoo, Miss.	7	2

WEST.

Location.	Variation. °	′	Location.	Variation. °	′
Augusta, Me.	14	34	Newburgh, N. Y.	8	
Bangor, Me.	16		Newport, R. I.	10	4
Batavia, N. Y.	4	40	Norfolk, Va.	2	35
Belfast, Me.	15	22	Ogdensburgh, N. Y.	9	25
Bridgeport, Conn.	8	12	Oswego, N. Y.	6	8
Calais, Me.	18		Ottawa, Can.	9	38
Concord, N. H.	11	42	Pittsburgh, Pa.	1	28
Dover, Del.	4	12	Raleigh, N. C.		24
Fall River, Mass.	10	30	Richmond, Va.	1	48
Hamilton, Can.	2	55	Rochester, N. Y.	5	20
Harrisburg, Pa.	4	18	Saratoga, N. Y.	9	40
Hudson, N. Y.	8	48	Stamford, Conn.	8	
Lewiston, Me.	14		Syracuse, N. Y.	7	
Lowell, Mass.	11	15	Toronto, Can.	3	50
Montpelier, Vt.	12	5	Trenton, N. J.	6	8
Montreal, Can.	12	20	Troy, N. Y.	9	25
New Bedford, Mass.	10	30	Utica, N. Y.	8	
New London, Conn.	9	15	Wilmington, Del.	4	52
Newark, N. J.	7	18	Wilmington, N. C.		18

Dip of Horizon.

Approximate, $57.4\sqrt{H} =$ *dip in seconds*, varying with temperature of air. H *representing height of observer's eye in feet.*

$.667\,n^2 = H$: $.498\,s^2 = H$: $1.42\sqrt{H} = s$: $1.23\sqrt{H} = n$.
n representing distance in geographical miles and s in statute.

Measurement of Heights with a Sextant.

Multiplier.	Angle.	Multiplier.	Angle.	Multiplier.	Angle.	Multiplier.	Angle.	Multiplier.	Angle.
	° ′		° ′		° ′		° ′		° ′
1	45 0	2.5	68 11	4	75 58	5.5	79 42	8	82 52
1.5	56 18	3	71 34	4.5	77 29	6	80 32	9	83 40
2	63 26	3.5	74 4	5	78 41	7	81 52	10	84 17

Operation.—Set sextant to any angle in table, and height will equal distance multiplied by number opposite to it.

ILLUSTRATION.—When sextant is set at 80° 32′, and horizontal distance from object in a vertical line is 100 feet, what is its height?

$100 \times 6 = 600$ *feet.*

By Trigonometry: 1 : 100 :: 5.997 (*tan.* angle) : 599.7 *feet.*

To Reduce a Sounding to Low Water.

$\frac{h}{2}\left(1 \mp \cos.\frac{180\,t}{t}\right) = h'$. *h representing vertical rise of tide, and h sounding or depth at low water, both in feet; t time between high and low water, and t′ time from time of sounding to low water, in hours.* — *cos. when* $\frac{180\,t}{t} < 90°$, *and* + *cos. when* $> 90°$.

ILLUSTRATION.—Low water occurring at 3.45, and high water at 10.15 P.M., a sounding taken at 5.30 P.M. was 18.25 feet; what was depth at low water, vertical rise being 10 feet?

$h = 10$ *feet*; $t' = 5h.\ 30m. - 3h.\ 45m. = 1h.\ 45m. = 1.75$ *hours.*
$t = 10h.\ 15m. - 3h.\ 45m. = 6h.\ 30m. = 6.5$ *hours.*

Then $\frac{10}{2}\left(1 \mp \cos.\frac{180 \times 1.75}{6.5}\right) = 5\,(1 - 48°\ 27'\ 24'') = 5 \times (1 - .663\,186) = 1.684\,07$ *feet.*

Sounding 18.25 *feet* — Reduction 1.684 07 *feet* = 16.565 93 *feet.*

Lengths of a Degree of Longitude on parallels of Latitude, for each of its Degrees from Equator to Pole.

Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.	Lat.	Miles.
1°	59.99	16°	57.67	31°	51.43	46°	41.68	61°	29.09	76°	14.52
2	59.96	17	57.38	32	50.88	47	40.92	62	28.17	77	13.5
3	59.92	18	57.06	33	50.32	48	40.15	63	27.74	78	12.48
4	59.85	19	56.73	34	49.74	49	39.36	64	26.3	79	11.45
5	59.77	20	56.38	35	49.15	50	38.57	65	25.36	80	10.42
6	59.67	21	56.01	36	48.54	51	37.76	66	24.4	81	9.38
7	59.55	22	55.63	37	47.92	52	36.94	67	23.44	82	8.35
8	59.42	23	55.23	38	47.28	53	36.11	68	22.48	83	7.31
9	59.26	24	54.81	39	46.63	54	35.27	69	21.5	84	6.27
10	59.09	25	54.38	40	45.96	55	34.41	70	20.52	85	5.23
11	58.89	26	53.93	41	45.28	56	33.45	71	19.53	86	4.18
12	58.69	27	53.46	42	44.59	57	32.68	72	18.54	87	3.14
13	58.46	28	52.97	43	43.88	58	31.79	73	17.54	88	2
14	58.22	29	52.48	44	43.16	59	30.9	74	16.54	89	1.05
15	57.95	30	51.96	45	42.43	60	30	75	15.53	90	.00

NOTE.—Degrees of longitude are to each other in length as Cosines of their latitudes.

Elements of Figure of the Earth.

Capt. A. R. Clarke, 1866.

	Feet.	Miles.
Major semi-axis of Equator (longitude 15° 34′ E.)	20 926 350	3 963.324.
Minor " " " " (" 105° 34′ E.)	20 919 972	3 962.115.
Polar " "	20 853 429	3 949.513.
Equatorial semi-axis	20 926 062	3 963.269.
Circumference, mean	——	24 898.562.
Diameter, "	——	7 916.

BOARD AND TIMBER MEASURE.

BOARD MEASURE.

In *Board Measure*, all boards are assumed to be 1 inch in thickness.

To Compute Measure or Surface.

When all Dimensions are in Feet.

RULE.—Multiply length by breadth, and product will give surface in square feet.

When either of Dimensions are in Inches.

RULE.—Multiply as above, and divide product by 12.

When all Dimensions are in Inches.

RULE.—Multiply as before, and divide product by 144.

EXAMPLE.—What are number of square feet in a board 15 feet in length and 16 inches in width?

$15 \times 16 = 240$, and $240 \div 12 = 20$ *feet.*

TIMBER MEASURE.

To Compute Volume of Round Timber.

When all Dimensions are in Feet.

RULE.—Add together squares of diameters of greater and lesser ends, and product of the two diameters; multiply sum by .7854, and product by one third of length.

Or, $a + a' + a'' \times \frac{l}{3} = V$, and $c^2 + c'^2 + \overline{c \times c'} \times .07958 \times \frac{l}{3} = V$. *a and a′ representing areas of ends, a″ area of mean proportional, l length, and c and c′ circumference of ends.*

NOTE.—Mean proportional is square root of product of areas of both ends.

ILLUSTRATION.—Diameters of a log are 2 and 1.5 feet, and length 15 feet.

$2^2 + 1.5^2 = \overline{4 + 2.25} + \overline{2 \times 1.5} = 9.25$, which $\times$ .7854 and $\frac{15}{3} = 36.32$ *cube feet.*

When Length in Feet, and Areas or Circumferences in Inches.

RULE.—Proceed as above, and divide by 144.

When all Dimensions are in Inches.

RULE.—Proceed as before, and divide by 1728.

NOTE.—Ordinary rule of Hutton, Ordnance Manual of U. S., and Molesworth, of $l \times \overline{c \div 4}^2$, gives a result of about .25 less than exact volume, or what it would be if the log was hewn or sawed to a square. *c representing mean circumferences.*

To Compute Volume of Squared Timber.

When all Dimensions are in Feet.

RULE.—Multiply product of breadth by depth, by length, and product will give volume in cube feet.

When either Dimension is in Inches.

RULE.—Multiply as above, and divide product by 12.

When any two Dimensions are in Inches.

RULE.—Multiply as before, and divide by 144.

EXAMPLE.—A piece of timber is 15 inches square, and 20 feet in length; required its volume in cube feet.

$$\frac{15 \times 15 \times 20}{144} = 31.25 \text{ cube feet.}$$

Allowance is to be made for bark, by deducting from each girth from .5 inch in logs with thin bark, to 2 inches in logs with thick bark.

Measures of Timber.—(*English.*)

100 superficial feet of planking = 1 square.
120 deals = 1 hundred.
50 cube feet of squared timber = 1 load.
40 feet of unhewn timber = 1 load.
600 superficial feet of inch planking = 1 load.

Deals.

Deals.—Boards exceeding 7 ins. in width, and if less than 6 feet in length, are termed deal ends.

Battens are similar to deals, but only 7 inches in width.

Balk.—Roughly squared log or trunk of a tree.

Planks are boards 12 ins. in width.

Local Standards.

Country.	Long.	Broad.	Thick.	Volume.
	Ft.	Ins.	Ins.	Cub. ft.
Russia and Prussia	12	11	1.5	1.375
Sweden	14	9	3	2.625
Norway	12	9	3	2.25
Christiana	11	9	1.25	.859
Quebec	12	11	2.5	2.292

100 Petersburgh standard deals equal 60 Quebec deals.

SPARS AND POLES.

Pine and *Spruce Spars*, from 10 to 4.5 inches in diameter inclusive, are to be measured by taking their diameter, clear of bark, at one third of their length from abut or large end.

Spars are usually purchased by the inch diameter; all under 4 inches are termed *Poles.*

Spars of 7 inches and less should have 5 feet in length for every inch of diameter, and those above 7 inches should have 4 feet in length for every inch of diameter.

Loss or Waste in Hewing or Sawing of Timber.

(*C. Mackrow.*)

Oak, English	200 per cent.
" African	100 " "
" Dantzic	50 " "
" American	10 " "
Yellow Pine from planks	10 per cent.
Teak	15 " "
Elm, English	200 " "
" American	15 " "

CISTERNS.

Capacity of Cisterns in Cube Feet and Gallons.

For each 10 Inches in Depth.

Diam.	Cub. ft.	Gallons.	Diam.	Cub. ft.	Gallons.	Diam.	Cub. ft.	Gallons.
Feet.			Feet.			Feet.		
2	2.618	19.58	9.5	59.068	441.8	17	189.15	1414.94
2.5	4.091	30.6	10	65.449	489.6	17.5	200.432	1499.33
3	5.89	44.07	10.5	72.158	539.78	18	212.056	1586.28
3.5	8.018	59.97	11	79.194	592.4	19	236.274	1767.45
4	10.472	78.33	11.5	86.558	647.5	20	261.797	1958.3
4.5	13.254	99.14	12	94.248	705	21	288.632	2159.11
5	16.362	122.4	12.5	102.265	764.99	22	316.776	2369.64
5.5	19.798	148.1	13	110.61	827.4	23	346.23	2589.97
6	23.562	176.24	13.5	119.282	892.29	24	376.992	2820.09
6.5	27.652	206.84	14	128.281	959.6	25	409.062	3059.8
7	32.07	239.88	14.5	137.608	1029.38	26	442.44	3309.67
7.5	36.816	275.4	15	147.262	1101.6	27	471.13	3569.17
8	41.888	313.33	15.5	157.243	1176.26	28	513.126	3838.44
8.5	47.288	353.72	16	167.552	1253.37	29	550.432	4117.51
9	53 014	396.55	16.5	178.187	1332.93	30	589.048	4406.08

Excavation and Lining of Wells or Cisterns.

For each 10 Inches in Depth.

Diameter.	Excavation.	Bricks. Number.	Bricks. Laid dry.	Masonry. 8 inches thick.	Masonry. 1 foot thick.	Diameter.	Excavation.	Bricks. Number.	Bricks. Laid dry.	Masonry. 8 inches thick.	Masonry. 1 foot thick.
Feet.	Cub. ft.		Cub. ft.	Cub. ft.	Cub. ft.	Feet.	Cub. ft.		Cub. ft.	Cub. ft.	Cub. ft.
3	12.29	126	5.24	6.4	10.47	8.5	63.29	356	14.83	16	24.87
3.5	15.29	147	6.11	7.27	11.78	9	69.89	377	15.71	16.87	26.18
4	18.62	168	6.98	8.14	13.09	9.5	76.81	398	16.58	17.75	27.49
4.5	22.27	188	7.85	9.02	14.4	10	84.07	419	17.45	18.62	28.8
5	26.25	209	8.73	9.89	15.71	10.5	91.65	440	18.33	19.49	30.11
5.5	30.56	230	9.6	10.76	17.02	11	99.56	461	19.2	20.36	31.42
6	35.2	251	10.47	11.64	18.33	12	116.36	503	20.94	22.11	34.03
6.5	40.16	272	11.34	12.51	19.63	13	134.46	545	22.69	23.85	36.65
7	45.45	293	12.22	13.38	20.94	14	153.88	586	24.43	25.6	39.27
7.5	51.07	314	13.09	14.25	22.25	15	174.61	628	26.18	27.34	41.89
8	57.02	335	13.96	15.13	23.56	16	196.64	670	27.92	29.09	44.51

Number of bricks and width of curb are taken at dimensions of ordinary brick—viz., 8 by 4 by 2.25 ins. = 72 cube ins.

In computing number of bricks required, an addition of 5 per cent. should be added for waste. It is to be considered, also, that diameter of excavation necessarily exceeds that of masonry.

SHINGLES.

Usually of white Cedar and Cypress; 27 inches in length and 6 to 7 inches in width, dressed to light .25 inch at point and .3125 inch at abut.

Laid in three thicknesses and courses of about 8 inches, so that less than .33 of a shingle is exposed to air, or about 2.25 shingles are required per square foot of roof.

Shingles, alike to Slates, are laid upon boards or battens.

SLATES AND SLATING.

A *Square* of Slate or Slating is 100 superficial feet.

Gauge is distance between the courses of the slates.

Lap is distance which each slate overlaps the slate lengthwise next but one below it, and it varies from 2 to 4 inches. Standard is assumed to be 3 inches.

Margin is width of course exposed or distance between tails of the slates.

Pitch of a slate roof should not be less than 1 in height to 4 of length.

To Compute Surface of a Slate when laid, and Number of Squares of Slating.

RULE.—Subtract lap from length* of slate, and half remainder will give length of surface exposed, which, when multiplied by width of slate, will give surface required.

Divide 14 400 (area of a square in inches) by surface thus obtained, and quotient will give number of slates required for a square.

EXAMPLE.—A slate is 24 × 12 inches, and lap is 3 inches; what will be number required for a square?

$24-3=21$, and $21 \div 2 = 10.5$, which $\times\ 12 = 126$ *inches;* and $14\,400 \div 126 = 114.29$ *slates.*

Dimensions of Slates.

[AMERICAN.]

Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
14 × 7	14 × 10	16 × 10	18 × 11	20 × 11	22 × 12	24 × 13
14 × 8	16 × 8	18 × 9	18 × 12	20 × 12	22 × 13	24 × 14
14 × 9	16 × 9	18 × 10	20 × 10	22 × 11	24 × 12	24 × 16

ENGLISH.

	Ins.		Ins.		Ins.
Doubles	13 × 10	Ladies	12 × 8	Marchioness ..	22 × 22
"	13 × 7	"	14 × 8	Duchess	24 × 12
Small doubles .	11 × 6	"	14 × 12	Imperial	30 × 24
"	10 × 5	"	15 × 8	Rags	36 × 24
Plantations ..	12 × 10	"	16 × 8	Queens	36 × 24
"	13 × 10	"	16 × 10	Empress	26 × 15
Viscountess ...	18 × 10	Countess	20 × 10	Princess	24 × 14

Thickness of slates ranges from .125 to .3125 of an inch, and their weight varies from 2 to 4.53 lbs. per sq. foot.

Weight of One Square Foot of Slating.

.125 in. thick on laths........	4.75 lbs.	.25 in. thick on laths........	9.25 lbs.
" " " " 1 in. boards..	6.75 "	" " " " 1 in. boards..	11.25 "
.1875 in. thick on laths.......	7 "	.3125 in. thick on laths......	11.15 "
" " " " 1 in. boards.	9 "	" " " " 1 in. boards,	14.10 "

Slate weighs from 167 to 181 lbs. per cube foot, and in consequence of laps, it requires an average of nearly 2.5 square feet of slate to make one of slating.

Weights per 1000 *and Number Required to Cover a Square.*

		Lbs.	No.			Lbs.	No.
Doubles	13 × 6	1680	480	Countess ...	20 × 10	6720	171
Ladies	15 × 8	2800	240	Duchess ...	24 × 12	4480	125

* Length of a slate is taken from nail-hole to tail.

PILING OF SHOT AND SHELLS.

To Compute Number of Shot.

Triangular Pile. RULE.—Multiply continually together, number of shot in one side of bottom course, and that number increased by 1, and again by 2, and one sixth of product will give number.

EXAMPLE.—What is number of shot in a triangular pile, each side of base containing 30 shot?

$$\frac{30 \times \overline{30+1} \times \overline{30+2}}{6} = \frac{29760}{6} = 4960 \textit{ shot.}$$

Square Pile. RULE.—Multiply continually together, number in one side of bottom course, and that number increased by 1, double same number increased by 1, and one sixth of product will give number.

EXAMPLE.—How many shells are there in a square pile of 30 courses?

$$\frac{30 \times \overline{30+1} \times \overline{30 \times 2+1}}{6} = \frac{56730}{6} = 9455 \textit{ shells.}$$

Oblong Pile. RULE.—From 3 times number in length of base course subtract one less than number in breadth of it; multiply remainder by number in breadth, and again by breadth, increased by 1, and one sixth of product will give number.

EXAMPLE.—Required number of shells in an oblong pile, numbers in base course being 16 and 7?

$$\frac{16 \times 3 - \overline{7-1} \times 7 \times \overline{7+1}}{6} = \frac{2352}{6} = 392 \textit{ shells.}$$

Incomplete Pile. RULE.—From number in pile, considered as complete, subtract number conceived to be in that portion of pile which is wanting, and remainder will give number.

FRAUDULENT BALANCES.

To Detect Them.—After an equilibrium has been established between weight and article weighed, transpose them, and weight will preponderate if article weighed is lighter than weight, and contrariwise if it is heavier.

To Ascertain True Weight. RULE.—Ascertain weight which will produce equilibrium after article to be weighed and weight have been transposed; reduce these weights to same denomination, multiply them together, and square root of their product will give true weight.

EXAMPLE.—If first weight is 32 lbs., and second, or weight of equilibrium after transposition, is 24 lbs. 8 oz., what is true weight?

24 lbs. 8 oz. $= 24.5$ lbs.

Then $32 \times 24.5 = 784$, and $\sqrt{784} = 28$ lbs.

Or, when a *represents longest arm*, b " *shortest arm*, | A *greatest weight*, and B *least weight*.

Then $Wa = Ab$, and $Wb = Ba$; multiplying these two equations, $W^2ab = ABab$, or $W^2 = AB$, and $W = \sqrt{AB}$.

ILLUSTRATION.—$A = 32$; $B = 24.5$; $W = 28$. Assume length of longest arm $= 10$.

Then $32 : 28 :: 10 : 8.75$.

Hence, $a = 10$, $b = 8.75$, or $28^2 = 32 \times 24.5$, and $\sqrt{32 \times 24.5} = 28$.

Weighing without Scales.

To Ascertain Weight of a Bar, Beam, etc., by Aid of a known Weight.

OPERATION.—Balance bar, etc., over a fulcrum, and note distance between it and end of its longest arm. Suspend a known weight from longest arm, and move bar, etc., upon fulcrum, so that bar with attached weight will be in equilibrio; subtract distance between the two positions of fulcrum from longest arm first obtained; multiply this remainder by weight suspended, divide product by distance between fulcrums, and quotient will give weight.

EXAMPLE.—A piece of tapered timber 24 feet in length is balanced over a fulcrum when 13 feet from less end; but when the body of a man weighing 210 lbs. is suspended from extreme of longest arm, the piece and weight are balanced when fulcrum is 12 feet from this end. What is weight of the timber?

$13 - 12 = 1$, and $13 - 1 = 12$ *feet*. Then $12 \times 210 \div 1 = 2520$ *lbs*.

PAINTING.

1 pound of paint will cover about 4 square yards for a first coat and about 6 yards for each additional coat.

Proportions of Colors for ordinary Paints.—By Weight.

COLORS.	White Lead.	Lamp-black.	Red Lead.	Red Ochre.	Verdigris.	Spanish Brown.	COLORS.	White Lead.	Lamp-black.	Red Lead.	Red Ochre.	Verdigris.	Spanish Brown.
White	100	—	—	—	—	—	Lead	98	2	—	—	—	—
Black.......	—	100	—	—	—	—	Red	—	—	50	50	—	—
Green	25	—	—	—	75	—	Chocolate..	—	4	—	—	—	96

These are the colors alone, to which boiled linseed oil, litharge, Japan varnish, and spirits turpentine are to be added according to the application of the paint.

Lamp black and litharge are ground separately with oil, then stirred into the lead and oil.

Thus for black paint: Lamp-black 25 parts, litharge 1, Japan varnish 1, boiled linseed oil 72, and spirits turpentine 1.

Tar Paint.—Coal tar 9 gallons, slaked lime 13 lbs., turpentine or naphtha 2 or 3 quarts.

A GALLON OF PAINT WILL COVER	Superficial feet.	A GALLON OF PAINT WILL COVER	Superficial feet.
On stone or brick, about	190 to 225	On well-painted surface or iron	600
On composite, etc., from	300 " 375	One gallon tar, first coat......	90
On wood, from	375 " 525	" " " second coat ...	160

Boiled Oil.—Raw linseed oil 91 parts, copperas 3, and litharge 6.

Put litharge and copperas in a cloth bag and suspend in middle of a kettle. Boil oil four hours and a half over a slow fire, then let it stand and deposit the sediment.

White Paint.

	Inside work.	Outside work.		Inside work.	Outside work.
White lead, in oil ..	80	80	Raw oil	—	9
Boiled oil.........	14.5	9	Spirits turpentine.	8	4

New wood-work requires 1 lb. to square yard for three coats.

Coats for 100 Square Yards New White Pine.

INSIDE.	White lead.	Raw oil.	Turpentine.	Drier.	OUTSIDE.	White lead.	Raw oil.	Boiled oil.	Turpentine.
	Lbs.	Pts.	Pts.	Lbs.		Lbs.	Pts.	Pts.	Pts.
Priming	16	—	6	.25	Priming....	18.5	2	2	—
2d coat.....	15	3.5	1.5	.25	2d and 3d coats	15	2	2	.5
3d "	13	2.5	1.5	.25					

.1 lb. of drier with priming and coating for outside.

HYDROMETERS.

U. S. Hydrometer (Tralle's) ranges from 0 (water) to 100 (pure spirit); it has not any subdivision or standard termed "Proof," but 50, upon stem of instrument, at a temperature of 60°, is basis upon which computations of duties are made.

In connection with this instrument, a Table of Corrections, for differences in temperature of spirits, becomes necessary; and one is furnished by the Treasury Department, from which all computations of value of a spirit are made.

ILLUSTRATION.—A cask contains 100 gallons of whiskey at 70°, and hydrometer sinks in the spirit to 25 upon its stem.

Then, by table, under 70°, and opposite to 25, is 22.99, showing that there are 22.99 gallons of pure spirit in the 100.

Commercial Hydrometer (Gendar's) has a "Proof" at 60°, which is equal to 50 upon U. S. Instrument and its gradations, run up to 100 with it, and down to 10 below proof, at 0 upon U. S. Instrument; or 0 of the Commercial Instrument is at 50 upon U. S. Instrument, from which it progresses numerically each way, each of its divisions being equal to two of latter.

In testing spirits, Commercial standard of value is fixed at proof; hence any difference, whether higher or lower, is added or subtracted, as case may be, to or from value assigned to proof.

A scale of Corrections for temperature being necessary, one is furnished with a Thermometer.

Application of Thermometer.—Elevation of the mercury indicates correction to be added or subtracted, to or from indication upon stem of hydrometer.
When elevation is above 60°, subtract correction; and when below, add it.

ILLUSTRATION.—A hydrometer in a spirit indicates upon its stem 50 below proof, and thermometer indicates 4 above 60° in appropriate column.

Then $50 - 4 = 46 =$ *strength below proof.*

To Compute Strength of a Spirit, or Volume of its Pure Spirit, by Commercial Hydrometer, and Convert it to Indication of a U. S. Hydrometer.

When Spirit is above Proof. RULE.—Add 100 to indication, and divide sum by 2.

When Spirit is below Proof. RULE.—Subtract indication from 100, and divide remainder by 2.

EXAMPLE.—A spirit is 11 above proof by a Commercial Hydrometer; what proportion of pure spirit does it contain?

$\overline{11 + 100} \div 2 = 55.5$ *per cent.*

To Compute Strength, etc., by a U. S. Hydrometer.

When Spirit is above Proof. RULE.—Multiply indication by 2, and subtract 100.

When Spirit is below Proof. RULE.—Multiply indication by 2, and subtract it from 100.

EXAMPLE.—A spirit is 55.5; what is its per centage above proof?

$\overline{55.5 \times 2} - 100 = 11$ *per cent.*

Commercial practice of reducing indications of a hydrometer is as follows:

Multiply number of gallons of spirit by per centage or number of degrees above or below proof, divide by 100, and quotient will give number of gallons to be added or subtracted, as case may be.

ILLUSTRATION.—50 gallons of whiskey are 11 per cent. above proof.

Then $50 \times 11 \div 100 = 5.5$, which added to $50 = 55.5$ *gallons.*

HYGROMETER.

Dew-point.—When air is gradually lowered in its temperature at a constant pressure, its density increases, and ratio of increase is sensibly same for the vapor as for the air with which it is combined, until a point is reached at which the density of the vapor becomes equal to the maximum density corresponding to the temperature.

This temperature is termed *dew-point* of given mass, and any further reduction of it will induce the condensation of a portion of the vapor in form of dew, rain, snow, or frost, according as temperature of surface is above or below freezing point.

Mason's or like Hygrometer.

To Ascertain Dew-point.

RULE. — Subtract absolute dryness from temperature of air, and remainder is dew-point.

EXAMPLE.—Temperature of air 57°, and absolute dryness 7°.

Hence $57^\circ - 7^\circ = 50^\circ$ *dew-point.*

To Ascertain Absolute Existing Dryness.

RULE.—Subtract temperature of wet bulb from temperature of air, as indicated by a dry bulb, add excess of dryness from following table, multiply sum by 2, and product will give absolute dryness in degrees.

EXAMPLE.—Temperature of air 57°, wet bulb 54°.

Then $57^\circ - 54^\circ = 3^\circ$, and $3^\circ + .5^\circ$ (from table) $\times 2 = 7^\circ$ *absolute dryness.*

Observed Dryness.	Excess of Dryness.	Observed Dryness.	Excess of Dryness.	Observed Dryness.	Excess of Dryness.	Observed Dryness.	Excess of Dryness.	Observed Dryness.	Excess of Dryness.
°	°	°	°	°	°	°	°	°	°
.5	.083	5	.833	9.5	1.583	14	2.333	18.5	3.083
1	.166	5.5	.9165	10	1.666	14.5	2.4165	19	3.166
1.5	.2495	6	1	10.5	1.7495	15	2.5	19.5	3.2495
2	.333	6.5	1.083	11	1.833	15.5	2.583	20	3.333
2.5	.4165	7	1.166	11.5	1.9165	16	2.666	20.5	3.4165
3	.5	7.5	1.2495	12	2	16.5	2.7495	21	3.5
3.5	.583	8	1.333	12.5	2.083	17	2.833	21.5	3.583
4	.666	8.5	1.4165	13	2.166	17.5	2.9165	22	3.666
4.5	.7495	9	1.5	13.5	2.2495	18	3	22.5	3.7495

To Compute Volume of Vapor in Atmosphere.

By a Hygrometer.

When temperature of atmosphere in shade, and of dew point are given.—If temperature of air and *dew-point* correspond, which is the case when both thermometers are alike, and air consequently saturated with moisture, then in table* opposite to temperature will be found corresponding weight of a cube foot of vapor in grains.

ILLUSTRATION.—Assume temperature of air and dew-point 70°. Then opposite temperature weight of a cube foot of vapor $= 8.392$ *grains.*

But if temperature of air is different from dew-point, a correction is necessary to obtain exact weight.

ILLUSTRATION.—Assume dew-point 70° as before, but temperature of air in shade 80°, then the vapor has suffered an expansion due to an excess of 10°, which requires a correction.

In table of corrections for 10° is 1.0208. Then divide 8.392 grains at dew point—viz., 70° by correction corresponding to degrees of absolute dryness—viz., 10°.

$\frac{8.392}{1.0208} = 8.221$ *grains of existing vapor*, which, subtracted from weight of vapor corresponding to temperature of 80°, will give number of grains required for saturation at that temperature.

11.333 grains at temperature of $80^\circ - 8.221$ contained in the air $= 3.112$ required for saturation.

---

* For table, see Mason's as published by Pike & Sons, New York, and compared with Sir John Leslie's and Professor Daniel's.

To ascertain relations of these conditions on natural scale of humidity (complete saturation being 1000), divide weight of vapor at dew-point by weight at temperature of air, and quotient will give degrees of saturation.

ILLUSTRATION.—Dew-point $= 70°$, weight $= 8.392$.
Then $8.392 \div 11.333$ (at $80°$) $= .7405$ degrees of humidity; saturation $= 1000$.

To Compute Weight of Vapor in a Cube Foot of Air.

See Pressures, Temperatures, Volumes, and Density of Steam, p. 708.

Thus, Required weight of vapor in a cube foot of saturated air at 212°.

At a temperature of 212° density or weight of 1 cube foot of air $= .038$ *lb.*

If density is required for any temperatures not in table, see rule, p. 706.

Humidity.—Condition of air in respect to its moisture involves amount of vapor present in air and ratio of it to amount which would saturate it at its temperature, and it is this element which is denoted by term *humidity*, and it is expressed as a per centage; thus, if weight of vapor present is .7 of that required for saturation, the humidity is 70.

Dry Air is air, humidity of which is below zero, but it is customary to term it dry when its humidity is below the average proportion.

NOTE.—Air in a highly heated space contains as much vapor (when weight of it is equal) as a like volume of external air, but it is drier as its capacity for vapor is greater.

SUN-DIAL.

To Set a Sun-dial.

Set column on which dial is to be placed perpendicular to horizon. Ascertain by spirit level that upper surface is perfectly horizontal; screw on plate loosely by means of centre screw, and bring gnomon as nearly as practicable to its proper direction.

On a bright day set dial at 9 A.M. and 3 P.M. exactly, with a correctly regulated watch; observe difference between them, and correct dial to half difference. Proceed in same manner till watch and dial are found to agree perfectly. Then fix plate firmly in that situation, and dial will be correctly set.

This is obvious; for, if there were any defects, the Sun's shadow would not agree with time indicated by watch, both before and after he passed meridian. Take care, however, to allow for equation of time, or you may set dial wrong. Best day in the year to set a dial is 15th of June, as there is no equation to allow for, and no error can arise from change of declination. A dial may be set without a watch, by drawing a circle around centre, and marking spot where top of shadow of an upright pin or piece of wire, placed in centre, just touches circle in A.M., and again in P.M. A line should be drawn from one spot to the other, and bisected exactly; then a line drawn from centre of dial through that bisection will be a true meridian line, on which the XII hours' mark should be set.

CHAINING OVER AN ELEVATION.

$$l\,C = L, \text{ and } C = \textit{cos. angle.}$$

l representing length of line chained, C cos. angle of elevation with horizon, and L length of line reduced to horizontal.

ILLUSTRATION.—Length of an elevation at an angle of 30° 17′ is 100 feet; what is horizontal distance?

By Table of Cosines, $30° 17' = .863\,54$. Hence, $100 \times .863\,54 = 86.354$ *feet.*

To set out a Right Angle with a Chain, Tape-line, etc.

Take 40 links on chain or feet of line for base, 30 links or feet for perpendicular, and 50 for hypothenuse, or in this ratio for any length or distance.

USEFUL NUMBERS IN SURVEYING.

For Converting	Multiplier.	Converse.	For Converting	Multiplier.	Converse.
Feet into links..	1.515	.66	Square feet into acres..	.000 022 9	43 560
Yards " " ..	4.545	.22	Square yards " " ..	.000 206 6	4 840

CHRONOLOGY.

Solar day is measured by rotation of the Earth upon its axis with respect to the Sun.

Motion of the Earth, on account of ellipticity of its orbit, and of perturbations produced by the planets, is subject to an acceleration and retardation. To correct this fluctuation, timepieces are adjusted to an average or mean solar day (*mean time*), which is divided into hours, minutes, and seconds.

In *Civil* computations day commences at midnight, or A.M., and is divided into two portions of 12 hours each.

In *Astronomical* computations and in *Nautical* time day commences at M., or 12 hours later than the civil day, and it is counted throughout the 24 hours.

Solar Year, termed also *Equinoctial*, *Tropical*, *Civil*, or *Calendar Year*, is the time in which the Sun returns from one Vernal Equinox to another; and its average time, termed a *Mean Solar Year*, is 365.242 218 solar days, or 365 *days*, 5 *hours*, 48 *minutes*, and 47.6 *seconds*.

Year is divided into 12 *Calendar* months, varying from 28 to 31 days.

Mean Lunar Month, or lunation of the Moon, is 29 days, 12 hours, 44 minutes, 2 seconds, and 5.24 thirds.*

Bissextile or *Leap Year* consists of 366 days; correction of one year in four is termed *Julian*; hence a mean *Julian* year is 365.25 days.

In year 1582 error of Julian computation of a year had amounted to a period of 10 days, which, by order of Pope Gregory VIII., was suppressed in the Calendar, and 5th of October reckoned as 15th.

Error of Julian computation, .007 76 days, is about 1 day in 128.79 years, and adoption of this period as a basis of intercalation is termed *Gregorian Calendar*, or *New Style*,† *Julian Calendar* being termed *Old Style*.

Error of Gregorian year (365.2425 days) amounts to 1 day in 3571.4286 years.

New Style was adopted in England in 1752 by reckoning 3d of September as 14th.

By an English law, the years 1900, 2100, 2200, etc., and any other 100th year, excepting only every 400th year, commencing at 2000, are not to be reckoned bissextile years.

Dominical or *Sunday Letter* is one of the first seven letters of alphabet, and is used for purpose of determining day of week corresponding to any given date. In *Ecclesiastical Calendar* letter A is placed opposite to 1st day of year, January 1st; B to second; and so on through the seven letters; then the letter which falls opposite to first Sunday in year will also fall opposite to every following Sunday in that year. See table, p. 73.

NOTE.—In bissextile years two Dominical letters are used, one before and the other after the intercalary day.

In *Ecclesiastical Year* the intercalary day is reckoned upon 24th of February; hence 24th and 25th days are denoted by same letter, the dominical letter being set back one place.

In *Civil Year* the intercalary day is added at end of February, the change of letter taking place at 1st of March.

Dominical Cycle is a period of 400 years, when the same order of dominical letters and days of the week will return.

Cycle of the Sun, or *Sunday Cycle*, is the 28 years before same order of Dominical letters return to same days of month, and it is considered as having commenced 9 years before the era of Julian Calendar.

To Compute Cycle of the Sun.

RULE.—Add 9 to given year; divide sum by 28; quotient is number of cycles that have elapsed, and remainder is number or years of cycle.

NOTE.—Use of this computation is determination of dominical letter for any given year of Julian Calendar for each of the 28 years of a cycle.

* Ferguson. † Now adopted in every Christian country except Russia and Greece.

By adoption of *Gregorian Calendar*, order of the letters is necessarily interrupted by suppression of the century bissextile years in 1900, 2100, etc., and a table of Dominical letters must necessarily be reconstructed for following century.

Lunar Cycle, or *Golden Number*, is a period of 19 years, after which the new moons fall on same days of the month of Julian year, within 1.5 hours.

Year of birth of Jesus Christ is reckoned first of the Lunar Cycle.

To Compute Lunar Cycle, or Golden Number.

RULE.—Add 1 to given year; divide sum by 19, and remainder is Golden Number.

NOTE.—If 0 remain, it is 19.

EXAMPLE.—What is Golden Number for 1879?

$1879 + 1 \div 19 = 98$, and remainder $= 18 =$ *Golden Number.*

Epact for any year is a number designed to represent age of the moon on 1st day of January of that year. See table, p. 73.

To Compute the Roman Indiction.

RULE.—Add 3 to given year; divide sum by 15, and remainder is Indiction.

NOTE.—If 0 remain, Indiction is 15.

Number of Direction is the number of days that Easter-day occurs after 21st of March.

Easter-day is first Sunday after first full moon which occurs upon or next after 21st of March; and if full moon occurs upon a Sunday, then Easter-day is Sunday after, and it is ascertained by adding number of direction to 21st of March. It is therefore March N + 21, or April N — 10.

ILLUSTRATION.—If Number of Direction is 19, then for March, $19 + 21 = 40$, and $40 - 31 = 9 =$ *9th of April;*

again for April, $19 - 10 = 9 =$ *9th of April.*

NOTE.—Moon upon which Easter immediately depends is termed *Paschal Moon.*

Full Moon is 14th day of moon, that is, 13 days after preceding day of new moon.

Days of the Roman Calendar.

Calends were the first 6 days of a month, *Nones* following 9 days, and *Ides* remaining days.

In March, May, July, and October, *Ides* fell upon 15th and *Nones* began upon 7th. In other months *Ides* commenced upon 13th and *Nones* upon 5th.

For Roman Indiction and Julian Period see p. 26.

Chronology.

B.C.

4004. Creation of World (according to Julius Africanus, Sept. 1, 5508; Samaritan Pentateuch, 4700; Septuagint, 5872; Josephus, 4658; Talmudists, 5344; Scaliger, 3950; Petavius, 3984; Hales, 5411).

2348 Deluge (according to Hales, 3154).
2247. Bricks made and Cement first used. Tower of Babel finished.
2203. Chinese Monarchy.
2090. First Egyptian Pyramid and Canal.
1920. Gold and Silver Money first introduced.
1891. Letters first used in Egypt.
1822. Memnon invents the Egyptian Alphabet.
1490. Crockery introduced.
1240. Axe, Wedge, Wimble, Lever, Masts and Sails invented by Daedalus of Athens.
1180. Troy destroyed.
1120. Mariner's Compass discovered in China.
753. Foundation of Rome.
640. Thales asserts Earth to be spherical.
605. Geometry, Maps, etc., first introduced.
576. Money coined at Rome.
562. First Comedy performed at Athens.
480. First recorded Map by Aristagoras.
420. First Theatre built at Athens.
336. Calippus calculates the revolution of Eclipses.
320. Aristotle writes first work on Mechanics.
310. Aqueducts and Baths introduced in Rome.
306. First Light house in Alexandria.
289. First Sun dial.
267. Ptolemy constructs a Canal from the Nile to the Red Sea.
224. Archimedes demonstrates the Properties of Mechanical Powers and the Art of measuring Surfaces, Solids, and Sections.
219. Hannibal crossed the Alps.
219. Surveying first introduced.
202. Printing introduced in China.

B.C.

198. Books with leaves of vellum first introduced by Attalus.
170. Paper invented in China.
168. An eclipse of the Moon which was predicted by Q. S. Gallus.
162. Hipparchus locates the first degree of Longitude and the Latitude at Ferro.
159. Clepsydra, or Water-clock, invented.
146. Carthage destroyed.
70. First Water-mill described.
51. Cæsar invaded Britain.
45. First Julian Year by Cæsar.
8. Augustus corrects the Calendar.

A.D.

69. Destruction of Jerusalem.
79. Destruction of Herculaneum and Pompeii.
214. Grist-mills introduced.
622. Year of Hegira, commencing 16th July; Glazed windows first introduced into England in this cent'y.
667. Glass discovered.
670. Stone buildings introduced into England.
842. Lands first enclosed in England.
933. Printing said to have been invented by the Chinese.
991. Arabic Numerals introduced.
1066. Battle of Hastings.
1111. Mariner's Compass discovered.
1180. Destruction of Troy. Mariner's Compass introduced in Europe.
1368. Chimneys first introduced into Rome from Padua.
1383. Cannon introduced.
1390. Woollens first made.
1434. Printing invented at Mayence.
1460. Wood-engraving invented and First Almanac.
1471. Printing in England by Caxton.
1477. Watches first introduced at Nuremberg.
1492. America discovered.
1497. Vasco de Gama discovers passage to India.
1500. Variation of Mariner's Compass observed.
1522. F. de Magellan circumnavigates the Globe.
1530. Incas conquered by Pizarro.
1545. Needles first introduced.
1586. Potato introduced into Ireland from America.
1590. Telescopes invented by Jansen and used in London in 1608.
1616. Tobacco first introduced into Virginia.
1620. Thermometer invented by Drebel.
1627. Barometer invented.
1629. First Printing-press in America.
1639. First Printing-office in America at Cambridge.
1647. Otto Van Gueriche constructed first electric machine.
1650. Railroads with wooden rails introduced near Newcastle.
1652. First Newspaper Advertisement.
1704. First Newspaper in America.
1705. Blankets first made at Bristol, England.
1752. Benjamin Franklin demonstrated identity of the electric spark and lightning, by aid of a kite.
1752. New Style, introduced into Britain; Sept. 3 reckoned Sept. 14.
1753. First Steam-engine in America.
1769. James Watt—First design and patent of a Steam-engine with separate vessel of condensation.
1772. Oliver Evans—Designed the Non-condensing Engine. 1792. Applied for a patent for it. 1801. Constructed and operated it.
1774. Spinning-jenny invented by Robert Arkwright.
1776. Iron Railway at Sheffield, England.
1783. First Balloon ascension, and Vessel's bottoms coppered.
1790. Water-lines first introduced in models of Vessels in the U. S.
1797. John Fitch—Propelled a yawl-boat by application of Steam to side-wheels, and also to a screw-propeller, upon Collect Pond, New York.
1807. Robert Fulton — First Passenger Steamboat.
1824. Compound marine steam-engines first introduced by James P. Allan, New York.
1825. Introduction of steam towing by Mowatt, Bros. & Co., of New York, by steam-boat "Henry Eckford," New York to Albany.*
1826. Voltaic Battery discovered by Alex. Volta, and First Horse-railroad.
1827. First Railroad in U. S., from Quincy to Neponset.
1829. First Lucifer Match and first Locomotive in America.
1830. Liverpool and Manchester Railroad opened. First Steel Pen and first Iron Steamer.
1832. S. F. B. Morse invents the Magnetic Telegraph.
1836. Robert L. Stevens first burned Anthracite Coal in furnace of boiler of steamboat "Passaic."
1840. First steam-boiler constructed for burning Anthracite Coal in steamboat "North America," N. Y.
1844. Telegraph line from Washington to Baltimore, Md.
1846. First complete Sewing-machine. Elias Howe, inventor.
1866. Submarine Telegraph laid from Valencia to Newfoundland, N.S.

* Witnessed by author.

Dates of Day of Week, corresponding to Day determined by following Table.

February, March, November.	February,* August.	May.	January, October.	January,* April, July.	September, December.	June.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Thus, if Monday is the day determined by the year given, the following dates are the Mondays in that year.

Epacts, Dominical Letters, and an Almanac, from 1800 to 1901.

USE OF TABLE. —To ascertain day of the week on which any given day of the month falls in any year from 1800 to 1901.

ILLUSTRATION.—The great fire occurred in New York on 16th of December, 1835; what was day of the week?

Opposite 1835 is Sunday; and by following table, under December, it is ascertained that 13th was Sunday; *consequently, 16th was Wednesday.*

Years.	Days.	Dom. Letters.	Epact.	Years.	Days.	Dom. Letters.	Epact.	Years.	Days.	Dom. Letters.	Epact.
1800	Saturday.	E	4	1834	Saturday.	E	20	1868	Sunday.*	ED	6
1801	Sunday.	D	15	1835	Sunday.	D	1	1869	Monday.	C	17
1802	Monday.	C	26	1836	Tuesday.*	CB	12	1870	Tuesday.	B	28
1803	Tuesday.	B	7	1837	Wednesd.	A	23	1871	Wednesd.	A	9
1804	Thursday.*	AG	18	1838	Thursday.	G	4	1872	Friday.*	GF	20
1805	Friday.	F	29	1839	Friday.	F	15	1873	Saturday.	E	1
1806	Saturday.	E	11	1840	Sunday.*	ED	26	1874	Sunday.	D	12
1807	Sunday.	D	22	1841	Monday.	C	7	1875	Monday.	C	23
1808	Tuesday.*	CB	3	1842	Tuesday.	B	18	1876	Wednesd.*	BA	4
1809	Wednesd.	A	14	1843	Wednesd.	A	29	1877	Thursday.	G	15
1810	Thursday.	G	25	1844	Friday.*	GF	11	1878	Friday.	F	26
1811	Friday.	F	6	1845	Saturday.	E	22	1879	Saturday.	E	7
1812	Sunday.*	ED	17	1846	Sunday.	D	3	1880	Monday.*	DC	18
1813	Monday.	C	28	1847	Monday.	C	14	1881	Tuesday.	B	29
1814	Tuesday.	B	9	1848	Wednesd.*	BA	25	1882	Wednesd.	A	11
1815	Wednesd.	A	20	1849	Thursday.	G	6	1883	Thursday.	G	22
1816	Friday.*	GF	1	1850	Friday.	F	17	1884	Saturday.*	FE	3
1817	Saturday.	E	12	1851	Saturday.	E	28	1885	Sunday.	D	14
1818	Sunday.	D	23	1852	Monday.*	DC	9	1886	Monday.	C	25
1819	Monday.	C	4	1853	Tuesday.	B	20	1887	Tuesday.	B	6
1820	Wednesd.*	BA	15	1854	Wednesd.	A	1	1888	Thursday.*	AG	17
1821	Thursday.	G	26	1855	Thursday.	G	12	1889	Friday.	F	28
1822	Friday.	F	7	1856	Saturday.*	FE	23	1890	Saturday.	E	9
1823	Saturday.	E	18	1857	Sunday.	D	4	1891	Sunday.	D	20
1824	Monday.*	DC	29	1858	Monday.	C	15	1892	Tuesday.*	CB	1
1825	Tuesday.	B	11	1859	Tuesday.	B	26	1893	Wednesd.	A	12
1826	Wednesd.	A	22	1860	Thursday.*	AG	7	1894	Thursday.	G	23
1827	Thursday.	G	3	1861	Friday.	F	18	1895	Friday.	F	4
1828	Saturday.*	FE	14	1862	Saturday.	E	29	1896	Sunday.*	ED	15
1829	Sunday.	D	25	1863	Sunday.	D	11	1897	Monday.	C	26
1830	Monday.	C	6	1864	Tuesday.*	CB	22	1898	Tuesday.	B	7
1831	Tuesday.	B	17	1865	Wednesd.	A	3	1899	Wednesd.	A	18
1832	Thursday.*	AG	28	1866	Thursday.	G	14	1900	Thursday.	G	29
1833	Friday.	F	9	1867	Friday.	F	25	1901	Friday.	F	11

* In leap-year, January and February must be taken in columns marked *.

To Ascertain Year or Years of Coincidences of a given Day of the Week with a given Day of a Month.

Look in preceding table and ascertain day of week opposite to year of occurrence, and every year in which same day is given will be year of coincidences required.

ILLUSTRATION.—If a child was born on Saturday, 19th Sept. 1829, when could and can his birthdays be celebrated, that occurred or are to occur on same day of week and date of month?

Opposite to 1829 is Sunday, and in preceding table the Sundays for September of that year were 6th, 13th, 20th; hence, if 20th was Sunday, the 19th was Saturday.

Hence, every year in table opposite to which is Sunday are the years of the coincidence required, as 1835, 1840, 1846, 1857, 1863, 1868, 1874, 1885, etc.

MOON'S AGE.

To Compute Moon's Age.

RULE.—To day of month add *Epact* and *Number* of month; from sum subtract 29 days, 12 hours, 44 min. and 2 sec., as often as sum exceeds this period, and result will give Moon's age approximately at 6 o'clock A.M. in United States, east of Mississippi River.

Numbers of the Months.

	d. h.		d. h.		d. h.		d. h.
January......	5	April.........	1 21	July.........	4 7	October......	7 16
February.....	1 22	May..........	2 8	August.......	5 18	November....	9 4
March........	9	June.........	3 19	September...	7 5	December....	9 15

EXAMPLE.—Required age of Moon on 25th February, 1877?

Given day 25 + epact 15 + number of month 1.22 = 41 *d.* 22 *h.* — 29 *d.* 12 *h.* 44 *m.* 2 *sec.* = 12 *d.* 9 *h.* 15 *min.* 58 *sec.*

In Leap-years add 1 day to result after 28th February.

To Compute Age of Moon at Mean Noon at any other Location than that Given.

RULE.—Ascertain age, and add or subtract difference of longitude or time, according as place may be West or East of it, to or from time given.

Or, when time of new Moon is ascertained for a location, and it is required to ascertain it for any other, *add* difference of longitude or time of the place, if East, and *subtract* it if it is West of it.

Moon's Southing, as usually given in United States Almanacs, both Civil and Nautical, is computed for Washington.

To Compute Time of High-water by Aid of American Nautical Almanac.

RULE.—Ascertain time of transit of Moon for Greenwich, preceding time of the high-water required.

For any other location (west of Greenwich), multiply the time in column "diff. for one hour" by longitude of location west of Greenwich, expressed in hours, and add product to time of transit.

NOTE.—It is frequently necessary to take the transit for preceding astronomical day, as the latter does not end until noon of day under computation.

EXAMPLE.—Required time of high-water at New York on 25th of August, 1864.

Longitude of New York from Greenwich = 4 *h.* 56 *m.* 1.65 *sec.*, which, multiplied by 2.17 *min.*, the difference for 1 hour = 10.71 *min.* for correction to be added to time of transit, to obtain time of transit at New York.

Time of transit, 18 *h.* 38.8 *m.*; then 18 *h.* 38.8 *m.* + 10.71 *m.* = 18 *hours* 49.51 *min.*

Time of transit at New York,	24 *d.* 18 *h.* 50 *m.*
Establishment of the Port,	8 13
	25 *d.* 3 *h.* 3 *m.* = *time of high-water.*

NOTE.—Time of 25th at 3 *h.* 3 *m.* Astronomical computation = 25th at 3 *h.* 3 *m.* P.M. Civil Time.

To Compute Time of High-water at Full and Change of Moon.

Time of High-water and Age of Moon on any Day being given.

RULE.—Note age of Moon, and opposite to it, in last column of following table, take time, which subtract from time of high-water at this age of Moon, added to 12 *h.* 26 *m.*, or 24 *h.* 52 *m.*, as case may require (when sum to be subtracted is greatest), and remainder is time required.

EXAMPLE.—What is time of high-water at full and change of Moon at New York?

Time of high water at Governor's Island on 25th of Jan. 1864, was 9 *h.* 20 *m.* A.M. civil time. Age of Moon at 12 M. on that day was 16 *d.* 8 *h.* 59 *m.*

Opposite to 16 *days*, in following table, is 13 *h.* 28 *m.*, and difference between 16 *d.* and 16 *d.* 12 *h.* = (16.5 — 16, or 13.53 — 13.28) is 25 *m.*; hence, if 12 *h.* = 25 *m.*, 16 *d.* 8 *h.* 59 *m.* — 16 *d.* = 8 *h.* 59 *m.* = 18.71 or 19 *m.*, which, added to 13 *h.* 28 *m.* = 13 *h.* 47 *m.*

Then 9 *h.* 20 *m.* + 12 *h.* 26 *m.* (as sum to be subtracted is greater than time) — 13 *h.* 47 *m.* = 21 *h.* 46 *m.* — 13 *h.* 47 *m.* = 7 *h.* 59 *m.*

This is a difference of but 13 minutes from *Establishment* of Port.

Time after apparent Noon before Moon next passes Meridian, Age at Noon being given.

(S. H. Wright, A.M., Ph.D.)

Age of Moon.	Moon at Meridian	Age of Moon.	Moon at Meridian.	Age of Moon.	Moon at Meridian.	Age of Moon.	Moon at Meridian.	Age of Moon.	Moon at Meridian.
Days.	H. M. P. M.	Days.	H. M. P. M.	Days.	H. M. P. M.	Days.	H. M. A. M.	Days.	H. M. A. M.
.0	0	6	5 3	12	10 6	18	15 8	24	20 11
.5	25	6.5	5 28	12.5	10 31	18.5	15 34	24.5	20 37
1	50	7	5 53	13	10 56	19	15 59	25	21 2
1.5	1 16	7.5	6 19	13.5	11 21	19.5	16 24	25.5	21 27
2	1 41	8	6 44	14	11 47 A. M.	20	16 49	26	21 52
2.5	2 6	8.5	7 9	14.5	12 12	20.5	17 15	26.5	22 17
3	2 31	9	7 34	15	12 37	21	17 40	27	22 43
3.5	2 57	9.5	7 59	15.5	13 2	21.5	18 5	27.5	23 8
4	3 22	10	8 25	16	13 28	22	18 30	28	23 33
4.5	3 47	10.5	8 50	16.5	13 53	22.5	18 56	28.5	23 58
5	4 12	11	9 15	17	14 28	23	19 21	29	24 24
5.5	4 38	11.5	9 40	17.5	14 43	23.5	19 46	29.5	24 48

Tidal Phenomena.

The elevation of a tidal wave towards the Moon slightly exceeds that of the opposite one, and the intensity of it diminishes from Equator to the Poles.

The Sun by its action twice elevates and depresses the sea every day, following the action of the Moon, but with less effect.

Spring Tides arise from the combined action of the Sun and Moon when they are on both sides of the Earth.

Neap Tides are the consequence of the divided action of the Sun and Moon, when they are on opposite sides of the Earth, and the greatest elevations and depressions do not occur until the 2d or 3d day after a full or a new Moon.

When Sun and Moon are in conjunction, and the time is near to the Equinoxes, the tides are fullest. The mean effect of the Moon on the tidal wave is 4.5 times that of the Sun. If, therefore, the Moon caused a tide of 6 feet, the Sun will cause one of 1.33 *feet*; hence a spring tide will be 7.33 *feet*, and a neap tide 4.67 *feet*.

Particular locations as to contour of shores, straits, capes, and rivers, lengths and depths of channels, shoals, etc., disturb these general rules.

LATITUDE AND LONGITUDE.

Latitude and Longitude of Principal Locations and Observatories.

Compiled from Records of U. S. Coast and Geodetic Survey and Topographical Engineer Corps, Imperial Gazetteer, and Bowditch's Navigator.

Longitude computed from Meridian of Greenwich.

A., *represents Academy;* Az., *Azimuth;* A. S., *Astronomical Station;* C., *College;* Cap., *Capitol;* Ch., *Church;* C. H., *City Hall;* C. S., *Coast Survey;* Ct., *Court-house;* Cy., *Chimney;* F.S., *Flagstaff;* G.S., *Geodetic Station;* Hos., *Hospital;* L. *Lighthouse;* Obs., *Observatory;* S.H., *State-house;* Sp. *Spire;* Sq., *Square;* S.S., *Signal Station;* T., *Telegraph;* T.H., *Town Hall;* U., *University;* Un., *Union;* B. *Baptist;* Con., *Congregational;* E., *Episcopal;* P., *Presby.;* and M.Ch., *Meth. Churches.*

Location.	Latitude.			Longitude.		
NORTH AND SOUTH AMERICA.		N.			W.	
	°	'	"	°	'	"
Acapulco .... Mex.	16	50	19	99	49	0
Albany, P.Ch.... N.Y.	42	39	3	73	45	24
Ann Arbor......Mich.	42	16	48	83	43	3
Annapolis........ Md.	38	58	42	76	29	6
Apalachicola, F.S. Fla.	29	43	30	84	59	
Astoria, F.S.......Or.	46	11	19	123	49	42
Atlanta, C. H......Ga.	33	44	57	84	23	22
Auburn.........N.Y.	42	55		76	28	
Augusta..........Ga.	33	28		81	54	
Augusta, B.Ch.... Me.	44	18	52	69	46	37
Austin...........Tex.	30	16	21	97	44	12
Balize............La.	29	8	5	89	1	4
Baltimore, Mon't. Md.	39	17	48	76	36	59
Bangor, Tho's Hill. Me.	44	48	23	68	46	59
Barbadoes, S.Pt.. W.I.	13	3		59	37	
Barnegat, L......N. J.	39	46		74	6	
Bath, W.S.Ch.....Me.	43	54	55	69	49	
Baton Rouge......La.	30	26		91	18	
Beaufort, Ct......N.C.	34	43	5	76	39	48
Beaufort, E.Ch...S.C.	32	26	2	80	40	27
Belfast, M.Ch.....Me.	44	25	29	69		19
Benicia, Ch...... Cal.	38	3	5	122	9	23
Benington ....Vt.	42	40		73	18	
Bismarck, S. S...Neb.	46	48		100	38	
Boston, L........Mass.	42	19	6	70	53	6
Boston, S.H..... "	42	21	30	71	3	30
Brazos Santiago..Tex.	26	6		97	12	
Bridgeport......Conn.	41	10	30	73	11	4
Bristol ....R. I.	41	40	11	71	16	5
Brooklyn, C.H...N.Y.	40	41	31	73	59	27
Brownsville, S.S..Tex.	26			97	30	
Brunswick, A.....Ga	31	8	51	81	29	26
Brunswick, C.Sp..Me.	43	54	29	69	57	24
Buffalo, L........N.Y.	42	50		78	59	
Burlington ......N. J.	40	4	52	74	52	37
Burlington, C......Vt.	44	28	52	78	10	
Burlington, Pub.Sq., Ia.	40	48	22	91	6	25
Bushnell ....Neb.	41	13	54	103	52	57
Cairo ....Ill.	36	59	48	89	11	14
Calais, C.S. Obs... Me.	45	11	5	67	16	5
		S.				
Callao, F.S.......Peru	12	4		77	13	
		N.				
Cambridge, Obs., Mass.	42	22	52	71	7	43
Camden..........S.C.	34	17		80	33	
Campeachy..Yucatan	19	49		90	33	

Location.	Latitude.			Longitude.		
NORTH AND SOUTH AMERICA.		N.			W.	
	°	'	"	°	'	"
Canandaigua ....N.Y.	42	54	9	77	17	
Cape Ann, S. L..Mass.	42	38	11	70	34	10
Cape Breton......Va.	45	57		59	48	5
Cape Canaveral...Fla.	28	27	30	80	33	
Cape Cod, L.P.L...Ms.	42	2		70	9	48
Cape Fear.......N.C.	33	48		77	57	
Cape Flattery, L..W.T.	48	23	15	124	43	54
Cape Florida, L...Fla.	25	39	54	80	9	2
Cape Hancock, Colo.R.	46	16	35	124	1	45
Cape Hatteras, L..N.C.	35	15	2	75	30	54
Cape Henlopen, L., Del.	38	46	6	75	4	7
Cape Henry, L....Va.	36	55	30	76	0	2
		S.				
Cape Horn, S.Pt., Hermit's Island.......	55	59		67	16	
		N.				
Cape May, L......N.J.	38	55	48	74	57	18
Cape Race.......N.S.	46	39	24	53	4	3
Cape Sable.......N.S.	43	24		65	36	
Cape Sable, C.S...Fla.	25	0	53	81	15	
		S.				
Cape St. Roque, Brazil	5	28		35	17	
		N.				
Carthagena......N.G.	10	26		75	38	
Castine...........Me.	44	22	30	68	45	
Cedar Keys, Depot Isl.	29	7	30	83	2	45
Chagres.........N.G.	9	20		80	1	21
Charleston, C.Ch., S.C.	32	46	44	79	55	39
Charlestown, Mon., Ms.	42	22	36	71	3	18
Cheboygan, L...Mich.	45	40	9	84	24	37
Chicago, C.Ch.....Ill.	41	53	48	87	37	47
Chickasaw......Miss.	35	53	30	88	6	25
Cincinnati, Obs.....O.	39	6	26	84	29	45
Cleveland, Hos..... "	41	30	25	81	40	30
Colorado Springs..Col.	38	50		104	49	8
Columbia, S.H....S.C.	33	59	58	81	2	3
Columbus, Cap.....O.	39	57	40	82	59	40
Concord, S. H....N. H.	43	12	29	71	29	
Corpus Christi...Tex.	27	47	18	97	27	2
Council Bluffs..Neb.T.	41	30		95	48	
Crescent City, L...Cal.	41	44	34	124	11	22
Cumberland......Md.	39	39	14	78	45	25
Darien, W.H......Ga.	31	21	54	81	25	39
Davenport, S. S....Ia.	41	32		90	38	
Dayton............O.	39	44		84	11	
Deadwood, S.S...Dak.	44	22		103	34	
Decatur, S. S.....Tex.	33	10		97	30	

Latitude and Longitude—*Continued.*

Location.	Latitude.	Longitude.
NORTH AND SOUTH AMERICA.	N.	W.
	° ′ ″	° ′ ″
Denver, S. H. Sp.. Col.	39 45	104 59 33
Des Moines, C. H... Ia.	41 35	93 37 16
Detroit, St. P. Ch., Mich.	42 19 46	83 2 23
Dover .... Del.	39 10	75 30
Dover .... N. H.	43 13	70 54
Dubuque.......... Ia.	42 29 55	90 39 57
Duluth, S. S...... Min.	46 48	92 8
Eastport, Con. Ch.. Me.	44 54 15	66 59 14
Edenton, C. H.... N. C.	36 3 24	76 36 31
Elizabeth City, Ct. "	36 17 58	76 13 23
Erie, L......... Penn.	42 8 43	80 4 12
Eureka, M. Ch.... Cal.	40 48 11	124 9 41
Falls St. Anth'y.. Minn.	44 58 40	93 10 30
Fernandina, A. S.. Fla.	30 40 18	81 27 47
Florence......... Ala.	34 47 13	87 41 40
Fort Gibson.... Ind. T.	35 47 35	95 15 10
Fort Henry..... Tenn.	36 30 22	88 3 40
Fort Laramie.. Neb. T.	42 12 10	104 47 43
Fort Leavenworth, Ks.	39 21 14	94 44
Frankfort........ Ky.	38 14	84 40
Frederick........ Md.	39 24	77 18
Fredericksburg, E. Ch., Va.	38 18 6	77 27 38
Fredericton...... N. B.	46 3	66 38 15
Galveston, Cath'l.. Tex.	29 18 17	94 47 26
Georgetown...... Ber.	32 22 2	64 37 6
Georgetown, E. Ch., S. C.	33 22 8	79 16 49
Gloucester, U. Ch... Ms.	42 36 46	70 39 59
Grand Haven, S. S., Mich.	43 5	86 18
Halifax, Obs...... N. S.	44 39 4	63 35
Harrisburg..... Penn.	40 16	76 50
Hartford, S. H... Conn.	41 45 59	72 40 45
Havana, Moro... Cuba	23 9	82 21 23
Hole in the Wall, L., Bahamas	25 51 5	77 10 6
Holmes's Hole, Ch., Ms.	41 27 13	70 35 59
Hudson......... N. Y.	42 14	73 46
Huntsville....... Ala.	34 36	86 57
Indianapolis..... Ind.	39 55	86 5
Indianola, G. S.... Tex.	28 32 28	96 31 1
Jackson......... Miss.	32 23	90 8
Jacksonville, M. Ch., Fla.	30 19 43	81 39 14
Jalapa.......... Mex.	19 30 8	96 54 30
Jefferson City.... Mo.	38 36	92 8
Jersey City, Gas Ch'y.	40 43 28	74 2 24
Kalama, M. Ch... W. T.	46 26	122 50 39
Keokuk, S. S....... Ia.	40 23	91 25
Key West, T. Obs., Fla.	24 33 31	81 48 31
Kingston..... Jamaica	17 58	76 46
Kingston, C. H... C. W.	44 8	76 28 37
Knoxville...... Tenn.	35 59	83 54
La Crosse, Ct. S... Wis.	43 58 50	91 14 48
Lancaster...... Penn.	40 2 36	76 20 33
Lavaca, A. S...... Tex.	28 37 36	96 37 21
Leavenworth, S. S.. Ks.	39 29	94 58
Lexington........ Ky.	38 6	84 18
	S.	
Lima........... Peru	12 3	77 6
	N.	
Little Rock...... Ark.	34 40	92 12
Lockport........ N. Y.	43 11	78 46
Los Angeles...... Cal.	34 3 5	118 14 32
Louisville........ Ky.	38 3	85 30
Lowell, St. Ann's Ch., Mass.	42 38 46	71 19 2
Machias, Th....... Me.	44 43 1	67 27 21
Macon, Arsenal.... Ga.	32 50 25	83 37 36
Madison, Dome... Wis.	43 4 33	89 24 3
Marblehead, L... Mass.	42 30 14	70 50 39
Martinique, S. P't. W. I.	14 27	60 55
Matagorda, G. S... Tex.	28 41 29	95 57 56
Matamoras....... "	25 52 50	97 27 50
Matanzas........ Cuba	23 3	81 40
Memphis, S. S... Tenn.	35 7	90 7
Mexico.......... Mex.	19 25 45	99 5 6
Milwaukee...... Mich.	43 2 24	87 54 4
Minneapolis, U. C., Min.	44 58 38	93 14 8
Mississippi City, G. S., Miss.	30 22 54	89 1 57
Mobile, E. Ch..... Ala.	30 41 26	88 2 28
Monterey, Az. S... Cal.	36 35 21	121 52 59
	S.	
Montevideo... Rat Is'd	34 53	56 13
	N.	
Montgomery, S. H., Ala.	32 22 45	86 18
Montreal........ C. E.	45 31	73 32 56
Mound City....... Ill.	37 4 47	89 12
Nantucket, L.... Mass.	41 23 24	70 2 24
Nantucket, S. Tower, Mass.	41 16 57	70 5 57
Nashville, U.... Tenn.	36 9 33	86 49 3
Nassau, L....... N. P.	25 5 2	77 21 2
Natchez........ Miss.	31 34	91 24 42
Nebraska, Junction of Forks of Platte Riv.	41 5 5	101 21 24
New Bedford, B. Ch., Mass.	41 38 10	70 55 36
New Haven, Col., Conn.	41 18 28	72 55 45
New London, P. Ch. "	41 21 16	72 5 29
New Orleans, Mint, La.	29 57 46	90 3 28
NEW YORK, C. H., N. Y.	40 42 44	74 24
Newbern, E. Sp... N. C.	35 6 21	77 5
Newburg, A. Sp., N. Y.	41 30 6	74 33
Newburyport, L., Mass.	42 48 30	70 52 28
New Castle, E. Ch., Del.	39 39 36	75 33 48
Newport, Sp...... R. I.	41 29 12	71 18 49
Norfolk, C. H...... Va.	36 50 47	76 7 22
Norwalk....... Conn.	41 2 50	73 25 35
Norwich........ "	41 33	72 7
Ocracoke, L...... N. C.	35 6 28	75 58 51
Ogdensburg, L... N. Y.	44 45	75 30
Old Point Comfort, Va.	37 2	76 18 6
Olympia...... Wash. T.	47 3	122 55
Omaha, P. Ch.... Neb.	41 15 43	95 56 14
Oswego, S. S..... N. Y.	43 28 32	76 35 5
Ottawa.......... Can.	45 23	75 42
Panama, Cath'l... N. G.	8 57 9	79 27 17
Parkersburg.... W. Va.	39 16 2	81 34 12
Pascagoula...... Miss.	30 20 42	88 32 45
Pensacola, Sq're.. Fla.	30 24 33	87 12 53
Petersburg, C. H... Va.	37 13 47	77 24 16

Latitude and Longitude—*Continued.*

Location.	Latitude.	Longitude.
NORTH AND SOUTH AMERICA.	N.	W.
	° ′ ″	° ′ ″
Philadelphia, S. H., Pa.	39 56 53	75 9 3
Pike's Peak, S.S. . Col.	38 48	104 59
Pittsburg.......Penn.	40 32	80 2
Plattsburg, Sp....N.Y.	44 41 57	73 26 54
Plymouth, Pier ...Ms.	41 58 44	70 39 12
Point Hudson....W.T.	48 7 3	122 44 33
Port au Prince...W. I.	18 33	72 16 3
Port Townshend, A.S., Wash. T.	48 6 56	122 44 58
Portland, C.H.....Me	43 39 28	70 15 1
Portland, S. S.......O.	45 30	122 27 30
Porto Bello......N.G.	9 34	79 40
Porto Cabello, Maracaibo	10 28	68 7
Portsmouth, L...N. H.	43 4 16	70 42 34
Prairie du Chien..Wis.	43 2	91 8 35
Princeton, S.Cap., N.J.	40 20 40	74 39 55
Providence, U.Ch., R.I.	41 49 26	71 24 19
Provincetown, Sp., Mass.	42 3	70 11 18
Puebla de los Angelos, Mex.	19 15	98 2 21
Quebec, Citadel..Can'a	46 49 12	71 12 15
Queenstown .... "	43 9	79 8
Raleigh, Square..N.C.	35 46 50	78 38 5
Richmond, Cap....Va.	37 32 16	77 26 4
	S.	
Rio de Janeiro, S. Loaf.	22 56	43 9
	N.	
Rochester, R. H. . N.Y.	43 8 17	77 51
Rockland, E.Ch...Me.	44 6 6	69 6 52
Sackett's Harbor, N.Y.	43 55	75 57
Sacramento.......Cal.	38 34 41	121 27 44
Salem, So.......Mass.	42 31 10	70 53 58
Salt Lake City, Obs., Utah	40 46 4	111 53 47
Saltillo..........Mex.	25 26 22	101 4 45
San Antonio.....Tex.	29 25 22	98 29 15
San Buenaventura, G. S............Cal.	34 15 46	119 15 56
San Diego, A.S.... "	32 43 68	117 9 40
San Francisco, C. S. Station.........Cal.	37 48	122 23 19
San José, Sp...... "	37 19 58	121 53 39
San Luis Obispo.. "	35 10 38	120 43 31
San Pedro........ "	33 43 20	118 16 3
Sandusky, L........O.	41 32 30	82 42 15
Sandy Hook, L...N.J.	40 27 40	74 9
Santa Barbara, M.Ch., Cal.	34 26 10	119 42 42
Santa Clara, C.Ch.. "	37 20 49	121 26 56
Santa Cruz, F. S.. "	36 57 31	122 1 29
Santa Fé......N. Mex.	35 41 6	106 1 22
Savannah, Sp......Ga.	32 4 52	81 5 26
Schenectady.....N.Y.	42 48	73 55
Sherman, R. R. D., Wy.	41 7 50	105 23 33
Shreveport, S.S....La.	32 30	93 45
Smithville, G.S...N.C.	33 54 58	78 1 8
Springfield......Mass.		
Springfield, S.H....Ill.	39 47 57	89 39 20
Springfield, S.S.... "	42 6	72 36
St. Augustine.....Fla.	29 48 30	81 35

Location.	Latitude.	Longitude.
NORTH AND SOUTH AMERICA.	N.	W.
	° ′ ″	° ′ ″
St. Augustine, P. Ch., Fla.	29 53 20	81 18 41
St. Bartholomew, S. Point..........W. I.	17 53 30	62 56 54
St. Christopher, N. Pt., W. I.	17 24	62 50
St. Croix, Obs.... "	17 44 30	64 40 42
St. Domingo..... "	18 29	69 52
St. Eustatia, Town. "	17 29	63
St. Jago de Cuba, Entrance........W. I.	19 58	75 52
St. John.........N. B.	45 14 6	66 3 30
St. Joseph........Mo.	23 3 13	109 40 44
St. Louis, W.U..... "	38 8 3	90 12 4
St. Mark's, Fort..Fla.	30 9 1	84 12 30
St. Martin's, Fort, W. I.	18 5	63 3
St. Mary's, M. H...Ga.	30 43 12	81 32 53
St. Paul........Minn.	44 52 46	95 4 54
St. Thomas, Fort Ch'n. W. I.	18 21	64 55 18
St. Vincent's, S. Point. W. I.	13 9	61 14
Staunton..........Va.	38 8 51	79 4 15
Stockton, S.S.....Tex.	30 50	102 50
Stonington, L...Conn.	41 19 36	71 54
Sweetwater River, Mouth of....Neb.T.	42 27 18	107 45 27
Sydney, S.S......N.S.	46 12	60 12
Syracuse........N.Y.	43 3	76 9 16
Tallahassee.......Fla.	30 28	84 36
Tampa Bay, E.Key "	27 30	82 45 15
Tampico, Bar....Mex.	22 15 30	97 51 51
Taunton, T.C.Ch., Mass.	41 54 11	71 5 55
Tobago, N. E. P'r. W. I.	11 20	60 27
Toronto..........Can.	43 39 35	79 23 21
Trenton, P.Ch ...N. J.	40 13 10	74 45 50
Trinidad, Fort...W. I.	10 39	61 32
Troy, D.Ch.......N.Y.	42 43	73 2 16
Tuscaloosa.......Ala.	33 12	87 42
Utica, Dut.Ch....N.Y.	43 6 49	75 13
	S.	
Valparaiso, Fort..Chili	33 2	71 41
	N.	
Vandalia..........Ill.	38 50	89 2
Vera Cruz.......Mex.	19 11 52	96 8 36
Vicksburg, S. S...Miss.	32 23	90 54
Victoria..........Tex.	28 46 57	97 1
Vincennes........Ind.	38 43	87 25
Virginia City, S.S., M.T.	45 20	112 3
WASHINGTON...Capitol	38 53 20	77 36
Watertown, Ars'l..Ms.	42 21 41	71 9 45
West Point......N. Y	41 23 26	73 57 1
Wheeling.........Va.	40 7	80 42
Wilmington, E. Ch., N. C.	34 14 2	77 56 38
Wilmington, T.H. . Del.	39 44 27	75 33 3
Worcester, Ant. H. . Ms.	42 16 17	71 48 13
Yankton, S.S.....Dak.	42 45	97 30
Yazoo..........Miss.	33 5	90 20
York...........Penn.	39 58	76 40
Yorktown........Va.	37 13	76 34

Latitude and Longitude—*Continued.*

LOCATION.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N.	E.
	° ′ ″	° ′ ″
Aleppo	36 11	37 10
Alexandria, L.	31 12	29 53
Algiers, L.	36 47	3 4
Amsterdam	52 22	4 53
Antwerp	51 13	4 24
Archangel	64 32	40 33
Athens	37 58	23 44
Barcelona	41 23	2 11
	S.	
Batavia, Obs.	6 8	106 50
Bencoolen, Fort, Su'a.	3 48	102 19
	N.	
Berlin, Obs.	52 30 16	13 23 45
Bombay, F.S.	18 56	72 54
	S.	
Botany Bay, C. Roads.	34 2	151 13
	N.	
Bremen	53 5	8 49
		W.
Bristol	51 27	2 35
		E.
Brussels, Obs.	50 51 11	4 22
Bussorah	30 30	48
		W.
Cadiz	36 32	6 18
		E.
Cairo	30 3	31 18
Calais	50 58	1 51
Calcutta	22 34	88 20
Candia	35 31	25 8
Canton	23 7	113 14
		W.
Cape Clear	51 26	9 29
	S.	E.
Cape of G. Hope, Obs.	33 56 3	18 28 45
Cape St. Mary, Mad'r.	25 39	45 7
	N.	
Ceylon, Port Pedro	9 49	80 23
Christiana	59 55	10 43
	S.	
Congo River	6 8	12 9
	N.	
Constantinople, St. S.	41 1	28 59
Copenhagen	55 41	12 34
Corinth	37 54	22 52
Cronstadt	59 59	29 47
Dover	51 8	1 19
		W.
Dublin	53 23 12	6 20 30
Edinburgh	55 57	3 12
	S.	
Falkland Islands, St. Helena, Obs.	15 55	5 45
	N.	
Fayal, S.E. Point	38 30	28 42
	S.	E.
Feejee Group, Ovolau, Obs.	17 41	178 53
	N.	
Florence	43 46	11 16
		W.
Funchal, Madeira	32 38	16 55
		E.
Geneva	46 11 59	6 9 15

LOCATION.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N.	E.
	° ′ ″	° ′ ″
Genoa	44 24	8 53
		W.
Gibraltar	36 7	5 22
Glasgow	55 52	4 16
GREENWICH	51 28 38	—
		E.
Hamburg	53 33	9 58
Havre	49 29	6
	S.	W.
Hawaii or Owyhee	20 23	155 54
	N.	E.
Hongkong	22 16 30	114 14 45
Honolulu	21 18 12	157 30 36
		W.
Hood Isl'd, Gallapagos.	1 23	89 46
	S.	E.
Hood's Island, Marquesas	9 26	138 57
	N.	
Jeddo or Tokio	35 40	139 40
Jerusalem	31 48	37 20
Leghorn, L.	43 32	10 18
Leipsic	51 20 20	12 22
Leyden	52 9 28	4 29 15
		W.
Lisbon	38 42	9 9
Liverpool, Obs.	53 24 48	3
		E.
Madras	14 4 9	80 15 45
		W.
Madrid	40 25	3 42
		E.
Majorca, Castle	39 34	2 23
		W.
Malaga	36 43	4 26
		E.
Malta, Valetta	35 54	14 30
Manila	14 36	121 2
Marseilles	43 18	5 22
Messina, L.	38 12	15 35
Mocha	13 20	43 12
Moscow	55 40	35 33
Muscat	23 37	58 35
Naples, L.	40 50	14 16
		W.
New Castle	54 58	1 37
	S.	E.
New Hebrides, Table Island	15 28	167 7
	N.	
Niphon, Cape Idron, Japan	34 36 3	138 50 35
Odessa	46 28	30 44
Palermo, L.	38 8	13 22
Paris, Obs.	48 50 13	2 20
Pekin	39 54	116 28
		W.
Plymouth	50 21	4 9
	S.	E.
Port Jackson, N.S.W.	35 51 32	151 18
	N.	W.
Porto Praya, Cape Verd Islands	14 54	23 3
	S.	E.
Prince of Wales Island	10 46	142 12

Latitude and Longitude—*Continued.*

Location.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N. ° ′ ″	W. ° ′ ″
Queenstown	51 47	8 19
		E.
Rome, St. Peter's	41 54	12 27
Rotterdam	5 54	4 29
		W.
Santa Cruz.....Ten'fe	28 28	16 16
Scilly, St. Agnes, L.	49 54	6 21
Senegal, Fort	16 1	16 32
		E.
Sevastopol	44 37	33 30
Seville	36 59	5 58
Siam	14 55	100
	S.	W.
Sierra Leone	8 30	13 18
	N.	E.
Singapore	1 17	103 50
Smyrna	38 26	27 7
		W.
Southampton	51	1 30
	S.	
St. Helena	15 55	5 45

Location.	Latitude.	Longitude.
EUROPE, ASIA, AFRICA, AND THE OCEANS.	N. ° ′ ″	E. ° ′ ″
St. Petersburg	59 56	30 19
Suez	29 59	32 34
Surat, Castle	21 11	72 47
	S.	
Sydney........N.S.W.	33 33 42	151 23
		W.
Tahiti or Otaheite	17 45	149 30
	N.	E.
Tangier	35 47	5 54
Toulon	43 7	5 22
Tripoli	34 54	13 11
Tunis, City	36 47	10 6
Venice	40 50	14 26
Vienna	48 13	16 23
Warsaw, Obs.	52 13 5	21 2 9
	S.	
Wellington...New Z'd	41 14	174 44
	N.	
Yokohama	35 26	139 39
	S.	
Zanzibar Island, Sp.	6 28	39 33

Observatories.—*Not included in previous Table.*

Longitude given in Time.

Location.	Latitude.	Longitude.
	N. ° ′ ″ ‴	W. h. m. s.
Albany, Dudley	42 39 49.55	4 54 59.52
Alleghany, Penn.	40 27 36	5 20 2.9
Birr Castle, Earl of Rosse	53 5 47	31 40.9
Cambridge, U. S.	42 22 52	4 44 30.9
		E.
Cambridge, Eng.	52 12 51.6	22.75
	S.	
Cape of G. Hope	33 56 3	1 13 55
	N.	
Copenhagen, Un'y.	55 40 53	50 19.8
		W.
Crescent City, A. S., Cal.	41 44 43	8 16 49.1
Dublin	53 23 13	25 22
Edinburgh	55 57 23.2	12 43.6
		E.
Florence	43 46 41.4	45 3.6
Geneva	46 11 59.4	24 37.7
		W.
Georgetown, U.S.	38 53 39	5 8 12.5
Gibbes's, Charleston, U. S.	32 47 7	5 19 44.7
GREENWICH	51 28 38	—
		E.
Hamburg	53 33 5	39 54.1
Leipsic	51 20 20.1	49 28.5
Leyden	52 9 28.2	17 57.5
		W.
Liverpool	53 24 47.8	12 0.11
L. M. Rutherfurd, New York	40 43 49	4 55 57

Location.	Latitude.	Longitude.
	N. ° ′ ″ ‴	E. h. m. s.
Madras	13 4 8.1	5 20 57.3
Marseilles	43 17 50	21 29
		W.
Mitchell's, Cin., O.	39 6 26	5 37 59
		E.
Moscow	55 45 19.8	2 30 16.96
Munich, Bogenh'n	48 8 45	46 26.5
Palermo	38 6 44	53 24.17
		W.
Portsmouth	50 48 3	4 23.9
Quebec	46 48 30	4 44 49.02
		E.
Rome, College	41 53 52.2	49 54.7
		W.
Salt Lake City, Utah	40 46 4	7 27 35.1
San Francisco, Sq., Cal.	37 47 55	8 9 38.1
	S.	
Santiago de Chili	33 26 24.8	4 42 18.9
	N.	
St. Croix, W. I.	17 44 30	4 18 42.8
		E.
St. Petersburg, A.	59 56 29.7	2 1 13.5
Stockholm	59 20 31	1 12 24.8
	S.	
Sydney	33 51 41.1	10 4 59.86
	N.	W.
Tifft's, Key West, Fla.	24 33 31	5 27 14.1
		E.
Unkrechtsberg, Olmutz	49 35 40	1 9 0.1
		W.
Washington	38 53 39	5 8 12.03
West Point, N. Y.	41 23 26	4 55 48

DIFFERENCE IN TIME.

Difference in Time at following Locations.

Longitude computed both from New York and Greenwich.

Exact Difference of Time between New York and Greenwich is 4 *h.* 56 *m.* 1.6 *sec.*, but in following table 2 seconds are given when the decimal in any reduction exceeds .5 seconds.

F *representing Fast, and* S *Slow.*

LOCATION.	New York. h. m. s.	Greenwich. h. m. s.	LOCATION.	New York. h. m. s.	Greenwich. h. m. s.
Acapulco	1 43 15 S.	6 39 17 S.	Cedar Keys	36 9 S.	5 32 11 S.
Albany	1 F.	4 55 2	Chagres	24 3	5 20 5
Alexandria..Egypt	6 55 34	1 59 32 F.	Charleston	23 41	5 19 43
Algiers	5 8 18	12 16	Charlestown	11 48 F.	4 44 13
Amsterdam	5 16 5	19 32	Cheboygan	41 37 S.	5 37 38
Antwerp	5 13 38	17 36	Chicago	54 30	5 50 31
Apalachicola	43 54 S.	5 39 56 S.	Chickasaw	56 24	5 52 26
Astoria	3 19 17	8 15 19	Cincinnati	41 57	5 37 59
Atlanta	41 32	5 37 33	Cleveland	30 40	5 26 42
Auburn	9 50	5 5 52	Colorado Springs..	2 3 15	6 59 17
Augusta........Ga.	31 34	5 27 36	Columbia	28 7	5 24 8
Augusta........Me.	16 55 F.	4 39 6	Columbus	35 57	5 31 59
Austin	1 34 55 S.	6 30 27	Concord	10 6 F.	4 45 56
Baltimore	10 26	5 6 28	Constantinople	6 51 58	1 55 56 F.
Bangor	20 54 F.	4 35 8	Copenhagen	5 46 18	50 16
Barbadoes, S. Pt.	57 34	3 58 28	Corpus Christi	1 33 47 S.	6 29 48 S.
Barnegat, L.	22 S.	4 56 24	Council Bluffs	1 27 10	6 23 12
Bath	16 46 F.	4 39 16	Crescent City	3 20 44	8 16 45
Baton Rouge	1 9 10 S.	6 5 12	Darien	29 41	5 25 43
Beaufort......N.C.	10 38	5 6 39	Davenport	1 12 30	6 2 32
Beaufort......S.C.	26 39	5 22 40	Dayton	40 42	5 36 44
Belfast	20 1 F.	4 36 1	Deadwood	1 58 30	6 54 32
Benicia	3 12 36 S.	8 8 38	Denver	2 3 57	6 59 58
Berlin	5 49 37 F.	53 35 F.	Detroit	36 76	5 32 10
Bismarck	1 46 30 S.	6 42 32 S.	Dover........Del.	35 58	5 2
Bombay, F.S.	9 47 38 F.	4 51 36 F	Dover........N. H.	12 26 F.	4 43 36
Boston, S.H.	11 47.6	4 44 14 S.	Dublin	4 30 40	25 22
Bremen	5 31 18	35 16 F.	Dubuque	1 6 38 S.	6 2 40
Bridgeport	3 17	4 52 44 S.	Duluth	1 12 10	6 8 32
Brooklyn, N. Yard.	4	4 55 58	Eastport	28 6 F.	4 27 56
Brunswick ....Me.	16 12	4 39 50	Edenton	10 24 S.	5 6 26
Brunswick.....Ga.	29 56 S.	5 25 58	Edinburgh	4 43 14 F.	12 48
Brussels	5 13 30 F.	17 28 F	Elizabeth City, N.C.	8 52 S	5 4 54
Buenos Ayres	1 2 34	3 53 28 S.	Erie	24 15	5 20 17
Buffalo, L.	19 54 S.	5 15 56	Eureka	3 20 37	8 16 39
Burlington......Ia.	1 8 24	6 4 26	Falls St. Anthony..	1 16 40	6 12 42
Burlington....N. J.	3 29	4 59 30	Fernandina	29 50	5 25 51
Burlington.....Vt.	16 38	5 12 40	Fire Island, L.	3 10 F.	4 52 51
Bushnell......Neb.	1 59 30	6 55 32	Florence......Ala.	54 45 S.	5 50 47
Cadiz	4 30 50 F.	25 12	Fort Gibson	1 24 59	6 21 1
Cairo	7 1 14	2 5 12 F.	Fort Henry..Tenn.	56 13	5 52 15
Cairo..........Ill.	1 43 S.	5 56 45 S.	Fort Laramie	2 3 9	6 59 11
Calais.........Me.	26 57 F.	4 29 4	Fort Leavenworth.	1 22 54	6 18 56
Calcutta	10 49 22	5 53 20 F	Frederick	13 10	5 9 12
Callao	12 50 S.	5 8 52 S.	Fredericksb'g..Va.	13 49	5 9 51
Cambridge ..Mass.	11 30 F.	4 44 31	Fredericton...N.B.	29 29 F.	4 26 33
Canton	12 28 58	7 32 56 F.	Funchal	3 48 22	1 7 40
Cape Girardeau	1 2 10 S.	5 58 12 S	Galveston	1 23 8 S.	6 19 10
Cape of Good Hope.	6 9 57 F.	1 13 55 F.	Geneva	5 20 39 F.	24 37 F.
Cape Horn	26 58	4 29 4 S.	Geneva.......N.Y.	12 14 S.	5 8 16 S.
Cape May	2 56 S.	4 58 58	Genoa	5 31 34 F.	35 32 F.
Cape Race	1 23 46 F.	3 32 16	Georgetown...Ber.	37 33	4 18 28 S.
Carthagena	6 30 S.	5 2 32	Georgetown...S.C.	21 6 S.	5 17 7
Castine	21 2 F.	4 35	Gibraltar	4 34 34 F.	21 28

Difference in Time—*Continued.*

Location.	New York. h. m. s.	Greenwich. h. m. s.
Glasgow	4 38 58 F.	17 4 S.
Gloucester	13 22	4 42 40
Grafton	24 5 S.	5 20 7
Grand Haven	49 10	5 45 12
GREENWICH	4 56 1.6	—
Halifax	41 42 F.	4 14 40
Hamburg	5 35 54	39 52 F
Harrisburg	11 18 S.	5 7 20 S.
Hartford	5 19 F.	4 50 43
Havana, Morro	33 24 S.	5 29 26
Havre	4 56 26 F.	24 F.
Hawaii or Owyhee	5 27 34 S.	10 23 36 S.
Hongkong	12 27 1 F.	7 36 59 F.
Honolulu	15 27 30	10 31 28
Hudson	1 12	4 54 40 S.
Huntsville	51 46 S.	5 47 48
Indianapolis	48 18	5 44 20
Indianola	1 30 2	6 26 4
Jackson	1 4 30	6 0 32
Jacksonville	30 35	5 26 37
Jalapa	1 31 36	6 27 38
Jeddo or Tokio	14 16 2 F.	9 20 F.
Jefferson City	1 12 30 S.	6 8 32 S.
Jersey City	8	4 56 10
Jerusalem	7 25 22 F.	2 29 20 F.
Kalama	3 15 21 S.	8 11 23 S.
Keokuk	1 9 38	6 5 40
Key West	31 13	5 27 14
Kingston....Can.	9 53	5 5 54
Kingston....Jam.	11 2	5 7 4
Knoxville	39 34	5 35 36
La Crosse	1 8 58	6 4 59
La Guayra	27 54 F.	4 28 8
Lancaster	19 21 S.	5 15 22
Lavaca	1 30 27	6 26 29
Leavenworth	1 23 40	6 19 52
Leghorn	5 37 14 F.	41 12 F.
Lexington	41 10 S.	5 37 12 S.
Lima	12 22	5 8 24
Lisbon	4 19 26 F.	36 36 F.
Little Rock	1 12 46 S.	6 8 48 S.
Liverpool	4 44 2 F.	12
Lockport	19 2 S.	5 15 4
Los Angeles	2 56 16	7 52 18
Louisville	45 58	5 42
Lowell	10 45 F.	4 45 16
Machias Bay	26 12	4 29 49
Macon	38 28 S.	5 34 30
Madison	1 1 35	5 57 36
Madrid	4 41 14 F.	14 48
Malaga	4 38 18	17 44
Malta	5 54 2	58 F.
Manila	13 10	8 4 8
Maracaibo	9 2	4 47 S.
Marblehead, L.	12 41	4 43 21
Marseilles	5 17 20	21 28 F.
Martinique	52 22	4 3 40 S.
Matagorda	1 27 50 S.	6 23 52
Matamoras	1 33 50	6 29 51
Matanzas	1 30 38	5 26 40
Memphis	1 4 26	6 0 28
Mexico	1 40 19	6 36 20
Milwaukee	55 35 S.	5 51 36 S.
Minneapolis	1 16 55	6 12 57
Mississippi City	1 6	5 56 8
Mobile	56 7	5 52 9
Montauk Point	18 22 F.	4 37 40
Monterey	3 11 30 S.	8 7 32
Montevideo	1 11 10 F.	3 44 52
Montgomery	49 10 S.	5 45 12
Montreal	1 50 F.	4 54 12
Montserrat	47 14	4 8 48
Moscow	7 18 14	2 22 12 F.
Mound City	1 26 S.	5 56 28 S.
Nantucket	15 38 F.	4 40 24
Naples	5 53 6	57 4 F.
Nashville	51 15 S.	5 47 16 S.
Nassau	13 23	5 9 24
Natchez	1 9 37	6 5 39
Nebraska	1 49 24	6 45 26
New Bedford	12 19 F.	4 43 42
New Haven	4 19	4 51 43
New London	7 40	4 48 22
New Orleans	1 4 12 S.	6 14
NEW YORK	—	4 56 1.6
Newbern	12 18	5 8 20
Newburg	1	4 56 2
Newburyport	12 32 F.	4 43 30
New Castle	4 49 34	6 28
New Castle...Del.	6 13 S.	5 2 15
Newport	10 46 F.	4 45 15
Norfolk	9 8 S.	5 5 9
Norwalk	2 19 F.	4 53 42
Norwich	7 34	4 48 28
Ocracoke	7 54 S.	5 3 55
Odessa	6 58 58 F.	2 2 56 F.
Ogdensburg	5 58 S.	5 2 S.
Old Point Comfort	9 11	5 5 12
Olympia	3 15 38	8 11 40
Omaha	1 27 43	6 23 45
Oswego	10 19	5 6 20
Ottawa	6 46	5 2 48
Paducah	58 22	5 54 24
Palermo	5 49 30 F.	53 28 F.
Panama	1 21 48 S.	5 17 49 S.
Paris	5 5 22 F.	9 20 F.
Parkersburg	30 15 S.	5 26 37 S.
Pekin	12 41 54 F.	7 45 52 F.
Pensacola	52 50 S.	5 48 52 S.
Petersburg	13 35.	5 9 37
Philadelphia	4 34 6	5 36.2
Pike's Peak	2 3 50	6 59 52
Pittsburg	24 6	5 20 8
Plattsburg	2 14 F.	4 53 48
Plymouth	4 39 26	16 36
Plymouth...Mass.	13 25	4 42 37
Port Au Prince, St. Domingo	16 34	4 39 28
Port Townshend	3 14 58 S.	8 11
Portland	15 2 F.	4 41
Portland......Or.	3 13 48 S.	8 9 50
Porto Praya	3 23 50 F.	1 32 12
Porto Rico	33 26	4 22 36
Portsmouth	13 11	4 42 51

Difference in Time—*Continued.*

Location.	New York.	Greenwich.	Location.	New York.	Greenwich.
	h. m. s.	h. m. s.		h. m. s.	h. m. s.
Prairie du Chien	1 8 33 S.	6 4 34 S.	St. Louis	4 47 S.	6 48 S.
Princeton	2 38	4 58 40	St. Mark's	40 48	5 36 50
Providence	10 24 F.	4 45 37	St. Mary's	30 10	5 26 12
Provincetown	15 16	4 40 45	St. Paul	11 24 18	6 20 20
Quebec	11 13	4 44 49	St. Petersburg	6 57 18 F.	2 1 16 F.
Queenstown, L.	4 42 46	33 16	St. Thomas, Fort	36 20	4 19 41 S.
Raleigh	18 31 S.	5 14 32	Staunton	20 15 S.	5 16 17
Richmond	13 43	5 9 44	Stockholm	6 8 26 F.	1 12 25 F.
Rio de Janeiro	2 3 26 F.	2 52 36	Stonington	8 26	4 37 36 S.
Rochester	15 22 S.	5 11 24	Suez	7 6 18	2 10 16 F.
Rockland	19 34 F.	4 36 27	Sweetwater River, Mouth of	2 15 S.	7 11 2 S.
Rome	5 45 50	49 48 F.	Sydney ....N.S.	8 2 F.	4 48
Rotterdam	5 13 58	17 56	Sydney...N.S.W.	15 1 34	10 5 32 F.
Sackett's Harbor	7 46 S.	5 3 48 S.	Syracuse	8 35 S.	5 4 37 S.
Sacramento	3 9 49	8 5 51	Tahiti or Otaheite	14 44 2 F.	9 48 F.
Salem	12 26 F.	4 43 36	Tallahassee	42 22 S.	5 38 24 S.
Salt Lake City	2 31 34 S.	7 27 35	Tampa Bay	34 59	5 31 1
Saltillo	1 48 17	6 44 19	Tampico Bar	1 35 25	6 31 27
San Antonio	1 37 55	6 33 57	Taunton	11 38 F.	4 44 24
San Buenaventura	3 1 2	7 57 4	Toronto	21 32 S.	5 17 33
San Diego	2 52 37	7 47 39	Toulon	5 17 30 F.	21 28 F.
San Francisco, C. S. S.	3 13 32	8 9 33	Trenton	3 2 S.	4 59 3 S.
San Francisco, P.	3 13 50	8 9 51	Tripoli	5 48 36 F.	52 44 F.
San José	3 11 33	8 7 35	Troy	3 54	4 52 9 S.
Sandusky	34 47	5 30 49	Tunis	5 36 26	40 24 F.
Sandy Hook	1 F.	4 56 1	Turk's Island	11 22	4 44 40 S.
Santa Barbara	3 2 49 S.	7 58 50	Tuscaloosa	54 46 S.	5 50 48
Santa Clara	3 9 46	8 5 48	Utica	4 50	5 52
Santa Cruz	3 12 4	8 8 6	Valparaiso	9 18	4 46 44
Santa Cruz, Ten'fe	3 50 58 F.	1 5 4	Vandalia	1 6	5 56 8
Santa Fé	2 8 4 S.	7 4 5	Venice	5 53 46 F.	57 44 F.
Savannah	28 20	5 24 22	Vera Cruz	1 28 33 S.	6 24 34 S.
Schenectady	22 F.	4 55 40	Vicksburg	1 7 34	6 3 36
Seville	4 32 10	23 52	Victoria.....Tex.	1 32 2	6 28 4
Sherman	2 5 33 S.	7 1 34	Vienna	6 1 34 F.	1 5 32 F.
Shreveport	1 18 58	6 15	Vincennes	53 38 S.	5 49 40 S.
Siam	11 36 2 F.	6 40 F.	Virginia City	2 32 10	7 28 12
Sierra Leone	4 2 50 S.	53 12 S.	Warsaw	6 20 11 F.	1 24 9 F.
Singapore	11 51 22 F.	6 55 20 F.	Washington, Obs.	12 10 S	5 8 12 S.
Smithville	16 3 S.	5 12 5 S.	West Point	14 F.	4 55 48
Smyrna	6 44 30 F.	1 48 28 F.	Wheeling	26 46 S.	5 22 48
Southampton	4 50 2	6 S.	Wilmington..Del.	6 11	5 2 12
Springfield....Ill.	1 2 36 S.	5 58 37	Wilmington..N.C.	15 45	5 11 47
Springfield..Mass.	5 39 F.	4 50 24	Worcester	8 49 F.	4 47 13
St. Augustine	29 13 S.	5 25 15	Yankton	1 33 58 S.	6 30
St. Croix, Obs.	37 19 F.	4 18 43	Yazoo	1 5 18	6 1 20
St. Helena	4 33 2	23	Yeddo	14 14 42 F.	9 18 40 F.
St. Jago de Cuba	7 26 S.	5 3 28	Yokohama	14 14 43	9 18 41
St. John	31 48 F.	4 24 14	York	10 38 S.	5 6 40 S.
St. Joseph....Mo.	2 22 41 S.	7 18 43	Yorktown	10 14	5 6 16

To Compute Difference of Time between New York and Greenwich and any Location not given in Table.

Rule. — Reduce longitude of location to time, and if it is W. of assumed meridian it is *Slow;* if E., it is *Fast.*

If difference for New York is required, and it exceeds 4 *h.* 56 *m.* 2 *sec.*, subtract this time, and remainder will give difference of time, S.; and if it (4 *h.* 56 *m.* 2 *sec.*) does not exceed it, subtract difference from it, and remainder will give difference of time, F.

TIDES.

Tide-Table for Coast of United States,

Showing Time of High-water at Full and New Moon, termed Establishment of the Port, being Mean Interval between Time of Moon's Transit and Time of High-water. (U. S. Coast and Geodetic Survey.)

Locations and Time.	h.	m.	Spring. Feet.	Neap. Feet.
COAST FROM EASTPORT TO NEW YORK.				
Eastport.........Me.	11	30	15	
Campo Bello*.... "	11		25	
Portland......... "	11	25	9.9	7.6
Cape Ann*....... "	11	30	11	
Portsmouth.....N.H.	11	23	9.9	7.2
Newburyport...Mass.	11	22	9.1	6.6
Salem.......... "	11	13	10.6	7.6
Cape Cod*...... "	11	30	6	
Boston Light ... "	11	12	10.9	8.1
Boston† .... "	11	27	10.3	8.5
Nantucket...... "	12	24	3.6	2.6
Edgartown ..... "	12	16	2.5	1.6
Holmes's Hole.. "	11	43	1.8	1.3
Tarpaulin Cove . "	8	4	2.8	1.8
Wood's Hole, n. side.	7	50	4.7	3.1
N. Bedford (Dumping Rock)	7	57	4.6	2.8
New York‡..... N.Y.	8	13	5.4	3.4
Albany*........ "	3	30	1	
LONG ISLAND SOUND.				
Newport........ R. I.	7	45	4.6	3.1
Point Judith..... "	7	32	3.7	2.6
Montauk Point.. N.Y.	8	20	2.4	1.8
Watch Hill ......R.I.	9	.	3.1	2.4
Providence*..... "	8	25	5	
Stonington ....Ct.	9	7	3.2	2.2
Little Gull Isl'd. N.Y.	9	38	2.9	2.3
New London......Ct.	9	28	3.1	2.1
New Haven....... "	11	16	6.2	5.2
Bridgeport .... "	11	11	8	4.7
Oyster Bay..... N.Y.	11	7	9.2	5.4
Sand's Point.... "	11	13	8.9	6.4
New Rochelle... "	11	22	8.6	6.6
Throg's Neck... "	11	20	9.2	6.1
Hell Gate*...... "	9	35	6	
COAST OF NEW JERSEY.				
Cold Spring Inlet. N.J.	7	32	5.4	3.6
Sandy Hook.....N.J.	7	29	5.6	4
Amboy.......... "	8	15	5	
Cape May Landing "	8	19	6	4.3
Egg Harbor*..... "	9	34	5	
DELAWARE BAY AND RIVER.				
Delaware Breakwater	8		4.5	3
Higbee's (Cape May)..	8	33	6.2	3.9
Egg Isl'd Light ..N.J.	9	4	7	5.1
New Castle ...... Del.	11	53	6.9	6.6
Philadelphia....Penn.	13	44	6.8	5.1

Locations and Time.	h.	m.	Spring. Feet.	Neap. Feet.
CHESAPEAKE BAY AND RIVERS.				
Old Pt. Comfort§..Va.	8	17	3	2
Cape Henry*..... "	7	51	6	
Point Lookout... Md.	12	58	1.9	.7
Annapolis....... "	17	4	1	.8
Bodkin Light.... "	18	8	1.3	.8
Baltimore....... "	18	59	1.5	.9
James R. (City Pt.), Va.	14	37	3	2.5
Richmond........ "	16	58	3.4	2.3
COASTS OF N. AND S. CAROLINA, GEORGIA, AND FLORIDA.				
Hatteras Inlet .. N.C.	7	4	2.2	1.8
Cape Hatteras ... "	9	1	5	
Beaufort .... "	7	26	3.3	2.2
Smithville (C. Fear) "	7	19	5.5	3.8
Charleston (C. H. Wharf) ...... S.C.	7	26	6	4.1
Fort Pulaski......Ga.	7	20	8	5.9
Savannah .... "	8	13	7.6	5.5
St. Augustine.... Fla.	8	21	4.9	3.6
Cape Florida .... "	8	84	1.8	1.2
Key West .... "	9	22	1.6	1
Tampa Bay ..... "	11	21	1.8	1
Cedar Keys...... "	13	15	3.2	1.6
WESTERN COAST.				
San Diego ....Cal.	9	38	5	2.3
San Pedro....... "	9	39	4.7	2.2
Cuyler's Harbor . "	9	25	5.1	2.8
San Luis Obispo.. "	10	8	4.8	2.4
Monterey........ "	10	22	4.3	2.5
South Farallone . "	10	37	4.4	2.8
San Francisco... "	12	6	4.3	2.8
Mare Island ..... "	13	40	5.2	4.1
Benicia .... "	14	10	5.1	3.7
Ravenswood..... "	12	36	7.3	4.9
Bodega.......... "	11	17	4.7	2.7
Humboldt Bay... "	12	2	5.5	3.5
Astoria...........Or.	12	42	7.4	4.6
Nee-ah Harbor, Wash.	12	33	7.4	4.8
Port Townshend "	3	49	5.5	4
MISCELLANEOUS.				
Bay of Fundy*..N.S.	12		60	
Blue Hill Bay*.. "	11		12	
St. John's*...... "	12		30	
Kingston*......Jam.	2	30	2	
Halifax*........N. S.			7.5	.5
Pensacola*.......Fla.			1.5	.4
Galveston*......Tex.			1.6	.8

* Refers to rise and fall of tide alone. † ‡ § ‖ see p. 85.

NOTE.—Mean interval has been increased 12 *h.* 26 *min.* (half a mean lunar day) for some ports in Delaware River and Chesapeake Bay, to give succession of times from the mouth; hence 12 *h.* 26 *min.* is to be subtracted from the *Establishments* which are greater than that, to give the interval required.

Bench Marks referred to in preceding Table.

† BOSTON.—Top of wall or quay, at entrance to dry-dock in Charlestown navy-yard, 14.76 feet above mean low-water.

‡ NEW YORK.—Lower edge of a straight line, cut in a stone wall, at head of wooden wharf on Governor's Island, 14.56 feet above mean low-water.

§ OLD POINT COMFORT, Va.—A line cut in wall of light-house, one foot from ground, on southwest side, 11 feet above mean low-water.

‖ CHARLESTON, S. C.—Outer and lower edge of embrasure of gun No. 3, at Castle Pinckney, 10.13 feet above mean low-water.

Establishment of the Port for several Locations in Europe, etc.

PORT.	TIME.	PORT.	TIME.	PORT.	TIME.
	h. m.		h. m.		h. m.
Amsterdam	3	Chatham	1 2	Liverpool	11 16
Antwerp	4 25	Cherbourg	7 49	London Bridge	2 7
Beachy Head..Eng.	11 50	Clear Cape	4	Newcastle	1 22
Belfast	10 43	Cowes	10 46	Portsmouth D.-yard, Eng.	11 41
Bordeaux	6 50	Dover Pier	11 12	Quebec	8
Bremen	6	Dublin Bar	11 12	Ramsgate Pier	10 27
Brest Harbor	3 47	Funchal	11 30	Rye Bay......Eng.	11 20
Bristol	7 21	Gravesend.....Eng.	1 14	Sheerness	57
Bristol Quay	6 27	Greenock	8	Sierra Leone	8 15
Cadiz	1 40	Holyhead	10 11	Southampton..Eng.	11 40
Calais	11 49	Hull..........Eng.	6 29	Thames R., mo'th "	12
Calf of Man	11 5	Land's End	3 57	Woolwich...... "	2 15
Cape St. Vincent	2 30	Lisbon	2 30		

Rise and Fall of Tides in Gulf of Mexico.

LOCATIONS.	Mean.	Spring.	Neap.	LOCATIONS.	Mean.	Spring.	Neap.
	Feet.	Feet.	Feet.		Feet.	Feet.	Feet.
St. George's Island....Fla.	1.1	1.8	.6	Isle Dernière..........La.	1.4	1.2	.7
Fort Morgan (Mobile Bay)............Ala.	1	1.5	.4	Entrance to Lake Calcasieu............La.	1.5	1.1	.6
Cat Island..........Miss.	1.3	1.9	.6	Aransas Pass........ "	1.1	1.8	.6
Southwest Pass.......La.	1.1	1.4	.5	Brazos Santiago..... "	.9	1.2	.5

Tides of Gulf of Mexico.

On Coast of Florida, from Cape Florida to St. George's Island, near Cape San Blas, the tides are of the ordinary kind, but with a large daily inequality. From St. George's Island, Apalachicola entrance, to Dernière Isle, the tides are usually of the single-day class, ebbing and flowing but once in 24 (lunar) hours. At Calcasieu entrance, double tides reappear, and except for some days about the period of Moon's greatest declination, tides are double at Galveston, Texas. At Aransas and Brazos Santiago the single-day tides are as perfectly well marked as at St. George's, Pensacola, Fort Morgan, Cat Island, and the mouths of the Mississippi. For some 3 to 5 days, however, about the time when the Moon's declination is nothing, there are generally two tides at all these places in 24 hours, the rise and fall being quite small.

Highest high and lowest low waters occur when greatest declination of Moon happens at full or change. Least tides when Moon's declination is nothing at first or last quarter.

Tides of Pacific Coast.

On Pacific coast there is, as a general rule, one large and one small tide during each day, heights of two successive high-waters occurring, one A.M., and other P.M. of same 24 hours, and intervals from next preceding transit of Moon are very different. These inequalities depend upon Moon's declination. When Moon's declination is nothing, they disappear, and when it is greatest, either North or South, they are greatest. The inequalities for low water are not same as for high, though they disappear, and have greatest value at nearly same time.

When Moon's declination is North, highest of two high tides of the 24 hours occurs at San Francisco, about 11.5 hours after Moon's southing (transit); and when declination is South, lowest of the two high tides occurs about this interval.

Lowest of two low-waters of the day is the one which follows next highest high-water.

STEAMING DISTANCES.

Distances between various Ports of United States and Canada.

By Lake, River, and Canal.

Locations.	Lake and River.	Canal.	Total.	Locations.	Lake and River.	Canal.	Total.
	Miles.	Miles.	Miles.		Miles.	Miles.	Miles.
Duluth to Buffalo	1024	1	1025	Chicago to New York, via Oswego	1195	232	1427
Chicago to Buffalo	925	—	925	Chicago to Montreal	1190	71	1261
Duluth to Oswego	1133	27	1160	Buffalo to Colborne, via Welland Canal	—	26.77	26.77
Chicago to Oswego	1034	26	1060	Buffalo to New York	142	352	494
Duluth to New York, via Buffalo	1166	353	1519	Welland Canal to Montreal	304.5	70.5	375
via Oswego	1294	233	1527	Montreal to Kingston	126.25	120	246.25
Duluth to Montreal	1289	72	1361	Ottawa to Kingston	—	126.25	126.25
Chicago to New York, via Buffalo	1067	352	1419				

Distances between various Ports and New York and London.

Not included in preceding Table.

Ports.	Miles. N.Y.	Miles. Lond.	Ports.	Miles. N.Y.	Miles. Lond.	Ports.	Miles. N.Y.	Miles. Lond.
Alexandria	4893	3 102	Cape Race	1 004	2 249	New Orleans	1790	4 730
Amsterdam	3291	262	Cowes	3 092	200	Norfolk	308	3 447
Barbadoes	1855	3 812	Funchal	2 760	1 303	Pensacola	1623	4 654
Batavia	8972	11 492	Galway	2 720	721	Philadelphia	262	3 404
Bermudas	682	3 142	Gibraltar	3 260	1 325	Quebec	1360	3 080
Bombay	8522	10 703	Glasgow	2 913	765	Queenstown	2780	551
Boston	356	3 030	Halifax	590	2 706	Rio Janeiro	4970	5 200
Bremen	3428	408	Havana	1 161	4 107	St. Johns	1064	2 214
Bristol	2979	501	Hobart Town	9 187	11 308	Southampton	3103	211
Buenos Ayres	6010	6 280	Kingston, Jam.	1 456	4 305	Swan River	8480	10 661
Cadiz	3125	1 115	Lima	10 050	10 149	Tortugas	1151	4 182
Calcutta	9350	11 531	Madras	8 707	10 888	Washington	461	3 612

Distances between various Ports of England, Canada, United States, etc.

Not included in preceding Table.

Ports.	Miles.	Ports.	Miles.	Ports.	Miles.
Halifax to		Liverpool to		Panama to	
Liverpool	2 563	Havana	4100	San Diego	2897
St. Thomas	1 563	Portland	2770	Monterey	3198
St Johns, N. F.	520	Baltimore	3400	San Francisco	3240
Quebec to Glasgow	2 563	N. Orleans to Havana	570	San Francisco to	
Liverpool to		Cape Race to		San Juan del Sud	2685
Boston	2 955	Fastnet	1711	Acapulco	1841
Quebec	2 855	Halifax	457	Manzanilla	1543
Philadelphia	3 147	Boston	835	San Diego	474
Callao	11 379	St. Johns, N. F., to		Monterey	105
Fastnet	283	Quebec	891	Humboldt	200
Cape Race	1 992	Boston	890	Columbia R. Bar	530
Aspinwall	4 650	Greenock	1848	Vancouver	638
Port Said	3 290	Bermudas to Nassau	804	Portland	650
Melbourne	13 290	Panama to		Port Townshend	732
Rio Janeiro	5 125	San Juan del Sud	570	Victoria	715
San Francisco	13 800	Gulf of Fonseca	739	Yokohama	4750
via Panama	7 378	Acapulco	1416	Honolulu	2080
via Tehuantepec	6 400	Manzanilla	1724	Honolulu to Callao	5145

Distances in Nautical Miles between Principal Ports of the World.

ILLUSTRATION OF TABLE.—Canton to San Francisco, 6090 miles.

New York to	Nautical Miles.
Canton *via* Pacific R. R.	10 500
" " Panama	11 500
" " Suez Canal	11 500
Callao *via* Panama	3 500
San Blas *via* Panama	3 860
Yokohama *via* San Francisco,	8 102
Calcutta *via* Panama	13 400
Melbourne *via* Cape Horn	12 720
Minatitlan	1 962
Puerto Cabello (Honduras)	2 114
Cape Race	1 004
Aspinwall	2 000
Fernandina	830
Calcutta	17 500
Valparaiso†	12 900
Galway	2 371
Callao†	13 500
San Blas†	17 800
St. Thomas	1 563
Brest	3 000
Hamburg	3 590
Port Said	3 365
New Orleans to Tehuantepec	787
Aspinwall	1 400

London to	Nautical Miles.
Canton *via* Pacific R. R.	13 500
Canton *via* Suez Canal	10 000
Manila *via* Suez Canal	9 650
Hongkong *via* Yokohama	9 780
Queenstown	605
Bermuda	3 055
Rio de Janeiro	5 200
Port Said	3 365
Liverpool to	
Melbourne *via* Cape Horn	11 355
Southampton to Gibraltar	1 160
Rio de Janeiro	5 060
Hongkong to Yokohama	1 620
Shanghai	1 800
Honolulu	4 356

	Nautical Miles.
Shanghai to Yokohama	1210
Sydney to Cape Horn	5470
Aspinwall to Panama	51
Portsmouth to	
Jamaica	4050
Halifax to Galway	2165
Boston to Galway	2385

	St. Petersburg	Port Jackson	New York	New Orleans	Marseilles	London	Liverpool	Lisbon	Havre	Hamburg	Gibraltar	Copenhagen	Constantinople	Cape Horn	Cape of Good Hope	Canton	Batavia
Canton																	1900
Cape of Good Hope																7000	5200
Cape Horn															3610	8840†	8820* / 8720†
Constantinople														8400	7040	14020*	12250*
Copenhagen													3673	8150	6900	13940*	12140*
Gibraltar												1953	1920	6470	5123	12140*	10330*
Hamburg											1657	492	3577	7810	6560	13600*	11800*
Havre										492	1189	830	3109	7460	6130	13140*	11340*
Lisbon									1040	1498	293	1835	2213	6020	4700	11700*	9900*
Liverpool								1010	550	992	1312	1329	3224	7400	6080	13100*	11310*
London							689	1196	190	417	1358	682	3278	7510	6260	13300*	11500*
Marseilles						2036	1846	973	1896	2336	680	2676	1470	7150	5800	12800*	11000*
New Orleans					5165	4730	4700	4370	4570	5030	4485	5370	6400	7600	7260	16440†	12470*
New York				1790	3850	3295	2986	2980	3068	3540	3260	3873	5100	7232	6790	16070†	12000*
Port Jackson			12900†	13260†	11800*	12250*	12100	10690*	12120*	12550*	11110*	12890*	13020*	5663	5990	4680	3910
St. Petersburg		13620	4100	6100	3400	1412	2060	2565	1560	1222	2653	730	4573	8880	7630	14670*	12870*
San Francisco	15250†	6454	13610†	13980	13500†	13880†	13800†	12400†	13840†	14200†	12850†	14520†	14770†	6380	9990†	6090	7650

* Around Cape of Good Hope. † Around Cape Horn.

Distances between Principal Ports of United States, etc.

ILLUSTRATION OF TABLE.—Sandy Hook to New Orleans, 1677 miles.

Nautical Miles.

New York to Matamoras	1 969
" " Coatzacoalcos	1 946
" " New Orleans *via* Florida Canal	1 293
" " Greytown	2 021
" " Bermudas	680
" " Nassau	962
" " Panama	2 047
" " " *via* Cape Horn	11 329
" " Minatitlan	1 962
" " Puerto Cabello (H.)	2 114
" " Cape Race	1 004
New Orleans to Minatitlan	816
" " Puerto Cabello (H.)	945
" " Tampico	717
" " Galveston	379
" " Matamoras	580
" " Coatzacoalcos	806
" " Greytown	1 308
Fernandina to Cedar Keys	154
Pensacola to Tortugas	472

By Rivers Exclusively, in Statute Miles.

Pittsburg	to Cincinnati	500
Cincinnati	" Louisville	150
Louisville	" Cairo	400
Cairo	" St. Louis	180
St. Louis	" Cincinnati	730
Cairo	" Memphis	240
"	" New Orleans	1040
Memphis	" Vicksburg	400
"	" Port Hudson	650
"	" New Orleans	800
Louisville	" " "	1450
Natchez	" " "	320

By Land, in Statute Miles.

New York to San Francisco	3344
New Orleans to Cairo	548
St. Louis to Omaha	479
New York to Omaha	1393
" Galveston	1900

	Vera Cruz	New Orleans	Havana	Key West	Savannah	Charleston	Cape Hatteras	Richmond	Norfolk	Washington	Baltimore	Cape Henry	Philadelphia	Cape Henlopen	New York	Sandy Hook	Nantucket Light	Boston	Halifax
Boston																			383
Nantucket Light																		103	365
Sandy Hook																	257	338	572
New York																18	275	356	590
Cape Henlopen															168	150	326	407	641
Philadelphia														94	262	244	420	501	735
Cape Henry													246	152	275	257	430	511	740
Baltimore												166	412	318	441	423	596	677	906
Washington											183	186	432	338	461	443	616	697	926
Norfolk										190	158	33	279	185	308	290	463	544	773
Richmond									101	275	223	129	375	281	404	386	559	640	869
Cape Hatteras								253	157	310	290	124	316	222	348	330	489	570	796
Charleston							265	527	431	584	564	398	597	503	623	605	761	842	1066
Savannah						98	341	603	507	660	640	474	673	579	699	681	837	918	1142
Key West					564	594	764	1026	930	1083	1063	897	1096	1002	1122	1104	1260	1341	1552
Havana				71	609	639	809	1071	975	1128	1108	942	1141	1047	1167	1149	1305	1386	1597
New Orleans			570	573	1137	1167	1337	1599	1503	1656	1636	1470	1669	1575	1790	1677	1833	1914	2125
Vera Cruz		782	877	859	1413	1443	1613	1875	1779	1932	1912	1746	1945	1851	1971	1953	2109	2190	2401
Puerto Bello	1558	1410	972	1142	1650	1643	1682	1928	1832	1985	1965	1799	1990	1896	2005	1987	2057	2138	2293

For relative distances by Statute miles between Western cities of U. S. and New York, Boston, etc., see Miscellaneous Notes, page 184, and for actual distances see page 187.

FRACTIONS.

A FRACTION, or broken number, is one or more parts of a UNIT.

ILLUSTRATION.—12 inches are 1 foot. Here, 1 *foot* is unit, and 12 *inches* its parts; 3 *inches* therefore, are *one fourth* of a foot, for 3 is fourth or quarter of 12.

A *Vulgar Fraction* is a fraction expressed by *two numbers* placed one above the other, with a line between them; as, 50 cents is the $\frac{1}{2}$ of a dollar.

Upper number is termed *Numerator*, the lower *Denominator*. *Terms* of a fraction express numerator and denominator; as, 6 and 9 are terms of $\frac{6}{9}$.

A *Proper* fraction has numerator equal to, or less than denominator; as, $\frac{1}{4}$, etc.

An *Improper* fraction is reverse of a proper one; as, $\frac{2}{1}$, etc.

A *Mixed* fraction is a compound of a whole number and a fraction; as, $5\frac{7}{8}$, etc.

A *Compound* fraction is fraction of a fraction; as, $\frac{1}{2}$ of $\frac{3}{4}$, etc.

A *Complex* fraction is one that has a fraction for its numerator or denominator, or both; as, $\frac{\frac{1}{2}}{6}$, or $\frac{\frac{5}{8}}{4}$, or $\frac{2}{\frac{8}{4}}$, or $\frac{3\frac{1}{2}}{6}$, etc.

NOTE.—A Fraction denotes division, and its value is equal to quotient, obtained by dividing numerator by denominator; thus, $\frac{12}{4}$ is equal to 3, and $\frac{21}{5}$ is equal to $4\frac{1}{5}$.

Reduction of Fractions.

To Compute Common Measure or greatest Number that will divide Two or more Numbers without a Remainder.

RULE.—Divide greater number by less; then divide divisor by remainder; and so on, dividing always last divisor by last remainder, until there is no remainder, and last divisor is greatest common measure required.

EXAMPLE 1.—What is greatest common measure of 1908 and 936?

```
936) 1908 (2
     1872
       36) 936 (26
           72
           216.  Hence 36.
```

2.—How many squares can there be obtained in an area of 90 by 160 feet?

Here 10 is greatest common measure.

Hence, $\frac{160}{10} = 16$, and $\frac{90}{10} = 9$; therefore $16 \times 9 = 144$.

To Compute least Common Multiple of Two or more Numbers.

RULE.—Divide given numbers by any number that will divide the greatest number of them without a remainder, and set quotients with undivided numbers in a line beneath.

Divide second line in same manner, and so on, until there are no two numbers that can be divided; then the continued product of divisors and last quotients will give common multiple required.

EXAMPLE.—What is least common multiple of 40, 50, and 25?

```
5) 40 . 50 . 25
5)  8 . 10 .  5
2)  8 .  2 .  1
    4 .  1 .  1.
```

Then $5 \times 5 \times 2 \times 4 \times 1 \times 1 = 200$.

To Reduce a Fraction to its Lowest Term.

RULE.—Divide terms by any number or series of numbers that will divide them without a remainder, or by their greatest common measure.

EXAMPLE.—Reduce $\frac{720}{960}$ of a foot to its lowest terms.

$$\frac{720}{960} \div 10 = \frac{72}{96} \div 8 = \frac{9}{12} \div 3 = \frac{3}{4}, \text{ or 9 } ins.$$

To Reduce a Mixed Fraction to its Equivalent, an Improper Fraction.

RULE.—Multiply whole number by denominator of fraction and to product add numerator; then set that sum above denominator.

EXAMPLE I.—Reduce $23\frac{2}{6}$ to a fraction. $\frac{23 \times 6 + 2}{6} = \frac{140}{6} = \frac{70}{3}$.

2.—Reduce $\frac{123}{6}$ inches to its value in feet. $123 \div 6 = 20\frac{3}{6} = 1$ *foot* $8\frac{1}{2}$ *ins.*

To Reduce a Complex Fraction to a Simple one.

RULE.—Reduce the two parts both to a simple fraction, multiply numerator of reduced fraction by denominator of reduced denominator, and denominator of numerator fraction by numerator of denominator fraction.

EXAMPLE.—Simplify complex fraction $\frac{2\frac{2}{3}}{4\frac{4}{5}}$. $\frac{2\frac{2}{3}}{4\frac{4}{5}} = \frac{\frac{8}{3}}{\frac{24}{5}}$ $\frac{8 \times 5 = 40}{3 \times 24 = 72} = \frac{5}{9}$.

To Reduce a Whole Number to an Equivalent Fraction having a given Denominator.

RULE.—Multiply whole number by given denominator, and set product over said denominator.

EXAMPLE.—Reduce 8 to a fraction, denominator of which shall be 9.

$8 \times 9 = 72$; then $\frac{72}{9}$ *result required.*

To Reduce a Compound Fraction to an Equivalent Simple one.

RULE.—Multiply all numerators together for a numerator, and all denominators together for a denominator.

NOTE.—When there are terms that are common, they may be cancelled.

EXAMPLE.—Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$ to a simple fraction.

$\frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$. Or, $\frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{4}$, *by cancelling* 2's *and* 3's.

To Reduce Fractions of different Denominations to Equivalents having a Common Denominator.

RULE.—Multiply each numerator by all denominators except its own for new numerators; and multiply all denominators together for a common denominator.

NOTE.—In this, as in all other operations, whole numbers, mixed or compound fractions, must first be reduced to form of simple fractions.

2. When many of denominators are same, or are multiples of each other, ascertain their least common multiple, and then multiply the terms of each fraction by quotient of least common multiple divided by its denominator.

EXAMPLE.—Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

$$\left.\begin{array}{l} 1 \times 3 \times 4 = 12 \\ 2 \times 2 \times 4 = 16 \\ 3 \times 2 \times 3 = 18 \end{array}\right\} = \frac{12}{24} = \frac{16}{24} = \frac{18}{24}, \text{ or } \frac{6}{12}, \frac{8}{12} \text{ and } \frac{9}{12}.$$
$$2 \times 3 \times 4 = 24$$

Addition.

RULE.—If fractions have a common denominator, add all numerators together, and place sum over denominator.

NOTE.—If fractions have not a common denominator, they must be reduced to one. Also, compound and complex must be reduced to simple fractions.

EXAMPLE I.—Add $\frac{1}{4}$ and $\frac{3}{4}$ together. $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$.

2.—Add $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{6}{10}$ to $2\frac{1}{8}$ of $\frac{3}{4}$.

$\frac{1}{2} \times \frac{3}{4} \times \frac{6}{10} = \frac{18}{80}$. $2\frac{1}{8}$ of $\frac{3}{4} = \frac{17}{8} \times \frac{3}{4} = \frac{51}{32}$.

Then, $\frac{18}{80} + \frac{51}{32} = \frac{4080}{2560} + \frac{576}{2560} = 1\frac{131}{160}$, *reduced to equivalent fractions having a common denominator and thence to its lowest terms.*

Subtraction.

RULE.—Prepare fractions same as for other operations, when necessary; then subtract one numerator from the other, and set remainder over common denominator.

EXAMPLE.—What is difference between $\frac{3}{9}$ and $\frac{6}{8}$?

$$\left.\begin{array}{l} 6 \times 9 = 54 \\ 3 \times 8 = 24 \\ 8 \times 9 = 72 \end{array}\right\} = \frac{54}{72} - \frac{24}{72} = \frac{30}{72} = \frac{15}{36} = \frac{5}{12}.$$

Multiplication.

RULE.—Prepare fractions as previously required; multiply all numerators together for a new numerator, and all denominators together for a new denominator.

EXAMPLE 1.—What is product of $\frac{3}{4}$ and $\frac{3}{9}$? $\frac{3}{4} \times \frac{3}{9} = \frac{9}{36} = \frac{1}{4}$.

2.—What is product of 6 and $\frac{2}{3}$ of 5? $\frac{6}{1} \times \frac{2}{3}$ of $5 = \frac{6}{1} \times \frac{10}{3} = \frac{60}{3} = 20$.

Division.

RULE.—Prepare fractions as before; then divide numerator by the numerator, and denominator by the denominator, if they will exactly divide; but if not, invert the terms of divisor, and multiply dividend by it, as in multiplication.

EXAMPLE 1.—Divide $\frac{25}{9}$ by $\frac{5}{3}$. $\frac{25}{9} \div \frac{5}{3} = \frac{5}{3} = 1\frac{2}{3}$.

2.—Divide $\frac{5}{9}$ by $\frac{2}{15}$. $\frac{5}{9} \div \frac{2}{15} = \frac{5}{9} \times \frac{15}{2} = \frac{15}{9} \times \frac{5}{2} = \frac{75}{18} = \frac{25}{6} = 4\frac{1}{6}$.

Application of Reduction of Fractions.

To Compute Value of a Fraction in Parts of a Whole Number.

RULE.—Multiply whole number by numerator, and divide by denominator; then, if anything remains, multiply it by the parts in next inferior denomination, and divide by denominator, as before, and so on as far as necessary; so shall the quotients placed in order be value of fraction required.

EXAMPLE 1.—What is value of $\frac{1}{2}$ of $\frac{2}{3}$ of 9?

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{2}{6}, \text{ and } \frac{2}{6} \times \frac{9}{1} = \frac{18}{6} = 3.$$

2.—Reduce $\frac{3}{4}$ of a pound to an avoirdupois ounce.

4) 3 (0 *lbs.*
16 *ounces in a lb.*
4) 48 (12 *ounces.*

To Reduce a Fraction from one Denomination to another.

RULE.—Multiply number of required denomination contained in given denomination by numerator *if reduction is to be to a less name*, but by denominator *if to a greater*.

EXAMPLE 1.—Reduce $\frac{1}{4}$ of a dollar to fraction of a cent.

$$\frac{1}{4} \times 100 = \frac{100}{4} = \frac{25}{1}.$$

2.—Reduce $\frac{1}{6}$ of an avoirdupois pound to fraction of an ounce.

$$\frac{1}{6} \times 16 = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}.$$

3.—Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of a mile to the fraction of a foot.

$$\frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} \times 5280 = \frac{31680}{12} = \frac{2640}{1}.$$

For Rule of Three in Vulgar Fractions, see Decimals, page 94.

DECIMALS.

A DECIMAL is a fraction, having for its denominator a UNIT with as many ciphers annexed as the numerator has places; it is usually expressed by writing the numerator only, with a point at the left of it. Thus, $\frac{4}{10}$ is .4; $\frac{85}{100}$ is .85; $\frac{0075}{10000}$ is .0075; and $\frac{125}{100000}$ is .00125. When there is a deficiency of figures in the numerator, prefix ciphers to make up as many places as there are ciphers in denominator.

Mixed numbers consist of a whole number and a fraction; as, 3.25, which is the same as $3\frac{25}{100}$, or $\frac{325}{100}$.

Ciphers on right hand make no alteration in their value; for .4, .40, .400 are decimals of same value, each being $\frac{4}{10}$, or $\frac{2}{5}$.

Addition.

RULE.—Set numbers under each other according to value of their places, as in whole numbers, in which position the decimal points will stand directly under each other; then begin at right hand, add up all the columns of numbers as in integers, and place the point directly below all the other points.

EXAMPLE.—Add together 25.125 and 293.7325.

$$\begin{array}{r} 25.125 \\ \underline{293.7325} \\ 318.8575 \end{array}\ \textit{sum.}$$

Subtraction.

RULE.—Set numbers under each other as in addition; then subtract as in whole numbers, and point off decimals as in last rule.

EXAMPLE.—Subtract 15.15 from 89.1759.

$$\begin{array}{r} 89.1759 \\ \underline{15.15} \\ 74.0259 \end{array}\ \textit{remainder.}$$

Multiplication.

RULE.—Set the factors, and multiply them together same as if they were whole numbers; then point off in product just as many places of decimals as there are decimals in both factors. But if there are not so many figures in product, supply deficiency by prefixing ciphers.

EXAMPLE.—Multiply 1.56 by .75.

$$\begin{array}{r} 1.56 \\ \underline{.75} \\ 780 \\ \underline{1092} \\ 1.1700 \end{array}\ \textit{product.}$$

By Contraction.

To Contract the Operation so as to retain only as many Decimal places in Product as may be required.

RULE.—Set unit's place of multiplier under figure of multiplicand, the place of which is same as is to be retained for the last in product, and dispose of the rest of figures in contrary order to which they are usually placed.

In multiplying, reject all figures that are more to right hand than each multiplying figure, and set down the products, so that their right-hand figures may fall in a column directly below each other, and increase first figure in every line with what would have arisen from figures omitted; thus, add 1 for every result from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, 4 from 35 to 44, etc., and the sum of all the lines will be the product as required.

EXAMPLE.—Multiply 13.574 93 by 46.2051, and retain only four places of decimals in the product.

$$\begin{array}{rl} 13.574\ 93 & \\ \underline{1\ 502.64} & \\ 54\ 299\ 72 & \\ 8\ 144\ 96 + 2 & \textit{for}\ 18 \\ 271\ 50 + 2 & \text{"}\ \ 18 \\ 6\ 79 + 4 & \text{"}\ \ 35 \\ \underline{14 + 1} & \text{"}\ \ \ 5 \\ 627.23\ 11 & \end{array}$$

NOTE.—When exact result is required, increase last figure with what would have arisen from all the figures omitted.

Division.

Rule.—Divide as in whole numbers, and point off in quotient as many places for decimals as decimal places in dividend exceed those in divisor; but if there are not so many places, supply deficiency by prefixing ciphers.

Example. Divide 53 by 6.75. 6.75) 53.000 00 (= 7.851+.

Here 5 ciphers are annexed to dividend to extend division.

By Contraction.

Rule.—Take only as many figures of divisor as will be equal to number of figures, both integers and decimals, to be in quotient, and ascertain how many times they may be contained in first figures of dividend, as usual.

Let each remainder be a new dividend; and for every such dividend leave out one figure more on right-hand side of divisor, carrying for figures cut off as in Contraction of Multiplication.

Note.—When there are not so many figures in divisor as there are required to be in quotient, continue first operation until number of figures in divisor are equal to those remaining to be found in quotient, after which begin the contraction.

Example.—Divide 2508.928 06 by 92.410 35, so as to have only four places of decimals in quotient.

```
92.4103|5) 2508.928|06 (27.1498     13.849      912
           1848 207 + 1              9 241      832 + 4
            660 721                  4 608       60
            646 872 + 2              3 696       74 + 2
             13 849                    912        6
```

Reduction of Decimals.

To Reduce a Vulgar Fraction to its Equivalent Decimal.

Rule.—Divide numerator by denominator, annexing ciphers to numerator to extent that may be necessary.

Example.—Reduce $\frac{4}{5}$ to a decimal. 5) 4.0 / .8

To Compute Value of a Decimal in Terms of an Inferior Denomination.

Rule.—Multiply decimal by number of parts in next lower denomination, and cut off as many places for a remainder, to right hand, as there are places in given decimal.

Multiply that remainder by the parts in next lower denomination, again cutting off for a remainder, and so on through all the parts of integer.

Example 1.—What is value of .875 dollars?

```
         .875
          100
Cents,  87.500
           10
Mills,   5.000 = 87 cents 5 mills.
```

2.—What is volume of .140 cube feet in inches?

```
   .140
   1728 cube inches in a cube foot.
241 920 cube ins.
```

3.—What is value of .00129 of a foot? .01548 *ins.*

To Reduce a Decimal to an Equivalent Decimal of a Higher Denomination.

Rule.—Divide by number of parts in next higher denomination, continuing operation as far as required.

Example 1.—Reduce 1 inch to decimal of a foot.

```
12|1.000 00
  | .083 33 + foot.
```

2.—Reduce 14″ 12‴ to decimal of a minute.

```
   14″ 12‴
    60
60| 852.‴
60|  14.2″
  |  .236 66′ + minute.
```

When there are several numbers, to be reduced all to decimal of highest.
RULE.—Reduce them all to lowest denomination, and proceed as for one denomination.

EXAMPLE.—Reduce 5 feet 10 inches and 3 barleycorns to decimal of a yard.

Feet.	Ins.	Bc.
5	10	3
12		
70		
3		
3) 213.		
12) 71.		
3) 5.9166		
1.9722+ *yards.*		

Rule of Three.

RULE.—Prepare the terms by reducing vulgar fractions to decimals, compound numbers to decimals of the highest denomination, first and third terms to same denomination; then proceed as in whole numbers.

EXAMPLE.—If .5 of a ton of iron cost .75 of a dollar, what will .625 of a ton cost?

$$.5 : .75 :: .625$$
$$.625$$
$$.5)\ .46875$$
$$.9375, \textit{ dollar.}$$

DUODECIMALS.

In Duodecimals, or Cross Multiplication, the dimensions are taken in feet, inches, and twelfths of an inch.

RULE.—Set dimensions to be multiplied together one under the other, feet under feet, inches under inches, etc.

Multiply each term of multiplicand, beginning at lowest, by feet in multiplier, and set result of each immediately under its corresponding term, carrying 1 for every 12 from one term to the other. In like manner, multiply all multiplicand by inches of multiplier, and then by twelfth parts, setting result of each term one place farther to right hand for every multiplier. And sum of products will give result.

EXAMPLE.—How many square inches are there in a board 35 feet 4.5 inches long and 12 feet $3\frac{1}{3}$ inches wide?

Feet.	Ins.	Twelfths.		
35	4	6		
12	3	4		
424	6	0		
8	10	1	6	
	11	9	6	0
434	3	11	0	0

Value of Duodecimals in Square Feet and Inches.

	Sq. Ft.		Sq. Ins.
1 Foot .... =	1	or	144.
1 Inch .... =	$\frac{1}{12}$	"	12.
1 Twelfth .... =	$\frac{1}{144}$	"	1.

	Sq. Ft.		Sq. Ins.
$\frac{1}{12}$ of 1 twelfth =	$\frac{1}{1728}$	or	.083 333, etc.
$\frac{1}{12}$ of $\frac{1}{12}$ of " =	$\frac{1}{20736}$	"	.006 944, etc.

ILLUSTRATION.—What number of square inches are there in a floor 100 feet 6 inches long and 25 feet 6 inches and 6 twelfths broad?

2566 *feet* 11 *ins.* 3 *twelfths* = 2566 *feet* 135 *ins.*

MEAN PROPORTION.

MEAN PROPORTION is proportion to two given numbers or terms.

RULE.—Multiply two numbers or terms together, and extract square root of their product.

EXAMPLE.—What is mean proportionate velocity to 16 and 81?

$16 \times 81 = 1296$, and $\sqrt{1296} = 36$ *mean velocity.*

RULE OF THREE.

RULE OF THREE.—It is so termed because three terms or numbers are given to ascertain a fourth.

It is either DIRECT or INVERSE.

It is Direct when more requires more, or less requires less; thus, if 3 barrels of flour cost $18, what will 10 barrels cost?

In this case Proportion is *Direct*, and stating must be,

As 3 : 10 :: 18 : 60.

It is Inverse when more requires less, or less requires more; thus, if 6 men build a certain quantity of wall in 10 days, in how many days will 8 men build like quantity? Or, if 3 men dig 100 feet of trench in 7 days, in how many days will 2 men perform same work?

Here the Proportion is *Inverse*, and stating must be,

As 8 : 6 :: 10 : 7.5, and 2 : 3 :: 7 : 10.5.

The fourth term is always ascertained by multiplying 2d and 3d terms together, and dividing their product by 1st term.

Of the three given numbers necessary for the stating, two of them contain the supposition, and the third a demand.

RULE.—State question by setting down in a straight line the three necessary numbers in following manner:

Let third term be that of *supposition*, of same denomination as the result, or 4th term is to be, making *demanding* number 2d term, and the other number 1st term when question is in *Direct Proportion*, but contrariwise if in *Inverse Proportion;* that is, let demanding number be 1st term.

Multiply 2d and 3d terms together, and divide by 1st, and product will give result, or 4th term sought, of same denomination as 2d term.

NOTE.—If first and third terms are of different denominations, reduce them to same. If, after division, there is any remainder, reduce it to next lower denomination, divide by divisor as before, and quotient will be of this last denomination.

Sometimes two or more statings are necessary, which may always be known by nature of question.

EXAMPLE 1.—If 20 tons of iron cost $225, what will 500 tons cost?

```
Tons. Tons.  Dolls.
 20 : 500 :: 225
             500
       2|0) 11 250|0
             5 625 dollars.
```

2.—A wall that is to be built to height of 36 feet, was raised 9 feet by 16 men in 6 days; how many men could finish it in 4 days at same rate of working?

Days. Days. Men. Men.
4 : 6 :: 16 : 24

Then, if 9 feet requires 24 men, what will 27 men require?

9 : 27 :: 24 : 72 *men.*

COMPOUND PROPORTION.

COMPOUND PROPORTION is rule by means of which such questions as would require two or more statings in simple proportion (Rule of Three) can be resolved in one.

As rule, however, is but little used, and not easily acquired, it is deemed preferable to omit it here, and to show the operation by two or more statings in Simple Proportion.

ILLUSTRATION 1.—How many men can dig a trench 135 feet long in 8 days, when 16 men can dig 54 feet in 6 days?

```
                        Feet. Feet.  Men. Men.
First................As   54 : 135 :: 16 : 40
                        Days. Days.  Men. Men.
Second...............As    8 : 6   :: 40 : 30
```

2.—If a man travel 130 miles in 3 days of 12 hours each, how many days of 10 hours each would he require to travel 360 miles?

		Miles.		Miles.		Days.		Days.
First..............	As	130	:	360	::	3	:	8.307+
		Hours.		Hours.		Days.		Days.
Second.............	As	10	:	12	::	8.307	:	9.9684

3.—If 12 men in 15 days of 12 hours build a wall 30 feet long, 6 wide, and 3 deep, in how many days of 8 hours will 60 men build a wall 300 feet long, 8 wide, and 6 deep? 120 *days.*

By Cancellation.

Rule.—On right of a vertical line put the number of same denomination as that of required answer.

Examine each simple proportion separately, and if its terms demand *a greater result than 3d term*, put *larger number on right and lesser on left of line*; but if its terms demand a *less result than 3d term*, put *smaller number on right and larger on left of line.*

Then *Cancel* the numbers divisible by a common divisor, and evolve the 4th term or result required.

Take Illustration 1, page 95: 3d term, or term of supposition of same denomination as required result, 16 *men.*

Statement.

	16
54	135
8	6

135 *feet* require more men than 54 *feet*, and 8 *days* less men than 6 *days*.

$2 \times 5 \times 3 = 30$ *men.*

Result by Cancellation.

		$\cancel{16}$	2
2	$\cancel{54}$	$\cancel{135}$	5
	$\cancel{8}$	$\cancel{6}$	3

Illustration 3.—3d term, 15 *days.*

Statement.

	15
60	12
8	12
30	300
6	8
3	6

60 men require less days than 12 men, 8 hours more days than 12 hours, 300 feet more days than 30 feet, 8 feet more days than 6 feet, and 6 feet more days than 3 feet.

$3 \times 4 \times 10 = 120$ *days.*

Result by Cancellation.

		$\cancel{15}$	
4	$\cancel{60}$	$\cancel{12}$	3
	$\cancel{8}$	$\cancel{12}$	4
	$\cancel{30}$	$\cancel{300}$	10
	$\cancel{6}$	$\cancel{8}$	
	$\cancel{3}$	$\cancel{6}$	

INVOLUTION.

Involution is multiplying any number into itself a certain number of times. Products obtained are termed *Powers*. The number is termed the *Root*, or first power.

When a number is multiplied by itself once, product is *square* of that number; twice, *cube*; three times, *biquadrate*; etc. Thus, of the number 5.

5 *is the Root*, or 1st *power*.

$5 \times 5 = 25$ " *Square*, or 2d *power*, and is expressed 5^2.

$5 \times 5 \times 5 = 125$ " *Cube*, or 3d *power*, and is expressed 5^3.

$5 \times 5 \times 5 \times 5 = 625$ " *Biquadrate*, or 4th *power*, and is expressed 5^4.

The lesser figure set superior to number denotes the power, and is termed the *Index* or *Exponent*.

Illustration 1.—What is cube of 9? 729.

2.—What is cube of $\frac{3}{4}$? $\frac{27}{64}$.

3.—What is 4th power of 1.5? 5.0625.

EVOLUTION.

Evolution is ascertaining *Root* of any number.

Sign $\sqrt{\ }$ placed before any number indicates that *square root* of that number is required or shown.

Same character expresses any other root by placing the index above it.

Thus, $\sqrt{25} = 5$; $4 + 2 = \sqrt{36}$.

$\sqrt[3]{27} = 3$, and $\sqrt[3]{64} = 4$.

Roots which only approximate are termed Surd Roots.

To Extract Square Root.

RULE.—Point off given number from units' place, into periods of two figures each.

Ascertain greatest square in left-hand period, and place its root in quotient; subtract square number from this period, and to remainder bring down next period for a dividend.

Double this root for a divisor; ascertain how many times it is contained in dividend, exclusive of right-hand figure, which, when multiplied by number to be put to right hand of this divisor, product will be equal to, or next less than dividend; place result in quotient, and also at right hand of divisor.

Multiply divisor by last quotient figure, and subtract product from dividend; bring down next period, and proceed as before.

NOTE.—Mixed decimals must be pointed off both ways from units.

EXAMPLE 1.—What is square root of 2?

1	2.000000 (1.41+.
1	1
24	100
4	96
281	400
1	281
282	11900

2. What is square root of 144?

1	144 (12
1	1
22	044
	44

Square Roots of Fractions.

RULE.—Reduce fractions to their lowest terms, and that fraction to a decimal, and proceed as in whole numbers and decimals.

NOTE.—When terms of fractions are squares, take root of each and set one above the other; as $\frac{5}{6}$ is square root of $\frac{25}{36}$.

EXAMPLE.—What is square root of $\frac{9}{12}$? .866 0254.

To Compute 4th or 8th Root of a Number, etc.

RULE.—For the 4th root extract square root twice, and for 8th root thrice, etc.

To Extract Cube Root.

RULE.—From table of roots (page 272) take nearest cube to given number, and term it the assumed cube.

Then, as given number added to twice assumed cube, is to assumed cube added to twice given number, so is root of assumed cube to required root, *nearly;* and by using in like manner the root thus found as an assumed cube, and proceeding in like manner, another root will be found still nearer; and in like manner as far as may be deemed necessary.

EXAMPLE.—What is cube root of 10517.9?

Nearest cube, page 272; 10648, root 22.

10648.	10517.9
2	2
21296	21035.8
10517.9	10648.
31813.9 :	31683.8 :: 22 : 21.9+.

To Ascertain or to Compute the Square or Cube Roots of Roots, Whole Numbers, and of Integers and Decimals, see Table of Squares and Cubes, and Rules, pp. 272, 300.

To Extract any Root whatever.

Let P represent number.
n " index of the power.

Let A represent assumed power, r its root.
R " required root of P.

Then, as sum of $n+1 \times A$ and $n-1 \times P$ is to sum of $n+1 \times P$ and $n-1 \times A$, so is assumed root r to required root R.

ILLUSTRATION.—What is cube root of 1500?

Nearest cube, page 272, is 1331, root 11.

$$P = 1500,\ n = 3,\ A = 1331,\ r = 11;$$

$$\text{then, } n+1 \times A = 5324,\ n+1 \times P = 6000$$

$$n-1 \times P = 3000,\ n-1 \times A = 2662$$

$$8324 \qquad 8662 :: 11 : 11.446+.$$

I

To Compute the Root of an Even Power greater than any given in Table of Square and Cube Roots.

RULE.—Extract square or cube root of it, which will reduce it to half the given power; then square or cube root of that power; and so on until required root is obtained.

EXAMPLE 1.—Suppose a 12th power is given; the square root of that reduces it to a 6th power, and the square root of 6th power to a cube.

2.—What is biquadrate, or 4th root, of 2 560 000?

$$\sqrt{2\,560\,000} = 1600, \text{ and } \sqrt{1600} = 40.$$

NOTE.—For other rules for extraction of roots see pp. 301–4.

PROPERTIES OF NUMBERS.

1. A *Prime Number* is that which can only be measured (divided without a remainder) by 1 or unity.
2. A *Composite Number* is that which can be measured by some number greater than unity.
3. A *Perfect Number* is that which is equal to the sum of all its divisors or aliquot parts; as $6 = \frac{6}{6}, \frac{6}{3}, \frac{6}{2}$.
4. If sum of the digits constituting any number be divisible by 3 or 9, the whole is divisible by them.
5. A square number cannot terminate with an *odd* number of ciphers.
6. No square number can terminate with two equal digits, except two ciphers or two fours.
7. No number, the last digit of which is 2, 3, 7, or 8, is a square number.

Powers of the first Nine Numbers.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2 187	6 561	19 683
4	16	64	256	1 024	4 096	16 384	65 536	262 144
5	25	125	625	3 125	15 625	78 125	390 625	1 953 125
6	36	216	1296	7 776	46 656	279 936	1 679 616	10 077 696
7	49	343	2401	16 807	117 649	823 543	5 764 801	40 353 607
8	64	512	4096	32 768	262 144	2 097 152	16 777 216	134 217 728
9	81	729	6561	59 049	531 441	4 782 969	43 046 721	387 420 489

POSITION.

POSITION is of two kinds, SINGLE and DOUBLE, and it is determined by number of SUPPOSITIONS.

Single Position.

RULE.—Take any number, and proceed with it as if it were the correct one; then, as result is to given sum, so is supposed number to number required.

EXAMPLE 1.—A commander of a vessel, after sending away in boats $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{4}$ of his crew, had left 300; what number had he in command?

Suppose he had .... 600.

$\frac{1}{3}$ of 600 is 200
$\frac{1}{6}$ of 600 is 100
$\frac{1}{4}$ of 600 is 150   450

150 : 300 :: 600 : 1200 *men.*

2. — A person asked his age, replied, if $\frac{3}{4}$ of my age be multiplied by 2, and that product added to half the years I have lived, the sum will be 75. How old was he?

37.5 *years.*

Double Position.

RULE.—Assume any two numbers, and proceed with each according to conditions of question; multiply results or *errors* by contrary supposition; that is, first position by last error, and last position by first error.

If errors are too great, mark them +; and if too little, —.

Then, if errors are *alike*, divide *difference* of products by *difference* of errors; but if they are *unlike*, divide *sum* of the products by *sum* of errors.

EXAMPLE 1.—A asked B how much his boat cost; he replied, that if it cost him 6 times as much as it did, and $30 more, it would have cost him $300. What was price of the boat?

Suppose it cost him	60	30
	6 *times.*	6 *times.*
	360	180
and	30 *more*	and 30 *more*
	390	210
	300	300
	90+	90—
	30 *2d position.*	60 *1st position.*
90	2700	5400
90	5400	
180)	8100 (45 *dollars.*	

2.—What is length of a fish when the head is 9 inches long, tail as long as its head and half its body, and body as long as both head and tail? 6 *feet.*

FELLOWSHIP.

FELLOWSHIP is a method of ascertaining gains or losses of individuals engaged in joint operations.

Single Fellowship.

RULE.—As the whole stock is to the whole gain or loss, so is each share to the gain or loss on that share.

EXAMPLE.—Two men drew a prize in a lottery of $9500. A paid $3, and B $2 for the ticket; how much is each share?

5 : 9500 :: 3 : 5700, A's *share.*
5 : 9500 :: 2 : 3800, B's *share.*

Double Fellowship,

Or Fellowship with Time.

RULE.—Multiply each share by time of its interest; then, as sum of products is to product of each interest, so is whole gain or loss to each share of gain or loss.

EXAMPLE.—A cutter's company take a prize of $10 000, which is to be divided according to their rate of pay and time of service on board. The officers have been on board 6 months, and the crew 3 months; pay of lieutenants is $100, ensigns $50, and crew $10 per month; and there are 2 lieutenants, 4 ensigns, and 50 men; what is each one's share?

2 lieutenants	$100 = 200 × 6 = 1200
4 ensigns	50 = 200 × 6 = 1200
50 men	10 = 500 × 3 = 1500
	3900

Lieutenants	3900 : 1200 :: 10 000 : 3076.92 ÷ 2 = 1538.46	*dolls.*
Ensigns	3900 : 1200 :: 10 000 : 3076.92 ÷ 4 = 769.23	"
Men	3900 : 1500 :: 10 000 : 3846.16 ÷ 50 = 76.92	"

PERMUTATION.

PERMUTATION is a rule for ascertaining how many different ways any given number of numbers of things may be varied in their position.

Permutation of the three letters *abc*, taken *all together*, are 6; taken *two* and *two*, are 6; and taken *singly*, are 3.

RULE.—Multiply all the terms continually together, and last product will give result.

EXAMPLE 1.—How many variations will the nine digits admit of?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362\,880.$$

2.—How many years would there be required to elapse before 10 persons could be seated in a varied position collectively, each day at dinner, including one day in every 4 years for a leap year? 9935 *years*, 42 *days*.

When only part of the Numbers or Elements are taken at once. RULE.—Take a series of numbers, beginning with number of things given, decreasing by 1, until number of terms equals number of things or quantities to be taken at a time, and product of all the terms will give sum required.

EXAMPLE 1.—How many changes can be made with 2 events in 5?

$5 - 1 = 4$, and $4 \times 5 = 2$ *terms*. Hence, $5 \times 4 = 20$ *changes*.

2.—How many changes of 2 will 3 playing cards admit of?

$3 - 1 = 2$, and $2 \times 3 = 2$ *terms*. Hence, $2 \times 3 = 6$ *changes*.

3.—How many changes can be rung with 4 bells (taken 4 and 4 together) out of 6?

$4 - 1 = 3$, and $3 \times 4 \times 5 \times 6 = 4$ *terms* or *changes*.
Hence, $3 \times 4 \times 5 \times 6 = 360$ *changes*.

When several of the Elements are alike. RULE.—Ascertain the permutations of all the numbers or things, and of all that can be made of each separate kind or division; divide number of permutations of whole by product of the several partial permutations, and quotient will give number of permutations.

EXAMPLE.—How many permutations can be made out of the letters of the word persevere (9 letters, having 4 e's and 2 r's)?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362\,880$;
$1 \times 2 \times 3 \times 4 = 24$ for the e's; $1 \times 2 = 2$ for the r's, and $24 \times 2 = 48$.
Hence, $362\,880 \div 48 = 7560$.

Or, Add logarithms of all the terms together, and number for the sum will give result.

EXAMPLE 1.—How many permutations can be made with three letters or figures?

$$\begin{aligned} \text{Log. } 1 &= .00 \\ 2 &= .301\,03 \\ 3 &= .477\,121\,3 \\ \hline & .778\,151\,3 = \text{log. of number } 6. \end{aligned}$$

2.—How many variations will 15 numbers in 16 places admit of?

Add logarithms of numbers 1 to 16 and take logarithm of their sum—

viz., $13.320\,661\,97 = 20\,922\,789\,888\,000$.

Number of positions of the blocks in the "15 puzzle" is as above for their 16 permutations.

Permutations,

Whereby any questions of Permutation may be solved by Inspection, number of terms not exceeding 20.

1	1	5	120	9	362 880	13	6 227 020 800	17	355 687 428 096 000
2	2	6	720	10	3 628 800	14	87 178 291 200	18	6 402 373 705 728 000
3	6	7	5 040	11	39 916 800	15	1 307 674 368 000	19	121 645 100 408 832 000
4	24	8	40 320	12	479 001 600	16	20 922 789 888 000	20	2 432 902 003 176 640 000

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is a series of numbers increasing or decreasing by a constant number or difference; as, 1, 3, 5, 7, 12, 9, 6, 3. The numbers which form the series are designated *Terms;* the first and last are termed *Extremes*, and the others *Means.*

When any three of following elements are given, the remaining two can be ascertained—viz., *First* term, *Last* term, *Number* of terms, *Common Difference*, and *Sum* of all the terms.

To Compute First Term.

When Last term, Number of terms, and Sum of series are given. RULE.—From quotient of twice sum of series, divided by number of terms, subtract last term.

Or, $\frac{l-d}{n-1}$; $\frac{S}{n}-\frac{d\,\overline{n-1}}{2}$; and $\sqrt{(l+.5\,d)^2-2\,d\,S}\pm.5\,d=a.$ *a representing 1st, l last, n number of, and S sum of all terms, and d common difference.*

ILLUSTRATION.—A man travelled 390 miles in 12 days, travelling 60 miles last day. How far did he travel first day?

$$\frac{390\times 2}{12}=65, \text{ and } 65-60=5 \textit{ first term.}$$

To Compute Last Term.

When First term, Common Difference, and Number of terms are given. RULE.—Multiply the number of terms less 1, by common difference, and to product add first term.

EXAMPLE.—A man travelled for 12 days, at the rate of 5 miles first day, 10 second, and so on; how far did he travel the last day?

$$12-1\times 5=55, \text{ and } 55+5=60 \textit{ miles.}$$

When First term, Number of terms, and Sum of series are given. RULE.—Divide twice sum of series by number of terms, and from quotient subtract first term.

Or, $\frac{2\,S}{n}-a$; $\sqrt{2\,d\,S+(a-.5\,d)^2}\pm.5\,d$; and $\frac{S}{n}+\frac{d\,(n-1)}{2}=l.$

ILLUSTRATION.—A man travelled 360 miles in 12 days, commencing with 5 miles first day; how far did he travel last day?

$$\frac{360\times 2}{12}=65, \text{ and } 65-5=60 \textit{ miles.}$$

To Compute Number of Terms.

When Common Difference and Extremes, or First and Last term, are given. RULE.—Divide difference of extremes by common difference, and add 1 to quotient.

EXAMPLE.—A man travelled 3 miles first day, 5 second, 7 third, and so on, till he went 57 miles in one day; how many days had he travelled at close of last day?

$$57-3\div 2=27, \text{ and } 27+1=28 \textit{ days.}$$

When Sum of series and Extremes are given. RULE.—Divide twice sum of series by sum of first and last terms.

$$\text{Or, } \frac{l-a}{d}+1; \quad \sqrt{\frac{2\,S}{d}+\left(\frac{2\,a-d}{2\,d}\right)^2}\pm\frac{d-2\,a}{2\,d};$$

$$\text{and } \sqrt{\left(\frac{2\,l+d}{2\,d}\right)^2-\frac{2\,S}{d}}\pm\frac{2\,l+d}{2\,d}=n.$$

ILLUSTRATION.—A man travelled 840 miles, walking 3 miles first day and 57 last day; how many days was he travelling?

$$\frac{840\times 2}{3+57}=\frac{1680}{60}=28 \textit{ days.}$$

To Compute Common Difference.

When Number of terms and Extremes are given. RULE.—Divide difference of extremes by 1 less than number of terms.

$$\text{Or, } \frac{2\,S-2\,an}{n\,(n-1)}; \qquad \frac{l+a\times\overline{l-a}}{2\,S-l-a}; \qquad \text{and } \frac{2\,nl-2\,S}{n\,(n-1)}=d.$$

ILLUSTRATION.—Extremes are 3 and 15, and number of terms 7; what is common difference?

$$15-3\div(7-1)=\frac{12}{6}, \text{ and } \frac{12}{6}=2 \textit{ com. dif.}$$

To Compute Sum of the Series or of all Terms.

When Extremes and Number of terms are given. RULE.—Multiply number of terms by half sum of extremes.

$$\text{Or, } 2\,a+\overline{d\,(n-1)}\times .5\,n; \qquad \frac{l+a\times(l-a)}{2\,d}+\frac{l+a}{2};$$
$$\text{and } 2\,l-(d\times\overline{n-1})\times .5\,n=S.$$

ILLUSTRATION.—How many times does hammer of a clock strike in 12 hours?

$$12\times\overline{12+1}=156, \text{ and } 156\div2=78 \textit{ times.}$$

To Compute any Number of Arithmetical Means or Terms between two Extremes.

RULE. — Subtract less extreme from greater, and divide difference by 1 more than number of means or terms required to be ascertained, and then proceed as in rule.

To Compute Two Arithmetical Means or Terms between two given Extremes.

RULE.—Subtract less extreme from greater, and divide difference by 3, quotient will be common difference, which being added to less extreme, or taken from greater, will give means.

EXAMPLE 1.—Compute two arithmetical means between 4 and 16.

$$16-4\div3=4 \textit{ com. dif.}$$
$$4+4=8 \textit{ one mean.}$$
$$16-4=12 \textit{ second mean.}$$

2.—Compute four arithmetical means between 5 and 30.

$$30-5=25, \text{ and } 25\div\overline{4+1}=5=\textit{com. dif.}$$

$5+5=10=1st$ *mean.* | $15+5=20=3d$ *mean.*
$10+5=15=2d$ " | $20+5=25=4th$ "

Miscellaneous Illustrations.

1. A steamer having been purchased upon following terms—viz.: \$5000 upon transfer of bill of sale and balance in monthly instalments, commencing at \$4500 for first month, and decreasing \$500 in each month, until whole sum is paid.

1st. How many months must elapse before final payment?

2d. What was amount of purchase-money, or sum of series?

Here are *first* and *last terms*—viz., 500 and 5000, and *common difference*, 500. *Hence*, To compute number of terms and amount of purchase,

$5000-\overline{500}\div500=9$, and $9+1=10=$ *number of terms or months*, and $10\times\frac{5000+500}{2}=10\times2750=\$27\,500$, *amount of purchase.*

2. If 100 stones are placed in a right line, one yard apart; how many yards must a person walk, to take them up one at a time and put them into a basket, one yard from first stone?

First term 2, last term 200, and number of terms 100.

$$\text{Hence, } 100\times\frac{200+2}{2}=10\,100 \textit{ yards.}$$

3. If in the sinking of curb of a well, \$3 is to be given for first foot in depth, \$5 for second, \$7 for third, and increasing in like manner to a depth of 20 feet, what would it cost?

First term 3, *common difference* 2, *and number of terms* 20.

Hence, $20 - 1 \times 2 + 3 = 41$, *last term.*

Then, $3 + 41 \times \frac{20}{2} = 440$, *sum of all terms, or cost of curb.*

4. If a contractor engaged to sink a curb to depth of 20 feet for \$400, and the contract was annulled when he had reached a depth of 8 feet; how much had he earned?

$400 \div 20 =$ *number of terms.* But inasmuch as 400 may be divided into 20 terms in arithmetical proportion in many different ways, according to value of 1st term, it becomes necessary to assume the value of the first foot as value of 1st term.

Assuming it at \$5, the required proportion will be, *1st term* 5, *number of terms* 20, *sum of series* 400.

Hence, $\frac{\overline{400 - 5 \times 20} \times 2}{20 \times (20 - 1)} = \frac{600}{380} = 1\frac{11}{19}$, *common difference.*

Then, $5 + 1\frac{11}{19} \times 7 = 16\frac{1}{19} =$ *1st term + product of common difference and 8th term less* 1, which added to $5 = 21\frac{1}{19}$, and $\times 4 =$ *half number of terms for which cost is sought* $= 84\frac{4}{19}$ *dollars, sum earned.*

GEOMETRICAL PROGRESSION.

Geometrical Progression is any series of numbers continually increasing by a constant multiplier, or decreasing by a constant divisor, as 1, 2, 4, 8, 16, etc., and 15, 7.5, 3.75, etc.

The constant multiplier or divisor is the *Ratio.*

When any three of following elements are given, remaining two can be computed, viz.: *First* term, *Last* term, *Number* of Terms, *Ratio*, and *Sum* of all Terms.

To Compute First Term.

When Ratio, Last Term, and Number of Terms are given. Rule. — Divide last term by ratio raised to a power denoted by number of terms less 1.

Or, $\frac{S\,(r - 1)}{r^n - 1}$ and $r\,l - S\,(r - 1) = a$. *a representing* 1st *term, l last, n number of, S sum of all terms, and r ratio.*

Illustration. — Last term is 4374, number of terms 8, and ratio 3; what is first term?

$$\frac{4374}{3^{8-1}} = \frac{4374}{2187} = 2, \textit{first term.}$$

To Compute Last Term.

When First Term and Ratio are Equal. Rule.—Write a few of leading terms of series and place their indices over them, beginning with a unit. Add together the most convenient and least number of indices to make the index to term required. Multiply terms of the series of these indices together, and product will give term required.

Or, Multiply first term by ratio raised to a power, denoted by number of terms less 1.

Example 1.—First term is 2, ratio 2, and number of terms 13; what is last term?

Indices, 1 2 3 4 5
Terms, 2, 4, 8, 16, 32.

Then, $5 + 5 + 3 = 13 =$ *sum of indices*, and $32 \times 32 \times 8 = 8192 =$ *last term.*

Or, $2 \times 2^{13-1} = 8192$. Also by inspection of table, page 105, 13th term = 8192.

2.—The price of 12 horses being 4 cents for first, 16 for second, and 64 for third, and so on; what is price of last horse?

Indices, 1 2 3 4
Terms, 4, 16, 64, 256.

Then, $4 + 4 + 4 = 12 =$ *sum of indices*, and $256 \times 256 \times 256 = 256^3 = \$167\,772.16$.

When First Term and Ratio are Different. RULE.—Write a few of leading terms of series, and place their indices over them, beginning with a cipher. Add together the most convenient indices to make an index less by 1 than term sought.

Multiply terms of these series belonging to these indices together, and take product for a dividend.

Or, Raise first term to a power, index of which is 1 less than number of terms multiplied; take result for a divisor; proceed with their division, and quotient will give term required.

EXAMPLE 1.—First term is 1, ratio 2, and number of terms 23; what is the last term?

Indices, 0 1 2 3 4 5
Terms, 1, 2, 4, 8, 16, 32.

Then, $5 + 5 + 5 + 5 + 2 = 22 =$ *sum of indices*, and $32 \times 32 \times 32 \times 32 \times 4 = 4\,194\,304$, and $4\,194\,304 \div$ the 5th power $(0 - 1)$ of $1 = 1 = 4\,194\,304$.

Or, $1 \times 2^{23-1} = 4\,194\,304$. By inspection of table, page 105, 23d term $= 4\,194\,304$.

2.—If 1 cent had been put out at interest in 1630, what would it have amounted to in 1834, if it had doubled its value every 12 years?

$1834 - 1630 = 204$, which $\div 12 = 17$, and $17 + 1 = 18 =$ *number of terms.*

Indices, 0 1 2 3 4 7
Terms, 1, 2, 4, 8, 16, 128.

Then, $7 + 4 + 3 + 2 + 1 = 17$, and $128 \times 16 \times 8 \times 4 \times 2 \times 1 = 131\,072$, and $131\,072 \div 1$, the 4th power $(5 - 1)$ of $1 = \$1\,310.72$.

When First Term, Ratio, and Sum of the series are given. RULE.—From sum of series subtract quotient of first term subtracted from sum of series, divided by ratio.

Or, $a \times r^{n-1} = l$.

EXAMPLE.—First term is 2, ratio 3, and sum of series 2186; what is last term?

$$2186 - \frac{2186 - 2}{3} = 2186 - 728 = 1458, \textit{ last term.}$$

To Compute Number of Terms.

When Ratio, First, and Last Terms are given. RULE.—Divide logarithm of quotient of product of ratio and last term, divided by first term, by logarithm of ratio.

$$\text{Or, } \frac{\log.\,(a + S\,\overline{r-1}) - \log.\,a}{\log.\,r}; \quad \frac{\log.\,l - \log.\,a}{\log.\,(S - a) - \log.\,(S - l)} + 1;$$

$$\text{and } \frac{\log.\,l - \log.\,(r\,l - \overline{r-1}\,S)}{\log.\,r} + 1 = n.$$

EXAMPLE.—Ratio is 2, and first and last terms are 1 and 131 072; what is number of terms?

$$\log.\,\frac{2 \times 131\,072}{1} = \log.\,262\,144 = 5.418\,54, \text{ and } 5.418\,54 \div \log.\text{ of } 2 = \frac{5.418\,54}{.301\,03} = 18.$$

To Compute Sum of Series.

When First Term, Ratio, and Number of Terms are given. RULE.—Raise ratio to a power index of which is equal to number of terms, from which subtract 1; then divide remainder by ratio less 1, and multiply quotient by first term.

$$\text{Or, } \frac{r\,l - a}{r - 1}; \quad \frac{l^{n}\sqrt[n-1]{l} - a^{n}\sqrt[n-1]{a}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}} \text{ and } \frac{l\,\overline{r^n - 1}}{\overline{r-1}\,r^{n-1}} = S.$$

ILLUSTRATION 1.—First term is 2, ratio 2, and number of terms 13; what is sum of series?

$2^{13-1} = 8192 - 1 = 8191$, and $8191 \div (2-1) = 8191$, and $8191 \times 2 = 16\,382$.

2.—If a man were to buy 12 horses, giving 2 cents for first horse, 6 cents for second, and so on, what would they cost him? \$5314.40.

To Compute Ratio.

When First Term, Last Term, and Numbers of Terms are given. RULE.—Divide last term by first, and quotient will be equal to ratio raised to power denoted by 1 less than number of terms; then extract root of this quotient.

$$\text{Or, } \frac{S-a}{S-l} = r.$$

ILLUSTRATION.—First term is 2, last term 4374, and number of terms 8; what is ratio?

$$\frac{4374}{2} = 2187, \text{ and } \sqrt[8-1]{2187} = 3, \textit{ ratio.}$$

Miscellaneous Illustrations.

1. What is 9th term in geometrical progression 3, 9, 27, 81, etc.? and what is sum of terms?

1st term = 3, number of terms 9, and ratio 3.

Hence, by rule to compute last term, 1st term and ratio being equal—

Indices, 1 2 3 4
Terms, 3, 9, 27, 81.

Then, $2+3+4 = 9 =$ *sum of indices*, and $9 \times 27 \times 81 = 19\,683 =$ *last term.*

By rule to compute sum of terms—

$$\frac{3^9 - 1}{3-1} \times 3 = \frac{19\,682}{2} = 9841 \times 3 = 29\,523, \textit{ sum of terms.}$$

2. First term is 1, ratio 2, and last term 131 072; what is sum of series?

$131\,072 \times 2 - 1 = 262\,143$, and $262\,143 \div 2 - 1 = 262\,143$.

3. What are the proportional terms between 2 and 2048?

$$4 + 2 = 6, \text{ and } 6 - 1 = 5, \text{ and } \sqrt[5]{\frac{2048}{2}} = 4.$$

Hence, 2 : 8 : 32 : 128 : 512 : 2048.

4. Sum of series is 6560, ratio 3, and number of terms 8; what is first term?

$$6560 \times \frac{3-1}{3^8 - 1} = 6560 \times \frac{2}{6560} = 2, \textit{ first term.}$$

Geometrical Progressions,

Whereby any questions of Geometrical Progression and of Double Ratio may be solved by Inspection, number of terms not exceeding 56.

1	1	15	16 384	29	268 435 456	43	4 398 046 511 104
2	2	16	32 768	30	536 870 912	44	8 796 093 022 208
3	4	17	65 536	31	1 073 741 824	45	17 592 186 044 416
4	8	18	131 072	32	2 147 483 648	46	35 184 372 088 832
5	16	19	262 144	33	4 294 967 296	47	70 368 744 177 664
6	32	20	524 288	34	8 589 934 592	48	140 737 488 355 328
7	64	21	1 048 576	35	17 179 869 184	49	281 474 976 710 656
8	128	22	2 097 152	36	34 359 738 368	50	562 949 953 421 312
9	256	23	4 194 304	37	68 719 476 736	51	1 125 899 906 842 624
10	512	24	8 388 608	38	137 438 953 472	52	2 251 799 813 685 248
11	1024	25	16 777 216	39	274 877 906 944	53	4 503 599 627 370 496
12	2048	26	33 554 432	40	549 755 813 888	54	9 007 199 254 740 992
13	4096	27	67 108 864	41	1 099 511 627 776	55	18 014 398 509 481 984
14	8192	28	134 217 728	42	2 199 023 255 552	56	36 028 797 018 963 968

ILLUSTRATIONS.—12th power of 2 = 4096, and 7th root of 128 = 2.

ALLIGATION.

Alligation is a method of finding mean rate or quality of different materials when mixed together.

To Compute Mean Price of a Mixture.

When Prices and Quantities are known. Rule.—Multiply each quantity by its rate, divide sum of products by sum of quantities, and quotient will give rate of the composition.

Example.—If 10 lbs. of copper at 20 cents per lb., 1 lb. of tin at 5 cents, and 1 lb. of lead at 4 cents, be mixed together, what is value of composition?

$$\begin{array}{r} 10 \times 20 = 200 \\ 1 \times 5 = 5 \\ \underline{1} \times 4 = \underline{4} \\ 12 \quad) \; 209 \; (17.416 \textit{ cents.} \end{array}$$

To Compute Quantity of each Article.

When Prices and Mean Price are given. Rule.—Write prices of ingredients, one under the other in order of their values, beginning with least, and set mean price at left. Connect with a line each price that is *less* than mean rate with one or more that is *greater.*

Write difference between mixture rate and that of each of simples opposite price with which it is *connected;* then sum of differences against any price will express quantity to be taken of that price.

Example.—How much gunpowder, at 72, 54, and 48 cents per pound, will compose a mixture worth 60 cents a pound?

$$60 \begin{cases} 48 \\ 54 \\ 72 \end{cases} \qquad \begin{array}{r} 12, \text{ at } 48 \textit{ cents.} \\ 12, \text{ at } 54 \textit{ cents.} \\ 12 + 6 = 18, \text{ at } 72 \textit{ cents.} \end{array}$$

Here, $72 - 60 = 12$ at 48, $72 - 60 = 12$ at 54, $60 - 48 = 12$, and $60 - 54 = 6 = 12 + 6 = 18$ at 72.

Then $12 \times 48 + 12 \times 54 + 18 \times 72 = 2520$, and $2520 \div \overline{12 + 12 + 12 + 6} = 60$ *cents.*

Note.—Should it be required to mix a definite quantity of any one article, the quantities of each, determined by above rule, must be increased or decreased in proportion they bear to defined quantity.

Thus, had it been required to mix 18 pounds at 48 cents, result would be 18 at 48, 18 at 54, and 27 at 72 cents per pound.

When the whole Composition is limited. Rule.—As sum of relative quantities, as ascertained by above rule, is to whole quantity required, so is each quantity so ascertained to required quantity of each.

Example.—Required 100 pounds of above mixture

Then, $12 + 12 + 18 = 42$. Then, $42 : 100 :: 12 : 28.571$ *pounds.*
$42 : 100 :: 12 : 28.571$ *pounds.*
$42 : 100 :: 18 : 42.857$ *pounds.*

When Price of Several Articles and Quantity of one of them is given. Rule.—Ascertain proportionate quantities of ingredients by previous rule.

Then, as number opposite ingredients, quantity of which is given, is to given quantity; so is number opposite to each ingredient to quantity required of that ingredient.

Example.—Having 35 lbs. of tobacco, worth 60 cents per pound, how much of other qualities, worth 65, 70, and 75 cents per pound, must be mixed with it, so as to sell mixture at 68 cents per pound?

By previous rule, it is ascertained there must be 7 lbs. at 60, 2 at 65, 3 at 70, and 8 at 75 cents; but as there are 35 lbs. at 60 cents to be taken, other quantities and kinds must be increased in like manner.

Hence, $7 : 35 :: 2 : 10 = 10$ at 65 *cents.*
$7 : 35 :: 3 : 15 = 15$ " 70 *cents.*
$7 : 35 :: 8 : 40 = 40$ " 75 *cents.*

SIMPLE INTEREST.

To Compute Interest on any Given Sum for a Period of One or more Years.

RULE.—Multiply given sum or *principal* by rate per cent. and number of years; point off two figures to right of product, and result will give interest in dollars and cents for 1 year.

EXAMPLE.—What is interest upon \$ 1050 for 5 years at 7 per cent.?

$$1050 \times 7 \times 5 = 36750, \text{ and } 367.50 = \$\,367.50.$$

When Time is less than One Year. RULE.—Proceed as before, multiplying by number of months or days, and dividing by following units—viz., 12 for months, and 365 or 366, as the case may be, for days.

EXAMPLE.—What is interest upon \$1050 for 5 months and 30 days at 7 per cent.?

5 months and 30 days = 183 *days*. $\frac{1050 \times 7 \times 183}{365} = 3685$, and $36.85 = \$\,36.85$.

The operation of computing interest may be performed thus:

Assuming interest upon any sum at 6 per cent. = 1 per cent. for 2 months.

Interest at 5 per cent. is $\frac{1}{6}$th less than at 6 per cent.

Interest at 7 per cent. is $\frac{1}{6}$th greater than at 6 per cent.

Taking preceding example—2 months	= 1	per cent. =	10.50
2 "	= 1	"	10.50
1 "	= $\frac{1}{2}$	"	5.25
30 days	= 1 month	=	5.25
			31.50
Add $\frac{1}{6}$ for 7 per cent.		=	5.25
			\$36.75

NOTE.—Difference between this amount and preceding arises from 183 days being taken in one case, and half a year, or 182.5 days, in the other.

In every computation of interest there are four elements—viz., Principal, Time, Rate, and Interest or Amount, any three of which being given, remaining one can be ascertained.

To Compute Principal.

When Time, Rate per Cent., and Interest are given. RULE.—Divide given interest by interest of \$1, etc., for given rate and time.

EXAMPLE.—What sum of money at 6 per cent. will in 14 months produce \$ 14?

$$14 \div .07 = 200 \textit{ dollars}.$$

To Compute Rate per Cent.

When Principal, Interest, and Time are given. RULE.—Divide given interest by interest of given sum, for time, at 1 per cent.

EXAMPLE. — If \$ 32.66 was discounted from a note of \$ 400 for 14 months, what was that per cent.?

Interest on 400 for 14 months at 1 per cent. = 4.66.

Then $32.66 \div 4.66 = 7$ *per cent.*

To Compute Time.

When Principal, Rate per Cent., and Interest are given. RULE.—Divide given interest by interest of sum, at rate per cent. for one year.

EXAMPLE.—In what time will \$ 108 produce \$ 11.34, at 7 per cent.?

Interest on 108 for one year is 7.56.

$$11.34 \div 7.56 = 1.5 \textit{ years}.$$

ILLUSTRATION 1. — If an amount of \$ 2175 is returned for a period of 15 months, rate of interest having been 7 per cent., what was principal invested? \$ 2000.

2.—If \$ 1000 in 18 months will produce \$ 1090, what is rate? 6 *per cent.*

COMPOUND INTEREST.

If any Principal be multiplied by number (in following table) opposite years, and under rate per cent., sum will be amount of that principal at compound interest for time and rate taken.

EXAMPLE.—What is amount of $500 for 10 years at 6 per cent.?

Tabular number.... 1.790 84, and 1.790 84 × 500 = 895.42 *dollars.*

Years.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.	Years.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.
1	1.03	1.04	1.05	1.06	13	1.468 53	1.665 07	1.885 64	2.132 92
2	1.0609	1.0816	1.1025	1.1236	14	1.515 29	1.731 67	1.979 93	2.2609
3	1.092 73	1.124 86	1.157 62	1.191 01	15	1.557 97	1.800 95	2.078 92	2.396 55
4	1.125 51	1.169 86	1.2155	1.262 47	16	1.604 71	1.872 98	2.182 87	2.540 35
5	1.159 27	1.216 68	1.276 98	1.338 22	17	1.652 85	1.947 99	2 292 01	2.692 77
6	1.194 05	1.265 32	1.34	1.418 51	18	1.702 44	2.025 81	2.406 61	2.854 33
7	1.229 87	1.315 93	1.4071	1.503 63	19	1 7535	2.100 84	2.526 95	3.025 59
8	1.266 77	1.368 57	1.477 45	1.593 84	20	1.806 11	2.191 13	2 653 29	3.207 13
9	1.304 77	1.423 31	1.551 32	1.689 47	21	1.860 29	2.278 76	2.785 96	3.399 56
10	1.343 92	1.480 24	1.628 89	1.790 84	22	1.9161	2.369 92	2 925 26	3.603 53
11	1.384 24	1.539 45	1.710 33	1.898 29	23	1.9736	2.464 21	3.071 52	3.819 74
12	1.425 76	1.601 03	1.795 85	2.012 19	24	2.032 79	2.5633	3.225 09	4.048 73

For any other Rate or Period.—Multiply logarithm of rate + 1 by period, and number for logarithm will give tabular amount as above.

ILLUSTRATION.—What is tabular number for 4 per cent. for 10 years?

Log. of 1.04 = .017 033 3, which × 10 = .170 333, and number for log. = 1.480 24.

Time in Years in which a Sum of Money will be doubled at Several Rates of Interest.

Rate.	Time.	Rate.	Time.	Rate.	Time.	Rate.	Time.
Per cent.		Per cent.		Per cent.		Per cent.	
1	69.68	4	17.67	7	10.34	10	7.27
2	35	5	14.21	8	9.01	20	3.8
3	23.44	6	11.88	9	8.04	30	2.64

Value of $1, etc., Computed Semi-annually for a Period of 12 Years.

Years.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.	Years.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.
.5	1.015	1.02	1.025	1.03	6.5	1.2134	1.2936	1.3785	1.4684
1	1.0302	1.0404	1.0506	1.0609	7	1.2317	1.3195	1.413	1.5102
1.5	1.0457	1.0612	1.0769	1.0927	7.5	1.2502	1.3459	1.4483	1.558
2	1.0614	1.0824	1.1038	1.1255	8	1.269	1.3728	1.4845	1.6047
2.5	1.0773	1.1041	1.1314	1.1593	8.5	1.288	1.4002	1.5216	1.6528
3	1.0934	1.1262	1.1597	1.1941	9	1.3073	1.4282	1.5597	1.7024
3.5	1.1098	1.1487	1.1887	1.2299	9.5	1.3269	1.4568	1.5987	1.7535
4	1.1265	1.1717	1.2184	1.2668	10	1.3469	1.486	1.6386	1.8061
4.5	1.1434	1.1951	1.2489	1.3048	10.5	1.3671	1.5157	1 6796	1.8603
5	1.1604	1.219	1.2801	1.3439	11	1.3876	1 546	1.7216	1.9161
5.5	1.178	1.2434	1.3121	1.3842	11.5	1.4084	1.5769	1.7606	1.9736
6	1.1956	1.2689	1.3449	1.4258	12	1.4295	1.6084	1.8087	2.0356

ILLUSTRATION.—What is amount of $500 at semi-annual interest of 5 per cent. compounded for 10 years?

Tabular number 1.6386. Then, 500 × 1.628 89 = $814.44.5.

To Compute Interest on any Given Sum.

For a Period of Years. $P(1+r)^n = A$; $\frac{A}{(1+r)^n} = P$; $\sqrt[n]{\frac{A}{P}} - 1 = r$,

and $\frac{\log. A - \log. P}{\log. (1+r)} = n$. P *representing principal,* r *rate per cent. per annum,* n *number of years, and* A *amount of principal and interest.*

ILLUSTRATION.—Assume as preceding, \$500 at 5 per cent. for 10 years.

$500 \times 1.05^{10} = 500 \times 1.628\,89 = \$814.44.5$, *amount.* $\frac{814.44.5}{(1+.05)^{10}} = 500$, *principal.*

$\sqrt[10]{\frac{814.44.5}{500}} - 1 = .05$, *rate.* $\frac{\log.\ 814.44.5 - \log.\ 500}{\log.\ (1+.05)} = 10$, *number of years.*

For any Period.—Assume elements of preceding case, interest payable semi-annually. $10 \times 2 = 20$, *number* of payments; $\frac{.05}{2} = .025$, *rate.*

Then, $500 \times 1.025^{20} = 500 \times 1.638\,62 = \$819.31$.

When term of payments and rate are not given in table.

$$\left[\log.\left(\frac{r}{2}+1\right) \times n\,P\right] = \log.\ A.$$

ILLUSTRATION.—Assume \$1000 for 30 years, at 7 per cent. half-yearly.

$$\log.\ \frac{.07}{2} + 1 = .014\,940\,3, \text{ and } \log.\ \overline{.014\,940\,3 \times 30 \times 1000} = \$2806.78.$$

DISCOUNT OR REBATE.

DISCOUNT or REBATE is a deduction upon money paid before it is due.

To Compute Rebate upon any Sum.

RULE.—Multiply amount by rate per cent. and by time, and divide product by sum of product of rate per cent. and time, added to 100.

EXAMPLE 1.—What is discount upon \$12 075 for 3 years, 5 months, and 15 days, at 6 per cent.?

3 years 5 months and 15 days = 3.4574 *years.*

$$\frac{12\,075 \times 6 \times 3.4574}{100 + (6 \times 3.4574)} = \frac{250\,488.63}{120.7444} = 2074.53 = \$2074.53.$$

2.—What is present value of a note for \$963.75, payable in 7 months, at 6 per cent.?

6 *rate.* 7 months $= \frac{7}{12}$ of 1 year $= 6 \times 7 \div 12 = 3.5$, and $3.5 + 100 = 103.5 = 1.035$.

$$963.75 \div 1.035 = \$931.16.$$

To Compute the Sum for a given Time and Rate, to yield a Certain Sum.

RULE.—Divide given sum by proceeds of \$1 for given time and rate.

EXAMPLE.—For what sum should a note be drawn at 90 days, that when discounted at 6 per cent. it will net \$200?

Discount on \$1 for 90 + 3 days at 6 per cent. = \$.0155.

Hence, \$1 — .0155 = .9845, *proceeds*, and \$200 ÷ .9845 = \$203.14.9.

EQUATION OF PAYMENTS.

RULE.—Multiply each sum by its time of payment in days, and divide sum of products by sum of payments.

EXAMPLE.—A owes B \$300 in 15 days, \$60 in 12 days, and \$350 in 20 days; when is the whole due?

$$\begin{array}{rl} 300 \times 15 = & 4\,500 \\ 60 \times 12 = & 720 \\ 350 \times 20 = & 7\,000 \\ \hline 710 \quad) & 12\,220\ (17+ \textit{days.} \end{array}$$

ANNUITIES.

To Compute Amount of Annuity.

When Time and Ratio of Interest are Given. RULE.—Raise the *ratio* to a power denoted by time, from which subtract 1; divide remainder by *ratio* less 1, and quotient, multiplied by annuity, will give amount.

NOTE.—$1 added to given rate per cent. is ratio, and preceding table in Compound Interest is a table of ratios.

EXAMPLE.—What is amount of an annual pension of $100, interest 5 per cent., which has remained unpaid for four years?

1.05 ratio; then $1.05^4 - 1 = 1.215\,506\,25 - 1 = .215\,506\,25$, and $.215\,506\,25 \div (1.05 - 1)\,.05 = 4.310\,125$, which $\times$ 100 = $431.01.25.

To Compute Present Worth of an Annuity.

When Time and Rate of Interest are Given. RULE.—Ascertain amount of it for whole time; divide by ratio, involved to time, and result will give worth.

EXAMPLE.—What is present worth of a pension or salary of $500, to continue 10 years at 6 per cent. compound interest?

$500, by last rule, is worth $6590.3975, which, divided by 1.06^{10} (by table, page 108, is 1.790 84) = $3680.05.

Or, Multiply tabular amount in following table by given annuity, and product will give present worth.

ILLUSTRATION 1.—As above; 10 years at 6 per cent. = 7.360 08, and 7.360 08 $\times$ 500 = 3.68.004 *dollars.*

2. What is present worth of $150 due in one year at 6 per cent. interest per annum?

$.943\,39 \times 150 =$ $141.50.85.

Present Worth of an Annuity of $1, at 4, 5, and 6 Per Cent. Compound Interest for Periods under 25 Years.

Years.	4 Per Cent.	5 Per Cent.	6 Per Cent.	Years.	4 Per Cent.	5 Per Cent.	6 Per Cent.
1	.961 54	.952 38	.943 39	13	9.985 62	9.393 57	8.852 68
2	1.886 09	1.859 41	1.833 39	14	10.563 07	9.898 64	9.294 98
3	2.775 1	2.723 25	2.673 01	15	11.118 43	10.379 66	9.712 25
4	3.629 9	3.545 95	3.465 1	16	11.651 28	10.837 78	10.105 89
5	4.452 03	4.329 48	4.212 36	17	12.166 26	11.274 07	10.477 26
6	5.242 15	5.075 69	4.917 32	18	12.659 26	11.689 58	10.827 6
7	6.002 03	5.786 37	5.582 38	19	13.133 88	12.085 32	11.158 11
8	6.731 76	6.463 21	6.209 79	20	13.590 29	12.462 21	11.469 92
9	7.436 4	7.107 82	6.801 69	21	14.029 12	12.821 15	11.764 07
10	8.110 85	7.721 73	7.360 08	22	14.451 12	13.163	12.041 58
11	8.760 44	8.306 41	7.886 87	23	14.856 82	13.488 07	12.303 38
12	9.385 05	8.863 25	8.383 84	24	15.246 95	13.798 64	12.550 35

For a Rate of Interest and Term of Years not given in either Table.

$$\frac{P}{r}\left[1 - \frac{1}{(1+r)^n}\right] = A. \quad \textit{Notation as preceding.}$$

ILLUSTRATION.—Take $1 at 4 per cent. for 24 years.

Log. 1.04 = .017 033, which $\times$ 24 = .408 799. log. .408 799 = 2.5633 = *ratio raised to power of* 24.

$$\text{Then, } \frac{1}{.04} \times \left(1 - \frac{1}{2.5633}\right) = 25 \times \overline{1 - .390\,122} = \$15.24.695.$$

To Compute Yearly Amount that will Liquidate a Debt in a Given Number of Years at Compound Interest.

$\frac{P\,r\,(1+r)^n}{(1+r)^n - 1} = A.$ ILLUSTRATION.—What is amount of an annual payment that will liquidate a debt of $100 in 6 years at 5 per cent. compound interest?

$(1+.05)^6$ per table, page 108, $=1.34$. $\frac{100\times.05\,(1+.05)^6}{(1+.05)^6-1}=\frac{5\times1.34}{1.34-1}=\frac{6.7}{.34}=\$19.76.$

When Annuities do not commence till a certain period of time, they are said to be in Reversion.

To Compute Present Worth of an Annuity in Reversion.

RULE.—Take two amounts under rate in above table—viz., that opposite sum of two given times and that of time of reversion; multiply their difference by annuity, and product will give present worth.

EXAMPLE.—What is present worth of the reversion of a lease of $40 per annum, to continue for 6 years, but not to commence until end of 2 years, at rate of 6 per cent.?

$6+2=8$ years 6.209 79
2 " 1.833 39
4.376 40 $\times$ 40 = $175.05.6.

Amount of Annuity of $1, etc., Compound Interest, from 1 to 20 Years.

Years.	4 Per Cent.	5 Per Cent.	6 Per Cent.	7 Per Cent.	Years.	4 Per Cent.	5 Per Cent.	6 Per Cent.	7 Per Cent.
1	1.	1.	1.	1.	11	13.486 35	14.206 79	14.971 64	15.783 6
2	2.04	2.05	2.06	2.07	12	15.025 8	15.917 13	16.869 94	17.888 45
3	3.121 6	3.152 5	3.183 6	3.214 9	13	16.626 84	17.712 98	18.882 14	20.140 64
4	4.246 46	4.310 12	4.374 62	4.439 94	14	18.291 91	19.598 63	21.015 07	22.550 49
5	5.416 32	5.525 63	5.637 09	5.750 74	15	20.023 59	21.578 56	23.275 97	25.129 02
6	6.632 97	6.801 91	6.975 32	7.153 29	16	21.824 53	23.657 49	25.672 53	27.888 05
7	7.898 29	8.142 01	8.393 84	8.654 02	17	23.697 51	25.840 37	28.212 88	30.840 22
8	9.214 23	9.549 11	9.897 47	10.259 8	18	25.645 41	28.132 38	30.905 65	33.999 03
9	10.582 79	11.026 56	11.491 32	11.977 99	19	27.671 23	30.539	33.759 99	37.378 96
10	12.006 11	12.577 89	13.180 79	13.816 45	20	29.778 08	33.065 95	36.785 59	40.995 49

ILLUSTRATION.—What is amount of $ 1000 for 20 years at 5 per cent.?

5 per cent. for 20 years = 33.065 95; hence, 1000 $\times$ 33.065 95 = $ 33.06.595.

To Compute Amount of an Annuity for any Period and Rate.

RULE.—From table for Compound Interest, page 108, take value for rate per cent. for 1 year, and raise it to a power determined by time in years, from which subtract 1, divide remainder by rate, and quotient multiplied by annuity will give amount required.

EXAMPLE.—What will an annuity of $ 50, payable yearly, amount to in 4 years, at 5 per cent.?

By table, page 108, $1.05^4=1.2155$.

$1.2155-1\div(1.05-1)=4.31$, and $4.31\times50=\$215.50$.

For Half-yearly and Quarterly Payments.

Multiply annuity for given time by amount in following table:

Rate per cent.	Half-yearly.	Quarterly.	Rate per cent.	Half-yearly.	Quarterly.
3	1.007 445	1.011 181	5.5	1.013 567	1.020 395
3.5	1.008 675	1.013 031	6	1.014 781	1.022 227
4	1.009 902	1.014 877	6.5	1.015 993	1.024 055
4.5	1.011 126	1.016 729	7	1.017 204	1.025 88
5	1.012 348	1.018 559	7.5	1.018 414	1.027 704

ILLUSTRATION 1.—Annuity as determined in previous case = $215.50.

Hence, 215.50 $\times$ 1.012 348 from above table = $218.16 *for half yearly payments.*

2. A person 30 years of age has an annuity for 10 years, present worth of it being $1000, provided he may live for 10 years. What is annuity worth, assuming that 60 persons out of every 3550, between the ages of 30 and 40, die annually?

3550 — 600 (60 $\times$ 10) = 2950 *would therefore be living.*
And, 3550 : 2950 :: 1000 = $830.98.

PERPETUITIES.

Perpetuities are such Annuities as continue forever.

To Compute Value of a Perpetual Annuity.

Rule.—Divide annuity by rate per cent., and multiply quotient by unit in preceding table.

Example.—What is present worth of an annuity for \$100, payable semi-annually, at 5 per cent.?

$100 \div .05 = 2$, and $2 \times 1.012\,348$, from preceding table $= 2.024.70$.

To Compute Value of a Perpetuity in Reversion.

Rule.—Subtract present worth of annuity for time of reversion from worth of annuity, to commence immediately.

Example.—What is present worth of an estate of \$50 per annum, at 5 per cent., to commence in 4 years?

$50 \div .05$ .... $= 1000$
\$50, for 4 years, at 5 per cent. $= 3.545\,95$ (from table, page 110) $\times 50 = 177.2975$
822.7025
which in 4 years, at 5 per cent. compound interest, would produce \$1000.

COMBINATION.

Combination is a rule for ascertaining how often a less number of numbers or things can be chosen varied from a greater, or how many different collections may be formed without regard to order of each collection.

Combinations of any number of things signify the different collections which may be formed of their quantities, without regard to the *order* of their arrangement.

Thus, 3 letters, a, b, c, taken *all together*, form but one combination, abc. Taken *two* and *two*, they form 3 combinations, as ab, ac, bc.

Note.—Class of the combination is determined by number of elements or things to be taken; if two are taken, the combination is of 2d class, and so on.

Rule.—Multiply together natural series 1, 2, 3, etc., up to the number to be taken at a time. Take a series of as many terms, decreasing by 1, from number out of which combination is to be made, ascertain their continued product, and divide this last product by former.

Example 1.—How many single combinations, as ab, ac, may be made of 2 letters out of 3?

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6} \div \frac{6}{2} = 3.$$

2.—How many combinations may be made of 7 letters out of 12?

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6} = \frac{5040}{3\,991\,680}, \text{ and } \frac{3\,991\,680}{5040} = 792.$$

3.—How many different hands of cards may be held, as at whist, combinations 13 out of 52? 635 013 559 600.

When two Numbers or Things are Combined.

Rule.—Multiply together natural series 1, 2, 3, etc., to one less term than number of combinations; ascertain their continued product, and proceed as before.

Example.—There are 3 cards in a box, out of which two are to be drawn in a required order. How many combinations are there?

Here there are 2 terms; hence, $2 - 1 = 1$, and $\frac{1}{3 \times 2} = \frac{1}{6} = 6 \div 1 = 6$.

To Compute Number of Ways in which any Number of Distinct Objects can be Divided among any Number.

Rule.—Multiply together numbers equal to number given, as often as objects are to be divided among them.

Example.—In how many different ways can 10 different cards be divided among 3 persons? $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ or $3^{10} = 59\,049$.

Combinations with Repetitions.

In this case the repetition of a term is considered a new combination. Thus, 1, 2, admits of but one combination, if not repeated; if repeated, however, it admits of three combinations, as 1, 1; 1, 2; 2, 2.

RULE.—To number of terms of series add number of class of combination, less 1; multiply sum by successive decreasing terms of series, down to last term of series; then divide this product by number of permutations of the terms, denoted by class of combination.

EXAMPLE.—How many different combinations of numbers of 6 figures can be made out of 11?

$11 + (6 - 1) = 16 =$ *sum of number of terms, and number of class, less* 1.
$16 \times 15 \times 14 \times 13 \times 12 \times 11 = 5\,765\,760 =$ *product of sum, and successive terms to last term.*
$1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ *permutations of class of combination.*

$$\text{Then, } \frac{5\,765\,760}{720} = 8008.$$

Variations with Repetitions.

Every different arrangement of individual number or things, including repetitions, is termed a Variation.
Class of Variation is denoted by number of individual things taken at a time.

RULE.—Raise number denoting the individual things to a power, the exponent of which is number expressing class of variation.

EXAMPLE 1.—How many variations with 4 repetitions can be made out of 5 figures? $5^4 = 625$.

2.—How many different combinations of 4 places of figures can be made out of the 9 digits?

$$9 + (4 - 1) = 12, \text{ and } \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4} = \frac{11\,880}{24} = 495.$$

Combination without Repetitions.

RULE.—From number of terms of series subtract number of class of combination, less 1; multiply this remainder by successive increasing terms of series, up to last term of series; then divide this product by number of permutations of the terms, denoted by class of combination.

EXAMPLE 1.—How many combinations can be made of 4 letters out of 10, excluding any repetition of them in any second combination?

$10 - (4 - 1) = 7 =$ *number of terms — number of class, less* 1.
$7 \times 8 \times 9 \times 10 = 5040 =$ *prod. of remainder* 7, *and successive terms up to last term.*
$1 \times 2 \times 3 \times 4 = 24 =$ *permutations of class of combination.*

$$\text{Then, } \frac{5040}{24} = 210.$$

2.—How many combinations of the 5th class, without repetitions, can be made of 12 different articles?

$$12 - (5 - 1) = 8, \text{ and } \frac{8 \times 9 \times 10 \times 11 \times 12}{1 \times 2 \times 3 \times 4 \times 5} = \frac{95\,040}{120} = 792.$$

CIRCULAR MEASURE.

Unit of Circular Measure is an angle which is subtended at centre of a circle by an arc equal to radius of that circle, being equal to

$$\frac{180^\circ}{3.1416} = 57.296^\circ.$$

Circular measure of an angle is equal to a fraction which has for its numerator the arc subtended by that angle at centre of any circle, and for its denominator the radius of that circle.

To Compute Circular Measure of an Angle.

RULE.—Multiply measure of angle in degrees by 3.1416, and divide by 180.

EXAMPLE.—What is circular measure of 24° 10′ 8″?

$$\frac{24^\circ\ 10'\ 8'' \times 3.1416}{180} = \frac{87\,008 \times 3.1416}{180 \times 60 \times 60} = .04218.$$

To Compute Measure of an Angle, its Circular Measure being Given.

RULE.—Multiply circular measure of angle by 180, and divide by 3.1416.

PROBABILITY.

Probability of any event is the *ratio* of the favorable cases, to all the cases which are similarly circumstanced with regard to the occurrence. If an event have 3 chances for occurring and 2 for failing, sum of chances being 5, the fraction $\frac{3}{5}$ will represent probability of its occurring and is taken as measure of it. Thus, from a receptacle containing 1 white and 2 black balls, the probability of drawing a white ball, by abstraction of 1, is $\frac{1}{3}$; probability of throwing ace with a die is $\frac{1}{6}$: in other words, the odds are 2 to 1 against first, and 5 to 1 against second.

If $m + n$ = whole number of chances, *m representing number which are favorable, and n unfavorable.* Therefore $\frac{m}{m+n}$ = *probability of event.*

Probabilities of two or more single events being known, probability of their occurring in succession may be determined by multiplying together the probabilities of their events, considered singly.

Thus, probability of one event in two is expressed by $\frac{1}{2}$; of its occurring twice in succession, $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$; of thrice in succession, $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{8}$, etc.

ILLUSTRATION 1.—If a cent is thrown twice into the air, the probability of its falling with its head up, twice in succession, is as 1 to 4. Thus, it may fall:

1. Head up twice in succession.
2. Head up 1st time and wreath 2d time.
3. Wreath up 1st time and head 2d time.
4. Wreath up twice in succession.

Hence, $\frac{1}{1+3} = .25 = \frac{1}{.25} = 4$ *times.*

These are the only results possible, and being all similarly circumstanced as to probability, the probability of each case is as 1 to 4, or odds are as 3 to 1.

Probability of either head or wreath being up twice in succession is as 1 to 1, or chances are even, because 1st and 4th cases favor such a result; probability of head once and wreath once in any order is as 1 to 2, because 2d and 3d cases favor such a result; and probability of head or wreath once is as 3 to 4, or odds are as 3 to 1, because 1st, 2d, and 3d, or 2d, 3d, and 4th cases favor such a result.

NOTE.—1 to 2 is an equal chance, for 1 out of 2 chances = 1 to 1, being an equal chance; again, 1 to 5 is 4 to 1, for 1 out of 5 chances is 1 to 4.

2.—If there are 4 white balls and 6 black in a bag, what is the chance of a person drawing out 2 black at two successive trials?

This is a combination without repetition. Hence, $6 - (2 - 1) = 5$,

and $\frac{5 \times 6}{1 \times 2} = \frac{30}{2} = \frac{15}{1}$, which $\times$ 2 for successive trials $= \frac{15}{2}$ or $\frac{2}{15}$.

3.—Suppose with two bags, one containing 5 white balls and 2 black, and the other 7 white and 3 black.

Number of cases possible in one drawing from each bag is $(5 + 2) \times (7 + 3) = 7 \times 10 = 70$, because every ball in one bag may be drawn alike to one in the other.

Number of cases which favor drawing of a white ball from both bags is $5 \times 7 = 35$, for every one of the 5 white balls in one bag may be drawn in combination with every one of the 7 in the other. For a like cause, number of cases which favor drawing of a white ball from 1st bag and a black one from 2d is $5 \times 3 = 15$; a black ball from 1st bag and a white ball from 2d is $7 \times 2 = 14$; and a black ball from both is $3 \times 2 = 6$.

Probability, therefore, of drawing is as

$\frac{5 \times 7}{70} = \frac{35}{70} = \frac{1}{2} = 1$ to 1, *a white ball from both bags.* $\frac{5 \times 3}{70} = \frac{15}{70} = \frac{3}{14} = 3$ to 11, *a white ball from 1st, and a black from 2d.* $\frac{7 \times 2}{70} = \frac{14}{70} = \frac{1}{5} = 1$ to 4, *a black ball from 1st, and a white from 2d.* $\frac{3 \times 2}{70} = \frac{6}{70} = \frac{3}{35} = 3$ to 32, *a black ball from both.* $\frac{5 \times 3 + 2 \times 7}{70} = \frac{29}{70} = 29$ to 41, *a white ball from one, and a black from other,* for both 2d and 3d cases favor this result; hence, $\frac{1}{5} + \frac{3}{14} = \frac{29}{70}$. $\frac{5 \times 7 + 5 \times 3 + 2 \times 7}{70} = \frac{64}{70} = \frac{32}{35} = 32$ to 3, *at least one white ball,* for the 1st, 2d, and 3d cases favor this result; hence, $\frac{1}{2} + \frac{3}{14} + \frac{1}{5} = \frac{32}{35}$.

Again, if number of white and black balls in each bag are same, say 5 white and 2 black, $5 + 2 \times \overline{5 + 2} = 49$, then probability of drawing is as

$\frac{5 \times 5}{49} = \frac{25}{49} = 25$ to 24, *a white ball from both.* $\frac{5 \times 2}{49} = \frac{10}{49} = 10$ to 39, *a white ball from 1st, and a black from 2d.* $\frac{2 \times 5}{49} = \frac{10}{49} = 10$ to 39, *a black ball from 1st, and a white from 2d.* $\frac{2 \times 2}{49} = \frac{4}{49} = 4$ to 45, *a black ball from both.*

4.—When two dice are thrown, probability that sum of numbers on upper sides is any given number, say 7, is as follows:

As every one of the six numbers on one die may come up alike to, or in combination with the other, number of throws is $6 \times 6 = 36$.

Number 7 may be a combination of $\left\{\begin{matrix} 1 \text{ and } 6 \\ 2 \quad “ \quad 5 \\ 3 \quad “ \quad 4 \end{matrix}\right\}$; and as these numbers may be upon either die, there are $3 \times 2 = 6$ throws in favor of the combination of 7; hence probability of throwing 7 is $\frac{6}{36} = \frac{1}{6}$, or as 1 to 5.

5.—Probability of a player's partner at Whist holding a given card is as follows:

Number of cards held by the other 3 players is $3 \times 13 = 39$; probability, therefore, that it is held by partner is $\frac{1}{39}$, but it may be one of the 13 cards which he holds; hence probability is $\frac{1}{39} \times 13 = \frac{13}{39} = \frac{1}{3}$, or as 1 to 2.

6.—Probability of a player's partner at Whist holding two given cards is as follows:

Number of combinations of 39 things, taken 2 and 2 together, is $\frac{39 \times 38}{1 \times 2} = 741$; therefore, probability that these 2 cards are in partner's hand is $\frac{1}{\frac{39 \times 38}{1 \times 2}} = \frac{1}{39 \times 19} = \frac{1}{741} = 1$ to 740; but they may be any 2 cards in partner's hand; therefore, since number of combinations of 13 cards, taken 2 and 2 together, is $\frac{13 \times 12}{1 \times 2} = \frac{156}{2} = 78$, probability required is $\frac{78}{741} = \frac{2}{19}$, or as 2 to 17.

Similarly, probability that he holds any 3 given cards is as $\frac{22}{703}$, or as 22 to 681.

Probabilities at a game of Whist upon following points are:

9 to 7, *that one hand has two honors, and two hands one;*
9 to 55, *that two hands have each two honors;*
3 to 29, *that each hand holds an honor;*
3 to 13, *that one hand has three honors, and one hand one;*
1 to 63, *that four honors are held by one hand.*

7.—If 3 half-dollars are thrown into the air, probability of any of the possible combinations of their falling is determined as follows:

$$\left(\frac{1}{2}+\frac{1}{2}\right)^3=\left(\frac{1}{2}\right)^3+\frac{3}{1}\left(\frac{1}{2}\right)^3+\frac{3\times 2}{1\times 2}\left(\frac{1}{2}\right)^3+\frac{3\times 2\times 1}{1\times 2\times 3}\left(\frac{1}{2}\right)^3$$

Hence, $\left(\frac{1}{2}\right)^3=.125=1$ to 7 in favor of 3 *heads.*

$\frac{3}{1}\left(\frac{1}{2}\right)^3=.375=3$ to 5 " " 2 *heads and* 1 *tail.*

$\frac{3\times 2}{1\times 2}\left(\frac{1}{2}\right)^3=.375=3$ to 5 " " 1 *head and* 2 *tails.*

$\frac{3\times 2\times 1}{1\times 2\times 3}\left(\frac{1}{2}\right)^3=.125=1$ to 7 " " 3 *tails.*

And in like manner, if 5 were thrown up, probability of any of their possible combinations would be determined as follows:

$$\left(\frac{1}{2}+\frac{1}{2}\right)^5=\left(\frac{1}{2}\right)^5+\frac{5}{1}\left(\frac{1}{2}\right)^5+\frac{5\times 4}{1\times 2}\left(\frac{1}{2}\right)^5+\frac{5\times 4\times 3}{1\times 2\times 3}\left(\frac{1}{2}\right)^5+\frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4}\left(\frac{1}{2}\right)^5+\frac{5\times 4\times 3\times 2\times 1}{1\times 2\times 3\times 4\times 5}\left(\frac{1}{2}\right)^5$$

Hence, $\left(\frac{1}{2}\right)^5=.03125=1$ to 31 in favor of 5 *heads;*

$\frac{5}{1}\left(\frac{1}{2}\right)^5=.15625=5$ to 27 " " 4 *heads and* 1 *tail;*

$\frac{5\times 4}{1\times 2}\left(\frac{1}{2}\right)^5=.3125=10$ to 22 " " 3 *heads and* 2 *tails;*

$\frac{5\times 4\times 3}{1\times 2\times 3}\left(\frac{1}{2}\right)^5=.3125=10$ to 22 " " 2 *heads and* 3 *tails;*

$\frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4}\left(\frac{1}{2}\right)^5=.15625=5$ to 27 " " 1 *head and* 4 *tails;*

$\frac{5\times 4\times 3\times 2\times 1}{1\times 2\times 3\times 4\times 5}\left(\frac{1}{2}\right)^5=.03125=1$ to 31 " " 5 *tails.*

All Wagers are founded upon the principle of product of the event, and contingent gain, being equal to amount at stake.

ILLUSTRATION 1.—Suppose 3 horses, A, B, and C, are entered for a race, and X wagers 12 to 5 against A, 11 to 6 against B, and 10 to 7 against C.

If A wins, X wins $6+7-12=1$.
" B " X " $5+7-11=1$.
" C " X " $5+6-10=1$.

Hence, X wins 1, whichever horse wins, from having taken field against each horse at odds named.

Odds given in favor of $\left\{\begin{array}{l}\text{A are 5 to 12}\\ \text{B " 6 " 11}\\ \text{C " 7 " 10}\end{array}\right\}$; corresponding probability is $\left\{\begin{array}{ll}\frac{5}{17} & \text{in favor of A,}\\ \frac{6}{17} & \text{" B,}\\ \frac{7}{17} & \text{" C,}\end{array}\right.$

and $\frac{5}{17}+\frac{6}{17}+\frac{7}{17}=\frac{18}{17}=1.06=1.06$ *to* 1 *in favor of taker of odds.*

2.—Odds given upon first seven favorite horses for Oaks Stakes of 1828 were so great, that probability in favor of taker of the odds when reduced was as follows:

1st, 5 to 2; 2d, 5 to 2; 3d, 4 to 1; 4th, 7 to 1; 5th, 14 to 1; 6th, 14 to 1; 7th, 15 to 1

$$= \frac{2}{7}+\frac{2}{7}+\frac{1}{5}+\frac{1}{8}+\frac{1}{15}+\frac{1}{15}+\frac{1}{16} = \frac{4}{7}+\frac{5}{15}+\frac{3}{16} = \frac{4}{7}+\frac{1}{3}+\frac{3}{16} = \left\{ \begin{array}{ll} 4\times3\times16 = & 192 \\ 1\times7\times16 = & 112 \\ 3\times7\times\ \ 3 = & \underline{\ 63} \\ \hline 7\times3\times16 & 336 \end{array}\right.$$

$= 367 \div 336 = 1.092 = 1.092$ *to* 1, *in favor of taker of odds*, yet neither of the horses upon which these odds were given won.

3.—If odds are 3 to 1 against a horse in a race, and 6 to 1 against another horse in a second race, probability of 1st horse winning is $\frac{1}{4}$, and of other $\frac{1}{7}$. Therefore probability of both races being won is $\frac{1}{28}$, and odds against it 27 to 1, or 1000 to 37.037. Odds upon such an event were given in 1828 at 1000 to 60, or 16.67 to 1.

4.—Two persons play for a certain stake, to be won by winner of three games or results. One having won one and the other two, they decide to divide the sum, proportionate to their interest. How much of it should each one receive?

Operation.—If winner of two games should win game to be played, he would be entitled to the whole sum; if he lost, he would be entitled to half of it. Now as one event is as probable as the other, $\frac{1}{1}+\frac{1}{2}=\frac{3}{2}$, half of which $=\frac{3}{4}$, or *share of winner of two games.*

When events are wholly independent, so that occurrence of one does not affect that of the other, probability that both will occur is product of probabilities that each will occur.

Note.—It is indifferent whether events are to occur together or consecutively.

Illustration 1.—Assume three boxes, each containing white and black balls as follows:

6 white, 5 black; 7 white, 2 black; 8 white, 10 black. What is chance of drawing from them a white, black, and a white ball?

Probabilities are $\frac{6}{11}$, $\frac{2}{9}$, and $\frac{8}{18}$, product of which $=\frac{6+2+8}{297}=17.625$ to 1.

2.—A gives an answer correctly 3 times out of 4, B 4 times out of 5, and C 6 out of 7. What is probability of an event which A and B declare correct and C denies?

Operation.—Compound probability that A and B answer correctly and C denies (all 3 of which are in favor of event) is $\frac{3}{4}\times\frac{4}{5}\times\frac{1}{7}=\frac{12}{140}=\frac{3}{35}$.

Compound probability that A and B deny and C is correct (all 3 of which are against event) is $\frac{1}{4}\times\frac{1}{5}\times\frac{6}{7}=\frac{6}{140}=\frac{3}{70}$.

Then correct, divided by sum of correct and incorrect, $=\frac{3}{35}\div\left(\frac{3}{35}+\frac{3}{70}\right)=\frac{.8714}{.85714+.42857}=.68$ or $\frac{2}{3}$.

Odds between Results or Chances, and between any Number and Whole Number, at various Odds against each, also Value of each Chance in parts of 100.

Odds against each.	Value of Chance.	Odds against each.	Value of Chance.	Odds against each.	Value of Chance.	Odds against each.	Value of Chance.
Even	50	2 to 1	33.33	6.5 to 1	13.33	15 to 1	6.25
11 to 10	47.62	2.5 " 1	28.57	7 " 1	12.5	18 " 1	5.26
6 " 5	45.45	3 " 1	25	7.5 " 1	11.76	20 " 1	4.76
5 " 4	44.44	3.5 " 1	22.22	8 " 1	11.11	25 " 1	3.84
5.5 " 4	42.1	4 " 1	20	8.5 " 1	10.52	30 " 1	3.22
6 " 4	40	4.5 " 1	18.18	9 " 1	10	40 " 1	2.44
6.5 " 4	38.1	5 " 1	16.66	9.5 " 1	9.52	50 " 1	1.96
7 " 4	36.36	5.5 " 1	15.38	10 " 1	9.09	60 " 1	1.64
7.5 " 4	34.78	6 " 1	14.28	12 " 1	7.7	100 " 1	.99

Operation.—Divide 100, or unit, as case may be, by sum of odds, and multiply quotient by lesser chance or odds.

Illustration.—6 to 4. $6+4=10$, and $100\div10\times4=40$, *value of chance.*

WEIGHTS OF IRON, STEEL, COPPER, ETC.

Wrought Iron, Steel, Copper, and Brass Plates.

SOFT ROLLED. (*American Gauge.*)

No. of Gauge.	Thickness.	Per Square Foot. Iron.	Steel.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46 or $\frac{7}{16}$ full	18.4575	18.7036	20.838	19.688
000	.409 64	16.4368	16.6559	18.556 7	17.532 6
00	.364 8 or $\frac{3}{8}$ light	14.6376	14.8328	16.525 4	15.613 4
0	.324 86 or $\frac{1}{3}$ "	13.0351	13.2088	14.716 2	13.904
1	.289 3	11.6082	11.7629	13.105 3	12.382
2	.257 63 or $\frac{1}{4}$ full	10.3374	10.4752	11.670 6	11.026 6
3	.229 42	9.2055	9.3283	10.392 7	9.819 2
4	.204 31 or $\frac{1}{5}$ full	8.1979	8.3073	9.255 2	8.744 5
5	.181 94 or $\frac{3}{16}$ light	7.3004	7.3977	8.241 9	7.787
6	.162 02	6.5011	6.5878	7.339 5	6.934 5
7	.144 28	5.7892	5.8664	6.535 9	6.175 2
8	.128 49 or $\frac{1}{8}$ full	5.1557	5.2244	5.820 6	5.499 4
9	.114 43	4.5915	4.6527	5.183 7	4.897 6
10	.101 89 or $\frac{1}{10}$ full	4.0884	4.1428	4.615 6	4.360 9
11	.090 742	3.641	3.6896	4.110 6	3.883 8
12	.080 808	3.2424	3.2856	3.660 6	3.458 6
13	.071 961	2.8874	2.9259	3.259 8	3.079 9
14	.064 084	2.5714	2.6057	2.903	2.742 8
15	.057 068	2.2899	2.3204	2.585 2	2.442 5
16	.050 82 or $\frac{1}{20}$ full	2.0392	2.0664	2.302 1	2.175 1
17	.045 257	1.8159	1.8402	2.050 1	1.937
18	.040 303	1.6172	1.6387	1.825 7	1.725
19	.035 89	1.44	1.4593	1.625 8	1.536 1
20	.031 961	1.2824	1.2995	1.447 8	1.367 9
21	.028 462	1.142	1.1573	1.289 3	1.218 2
22	.025 347	1.017	1.0306	1.148 2	1.084 9
23	.022 571	.9057	.9177	1.022 5	.966 04
24	.021 1	.8065	.8173	.910 53	.860 28
25	.017 9	.7182	.7278	.810 87	.766 12
26	.015 94	.6396	.6481	.722 08	.682 23
27	.014 195	.5696	.5772	.643 03	.607 55
28	.012 641	.5072	.514	.572 64	.541 03
29	.011 257	.4517	.4577	.509 94	.481 8
30	.010 025	.4023	.4076	.454 13	.429 07
31	.008 928	.3582	.363	.404 44	.382 12
32	.007 95	.319	.3232	.360 14	.340 26
33	.007 08	.2841	.2879	.320 72	.303 02
34	.006 304	.2529	.2563	.285 57	.269 81
35	.005 614	.2253	.2283	.254 31	.240 28
36	.005	.2006	.2033	.226 5	.214
37	.004 453	.1787	.181	.201 72	.190 59
38	.003 965	.1591	.1612	.179 61	.169 7
39	.003 531	.1417	.1436	.159 95	.151 13
40	.003 144	.1261	.1278	.142 42	.134 56
Specific Gravities		7.704	7.806	8.698	8.218
Weights of a Cube Foot		481.75	487.75	543.6	513.6
" " Inch		.278 7	.2823	.314 6	.297 2

Wrought Iron, Steel, Copper, and Brass Plates.

(Birmingham Gauge.)

No. of Gauge.	Thickness.	Per Square Foot. Iron.	Steel.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.454 or 7/16 full	18.2167	18.4596	20.5662	19.4312
000	.425	17.0531	17.2805	19.2525	18.19
00	.38 or 3/8 full	15.2475	15.4508	17.214	16.264
0	.34 or 1/3 "	13.6425	13.8244	15.402	14.552
1	.3	12.0375	12.198	13.59	12.84
2	.284	11.3955	11.5474	12.8652	12.1552
3	.259 or 1/4 full	10.3924	10.5309	11.7327	11.0852
4	.238	9.5497	9.6771	10.7814	10.1864
5	.22	8.8275	8.9452	9.966	9.416
6	.203 or 1/5 full	8.1454	8.254	9.1959	8.6884
7	.18 or 3/16 light	7.2225	7.3188	8.154	7.704
8	.165 or 1/6 "	6.6206	6.7089	7.4745	7.062
9	.148 or 1/7 full	5.9385	6.0177	6.7044	6.3344
10	.134	5.3767	5.4484	6.0702	5.7352
11	.12 or 1/8 light	4.815	4.8792	5.436	5.136
12	.109	4.3736	4.4319	4.9377	4.6652
13	.095 or 1/10 light	3.8119	3.8627	4.3035	4.066
14	.083	3.3304	3.3748	3.7599	3.5524
15	.072	2.889	2.9275	3.2616	3.0816
16	.065	2.6081	2.6429	2.9445	2.782
17	.058	2.3272	2.3583	2.6274	2.4824
18	.049 or 1/20 light	1.9661	1.9923	2.2197	2.0972
19	.042	1.6852	1.7077	1.9026	1.7976
20	.035	1.4044	1.4231	1.5855	1.498
21	.032	1.284	1.3011	1.4496	1.3696
22	.028	1.1235	1.1385	1.2684	1.1984
23	.025 or 1/40	1.0031	1.0165	1.1325	1.07
24	.022	.8827	.8945	.9966	.9416
25	.02 or 1/50	.8025	.8132	.906	.856
26	.018	.7222	.7319	.8154	.7704
27	.016	.642	.6506	.7248	.6848
28	.014	.5617	.5692	.6342	.5992
29	.013	.5216	.5286	.5889	.5564
30	.012	.4815	.4879	.5436	.5136
31	.01 or 1/100	.4012	.4066	.453	.428
32	.009	.3611	.3659	.4077	.3852
33	.008	.321	.3253	.3624	.3424
34	.007	.2809	.2846	.3171	.2996
35	.005 or 1/200	.2006	.2033	.2265	.214
36	.004 or 1/250	.1605	.1626	.1812	.1712

Thickness of Sheet Silver, Gold, etc.

By Birmingham Gauge for these Metals.

No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
1	.004	7	.015	13	.036	19	.064	25	.095	31	.133
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.01	10	.024	16	.051	22	.074	28	.12	34	.148
5	.013	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

Wrought Iron, Steel, Copper, and Brass Wire.

American Gauge. f. full, l. light.

No. of Gauge.	Diameter.	Per Lineal Foot. Iron.	Steel.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46 or $\frac{7}{16}$ f.	.560 74	.566 03	.640 513	.605 176
000	.409 64	.444 683	.448 879	.507 946	.479 908
00	.364 8 or $\frac{3}{8}$ l.	.352 659	.355 986	.402 83	.380 666
0	.324 86 or $\frac{5}{16}$ f.	.279 665	.282 303	.319 451	.301 816
1	.289 3	.221 789	.223 891	.253 342	.239 353
2	.257 63 or $\frac{1}{4}$ f.	.175 888	.177 548	.200 911	.189 818
3	.229 42	.139 48	.140 796	.159 323	.150 522
4	.204 31 or $\frac{1}{5}$ f.	.110 616	.111 66	.126 353	.119 376
5	.181 94 or $\frac{3}{16}$ l.	.087 72	.088 548	.100 2	.094 666
6	.162 02	.069 565	.070 221	.079 462	.075 075
7	.144 28	.055 165	.055 685	.063 013	.059 545
8	.128 49 or $\frac{1}{8}$ f.	.043 751	.044 164	.049 976	.047 219
9	.114 43	.034 699	.035 026	.039 636	.037 437
10	.101 89 or $\frac{1}{10}$ f.	.027 512	.027 772	.031 426	.029 687
11	.090 742	.021 82	.022 026	.024 924	.023 549
12	.080 808	.017 304	.017 468	.019 766	.018 676
13	.071 961	.013 722	.013 851	.015 674	.014 809
14	.064 084	.010 886	.010 989	.012 435	.011 746
15	.057 068	.008 631	.008 712	.009 859	.009 315
16	.050 82 or $\frac{1}{20}$ f.	.006 845	.006 909	.007 819	.007 587
17	.045 257	.005 427	.005 478	.006 199	.005 857
18	.040 303	.004 304	.004 344	.004 916	.004 645
19	.035 89	.003 413	.003 445	.003 899	.003 684
20	.031 961	.002 708	.002 734	.003 094	.002 92
21	.028 462	.002 147	.002 167	.002 452	.002 317
22	.025 347	.001 703	.001 719	.001 945	.001 838
23	.022 571	.001 35	.001 363	.001 542	.001 457
24	.020 1 or $\frac{1}{50}$ f.	.001 071	.001 081	.001 223	.001 155
25	.017 9	.000 849 1	.000 857 1	.000 969 9	.000 916 3
26	.015 94	.000 673 4	.000 679 7	.000 769 2	.000 726 7
27	.014 195	.000 534	.000 539 1	.000 609 9	.000 576 3
28	.012 641	.000 423 5	.000 427 5	.000 483 7	.000 457
29	.011 257	.000 335 8	.000 338 9	.000 383 5	.000 362 4
30	.010 025 or $\frac{1}{100}$ f.	.000 266 3	.000 268 8	.000 304 2	.000 287 4
31	.008 928	.000 211 3	.000 213 2	.000 241 3	.000 228
32	.007 95	.000 167 5	.000 169 1	.000 191 3	.000 180 8
33	.007 08	.000 132 8	.000 134 1	.000 151 7	.000 143 4
34	.006 304	.000 105 3	.000 106 3	.000 120 4	.000 113 7
35	.005 614	.000 083 66	.000 084 45	.000 095 6	.000 090 15
36	.005 or $\frac{1}{200}$	.000 066 25	.000 066 87	.000 075 7	.000 071 5
37	.004 453	.000 052 55	.000 053 04	.000 060 03	.000 056 71
38	.003 965	.000 041 66	.000 042 05	.000 047 58	.000 044 96
39	.003 531	.000 033 05	.000 033 36	.000 037 75	.000 035 66
40	.003 144	.000 026 2	.000 026 44	.000 029 92	.000 028 27

	Iron.	Steel.	Copper.	Brass.
Specific Gravities	7.774	7.847	8.88	8.386
Weights of a Cube Foot ..	485.87	490.45	554.988	524.16
" " Inch ..	.2812	.2838	.3212	.3033

Specific Gravities to determine the computations of these weights were made by author for Messrs. J. R. Browne & Sharpe, Providence, R. I.

Wrought Iron, Steel, Copper, and Brass Wire.

Birmingham Wire Gauge. f. full, l. light.

No. of Gauge.	Thickness.	Per Lineal Foot. Iron.	Steel.	Copper.	Brass.
	Inch.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.454 or $\frac{7}{16}$ f.	.546 207	.551 36	.623 913	.589 286
000	.425	.478 656	.483 172	.546 752	.516 407
00	.38 or $\frac{3}{8}$ f.	.382 66	.386 27	.437 099	.412 84
0	.34 or $\frac{1}{3}$ f.	.306 34	.309 23	.349 921	.330 5
1	.3	.238 5	.240 75	.272 43	.257 31
2	.284	.213 738	.215 755	.244 146	.230 596
3	.259 or $\frac{1}{4}$ f.	.177 765	.179 442	.203 054	.191 785
4	.238	.150 107	.151 523	.171 461	.161 945
5	.22	.128 26	.129 47	.146 507	.138 376
6	.203 or $\frac{1}{5}$ f.	.109 204	.110 234	.124 74	.117 817
7	.18 or $\frac{3}{16}$ l.	.085 86	.086 667	.098 075	.092 632
8	.165 or $\frac{1}{6}$ l.	.072 146	.072 827	.082 41	.077 836
9	.148 or $\frac{1}{7}$ f.	.058 046	.058 593	.066 303	.062 624
10	.134	.047 583	.048 032	.054 353	.051 336
11	.12 or $\frac{1}{8}$ l.	.038 16	.038 52	.043 589	.041 17
12	.109	.031 485	.031 782	.035 964	.033 968
13	.095 or $\frac{1}{10}$ l.	.023 916	.024 142	.027 319	.025 802
14	.083	.018 256	.018 428	.020 853	.019 696
15	.072	.013 728	.013 867	.015 692	.014 821
16	.065	.011 196	.011 302	.012 789	.012 079
17	.058	.008 915	.008 999	.010 183	.009 618
18	.049 or $\frac{1}{20}$ l.	.006 363	.006 423	.007 268	.006 864
19	.042	.004 675	.004 719	.005 34	.005 043
20	.035	.003 246	.003 277	.003 708	.003 502
21	.032	.002 714	.002 739	.003 1	.002 928
22	.028	.002 078	.002 097	.002 373	.002 241
23	.025 or $\frac{1}{40}$	.001 656	.001 672	.001 892	.001 787
24	.022	.001 283	.001 295	.001 465	.001 384
25	.02 or $\frac{1}{50}$	.001 06	.001 070	.001 211	.001 144
26	.018	.000 858 6	.000 866 7	.000 980 7	.000 926 3
27	.016	.000 678 4	.000 684 8	.000 774 9	.000 731 9
28	.014	.000 519 4	.000 524 3	.000 593 3	.000 560 4
29	.013	.000 447 9	.000 452 1	.000 511 6	.000 483 2
30	.012	.000 381 6	.000 385 2	.000 435 9	.000 411 7
31	.01 or $\frac{1}{100}$	.000 265	.000 267 5	.000 302 7	.000 285 9
32	.009	.000 214 7	.000 216 7	.000 245 2	.000 231 6
33	.008	.000 169 6	.000 171 2	.000 193 7	.000 183
34	.007	.000 129 9	.000 131 1	.000 148 3	.000 140 1
35	.005 or $\frac{1}{200}$	.000 066 25	.000 066 88	.000 075 68	.000 071 48
36	.004 or $\frac{1}{250}$	.000 042 4	.000 042 8	.000 048 43	.000 045 74

Thickness of Plates.

No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
1	.312 5	9	.156 25	17	.056 25	25	.023 44
2	.281 25	10	.140 625	18	.05	26	.021 875
3	.25	11	.125	19	.043 75	27	.020 312
4	.234 375	12	.112 5	20	.037 5	28	.018 75
5	.218 75	13	.1	21	.034 375	29	.017 19
6	.203 125	14	.087 5	22	.031 25	30	.015 625
7	.187 5	15	.075	23	.028 125	31	.014 06
8	.171 875	16	.062 5	24	.025	32	.012 5

WIRE GAUGES. (English.)

Warrington (Rylands Brothers).

No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
7/0	½	0	.326	6	.191	11	.117	17	.053
6/0	15/32	1	.3	7	.174	12	.1	18	.047
5/0	7/16	2	.274	8	.159	13	.09	19	.041
4/0	13/32	3	.25	9	.146	14	.079	20	.036
3/0	3/8	4	.229	10	.133	15	.069	21	.0315
2/0	11/32	5	.209	10.5	.125	16	.0625	22	.028

Sir Joseph Whitworth & Co.'s.

No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
1	.001	14	.014	34	.034	85	.085	240	.24
2	.002	15	.015	36	.036	90	.09	260	.26
3	.003	16	.016	38	.038	95	.09	280	.28
4	.004	17	.017	40	.04	100	.1	300	.3
5	.005	18	.018	45	.045	110	.11	325	.325
6	.006	19	.019	50	.05	120	.12	350	.35
7	.007	20	.02	55	.055	135	.135	375	.375
8	.008	22	.022	60	.06	150	.15	400	.4
9	.009	24	.024	65	.065	165	.165	425	.425
10	.01	26	.026	70	.07	180	.18	450	.45
11	.011	28	.028	75	.075	200	.2	475	.475
12	.012	30	.03	80	.08	220	.22	500	.5
13	.013	32	.032						

Sir Joseph Whitworth, in 1857, introduced a Standard Wire-Gauge, ranging from half an inch to a thousandth, and comprising 62 measurements. It commences with least thickness, and increases by thousandths of an inch up to half an inch. Smallest thickness, $\frac{1}{1000}$ of an inch, is No. 1; No. 2 is $\frac{2}{1000}$, and so on, increasing up to No. 20 by intervals of $\frac{1}{1000}$; from No. 20 to No. 40 by $\frac{2}{1000}$; and from No. 40 to No. 100 by $\frac{5}{1000}$. The thicknesses are designated or marked by their respective numbers in thousandths of an inch.

This gauge is entering into general use in England.

New Standard Wire Gauge of Great Britain, 1884.

No.	Inch.	No.	Inch.	No.	Inch.	No.	Inch.
7/0	.5	8	.160	22	.028	36	.0076
6/0	.464	9	.144	23	.024	37	.0068
5/0	.432	10	.128	24	.022	38	.006
4/0	.4	11	.116	25	.02	39	.0052
3/0	.372	12	.104	26	.018	40	.0048
2/0	.348	13	.092	27	.0164	41	.0044
0	.324	14	.08	28	.0148	42	.004
1	.3	15	.072	29	.0136	43	.0036
2	.276	16	.064	30	.0124	44	.0032
3	.252	17	.056	31	.0116	45	.0028
4	.232	18	.048	32	.0108	46	.0024
5	.212	19	.04	33	.01	47	.002
6	.192	20	.036	34	.0092	48	.0016
7	.176	21	.032	35	.0084	49	.0012

No. 50, .001 inch.

French (*Jauges de Fils de Fer*).

French wire-gauges, alike to the English, have been subjected to variation. Following table contains diameters of the numbers of the Limoges gauge.

Wire-Gauge (*Jauge de Limoges*).

Number.	Millimetre.	Inch.	Number.	Millimetre.	Inch.	Number.	Millimetre.	Inch.
0	.39	.0154	9	1.35	.0532	18	3.4	.134
1	.45	.0177	10	1.46	.0575	19	3.95	.156
2	.56	.0221	11	1.68	.0661	20	4.5	.177
3	.67	.0264	12	1.8	.0706	21	5.1	.201
4	.79	.0311	13	1.91	.0752	22	5.65	.222
5	.9	.0354	14	2.02	.0795	23	6.2	.244
6	1.01	.0398	15	2.14	.0843	24	6.8	.268
7	1.12	.0441	16	2.25	.0886			
8	1.24	.0488	17	2.84	.112			

For Galvanized Iron Wire.

Number.	Millimetre.	Inch.	Number.	Millimetre.	Inch.	Number.	Millimetre.	Inch.
1	.6	.0236	9	1.4	.0551	17	3.	.118
2	.7	.0276	10	1.5	.0591	18	3.4	.134
3	.8	.0315	11	1.6	.063	19	3.9	.154
4	.9	.0354	12	1.8	.0709	20	4.4	.173
5	1.	.0394	13	2.	.0787	21	4.9	.193
6	1.1	.0433	14	2.2	.0866	22	5.4	.213
7	1.2	.0473	15	2.4	.0945	23	5.9	.232
8	1.3	.0512	16	2.7	.106			

For Wire and Bars.

Mark.	Millimetre.	Mark.	Millimetre.	Mark.	Millimetre.	Mark.	Millimetre.	Mark.	Millimetre.
P	5	7	12	13	20	19	39	25	70
1	6	8	13	14	22	20	44	26	76
2	7	9	14	15	24	21	49	27	82
3	8	10	15	16	27	22	54	28	88
4	9	11	16	17	30	23	59	29	94
5	10	12	18	18	34	24	64	30	100
6	11								

Thickness of Gas Pipes.

Diameter.	Thickness.	Diameter.	Thickness.	Diameter.	Thickness.
1.5 to 3	.25	8 to 10	.5	14 to 15	.75
4 " 6	.375	12 " 13	.625	16 " 48	.875

Copper Wire Cord.

Circumference and Safe Load.

	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Ins.	Ins.
Circumference	.25	.375	.5	.625	.75	1	1.125	1.25
Safe load in Lbs.	34	50	75	112	168	224	336	448

Zinc-sheets.

Thickness and Weight per Square Foot.

Inch.	Inch.	Inch.
.0311 = 10 oz.	.0534 = 14 oz.	.0686 = 18 oz.
.0457 = 12 oz.	.0611 = 16 oz.	.0761 = 20 oz.

WEIGHT AND STRENGTH OF WIRE, IRON, ETC.

Weight and Strength of Warrington Iron Wire.

Manufactured by Rylands Brothers. (England.)

Weight per 100 Lineal Feet.

No.	Diameter.	Weight	Breaking Weight. Annealed.	Breaking Weight. Bright.	No.	Diameter.	Weight.	Breaking Weight. Annealed.	Breaking Weight. Bright.
Gauge.	Inch	Lbs.	Lbs.	Lbs.	Gauge.	Inch.	Lbs.	Lbs.	Lbs.
7/0	1/2	64.46	3490	5233	9	.146	5.5	298	447
6/0	15/32	56.66	3066	4603	10	.133	4.43	247	370
5/0	7/16	49.36	2673	4000	10.5	.125	4.03	218	327
4/0	13/32	42.53	2303	3457	11	.117	3.53	191	288
3/0	3/8	36.26	1963	2945	12	.1	2.66	145	217
2/0	11/32	30.46	1653	2473	13	.09	2.1	113	169
0	.326	27.36	1486	2226	14	.079	1.6	87	130
1	.3	23.3	1257	1885	15	.069	1.23	66	99
2	.274	19.36	1046	1572	16	.0625	.96	53	77
3	.25	16.13	873	1309	17	.053	.73	39	59
4	.229	13.53	732	1098	18	.047	.56	31	46
5	.209	11.26	610	913	19	.041	.43	23	35
6	.191	9.4	509	763	20	.036	.33	18	27
7	.174	7.8	422	633	21	.03125	.26	14	21
8	.159	6.53	353	519	22	.028	.2	11	16

To Compute Length of 100 Pounds of Wire of a Given Diameter.

RULE.—Divide following numbers by square of diameter, in parts of an inch, and quotient is length in feet.

37.68 for wrought iron.	33.42 for copper.	28 for silver.
37.45 for steel.	34.41 for brass.	15.3 for gold.

13.64 for platinum.

WINDOW GLASS.

Thickness and Weight per Square Foot.

No.	Thickness.	Weight.	No.	Thickness.	Weight.	No.	Thickness.	Weight.
	Inch.	Oz.		Inch.	Oz.		Inch.	Oz.
12	.059	12	17	.083	17	26	.125	26
13	.063	13	19	.091	19	32	.154	32
15	.071	15	21	.1	21	36	.167	36
16	.077	16	24	.111	24	42	.2	42

Terne Plates.

Terne Plates—Are of iron covered with an amalgam of lead.

Thickness and Weight of Galvanized Sheet Iron.

Sheet 2 Feet in Width by from 6 to 9 Feet in Length (M. Lefferts).

Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.	Wire Gauge.	Weight per Sq. Foot.
No.	Oz.	No.	Oz.	No.	Oz.	No.	Oz.	No.	Oz.	No.	Oz.
29	12	26	15	23	20	20	27	17	36	14	53
28	13	25	16	22	22	19	30	16	42	13	61
27	14	24	18	21	24	18	35	15	46	12	70

Wrought Iron.

Weight of Square Rolled Iron,

From .125 *Inch to* 10 *Inches.* ONE FOOT IN LENGTH.

Side.	Weight.	Side.	Weight.	Side.	Weight.	Side.	Weight.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.125	.053	2.125	15.263	4.125	57.517	6.25	132.04
.25	.211	.25	17.112	.25	61.055	.5	142.816
.375	.475	.375	19.066	.375	64.7	.75	154.012
.5	.845	.5	21.12	.5	68.448	7	165.632
.625	1.32	.625	23.292	.625	72 305	.25	177.672
.75	1.901	.75	25.56	.75	76.264	.5	190.136
.875	2.588	.875	27.939	.875	80.333	.75	203.024
1	3.38	3	30.416	5	84.48	8	216.336
.125	4.278	.125	33.01	.125	88.784	.25	230.068
.25	5.28	.75	35.704	.25	93.168	.5	244.22
.375	6.39	.375	38.503	.375	97.657	.75	258.8
.5	7.604	.5	41.408	.5	102.24	9	273.792
.625	8.926	.625	44.418	.625	106.953	.25	289.22
.75	10.352	.75	47.534	.75	111.756	.5	305.056
.875	11.883	.875	50.756	.875	116.671	.75	321.33
2	13.52	4	54.084	6	121.664	10.	327.92

ILLUSTRATION.—What is weight of a bar 1.5 inches, by 12 inches in length?

In column 1st, find 1.5; opposite to it is 7.604 lbs., which is 7 lbs. and .604 of a lb. If lesser denomination of ounces is required, result is obtained as follows:

Multiply remainder by 16, point off the decimals, and the figures remaining on left of the point will give number of ounces.

Thus, .604 of a lb. $= .604 \times 16 = .9.664 = 7$ *lbs.* 9.664 *ounces.*

To Compute Weight for less than a Foot in Length.

OPERATION.—What is weight of a bar 6.25 inches square and 10.5 inches long?

In column 7th, opposite to 6.25 is 132.04, which is weight for a foot in length.

6.25×12 inches $= 132.04$

6	ins.	$= .5$	$= 66.02$
3	"	$= .25$	$= 33.01$
1.5	"	$= .125$	$= 16.505$
			115.535 *lbs.*

Weight of Angle Iron,

From 1.25 *to* 4.5 *Inches.* ONE FOOT IN LENGTH.

Thickness measured in Middle of each Side.

L EQUAL SIDES.			L UNEQUAL SIDES.			L UNEQUAL SIDES.		
Sides.	Thickness.	Weight	Sides.	Thickness.	Weight.	Sides.	Thickness.	Weight.
Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.
1.25 × 1.25	.1875	1.5	3 × 2.5	.375	6.25	6 × 3.5	.625	18
1.5 × 1.5	.1875	2	3.5 × 3	.4375	7.75	6 × 4.5	.625	20
1.75 × 1.75	.25	3	3.5 × 3	.4375	9.6			
2 × 2	.25	3.5	4 × 3	.5	11	T		
2.25 × 2.25	.3125	4.5	4 × 3.5	.5	11.5	2 × 2.375*	.375	5.5
2.5 × 2.5	.3125	5	4 × 3.5	.5	11.75	2.5 × 2.875	.375	6.5
3 × 3	.375	7	4.5 × 3	.5	11.75	3.5 × 3.5	.4375	10.5
3.5 × 3.5	.4375	9	5 × 3	.5	12.65	4 × 3.5	.4375	13
4 × 4	.5	12.5	5 × 3	.5625	13.7		.75	
4.5 × 4.5	.5	14	5.5 × 3.5	.5	14.5	4 × 3.5	.75	13.5
4.5 × 4.5	.5625	16	5.5 × 3.5	.5625	15.6			

* This column gives depth of web added to the thickness of base or flange.

Weight of Round Rolled Iron,

From .125 *Inch to* 12 *Inches in Diameter.* ONE FOOT IN LENGTH.

Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.125	.041	2	10.616	4 .375	50.815	7.5	149.328
.25	.165	.125	11.988	.5	53.76	.75	159.456
.3125	.259	.25	13.44	.625	56.788	8	169.856
.375	.373	.375	14.975	.75	59.9	.25	180.696
.4375	.508	.5	16.588	5	66.35	.5	191.808
.5	.663	.625	18.293	.125	69.731	.75	203.26
.5625	.84	.75	20.076	.25	73.172	9	215.04
.625	1.043	.875	21.944	.375	76.7	.25	227.152
.6875	1.254	3	23.888	.5	80.304	.5	239.6
.75	1.493	.125	25.926	.625	84.001	.75	252.376
.875	2.032	.25	28.04	.75	87.776	10	265.4
1	2.654	.375	30.24	6	95.552	.25	278.924
.125	3.359	.5	32.512	.25	103.704	.5	292.688
.25	4.147	.625	34.886	.375	107.86	.75	306.8
.375	5.019	.75	37.332	.5	112.16	11	321.216
.5	5.972	.875	39.864	.625	116.484	.25	336.004
.625	7.01	4	42.464	.75	120.96	.5	351.104
.75	8.128	.125	45.174	7	130.048	.75	366.536
.875	9.333	.25	47.952	.25	139.544	12	382.208

Weight of Flat Rolled Iron,

From .5×.125 *Inch to* 5.5×4.5 *Inches.** ONE FOOT IN LENGTH.

Thickness.	Weight.
Inch.	Lbs.
	.5
.125	.211
.25	.422
.375	.634
.5	.845
	.625
.125	.264
.25	.528
.375	.792
.5	1.056
.625	1.32
	.75
.125	.316
.25	.633
.375	.95
.5	1.265
.625	1.584
.75	1.9
	.875
.125	.369
.25	.738
.375	1.108
.5	1.477
.625	1.846

Thickness.	Weight.
Ins.	Lbs.
	.875
.75	2.217
.875	2.583
	1
.125	.422
.25	.845
.375	1.267
.5	1.69
.625	2.112
.75	2.534
.875	2.956
	1.125
.125	.475
.25	.95
.375	1.425
.5	1.901
.625	2.375
.75	2.85
.875	3.326
1	3.802
	1.25
.125	.528
.25	1.056
.375	1.584

Thickness	Weight.
Ins.	Lbs.
	1.25
.5	2.112
.625	2.64
.75	3.168
.875	3.696
1	4.224
1.125	4.752
	1.375
.125	.58
.25	1.161
.375	1.742
.5	2.325
.625	2.904
.75	3.484
.875	4.065
1	4.646
1.125	5.227
1.25	5.808
1.375	6.389
	1.5
.125	.633
.25	1.266
.375	1.9
.5	2.535
.625	3.168

Thickness.	Weight.
Ins.	Lbs.
	1.5
.75	3.802
.875	4.435
1	5.069
1.125	5.703
1.25	6.337
1.375	6.97
	1.625
.125	.686
.25	1.372
.375	2.059
.5	2.746
.625	3.432
.75	4.119
.875	4.805
1	5.492
1.125	6.178
1.5	6.864
1.375	7.551
1.5	8.237
	1.75
.125	.739
.25	1.479
.375	2.218
.5	2.957

* For weights of square bars see preceding page.

Thickness.	Weight.
Ins.	Lbs.
	1.75
.625	3.696
.75	4.435
.875	5.178
1	5.914
1.125	6.653
1.25	7.393
1.375	8.132
1.5	8.871
1.625	9.61
	1.875
.125	.792
.25	1.584
.375	2.376
.5	3.168
.625	3.96
.75	4.752
.875	5.544
1	6.336
1.125	7.129
1.25	7.921
1.375	8.713
1.5	9.505
1.625	10.297
1.75	11.089
	2
.125	.845
.25	1.689
.375	2.534
.5	3.379
.625	4.224
.75	5.069
.875	5.914
1	6.758
1.125	7.604
1.25	8.448
1.375	9.294
1.5	10.138
1.625	10.983
1.75	11.828
1.875	12.673
	2.125
.125	.898
.25	1.795
.375	2.693
.5	3.591
.625	4.488
.75	5.386
.875	6.283
1	7.181
1.125	8.079
1.25	8.977
1.375	9.874
	1.125
1.5	10.772
1.625	11.67
1.75	12.567
1.875	13.465
2	14.362
	2.25
.125	.95
.25	1.9
.375	2.851
.5	3.802
.625	4.752
.75	5.703
.875	6.653
1	7.604
1.125	8.554
1.25	9.505
1.375	10.455
1.5	11.406
1.625	12.356
1.75	13.307
1.875	14.257
2	15.208
2.125	16.158
	2.375
.125	1.003
.25	2.006
.375	3.009
.5	4.013
.625	5.916
.75	6.019
.875	7.022
1	8.025
1.125	9.028
1.25	10.032
1.375	11.035
1.5	12.038
1.625	13.042
1.75	14.045
1.875	15.048
2	16.051
2.125	17.054
2.25	18.057
	2.5
.125	1.056
.25	2.112
.375	3.168
.5	4.224
.625	5.28
.75	6.336
.875	7.392
1	8.448
1.125	9.504
	2.5
1.25	10.56
1.375	11.616
1.5	12.672
1.625	13.728
1.75	14.784
1.875	15.84
2	16.896
2.125	17.952
2.25	19.008
2.375	20.064
	2.625
.125	1.109
.25	2.218
.375	3.327
.5	4.436
.625	5.545
.75	6.654
.875	7.763
1	8.872
1.125	9.981
1.25	11.09
1.375	12.199
1.5	13.308
1.625	14.417
1.75	15.526
1.875	16.635
2	17.744
2.125	18.853
2.25	19.962
2.375	21.071
2.5	22.18
	2.75
.125	1.162
.25	2.323
.375	3.485
.5	4.647
.625	5.808
.75	6.97
.875	8.132
1	9.294
1.125	10.455
1.25	11.617
1.375	12.779
1.5	13.94
1.625	15.102
1.75	16.264
1.875	17.425
2	18.587
2.125	19.749
2.25	20.91
2.375	22.072
2.5	23.234
2.625	24.395
	2.875
.125	1.215
.25	2.429
.375	3.644
.5	4.858
.625	6.072
.75	7.287
.875	8.502
1	9.716
1.125	10.931
1.25	12.145
1.375	13.36
1.5	14.574
1.625	15.789
1.75	17.003
1.875	18.218
2	19.432
2.125	20.647
2.25	21.861
2.375	23.076
2.5	24.29
2.625	25.505
2.75	26.719
	3
.125	1.267
.25	2.535
.375	3.802
.5	5.069
.625	6.337
.75	7.604
.875	8.871
1	10.138
1.125	11.406
1.25	12.673
1.375	13.94
1.5	15.208
1.625	16.475
1.75	17.742
1.875	19.01
2	20.277
2.25	22.811
2.5	25.346
2.75	27.881
	3.25
.125	1.373
.25	2.746
.375	4.119
.5	5.492
.625	6.865
.75	8.237
.875	9.61
1	10.983
1.125	12.356
1.25	13.73

Thickness.	Weight.
Ins.	Lbs.
	3.25
1.375	15.102
1.5	16.475
1.625	17.848
1.75	19.221
1.875	20.594
2	21.967
2.25	24.712
2.5	27.458
2.75	30.204
3	32.95
	3.5
.125	1.479
.25	2.957
.375	4.436
.5	5.914
.625	7.393
.75	8.871
.875	10.35
1	11.828
1.125	13.307
1.25	14.785
1.375	16.264
1.5	17.742
1.625	19.221
1.75	20.699
1.875	22.178
2	23.656
2.25	26.613
2.5	29.57
2.75	32.527
3	35.485
3.25	38.441
	3.75
.125	1.584
.25	3.168
.375	4.752
.5	6.336
.625	7.921
.75	9.505
.875	11.089
1	12.673
1.125	14.257
1 25	15.841
1.375	17.425
1.5	19.009
1.625	20.594
1.75	22.178

Thickness.	Weight.
Ins.	Lbs.
	3.75
1.875	23.762
2	25.346
2.25	28.514
2.5	31.682
2.75	34.851
3	38.019
3.25	41.187
3.5	44.355
	4
.125	1.69
.25	3.38
.5	6.759
.75	10.138
1	13.518
1.25	16.897
1.5	20.277
1.75	23.656
2	27.036
2.25	30.415
2.5	33.795
2.75	37.174
3	40.554
3.25	43.933
3.5	47.313
3.75	50.692
	4.25
.125	1.795
.25	3.591
.5	7.181
.75	10.772
1	14.364
1.25	17.953
1.5	21.544
1.75	25.135
2	28.725
2.25	32.316
2.5	35.907
2.75	39.497
3	43.088
3.25	46.679
3.5	50.269
3.75	53.86
4	57.45
	4.5
.25	3.802
.5	7.604

Thickness.	Weight.
Ins.	Lbs.
	4.5
.75	11.406
1	15.208
1.25	19.01
1.5	22.812
1.75	26.614
2	30.415
2.25	34.217
2.5	38.019
2.75	41.82
3	45.623
3.25	49.425
3.5	53.226
3.75	57.028
4	60.83
4.25	64.632
	4.75
.25	4.013
.5	8.026
.75	12.036
1	16.052
1.25	20.066
1.5	24.079
1.75	28.092
2	32.105
2.25	36.118
2.5	40.131
2.75	44.144
3	48.157
3.25	52.17
3.5	56.184
3.75	60.197
4	64.21
4.25	68.223
4.5	72.235
	5
.25	4.224
.5	8.449
.75	12.673
1	16.897
1.25	21.122
1.5	25.346
1.75	29.57
2	33.795
2.25	38.019
2.5	42.243
2.75	46.468
3	50.692

Thickness.	Weight.
Ins.	Lbs.
5	
3.25	54.916
3.5	59.14
3.75	63.365
4	67.589
4.25	71.813
4.5	76.038
4.75	80.262
5.25	
.25	4 436
.5	8 871
.75	13.307
1	17.742
1.25	22.178
1.5	26.613
1.75	31.049
2	35.484
2.25	39.92
2.5	44.355
2.75	48.791
3	53.226
3.25	57.662
3 5	62.097
3.75	66.533
4	70.968
4.25	75.404
4.5	79.839
4.75	84.275
5	88.71
5.5	
.25	4.647
.5	9.294
.75	13.94
1	18.587
1.25	23.234
1.5	27.881
1.75	32.527
2	37.174
2.25	41.821
2.5	46.468
2.75	51.114
3	55.761
3.25	60.408
3.5	65.055
3.75	69.701
4	74.348
4.25	78.995
4.5	83.642

Illustration.—What is weight of a bar of iron 5.25 ins. in breadth by .75 inch in thickness?

In column 7, as above, find 5.25; and below it, in column, .75; and opposite to that is 13.307, which is 13 lbs. and .307 of a pound.

For parts of a pound and of a foot, operate according to rule laid down for table, page 125.

Weight of Sheet Iron. (*English. D. K. Clark.*)

PER SQUARE FOOT (*at 480 lbs. per Cube Foot*).

As by Wire-gauge used in South Staffordshire, England.

Thickness.		Weight.	Square Feet in 1 ton.	Thickness.		Weight.	Square Feet in 1 ton.	Thickness.		Weight.	Square Feet in 1 ton.
No.	Inch.	Lbs.	No.	No.	Inch.	Lbs.	No.	No.	Inch.	Lbs.	No.
32	.0125	.5	4480	21	.0344	1.38	1623	10	.1406	5.63	398
31	.0141	.562	3986	20	.0375	1.5	1493	9	.1563	6.25	358
30	.0156	.625	3584	19	.0438	1.75	1280	8	.1719	6.88	326
29	.0172	.688	3256	18	.05	2	1120	7	.1875	7.5	299
28	.0188	.75	2987	17	.0563	2.25	996	6	.2031	8.13	276
27	.0203	.813	2755	16	.0625	2.5	896	5	.2188	8.75	256
26	.0219	.875	2560	15	.075	3	747	4	.2344	9.38	239
25	.0234	.938	2388	14	.0875	3.5	640	3	.25	10	224
24	.025	1	2240	13	.1	4	560	2	.2813	11.25	199
23	.0281	1.13	1982	12	.1125	4.5	498	1	.3125	12.5	179
22	.0313	1.25	1792	11	.125	5	448				

Weight of Hoop Iron. (*English.*)

PER LINEAL FOOT.

Width.	W. G.	Weight.	Width.	W. G.	Weight.	Width.	W. G.	Weight.
Ins.	No.	Lbs.	Ins.	No.	Lbs.	Ins.	No.	Lbs.
.625	21	.067	1.125	17	.21	1.75	14	.484
.75	20	.0875	1.25	16	.27	2	13	.634
.875	19	.1216	1.375	15	.33	2.25	13	.714
1	18	.1636	1.5	15	.36	2.5	12	.91

Weight of Black and Galvanized Sheet Iron.

(*Morton's Table, founded upon Sir Joseph Whitworth & Co.'s Standard Birmingham Wire-Gauge.*) (*D. K. Clark.*)

NOTE.—Numbers on Holtzapffel's wire-gauge are applied to thicknesses on Whitworth gauge.

Gauge and Weight of Black Sheets.			Approximate number of Sq. Ft. in 1 ton. Black.	Galvanized.	Gauge and Weight of Black Sheets.			Approximate number of Sq. Ft. in 1 ton. Black.	Galvanized.
No.	Inch.	Lbs.	Sq. Ft.	Sq. Ft.	No.	Inch.	Lbs.	Sq. Ft.	Sq. Ft.
1	.3	12	187	185	17	.06	2.4	933	876
2	.28	11.2	200	197	18	.05	2	1120	1038
3	.26	10.4	215	212	19	.04	1.6	1400	1274
4	.24	9.6	233	229	20	.036	1.4	1556	1403
5	.22	8.8	254	250	21	.032	1.28	1750	1558
6	.2	8	280	275	22	.028	1.12	2000	1753
7	.18	7.2	311	304	23	.024	.96	2333	2004
8	.165	6.6	339	331	24	.022	.88	2545	2159
9	.15	6	373	363	25	.02	.8	2800	2339
10	.135	5.4	415	403	26	.018	.72	3111	2553
11	.12	4.8	467	452	27	.016	.64	3500	2808
12	.11	4.4	509	491	28	.014	.56	4000	3122
13	.095	3.8	589	566	29	.013	.52	4308	3306
14	.085	3.4	659	630	30	.012	.48	4667	3513
15	.07	2.8	800	757	31	.01	.4	5600	4017
16	.065	2.6	862	813	32	.009	.36	6222	4327

Weight of English Angle and T Iron. (*D. K. Clark.*)

ONE FOOT IN LENGTH.

NOTE.—When base or web tapers in section, mean thickness is to be measured.

Thickness.	Sum of Width and Depth in Inches.										
	1.5	1.625	1.75	1.875	2	2.125	2.25	2.375	2.5	2.625	2.75
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.125	.57	.62	.68	.73	.78	.83	.88	.94	.99	1.04	1.09
.1875	.81	.89	.97	1.05	1.13	1.21	1.29	1.37	1.45	1.52	1.6
.25	1.04	1.15	1.25	1.36	1.46	1.56	1.67	1.77	1.88	1.98	2.08
.3125	1.24	1.37	1.5	1.63	1.76	1.89	2.02	2.15	2.28	2.41	2.54
	2.875	3	3.125	3.25	3.375	3.5	3.625	3.75	3.875	4	4.25
.125	1.14	1.2	1.25	1.3	1.45	1.41	1.46	1.51	1.56	1.62	1.72
.1875	1.68	1.76	1.84	1.91	1.99	2.07	2.15	2.23	2.3	2.38	2 54
.25	2.19	2.29	2.4	2.5	2.6	2.71	2.81	2.92	3.02	3.13	3.33
.3125	2.67	2.8	2.93	3.06	3.19	3.32	3.45	3.58	3.71	3.84	4.1
.375	3.13	3.28	3.44	3.59	3.75	3.91	4.06	4.22	4.38	4.53	4.84
.4375	3.57	3.75	3.93	4.11	4.29	4.48	4.66	4.84	5.02	5.2	5.56
	4.5	4.75	5	5.25	5.5	5.75	6	6.25	6.5	6.75	7
.1875	2.7	2.85	3.01	3.16	3.32	3.48	3.63	3.79	3.95	4.1	4.26
.25	3.54	3.75	3.96	4.17	4.38	4.58	4.79	5	5.21	5.42	5.63
.3125	4.36	4.62	4.88	5.14	5.4	5.66	5.92	6.18	6.45	6.71	6.97
.375	5.16	5.47	5.78	6.09	6.41	6.72	7.03	7.34	7.66	7.97	8 28
.4375	5.92	6.29	6.65	7.02	7.38	7.75	8.11	8.48	8.84	9.21	9.57
.5	6.67	7.08	7.5	7.92	8.33	8.75	9.17	9.58	10	10.42	10.83
.5625	7.38	7.85	8.32	8.79	9.26	9.73	10.2	10.66	11.13	12.6	12.07
	7.25	7.5	7.75	8	8.25	8.5	8.75	9	9.25	9.5	9.75
.25	5.83	6.04	6.25	6.46	6.67	6.88	7.08	7.29	7.5	7.71	7.92
.3125	7.23	7.49	7.75	8.01	8.27	8.53	8.79	9.05	9.31	9.57	9.83
.375	8.59	8.91	9.22	9.53	9.84	10.16	10.47	10.78	11.09	11.41	11.72
.4375	9.93	10.3	10.66	11.03	11.39	11.76	12.12	12.49	12.85	13.22	13.58
.5	11.25	11.67	12.08	12.5	12.92	13.33	13.75	14.17	14.58	15	15.42
.5625	12.54	13.01	13.48	13.94	14.41	14.88	15.35	15.82	16.29	16.76	17.23
.625	13.8	14.32	14.84	15.36	15.89	16.41	16.93	17.45	17.97	18.49	19.01
	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
.375	12.03	12.66	13.28	13.91	14.53						
.4375	13.95	14.67	15.4	16.13	16.86	17.59	18.31	19.04	19.77	20.5	21.22
.5	15.83	16.67	17.5	18.33	19.17	20	20.84	21.67	22.5	23.34	24.17
.5625	17.7	18.63	19.57	20.51	21.44	22.38	23.31	24.25	25.19	26.12	27.06
.625	19.53	20.57	21.61	22.66	23.7	24.74	25.78	26.83	27.87	28.91	29.95
.75	23.13	24.38	25.63	26.88	28.13	29.37	30.63	31.88	33.13	34.38	35.63
	12	12.5	13	13.5	14	15	16	17	18	19	20
.625	23.7	24.74	25.78	26.83	27.87	29.95	32.03	34.12	36.2	38.28	40.36
.75	28.13	29.37	30.63	31.88	33.13	35.63	38.13	40.63	41.13	43.63	46.13
.875	32.45	33.91	35.36	36.82	38.28	41.19	44.12	47.02	49.95	52.87	55.78
1	36.67	38.33	40	41.67	43.33	46.67	50	53 33	56.67	60	63.33

NOTE.—American rolled is slightly heavier.

Weight of Hoop Iron. (*D. K. Clark.*)

ONE FOOT IN LENGTH.

As by Wire-gauge used in South Staffordshire (England).

Thickness.		Width in Inches.										
		.625	.75	.875	1	1.125	1.25	1.375	1.5	1.625	1.75	2
No.	Inch.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
21	.0344	.0716	.0861	.1	.115	.129	.144	.158	.172	.197	.201	.229
20	.0375	.0781	.0938	.109	.125	.141	.156	.172	.188	.203	.219	.25
19	.0438	.0911	.109	.128	.146	.164	.182	.2	.219	.238	.257	.292
18	.05	.104	.125	.146	.167	.188	.208	.229	.25	.271	.292	.333
17	.0563	.117	.141	.164	.188	.211	.234	.258	.281	.305	.328	.375
16	.0625	.13	.156	.182	.208	.234	.26	.286	.313	.339	.365	.417
15	.075	.156	.188	.219	.25	.281	.313	.344	.375	.397	.438	.5
14	.0875	.183	.219	.256	.293	.329	.366	.402	.438	.475	.512	.585
13	.1	.208	.25	.292	.333	.375	.416	.458	.5	.543	.584	.667
12	.1125	.234	.281	.328	.375	.422	.469	.516	.563	.609	.656	.75
11	.125	.26	.313	.365	.417	.469	.521	.573	.625	.677	.729	.833
10	.1406	.293	.352	.41	.469	.527	.586	.645	.703	.762	.82	.938
9	.1563	.326	.391	.456	.522	.587	.652	.717	.783	.848	.913	1.04
8	.1719	.358	.43	.501	.573	.644	.716	.788	.859	.931	1	1.15
7	.1875	.391	.469	.547	.625	.703	.781	.859	.938	1.02	1.09	1.25
6	.2031	.423	.508	.593	.677	.762	.836	.931	1.02	1.1	1.19	1.35
5	.2188	.456	.547	.638	.729	.82	.912	1	1.09	1.19	1.28	1.46
4	.2344	.488	.586	.683	.781	.879	.977	1.07	1.17	1.27	1.37	1.56

CAST IRON.

To Compute Weight of a Cast Iron Bar or Rod.

Ascertain weight of a wrought iron bar or rod of same dimensions in preceding tables, or by computation, and from weight deduct $\frac{2}{27}$th part.

Or, As 1000 : .9257 :: weight of a wrought bar or rod : to weight required. Thus, what is weight of a piece of cast iron 4 × 3.75 × 12 inches?

In table, page 128, weight of a piece of wrought iron of these dimensions is 50.692 lbs. Then, 1000 : .9257 :: 50.692 : 46.93 *lbs.*

Braziers' and Sheathing Copper.

Braziers' Sheets, 2 × 4 feet from 5 to 25 lbs., 2.5 × 5 feet from 9 to 150 lbs., and 3 × 5 feet and 4 × 6 feet, from 16 to 300 lbs. per sheet.

Sheathing Copper, 14 × 48 inches, and from 14 to 34 oz. per square foot.

Yellow Metal, 14 × 48 inches, and from 16 to 34 oz. per square foot.

Weight of Corrugated Iron Roof Plates.

PER SQUARE FOOT. (*Birmingham Gauge.*)

No.	Black.	Galvanized.	No.	Black.	Galvanized.	No.	Black.	Galvanized.
	Oz.	Oz.		Oz.	Oz.		Oz.	Oz.
20	26	29	23	20	22	25	16	18
22	22	24	24	18	20	26	14	16

METALS.

To Compute Weight of Metals of any Dimensions or Form.

By rules in Mensuration of Solids (page 360), ascertain volume of the piece, multiply it by weight of a cube inch, and product will give weight in pounds.

Weight of Cast Iron Pipes or Cylinders.

From 1 to 70 Inches in Internal Diameter.

ONE FOOT IN LENGTH.

Diameter.	Thickn.	Weight.	Diameter.	Thickn.	Weight.	Diameter.	Thickn.	Weight.
Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.
1	.25	3.06	4.75	.375	18.84	11	.875	101.85
	.375	5.05		.5	25.72	11.5	.5	58.81
1.25	.25	3.68		.625	32.93		.625	74.28
	.3125	4.79		.75	40.43		.75	90.06
	.375	5.97	5	.375	19.76		.875	106.13
1.5	.375	6.89		.5	26.95	12	.5	61.26
	.4375	8.31		.625	34.46		.625	77.34
	.5	9.8		.75	42.27		.75	93.73
1.75	.375	7.81	5.5	.375	21.59		.875	110.42
	.4375	9.38		.5	29.4	12.5	.5	63.71
	.5	11.03		.625	37.52		.625	80.4
2	.375	8.73		.75	45.95		.75	97.4
	.4375	10.45	6	.375	23.43		.875	114.71
	.5	12.25		.5	31.86	13	.5	66.16
2.25	.375	9.65		.625	40.59		.625	83.47
	.4375	11.52		.75	49.62		.75	101.08
	.5	13.48	6.5	.375	25.27		.875	119
2.5	.375	10.57		.5	34.31	13.5	.5	68.61
	.4375	12.6		.625	43.65		.625	86.53
	.5	14.7		.75	53.3		.75	104.76
2.75	.375	11.49	7	.5	36.76		.875	123.29
	.4375	14.67		.5625	41.7	14	.5	71.06
	.5	15.93		.625	46.71		.625	89.6
3	.375	12.4		.75	56.97		.75	108.43
	.5	17.15	7.5	.5	39.21		.875	127.58
	.625	22.2		.5625	44.45	14.5	.5	73.51
	.75	27.57		.625	49.77		.625	92.66
3.25	.375	13.32		.75	60.65		.75	112.11
	.5	18.38	8	.5	41.66		.875	131.87
	.625	23.74		.5625	47.21	15	.5	75.96
	.75	29.4		.625	52.84		.625	95.72
3.5	.375	14.24		.75	64.32		.75	115.78
	.5	19.6	9	.5	46.56		.875	136.16
	.625	25.27		.5625	52.72	15.5	.5	78.47
	.75	31.24		.625	58.96		.625	98.78
3.75	.375	15.16		.75	71.67		.75	119.46
	.5	20.83	9.5	.5	49.01		.875	140.44
	.625	26.8		.5625	55.48	16	.625	101.85
	.75	33.08		.625	62.06		.75	123.14
4	.375	16.08		.75	75.35		.875	144.73
	.5	22.05	10	.5	51.45		1	166.63
	.625	28.33		.625	65.09	16.5	.625	104.9
	.75	34.92		.75	79.03		.75	126.75
4.25	.375	17		.875	93.27		.875	149.02
	.5	23.28	10.5	.5	53.91		1	171.53
	.625	29.86		.625	68.15	17	.625	107.97
	.75	36.76		.75	82.7		.75	130.48
4.5	.375	17.92		.875	97.56		.875	153.3
	.5	23.88	11	.5	56.36		1	176.43
	.625	31.4		.625	71.21	17.5	.625	111.03
	.75	38.59		.75	86.38		.75	134.16

Diameter.	Thickn.	Weight.	Diameter.	Thickn.	Weight.	Diameter.	Thickn.	Weight.
Ins.	Inch.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
17.5	.875	157.59	29	.75	218.7	40	.875	350.56
	1	181.33		.875	256.23		1	401.86
18	.625	114.1		1	294.05		1.125	453.46
	.75	137.84	30	.75	226.05		1.25	505.41
	.875	161.88		.875	264.8	42	.875	367.69
	1	186.23		1	303.86		1	421.45
19	.625	120.23		1.125	343.22		1.125	472.52
	.75	145.19	31	.75	233.41		1.25	529.87
	.875	170.46		.875	273.38	44	.875	384.88
	1	196.03		1	313.66		1	441.1
20	.625	126.35		1.125	354.24		1.125	497.58
	.75	152.54	32	.75	240.75		1.25	554.42
	.875	179.03		.875	281.95	46	.875	402.01
	1	205.84		1	323.46		1	460.07
21	.625	132.48		1.125	365.27		1.125	519 64
	.75	159.89	33	.75	248.11		1 25	578.88
	.875	187.61		.875	290.53	48	.875	419.17
	1	215.64		1	333.26		1	480.29
22	.625	138.61		1.125	376.29		1.125	541.69
	.75	167.24	34	.75	255.46		1.25	603.44
	.875	196.19		.875	299.11	50	.875	436.43
	1	225.44		1	343.06		1	499.89
23	.625	144.73		1.125	387.33		1.125	563.75
	.75	174.59	35	.75	262.81		1.25	627.93
	.875	204.76		.875	307.68	52	.875	453.49
	1	235.24		1	352.87		1	519.5
24	.625	150.86		1.125	398.35		1.125	585.81
	.75	181.95	36	.75	270.16		1.25	654.42
	.875	213.34		.875	316.26	55	.875	479.23
	1	245.04		1	362.67		1	548.9
25	.625	156.98		1.125	409.28		1.125	618.91
	.75	189.3		1.25	456.37		1.25	689.21
	.875	221.92	37	.75	277.51	58	1	578.29
	1	254.85		.875	324.84		1.125	651.96
26	.625	163.11		1	372.47		1.25	725.93
	.75	196.65		1.125	420.4		1.375	800.22
	.875	230.5		1.25	468.65	60	1	597.92
	1	264.65	38	.75	284.86		1.125	674.01
27	.625	169.23		.875	333.41		1.25	750.45
	.75	204		1	382.27		1.375	827.17
	.875	239.07		1.125	431.41	65	1	646.93
	1	274.45		1.25	480.89		1.125	729.18
28	.625	175.36	39	.75	292.21		1.25	811.73
	.75	211.35		.875	341.97		1.375	894.6
	.875	247.65		1	392.08	70	1	695.92
	1	284.25		1.125	442.44		1.25	872.98
29	.625	181.49		1.25	493.14		1.5	1051.25

Equivalent Length of Pipe for a Socket.

$7 + \frac{d}{15} = l.$ *d representing diameter of pipe and l length in inches.*

Additional weight of two flanges for any diameter is computed equal to a lineal foot of the pipe.

NOTE.—These weights do not include any allowance for spigot and socket ends.

2.—For rule to compute thicknesses of pipes, flanges, etc., see page 560.

Weight of Flat Rolled Bar Steel. (*D. K. Clark.*)

From .5 Inch to 8 Inches in Width. ONE FOOT IN LENGTH.

WIDTH IN INCHES.

Thickness.	.5	.625	.75	.875	1	1.25	1.5	1.75	2	2.25	2.5	2.75
Inch.	Lb.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1/4	.425	.533	.64	.743	.85	1.06	1.28	1.49	1.7	1.91	2.13	2.34
5/16	.531	.665	.8	.929	1.06	1.33	1.59	1.86	2.13	2.39	2.66	2.92
3/8	.638	.798	.96	1.11	1.28	1.59	1.91	2.23	2.55	2.87	3.19	3.51
7/16	.744	.931	1.12	1.3	1.49	1.86	2.23	2.6	2.98	3.35	3.72	4.09
1/2	.85	1.06	1.28	1.49	1.7	2.13	2.55	2.98	3.4	3.83	4.25	4.68
9/16	—	1.2	1.44	1.67	1.91	2.39	2.87	3.35	3.83	4.3	4.78	5.26
5/8	—	1.33	1.6	1.86	2.12	2.66	3.19	3.72	4.25	4.78	5.31	5.84
11/16	—	—	1.76	2.04	2.34	2.92	3.51	4.09	4.68	5.26	5.84	6.43
3/4	—	—	1.92	2.23	2.55	3.19	3.83	4.46	5.1	5.74	6.38	7.01
13/16	—	—	—	2.41	2.76	3.45	4.14	4.83	5.53	6.22	6.91	7.6
7/8	—	—	—	2.6	2.98	3.72	4.46	5.21	5.95	6.69	7.44	8.18
15/16	—	—	—	—	3.19	3.98	4.78	5.58	6.38	7.17	7.97	8.77
1	—	—	—	—	3.4	4.25	5.1	5.95	6.8	7.65	8.5	9.35

WIDTH IN INCHES.

Thickness.	3	3.25	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1/4	2.55	2.76	2.98	3.4	3.82	4.26	4.68	5.1	5.52	5.96	6.38	6.8
5/16	3.19	3.45	3.72	4.25	4.78	5.32	5.84	6.38	6.9	7.44	7.97	8.5
3/8	3.83	4.14	4.46	5.1	5.74	6.38	7.02	7.66	8.28	8.92	9.56	10.2
7/16	4.46	4.83	5.21	5.95	6.7	7.44	8.18	8.92	9.66	10.4	11.2	11.9
1/2	5.1	5.53	5.95	6.8	7.66	8.5	9.36	10.2	11.1	11.9	12.8	13.6
9/16	5.74	6.22	6.69	7.65	8.6	9.56	10.5	11.5	12.4	13.4	14.3	15.3
5/8	6.38	6.91	7.44	8.5	9.56	10.6	11.7	12.8	13.8	14.9	15.9	17
11/16	7.01	7.6	8.18	9.35	10.5	11.7	12.9	14	15.2	16.4	17.5	18.7
3/4	7.65	8.29	8.93	10.2	11.5	12.8	14	15.3	16.6	17.9	19.1	20.4
13/16	8.29	8.98	9.67	11.1	12.4	13.8	15.2	16.6	18	19.3	20.7	22.2
7/8	8.93	9.67	10.4	11.9	13.4	14.9	16.4	17.9	19.4	20.8	22.3	23.8
15/16	9.56	10.4	11.2	12.8	14.3	15.9	17.5	19.1	20.8	22.4	23.9	25.6
1	10.2	11.1	11.9	13.6	15.3	17	18.7	20.4	22.1	23.8	25.5	27.2

Weight of Rolled Square Steel.

From .125 Inch to 6 Inches Square. ONE FOOT IN LENGTH.

Side.	Weight.	Side.	Weight.	Side.	Weight.	Side.	Weight.	Side.	Weight.
Inch.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.125	.053	.75	1.92	1.375	6.43	2.125	15.4	3.75	47.8
.1875	.119	.8125	2.24	1.4375	7.03	2.25	17.2	4	54.4
.25	.212	.875	2.6	1.5	7.65	2.375	19.2	4.25	61.4
.3125	.333	.9375	3.06	1.5625	8.3	2.5	21.2	4.5	68.9
.375	.478	1	3.4	1.625	8.98	2.625	23.5	4.75	76.7
.4375	.651	1.0625	3.83	1.6875	9.79	2.75	25.7	5	85
.5	.85	1.125	4.3	1.75	10.4	2.875	28.2	5.25	93.7
.5625	1.08	1.1875	4.79	1.8125	11.2	3	30.6	5.5	102.8
.625	1.33	1.25	5.31	1.875	11.9	3.25	35.9	5.75	112.4
.6875	1.61	1.3125	5.86	2	13.6	3.5	41.6	6	122.4

Weight of Round Rolled Steel.

From .125 Inch to 12 Inches Diameter. ONE FOOT IN LENGTH.

Diam.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diam.	Weight.	Diam.	Weight.
Inch.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.125	.0417	.875	2.04	1.625	7.05	2.875	22	5.75	88.3
.1875	.0939	.9375	2.35	1.6875	7.61	3	24.1	6	96.1
.25	.167	1	2.67	1.75	8.18	3.25	28.3	6.5	113.2
.3125	.26	1.0625	3	1.8125	8.77	3.5	32.7	7	130.8
.375	.375	1.125	3.38	1.875	9.38	3.75	34.2	7.5	136.8
.4375	.511	1.1875	3.76	2	10.7	4	42.7	8	170.8
.5	.667	1.25	4.17	2.125	12	4.25	48.3	8.5	193.2
.5625	.845	1.3125	4.6	2.25	13.6	4.5	54.6	9	218.4
.625	1.04	1.375	5.05	2.375	15.1	4.75	60.3	9.5	241.2
.6875	1.27	1.4375	5.18	2.5	16.7	5	66.8	10	267.2
.75	1.5	1.5	6.01	2.625	18.4	5.25	73.6	11	323
.8125	1.76	1.5625	6.52	2.75	20.2	5.5	80.8	12	352.8

Weight of Hexagonal, Octagonal, and Oval Steel.

ONE FOOT IN LENGTH.

HEXAGONAL.				OCTAGONAL.				OVAL.		
Diam. over Sides.	Weight.	Diam. over Sides.	Weight.	Diam. over Sides.	Weight.	Diam. over Sides.	Weight.	Diam. over Sides.	Area.	Weight.
Inch.	Lbs.	Ins.	Lbs.	Inch.	Lbs.	Ins.	Lbs.	Ins.	Sq. In.	Lbs.
3/8	.414	1	2.94	3/8	.396	1	2.82	3/4 × 3/8	.251	.853
1/2	.736	1 1/8	3.73	1/2	.704	1 1/8	3.56	7/8 × 1/2	.344	1.17
5/8	1.15	1 1/4	4.6	5/8	1.1	1 1/4	4.4	1 × 1/2	.446	1.52
3/4	1.66	1 3/8	5.57	3/4	1.58	1 3/8	5.32	1 1/4 × 5/8	.697	2.37
7/8	2.25	1 1/2	6.63	7/8	2.16	1 1/2	6.34	1 1/2 × 3/4	.884	3

Weight of a Square Foot of Sheet Copper.

Wire Gauge of Wm. Foster & Co. (England.)

Thickness.		Weight.	Thickness.		Weight.	Thickness.		Weight.
W. G.	Inch.	Lbs.	W. G.	Inch.	Lbs.	W. G.	Inch.	Lbs.
1	.306	14	11	.123	5.65	21	.034	1.55
2	.284	13	12	.109	5	22	.029	1.35
3	.262	12	13	.098	4.5	23	.025	1.15
4	.24	11	14	.088	4	24	.022	1
5	.222	10.15	15	.076	3.5	25	.019	.89
6	.203	9.3	16	.065	3	26	.017	.79
7	.186	8.5	17	.057	2.6	27	.015	.7
8	.168	7.7	18	.049	2.25	28	.013	.62
9	.153	7	19	.044	2	29	.012	.56
10	.138	6.3	20	.038	1.75	30	.011	.5

Weight of Composition Sheathing Nails.

No.	Length.	Number in a Pound.	No.	Length.	Number in a Pound.	No.	Length.	Number in a Pound.	No.	Length.	Number in a Pound.
	Inch.			Ins.			Ins.			Ins.	
1	.75	290	4	1.125	201	7	1.125	184	10	1.625	101
2	.875	260	5	1.25	199	8	1.25	168	11	1.75	74
3	1	212	6	1	190	9	1.5	110	12	2	64

Weight of Cast and Wrought Iron, Steel, Copper, and Brass, of a given Sectional Area.

PER LINEAL FOOT.

Sectional Area.	Wrought Iron.	Cast Iron.	Steel.	Copper.	Lead.	Brass.	Gun-metal.
Sq. Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.1	.336	.313	.339	.385	.492	.357	.38
.2	.671	.626	.677	.771	.984	.713	.759
.3	1.007	.939	1.016	1.156	1.476	1.07	1.139
.4	1.343	1.251	1.355	1.542	1.967	1.427	1.519
.5	1.678	1.564	1.694	1.927	2.461	1.783	1.894
.6	2.014	1.877	2.032	2.312	2.953	2.14	2.279
.7	2.35	2.19	2.371	2.698	3.445	2.497	2.658
.8	2.685	2.503	2.71	3.083	3.937	2.853	3.038
.9	3.021	2.816	3.049	3.469	4.429	3.21	3.418
1	3.357	3.129	3.387	3.854	4.922	3.567	3.798
1.1	3.692	3.442	3.726	4.24	5.414	3.923	4.177
1.2	4.028	3.754	4.065	4.625	5.906	4.28	4.557
1.3	4.364	4.067	4.404	5.01	6.398	4.636	4.937
1.4	4.699	4.38	4.742	5.396	6.89	4.993	5.317
1.5	5.035	4.693	5.081	5.781	7.383	5.35	5.696
1.6	5.371	5.006	5.42	6.167	7.875	5.707	6.076
1.7	5.706	5.319	5.759	6.552	8.367	6.063	6.456
1.8	6.042	5.632	6.097	6.937	8.859	6.42	6.836
1.9	6.378	5.945	6.436	7.323	9.351	6.777	7.215
2	6.714	6.258	6.775	7.708	9.843	7.133	7.595
2.1	7.049	6.57	7.114	8.094	10.33	7.49	7.97
2.2	7.385	6.883	7.452	8.474	10.83	7.847	8.35
2.3	7.721	7.196	7.791	8.864	11.32	8.203	8.73
2.4	8.056	7.509	8.13	9.25	11.81	8.56	9.11
2.5	8.392	7.822	8.469	9.635	12.3	8.917	9.49
2.6	8.728	8.135	8.807	10.02	12.8	9.273	9.87
2.7	9.063	8.448	9.146	10.41	13.29	9.63	10.25
2.8	9.399	8.76	9.485	10.79	13.78	9.98	10.63
2.9	9.734	9.073	9.824	11.18	14.27	10.34	11.01
3	10.07	9.386	10.16	11.56	14.76	10.7	11.39
3.1	10.41	9.699	10.5	11.95	15.26	11.06	11.77
3.2	10.74	10.01	10.84	12.33	15.75	11.41	12.15
3.3	11.08	10.32	11.18	12.72	16.24	11.77	12.53
3.4	11.41	10.64	11.52	13.1	16.73	12.13	12.91
3.5	11.75	10.95	11.86	13.49	17.22	12.48	13.29
3.6	12.08	11.26	12.19	13.87	17.72	12.84	13.67
3.7	12.42	11.58	12.53	14.26	18.21	13.2	14.05
3.8	12.76	11.89	12.87	14.64	18.7	13.55	14.43
3.9	13.09	12.2	13.21	15.03	19.19	13.91	14.81
4	13.43	12.51	13.55	15.42	19.69	14.27	15.19
4.1	13.76	12.83	13.89	15.8	20.18	14.62	15.57
4.2	14.1	13.14	14.23	16.19	20.67	14.98	15.95
4.3	14.43	13.45	14.57	16.57	21.16	15.34	16.33
4.4	14.77	13.77	14.91	16.96	21.65	15.69	16.71
4.5	15.11	14.08	15.24	17.34	22.15	16.05	17.09
4.6	15.44	14.39	15.58	17.73	22.64	16.41	17.47
4.7	15.78	14.7	15.92	18.11	23.13	16.76	17.85
4.8	16.11	15.02	16.26	18.5	23.62	17.12	18.23
4.9	16.45	15.33	16.6	18.88	24.12	17.48	18.61
5	16.78	15.64	16.94	19.27	24.61	17.83	18.99

Lap Welded Charcoal Iron Boiler Tubes.

Standard Dimensions.

National Tube Works Company.

Diameter.		Thickness.	Wire Gauge.	Circumference.		Transverse Areas.			Length per Sq. Foot of Surface.		Weight per Foot.
External.	Internal.			External.	Internal.	External.	Internal.	Metal.	External.	Internal.	
Ins.	Ins.	Ins.	No.	Ins.	Ins.	Sq. Ins.	Sq. Ins.	Sq. Ins.	Feet.	Feet.	Lbs.
1	.86	.072	15	3.14	2.69	.78	.57	.21	3.82	4.46	.71
1.125	.98	.072	15	3.53	3.08	.99	.76	.24	3.39	3.89	.8
1.25	1.11	.072	15	3.93	3.47	1.23	.96	.27	3.06	3.45	.89
1.32	1.15	.083	14	4.12	3.6	1.35	1.03	.32	2.91	3.33	1.08
1.375	1.21	.083	14	4.32	3.8	1.48	1.15	.34	2.78	3.16	1.13
1.5	1.33	.083	14	4.71	4.19	1.77	1.4	.37	2.55	2.86	1.24
1.625	1.43	.095	13	5.1	4.51	2.07	1.62	.46	2.35	2.66	1.53
1.75	1.56	.095	13	5.5	4.9	2.4	1.91	.49	2.18	2.45	1.66
1.875	1.68	.095	13	5.89	5.29	2.76	2.23	.53	2.04	2.27	1.78
2	1.81	.095	13	6.28	5.69	3.14	2.57	.57	1.91	2.11	1.91
2.125	1.93	.095	13	6.68	6.08	3.55	2.94	.61	1.8	1.97	2.04
2.25	2.06	.095	13	7.07	6.47	3.98	3.33	.64	1.7	1.85	2.16
2.375	2.16	.109	12	7.46	6.78	4.43	3.65	.78	1.61	1.77	2.61
2.5	2.28	.109	12	7.85	7.17	4.91	4.09	.82	1.53	1.67	2.75
2.75	2.53	.109	12	8.64	7.95	5.94	5.03	.9	1.39	1.51	3.04
2.875	2.66	.109	12	9.03	8.35	6.49	5.54	.95	1.33	1.44	3.18
3	2.78	.109	12	9.42	8.74	7.07	6.08	.99	1.27	1.37	3.33
3.25	3.01	.12	11	10.21	9.46	8.3	7.12	1.18	1.17	1.26	3.96
3.5	3.26	.12	11	11	10.24	9.62	8.35	1.27	1.09	1.17	4.28
3.75	3.51	.12	11	11.78	11.03	11.04	9.68	1.37	1.02	1.09	4.6
4	3.73	.134	10	12.57	11.72	12.57	10.94	1.63	.95	1.02	5.47
4.25	3.98	.134	10	13.35	12.51	14.19	12.45	1.73	.9	.96	5.82
4.5	4.23	.134	10	14.14	13.29	15.9	14.07	1.84	.85	.9	6.17
4.75	4.48	.134	10	14.92	14.08	17.72	15.78	1.94	.8	.85	6.53
5	4.7	.148	9	15.71	14.78	19.63	17.38	2.26	.76	.81	7.58
5.25	4.95	.148	9	16.49	15.56	21.65	19.27	2.37	.73	.77	7.97
5.5	5.2	.148	9	17.28	16.35	23.76	21.27	2.49	.7	.73	8.36
6	5.67	.165	8	18.85	17.81	28.27	25.25	3.02	.64	.67	10.16
7	6.67	.165	8	21.99	20.95	38.48	34.94	3.54	.55	.57	11.9
8	7.67	.165	8	25.13	24.1	50.27	46.2	4.06	.48	.50	13.65
9	8.64	.18	7	28.27	27.14	63.62	58.63	4.99	.42	.44	16.76
10	9.59	.203	6	31.42	30.14	78.54	72.29	6.25	.38	.4	20.99
11	10.56	.22	5	34.56	33.17	95.03	87.58	7.45	.35	.36	25.03
12	11.54	.229	4.5	37.7	36.26	113.1	104.63	8.47	.32	.33	28.46
13	12.52	.238	4	40.84	39.34	132.73	123.19	9.54	.29	.3	32.06
14	13.5	.248	3.5	43.98	42.42	153.94	143.22	10.71	.27	.28	36
15	14.48	.259	3	47.12	45.5	176.71	164.72	11.99	.25	.26	40.3
16	15.43	.284	2	50.26	48.48	201.06	187.04	14.02	.24	.25	47.11
17	16.4	.3	1	53.41	51.52	226.98	211.24	15.74	.22	.23	52.89
18	17.32	.34	0	56.55	54.41	254.47	235.61	18.86	.21	.22	63.32

Note.—In estimating effective heating or evaporating surface of Tubes, as heating liquids by steam, superheating steam, or transferring heat from one liquid or one gas to another, mean surface of Tubes is to be computed.

Iron Welded Steam, Gas, and Water Pipe.

STANDARD DIMENSIONS.

National Tube Works Company.

Diameter.			Thickness.	Circumference.		Transverse Areas.			Length of Pipe per Sq. Foot of Surface.		Length of Pipe containing one Cubic Foot.	Weight per Foot.
Internal.	External.	Actual Int'nal.		External.	Internal.	External.	Internal.	Metal	External.	Internal.		
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Sq. Ins.	Sq. Ins.	Sq. Ins.	Feet.	Feet.	Feet.	Lbs.
.125	.4	.27	.07	1.27	.85	.13	.06	.07	9.44	14.15		.24
.25	.54	.36	.09	1.7	1.14	.23	.1	.12	7.07	10.49		.42
.375	.67	.49	.09	2.12	1.55	.36	.19	.17	5.66	7.73	751.2	.56
.5	.84	.62	.11	2.64	1.96	.55	.3	.25	4.55	6.13	472.4	.84
.75	1 05	.82	.11	3.3	2.59	.87	.53	.33	3.64	4.63	270	1.11
1	1.31	1.05	.13	4.13	3.29	1.36	.86	.49	2.9	3.64	166 9	1.67
1.25	1.66	1.38	.14	5.21	4.33	2.16	1.5	.67	2.3	2.77	96.2	2.24
1.5	1.9	1.61	.14	5.97	5.06	2.83	2.04	.8	2.01	2.37	70.7	2.68
2	2.37	2.07	.15	7.46	6.49	4.43	3.36	1.07	1.61	1.85	42.9	3.61
2.5	2.87	2.47	.20	9.03	7.75	6.49	4.78	1.71	1.33	1.55	30.1	5.74
3	3.5	3.07	.22	11	9.64	9.62	7.39	2.24	1.09	1.24	19.5	7.54
3 5	4	3.55	.23	12.57	11.15	12.57	9.89	2.68	.95	1.08	14.6	9
4	4.5	4.03	.24	14.14	12.65	15.9	12.73	3.17	.85	.95	11.3	10.66
4.5	5	4.51	.25	15.71	14.16	19.63	15.96	3.67	.76	.85	9	12.34
5	5.56	5.04	.26	17.48	15.85	24.31	19.99	4.32	.69	.76	7.2	14.5
6	6 62	6.06	.28	20.81	19.05	34.47	28.89	5.58	.58	.63	5	18.76
7	7.62	7.02	.30	23.95	22.06	45.66	38.74	6.93	.5	.54	3.7	23.27
8	8.62	7.98	.32	27.1	25.08	58.43	50.04	8.39	.44	.48	2.9	28.18
9	9.62	8.94	.34	30.24	28.08	72.76	62.73	10.03	.4	.43	2.3	33.7
10	10.75	10.02	.37	33.77	31.48	90.76	78.84	11.92	.35	.38	1.8	40.06
11	11.75	11	.37	36.91	34.56	108.43	95.03	13.4	.32	.35	1.5	45.02
12	12.75	12	.37	40.05	37.7	127.68	113.1	14.58	.3	.32	1.3	48.98
13	14	13.25	.37	43.98	41.63	153.94	137.89	16.05	.27	.29	1	53.92
14	15	14.25	.37	47.12	44.77	176.71	159.48	17.23	.25	.27	.9	57.89
15	16	15.43	.28	50.26	48.48	201.06	187.04	14.02	.24	.25	.8	47.11
16	17	16.4	.3	53.4	51.52	226.98	211.24	15.74	.22	.23	.7	52.89
17	18	17.32	.34	56.54	54.41	254.46	235.60	18.86	.21	.22	.6	63.32

STEEL LOCOMOTIVE TUBES.

Lap Welded Semi-Steel Locomotive Tubes.

STANDARD DIMENSIONS.

National Tube Works Company.

Diameter.		Thickness.	Wire Gauge.	Circumference.		Transverse Areas.			Length per Sq. Foot of Surface.		Weight per Foot.
External.	Internal.			External.	Internal.	External.	Internal.	Metal.	External.	Internal.	
Ins.	Ins.	Ins.	No.	Ins.	Ins.	Sq. Ins.	Sq. Ins.	Sq. Ins	Feet.	Feet.	Lbs.
1	.834	.083	14	3.142	2.62	.785	.546	.239	3.82	4.58	.81
1.25	1.084	.083	14	3.927	3.405	1.227	.923	.304	3.056	3.524	1.03
1.5	1.31	.095	13	4.712	4.115	1.767	1.348	.419	2.546	2.916	1.42
1.75	1.532	.109	12	5.498	4.813	2.405	1.843	.562	2.183	2.493	1.91
2	1.782	.109	12	6.283	5.598	3.142	2.494	.648	1.91	2.144	2.2
2.25	2.032	.109	12	7.069	6.384	3 976	3.243	.733	1.698	1.88	2.49
2.5	2.26	.12	11	7.854	7.1	4 909	4.011	.898	1.528	1.69	3.05
2.75	2.51	.12	11	8.639	7 885	5.94	4.948	.992	1.389	1.522	3.37
3	2.76	.12	11	9.425	8.67	7.069	5.983	1.086	1.273	1.384	3.68

Weight of Lead and Tin Lined Pipe per Foot.

From .375 Inch to 5 Inches in Diameter. (Tatham & Bros.)

WASTE-PIPE.				BLOCK-TIN PIPE.					
Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.
Ins.	Lbs.	Ins.	Lbs.	Inch.	Lb.	Inch.	Lbs.	Ins.	Lbs.
1.5	2	4	8	.375	.3594	.625	.5	1.25	1.25
2	3	4.5	6	.375	.375	.625	.625	1.25	1.5
3	3.5	4.5	8	.375	.5	.75	.625	1.5	2
3	5	5	8	.5	.375	.75	.75	1.5	2.5
4	5	5	10	.5	.5	1	.9375	2	2.5
4	6	5	12	.5	.625	1	1.125	2	3

WATER-PIPE.

From .375 Inch to 5 Inches in Diameter.

Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.
Inch.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.
.375	.08	.625	.625	.25	3.5	1.25	.19	4.75	2.5	.3125	14
.375	.12	1	.75	.1	1.25	1.25	.25	6	2.5	.375	17
.375	.16	1.25	.75	.12	1.75	1.5	.12	3	3	.1875	9
.375	.19	1.5	.75	.16	2.25	1.5	.14	3.5	3	.25	12
.375	.34	2.5	.75	.2	3	1.5	.17	4.25	3	.3125	16
.5	.07	.0545	.75	.23	3.5	1.5	.19	5	3	.375	20
.5	.09	.75	.75	.3	4.75	1.5	.23	6.5	3.5	.1875	9.5
.5	.11	1	1	.1	1.5	1.5	.27	8	3.5	.25	15
.5	.13	1.25	1	.11	2	1.75	.13	4	3.5	.3125	18.5
.5	.16	1.75	1	.14	2.5	1.75	.17	5	3.5	.375	22
.5	.19	2	1	.17	3.25	1.75	.21	6.5	4	.1875	12.5
.5	.25	3	1	.21	4	1.75	.27	8.5	4	.25	16
.625	.08	.0727	1	.24	4.75	2	.15	4.75	4	.3125	21
.625	.09	1	1	.3	6	2	.18	6	4	.375	25
.625	.13	1.5	1.25	.1	2	2	.22	7	4.5	.1875	14
.625	.16	2	1.25	.12	2.5	2	.27	9	4.5	.25	18
.625	.2	2.5	1.25	.14	3	2.5	.1875	8	5	.25	20
.625	.22	2.75	1.25	.16	3.75	2.5	.25	11	5	.375	31

Marks and Weight of Tin-plates. *(English.)*

MARK OR BRAND.	Plates per Box.	Dimensions.	Weight per Box.	MARK OR BRAND.	Plates per Box.	Dimensions.	Weight per Box.
	No.	Ins.	No.		No.	Ins.	No.
1 C or 1 Com.	225	13.75×10	112	DXXXX.....	100	16.75×12.5	189
2 C.........	225	13.25× 9.75	105	SDC.........	200	15 ×11	168
3 C.........	225	12.75× 9.5	98	SDX.........	200	15 ×11	188
H C.........	225	13.75×10	119	SDXX.......	200	15 ×11	209
H X.........	225	13.75×10	157	SDXXX......	200	15 ×11	230
1 X.........	225	13.75×10	140	SDXXXX.....	200	15 ×11	251
2 X.........	225	13.25× 9.75	133	SDXXXXX..	200	15 ×11	272
3 X.........	225	12.75× 9.5	126	SDXXXXXX.	200	15 ×11	293
1 XX.......	225	13.75×10	161	Leaded IC...	112	20 ×14	112
1 XXX......	225	13.75×10	182	" IX...	112	20 ×14	140
1 XXXX....	225	13.75×10	203	ICW.........	225	13.75×10	112
1 XXXXX..	225	13.75×10	224	IXW.........	225	13.75×10	140
1 XXXXXX.	225	13.75×10	245	CSDW.......	200	15 ×11	168
DC.........	100	16.75×12.5	98	CIIW........	100	16.75×12.5	105
DX.........	100	16.75×12.5	126	XIIW........	100	16.75×12.5	126
DXX.......	100	16.75×12.5	147	TT..........	450	13.75×10	112
DXXX.	100	16.75×12.5	168	XTT.........	450	13.75×10	126

When the plates are 14 by 20 inches, there are 112 in a box.

Weight of Seamless Drawn Copper Tubes.

American Tube Works. (*Boston.*)

BY EXTERNAL DIAMETER. ONE FOOT IN LENGTH.

Stubs' W. G. From .25 Inch to 12 Ins.—f full, l light.

No.	20	19	18	17	16	15	14	13	12	11
Ins.	1/32 f	3/64 l	3/64 f	1/16 l	1/16 f	5/64 l	5/64 f	3/32 f	7/64	1/8 l
Diamet'r.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.25	.09	.1	.12	.13	.14	.15	.17	.18	.19	.19
.375	.14	.16	.19	.23	.24	.26	.29	.32	.35	.37
.5	.2	.23	.27	.31	.34	.37	.42	.47	.52	.56
.625	.25	.29	.34	.4	.44	.48	.55	.61	.69	.74
.75	.3	.36	.42	.49	.54	.59	.67	.76	.85	.92
.875	.36	.42	.49	.58	.64	.7	.8	.9	1.02	1.11
1	.41	.48	.57	.67	.74	.81	.93	1.05	1.18	1.29
1.125	.46	.55	.64	.76	.83	.92	1.05	1.19	1.35	1.47
1.25	.52	.61	.71	.84	.93	1.03	1.18	1.34	1.52	1.65
1.375	.57	.68	.79	.93	1.03	1.14	1.31	1.48	1.68	1.84
1.5	.62	.74	.86	1.02	1.13	1.25	1.43	1.63	1.85	2.02
1.625	.68	.8	.94	1.11	1.23	1.36	1.56	1.77	2.02	2.2
1.75	.73	.87	1.01	1.2	1.33	1.47	1.69	1.92	2.18	2.39
1.875	.78	.93	1.09	1.29	1.43	1.58	1.81	2.06	2.35	2.57
2	.84	1	1.16	1.37	1.53	1.69	1.94	2.21	2.51	2.75
2.125	.89	1.06	1.24	1.46	1.63	1.8	2.07	2.35	2.68	2.93
2.25	.94	1.13	1.31	1.55	1.73	1.91	2.19	2.5	2.85	3.12
2.375	1	1.19	1.39	1.64	1.82	2.02	2.32	2.64	3.01	3.3
2.5	1.05	1.25	1.46	1.73	1.92	2.13	2.45	2.79	3.18	3.48
2.625	1.1	1.32	1.54	1.82	2.02	2.23	2.57	2.93	3.35	3.67
2.75	1.16	1.38	1.61	1.9	2.12	2.34	2.7	3.08	3.51	3.85
2.875	1.21	1.45	1.68	1.99	2.22	2.45	2.83	3.22	3.68	4.03
3	1.26	1.51	1.76	2.08	2.32	2.56	2.95	3.37	3.84	4.22
3.25	1.37	1.64	1.91	2.26	2.52	2.78	3.21	3.66	4.18	4.58
3.5	1.48	1.77	2.06	2.43	2.72	3	3.46	3.95	4.51	4.95
3.75	1.58	1.9	2.21	2.61	2.92	3.22	3.71	4.24	4.84	5.31
4	1.69	2.02	2.36	2.79	3.11	3.44	3.97	4.53	5.17	5.68
4.25	1.8	2.15	2.51	3.14	3.31	3.66	4.22	4.82	5.51	6.05
4.5	1.9	2.28	2.65	3.32	3.51	3.88	4.47	5.11	5.84	6.41
4.75	2.01	2.41	2.8	3.49	3.71	4.1	4.73	5.4	6.17	6.78
5	2.12	2.54	2.95	3.67	3.91	4.32	4.98	5.69	6.5	7.14
5.25	2.23	2.66	3.1	3.85	4.11	4.54	5.23	5.98	6.84	7.51
5.5	2.34	2.79	3.25	3.85	4.3	4.76	5.49	6.27	7.17	7.87
5.75	2.44	2.92	3.4	4.02	4.5	4.98	5.74	6.56	7.5	8.24
6	2.55	3.05	3.55	4.2	4.7	5.2	5.99	6.85	7.83	8.61
6.25	2.66	3.18	3.7	4.38	4.9	5.41	6.25	7.14	8.17	8.97
6.5	2.76	3.31	3.85	4.55	5.1	5.63	6.5	7.43	8.5	9.34
6.75	2.87	3.44	4	4.73	5.3	5.85	6.75	7.72	8.83	9.7
7	2.98	3.56	4.15	4.91	5.49	6.07	7.01	8.01	9.16	10.07
7.25	3.09	3.69	4.3	5.09	5.69	6.29	7.26	8.30	9.5	10.44
7.5	3.19	3.82	4.45	5.26	5.89	6.51	7.51	8.59	9.83	10.8
8	3.41	4.08	4.74	5.62	6.29	6.95	8.02	9.17	10.49	11.53
8.5	3.62	4.33	5.04	5.97	6.68	7.39	8.52	9.75	11.16	12.26
9	3.83	4.59	5.34	6.33	7.08	7.83	9.03	10.33	11.82	13
9.5	4.05	4.85	5.64	6.68	7.48	8.26	9.54	10.91	12.49	13.73
10	4.26	5.11	5.94	7.03	7.87	8.7	10.05	11.49	13.15	14.46
10.5	4.47	5.37	6.24	7.39	8.27	9.14	10.55	12.07	13.82	15.19
11	4.69	5.62	6.54	7.74	8.67	9.58	11.06	12.65	14.48	15.92
11.5	4.9	5.88	6.84	8.1	9.06	10.02	11.56	13.23	15.15	16.66
12	5.11	6.13	7.13	8.45	9.46	10.45	12.07	13.81	15.81	17.29

No.	10	9	8	7	6	5	4	3	2	1
Ins.	9/64 l	9/64 f	11/64 l	3/16 l	13/64	7/32 f	15/64 f	1/4 f	9/32 f	19/64 f
Diamet'r.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
·375	·4	·41	·42	·44	—	—	—	—	—	—
·5	·61	·64	·67	·71	·73	·75	·76	—	—	—
.625	.81	.86	.92	.99	1.04	1.09	1.12	1.13	1.18	—
·75	1.01	1.09	1.17	1.26	1.35	1.42	1.49	1.53	1.61	1.63
.875	1.22	1.31	1.42	1.53	1.66	1.76	1.85	1.92	2.04	2.09
1	1.42	1.54	1.67	1.81	1.97	2.09	2.21	2.32	2.48	2.55
1.125	1.63	1.78	1.93	2.08	2.28	2.43	2.58	2.71	2.91	3
1.25	1.83	2	2.18	2.36	2.59	2.76	2.94	3.11	3.34	3.46
1.375	2.03	2.22	2.43	2.63	2.9	3.1	3.3	3.5	3.77	3.92
1.5	2.24	2.44	2.68	2.91	3.21	3.43	3.67	3.9	4.21	4.38
1.625	2.44	2.67	2.93	3.18	3.52	3.77	4.03	4.29	4.64	4.83
1.75	2.65	2.89	3.18	3.45	3.83	4.11	4.39	4.69	5.07	5.29
1.875	2.85	3.12	3.44	3.73	4.14	4.44	4.76	5.08	5.51	5.75
2	3.06	3.34	3.69	4	4.45	4.78	5.12	5.48	5.94	6.21
2.125	3.26	3.57	3.94	4.28	4.75	5.11	5.48	5.87	6.37	6.66
2.25	3.46	3.8	4.19	4.55	5.06	5.45	5.84	6.27	6.81	7.12
2.375	3.67	4.02	4.44	4.82	5.37	5.78	6.21	6.66	7.24	7.57
2.5	3.87	4.25	4.69	5.1	5.68	6.12	6.57	7.06	7.67	8.04
2.625	4.08	4.47	4.95	5.37	6	6.45	6.93	7.45	8.1	8.49
2.75	4.28	4.7	5.2	5.65	6.3	6.79	7.29	7.85	8.54	8.95
2.875	4.48	4.92	5.45	5.92	6.61	7.12	7.66	8.24	8.97	9.41
3	4.69	5.15	5.7	6.2	6 92	7.46	8.02	8.64	9.4	9.87
3.25	5.1	5.6	6.2	6.74	7.54	8.13	8.75	9.43	10.27	10.78
3.5	5.51	6.05	6.71	7.29	8.16	8.8	9.47	10.22	11.14	11.7
3.75	5.91	6.5	7.21	7.84	8.78	9.47	10.2	11.01	12	12.61
4	6.32	6.95	7.71	8.39	9 4	10.14	10.92	11.8	12.87	13.53
4.25	6.73	7.4	8.22	8.94	10.02	10.81	11.65	12.59	13.73	14.44
4.5	7.14	7.85	8.72	9.49	10 64	11.48	12.37	13.38	14.6	15.36
4.75	7.55	8.3	9.22	10.04	11.26	12.16	13.1	14.17	15.46	16.27
5	7.96	8.75	9.73	10.58	11.88	12.83	13.83	14.96	16.33	17.19
5.25	8.36	9.21	10.23	11.13	12.49	13.5	14.55	15.75	17.2	18.1
5.5	8.77	9.66	10.73	11.68	13.11	14.17	15.28	16.54	18.06	19.02
5.75	9.18	10.11	11.24	12.23	13.73	14.84	16	17.33	18.93	19 93
6	9.59	10.56	11.74	12.78	14.35	15.51	16.73	18.12	19.79	20.85
6.25	10	11.01	12.24	13.33	14.97	16.18	17.46	18.91	20.66	21.76
6.5	10.41	11.46	12.75	13.88	15.59	16.85	18.18	19.7	21.53	22.68
6.75	10.82	11.91	13.25	14.42	16.21	17.52	18.91	20.49	22.39	23.59
7	11.22	12.36	13.75	14.97	16.83	18.19	19.63	21.28	23.26	24.51
7.25	11.63	12.81	14.26	15.52	17.45	18.86	20.36	22.07	24.13	25.42
7.5	12.04	13.26	14.76	16.07	18.07	19.54	21.08	22.86	25	26.34
7.75	12.45	13.71	15.26	16.62	18.68	20.21	21.81	23.65	25.86	27.25
8	12.86	14.17	15.77	17.17	19.3	20.88	22.54	24.44	26.72	28.17
8.25	13.27	14.62	16.27	17.71	19 92	21.55	23.26	25.23	27.59	29.08
8.5	13.67	15.07	16.77	18.26	20.54	22.22	23.99	26.02	28.45	30
8.75	14.08	15.52	17.28	18.81	21.16	22.89	24.71	26.81	29.32	30.91
9	14.49	15.97	17.78	19.36	21.78	23.56	25.44	27.6	30.18	31.83
9.25	14.9	16.42	18.28	19.91	22.4	24.23	26.17	28.39	31.05	32.74
9.5	15.31	16.87	18.79	20.46	23 02	24.9	26.89	29.18	31.92	33.66
9.75	15.72	17.32	19.29	21.01	23.64	25.57	27.62	29.97	32.78	34.57
10	16.12	17.77	19.79	21.55	24.26	26.24	28.34	30.76	33.65	35.49
10.5	16.94	18.68	20.8	22.65	25.5	27.59	29.79	32.34	35.38	37.32
11	17.76	19 58	21.81	23.75	26.73	28.93	31.25	33 92	37.11	39.15
11.5	18.57	20.48	22.81	24.84	27.97	30.27	32.7	35.5	38.84	40.98
12	19.39	21.38	23.82	25.94	29.21	31.61	34.15	37.08	40.58	42.81

By Internal Diameter.

Add following Units to Weights for External Diameter in preceding tables.

No.	1	2	3	4	5	6	7	8	9	10
	2.21	1.97	1.66	1.38	1.18	1.01	.78	.67	.53	.43
No.	11	12	13	14	15	16	17	18	19	20
	.35	.29	.22	.17	.13	.11	.08	.06	.05	.03

ILLUSTRATION.—What is weight of a copper tube 6 ins. in internal diameter, No. 3 gauge, and one foot in length?

By preceding table 6 ins. external, No. 3 gauge = 18.12. and 18.12 + 1.66 = 19.78 *lbs.*

WEIGHT OF BRASS TUBES.

To Compute Weight of Brass Tubes.

American Tube Works. (*Boston.*)

RULE.—Deduct 5 per cent. from weight of Copper tubes.

EXAMPLE. — What is weight of a brass tube 6 ins. in external diameter, No. 3 gauge, and one foot in length?

By preceding table 6 ins. = 18.12, from which deduct 5 per cent. = 17.21 lbs.

By Internal Diameter.

RULE.—Proceed as above for internal diameter of copper tube, and deduct 5 per cent.

EXAMPLE.—Weight of a copper tube 6 ins. internal diameter, No. 3 gauge, and 1 foot in length = 19.78 *lbs.*

Hence, 19.78 — 5 per cent. = 18.79 *lbs.*

NOTE.—Diameter of Tubes, as for Boilers, is given externally, and that for Pipes internally.

Weights of English as given by D. K. Clark are essentially alike to the preceding.

Brass Tubes Corresponding with and Fitted for Iron Tubes or Pipes.

American Tube Works. (*Boston.*)

WEIGHT PER LINEAL FOOT.

Diameter of Iron Pipe. Internal.	Diameter of Iron Pipe. External.	Weight.	Diameter of Iron Pipe. Internal.	Diameter of Iron Pipe. External.	Weight.	Diameter of Iron Pipe. Internal.	Diameter of Iron Pipe. External.	Weight.
Inch.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.
.125	.375	.25	1	1.3125	1.7	3	3.5	8.3
.25	.5625	.43	1.25	1.625	2.5	3.5	4	10.9
.375	.6875	.63	1.5	1.875	3	4	4.5	12.7
.5	.8125	.9	2	2.375	4	5	5.5	15.7
.75	1.0625	1.25	2.5	2.875	4.87			

Weight of Sheet Brass.

ONE SQUARE FOOT. (*Holtzapffel's Gauge.*)

Thickness. No.	Thickness. Inch.	Weight. Lbs.	Thickness. No.	Thickness. Inch.	Weight. Lbs.	Thickness. No.	Thickness. Inch.	Weight. Lbs.	Thickness. No.	Thickness. Inch.	Weight. Lbs.
3	.259	10.9	9	.148	6.23	15	.072	3.03	21	.032	1.35
4	.238	10	10	.134	5.64	16	.065	2.74	22	.028	1.18
5	.22	9.26	11	.12	5.05	17	.058	2.44	23	.025	1.05
6	.203	8.55	12	.109	4.59	18	.049	2.06	24	.022	.926
7	.18	7.58	13	.095	4	19	.042	1.77	25	.02	.842
8	.165	6.95	14	.083	3.49	20	.035	1.47			

Weight of Wrought Iron Tubes. (*English.*)

EXTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzapffel's Wire-Gauge. f full, l light.

No.	—	—	4	5	6	7	8	9
Ins.	.3125 5/16	.281 9/32	.238 15/64 *f*	.22 7/32	.203 13/64	.18 3/16 *l*	.165 11/64 *l*	.148 9/64 *f*
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
7	21.9	19.8	16.9	15.6	14.5	12.9	11.8	10.6
7.5	23.5	21.3	18.1	16.8	15.5	13.8	12.7	11.4
8	25.2	22.7	19.3	17.9	16.6	14.7	13.5	12.2
8.5	26.8	24.2	20.6	19.1	17.6	15.7	14.4	12.9
9	28.4	25.7	21.8	20.2	18.7	16.6	15.3	13.7
9.5	30.1	27.1	23.1	21.4	19.8	17.6	16.1	14.5
10	31.7	28.6	24.3	22.5	20.8	18.5	17	15.3

No.	7	8	9	10	11	12	13	14	15
Ins.	.18 3/16 *l*	.165 11/64 *l*	.148 9/64 *f*	.134 9/64 *l*	.12 1/8 *l*	.109 7/64	.095 3/32 *f*	.083 5/64 *f*	.072 5/64 *l*
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1	1.55	1.44	1.32	1.22	1.11	1.02	.9	.797	.7
1.125	1.78	1.66	1.51	1.39	1.26	1.16	1.3	.906	.794
1.25	2.02	1.88	1.71	1.57	1.42	1.3	1.15	1.01	.888
1.375	2.25	2.09	1.9	1.74	1.58	1.45	1.27	1.12	.983
1.5	2.49	2.31	2.1	1.92	1.73	1.59	1.4	1.23	1.08
1.625	2.72	2.52	2.29	2.09	1.89	1.73	1.52	1.34	1.17
1.75	2.96	2.74	2.48	2.27	2.05	1.87	1.65	1.45	1.27
1.875	3.19	2.96	2.68	2.45	2.21	2.02	1.77	1.56	1.36
2	3.43	3.17	2.87	2.62	2.36	2.16	1.9	1.67	1.45
2.125	3.67	3.39	3.06	2.8	2.52	2.3	2.02	1.78	1.55
2.25	3.9	3.6	3.26	2.97	2.68	2.44	2.14	1.88	1.64
2.375	4.14	3.82	3.45	3.15	2.83	2.59	2.27	1.99	1.74
2.5	4.37	4.04	3.65	3.32	2.99	2.73	2.39	2.1	1.83
2.625	4.61	4.25	3.84	3.5	3.15	2.87	2.52	2.21	1.93
2.75	4.84	4.47	4.03	3.67	3.31	3.02	2.64	2.32	2.02
2.875	5.08	4.68	4.23	3.85	3.46	3.16	2.77	2.43	2.11
3	5.32	4.9	4.42	4.02	3.62	3.3	2.89	2.54	2.21
3.25	5.79	5.33	4.81	4.37	3.94	3.59	3.14	2.75	2.4
3.5	6.26	5.76	5.2	4.72	4.25	3.87	3.39	2.97	2.59
3.75	6.73	6.19	5.58	5.07	4.57	4.16	3.64	3.19	2.77
4	7.2	6.63	5.97	5.43	4.88	4.44	3.89	3.4	2.96
4.25	7.67	7.06	6.36	5.78	5.2	4.73	4.13	3.62	3.15
4.5	8.14	7.49	6.45	6.13	5.51	5.01	4.38	3.84	3.34
4.75	8.61	7.91	7.13	6.48	5.82	5.3	4.63	4.06	3.53
5	9.08	8.35	7.52	6.83	6.13	5.58	4.88	4.27	3.72
5.25	9.56	8.79	7.91	7.18	6.44	5.87	5.13	4.49	3.9
5.5	10	9.22	8.3	7.53	6.76	6.15	5.38	4.71	4.09
5.75	10.5	9.65	8.68	7.88	7.07	6.44	5.63	4.93	4.28
6	11	10.1	9.07	8.23	7.39	6.73	5.87	5.14	4.47
6.25	11.4	10.5	9.46	8.58	7.7	7.01	6.12	5.36	4.66
6.5	11.9	10.9	9.85	8.93	8.02	7.3	6.37	5.58	4.85
6.75	12.4	11.4	10.2	9.28	8.33	7.58	6.62	5.79	5.03
7	12.9	11.8	10.6	9.63	8.64	7.87	6.87	6.01	5.22
7.25	13.3	12.2	11	9.99	8.96	8.15	7.12	6.23	5.41
7.5	13.8	12.7	11.4	10.3	9.27	8.44	7.37	6.45	5.6
7.75	14.3	13.1	11.8	10.7	9.59	8.72	7.62	6.66	5.79
8	14.7	13.5	12.2	11	9.9	9.01	7.86	6.88	5.98

Weight of Seamless Drawn Copper Tubes. (*English.*)

For Diameters and Thicknesses not given in preceding Tables. (*D. K. Clark.*)

INTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzapffel's Wire-Gauge. f full, l light.

Specific Weight = 1.16. Wrought Iron = 1.

No.	0000	000	00	0	No.	0000	000	00	0
Ins.	.454 29/64	.425 27/64 f	.38 3/8 f	.34 11/32	Ins.	.454 29/64	.425 27/64 f	.38 3/8 f	.34 11/32
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Diam.	Lbs.	Lbs.	Lbs.	Lbs.
.75	—	—	—	4.5	5.75	34.2	31.9	28.3	25.2
.875	—	—	5.79	5.02	6	35.6	33.2	29.5	26.2
1	8.02	7.36	6.37	5.53	6.5	38.4	35.8	31.8	28.3
1.125	8.71	8	6.95	6.05	7	41.1	38.3	34.1	30.3
1.25	9.4	8.65	7.52	6.57	7.5	43.9	40.9	36.4	32.4
1.375	10.1	9.3	8.1	7.08	8	46.6	43.5	38.7	34.5
1.5	10.8	9.94	8.68	7.6	9	52.1	48.7	43.3	38.6
1.625	11.5	10.6	9.26	8.12	10	57.7	53.8	47.9	42.7
1.75	12.1	11.2	9.83	8.63	11	63.2	59	52.5	46.8
1.875	12.8	11.9	10.4	9.15	12	68.7	64.2	57.2	51
2	13.5	12.5	11	9.66	13	74.2	69.3	61.8	55.1
2.125	14.2	13.3	11.6	10.2	14	79.7	74.5	66.4	59.2
2.25	14.9	13.8	12.1	10.7	15	85.2	79.6	71	63.4
2.375	15.6	14.5	12.7	11.2	16	90.7	84.8	75.6	67.7
2.5	16.3	15.1	13.3	11.7	17	96.3	90	80.2	71.8
2.625	17	15.8	13.9	12.2	18	101.8	95.1	84.9	76
2.75	17.7	16.4	14.5	12.8	19	107.3	100.3	89.5	80.1
3	19.1	17.7	15.6	13.8	20	112.8	105.5	94.1	84.2
3.25	20.4	19	16.8	14.8	21	118.3	110.7	98.7	88.3
3.5	21.8	20.3	17.9	15.9	22	123.8	115.8	103.3	92.5
3.75	23.2	21.6	19.1	16.9	23	129.3	120.9	107.9	96.6
4	24.6	22.9	20.2	17.9	24	134.8	126.1	112.6	100.6
4.25	25.9	24.2	21.4	19	26	146	136.4	121.8	108.8
4.5	27.3	25.4	22.5	20	28	157.2	146.7	131	117.1
4.75	28.7	26.7	23.7	21	30	168.4	157.1	140.2	125.4
5	30.1	28	24.8	22.1	32	179.6	167.4	149.5	133.6
5.25	31.5	29.3	26	23.1	34	190.7	177.7	158.7	141.9
5.5	32.8	30.6	27.1	24.1	36	201.9	188	167.9	150.1

For Diameters from 13 to 24 Inches.

No.	1	2	3	4	5	6	7	8	9	10
Ins.	.3 19/64 f	.284 9/32 f	.259 1/4 f	.238 15/64 f	.22 7/32 f	.203 13/64	.18 3/16 l	.165 11/64 l	.148 9/64 f	.134 9/64 l
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
13	48.5	45.8	41.7	38.3	35.3	32.6	28.8	26.4	23.6	21.4
14	52.1	49.3	44.9	41.2	38	35.1	31	28.4	25.4	23
15	55.8	52.7	48	44.1	40.7	37.6	33.2	30.4	27.2	24.6
16	59.4	56.2	51.2	46.9	43.4	40	35.4	32.4	29	26.3
17	63	59.6	54.3	49.8	46	42.5	37.5	34.4	30.8	27.9
18	66.7	63.1	57.4	52.7	48.7	45	39.7	36.4	32.6	29.5
19	70.3	66.5	60.6	55.6	51.4	47.4	41.9	38.4	34.4	31.2
20	74	70	63.7	58.5	54	49.9	44.1	40.4	36.2	32.8
21	77.6	73.4	66.9	61.4	56.7	52.4	46.3	42.4	38	34.4
22	81.3	76.9	70	64.3	59.4	54.9	48.5	44.4	39.8	36
23	84.9	80.3	73.2	67.2	62.1	57.3	50.7	46.4	41.6	37.7
24	88.6	83.8	76.3	70.1	64.7	59.8	52.9	48.5	43.4	39.3

For Diameters from 13 to 24 Inches.

No.	11	12	13	14	15	16	17	18	19	20
Ins.	.12 1/8 *l*	.109 7/64	.095 3/32 *f*	.083 5/64 *f*	.072 5/64 *l*	.065 1/16 *f*	.058 1/16 *l*	.049 3/64 *f*	.042 3/64 *l*	.035 1/32 *f*
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
13	19.1	17.4	15.1	13.2	11.4	10.3	9.2	7.77	6.65	5.55
14	20.6	18.7	16.3	14.2	12.3	11.1	9.9	8.37	7.16	5.98
15	22.1	20	17.4	15.2	13.2	11.9	10.6	8.96	7.67	6.4
16	23.5	21.3	18.6	16.2	14.1	12.7	11.3	9.56	8.18	6.82
17	25	22.7	19.7	17.2	14.9	13.5	12.1	10.2	8.69	7.27
18	26.4	24	20.9	18.2	15.8	14.3	12.7	10.7	9.2	7.69
19	27.9	25.3	22	19.2	16.7	15.1	13.4	11.3	9.71	8.12
20	29.3	26.6	23.2	20.2	17.6	15.9	14.1	11.9	10.2	8.54
21	30.8	27.9	24.3	21.3	18.4	16.6	14.8	12.5	10.7	8.96
22	32.3	29.3	25.5	22.3	19.3	17.4	15.5	13.1	11.2	9.39
23	33.7	30.6	26.7	23.3	20.2	18.2	16.2	13.7	11.8	9.81
24	35.2	31.9	27.8	24 3	21.1	19	16.9	14.3	12.3	10.2

Weight of Wrought Iron Tubes. (*English.*)

For Diameters and Thicknesses not given in preceding Tables. (*D. K. Clark.*)

INTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzapffel's Wire-Gauge. f full, l light.

No.								4	5	6	7
Ins.	THICKNESS IN INCHES.							.238 15/64 *f*	.22 7/32 *f*	.203 13/64	.18 3/16 *l*
	5/8	9/16	1/2	7/16	3/8	5/16	1/4				
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
19	128.5	115.2	102.1	89.1	76.1	63.2	50.4	48	44.2	40.8	36.2
20	135	121.1	107.3	93.6	80	66.5	53	50.4	46.5	42.9	38
21	141.5	127	112.6	98.2	83.9	69.7	55.6	52.9	48.8	45.1	39.9
22	148.1	132.9	117.8	102.8	87.9	73	58.3	55.4	51.1	47.2	41.8
23	154.6	138.8	123.1	107.4	91.8	76.3	60.9	57.9	53.4	49.3	43.7
24	161.2	144.7	128.3	112	95.7	79.6	63.5	60.4	55.7	51.5	45.6
26	174.3	156.5	138.8	121.1	103.6	86.1	68.7	65.4	60.3	55.7	49.3
28	187.4	168.3	149.2	130.3	111.4	92.7	74	70.4	64.9	60	53.1
30	200.4	180	159.7	139.5	119.3	99.2	79.2	75.4	69.5	64.2	56.8
32	213.5	191.8	170.2	148.6	127.1	105.7	84.4	80.4	74.1	68.5	60.6
34	226.6	203.6	180.6	157.8	135	112.3	89.7	85.4	78.7	72.8	64.4
36	239.7	215.4	191.1	167	142.9	118.8	94 9	90.4	83.4	77	68.1

No.	8	9	10	11	12	13	14	15	16	17	18
Ins.	.165 11/64 *l*	.148 9/64 *f*	.134 9/64 *l*	.12 1/8 *l*	.109 7/64	.095 3/32 *f*	.083 5/64 *f*	.072 5/64 *l*	.065 1/16 *f*	.058 1/16 *l*	.049 3/64 *f*
Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
19	33.1	29.7	26.9	24	21.8	19	16.6	14.4	13	11.6	9.78
20	34.8	31.2	28.3	25.3	22.9	20	17.5	15.1	13.7	12.2	10.3
21	36.6	32.8	29.7	26.5	24.1	21	18.3	15.9	14.3	12.8	10.8
22	38.3	34.3	31.1	27.8	25.2	22	19.2	16.6	15	13.4	11.3
23	40	35.9	32.5	29.1	26.4	23	20.1	17.4	15.7	14	11.8
24	41.8	37.4	33.9	30.3	27.5	24	20.9	18.1	16.4	14 6	12.6
26	45.2	40.5	36.7	32.8	29.8	26	22.6	19.7	17.7	15.8	13 4
28	48.7	43.6	39.5	35.3	32.1	28	24.4	21.2	19.1	17	14 4
30	52.1	46.7	42.3	37.8	34.4	30	26.1	22.7	20.5	18.3	15.4
32	55.5	49.8	45.1	40.4	36.7	32	27.9	24.2	21.8	19.5	16.5
34	59	52.9	48	42.9	39	34	29.7	25.8	23 2	20.7	17.5
36	62.4	56	50.8	45.4	41.3	36	31.4	27.3	24.6	21.9	18.6

Weight of a Square Foot of Wrought and Cast Iron, Steel, Copper, Lead, Brass, and Zinc Plates.

From .0625 to 1 Inch in Thickness.

Thickness.	Wrought Iron.	Cast Iron.	Steel.	Copper.	Lead.	Brass.	Gun-metal.	Zinc.
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.0625	2.517	2.346	2.541	2.89	3.691	2.675	2.848	2.34
.125	5.035	4.693	5.081	5.781	7.382	5.35	5.696	4.68
.1875	7.552	7.039	7.622	8.672	11.074	8.025	8.545	7.02
.25	10.07	9.386	10.163	11.562	14.765	10.7	11.393	9.36
.3125	12.588	11.733	12.703	14.453	18.456	13.375	14.241	11.7
.375	15.106	14.079	15.244	17.344	22.148	16.05	17.089	14.04
.4375	17.623	16.426	17.785	20.234	25.839	18.725	19.938	16.34
.5	20.141	18.773	20.326	23.125	29.53	21.4	22.786	18.72
.5625	22.659	21.119	22.866	26.016	33.222	24.075	25.634	21.06
.625	25.176	23.466	25.407	28.906	36.913	26.75	28.483	23.4
.6875	27.694	25.812	27.948	31.797	40.604	29.425	31.331	25.74
.75	30.211	28.159	30.488	34.688	44.296	32.1	34.179	28.68
.8125	32.729	30.505	33.029	37.578	47.987	34.775	37.027	30.42
.875	35.247	32.852	35.57	40.469	51.678	36.656	39.875	32.76
.9375	37.764	35.199	38.11	43.359	55.37	39.331	42.723	35.1
1	40.282	37.545	40.651	46.25	59.061	42.8	45.572	37.44

From One Twentieth Inch to Two Inches in Thickness.

Thickness.	Wrought Iron.	Cast Iron.	Steel.	Copper.	Lead.	Brass.	Gun-metal.	Zinc.
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.05	2.014	1.877	2.033	2.312	2.593	2.14	2.279	1.872
.1	4.028	3.754	4.065	4.625	5.906	4.28	4.557	3.744
.15	6.042	5.632	6.098	6.938	8.859	6.42	6.836	5.616
.2	8.056	7.509	8.13	9.25	11.812	8.56	9.114	7.488
.25	10.071	9.386	10.163	11.562	14.765	10.7	11.393	9.36
.3	12.085	11.264	12.195	13.875	17.718	12.84	13.672	11.232
.35	14.099	13.141	14.228	16.187	20.671	14.98	15.95	13.104
.4	16.113	15.018	16.26	18.5	23.624	17.12	18.229	14.976
.45	18.127	16.895	18.293	20.812	26.577	19.26	20.507	16.848
.5	20.141	18.773	20.325	23.125	29.53	21.4	22.786	18.72
.55	22.155	20.65	22.358	25.437	32.484	23.54	25.065	20.592
.6	24.169	22.527	24.391	27.75	35.437	25.68	27.343	22.464
.65	26.183	24.409	26.423	30.063	38.39	27.82	29.622	24.336
.7	28.197	26.281	28.456	32.375	41.343	29.96	31.9	26.208
.75	30.211	28.154	30.488	34.687	44.296	32.1	34.179	28.08
.8	32.226	30.035	32.521	37	47.249	34.24	36.458	29.95
.85	34.24	31.912	34.553	39.312	50.202	36.38	38.736	31.824
.9	36.254	33.79	36.586	41.625	53.154	38.52	41.015	33.696
.95	38.268	35.668	38.628	43.937	56.108	40.66	43.293	35.568
1	40.282	37.545	40.651	46.25	59.061	42.8	45.572	37.44
1.125	45.317	42.238	45.732	52.031	66.443	48.15	51.268	42.12
1.25	50.352	46.931	50.814	57.813	73.826	53.5	56.965	46.8
1.3125	52.87	49.278	53.354	60.703	77.517	56.17	59.813	49.14
1.375	55.387	51.624	55.895	63.594	81.209	58.85	62.661	51.48
1.4375	57.905	53.971	58.436	66.484	84.9	61.53	65.51	53.82
1.5	60.422	56.317	60.976	69.375	88.591	64.2	68.358	56.16
1.5625	62.94	58.663	63.517	72.266	92.283	66.88	71.206	58.5
1.625	65.458	61.011	66.058	75.156	95.974	69.55	74.054	60.84
1.75	70.493	65.704	71.139	80.938	103.356	74.9	79.751	65.52
1.875	75.528	70.397	76.22	86.719	110.739	80.25	85.447	70.2
2	80.564	75.09	81.3	92.5	118.122	85.6	91.144	74.88

Standard Cast Iron Water Pipes. (*English.*)

For a Head of 200 Feet.

Diameter.	Thickness.	Depth of Socket.	Thickness of Socket.	Packing.	Weight per Yard.*	Lead Joint.	Diameter.	Thickness.	Depth of Socket.	Thickness of Socket.	Packing.	Weight per Yard.*	Lead Joint.
Ins.	Inch.	Ins.	Inch.	Inch.	Lbs.	Lbs.	Ins.	Inch.	Ins.	Inch.	Inch.	Lbs.	Lbs.
3	·3125	3·5	.625	.25	36	.8	8	·4375	3·75	.625	·375	113	3·3
4	·3125	3	.625	.25	51	1.2	9	·4375	3·75	·75	·375	128	4.6
5	·375	3	.625	·375	61	2	10	·5	4	·75	·375	168	4·9
6	·375	3·75	.625	·375	75	2.7	11	·5	4	·75	·375	175	5·3
7	·375	3·75	.625	·375	85	2.9	12	·5625	4	.875	·375	213	5·7

* Measured as laid.

To Compute Weight of Metal Pipes.

$\overline{D^2 - d^2}\,C$. D *and* d *representing external and internal diameters in inches, and* C *coefficient.*

Cast Iron 2.45. Wrought Iron 2.64. Brass 2.82. Copper 3.03. Lead 3.86.

To Compute Weight of Metal Tubes and Pipes per Lineal Foot.

From .5 Inch to 6 Inches Internal Diameter.

Diam.	Area of Plate.	Diam.	Area of Plate.	Diam.	Area of Plate.	Diam.	Area of Plate.
Ins.	Sq. Foot.	Ins.	Sq. Foot.	Ins.	Sq. Feet.	Ins.	Sq. Feet.
·5	.1309	1.3125	·3436	2.75	·7199	4.5	1.1781
.5625	.1473	1.375	·36	2.875	·7526	4.625	1.2108
.625	.1636	1.4375	·3764	3	·7854	4.75	1.2435
.6875	.18	1.5	·3927	3.125	.8181	4.875	1.2763
·75	.1964	1.625	·4254	3.25	.8508	5	1.309
.8125	.2127	1.75	·4581	3.375	.8836	5.125	1.3417
.875	.2291	1.875	·4909	3.5	·9163	5.25	1.3744
·9375	.2454	2	·5236	3.625	·949	5.375	1.4072
1	.2618	2.125	·5543	3.75	·9818	5.5	1.4399
1.0625	.2782	2.25	·587	4	1.0472	5.625	1.4726
1.125	.2945	2.375	.6198	4.125	1.0799	5.75	1.5053
1.1875	·3105	2.5	.6545	4.25	1.1126	5.875	1.5381
1.25	·3272	2.625	.6872	4.375	1.1454	6	1.5708

Application of Table.

When Thickness of Metal is given in Divisions of an Inch.

To internal diameter of tube or pipe add thickness of metal; take area of the plate in square feet, from table for a diameter equal to sum of diameter and thickness of tube or pipe, and multiply it by weight of a square foot of metal for given thickness (see table, page 146), and again by its length in feet.

Illustration.—Required weight of 10 feet of copper tube 1 inch in diameter and .125 of an inch in thickness.

$1 + .125 = 1.125 \times 3.1416 \div 12 = .2945$ *square feet for* 1 *foot of length.*

Weight of 1 square foot of copper .125th of an inch in thickness, per table, page 135, $= 5.781$ *lbs.*; then, .2945 (from table above) $\times 5.781 \times 10 = 17.025$ *lbs.*

When Thickness of Metal is given in Numbers of a Wire-Gauge.

To internal diameter of tube or pipe add thickness of number from table, pp. 120 or 121; multiply sum by 3.1416, divide product by 12, and quotient will give area of plate in square feet. Then proceed as before.

ILLUSTRATION.—Required weight of 10 feet of copper pipe 2 inches in diameter and No. 2 American wire-gauge in thickness.

$2 + .25763 \times 3.1416 \div 12 = 2\ 25763 \times 3.1416 \div 12 = .591$ *square feet*; then, $.591 \times 11.6706$ (weight from table, page 118) $= 6.897$ *lbs.*

Weight of Riveted Iron and Copper Pipes,

From 5 to 30 Inches in Diameter.

ONE FOOT IN LENGTH.

Diameter.	Thickness.	Iron.	Copper.	Diameter.	Thickness.	Iron.	Copper.
Ins.	Inch.	Lbs.	Lbs.	Ins.	Inch.	Lbs.	Lbs.
5	.125	7.12	8.14	9	.25	25.01	28.58
	.1875	10.68	12.21		.25	26.33	30.09
	.25	14.25	16.28	10	.25	27.75	31.71
5.5	.125	7.78	8.89	10.5	.25	29.19	33 22
	.1875	11.66	13.33	11	.25	30.49	34.85
	.25	15.56	17.78	12	.25	33.13	37.86
6	.125	8.44	9.64	13	.25	35.88	41
	.1875	12.65	14.46	14	.25	38.52	44.02
	.25	16.88	19.29	15	.25	41.26	47.15
6.5	.125	9.1	10.4		.3125	51.57	58.94
	.1875	13.65	15.6	16	.25	43.9	50.17
	.25	18.2	20.8		.3125	54.87	62.71
7	.125	9.78	11.18	17	.25	46.53	53.18
	.1875	14.68	16.78		.3125	58.17	66.48
	.25	19.57	22.37	18	.25	49.17	56.2
7.5	.125	10.49	11.99		.3125	61.47	70.25
	.1875	15.73	17.98	20	.3125	68.07	77.79
	.25	20.89	23.87	24	.3125	81.33	92 95
8	.1875	16.7	19 08	25	.3125	84.57	96.65
	.25	22.26	25.44	28	.3125	94.56	107.95
8.5	.25	23.59	26.96	30	.3125	101.14	115.59

Above weights include laps of sheets for riveting and calking.

Weights of the rivets are not added, as number per lineal foot of pipe depends upon the distance they are placed apart, and their diameter and length depend upon thickness of metal of the pipe.

Weight of Copper Rods or Bolts,

From .125 Inch to 4 Inches in Diameter.

ONE FOOT IN LENGTH.

Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.
Inch.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.125	.047	.8125	1.998	1.5	6.811	2.75	22.891
.1875	.106	.875	2.318	.5625	7.39	.875	25.019
.25	.189	.9375	2.66	.625	7.993	3	27.243
.3125	.296	1	3.03	.75	9.27	.125	29.559
.375	.426	1.0625	3.42	.875	10.642	.25	31.972
.4375	.579	.125	3.831	2	12.108	.375	34.481
.5	.757	.1875	4.269	.125	13.668	.5	37.081
.5625	.958	.25	4.723	.25	15.325	.625	39.777
.625	1.182	.3125	5.21	.375	17.075	.75	42.568
.6875	1.431	.375	5.723	.5	18.916	.875	45.455
.75	1.703	.4375	6.255	.625	20.856	4	48.433

Weight of Metals of a Given Sectional Area.

From .1 Square Inch to 10 Square Inches.

PER LINEAL FOOT. (*D. K. Clark.*)

Sect. Area.	Wrought Iron. 1.	Cast Iron. .9375.	Steel. 1.02.	Brass. 1.052.	Gun-metal. 1.092.	Sect. Area.	Wrought Iron. 1.	Cast Iron. .9375.	Steel. 1.02.	Brass. 1.052.	Gun-metal. 1.092.
Sq.Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Sq.Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.1	.33	.31	.34	.35	.36	5.1	17	15.9	17.3	17.9	18.6
.2	.67	.62	.68	.7	.73	5.2	17.3	16.3	17.7	18.2	18.9
.3	1	.94	1.02	1.05	1.09	5.3	17.7	16.6	18	18.6	19.3
.4	1.33	1.25	1.36	1.43	1.46	5.4	18	16.9	18.4	18.9	19.7
.5	1.67	1.56	1.7	1.75	1.82	5.5	18.3	17.2	18.7	19.3	20
.6	2	1.88	2.04	2.11	2.18	5.6	18.7	17.5	19	19.6	20.4
.7	2.33	2.19	2.38	2.46	2.55	5.7	19	17.8	19.4	20	20.8
.8	2.67	2.5	2.72	2.81	2.91	5.8	19.3	18.1	19.7	20.3	21.1
.9	3	2.81	3.06	3.16	3.28	5.9	19.7	18.4	20.1	20.7	21.5
1	3.33	3.15	3.4	3.51	3.64	6	20	18.8	20.4	21	21.8
1.1	3.67	3.44	3.74	3.86	4	6.1	20.3	19.1	20.7	21.4	22.2
1.2	4	3.75	4.08	4.21	4.37	6.2	20.7	19.4	21.1	21.7	22.6
1.3	4.33	4.06	4.42	4.56	4.73	6.3	21	19.7	21.4	22.1	22.9
1.4	4.67	4.38	4.76	4.91	5.1	6.4	21.3	20	21.8	22.4	23.3
1.5	5	4.69	5.1	5.26	5.46	6.5	21.7	20.3	22.1	22.8	23.7
1.6	5.33	5	5.44	5.61	5.82	6.6	22	20.6	22.4	23.1	24
1.7	5.67	5.31	5.78	5.96	6.19	6.7	22.3	20.9	22.8	23.5	24.4
1.8	6	5.63	6.12	6.31	6.55	6.8	22.7	21.3	23.1	23.9	24.8
1.9	6.33	5.94	6.46	6.66	6.92	6.9	23	21.6	23.5	24.2	25.1
2	6.67	6.25	6.8	7.01	7.28	7	23.3	21.9	23.8	24.6	25.5
2.1	7	6.56	7.14	7.36	7.64	7.1	23.7	22.2	24.1	24.9	25.8
2.2	7.33	6.88	7.48	7.72	8.01	7.2	24	22.5	24.5	25.3	26.2
2.3	7.67	7.19	7.82	8.07	8.37	7.3	24.3	22.8	24.8	25.6	26.6
2.4	8	7.5	8.16	8.42	8.74	7.4	24.7	23.1	25.2	26	26.9
2.5	8.33	7.81	8.5	8.77	9.1	7.5	25	23.4	25.5	26.3	27.3
2.6	8.67	8.13	8.84	9.12	9.46	7.6	25.3	23.8	25.9	26.7	27.7
2.7	9	8.44	9.18	9.47	9.83	7.7	25.7	24.1	26.2	27	28
2.8	9.33	8.75	9.52	9.82	10.2	7.8	26	24.4	26.5	27.4	28.4
2.9	9.67	9.06	9.86	10.2	10.6	7.9	26.3	24.7	26.9	27.7	28.8
3	10	9.38	10.2	10.5	10.9	8	26.7	25	27.2	28.1	29.1
3.1	10.3	9.69	10.5	10.9	11.3	8.1	27	25.3	27.5	28.4	29.5
3.2	10.7	10	10.9	11.2	11.7	8.2	27.3	25.6	27.9	28.8	29.9
3.3	11	10.3	11.2	11.6	12	8.3	27.7	25.9	28.2	29.1	30.2
3.4	11.3	10.6	11.6	11.9	12.4	8.4	28	26.3	28.6	29.5	30.6
3.5	11.7	10.9	11.9	12.3	12.7	8.5	28.3	26.6	28.9	29.8	30.9
3.6	12	11.3	12.2	12.6	13.1	8.6	28.7	26.9	29.2	30.2	31.3
3.7	12.3	11.6	12.6	13	13.5	8.7	29	27.2	29.6	30.5	31.7
3.8	12.7	11.9	12.9	13.3	13.8	8.8	29.3	27.5	29.9	30.9	32
3.9	13	12.2	13.3	13.7	14.2	8.9	29.7	27.8	30.3	31.2	32.4
4	13.3	12.5	13.6	14	14.6	9	30	28.1	30.6	31.6	32.8
4.1	13.7	12.8	13.9	14.4	14.9	9.1	30.3	28.4	30.9	31.9	33.1
4.2	14	13.1	14.3	14.7	15.3	9.2	30.7	28.8	31.3	32.3	33.5
4.3	14.3	13.4	14.6	15.1	15.7	9.3	31	29.1	31.6	32.6	33.9
4.4	14.7	13.8	15	15.4	16	9.4	31.3	29.4	32	33	34.2
4.5	15	14.1	15.3	15.8	16.4	9.5	31.7	29.7	32.3	33.3	34.6
4.6	15.3	14.4	15.6	16.1	16.7	9.6	32	30	32.6	33.7	34.9
4.7	15.7	14.7	16	16.5	17.1	9.7	32.3	30.3	33	34	35.3
4.8	16	15	16.3	16.8	17.5	9.8	32.7	30.6	33.3	34.4	35.7
4.9	16.3	15.3	16.7	17.2	17.8	9.9	33	30.9	33.7	34.7	36
5	16.7	15.6	17	17.5	18.2	10	33.3	31.3	34	35.1	36.4

Weight of Lead Pipe. (*English.*)

ONE FOOT IN LENGTH.

Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.	Diam.	Thickness	Weight.	Diam.	Thickness.	Weight.
Inch.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.	Ins.	Inch.	Lbs.
.5	.097	.93	1	.136	2.4	1.75	.166	5	3	.275	14
	.112	1.07		.156	2.8		.199	6	3.5	.225	13
	.124	1.2		.2	3.73		.228	7		.273	16
	.146	1.47		.225	4.27		.256	8	4	.257	17
.625	.089	1	1.25	.139	3	2	.178	6		.3125	20.5
	.101	1.13		.16	3.5		.204	7		.327	22
	.121	1.4		.18	4		.231	8	4.25	.3125	22.04
	.14	2		.193	4.33		.266	9.33	4.5	.232	17
.75	.112	1.6	1.5	.156	4	2.5	.2	8.4		.295	22
	.147	1.87		.179	4.67		.227	9.6		.3125	23.25
	.181	2.13		.224	6		.261	11.2	4.75	.3125	24.45
	.215	2.4		.257	7	3	.218	11.2	5	.3125	25.66

Dimensions of Copper Pipes and Composition Cocks.

From 1 *Inch to* 23 *Inches in Diameter.*

Diam. of Pipe and Cock.	Flange Diameter.		Thickness.	Bolts.		Diam. of Pipe and Cock.	Flange Diam.	Thickness.	Bolts.	
	Pipe.	Cock.		No.	Diam.		Pipe.		No.	Diam.
Ins.	Ins.	Ins.	Inch.		Inch.	Ins.	Ins.	Inch.		Inch.
1	3.375	3.5	.375	3	.5	9	12.75	.625	9	.625
1.25	3.625	3.75	.375	3	.5	9.25	13.125	.625	10	.625
1.5	3.875	4.25	.375	3	.5	9.5	13.375	.6875	10	.625
1.75	4.125	4.375	.4375	4	.5	9.75	13.625	.6875	10	.625
2	4.375	4.75	.4375	4	.5	10	13.875	.6875	10	.625
2.25	4.625	5.25	.4375	5	.5	10.5	14.5	.6875	10	.625
2.5	4.875	5.5	.4375	5	.5	11	15	.6875	10	.625
2.75	5.25	5.75	.4375	5	.5	11.5	15.625	.75	10	.75
3	6	6.25	.5	5	.625	12	16.125	.75	10	.75
3.25	6.125	6.625	.5	6	.625	12.5	16.625	.75	10	.75
3.5	6.375	6.875	.5	6	.625	13	17.25	.75	10	.75
3.75	6.625	7.25	.5	6	.625	13.5	17.875	.75	10	.75
4	6.875	7.375	.5	6	.625	14	18.375	.75	10	.75
4.25	7.125	7.625	.5	6	.625	14.5	18.875	.75	10	.75
4.5	7.375	8.25	.5	6	.625	15	19.5	.75	10	.75
4.75	7.625	8.5	.5	6	.625	15.5	20	.75	10	.75
5	8	9	.5	6	.625	16	20.5	.75	10	.75
5.25	8.25	9.25	.5	6	.625	16.5	21.125	.75	10	.75
5.5	8.5	9.5	.5	6	.625	17	21.625	.75	11	.75
5.75	9	9.875	.5	6	.625	17.5	22.125	.75	11	.75
6	9.25		.625	8	.625	18	22.75	.75	11	.75
6.25	9.75		.625	8	.625	18.5	23.25	.75	11	.75
6.5	10		.625	8	.625	19	23.75	.75	12	.75
6.75	10		.625	8	.625	19.5	24.375	.75	12	.75
7	10.5		.625	8	.625	20	24.875	.75	12	.75
7.25	10.75		.625	8	.625	20.5	25.375	.75	13	.75
7.5	11.125		.625	8	.625	21	26	.75	13	.75
7.75	11.375		.625	8	.625	21.5	26.5	.75	13	.75
8	11.625		.625	9	.625	22	27	.75	13	.75
8.25	12		.625	9	.625	22.5	27.625	.75	14	.75
8.5	12.25		.625	9	.625	23	28.125	.75	14	.75
8.75	12.5		.625	9	.625					

Weight of Sheet Lead.

PER SQUARE FOOT.

Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.
Inch.	Lbs.	Inch.	Lbs.	Inch.	Lbs.	Inch.	Lbs.
.017	1	.068	4	.118	7	.169	10
.034	2	.085	5	.135	8	.186	11
.051	3	.101	6	.152	9	.203	12

Weight of Tin Pipe.

ONE FOOT IN LENGTH.

Diam. External.	THICKNESS. 3/32 inch.	THICKNESS. 1/8 inch.	Diam. External.	THICKNESS. 3/32 inch.	THICKNESS. 1/8 inch.	Diam. External.	THICKN. 1/4 inch.	Diam. External.	THICKN. 1/4 inch.
Inch.	Lb.	Lbs.	Ins.	Lbs.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.25	.148	—	1.25	1.095	1.417	2.25	5.04	3.25	7.56
.5	.384	.472	1.5	1.328	1.732	2.5	5.67	3.5	8.19
.75	.62	.787	1.75	1.564	2.047	2.75	6.3	3.75	8 82
1	.856	1.103	2	1.802	2.362	3	6.93	4	9.45

Weight of Lead Encased Tin Pipes.

Diameter.	Light Weights.			For Supply of Water Head.* 50 feet and under.	51 to 250 feet.	251 to 500 feet.
Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.375	1	1.5	2	2.5 to 4	3 to 4.5	3.5 to 5
.5	2	2.5	3	3.5 " 5	4 " 6	4.5 " 7
.625	3	3.5	4	4.5 " 7	5.25 " 8	6 " 9
.75	3.5	4	4.5	5.5 " 8	6 " 9	7 " 10
1	4.5	5	5.5	7.25 " 10	8 " 11	9 " 12
1.25	6.5	7	8	9 " 12.5	10 " 14	12 " 16
1.5	8	9	10	11 " 16	12.5 " 18	14 " 21
2	11	13	—	16 " 23	18.5 " 26	21 " 30

* The extreme weights are for extra heavy pipe with less proportion of tin.

Dimensions and Weight of Sheet Zinc. (*Vielle-Montagne.*)

PER SQUARE FOOT.

No.	Thickness.		2×.5 metres; area, 1 square metre. 6.56×1 64 feet; area, 10.76 square feet.		2×.65 metres; area, 1.3 sq. metres. 6.56×2.13 feet; area, 13.99 square feet.		2×.8 metres; area, 1.6 sq. metres. 6 56×2.62 ft.; area, 17.22 square feet.		Weight.
	Millim.	Inch.	Kilom.	Lbs.	Kilom.	Lbs.	Kilom.	Lbs.	Lbs.
9	.41	.0161	2.9	6.39	3.7	8.16	4.6	10.14	.589
10	.51	.0201	3.45	7.61	4.45	9.81	5.5	12.12	.704
11	.6	.0236	4.05	8.93	5.3	11.68	6.5	14.33	.832
12	.69	.0272	4.65	10.25	6.1	13.45	7.5	16.53	.96
13	.78	.0307	5.3	11.68	6.9	15.21	8.5	18.74	1.088
14	.87	.0343	5.95	13.12	7.7	16.94	9.5	20.94	1.216
15	.96	.0378	6.55	14.44	8.55	18.85	10.5	23.15	1.344
16	1.1	.0433	7.5	16.53	9.75	21.5	12	26.46	1.536
17	1.23	.0485	8.45	18.63	10.95	24.14	13.5	29.97	1.74
18	1.36	.0536	9.35	20.61	12.2	26.9	15	33.07	1.92
19	1.48	.0583	10.3	22.71	13.4	29.54	16.5	36.38	2.112
20	1.66	.0654	11.25	24.8	14.6	32.19	18	39.68	2.304
21	1.85	.0729	12.5	27.56	16.25	35.82	20	44.09	2.56
22	2.02	.0795	13.75	30.31	17.9	39.46	22	48.5	2.816
23	2.19	.0862	15	33 07	19.5	42.99	24	52.91	3.073
24	2.37	.0933	16.25	35.82	21.1	46.52	26	57.32	3.329
25	2.52	.0992	17.5	38.58	22.75	50.15	28	61.73	3.585
26	2.66	.1047	18.8	41.44	24.4	53.79	31	68.34	3.969

Table—(*Continued*).

Special Sizes for Sheathing Ships.

No.	Thickness.		Dimensions of Sheets. 1.15 × .35 metres; area, .402 sq. metre. 3.77 × 1.15 feet; area, 4.33 sq. feet.		Dimensions of Sheets. 1.3 × .4 metres; area, .52 sq. metre. 4.26 × 1.31 feet; area, 5.6 sq. feet.		Weight per Sq. Foot.
	Millim.	Inch.	Kilom.	Lbs.	Kilom.	Lbs	Lbs.
15	.96	.0378	2.65	5.84	3.4	7.5	1.344
16	1.1	.0433	3	6.61	3.9	8.6	1.536
17	1.23	.0485	3.4	7.5	4.4	9.7	1.74
18	1.36	.0536	3.75	8.27	4.9	10.8	1.92
19	1.48	.0583	4.15	9.15	5.35	11.79	2.112
20	1.66	.0654	4.55	10.03	5.85	12.9	2.304

NOTE.—A deviation of 25 dekagrammes, or about half a pound, more or less, from the proper weight of each number of sheet, is allowed.

Nos. 1 to 9 are employed for perforated articles, as sieves, and for *articles de Paris*. Nos. 10 to 12 are used in manufacture of lamps, lanterns, and tin-ware generally, and for stamped ornaments. The last numbers are used for lining reservoirs, and for baths and pumps.

Ship and Railroad Spikes.

DIMENSIONS AND NUMBER PER POUND. (*P. C. Page, Mass.*)

Ship Spikes.

¼ In. Sq. Length.	¼ In. Sq. No. in Pound.	5/16 In. Sq. Length.	5/16 In. Sq. No. in Pound.	7/16 In. Sq. Length.	7/16 In. Sq. No. in Pound.	½ In. Sq. Length.	½ In. Sq. No. in Pound.	9/16 In. Sq. Length.	9/16 In. Sq. No. in Pound.	5/8 In. Sq. Length.	5/8 In. Sq. No. in Pound.	¾ In. Sq. Length.	¾ In. Sq. No. in Pound.
Ins.		Ins.		Ins.		Ins.		Ins.		Ins.		Ins.	
3	19	3	10	4	5.4	5	3.4	6	2.2	8	1.4	10	.8
3.5	15.8	3.5	9.6	4.5	5	5.5	3.1	6.5	2	9	1.2	15	.6
4	13.2	4	8	5	4.6	6	3	7	1.9	10	1.1	—	—
4.5	12.2	4.5	6	5.5	4.2	6.5	2.8	7.5	1.8	11	1	—	—
5	10.2	5	5.8	6	4	7	2.6	8	1.7	—	—	—	—
—	—	6	5.2	6.5	3.2	7.5	2.4	8.5	1.6	—	—	—	—
—	—	—	—	—	—	8	2.2	9	1.5	—	—	—	—
—	—	—	—	—	—	—	—	10	1.4	—	—	—	—

Railroad Spikes . . . . . .5 inch square × 5.5 ins. 2 per lb.
" " . . . . . .5625 " " × 5.5 " 1.6 "

Spikes and Horseshoes.

LENGTH AND NUMBER PER POUND. (*H. Burden, Troy, N. Y.*)

Boat Spikes. Length.	Boat Spikes. No. in Lb.	Boat Spikes. Length.	Boat Spikes. No. in Lb.	Ship Spikes. Length.	Ship Spikes. No. in Lb.	Ship Spikes. Length.	Ship Spikes. No. in Lb.	Hook Head. Length.	Hook Head. No. in Lb.	Horseshoes. Length.	Horseshoes. No. in Lb.
Ins.		Ins.		Ins.		Ins.		Ins.		Ins.	
3	17.5	6.5	4.78	4	8	7.5	2.5	4 × .375	5.55	1	.84
3.5	14.68	7	3.62	4.5	6.5	8	1.74	4.5 × .4375	4.14	2	.75
4	12.57	7.5	3.37	5	4.37	8.5	1.63	5 × .5	2.52	3	.65
4.5	9.2	8	2.95	5.5	4.3	9	1.55	5.5 × .5	2.41	4	.56
5	7.2	8.5	2.9	6	4.2	10	1.15	5.5 × .5625	1.87	5	.39
5.5	6.3	9	2.1	6.5	3.77	—	—	6 × .5625	1.72	—	—
6	4.97	10	1.98	7	2.75	—	—	6 × .625	1.38	—	—

Weight and Volume of Cast Iron and Lead Balls.

From 1 Inch to 20 Inches in Diameter.

Diameter.	Volume.	Cast Iron.	Lead.	Diameter.	Volume.	Cast Iron.	Lead.
Ins.	Cube Ins.	Lbs.	Lbs.	Ins.	Cube Ins.	Lbs.	Lbs.
1	.523	.136	.215	9	381.703	99.51	156.553
1.5	1.767	.461	.725	9.5	448.92	117.034	184.121
2	4.189	1.092	1.718	10	523.599	136.502	214.749
2.5	8.181	2.133	3.355	10.5	606.132	158.043	248.587
3	14.137	3.685	5.798	11	696.91	181.765	285.832
3.5	22.449	5.852	9.207	11.5	796.33	207.635	326.591
4	33.51	8.736	13.744	12	904.778	235.876	371.096
4.5	47.713	12.439	19.569	12.5	1022.656	266.647	419.512
5	65 45	17.063	26.843	13	1150.346	299.623	471.806
5.5	87.114	22.721	35.729	14	1436.754	374.563	589.273
6	113.097	29.484	46.385	15	1767.145	460.696	724.781
6.5	143.793	37.453	58.976	16	2144.66	559.114	879.616
7	179.594	46.82	73.659	17	2572.44	670.717	1055.066
7.5	220.893	57.587	90.598	18	3053.627	796.082	1252.422
8	268.082	69.889	109.952	19	3591.363	936.271	1472.97
8.5	321.555	83.84	131.883	20	4188.79	1092.02	1717.995

NOTE.—To compute weight of balls of other metals, multiply weight given in table by following multipliers:

For Wrought Iron	1.067.	Brass	1.12.
Steel	1.088.	Gun-metal	1.165.

Weight and Diameter of Cast Iron Balls.

Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight	Diameter.
Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.
1	1.94	12	4.45	50	7.16	224	11.8	1344	21.44
2	2.45	14	4.68	56	7.43	336	13.51	1568	22.57
3	2.8	16	4.89	60	7.6	448	14.87	1792	23.6
4	3.08	18	5.09	70	8.01	560	16.02	2016	24.54
5	3.32	20	5.27	80	8.37	672	17.02	2240	25.42
6	3.53	25	5.68	90	8.71	784	17.91	2800	27.38
7	3.72	28	5.9	100	9.02	896	18.73	3360	29.1
8	3.89	30	6.04	112	9.37	1008	19.48	3920	30.64
9	4.04	40	6.64	168	10.72	1120	20.17	4480	32.03

Length of Horseshoe Nails.

By Numbers.

No. 5	1.5 Ins.	No. 7	1.875 Ins.	No. 9	2.25 Ins.
" 6	1.75 "	" 8	2 "	" 10	2.5 "

Lengths of Iron Nails, and Number in a Lb.

Size.	L'gth.	No.	Size.	L'gth.	No.	Size.	L'gth.	No.	Size.	L'gth.	No.	Size.	L'gth.	No.
	Ins.			Ins.			Ins.			Ins.			Ins.	
3*d.*	1.25	420	5*d.*	1.75	220	8*d.*	2.5	100	12*d.*	3.25	52	30*d.*	4	24
4	1.5	270	6	2	175	10	3	65	20	3.5	28	40	4.25	20

Wrought Iron Cut Nails, Tacks, Spikes, etc.

(Cumberland Nail and Iron Co.)

Lengths and Number per Lb.

Ordinary.

Size.	Length.	No. per Lb.
	Ins.	
2d	.875	716
3 fine	1.0625	588
3	1.0625	448
4	1.375	336
5	1.75	216
6	2	166
7	2.25	118
8	2.5	94
10	2.75	72
12	3.5	50
20	3.75	32
30	4.25	20
40	4.75	17
50	5	14
60	5.5	10

Light.

4d	1.375	373
5	1.75	272
6	2	196

Brads.

6d	2	163
8	2.5	96
10	2.75	74
12	3.125	50

Fence.

6d	2	96
7	2.25	66
8	2.5	56
10	2.75	50
—	3	40

Finishing.

Size.	Length.	No. per Lb.
	Ins.	
4d	1.375	384
5	1.75	256
6	2	204
8	2.5	102
10	3	80
12	3.625	65
20	3.875	46

Core.

6d	2	143
8	2.5	68
10	2.333	60
12	3.125	42
20	3.75	25
30	4.25	18
40	4.75	14
WH	2.5	69
WHL	2.25	72

Clinch.

6d	2	152
7	2.25	133
8	2.5	92
10	2.75	72
—	3	60
—	3.25	43

Slate.

3d	1.625	288
4	1.4375	244
5	1.75	187
6	2	146

Shingle.

Size.	Length.	No. per Lb.
	Ins.	
5d	1.75	178
8	2.5	74
9	2.75	60
10	3	52

Tacks.

1 oz.	.125	16 000
1.5	.1875	10 666
2	.25	8 000
2.5	.3125	6 400
3	.375	5 333
4	.4375	4 000
6	.5625	2 666
8	.625	2 000
10	.6875	1 600
12	.75	1 333
14	.8125	1 143
16	.875	1 000
18	.9375	888
20	1	800

Boat.

Size.	No. per Lb.
Ins.	
1.5	206

Spikes.

3.5	19
4	15
4.5	13
5	10
5.5	9
6	7

Railroad Spikes.

Number in a Keg of 150 lbs.

Length.	No.	Length.	No.	Length.	No.	Length.	No.
No.		Ins.		Ins.		Ins.	
3 × .375	930	3.5 × .4375	675	4 × .5	450	5 × .5625	300
3.5 × .375	890	4 × .4375	540	4.5 × .5	400	5.5 × .5625	280
4 × .375	760	4.5 × .4375	510	5 × .5	340		

5.5 × .5625 standard for a gauge of 4 feet 8.5 ins.

Ship and Boat Spikes.

Number in a Keg of 150 lbs.

Length.	No.	Length.	No.	Length.	No.	Length.	No.
Ins.		Ins.		Ins.		Ins.	
4 × .25	1650	5 × .3125	930	8 × .375	455	10 × .4375	270
4.5 × .25	1464	6 × .3125	868	9 × .375	424	8 × .5	256
5 × .25	1380	7 × .3125	662	10 × .375	390	9 × .5	240
6 × .25	1292	6 × .375	570	8 × .4375	384	10 × .5	222
7 × .25	1161	7 × .375	482	9 × .4375	300	11 × .5	203

Weight of Various Metals.

Per Cube Inch and Foot.

Metals.	Spec. Gravity.	W'ght in an Inch.	Ins. in a Lb.	Weight in a Foot.	Metals.	Specific Gravity.	W'ght in an Inch.	Ins. in a Lb.	Weight in a Foot.
		Lb.		Lbs.			Lb.		Lb.
Wrought-iron plates	7734	.2797	3.57	483.38	Brass, rolled.	8 217	.2972	3.37	513.6
" wire.	7774	.2812	3.55	485.87	" cast...	8 080	.2922	3.42	505
Cast iron....	7209	.2607	3.84	450.54	Lead, rolled .	11 340	.4101	2.44	708.73
Steel plates..	7804	.2823	3.54	487.8	Tin, cast.....	7 292	.2673	3.74	462
" wire...	7847	.2838	3.52	490.45	Zinc, rolled..	7 188	.26	3.85	449.28
Copper, rolled {...	8697	.3146	3.19	543.6	Aluminium, cast	2 560	.0926	10.8	160
{...	8880	.3212	3.11	555					
Gun-metal, cast.....	8750	.3165	3.16	546.875	Silver.......	10 480	.3791	2.64	655

English. (*D. K. Clark.*)

Wrought iron	7.698	.278	3.6	480	Tin.........	7.409	.268	3.74	462
Cast iron....	7.217	.26	3.84	450	Zinc........	7.008	.253	3.95	437
Steel........	7.852	.283	3.53	489.6	Lead........	11.418	.412	2.43	712
Copper plates	8.805	.318	3.15	549	Brass, cast...	8.099	.292	3.42	505
Gun-metal...	8.404	.304	2.02	524	" wire..	8.548	.308	3.24	533

WROUGHT AND CAST IRON.

To Compute Weight of Wrought or Cast Iron.

Rule.—Ascertain number of cube inches in piece; multiply sum by .2816* for wrought iron and .2607* for cast, and product will give weight in pounds.

Or, for cast iron multiply weight of pattern, if of pine, by from 18 to 20, according to its degree of dryness.

Example.—What is weight of a cube of wrought iron 10 inches square by 15 inches in length?

$$10 \times 10 \times 15 \times .2816 = 422.4 \text{ lbs.}$$

COPPER.

To Compute Weight of Copper.

Rule.—Ascertain number of cube inches in piece; multiply sum by .321 18,* and product will give weight in pounds.

Sheathing and Braziers' Sheets.

For dimensions and weights see Measures and Weights, pages 118–121, 131, 142.

LEAD.

To Compute Weight of Lead.

Rule.—Ascertain number of cube inches in piece; multiply sum by .410 15,* and product will give weight in pounds.

Example.—What is weight of a leaden pipe 12 feet long, 3.75 inches in diameter, and 1 inch thick?

By Rule in Mensuration of Surfaces, to ascertain Area of Cylindrical Rings.

Area of $(3.75 + 1 + 1) = 25.967$
" " $3.75 \qquad = 11.044$

Difference, 14.923 (*area of ring*) $\times$ 144 (12 feet) $= 2148.912 \times .410\,15 = 881.376$ *lbs.*

BRASS.

To Compute Weight of Ordinary Brass Castings.

Rule.—Ascertain number of cube inches in piece; multiply sum by .2922,* and product will give weight in pounds.

* Weights of a cube inch as here given are for the ordinary metals; when, however, the specific gravity of the metal under consideration is accurately known, the weight of a cube inch of it should be substituted for the units here given.

Dimensions and Weights of Wrought Iron Bolts and Nuts.

SQUARE AND HEXAGONAL HEADS AND NUTS.

Rough, *and from* .25 *Inch to* 4 *Inches in Diameter.*

Square Head and Nut.

Diameter of Bolt.	Width. Head.	Width. Nut.	Diagonal. Head.	Diagonal. Nut.	Depth. Head.	Depth. Nut.	Weight. Head and Nut.	Weight. Bolt per Inch.	Threads per Inch.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Lbs.	Lbs.	No.
.25	.36	.49	.51	.69	.25	.25	.024	.014	20
.3125	.45	.58	.64	.82	.3	.3125	.043	.022	18
.375	.54	.67	.76	.95	.34	.375	.068	.031	16
.4375	.63	.76	.89	1.07	.4	.4375	.104	.042	14
.5	.72	.84	1.02	1.19	.44	.5	.145	.055	13
.5625	.82	.94	1.16	1.33	.48	.5625	.204	.07	12
.625	.91	1.03	1.29	1.46	.53	.625	.273	.086	11
.6875	1	1.12	1.41	1.58	.58	.6875	.356	.104	11
.75	1.09	1.21	1.54	1.71	.63	.75	.454	.124	10
.8125	1.18	1.3	1.67	1.84	.67	.8125	.565	.145	10
.875	1.27	1.39	1.8	1.96	.72	.875	.696	.168	9
1	1.45	1.57	2.05	2.22	.81	1	1.013	.22	8
1.125	1.63	1.75	2.3	2.47	.9	1.125	1.416	.278	7
1.25	1.81	1.94	2.56	2.74	1	1.25	1.923	.344	7
1.375	1.99	2.12	2.81	3	1.1	1.375	2.543	.416	6
1.5	2.17	2.3	3.07	3.25	1.18	1.5	3.234	.495	6
1.625	2.36	2.48	3.34	3.51	1.28	1.625	4.105	.581	5.5
1.75	2.54	2.66	3.59	3.76	1.37	1.75	5.087	.674	5
1.875	2.72	2.84	3.85	4.02	1.46	1.875	6.182	.773	5
2	2.9	3.02	4.1	4.27	1.56	2	7.491	.88	4.5
2.125	3.08	3.21	4.35	4.54	1.65	2.125	8.936	.993	4.5
2.25	3.26	3.39	4.61	4.79	1.75	2.25	10.543	1.113	4.5
2.375	3.44	3.57	4.86	5.05	1.84	2.375	12.335	1.24	4.375
2.5	3.62	3.75	5.12	5.3	1.94	2.5	14.359	1.375	4.25
2.625	3.81	3.93	5.49	5.56	2.03	2.625	16.549	1.515	4
2.75	3.99	4.11	5.64	5.81	2.12	2.75	18.897	1.663	4
2.875	4.17	4.29	5.9	6.07	2.22	2.875	21.545	1.818	3.75
3	4.35	4.47	6.15	6.32	2.31	3	24.464	1.979	3.5
3.25	4.71	4.84	6.66	6.84	2.5	3.25	30.922	2.323	3.5
3.5	5.07	5.2	7.17	7.35	2.68	3.5	38.391	2.694	3.25
3.75	5.44	5.56	7.69	7.86	2.87	3.75	47.168	3.093	3
4	5.8	5.92	8.2	8.37	3.06	4	56.882	3.518	3

FINISHED.—Deduct .0625 from diameters of bolts and depths of all heads and nuts.

Screws with square threads have but one half number of threads of those with triangular threads.

NOTE.—The loss of tensile strength of a bolt by cutting of thread is, for one of 1.25 ins. diameter, 8 per cent. The safe stress or capacity of a wrought iron bolt and nut may be taken at 5000 lbs. per square inch.

Preceding width, depth, etc., are for work to exact dimensions, whether forged or finished.

To Compute Weight of a Bolt and Nut.

Operation.—Ascertain from table weight of head and nut for given diameter of bolt, and add thereto weight of bolt per inch of its length, multiplied by full length of its body from inside of its head to end.

NOTE.—Length of a bolt and nut for *measurement*, as such, is taken from inside of head to inside of nut, or its greatest capacity when in position.

ILLUSTRATION.—A wrought iron bolt and nut with a square head and nut is 1 inch in diameter and 10 inches in length; what is its weight?

Weight of head and nut ....1.013 *lbs.*
" bolt per inch of length .22 × 10 = 2.2 "
3.213 "

Hexagonal Head and Nut.

Diameter of Bolt.	Width. Head.	Width. Nut.	Diagonal. Head.	Diagonal. Nut.	Depth. Head.	Depth. Nut.	Weight. Head and Nut.	Weight. Bolt per Inch.	Threads per Inch.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Lbs.	Lbs.	No.
.25	.375	.5	.43	.58	.25	.25	.022	.014	20
.3125	.4375	.5625	.5	.65	.3	.3125	.037	.022	18
.375	.5625	.6875	.65	.79	.34	.375	.062	.031	16
.4375	.625	.75	.72	.87	.4	.4375	.094	.042	14
.5	.75	.875	.87	1	.44	.5	.134	.055	13
.5625	.8125	.9375	.94	1.08	.48	.5625	.18	.07	12
.625	.9375	1.0625	1.08	1.23	.53	.625	.249	.086	11
.6875	1	1.125	1.16	1.3	.58	.6875	.318	.104	11
.75	1.125	1.25	1.3	1.44	.63	.75	.413	.124	10
.8125	1.25	1.375	1.44	1.59	.67	.8125	.522	.145	10
.875	1.3125	1.4375	1.52	1.66	.72	.875	.639	.168	9
1	1.5	1.625	1.73	1.88	.81	1	.931	.22	8
1.125	1.6875	1.8125	1.95	2.09	.9	1.125	1.299	.278	7
1.25	1.875	2	2.17	2.31	1	1.25	1.759	.344	7
1.375	2	2.1875	2.31	2.53	1.1	1.375	2.263	.416	6
1.5	2.25	2.375	2.6	2.74	1.18	1.5	2.958	.495	6
1.625	2.4375	2.5625	2.81	2.96	1.28	1.625	3.741	.581	5.5
1.75	2.625	2.75	3.03	3.18	1.37	1.75	4.654	.674	5
1.875	2.8125	2.9375	3.25	3.39	1.46	1.875	5.675	.773	5
2	3	3.125	3.46	3.61	1.56	2	6.854	.88	4.5
2.125	3.1875	3.3125	3.68	3.83	1.65	2.125	8.163	.993	4.5
2.25	3.375	3.5	3.9	4.04	1.75	2.25	9.658	1.113	4.5
2.375	3.5625	3.6875	4.11	4.26	1.84	2.375	11.263	1.24	4.375
2.5	3.75	3.875	4.33	4.47	1.94	2.5	13.149	1.375	4.25
2.625	3.9375	4.0625	4.55	4.69	2.03	2.625	15.15	1.515	4
2.75	4.125	4.25	4.77	4.91	2.12	2.75	17.285	1.663	4
2.875	4.3125	4.4375	4.99	5.12	2.22	2.875	19.751	1.818	3.75
3	4.5	4.625	5.2	5.34	2.31	3	22.378	1.979	3.5
3.25	4.875	5	5.63	5.77	2.5	3.25	28.258	2.323	3.5
3.5	5.25	5.375	6.06	6.21	2.68	3.5	35.081	2.694	3.25
3.75	5.625	5.75	6.5	6.64	2.87	3.75	43.178	3.093	3
4	6	6.125	6.93	7.07	3.06	4	51.942	3.518	3

FINISHED.—Deduct .0625 from diameters of bolts and depths of all heads and nuts.

For Wood or Carpentry.

Head and Nut (*Square*), 1.73 diameter of bolt. *Depth of Head*, .75, and of *Nut*, .9.

Washer.—Thickness, .35 to .4 of diameter of bolt, on *Pine* 3.5 diameter, and *Oak* 2.5.

English.

Molesworth gives following elements of Thread of Bolts:

Angle of thread, 55°. *Depth of thread* = Pitch of screw.

Number of threads per Inch.—*Square*, half number of those in angular threads.

Depth of thread.—.64 pitch for angular and .475 for square threads.

French Standard Bolts and Nuts. (*Armengaud's.*)

HEXAGONAL HEADS AND NUTS.

Equilateral Triangular Thread. Diameter of Bolt.		Diameter at Base of Thread.	Threads per Inch.	Thickness. Head.	Thickness. Nut.	Breadth across Flats.	Safe Tensile Stress.	*Square Thread.* Diameter of Bolt.		Depth of Thread.	Threads per Inch.	Thickness of Nut.	Safe Tensile Stress.
Mm.	Ins.	Ins.	No.	Ins.	Ins.	Ins.	Lbs.	Mm.	Ins.	Ins.	No.	Ins.	Lbs.
5	.2	.13	18.1	.24	.2	.55	44	20	.79	.072	6.57	1.82	717
7.5	.3	.22	16	.3	.3	.68	99	25	.98	.081	5.97	2.01	1 142
10	.39	.31	14.1	.38	.39	.88	178	30	1.18	.093	5.4	2.22	1 635
12.5	.49	.39	12.7	.44	.49	1.04	277	35	1.38	.1	4.93	2.41	2 218
15	.59	.48	11.5	.52	.59	1.2	400	40	1.57	.106	4.53	2.63	2 912
17.5	.69	.58	10.6	.58	.69	1.4	545	45	1.77	.114	4.2	2.85	3 674
20	.79	.66	9.8	.66	.79	1.5	713	50	1.97	.128	3.91	3.07	4 547
22.5	.89	.76	9.1	.72	.89	1.68	902	55	2.17	.13	3.65	3.3	5 288
25	.98	.84	8.5	.8	.98	1.84	1 120	60	2.36	.14	3.43	3.5	6 540
30	1.18	1.02	7.5	.94	1.18	2.16	1 635	65	2.56	.15	3.23	3.7	7 660
35	1.38	1.2	6.7	1.08	1.38	2.48	2 218	70	2.76	.158	3.06	3.92	8 893
40	1.58	1.4	6	1.22	1.58	2.8	2 912	75	2.95	.166	2.92	4.13	10 214
45	1.77	1.56	5.5	1.36	1.77	3.2	3 674	80	3.15	.174	2.76	4.36	11 603
50	1.97	1.74	5.1	1.5	1.97	3.44	4 547	85	3.35	.183	2.63	4.58	13 100
55	2.17	1.92	4.7	1.64	2.17	3.76	5 288	90	3.54	.192	2.51	4.78	14 794
60	2.36	2.08	4.4	1.74	2.36	4.08	6 540	95	3.74	.2	2.41	5	16 352
65	2.56	2.26	4.1	1.92	2.56	4.4	7 660	100	3.94	.209	2.31	5.22	18 144
70	2.76	2.44	3.8	2.06	2.76	4.7	8 893	105	4.13	.22	2.22	5.43	20 000
75	2.95	2.6	3.5	2.2	2.95	5	10 214	110	4.33	.226	2.13	5.66	21 950
80	3.15	2.78	3.4	2.34	3.15	5.35	11 468	115	4.53	.23	2.06	5.87	23 990

English Bolts and Nuts. (*Whitworth's.*)

Hexagonal Heads and Nuts, and Triangular Threads.

Diameter. Bolt.	Diameter. Base of Thread.	Threads per Inch.	Depth. Head.	Depth. Nut.	Width of Head and Nut.	Diameter. Bolt.	Diameter. Base of Thread.	Threads per Inch.	Depth. Head.	Depth. Nut.	Width of Head and Nut.
Ins.	Inch.	No.	Inch.	Ins.	Ins.	Ins.	Ins.	No.	Ins.	Ins.	Ins.
.125	.093	40	.109	.125	.338	1.25	1.067	7	1.094	1.25	2.048
.1875	.134	24	.164	.1875	.448	1.375	1.161	6	1.203	1.375	2.215
.2187	—	24	—	—	—	1.5	1.286	6	1.312	1.5	2.413
.25	.186	20	.219	.25	.525	1.625	1.369	5	1.422	1.625	2.576
.3125	.241	18	.273	.3125	.601	1.75	1.494	5	1.531	1.75	2.758
.375	.295	16	.328	.375	.709	1.875	1.59	4.5	1.641	1.875	3.018
.4375	.346	14	.383	.4375	.82	2	1.715	4.5	1.75	2	3.149
.5	.393	12	.437	.5	.919	2.125	1.84	4.5	1.859	2.125	3.337
.5625	.456	12	.492	.5625	1.011	2.25	1.93	4	1.969	2.25	3.546
.625	.508	11	.547	.625	1.101	2.375	2.055	4	2.078	2.375	3.75
.6875	.571	11	.601	.6875	1.201	2.5	2.18	4	2.187	2.5	3.894
.75	.622	10	.656	.75	1.301	2.625	2.305	4	2.297	2.625	4.049
.8125	.684	10	.711	.8125	1.39	2.75	2.384	3.5	2.406	2.75	4.181
.875	.733	9	.766	.875	1.479	2.875	2.509	3.5	2.516	2.875	4.346
.9375	.795	9	.82	.9375	1.574	3	2.634	3.5	2.625	3	4.531
1	.84	8	.875	1	1.67	3.25	2.84	3.25	—	—	—
1.125	.942	7	.984	1.125	1.86	3.5	3.06	3.25	—	—	—

Square Heads and Nuts. (*Whitworth's.*)

Diameter. Bolt.	Diameter. Base of Thread.	Threads per Inch.	Diameter. Bolt.	Diameter. Base of Thread.	Threads per Inch.	Diameter. Bolt.	Diameter. Base of Thread.	Threads per Inch.
Ins.	Ins.	No.	Ins.	Ins.	No.	Ins.	Ins.	No.
3.75	3.25	3	4.5	3.875	2.875	5.25	4.4375	2.625
4	3.5	3	4.75	4.0625	2.75	5.5	4.625	2.625
4.25	3.75	2.875	5	4.25	2.75	6	4.875	2.5

Weight of Heads and Nuts in Lbs. (*Molesworth.*)

Hexagonal, 1.07 D^3. *Square*, 1.35 ^{3} D^3. *D representing diameter of bolt in inches.*

Retentiveness of Wrought Iron Spikes and Nails.

Deduced from Experiments of Johnson and Bevan.

SPIKES.

Spike.	Wood.	Breadth.	Depth.	Depth of Insertion.	Force required to draw it.	Ratio of force to weight.	Remarks.
		Ins.	Ins.	Ins.	Lbs.		
Square......	Hemlock†	.39	.3	3.5	1297	1.58	Seasoned in part.
" *....	Chestnut	.37	.38	3.5	1873	2.16	Unseasoned.
" *....	Yellow pine	.375	.375	3.375	2052	2.37	Seasoned.
" *....	White oak	.375	.375	3.375	3910	4.52	"
"	Locust	.4	.4	3.5	5967	6.33	"
Flat narrow..	Chestnut	.39	.25	3.5	2223	3.93	Unseasoned.
" "	White oak	.39	.25	3.5	3990	7.05	Seasoned.
" "	Locust	.39	.25	3.5	5673	9.32	"
" broad..	Chestnut	.539	.288	3.5	2394	2.66	Unseasoned.
" "	White oak	.539	.288	3.5	5330	5.71	Seasoned.
" "	Locust	.539	.288	3.5	7040	7.84	"
Square } Draw filed.	Hemlock†	.4	.39	3.5	1638	1.75	Seasoned in part.
" } Draw filed.	Chestnut†	.4	.39	3.5	1790	1.81	Unseasoned.
" } Draw filed.	Locust†	.4	.39	3.5	3990	4.17	Seasoned in part.
Round and grooved..	Ash	Diam.	.5	3.5	2052	2.21	Seasoned.
"	"	"	.5	3.5	2451	2.41	"
"	White oak	"	.48	3.5	3876	3.2	"

* Burden's patent. † Soaked in water after the spikes were driven.

NAILS.

Nail.	Length.	Depth of Insertion.	Force required to draw it. Pine.	Hemlock.	Elm.	Oak.	Beech.	Pressure required to force them into Pine.
	Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Sixpenny	2	1	187	312	327	507	667	235
"	2	1.5	327	539	571	675	889	400
"	2	2	530	857	899	1394	1834	610

General Remarks.

With a given breadth of face, a decrease of depth will increase retention. In soft woods, a blunt-pointed spike forces the fibres downwards and backwards so as to leave the fibres longitudinally in contact with the faces of the spike.

To obtain greatest effect, fibres of the wood should press faces of the spike in direction of their length; thus, a round blunt bolt, driven into a hole of a less diameter, has a retention equal to that of any other form, when wholly driven, as without boring.

The retention of a spike, whether square or flat, in unseasoned chestnut, from two to four inches in length of insertion, is about 800 lbs. per square inch of the two surfaces which laterally compress the faces of the spike.

When wood was soaked in water, after spikes were driven, order of their retentive power was Locust, White oak, Chestnut, Hemlock, and Yellow Pine.

Gas Pipe Threads.

Diameter in Inches..	.125	.25	.375	.5	.75	1	1.25	1.5	1.75	2
Threads per Inch...	28	19	19	14	14	11	11	11	11	11

ANGLES AND DISTANCES.

Angles and Distances corresponding to Opening of a Rule of Two Feet.

Angle.	Distance.	Angle.	Distance.	Angle.	Distance.	Angle.	Distance.	Angle.	Distance.
°	Ins.	°	Ins.	°	Ins.	°	Ins.	°	Ins.
1	.2	19	3.96	37	7.61	55	11.08	73	14.28
2	.42	20	4.17	38	7.81	56	11.27	74	14.44
3	.63	21	4.37	39	8.01	57	11.45	75	14.61
4	.84	22	4.58	40	8.2	58	11.64	76	14.78
5	1.05	23	4.78	41	8.4	59	11.82	77	14.94
6	1.26	24	4.99	42	8.6	60	12	78	15.11
7	1.47	25	5.19	43	8.8	61	12.18	79	15.27
8	1.67	26	5.4	44	8.99	62	12.36	80	15.43
9	1.88	27	5.6	45	9.18	63	12.54	81	15.59
10	2.09	28	5.81	46	9.38	64	12.72	82	15.75
11	2.3	29	6.01	47	9.57	65	12.9	83	15.9
12	2.51	30	6.21	48	9.76	66	13.07	84	16.06
13	2.72	31	6.41	49	9.95	67	13.25	85	16.21
14	2.92	32	6.62	50	10.14	68	13.42	86	16.37
15	3.13	33	6.82	51	10.33	69	13.59	87	16.52
16	3.34	34	7.02	52	10.52	70	13.77	88	16.67
17	3.55	35	7.22	53	10.71	71	13.94	89	16.82
18	3.75	36	7.42	54	10.9	72	14.11	90	16.97

Distances and Angles corresponding to Opening of a Rule of Two Feet.

Distance.	Angle.	Distance.	Angle.	Distance.	Angle.	Distance.	Angle.	Distance.	Angle.
Ins.	°	Ins.	°	Ins.	°	Ins.	°	Ins.	°
.25	1.12	3	14.22	6.5	31.26	10	49.14	13.5	68.28
.375	1.48	3.25	15.34	6.75	32.4	10.25	50.34	13.75	69.54
.5	2.24	3.5	16.46	7	33.54	10.5	51.54	14	71.22
.625	2.59	3.75	17.58	7.25	35.09	10.75	53.14	14.25	72.5
.75	3.35	4	19.11	7.5	36.24	11	54.34	14.5	74.2
.875	4.12	4.25	20.24	7.75	37.4	11.25	55.54	14.75	75.5
1	4.48	4.5	21.37	8	38.56	11.5	57.16	15	77.22
1.25	5.58	4.75	22.5	8.25	40.12	11.75	58.38	15.25	78.54
1.5	7.1	5	24.4	8.5	41.28	12	60	15.5	80.28
1.75	8.22	5.25	25.16	8.75	42.46	12.25	61.23	15.75	82.2
2	9.34	5.5	26.3	9	44.2	12.5	62.46	16	83.36
2.25	10.46	5.75	27.44	9.25	45.2	12.75	64.1	16.25	85.14
2.5	11.58	6	28.58	9.5	46.38	13	65.36	16.5	86.52
2.75	13.1	6.25	30.12	9.75	47.56	13.25	67.02	16.75	88 32

WIRE ROPE.

Wire rope of same strength as new *Hemp rope* will run on sheaves of like diameter; but greater diameter of sheaves, less the wear. Short bends should be avoided, and wear increases with the speed. Adhesion is same as that of hemp rope. It should not be coiled, but should be wound as upon a reel.

When substituting wire rope for hemp, it is well to allow for former same weight per foot which experience has approved of for latter. As a general rule, one wire rope will outlast three of hemp. To guard against rust, stationary rope should be coated once a year with linseed-oil, or well painted or tarred. Running rope in use does not require any protection.

Where great pliability is required, centre or core of rope should be of hemp.

Annealing wire, in rendering it more pliable than when unannealed, reduces its elasticity and consequent strength from 25 to 50 per cent.

Running rope is made of finer wire than standing rope.

For safe working load, deduct one fifth to one seventh of ultimate strength, according to speed and vibration. It is better to increase load than speed, as it increases wear.

Standing rigging of a vessel of wire rope is one fourth less in weight than when of hemp.

Rope of 19 wires to a strand is more pliable than one of 7 and 12 wires, and hence it is better suited to operation over small drums, for hoisting, etc.

Ultimate strength of iron ropes is 4480 lbs. for each pound in weight per fathom, and for galvanized steel 6720 lbs.

Strength per square inch of section of a rope is about 53 per cent. of an equal section of solid metal of same tensile strength per square inch.

Steel ropes may be one third less in weight than iron for same load. Their durability is much greater, especially when required to run rapidly over sheaves. Hemp should be one third heavier than iron.

Steel wire No. 14 W. G. = .083 inch, weight 2 lbs. per yard, will bear a stress of 2000 lbs.

The combined sectional area of the wires in a cable is to the area of the cable as 1 to 1.3. Hence, to ascertain areas of the wires in a cable multiply diameter by .77, and for areas of the voids, multiply area of cable by .23.

In short transmissions, it is necessary to connect rope quite taut, and an additional diameter of two numbers of rope must be given to it.

In long transmissions, when there is an insufficiency of height to admit of a proper deflection of rope, and it becomes necessary to connect it very taut, an additional diameter of one number of rope must be given to it.

When distance exceeds 350 feet, transmission should be divided into two or more equal lengths by aid of intermediate wheels.

Rope Nos. 7 and 8 (*Roebling's*) are made with Nos. 1 and 2 as strands, and twisting six of them around a hemp centre.

Results of an Experiment with Galvanized Wire.

A strand of 2-inch wire rope broke with a strain of 13 564 lbs., and a piece of a like rope, when galvanized, withstood a strain of 14 796 lbs. before breaking.

Elements of Running and Standing Wire Rope.

J. A. Roebling's Sons Co.

19 *Wires in a Strand.*

		Iron.				Cast Steel.			
Diam.	Circum.	Breaking Weight.	Safe Load.	Circum. of Hemp Rope.	Weight per Foot.	Breaking Weight.	Safe Load.	Circum. of Hemp Rope.	Weight per Foot.
Ins.	Ins.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Lbs.
.5	1.5	6 960	1 000	3.5	.35	11 000	2 000	4.5	.35
.5625	1.625	8 480	1 500	4	.44	13 000	3 000	4.75	.44
.625	2	10 260	2 500	4.5	.7	18 000	4 000	5.25	.65
.75	2.25	17 280	3 500	5	.88	26 000	6 000	6.5	.83
.875	2.75	23 000	5 000	6	1.2	40 000	8 000	8	1.14
1	3.125	32 000	6 000	7	1.58	48 000	10 000	9.25	1.5
1.125	3.5	40 000	8 000	8	2	60 000	12 000	10	1.95
1.25	4	54 000	11 000	9.5	2.5	78 000	16 000	11.5	2.44
1.5	4.375	70 000	14 000	10.75	3.65	110 000	22 000	13	3.1
1.625	5	88 000	18 000	12	4.1	128 000	26 000	14.5	4.1
1.75	5.5	108 000	22 000	13	5.25	156 000	34 000	15.75	5.08
2	6	130 000	26 000	14.5	6.3	200 000	40 000	19	6.02
2.25	6.75	148 000	30 000	15.5	8	260 000	44 000	22.5	7.8

Transmission and Standing Rope.

7 *Wires in a Strand.*

		Iron.				Cast Steel.			
Diam.	Circum.	Breaking Weight.	Safe Load.	Circum. of Hemp Rope.	Weight per Foot.	Breaking Weight.	Safe Load.	Circum. of Hemp Rope.	Weight per Foot.
Ins.	Ins.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Lbs.
.28	.875	2 000	500	2	.125	4 200	1 050	2.375	.125
.3125	1	2 760	690	2.25	.16	5 000	1 250	3	.16
.375	1.125	3 300	825	2.5	.19	8 000	2 000	3.75	.19
.4375	1.25	4 260	1 065	2.75	.23	10 000	2 400	4.125	.23
.5	1.375	5 660	1 415	3.25	.31	11 000	2 500	4.75	.31
.5625	1.625	8 200	2 050	4	.41	12 000	3 500	5	.41
.625	1.875	11 600	2 900	4.75	.51	20 000	4 500	5.5	.51
.6875	2.125	15 200	3 800	5	.68	27 000	6 000	6.5	.68
.75	2.375	17 600	4 400	5.25	.86	34 000	7 000	7.25	.86
.875	2.625	24 600	6 150	6.25	1.12	44 000	10 000	8.5	1.12
1	3	32 000	8 000	7	1.5	60 000	13 000	10	1.5
1.125	3.375	40 000	10 000	8	1.82	72 000	16 000	10.75	1.82
1.25	3.75	50 000	12 500	9.25	2.28	90 000	20 000	12	2.28
1.375	4.25	60 000	15 000	10	2.77	110 000	25 000	13	2.77
1.5	4.625	72 000	18 000	10.75	3.37	134 000	32 000	15	3.37

Note.—When made with wire centre instead of hemp, weight is 10 per cent. more.

Galvanized Charcoal Iron Wire.

Vessels' Rigging and Derrick Guys.

12 *Wires in a Strand.*

Circum.	Circum. of Hemp Rope.	Breaking Weight.	Safe Load.	Weight per Foot.	Circum.	Circum. of Hemp Rope.	Breaking Weight.	Safe Load.	Weight per Foot.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.	Lbs.
3	6	24 000	6 000	1.33	4.5	9	60 000	15 000	3
3.25	6.5	28 000	7 000	1.58	4.75	9.5	66 000	16 500	3.46
3.5	7	32 000	8 000	1.83	5	10	70 000	17 500	3.66
3.75	7.5	40 000	10 000	2	5.25	10.5	80 000	20 000	4.12
4	8	46 000	11 500	2.46	5.5	11	86 000	21 500	4.46
4.25	8.5	52 000	13 000	2.66	6	12	100 000	25 000	4.83

Galvanized Charcoal Iron.

Vessels' Rigging and Derrick Guys.

(*J. A. Roebling's Sons Co.*)

7 Wires in a Strand.

Circum.	Circum. of Hemp Rope.	Breaking Weight.	Safe Load.	Weight per Foot.	Circum.	Circum. of Hemp Rope.	Breaking Weight.	Safe Load.	Weight per Foot.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.	Lbs.
1	2	4 000	1 000	.125	3.5	7	32 000	8 000	1.79
1.25	2.5	5 000	1 250	.25	3.75	7.5	40 000	10 000	2
1.5	3	7 000	1 750	.334	4	8	46 000	11 500	2.46
1.75	3.5	10 000	2 500	.427	4.25	8.5	52 000	13 000	2.67
2	4	14 000	3 500	.583	4.5	9	60 000	15 000	3
2.25	4.5	16 000	4 000	.708	4.75	9.5	66 000	16 500	3.42
2.5	5	18 000	4 500	.875	5	10	70 000	17 500	3.66
2.75	5.5	20 000	5 000	1.17	5.25	10.5	80 000	20 000	4.08
3	6	24 000	6 000	1.33	5.5	11	86 000	21 500	4.41
3.25	6.5	28 000	7 000	1.58	6	12	100 000	25 000	4.81

Gauge, Weight, and Length of Iron Wire.

Gauge.	Diam.	Weight per 100 Feet.	Weight of one Mile.	63 lbs. Bundle.	Area.	Gauge.	Diam.	Weight per 100 Feet.	Weight of one Mile.	63 lbs. Bundle.	Area.
No.	Inch.	Lbs.	Lbs.	Feet.	Sq. Inch.	No.	Inch.	Lbs.	Lbs.	Feet.	Sq. Inch.
6/0	.46	56.1	2962	112	.166 19	16	.063	1.05	55	6 000	.003 117
5/0	.43	49.01	2588	129	.145 22	17	.054	.77	41	8 182	.002 29
4/0	.393	40.94	2162	154	.121 304	18	.047	.58	31	10 862	.001 734
3/0	.362	34.73	1834	181	.102 921	19	.041	.45	24	14 000	.001 32
2/0	.331	29.04	1533	217	.086 049	20	.035	.32	17	19 687	.000 962
1/0	.307	27.66	1460	228	.074 023	21	.032	.27	14	23 333	.000 804
1	.283	21.23	1121	296	.062 901	22	.028	.21	11	30 000	.000 615
2	.263	18.34	968	343	.054 325	23	.025	.175	9.24	36 000	.000 491
3	.244	15.78	833	399	.046 759	24	.023	.14	7.39	45 000	.000 415
4	.225	13.39	707	470	.039 76	25	.02	.116	6.124	54 310	.000 314
5	.207	11.35	599	555	.033 653	26	.018	.093	4.91	67 742	.000 254
6	.192	9.73	514	647	.028 952	27	.017	.083	4.382	75 903	.000 227
7	.177	8.03	439	759	.024 605	28	.016	.074	3.907	85 135	.000 201
8	.162	6.96	367	905	.020 612	29	.015	.061	3.22	103 278	.000 176
9	.148	5.08	306	1086	.017 203	30	.014	.054	2.851	116 666	.000 154
10	.135	4.83	255	1304	.014 313	31	.0135	.05	2.64	126 000	.000 133
11	.12	3.82	202	1649	.011 309	32	.013	.046	2.428	136 956	.000 132
12	.105	2.92	154	2158	.008 659	33	.011	.037	1.953	170 270	.000 095
13	.092	2.24	118	2813	.006 647	34	.01	.03	1.584	210 000	.000 078
14	.08	1.69	89	3728	.005 026	35	.0095	.025	1.32	252 000	.000 071
15	.072	1.37	72	4598	.004 071	36	.009	.021	1.161	286 363	.000 064

Galvanized Steel Cables for Suspension Bridges.

Diameter.	Ultimate Strength.	Weight per Foot.	Diameter.	Ultimate Strength.	Weight per Foot.	Diameter.	Ultimate Strength.	Weight per Foot.
Ins.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.
1.5	130 000	3.7	1.875	200 000	5.8	2.375	360 000	10
1.625	150 000	4.35	2	220 000	6.5	2.5	400 000	11.3
1.75	190 000	5.6	2.25	310 000	8.64	2.625	440 000	13

Weight and Strength of Single Strand and Cable laid Fence Wire. (*F. Morton & Co.*)

Strands.	No.	Single Wire of equal Diameter.		Length per 1000 lbs.		Strands.	No.	Single Wire of equal Diameter.		Length per 1000 lbs.	
				Of a Strand.	Of Rope.					Of a Strand.	Of Rope.
No.		No.	Inch.	Feet.	Feet.	No.		No.	Inch.	Feet.	Feet.
3	2A	8	.159	20 090	15 270	7	00	4	.229	8300	7366
4	2	7	.174	14 730	12 790	7	3/0	3	.25	8036	6228
7	1	6	.191	13 125	10 580	7	4/0	2	.274	7500	5156
7	0	5	.209	10 446	8 928	7	5/0	1	.3	5090	4286

No. and diameter of wire is that of Ryland's Bros., pp. 122–4.

Hemp, Iron, and Steel. (*R. S. Newall & Co.*)

ROUND.

HEMP.		IRON.		STEEL.		Tensile Strength.	
Circumference.	Weight per Foot.	Circumference.	Weight per Foot.	Circumference.	Weight per Foot.	Safe Load.	Ultimate Strength.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Lbs.	Lbs.
2.75	.33	1	.16	—	—	672	4 480
		1.5	.25	1	.16	1 008	6 720
3.75	.66	1.625	.33	—	—	1 344	8 960
		1.75	.42	1.5	.25	1 680	11 200
4.5	.83	1.875	.5	—	—	2 016	13 440
		2	.58	1.625	.33	2 352	15 680
5.5	1.16	2.125	.66	1.75	.42	2 688	17 920
		2.25	.75	—	—	3 024	20 160
6	1.5	2.375	.83	1.875	.5	3 360	22 400
		2.5	.92	—	—	3 696	24 640
6.5	1.66	2.625	1	2	.58	4 032	26 680
		2.75	1.08	2.125	.66	4 368	29 120
7	2	2.875	1.16	2.25	.75	4 704	31 360
		3	1.25	—	—	5 040	33 600
7.5	2.33	3.125	1.33	2.375	.83	5 376	36 840
		3.25	1.41	—	—	5 672	38 080
8	2.66	3.375	1.5	2.5	.92	6 048	40 320
		3.5	1.66	2.625	1	6 720	44 800
8.5	3	3.625	1.83	2.75	1.08	7 392	49 280
		3.75	2	—	—	8 064	53 760
9.5	3.66	3.875	2.16	3.25	1.33	8 736	58 240
10	4.33	4	2.33	—	—	9 408	62 720
		4.25	2.5	3.375	1.5	10 080	67 200
11	5	4.375	2.66	—	—	10 752	71 680
		4.5	3	3.5	1.66	12 096	80 640
12	5.66	4.625	3.33	3.75	2	13 440	89 600
			FLAT.				
Dimensions.		Dimensions.		Dimensions.			
4 × .5	3.33	2.25 × .5	1.85	—	—	4 928	44 800
5 × 1.25	4	2.5 × .5	2.16	—	—	5 824	51 520
5.5 × 1.375	4.33	2.75 × .625	2.5	—	—	6 720	60 480
5.75 × 1.5	4.66	3 × .625	2.66	2 × .5	1.66	7 168	62 720
6 × 1.5	5	3.25 × .625	3	2.25 × .5	1.83	8 064	71 680
7 × 1.875	6	3.5 × .625	3.33	2.25 × .5	2	8 960	80 640
8.25 × 2.125	6.66	3.75 × .6875	3.66	2.5 × .5	2.16	9 850	89 600
8.5 × 2.25	7.5	4 × .6875	4.16	2.75 × .375	2.5	11 200	100 800
9 × 2.5	8.33	4.25 × .75	4.66	3 × .375	2.66	12 544	112 000
9.5 × 2.375	9.16	4.5 × .75	5.33	3.25 × .375	3	14 336	125 440
10 × 2.5	10	4.625 × .75	5.66	3.5 × .375	3.23	15 232	134 400

From preceding tables following results are determined:

	Ultimate Strength per Lb. Weight per Foot.	Safe Load per Lb. Weight per Foot.	Safe Load per Square of Circumference in Inches.
	Lbs.	Lbs.	Lbs.
Hemp...........	15 000	4550	100
Iron............	22 000	4500	600
Steel...........	{30 000 45 500	{6000 8000	{1000 1300

ROUND AND FLAT MINING ROPES.

(MM. Harmegnies, Dumont & Co., Anzin, France.)

For a Depth of 400 Metres or 440 Yards.

Round.				Flat.				
No.	Diameter.	Weight per Foot.	Safe Load.	No. of Strands.	Width.	Thickness.	Weight per Foot.	Safe Load.
	Ins.	Lbs.	Lbs.		Ins.	Ins.	Lbs.	Lbs.
17	.51	2.16	560	9	2.4	.55	2	3 360
16	.59	1.66	1120	6	2.8	.59	2.13	4 032
15	.63	1.26	1680	[illegible]	3.2	.63	2.66	4 480
14	.71	1	2240	[illegible]	3.2	.67	3	5 600
13	.83	.83	3360	[illegible]	3.5	.79	3.33	6 720
12	.98	.66	4480	[illegible]	4.3	.67	3.66	7 840
11	1.1	.5	5600	[illegible]	3.9	.83	4	8 960
10	1.3	.33	6720	8	4.7	.79	4.33	10 080
				8	5.1	.87	5.33	11 200

Ropes and Chains of Equal Strength.

Diameter of Iron Chain.	CIRCUMFERENCE. Hemp Rope.	Crucible Steel Rope.	Charcoal Iron Rope.	WEIGHT PER FOOT. Steel Rope.	Iron Rope.	Hemp Rope.	Iron Chain.	Safe Load.
Ins.	Ins.	Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Tons.
.218 75	2.75	—	1	—	.14	.34	.5	.3
.25	3	—	1.18	—	.21	.46	.65	.4
.281 25	3.5	1	1.39	.17	.28	.67	.81	.5
.312 5	4.25	1.26	1.57	.25	.33	.75	.96	.6
.375	4.5	1.45	1.77	.3	.45	.83	1.38	.8
.437 5	5	1.57	1.97	.35	.57	1.16	1.76	1
.468 75	5.5	1.77	2.19	.45	.7	1.2	2.2	1.3
.5	5.75	1.96	2.36	.59	.83	1.6	2.63	1.5
.625	6.75	2.36	2.75	.85	1.08	2	4.21	2.3
.687 5	7.75	2.75	3.14	1.1	1.43	2.65	4.83	3.1
.75	8.75	2.95	3.53	1.28	1.8	3.35	5.75	3.8
.875	9.75	3.14	3.93	1.45	2.3	4.6	7.5	4.8
.937 5	10.5	3.53	4.32	1.83	2.94	4.92	9.33	5.9
1.062 5	11.75	3.93	4.71	2.33	3.56	5.83	10.6	7
1.125	12.75	4.32	5.1	2.98	4	6.2	11.9	8.2
1.25	14.75	4.71	5.5	3.58	4.8	8.7	14.5	9.5
1.375	15.25	4.81	5.89	3.65	5.6	9	17.6	11
1.5	15.75	5.1	6.28	4.04	6.3	10.1	20	12.5
1.625	17.75	5.8	7.07	5.65	7.95	13.7	22.3	15.9
1.75	19.5	6.35	7.85	6.5	9.81	16.4	24.3	19.6

By experiments of *U. S. Navy, hemp rope of this circumference has a breaking weight of* 71 309 *lbs., and a wire rope of* 5.34 *ins. has equivalent strength.*

Weight of Hemp and Wire Rope. (*Molesworth.*)

In Lbs. per Fathom.

Circumference.	Hemp. Common.	Hemp. Good.	Wire. Iron.	Wire. Steel.	Circumference.	Hemp. Common.	Hemp. Good.
Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Ins.	Lbs.	Lbs.
1	.18	.24	.87	.89	5	4.5	6
1.5	.41	.54	1.96	2	5.5	5.45	7.26
1.75	.55	.74	2.66	2.73	6	6.48	8.64
2	.72	.96	3.48	3.56	6.5	7.61	10.14
2.25	.91	1.22	4.4	4.51	7	8.82	11.76
2.5	1.13	1.5	5.44	5.56	7.5	10.13	13.5
2.75	1.36	1.82	6.58	6.73	8	11.52	15.36
3	1.62	2.16	7.83	8.01	8.5	13.05	17.34
3.25	1.9	2.54	9.19	9.4	9	14.58	19.44
3.5	2.21	2.94	10.66	10.9	10	18	24
3.75	2.53	3.38	12.23	12.52	12	26	34.56
4	2.88	3.84	13.92	14.24	15	40.52	54

To Compute Stress upon a Rope set at an Inclination.

Rule.—Multiply sine of angle of elevation by strain in lbs., add an allowance for rolling friction and weight of rope, and multiply by factor of safety.

Factor of safety.—For standing rope 4, for running 5, and for inclined planes from 5 to 7.

Illustration.—Inclination of rope 92.5 feet in 100, velocity 1500 feet per minute, and strain 2000 lbs.; what should be diam. of iron rope, 7 wires to a strand?

Angle of 92.5 feet in $100 = 43^\circ$, and sine of $43^\circ = .682$. $.682 \times 2000 = 1364$, to which is to be added rolling friction and weight of rope, assumed to be 11; hence, $1364 + 11 = 1375$.

Factor of safety assumed at 6, consequently $1375 \times 6 = 8250$ *lbs., capacity or breaking weight or stress of rope.*

By table, page 162, 8200 lbs. is breaking weight of a wire rope of 7 strands, .625 *inch in diam.*

To Compute Tension of a Rope.

$\frac{\text{HP}}{v} = t.$ *v representing velocity of rope in feet per minute,* HP *horses' power, and t tension in lbs.*

Illustration.—Assume wheel 7 feet in diameter, revolution 140 per minute, and HP as per preceding table, 29.6.

$$\text{Then } \frac{29.6 \times 33\,000}{7 \times 3.1416 \times 140} = \frac{976\,800}{3079} = 317.2 \textit{ lbs.}$$

To Compute Operative Deflection of a Rope.

$\frac{D^2 w}{10.7\, t} = d.$ D *representing distance between centres of wheels or drums in feet, w weight of rope in feet per lb., t tension, or power required to produce required power or tension of rope when at rest, and d deflection in feet.*

Illustration.—Take elements of preceding case: diam. of wire rope of 7 strands $= .5625$ inch, and by table, page 162, $w = .41$ lb., and $D = 300$ feet.

$$\text{Then } \frac{300^2 \times .41}{10.7 \times 317.2} = 10.87 \textit{ feet.}$$

Capacity.—At the Falls of the river Rhine there is a wire rope in operation that transmits the power of 600 horses for a distance exceeding one mile.

Endless Ropes.

Wire Ropes, when practicable and proper for application, can be used for transmission of power at a less cost than belting or shafting.

Transmission of Power.

Diameter of Wheel.	Revolutions per Minute.	Diameter of Rope.	Horse Power.	Diameter of Wheel.	Revolutions per Minute.	Diameter of Rope.	Horse Power.	Diameter of Wheel.	Revolutions per Minute.	Diameter of Rope.	Horse Power.
Feet.		Ins.		Feet.		Ins.		Feet.		Ins.	
4	80	.375	3.3	7	100	.5625	21.1	11	140	.6875	132.1
4	100	.375	4.1	7	140	.5625	29.6	12	80	.75	99.3
4	120	.375	5	8	80	.625	22	12	100	.75	124.1
4	140	.375	5.8	8	100	.625	27.5	12	140	.75	173.7
5	80	.4375	6.9	8	140	.625	38.5	13	80	.75	122.6
5	100	.4375	8.6	9	80	.625	41.5	13	100	.75	153.2
5	120	.4375	10.3	9	100	.625	51.9	13	120	.75	183.9
5	140	.4375	12.1	9	140	.625	72.6	14	80	.875	148
6	80	.5	10.7	10	80	.6875	58.4	14	100	.875	176
6	100	.5	13.4	10	100	.6875	73	14	120	.875	222
6	120	.5	16.1	10	140	.6875	102.2	15	80	.875	217
6	140	.5	18.7	11	80	.6875	75.5	15	100	.875	259
7	80	.5625	16.9	11	100	.6875	94.4	15	120	.875	300

Wire Rope and Equivalent Belt.

In substituting wire rope for an ordinary flat belt, the diameter is determined by rule in practice for estimating power transmitted by a belt—viz.,

One horse power for every 70 square feet of running belt surface per minute. Thus, a belt 15 inches wide running at rate of 1400 feet per minute, its power would be equal to $(1400 \times 15) \div (70 \times 12) = 25$ horses' power.

The same result is obtained by the use of a wire rope .5625 inch in diameter, running over a wheel 6 feet in diameter, making 130 revolutions per minute.

Average life of iron wire rope with good care is from 3 to 5 years, and that of steel rope is greater. Wear increases rapidly with velocity.

General Notes.—Hemp and Wire Ropes.

White Rope, 2 inches in circumference, of different manufactures, parted at a stress of from 4413 to 6160 lbs.

Specimens of Italian, Russian, and French manufacture parted with an average stress of 5128 lbs. = 1633 lbs. per square inch of rope.

Bearing capacity of a hemp rope is proportional to its thickness, number of its strands, slackness with which they are twisted, and quality of the hemp.

Hemp and Wire Ropes.—Ultimate Strength is 2240 lbs. per lb. per fathom for round hemp, 4480 lbs. for iron, and 6720 to 7840 lbs. for steel.

Working Load is 336 lbs. per lb. weight per fathom for round hemp, 672 lbs. for iron, and 1120 lbs. for steel.

Or, .83 times square of circumference in inches for round hemp, 5 times square of circumference for iron, and 9 times square of circumference for steel. (*D. K. Clark.*)

Steel Ropes may be one third less in weight than iron for like working load, and *Hemp Ropes* should be one third heavier than iron for like working load.

IRON WIRE AND UNITED STATES NAVY HEMP ROPE.

Wire 6 Strands, Hemp Core. Rope 4 Strands.

WIRE.					HEMP.			
Circumference. Actual.	Circumference. Nominal.	Core.	Wires.	Breaking Weight.	Circumference. Actual.	Circumference. Nominal.	Yarns.	Breaking Weight.
Ins.	Ins.	Ins.	No.	Lbs.	Ins.	Ins.	No.	Lbs.
7	7	2.35	108	187 400	12	13.25	1168	75 966
6	6	2.25	108	104 050	11	12.25	1036	77 633
4.937	4.9	1.57	114	65 409	10.5	11.875	928	76 933
4.375	4.5	1.57	114	55 316	10	11.375	876	70 533
3.5	3.36	1.27	114	34 480	9 5	10.5	800	58 766
3.187	2.98	1.17	114	28 606	9	10.312	712	56 466
2.75	2.68	.78	114	21 846	8.5	9.437	640	42 866
2.5	2.45	.78	114	15 692	8	8.812	560	38 500
2.375	2.4	.78	42	15 718	7.5	8.437	484	40 000
2	2.06	.39	114	10 925	7	7.812	436	32 166

Weight and Strength of Stud-link Chain Cable.

(English.)

Dimensions. Diam. of each Side.	Dimensions. Length of Link.	Dimensions. Width of Link.	Weight per Fathom.	Admiralty Proof-stress (adopted by Lloyds').	Dimensions. Diam. of each Side.	Dimensions. Length of Link.	Dimensions. Width of Link.	Weight per Fathom.	Admiralty Proof-stress (adopted by Lloyds').
Ins.	Ins.	Ins.	Lbs.	Tons.	Ins.	Ins.	Ins.	Lbs.	Tons.
.4375	2.625	1.575	11.3	3.5	1.5	9	5.4	121	40.5
.5	3	1.8	13.4	4.5	1.625	9.75	5.85	142	47.5
.5625	3.375	2.025	17.2	5.5	1.75	10.5	6.3	164.6	55.125
.625	3.75	2.25	21	7	1.875	11.25	6.75	189	63.25
.6875	4.125	2.475	25.4	8.5	2	12	7.2	215	72
.75	4.5	2.7	30.2	10.125	2.125	12.75	7.65	242.8	81.25
.875	5.25	3.15	41.2	13.75	2.25	13.5	8.1	276.2	91.125
1	6	3.6	53.8	18	2.375	14.25	8.55	303.2	101.5
1.125	6.75	4.05	69	22.75	2.5	15	9	336	112.5
1.25	7.5	4.5	84	28.125	2.75	16.5	9.9	406.6	136.125
1.375	8.25	4.95	101.6	34					

NOTE 1.—*Safe Working-stress* is taken at half Proof-stress, 3.82 tons per sq. inch of section.

2.—*Proof-stress* and *Safe Working-stress* for close-link chains are respectively two-thirds of those of stud-link chains.

3.—*Proof-stress* averages 72 per cent. ultimate strength, and *Ultimate Strength* averages 8 tons per square inch of section of rod or one side of a link.

Weight of close-link chain is about three times weight of bar from which it is made, for equal lengths.

Karl von Ott, comparing weight, cost, and strength of the three materials, hemp, iron wire, and chain iron, concludes that the proportion between cost of hemp rope, wire rope, and chain is as 2 : 1 : 3, and that, therefore, for equal resistances, wire rope is only half the cost of hemp rope, and a third of cost of chains.

Safe Working Load of Chains. *(Molesworth).*

Diameter of Iron.	Load.	Diameter of Iron.	Load.	Diameter of Iron.	Load.	Diameter of Iron.	Load.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
.375	2240	.6875	7 390	.9375	13 700	1.1875	22 400
.5	3800	.75	8 960	1	15 680	1.25	24 640
.5625	4900	.8125	10 280	1.0625	17 920	1.3125	26 680
.625	6270	.875	12 320	1.125	20 160	1.375	30 240

Breaking Strain and Proof of Chain Cables.

Diam. of Chain.	Breaking Strain.	Diam. of Chain.	Breaking Strain.	Diam. of Chain.	Breaking Strain.	Diam. of Chain.	Breaking Strain.
Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.
1	67 700	1.1875	92 940	1.5	143 100	2	243 180
1.0625	75 640	1.25	102 160	1.625	165 920	2.125	272 580
1.125	84 100	1.375	121 840	1.75	216 120	2.25	303 280

Proof-stress is 50 per cent. of estimated strength of weakest *link* and 46 per cent. of strongest.

Comparison of Wire Ropes and Tarred Hemp Rope, Hawsers, and Cables.

COARSE LAID.							FINE LAID.				
			Ropes.		Haws'rs.	Cables.			Ropes.	Haws'rs.	Cables.
Diameter.	Circum.	Safe Load.	Three Strands.	Four Strands.	Three Strands.	Three Strands.	Diameter.	Safe Load.	Four Strands.	Three Strands.	Three Strands.
Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.
.25	.78	425	1.25	—	—	—	.5	1 875	3.12	2.87	—
.3125	1	690	2.43	2.25	3.32	—	.5625	2 420	3.56	3.25	4.87
.375	1.25	825	2.68	2.375	3.5	—	.625	2 900	3.93	3.62	5.25
.5	1.375	1 600	2.87	2.62	3.87	—	.75	4 320	4.81	4.37	6.37
.5625	1.75	2 800	3.81	3.5	5.18	—	.875	5 700	5.5	5	7.25
.6875	2.125	3 800	4.75	4.25	6.12	—	1	8 200	7.25	6.25	8.75
.75	2.375	4 400	5.25	4.87	7	—	1.125	10 100	8.18	7	9.5
.875	2.625	6 150	6.12	5.75	8	8	1.25	13 600	8.81	8.06	11
1	3	8 400	6.62	6.12	8.62	8.62	1.5	17 500	10	9.75	12.5
1.25	3.75	13 400	8.81	8.5	10.93	10.93	1.625	21 800	11.18	10.93	—
1.375	4.25	16 800	9.87	9.56	12.25	12.12	1.75	27 000	12.5	12.12	—
1.5	4.625	20 160	10.75	10.5	13	13.12	1.875	32 500	—	—	—
1.625	5	24 600	—	11.87	11.56	11.75	2	37 000	—	—	—

In above table, determination of circumference of rope, etc., is based upon Breaking Weight or Tensile resistance of wire being reduced by one fourth, and ultimate resistances of rope, etc., are reduced one third.

Result of Experiments upon Wire Rope at U. S. Navy Yard, Washington. (*J. A. Roebling's Sons.*)

Circumference. Actual.	Circumference. Nominal.	Wire in each Strand.	Diam. of Wire by W. G.	Weight per Foot.	Breaking Weight.	Circumference. Actual.	Circumference. Nominal.	Wire in each Strand.	Diam. of Wire by W. G.	Weight per Foot.	Breaking Weight.
Ins.	Ins.	No.	No.	Lbs.	Lbs.	Ins.	Ins.	No.	No.	Lbs.	Lbs.
4.9375	4.9	19	11	3.14	65 409	2.375	2.4	7	13	.14	15 718
4.375	4.5	19	13	2.15	55 316	2.1875	2.12	7	14	.11	14 478
3.9375	3.91	19	14	2.0875	44 420	2	2.06	19	19	.1	10 925
3.5	3.36	19	14	1.1525	34 840	1.9375	1.9	7	14	.1	10 118
3.1875	2.98	19	15	1.09	28 606	1.75	1.85	7	17	.07	7 880
2.75	2.68	19	17	1.0275	21 846	1.4375	1.45	19	20	.06	5 687
2.6875	2.56	7	13	1.0225	18 810	1.3125	1.31	7	18	.05	4 428
2.5	2.45	19	18	.14	15 692	1.125	1.11	7	19	.035	3 729

To Compute Circumference of Wire Rope with Hemp Core, of Corresponding Strength to Hemp Rope, and of Hemp Rope to Circumference of Wire Rope.

RULE 1.—Multiply square of circumference of hemp rope by .223 for iron wire and .12 for steel, and extract square root of product.

2.—Multiply square of circumference of hemp-core wire rope by 4.5 for iron wire and 8.4 for steel wire.

EXAMPLE.—What are the circumferences of an iron and steel wire rope corresponding to one of hemp-core, having a circumference of 8 ins.?

$\sqrt{8^2 \times .223} = 3.78$ *ins. iron*, and $\sqrt{8^2 \times .12} = 2.77$ *ins. steel.*

ROPES, HAWSERS, AND CABLES.

Ropes of hemp fibres are laid with three or four strands of twisted fibres, and are made up to a circumference of 12 ins., and those of four strands up to 8 ins. are fully 16 *per cent.* stronger than those of three strands.

Hawsers are laid with three or four strands of rope. *Cables* are laid with but three strands of rope. Hawsers and Cables, from having a less proportionate number of fibres, and from the irregularity of the resistance of their fibres in consequence of the twisting of them, have less strength than ropes, difference varying from 35 to 45 *per cent.*, being greatest with least circumference, and those of three strands up to 12 ins. are fully 10 *per cent.* stronger than those having four strands.

Tarred ropes, hawsers, etc., have 25 *per cent.* less strength than white ropes; this is in consequence of the injury fibres receive from the high temperature of the tar, viz. 290°.

Tarred hemp and Manila ropes are of about equal strength, and have from 25 to 30 *per cent.* less strength than white ropes.

White ropes are more durable than tarred.

The greater degree of twisting given to fibres of a rope, etc., less its strength, as exterior, alone resists greater portion of strain.

Ultimate strength of ropes varies from 7000 to 12 000 lbs. per square inch of section, according as they are wetted, tarred, or dry. One sixth of ultimate strength is a safe working load = 1166 to 2000 lbs. per square inch.

Units for computing Safe Strain that may be borne by New Ropes, Hawsers, and Cables. (*U. S. Navy.*)

Description.	Circumference.	Ropes. White. 3 strands.	Ropes. White. 4 strands.	Ropes. Tarred. 3 str'ds	Ropes. Tarred. 4 str'ds	Hawsers. White. 3 str'ds.	Hawsers. Tarred. 3 str'ds.	Cables. White. 3 str'ds.	Cables. Tarred. 3 str'ds.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
White	2.5 to 6	1140	1330	—	—	600	—	—	—
"	6 " 8	1090	1260	—	—	570	—	510	—
"	8 " 12	1045	880	—	—	530	—	530	—
"	12 " 18	—	—	—	—	550	—	550	—
"	18 " 26	—	—	—	—	—	—	560	—
Tarred	2.5 " 5	—	—	855	1005	—	460	—	—
"	5 " 8	—	—	825	940	—	480	—	—
"	8 " 12	—	—	780	820	—	505	—	505
"	12 " 18	—	—	—	—	—	—	—	525
"	18 " 26	—	—	—	—	—	—	—	550
Manila	2.5 " 6	810	950	—	—	440	—	—	—
"	6 " 12	760	835	—	—	465	—	510	—
"	12 " 18	—	—	—	—	—	—	535	—
"	18 " 26	—	—	—	—	—	—	560	—

Illustration.—What weight can be borne *with safety* by a Manila rope of 3 strands, having a circumference of 6 inches? (*See Rule, page* 167.)

$$6^2 \times 760 = 27\,360 \text{ lbs.}$$

When it is required to ascertain weight or strain that can be borne by ropes, etc., in general use, preceding Units should be reduced from *one third to two thirds*, in order to meet their condition or reduction of their strength by chafing and exposure to weather. Molesworth's table is based upon a reduction of three fourths.

Illustration.—What weight can be borne by a tarred hawser of 3 strands, 10 inches in circumference, in *general use?*

$$10^2 \times (505 - \overline{505 \div 3}) = 100 \times 366.67 = 33\,667 \text{ lbs.}$$

Destructive Strength of Tarred Hemp Ropes.

(*D. K. Clark.*)

Circum.	Diam.	Register. Common Cold.	Register. Russian Warm.	Circum.	Diam.	Register. Common Cold.	Register. Russian Warm.
Ins.	Ins.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.
3	.95	7 390	8 620	5.5	1.75	24 800	29 120
3.5	1.11	11 200	11 760	6	1.91	28 985	33 150
4	1.27	13 100	15 340	6.5	2.07	34 030	40 550
4.5	1.43	16 330	19 440	7	2.24	40 320	47 041
5	1.59	19 580	23 990	8	2.54	52 480	61 420

Specimens furnished by National Association of Rope and Twine Spinners,
As tested by Mr. Kirkaldy.

Rope.	Circumference.	Weight per Lb.	Extreme Strength.	Breaking Weight per lb. per Fathom.	Extension in 50 ins. Length at Stress per lb. Weight per Fathom of 1000 lbs.	2000 lbs.	3000 lbs.
	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Ins.
Russian rope . . . 48 thr'ds.	5.26	.926	11 088	1933	5.29	—	—
Machine yarn. . . 50 "	5.37	.891	11 514	2152	4.53	6.56	—
Hand-spun yarn, 51 "	5.39	1.006	18 278	3024	4.46	5.91	6.63

Breaking Strength of Tarred Hemp Ropes. (*Mr. Glynn.*)

Circum.	Diam.	Old Method. Common Hemp.	Old Method. Best Russian.	By Register. Cold.	By Register. Warm.	Circum.	Diam.	Old Method. Common Hemp.	Old Method. Best Russian.	By Register. Cold.	By Register. Warm.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
3	.95	5 056	6 248	7 392	8 624	5.5	1.75	15 456	18 414	24 797	29 120
3.5	1.11	7 466	8 668	11 200	11 760	6	1.91	18 144	21 610	28 986	33 150
4	1.27	8 780	10 460	13 104	17 810	6.5	2.07	20 518	23 610	34 630	40 544
4.5	1.43	10 300	12 432	16 330	19 443	7	2.24	22 938	27 462	40 320	47 040
5	1.59	13 328	15 859	20 496	23 990	8	2.54	26 680	32 032	52 483	61 420

To Compute Strain that may be borne with safety by new Ropes, Hawsers, and Cables.

Deduced from experiments of Russian Government upon relative strength of different Circumferences of Ropes, Hawsers, etc.

U. S. Navy test is 4200 *lbs. for a White rope of three strands of best Riga hemp, of* 1.75 *inches in circumference* (= 17 000 *lbs. per square inch of fibre*), *but in preceding table* (page 166) 14 000 *lbs. is taken as unit of strain that may be borne with safety.*

Rule.—Square circumference of rope, hawser, etc., and multiply it by Units in table.

To Compute Circumference of a Rope, Hawser, or Cable for a Given Strain.

Rule.—Divide strain in pounds by appropriate units in preceding table, and square root of product will give circumference of rope, etc., in ins.

Example 1.—Stress to be borne in safety is 165 550 lbs.; what should be circumference of a tarred cable to withstand it?

$$165\,552 \div 550 = 301, \text{ and } \sqrt{301} = 17.35 \textit{ ins.}$$

2.—What should be circumference of a Manila cable to withstand a strain, in *general use*, of 149 336 lbs.?
Assuming circumference to exceed 18 ins., unit = 560.

$$149\,336 \div (560 - \overline{560 \div 3}) = 400, \text{ and } \sqrt{400} = 20 \textit{ ins.}$$

To Compute Weight of Ropes, Hawsers, and Cables.

RULE.—Square circumference, and multiply it by appropriate unit in following table, and product will give weight per foot in lbs.:

	Ropes.	Hawsers.	Cables.		Ropes.	Hawsers.	Cables.
3-strand Hemp	.032	.031	.031	4-strand Hemp	.033	—	—
3-strand tarred Hemp	.042	.041	.041	4-strand tarred Hemp	.048	—	—
3-strand Manila	.032	.031	.031	4-strand Manila	.035	.034	.034

Units for Thread Ropes is same as that for Ropes of like material.

EXAMPLE.—What is weight of a coil of 10-inch Manila hawser of 4 strands of 120 fathoms?

$10^2 \times .034 = 3.4$, and $120 \times 6 \times 3.4 = 2448$ *lbs.*

Weight and Strength of Hemp and Wire Ropes.

(Molesworth.)

$$C^2 y = W; \qquad C^2 k = L; \qquad C^2 x = S; \qquad \text{and} \sqrt{\frac{L}{k}} = C.$$

C *representing circumference in ins.*, W *weight of rope in lbs. per fathom*, L *working load in tons, and* S *destructive stress in tons.*

VALUES OF y, x, AND k.

Ropes.	y	x	k	Ropes.	y	x	k
Hawser, hemp	.131	—	—	Warm register, hemp	—	.7	.116
Cable "	.117	—	—	Manila hawser	.177	.27	.045
Tarred hawser, hemp	.235	.22	.037	" cable	.155	.19	.033
" cable, "	.207	.15	.025	Iron rope	.87	1.8	.29
Cold register, "	—	.6	.1	Steel "	.89	2.8	.45

To Compute Circumference of Hemp or Wire Rope for Fore or Main Standing Rigging. *(U. S. Navy.)*

RULE.—To length of mast between partners and deck, add half extreme breadth of beam of vessel and divide sum by half extreme breadth. Multiply quotient by half square root of tonnage (OM) and extract square root of product.

For Mizzen, take .74 of Fore and Main.

EXAMPLE.—Required circumference of hemp rope, for main-mast of a vessel having a breadth of beam of 45 feet and a burden of 3213 tons?

Extreme length of mast .... 94.4 feet.
Depth of hold, or total bury of mast, 21.4 feet.
Head .... 15 " 36.4 "
Breadth of beam, 45 feet. 58 "

$$58 + \frac{45}{2} \div \frac{45}{2} = 3.58, \text{ and } \sqrt{\left(3.58 \times \frac{\sqrt{3213}}{2}\right)} = \sqrt{101.46} = 10.11 \text{ ins.}$$

Then if circumference for a wire rope is required, see table, page 164.

Thus, a hemp rope 10 ins. in circumference has equivalent strength of an iron wire rope of 4 ins. and a steel rope of 3.25+ ins.

Galvanized Iron Wire.—Experiments at Navy Yard, Washington, gave for flexibility a mean loss of 30 per cent., and for tensile strength a like loss of 13.5 per cent.

Relative Dimensions of Hemp Rope and Iron and Steel Wire Rope. *(U. S. Navy.)*

Circumference in Inches.

Hemp.	2.5	3.125	4	4.5	5.25	6.5	7.75	8.5	9.5	11	11.75	13.5	16.5
Iron ..	1.25	1.625	2	2.125	2.5	3	3.5	4	4.5	5	5.5	6	7
Steel ..	.875	1.125	1.5	1.625	1.875	2.125	2.5	2.75	3.25	3.5	4	4.375	5.25

ANCHORS, CABLES, ETC.

Anchors, Chains, etc., for a Given Tonnage.

(*American Shipmasters' Association.*)

SAILS.

Tonnage computed as per Rule.*	Anchors. Bowers. Without Stock.	Anchors. Bowers. Admiralty Test.	Anchors. Including Stock. Stream.	Anchors. Including Stock. Kedge.	Anchors. Including Stock. 2d Kedge.	Chain Cable.—Stud. Diameter.	Chain Cable.—Stud. Length.	Chain Cable.—Stud. Admiralty Test.	Weight per Fathom. Stud.	Weight per Fathom. Short Link.	Weight per Fathom. English.†
	Lbs.	Tons.	Lbs.	Lbs.	Lbs.	Ins.	Faths.	Tons.	Lbs.	Lbs.	
75	616	7	168	84	—	.8125	90	11	40	42	35
100	728	8	196	112	—	.875	105	13	44	48	—
125	840	9	224	112	—	.9375	105	15	51	55	48
150	952	10	280	140	—	1	120	17.5	59	63	54
175	1036	11	336	168	—	1.0625	120	20	66	70	—
200	1120	12	392	196	—	1.125	120	22.5	75	79	68
250	1288	13	448	224	112	1.1875	135	25	82	88	—
300	1456	14	504	252	126	1.25	135	28	91	98	84
350	1624	15.5	560	280	140	1.3125	150	31	100	106	—
400	1848	17	616	308	154	1.3125	150	31	100	106	—
450	1904	18.5	672	336	168	1.375	165	37	115	118	102
500	2016	20	784	392	196	1.4375	165	40	120	—	—
600	2352	22	896	448	224	1.5	180	44	132	—	122
700	2688	24	1008	504	252	1.5625	180	47	145	—	—
800	3024	26	1120	560	280	1.625	180	51	156	—	143
900	3248	28	1232	616	308	1.6875	180	55	162	—	—
1000	3584	29.5	1344	672	336	1.75	180	59	175	—	166
1200	3808	31	1456	738	364	1.875	180	63	189	—	191
1400	4032	32.5	1568	784	392	1.9375	180	67	205	—	—
1600	4256	34	1680	840	420	2	180	72	219	—	—
1800	4480	35.5	1792	896	448	2	180	72	240	—	217
2000	4704	37	1904	952	504	2.0625	180	81	—	—	—
2500	5040	39	2128	1120	560	2.125	180	86	—	—	244
3000	5376	41	2353	1232	616	2.1875	180	96	—	—	—

† Brown, Lennox, & Co.

To Compute Tonnage.

Take dimensions as follows: *Length.*—From after-side of stem to forward-side of stern-post, measured on spar or upper deck in vessels having two decks and under, and on main deck in vessels having three or more decks. *Breadth.*—Extreme at widest point. *Depth.*—At forward coaming of main hatch, from top of ceiling at side of keelson to under side of deck.

Then multiply these dimensions together, divide product by 100, and take .75 of quotient.

All vessels to have 2 bowers and 1 each stream and kedge anchor, and for a tonnage exceeding 1400 a third bower is recommended.

Hawsers and *Warps* to be 90 fathoms in length.

Shrouds.

SQUARE-RIGGED. *Hemp.*—5.75 ins. in diameter for a tonnage of 75, increasing progressively up to 12.75 ins. for 3000 tons.

FORE-AND-AFT RIGGED. From .25 to 1 inch in diameter progressively greater than for square-rigged.

Wire.—One half diameter of hemp, increasing very slightly as tonnage increases. Thus, for 3000 tons, 12.75 ins. for hemp and 6.875 ins. for wire.

(American Shipmasters' Association.)

STEAM.

Tonnage computed as per Rule preceding.	Anchors. Bowers. Without Stock.	Anchors. Bowers. Admiralty Test.	Anchors. Including Stock. Stream.	Anchors. Including Stock. Kedge.	Anchors. Including Stock. 2d Kedge.	Chain Cable—Stud. Diameter.	Chain Cable—Stud. Length.	Chain Cable—Stud. Admiralty Test.	Chain Cable—Stud. Diam. Stream.	Weight per Fath. Stud.	Weight per Fath. Short Link.	Weight per Fath. English.*
	Lbs.	Tons.	Lbs.	Lbs.	Lbs.	Ins.	Faths.	Tons.	Ins.	Lbs.	Lbs.	
100	336	4.9	112	—	—	.6875	105	8.1	.5	—	—	25
150	448	6.4	196	—	—	.8125	120	11.9	.5625	40	42	35
200	616	7.6	224	—	—	.875	120	13.8	.5625	44	48	—
250	672	8.2	280	—	—	.9375	120	15.8	.625	51	55	48
300	812	9.5	308	—	—	1	120	18	.625	59	63	54
350	924	10.4	336	—	—	1.0625	120	20.3	.6875	66	70	—
400	1120	12	532	252	—	1.125	135	22.8	.6875	75	79	68
450	1344	13.9	560	280	—	1.1875	135	25.4	.75	82	88	—
500	1512	15.2	672	336	—	1.25	150	28.1	.75	91	98	84
600	1708	16.7	738	364	—	1.3125	150	31	.8125	100	106	—
700	1876	18	784	392	—	1.375	165	34	.8125	115	118	104
800	2026	19	896	448	224	1.4375	165	37.2	.875	120	—	—
900	2352	21.6	1008	504	252	1.5	180	40.5	.875	132	—	122
1000	2632	23.5	1120	560	280	1.5625	180	44	.9375	145	—	—
1200	2856	25.2	1176	588	308	1.625	180	47.5	.9375	156	—	143
1400	3108	26.9	1232	616	308	1.6875	180	51.2	1	162	—	—
1600	3360	28.6	1344	672	336	1.75	180	55.1	1	175	—	166
1800	3584	30.1	1456	738	364	1.8125	180	59.1	1.0625	189	—	—
2000	3808	31.6	1512	766	364	1.875	180	63.3	1.0625	205	—	191
2300	4088	33.4	1568	784	392	1.9375	180	67.6	1.125	215	—	—
2600	4256	34.5	1624	812	392	2	270	72	1.125	240	—	217
3000	4480	35.7	1680	840	420	2.0625	270	76.6	1.1875	—	—	—
3500	4592	37	1792	896	476	2.125	270	81.3	1.1875	—	—	244
4000	4816	38	1960	952	504	2.1875	270	86.1	1.25	—	—	—
4500	5040	39.2	2128	1064	532	2.25	270	91.1	1.25	—	—	—
5000	5264	41	2352	1120	560	2.3125	270	96	1.3125	—	—	—

* Brown, Lennox, & Co.

ANCHORS AND KEDGES.

(U. S. Navy.)

To Compute Weight of a Bower Anchor for a Vessel of a given Character and Rate.

RULE.—Multiply approximate displacement in tons, by unit in following table, and product will give weight in lbs., inclusive of stock.

Units to determine Weights and Number of Anchors or Kedges.

Displacement of Vessel in Tons.	Unit.	Bower.	Sheet.	Stream	Kedge.	Displacement of Vessel in Tons.	Unit.	Bower.	Sheet.	Kedge.
Over 3700	1.75	2	2	1	4	Over 1500 ..	2.5	2	2	3
" 2400	2	2	2	1	3	" 900 ..	2.75	2	1	3
" 1900	2.25	2	2	1	3	900 and under	3	2	1	2

EXAMPLE.—Tonnage of a bark-rigged steamer is 1500.

$1500 \times 2.5 = 3750$ *lbs., weight of anchor.*

Bower and Sheet Anchors should be alike in weight.

Stream Anchors and *Kedges* are proportional to weight of bowers. Thus, *Stream Anchor* .25 weight. *Kedges.*—If 1, .125 weight; if 2, .16 and .1 weight; if 3, .16, .125, and .1 weight.

To Compute Diameter of a Chain Cable corresponding to a Given Weight of Anchor.

(*U. S. Navy.*)

RULE.—Cut off the two right-hand figures of the anchor's weight in lbs., multiply square root of remainder by 4, and result will give diameter of chain in sixteenths of an inch.

EXAMPLE.—The weight of an anchor is 2500 lbs.

$\sqrt{25.00} \times 4 = 20$ *sixteenths* $= 1.25$ *ins.*

NOTE.—Diam. of a messenger should be .66 that of the cable to which it is applied.

Lengths of Chain Cables for each Anchor.

(*U. S. Navy.*)

Weight of Anchor.	Bower.	Sheet.	Stream.	Weight of Anchor.	Bower.	Sheet.	Stream.
Lbs.	Fathoms.	Fathoms.	Fathoms.	Lbs.	Fathoms.	Fathoms.	Fathoms.
Under 800	60	60	60	Over 2000	120	120	90
Over 800	90	90	60	" 3000	120	120	90
" 1200	90	90	75	" 5000	120	120	105
" 1600	105	105	75	" 7500	135	135	105

ANCHORS.

From Experiments of a Joint Committee of Representatives of Ship-owners and Admiralty of Great Britain.

An anchor of ordinary or Admiralty pattern, Trotman or Porter's improved (pivot fluke), Honiball, Porter's, Aylin's, Rodgers's, Mitcheson's, and Lennox's, each weighing, inclusive of stock, 27 000 lbs., withstood without injury a proof strain of 45 000 lbs.

Breaking weights between a Porter and Admiralty anchor, as tested at Woolwich Dock-yard, were as 43 to 14.

Comparative Resistance to Dragging.

Trotman's dragged Aylin's, Honiball's Mitcheson's and Lennox's; Aylin's and Mitcheson's dragged Rodgers's; and Rodgers's and Lennox's dragged Admiralty's.

TONNAGE OF VESSELS.

To Compute Tonnage of Vessels.

For Laws of United States of America, with amendments of 1882 relative to Steam-vessels, see Mechanics' Tables, with rule and illustrated diagrams, by Chas. H. Haswell, 3d edition, Harper & Bros., New York, 1878.

English Registered Tonnage. (*New Measurement.*)

Divide length of upper deck between after-part of stem and fore-part of stern-post into 6 equal parts, and note foremost, middle, and aftermost points of division. Measure depths at these three points in feet and tenths of a foot; also depths from under-side of upper deck to ceiling of limber-strake; or in case of a break in the upper deck, from a line stretched in continuation of the deck. For breadths, divide each depth into 5 equal parts, and measure the inside breadths at following points, *viz.*:—At .2 and .8 from upper deck of foremost and aftermost depths; and from .4 and .8 from upper deck of amidship depth. Take length at half amidship depth from after-part of stem to fore-part of stern-post.

Then, to twice amidship depth add foremost and aftermost depths for *sum of depths*, and add together foremost upper and lower breadths, 3 times upper breadth with lower breadth at amidship, and upper and twice lower breadth at after division for *sum of breadths.*

Multiply together sum of depths, sum of breadths, and length, and divide product by 3500, which will give number of tons.

If the vessel has a poop or half-deck, or a break in upper deck, measure inside mean length, breadth, and height of such part thereof as may be included within the bulkhead; multiply these three measurements together, divide product by 92.4, and quotient will give number of tons to be added to result as above ascertained.

For Open Vessels.—Depths are to be taken from upper edge of upper strake.

For Steam Vessels.—Tonnage due to engine-room is deducted from total tonnage computed by above rule. To determine this, measure inside of the engine room from foremost to aftermost bulkhead; then multiply this length by amidship depth of vessel, and product by inside amidship breadth at .4 of depth from deck, and divide final product by 92.4.

The volume of the poop, deck houses, and other permanently enclosed spaces, available for cargo or passengers, is to be measured and included in the tonnage, but following deductions are allowed, the remainder being the *Register tonnage*.

Deductions.—Houses for the shelter of passengers only; space allotted to crew (12 square feet in surface and 72 cube feet in volume for each person); and space occupied by propelling power.

Approximate Rule.

Gross Register.—Tonnage of a vessel expresses her entire cubical volume in tons of 100 cube feet each, and is ascertained by following formula:

$\frac{L\,B\,D}{100} =$ *Gross tonnage*, and $\frac{L\,B\,D}{100}\,c =$ *Register tonnage.* L *representing length of keel between perpendiculars,* B *breadth of vessel, and* D *depth of hold, all in feet.*

Builders' Measurement.

$$\frac{(L - .6\,B) \times B \times .5\,B}{94} = \textit{Tonnage.}$$

Fore-perpendicular is taken at fore-part of stem at height of upper deck.
Aft-perpendicular is taken at back of stern post at height of upper deck.
In three-deckers, middle deck is taken instead of upper deck.
Breadth is taken as extreme breadth at height of the wales, subtracting difference between thickness of wales and bottom plank. Deductions to be made for rake of stem and stern.

Iron Vessels. $\frac{18}{10\,000}\left(\frac{\textit{Girth} + \textit{Breadth}}{2}\right)^2 \times \textit{length} = \textit{Gross tonnage.}$

Length measured on upper deck, between outside of outer plank at stem and the after side of stern post and rabbet of stern-post, at point where counter plank crosses it. Girth measured by a chain passed under bottom from upper deck at extreme breadth, on one side, to corresponding point on the other.

Register tonnage $= \frac{L \times B \times D}{100} \times C.$ C *representing a coefficient for vessels as follows:*

Ships of usual form	.7	Yachts above 60 tons		.5
Clippers and Steamers, 2 decks	.65	Small vessels	sharp	.45
Clippers and Steamers, 3 "	.68	Small vessels	very sharp	.4

Units for Measurement and Dead-weight Cargoes.

(C. Mackrow, M. S. N. A.)

To Compute Approximately for an Average Length of Voyage the Measurement Cargo, at 40 feet per Ton, which a Vessel can carry.

RULE.—Multiply number of register tons by unit 1.875, and product will give approximate measurement cargo.

To Compute Approximately Dead-weight Cargo in Tons which a Vessel can carry on an Average Length of Voyage.

RULE.—Multiply number of register tons by 1.5, and product will give approximate dead-weight cargo required.

With regard to cargoes of coasters and colliers, as ascertained above, about 10 per cent. may be added to said results, while about 10 per cent. may be deducted in cases of larger vessels on longer voyages.

In case of measurement cargoes of steam-vessels, spaces occupied by machinery, fuel, and passenger cabins under the deck must be deducted from space or tonnage under deck before application of measurement unit thereto.

In case of dead-weight cargoes, weight of machinery, water in boilers, and fuel must be deducted from whole dead weight, as ascertained above by application of dead-weight unit.

The deductions necessary for provisions, stores, etc., are allowed for in selection of the two units.

To Ascertain Weight of Cargo for an Average Length of Voyage. (*Moorsom.*)

Deduct tonnage of spaces of passenger accommodations from net register tonnage, and multiply remainder by 1.5.

Average space for each ton weight of cargo on such a voyage 67 cube feet.

Freight Tonnage or Measurement Cargo.

Freight Tonnage or *Measurement Cargo* is 40 cube feet of space for cargo, and it is about 1.875 times net register tonnage less that for passenger space.

Royal Thames Yacht Club.

Measure length of yacht in a straight line at deck from fore-part of stem to after-part of stern post, from which deduct extreme breadth (measured from outside of outside planking), both in feet; remainder is length for tonnage. Multiply length for tonnage by extreme breadth, that product by half extreme breadth, divide result by 94, and quotient will give tonnage.

If any part of stem or stern-post projects beyond length as taken above, such projection or projections shall, for purpose of computing tonnage, be added to length taken as before mentioned.

All fractional parts of a ton are to be considered as a ton.

Measurements to be taken either above or below main wales.

$$\frac{L - B \times B \times .5\,B}{94} = \textit{Tons.}$$ L *representing length and* B *breadth, in feet.*

Corinthian and New Thames Yacht Club.

Measure length and breadth as in foregoing rule, and depth to top of covering board; multiply length, breadth, and depth together, divide result by 200, and quotient will give tonnage.

$$\frac{L \times B \times D}{200} = \textit{Tons.}$$

Suez Canal Tonnage.

Gross Tonnage.—Spaces under tonnage deck, below tonnage and uppermost deck, all covered or closed-in spaces, such as poop, forecastle, officers' cabins, galley, cook, deck, and wheel houses, and all inclosed or covered-in spaces for working the vessel.

From which are to be deducted berthing accommodations for crew, not including spaces for stewards and passengers' servants; berthing accommodations for officers, except captain; galleys, cook-houses, etc., used exclusively for crew, and inclosed spaces above uppermost deck, designed for working the vessel. In none of these spaces can passengers be berthed or cargo carried, and total deduction under all of these spaces must not exceed 5 per cent. of gross tonnage.

In steamers with standing coal-bunkers, English rule may be followed, or owner may elect to have tonnage of his vessel computed by "Danube rule," which is an allowance of 50 per cent. above space allowed to machinery in side-wheel steamers and 75 in screw steamers.

In no case, however, except with tow-boats, must deduction for propelling power exceed 50 per cent. of gross tonnage.

WORKS OF MAGNITUDE.

American.

Aqueducts, Roads, and Railroads.

Croton Aqueduct, N. Y. — Has a section of 53.34 square feet and capacity of 100 000 000 to 118 000 000 gallons per day. and from Dam to Receiving Reservoir is 38.134 miles in length.

Aqueduct, Washington.—Cylinder of masonry 9 feet in diameter. Stone arch over Cabin John's Creek, 220 feet span, 57.25 feet rise.

National Road.—Over the Alleghany Mountains, Cumberland to Illinois Town, 650.625 miles in length, and 80 feet in width. Macadamized for a width of 30 feet.

Illinois Central Railroad.—Chicago to Cairo, length 365 miles, Centralia to Dunleith 344 miles, total 709 miles.

Bridges.

Suspension Bridge, Niagara River.—Wire. Span 1042 feet 10 ins.

Suspension Bridge, New York and Brooklyn. — Length of river span 1595 feet 6 ins.; of each land span 930 feet; length of Brooklyn approach 971 feet; of N. Y. approach 1562 feet 6 ins.; total length of bridge 5989 feet; width 85 feet; number of cables 4; diameter of each cable 15.5 ins.; each consisting of 6300 parallel steel wires No. 7 gauge, closely laid and wrapped to a solid cylinder; ultimate strength of each cable 11 200 tons; depth of tower foundation below high water, Brooklyn, 45 feet—New York 78 feet; towers at high-water line 140×59 feet; towers at roof course 136×53 feet; total height of towers above high water 277 feet; clear height of bridge in centre of river span above high water, at 50°, 135 feet; height of floor at towers above high water 119 feet 3 ins.; grade of roadway 3 feet in 100; anchorages, at base 129×119 feet, at top 117×104 feet; weight of each anchor-plate 23 tons.

Iron Pipe Bridge over Rock Creek.—200 feet span, 20 feet rise. Arch of 2 lateral courses of cast-iron pipe, 4 feet internal diameter, and 1 inch thick. These pipes conveying the water not only sustain themselves over the great span, but support a street road and railway.

Iron Bridge over Kentucky River near Shakers' Ferry, Md.—3 spans, each 375 feet, and 275.5 feet above low water.

Bridge on line of New York, Erie, and Western Railroad across the Kinzua.—Of iron; length 2060 feet; central span 301 feet in height.

Iron Truss.—Cincinnati and Southern Railway, over Ohio River, 519 feet.

Foreign.

Pyramids, Statues, etc.

Pyramid of Cheops, Egypt.—Length of side at base 762 feet; height to present summit 453.3 feet; to original summit 485.2 feet; inclined length 568.25 feet; angle of side 51° 51′ 14″; area of each face = square of height; weight 5 272 600 tons; built 2170 years B.C.

Peter the Great, St. Petersburg, Russia.—Bronze; height of horse 17 feet; of man 11 feet; base of rock 42 feet at bottom, 36 at top, 21 wide, and 17 high, weighing 1100 tons.

Liberty, New York Harbor. — 110 feet in height from head to foot and 140 feet to flambeau; including base, 309 feet. Weight of statue 150 tons.

Daibutsu, of stone, Japan. — Sitting posture; height 44 feet; circumference 87 feet; face 8.5 feet; circumference of thumb 3.5 feet.

Colossus of Rhodes.—Height, 105 feet.

Bridge.

Britannia Tubular Bridge. — Of iron, with a double line of Railway, 964 feet in length, with two approaches of 230 feet each. Weight 3658 tons.

Monoliths.

Obelisk at Karnak, Egypt.—Of granite, 108 feet 10 ins.; pedestal 13 feet 2 ins.; weight 400 tons.

Obelisk in Central Park, N. Y.—Of granite, 68 feet 11 ins.; weight 168 tons.

U. S. Treasury, Washington.—Some stones of, are heavier than any in the Pyramids of Egypt.

Steam Hammers.

At workshops of Herr Krupp, at Essen, there is a steam hammer weighing 50 tons having a fall of 3 metres; and at Creusot there is a hammer weighing between 75 and 80 tons having a fall of 5 metres.

Crane.

At Creusot there is a steam crane having a capacity to lift and revolve with 150 tons.

Chimneys.

J. Townsend's chemical works, Glasgow, diameter at foundation 50 feet; at top 12 feet 8 ins.; height from foundation 488 feet; from ground 474 feet.

New York Steam Heating Co., 220 feet in height.

Pillar.

At a gate near Delhi is a wrought-iron pillar having diameters of 16.4 ins. at 22 feet in its height above ground and 12 ins. at its top. It is estimated from the result of excavations at its base to be 60 feet in length or height and to weigh 17 tons. Its period of structure is assigned to the 3d or 4th century A.D.

Roofs.

Midland Railway Station, London.	240 ft.	Union Railway Station, Glasgow.	195 ft.
Imperial Riding-School, Moscow.	235 "	Grand Central Station, N. Y.....	200 "

Diameters of Domes.

Domes.	Feet.	Domes.	Feet.	Domes.	Feet.
Capitol, Washington	124.75	St. Paul's, London.	112	Midl'nd Rail'y, Lon.	240
Glasgow W. Railw'y	198	St. Peter's, Rome..	139	Great North'n, Eng.	210

Lengths of Tunnels.

Tunnels.	Feet.	Tunnels.	Feet.	Tunnels.	Feet.
Blaizy	13 455	Gunpowder, Md...	36 500	Nerthe.........	15 153
Blue Ridge......	4 280	Sutro............	20 028	Nochistongo....	21 659
Hoosac..........	25 031	Semmering......	5 630	Riquivel........	18 623

Thames and Medway, 11 880 feet. Weehawken, 4000 feet.

Mont Cenis 7.5 miles 242 yards, rises 1 in 45, and descends 1 in 2000.

St. Gothard Tunnels and Roads 9 miles 477 yards; tunnels 116 156.5 feet, and rises 1 in 233 in whole length; 26.5 feet in width; 19 feet 10 inches in height. Maximum grade 2.7 feet per 100.

Miscellaneous.

Fortress Monroe, Old Point Comfort, Va.—Largest fortress.

Telegraph Wire.—Span over river Kistnah between Bezorah and Sectanagran, 6000 feet in length.

Deer Park, Copenhagen.—4200 acres.

Oxford College, England.—Largest University; said to have been founded by Alfred.

Cathedral. St. Peter's, Rome.—Width of front 216 feet; of the cross 251 feet; total height 469.5 feet.

Steamer Great Eastern.—Of iron, 680 feet in length; 83 feet width of beam; 60 feet depth of hold; 22 927 tons; built at Millwall, England, 1857.

Chinese Wall.—25 feet at base; 15 at top; height, with a parapet of 5 feet, 20 feet; length 1250 miles.

Artesian Well, Perth.—3050 feet in depth; temperature of water 99°; volume of discharge 18 000 gallons per day.

Weights of Bells.

Bells.	Lbs.	Bells.	Lbs.	Bells.	Lbs.
Pekin	120 000	Oxford, "Great Tom," Eng.	17 024	St. Peter's, Rome	18 000
Lewiston, Me.	10 233	Olmutz, Bohemia	40 320	Vienna	40 200
Montreal, Can.	28 560	Rouen, France	40 000	Westmi'ster, "Big Ben," England	35 620
Moscow, Russia	443 772	St. Paul's, Eng.	42 000	York "	24 080
Erfurt, Saxony	30 800	St. Ivan's, Moscow	127 830	State House, Phila.	13 000
Notre Dame, Paris	28 670				

Rangoon, Burmah, 201 600 lbs.

Capacity of Principal Churches and Opera Houses.

Estimating a person to occupy an Area of 19.7 Ins. Square.

Churches.

St. Peter's	54 000	St. John, Lateran	22 900
Milan Cathedral	37 000	Notre Dame, Paris	21 000
St. Paul's, Rome	32 000	Pisa Cathedral	13 000
St. Paul's, London	25 600	St. Stephen's, Vienna	12 400
St. Petronio, Bologna	24 400	St. Dominic's, Bologna	12 000
Florence Cathedral	24 300	St. Peter's, Bologna	11 400
Antwerp Cathedral	24 000	Cathedral of Sienna	11 000
St. Sophia's, Constantinople	23 000	St. Mark's, Venice	7 000

Opera Houses and Theatres.

Carlo Felice, Genoa	2560	Teatro del Liceo, Barcelona	4000
Opera House, Munich	2370	Covent Garden, London	2684
Alexander, St. Petersburg	2332	Opera House, Berlin	1636
San Carlos, Naples	2240	New York Academy	2526
Imperial, St Petersburg	2160	" " Windsor	3400
La Scala, Milan	2113	Philadelphia Academy	3124
Academy of Paris	2092	Chicago "	3000

Heights of Columns, Towers, Domes, Spires, etc.

Locations.		Feet.
CHIMNEYS.		
Townsend's	Glasgow	474
St. Rollox	"	455.5
Musprat's	Liverpool	406
Gas Works	Edinburgh	341.5
New England Glass Co.	Boston	230
Steam Heating Co.	New York	220
COLUMNS.		
Alexander	St. Peters'g	175
Bunker Hill	Mass.	221
City	London	202
July	Paris	157
Napoleon	"	132
Nelson's	Dublin	134
Nelson's	London	171
Place Vendôme	Paris	136
Pompey's Pillar	Egypt	114
Trajan	Rome	145
Washington	Wash'gton	555
York	London	138
TOWERS AND DOMES.		
Babel		680
Balbec		500
Capitol	Wash'gton	287.5
Cathedral	Antwerp	404.8
"	Cologne	524.9
"	Cremona	392
"	Escurial	200

Locations.		Feet.
TOWERS AND DOMES.		
Cathedral	Florence	390.5
"	Magdeb'rg	339.9
"	Milan	438
"	Petersburg	363
Leaning	Pisa	188
Porcelain	China	200
St. Mark's	Venice	328
St. Nicholas	Hamburg	473
St. Paul's	London	355.1
St. Stephen	Vienna	443.8
Strasburg		486
Utrecht		464
Votive Church	Vienna	314.9
SPIRES.		
Cathedral	New York	325
"	Strasburg	465.9
Grace Church	New York	216
Freiburg		410
Salisbury		450
St. John's	New York	210
St. Paul's	"	200
St. Mary's	Lübeck	404
St. Peter's	Rome	469.5
Trinity Church	New York	286
Balustrade of Notre Dame	Paris	216
Towers of ditto	"	232.9
Hôtel des Invalides	"	344

Areas of Lakes in Europe, Asia, and Africa.

Lakes.	Sq. Miles.	Lakes.	Sq. Miles.	Lakes.	Sq. Miles.
Geneva	400	Dembia, Abyssinia.	13 000	Lough Neagh, Irel'd	80
Tchad, Africa	11 600	Loch Lomond	27	Tonting, China	1200

Lengths of Bridges.

Bridges.	Feet.	Bridges.	Feet.	Bridges.	Feet.
Avignon	1710	Lyons	1560	Potomac	5300
Badajoz	1874	Menai	1050	Riga	2600
Belfast	2500	N. Y. and Brooklyn spans and approaches	5989	St. Lawrence Riv'r	9144
Blackfriars	995			Strasburg	3390
Boston	3483			Vauxhall	860
London	950	Pont St. Esprit	3060	Westminster	1223

Lengths of Spans of Bridges.

Bridges.	Feet.	Bridges.	Feet.	Bridges.	Feet.
Britannia	460	Niag'a at the Falls	1268	Schuylkill	340
Conway	400	" at Queenstown	1040	Southwark	240
Menai	580			Wheeling	1010

Canals.

Lengths.—Lake Erie to Albany 352 miles; Chesapeake and Ohio 307; Schuylkill 108; Delaware and Hudson 109; Rideau 132; London to Liverpool 265; Caledonia 25; Liverpool and Leeds 127.5; Rhone to Rhine 203.

Capacity of Locks of Erie 240 tons, and of Welland 1500.

Welland 26.77 miles. Lake Erie to Montreal *via* Canal 70.5; Lake and River 375 miles.

Montreal to Kingston.—Canal 120 miles; River 126.25. Suez, see page 183.

Breakwaters.

Delaware.—Average depth of water 29.4 feet below low-water level; range of tide 6.66 feet; Outer slope 45°; Inner slopes 1.5, 5, 3, and 1.3 to 1; length of base 172.12 feet.

Plymouth.—Outer slopes 1.75 to 1 from bottom to 7 feet 6 ins. below low-water line; 4 to 1 to low-water line; 16 to 1 to 4 feet 6 ins. above low-water line; 5 to 1 to high water; Inner slope 1.5 to 1 above low water line; 2 to 1 below low-water line.

Depth of water at high tide 46.5 feet; at low tide 30 feet.

Body of breakwater cased with large squared stones cramped together.

Portland.—Depth of high water 58 feet; of low water 51 feet; Outer slopes 1 to 1 from bottom to 20 feet below low water; 2 to 1 to 12 feet below low water; 6 to 1 to low-water line; 4 to 1 to high-water line; Inner slope 1.25 to 1.

Body of breakwater, rubble, with crest wall of ashlar.

Dover.—Depth of high-water line 61 feet; of low-water line 42 feet.

Body of breakwater, concrete blocks faced with granite; batter 3 inches to the foot, stepped up in each course.

Marseilles.—Depth of water 33 feet; Outer casing of beton 25.5 tons each; average thickness of casing from 14 to 20 feet; slope 1 to 1 from bottom to water line; 2.5 to 1 above water-line; all other slopes .33 to 1; Inner casing of first-class rubble (of stones 2 to 5 tons weight), about 12 feet thick; Hearting, second-class rubble (of stones .5 to 2 tons weight), about 6 feet thick; Nucleus, of quarry rubbish.

Algiers.—Depth of water 50 feet; rubble base carried up to 33 feet from surface of water; the remainder composed of large beton blocks 25.5 tons each; slopes of rubble base 1 to 1; Outer slope of beton blocks 1.25 to 1; Inner slope of beton blocks 1 to 1.

Port Said (Suez Canal).—Concrete blocks, 10 cubic metres each, composed of 1 of hydraulic lime to 13 of sand, mixed with sea water; 4 days in the mold and dried for 4 months before being put in position. In some instances the composition of beton blocks is .33 lime or cement to .66 sand and broken stone, about the size of ballasting.

Rubble or Block Filling.—Proportion of interstices to volume of breakwater finished: First class rubble of 2 to 5 tons, .25; second-class rubble of .5 to 2 tons, .2; third-class rubble, quarry chips, etc., .16; beton blocks, 15 to 25 tons, .33.

Note.—For force of water, see Waves of the Sea, page 853.

Areas, Depths, and Heights of Great Northern Lakes of United States.

Lakes.	Length.	Breadth.	Mean Depth.	Height above Sea.	Area.
	Miles.	Miles.	Feet.	Feet.	Sq. Miles.
Erie	250	80	200	564	6 000
Huron	200	160	120	574	20 000
Michigan	360	109	900	587	20 000
Ontario	180	65	500	234	6 000
Superior *	400	160	288	635	32 000

* Greatest depth 5400 feet.

Elevation Above Tide-water at Albany. — Lake Erie 570.6 feet; Hudson River 2.46 feet.

Mean Depths and Areas of the Oceans and Seas.

(*Herr Krümmel.*)

	Fathoms.	Area Sq. Miles.		Fathoms.	Area Sq. Miles.
Atlantic	2013	29 514 275	Gulf of Mexico	1001	1 765 910
Archipelago	487	3 046 600	" " St. Lawrence	160	101 075
Azof	—	8 800	Indian	1829	28 369 595
Baltic Sea	36	159 690	Japan	1200	383 205
Black Sea	—	150 000	Mediterranean	729	1 109 230
Behring's Straits	550	864 555	North Sea	48	210 505
Caspian Sea	—	120 000	North Ice Sea	845	5 264 600
China (East) Sea	66	472 210	Persian Gulf	20	90 100
Dead Sea	—	370	Pacific	3887	60 343 690
English Channel, etc.	47	78 416	Red Sea	243	170 820

Mean depth of Ocean surrounding land 1877 fathoms = 2.19 miles.

In his subsequent computations he estimates ocean area at 143 703 000 square miles and determines area of land to water as 1 to 2.75, and that mean height of land = 1377 feet, or one eighth that of Ocean.

Heights of Mountains, Volcanoes, and Passes above Level of Sea.

Mountains.	Feet.	Mountains.	Feet.	Mountains.	Feet.
EUROPE.		Mount Everest (Himalaya, highest)	29 003	Mount Pitt	9 549
Azores Pico	7 613	Mount Libanus	9 523	Mount Washington	6 426
Barthelemy, France	7 365	Petcha	15 000	Nevado de Sorata	25 248
Ben Lomond	3 240	Sinai	7 496	Orizaba	18 879
Ben Nevis	4 380	AFRICA.		Potosi	18 000
Elbrus, Caucasus	17 776	Atlas	10 400	Sierra Nevada	15 700
Guadarama, Spain	8 520	Compass, Cape of Good Hope	10 000	Tahiti	10 895
Hecla	5 147	Dianai Peak, St Helena	2 700	White Mountains	6 230
Ida	4 960	Kilimanjaro	20 000	VOLCANOES.	
Jungfrau, Switz'd.	13 725	Ruivo, Madeira	5 160	Cotopaxi	18 887
Mont Blanc	15 797	Teneriffe Peak	12 300	Etna	10 874
" Cenis	6 780	AMERICA.		Hecla	5 000
Mont d' Or, France.	6 510	Aconcagua (highest in America)	23 910	Popocatapetl	17 784
Mulahassen, Gren'a.	11 663	Blue Mount, Jam'a.	8 000	Sahama	22 350
Nephin, Ireland	2 634	Catskill	3 804	St. Helen's, Oregon.	13 320
Olympus	6 510	Chimborazo	21 441	Vesuvius	3 930
Parnassus	6 000	Correde, Potosi	16 036	PASSES.	
Plynlimmon, Wales.	2 463	Crows' Nest, Highlands, N. Y.	1 370	Cordilleras	13 525 / 15 225
The Cylinder, Pyr.	10 930	Great Peak, New Mexico	19 788	Mont Cenis	6 778
Wetterhorn	12 154	Mauna Rauh, Owy'e	18 400	" Cervis	11 100
ASIA.				Pont d' Or	9 843
Ararat	17 100			St. Bernard, Great	8 172
Caucasus	16 433			" Little	7 192
Dhawalagheri	28 077			St. Gothard	6 808
Geta, Java	8 500			Simplon	6 578
Mount Lebanon	12 000				

Dimensions of Canal Locks.—(*U. S.*)

Canal.	Length.	Breadth.	Depth.	Length of Canal.
	Feet.	Ft.	Ft.	Miles.
Albemarle and Chesapeake	220	40	6	14
Black River,	90	15	4	77
Crook'd L'ke,				8
Chenango,				97
Chemung,				33
and Genesee Valley				113.75
Chesapeake and Delaware	220	24	9	14

Canal.	Length.	Breadth.	Depth.	Length of Canal.
	Feet.	Feet.	Feet.	Miles.
Champlain	110	18	5	66.75
Cayuga and Seneca	110	18	7	24.75
Delaware and Raritan	220	24	7	43
Dismal Swamp	90	17.5	5.5	44
Erie	110	18	7	352
Falls of Ohio, Ky.	350	80	2–60	—
Oswego	110	18	4	38
Welland, Canada	270	45	14	28

Length of vessel that can be transported is somewhat less than lengths of locks.

Suez Canal.—Width 196 to 328 feet at surface, 72 at bottom, and 26 deep. Length 99 miles.

Heights of obtained Elevations, and various Places and Points above the Sea.

Locations.	Feet.
Aconcagua, Chili	23 910
Antisana, highest established elevation (Farmhouse)	13 434
Balloon (Gay Lussac)	22 900
" (Green, 1837)	27 000
" (Glaisher and Coxwell)	37 000
Brazil, Quito, and	6 000
Mexico plains	8 000
Condor's flight	29 500
Eagle's "	16 500
Everest, Himalaya	29 003

Locations.	Feet.
Geneva city	1 220
Geneva Lake	1 096
Gibraltar	1 439
Humboldt's highest elevation	19 400
Isthmus of Darien	645
Jungfrau, Switz'd	13 725
La Paz, Bolivia	12 225
Laguna, Teneriffe	2 000
London, city	64
Madrid	2 200
Mexico, city of	7 525
Mont Blanc, Alps	15 797

Locations.	Feet.
Mont Rosa, Alps	15 155
Mount Adams	5 930
Mount Katahdin	5 360
Mount Pitt	9 549
Mount Washington	6 426
Paris, city	115
Pont d' Oro, Pyr's	9 843
Posthouse, Ap., Peru	14 377
Potosi, Bolivia	13 223
Quito	13 500
St. Bernard's Mon'y	8 040
Vegetation	17 000
White Mountain	6 230

Lengths of Rivers.

Rivers.	Miles.
EUROPE.	
Danube	1800
Dnieper	1243
Douro	400
Dwina	1035
Elbe	780
Garonne	442
Loire	545
Po	420
Rhine	760
Rhone	510
Seine	450
Shannon	250
Tagus	510
Thames	220
Tiber	190
Vistula	630
Volga, Russia	2400
ASIA.	
Amoor	2500
Euphrates	1786
Ganges	1514
Hoang Ho	3040
Indus	1800
Jordan	176
Lena	2762
Tigris	1160
Yenesei and Selenga	3580
Yang-Tse	3314
AFRICA.	
Gambia	700
Niger	2400
Nile	4000
NORTH AMERICA.	
Arkansas	2070
Colorado	1050
Columbia	1200
Connecticut	410
Delaware	420
Hudson and Mohawk	325
Kansas	1400
La Platte	850
Mackenzie	2440
Mississippi	1350
Missouri	3030
Ohio and Allegheny	1480
Potomac	420
Red	1520
Rio Bravo	2300
Rio Grande	1800
St. Lawrence	2172
Susquehanna	620
Tennessee	790
SOUTH AMERICA.	
Amazon	4000
Essequibo	520
Magdalena	900
Orinoco	1600
Platte	2300
Rio Madeira	2300
Rio Negro	1650
Uruguay	1100

Large Trees in California.

"*Keystone State.*"—Calavera Grove, is 325 feet in height.

"*Father of the Forest.*"—Felled, is 385 feet in length, and a man on horseback can ride erect 90 feet inside of its trunk.

"*Mother of the Forest.*"—Is 315 feet in height, 84 feet in circumference (26.75 feet in diameter) inside of its bark, and is computed to contain 537 000 feet of sound 1 inch lumber.

Sea Depths.

	Feet.		Feet.		Feet.
Baltic Sea	120	Coast of Spain	6 000	Off Cape Canaveral	2400
Adriatic	130	West of St. Helena	27 000	" Charleston	4200
English Channel	300	Tortugas to Cuba	4 200	" Cape Hatteras	3120
Straits of Gibraltar	100	Gulf of Florida	3 720	" Cape Henry	4200
Eastward of "	3000	Off Cape Florida	1 950	" Sandy Hook	2400

Estimated depth of Atlantic........................ 26 000 feet.
" " Pacific........................ 29 000 "
250 miles off Cape Cod, no bottom at 7800 feet.

Cascades and Waterfalls.

Location.	Feet.	Location.	Feet.	Location.	Feet.
Arve, Savoy	1600	Genesee, N. Y.	100	Niagara	164
Cascade, Alps	2400	Lidford, England	100	Great Fall	152
Cataracts of the Nile	30, 34, 40	Lulea, Sweden	600	Passaic	74
		Mohawk	68	Potomac	74
Chachia, Asia	362	Missouri	50, 80, 94	Ribbon, Yosemite Valley	3300
Foyers, Scotland	197			Ruican, Norway	800
Garisha, India	1000	Montmorenci	250	Staubbach, Switz'd.	798
Gavarny, Pyrenees	1260	Nant d'Apresias	800	Tendon, France	125

Yosemite Valley............2600 feet.

Expansion and Contraction of Building Stones for each Degree of Temperature. (*Lieut. W. H. C Bartlett, U. S. E.*)

	For One Inch.		For One Inch.
Granite	.000 004 825	Sandstone	.000 009 532
Marble	.000 005 668	Whitepine	.000 002 55

Resistance of Stones, etc., to the Effects of Freezing.

Various experiments show that the power of stones, etc., to resist effects of freezing is a fair exponent of that to resist compression.

Magnetic Bearings of New York.

The Avenues of the City of New York bear 28° 50′ 30″ East of North.

Filters for Waterworks.

1 square yard of filter for each 840 U. S. and 700 Imp'l gallons in 24 hours; formed of 2.5 feet of fine sand or gravel and 6 inches of common sand or shells.

Led off by perforated pipes laid in lowest stratum.

Distances between New York, Boston, Philadelphia, Baltimore, and Western Cities of U. S.

Assuming Boston as standard, New York averages 12 per cent. nearer to these cities, Philadelphia 18 per cent., and Baltimore 22 per cent.

Between New York and Chicago the line of the Pennsylvania Railroad is 47 miles shorter than that by the Erie and its connections, 50 miles shorter than that by the N. Y. Central and Hudson River and its connections, and 114 miles shorter than that by the Baltimore and Ohio and its connections.

For Distances between these and other cities of the U. S., see page 88.

Weather-foretelling Plants. (*Hanneman.*)

If Rain is imminent.—Chickweed,* *Stellaria media;* its flowers droop and do not open. Crowfoot anemone, *Anemone ranunculoides;* its blossoms close. Bladder Ketmia, *Hibiscus trionum;* its blossoms do not open. Thistle, *Carduus acaulis;* its flowers close. Clover, *Trifolium pratense*, and its allied kinds, and Whitlow grass, *Draba verna;* all droop their leaves. Nipplewort, *Lampsana communis;* its blossoms will not close for the night. Yellow Bedstraw, *Galium verum;* it swells, and exhales strongly; and Birch, *Betula alba*, exhales and scents the air.

Indications of Rain.—Marigold, *Calendula pluvialis;* when its flowers do not open by 7 A. M. Hog Thistle, *Sonchus arvensis* and *oleraceus;* when its blossoms open.

Rain of short duration.—Chickweed, *Stellaria media;* if its leaves open but partially.

If cloudy.—Wind-flower, or Wood Anemone, *Anemone memorasa;* its flowers droop.

Termination of Rain.—Clover, *Trifolium pratense;* if it contracts its leaves. Birdweed and Pimpernel, *Convolvulus* and *Anagallis arvensis;* if they spread their leaves.

Uniform Weather.—Marigold, *Calendula pluvialis;* if its flowers open early in the A. M. and remain open until 4 P. M.

Clear Weather.—Wind-flower, or Wood Anemone, *Anemone memorasa;* if it bears its flowers erect. Hog Thistle, *Sonchus arvensis* and *oleraceus;* if the heads of its blossoms close at and remain closed during the night.

Treatment and Antidotes to Severe Ordinary Poisons.

Antidotes in very small doses.

Chloroform and Ether.—Cold affusions on head and neck, and ammonia to nostrils. *Antidote.*—Camphor, petroleum, sulphur.

Toadstools.—(Inedible mushroom). *Antidote.*—Same as for chloroform.

Arsenic or *Fly Powder.*—Emetic; after free vomiting give calcined magnesia freely. If poison has passed out of stomach, give castor oil.
Antidote.—Camphor, nux vomica, ipecacuanha.

Acetate of Lead (Sugar of lead).—Mustard emetic, followed by salts, Large draughts of milk with white of eggs.
Antidote.—Alum, sulphuric acid alike to lemonade, belladonna, strychnine.

Corrosive Sublimate (Bug poison).—White of eggs in 1 quart of cold water, give cupful every two minutes. Induce vomiting without aid of emetics. Soapsuds and wheat flour is a substitute for white of eggs.
Antidote.—Nitric acid, camphor, opium, sulphate of zinc.

Phosphorus Matches.—*Rat Paste.*—Two teaspoonfuls of calcined magnesia, followed by mucilaginous drinks. *Antidote.*—Camphor, coffee, nux vomica.

Carbonic Acid (Charcoal fumes), *Chlorine*, *Nitrous Oxide*, or *Ordinary Gas.*—Fresh air, artificial respiration, ammonia, ether, or vapor of hot water.
Antidote.—Camphor, coffee, nux vomica.

Belladonna (Nightshade).—Emetic and stomach pump, morphine and strong coffee. *Antidote.*—Camphor.

Opium.—Stomach pump or emetic of sulphate of zinc, 20 or 30 grains, or mustard or salt. Keep patient in motion. Cold water to head and chest.
Antidote.—Strong coffee freely and by injection, camphor, ether, and nux vomica.

Strychnine (Nux vomica).—Stomach pump or emetic, chloroform, camphor, animal charcoal, lard, or fat.
Antidote.—Wine, coffee, camphor, opium freely, and alcohol in small doses.

Vegetable Poisons.—As a rule, an emetic of mustard and drink freely of warm water.

* Spreads its leaves about 9 A. M., and they remain open until noon.

Veterinary.

Horses.—*Cathartic Ball.*—Cape Aloes, 6 to 10 drs.; Castile Soap, 1 dr.; Spirit of Wine, 1 dr.; Sirup to form a ball. If Calomel is required, add from 20 to 50 grains. During its operation, feed upon mashes and give plenty of water.

Cattle.—*Cathartic.*—Cape Aloes, 4 drs. to 1 oz.; Epsom Salts, 4 to 6 oz.; Ginger, 3 drs. Mix, and give in a quart of gruel. For Calves, one third will be sufficient.

Horses and Cattle.—*Tonic.*—Sulphate of Copper, 1 oz. to 12 drs.; Sugar, .5 oz. Mix, and divide into 8 powders, and give one or two daily in food.

Cordial.—Opium, 1 dr.; Ginger, 2 drs.; Allspice, 3 drs., and Caraway Seeds, 4 drs., all powdered. Make into a ball with sirup, or give as a drench in gruel.

Cordial Astringent Drench, for Diarrhœa, Purging, or Scouring.—Tincture of Opium, .5 oz.; Allspice, 2.5 drs.; powdered Caraway, .5 oz.; Catechu Powder, 2 drs.; strong Ale or Gruel, 1 pint. Give every morning till purging ceases. For Sheep .25 this quantity.

Alterative.—Ethiop's Mineral, .5 oz.; Cream of Tartar, 1 oz.; Nitre, 2 drs. Divide into from 16 to 24 doses, one morning and evening in all cutaneous diseases.

Diuretic Ball.—Hard Soap and Turpentine, each 4 drs.; Oil of Juniper, 20 drops; and powdered Resin to form a ball.

For Dropsy, Water Farcy, Broken Wind, or Febrile Diseases, add to above, Allspice and Ginger, each 2 drs. Divide into 4 balls, and give one morning and evening.

Alterative or Condition Powder.—Resin and Nitre, each 2 oz.; levigated Antimony, 1 oz. Mix for 8 or 10 doses, and give one morning and evening. When given to Cattle, add Glauber Salts, 1 lb.

Fever Ball.—Cape Aloes, 2 oz.; Nitre, 4 oz.; Sirup to form a mass. Divide into 12 balls, and give one morning and evening until bowels are relaxed; then give an Alterative Powder or Worm Ball.

Hoof Ointment.—Tar and Tallow, each 1 lb.; Turpentine .5 lb. Melt and mix.

Dogs.—*Cathartic.*—Cape Aloes, .5 dr. to 1 oz.; Calomel, 2 to 3 grs.; Oil of Caraway, 6 drops; Sirup to form a ball. Repeat every 5 hours till it operates.

Emetic.—2 to 4 grs. of Tartar Emetic in a meat ball, or a teaspoonful or two of common salt. Give twice a week if required.

Distemper Powder.—Antimonial Powder, 2, 3, or 4 grs.; Nitre, 5, 10, or 15 grs.; powdered Ipecacuanha, 2, 3, or 4 grs. Make into a ball, and give two or three times a day. If there is much cough, add from .5 gr. to 1 gr. of Digitalis, and every 3 or 4 days give an Emetic.

Mange Ointment.—Powdered Aloes, 2 drs.; White Hellebore, 4 drs.; Sulphur, 4 oz.; Lard, 6 oz.—*Red Mange*, add 1 oz. of Mercurial Ointment, and apply a muzzle.

NOTE.—Physic, except in urgent cases, should be given in morning, and upon an empty stomach; and, if required to be repeated, there should be an interval of several days between each dose.

Age of Horses.

To Ascertain a Horse's Age.

A foal of six months has six grinders in each jaw, three in each side, and also six nippers or front teeth, with a cavity in each.

At age of one year, cavities in front teeth begin to decrease, and he has four grinders upon each side, one of permanent and remainder of milk set.

At age of two years he loses the first milk grinders above and below, and front teeth have their cavities filled up alike to teeth of horses of eight years of age.

At age of three years, or two and a half, he casts his two front uppers, and in a short time after the two next.

At four, grinders are six upon each side; and, about four and a half, his nippers are permanent by replacing of remaining two corner teeth; tushes then appear, and he is no longer a colt.

At five, a horse has his tushes, and there is a black-colored cavity in centre of all his lower nippers.

At six, this black cavity is obliterated in the two front lower nippers.

At seven, cavities of next two are filled up, and tushes blunted; and at eight, that of the two corner teeth. Horse may now be said to be aged. Cavities in nippers of upper jaw are not obliterated till horse is about ten years old, after which time tushes become round, and nippers project and change their surface.

Distances between Principal Cities of East and West.

In Miles.

Cities.	Boston.	New York.	Phila-delphia.	Balti-more.	Cities.	Boston.	New York.	Phila-delphia.	Balti-more.
Burlington, Ia.	1216	1106	1030	995	Louisville.....	1161	870	794	706
Chicago.......	1009	900	823	802	Memphis......	1438	1247	1171	1083
Cincinnati....	927	743	667	576	Milwaukee....	998	947	908	887
Cleveland.....	671	580	504	483	Omaha.......	1503	1393	1317	1294
Columbus, O..	807	623	547	512	St. Joseph.....	1478	1356	1280	1223
Detroit.......	724	673	682	661	St. Louis......	1212	1050	973	917
Indianapolis..	951	810	735	700	St. Paul.......	1418	1308	1232	1211
Kansas City...	1487	1324	1248	1192	Toledo........	784	693	617	596

Population of Principal Cities (1882).

London.........	3 832 440	Marseilles........	357 530	Stockholm.......	168 770
Paris...........	2 225 910	St. Louis.........	350 518	Brussels.........	161 820
Berlin..........	1 222 500	Warsaw...........	339 400	Cleveland........	160 146
New York......	1 206 299	Baltimore........	332 313	Pittsburgh........	156 389
Vienna.........	1 103 110	Milan.............	321 440	Buffalo...........	155 134
St. Petersburg...	876 570	Amsterdam.......	317 010	Antwerp..........	150 650
Philadelphia....	847 170	Rome.............	300 470	Washington......	147 293
Moscow.........	611 970	Lisbon...........	246 300	Cologne..........	144 770
Constantinople..	600 000	Palermo..........	244 990	Frankfort........	136 820
Chicago.........	583 185	Copenhagen......	234 850	Newark..........	136 508
Brooklyn.......	566 663	San Francisco....	233 959	Venice...........	132 830
Hamburg.......	410 120	Munich..........	230 200	Louisville........	123 758
Naples.........	403 110	Cincinnati........	225 139	Jersey City.......	120 722
Lyons..........	372 890	Bucharest........	221 800	Detroit...........	116 340
Madrid.........	367 280	Dresden..........	220 820	Milwaukee.......	115 587
Boston.........	362 839	New Orleans.....	216 190	Providence.......	104 857
Buda-Pesth.....	360 580	Florence.........	169 000	Rouen...........	104 010

Treatment of Drowning Persons.

Practice adopted by Board of Health, New York.

Place patient face downward, with one of his wrists under his forehead. Cleanse his mouth. If he does not breathe, turn him on his back with shoulders raised on a support. Grasp tongue gently but firmly with fingers covered with end of a handkerchief or cloth, draw it out beyond lips, and retain it in this position.

To Produce and Imitate Movements of Breathing.—Raise patient's extended arms upward to sides of his head, pull them steadily, firmly, slowly, outwards. Turn down elbows by patient's sides, and bring arms closely and firmly across pit of stomach, and press them and sides and front of chest gently but strongly for a moment, then quickly begin to repeat first movement.

Let these two movements be made very deliberately and without ceasing until patient breathes, and let the two movements be repeated about twelve or fifteen times in a minute, but not more rapidly, bearing in mind that to thoroughly fill the lungs with air is the object of first or upward and outward movement, and to expel as much air as practicable is object of second or downward motion and pressure. This artificial respiration should be maintained for forty minutes or more, when the patient appears not to breathe; and after natural breathing begins, let same motion be very gently continued, and give proper stimulants in intervals.

What Else is to be Done, and What is Not to be Done, while the Movements are Being Made.—If help and blankets are at hand, have body stripped, wrapped in blankets, but not allow movements to be stopped. Briskly rub feet and legs, pressing them firmly and rubbing upward, while the movements of the arms and chest are in progress. Apply hartshorn, or like stimulus, or a feather within the nostrils occasionally, and sprinkle or lightly dash cold water upon face and neck. The legs and feet should be rubbed and wrapped in hot blankets, if blue or cold, or if weather is cold.

What to Do when Patient Begins to Breathe.—Give stimulants by teaspoonful two or three times a minute, until beating of pulse can be felt at wrist, but be careful and not give more of stimulant than is necessary. Warmth should be kept up in feet and legs, and as soon as patient breathes naturally, let him be carefully removed to an enclosure, and placed in bed, under medical care. ....

MISCELLANEOUS ELEMENTS.

Earth.

Polar diameter 7899.3 *miles.* Mean density or specific gravity of mass 5.672. Mass 5 272 600 000 000 000 000 000 *tons.* Apparent diameter as seen from Sun 17 seconds.

Sun.

Heat of Sun equal to 322 794 thermal units per minute for each sq. foot of photosphere or solar surface.

Diameter of Sun 882 000 miles, tangential velocity 1.25 miles per second or 4.41 times greater than that of the Earth.

Distance from Earth 91.5 to 92 millions of miles.

Mason and Dixon's Line.

39° 43′ 26.3″ N. mean latitude. 68.895 miles.

Area and Population. (*Behm and Wagner.*)

Divisions.	Area.	Population.	Divisions.	Area.	Population.
	Sq. Miles.			Sq. Miles.	
America	14 491 000	95 495 500	Oceanica	4 500 000	4 031 000
Europe	3 760 000	315 929 000	Greenland, Iceland		82 000
Asia	16 313 000	834 707 000			
Africa	10 936 000	205 679 000	Total	50 000 000	1 455 923 500

Countries.

Austria, Hungary	38 000 000	Germany	43 900 000	India, British	240 298 000
		Great Britain	34 000 000	Canada	3 839 000
China	434 626 000	Russia	65 000 000	Mexico	9 485 000
France	37 000 000	Territories	22 000 000	Brazil	11 106 000
United States	50 000 000	Turkey	8 866 000		
Indians	300 000	" in Asia	16 320 000		

About one thirtieth of whole population are born every year, and nearly an equal number die in same time; making about one birth and one death per second.

Earlier authority estimated population at 1 288 000 000, divided as follows:

Caucasians	360 000 000	Malays and Indo Amer's	177 000 000	Mohammedans	190 000 000
Mongolians	552 000 000			Pagans	300 000 000
Ethiopians	190 000 000	Protestants	80 000 000	Catholics, Rom. & Greek	250 000 000
Asiatics	60 000 000	Israelites	5 000 000		

Descent of Western Rivers.

Slope of rivers flowing into Mississippi from East is about 3 inches per mile; and from West 6 inches.

Mean descent of Ohio River from Pittsburgh to Mississippi, 975 miles, is about 5.2 inches per mile; and that of Mississippi to Gulf of Mexico, 1180 miles, about 2.8 inches.

Transmission of Horse Power.

Largest, and perhaps most successful, wire rope transmission is one at Schaffhausen, at Falls of the Rhine. Here, power of a number of turbines, amounting to over 600 IP, is conveyed across the stream, and thence a mile to a town, where it is distributed and utilized.

At mines of Falun, Sweden, a power of over 100 horses is transmitted in like manner for a distance of three miles.

Acids.

Acetic Acid (Vinegar), acid of *Malt beer*, etc. *Tartaric Acid*, acid of *Grape wine*. *Lactic Acid*, acid of *Milk*, *Millet beer*, and *Cider*.

Manures.

Relative Fertilizing Properties of Various Manures.

Peruvian Guano	1	Horse	.048	Farm-yard	.0298
Human, mixed	.069	Swine	.044	Cow	.0259

Or, 1 lb. guano = 14.5 human, 21 horse, 22.5 swine, 33.5 farm-yard, and 38.5 cow.

Relative Value, Covered and Uncovered, on an Acre of Ground.

Covered	11 tons	1665 lbs. potatoes,	61 lbs. wheat,	215 lbs. straw
Uncovered	7 "	1397 " "	61.5 " "	156 " "

Yield of Oil of Several Seeds.

	Per Cent.		Per Cent.		Per Cent.		Per Cent.		Per Cent.
Poppy..	56 to 63	Castor..	25	Sunflower.	15	Hemp.	14 to 25	Linseed.	11 to 22

Thickness of Walls of Buildings. *(English.)* *(Moleswort.)*

OUTER WALLS.	Maximum Height of Wall.	Width of Footings.	Minimum Width of Walls. Ground Floor.	1st Floor.	2d Floor.	3d Floor.	4th Floor.	5th Floor.	6th Floor.
	Feet.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
1st class dwelling.	85	38.5	21.5	21.5	17.5	17.5	17.5	13	13
2d " "	70	30.5	17.5	17.5	17.5	13	13	13	—
3d " "	52	30.5	17.5	13	13	13	13	—	—
4th " "	38	21.5	13	13	8.5	8.5	—	—	—
PARTY WALLS.									
1st class dwelling.	85	38.5	21.5	21.5	17.5	17.5	17.5	13	13
2d " "	70	30.5	17.5	17.5	17.5	13	13	13	—
3d " "	52	30.5	17.5	13	13	13	8.5	—	—
4th " "	38	21.5	13	8.5	8.5	8.5	—	—	—

If walls are more than 70 feet in length, those of lower stories must be widened by half a brick.

Warehouses 1st Class.	Minimum Width of Wall.	Warehouses 2d Class.	Minimum Width of Wall.
For a height of 36 feet from topmost ceiling............	Ins. 17.5	For a height of 22 feet below topmost ceiling............	Ins. 13
For a height of 40 feet lower..	21.5	For a height of 36 feet lower..	17.5
" " 24 feet lower..	26	" " 8 feet lower..	21.5
For footings................	43.5	For footings................	34.5
3d Class.		**4th Class.**	
For a height of 28 feet below topmost ceiling............	13	For a height of 9 feet below topmost ceiling............	8.5
For a height of 16 feet lower..	17.5	For a height of 13 feet below..	13
For footings................	30.5	For footings................	21.5

Wooden Roofs. *(English.)*

Span in Feet.	Principal Beam.	Tie Beam.	King Posts.	Queen Posts.	Small Queens.	Straining Beam.	Struts.
20	4 × 4	9 × 4	4 × 4	—	—	—	3 × 3
25	5 × 4	10 × 5	5 × 5	—	—	—	5 × 3
30	6 × 4	11 × 6	6 × 6	—	—	—	6 × 3
35	5 × 4	11 × 4	—	4 × 4	—	7 × 4	4 × 2
45	6 × 5	13 × 6	—	6 × 6	—	7 × 6	5 × 3
50	8 × 6	13 × 8	—	8 × 8	8 × 4	9 × 6	5 × 3
55	8 × 7	14 × 9	—	9 × 8	9 × 4	10 × 6	5.5 × 3
60	8 × 8	15 × 10	—	10 × 8	10 × 4	11 × 6	6 × 3

Mineral Constituents absorbed or removed from an Acre of Soil by several Crops. *(Johnson.)*

CROPS.	Wheat, 25 bushels.	Barley, 40 bushels.	Turnips, 20 tons.	Hay, 1.5 tons.	CROPS.	Wheat, 25 bushels.	Barley, 40 bushels.	Turnips, 20 tons.	Hay, 1.5 tons.
	Lbs.	Lbs.	Lbs.	Lbs.		Lbs.	Lbs.	Lbs.	Lbs.
Potassa......	29.6	17.5	47.1	38.2	Sulphuric Acid...	10.6	2.7	13.3	9.2
Soda.........	3	5.2	8.2	12					
Lime.........	12.9	17	29.9	44.5	Chlorine.....	2	16	3.6	4.1
Magnesia....	10.6	9.2	19.7	7.1	Silica........	118.1	129.5	247.8	78.2
Oxide of Iron.	2.6	2.1	7.1	.6	Alumina.....	—	2.4	—	—
Phosphoric Acid....	20.6	25.8	46.3	15.1	Total....	210	213	423	209

Average Quantity of Tannin in Several Substances.

(*Morfit.*)

Catechu.	Per Cent.	*Oak.*	Per Cent.	*Sumac.*	Per Cent.
Bombay	55	Young, inner b'k	15.2	Sicily and Malaga	16
Bengal	44	" entire b'k	6	Virginia	10
Kino	75	" spring-cut bark	22	Carolina	5
Nutgalls.		" root bark	8.9	*Willow.*	
Aleppo	65	*Chestnut.*		Inner bark	16
Chinese	69	Amer. rose, bark	8	Weeping	16
Oak.		Horse, "	2	*Sycamore* bark	16
Old, inner bark	14.2 / 21	*Sassafras*, root bark	58	*Tan shrub* "	13
				Cherry-tree	24

Alder bark.................. 36 per cent.

To Convert Chemical Formulæ into a Mathematical Expression.

Rule.—Multiply together equivalent and exponent of each substance, and product will give proportion in compound by weight. Divide 1000 by sum of their products, and multiply this quotient by each of these products, and products will give respective proportion of each part by weight in 1000.

Example.—Chemical formula for alcohol is $C_4 H_6 O_2$. Required their proportional parts by weight in 1000?

$$\left.\begin{array}{lll} C_4 \text{ Carbon} & = 6.1 \times 4 = 24.4 \\ H_6 \text{ Hydrogen} & = 1 \times 6 = 6 \\ O_2 \text{ Oxygen} & = 8 \times 2 = 16 \end{array}\right\} \times 21.55 \left\{\begin{array}{l} 525.82 \\ 129.3 \\ 344.8 \end{array}\right\} \text{by weight.}$$

$$1000 \div 46.4 = 21.55 \quad 999.92$$

Elementary Bodies, with their Symbols and Equivalents.

Body.	Symb.	Equiv.	Body.	Symb.	Equiv.	Body.	Symb.	Equiv.
Aluminium	Al	13.7	Gold	Au	196.6	Platinum	Pt	98.8
Antimony	Sb	64.6	Hydrogen	H	1	Potassium	K	39.2
Arsenic	As	37.7	Iodine	I	126.5	Rhodium	R	52.2
Barium	Ba	68.6	Iridium	Ir	98.5	Selenium	Se	40
Bismuth	Bi	71.5	Iron	Fe	28	Silicon	Si	22
Boron	B	11	Lead	Pb	103.7	Silver	Ag	108.3
Bromine	Br	78.4	Lithium	L	7	Sodium	Na	23.5
Cadmium	Cd	55.8	Magnesium	Mg	12.7	Strontium	Sr	43.8
Calcium	Ca	20.5	Manganese	Mn	26	Sulphur	S	16.1
Carbon	C	6.1	Mercury	Hg	200	Tellurium	Te	64.2
Chlorine	Cl	35.5	Molybdenum	Mo	47.9	Tin	Sn	58.9
Chromium	Cr	26.2	Nickel	Ni	29.5	Titanium	Ti	24.5
Cobalt	Co	29.5	Nitrogen	N	14.2	Tungsten	W	92
Columbium	Ta	184.8	Osmium	Os	99.7	Uranium	U	60
Copper	Cu	31.7	Oxygen	O	8	Yttrium	Y	32
Fluorine	F	18.7	Palladium	Pd	53.3	Zinc	Zn	32.3
Glucinum	G	6.9	Phosphorus	P	15.9	Zirconium	Zr	34

Analysis of certain Organic Substances by Weight.

Body.	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Body.	Carbon.	Hydrogen.	Oxygen.	Nitrogen.
Albumen	52.9	7.5	23.9	15.7	Morphine	72.3	6.4	16.3	5
Alcohol	52.7	12.9	34.4	—	Narcotine	65	5.5	27	2.5
Atmospheric air	—	—	77	23	Oil, Castor	74	10.3	15.7	—
Camphor	73.4	10.7	15.6	.3	Linseed	76	11.3	12.7	—
Caoutchouc	87.2	12.8	—	—	Spermaceti	78	11.8	10.2	—
Casein	59.8	7.4	11.4	21.4	Quinine	75.8	7.5	8.6	8.1
Fibrin	53.4	7	19.7	19.9	Starch	44.2	6.7	49.1	—
Gelatine	47.9	7.9	27.2	17	Strychnine	76.4	6.7	11.1	5.8
Gum	42.7	6.4	50.9	—	Sugar	42.2	6.6	51.2	—
Hordein	44.2	6.4	47.6	1.8	Tannin	52.6	3.8	43.6	—
Lignin	52.5	5.7	41.8	—	Urea	18.9	9.7	26.2	45.2

Dilution Per Cent. Necessary to Reduce Spirituous Liquors.

Water to be added to 100 volumes of spirit when of following strength:

Strength Required.	90	85	80	75	70	65	60	55	50
Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
85	5.9	—	—	—	—	—	—	—	—
80	12.5	6.3	—	—	—	—	—	—	—
75	20	13.3	6.7	—	—	—	—	—	—
70	28.6	21.4	14.3	7.1	—	—	—	—	—
65	38.5	30.8	23.1	15.4	7.7	—	—	—	—
60	50	41.7	33.3	25	16.7	8.3	—	—	—
55	63.6	54.5	45.5	36.4	27.4	18.2	9.1	—	—
50	80	70	60	50	40	30	20	10	—
40	125	112.5	100	87.5	75	62.5	50	37.5	25
30	200	183.3	166.7	150	133.3	116.7	100	83.3	66.7

ILLUSTRATION.—100 volumes of spirituous liquor having 90 per cent. of spirit contains: alcohol 90, water 10, = 100.

To reduce it to 30 per cent. there is required 200 volumes of water.

Hence $200 + 10 = 210$, and $\frac{90}{210} = \frac{30}{70} = \frac{30 \text{ spirit,}}{70 \text{ water,}}$ or 30 *per cent.*

Proportion of Alcohol Per Cent.

In 100 Parts of Spirit, by Weight or Volume, at 60°.

Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.
0	1	20	.972	50	.918	80	.848
5	.991	30	.958	60	.896	90	.823
10	.984	40	.94	70	.872	100	.794

In 100 Parts of Alcohol and Water, by Weight, at 60°.

Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.	Alcohol.	Specific Gravity.
0	1	1.99	.996	5.01	.991	7.99	.987
.53	.999	3.92	.994	6.02	.99	9.05	.985
1.02	.998	4.02	.993	7.02	.988	10.07	.984

Tides of Atlantic and Pacific Oceans at Isthmus of Panama. (*Totten.*)

Atlantic, Navy Bay.—Highest tide 1.5 *feet;* lowest 63 *feet.*

Pacific, Panama Bay.—Highest tide 17.72 to 21.3 *feet;* lowest 9.7 *feet.*

Areas of U. S. Coal Fields.

STATE.	Sq. Miles.	STATE.	Sq. Miles.	STATE.	Sq. Miles.
Illinois	44 000	Ohio	11 900	Tennessee	4300
Virginia	21 000	Indiana	7 700	Alabama	3400
Pennsylvania*	15 437	Missouri†	6 000	Maryland	550
Kentucky	13 500	Michigan†	5 000	Georgia	150

* Bituminous and Anthracite. † Anthracite.

Extremes of Heat in Various Countries.

England .... 96°	Denmark }	Greece ...... 105°	Egypt ...... 116.1°
France ..... 106.5°	Sweden } .. 99.5°	Italy .... 104°	Africa ...... 133.4°
Holland } . 102°	Norway }	Spain .... 102°	Asia .... 120°
Belgium }	Russia ...... 102°	Tunis .... 112.5°	Suez .... 126.5°

Germany .... 103° | Manilla ...... 113.5°

Extremes of temperature upon the Earth 240°.

Extremes of Cold in Various Countries.

England ...—5°	Denmark }	France ....—24°	Italy ....—10°
Holland } —12°	Sweden } —67°	Russia ....—46°	Fort Reliance, N. A..—70°
Belgium }	Norway }	Germany ..—32°	Semipalatinsk, " ..—76°

Mean Temperatures of Various Localities.

London	51°	Rome	60°	Poles	—13°	Polar Regions	36°
Edinburgh	41°	Equator	82°	Torrid Zone	75°	Globe	50°

Line of Perpetual Congelation, or Snow Line.

Latitude.	Height.	Latitude.	Height.	Latitude.	Height.	Latitude.	Height.
°	Feet.	°	Feet.	°	Feet.	°	Feet.
10	14 764	30	11 484	50	6334	70	1278
15	14 760	35	10 287	55	5020	75	1016
20	13 478	40	9 000	60	3818	80	451
25	12 557	45	7 670	65	2230	85	327

At the Equator it is 15 260 feet; at the Alps 8120 feet; and in Iceland 3084 feet. At Polar Regions ice is constant at surface of the Earth.

Limits of Vegetation in Temperate Zone.

The Vine ceases to grow at about 2300 feet above level of the sea. Indian Corn at 2800, Oak at 3350, Walnut at 3600, Ash at 4800, Yellow Pine at 6200, and Fir at 6700.

Periods of Gestation and Number of Young.

	Weeks.	No.		Weeks.	No.		Weeks.	No.		Weeks.	No.
Elephant	100	1	Cow	41	1	Sheep	21	2	Dog	9	6
Horse	43 / 50	1	Buffalo	40	1	Goat	22	2	Fox	9	5
			Stag	36	1	Beaver	17	3	Cat	8	6
Camel	45	1	Bear	30	2	Pig	17	12	Rat	5	8
Ass	43	1	Deer	24	2	Wolf	10	5	Squirrel	4	6
			Rabbit	4	6	Guinea Pig	3	3			

Periods of Incubation of Birds.

Swan, 42 days; Parrot, 40 days; Goose and Pheasant, 25 days; Duck, Turkey, and Peafowl, 28 days; Hens of all gallinaceous birds, 21 days; Pigeon and Canary, 14 days. Temperature of incubation is 104°.

Ages of Animals, etc.

Whale, estimated 1000 years; Elephant, 400; Swan, 300; Camel, 100; Eagle, 100; Raven, 100; Tortoise, 100 to ——; Lion, 70; Dolphin, 30; Horse, 30; Porpoise, 30; Bear, 20; Cow, 20; Deer, 20; Rhinoceros, 20; Swine, 20; Wolf, 20; Cat, 15; Fox, 15; Dog, 15; Sheep, 10; Hare, Rabbit, and Squirrel, 7.

Relative Weights of Brain.

Man, 154.33; Mammifers, 29.88; Birds, 26.22; Reptiles, 4.2; Fish, 1.

Buoyancy of Casks.

Buoyancy of a cask in fresh water in lbs. = 11.97 times volume of it in U. S. gallons and 10 times in Imperial gallons, less weight of cask.

Transportation of Horses and Cattle.

Space required on board of a Marine Transport is: for Horses, 30 ins. by 9 feet; Beeves, 32 ins. by 9 feet. Provender required *per diem* is: for Horses, Hay, 15 lbs.; Oats, 6 quarts; Water, 4 gallons. Beeves, Hay, 18 lbs.; Water, 6 gallons.

Rock and Earth Excavation and Embankment.

Number of Cube Feet of various Earths in a Ton.

Loose Earth	24	Clay	18.6	Clay with Gravel	14.4
Coarse Sand	18.6	Earth with Gravel	17.8	Common Soil	15.6

The volume of Earth and Sand in embankment exceeds that in a primary excavation in following proportions:

Rock, large	1.5	Rock, ballast	1.2	Clay	.111
" medium	1.25	Sand	.143	Gravel	.09

Clay and Earth will subside about .12.

Hills or Plants in an Area of One Acre.

From 1 to 40 feet apart from centres.

Feet apart.	No.	Feet apart.	No.	Feet apart.	No.	Feet apart.	No.
1	43 560	5	1742	9	538	16	171
1.5	19 360	5.5	1440	9.5	482	17	151
2	10 890	6	1210	10	435	18	135
2.5	6 969	6.5	1031	10.5	361	20	108
3	4 840	7	889	12	302	25	69
3.5	3 556	7.5	775	13	258	30	48
4	2 722	8	680	14	223	35	35
4.5	2 151	8.5	692	15	193	40	27

Number of several Seeds in a Bushel, and Number per Square Foot per Acre.

	No.	Sq. Foot.		No.	Sq. Foot.
Timothy........	41 823 360	960	Rye............	888 390	20.4
Clover..........	16 400 960	376	Wheat..........	556 290	12.8

Volumes.

Permanent gases, as air, etc., are diminished in their volume in a ratio direct with that of pressure applied to them. With vapor, as steam, etc., this rule is varied in consequence of presence of the temperature of vaporization.

Minerals.

Relative Hardness of some Minerals.

Talc............	1	Barytes.......	3.5	Opal..........	6	Emerald......	8
Gypsum..........	2	Fluor-spar....	4	Quartz.........	7	Topaz.........	8
Mica............	2.5	Feldspar	6	Tourmalin	7	Ruby..........	9
Carbonate of lime.	3	Lapis Lazuli .	6	Garnet.......	7.5	Diamond......	10

Weight of Diamonds.

	Carats.		Carats.		Carats.
Mattam.............	367	Regent or Pitt.....	136.75	Dresden..........	76.5
Grand Mogul*.......	279.9	Star of the South†..	125	Saucy............	53.5
Orloff...............	194.25	Koh-i-Noor‡.......	106.06	Eugenie, brilliant .	51
Florentine, brilliant .	139.5	Piggott	82.25	Hope (blue).......	48.5
Crown of Portugal...	138.5	Napac	78.625	Polar Star	40.25

* Rough 900. † Rough 254.5. ‡ Originally 793.

Heat of the Sun.

Sir Isaac Newton.............	3 138 740°	Waterston	16 000 000°
Capt. John Ericsson...........	4 909 860°	Soret.....................	10 443 323°

Sundry others ranging from 2520° to 183 600°.

Moon.—Distance of Moon from Earth 237 000 miles.

Frigorific Mixture.

Lowest temperature yet procured. Faraday obtained 166° by evaporation of a mixture of solid carbonic acid and sulphuric ether.

Current of Rivers.

A fall of .1 of an inch in a mile will produce a current in rivers.

Sandstones.

Structures of sandstone erected in England in 12th century are yet in good condition.

Canal Transportation.

Erie Canal and Hudson River.—From Buffalo to New York, 495 miles, cost of transportation 2.46 mills per ton (inclusive of tolls) per mile. Transportation of wheat costs when it reaches New York 4.72 cents per bushel, and .61 cents per bushel for elevating and trimming.

Towing.—Erie Canal.—Four mules will tow 230 tons of freight down and 100 tons back, involving a period of 30 days, at a cost of 8 cents per mile for a course of 690 miles.

Matter.

Unit of the Physicist is a molecule, and a mass of matter is composed of them, having same physical properties as parent mass.

It exists in three forms, known as solid, liquid, and gaseous. Solids have individuality of form, and they press downward alone. Liquids have not individuality of form, except in spherical form of a drop, and they press downward and sideward. Gases are wholly deficient in form, expanding in all directions, and consequently they press upward, downward, and sideward.

Liquids are compressible to a very moderate degree. Water has been forced through pores of silver, and it may be compressed by a pressure of one pound per square inch to the 3 300 000th part of its volume.

Gases may be liquefied by pressure or by reduction of their temperature.

Combustible matter (as coal) may be burned, a structure (as a house) may be destroyed as such, and the fluid (of an ink) may be evaporated, yet the matter of which coal and house were composed, although dissipated, exists, and the water and coloring matter of the ink are yet in existence.

Spaces between the particles of a body are termed pores.

All matter is porous. Polished marble will absorb moisture, as evidenced in its discoloration by presence of a colored fluid, as ink, etc.

Silica is the base of the mineral world, and *Carbon* of the organized.

Minuteness of Matter.

A piece of metal, stone, or earth, divided to a powder, a particle of it, however minute, is yet a piece of the original material from which it was separated, retaining its identity, and is termed a molecule.

It is estimated there are 120 000 000 corpuscles in a drop of blood of the musk-deer.

Thread of a spider's web is of a cable form, is but one sixth diameter of a fibre of silk, and 4 miles of it is estimated to have a weight of but 1 grain

One imperial gallon (277.24 cube ins.) of water will be colored by mixture therein of a grain of carmine or indigo.

A grain of platinum can be drawn out the length of a mile.

Film of a soap and-water bubble is estimated to be but the 300 000th part of an inch in thickness.

It is computed that it would require 12 000 of the insect known as the twilight monad to fill up a line one inch in length.

A drop of water, or a minute volume of gas, however much expanded—even to the volume of the Earth—would present distinct molecules.

Gold leaf is the 280 000th part of an inch in thickness.

A thread of silk is 2500th of an inch in diameter.

A cube inch of chalk in some places in vicinity of Paris contains 100 000 of shells of the foraminifera.

There are animalcules so small that it requires 75 000 000 of them to weigh a grain.

Velocity, Weight, and Volume of Molecules.

Velocity.—Collisions among the particles of *Hydrogen* are estimated to occur at the rate of 17 million million-million per second, and in *Oxygen* less than half this number.

Weight.—A million million million-million molecules of *Hydrogen* are estimated to weigh but 60 grains.

Volume.—19 million-million-million molecules of *Hydrogen* have a volume of .061 cube ins. *Diameter.*—Five millions in a line would measure but .1 inch.

Charcoal, Alcohol.

Charcoal as yet has not been liquefied, nor has Alcohol been solidified.

Metals.

Metals have five degrees of lustre—*splendent*, *shining*, *glistening*, *glimmering*, and *dull.*

All metals can be vaporized, or exist as a gas, by application to them of their appropriate temperature of conversion.

Repeated hammering of a metal renders it brittle; reheating it restores its tenacity.

Repeated melting of iron renders it harder, and up to twelfth time it becomes stronger.

Platinum is the most ductile of all metals.

Impenetrability.

Impenetrability expresses the inability of two or more bodies to occupy same space at same time.

A mixture of two or more fluids may compose a less volume than that due to sum of their original volume, in consequence of a denser or closer occupation of their molecules. This is evident in the mixture of alcohol and water in the proportion of 16.5 volumes of former to 25 of latter, when there is a loss of one volume.

Elasticity.

Elasticity is the term for the capacity of a body to recover its former volume, after being subjected to compression by percussion or deflection.

Glass, ivory, and steel are the most elastic of all bodies, and clay and putty are illustrations of bodies almost devoid of elasticity. Caoutchouc (India rubber) is but moderately elastic; it possesses contractility, however, in a great degree.

Momentum.

Momentum is quantity of motion, and is product of mass and its velocity. Thus, the momentum of a cannon-ball is product of its velocity in feet per second and its weight, and is denominated *foot-pounds.*

A foot-pound is the power that will raise one pound one foot.

Sound.

Velocity of sound is proportionate to its volume; thus, report of a blast with 2000 lbs. of powder passed 967 feet in one second, and one of 1200 lbs. 1210 feet. It passes in water with a velocity of 4708 feet per second. Conversation in a low tone has been maintained through cast-iron water pipes for a distance of 3120 feet, and its velocity is from 4 to 16 times greater in metals and wood than air.

Light.

Sun's rays have a velocity of 185 000 miles per second, equal to 7.5 times around the Earth.

Color Blindness

Is absence of elementary sensation corresponding to red.

Luminous Point.

To produce a visual circle, a luminous point must have a velocity of 10 feet in a second, the diameter not exceeding 15 ins.

All solid bodies become luminous at 800 degrees of heat.

Mirage.

When air near to surface of Earth becomes so highly heated, as upon a sandy plain, that its density within a defined distance from it increases upwards, a line of vision directed obliquely downwards will be rendered by refraction, gradually increasing, more and more nearly horizontal as it advances, until its direction is so great as to produce a total reflection, and the reflected ray then, by successive refractions, is gradually elevated until it meets the eye of the observer.

Looming is inverted mirage, frequently seen over calm water, and is effect of lower or surface stratum of air being colder than that above it.

Snow Flakes.

96 forms of snow flakes have been observed.

Melted Snow

Produces from .25 to .125 of its bulk in water.

Strength of Ice.

Two inches thick will support men in single file on planks 6 feet apart; 4 inches will support cavalry, light guns, and carts; and 6 inches wagons drawn by horses.

Temperature.

Sulphuric acid and water produce a much greater proportionate contraction than alcohol and water. Both of these mixtures, however low their temperature, produce heat which is in a direct proportion to their diminution in volume.

At the depth of 45 feet, the temperature of the Earth is uniform throughout the year.

Temperature of Earth increases about 1° for every 50 to 60 feet of depth, and its crust is estimated at 30 miles.

A body at Equator weighs two hundred and eighty-nine parts less than at the Poles.

Colors for Drawings.

Material.	Colors.	Material.	Colors.
Brass	Gamboge.	Granite	Indian Ink, light.
Brick	Carmine.	Lead........	" " and Prussian blue.
Cast Iron.	Neutral tint.	Steel........	Light blue and Lake.
Clay	Burned umber.	Water.......	Cobalt or Verdigris.
Earth	" " light.	Woods	Burned Sienna, deep and light, for dark and light wood.
Concrete .	Sepia with dark spots.		
Copper...	Lake and Bur'd Sienna.	Wr'ght Iron .	Prussian Blue, light.

Birds and Insects.—(*M. De Lacy.*)

Elements of Flight.—Resistance of air to a body in motion is in ratio of surface of body and as square of its velocity.

Wing Surface.—Extent or area of winged surface is in an inverse ratio to weight of bird or insect.

A Stag beetle weighs 460 times more than a Gnat, and has but one fourteenth of its wing surface; 150 times more than a Lady Bird (bug), and has but one fifth. An Australian Crane weighs 339 times more than a sparrow, and has but one seventh; 3 000 000 times more than a Gnat, and has but one hundred and fortieth. A Stork weighs eight times more than a Pigeon, and has but one half. A Pigeon weighs ten times more than a Sparrow, and has but one half; 97 000 times more than a Gnat, and has but one fortieth.

A resisting surface of 30 sq. yards will enable a man of ordinary weight to descend safely from a great elevation.

Strength of Insects.—Insects are relatively strongest of all animals. A Cricket can leap 80 times its length, and a Flea 200 times.

Application for Stings and Burns.

Sting of Insects.—Ammonia, or Soda moistened with water, and applied as a paste.

Burns.—Hot alcohol or turpentine, and afterwards bathed with lime water and sweet oil. Cold water not to be applied.

To Preserve Meat.

Meat of any kind may be preserved in a temperature of from 80° to 100°, for a period of ten days, after it has been soaked in a solution of 1 pint of salt dissolved in 4 gallons of cold water and .5 gallon of a solution of bisulphate of calcium.

By repeating this process, preservation may be extended by addition of a solution of gelatin or white of an egg to the salt and water.

To Detect Starch in Milk.

Add a few drops of acetic acid to a small quantity of milk; boil it, and after it has cooled filter the whey. If starch is present, a drop of iodine solution will produce a blue tint.

This process is so delicate that it will show the presence of a milligram of starch in a cube centimeter of whey (1 grain of starch in 2.16 fluid-ounces).

Retaining Walls of Iron Piles.

Sheet Piles.—7 feet from centres, 18 ins. in width and 2 ins. in thickness, strengthened with 2 *ribs* 8 ins. in depth.

Plates.—7 feet in length by 5 feet in width and 1 inch in thickness, with one diagonal feather 1 by 6 ins.

Tie-rods 2 ins. in diameter.

Stone Sawing.

Diamond Stone Sawing.—(*Emerson.*) Alabama marble 6 feet × 2.5 feet in 22 minutes = 41 sq. feet per hour.

Wood Sawing.

7722 feet of poplar, board measure, from 9 round logs in 1 hour. Engine 12 ins diameter by 24 ins. stroke.

Cost of Dredging.

Actual cost, if on an extended work, inclusive of Delivery, if dredging into or on a vessel alongside of dredger.—(Trautwine.)

Labor at $ 1 per day and Repairs of Plant included.

Depth.	Cents.	Depth.	Cents.	Depth.	Cents.	Depth.	Cents.
Feet.	Cube Yards.	Feet.	Cube Yards.	Feet.	Cube Yards.	Feet.	Cube Yards.
10	6	20	8	25	10	35	18
15	7	22	9	30	13	40	25

Discharge of Scows or Camels.—Towing .25 mile 4 cents per cube yard, .5 mile 6 cents, .75 mile 8 cents, and 1 mile 10 cents.

NOTE. — A *Scow* is a flat-bottomed vessel or boat. A *Camel* is a shallow, flat-bottomed and decked vessel, designed for the transportation of heavy freight or the sustaining of attached bodies, as a vessel, by its buoyancy.

Dredging.

A steam dredge will raise 6 cube yards, or 8.5 tons, per hour per IP.

Metal Boring and Turning.

BORING.—*Cast iron.*—Divide 25 by the diameter of the cylinder in inches for the revolutions per minute.

Wrought iron.—The speed is one fourth to one fifth greater than for cast iron.

Brass.—The speed is about twice that for cast iron.

TURNING.—*Cast iron.*—The speed is twice that of boring.

Wrought iron.—The speed is one fourth to one fifth greater than that for cast iron.

Brass.—The speed is twice that of boring.

Vertical boring.—The speed may be twice that of horizontal boring.

The *feed* depends upon the stability of the machine and depth of the cut.

Well Boring.

At Coventry, Eng., 750 000 galls. of water per day are obtained by two borings of 6 and 8 ins., at depths of 200 and 300 feet.

At Liverpool, Eng., 3 000 000 galls. of water per day are obtained by a bore 6 ins. in diameter and 161 feet in depth.

This large yield is ascribed to the existence of a *fault* near to it, and extending to a depth of 484 feet.

At Kentish Town, Eng., a well is bored to the depth of 1302 feet.

At Passy, France, a well with a bore of 1 meter in diameter is sunk to a depth of 1804 feet, and for a diameter of 2 feet 4 ins. it is further sunk to a depth of 109 feet 10 ins., or 1903 feet 10 ins., from which a yield of 5 582 000 galls. of water are obtained per day.

Tempering Boring Instruments.

Heat the tool to a blood-red heat; hammer it until it is nearly cold; reheat it to a blood-red heat, and plunge it into a mixture of 2 oz. each of vitriol, soda, sal-ammoniac, and spirits of nitre, 1 oz. of oil of vitriol, .5 oz. of saltpetre, and 3 galls. of water, retaining it there until it is cool.

Circular Saws.

Revolutions per Minute.—8 ins. 4500, 10 ins. 3600, and 36 ins. 1000.

Masonry.

Concrete or Beton should be thrown, or let fall from a height of at least 10 feet, or well beaten down.

The average weight of brickwork in mortar is about 102 lbs. per cube foot.

Plastering.

In measuring Plasterers' work all openings, as doors, windows, etc., are computed at one half of their areas, and cornices are measured upon their extreme edges, including that cut off by mitring.

Glazing.

In Glaziers' work, oval and round windows are measured as squares.

Corn Measure.

Two cube feet of corn in ear will make a bushel of corn when shelled.

Tenacity of Iron Bolts in Woods.

Diameter 1.125 ins. and 12 ins. in length required for Hemlock 8 tons, and for Pine 6 tons to withdraw them.

Length of Gun Barrels. (*C. T. Coathupe.*)

The length of the barrel of a gun, to shoot well, measured from vent-hole, should not be less than 44 times diameter of its bore, nor more than 47.

Hay and Straw.

Hay, loose, 5 lbs. per cube foot. Ordinarily pressed, as in a stack or mow, 8 lbs. Close pressed, as in a bale, 12 to 14 lbs.

Ordinarily pressed, as in a wagon load, 450 to 500 cube feet will weigh a ton.

Straw in a bale 10 to 12 lbs. per cube foot.

Natural Powers.

Sun.—The power or work performed by the Sun's evaporation is estimated at 90 000 000 000 HP.

Niagara.—Volume of water discharged over the falls is estimated at 33 000 000 tons per hour, and the entire fall from Lake Erie at Buffalo to Lake Ontario is 323.35 feet.

Velocity of Stars.

According to computation of Mr. Trautwine a Star passes a range in 3′ 55.91″ less time each day.

Service Train of a Quartermaster.

Quartermaster's train of an army averages 1 wagon to every 24 men; and a well-equipped army in the field, with artillery, cavalry, and trains, requires 1 horse or mule, upon the average, to every 2 men.

Tides.

The difference in time between high water averages about 49 minutes each day.

Atlantic and Pacific Oceans.—Rise and fall of tide in Atlantic at Aspinwall 2 feet, in Pacific at Panama 24 feet.

Dimensions of Drawings for Patents.

United States, 8.5 × 12 inches.

Latitude.

One minute of latitude, mean level of Sea, nearly 6076 feet = 1.1508 Statute miles.

Artesian Well.

White Plains, Nev., Depth 2500 feet.

Foundation Piles.

A pile, if driven to a fair refusal by a ram of 1 ton, falling 30 feet, will bear 1 ton weight for each sq. foot of its external or frictional surface, or a safe load of 750 lbs. per sq. foot of surface.

Earth.

Density of its mass 5.67.

Tripolith.

A new building material, compounded of Coke, Sulphate of Lime, and Oxide of Iron. It has increased tensile strength after exposure to the air, being much in excess of that of lime and cement.

Gas and Electric Light.

Gas light of 16 candle power costs 5 cent per hour; Electric, 4.15 cents.

Niagara.

Discovered, 1678. Falls have receded 76 feet in 175 years. Height, American Falls, 164 feet; Horseshoe, 158 feet.

Suspension Bridges.

Lengths of Spans in Feet.

You-Man, China	330	Niagara	822
Schuylkill (Phila.)	342	Lewistown and Queenstown	1040
Hammersmith, Eng.	422	Cincinnati	1057
Pesth (Danube)	660	Niagara Falls	1280

New York and Brooklyn, 930, 1595.5, and 930; clear height of Bridge above high water at 90°, 135 feet.

U. S. ENSIGN, PENNANTS, AND FLAGS.

Ensign.—*Head* (*Depth*, or *Hoist*).—Ten nineteenths of its length.

Field.—Thirteen horizontal stripes of equal breadth, alternately red and white, beginning with red.

Union.—A blue field in upper quarter, next the head, .4 of length of field, and seven stripes in depth, with white stars ranged in equidistant, horizontal, and vertical lines, equal in number to number of States of the Union.

Pennants (Narrow).—*Head.*—6.24 ins. to a length of 70 feet; 5.04 ins. to a length of 40 feet; 4.2 ins. to a length of 35 feet. *Night*, 3.6 ins. to a length of 20 feet, and 3 ins. to a length of 9 feet.—*Boat*, 2.52 ins. to a length of 6 feet.

Union.—A blue field at head, one fourth the length, with 13 white stars in a horizontal line. *Field.*—A red and white stripe uniformly tapered to a point, red uppermost. *Night and Boat Pennants.*—Union to have but 7 stars.

Union Jack.—Alike to the Union of an Ensign in dimensions and stars.

Flags.—President.—Rectangle, with arms of the U. S. in centre of a blue field.

Secretary of Navy.—Rectangle, with a vertical white foul anchor in centre of a blue field.

Admiral.—Rectangle, with 4 white stars in centre of a blue field, set as a square.

Vice-Admiral.—Same as Admiral's, with 3 white stars set as an equilateral triangle.

Rear-Admiral.—Same as Admiral's, with two white stars set vertical.

If two or more rear-admirals in command afloat should meet, their seniority is to be indicated respectively by a Blue flag, a Red with White stars, and a White with Blue stars, and another or all others, a White flag with Blue stars.

Commodore's (*Broad Pennant*).—Blue, Red, or White, according to rank, with one star in centre of field, being white in blue and red pennants, and blue in white.

Swallow-tailed, angle at tail, bisected by a line drawn at a right angle from centre of depth or hoist, and at a distance from head of three fifths of length of pennant; the lower side rectangular with head or hoist; upper side tapered, running the width of pennant at the tails .1 the hoist. *Head.*—.6 *length.* *Fly* 1.66 *hoist.*

Divisional Marks.—Triangle, 1st Blue, 2d Red, 3d White. Blue vertical. *Reserve Division.*—Yellow Red vertical. Division mark is worn by Commander of a division of a squadron at mizzen, when not authorized to wear Broad Pennant of a Commander or Flag of an Admiral. *Fly* .8 *hoist.*

Signal Numbers.—*Fly* 1.25 *hoist.* *Signal Pennants, Fly* 4.6 *hoist.* *Repeaters* 1.89 *hoist.*

International, Signal Number, Square, Signal Pennants. Fly .3 *hoist.*

Alimentary Principles.

Primary division of Food is into Organic and Inorganic.

Organic is subdivided into Nitrogenous and Non-Nitrogenous; Inorganic is composed of water and various saline principles. The former elements are destined for growth and maintenance of the body, and are termed "plastic elements of nutrition." The latter are designed for undergoing oxidation, and thus become source of heat, and are termed "elements of respiration," or "Calorificient."

Although Fat is non-nitrogenous, it is so mixed with nitrogenous matter that it becomes a nutrient as well as a calorificient.

Alimentary Principles. — 1. Water; 2. Sugar; 3. Gum; 4. Starch; 5. Pectine; 6. Acetic Acid; 7. Alcohol; 8. Oil or Fat. *Vegetable and Animal.*—9. Albumen; 10. Fibrine; 11. Caseine; 12. Gluten; 13. Gelatine; 14. Chloride of Sodium.

These alimentary principles, by their mixture or union, form our ordinary foods, which, by way of distinction, may be denominated *compound aliments*; thus, meat is composed of fibrine, albumen, gelatine, fat, etc.; wheat consists of starch, gluten, sugar, gum, etc.

Analysis of Meats, Fish, Vegetables, etc.

Food.	Water.	Nitrogenous Matter.	Fat.	Saline Matter.	Non-Nitrogenous Matter.	Sugar.	Cellulose.	Ash, etc.
Arrowroot	18	—	—	—	82	—	—	—
Barley Meal	15	6.3	2.4	2	69.4	4.9	—	—
Beans, White	9.9	25.5	2.8	—	55.7	—	2.9	3.2
Beef, roast	54	27.6	15.45	2.95	—	—	—	—
fat	51	14.8	29.8	4.4	—	—	—	—
lean	72	19.3	3.6	5.1	—	—	—	—
salt	49.1	29.6	.2	21.1	—	—	—	—
Beer and Porter	91	.1	—	.2	8.7	—	—	—
Buckwheat	13	13.1	3	.4	64.5	—	3.5	2.5
Butter and Fats	15	—	83	2	—	—	—	—
Cabbage	91	2	.5	.7	5.8	—	—	—
Carrots	83	1.3	.2	1	7.4	6.1	—	1
Cheese	36.8	33.5	24.3	5.4	—	—	—	—
Corn Meal	14	11.1	8.1	1.7	57.6	.4	5.9	1.2
Cream	66	2.7	26.7	1.8	—	2.8	—	—
Egg	74	14	10.5	1.5	—	—	—	—
yolk	52	16	30.7	1.3	—	—	—	—
Fish, white flesh	78	18.1	2.9	1	—	—	—	—
Eels	75	9.9	13.8	1.3	—	—	—	—
Lobster, flesh	76.6	19.17	1.17	1.8	1.26	—	—	—
Oysters	80.39	14.01	1.52	2.7	1.38	—	—	—
Liver, Calf's	72.33	20.55	5.58	1.54	—	—	—	—
Milk, Cow's	86	4.1	3.9	.8	—	5.2	—	—
Mutton, fat	53	12.4	31.1	3.5	—	—	—	—
Oatmeal	15	12.6	5.6	3	58.4	5.4	—	—
Oats	21	14.4	5.5	—	48.2	—	7.6	3.3
Parsnips	82	1.1	.5	1	9.6	5.8	—	—
Peas	15	23	2.1	2.5	50.2	2	3.1	2.1
Pork, fat	39	9.8	48.9	2.3	—	—	—	—
Bacon, dry	15	8.8	73.3	2.9	—	—	—	—
Potatoes	75	2.1	.2	.7	16.8	3.2	1	1
Poultry	74	21	3.8	1.2	—	—	—	—
Rice	13	6.3	.7	.5	78.1	.4	—	1
Rye Meal	15	8	2	1.8	69.5	3.7	—	—
Sugar	5	—	—	—	95	—	—	—
Tripe	68	13.2	16.4	2.4	—	—	—	—
Turnips	91	1.2	—	.6	4.3	2.1	—	.8
Veal	63	16.5	15.8	4.7	—	—	—	—
Wheat Flour	15	10.8	2	1.7	61.1	4.2	3.5	1.7
Bread*	37	8.1	1.6	2.3	45.4	3.6	—	2
Bran	13	18	6	—	60	—	—	3

* Water absorbed by flour varies from 40 to 60 per cent. of weight of flour, the best quality absorbing most. 100 lbs. flour yield 130 lbs. bread.

Analysis of Different Foods

In their Natural Condition.

	Nitrates.	Carbonates.	Phosphates.	Water.		Nitrates.	Carbonates.	Phosphates.	Water.
Apples	5	10	1	84	Milk of cow	5	8	1	86
Barley	17	69.5	3.5	10	Mutton	12.5	40	4.5	43
Beans	24	57.7	3.5	14.8	Oats	17	66.4	3	13.6
Beef	15	30	5	50	Parsnips	9.2	7	1	82.8
Buckwheat	8.6	75.4	1.8	14.2	Pork	10	50	1.5	38.5
Cabbage	4	5	1	90	Potatoes	2.4	22.5	.9	74.2
Chicken	19	3.5	4.5	73	" sweet	1.5	28.4	2.6	67.5
Corn, North'n	12	73	1	14	Rice	6.5	79.5	.5	13.5
" South'n	35	48	3	14	Turnips	5	4	.5	90.5
Cucumbers	1.5	1	.5	97	Veal	16	16.5	4.5	63
Lamb	11	35.5	3.5	50	Wheat	15	69.2	1.6	14.2

Nitrates—Are that class which supplies waste of muscle.

Carbonates—Are that class which supplies lungs with fuel, and thus furnishes heat to the system, and supplies fat or adipose substances.

Phosphates—Are that class which supplies bones, brains, and nerves, and gives vital power, both muscular and mental.

From above it appears, that Southern corn produces most muscle and least fat, and contains enough of phosphates to give vital power to brain, and make bones strong. Mutton is the meat which should be eaten with Southern corn.

The nitrates in all the fine bread which a man can eat will not sustain life beyond fifty days; but others, fed on unbolted flour bread, would continue to thrive for an indefinite period. It is immaterial whether the general quantity of food be reduced too low, or whether either of the muscle-making or heat-producing principles be withdrawn while the other is fully supplied. In either case the effect will be the same. A man will become weak, dwindle away and die, sooner or later, according to the deficiency; and if food is eaten which is deficient in either principle, the appetite will demand it in quantity till the deficient element is supplied. All food, beyond the amount necessary to supply the principle that is not deficient, is not only wasted, but burdens the system with efforts to dispose of it.

Analysis of Fruits.

Fruit.	Water.	Sugar.	Acid.	Albuminous substances.	Insoluble matter.	Pectous substances.	Ash.
Apple, white	85	7.6	1	.22	1.83	3.88	.47
Apricot, average	83.5	1.8	1.1	.51	4.7	7.55	.84
Blackberry	86.4	4.44	1.19	.51	5.26	1.72	.48
Cherry, red	75.4	13.1	.35	.9	5.83	3.73	.69
sour	80.5	8.77	1.28	.83	5.91	2.07	.64
black	79.7	10.7	.56	1	6.04	1.33	.67
Currant, red	85.4	5.6	1.7	.36	3.74	2.4	.8
Gooseberry, red	85.6	8	1.35	.44	2.92	1.26	.43
yellow	85.4	7	1.2	.46	3.17	2.4	.37
Grape, white	80	13.78	1	.83	2.48	1.44	.47
Peach, Dutch	85	1.58	.61	.46	5.49	6.4	.46
Pear, red	83.5	7.5	.07	.25	3.54	4.8	.34
Plum, yellow gage	80.8	2.96	.96	.48	3.98	10.48	.34
large "	79.7	3.4	.87	.4	3.91	11.3	.42
black blue	88.7	2	1.27	.4	6.86	.23	.54
" red	85.3	2.25	1.33	.43	4.23	5.85	.61
Italian, sweet	81.3	6.73	.84	.83	4.01	5.63	.66
Raspberry, wild	83.9	3.6	2	.55	8.37	1.28	.4
Strawberry, "	87	4	1.5	.6	5.5	.4	1
Banana	73.9	Sugar, Pectin, Salt, Acid, etc., 26.1.					

Sugar and Water in Various Products not Included in the Table. (*Per Cent.*)

	Sugar.		Water.		Water.
Sugar, crude	95	Molasses	23	Cabbage	91
Molasses	77	Lean beef	72	Ale and Beer	91
Buttermilk	6.4	Buttermilk	88	Coffee and Tea	100

Relative Values of Foods or Assimilating Quality to make an Equal Quantity of Flesh in Cattle or Sheep.

(*Ewart.*)

Article.	Cattle.	Sheep.	Article.	Cattle.	Sheep.
Turnips	800	400	Wheat bran	45	105
Carrots	630	—	Corn and Barley meal	35	—
Beets	600	300	Oatmeal	34	—
Parsnips and Swedes	600	200	Beanmeal	33	—
Meadow grass in bloom	400	—	Peameal	32	—
Vetches, pods open	360	90	Cabbage	—	500
Potatoes at maturity	280	200	Pea straw	—	200
Oat straw, cut green	125	—	Rye bran	—	109
Bean or Vetch straw	—	200	Oats	—	70
Meadow hay	100	100	Buckwheat	—	65
Vetch "	90	—	Barley	—	60
Linseed cake	50	—	Pease or Beans	—	54

NOTE.—When these values express weight in lbs., then such food will produce about 4 to 5 lbs. beef or mutton.

Nutritive Constituents and Values of Food in Grains per Pound.

Food.	Carbon.	Nitrogen.	Food.	Carbon.	Nitrogen.
Bakers' Bread	1975	88	Mutton	1900	189
Barley Meal	2563	68	New Milk	599	44
Beef	1854	184	Oatmeal	2831	136
Beer and Porter	274	1	Parsnips	554	12
Bullock's Liver	934	204	Pearl Barley	2660	91
Buttermilk	387	44	Potatoes	769	22
Carrots	508	14	Red Herrings	1435	217
Cheddar Cheese	3344	306	Rice	2732	68
Cocoa	3934	140	Rye Meal	2693	86
Dry Bacon	5987	95	Salt Butter	4585	—
Fat Pork	4113	106	Skim Cheese	1947	483
Flour, Seconds	2700	116	Skimmed Milk	438	43
Fresh Butter	6456	—	Split Pease	2698	248
Green Bacon	5426	76	Suet	4710	—
Green Vegetables	420	14	Sugar	2955	—
Indian Meal	3016	120	Turnips	263	13
Lard	4819	—	Whey	154	13
Molasses	2395	—	Whitefish	871	195

The *Full Daily Diet* of a man is held to be 12 oz. bread, 8 oz. potatoes, 6 oz. meat, 4 oz. boiled rice with milk, .375 pint of broth or pea soup, 1 pint milk, and 1 pint of beer.

Nutritive Values and Constituents of Milk.—(*Payen.*)

Animal.	Nitrogenous Matter and insoluble Salts.	Butter.	Lactic and soluble Salts.	Water.	Animal.	Nitrogenous Matter and insoluble Salts.	Butter.	Lactic and soluble Salts.	Water.
Goat	4.5	4.1	5.8	85.6	Ass	1.7	1.4	6.4	90.5
Cow	4.55	3.7	5.35	86.4	Mare	1.62	.2	8.75	89.43
Woman	3.35	3.34	3.77	89.54	Ewe	4.68	4.2	5.5	85.62

Weight of some Different Foods required to furnish 1220 Grains of Nitrogenous Matter.

	Lbs.		Lbs.		Lbs.		Lbs.
Cheese	.4	Meat, fat	1.3	Bacon, fat	1.8	Barley Meal	2.9
Pease	.7	Oatmeal	1.5	Bread	2.1	Milk	4.2
Meat, lean	.9	Corn Meal	1.6	Rye Meal	2.3	Potatoes	8.3
Fish, White	1	Wheat Flour	1.7	Rice	2.8	Parsnips	15.9

Turnips, 15.9 lbs.; Beer or Porter, 158.6 lbs.

Proportion of Sugar and Acid in Various Fruits.

(Fresenius.)

Fruit.	Sugar.	Acid.	Fruit.	Sugar.	Acid.
	Per Cent.	Per Cent.		Per Cent.	Per Cent.
Apple	8.4	.8	Plum	2.1	1.3
Apricot	1.8	1.1	Prune	6.3	.9
Blackberry	4.4	1.2	Raspberry	4	1.5
Currants	6.1	2	Red Pear	7.5	.1
Gooseberry	7.2	1.5	Sour Cherry	8.8	1.3
Grape	14.9	.7	Strawberry	5.7	1.3
Mulberry	9.2	1.9	Sweet Cherry	10.8	.6
Peach	1.6	.7	Whortleberry	5.8	1.3

Proportion of Oil in Various Air-dry Seeds. *(Berjot.)*

Beechnut	24	Mustard	30	Almond	40	Orange	40
Hemp	28	Flax	34	Colza	40, 45	Poppy	40, 50
Watermelon	36	Peanut	38				

Analysis of different Articles of Food, with Reference only to their Properties for giving Heat and Strength.

(Payen.) In 100 Parts.

Substances.	Carbon.	Nitrogen.	Substances.	Carbon.	Nitrogen.	Substances.	Carbon.	Nitrogen.
Alcohol	52	—	Coffee	9	1.1	Oil, Olive	98	—
Barley	40	1 9	Corn	44	1.7	Oysters	7.18	2.13
Beans	42	4.5	Eels	30.05	2	Pease	44	3.66
Beef, meat	11	3	Eggs	13.5	1.9	Potatoes	11	.33
Beer, strong	4.5	.08	Figs, dried	34	.92	Rice	41	1.8
Bread, stale	28	1.07	Herring, salted	23	3.11	Rye Flour	41	1.75
Buckwheat	42.5	2.2	Liver, Calf's	15.68	3.93	Salmon	16	2.09
Butter	83	.64	Lobster	10.06	2.93	Sardines	29	6
Carrots	5.5	.31	Mackerel	19.26	3.74	Tea	2.1	.2
Caviare	27.41	4.49	Milk, Cow's	8	.66	Truffles	9.45	1.35
Cheese, Chest'r	41.04	4.13	Nuts	10.65	1.4	Wheat	41	3
Chocolate	58	1.52	Oatmeal	44	1.95	" Flour	38.5	1.64
Cod-fish, salt'd	16	5.02				Wine	4	.015

NOTE.—Multiply figures representing nitrogen by 6.5, and equivalent amount of nitrogenous matter is obtained.

Human and Animal Sustenance.

Least Quantity of Food required to Sustain Life. (E. Smith, M. D.)

	Carbon. Grs.		Hydrogen. Grs.	
Adult Man,	4300	Mean, 4100.	200	Mean, 190.
Adult Woman,	3900		180	

An adult man, for his daily sustenance, requires about 1220 grs. nitrogenous matter or 200 of nitrogen, and bread contains 8.1 per cent. of it.

Hence, $\frac{1220}{.081} = 15062$ *grains which* $\div$ 7000 *in a lb.* = 2 *lbs.* 2.43 *oz.* of bread.

These quantities and proportions are also contained in about 16 lbs. of turnips.

Thus, by table of nutritive values, page 202, turnips have 263 grains of carbon and 13 of nitrogen.

Hence, $\frac{4300}{263}$ and $\frac{200}{13} = 16.35$ *lbs. for the necessary carbon and* 15.4 *lbs. for the nitrogen.*

Relative Value of Foods compared with 100 lbs. of Good Hay.

	Lbs.		Lbs.		Lbs.		Lbs.
Clover, green	400	Rye straw	442	Carrots	276	Corn	59
Corn, green	275	Oat straw	195	Barley	54	Linseed cake	69
Wheat straw	374	Cornstalks	400	Oats	57	Wheat bran	105

Weight of Articles of Food required to be consumed in the human system to develop a power equal to raising 140 lbs. to a height of 10 000 feet. (*Frankland.*)

Substances.	Weight.	Substances.	Weight.	Substances.	Weight.
	Lbs.		Lbs.		Lbs.
Cod-liver oil	.553	Rice	1.341	Salt Beef	3.65
Beef, fat	.555	Isinglass	1.379	Veal, lean	4.3
Bacon	.67	Sugar, lump	1.505	Porter	4.615
Butter	.693	Cream	2.062	Potatoes	5.068
Cocoa	.797	Egg, boiled	2.209	Fish	6.316
Fat of Pork	.97	Bread	2.345	Apples	7.815
Cheese	1.156	Salt Pork	2.826	Milk	8.021
Oatmeal	1.281	Ham, lean, boiled	3.001	Egg, white of	8.745
Arrowroot	1.287	Mackerel	3.124	Carrots	9.685
Wheat flour	1.311	Ale, bottled	3.461	Cabbage	12.02

Relative Value of Various Foods as Productive of Force when Oxidized in the Body.

Cabbage	1	Porter	2.6	Egg, hard boil'd	5.4	Oatmeal	9.3
Carrots	1.2	Veal, lean	2.8	Cream	5.9	Cheese	10.4
Skimmed Milk	1.2	Salt Beef	3.3	Egg, yolk	7.9	Fat of Pork	12.4
White of Egg	1.4	Poultry	3.3	Sugar	8	Cocoa	16.3
Milk	1.5	Lean Beef	3.4	Isinglass	8.7	Pemmican	16.9
Apples	1.5	Mackerel	3.8	Rice	8.9	Butter	17.3
Ale	1.8	Ham, lean	4	Pea Meal	9.	Bacon	17.94
Fish	1.9	Salt Pork	4.3	Wheat Flour	9.1	Fat of Beef	21.6
Potatoes	2.4	Bread, crumb	5.1	Arrowroot	9.3	Cod liver Oil	21.7

Nutritious Properties of different Vegetables and Oil-cake, compared with each other in Quantities.

Oil-cake	1	Rye	2.5	Clover hay	4	Cabbage	18
Peas and Beans	1.5	Bran, wheat	2.75 / 3	Hay	5	Wheat straw	26
Wheat, flour	2			Potatoes	14	Barley "	26
" grain	2.5	Corn	3	" old	20	Oat "	27.5
Oats	2.5	Barley	3	Carrots	17.5	Turnips	30

Illustration.—1 lb. of oil-cake is equal to 18 lbs. of cabbage.

Volume of Oxygen required to Oxidize 100 *parts of following Foods as consumed in the Body.*

Grape Sugar .. 106 | Starch........ 120 | Albumen........ 150 | Fat. 293

Hence, assuming capacity for oxidation as a measure, albumen has half value of fat as a food-producing element, and a greater value than either starch or sugar.

Proportion of Alcohol in 100 Parts of following Liquors. (*Brande.*)

Small Beer	1 and 1.08	Hermitage, red	12.32	Lisbon	18.94
Porter	3.5 and 5.26	Champagne	12.61	Lachryma	19.7
Cider	5.2 and 9.8	Amontillado	12.63	Teneriffe	19.79
Brown Stout	5.5 and 6.8	Frontignac	12.89	Currant Wine	20.55
Ale	6.87 and 10	Barsac	13.86	Madeira	22.27
Rhenish	7.58	Sauterne	14.22	Port	23
Moselle	8.7	Champagne Burg'dy	14.57	Sherry, old	23.86
Johannisberger	8.71	White Port	15	Marsala	25.09
Elder Wine	8.79	Bordeaux	15.1	Raisin Wine	25.12
Claret ordinaire	8.99	Malmsey	16.4	Madeira, Sercial	27.4
Tokay	9.33	Sherry	17.17	Cape Madeira	29.51
Rudesheimer	10.72	Malaga	17.2	Gin	51.6
Marcobrunner	11.6	Alba Flora	17.26	Brandy	53.39
Gooseberry Wine	11.84	Hermitage, white	17.43	Rum	53.68
Hockheimer	12.03	Cape Muscat	18.25	Irish Whiskey	53.9
Vin de Grave	12.08	Constantia, red	18.92	Scotch Whiskey	54.32

Proportion of Food Appropriated and Expended by following Animals.

	Oxen.	Sheep.	Swine.
Proportion appropriated	6.2	8	17.6
" in manure	36.5	31.9	16.9
" respired	57.3	60.1	65.5
	100	100	100

Specific Gravity of Milk and Percentage of Cream, etc.

Milk.	Specific Gravity.	Volume of Cream.	Volume of Curd.	Specific Gravity when skimmed.
Milk, pure*	1030	12	6.3	1032
" 10 per cent. water	1027	10.5	5.6	1029
" 20 " " "	1024	8.5	4.9	1026
" 30 " " "	1021	6	4.2	1023

* For a method of testing the purity of milk, see Pavy on Food (Philadelphia, 1874), page 196.

NOTE.—The average proportion of cream is 10, or 10 *per cent.*

Proportion Per cent. of Starch in sundry Vegetables.

Arrowroot	82	Wheat flour	66.3	Oatmeal	58.4	Potatoes	18.8
Rice	79.1	Corn meal	64.7	Pease	55.4	Turnips	5.1

Composition of Cheese of Different Countries.—(*Payen.*)

	Fat.	Nitrogen.	Salt.	Water.		Fat.	Nitrogen.	Salt.	Water.
Neufchatel	18.74	2.28	4.25	61.87	Chester	25.41	5.56	4.78	30.39
Parmesan	21.68	5.48	7.09	30.31	Gruyères	28.4	5.4	4.29	32.05
Brie	24.83	2.39	5.63	53.99	Marolles	28.73	3.73	5.93	40.07
Holland	25.06	4.1	6.21	41.41	Roquefort	32.31	5.07	4.45	26.53

Nutritive Equivalents. Computed from Amount of Nitrogen in Substances when Dried. Human Milk at 1.

Rice	.81	Bread, White	1.42	Cheese	3.31	Lamb	8.33
Potatoes	.84	Milk, Cows'	2.37	Eel	4.34	Egg, White	8.45
Corn	1	Pease	2.39	Mussel	5.28	Lobster	8.59
Rye	1.06	Lentils	2.76	Liver, Ox	5.7	Veal	8.73
Wheat	1.19	Egg, Yolk	3.05	Pigeon	7.56	Beef	8.8
Barley	1.25	Oysters	3.05	Mutton	7.73	Pork	8.93
Oats	1.38	Beans	3.2	Salmon	7.76	Ham	9.1

Herring, 9.14.

Thermometric Power and Mechanical Energy of 10 Grains of Various Substances in their Natural Condition, when Oxidized in the Animal Body into Carbonic Acid, Water, and Urea.—(*Frankland.*)

Substance.	Water raised 1°.	Lifted 1 foot high.	Substance.	Water raised 1°.	Lifted 1 foot high.	Substance.	Water raised 1°.	Lifted 1 foot high.
	Lbs.	Lbs.		Lbs.	Lbs.		Lbs.	Lbs.
Ale, Bass's	1.99	1.54	Cheese	11.2	8.65	Mackerel	4.14	3.2
Apples	1.48	1.29	Cocoa-nibs	17	7.3	Milk	1.64	1.25
Arrowroot	10.06	7.77	Cod-liver oil	11	18.12	Oatmeal	10.1	7.8
Beef, lean	3.66	2.83	Egg, h'd boil.	5.86	4.53	Pea meal	9.57	7.49
Bread	5.52	4.26	" yolk	8.5	6.56	Potatoes	2.56	1.99
Butter	18.68	14.42	" white	1.48	1.14	Porter	2.77	2.19
Cabbage	1.08	.83	Flour, wheat	9.87	7.62	Rice, ground	9.52	7.45
Carrots	1.33	1.03	Ham, boiled	4.3	3.32	Sugar, grape	8.42	6.51

Digestion.

Time required for Digestion of several Articles of Food.

(Beaumont, M.D.)

Food.	Time. h. m.	Food.	Time. h. m.
Apple, sweet and mellow	1 50	Heart, Animal, fried	4
sour and mellow	2	Lamb, boiled	2 30
sour and hard	2 50	Liver, Beef's, boiled	2
Barley, boiled	2	Meat and Vegetables, hashed	2 30
Bean, boiled	2 30	Milk, boiled or fresh	2
Bean and Green Corn, boiled	3 45		2 15
Beef, roasted rare	3	Mutton, roasted	3 15
roasted dry	3 30	broiled or boiled	3
Steak, broiled	3	Oyster	2 55
boiled	2 45	roasted	3 15
boiled, with mustard, etc.	3 30	stewed	3 30
Tendon, boiled	5 30	Parsnip, boiled	2 30
" fried	4	Pig, sucking, roasted	2 30
old salted, boiled	4 15	Feet, soured, boiled	1
Beet, boiled	3 45	Pork, fat and lean, roasted	5 15
Bread, Corn, baked	3 15	recently salted, boiled	4 30
Wheat, baked, fresh	3 30	" " fried	4 15
Butter, melted	3 30	" " broiled	3 15
Cabbage, crude	2 30	" " raw	3
crude, vinegar	2	Potato, boiled	3 30
crude, vin'r, boiled	4	baked	3 20
	4 30	roasted	2 30
Carrot, boiled	3 15	Rice, boiled	1
Cartilage, boiled	4 15	Sago, boiled	1 45
Cheese, old and strong	3 30	Sausage, Pork, broiled	3 20
Chicken, fricasseed	2 45	Soup, Barley	1 30
Custard, baked	2 45	Beef and Vegetables	4
Duck, roasted	4	Chicken	3
	4 30	Mutton or Oyster	3 30
Dumpling, Apple, boiled	3	Sponge-cake, baked	2 30
Egg	2	Suet, Beef, boiled	5 30
whipped	1 30	Mutton, boiled	4 30
boiled hard	3 30	Tapioca, boiled	2
" soft	3	Tripe, soured	1
fried	3 30	Turkey, roasted Wild	2 18
Fish, Cod or Flounder, fried	3 30	Turkey, roasted Domestic	2 30
Cod, cured, boiled	2	boiled	2 25
Salmon, salt'd and boil'd	4	Turnip, boiled	3 30
Trout, boiled or fried	1 30	Veal, roasted	4
Fowl, boiled or roasted	4	fried	4 50
Goose, roasted	3	Brain, boiled	1 45
Gelatine, boiled	2 30	Venison Steak, broiled	1 35

General Notes.

The per-centage of loss in the cooking of meats is as follows: Boiling 23; Baking 31; Roasting 34.

Potatoes possess anti-scorbutic power in a greater degree than any other of the succulent vegetables.

The average yearly consumption of wheat and wheat flour in Great Britain is 5.5 bushels per capita of its population.

The daily ration of an Esquimaux is 20 lbs. of flesh and blubber.—*(Sir John Ross.)*

An adult healthy man, according to Dr. Edward Smith, requires daily of

Phosphoric acid from	32 to 79 grains.	Potash	27 to 107 grains.
Chlorine	51 " 175 "	Soda	80 " 171 "
Or of common salt	85 " 291 "	Lime	2.3 " 6.3 "

and of Magnesia 2.5 to 3 grains.

A common fowl's egg contains 120 grains of Carbon and 17.75 of Nitrogen.

An ordinary working man requires for his daily sustenance

Oxygen	1.47	Starch	.66
Albuminous matter	.305	Salts	.04
Fat	.22	Water	4.535

= 7.23 lbs. avoirdupois.

Milk.—If the milk of an animal is taken at three immediately successive periods, that which is first received will not be as rich in milk-fat as the last.

In a Devon cow, milked in this manner, the first milk gave but 1.166 per cent. of fat, and the last, or that known as "strippings," 5.81 per cent.

Relative Richness of Milk of Several Animals.

Human Milk = 1.

	Milk-fat.	Casein.	Sugar.		Milk-fat.	Casein.	Sugar.
Cow	1.66	1.38	.69	Ass	.5	.38	.94
Mare	1.19	.75	.94	Sheep	2.52	2.1	.72
Goat	2	1.04	.69	Camel	1.4	—	.96

The condensation of milk reduces it to about one third of its original volume.

A Farm of second-rate quality, properly cultivated, will sustain 100 head of cattle per 100 acres, besides laboring stock (employed in cultivation of farm), and swine. —(*Ewart.*)

Thus, calves 25; do. 1 year 25; do. 2 years 25; cows 25.

Cane Sugar (Saccharose)—Is insoluble in absolute alcohol, and in diluted alcohol it is soluble only in proportion to its weakness. Loaf sugar, as a rule, is chemically pure.

Beet Root Sugar—Contains 85 to 96 per cent. of cane sugar, 1.6 to 5.1 of organic matter, and 2 to 4.3 of water.

Honey—Contains 32 per cent. of sugar (levulose), 25.5 of water, 27.9 of dextrine, and 14.6 of other matter, as mannite, wax, pollen, and insoluble matter.

Molasses—Contains 47 per cent. of cane sugar, 20.4 of fruit sugar, 2.6 of salts, 2.7 extractive and coloring matter, and 27.3 of water.

Flour.—Tests of flour, see A. W. Blyth, London, 1882, page 152.

Bread.—Wheat loses of water after 1 day 7.71 per cent., 3 days 8.86, and 7 days 14.05 per cent.

Sago.—2.5 lbs. per day will support a healthy man.

Fig—Contains nearly as much gluten as wheat bread (as 6 to 7), and in starch and sugar it is 16 per cent. richer.

Gooseberry (dry)—Is as nutritious as wheat bread.

Watermelon, *Vegetable marrow*, and *Cucumber*—Contain 94, 95, and 97 per cent. of water respectively.

Onion (dry)—Contains 25 to 30 per cent. of gluten. Potato containing but 5.

Cabbage, *Cauliflower*, *Broccoli*, and *Leaves* are generally rich in gluten, while the potato is poor.

Ratio of Flesh-formers of Tubers.

Per Cent.

Tubers.	Flesh-formers.	Starch, etc.	Ratio to Heat-giv'rs.	Tubers.	Flesh-formers.	Starch, etc.	Ratio to Heat-giv'rs.
Beet root	.4	13.4	1 : 30	Parsnip	1.2	8.7	1 : 10
Turnip	.5	4	1 : 8	Onion	1.5	4.8	1 : 3.5
Carrot	.5	5	1 : 10	Sweet Potato	1.5	20.2	1 : 13
Potato	1.2	18	1 : 16	Yam	2.2	16.3	1 : 7.5

GRAVITY OF BODIES.

Gravity acts equally on all bodies at equal distances from Earth's centre; its force diminishes as distance increases, and increases as distance diminishes.

Gravitating forces of bodies are to each other,

1. Directly as their masses.
2. Inversely as squares of their distances.

Gravity of a body, or its weight above Earth's surface, decreases as square of its distance from Earth's centre in semi-diameters of Earth.

Illustration 1.—If a body weighs 900 lbs. at surface of the Earth, what will it weigh 2000 miles above surface?—Earth's semi-diameter is 3963 miles (say 4000).

Then $2000 + 4000 = 6000 = 1.5$ *semi-diam's*, and $900 \div 1.5^2 = \frac{900}{2.25} = 400$ *lbs.*

Inversely, If a body weighs 400 lbs. at 2000 miles above Earth's surface, what will it weigh at surface?

$$400 \times 1.5^2 = 900 \text{ lbs.}$$

2.—A body at Earth's surface weighs 360 lbs.; how high must it be elevated to weigh 40 lbs.?

$\frac{360}{40} = 9$ *semi-diameters*, if gravity acted directly; but as it is inversely as square of the distance, then $\sqrt{9} = 3$ *semi-diameters* $- 3 \times 4000 = 12\,000$ *miles*.

3.—To what height must a body be raised to lose half its weight?

As $\sqrt{1} : \sqrt{2} :: 4000 : 5656$ = *as square root of one semi-diameter is to square root of two semi-diameters, so is one semi diameter to distance required.*

Hence $5656 - 4000 = 1656$ = *distance from Earth's surface.*

Diameters of two Globes being equal, and their densities different, weight of a body on their surfaces will be as their densities.

Their densities being equal and their diameters different, weight of them will be as their diameters.

Diameters and densities being different, weight will be as their product.

Illustration.—If a body weighs 10 lbs. at surface of Earth, what will it weigh at surface of Sun, densities being 392 and 100, and diameters 8000 and 883 000 miles?

$\overline{883\,000 \times 100} \div \overline{8000 \times 392} = 28.157$ = *quotient of product of diameter of Sun and its density, and product of diameter of Earth and its density.*

Then $28.157 \times 10 = 281.57$ *lbs.*

Note.—Gravity of a body is .003 46 less at Equator than at Poles.

SPECIFIC GRAVITY AND WEIGHT.

Specific Gravity or Weight of a body is the proportion it bears to the weight of another body of known density or of equal volume, and which is adopted as a standard.

If a body float on a fluid, the part immersed is to whole body as specific gravity of body is to specific gravity of fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to difference between its own weight and weight of its bulk of the fluid, less resistance of the fluid to its passage.

Water is well adapted for standard of gravity; and as a cube foot of it at 62° F. weighs 997.68 ounces avoirdupois, its weight is taken as the unit, or approximately 1000.

French standard temperature for comparison of density of solid bodies and determination of their specific gravities, is that of maximum density of water, at 4° C. or 39.1° F., and for gases and vapors under one atmosphere or .76 centimeters of mercury is 32° F. or 0° C., and specific gravity of a body is expressed by weight in kilogrammes of a cube decimeter of that body.

Densities of metals vary greatly.

Potassium, Sodium, Barium, and Lithium are lighter than water. Mercury is heaviest liquid and Platinum heaviest metal. Volcanic scoriæ is lighter than water.

Pomegranate and Lignum-vitæ are heaviest of woods. Pearl is heaviest of animal substances, and Flax and Cotton are heaviest of vegetable substances, former weighing nearly twice as much as water.

Zircon is heaviest of precious stones, being 4.5 times heavier than water. Garnet is 4 times heavier, Diamond 3.5 times, and Opal, lightest of all, is but twice as heavy as water.

To Ascertain Specific Gravity of a Solid Body heavier than Water.

Rule.—Weigh it both in and out of water, and note difference; then, as weight lost in water is to whole weight, so is 1000 to specific gravity of body.

Or, $\frac{W \times 1000}{W - w} = G$, W *and w representing weights out and in water, and* G *specific gravity.*

Example.—What is specific gravity of a stone which weighs in air 15 lbs., in water 10 lbs.?

$15 - 10 = 5$; then $5 : 15 :: 1000 : 3000$ *Spec. Grav.*

To Ascertain Specific Gravity of a Body lighter than Water.

Rule.—Annex to lighter body one that is heavier than water, or fluid used; weigh piece added and compound mass separately, both in and out of water, or fluid; ascertain how much each loses, by subtracting its weight from its weight in air, and subtract less of these differences from greater.

Then, as last remainder is to weight of light body in air, so is 1000 to specific gravity of body.

Example.—What is specific gravity of a piece of wood that weighs 20 lbs. in air; annexed to it is a piece of metal that weighs 24 lbs. in air and 21 lbs. in water, and the two pieces in water weigh 8 lbs.?

$20 + 24 - 8 = 44 - 8 = 36 =$ *loss of compound mass in water;*
$24 - 21 = \underline{3} =$ *loss of heavy body in water.*
$33 : 20 :: 1000 : 606 = 24$ *Spec. Grav.*

To Ascertain Specific Gravity of a Fluid.

Rule.—Take a body of known specific gravity, weigh it in and out of the fluid; then, as weight of body is to loss of weight, so is specific gravity of body to that of fluid.

Example.—What is specific gravity of a fluid in which a piece of copper (*spec. grav.* = 9000) weighs 70 lbs. in, and 80 lbs. out of it?

$80 : 80 - 70 = 10 :: 9000 : 1125$ *Spec. Grav.*

To Ascertain Specific Gravity of a Solid Body which is soluble in Water.

Rule.—Weigh it in a liquid in which it is not soluble, divide its weight out of the liquid by loss of its weight in the liquid, and multiply quotient by specific gravity of liquid; the product is specific gravity.

Example.—What is specific gravity of a piece of clay, which weighs 15 lbs. in air and 5 lbs. in a liquid of a specific gravity of 1500, in which it is insoluble?

$15 \div 10 \times 1500 = 2250$ *Spec. Grav*

SOLIDS.

SUBSTANCES.	Specific Gravity.	Weight of a Cube Inch.
Metals.		Lb.
Aluminum, cast........	2 560	.0926
" wrought....	2 670	.0906
" Bronze.....	7 700	.2785
Antimony..............	6 712	.2428
Arsenic................	5 763	.2084
Barium.................	470	.017
Bismuth...............	9 823	.3553
Boron...................	2 000	.0723
Brass.		
Sheet, cop. 75, zinc 25.	8 450	.3056
Yellow " 66, " 34.	8 300	.2997
Muntz " 60, " 40.	8 200	.2966
Plate..................	8 380	.3026
Cast..................	8 100	.2930
Wire..................	8 214	.2972
Bromine................	3 000	.1085
Bronze, gun metal......	8 750	.3165
" ordinary mean .	8 217	.2972
" cop. 84, tin 16 ..	8 832	.3194
" " 81, " 19 ..	8 700	.2929
" small bells, cop. 35, tin 65....	8 060	.291
" cop. 21, tin 74 ..	7 390	.2668
Cadmium..............	8 650	3129
Calcium................	1 580	.057
Chromium..............	5 900	.2134
Cinnabar...............	8 098	.2929
Cobalt	8 600	.3111
Columbium............	6 000	.217
Copper, cast...........	8 788	.3179
" plates..........	8 698	.3146
" wire and bolts..	8 880	.3212
" ordinary mean.	8 880	.3212
Gold, pure, cast	19 258	.6965
" hammered	19 361	.7003
" 22 carats fine.....	17 486	.6325
" 20 " "	15 709	.5682
Iridium..................	18 680	.6756
" hammered......	23 000	.8319
Iron, Cast, gun metal...	7 308	.264
" minimum	6 900	.2491
" maximum.........	7 500	.2707
" ordinary mean....	7 207	.2607
" mean, Eng........	7 217	.2609
" cast, hot blast.....	7 065	.2555
" " cold "	7 218	.2611
" Wrought bars.....	7 788	.2817
" " wire.......	7 774	.2811
" " rolled plates	7 704	.2787
" " average	7 698	.2779
" " Eng. rails ..	7 540	.2722
" " Lowmoor...	7 808	.2819
" " pure.......	8 140	.2938
" ordinary mean....	7 744	.2801
Lead, cast	11 352	.4106
" rolled............	11 388	.4119
Lithium................	590	.0213
Magnesium	1 750	.0633
Manganese.............	8 000	.2894
Mercury —40°	15 632	.5661
" +32°	13 598	.4918

SUBSTANCES.	Specific Gravity.	Weight of a Cube Inch.
Metals.		Lb.
Mercury 60°.........	13 569	.4908
" 212°..........	13 370	.4836
Molybdenum...........	8 600	.3111
Nickel..................	8 800	.3183
" cast..............	8 279	.2994
Osmium	10 000	.3613
Palladium	11 350	.4105
Platinum, hammered ...	20 337	.7356
" native.......	16 000	.5787
" rolled........	22 069	.7982
Potassium, 59°.........	865	.0313
Red lead................	8 940	.324
Rhodium	10 650	.3852
Rubidium	1 520	.055
Ruthenium	8 600	.3111
Selenium...............	4 500	.1627
Silver, pure, cast	10 474	.3788
" " hammered.	10 511	.3802
Sodium.................	970	.0351
Steel, minimum........	7 700	.2785
" maximum........	7 900	.2857
" plates, mean	7 806	.2823
" soft...............	7 833	.2833
" temper'd and hardened	7 818	.2828
" wire..............	7 847	.2838
" blistered	7 823	.283
" crucible..........	7 842	.2836
" cast	7 848	.2839
" Bessemer	7 852	.284
" ordinary mean....	7 834	.2916
Strontium	2 540	.0918
Tellurium	6 110	.221
Thalium................	11 850	.4286
Tin, Cornish, hammered.	7 390	.2673
" " pure	7 291	.2637
Titanium	5 300	.1917
Tungsten	17 000	.6149
Uranium	18 330	.6629
Wolfram...............	7 119	.2575
Zinc, cast..............	6 861	.2482
" rolled.......	7 191	.26
Woods (*Dry*).		Cube Foot.
Alder..................	800	50
Apple..................	793	49.562
Ash	845	52.812
	690	43.125
Bamboo	400	25
Baytree................	822	51.375
Beech	852	53.25
	690	43.125
Birch..................	567	35.437
	720	45
Blackwood, India	898	56.125
Boxwood, Brazil........	1 031	64.437
" France.......	1 328	83
" Holland......	912	57
Bullet-wood...........	928	58
Butternut..............	376	23.5

SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.
Woods (*Dry*).		Lbs.
Campeachy	.913	57.062
Cedar	561	35.062
" Indian	1315	82.157
Charcoal, pine	441	27.562
" fresh burned	380	23.75
" oak	1573	98.312
" soft wood	280	17.5
" triturated	1380	86.25
Cherry	715	44.687
Chestnut, sweet	610	38.125
Citron	726	45.375
Cocoa	1040	65
Cork	240	15
Cypress, Spanish	644	40.25
Dog-wood	756	47.25
Ebony, American	1331	83.187
" Indian	1209	75.562
Elder	695	43.437
Elm	570	35.625
	671	41.937
" rock	800	50
Erroul, India	1014	63.375
Filbert	600	37.5
Fir, Norway Spruce	512	32
" Dantzic	582	36.375
Fustic	970	60.625
Greenheart or Sipiri	1055	65.95
Gum, blue	843	52.687
" water	1000	62.5
Hackmatack	592	37
Hawthorn	910	56.875
Hazel	860	53.75
Hemlock	368	23
Hickory, pig-nut	792	49.5
" shell-bark	690	43.125
Holly	760	47.5
Iron-wood	990	61.875
Jasmine	770	48.125
Juniper	566	35.375
Khair, India	1171	73.187
Lancewood, mean	720	45
Larch	544	34
	560	35
Lemon	703	43.937
Lignum-vitæ	650	40.625
	1333	83.312
Lime	804	50.25
Linden	604	37.75
Locust	728	45.5
Logwood	913	57.062
Mahogany	720	45
	1063	66.437
" Honduras	560	35
" Spanish	852	53.25
Maple	750	46.875
" bird's-eye	576	36
Mastic	849	53.062
Mulberry	561	35.062
	897	56.062
Oak, African	823	51.437
" Canadian	872	54.5
" Dantzic	759	47.437

SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.
Woods (*Dry*).		Lbs.
Oak, English	858	53.625
	932	58.25
" green	1146	71.625
" heart, 60 years	1170	73.125
" live, green	1260	78.75
" " seasoned	1068	66.75
" white	860	53.75
Olive	680	42.5
Orange	705	44.062
Pear	661	41.312
Persimmon	710	44.375
Plum	785	49.062
Pine, pitch	660	41.25
" red	590	36.875
" white	554	34.625
" yellow	461	28.812
" Norway	740	46.25
Pomegranate	1354	84.625
Poon	580	36.25
Poplar	383	23.937
" white	529	33.062
Quince	705	44.062
Rosewood	728	45.5
Sassafras	482	30.125
Satinwood	885	55.312
Spruce	500	31.25
Sycamore	623	38.937
Tamarack	383	23.937
Teak (African oak)	657	41.062
	980	61.25
Walnut	671	41.937
" black	500	31.25
Willow	486	30.375
	585	36.562
Yew, Dutch	788	49.25
" Spanish	807	50.437
(*Well Seasoned.**)		
Ash	722	45.125
Beech	624	39
Cherry	606	37.875
Cypress	441	27.562
Hickory, red	838	52.375
Mahogany, St. Domingo	720	45
Pine, white	473	29.562
" yellow	541	33.812
Poplar	587	36.687
White Oak, upland	687	42.937
" " James River	759	42.437
Stones, Earths, etc.		
Alabaster, white	2730	170.625
" yellow	2699	168.687
Alum	1714	107.125
Amber	1078	67.375
Ambergris	866	—
Asbestos, starry	3073	192.062
Asphalte	2250	140.625
Barytes, sulphate	4000	250
	4865	304.062
Beton, N. Y. St. Con'g Co.	2305	144.06

* U. S. Ordnance Manual, 1841.

Substances.	Specific Gravity.	Weight of a Cube Foot.
		Lbs.
Stones, Earths, etc.		
Basalt	2740	171.25
	2864	179
Bitumen, red	1160	72.3
" brown	830	51.7
Borax	1714	107.125
Brick	1367	85.437
	1900	118.75
" pressed	2400	150
" fire	2201	137.562
" work in cement	1800	112.5
" " " mortar	1600	100
	2000	125
Carbon	3500	218.75
Cement, Portland	1300	81.25
" Roman	1560	97.25
Chalk	1520	95
	2784	174
Clay	1930	120.625
" with gravel	2480	155
Coal, Anthracite	1350	84.375
	1436	89.75
	1640	102.5
" Borneo	1290	80.625
" Cannel	1238	77.375
	1318	82.375
" Caking	1277	79.812
" Cherry	1276	79.75
" Chili	1290	80.625
" Derbyshire	1292	80.75
" Lancaster	1273	79.562
" Maryland	1355	84.687
" Newcastle	1270	79.375
" Rive de Gier	1300	81.25
" Scotch	1259	78.687
	1300	81.25
" Splint	1302	81.375
" Wales, mean	1315	82.187
Coke	1000	62.5
" Nat'l, Va.	746	46.64
Concrete, in cement	2200	137.5
" mean	2000	125
Earth,* common soil, dry	1216	76
" loose	1500	93.75
" moist sand	2050	128.125
" mold, fresh	2050	128.125
" rammed	1600	100
" rough sand	1920	120
" with gravel	2020	126.25
" Potters'	1900	118.75
" light vegetable	1400	87.5
Emery	4000	250
Feldspar	2600	162.5
Flint, black	2582	161.375
" white	2594	162.125
Fluorine	1320	82.5
Fuel, Warlich's	1150	71.875
" Lignite	1300	81.25
Glass, bottle	2732	170.75
" Crown	2487	155.437
" flint	2933	183.312
	3200	196

Substances.	Specific Gravity.	Weight of a Cube Foot.
		Lbs.
Stones, Earths, etc.		
Glass, green	2642	165.125
" optical	3450	215.625
" white	2892	180.75
" window	2642	165.125
" soluble	1250	78.125
Gniess, common	270	15.875
Granite, Egyptian red	2654	165.875
" Patapsco	2640	165
" Quincy	2652	165.75
" Scotch	2625	164.062
" Susquehanna	2704	169
" " gray	2800	175
Graphite	2200	137.5
Gravel, common	1749	109.312
Grindstone	2143	133.937
Gypsum, opaque	2168	135.5
Hone, white, razor	2876	179.75
Hornblende	3540	221.25
Iodine	4040	—
Lava, Vesuvius	1710	106.875
	2810	175.625
Lias	1350	146.875
Lime, quick	804	50.25
" hydraulic	2745	171.562
Limestone, white	3156	197.25
" green	3180	198.75
Magnesia, carbonate	2400	150
Magnetic ore	5094	317.6
Marble, Adelaide	2715	169.687
" African	2708	169.25
" Biscayan, black	2695	168.437
" Carrara	2716	169.75
" common	2686	167.875
" Egyptian	2668	166.75
" French	2649	165.562
" Italian, white	2708	169.25
" Parian	2838	177.375
" Vermont, white	2650	165.57
" Silesian	2730	170.625
Marl, mean	1750	109.375
" tough	2340	146.25
Masonry, rubble	2050	128.125
" Granite	2640	165
" Limestone	2640	165
" Sandstone	2160	135
" Brick	2240	140
" " rough work	1600	100
Mica	2800	175
Millstone	2484	155.25
" Quartz	1260	78.75
Mortar	1384	86.5
	1750	109.375
Mud	1630	101.875
" wet and fluid	1782	112
" " " pressed	1920	120
Nitre	1900	118.75
Oyster-shell	2092	130.75
Paving-stone	2416	151
Peat, Irish, light	278	17.375
" " dense	562	35.125
" very "	675	42.187

* Specific gravity of earth is estimated at from 1520 to 2200.

Substances.	Specific Gravity.	Weight of a Cube Foot.
		Lbs.
Stones, Earths, etc.		
Peat, black	1058	66.125
	1329	83.062
Phosphorus	1770	110.625
Plaster of Paris	1176	73.5
	3400	212.5
" " " dry	1400	87.5
Plumbago	2100	131.25
Porcelain, China	2300	143.75
Porphyry, red	2765	172.812
Pumice-stone	915	57.187
Quartz	2660	166.25
Red lead	8940	558.75
Resin	1089	68.062
Rock, crystal	2735	170.937
Rotten-stone	1981	123.812
Salt, common	2130	133.125
" rock	2200	137.5
Saltpetre	2090	130.625
Sand, coarse	1800	112.5
" common	1670	104.375
" damp and loose	1392	87
" dried " "	1560	97.5
" dry	1420	88.75
" mortar, Ft. Richm'd	1659	103.66
" " Brooklyn	1716	107.25
" silicious	1701	106.33
Sandstone, mean	2200	137.5
" Sydney	2237	139.81
Schorl	3170	198.125
Scoria, volcanic	830	51.875
Sewer pipe, mean	2250	140.625
Shale	2600	162.5
Slate	2672	167
	2900	181.25
" purple	2784	174
Smalt	2440	152.5
Soapstone	2730	170.625
Spar, calcareous	2735	170.937
" Feld, blue	2693	168.312
" " green	2704	169
" Fluor	3400	212.5
Specular ore	5251	328.187
Stalactite	2415	150.937
Stone, Bath, Engl.	1961	122.562
" Blue Hill	2640	165
" Bluestone (basalt)	2625	164.062
" Breakneck, N.Y.	2704	169
" Bristol, Engl.	2510	156.875
" Caen, Normandy	2076	129.75
" common	2520	157.5
" Craigleith, Scotl.	2316	144.75
" Kentish rag, "	2651	165.687
" Kip's Bay, N.Y.	2759	172
" Norfolk (Parliament House)	2304	144
" Portland, Engl.	2368	148
" Staten Isl'd, N.Y.	2976	186
" Sullivan Co., "	2688	168
Sulphur, native	2033	127.062
Terra Cotta	1952	122
Tile	1815	113.437
Trap	2720	170

Substances.	Specific Gravity.	Weight of a Cube Foot.
		Lbs.
Granite. *(Gen'l Gillmore, U. S. A.)*		
Duluth, Minn., dark	2780	173.7
Fall River, Mass., gray	2635	164.7
Garrison's, N. Y. "	2580	161.2
Jersey City, N. J., soap	3030	189.3
Keene, N. H., bluish gray	2656	166
Maine	2635	164.7
Millstone Pt., Conn.	2706	169.1
New London, "	2660	166.25
Quincy, Mass., light	2695	168.5
Richmond, Va.	2727	170.5
" " gray	2630	164.4
Staten Island, N. Y.	2861	178.8
Westchester Co., N. Y.	2655	165.9
Westerly, R. I., gray	2670	166.9
Limestone. *(Gen'l Gillmore, U. S. A.)*		
Bardstown, Ky., dark	2670	166.9
Caen, France	1900	118.8
Canajoharie, N. Y.	2685	167.8
Cooper Co., Mo., d'k drab	2320	141.3
Erie Co., N. Y., blue	2640	165
Garrison's, N. Y.	2635	164.7
Glens' Falls, "	2700	168.7
Joliet, Ill., white	2540	158.7
Kingston, N. Y.	2690	168.1
Lake Champlain, N. Y.	2750	171.9
Lime Island, Mich., drab	2500	156.3
Marblehead, Ohio, white	2400	150
Marquette, Mich., drab	2340	146.25
Sturgeon Bay, Wis., bluish drab	2780	173.7
Marble. *(Gen'l Gillmore, U. S. A.)*		
Dorset, Vt.	2635	164.7
East Chester, N. Y.	2875	179.7
Italian, common	2690	168.1
Mill Creek, Ill., drab	2570	171.9
North Bay, Wis., "	2800	175
Sandstone. *(Gen'l Gillmore, U. S. A.)*		
Albion, N. Y., brown	2420	151.25
Belleville, N. J., gray	2259	141.2
Berea, Ohio, drab	2110	131.9
Cleveland, " olive green	2240	140
Edinb'h, Sc'tl., Craigleith	2260	141.25
Fond du Lac, Wis., purple	2220	138.7
Fontenac, Minn., l'g't buff	2325	145.31
Haverstraw, N. Y., red	2130	133.1
Kasota, Minn., pink	2630	164.375
Little Falls, N. Y., brown	2250	140.6
Marquette, Mich., purple	2285	142.5
Masillon, O., yellow drab	2110	131.87
Medina, N. Y., pink	2410	150.6
Middletown, Ct., brown	2360	147.5
Seneca, Ohio, red "	2390	149.4
Vermillion, Ohio, drab	2160	135
Warrensburgh, Mo.	2140	133.75

Precious Stones.

	Spec. Grav.		Spec. Grav.		Spec. Grav.
Agate	2590	Emerald, aqua marine	2730	Onyx	2700
Amethyst	3920	Garnet	4189	Opal	2090
Carnelian	2613	" black	3750	Pearl, Oriental	2650
Chrysolite	2782	Jasper	2600	Ruby	3980
Diamond, Oriental	3521	Jet	1300	Sapphire	3994
" Brazilian	3444	Lapis lazuli	2960	Topaz	3500
" pure	3520	Malachite	4020	Tourmaline	3070
Emerald	3950			Turquoise	2750

SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.
Miscellaneous.		Lbs.
Amber	1090	68.125
Atmospheric Air	.001292	.080728
Beeswax	965	60.312
Bone	1900	118.75
Butter	942	58.875
Camphor	988	61.75
Caoutchouc	930	58.125
Cotton	950	59.375
Dynamite	1650	103.125
Egg	1090	—
Fat of Beef	923	57.687
" Hogs	936	58.5
" Mutton	923	57.687
Flax	1790	111.875
Gamboge	1222	—
Glycerine, 60°	1261	78.752
Grain, Barley	590	36.875
" Wheat	750	46.875
" Oats	500	31.25
Gum Arabic	1452	90.75
Gunpowder, loose	900	56.25
" shaken	1000	62.5
" solid	1550 1800	96.875 112.5
Gutta-percha	980	61.25
Hay, old compact	128.8	8.05
Horn	1689	105.562
Human body	1070	66.935
Ice, at 32°	922	57.5
Indigo	1009	63.062
Isinglass	1111	69.437
Ivory	1825	114.062
Lard	947	59.187
Leather	960	60
Mastic	1074	67.125
Myrrh	1360	85
Nitro-Glycerine	1600	100
Opium	1336	83.5
Potash	2100	131.25
Resin	1089	68.062
Snow	.0833	5.2
Soap, Castile	1071	56.937
Spermaceti	943	58.937
Starch	950	59.375
Sugar	1606	100.375
" .66	972 1326	60.25 82.875
Tallow	941	58.812
Wax	964 970	60.25 60.625

SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.
Liquids.		Lbs.
Acid, Acetic	1062	66.375
" Benzoic	667	41.687
" Citric	1034	64.625
" Concentrated	1521	95.062
" Fluoric	1500	93.75
" Muriatic	1200	75
" Nitric	1217	76.062
" Nitrous	1550	96.875
" Phosphoric	1558	97.375
" " solid	2800	175
" Sulphuric	1849	115.562
Alcohol, pure, 60°	794	49.622
" 95 per cent	816	51
" 80 "	863	53.937
" 50 "	934	58.375
" 40 "	951	59.437
" 25 "	970	60.625
" 10 "	986	61.625
" 5 "	992	62
" proof spirit,* 50 per cent., 60°	934	58.375
" proof spirit, 50 per cent., 80°	875	54.687
Ammonia, 27.9 per cent.	891	55.687
Aquafortis, double	1300	81.25
" single	1200	75
Beer	1034	64.625
Benzine	850	53.125
Bitumen, liquid	848	53
Blood (human)	1054	65.875
Brandy, .83 or .5 of spirit	924	57.75
Bromine	2966	185.375
Cider	1018	63.625
Ether, Acetic	866	54.125
" Muriatic	845	52.812
" Nitric	1110	69.375
" Sulphuric	715	44.687
Honey	1450	90.625
Milk	1032	64.5
Oil, Anise-seed	986	61.625
" Codfish	923	57.687
" Whale	923	57.687
" Linseed	940	58.75
" Naphtha	850	53.125
" Olive	915	57.187
" Palm	969	60.562
" Petroleum	880	55
" Rape	914	57.125
" Sunflower	926	57.875
" Turpentine	870	54.375

* Specific gravity of proof spirit according to Ure's Table for Sykes's Hydrometer, 920.

SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.	SUBSTANCES.	Specific Gravity.	Weight of a Cube Foot.
Liquids.		Lbs.	**Liquids.**		Lbs.
Spirit, rectified	824	51.5	Water, Dead Sea	1240	77.5
Steam, at 212°	.00061	.038*	" Mediterranean	1029	64.312
Tar	1015	63.437	" sea	1029	64.312
Vinegar	1080	67.5	" Black Sea	1016	63.5
Water, at 32°	998.7	62.418	" rain	1000	62.5
" " 39.1°	998.8	62.425	Wine, Burgundy	992	62
" " 62°†	997.7	62.355	" Champagne	997	64.375
" " 212°	956.4	59.64	" Madeira	1038	62.312
" distilled, at 39°	998	62.379	" Port	997	62.312

* .038 18. † 1 cube inch at standard temperature = 252.5954 grains.

Compression of following fluids under a pressure of 15 lbs. per square inch:

Alcohol.. .000 021 6 | Mercury.. .000 002 65 | Water.. .000 046 63 | Ether.. .000 061 58

Elastic Fluids.

1 Cube Foot of Atmospheric Air at 32° weighs .080 728 *lbs. Avoirdupois* = 565.096 *grains, and at 62°* 532.679 *grains.*

Its assumed Gravity of 1 is Unit for Elastic Fluids.

	Spec. Grav.		Spec. Grav.		Spec. Grav.
Acetic Ether	3.04	Nitric acid	1.217	Vapor.	
Ammonia	.589	" oxide	1.094	Alcohol	1.613
Atmos. air, at 32°	1	Nitrogen	.974	Bisulphuret of Carbon	2.64
Azote	.976	Nitrous acid	2.638	Bromine	5.4
Carbonic acid	1.53	Nitrous oxide	1.527	Chloric Ether	3.44
" oxide	.972	Olefiant gas	.9672	Chloroform	4.2
Carburet'd Hydrog.	.559	Oxygen	1.106	Ether	2.586
Chlorine	2.421	Phosphurett'd Hydrogen	1.77	Hydrochlor. Ether	2.255
Chloro-carbonic	3.389	Sulphuretted Hydrogen	1.17	Iodine	8.716
Chloroform	5.3	Sulphurous acid	2.21	Nitric acid	3.75
Cyanogen	1.815	Steam,‡ at 212°	.47295	Spirits of Turpentine	5.013
Gas, coal	.438 / .752	Smoke.		Sulphuric acid	2.7
Hydrochloric acid	1.278	Bitum. Coal	.102	" Ether	2.586
Hydrocyanic "	.942	Coke	.105	Sulphur	2.214
Hydrogen	.0692	Wood	.09	Water	.623
Muriatic acid	1.247				

‡ Weight of a cube foot 267.26 *grains*, and compared with water at 62° specific gravity = .000 612 3.

Weight of a Cube Foot of Gases at 32° F., and under Pressure of one Atmosphere, or 2116.4 lbs. per Square Foot.

	Lbs.		Lbs.		Lbs.
Air, at 32°	.080 728	Chlorine	.197	Hydrogen	.005 594
" " 62°	.076 097	Chloroform	.428	Nitrogen	.078 596
Alcohol	.130 2	Coal gas	.035 36	Olefiant gas	.079 5
Carbonic acid	.123 44	Ether, Sulphuric	.209 3	Oxygen	.089 256
Carburet. Hydrog.	.044 62	Gaseous steam	.050 22	Steam	.050 22

Sulphurous acid .1814 lbs.

To Compute Weight of a Body or Substance when Specific Gravity is given.

RULE.—Multiply specific gravity by unit or standard of body or substance, and product is the weight.

Or, Divide specific gravity of body or substance by 16, and quotient will give weight of a cube foot of it in lbs.

EXAMPLE.—Specific gravity is 2250; what is weight of a cube foot of it?

$2250 \times 62.5 = 140.625$ *lbs.*

Weights and Volumes of various Substances in Ordinary Use.

Substances.	Cube Foot.	Cube Inch.
Metals.	Lbs.	Lbs.
Brass .. { copper 67 / zinc 33 }	488.75	.2829
" gun metal	543.75	.3147
" sheets	513.6	.297
" wire	524.16	.3033
Copper, cast	547.25	.3179
" plates	543.625	.3167
Iron, cast	450.437	.2607
" gun metal	466.5	.27
" heavy forging	479.5	.2775
" plates	481.5	.2787
" wrought bars	486.75	.2816
Lead, cast	709.5	.4106
" rolled	711.75	.4119
Mercury, 60°	848.7487	.491174
Steel, plates	487.75	.2823
" soft	489.562	.2833
Tin	455.687	.2637
Zinc, cast	428.812	.2482
" rolled	449.437	.2601
Woods.		Cube Feet in a Ton.
Ash	52.812	42.414
Bay	51.375	43.601
Blue Gum	64.3	34.837
Cork	15	149.333
Cedar	35.062	63.886
Chestnut	38.125	58.754
Hickory, pig-nut	49.5	45.252
" shell-bark	43.125	51.942
Lignum-vitæ	83.312	26.886
Logwood	57.062	39.255
Mahoga'y, Hondur's {	35 66.437	64 33.714
Oak, Canadian	54.5	41.101
" English	58.25	38.455
" live, seasoned	66.75	33.558
" white, dry	53.75	41.674
" " upland	42.937	52.169
Pine, pitch	41.25	54.303
" red	36.875	60.745
" white	34.625	64.693
" well-seasoned	29.562	75.773
Pine, yellow	33.812	66.248

Substances.	Cube Foot.	Cube Feet in a Ton.
Woods.	Lbs.	
Spruce	31.25	71.68
Walnut, black, dry	31.25	71.68
Willow	36.562	61.265
" dry	30.375	73.744
Miscellaneous.		
Air	.075291	—
Basalt, mean	175	12.8
Brick, fire	137.562	16.284
" mean	102	21.961
Coal, anthracite .... {	89.75 102.5	24.958 21.854
" bitumin., mean	80	28
" Cannel	94.875	23.609
" Cumberland	84.687	26.451
" Welsh, mean	81.25	27.569
Coke	62.5	35.84
Cotton, bale, mean	14.5	154.48
" " pressed {	20 25	114 89.6
Earth, clay	120.625	18.569
" common soil	137.125	16.335
" " gravel	109.312	20.49
" dry, sand	120	18.667
" loose	93.75	23.893
" moist, sand	128.125	17.482
" mold	128.125	17.482
" mud	101.875	21.987
" with gravel	126.25	17.742
Granite, Quincy	165.75	13.514
" Susquehanna	169	13.254
Gypsum	135.5	16.531
Hay, bale	12	186.66
" hard pressed	25	89.6
Ice, at 32°	57.5	38.95
India rubber	56.437	39.69
" " vulcanized	—	—
Limestone	197.25	11.355
Marble, mean	167.875	13.343
Mortar, dry, mean	97.98	22.862
Plaster of Paris	73.5	30.476
Water, rain	62.5	35.84
" salt	64.312	34.83
" at 62°	62.355	35.955

To Compute Proportions of Two Ingredients in a Compound, or to Discover Adulteration in Metals.

RULE.—Take differences of each specific gravity of ingredients and specific gravity of compound, then multiply gravity of one by difference of other; and, as sum of products is to respective products, so is specific gravity of body to proportions of the ingredients.

EXAMPLE.—A compound of gold (*spec. grav.* = 18.888) and silver (*spec. grav.* = 10.535) has a specific gravity of 14; what is proportion of each metal?

$18.888 - 14 = 4.888 \times 10.535 = 51.495.$ $14 - 10.535 = 3.465 \times 18.888 = 65.447.$
$65.447 + 51.495 : 65.447 :: 14 : 7.835$ *gold*, $65.447 + 51.495 : 51.495 :: 14 : 6.165$ *silver*.

Weights of Various Substances per Cube Foot in Bulk.

	Lbs.		Lbs.		Lbs.
Lead, in pigs	567	Potters' clay	130	Coal, caking	50
Iron, "	360	Loam	126	Wheat	48
Marble, in blocks }	172	Gravel	109	Barley	38
Limestone, " }		Sand	95	Fruit and vegetables	22
Trap	170	Bricks, common	93	Cotton seeds	12
Granite, in blocks	164	Ice, at 32°	57.5	Cotton	10
Sandstone	141	Oak, seasoned	52	Hay, old	8

Ash, dry, 100 feet BM	.175 ton.	Earth, loose	93.75 lbs.
" white, " "	.141 "	Elm, dry, 100 feet BM	.13 ton.
Cement, struck bushel and packed*	100 lbs.	Gypsum, ground, str. bush.	70 lbs.
Cement, Portland, bushel	110 lbs.	" " well shaken	80 "
Cherry, dry, 100 BM	.156 ton.	Hemlock, dry, 100 feet BM.	.093 ton.
Chestnut, dry, 100 BM	.153 "	Hickory, " " "	.197 "
Coal, anthracite, 1 cub. yd. broken and loose	1.75 yds.	Masonry, Granite, dressed	165 lbs.
" " " 1 ton	41.5 cub. feet.	" " " rough	126 "
Coke, ton =	80 to 97 cub. feet.	" Limestone, dres'd	165 "
Earth, common soil	137.125 lbs.	" Sandstone	135 "
		" Brick, pressed	140 "
		" " com'n, rough	100 "

* One packed bushel = 1.43 loose.

Comparative Weight of Green and Seasoned Timber.

Timber.	Weight of a Cube Foot. Green.	Seasoned.	Timber.	Weight of a Cube Foot. Green.	Seasoned.
	Lbs.	Lbs.		Lbs.	Lbs.
American Pine	44.75	30.7	Cedar	32	28.25
Ash	58.18	50	English Oak	71.6	43.5
Beech	60	53.37	Riga Fir	48.75	35.5

Application of the Tables.

When Weight of a Solid or Liquid Substance is required. Rule.—Ascertain volume of substance in cube feet; multiply it by unit in second column of tables (its specific gravity), and divide product by 16; quotient will give weight in lbs.

When Volume is given or ascertained in Inches. Rule.—Multiply it by unit in third column of tables (weight of a cube inch), and product will give weight in lbs.

Example.—What is weight of a cube of Italian marble, sides being 3 feet?

$3^3 \times 2708 = 73\,116$ *oz.*, which $\div 16 = 4569.75$ *lbs.*

Or of a sphere of cast iron 2 inches in diameter?

$2^3 \times .5236 \times .2607$ *weight of a cube inch* $= 1.092$ *lbs.*

When Weight of an Elastic Fluid is required. Rule.—Multiply specific gravity of fluid by 532.679 (weight of a cube foot of air at 62° in grains), divide product by 7000 (grains in a lb. Avoirdupois), and quotient will give weight of a cube foot in lbs.

Example.—What is weight of a cube foot of hydrogen?

Specific gravity of hydrogen .0692.

$532.679 \times .0692 \div 7000 = .005\,265\,9$ *lbs.*

To Compute Weight of Cast Metal by Weight of Pattern.

When Pattern is of White Pine. Rule.—Multiply weight of pattern in lbs. by following multipliers, and product will give weight of casting:

Iron, 14; Brass, 15; Lead, 22; Tin, 14; Zinc, 13.5.

When there are Circular Cores or Prints. Multiply square of diameter of core or print by its length in inches, the product by .0175, and result is weight of pattern of core or print to be deducted from weight of pattern.

To Compute Weights of Ingredients, that of Compound being given.

RULE.—As specific gravity of compound is to weight of compound, so are each of the proportions to weight of its material.

EXAMPLE.—Weight, as above, being 28 lbs., what are weights of the ingredients?

$$14 : 28 :: \begin{cases} 7.835 : 15.67 \textit{ gold}, \\ 6.165 : 12.33 \textit{ silver}. \end{cases}$$

NOTE.—Specific gravity of alloys does not usually follow ratio of their components, it being sometimes greater and sometimes less than their mean.

To Compute Capacity of a Balloon.

RULE.—From specific gravity of air in grains per cube foot, subtract that of the gas with which it is inflated; multiply remainder by volume of balloon in cube feet; divide product by 7000, and from quotient subtract weight of balloon and its attachments.

EXAMPLE.—Diameter of a balloon is 26.6 feet, its weight is 100 lbs., and specific gravity of the gas with which it is inflated is .07 (air being assumed at 1); what is its capacity, specific gravity of air assumed at 527.04 grains.

$$\frac{527.04 - (527.04 \times .07)\ 36.89 \times 26.6^3 \times .5236}{7000} - 100 = 590.04 \textit{ lbs.}$$

To Compute Diameter of a Balloon.

Weight to be raised being given.—By inversion of preceding rule.

$\sqrt[3]{\dfrac{W \times 7000 + s - s'}{.5236}} = d'$. *s* and *s'* *representing weight of air and gas in grains per cube foot,* W *weight to be raised in lbs., and d diameter of balloon in feet.*

ILLUSTRATION.—Given elements in preceding case.

$$\text{Then } \sqrt[3]{\frac{590.04 + 100 \times 7000 \div 527.04 - 36.89}{.5236}} = \sqrt[3]{\frac{0854\ 69}{.5236}} = 26.6 \textit{ feet.}$$

Proof of Spirituous Liquors.

A cube inch of *Proof Spirits* weighs 234 grains; then, if an immersed cube inch of any heavy body weighs 234 grains less in spirits than air, it shows that the spirit in which it was weighed is *Proof.*

If it lose less of its weight, the spirit is above proof; and if it lose more, it is below proof.

ILLUSTRATION.—A cube inch of glass weighing 700 grains weighs 500 grains when weighed in a certain spirit; what is the proof of it?

$$700 - 500 = 200 = \textit{grains} = \textit{weight lost in spirit.}$$

Then $200 : 234 :: 1 : 1.17 =$ *ratio of proof of spirits compared to proof spirits, or* $1 = .17$ *above proof.*

NOTE.—For Hydrometers and Rules for ascertaining Proof of Spirits, see page 67; and for a very full treatise on Specific Gravities and on Floatation, see Jamieson's Mechanics of Fluids. Lond., 1837.

Shrinkage of Castings.

It is customary, in making of patterns for castings, to allow for shrinkage per lineal foot of pattern as follows:

Iron, small cylinders	= $\frac{1}{16}$ in. per ft.	Ditto in length	= $\frac{1}{8}$ in 16 ins.
" Pipes	= $\frac{1}{8}$ "	Brass, thin	= $\frac{1}{8}$ in 9 ins.
" Girders, beams, etc.	= $\frac{1}{8}$ in 15 ins.	" thick	= $\frac{1}{8}$ in 10 ins.
" Large cylinders, the contraction of diam. at top.	= $\frac{1}{16}$ per foot.	Zinc	= $\frac{5}{16}$ in a foot.
		Lead	= $\frac{5}{16}$ "
		Copper	= $\frac{3}{16}$ "
" Ditto at bottom	= $\frac{1}{12}$ "	Bismuth	= $\frac{5}{32}$ "

GEOMETRY.

Definitions.

Point has position, but not magnitude.
Line is length without breadth, and is either *Right*, *Curved*, or *Mixed*.
Right Line is shortest distance between two points.
Curved Line is one that continually changes its direction.
Mixed Line is composed of a right and a curved line.
Superficies has length and breadth only, and is plane or curved.
Solid has length, breadth, and thickness, or depth.

Angle is opening of two lines having different directions, and is either *Right*, *Acute*, or *Obtuse*.
Right Angle is made by a line perpendicular to another falling upon it.
Acute Angle is less than a right angle.
Obtuse Angle is greater than a right angle.

Triangle is a figure of three sides.
Equilateral Triangle has all its sides equal.
Isosceles Triangle has two of its sides equal.
Scalene Triangle has all its sides unequal.
Right-angled Triangle has one right angle.
Obtuse-angled Triangle has one obtuse angle.
Acute-angled Triangle has all its angles acute.
Oblique-angled Triangle has no right angle.

Quadrangle or *Quadrilateral* is a figure of four sides, and has following particular designations—viz.,
Parallelogram, having its opposite sides parallel.
Square, having length and breadth equal.
Rectangle, a parallelogram having a right angle.
Rhombus or *Lozenge*, having equal sides, but its angles not right angles.
Rhomboid, a parallelogram, its angles not being right angles.
Trapezium, having unequal sides.
Trapezoid, having only one pair of opposite sides parallel.
NOTE.—*Triangle is sometimes termed a Trigon, and a Square a Tetragon.*

Gnomon is space included between the lines forming two similar parallelograms, of which smaller is inscribed within larger, so as to have one angle in each common to both.

Polygons are plane figures having more than four sides, and are either *Regular* or *Irregular*, according as their sides and angles are equal or unequal, and they are named from number of their sides or angles. Thus:

Pentagon	has	five sides.	Nonagon	has	nine sides.
Hexagon	"	six "	Decagon	"	ten "
Heptagon	"	seven "	Undecagon	"	eleven "
Octagon	"	eight "	Dodecagon	"	twelve "

Circle is a plane figure bounded by a curved line, termed *Circumference* or *Periphery*.
Diameter is a right line passing through centre of a circle or sphere, and terminated at each end by periphery or surface.
Arc is any part of circumference of a circle.
Chord is a right line joining extremities of an arc.
Segment of a circle is any part bounded by an arc and its chord.
Radius of a circle is a line drawn from centre to circumference.
Sector is any part of a circle bounded by an arc and its two radii.
Semicircle is half a circle.
Quadrant is a quarter of a circle.
Zone is a part of a circle included between two parallel cords.
Lune is space between the intersecting arcs of two eccentric circles.

Secant is line running from centre of circle to extremity of tangent of arc.

Cosecant is secant of complement of an arc, or line running from centre of circle to extremity of cotangent of arc.

Sine of an arc is a line running from one extremity of an arc perpendicular to a diameter passing through other extremity, and sine of an angle is sine of arc that measures that angle.

Versed Sine of an arc or angle is part of diameter intercepted between sine and arc.

Cosine of an arc or angle is part of diameter intercepted between sine and centre.

Coversed Sine of an arc or angle is part of secondary radius intercepted between cosine and circumference.

Tangent is a right line that touches a circle without cutting it.

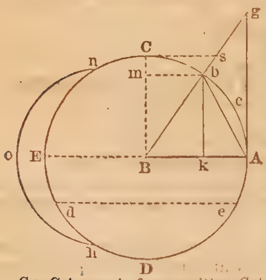
Cotangent is tangent of complement of arc.

Circumference of every circle is supposed to be divided into 360 equal parts, termed *Degrees*; each degree into 60 *Minutes*, and each minute into 60 *Seconds*, and so on.

Complement of an angle is what remains after subtracting angle from 90 degrees.

Supplement of an angle is what remains after subtracting angle from 180 degrees.

To exemplify these definitions, let A c b, *in following Figure, be an assumed arc of a circle described with radius* B A:



A c b, an Arc of circle A C E D.
A b, Chord of that arc.
B A, an Initial radius.
B C, a Secondary radius.
e D d, a Segment of the circle.
A B b, a Sector.
A D E, a Semicircle.
C B E, a Quadrant.
A e d E, a Zone.
n o h, a Lune.
B g, Secant of arc A c b; written Sec.
b k, Sine of arc A c b; written Sin.
A k, Versed Sine of arc A c b; written Versin.
B k or m b, Cosine of arc A c b.
A g, Tangent of arc A c b.
C B b, Complement, and b B E, Supplement of arc A c b.

C s, *Cotangent* of arc, written Cot. B s, *Cosecant* of arc; written Cosec.
m C, *Coversed sine* of arc, or, by convention, of angle A B b; written Coversin.

Vertex of a figure is its top or upper point. In Conic Sections it is point through which generating line of the conical surface always passes.

Altitude, or height of a figure, is a perpendicular let fall from its vertex to opposite side, termed base.

Measure of an angle is an arc of a circle contained between the two lines that form the angle, and is estimated by number of degrees in arc.

Segment is a part cut off by a plane, parallel to base.

Frustum is the part remaining after segment is cut off.

Perimeter of a figure is the sum of all its sides.

Problem is something proposed to be done.

Postulate is something required.

Theorem is something proposed to be demonstrated.

Lemma is something premised, to render what follows more easy.

Corollary is a truth consequent upon a preceding demonstration.

Scholium is a remark upon something going before it.

For other definitions see Mensuration of Surfaces and Solids, and Conic Sections.

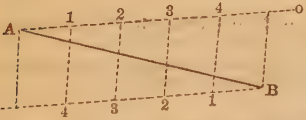
Lengths of following Elements, Radius = 1.

	Angle 45°.	Angle 60°.		Angle 45°.	Angle 60°.
Sine	.707 107	.866 025	Cosecant.	1.414 214	1.154 7
Cosine	.707 107	.5	Tangent.	1	1.732 05
Versed Sine. .	.292 893	.5	Cotangent . . .	1	.577 349
Coversed " . .	.292 893	.133 975	Chord.	.765 366	1
Secant	1.414 214	2	Arc.	.785 398	1.047 2

Scales.

To Divide a Line, as A B, with any required Number of Equal Parts.—Fig. 1.

1.

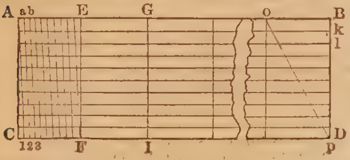


From A and B draw two parallel lines, A *o*, B *r*, to an indefinite length, and upon them point off required number of equal parts, as A 1, 2, 3, 4, and B 1, 2, 3, 4; join *o* B, 4 1, etc.

Or, point off on A *o*, join *o* B, and draw the other lines parallel thereto.

To Construct a Diagonal Scale, as A B.—Fig. 2.

2.



Divide a line into as many divisions as there are hundreds of feet, spaces of ten feet, feet, or inches required.

Draw perpendiculars from each division to a parallel line, C D. Divide them and one of divisions, A E, C F, into spaces of ten if for feet and hundredths, and twelve if for feet and inches; draw the lines A 1, *a* 2, *b* 3, etc., and they will complete scale.

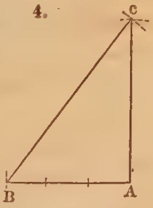
Thus: Line A B representing ten feet; A to E, E to G, etc., will measure one foot; A to *a*, C to 1, 1 to 2, etc., will measure 1-10th of a foot. The several lines A 1, *a* 2, etc., will measure upon lines *k*, *l*, etc., 1-100th of a foot; and *o p* will measure upon *k*, *l*, etc., divisions of 1-10th of a foot.

Lines.

To Draw a Perpendicular to a Right Line, as *o r*, Fig. 3, *c* A, Fig. 4, or from a Point external to it, as A, Fig. 5, and from any two Points, as *c d*, Fig. 6.

3.

4.



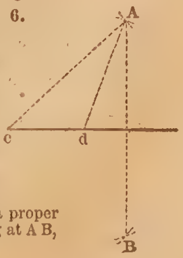
With any radius as *r* A, *r* B, cut line at A and B; then with a longer radius, as A *o*, B *o*, describe arcs cutting each other at *o*, and connect *o r*. (Fig. 3.)

Or, from A, set off A B equal to 3 parts by scale; from A B, with radii of 4 and 5 parts, describe arcs cutting at *c*, and connect *c* A. (Fig. 4.)

NOTE.—This method is useful where straight edges are inapplicable. Any multiples of numbers 3, 4, 5 may be taken with same effect, as 6, 8, 10, or 9, 12, 15.

5.

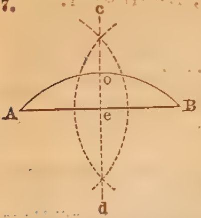
6.



From A, with a sufficient radius, cut line at *o c*, and from them describe arcs cutting at *r*, and connect A *r*. (Fig. 5.)

From any two points, as *c d*, at a proper distance apart, describe arcs cutting at A B, and connect them. (Fig. 6.)

7.



To Bisect a Right Line or an Arc of a Circle, and to Draw a Perpendicular to a Circular or Right Line, or a Radial Arc.—Fig. 7.

From A B as centres describe arcs cutting each other at *c* and *d*, connect *c d*, and line and arc are bisected at *e* and *o*.

Line *c d* is also perpendicular to a right line as A B, and radial to a circular arc as A *o* B.

8.

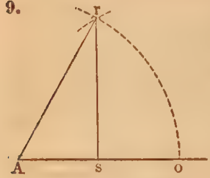
To Draw a Line Parallel to a Given Right Line, as c d, Fig. 8.

From A B describe arcs A c, A *d*, and draw a line parallel thereto, touching arcs c and *d*.

Angles.

To Describe Angles of 30° and 60°, Fig. 9, and 45°, Fig. 10.

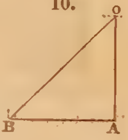
9.



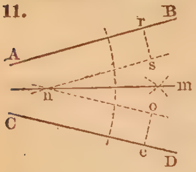
From A, with any radius, A *o*, describe *o r*, and from *o* with a like radius cut it at *r*, let fall perpendicular *r s*; then *o* A *r* = 60°, and A *r s* = 30°. (Fig. 9.)

Set off any distance, as A B, erect perpendicular A *o* = A B, and connect *o* B. (Fig. 10.)

10.



11.



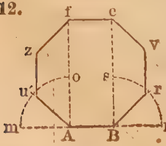
To Bisect Inclination of Two Lines, when Point of Intersection is Inaccessible.—Fig. 11.

Upon given lines, A B, C D, at any points draw perpendiculars *e o*, *s r*, of equal lengths, and from *o* and *s* draw parallels to their respective lines, cutting at *n*; bisect angle *o n s*, connect *n m*, and line will bisect lines as required.

Rectilineal Figures.

To Describe an Octagon upon a Line, as A B.—Fig. 12.

12.

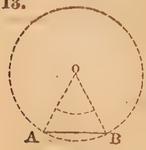


From points A B erect indefinite perpendiculars A *f*, B *e*; produce A B to *m* and *n*, and bisect angles *m* A *o* and *n* B *p* with A *u* and B *r*.

Make A *u* and B *r* equal to A B, and draw *u z*, *r v* parallel to A *f*, and equal to A B.

From *z* and *v*, as centres, with a radius equal to A B, describe arcs cutting A *f*, B *e*, in *f* and *e*. Connect *z f*, *f e*, and *e v*.

13.

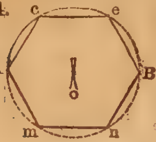


To Inscribe any Regular Polygon in a Circle, or to Divide Circumference into a given Number of Equal Parts.—Fig. 13.

If Circle is to contain a Heptagon.—Draw angle A *o* B at centre *o* for $360° \div 7 = 51° \; 42' \; 51''+$, or $51\frac{3}{7}$, then set off upon circumference distance A B or remaining angles A *o* B.

To Inscribe a Hexagon in a Circle.—Fig. 14.

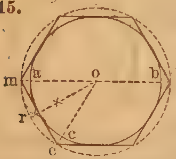
14.



Draw a diameter, A o B. From A and B as centres, with A o and B o, cut circle at c m and e n, and connect.

To Describe a Hexagon about a Circle.—Fig. 15.

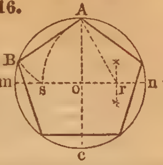
15.



Draw a diameter as a o b; and with a o cut circle at c; join a c, and bisect it with radius o r, through r draw e r parallel to c a, cutting diameter at m; then with radius o m describe circle, within which describe a hexagon as above.

To Inscribe a Pentagon in a Circle.—Fig. 16.

16.

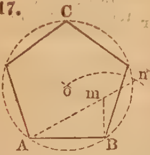


Draw diameters A c and m n, at right angles to each other; bisect o n in r, and with r A describe A s; from A with A s describe s B.

Connect A B, and distance is equal to one side of a pentagon.

To Describe a Pentagon upon a Line, as A B.—Fig. 17.

17.

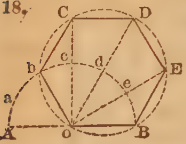


Draw B m perpendicular to A B, and equal to one half of it; extend A m until m n is equal to B m.

From A and B, with radius B n, describe arcs cutting each other in o; then from o, with radius o B, describe circle A C B, and line A B is equal to one side of a pentagon upon circle described.

To Describe a Regular Polygon of any required Number of Sides.—Fig. 18.

18.



From point o, with distance o B, describe semicircle B b A, which divide into as many equal parts, A a, a b, b c, etc., as the polygon is to have sides.

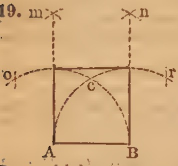
Thus, let a Hexagon be required:

From o to second point b of six divisions draw o b, and through other points, c, d, and e, draw o C, o D, etc.

Apply distance o B, from B to E, from E to D, from D to C, etc. Join these points, as b C, C D, etc.

To Construct a Square or a Rectangle on a given Line.—Fig. 19.

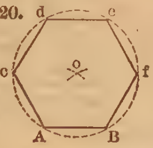
19.



On A B as centres, with A B as radius, describe arcs cutting at c; on c describe arcs cutting at o r; and on o r describe others, cutting at m n; draw A m, and B n, and join o r.

To Construct a Hexagon upon a given Line.—Fig. 20.

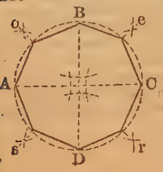
20.



From ends of line, A B, describe arcs cutting each other at o, and from o as a centre, with radius o A, describe a circle, and with same radius set off A c, e d, B f, f e, and connect them.

To Inscribe an Octagon in a Circle.—Fig. 21.

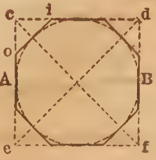
21.



Draw diameters, A C, B D, at right angles, bisect arcs, A B, B C, etc., at s, r, o, e, and join A o, o B, etc. (Fig. 21.)

To Describe an Octagon about a Circle.—Fig. 22.

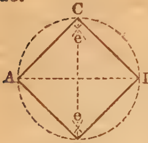
22.



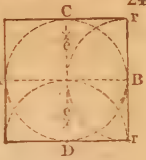
Describe a square about circle A B, draw diagonals c f, e d, draw o i, etc., perpendicular to diagonals and touching circle. (Fig. 22.)

To Inscribe a Square in a Circle.—Fig. 23.

23.



24.



Draw line A B through centre of circle; take any radius, as A *e*, and describe the arcs A *e e*, B *e e*; connect *e e*, continuing line to C and D; join A C, A D, etc. (Fig. 23.)

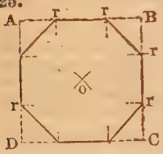
To Describe a Square about a Circle.—Fig. 24.

Draw line A B through centre of circle. Take any radius, as A *e*; describe arcs A *e e*, B *e e*; connect *e e*, continuing line to C D.

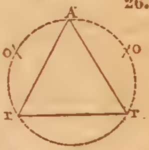
Describe B *r* and D *r*; draw and extend B *r* and D *r*, and sides A and C parallel to them. (Fig. 24.)

To Describe an Octagon in a Square.—Fig. 25.

25.



26.



Let A B C D be given square.

Describe A *o r r*, B *o r r*, etc.; join intersections *r r r r*, etc., and figure formed is octagon required. (Fig. 25.)

To Inscribe an Equilateral Triangle in a Circle.—Fig. 26.

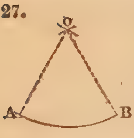
From point A, with A *o* equal to radius of circle, describe *o o*; from *o* and *o* describe *o r*, *o r*; join A *r*, *r r*, and *r* A. (Fig. 26.)

Note.—All figures of 10 or 20 sides are readily determined from side of a pentagon, being halved or quartered; and in like manner, all figures of 6, 12, or 24 sides are readily determined from radius of a circle, being equal to the side of a hexagon.

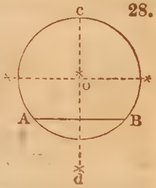
Circles.

To Describe an Arc of a Circle, through Two given Points, with a given Radius.—Fig. 27.

27.



28.

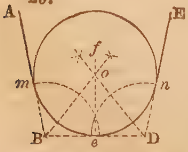


On A B as centres, with given radius, describe arcs cutting at *o*, and from *o* with same radius describe arc A B. (Fig. 27.)

To Ascertain Centre of a Circle or of an Arc of a Circle.—Fig. 28.

Draw chord A B, bisect it with perpendicular *c d*, then bisect *c d* for centre *o*. (Fig. 28.)

29.

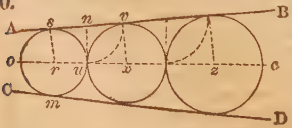


To Describe a Circular Segment that will both fill the angle between two diverging lines and touch them.—Fig. 29.

Bisect inclined lines, A B, D E, by line *e f*, and connect perpendicular thereto, B D, to define boundary of segment to be described. Bisect angles at B and D by lines cutting at *o*, and from *o*, with radius *o e*, describe arc *m e n*.

To Draw a Series of Circles between Two Inclined Lines, touching them and each other.—Fig. 30.

30.

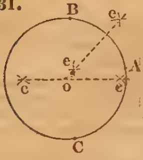


Bisect given lines A B, C D, by line *o c*. From a point *r* in this line erect *r s* perpendicular to A B, and on *r* describe circle *s m*, cutting centre line at *u*; from *u* erect *u n* perpendicular to centre line, cutting A B at *n*, and from *n* describe an arc *n u v*, cutting A B at *v*, erect *x v* parallel to *r s*, making *x* centre of next circle to be described, with radius *x u*, and so on.

Note.—Largest circle may be described first.

To Describe a Circle that shall pass through any three given Points, as A B C.—Figs. 31 and 32.

31.

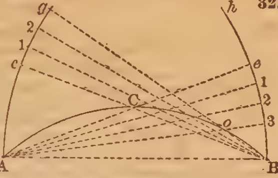


Upon points A and B, with any opening of a dividers, describe arcs cutting each other at *e e*.

On points B C describe two more cutting each other in points *c c*.

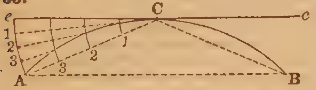
Draw lines *e e* and *c c*, and intersection of these lines, *o*, is centre of circle A B C. (Fig. 31.)

32.



When Centre is not attainable.—From A B as centres, describe arcs A *g*, B *h*; through C draw A *e*, B *c*. Divide A *e* and B *c* into any number of equal parts, also *c g* and B *h* into a like number. Draw A 1, 2, 3, etc., and B 1, 2, etc., and intersection of these lines as at *o* are points in the circle required. (Fig. 32.)

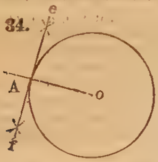
33.



Or, let A B C be given points, connect A B, A C, C B, and draw *e c* parallel to A B. Divide C A into a number of equal parts, as at 1, 2, and 3, and from C describe arcs through these points to meet right lines from C to points 1, 2, and 3, or A *e*, and these are points in a circle, to be drawn as before directed. (Fig. 33.)

To Draw a Tangent to a Circle from a given Point in Circumference.—Fig. 34.

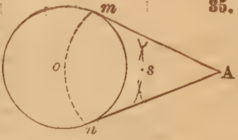
34.



Through point A draw radial line A *o*, and erect perpendicular *e f*. (Fig. 34.)

To Draw Tangents to a Circle from a Point without it.—Fig. 35.

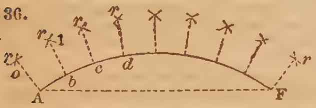
35.



From A draw A *o*, and bisect it at *s*; describe arc through *o*, cutting circle at *m n*; join A *m* or A *n*.

To Draw from or to Circumference of a Circle, Lines leading to an Inaccessible Centre.—Fig. 36.

36.

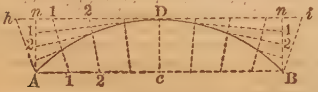


Divide whole or any given portion of circumference into desired number of parts; then, with any radius less than distance of two divisions, describe arcs cutting each other, as A *r*, *b r*, *c r*, *d r*, etc.; draw lines *b r*, *c r*, etc., and they will lead to centre.

To draw end lines, as A *r*, F *r*. From *b* describe arc *o*, and with radius *b* 1, from A or F as centres, cut arcs A *r*, etc., and lines A *r*, F *r*, will lead to centre.

To Describe an Arc, or Segment of a Circle, of a large Radius.—Fig. 37.

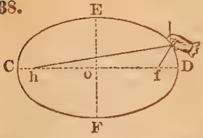
37.



Draw chord A *c* B; also line *h* D *i* parallel with chord, and at a distance equal to height of segment; bisect chord in *c*, and erect perpendicular *c* D; join A D, D B; draw A *h* and B *i* perpendicular to A D, B D; erect also perpendiculars A *n*, B *n*; divide A B and *h i* into any number of equal parts; draw lines 1 1, 2 2, etc., and divide lines A *n*, B *n*, each into half number of equal parts in A B; draw lines to D from each division in lines A *n*, B *n*, and at points of intersection with former lines describe arc or segment.

Ellipse.

To Describe an Ellipse to any Length and Breadth given.—Fig. 38.

38.


Let longest diameter be C D, and shortest E F. Take distance C o or o D, and with it, from points E and F, describe arcs h and f upon diameter C D.

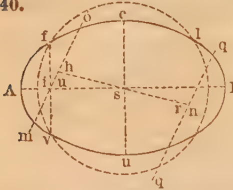
Insert pins at h and at f, and loop a string around them of such a length that when a pencil is introduced within it it will just reach to E or F. Bear upon string, sweep it around centre o, and it will describe ellipse.

NOTE.—It is a property of Ellipse that sum of two lines drawn from *foci* to meet in any point in curve is equal to transverse diameter.

39.


Bisect transverse axis A B at o, and on centre o erect perpendicular C D, making o D and o C each equal to half conjugate axis. From C or D, with radius A o, cut transverse axis at $s\,s$ for *foci*. Divide A o into any number of equal parts, as 1, 2, 3, etc. With radii A 1, B 1, on s and s as centres, describe arcs, and repeat this operation for all other divisions 1, 2, 3, etc., and these points of intersection will give line of curve.

To Ascertain Centre and Two Diameters of an Ellipse. —Fig. 40.

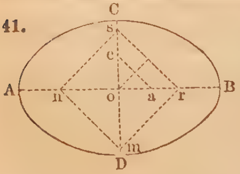
40.


Let A B $c\,u$ be diameters of an Ellipse.

Draw at pleasure two lines, $q\,q$, $o\,m$, parallel to each other, and equidistant from A and B; bisect them in points $h\,n$, and draw line $u\,r$; bisect it in s, and upon s, as a centre, describe a circle at pleasure, as $f\,l\,v$, cutting figure in points $f\,v$.

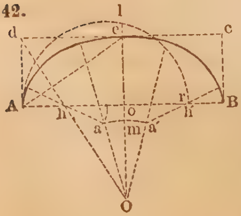
Draw right line $f\,v$; bisect it in i, and through points $i\,s$ draw greatest diameter A B, and through centre, s, draw least diameter $c\,u$, parallel to $f\,v$.

To Describe an Ellipse approximately by Circular Arcs. —Fig. 41.

41.


Set off differences of axes from centre o to a and c or o A and o C; draw $a\,c$ and bisect it, and set off its half to r; draw $r\,s$ parallel to $a\,c$, set off $o\,n$ equal to $o\,r$, connect $n\,s$, and draw parallels $r\,m$, $n\,m$; from m, with radii $m\,s$ and $s\,m$, describe arcs through C and D, and from n and r describe arcs through A and B.

NOTE.—This method is not satisfactory when conjugate axis is less than two thirds of transverse axis.

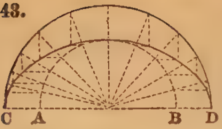
42.


With Arcs of Three Radii.—Fig. 42.

On transverse axis A B draw rectangle A B $c\,d$, on height $o\,e$; to diagonal A e draw perpendicular $d\,h$ O; set off $o\,r$ equal to $o\,e$, describe a semicircle on A r, and produce O e to l; set off $o\,m$ equal to $e\,l$, and on o describe an arc with radius O m; on A, with radius $o\,l$, cut this arc at a. Thus the five centres, O, a, a', h, h', are found, from which arcs are described to form ellipse.

NOTE.—This process answers for nearly all proportions of ellipses. It is used in striking vaults, stone bridges, etc.

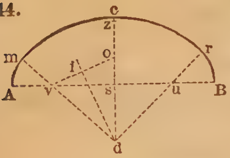
43.



To Construct an Ellipse from Two Circles.—Fig. 43.

Describe two semicircles, as A B, C D, diameters of which are respectively lengths of major and minor axes. The intersection of the horizontal and vertical lines drawn from any radial line will give a point in the curve C D.

44.

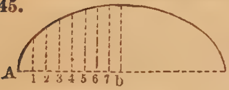


To Construct an Ellipse, when Two Diameters are Given.—Fig. 44.

Make *c o* and A *v* equal to each other, but less than half breadth. Draw *v o*, and from its centre *i* draw and extend perpendicular at *i* to *d*, draw *d v m*, make B *u* = A *v*, draw *d u r*, from *u* and *v* describe B *r* and A *m*, from *d* describe *m c r*, extend *c z* to *s*, and it will be centre for other half of figure.

To Construct an Ellipse by Ordinates.—Fig. 45.

45.



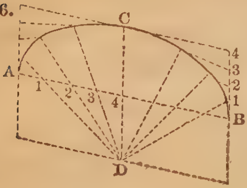
Divide semi-transverse axis, as A *b*, into 8 or 10 divisions, as may be convenient, and erect ordinates, the lengths of which are equal to semi-conjugate, multiplied by the units for each division as follows:

Divisions.

Eighths.		*Tenths.*		
1 — .484 12	5 — .927 03	1 — .453 89	5 — .866 02	9 — .994 99
2 — .661 44	6 — .968 24	2 — .6	6 — .916 51	10 — 1
3 — .780 63	7 — .992 16	3 — .714 14	7 — .993 94	
4 — .866 03	8 — 1	4 — .8	8 — .999 79	

To Construct an Ellipse when Diameters do not Intersect at Right Angles.—Fig. 46.

46.

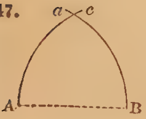


Let A B and C D be given diameters.

Draw boundary lines parallel to diameters, divide longest diameter into any number of equal parts, and divide shortest boundary lines into same number of equal parts.

From one end of shortest diameter, D, draw radial lines *through* divisions of longest diameter, and from opposite end, C, draw radial lines to divisions on shortest boundary lines; the intersection of these lines will give points in the curve.

47.



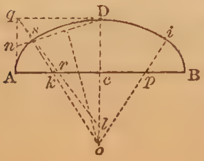
Arcs.

To Describe a Gothic Arc.—Fig. 47.

Take line A B. At points A and B draw arcs B *a* and A *c*, and it will describe arc required.

To Describe an Elliptic Arc, Chord and Height being given.—Fig. 48.

48.



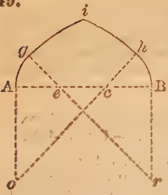
Bisect A B at *c*; erect perpendicular A *q*, and draw line *q* D equal and parallel to A *c*.

Bisect A *c* and A *q* in *r* and *n*; make *c l* equal to *c* D, and draw line *l r q*; draw also line *n s* D; bisect *s* D with a line at right angles, and cutting line *c* D at *o*; draw line *o q*; make *c p* equal to *c k*, and draw line *o p i*.

Then, from *o* as a centre, with radius *o* D, describe arc *s* D *i*; and from *k* and *p* as centres, with radius A *k*, describe arcs A *s* and B *i*.

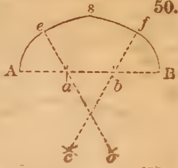
To Describe a Gothic Arc.—Figs. 49 and 50.

49.



Divide line A B into three equal parts, e c; from points A and B let fall perpendiculars A o and B r, equal in length to two of divisions of line A B; draw lines o h and r g from points e, c; with length of c B, describe arcs A g and B h, and from points o and r describe arcs g i and i h. (Fig. 49.)

50.



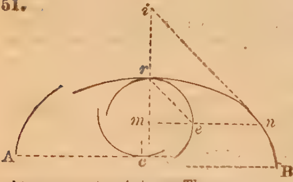
Or, divide line A B into three equal parts at a and b, and on points A, a, b, and B, with distance of two divisions, make four arcs intersecting at c and o.

Through points c, o, and divisions a, b, draw lines c f and o e, on points a and b describe arcs A e and B f, and on points c o arcs f s and e s. (Fig. 50.)

Cycloid and Epicycloid.

To Describe a Cycloid.—Fig. 51.

51.



When a circle, as a wheel, rolls over a straight right line, beginning as at A and ending at B, it completes one revolution, and measures a straight line, A B, exactly equal to circumference of circle c e r, which is termed the *generating circle*, and a point or pencil fixed at point r in circumference traces out a curvilinear path, A r B, termed a *cycloid*. A B is its *base* and c r its *axis*.

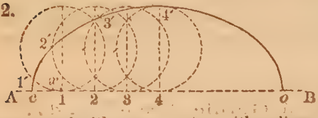
Place generating circle in middle of Cycloid, as in figure; draw a line, m n, parallel to base, cutting circle at e; and tangent n i to curve at point n. The fo… …wing are some of properties of Cycloid:

Horizontal line e n = arc of circle e.
Half base A c = half circumference c e r.
Arc of Cycloid r n = twice chord r e.
Half arc of cycloid A r = twice diameter of circle r c.

Or, whole arc of Cycloid A r B = four times axis c r.
Area of Cycloid A r B A = three times area of generating circle r c.
Tangent n i is parallel to chord e r.

To Describe Curve of a Cycloid.—Fig. 52.

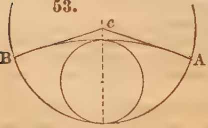
52.



On an indefinite line, A B, set off circumference of generating circle, divide this line into any number of equal parts (8 in figure), and at points of division erect perpendiculars thereto. Upon each of these lines describe a circle = generating circle. On c 1 take 1 x = .25 c 1, and with x as a centre, with radius x c = .75 c 1, describe an arc cutting circle at 1′; from 2 on next circle, with two distances of 1 1′, measured as chords, cut circle at 2′; from 3 on next circle, with three distances of 1 1′, cut circle at 3′, and proceed in like manner from each side until figure is complete.

To Describe an Interior Epicycloid or Hypocycloid.—Fig. 53.

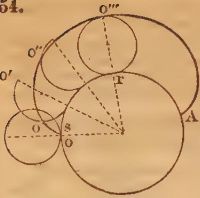
53.



If generating circle is rolled on inside of fundamental circle,* as in Fig. 53, it forms an *interior epicycloid*, or *hypocycloid*, A c B, which becomes in this case nearly a straight line. Other points of reference in figure correspond to those in Fig. 51. When diameter of generating circle is equal to half that of fundamental circle, epicycloid becomes a straight line, being diameter of the larger circle.

* See explanation, Fig. 54.

54.

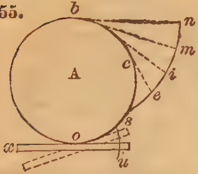


To Describe an Exterior Epicycloid.—Fig. 54.

An Epicycloid differs from a Cycloid in this, that it is generated by a point, o''', in one circle, $o\ r$, rolling upon circumference of another, A $r\ s$, instead of upon a right line or horizontal surface, former being *generating circle* and latter *fundamental circle.*

Generating circle is shown in four positions, in which its generating point is indicated by $o\ o'\ o''\ o'''$. A $o'''\ s$ is an Epicycloid.

55.



Involute.

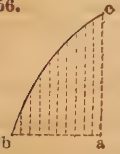
To Describe an Involute.—Fig. 55.

Assume A as centre of a circle, $b\ c\ o$; a cord laid partly upon its circumference, as $b\ e$; then the curve $e\ i\ m\ n$, described by a tracer at end of cord, when unwound from a circle, is an involute.

This curve can also be defined by a batten, x, rolling on a circle, as $s\ u$.

Parabola.

56.

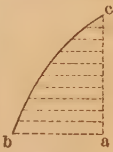


To Construct a Parabola by Ordinates or Abscissa.—Figs. 56 and 57.

By Ordinates.

Divide ordinate $a\ b$ into 10 equal parts, and erect perpendiculars, length of which will be determined by multiplying abscissa $a\ c$ by respective units for each perpendicular, as follows:

57.



Divisions.

1—.19	3—.51	5—.75	7—.91	9—.99
2—.36	4—.64	6—.84	8—.96	10—1

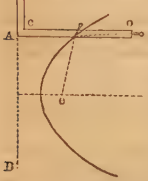
By Abscissa.

Divide abcissa $a\ c$ into 8 or 10 equal parts, as may be convenient, and draw ordinates thereto, the lengths of which will be determined by multiplying half ordinate $a\ b$ by respective units for each ordinate, as follows:

Divisions.

Eighths.		*Tenths.*	
1—.3535	5—.790 57	1—.316 23	6—.7746
2—.5	6—.866 02	2—.447 21	7—.836 66
3—.612 27	7—.935 41	3—.547 72	8—.894 43
4—.707 11	8—1	4—.632 45	9—.948 68
		5—.707 11	10—1

58.

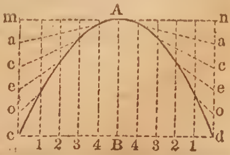


With a Square and Cord.—Fig. 58.

Place a straight edge to directrix A B, and apply to it a square, $c\ o$.

Attach to end o end of a cord equal to o A, and attach other end to focus e; slide square along straight edge, maintaining cord taut against edge of square, by a point or pencil, and curve will be traced. (Fig. 58.)

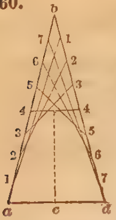
59.



When Height and Base are given.—Fig. 59.

Assume A B axis and $c\ d$ a double ordinate or base. Through A draw $m\ n$ parallel to $c\ d$, and through c and d draw $c\ m$, $d\ n$, parallel to axis A B. Divide $c\ m$, $d\ n$ into any number of equal parts, as at $a\ c\ e\ o$, also c B, B d, into a like number of parts. Through points 1, 2, 3, and 4 draw lines parallel to axis, and through $a\ c\ e\ o$ draw lines to vertex A, cutting these perpendiculars, and through these points curve may be traced. (Fig. 59.)

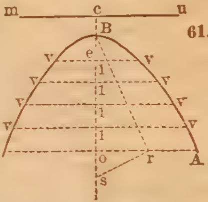
60.



To Describe Curve of a Parabola, Base and Height being given.—Fig. 60.

Draw an isosceles triangle, as *a b d*, base of which shall be equal to, and its height, *c b*, twice that of proposed parabola. Divide each side, *a b*, *d b*, into any number of equal parts; then draw lines, 1 1, 2 2, 3 3, etc., and their intersection will define curve. (Fig. 60.)

To Describe a Parabola, any Ordinate to Axis and its Abscissa being given.—Fig. 61.



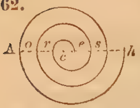
61.

Bisect ordinate, as A *o* in *r*; join B *r*, and draw *r s* perpendicular to it, meeting axis continued to *s*. Set off B *c*, B *e*, each equal to *o s*; draw *m c u* perpendicular to B *s*, then *m u* is directrix and B *e* focus; through *e* and any number of points, 1, 1, 1, etc., in axis, draw double ordinates *v* 1 *v*, and on centre *e*, with radii *e c*, 1 *c*, etc., cut respective ordinates at *v v*, etc., and trace curve through these points.

NOTE.—Line *v e v* passing through focus is parameter.

Spiral.

62.



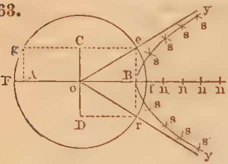
To Draw a Spiral about a given Point.—Fig. 62.

Assume *c* the centre. Draw A *h*, divide it into twice number of parts that there are to be revolutions of line. Upon *c* describe *r e*, *o s*, A *h*, and upon *e* describe *r s*, *o s*, etc.

Hyperbola.

To Describe a Hyperbola, Transverse and Conjugate Diameters being given.—Fig. 63.

63.



Let A B represent transverse diameter, and C D conjugate.

Draw C *e* parallel to A B, and *e r* parallel to C D; draw *o e*, and with radius *o e*, with *o* as a centre, describe circle F *e r*, cutting transverse axis produced in F and *f*; then will F and *f* be foci of figure.

In *o* B produced take any number of points, *n*, *n*, etc., and from F and *f* as centres, with A *n* and B *n* as radii, describe arcs cutting each other in *s*, *s*, etc. Through *s*, *s*, etc., draw curve *s s s s* B *s s s s*.

NOTE.—If straight lines, as *o e y* and *o r y*, are drawn from centre *o* through extremities *e r*, they will be asymptotes of hyperbola, property of which is to approach continually to curve, and yet never to touch it.

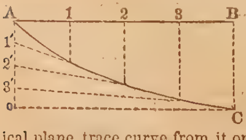
When Foci and Conjugate Axis are given.—Let F and *f* be foci, and C D conjugate axis, as in preceding figure.

Through C draw *g* C *e* parallel to F and *f*; then, with *o* as a centre and *o* F as a radius, describe an arc cutting *g* C *e* at *g* and *e*; from these points let fall perpendiculars upon line connecting F and *f*, and part intercepted between them, as A B, will be transverse axis.

Catenary.

To Delineate a Catenary, Span and Versed Sine being given.—Fig. 64. (*W. Hildenbrand.*)

64.



Divide half span, as A B, into any required number of equal parts, as 1, 2, 3, and let fall B C and A *o*, each equal to versed sine of curve; divide A *o* into like number of parts, 1′, 2′, 3′, as A B. Connect C 1′, C 2′, and C 3′, and points of intersection of perpendiculars let fall from A B will give points through which curve is to be drawn.

Or, suspend a finely linked chain against a vertical plane, trace curve from it on the plane in accordance with conditions of given length and height, or of given width or length of arc.

NOTE.—For other methods see D. R. Clark's Manual, pp. 18, 19.

Areas of Circles, from 1/64 to 150.

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
1/64	.000 192	3	7.0686	7	38.4846	14	153.938
1/32	.000 767	1/16	7.3662	1/8	39.8713	1/8	156.7
1/16	.003 068	1/8	7.6699	1/4	41.2826	1/4	159.485
1/8	.012 272	3/16	7.9798	3/8	42.7184	3/8	162.296
3/16	.027 612	1/4	8.2958	1/2	44.1787	1/2	165.13
1/4	.049 087	5/16	8.618	5/8	45.6636	5/8	167.99
5/16	.076 699	3/8	8.9462	3/4	47.1731	3/4	170.874
3/8	.110 447	7/16	9.2807	7/8	48.7071	7/8	173.782
7/16	.150 33	1/2	9.6211	8	50.2656	15	176.715
1/2	.196 35	9/16	9.968	1/8	51.8487	1/8	179.673
9/16	.248 505	5/8	10.3206	1/4	53.4563	1/4	182.655
5/8	.306 796	11/16	10.679	3/8	55.0884	3/8	185.661
11/16	.371 224	3/4	11.0447	1/2	56.7451	1/2	188.692
3/4	.441 787	13/16	11.416	5/8	58.4264	5/8	191.748
13/16	.518 487	7/8	11.7933	3/4	60.1322	3/4	194.828
7/8	.601 322	15/16	12.177	7/8	61.8625	7/8	197.933
15/16	.690 292	4	12.5664	9	63.6174	16	201.062
1	.7854	1/16	12.962	1/8	65.3968	1/8	204.216
1/16	.8866	1/8	13.3641	1/4	67.2008	1/4	207.395
1/8	.994 02	3/16	13.772	3/8	69.0293	3/8	210.598
3/16	1.1075	1/4	14.1863	1/2	70.8823	1/2	213.825
1/4	1.2272	5/16	14.606	5/8	72.7599	5/8	217.077
5/16	1.353	3/8	15.033	3/4	74.6621	3/4	220.354
3/8	1.4849	7/16	15.465	7/8	76.5888	7/8	223.655
7/16	1.6229	1/2	15.9043	10	78.54	17	226.981
1/2	1.7671	9/16	16.349	1/8	80.5158	1/8	230.331
9/16	1.9175	5/8	16.8002	1/4	82.5161	1/4	233.706
5/8	2.0739	11/16	17.257	3/8	84.5409	3/8	237.105
11/16	2.2365	3/4	17.7206	1/2	86.5903	1/2	240.529
3/4	2.4053	13/16	18.19	5/8	88.6643	5/8	243.977
13/16	2.58	7/8	18.6655	3/4	90.7628	3/4	247.45
7/8	2.7612	15/16	19.147	7/8	92.8858	7/8	250.948
15/16	2.9483	5	19.635	11	95.0334	18	254.47
2	3.1416	1/16	20.129	1/8	97.2055	1/8	258.016
1/16	3.338	1/8	20.629	1/4	99.4022	1/4	261.587
1/8	3.5466	3/16	21.135	3/8	101.6234	3/8	265.183
3/16	3.7584	1/4	21.6476	1/2	103.8691	1/2	268.803
1/4	3.9761	5/16	22.166	5/8	106.1394	5/8	272.448
5/16	4.2	3/8	22.6907	3/4	108.4343	3/4	276.117
3/8	4.4301	7/16	23.221	7/8	110.7537	7/8	279.811
7/16	4.7066	1/2	23.7583	12	113.098	19	283.529
1/2	4.9087	9/16	24.301	1/8	115.466	1/8	287.272
9/16	5.1573	5/8	24.8505	1/4	117.859	1/4	291.04
5/8	5.4119	11/16	25.406	3/8	120.277	3/8	294.832
11/16	5.6723	3/4	25.9673	1/2	122.719	1/2	298.648
3/4	5.9396	13/16	26.535	5/8	125.185	5/8	302.489
13/16	6.2126	7/8	27.1086	3/4	127.677	3/4	306.355
7/8	6.4918	15/16	27.688	7/8	130.192	7/8	310.245
15/16	6.7772	6	28.2744	13	132.733	20	314.16
		1/8	29.4648	1/8	135.297	1/8	318.099
		1/4	30.6797	1/4	137.887	1/4	322.063
		3/8	31.9191	3/8	140.501	3/8	326.051
		1/2	33.1831	1/2	143.139	1/2	330.064
		5/8	34.4717	5/8	145.802	5/8	334.102
		3/4	35.7848	3/4	148.49	3/4	338.164
		7/8	37.1224	7/8	151.202	7/8	342.25

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
21	346.361	28	615.754	35	962.115	42	1385.45
⅛	350.497	⅛	621.264	⅛	969	⅛	1393.7
¼	354.657	¼	626.798	¼	975.909	¼	1401.99
⅜	358.842	⅜	632.357	⅜	982.842	⅜	1410.3
½	363.051	½	637.941	½	989.8	½	1418.63
⅝	367.285	⅝	643.549	⅝	996.783	⅝	1426.99
¾	371.543	¾	649.182	¾	1003.79	¾	1435.37
⅞	375.826	⅞	654.84	⅞	1010.822	⅞	1443.77
22	380.134	29	660.521	36	1017.878	43	1452.2
⅛	384.466	⅛	666.228	⅛	1024.96	⅛	1460.66
¼	388.822	¼	671.959	¼	1032.065	¼	1469.14
⅜	393.203	⅜	677.714	⅜	1039.195	⅜	1477.64
½	397.609	½	683.494	½	1046.349	½	1486.17
⅝	402.038	⅝	689.299	⅝	1053.528	⅝	1494.73
¾	406.494	¾	695.128	¾	1060.732	¾	1503.3
⅞	410.973	⅞	700.982	⅞	1067.96	⅞	1511.91
23	415.477	30	706.86	37	1075.213	44	1520.53
⅛	420.004	⅛	712.763	⅛	1082.49	⅛	1529.19
¼	424.558	¼	718.69	¼	1089.792	¼	1537.86
⅜	429.135	⅜	724.642	⅜	1097.118	⅜	1546.56
½	433.737	½	730.618	½	1104.469	½	1555.29
⅝	438.364	⅝	736.619	⅝	1111.844	⅝	1564.04
¾	443.015	¾	742.645	¾	1119.244	¾	1572.81
⅞	447.69	⅞	748.695	⅞	1126.669	⅞	1581.61
24	452.39	31	754.769	38	1134.118	45	1590.43
⅛	457.115	⅛	760.869	⅛	1141.591	⅛	1599.28
¼	461.864	¼	766.992	¼	1149.089	¼	1608.16
⅜	466.638	⅜	773.14	⅜	1156.612	⅜	1617.05
½	471.436	½	779.313	½	1164.159	½	1625.97
⅝	476.259	⅝	785.51	⅝	1171.731	⅝	1634.92
¾	481.107	¾	791.732	¾	1179.327	¾	1643.89
⅞	485.979	⅞	797.979	⅞	1186.948	⅞	1652.89
25	490.875	32	804.25	39	1194.593	46	1661.91
⅛	495.796	⅛	810.545	⅛	1202.263	⅛	1670.95
¼	500.742	¼	816.865	¼	1209.958	¼	1680.02
⅜	505.712	⅜	823.21	⅜	1217.677	⅜	1689.11
½	510.706	½	829.579	½	1225.42	½	1698.23
⅝	515.726	⅝	835.972	⅝	1233.188	⅝	1707.37
¾	520.769	¾	842.391	¾	1240.981	¾	1716.54
⅞	525.838	⅞	848.833	⅞	1248.798	⅞	1725.73
26	530.93	33	855.301	40	1256.64	47	1734.95
⅛	536.048	⅛	861.792	⅛	1264.506	⅛	1744.19
¼	541.19	¼	868.309	¼	1272.397	¼	1753.45
⅜	546.356	⅜	874.85	⅜	1280.312	⅜	1762.74
½	551.547	½	881.415	½	1288.252	½	1772.06
⅝	556.763	⅝	888.005	⅝	1296.217	⅝	1781.4
¾	562.003	¾	894.62	¾	1304.206	¾	1790.76
⅞	567.267	⅞	901.259	⅞	1312.219	⅞	1800.15
27	572.557	34	907.922	41	1320.257	48	1809.56
⅛	577.87	⅛	914.611	⅛	1328.32	⅛	1819
¼	583.209	¼	921.323	¼	1336.407	¼	1828.46
⅜	588.571	⅜	928.061	⅜	1344.519	⅜	1837.95
½	593.959	½	934.822	½	1352.655	½	1847.46
⅝	599.371	⅝	941.609	⅝	1360.816	⅝	1856.99
¾	604.807	¾	948.42	¾	1369.001	¾	1866.55
⅞	610.268	⅞	955.255	⅞	1377.211	⅞	1876.14

DIAM.	AREA.	DIAM.	AREA.	DIAM.	AREA.	DIAM.	AREA.
49	1885.75	56	2463.01	63	3117.25	70	3848.46
⅛	1895.38	⅛	2474.02	⅛	3129.64	⅛	3862.22
¼	1905.04	¼	2485.05	¼	3142.04	¼	3876
⅜	1914.72	⅜	2496.11	⅜	3154.47	⅜	3889.8
½	1924.43	½	2507.19	½	3166.93	½	3903.63
⅝	1934.16	⅝	2518.3	⅝	3179.41	⅝	3917.49
¾	1943.91	¾	2529.43	¾	3191.91	¾	3931.37
⅞	1953.69	⅞	2540.58	⅞	3204.44	⅞	3945.27
50	1963.5	57	2551.76	64	3217[illegible]	71	3959.2
⅛	1973.33	⅛	2562.97	⅛	3229.58	⅛	3973.15
¼	1983.18	¼	2574.2	¼	3242.18	¼	3987.13
⅜	1993.06	⅜	2585.45	⅜	3254.81	⅜	4001.13
½	2002.97	½	2596.73	½	3267.46	½	4015.16
⅝	2012.89	⅝	2608.03	⅝	3280.14	⅝	4029.21
¾	2022.85	¾	2619.36	¾	3292.84	¾	4043.29
⅞	2032.82	⅞	2630.71	⅞	3305.56	⅞	4057.39
51	2042.83	58	2642.09	65	3318.31	72	4071.51
⅛	2052.85	⅛	2653.49	⅛	3331.09	⅛	4085.66
¼	2062.9	¼	2664.91	¼	3343.89	¼	4099.84
⅜	2072.98	⅜	2676.36	⅜	3356.71	⅜	4114.04
½	2083.08	½	2687.84	½	3369.56	½	4128.26
⅝	2093.2	⅝	2699.33	⅝	3382.44	⅝	4142.51
¾	2103.35	¾	2710.86	¾	3395.33	¾	4156.78
⅞	2113.52	⅞	2722.41	⅞	3408.26	⅞	4171.08
52	2123.72	59	2733.98	66	3421.2	73	4185.4
⅛	2133.94	⅛	2745.57	⅛	3434.17	⅛	4199.74
¼	2144.19	¼	2757.2	¼	3447.17	¼	4214.11
⅜	2154.46	⅜	2768.84	⅜	3460.19	⅜	4228.51
½	2164.76	½	2780.51	½	3473.24	½	4242.93
⅝	2175.08	⅝	2792.21	⅝	3486.3	⅝	4257.37
¾	2185.42	¾	2803.93	¾	3499.4	¾	4271.84
⅞	2195.79	⅞	2815.67	⅞	3512.52	⅞	4286.33
53	2206.19	60	2827.44	67	3525.66	74	4300.85
⅛	2216.61	⅛	2839.23	⅛	3538.83	⅛	4315.39
¼	2227.05	¼	2851.05	¼	3552.02	¼	4329.96
⅜	2237.52	⅜	2862.89	⅜	3565.24	⅜	4344.55
½	2248.01	½	2874.76	½	3578.48	½	4359.17
⅝	2258.53	⅝	2886.65	⅝	3591.74	⅝	4373.81
¾	2269.07	¾	2898.57	¾	3605.04	¾	4388.47
⅞	2279.64	⅞	2910.51	⅞	3618.35	⅞	4403.16
54	2290.23	61	2922.47	68	3631.69	75	4417.87
⅛	2300.84	⅛	2934.46	⅛	3645.05	⅛	4432.61
¼	2311.48	¼	2946.48	¼	3658.44	¼	4447.38
⅜	2322.15	⅜	2958.52	⅜	3671.86	⅜	4462.16
½	2332.83	½	2970.58	½	3685.29	½	4476.98
⅝	2343.55	⅝	2982.67	⅝	3698.76	⅝	4491.81
¾	2354.29	¾	2994.78	¾	3712.24	¾	4506.67
⅞	2365.05	⅞	3006.92	⅞	3725.75	⅞	4521.56
55	2375.83	62	3019.08	69	3739.29	76	4536.47
⅛	2386.65	⅛	3031.26	⅛	3752.85	⅛	4551.41
¼	2397.48	¼	3043.47	¼	3766.43	¼	4566.36
⅜	2408.34	⅜	3055.71	⅜	3780.04	⅜	4581.35
½	2419.23	½	3067.97	½	3793.68	½	4596.36
⅝	2430.14	⅝	3080.25	⅝	3807.34	⅝	4611.39
¾	2441.07	¾	3092.56	¾	3821.02	¾	4626.45
⅞	2452.03	⅞	3104.89	⅞	3834.73	⅞	4641.53

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
77	4656.64	84	5541.78	91	6503.9	98	7542.98
⅛	4671.77	⅛	5558.29	⅛	6521.78	⅛	7562.24
¼	4686.92	¼	5574.82	¼	6539.68	¼	7581.52
⅜	4702.1	⅜	5591.37	⅜	6557.61	⅜	7600.82
½	4717.31	½	5607.95	½	6575.56	½	7620.15
⅝	4732.54	⅝	5624.56	⅝	6593.54	⅝	7639.5
¾	4747.79	¾	5641.18	¾	6611.55	¾	7658.88
⅞	4763.07	⅞	5657.84	⅞	6629.57	⅞	7678.28
78	4778.37	85	5674.51	92	6647.63	99	7697.71
⅛	4793.7	⅛	5691.22	⅛	6665.7	⅛	7717.16
¼	4809.05	¼	5707.94	¼	6683 8	¼	7736.63
⅜	4824.43	⅜	5724.69	⅜	6701.93	⅜	7756.13
½	4839.83	½	5741.47	½	6720.08	½	7775.66
⅝	4855.26	⅝	5758.27	⅝	6738.25	⅝	7795.21
¾	4870.71	¾	5775.1	¾	6756.45	¾	7814.78
⅞	4886.18	⅞	5791.94	⅞	6774.68	⅞	7834.38
79	4901.68	86	5808.82	93	6792.92	100	7854
⅛	4917.21	⅛	5825.72	⅛	6811.2	¼	7893.32
¼	4932.75	¼	5842.64	¼	6829 49	½	7932.74
⅜	4948.33	⅜	5859.59	⅜	6847.82	¾	7972.25
½	4963.92	½	5876.56	½	6866.16	101	8011.87
⅝	4979.55	⅝	5893.55	⅝	6884.53	¼	8051.58
¾	4995.19	¾	5910.58	¾	6902.93	½	8091.39
⅞	5010.86	⅞	5927.62	⅞	6921.35	¾	8131.3
80	5026.56	87	5944.69	94	6939.79	102	8171.3
⅛	5042.28	⅛	5961.79	⅛	6958.26	¼	8211.41
¼	5058.03	¼	5978.91	¼	6976.76	½	8251.61
⅜	5073.79	⅜	5996.05	⅜	6995.28	¾	8291.91
½	5089.59	½	6013.22	½	7013.82	103	8332.31
⅝	5105.41	⅝	6030.41	⅝	7032.39	¼	8372.81
¾	5121.25	¾	6047.63	¾	7050.98	½	8413.4
⅞	5137.12	⅞	6064.87	⅞	7069.59	¾	8454.09
81	5153.01	88	6082.14	95	7088.23	104	8494.89
⅛	5168.93	⅛	6099 43	⅛	7106.9	¼	8535.78
¼	5184.87	¼	6116.74	¼	7125.59	½	8576.76
⅜	5200.83	⅜	6134.08	⅜	7144.31	¾	8617.85
½	5216.82	½	6151.45	½	7163.04	105	8659.03
⅝	5232.84	⅝	6168.84	⅝	7181.81	¼	8700.32
¾	5248.88	¾	6186.25	¾	7200.6	½	8741.7
⅞	5264.94	⅞	6203.69	⅞	7219.41	¾	8783.18
82	5281.03	89	6221.15	96	7238.25	106	8824.75
⅛	5297.14	⅛	6238.64	⅛	7257.11	¼	8866.43
¼	5313.28	¼	6256.15	¼	7275.99	½	8908.2
⅜	5329.44	⅜	6273.69	⅜	7294.91	¾	8950.07
½	5345.63	½	6291.25	½	7313.84	107	8992.04
⅝	5361.84	⅝	6308 84	⅝	7332.8	¼	9034.11
¾	5378.08	¾	6326.45	¾	7351.79	½	9076.28
⅞	5394.34	⅞	6344.08	⅞	7370.79	¾	9118.54
83	5410.62	90	6361.74	97	7389.83	108	9160.91
⅛	5426.93	⅛	6379.42	⅛	7408.89	¼	9203.37
¼	5443.26	¼	6397.13	¼	7427.97	½	9245.93
⅜	5459.62	⅜	6414.86	⅜	7447.08	¾	9288.58
½	5476.01	½	6432.62	½	7466.21	109	9331.34
⅝	5492.41	⅝	6450.4	⅝	7485.37	¼	9374.19
⅝	5508.84	¾	6468.21	¾	7504.55	½	9417.14
⅞	5525.3	⅞	6486.04	⅞	7523.75	¾	9460.19

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
110	9 503.34	120	11 309.76	130	13 273.26	140	15 393.84
¼	9 546.59	¼	11 356.93	¼	13 324.36	¼	15 448.87
½	9 589.93	½	11 404.2	½	13 375.56	½	15 503.99
¾	9 633.37	¾	11 451.57	¾	13 426.85	¾	15 559.22
111	9 676.91	121	11 499.04	131	13 478.25	141	15 614.54
¼	9 720.55	¼	11 546.61	¼	13 529.74	¼	15 669.96
½	9 764.29	½	11 594.27	½	13 581.33	½	15 725.48
¾	9 808.12	¾	11 642.03	¾	13 633.02	¾	15 781.09
112	9 852.06	122	11 689.89	132	13 684.81	142	15 836.81
¼	9 896.09	¼	11 737.85	¼	13 736.69	¼	15 892.62
½	9 940.22	½	11 785.91	½	13 788.68	½	15 948.53
¾	9 984.45	¾	11 834.06	¾	13 840.76	¾	16 004.54
113	10 028.77	123	11 882.32	133	13 892 94	143	16 060.64
¼	10 073.2	¼	11 930.67	¼	13 945.22	¼	16 116.85
½	10 117.72	½	11 979.12	½	13 997.6	½	16 173.15
¾	10 162.34	¾	12 027.66	¾	14 050.07	¾	16 229.55
114	10 207.06	124	12 076.31	134	14 102.64	144	16 286.05
¼	10 251.88	¼	12 125.05	¼	14 155.31	¼	16 342.65
½	10 296.79	½	12 173.9	½	14 208.08	½	16 399 35
¾	10 341.8	¾	12 222.84	¾	14 253.09	¾	16 456.14
115	10 386.91	125	12 271.87	135	14 313.91	145	16 513.03
¼	10 432.12	¼	12 321.01	¼	14 366.98	¼	16 570.02
½	10 477.43	½	12 370.25	½	14 420.14	½	16 627.11
¾	10 522.84	¾	12 419.58	¾	14 473.4	¾	16 684.3
116	10 568.34	126	12 469.01	136	14 526.76	146	16 741.59
¼	10 613.94	¼	12 518.54	¼	14 580.21	¼	16 798.97
½	10 659.65	½	12 568.17	½	14 633.77	½	16 856.45
¾	10 705.44	¾	12 618.09	¾	14 687.42	¾	16 914.03
117	10 751.34	127	12 667.72	137	14 741.17	147	16 971.71
¼	10 797.34	¼	12 717.64	¼	14 795.02	¼	17 029.48
½	10 843.43	½	12 767.66	½	14 848.97	½	17 087.36
¾	10 889.62	¾	12 817.78	¾	14 903.01	¾	17 145.33
118	10 935.91	128	12 867.99	138	14 957.16	148	17 203.4
¼	10 982.3	¼	12 918.31	¼	15 011.4	¼	17 261.57
½	11 028.78	½	12 968.72	½	15 065.74	½	17 319.84
¾	11 075.37	¾	13 019.23	¾	15 120.18	¾	17 378.2
119	11 122.05	129	13 069.84	139	15 174.71	149	17 436.67
¼	11 168.83	¼	13 120.55	¼	15 229.35	¼	17 495.23
½	11 215.71	½	13 171.35	½	15 284.08	½	17 553.89
¾	11 262.69	¾	13 222.26	¾	15 338.91	150	17 671.5

To Compute Area of a Circle greater than any in Table.

Rule.—Divide dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in table.

Take tabular area for this diameter, multiply it by square of divisor, and product will give area required.

Example.—What is area for a diameter of 1050?

$1050 \div 7 = 150$; *tab. area*, $150 = 17\,671.5$, which $\times 7^2 = 865\,903.5$, *area*.

To Compute Area of a Circle in Feet and Inches, etc., by preceding Table.

Rule.—Reduce dimension to inches or eighths, as the case may be, and take area in that term from table for that number.

Divide this number by 64 (square of 8) if it is in eighths, and quotient will give area in inches, and divide again by 144 (square of 12) if it is in inches, and quotient will give area in feet.

EXAMPLE.—What is area of 1 foot 6.375 ins.?

1 foot 6.375 ins. = 18.375 *ins.* = 147 *eighths.* Area of 147 = 16 971.71, which ÷ 64 = 265.181 25 *ins.*; and by 144 = 1.84 125 *feet.*

To Compute Area of a Circle Composed of an Integer and a Fraction.

RULE.—Double, treble, or quadruple dimension given, until fraction is increased to a whole number, or to one of those in the table, as ⅛, ¼, etc., provided it is practicable to do so.

Take area for this diameter; and if it is double of that for which area is required, take one fourth of it; if treble, take one sixteenth of it, etc.

EXAMPLE.—Required area for a circle of 2.1875 ins.

2.1875 × 2 = 4.375, area for which = 15.0331, which ÷ 4 = 3.758 *ins.*

When Diameter is composed of Integers and Fractions contained in Table.

RULE.—Point off a decimal to a diameter from table, and add twice as many figures or ciphers to the right of the area as there are figures cut off from the diameter.

EXAMPLE 1.—What is area of 9675 feet diameter?

Area of 96.75 = 7351.79; hence, area = 73 517 900 *feet.*

2.—What is area of 24 375 feet diameter?

Area of 2.4375 = 4.6664; hence, area = 466 640 000 *feet.*

To Ascertain Area of a Circle as 300, 3000, etc., not contained in Table.

RULE.—Take area of 3 or 30, and add twice the excess of ciphers to the result.

EXAMPLE.—What is area of a circle 3000 feet in diameter?

Area of 30 = 706.86, hence area of 3000 = 7 068 600 *feet.*

To Compute Area of a Circle by Logarithms.

RULE.—To twice log. of diameter add $\bar{1}.895\,091$ (log. of .7854), and sum is log. of area, for which take number.

EXAMPLE.—What is area of a circle 1200 feet in diameter?

Log. 1200 × 2 + $\bar{1}.895\,091$ = 6.158 362 + $\bar{1}.895\,091$ = 6.053 453, and number for which = 1 130 976 *feet.*

Areas of Birmingham Wire Gauge.

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
No.	Sq. Inch.	No.	Sq. Inch.	No.	Sq. Inch.	No.	Sq. Inch.
1	.070 686	10	.014 103	19	.001 385	28	.000 154
2	.063 347	11	.011 309	20	.000 962	29	.000 133
3	.052 685	12	.009 331	21	.000 804	30	.000 113
4	.044 488	13	.007 088	22	.000 616	31	.000 078
5	.038 013	14	.005 411	23	.000 491	32	.000 064
6	.032 365	15	.004 071	24	.000 38	33	.000 05
7	.025 447	16	.003 318	25	.000 314	34	.000 038
8	.021 382	17	.002 642	26	.000 254	35	.000 02
9	.017 203	18	.001 886	27	.000 201	36	.000 013

Circumferences of Circles, from $\frac{1}{64}$ to 150.

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
1/64	.049 09	3	9.4248	8	25.1328	15	47.124
1/32	.098 18	1/16	9.6211	1/8	25.5255	1/8	47.5167
1/16	.196 35	1/8	9.8175	1/4	25.9182	1/4	47.9094
1/8	.392 7	3/16	10.014	3/8	26.3109	3/8	48.3021
3/16	.589	1/4	10.2102	1/2	26.7036	1/2	48.6948
1/4	.785 4	5/16	10.406	5/8	27.0963	5/8	49.0875
5/16	.981 75	3/8	10.6029	3/4	27.489	3/4	49.4802
3/8	1.178 1	7/16	10.799	7/8	27.8817	7/8	49.8729
7/16	1.374 45	1/2	10.9956	9	28.2744	16	50.2656
1/2	1.570 8	9/16	11.191	1/8	28.6671	1/8	50.6583
9/16	1.767 15	5/8	11.3883	1/4	29.0598	1/4	51.051
5/8	1.963 5	11/16	11.584	3/8	29.4525	3/8	51.4437
11/16	2.159 85	3/4	11.781	1/2	29.8452	1/2	51.8364
3/4	2.356 2	13/16	11.977	5/8	30.2379	5/8	52.2291
13/16	2.552 55	7/8	12.1737	3/4	30.6306	3/4	52.6218
7/8	2.748 9	15/16	12.369	7/8	31.0233	7/8	53.0145
15/16	2.945 25	4	12.5664	10	31.416	17	53.4072
1	3.141 6	1/16	12.762	1/8	31.8087	1/8	53.7999
1/16	3.337 9	1/8	12.9591	1/4	32.2014	1/4	54.1926
1/8	3.534 3	3/16	13.155	3/8	32.5941	3/8	54.5853
3/16	3.730 6	1/4	13.3518	1/2	32.9868	1/2	54.978
1/4	3.927	5/16	13.547	5/8	33.3795	5/8	55.3707
5/16	4.123 3	3/8	13.7445	3/4	33.7722	3/4	55.7634
3/8	4.319 7	7/16	13 94	7/8	34.1649	7/8	56.1561
7/16	4.516	1/2	14.1372	11	34.5576	18	56.5488
1/2	4.712 4	9/16	14.333	1/8	34.9503	1/8	56.9415
9/16	4.908 7	5/8	14.5299	1/4	35.343	1/4	57.3342
5/8	5.105 1	11/16	14.725	3/8	35.7357	3/8	57.7269
11/16	5.301 4	3/4	14.9226	1/2	36.1284	1/2	58.1196
3/4	5.497 8	13/16	15.119	5/8	36.5211	5/8	58.5123
13/16	5.694 1	7/8	15.3153	3/4	36.9138	3/4	58.905
7/8	5.890 5	15/16	15.511	7/8	37.3065	7/8	59.2977
15/16	6.086 8	5	15.708	12	37.6992	19	59.6904
2	6.283 2	1/8	16.1007	1/8	38.0919	1/8	60.0831
1/16	6.479 5	1/4	16.4934	1/4	38.4846	1/4	60.4758
1/8	6.675 9	3/8	16.8861	3/8	38.8773	3/8	60.8685
3/16	6.872 2	1/2	17.2788	1/2	39.27	1/2	61.2612
1/4	7.068 6	5/8	17.6715	5/8	39.6627	5/8	61.6539
5/16	7.264 9	3/4	18.0642	3/4	40.0554	3/4	62.0466
3/8	7.461 3	7/8	18.4569	7/8	40.4481	7/8	62.4393
7/16	7.657 6	6	18.8496	13	40.8408	20	62.832
1/2	7.854	1/8	19.2423	1/8	41.2335	1/8	63.2247
9/16	8.050 3	1/4	19.635	1/4	41.6262	1/4	63.6174
5/8	8.246 7	3/8	20.0277	3/8	42.0189	3/8	64.0101
11/16	8.443	1/2	20.4204	1/2	42.4116	1/2	64.4028
3/4	8.639 4	5/8	20.8131	5/8	42.8043	5/8	64.7955
13/16	8.835 7	3/4	21.2058	3/4	43.197	3/4	65.1882
7/8	9.032 1	7/8	21.5985	7/8	43.5897	7/8	65.5809
15/16	9.228 4	7	21.9912	14	43.9824	21	65.9736
		1/8	22.3839	1/8	44.3751	1/8	66.3663
		1/4	22.7766	1/4	44.7678	1/4	66.759
		3/8	23.1693	3/8	45.1605	3/8	67.1517
		1/2	23.562	1/2	45.5532	1/2	67.5444
		5/8	23.9547	5/8	45.9459	5/8	67.9371
		3/4	24.3474	3/4	46.3386	3/4	68.3298
		7/8	24.7401	7/8	46.7313	7/8	68.7225

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
22	69.1152	29	91.1064	36	113.098	43	135.089
1/8	69.5079	1/8	91.4991	1/8	113.49	1/8	135.481
1/4	69.9006	1/4	91.8918	1/4	113.883	1/4	135.874
3/8	70.2933	3/8	92.2845	3/8	114.276	3/8	136.267
1/2	70.686	1/2	92.6772	1/2	114.668	1/2	136.66
5/8	71.0787	5/8	93.0699	5/8	115.061	5/8	137.052
3/4	71.4714	3/4	93.4626	3/4	115.454	3/4	137.445
7/8	71.8641	7/8	93.8553	7/8	115.846	7/8	137.838
23	72.2568	30	94.248	37	116.239	44	138.23
1/8	72.6495	1/8	94.6407	1/8	116.632	1/8	138.623
1/4	73.0422	1/4	95.0334	1/4	117.025	1/4	139.016
3/8	73.4349	3/8	95.4261	3/8	117.417	3/8	139.408
1/2	73.8276	1/2	95.8188	1/2	117.81	1/2	139.801
5/8	74.2203	5/8	96.2115	5/8	118.203	5/8	140.194
3/4	74.613	3/4	96.6042	3/4	118.595	3/4	140.587
7/8	75.0057	7/8	96.9969	7/8	118.988	7/8	140.979
24	75.3984	31	97.3896	38	119.381	45	141.372
1/8	75.7911	1/8	97.7823	1/8	119.773	1/8	141.765
1/4	76.1838	1/4	98.175	1/4	120.166	1/4	142.157
3/8	76.5765	3/8	98.5677	3/8	120.559	3/8	142.55
1/2	76.9692	1/2	98.9604	1/2	120.952	1/2	142.943
5/8	77.3619	5/8	99.3531	5/8	121.344	5/8	143.335
3/4	77.7546	3/4	99.7458	3/4	121.737	3/4	143.728
7/8	78.1473	7/8	100.1385	7/8	122.13	7/8	144.121
25	78.54	32	100.5312	39	122.522	46	144.514
1/8	78.9327	1/8	100.9239	1/8	122.915	1/8	144.906
1/4	79.3254	1/4	101.3166	1/4	123.308	1/4	145.299
3/8	79.7181	3/8	101.7093	3/8	123.7	3/8	145.692
1/2	80.1108	1/2	102.102	1/2	124.093	1/2	146.084
5/8	80.5035	5/8	102.4947	5/8	124.486	5/8	146.477
3/4	80.8962	3/4	102.8874	3/4	124.879	3/4	146.87
7/8	81.2889	7/8	103.2801	7/8	125.271	7/8	147.262
26	81.6816	33	103.673	40	125.664	47	147.655
1/8	82.0743	1/8	104.065	1/8	126.057	1/8	147.048
1/4	82.467	1/4	104.458	1/4	126.449	1/4	148.441
3/8	82.8597	3/8	104.851	3/8	126.842	3/8	148.833
1/2	83.2524	1/2	105.244	1/2	127.235	1/2	149.226
5/8	83.6451	5/8	105.636	5/8	127.627	5/8	149.619
3/4	84.0378	3/4	106.029	3/4	128.02	3/4	150.011
7/8	84.4305	7/8	106.422	7/8	128.413	7/8	150.404
27	84.8232	34	106.814	41	128.806	48	150.797
1/8	85.2159	1/8	107.207	1/8	129.198	1/8	151.189
1/4	85.6086	1/4	107.6	1/4	129.591	1/4	151.582
3/8	86.0013	3/8	107.992	3/8	129.984	3/8	151.975
1/2	86.394	1/2	108.385	1/2	130.376	1/2	152.368
5/8	86.7867	5/8	108.778	5/8	130.769	5/8	152.76
3/4	87.1794	3/4	109.171	3/4	131.162	3/4	153.153
7/8	87.5721	7/8	109.563	7/8	131.554	7/8	153.546
28	87.9648	35	109.956	42	131.947	49	153.938
1/8	88.3575	1/8	110.349	1/8	132.34	1/8	154.331
1/4	88.7502	1/4	110.741	1/4	132.733	1/4	154.724
3/8	89.1429	3/8	111.134	3/8	133.125	3/8	155.116
1/2	89.5356	1/2	111.527	1/2	133.518	1/2	155.509
5/8	89.9283	5/8	111.919	5/8	133.911	5/8	155.902
3/4	90.321	3/4	112.312	3/4	134.303	3/4	156.295
7/8	90.7137	7/8	112.705	7/8	134.696	7/8	156.687

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
50	157.08	57	179.071	64	201.062	71	223.054
⅛	157.473	⅛	179.464	⅛	201.455	⅛	223.446
¼	157.865	¼	179.857	¼	201.848	¼	223.839
⅜	158.258	⅜	180.249	⅜	202.24	⅜	224.232
½	158.651	½	180.642	½	202.633	½	224.624
⅝	159.043	⅝	181.035	⅝	203.026	⅝	225.017
¾	159.436	¾	181.427	¾	203.419	¾	225.41
⅞	159.829	⅞	181.82	⅞	203.811	⅞	225.802
51	160.222	58	182.213	65	204.204	72	226.195
⅛	160.614	⅛	182.605	⅛	204.597	⅛	226.588
¼	161.007	¼	182.998	¼	204.989	¼	226.981
⅜	161.4	⅜	183.391	⅜	205.382	⅜	227.373
½	161.792	½	183.784	½	205.775	½	227.766
⅝	162.185	⅝	184.176	⅝	206.167	⅝	228.159
¾	162.578	¾	184.569	¾	206.56	¾	228.551
⅞	162.97	⅞	184.962	⅞	206.953	⅞	228.944
52	163.363	59	185.354	66	207.346	73	229.337
⅛	163.756	⅛	185.747	⅛	207.738	⅛	229.729
¼	164.149	¼	186.14	¼	208.131	¼	230.122
⅜	164.541	⅜	186.532	⅜	208.524	⅜	230.515
½	164.934	½	186.925	½	208.916	½	230.908
⅝	165.327	⅝	187.318	⅝	209.309	⅝	231.3
¾	165.719	¾	187.711	¾	209.702	¾	231.693
⅞	166.112	⅞	188.103	⅞	210.094	⅞	232.086
53	166.505	60	188.496	67	210.487	74	232.478
⅛	166.897	⅛	188.889	⅛	210.88	⅛	232.871
¼	167.29	¼	189.281	¼	211.273	¼	233.264
⅜	167.683	⅜	189.674	⅜	211.665	⅜	233.656
½	168.076	½	190.067	½	212.058	½	234.049
⅝	168.468	⅝	190.459	⅝	212.451	⅝	234.442
¾	168.861	¾	190.852	¾	212.843	¾	234.835
⅞	169.254	⅞	191.245	⅞	213.236	⅞	235.227
54	169.646	61	191.638	68	213.629	75	235.62
⅛	170.039	⅛	192.03	⅛	214.021	⅛	236.013
¼	170.432	¼	192.423	¼	214.414	¼	236.405
⅜	170.824	⅜	192.816	⅜	214.807	⅜	236.798
½	171.217	½	193.208	½	215.2	½	237.191
⅝	171.61	⅝	193.601	⅝	215.592	⅝	237.583
¾	172.003	¾	193.994	¾	215.985	¾	237.976
⅞	172.395	⅞	194.386	⅞	216.378	⅞	238.369
55	172.788	62	194.779	69	216.77	76	238.762
⅛	173.181	⅛	195.172	⅛	217.163	⅛	239.154
¼	173.573	¼	195.565	¼	217.556	¼	239.547
⅜	173.966	⅜	195.957	⅜	217.948	⅜	239.94
½	174.359	½	196.35	½	218.341	½	240.332
⅝	174.751	⅝	196.743	⅝	218.734	⅝	240.725
¾	175.144	¾	197.135	¾	219.127	¾	241.118
⅞	175.537	⅞	197.528	⅞	219.519	⅞	241.51
56	175.93	63	197.921	70	219.912	77	241.903
⅛	176.322	⅛	198.313	⅛	220.305	⅛	242.296
¼	176.715	¼	198.706	¼	220.697	¼	242.689
⅜	177.108	⅜	199.099	⅜	221.09	⅜	243.081
½	177.5	½	199.492	½	221.483	½	243.474
⅝	177.893	⅝	199.884	⅝	221.875	⅝	243.867
¾	178.286	¾	200.277	¾	222.268	¾	244.259
⅞	178.678	⅞	200.67	⅞	222.661	⅞	244.652

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
78	245.045	85	267.036	92	289.027	99	311.018
1/8	245.437	1/8	267.429	1/8	289.42	1/8	311.411
1/4	245.83	1/4	267.821	1/4	289.813	1/4	311.804
3/8	246.223	3/8	268.214	3/8	290.205	3/8	312.196
1/2	246.616	1/2	268.607	1/2	290.598	1/2	312.589
5/8	247.008	5/8	268.999	5/8	290.991	5/8	312.982
3/4	247.401	3/4	269.392	3/4	291.383	3/4	313.375
7/8	247.794	7/8	269.785	7/8	291.776	7/8	313.767
79	248.186	86	270.178	93	292.169	100	314.16
1/8	248.579	1/8	270.57	1/8	292.562	1/4	314.945
1/4	248.972	1/4	270.963	1/4	292.954	1/2	315.731
3/8	249.364	3/8	271.356	3/8	293.347	3/4	316.516
1/2	249.757	1/2	271.748	1/2	293.74	101	317.302
5/8	250.15	5/8	272.141	5/8	294.132	1/4	318.087
3/4	250.543	3/4	272.534	3/4	294.525	1/2	318.872
7/8	250.935	7/8	272.926	7/8	294.918	3/4	319.658
80	251.328	87	273.319	94	295.31	102	320.443
1/8	251.721	1/8	273.712	1/8	295.703	1/4	321.229
1/4	252.113	1/4	274.105	1/4	296.096	1/2	322.014
3/8	252.506	3/8	274.497	3/8	296.488	3/4	322.799
1/2	252.899	1/2	274.89	1/2	296.881	103	323.585
5/8	253.291	5/8	275.283	5/8	297.274	1/4	324.37
3/4	253.684	3/4	275.675	3/4	297.667	1/2	325.156
7/8	254.077	7/8	276.068	7/8	298.059	3/4	325.941
81	254.47	88	276.461	95	298.452	104	326.726
1/8	254.862	1/8	276.853	1/8	298.845	1/4	327.512
1/4	255.255	1/4	277.246	1/4	299.237	1/2	328.297
3/8	255.648	3/8	277.629	3/8	299.63	3/4	329.083
1/2	256.04	1/2	278.032	1/2	300.023	105	329.868
5/8	256.433	5/8	278.424	5/8	300.415	1/4	330.653
3/4	256.826	3/4	278.817	3/4	300.808	1/2	331.439
7/8	257.218	7/8	279.21	7/8	301.201	3/4	332.224
82	257.611	89	279.602	96	301.594	106	333.01
1/8	258.004	1/8	279.995	1/8	301.986	1/4	333.795
1/4	258.397	1/4	280.388	1/4	302.379	1/2	334.58
3/8	258.789	3/8	280.78	3/8	302.772	3/4	335.366
1/2	259.182	1/2	281.173	1/2	303.164	107	336.151
5/8	259.575	5/8	281.566	5/8	303.557	1/4	336.937
3/4	259.967	3/4	281.959	3/4	303.95	1/2	337.722
7/8	260.36	7/8	282.351	7/8	304.342	3/4	338.507
83	260.753	90	282.744	97	304.735	108	339.293
1/8	261.145	1/8	283.137	1/8	305.128	1/4	340.078
1/4	261.538	1/4	283.529	1/4	305.521	1/2	340.864
3/8	261.931	3/8	283.922	3/8	305.913	3/4	341.649
1/2	262.324	1/2	284.315	1/2	306.306	109	342.434
5/8	262.716	5/8	284.707	5/8	306.699	1/4	343.22
3/4	263.109	3/4	285.1	3/4	307.091	1/2	344.005
7/8	263.502	7/8	285.493	7/8	307.484	3/4	344.791
84	263.894	91	285.886	98	307.877	110	345.576
1/8	264.287	1/8	286.278	1/8	308.27	1/4	346.361
1/4	264.68	1/4	286.671	1/4	308.662	1/2	347.147
3/8	265.072	3/8	287.064	3/8	309.055	3/4	347.932
1/2	265.465	1/2	287.456	1/2	309.448	111	348.718
5/8	265.858	5/8	287.849	5/8	309.84	1/4	349.503
3/4	266.251	3/4	288.242	3/4	310.233	1/2	350.288
7/8	266.643	7/8	288.634	7/8	310.626	3/4	351.074

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
112	351.859	121	380.134	130	408.408	139	436.682
¼	352.645	¼	380.919	¼	409.192	¼	437.467
½	353.43	½	381.704	½	409.979	½	438.253
¾	354.215	¾	382.49	¾	410.763	¾	439.037
113	355.001	122	383.275	131	411.55	140	439.824
¼	355.786	¼	384.061	¼	412.334	¼	440.608
½	356.572	½	384.846	½	413.12	½	441.395
¾	357.357	¾	385.631	¾	413.905	¾	442.179
114	358.142	123	386.417	132	414.691	141	442.966
¼	358.928	¼	387.202	¼	415.476	¼	443.75
½	359.713	½	387.988	½	416.262	½	444.536
¾	360.499	¾	388.773	¾	417.046	¾	445.321
115	361.284	124	389.558	133	417.833	142	446.107
¼	362.069	¼	390.344	¼	418.617	¼	446.891
½	362.855	½	391.129	½	419.404	½	447.678
¾	363.64	¾	391.915	¾	420.188	¾	448.462
116	364.426	125	392.7	134	420.974	143	449.249
¼	365.211	¼	393.484	¼	421.759	¼	450.033
½	365.996	½	394.271	½	422.545	½	450.82
¾	366.782	¾	395.055	¾	423.33	¾	451.604
117	367.567	126	395.842	135	424.116	144	452.39
¼	368.353	¼	396.626	¼	424.9	¼	453.175
½	369.138	½	397.412	½	425.687	½	453.961
¾	369.923	¾	398.197	¾	426.471	¾	454.745
118	370.709	127	398.983	136	427.258	145	455.532
¼	371.494	¼	399.768	¼	428.042	¼	456.316
½	372.28	½	400.554	½	428.828	½	457.103
¾	373.065	¾	401.338	¾	429.613	146	458.674
119	373.85	128	402.125	137	430.399	½	460.244
¼	374.636	¼	402.909	¼	431.183	147	461.815
½	375.421	½	403.696	½	431.97	½	463.386
¾	376.207	¾	404.48	¾	432.754	148	464.957
120	376.992	129	405.266	138	433.541	½	466.528
¼	377.777	¼	406.051	¼	434.325	149	468.098
½	378.563	½	406.837	½	435.112	½	469.669
¾	379.348	¾	407.622	¾	435.896	150	471.24

To Compute Circumference of a Diameter greater than any in preceding Table.

Rule.—Divide dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter in table.

Take tabular circumference for this dimension, multiply it by divisor, according as it was divided, and product will give circumference required.

Example.—What is circumference for a diameter of 1050?

1050 ÷ 7 = 150; *tab. circum.*, 150 = 471.24, which × 7 = 3298.68, *circumference.*

To Compute Circumference of a Diameter in Feet and Inches, etc., by preceding Table.

Rule.—Reduce dimension to inches or eighths, as the case may be, and take circumference in that term from table for that number.

Divide this number by 8 if it is in eighths, and by 12 if in inches, and quotient will give circumference in feet.

EXAMPLE.—Required circumference of a circle of 1 foot 6.375 ins.

1 foot 6.375 ins. = 18.375 *ins.* = 147 *eighths.* Circum. of 147 = 461.815, which ÷ 8 = 57.727 *ins.*; and by 12 = 4.8106 *feet.*

To Compute Circumference for a Diameter composed of an Integer and a Fraction.

RULE.—Double, treble, or quadruple dimension given, until fraction is increased to a whole number or to one of those in the table, as ⅛, ¼, etc., provided it is practicable to do so.

Take circumference for this diameter; and if it is double of that for which circumference is required, take one half of it; if treble, take one third of it; and if quadruple, one fourth of it.

EXAMPLE.—Required circumference of 2.218 75 ins.

2.218 75 × 2 = 4.4375, which × 2 = 8.875; circum. for which = 27.8817, which ÷ 4 = 6.9704 *ins.*

When Diameter consists of Integers and Fractions contained in Table.

RULE.—Point a decimal to a diameter in table, take circumference from table, and add as many figures to the right as there are figures cut off.

EXAMPLE.—What is circumference of a circle 9675 feet in diameter?

Circumference of 96.75 = 303.95; hence, circumference of 9675 = 30 395 *feet.*

To Ascertain Circumference for a Diameter, as 500, 5000, etc., not contained in Table.

Rule.—Take circumference of 5 or 50 from table, and add the excess of ciphers to the result.

EXAMPLE.—What is circumference of a circle 8000 feet in diameter?

Circumference of 80 = 251.38; hence, circumference of 8000 = 25 138 *feet.*

To Compute Circumference of a Circle by Logarithms.

RULE.—To log. of diameter add .497 01 (log. of 3.1416), and sum is log. of circumference, from which take number.

EXAMPLE.—What is circumference of a circle 1200 feet in diameter?

Log. 1200 = 3.079 18 + .497 01 = 3.576 19, and number for which = 3769.91 *feet.*

Circumferences of Birmingham Wire Gauge.

Diam. No.	Circum. Ins.	Diam. No.	Circum. Ins.	Diam. No.	Circum. Ins.	Diam. No.	Circum. Ins.
1	.942 48	10	.420 97	19	.131 95	28	.043 98
2	.892 21	11	.376 99	20	.109 95	29	.040 84
3	.813 67	12	.342 43	21	.100 53	30	.037 7
4	.747 7	13	.298 45	22	.087 96	31	.031 41
5	.691 15	14	.260 75	23	.078 54	32	.028 27
6	.637 74	15	.226 19	24	.069 11	33	.025 13
7	.565 49	16	.204 2	25	.062 83	34	.021 99
8	.518 36	17	.182 21	26	.056 55	35	.015 71
9	.464 95	18	.153 94	27	.050 26	36	.012 57

Areas and Circumferences. *(Advancing by Tenths.)*

DIAM.	AREA.	CIRCUM.	DIAM.	AREA.	CIRCUM.
.1	.007854	.31416	.6	24.6301	17.593
.2	.031416	.62832	.7	25.5176	17.9071
.3	.070686	.94248	.8	26.4209	18.2213
.4	.125664	1.2566	.9	27.3398	18.5354
.5	.19635	1.5708	6	28.2744	18.8496
.6	.282744	1.885	.1	29.2247	19.1638
.7	.384846	2.1991	.2	30.1908	19.4779
.8	.502656	2.5133	.3	31.1725	19.7921
.9	.636174	2.8274	.4	32.17	20.1062
1	.7854	3.1416	.5	33.1831	20.4204
.1	.9503	3.4558	.6	34.212	20.7346
.2	1.131	3.7699	.7	35.2566	21.0487
.3	1.3273	4.0841	.8	36.3169	21.3629
.4	1.5394	4.3982	.9	37.3929	21.677
.5	1.7671	4.7124	7	38.4846	21.9912
.6	2.0106	5.0266	.1	39.592	22.3054
.7	2.2698	5.3407	.2	40.7151	22.6195
.8	2.5447	5.6549	.3	41.854	22.9337
.9	2.8353	5.969	.4	43.0085	23.2478
2	3.1416	6.2832	.5	44.1787	23.562
.1	3.4636	6.5974	.6	45.3647	23.8762
.2	3.8013	6.9115	.7	46.5664	24.1903
.3	4.1548	7.2257	.8	47.7837	24.5045
.4	4.5239	7.5398	.9	49.0168	24.8186
.5	4.9087	7.854	8	50.2656	25.1328
.6	5.3093	8.1682	.1	51.5301	25.447
.7	5.7256	8.4823	.2	52.8103	25.7611
.8	6.1575	8.7965	.3	54.1062	26.0753
.9	6.6052	9.1106	.4	55.4178	26.3894
3	7.0686	9.4248	.5	56.7451	26.7036
.1	7.5477	9.739	.6	58.0882	27.0178
.2	8.0425	10.0531	.7	59.4469	27.3319
.3	8.553	10.3673	.8	60.8214	27.6461
.4	9.0792	10.6814	.9	62.2115	27.9602
.5	9.6211	10.9956	9	63.6174	28.2744
.6	10.1788	11.3098	.1	65.039	28.5886
.7	10.7521	11.6239	.2	66.4763	28.9027
.8	11.3412	11.9381	.3	67.9292	29.2169
.9	11.9459	12.2522	.4	69.3979	29.531
4	12.5664	12.5664	.5	70.8823	29.8452
.1	13.2026	12.8806	.6	72.3825	30.1594
.2	13.8545	13.1947	.7	73.8983	30.4735
.3	14.522	13.5089	.8	75.4298	30.7877
.4	15.2053	13.823	.9	76.9771	31.1018
.5	15.9043	14.1372	10	78.54	31.416
.6	16.6191	14.4514	.1	80.1187	31.7302
.7	17.3495	14.7655	.2	81.713	32.0443
.8	18.0956	15.0797	.3	83.3231	32.3585
.9	18.8575	15.3938	.4	84.9489	32.6726
5	19.635	15.708	.5	86.5903	32.9868
.1	20.4283	16.0222	.6	88.2475	33.301
.2	21.2372	16.3363	.7	89.9204	33.6151
.3	22.0619	16.6505	.8	91.6091	33.9293
.4	22.9023	16.9646	.9	93.3134	34.2434
.5	23.7583	17.2788			

DIAM.	AREA.	CIRCUM.	DIAM.	AREA.	CIRCUM.
11	95.0334	34.5576	.5	213.8251	51.8364
.1	96.7691	34.8718	.6	216.4248	52.1505
.2	98.5206	35.1859	.7	219.0402	52.4647
.3	100.2877	35.5001	.8	221.6713	52.7789
.4	102.0706	35.8142	.9	224.3181	53.093
.5	103.8691	36.1284	17	226.9806	53.4072
.6	105.6834	36.4426	.1	229.6588	53.7214
.7	107.5134	36.7567	.2	232.3527	54.0355
.8	109.3591	37.0709	.3	235.0624	54.3497
.9	111.2205	37.385	.4	237.7877	54.6638
12	113.0976	37.6992	.5	240.5287	54.978
.1	114.9904	38.0134	.6	243.2855	55.2922
.2	116.8989	38.3275	.7	246.058	55.6063
.3	118.8232	38.6417	.8	248.8461	55.9205
.4	120.7631	38.9558	.9	251.65	56.2346
.5	122.7187	39.27	18	254.4696	56.5488
.6	124.6901	39.5842	.1	257.3049	56.863
.7	126.6772	39.8983	.2	260.1559	57.1771
.8	128.6799	40.2125	.3	263.0226	57.4913
.9	130.6984	40.5266	.4	265.905	57.8054
13	132.7326	40.8408	.5	268.8031	58.1196
.1	134.7825	41.155	.6	271.717	58.4338
.2	136.8481	41.4691	.7	274.6465	58.7479
.3	138.9294	41.7833	.8	277.5918	59.0621
.4	141.0264	42.0974	.9	280.5527	59.3762
.5	143.1391	42.4116	19	283.5294	59 6904
.6	145.2676	42.7258	.1	286.5218	60.0046
.7	147.4117	43.0399	.2	289.5299	60.3187
.8	149.5716	43.3541	.3	292.5536	60.6329
.9	151.7471	43.6682	.4	295.5931	60.947
14	153.9384	43.9824	.5	298.6483	61.2612
.1	156.1454	44.2966	.6	301.7193	61.5754
.2	158.3681	44.6107	.7	304.806	61.8895
.3	160.6064	44.9249	.8	307.9082	62.2037
.4	162.8605	45.239	.9	311.0263	62.5178
.5	165.1303	45.5532	20	314.16	62.832
.6	167.4159	45.8674	.1	317.3094	63.1462
.7	169.7171	46.1815	.2	320 4746	63.4603
.8	172 034	46.4957	.3	323.6555	63.7745
.9	174.3667	46.8098	.4	326.8521	64.0886
15	176.715	47.124	.5	330.0643	64.4028
.1	179.0791	47.4382	.6	333.2923	64.717
.2	181.4588	47.7523	.7	336.536	65.0311
.3	183.8543	48.0665	.8	339.7955	65.3453
.4	186.2655	48.3806	.9	343.0706	65.6594
.5	188.6924	48.6948	21	346.3614	65.9736
.6	191.1349	49.009	.1	349.6679	66.2878
.7	193.5932	49.3231	.2	352.9902	66.6019
.8	196.0673	49.6373	.3	356.3281	66.9161
.9	198.557	49.9514	.4	359.6818	67.2302
16	201.0624	50.2656	.5	363.0511	67.5444
.1	203.5835	50.5797	.6	366.4362	67.8586
.2	206.1204	50.8939	.7	369.837	68.1727
.3	208.6729	51.2081	.8	373.2535	68.4869
.4	211.2412	51.5222	.9	376.6857	68.801

Diam.	Area.	Circum.	Diam.	Area.	Circum.
22	380.1336	69.1152	.5	593.9587	86.394
.1	383.5972	69.4294	.6	598.2863	86.7082
.2	387.0765	69.7435	.7	602.6296	87.0223
.3	390.5716	70.0577	.8	606.9885	87.3365
.4	394.0823	70.3718	.9	611.3632	87.6506
.5	397.6087	70.686	28	615.7536	87.9648
.6	401.1509	71.0002	.1	620.1597	88.279
.7	404.7088	71.3143	.2	624.5815	88.5931
.8	408.2823	71.6285	.3	629.019	88.9073
.9	411.8716	71.9426	.4	633.4722	89.2214
23	415.4766	72.2568	.5	637.9411	89.5356
.1	419.0973	72.571	.6	642.4258	89.8498
.2	422.7337	72.8851	.7	646.9261	90.1639
.3	426.3858	73.1993	.8	651.4422	90.4781
.4	430.0536	73.5134	.9	655.9739	90.7922
.5	433.7371	73.8276	29	660.5214	91.1064
.6	437.4364	74.1418	.1	665.0846	91.4206
.7	441.1513	74.4559	.2	669.6635	91.7347
.8	444.882	74.7701	.3	674.258	92.0489
.9	448.6283	75.0842	.4	678.8683	92.363
24	452.3904	75.3984	.5	683.4943	92.6772
.1	456.1682	75.7126	.6	688.1361	92.9914
.2	459.9617	76 0267	.7	692.7935	93.3055
.3	463.7708	76.3409	.8	697.4666	93.6197
.4	467.5957	76.655	.9	702.1555	93.9338
.5	471.4363	76.9692	30	706.86	94.248
.6	475.2927	77.2834	.1	711.5803	94.5622
.7	479.1647	77.5975	.2	716.3162	94.8763
.8	483.0524	77.9117	.3	721.0679	95.1905
.9	486.9559	78.2258	.4	725.8353	95.5046
25	490.875	78.54	.5	730.6183	95.8188
.1	494.8099	78.8542	.6	735.4171	96.133
.2	498.7604	79.1683	.7	740.2316	96.4471
.3	502.7267	79.4825	.8	745.0619	96.7613
.4	506.7087	79.7966	.9	749.9078	97.0754
.5	510.7063	80.1108	31	754.7694	97.3896
.6	514.7196	80.425	.1	759.6467	97.7038
.7	518.7488	80.7391	.2	764.5398	98.0179
.8	522.7937	81.0533	.3	769.4485	98.3321
.9	526.8542	81.3674	.4	774.373	98.6462
26	530.9304	81.6816	.5	779.3131	98.9604
.1	535.0223	81.9958	.6	784.269	99.2746
.2	539.13	82.3099	.7	789.2406	99.5887
.3	543.2533	82.6241	.8	794.2279	99.9029
.4	547.3924	82.9382	.9	799.2309	100.217
.5	551.5471	83.2524	32	804.2496	100.5312
.6	555.7176	83.5666	.1	809.284	100.8454
.7	559.9038	83.8807	.2	814.3341	101.1595
.8	564.1057	84.1949	.3	819.4	101.4737
.9	568.3233	84.509	.4	824.4815	101.7878
27	572.5566	84.8232	.5	829.5787	102.102
.1	576.8056	85.1374	.6	834.6917	102.4162
.2	581.0703	85.4515	.7	839.8204	102.7303
.3	585.3508	85.7657	.8	844 9647	103.0445
.4	589.6469	86.0798	.9	850.1248	103.3586

Diam.	Area.	Circum.	Diam.	Area.	Circum.
33	855.3006	103.6728	.5	1164.1591	120.9516
.1	860.4921	103.987	.6	1170.2146	121.2658
.2	865.6993	104.3011	.7	1176.2857	121.5799
.3	870.9222	104.6153	.8	1182.3726	121.8941
.4	876.1608	104.9294	.9	1188.4751	122.2082
.5	881.4151	105.2436	39	1194.5934	122.5224
.6	886.6852	105.5578	.1	1200.7274	122.8366
.7	891.9709	105.8719	.2	1206.8771	123.1507
.8	897.2724	106.1861	.3	1213.0424	123.4649
.9	902.5895	106.5002	.4	1219.2235	123.779
34	907.9224	106.8144	.5	1225.4203	124.0932
.1	913.271	107.1286	.6	1231.6329	124.4074
.2	918.6353	107.4427	.7	1237.8611	124.7215
.3	924.0152	107.7569	.8	1244.105	125.0357
.4	929.4109	108.071	.9	1250.3647	125.3498
.5	934.8223	108.3852	40	1256.64	125.664
.6	940.2495	108.6994	.1	1262.9311	125.9782
.7	945.6923	109.0135	.2	1269.2378	126.2923
.8	951.1508	109.3277	.3	1275.5603	126.6065
.9	956.6251	109.6418	.4	1281.8985	126.9206
35	962.115	109.956	.5	1288.2523	127.2348
.1	967.6207	110.2702	.6	1294.6219	127.549
.2	973.142	110.5843	.7	1301.0072	127.8631
.3	978.6791	110.8985	.8	1307.4083	128.1773
.4	984.2319	111.2126	.9	1313 825	128.4914
.5	989.8003	111.5268	41	1320.2574	128.8056
.6	995.3845	111.841	.1	1326.7055	129.1198
.7	1000.9844	112.1551	.2	1333.1694	129.4339
.8	1006.6001	112.4693	.3	1339.6489	129.7481
.9	1012.2314	112.7834	.4	1346.1442	130.0622
36	1017.8784	113.0976	.5	1352.6551	130.3764
.1	1023.5411	113.4118	.6	1359.1818	130.6906
.2	1029.2196	113.7259	.7	1365.7242	131.0047
.3	1034.9137	114.0401	.8	1372.2823	131.3189
.4	1040.6236	114.3542	.9	1378.8561	131.633
.5	1046.3491	114.6684	42	1385.4456	131.9472
.6	1052.0904	114.9826	.1	1392.0508	132.2614
.7	1057.8474	115.2967	.2	1398.6717	132.5755
.8	1063.6201	115.6109	.3	1405.3084	132.8897
.9	1069.4085	115.925	.4	1411.9607	133.2038
37	1075.2126	116.2392	.5	1418.6287	133.518
.1	1081.0324	116.5534	.6	1425.3125	133.8322
.2	1086.8679	116.8675	.7	1432.012	134.1463
.3	1092.7192	117.1817	.8	1438.7271	134.4605
.4	1098.5861	117.4958	.9	1445.458	134.7746
.5	1104.4687	117.81	43	1452.2046	135.0888
.6	1110.3671	118.1242	.1	1458.9669	135.403
.7	1116.2812	118.4383	.2	1465.7449	135.7171
.8	1122.2109	118.7525	.3	1472.5386	136.0313
.9	1128.1564	119.0666	.4	1479.348	136.3454
38	1134.1176	119.3808	.5	1486.1731	136.6596
.1	1140.0945	119.695	.6	1493.014	136.9738
.2	1146.0871	120.0091	.7	1499.8705	137.2879
.3	1152.0954	120.3233	.8	1506.7428	137.6021
.4	1158.1194	120.6374	.9	1513.6307	137.9162

Diam.	Area.	Circum.	Diam.	Area.	Circum.
44	1520.5344	138.2304	.5	1924.4263	155.5092
.1	1527.4538	138.5446	.6	1932.2097	155.8234
.2	1534.3889	138.8587	.7	1940.0087	156.1375
.3	1541.3396	139.1729	.8	1947.8234	156.4517
.4	1548.3061	139.487	.9	1955.6539	156.7658
.5	1555.2883	139.8012	50	1963.5	157.08
.6	1562.2863	140.1154	.1	1971.3619	157.3942
.7	1569.2999	140.4295	.2	1979.2394	157.7083
.8	1576.3292	140.7437	.3	1987.1327	158.0225
.9	1583.3743	141.0578	.4	1995.0417	158.3366
45	1590.435	141.372	.5	2002.9663	158.6509
.1	1597.5115	141.6862	.6	2010.9067	158.965
.2	1604.6036	142.0003	.7	2018.8628	159.2791
.3	1611.7115	142.3145	.8	2026.8347	159.5933
.4	1618.8351	142.6286	.9	2034.8222	159.9074
.5	1625.9743	142.9428	51	2042.8254	160.2216
.6	1633.1293	143.257	.1	2050.8443	160.5358
.7	1640.3	143.5711	.2	2058.879	160.8499
.8	1647.4865	143.8853	.3	2066.9293	161.1641
.9	1654.6886	144.1994	.4	2074.9954	161.4782
46	1661.9064	144.5136	.5	2083.0771	161.7924
.1	1669.1399	144.8278	.6	2091.1746	162.1066
.2	1676.3892	145.1419	.7	2099.2878	162.4207
.3	1683.6541	145.4561	.8	2107.4167	162.7349
.4	1690.9348	145.7702	.9	2115.5613	163.049
.5	1698.2311	146.0844	52	2123.7216	163.3632
.6	1705.5432	146.3986	.1	2131.8976	163.6774
.7	1712.871	146.7127	.2	2140.0893	163.9915
.8	1720.2145	147.0269	.3	2148.2968	164.3057
.9	1727.5737	147.341	.4	2156.5199	164.6198
47	1734.9486	147.6552	.5	2164.7587	164.934
.1	1742.3392	147.9694	.6	2173.0133	165.2482
.2	1749.7455	148.2835	.7	2181.2836	165.5623
.3	1757.1676	148.5977	.8	2189.5695	165.8765
.4	1764.6053	148.9118	.9	2197.8712	166.1906
.5	1772.0587	149.226	53	2206.1886	166.5048
.6	1779.5279	149.5402	.1	2214.5217	166.819
.7	1787.0128	149.8543	.2	2222.8705	167.1331
.8	1794.5133	150.1685	.3	2231.235	167.4473
.9	1802.0296	150.4826	.4	2239.6152	167.7614
48	1809.5616	150.7968	.5	2248.0111	168.0756
.1	1817.1093	151.111	.6	2256.4228	168.3898
.2	1824.6727	151.4251	.7	2264.8501	168.7039
.3	1832.2518	151.7393	.8	2273.2932	169.0181
.4	1839.8466	152.0534	.9	2281.7519	169.3322
.5	1847.4571	152.3676	54	2290.2264	169.6464
.6	1855.0834	152.6818	.1	2298.7166	169.9606
.7	1862.7253	152.9959	.2	2307.2225	170.2747
.8	1870.383	153.3101	.3	2315.744	170.5889
.9	1878.0563	153.6242	.4	2324.2813	170.903
49	1885.7454	153 9384	.5	2332.8343	171.2172
.1	1893.4502	154.2526	.6	2341.4031	171.5314
.2	1901.1707	154.5667	.7	2349 9875	171.8455
.3	1908.9068	154.8809	.8	2358 5876	172.1597
.4	1916.6587	155.195	.9	2367.2035	172.4738

Diam.	Area.	Circum.	Diam.	Area.	Circum.
55	2375.835	172.788	.5	2874.7603	190.0668
.1	2384.4823	173.1022	.6	2884.2715	190.381
.2	2393.1452	173.4163	.7	2893.7984	190.6951
.3	2401.8239	173.7305	.8	2903.3411	191.0093
.4	2410.5183	174.0446	.9	2912.8994	191.3234
.5	2419.2283	174.3588	61	2922.4734	191.6376
.6	2427.9541	174.673	.1	2932.0631	191.9518
.7	2436.6957	174.9871	.2	2941.6686	192.2659
.8	2445.4529	175.3013	.3	2951.2897	192.5801
.9	2454.2258	175.6154	.4	2960.9266	192.8942
56	2463.0144	175.9296	.5	2970.5791	193.2084
.1	2471.8187	176.2438	.6	2980.2474	193.5226
.2	2480.6388	176.5579	.7	2989.9314	193.8367
.3	2489.4745	176.8721	.8	2999.6311	194.1509
.4	2498.326	177.1862	.9	3009.3465	194.465
.5	2507.1931	177.5004	62	3019.0776	194.7792
.6	2516.076	177.8146	.1	3028.8244	195.0934
.7	2524.9736	178.1287	.2	3038.5869	195.4075
.8	2533.8889	178.4429	.3	3048.3652	195.7217
.9	2542.8189	178.757	.4	3058.1591	196.0358
57	2551.7646	179.0712	.5	3067.9687	196.35
.1	2560.726	179.3854	.6	3077.7941	196.6642
.2	2569.7031	179.6995	.7	3087.6341	196.9783
.3	2578.696	180.0137	.8	3097.4919	197.2925
.4	2587.7045	180.3278	.9	3107.3644	197.6066
.5	2596.7287	180.642	63	3117.2526	197.9208
.6	2605.7687	180.9562	.1	3127.1565	198.235
.7	2614.8244	181.2703	.2	3137.0761	198.5491
.8	2623.8957	181.5845	.3	3147.0114	198.8633
.9	2632.9828	181.8986	.4	3156.9624	199.1774
58	2642.0856	182.2128	.5	3166.9291	199.4916
.1	2651.2041	182.527	.6	3176.9116	199.8058
.2	2660.3383	182.8411	.7	3186.9097	200.1199
.3	2669.4882	183.1553	.8	3196.9236	200.4341
.4	2678.6538	183.4694	.9	3206.9531	200.7482
.5	2687.8351	183.7836	64	3216.9984	201.0624
.6	2697.0322	184.0978	.1	3227.0594	201.3766
.7	2706.2449	184.4119	.2	3237.1361	201.6907
.8	2715.4734	184.7261	.3	3247.2284	202.0049
.9	2724.7175	185.0402	.4	3257.3365	202.319
59	2733.9774	185.3544	.5	3267.4603	202.6332
.1	2743.253	185.6686	.6	3277.5999	202.9474
.2	2752.5443	185.9827	.7	3287.7551	203.2615
.3	2761.8512	186.2969	.8	3297.9261	203.5757
.4	2771.1739	186.611	.9	3308.1127	203.8898
.5	2780.5123	186.9252	65	3318.315	204.204
.6	2789.8665	187.2394	.1	3328.5331	204.5182
.7	2799.2363	187.5535	.2	3338.7668	204.8323
.8	2808.6218	187.8677	.3	3349.0163	205.1465
.9	2818.0231	188.1818	.4	3359.2815	205.4606
60	2827.44	188.496	.5	3369.5623	205.7748
.1	2836.8727	188.8102	.6	3379.8589	206.089
.2	2846.321	189.1243	.7	3390.1712	206.4031
.3	2855.7851	189.4385	.8	3400.4993	206.7173
.4	2865.2649	189.7526	.9	3410.843	207.0314

DIAM.	AREA.	CIRCUM.	DIAM.	AREA.	CIRCUM.
66	3421.2024	207.3456	.5	4015.1611	224.6244
.1	3431.5775	207.6598	.6	4026.4002	224.9386
.2	3441.9684	207.9739	.7	4037.655	225.2527
.3	3452.3749	208.2881	.8	4048.9255	225.5669
.4	3462.7972	208.6022	.9	4060.2117	225.881
.5	3473.2351	208.9164	72	4071.5136	226.1952
.6	3483.6888	209.2306	.1	4082.8312	226.5094
.7	3494.1582	209.5447	.2	4094.1645	226.8235
.8	3504.6433	209.8589	.3	4105.5136	227.1377
.9	3515.1441	210.173	.4	4116.8783	227.4518
67	3525.6606	210.4872	.5	4128.2587	227.766
.1	3536.1928	210.8014	.6	4139.655	228.0802
.2	3546.7407	211.1155	.7	4151.0668	228.3943
.3	3557.3044	211.4297	.8	4162.4943	228.7085
.4	3567.8837	211.7438	.9	4173.9376	229 0226
.5	3578.4787	212.058	73	4185.3966	229.3368
.6	3589.0895	212.3722	.1	4196.8713	229.651
.7	3599.716	212.6863	.2	4208.3617	229.9651
.8	3610.3581	213.0005	.3	4219.8678	230.2793
.9	3621.016	213.3146	.4	4231.3896	230.5934
68	3631.6896	213.6288	.5	4242.9271	230.9076
.1	3642.3789	213.943	.6	4254.4804	231.2218
.2	3653.0839	214.2571	.7	4266.0493	231.5359
.3	3663.805	214.5713	.8	4277.634	231.8501
.4	3674.541	214.8854	.9	4289.2343	232.1642
.5	3685.2931	215.1996	74	4300.8504	232.4784
.6	3696.061	215.5138	.1	4312.4822	232.7926
.7	3706.8445	215.8279	.2	4324.1297	233.1067
.8	3717.6438	216.1421	.3	4335.7928	233.4209
.9	3728.4587	216.4562	.4	4347.4717	233.735
69	3739.2894	216.7704	.5	4359.1663	234.0492
.1	3750.1358	217.0846	.6	4370.8767	234.3634
.2	3760.9979	217.3987	.7	4382 6027	234.6775
.3	3771.8756	217.7129	.8	4394.3444	234.9917
.4	3782.7691	218.027	.9	4406.1019	235.3058
.5	3793.6783	218.3412	75	4417.875	235.62
.6	3804.6033	218.6554	.1	4429.6639	235.9342
.7	3815.5439	218.9695	.2	4441.4684	236.2483
.8	3826.5002	219.2837	.3	4453.2887	236.5625
.9	3837.4722	219.5978	.4	4465.1247	236.8766
70	3848.46	219.912	.5	4476.9763	237.1908
.1	3859.4635	220.2262	.6	4488.8437	237.505
.2	3870.4826	220.5403	.7	4500.7268	237.8191
.3	3881.5175	220.8545	.8	4512.6257	238.1333
.4	3892.5681	221.1686	.9	4524.5402	238.4474
.5	3903.6343	221.4828	76	4536.4704	238.7616
.6	3914.7163	221.797	.1	4548.4163	239.0758
.7	3925.814	222.1111	.2	4560.378	239.3899
.8	3936.9275	222.4253	.3	4572.3553	239.7041
.9	3948.9566	222.7394	.4	4584.3484	240.0182
71	3959.2014	223.0536	.5	4596.3571	240.3324
.1	3970.3619	223.3678	.6	4608.3816	240.6466
.2	3981.5382	223.6819	.7	4620.4218	240.9607
.3	3992.7301	223.9961	.8	4632.4777	241.2749
.4	4003.9378	224.3102	.9	4644.5493	241.589

Diam.	Area.	Circum.	Diam.	Area.	Circum.
77	4656.6366	241.9032	.5	5345.6287	259.182
.1	4668.7396	242.2174	.6	5358.5957	259.4962
.2	4680.8583	242.5315	.7	5371.5784	259.8103
.3	4692.9928	242.8457	.8	5384.5767	260.1245
.4	4705.1429	243.1598	.9	5397.5908	260.4386
.5	4717.3087	243.474	83	5410.6206	260.7528
.6	4729.4903	243.7882	.1	5423.6661	261.067
.7	4741.6876	244.1023	.2	5436.7273	261.3811
.8	4753.9005	244.4165	.3	5449.8042	261.6953
.9	4766.1292	244.7306	.4	5462.8968	262.0094
78	4778.3736	245.0448	.5	5476.0051	262.3236
.1	4790.6337	245.359	.6	5489.1292	262.6378
.2	4802.9095	245.6731	.7	5502.2689	262.9519
.3	4815.201	245.9873	.8	5515.4244	263.2661
.4	4827.5082	246.3014	.9	5528.5955	263.5802
.5	4839.8311	246.6156	84	5541.7824	263.8944
.6	4852.1698	246.9298	.1	5554.985	264.2086
.7	4864.5241	247.2439	.2	5568.2033	264.5227
.8	4876.8942	247.5581	.3	5581.4372	264.8369
.9	4889.2799	247.8722	.4	5594 6869	265.151
79	4901.6814	248.1864	.5	5607.9523	265.4652
.1	4914.0986	248.5006	.6	5621.2335	265.7794
.2	4926.5315	248.8147	.7	5634.5303	266.0935
.3	4938.98	249.1289	.8	5647.8428	266.4077
.4	4951.4443	249.443	.9	5661.1711	266.7218
.5	4963.9243	249.7572	85	5674.515	267.036
.6	4976.4201	250.0714	.1	5687.8747	267.3502
.7	4988.9315	250.3855	.2	5701.25	267.6643
.8	5001.4586	250.6997	.3	5714.6411	267.9785
.9	5014.0015	251.0138	.4	5728.0479	268.2926
80	5026.56	251.328	.5	5741.4703	268.6068
.1	5039.1343	251.6422	.6	5754.9085	268.921
.2	5051.7242	251.9563	.7	5768.3624	269.2351
.3	5064.3299	252.2705	.8	5781.8321	269.5493
.4	5076.9513	252.5846	.9	5795.3174	269.8634
.5	5089.5883	252.8988	86	5808.8184	270.1776
.6	5102.2411	253.213	.1	5822.3351	270.4918
.7	5114.9096	253.5271	.2	5835.8676	270.8059
.8	5127.5939	253.8413	.3	5849.4157	271.1201
.9	5140.2938	254.1554	.4	5862.9796	271.4342
81	5153.0094	254.4696	.5	5876.5591	271.7484
.1	5165.7407	254.7838	.6	5890.1544	272.0626
.2	5178.4878	255.0979	.7	5903.7654	272.3767
.3	5191.2505	255.4121	.8	5917.3921	272.6909
.4	5204.0289	255.7262	.9	5931.0345	273.005
.5	5216.8231	256.0404	87	5944.6926	273.3192
.6	5229.633	256.3546	.1	5958.3644	273.6334
.7	5242.4586	256.6687	.2	5972.0559	273 9475
.8	5255.2999	256.9829	.3	5985.7612	274.2617
.9	5268.1569	257.297	.4	5999.4821	274.5758
82	5281.0296	257.6112	.5	6013.2187	274.89
.1	5293.918	257.9254	.6	6026.9711	275.2042
.2	5306.8221	258.2395	.7	6040.7392	275.5183
.3	5319.742	258.5537	.8	6054.5229	275.8325
.4	5332.6775	258 8678	.9	6068.3224	276.1466

DIAM.	AREA.	CIRCUM.	DIAM.	AREA.	CIRCUM.
88	6082.1376	276.4608	.5	6866.1631	293.7396
.1	6095.9685	276.775	.6	6880.858	294.0538
.2	6109.8151	277.0891	.7	6895.5685	294.3679
.3	6123.6774	277.4033	.8	6910.2948	294.6821
.4	6137.5554	277.7174	.9	6925.0367	294.9962
.5	6151.4491	278.0316	94	6939.7944	295.3104
.6	6165.3586	278.3458	.1	6954.5678	295.6246
.7	6179.2837	278.6599	.2	6969.3569	295.9387
.8	6193.2246	278.9741	.3	6984.1616	296.2529
.9	6207.1811	279.2882	.4	6998.9821	296.567
89	6221.1534	279.6024	.5	7013.8183	296.8812
.1	6235.1414	279.9166	.6	7028.6703	297.1954
.2	6249.1451	280.2307	.7	7043.5379	297.5095
.3	6263.1644	280.5449	.8	7058.4212	297.8237
.4	6277.1995	280.859	.9	7073.3203	298.1378
.5	6291.2503	281.1732	95	7088.235	298.452
.6	6305.3169	281.4874	.1	7103.1655	298.7662
.7	6319.3991	281.8015	.2	7118.1116	299.0803
.8	6333.497	282.1157	.3	7133.0735	299.3945
.9	6347.6107	282.4298	.4	7148.0511	299.7086
90	6361.74	282.744	.5	7163.0443	300.0228
.1	6375.8851	283 0582	.6	7178.0533	300.337
.2	6390.0458	283.3723	.7	7193.078	300.6511
.3	6404.2223	283.6865	.8	7208.1185	300.9653
.4	6418.4144	284.0006	.9	7223.1746	301.2794
.5	6432.6223	284.3148	96	7238.2464	301.5936
.6	6446.8459	284.629	.1	7253.3339	301.9078
.7	6461.0852	284.9431	.2	7268.4372	302.2219
.8	6475.3403	285.2573	.3	7283.5561	302.5361
.9	6489.611	285.5714	.4	7298.6908	302.8502
91	6503.8974	285.8856	.5	7313.8411	303.1644
.1	6518.1995	286.1998	.6	7329 0072	303.4786
.2	6532.5174	286.5139	.7	7344.189	303.7927
.3	6546.8509	286.8281	.8	7359.3865	304.1069
.4	6561.2002	287.1422	.9	7374.5997	304.421
.5	6575.5651	287.4564	97	7389.8286	304.7352
.6	6589.9458	287.7706	.1	7405.0732	305.0494
.7	6604.3422	288.0847	.2	7420.3335	305.3635
.8	6618.7543	288.3989	.3	7435.6096	305.6777
.9	6633.1821	288.713	.4	7450.9013	305.9918
92	6647.6256	289.0272	.5	7466.2087	306.306
.1	6662.0848	289.3414	.6	7481.5319	306.6202
.2	6676.5598	289.6555	.7	7496.8708	306.9343
.3	6691.0504	289.9697	.8	7512.2253	307.2485
.4	6705.5567	290.2838	.9	7527.5956	307.5626
.5	6720.0787	290.598	98	7542.9816	307.8768
.6	6734.6165	290.9121	.1	7558.3833	308.191
.7	6749.17	291.2263	.2	7573.8007	308.5051
.8	6763.7391	291.5405	.3	7589.2338	308.8193
.9	6778.324	291.8546	.4	7604.6826	309.1334
93	6792.9246	292.1688	.5	7620.1471	309.4476
.1	6807.5409	292.483	.6	7635.6274	309.7618
.2	6822.1729	292.7971	.7	7651.1233	310.0759
.3	6836.8206	293.1113	.8	7666.635	310.3901
.4	6851.484	293 4254	.9	7682.1623	310.7042

Diam.	Area.	Circum.	Diam.	Area.	Circum.
99	7697.7054	311.0184	.5	7775.6563	312.5892
.1	7713.2642	311.3326	.6	7791.2937	312.9034
.2	7728.8337	311.6467	.7	7806.9467	313.2175
.3	7744.4288	311.9609	.8	7822.6154	313.5317
.4	7760.0347	312.275	.9	7838.2999	313.8458

To Compute Area or Circumference of a Diameter greater than any in preceding Table.

See Rules, pages 235-6 and 241-2.

Or, *If Diameter exceeds* 100 *and is less than* 1001.

Put a decimal point, and take out area or circumference as for a Whole Number by removing decimal point, if for an area, two places to right, and if for a circumference, one place.

Example.—What is area and what circumference of a circle 967 feet in diameter?

Area of 96.7 is 7344.189; hence, for 967 it is 734 418.9; and circumference of 96.7 is 303.7927, and for 967 it is 3037.927.

To Compute Area and Circumference of a Circle by Logarithms.

See Rules, pages 236, 242.

Areas and Circumferences of Circles.

From 1 to 50 Feet (*advancing by an Inch*).

Or, From 1 to 50 Inches (*advancing by a Twelfth*).

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
1 *ft.*	.7854	3.1416	3 *ft.*	7.0686	9.4248
1	.9217	3.4034	1	7.4668	9.6866
2	1.069	3.6652	2	7.8758	9.9484
3	1.2272	3.927	3	8.2958	10.2102
4	1.3963	4.1888	4	8.7267	10.472
5	1.5763	4.4506	5	9.1685	10.7338
6	1.7671	4.7124	6	9.6211	10.9956
7	1.969	4.9742	7	10.0848	11.2574
8	2.1817	5.236	8	10.5593	11.5192
9	2.4053	5.4978	9	11.0447	11.781
10	2.6398	5.7596	10	11.541	12 0428
11	2.8853	6.0214	11	12.0483	12.3046
2 *ft.*	3.1416	6.2832	4 *ft.*	12.5664	12.5664
1	3.4088	6 545	1	13.0955	12.8282
2	3.687	6.8068	2	13.6354	13.09
3	3.9761	7.0686	3	14.1863	13.3518
4	4.2761	7.3304	4	14.7481	13.6136
5	4.5869	7.5922	5	15.3208	13.8754
6	4.9087	7.854	6	15.9043	14.1372
7	5.2415	8.1158	7	16.4989	14.499
8	5.5852	8.3776	8	17.1043	14.6608
9	5.9396	8.6394	9	17.7206	14.9226
10	6.305	8.9012	10	18.3478	15.1844
11	6.6814	9.163	11	18.9859	15.4462

Diam	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
5 *ft.*	19.635	15.708	6	70.8823	29.8452
1	20.2949	15.9698	7	72.1314	30.107
2	20.9658	16.2316	8	73.3913	30.3688
3	21.6476	16.4934	9	74.6621	30.6306
4	22.3403	16.7552	10	75.9439	30.8924
5	23.0439	17.017	11	77.2365	31.1542
6	23.7583	17.2788	10 *ft.*	78.54	31.416
7	24.4837	17.5406	1	79.8545	31.6778
8	25.22	17.8024	2	81.1798	31.9396
9	25.9673	18.0642	3	82.5161	32.2014
10	26.7254	18.326	4	83.8633	32.4632
11	27.4944	18.5878	5	85.2214	32.725
6 *ft.*	28.2744	18.8496	6	86.5903	32.9868
1	29.0653	19.1114	7	87.9703	33.2486
2	29.867	19.3732	8	89.3611	33.5104
3	30.6797	19.635	9	90.7628	33.7722
4	31.5033	19.8968	10	92.1754	34.034
5	32.3378	20.1586	11	93.599	34.2958
6	33.1831	20.4204	11 *ft.*	95.0334	34.5576
7	34.0394	20.6822	1	96.4787	34.8194
8	34.9067	20.944	2	97.935	35.0812
9	35.7848	21.2058	3	99 4022	35.343
10	36.6738	21.4676	4	100.8803	35.6048
11	37.5738	21.7294	5	102.3693	35.8666
7 *ft.*	38.4846	21.9912	6	103 8691	36.1284
1	39.4064	22.253	7	105.38	36.3902
2	40.339	22.5148	8	106.9017	36.652
3	41.2826	22.7766	9	108.4343	36.9138
4	42.2371	23.0384	10	109.9778	37.1756
5	43.2025	23.3002	11	111.5323	37.4374
6	44.1787	23.562	12 *ft.*	113.0976	37.6992
7	45.1659	23.8238	1	114.6739	37.961
8	46.1641	24.0856	2	116.261	38.2228
9	47.1731	24.3474	3	117.8591	38.4846
10	48.193	24.6092	4	119.468	38.7464
11	49.2238	24.871	5	121.088	39.0082
8 *ft.*	50.2656	25.1328	6	122.7187	39.27
1	51.3183	25.3946	7	124.3605	39.5318
2	52.3818	25.6564	8	126.0131	39.7936
3	53.4563	25.9182	9	127.6766	40.0554
4	54.5417	26.18	10	129.351	40.3172
5	55.638	26.4418	11	131.0366	40.579
6	56.7451	26.7036	13 *ft.*	132.7326	40.8408
7	57.8632	26.9654	1	134.4398	41.1026
8	58.9923	27.2272	2	136.1578	41.3644
9	60.1322	27.489	3	137.8868	41.6262
10	61.283	27.7508	4	139 6267	41.888
11	62.4448	28.0126	5	141.3774	42.1498
9 *ft.*	63.6174	28.2744	6	143.1391	42.4116
1	64.801	28.5362	7	144.9117	42.6734
2	65.9954	28.798	8	146.6953	42.9352
3	67.2008	29.0598	9	148.4897	43.197
4	68.417	29.3216	10	150.295	43.4588
5	69.6442	29.5834	11	152.1113	43.7206

Diam.	Area. Feet.	Circum. Feet.
14 *ft.*	153.9384	43.9824
1	155.7764	44.2442
2	157.6254	44.506
3	159.4853	44.7678
4	161.3561	45.0296
5	163.2378	45.2914
6	165.1303	45.5532
7	167.0338	45.815
8	168.9483	46.0768
9	170.8736	46.3386
10	172.8098	46.6004
11	174.7569	46.8622
15 *ft.*	176.715	47.124
1	178.684	47.3858
2	180.6638	47.6476
3	182.6546	47.9094
4	184.6563	48.1712
5	186.6689	48.433
6	188 6924	48.6948
7	190.7267	48.9566
8	192.7721	49.2184
9	194.8283	49.4802
10	196.8954	49.742
11	198.9734	50.0038
16 *ft.*	201.0624	50.2656
1	203.1622	50.5274
2	205.273	50.7892
3	207.3947	51.051
4	209.5273	51.3128
5	211.6707	51.5746
6	213.8252	51.8364
7	215.9904	52.0982
8	218.1667	52.36
9	220.3538	52.6218
10	222.5518	52.8836
11	224.7607	53.1454
17 *ft.*	226.9806	53.4072
1	229.2113	53.669
2	231.453	53.9308
3	233.7056	54.1926
4	235.9691	54.4544
5	238.2434	54.7162
6	240.5287	54.978
7	242.8249	55.2398
8	245.1321	55.5016
9	247.4501	55.7634
10	249.779	56.0252
11	252.1188	56.287
18 *ft.*	254.4696	56.5488
1	256.8312	56.8106
2	259.2038	57.0724
3	261.5873	57.3342
4	263.9817	57.596
5	266.3869	57.8578
6	268.8031	58.1196
7	271.2302	58.3814
8	273.6683	58.6432
9	276.1172	58.905
10	278.577	59.1668
11	281.0477	59.4286
19 *ft.*	283.5294	59.6904
1	286.0219	59.9522
2	288.5255	60.214
3	291.0398	60.4758
4	293.5651	60.7376
5	296.1012	60.9994
6	298.6483	61.2612
7	301.2064	61.523
8	303.7753	61.7848
9	306.3551	62.0466
10	308.9458	62.3084
11	311.5475	62.5702
20 *ft.*	314.16	62.832
1	316.7834	63.0938
2	319.4178	63.3556
3	322.0631	63.6174
4	324.7193	63.8792
5	327.3864	64.141
6	330.0643	64.4028
7	332.7532	64.6646
8	335.4531	64.9264
9	338.1638	65.1882
10	340.8854	65.45
11	343.618	65.7118
21 *ft.*	346.3614	65.9736
1	349.1157	66.2354
2	351.881	66.4972
3	354.6572	66.759
4	357.4442	67.0208
5	360.2422	67.2826
6	363 0511	67.5444
7	365.8709	67.8062
8	368.7017	68.068
9	371.5433	68.3298
10	374.3958	68.5916
11	377.2592	68.8534
22 *ft.*	380.1336	69.1152
1	383.0188	69.377
2	385 915	69.6388
3	388.8221	69.9006
4	391.74	70.1624
5	394.6689	70.4242
6	397.6087	70.686
7	400.5594	70.9478
8	403.5211	71.2096
9	406.4936	71.4714
10	409.477	71.7332
11	412.4713	71.995

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
23 *ft.*	415.4766	72.2568	6	593.9587	86.394
1	418.4927	72.5186	7	597.5639	86.6558
2	421.5198	72.7804	8	601.18	86.9176
3	424.5578	73.0422	9	604.8071	87.1794
4	427.6067	73.304	10	608.445	87.4412
5	430.6664	73.5658	11	612.0938	87.703
6	433.7371	73.8276	28 *ft.*	615.7536	87.9648
7	436.8187	74.0894	1	619.4242	88.2266
8	439.91	74.3512	2	623.1058	88.4884
9	443.0147	74.613	3	626.7983	88.7502
10	446.129	74.8748	4	630.5016	89.012
11	449.2542	75.1366	5	634.2159	89.2738
24 *ft.*	452.3904	75.3984	6	637.9411	89.5356
1	455.5374	75.6602	7	641.6772	89.7974
2	458.6954	75.922	8	645.4243	90.0592
3	461.8643	76.1838	9	649.1822	90.321
4	465.044	76.4456	10	652.951	90.5828
5	468.2347	76.7074	11	656.7307	90.8446
6	471.4363	76.9692	29 *ft.*	660.5214	91.1064
7	474.6488	77.231	1	664.3229	91.3682
8	477.8723	77.4928	2	668.1354	91.63
9	481.1066	77.7546	3	671.9588	91.8918
10	484.3518	78.0164	4	675.7931	92.1536
11	487.6076	78.2782	5	679.6382	92.4154
25 *ft.*	490.875	78.54	6	683.4943	92.6772
1	494.1529	78.8018	7	687.3613	92.939
2	497.4418	79.0636	8	691.2393	93.2008
3	500.7416	79.3254	9	695.1281	93.4626
4	504.0523	79.5872	10	699.0278	93.7244
5	507.3738	79.849	11	702.9384	93.9862
6	510.7063	80.1108	30 *ft.*	706.86	94.248
7	514.0485	80.3726	1	710.7924	94.5098
8	517.404	80.6344	2	714.7358	94.7716
9	520.7693	80.8962	3	718.6901	95.0334
10	524.1454	81.158	4	722.6553	95.2952
11	527.5324	81.4198	5	726.6313	95.557
26 *ft.*	530.9304	81.6816	6	730.6183	95.8188
1	534.3397	81.9434	7	734.6162	96.0806
2	537.759	82.2052	8	738.6251	96.3424
3	541.1897	82.467	9	742.6448	96.6042
4	544.6313	82.7288	10	746.6754	96.866
5	548.0837	82.9906	11	750.7164	97.1278
6	551.5471	83.2524	31 *ft.*	754.7694	97.3896
7	555.0214	83.5142	1	758.8327	97.6514
8	558.5066	83.776	2	762.907	97.9132
9	562.0028	84.0378	3	766.9922	98.175
10	565.5098	84.2996	4	771.0883	98.4368
11	569.0277	84.5614	5	775.1952	98.6986
27 *ft.*	572.5566	84.8232	6	779.3131	98.9604
1	576.0963	85.085	7	783.4419	99.2222
2	579.6467	85.3468	8	787.5817	99.484
3	583.2086	85.6086	9	791.7323	99.7458
4	586.781	85.8704	10	795.8938	100.0076
5	590.3644	86.1322	11	800.0662	100.2694

Diam.	Area.	Circum.	Diam.	Area.	Circum.
	Feet.	Feet.		Feet.	Feet.
32 *ft.*	804.2496	100.5312	6	1046.3491	114.6684
1	808.4439	100.793	7	1051.1324	114.9302
2	812.649	101.0548	8	1055.9266	115.192
3	816.8651	101.3166	9	1060.7318	115.4538
4	821.092	101.5784	10	1065.5478	115.7156
5	825.3299	101.8402	11	1070.3747	115.9774
6	829.5787	102.102	37 *ft.*	1075.2126	116.2392
7	833.8384	102.3638	1	1080.0613	116.501
8	838.1091	102.6256	2	1084.921	116.7628
9	842.3906	102.8874	3	1089.7916	117.0246
10	846.683	103.1492	4	1094.6731	117.2864
11	850.9863	103.411	5	1099.5654	117.5482
33 *ft.*	855.3006	103.6728	6	1104.4687	117.81
1	859.6257	103 9346	7	1109.3829	118.0718
2	863.9618	104.1964	8	1114.308	118.3336
3	868.3088	104.4582	9	1119.2441	118 5954
4	872.6667	104.72	10	1124.191	118.8572
5	877.0354	104.9818	11	1129.1489	119.119
6	881.4151	105.2436	38 *ft.*	1134.1176	119 3808
7	885.8057	105.5054	1	1139.0972	119.6426
8	890.2073	105.7672	2	1144 0878	119 9044
9	894.6197	106.029	3	1149 0893	120.1662
10	899.043	106.2908	4	1154.1017	120.428
11	903.4772	106.5526	5	1159.1249	120.6898
34 *ft.*	907.9224	106.8144	6	1164.1591	120.9516
1	912.3784	107.0762	7	1169.2042	121.2134
2	916.8454	107.338	8	1174.2603	121.4758
3	921.3233	107.5998	9	1179 3272	121.737
4	925.812	107.8616	10	1184.405	121.9988
5	930.3117	108.1234	11	1189.4937	122.2606
6	934.8223	108.3852	39 *ft.*	1194.5934	122.5224
7	939.3439	108.647	1	1199.7039	122.7848
8	943.8763	108.9088	2	1204.8254	123.046
9	948.4196	109.1706	3	1209.9578	123.3078
10	952.9738	109 4324	4	1215.101	123.5696
11	957.5392	109.6942	5	1220.2552	123.8314
35 *ft.*	962.115	109.956	6	1225.4203	124.0932
1	966.7019	110.2178	7	1230.5963	124.355
2	971.2998	110.4796	8	1235.7833	124.6168
3	975.9086	110.7414	9	1240.9811	124.8786
4	980.5287	111.0032	10	1246.1898	125.1404
5	985.1588	111.265	11	1251.4094	125 4022
6	989.8005	111.5268	40 *ft.*	1256.64	125.664
7	994.4527	111.7886	1	1261.8814	125.9258
8	999.116	112.0504	2	1267.1338	126.1876
9	1003.7903	112.3122	3	1272.3971	126.4494
10	1008.4754	112.574	4	1277.6712	126.7112
11	1013.1714	112.8358	5	1282.9563	126.973
36 *ft.*	1017.8784	113.0976	6	1288.2523	127.2348
1	1022.5962	113.3594	7	1293.5592	127.4966
2	1027.325	113.6212	8	1298.877	127.7584
3	1032.0647	113.883	9	1304.2058	128.0202
4	1036.8153	114.1448	10	1309.5454	128.282
5	1041.5767	114.4066	11	1314.8959	128.5438

Diam.	Area.	Circum.
	Feet.	Feet.
41 *ft.*	1320.2574	128.8056
1	1325.6297	129.0674
2	1331.013	129.3292
3	1336.4072	129.591
4	1341.8123	129.8528
5	1347.2282	130.1146
6	1352.6551	130.3764
7	1358.0929	130.6382
8	1363.5416	130.9
9	1369.0013	131.1618
10	1374.4718	131.4236
11	1379.9532	131.6854
42 *ft.*	1385.4456	131.9472
1	1390.9488	132.209
2	1396.463	132.4708
3	1401.9881	132.7326
4	1407.5241	132.9944
5	1413.0709	133.2562
6	1418.6287	133.518
7	1424.1974	133.7798
8	1429.777	134.0416
9	1435.3676	134.3034
10	1440.969	134.5652
11	1446.5813	134.827
43 *ft.*	1452.2046	135.0888
1	1457.8387	135.3506
2	1463.4838	135.6124
3	1469.1398	135.8742
4	1474.8066	136.136
5	1480.4844	136.3978
6	1486.1731	136.6596
7	1491.8717	136.9214
8	1497.5833	137.1832
9	1503.3047	137.445
10	1509.037	137.7068
11	1514.7802	137.9686
44 *ft.*	1520.5344	138.2304
1	1526.2994	138.4922
2	1532.0754	138.754
3	1537.8623	139.0158
4	1543.66	139.2776
5	1549.4687	139.5394
6	1555.2883	139.8012
7	1561.1188	140.063
8	1566.9603	140.3248
9	1572.8126	140.5866
10	1578.6756	140.8484
11	1584.5499	141.1102
45 *ft.*	1590.435	141.372
1	1596.3309	141.6338
2	1602.2378	141.8956
3	1608.1556	142.1574
4	1614.0843	142.4192
5	1620.0238	142.681
6	1625.9743	142.9428
7	1631.9357	143.2046
8	1637.9081	143.4664
9	1643.8913	143.7282
10	1649.8854	143.99
11	1655.8904	144.2518
46 *ft.*	1661.9064	144.5136
1	1667.9332	144.7754
2	1673.971	145.0372
3	1680.0197	145.299
4	1686.0792	145.5608
5	1692.1497	145.8226
6	1698.2311	146.0844
7	1704.3195	146.3462
8	1710.4267	146.608
9	1716.5408	146.8698
10	1722.6658	147.1316
11	1728.8017	147.3934
47 *ft.*	1734.9486	147.6552
1	1741.1063	147.917
2	1747.275	148.1788
3	1753.4546	148.4406
4	1759.6451	148.7024
5	1765.8464	148.9642
6	1772.0587	149.226
7	1778.2819	149.4878
8	1784.516	149.7496
9	1790.7611	150.0114
10	1797.017	150.2732
11	1803.2838	150.535
48 *ft.*	1809.5616	150.7968
1	1815.8502	151.0586
2	1822.1498	151.3204
3	1828.4603	151.5822
4	1834.7817	151.844
5	1841.1139	152.1058
6	1847.4571	152.3676
7	1853.8112	152.6294
8	1860.1763	152.8912
9	1866.5522	153.153
10	1872.939	153.4148
11	1879.3367	153.6766
49 *ft.*	1885.7454	153.9384
1	1892.1649	154.2002
2	1898.5954	154.462
3	1905.0368	154.7238
4	1911.4897	154.9856
5	1917.9522	155.2474
6	1924.4263	155.5092
7	1930.9113	155.771
8	1937.4073	156.0328
9	1943.9142	156.2946
10	1950.4318	156.5564
11	1956.9604	156.8182
50 *ft.*	1963.5	157.08

Sides of Squares—equal in Area to a Circle.

Diameter from 1 to 100.

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
1	.8862	14	12.4072	27	23.9281	40	35.4491
¼	1.1078	¼	12.6287	¼	24.1497	¼	35.6706
½	1.3293	½	12.8503	½	24.3712	½	35 8922
¾	1.5509	¾	13.0718	¾	24.5928	¾	36.1137
2	1.7724	15	13.2934	28	24.8144	41	36.3353
¼	1.994	¼	13.515	¼	25.0359	¼	36.5569
½	2.2156	½	13.7365	½	25.2575	½	36.7784
¾	2.4371	¾	13.9581	¾	25.479	¾	37
3	2.6587	16	14.1796	29	25.7006	42	37.2215
¼	2.8802	¼	14.4012	¼	25.9221	¼	37.4431
½	3.1018	½	14.6227	½	26.1437	½	37.6646
¾	3.3233	¾	14.8443	¾	26.3653	¾	37.8862
4	3.5449	17	15.0659	30	26.5868	43	38.1078
¼	3.7665	¼	15.2874	¼	26.8084	¼	38.3293
½	3.988	½	15.509	½	27.0299	½	38.5509
¾	4.2096	¾	15.7305	¾	27.2515	¾	38.7724
5	4.4311	18	15.9521	31	27.473	44	38.994
¼	4.6527	¼	16.1736	¼	27.6946	¼	39.2155
½	4.8742	½	16.3952	½	27.9161	½	39.4371
¾	5.0958	¾	16.6168	¾	28.1377	¾	39.6587
6	5.3174	19	16.8383	32	28.3593	45	39.8802
¼	5.5389	¼	17.0599	¼	28.5808	¼	40.1018
½	5.7605	½	17.2814	½	28.8024	½	40.3233
¾	5.982	¾	17.503	¾	29.0239	¾	40.5449
7	6.2036	20	17.7245	33	29.2455	46	40.7664
¼	6.4251	¼	17.9461	¼	29.467	¼	40.988
½	6.6467	½	18.1677	½	29.6886	½	41.2096
¾	6.8683	¾	18.3892	¾	29.9102	¾	41.4311
8	7.0898	21	18.6108	34	30.1317	47	41.6527
¼	7.3114	¼	18.8323	¼	30.3533	¼	41.8742
½	7.5329	½	19.0539	½	30.5748	½	42.0958
¾	7.7545	¾	19.2754	¾	30.7964	¾	42.3173
9	7.976	22	19.497	35	31.0179	48	42.539
¼	8.1976	¼	19.7185	¼	31.2395	¼	42.7604
½	8.4192	½	19.9401	½	31.4611	½	42.982
¾	8.6407	¾	20.1617	¾	31.6826	¾	43.2036
10	8.8623	23	20.3832	36	31.9042	49	43.4251
¼	9.0838	¼	20.6048	¼	32.1257	¼	43.6467
½	9.3054	½	20.8263	½	32.3473	½	43.8682
¾	9.5269	¾	21.0479	¾	32.5688	¾	44.0898
11	9.7485	24	21.2694	37	32.7904	50	44.3113
¼	9.97	¼	21.491	¼	33.0112	¼	44.5329
½	10.1916	½	21.7126	½	33.2335	½	44.7545
¾	10.4132	¾	21.9341	¾	33.4551	¾	44.976
12	10.6347	25	22.1557	38	33.6766	51	45.1976
¼	10.8563	¼	22.3772	¼	33.8982	¼	45.4191
½	11.0778	½	22.5988	½	34.1197	½	45.6407
¾	11.2994	¾	22.8203	¾	34.3413	¾	45.8622
13	11.5209	26	23 0419	39	34.5628	52	46.0838
¼	11.7425	¼	23.2634	¼	34.7884	¼	46.3054
½	11.9641	½	23.485	½	35.006	½	46.5269
¾	12.1856	¾	23.7066	¾	35.2275	¾	46.7485

Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.	Diam.	Side of Sq.
53	46.97	65	57.6047	77	68.2395	89	78.8742
¼	47.1916	¼	57.8263	¼	68.461	¼	79.0957
½	47.4131	½	58.0479	½	68.6826	½	79.3173
¾	47.6347	¾	58.2694	¾	68.9041	¾	79.5389
54	47.8562	66	58.491	78	69.1257	90	79.7604
¼	48.0778	¼	58.7125	¼	69.3473	¼	79.982
½	48.2994	½	58.9341	½	69.5688	½	80.2035
¾	48.5209	¾	59.1556	¾	69.7904	¾	80.4251
55	48.7425	67	59.3772	79	70.0119	91	80.6467
¼	48.964	¼	59.5988	¼	70.2335	¼	80.8682
½	49.1856	½	59.8203	½	70.455	½	81.0898
¾	49.4071	¾	60.0419	¾	70.6766	¾	81.3113
56	49.6287	68	60.2634	80	70.8981	92	81.5329
¼	49.8503	¼	60.485	¼	71.1197	¼	81.7544
½	50.0718	½	60.7065	½	71.3413	½	81.976
¾	50.2934	¾	60.9281	¾	71.5628	¾	82.1975
57	50.5149	69	61.1497	81	71.7844	93	82.4191
¼	50.7365	¼	61.3712	¼	72.0059	¼	82.6407
½	50.958	½	61.5928	½	72.2275	½	82.8622
¾	51.1796	¾	61.8143	¾	72.4491	¾	83.0838
58	51.4012	70	62.0359	82	72.6706	94	83.3053
¼	51.6227	¼	62.2574	¼	72.8921	¼	83.5269
½	51.8443	½	62.479	½	73.1137	½	83.7484
¾	52.0658	¾	62.7006	¾	73.3353	¾	83.97
59	52.2874	71	62.9221	83	73.5568	95	84.1916
¼	52.5089	¼	63.1437	¼	73.7784	¼	84.4131
½	52.7305	½	63.3652	½	73.9999	½	84.6347
¾	52.9521	¾	63.5868	¾	74.2215	¾	84.8562
60	53.1736	72	63.8083	84	74.4431	96	85.0778
¼	53.3952	¼	64.0299	¼	74.6647	¼	85.2993
½	53.6167	½	64.2514	½	74.8862	½	85.5209
¾	53.8383	¾	64.4730	¾	75.1077	¾	85.7425
61	54.0598	73	64.6946	85	75.3293	97	85.9646
¼	54.2814	¼	64.9161	¼	75.5508	¼	86.185
½	54.503	½	65.1377	½	75.7724	½	86.4071
¾	54.7245	¾	65.3592	¾	75.9934	¾	86.6289
62	54.9461	74	65.5808	86	76.2155	98	86.8502
¼	55.1676	¼	65.8023	¼	76.4371	¼	87.0718
½	55.3892	½	66.0239	½	76.6586	½	87.2933
¾	55.6107	¾	66.2455	¾	76.8802	¾	87.5449
63	55.8323	75	66.467	87	77.1017	99	87.7364
¼	56.0538	¼	66.6886	¼	77.3233	¼	87.958
½	56.2754	½	66.9104	½	77.5449	½	88.1796
¾	56.497	¾	67.1317	¾	77.7664	¾	88.4011
64	56.7185	76	67.3532	88	77.988		
¼	56.9401	¼	67.5748	¼	78.2095		
½	57.1616	½	67.7964	½	78.4316		
¾	57.3832	¾	68.0179	¾	78.6526	100	88.6227

Application of Table.

To Ascertain a Square that has same Area as a Given Circle.

ILLUS.—If side of a square that has same area as a circle of 73.25 ins. is required. By Table of Areas, page 233, opposite to 73.25 is 4214.11; and in this table is 64.9161, which is side of a square having same area as a circle of that diameter.

Lengths of Circular Arcs, up to a Semicircle.

Diameter of a Circle = 1, *and divided into* 1000 *equal Parts.*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.02645	.15	1.05896	.2	1.10348	.25	1.15912	.3	1.22495
.101	1.02698	.151	1.05973	.201	1.10447	.251	1.16033	.301	1.22635
.102	1.02752	.152	1.06051	.202	1.10548	.252	1.16157	.302	1.22776
.103	1.02806	.153	1.0613	.203	1.1065	.253	1.16279	.303	1.22918
.104	1.0286	.154	1.06209	.204	1.10752	.254	1.16402	.304	1.23061
.105	1.02914	.155	1.06288	.205	1.10855	.255	1.16526	.305	1.23205
.106	1.0297	.156	1.06368	.206	1.10958	.256	1.16649	.306	1.23349
.107	1.03026	.157	1.06449	.207	1.11062	.257	1.16774	.307	1.23494
.108	1.03082	.158	1.0653	.208	1.11165	.258	1 16899	.308	1.23636
.109	1.03139	.159	1.06611	.209	1.11269	.259	1.17024	.309	1.2378
.11	1.03196	.16	1.06693	.21	1.11374	.26	1.1715	.31	1.23925
.111	1.03254	.161	1.06775	.211	1.11479	.261	1.17275	.311	1.2407
.112	1.03312	.162	1.06858	.212	1.11584	.262	1.17401	.312	1.24216
.113	1.03371	.163	1.06941	.213	1.11692	.263	1.17527	.313	1.2436
.114	1.0343	.164	1.07025	.214	1.11796	.264	1.17655	.314	1.24506
.115	1.0349	.165	1.07109	.215	1.11904	.265	1.17784	.315	1.24654
.116	1.03551	.166	1.07194	.216	1.12011	.266	1.17912	.316	1.24801
.117	1.03611	.167	1.07279	.217	1.12118	.267	1.1804	.317	1.24946
.118	1.03672	.168	1.07365	.218	1.12225	.268	1.18162	.318	1.25095
.119	1.03734	.169	1.07451	.219	1.12334	.269	1.18294	.319	1.25243
.12	1.03797	.17	1.07537	.22	1.12445	.27	1.18428	.32	1.25391
.121	1.0386	.171	1.07624	.221	1.12556	.271	1.18557	.321	1.25539
.122	1.03923	.172	1.07711	.222	1.12663	.272	1.18688	.322	1.25686
.123	1.03987	.173	1.07799	.223	1.12774	.273	1.18819	.323	1.25836
.124	1.04051	.174	1.07888	.224	1.12885	.274	1.18969	.324	1.25987
.125	1.04116	.175	1.07977	.225	1.12997	.275	1.19082	.325	1.26137
.126	1.04181	.176	1.08066	.226	1.13108	.276	1.19214	.326	1.26286
.127	1.04247	.177	1.08156	.227	1.13219	.277	1.19345	.327	1.26437
.128	1.04313	.178	1.08246	.228	1.13331	.278	1.19477	.328	1.26588
.129	1.0438	.179	1.08337	.229	1.13444	.279	1.1961	.329	1.2674
.13	1.04447	.18	1.08428	.23	1.13557	.28	1.19743	.33	1.26892
.131	1.04515	.181	1.08519	.231	1.13671	.281	1.19887	.331	1.27044
.132	1.04584	.182	1.08611	.232	1.13786	.282	1.20011	.332	1.27196
.133	1.04652	.183	1.08704	.233	1.13903	.283	1.20146	.333	1.27349
.134	1.04722	.184	1.08797	.234	1.1402	.284	1.20282	.334	1.27502
.135	1.04792	.185	1.0889	.235	1.14136	.285	1.20419	.335	1.27656
.136	1.04862	.186	1.08984	.236	1.14247	.286	1.20558	.336	1.2781
.137	1.04932	.187	1.09079	.237	1.14363	.287	1.20696	.337	1.27964
.138	1.05003	.188	1.09174	.238	1.1448	.288	1.20828	.338	1.28118
.139	1.05075	.189	1.09269	.239	1.14597	.289	1.20967	.339	1.28273
.14	1.05147	.19	1.09365	.24	1.14714	.29	1.21202	.34	1.28428
.141	1.0522	.191	1.09461	.241	1.14831	.291	1.21239	.341	1.28583
.142	1.05293	.192	1.09557	.242	1.14949	.292	1.21381	.342	1.28739
.143	1.05367	.193	1.09654	.243	1.15067	.293	1.2152	.343	1.28895
.144	1.05441	.194	1.09752	.244	1.15186	.294	1.21658	.344	1.29052
.145	1.05516	.195	1.0985	.245	1.15308	.295	1.21794	.345	1.29209
.146	1.05591	.196	1.09949	.246	1.15429	.296	1.21926	.346	1.29366
.147	1.05667	.197	1.10048	.247	1.15549	.297	1.22061	.347	1.29523
.148	1.05743	.198	1.10147	.248	1.1567	.298	1.22203	.348	1.29681
.149	1.05819	.199	1.10247	.249	1.15791	.299	1.22347	.349	1.29839

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.35	1.299 97	.38	1.348 99	.41	1.400 77	.44	1.455 12	.47	1.511 85
.351	1.301 56	.381	1.350 68	.411	1.402 54	.441	1.456 97	.471	1.513 78
.352	1.303 15	.382	1.352 37	.412	1.404 32	.442	1.458 83	.472	1.515 71
.353	1.304 74	.383	1.354 06	.413	1.406 1	.443	1.460 69	.473	1.517 64
.354	1.306 34	.384	1.355 75	.414	1.407 88	.444	1.462 55	.474	1.519 58
.355	1.307 94	.385	1.357 44	.415	1.409 66	.445	1.464 41	.475	1.521 52
.356	1.309 54	.386	1.359 14	.416	1.411 45	.446	1.466 28	.476	1.523 46
.357	1.311 15	.387	1.360 84	.417	1.413 24	.447	1.468 15	.477	1.525 41
.358	1.312 76	.388	1.362 54	.418	1.415 03	.448	1.470 02	.478	1.527 36
.359	1.314 37	.389	1.364 25	.419	1.416 82	.449	1.471 89	.479	1.529 31
.36	1.315 99	.39	1.365 96	.42	1.418 61	.45	1.473 77	.48	1.531 26
.361	1.317 61	.391	1.367 67	.421	1.420 41	.451	1.475 65	.481	1.533 22
.362	1.319 23	.392	1.369 39	.422	1.422 22	.452	1.477 53	.482	1.535 18
.363	1.320 86	.393	1.371 11	.423	1.424 02	.453	1.479 42	.483	1.537 14
.364	1.322 49	.394	1.372 83	.424	1.425 83	.454	1.481 31	.484	1.539 1
.365	1.324 13	.395	1.374 55	.425	1.427 64	.455	1.483 2	.485	1.541 06
.366	1.325 77	.396	1.376 28	.426	1.429 45	.456	1.485 09	.486	1.543 02
.367	1.327 41	.397	1.378 01	.427	1.431 27	.457	1.486 99	.487	1.544 99
.368	1.329 05	.398	1.379 74	.428	1.433 09	.458	1.488 89	.488	1.546 96
.369	1.330 69	.399	1.381 48	.429	1.434 91	.459	1.490 79	.489	1.548 93
.37	1.332 34	.4	1.383 22	.43	1.436 73	.46	1.492 69	.49	1.550 9
.371	1.333 99	.401	1.384 96	.431	1.438 56	.461	1.494 6	.491	1.552 88
.372	1.335 64	.402	1.386 71	.432	1.440 39	.462	1.496 51	.492	1.554 86
.373	1.337 3	.403	1.388 46	.433	1.442 22	.463	1.498 42	.493	1.556 85
.374	1.338 96	.404	1.390 21	.434	1.444 05	.464	1.500 33	.494	1.558 54
.375	1.340 63	.405	1.391 96	.435	1.445 89	.465	1.502 24	.495	1.560 83
.376	1.342 29	.406	1.393 72	.436	1.447 73	.466	1.504 16	.496	1.562 82
.377	1.343 96	.407	1.395 48	.437	1.449 57	.467	1.506 08	.497	1.564 81
.378	1.345 63	.408	1.397 24	.438	1.451 42	.468	1.508	.498	1.566 8
.379	1.347 31	.409	1.399	.439	1.453 27	.469	1.509 92	.499	1.568 79
								.5	1.570 79

To Ascertain Length of an Arc of a Circle by preceding Table.

RULE.—Divide height by base, find quotient in column of heights, take length for that height opposite to it in next column on the right hand. Multiply length thus obtained by base of arc, and product will give length.

EXAMPLE.—What is length of an arc of a circle, base or span of it being 100 feet, and height 25?

$25 \div 100 = .25$; and .25, per table, $= 1.159\,12$, *length of base, which, multiplied by* $100 = 115.912$ *feet.*

When, in division of a height by base, the quotient has a remainder after third place of decimals, and great accuracy is required.

RULE.—Take length for first three figures, subtract it from next following length; multiply remainder by this fractional remainder, add product to first length, and sum will give length for whole quotient.

EXAMPLE.—What is length of an arc of a circle, base of which is 35 feet, and height or versed sine 8 feet?

$8 \div 35 = .228\,571\,4$; *tabular length for* $.228 = 1.133\,31$, and for $.229 = 1.134\,44$, *the difference between which is* $.001\,13$. Then $.5714 \times .001\,13 = .000\,645\,682$.

$$\begin{aligned} \text{Hence } .228 \quad &= 1.133\,31, \\ \text{and } .000\,571\,4 &= \underline{.000\,645\,682} \\ &\quad 1.133\,955\,682, \end{aligned}$$

the sum by which base of arc is to be multiplied; and $1.133\,955\,682 \times 35 = 39.688\,45$ *feet.*

Lengths of Circular Arcs from 1° to 180°.

(*Radius* = 1.)

Degrees.	Length.	Degrees.	Length.	Degrees.	Length.	Degrees.	Length.
1	.0174	46	.8028	91	1.5882	136	2.3736
2	.0349	47	.8203	92	1.6057	137	2.3911
3	.0524	48	.8377	93	1.6231	138	2.4085
4	.0698	49	.8552	94	1.6406	139	2.426
5	.0873	50	.8727	95	1.6581	140	2.4435
6	.0147	51	.8901	96	1.6755	141	2.4609
7	.0222	52	.9076	97	1.693	142	2.4784
8	.0396	53	.925	98	1.7104	143	2.4958
9	.0571	54	.9424	99	1.7279	144	2.5133
10	.1745	55	.9599	100	1.7453	145	2.5307
11	.192	56	.9774	101	1.7628	146	2.5482
12	.2094	57	.9948	102	1.7802	147	2.5656
13	.2269	58	1.0123	103	1.7977	148	2.5831
14	.2443	59	1.0297	104	1.8151	149	2.6005
15	.2618	60	1.0472	105	1.8326	150	2.618
16	.2792	61	1.0646	106	1.85	151	2.6354
17	.2967	62	1.0821	107	1.8675	152	2.6529
18	.3141	63	1.0995	108	1.8849	153	2.6703
19	.3316	64	1.117	109	1.9024	154	2.6878
20	.3491	65	1.1345	110	1.9199	155	2.7053
21	.3665	66	1.1519	111	1.9373	156	2.7227
22	.384	67	1.1694	112	1.9548	157	2.7402
23	.4014	68	1.1868	113	1.9722	158	2.7576
24	.4189	69	1.2043	114	1.9897	159	2.7751
25	.4363	70	1.2217	115	2.0071	160	2.7925
26	.4538	71	1.2392	116	2.0246	161	2.81
27	.4712	72	1.2566	117	2.042	162	2.8274
28	.4887	73	1.2741	118	2.0595	163	2.8449
29	.5061	74	1.2915	119	2.0769	164	2.8623
30	.5236	75	1.309	120	2.0944	165	2.8798
31	.541	76	1.3264	121	2.1118	166	2.8972
32	.5585	77	1.3439	122	2.1293	167	2.9147
33	.5759	78	1.3613	123	2.1467	168	2.9321
34	.5934	79	1.3788	124	2.1642	169	2.9496
35	.6109	80	1.3963	125	2.1817	170	2.967
36	.6283	81	1.4137	126	2.1991	171	2.9845
37	.6458	82	1.4312	127	2.2166	172	3.002
38	.6632	83	1.4486	128	2.2304	173	3.0194
39	.6807	84	1.4661	129	2.2515	174	3.0369
40	.6981	85	1.4835	130	2.2689	175	3.0543
41	.7156	86	1.501	131	2.2864	176	3.0718
42	.733	87	1.5184	132	2.3038	177	3.0892
43	.7505	88	1.5359	133	2.3213	178	3.1067
44	.7679	89	1.5533	134	2.3387	179	3.1241
45	.7854	90	1.5708	135	2.3562	180	3.1416

To Ascertain Length of a Circular Arc by Table.

RULE.—From column opposite to degrees of arc, take length, and multiply it by radius of circle.

EXAMPLE.—Number of degrees in an arc are 45°, and diameter of circle 5 feet.

Then .7854 tab. length $\times 5 \div 2 = 1.9635$ *feet.*

Lengths of Elliptic Arcs.

Up to a Semi-ellipse.

Transverse Diameter = 1, *and divided into* 1000 *equal Parts.*

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.1	1.04162	.15	1.0933	.2	1.15014	.25	1.21136	.3	1.27669
.101	1.04262	.151	1.09448	.201	1.15131	.251	1.21263	.301	1.27803
.102	1.04362	.152	1.09558	.202	1.15248	.252	1.2139	.302	1.27937
.103	1.04462	.153	1.09669	.203	1.15366	.253	1.21517	.303	1.28071
.104	1.04562	.154	1.0978	.204	1.15484	.254	1.21644	.304	1.28205
.105	1.04662	.155	1.09891	.205	1.15602	.255	1.21772	.305	1.28339
.106	1.04762	.156	1.10002	.206	1.1572	.256	1.219	.306	1.28474
.107	1.04862	.157	1.10113	.207	1.15838	.257	1.22028	.307	1.28609
.108	1.04962	.158	1.10224	.208	1.15957	.258	1.22156	.308	1.28744
.109	1.05063	.159	1.10335	.209	1.16076	.259	1.22284	.309	1.28879
.11	1.05164	.16	1.10447	.21	1.16196	.26	1.22412	.31	1.29014
.111	1.05265	.161	1.1056	.211	1.16316	.261	1.22541	.311	1.29149
.112	1.05366	.162	1.10672	.212	1.16436	.262	1.2267	.312	1.29285
.113	1.05467	.163	1.10784	.213	1.16557	.263	1.22799	.313	1.29421
.114	1.05568	.164	1.10896	.214	1.16678	.264	1.22928	.314	1.29557
.115	1.05669	.165	1.11008	.215	1.16799	.265	1.23057	.315	1.29603
.116	1.0577	.166	1.1112	.216	1.1692	.266	1.23186	.316	1.29829
.117	1.05872	.167	1.11232	.217	1.17041	.267	1.23315	.317	1.29965
.118	1.05974	.168	1.11344	.218	1.17163	.268	1.23445	.318	1.30102
.119	1.06076	.169	1.11456	.219	1.17285	.269	1.23575	.319	1.30239
.12	1.06178	.17	1.11569	.22	1.17407	.27	1.23705	.32	1.30376
.121	1.0628	.171	1.11682	.221	1.17529	.271	1.23835	.321	1.30513
.122	1.06382	.172	1.11795	.222	1.17651	.272	1.23966	.322	1.3065
.123	1.06484	.173	1.11908	.223	1.17774	.273	1.24097	.323	1.30787
.124	1.06586	.174	1.12021	.224	1.17897	.274	1.24228	.324	1.30924
.125	1.06689	.175	1.12134	.225	1.1802	.275	1.24359	.325	1.31061
.126	1.06792	.176	1.12247	.226	1.18143	.276	1.2448	.326	1.31198
.127	1.06895	.177	1.1236	.227	1.18266	.277	1.24612	.327	1.31335
.128	1.06998	.178	1.12473	.228	1.1839	.278	1.24744	.328	1.31472
.129	1.07001	.179	1.12586	.229	1.18514	.279	1.24876	.329	1.3161
.13	1.07204	.18	1.12699	.23	1.18638	.28	1.25001	.33	1.31748
.131	1.07308	.181	1.12813	.231	1.18762	.281	1.25142	.331	1.31886
.132	1.07412	.182	1.12927	.232	1.18886	.282	1.25274	.332	1.32024
.133	1.07516	.183	1.13041	.233	1.1901	.283	1.25406	.333	1.32162
.134	1.07621	.184	1.13155	.234	1.19134	.284	1.25538	.334	1.323
.135	1.07726	.185	1.13269	.235	1.19258	.285	1.2567	.335	1.32438
.136	1.07831	.186	1.13383	.236	1.19382	.286	1.25803	.336	1.32576
.137	1.07937	.187	1.13497	.237	1.19506	.287	1.25936	.337	1.32715
.138	1.08043	.188	1.13611	.238	1.1963	.288	1.26069	.338	1.32854
.139	1.08149	.189	1.13726	.239	1.19755	.289	1.26202	.339	1.32993
.14	1.08255	.19	1.13841	.24	1.1988	.29	1.26335	.34	1.33132
.141	1.08362	.191	1.13956	.241	1.20005	.291	1.26468	.341	1.33272
.142	1.08469	.192	1.14071	.242	1.2013	.292	1.26601	.342	1.33412
.143	1.08576	.193	1.14186	.243	1.20255	.293	1.26734	.343	1.33552
.144	1.08684	.194	1.14301	.244	1.2038	.294	1.26867	.344	1.33692
.145	1.08792	.195	1.14416	.245	1.20506	.295	1.27	.345	1.33833
.146	1.08901	.196	1.14531	.246	1.20632	.296	1.27133	.346	1.33974
.147	1.0901	.197	1.14646	.247	1.20758	.297	1.27267	.347	1.34115
.148	1.09119	.198	1.14762	.248	1.20884	.298	1.27401	.348	1.34256
.149	1.09228	.199	1.14888	.249	1.2101	.299	1.27535	.349	1.34397

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.35	1.34539	.405	1.42533	.46	1.50842	.515	1.59408	.57	1.68036
.351	1.34681	.406	1.42681	.461	1.50996	.516	1.59564	.571	1.68195
.352	1.34823	.407	1.42829	.462	1.5115	.517	1.5972	.572	1.68354
.353	1.34965	.408	1.42977	.463	1.51304	.518	1.59876	.573	1.68513
.354	1.35108	.409	1.43125	.464	1.51458	.519	1.60032	.574	1.68672
.355	1.35251	.41	1.43273	.465	1.51612	.52	1.60188	.575	1.68831
.356	1.35394	.411	1.43421	.466	1.51766	.521	1.60344	.576	1.6899
.357	1.35537	.412	1.43569	.467	1.5192	.522	1.605	.577	1.69149
.358	1.3568	.413	1.43718	.468	1.52074	.523	1.60656	.578	1.69308
.359	1.35823	.414	1.43867	.469	1.52229	.524	1.60812	.579	1.69467
.36	1.35967	.415	1.44016	.47	1.52384	.525	1.60968	.58	1.69626
.361	1.36111	.416	1.44165	.471	1.52539	.526	1.61124	.581	1.69785
.362	1.36255	.417	1.44314	.472	1.52691	.527	1.6128	.582	1.69945
.363	1.36399	.418	1.44463	.473	1.52849	.528	1.61436	.583	1.70105
.364	1.36543	.419	1.44613	.474	1.53004	.529	1.61592	.584	1.70264
.365	1.36688	.42	1.44763	.475	1.53159	.53	1.61748	.585	1.70424
.366	1.36833	.421	1.44913	.476	1.53314	.531	1.61904	.586	1.70584
.367	1.36978	.422	1.45064	.477	1.53469	.532	1.6206	.587	1.70745
.368	1.37123	.423	1.45214	.478	1.53625	.533	1.62216	.588	1.70905
.369	1.37268	.424	1.45364	.479	1.53781	.534	1.62372	.589	1.71065
.37	1.37414	.425	1.45515	.48	1.53937	.535	1.62528	.59	1.71225
.371	1.37662	.426	1.45665	.481	1.54093	.536	1.62684	.591	1.71286
.372	1.37708	.427	1.45815	.482	1.54249	.537	1.6284	.592	1.71546
.373	1.37854	.428	1.45966	.483	1.54405	.538	1.62996	.593	1.71707
.374	1.38	.429	1.46167	.484	1.54561	.539	1.63152	.594	1.71868
.375	1.38146	.43	1.46268	.485	1.54718	.54	1.63309	.595	1.72029
.376	1.38292	.431	1.46419	.486	1.54875	.541	1.63465	.596	1.7219
.377	1.38439	.432	1.4657	.487	1.55032	.542	1.63623	.597	1.7235
.378	1.38585	.433	1.46721	.488	1.55189	.543	1.6378	.598	1.72511
.379	1.38732	.434	1.46872	.489	1.55346	.544	1.63937	.599	1.72672
.38	1.38879	.435	1.47023	.49	1.55503	.545	1.64094	.6	1.72833
.381	1.39024	.436	1.47174	.491	1.5566	.546	1.64251	.601	1.72994
.382	1.39169	.437	1.47326	.492	1.55817	.547	1.64408	.602	1.73155
.383	1.39314	.438	1.47478	.493	1.55974	.548	1.64565	.603	1.73316
.384	1.39459	.439	1.4763	.494	1.56131	.549	1.64722	.604	1.73477
.385	1.39605	.44	1.47782	.495	1.56289	.55	1.64879	.605	1.73638
.386	1.39751	.441	1.47934	.496	1.56447	.551	1.65036	.606	1.73799
.387	1.39897	.442	1.48086	.497	1.56605	.552	1.65193	.607	1.7396
.388	1.40043	.443	1.48238	.498	1.56763	.553	1.6535	.608	1.74121
.389	1.40189	.444	1.48391	.499	1.56921	.554	1.65507	.609	1.74283
.39	1.40335	.445	1.48544	.5	1.57089	.555	1.65665	.61	1.74444
.391	1.40481	.446	1.48697	.501	1.57234	.556	1.65823	.611	1.74605
.392	1.40627	.447	1.4885	.502	1.57389	.557	1.65981	.612	1.74767
.393	1.40773	.448	1.49003	.503	1.57544	.558	1.66139	.613	1.74929
.394	1.40919	.449	1.49157	.504	1.57699	.559	1.66297	.614	1.75091
.395	1.41065	.45	1.49311	.505	1.57854	.56	1.66455	.615	1.75252
.396	1.41211	.451	1.49465	.506	1.58009	.561	1.66613	.616	1.75414
.397	1.41357	.452	1.49618	.507	1.58164	.562	1.66771	.617	1.75576
.398	1.41504	.453	1.49771	.508	1.58319	.563	1.66929	.618	1.75738
.399	1.41651	.454	1.49924	.509	1.58474	.564	1.67087	.619	1.759
.4	1.41798	.455	1.50077	.51	1.58629	.565	1.67245	.620	1.76062
.401	1.41945	.456	1.5023	.511	1.58784	.566	1.67403	.621	1.76224
.402	1.42092	.457	1.50383	.512	1.5894	.567	1.67561	.622	1.76386
.403	1.42239	.458	1.50536	.513	1.59096	.568	1.67719	.623	1.76548
.404	1.42386	.459	1.50689	.514	1.59252	.569	1.67877	.624	1.7671

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.625	1.768 72	.68	1.858 74	.735	1.950 59	.79	2.044 62	.845	2.141 55
.626	1.770 34	.681	1.860 39	.736	1.952 28	.791	2.046 35	.846	2.143 34
.627	1.771 97	.682	1.862 05	.737	1.953 97	.792	2.048 09	.847	2.145 13
.628	1.773 59	.683	1.863 7	.738	1.955 66	.793	2.049 83	.848	2.146 92
.629	1.775 21	.684	1.865 35	.739	1.957 35	.794	2.051 57	.849	2.148 71
.63	1.776 84	.685	1.867	.74	1.959 94	.795	2.053 31	.85	2.150 5
.631	1.778 47	.686	1.868 66	.741	1.960 74	.796	2.055 05	.851	2.152 29
.632	1.780 09	.687	1.870 31	.742	1.962 44	.797	2.056 79	.852	2.154 09
.633	1.781 72	.688	1.871 96	.743	1.964 14	.798	2.058 53	.853	2.155 89
.634	1.783 35	.689	1.873 62	.744	1.965 83	.799	2.060 27	.854	2.157 7
.635	1.784 98	.69	1.875 27	.745	1.967 53	.8	2.062 02	.855	2.159 5
.636	1.786 6	.691	1.876 93	.746	1.969 23	.801	2.063 77	.856	2.161 3
.637	1.788 23	.692	1.878 59	.747	1.970 93	.802	2.065 52	.857	2.163 09
.638	1.789 86	.693	1.880 24	.748	1.972 62	.803	2.067 27	.858	2.164 89
.639	1.791 49	.694	1.881 9	.749	1.974 32	.804	2.069 01	.859	2.166 68
.64	1.793 12	.695	1.883 56	.75	1.976 02	.805	2.070 76	.86	2.168 48
.641	1.794 75	.696	1.885 22	.751	1.977 72	.806	2.072 51	.861	2.170 28
.642	1.700 38	.697	1.886 88	.752	1.979 43	.807	2.074 27	.862	2.172 09
.643	1.798 01	.698	1.888 54	.753	1.981 13	.808	2.076 02	.863	2.173 89
.644	1.799 64	.699	1.890 2	.754	1.982 83	.809	2.077 77	.864	2.175 7
.645	1.801 27	.7	1.891 86	.755	1.984 53	.81	2.079 53	.865	2.177 51
.646	1.802 9	.701	1.893 52	.756	1.986 23	.811	2.081 28	.866	2.179 32
.647	1.804 54	.702	1.895 19	.757	1.987 94	.812	2.083 04	.867	2.181 13
.648	1.806 17	.703	1.896 85	.758	1.989 64	.813	2.084 8	.868	2.182 94
.649	1.807 8	.704	1.898 51	.759	1.991 34	.814	2.086 56	.869	2.184 75
.65	1.809 43	.705	1.900 17	.76	1.993 05	.815	2.088 32	.87	2.186 56
.651	1.811 07	.706	1.901 84	.761	1.994 76	.816	2.090 08	.871	2.188 37
.652	1.812 71	.707	1.903 5	.762	1.996 47	.817	2.091 98	.872	2.190 18
.653	1.814 35	.708	1.905 17	.763	1.998 18	.818	2.093 6	.873	2.192
.654	1.815 99	.709	1.906 84	.764	1.999 89	.819	2.095 36	.874	2.193 82
.655	1.817 63	.71	1.908 52	.765	2.001 6	.82	2.097 12	.875	2.195 64
.656	1.819 28	.711	1.910 19	.766	2.003 31	.821	2.098 88	.876	2.197 46
.657	1.820 91	.712	1.911 87	.767	2.005 02	.822	2.100 65	.877	2.199 28
.658	1.822 55	.713	1.913 55	.768	2.006 73	.823	2.102 42	.878	2.201 1
.659	1.824 19	.714	1.915 23	.769	2.008 44	.824	2.104 19	.879	2.202 92
.66	1.825 83	.715	1.916 91	.77	2.010 16	.825	2.105 96	.88	2.204 74
.661	1.827 47	.716	1.918 59	.771	2.011 87	.826	2.107 73	.881	2.206 56
.662	1.829 11	.717	1.920 27	.772	2.013 59	.827	2.109 5	.882	2.208 39
.663	1.830 75	.718	1.921 95	.773	2.015 31	.828	2.111 27	.883	2.210 22
.664	1.832 4	.719	1.923 63	.774	2.017 02	.829	2.113 04	.884	2.212 05
.665	1.834 04	.72	1.925 31	.775	2.018 74	.83	2.114 81	.885	2.213 88
.666	1.835 68	.721	1.927	.776	2.020 45	.831	2.116 59	.886	2.215 71
.667	1.837 33	.722	1.928 68	.777	2.022 17	.832	2.118 37	.887	2.217 54
.668	1.838 97	.723	1.930 36	.778	2.023 89	.833	2.120 15	.888	2.219 37
.669	1.840 61	.724	1.932 04	.779	2.025 61	.834	2.121 93	.889	2.221 2
.67	1.842 26	.725	1.933 73	.78	2.027 33	.835	2.123 71	.89	2.223 03
.671	1.843 91	.726	1.935 41	.781	2.029 07	.836	2.125 49	.891	2.224 86
.672	1.845 56	.727	1.937 1	.782	2.030 8	.837	2.127 27	.892	2.226 7
.673	1.847 2	.728	1.938 78	.783	2.032 52	.838	2.129 05	.893	2.228 54
.674	1.848 85	.729	1.940 46	.784	2.034 25	.839	2.130 83	.894	2.230 38
.675	1.850 5	.73	1.942 15	.785	2.035 98	.84	2.132 61	.895	2.232 22
.676	1.852 15	.731	1.943 83	.786	2.037 71	.841	2.134 39	.896	2.234 06
.677	1.853 79	.732	1.945 52	.787	2.039 44	.842	2.136 18	.897	2.235 9
.678	1.855 44	.733	1.947 21	.788	2.041 17	.843	2.137 97	.898	2.237 74
.679	1.857 09	.734	1.948 9	.789	2.042 9	.844	2.139 76	.899	2.239 58

H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.	H'ght.	Length.
.9	2.241 42	.92	2.278 03	.94	2.314 79	.96	2.352 41	.98	2.390 55
.901	2.243 25	.921	2.279 87	.941	2.316 66	.961	2.354 31	.981	2.392 47
.902	2.245 08	.922	2.281 7	.942	2.318 52	.962	2.356 21	.982	2.394 39
.903	2.246 91	.923	2.283 54	.943	2.320 38	.963	2.358 1	.983	2.396 31
.904	2.248 74	.924	2.285 37	.944	2.322 24	.964	2.36	.984	2.398 23
.905	2.250 57	.925	2.287 2	.945	2.324 11	.965	2.361 91	.985	2.400 16
.906	2.2524	.926	2.289 03	.946	2.325 98	.966	2.363 81	.986	2.402 08
.907	2.254 23	.927	2.290 86	.947	2.327 85	.967	2.365 71	.987	2.404
.908	2.256 06	.928	2.292 7	.948	2.329 72	.968	2.367 62	.988	2.405 92
.909	2.257 89	.929	2.294 53	.949	2.331 6	.969	2.369 52	.989	2.407 84
								.99	2.409 76
.91	2.259 72	.93	2.296 36	.95	2.333 48	.97	2.371 43	.991	2.411 69
.911	2.261 55	.931	2.298 2	.951	2.335 37	.971	2.373 34	.992	2.413 62
.912	2.263 38	.932	2.300 04	.952	2.337 26	.972	2.375 25	.993	2.415 56
.913	2.265 21	.933	2.301 88	.953	2.339 15	.973	2.377 16	.994	2.417 49
.914	2.267 04	.934	2.303 73	.954	2.341 04	.974	2.379 08	.995	2.419 43
.915	2.268 88	.935	2.305 57	.955	2.342 93	.975	2.381	.996	2.421 36
.916	2.270 71	.936	2.307 41	.956	2.344 83	.976	2.382 91	.997	2.423 29
.917	2.272 54	.937	2.309 26	.957	2.346 73	.977	2.384 82	.998	2.425 22
.918	2.274 37	.938	2.311 11	.958	2.348 62	.978	2.386 73	.999	2.427 15
.919	2.2762	.939	2.312 95	.959	2.350 51	.979	2.388 64	1.	2.429 08

To Ascertain Length of an Elliptic Arc (right Semi-Ellipse) by preceding Table.

Rule.—Divide height by base, find quotient in column of heights, and take length for that height from next right-hand column. Multiply length thus obtained by base of arc, and product will give length.

Example.—What is length of arc of a semi-ellipse, base being 70 feet, and height 30.10 feet?

$30.10 \div 70 = .43$; and 43, *per table*, $= 1.462\,68$.

Then $1.462\,68 \times 70 = 102.3876$ *feet.*

When Curve is not that of a right Semi-Ellipse, Height being half of Transverse Diameter.

Rule.—Divide half base by twice height, then proceed as in preceding example; multiply tabular length by twice height, and product will give length.

Example.—What is length of arc of a semi-ellipse, height being 35 feet, and base 60 feet?

$60 \div 2 = 30$, and $30 \div \overline{35 \times 2} = .428$, *tabular length of which* $= 1.459\,66$.

Then $1.459\,66 \times 35 \times 2 = 102.1762$ *feet.*

When, in Division of a Height by Base, Quotient has a Remainder after third Place of Decimals, and great Accuracy is required,

Rule.—Take length for first three figures, subtract it from next following length; multiply remainder by this fractional remainder, add product to first length, and sum will give length for whole quotient.

Example.—What is length of an arc of a semi-ellipse, base being 171.3 feet and height 125 feet?

$171.3 \div 2 = 85.65$, and $125 \times 2 = 250$. $171.3 \div 250 = .3426$; *tabular length for* $.342 = 1.334\,12$, and for $.343 = 1.335\,52$, *the difference between which* is .0014.

Then $.6 \times .0014 = .0084$.

$$\begin{aligned} \text{Hence, } .342 &= 1.334\,12 \\ .0006 &= \underline{.008\,4} \\ &\ 1.342\,52, \end{aligned}$$

the sum by which base of arc is to be multiplied; and $1.342\,52 \times 171.3 = 229.973\,676$ *feet.*

Areas of Segments of a Circle.

The Diameter of a Circle = 1, and divided into 1000 equal Parts.

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
.001	.00004	.052	.01556	.103	.04269	.154	.07675	.205	.11584
.002	.00012	.053	.01601	.104	.0431	.155	.07747	.206	.11665
.003	.00022	.054	.01646	.105	.04391	.156	.0782	.207	.11746
.004	.00034	.055	.01691	.106	.04452	.157	.07892	.208	.11827
.005	.00047	.056	.01737	.107	.04514	.158	.07965	.209	.11908
.006	.00062	.057	.01783	.108	.04575	.159	.08038	.21	.1199
.007	.00078	.058	.0183	.109	.04638	.16	.08111	.211	.12071
.008	.00095	.059	.01877	.11	.047	.161	.08185	.212	.12153
.009	.00113	.06	.01924	.111	.04763	.162	.08258	.213	.12235
.01	.00133	.061	.01972	.112	.04826	.163	.08332	.214	.12317
.011	.00153	.062	.0202	.113	.04889	.164	.08406	.215	.12399
.012	.00175	.063	.02068	.114	.04953	.165	.0848	.216	.12481
.013	.00197	.064	.02117	.115	.05016	.166	.08554	.217	.12563
.014	.0022	.065	.02165	.116	.0508	.167	.08629	.218	.12646
.015	.00244	.066	.02215	.117	.05145	.168	.08704	.219	.12728
.016	.00268	.067	.02265	.118	.05209	.169	.08779	.22	.12811
.017	.00294	.068	.02315	.119	.05274	.17	.08853	.221	.12894
.018	.0032	.069	.02336	.12	.05338	.171	.08929	.222	.12977
.019	.00347	.07	.02417	.121	.05404	.172	.09004	.223	.1306
.02	.00375	.071	.02468	.122	.05469	.173	.0908	.224	.13144
.021	.00403	.072	.02519	.123	.05534	.174	.09155	.225	.13227
.022	.00432	.073	.02571	.124	.056	.175	.09231	.226	.13311
.023	.00462	.074	.02624	.125	.05666	.176	.09307	.227	.13394
.024	.00492	.075	.02676	.126	.05733	.177	.09384	.228	.13478
.025	.00523	.076	.02729	.127	.05799	.178	.0946	.229	.13562
.026	.00555	.077	.02782	.128	.05866	.179	.09537	.23	.13646
.027	.00587	.078	.02835	.129	.05933	.18	.09613	.231	.13731
.028	.00619	.079	.02889	.13	.06	.181	.0969	.232	.13815
.029	.00653	.08	.02943	.131	.06067	.182	.09767	.233	.139
.03	.00686	.081	.02997	.132	.06135	.183	.09845	.234	.13984
.031	.00721	.082	.03052	.133	.06203	.184	.09922	.235	.14069
.032	.00756	.083	.03107	.134	.06271	.185	.1	.236	.14154
.033	.00791	.084	.03162	.135	.06339	.186	.10077	.237	.14239
.034	.00827	.085	.03218	.136	.06407	.187	.10155	.238	.14324
.035	.00864	.086	.03274	.137	.06476	.188	.10233	.239	.14409
.036	.00901	.087	.0333	.138	.06545	.189	.10312	.24	.14494
.037	.00938	.088	.03387	.139	.06614	.19	.1039	.241	.1458
.038	.00976	.089	.03444	.14	.06683	.191	.10468	.242	.14665
.039	.01015	.09	.03501	.141	.06753	.192	.10547	.243	.14751
.04	.01054	.091	.03558	.142	.06822	.193	.10626	.244	.14837
.041	.01093	.092	.03616	.143	.06892	.194	.10705	.245	.14923
.042	.01133	.093	.03674	.144	.06962	.195	.10784	.246	.15009
.043	.01173	.094	.03732	.145	.07033	.196	.10864	.247	.15095
.044	.01214	.095	.0379	.146	.07103	.197	.10943	.248	.15182
.045	.01255	.096	.03849	.147	.07174	.198	.11023	.249	.15268
.046	.01297	.097	.03908	.148	.07245	.199	.11102	.25	.15355
.047	.01339	.098	.03968	.149	.07316	.2	.11182	.251	.15441
.048	.01382	.099	.04027	.15	.07387	.201	.11262	.252	.15528
.049	.01425	.1	.04087	.151	.07459	.202	.11343	.253	.15615
.05	.01468	.101	.04148	.152	.07531	.203	.11423	.254	.15702
.051	.01512	.102	.04208	.153	.07603	.204	.11503	.255	.15789

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
.256	.158 76	.305	.202 76	.354	.248 8	.403	.296 31	.452	.344 77
.257	.159 64	.306	.203 68	.355	.249 76	.404	.297 29	.453	.345 77
.258	.160 51	.307	.204 6	.356	.250 71	.405	.298 27	.454	.346 76
.259	.161 39	.308	.205 53	.357	.251 67	.406	.299 25	.455	.347 76
.26	.162 26	.309	.206 45	.358	.252 63	.407	.300 24	.456	.348 75
.261	.163 14	.31	.207 38	.359	.253 59	.408	.301 22	.457	.349 75
.262	.164 02	.311	.208 3	.36	.254 55	.409	.302 2	.458	.350 75
.263	.164 9	.312	.209 23	.361	.255 51	.41	.303 19	.459	.351 74
.264	.165 78	.313	.210 15	.362	.256 47	.411	.304 17	.46	.352 74
.265	.166 66	.314	.211 08	.363	.257 43	.412	.305 15	.461	.353 74
.266	.167 55	.315	.212 01	.364	.258 39	.413	.306 14	.462	.354 74
.267	.168 44	.316	.212 94	.365	.259 36	.414	.307 12	.463	.355 73
.268	.169 31	.317	.213 87	.366	.260 32	.415	.308 11	.464	.356 73
.269	.170 2	.318	.214 8	.367	.261 28	.416	.309 09	.465	.357 73
.27	.171 09	.319	.215 73	.368	.262 25	.417	.310 08	.466	.358 72
.271	.171 97	.32	.216 67	.369	.263 21	.418	.311 07	.467	.359 72
.272	.172 87	.321	.217 6	.37	.264 18	.419	.312 05	.468	.360 72
.273	.173 76	.322	.218 53	.371	.265 14	.42	.313 04	.469	.361 72
.274	.174 65	.323	.219 47	.372	.266 11	.421	.314 03	.47	.362 72
.275	.175 54	.324	.220 4	.373	.267 08	.422	.315 02	.471	.363 71
.276	.176 43	.325	.221 34	.374	.268 04	.423	.316	.472	.364 71
.277	.177 33	.326	.222 28	.375	.269 01	.424	.316 99	.473	.365 71
.278	.178 22	.327	.223 21	.376	.269 98	.425	.317 98	.474	.366 71
.279	.179 12	.328	.224 15	.377	.270 95	.426	.318 97	.475	.367 71
.28	.180 02	.329	.225 09	.378	.271 92	.427	.319 96	.476	.368 71
.281	.180 92	.33	.226 03	.379	.272 89	.428	.320 95	.477	.369 71
.282	.181 82	.331	.226 97	.38	.273 86	.429	.321 94	.478	.370 71
.283	.182 72	.332	.227 91	.381	.274 83	.43	.322 93	.479	.371 7
.284	.183 61	.333	.228 86	.382	.275 80	.431	.323 91	.48	.372 7
.285	.184 52	.334	.229 8	.383	.276 77	.432	.324 9	.481	.373 7
.286	.185 42	.335	.230 74	.384	.277 75	.433	.325 9	.482	.374 7
.287	.186 33	.336	.231 69	.385	.278 72	.434	.326 89	.483	.375 7
.288	.187 23	.337	.232 63	.386	.279 69	.435	.327 88	.484	.376 7
.289	.188 14	.338	.233 58	.387	.280 67	.436	.328 87	.485	.377 7
.29	.189 05	.339	.234 53	.388	.281 64	.437	.329 87	.486	.378 7
.291	.189 95	.34	.235 47	.389	.282 62	.438	.330 86	.487	.379 7
.292	.190 86	.341	.236 42	.39	.283 59	.439	.331 85	.488	.380 7
.293	.191 77	.342	.237 37	.391	.284 57	.44	.332 84	.489	.381 7
.294	.192 68	.343	.238 32	.392	.285 54	.441	.333 84	.49	.382 7
.295	.193 6	.344	.239 27	.393	.286 52	.442	.334 83	.491	.383 7
.296	.194 51	.345	.240 22	.394	.287 5	.443	.335 82	.492	.384 7
.297	.195 42	.346	.241 17	.395	.288 48	.444	.336 82	.493	.385 7
.298	.196 34	.347	.242 12	.396	.289 45	.445	.337 81	.494	.386 7
.299	.197 25	.348	.243 07	.397	.290 43	.446	.338 8	.495	.387 7
.3	.198 17	.349	.244 03	.398	.291 41	.447	.339 8	.496	.388 7
.301	.199 08	.35	.244 98	.399	.292 39	.448	.340 79	.497	.389 7
.302	.2	.351	.245 93	.4	.293 37	.449	.341 79	.498	.390 7
.303	.200 92	.352	.246 89	.401	.294 35	.45	.342 78	.499	.391 7
.304	.201 84	.353	.247 84	.402	.295 33	.451	.343 78	.5	.392 7

To Compute Area of a Segment of a Circle by preceding Table.

Rule.—Divide height or versed sine by diameter of circle; find quotient in column of versed sines. Take area for versed sine opposite to it in next column on right hand, multiply it by square of diameter, and it will give area.

EXAMPLE.—Required area of a segment of a circle, its height being 10 feet and diameter of circle 50.

$10 \div 50 = .2$, and .2, *per table*, $= .111\ 82$; then $.111\ 82 \times 50^2 = 279.55$ *feet.*

When, in Division of a Height by Base, Quotient has a Remainder after Third Place of Decimals, and great Accuracy is required.

RULE.—Take area for first three figures, subtract it from next following area, multiply remainder by said fraction, add product to first area, and sum will give area for whole quotient.

EXAMPLE.—What is area of a segment of a circle, diameter of which is 10 feet, and height of it 1.575?

$1.575 \div 10 = .1575$; *tabular area* for $.157 = .078\ 92$, and for $.158 = .079\ 65$, *the difference between which is* .000 73.

Then $.5 \times .000\ 73 = .000\ 365$.

Hence, $.157 = .078\ 92$
$.0005 = .000\ 365$

.079 285, *sum by which square of diameter of circle is to be multiplied*; and $.079\ 285 \times 10^2 = 7.9285$ *feet.*

Areas of Zones of a Circle.

The Diameter of a Circle = 1, and divided into 1000 equal Parts.

H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.
.001	.001	.035	.034 97	.069	.068 78	.103	.102 27	.137	.135 27
.002	.002	.036	.035 97	.07	.069 77	.104	.103 25	.138	.136 23
.003	.003	.037	.036 97	.071	.070 76	.105	.104 22	.139	.137 19
.004	.004	.038	.037 96	.072	.071 75	.106	.105 2	.14	.138 15
.005	.005	.039	.038 96	.073	.072 74	.107	.106 18	.141	.139 11
.006	.006	.04	.039 96	.074	.073 73	.108	.107 15	.142	.140 07
.007	.007	.041	.040 95	.075	.074 72	.109	.108 13	.143	.141 03
.008	.008	.042	.041 95	.076	.075 5	.11	.109 11	.144	.141 98
.009	.009	.043	.042 95	.077	.076 69	.111	.110 08	.145	.142 94
.01	.01	.044	.043 94	.078	.077 68	.112	.111 06	.146	.143 9
.011	.011	.045	.044 94	.079	.078 67	.113	.112 03	.147	.144 85
.012	.012	.046	.045 93	.08	.079 66	.114	.113	.148	.145 81
.013	.013	.047	.046 93	.081	.080 64	.115	.113 98	.149	.146 77
.014	.014	.048	.047 93	.082	.081 63	.116	.114 95	.15	.147 72
.015	.015	.049	.048 92	.083	.082 62	.117	.115 92	.151	.148 67
.016	.016	.05	.049 92	.084	.083 6	.118	.116 9	.152	.149 62
.017	.017	.051	.050 91	.085	.084 59	.119	.117 87	.153	.150 58
.018	.018	.052	.051 9	.086	.085 57	.12	.118 84	.154	.151 53
.019	.019	.053	.052 9	.087	.086 56	.121	.119 81	.155	.152 48
.02	.02	.054	.053 89	.088	.087 54	.122	.120 78	.156	.153 43
.021	.021	.055	.054 89	.089	.088 53	.123	.121 75	.157	.154 38
.022	.022	.056	.055 88	.09	.089 51	.124	.122 72	.158	.155 33
.023	.023	.057	.056 88	.091	.090 5	.125	.123 69	.159	.156 28
.024	.024	.058	.057 87	.092	.091 48	.126	.124 69	.16	.157 23
.025	.025	.059	.058 86	.093	.092 46	.127	.125 62	.161	.158 17
.026	.025 99	.06	.059 86	.094	.093 44	.128	.126 59	.162	.159 12
.027	.026 99	.061	.060 85	.095	.094 43	.129	.127 55	.163	.160 06
.028	.027 99	.062	.061 84	.096	.095 4	.13	.128 52	.164	.161 01
.029	.028 98	.063	.062 83	.097	.096 39	.131	.129 49	.165	.161 95
.03	.029 98	.064	.063 82	.098	.097 37	.132	.130 45	.166	.162 9
.031	.030 98	.065	.064 82	.099	.098 35	.133	.131 41	.167	.163 84
.032	.031 98	.066	.065 8	.1	.099 33	.134	.132 38	.168	.164 78
.033	.032 98	.067	.066 8	.101	.100 31	.135	.133 34	.169	.165 72
.034	.033 97	.068	.067 8	.102	.101 29	.136	.134 3	.17	.166 67

H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.
.171	.16761	.226	.21805	.281	.26541	.336	.30864	.391	.34632
.172	.16855	.227	.21894	.282	.26624	.337	.30938	.392	.34694
.173	.16948	.228	.21983	.283	.26706	.338	.31012	.393	.34756
.174	.17042	.229	.22072	.284	.26789	.339	.31085	.394	.34818
.175	.17136	.23	.22161	.285	.26871	.34	.31159	.395	.34879
.176	.1723	.231	.2225	.286	.26953	.341	.31232	.396	.3494
.177	.17323	.232	.22335	.287	.27035	.342	.31305	.397	.35001
.178	.17417	.233	.22427	.288	.27117	.343	.31378	.398	.35062
.179	.1751	.234	.22515	.289	.27199	.344	.3145	.399	.35122
.18	.17603	.235	.22604	.29	.2728	.345	.31523	.4	.35182
.181	.17697	.236	.22692	.291	.27362	.346	.31595	.401	.35242
.182	.1779	.237	.2278	.292	.27443	.347	.31667	.402	.35302
.183	.17883	.238	.22868	.293	.27524	.348	.31739	.403	.35361
.184	.17976	.239	.22956	.294	.27605	.349	.31811	.404	.3542
.185	.18069	.24	.23044	.295	.27686	.35	.31882	.405	.35479
.186	.18162	.241	.23131	.296	.27766	.351	.31954	.406	.35538
.187	.18254	.242	.23219	.297	.27847	.352	.32025	.407	.35596
.188	.18347	.243	.23306	.298	.27927	.353	.32096	.408	.35654
.189	.1844	.244	.23394	.299	.28007	.354	.32167	.409	.35711
.19	.18532	.245	.23481	.3	.28088	.355	.32237	.41	.35769
.191	.18625	.246	.23568	.301	.28167	.356	.32307	.411	.35826
.192	.18717	.247	.23655	.302	.28247	.357	.32377	.412	.35883
.193	.18809	.248	.23742	.303	.28327	.358	.32447	.413	.35939
.194	.18902	.249	.23829	.304	.28406	.359	.32517	.414	.35995
.195	.18994	.25	.23915	.305	.28486	.36	.32587	.415	.36051
.196	.19086	.251	.24002	.306	.28565	.361	.32656	.416	.36107
.197	.19178	.252	.24089	.307	.28644	.362	.32725	.417	.36162
.198	.1927	.253	.24175	.308	.28723	.363	.32794	.418	.36217
.199	.19361	.254	.24261	.309	.28801	.364	.32862	.419	.36272
.2	.19453	.255	.24347	.31	.2888	.365	.32931	.42	.36326
.201	.19545	.256	.24433	.311	.28958	.366	.32999	.421	.3638
.202	.19636	.257	.24519	.312	.29036	.367	.33067	.422	.36434
.203	.19728	.258	.24604	.313	.29115	.368	.33135	.423	.36488
.204	.19819	.259	.2469	.314	.29192	.369	.33203	.424	.36541
.205	.1991	.26	.24775	.315	.2927	.37	.3327	.425	.36594
.206	.20001	.261	.24861	.316	.29348	.371	.33337	.426	.36646
.207	.20092	.262	.24946	.317	.29425	.372	.33404	.427	.36698
.208	.20183	.263	.25021	.318	.29502	.373	.33471	.428	.3675
.209	.20274	.264	.25116	.319	.2958	.374	.33537	.429	.36802
.21	.20365	.265	.25201	.32	.29656	.375	.33604	.43	.36853
.211	.20456	.266	.25285	.321	.29733	.376	.3367	.431	.36904
.212	.20546	.267	.2537	.322	.2981	.377	.33735	.432	.36954
.213	.20637	.268	.25455	.323	.29886	.378	.33801	.433	.37005
.214	.20727	.269	.25539	.324	.29962	.379	.33866	.434	.37054
.215	.20818	.27	.25623	.325	.30039	.38	.33931	.435	.37104
.216	.20908	.271	.25707	.326	.30114	.381	.33996	.436	.37153
.217	.20998	.272	.25791	.327	.3019	.382	.34061	.437	.37202
.218	.21088	.273	.25875	.328	.30266	.383	.34125	.438	.3725
.219	.21178	.274	.25959	.329	.30341	.384	.3419	.439	.37298
.22	.21268	.275	.26043	.33	.30416	.385	.34253	.44	.37346
.221	.21358	.276	.26126	.331	.30491	.386	.34317	.441	.37393
.222	.21447	.277	.26209	.332	.30566	.387	.3438	.442	.3744
.223	.21537	.278	.26293	.333	.30641	.388	.34444	.443	.37487
.224	.21626	.279	.26376	.334	.30715	.389	.34507	.444	.37533
.225	.21716	.28	.26459	.335	.3079	.39	.34569	.445	.37579

H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.	H'ght.	Area.
.446	.376 24	.457	.380 96	.468	.385 14	.479	.388 67	.49	.391 37
.447	.376 69	.458	.381 37	.469	.385 49	.48	.388 95	.491	.391 56
.448	.377 14	.459	.381 77	.47	.385 83	.481	.389 23	.492	.391 75
.449	.377 58	.46	.382 16	.471	.386 17	.482	.389 5	.493	.391 92
.45	.378 02	.461	.382 55	.472	.386 5	.483	.389 76	.494	.392 08
.451	.378 45	.462	.382 94	.473	.386 83	.484	.390 01	.495	.392 23
.452	.378 88	.463	.383 32	.474	.387 15	.485	.390 26	.496	.392 36
.453	.379 31	.464	.383 69	.475	.387 47	.486	.390 5	.497	.392 48
.454	.379 73	.465	.384 06	.476	.387 78	.487	.390 73	.498	.392 58
.455	.380 14	.466	.384 43	.477	.388 08	.488	.390 95	.499	.392 66
.456	.380 56	.467	.384 79	.478	.388 38	.489	.391 17	.5	.392 7

This Table is computed only for Zones, longest Chord of which is Diameter.

To Compute Area of a Zone by preceding Table.

When Zone is Less than a Semicircle.

RULE.—Divide height by diameter, find quotient in column of heights. Take area for height opposite to it in next column on right hand, multiply it by square of longest chord, and product will give area of zone.

EXAMPLE.—Required area of a Zone, diameter of which is 50, and its height 15.

$15 \div 50 = .3$; and .3, *as per table*, $= .280\,88$.

Hence $.280\,88 \times 50^2 = 702.2$ *area.*

When Zone is Greater than a Semicircle.

RULE.—Take height on each side of diameter of circle, and ascertain, by preceding Rule, their respective areas; add areas of these two portions together, and sum will give area.

EXAMPLE.—Required area of a zone, diameter of circle being 50, and heights of zone on each side of diameter of circle 20 and 15.

$20 \div 50 = .4$; .4, *as per table*, $= .351\,82$; and $.351\,82 \times 50^2 = 879.55$.
$15 \div 50 = .3$; .3, *as per table*, $= .280\,88$; and $.280\,88 \times 50^2 = 702.2$.

Hence $879.55 + 702.2 = 1581.75$ *area.*

When, in Division of a Height by Chord, Quotient has a Remainder after Third Place of Decimals, and great Accuracy is required.

RULE.—Take area for first three figures, subtract it from the next following area, multiply remainder by said fraction, and add product to first area; sum will give area for whole quotient.

EXAMPLE.—What is area of a zone of a circle, greater chord being 100 feet, and breadth of it 14 feet 3 ins.?

14 feet 3 ins. $= 14.25$, and $14.25 \div 100 = .1425$; *tabular length for* $.142 = .140\,07$, *and for* $.143 = .141\,03$, *difference between which is* .000 96.

Then $.5 \times .000\,96 = .000\,48$. Hence $.142 = .140\,07$
$.0005 = .000\,48$

$.140\,55$, *sum by which square of greater chord is to be multiplied;* and $.140\,55 \times 100^2 = 1405.5$ *feet.*

Squares, Cubes, and Square and Cube Roots,

From 1 to 1600.

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1	1
2	4	8	1.414 213 6	1.259 921
3	9	27	1.732 050 8	1.442 249 6
4	16	64	2	1.587 401 1
5	25	125	2.236 068	1.709 975 9
6	36	216	2.449 489 7	1.817 120 6
7	49	343	2.645 751 3	1.912 931 2
8	64	512	2.828 427 1	2
9	81	729	3	2.080 083 7
10	1 00	1 000	3.162 277 7	2.154 434 7
11	1 21	1 331	3.316 624 8	2.223 980 1
12	1 44	1 728	3.464 101 6	2.289 428 6
13	1 69	2 197	3.605 551 3	2.351 334 7
14	1 96	2 744	3.741 657 4	2.410 142 2
15	2 25	3 375	3.872 983 3	2.466 212 1
16	2 56	4 096	4	2.519 842 1
17	2 89	4 913	4.123 105 6	2.571 281 6
18	3 24	5 832	4.242 640 7	2.620 741 4
19	3 61	6 859	4.358 598 9	2.668 401 6
20	4 00	8 000	4.472 136	2.714 417 7
21	4 41	9 261	4.582 575 7	2.758 924 3
22	4 84	10 648	4.690 415 8	2.802 039 3
23	5 29	12 167	4.795 831 5	2.843 867
24	5 76	13 824	4.898 979 5	2.884 499 1
25	6 25	15 625	5	2.924 017 7
26	6 76	17 576	5.099 019 5	2.962 496
27	7 29	19 683	5.196 152 4	3
28	7 84	21 952	5.291 502 6	3.036 588 9
29	8 41	24 389	5.385 164 8	3.072 316 8
30	9 00	27 000	5.477 225 6	3.107 232 5
31	9 61	29 791	5.567 764 4	3.141 380 6
32	10 24	32 768	5.656 854 2	3.174 802 1
33	10 89	35 937	5.744 562 6	3.207 534 3
34	11 56	39 304	5.830 951 9	3.239 611 8
35	12 25	42 875	5.916 079 8	3.271 066 3
36	12 96	46 656	6	3.301 927 2
37	13 69	50 653	6.082 762 5	3.332 221 8
38	14 44	54 872	6.164 414	3.361 975 4
39	15 21	59 319	6.244 998	3.391 211 4
40	16 00	64 000	6.324 555 3	3.419 951 9
41	16 81	68 921	6.403 124 2	3.448 217 2
42	17 64	74 088	6.480 740 7	3.476 026 6
43	18 49	79 507	6.557 438 5	3.503 398 1
44	19 36	85 184	6.633 249 6	3.530 348 3
45	20 25	91 125	6.708 203 9	3.556 893 3
46	21 16	97 336	6.782 33	3.583 047 9
47	22 09	103 823	6.855 654 6	3.608 826 1
48	23 04	110 592	6.928 203 2	3.634 241 1
49	24 01	117 649	7	3.659 305 7
50	25 00	125 000	7.071 067 8	3.684 031 4
51	26 01	132 651	7.141 428 4	3.708 429 8
52	27 04	140 608	7.211 102 6	3.732 511 1
53	28 09	148 877	7.280 109 9	3.756 285 8
54	29 16	157 464	7.348 469 2	3.779 763 1

Number.	Square.	Cube.	Square Root.	Cube Root.
55	30 25	166 375	7.416 198 5	3.802 952 5
56	31 36	175 616	7.483 314 8	3.825 862 4
57	32 49	185 193	7.549 834 4	3.848 501 1
58	33 64	195 112	7.615 773 1	3.870 876 6
59	34 81	205 379	7.681 145 7	3.892 996 5
60	36 00	216 000	7.745 966 7	3.914 867 6
61	37 21	226 981	7.810 249 7	3.936 497 2
62	38 44	238 328	7.874 007 9	3.957 891 5
63	39 69	250 047	7.937 253 9	3.979 057 1
64	40 96	262 144	8	4
65	42 25	274 625	8.062 257 7	4.020 725 6
66	43 56	287 496	8.124 038 4	4.041 240 1
67	44 89	300 763	8.185 352 8	4.061 548
68	46 24	314 432	8.246 211 3	4.081 655 1
69	47 61	328 509	8.306 623 9	4.101 566 1
70	49 00	343 000	8.366 600 3	4.121 285 3
71	50 41	357 911	8.426 149 8	4.140 817 8
72	51 84	373 248	8.485 281 4	4.160 167 6
73	53 29	389 017	8.544 003 7	4.179 339
74	54 76	405 224	8.602 325 3	4.198 336 4
75	56 25	421 875	8.660 254	4.217 163 3
76	57 76	438 976	8.717 797 9	4.235 823 6
77	59 29	456 533	8.774 964 4	4.254 321
78	60 84	474 552	8.831 760 9	4.272 658 6
79	62 41	493 039	8.888 194 4	4.290 840 4
80	64 00	512 000	8.944 271 9	4.308 869 5
81	65 61	531 441	9	4.326 748 7
82	67 24	551 368	9.055 385 1	4.344 481 5
83	68 89	571 787	9.110 433 6	4.362 070 7
84	70 56	592 704	9.165 151 4	4.379 519 1
85	72 25	614 125	9.219 544 5	4.396 829 6
86	73 96	636 056	9.273 618 5	4.414 004 9
87	75 69	658 503	9.327 379 1	4.431 047 6
88	77 44	681 472	9.380 831 5	4.447 960 2
89	79 21	704 969	9.433 981 1	4.464 745 1
90	81 00	729 000	9.486 833	4.481 404 7
91	82 81	753 571	9.539 392	4.497 941 4
92	84 64	778 688	9.591 663	4.514 357 4
93	86 49	804 357	9.643 650 8	4.530 654 9
94	88 36	830 584	9.695 359 7	4.546 835 9
95	90 25	857 375	9.746 794 3	4.562 902 6
96	92 16	884 736	9.797 959	4.578 857
97	94 09	912 673	9.848 857 8	4.594 700 9
98	96 04	941 192	9.899 494 9	4.610 436 3
99	98 01	970 299	9.949 874 4	4.626 065
100	1 00 00	1 000 000	10	4.641 588 8
101	1 02 01	1 030 301	10.049 875 6	4.657 009 5
102	1 04 04	1 061 208	10.099 504 9	4.672 328 7
103	1 06 09	1 092 727	10.148 891 6	4.687 548 2
104	1 08 16	1 124 864	10.198 039	4.702 669 4
105	1 10 25	1 157 625	10.246 950 8	4.717 694
106	1 12 36	1 191 016	10.295 630 1	4.732 623 5
107	1 14 49	1 225 043	10.344 080 4	4.747 459 4
108	1 16 64	1 259 712	10.392 304 8	4.762 203 2
109	1 18 81	1 295 029	10.440 306 5	4.776 856 2
110	1 21 00	1 331 000	10.488 088 5	4.791 419 9

Number.	Square.	Cube.	Square Root.	Cube Root.
111	1 23 21	1 367 631	10.535 653 8	4.805 895 5
112	1 25 44	1 404 928	10.583 005 2	4.820 284 5
113	1 27 69	1 442 897	10.630 145 8	4.834 588 1
114	1 29 96	1 481 544	10.677 078 3	4.848 807 6
115	1 32 25	1 520 875	10.723 805 3	4.862 944 2
116	1 34 56	1 560 896	10.770 329 6	4.876 999
117	1 36 89	1 601 613	10.816 653 8	4.890 973 2
118	1 39 24	1 643 032	10.862 780 5	4.904 868 1
119	1 41 61	1 685 159	10.908 712 1	4.918 684 7
120	1 44 00	1 728 000	10.954 451 2	4.932 424 2
121	1 46 41	1 771 561	11	4.946 087 4
122	1 48 84	1 815 848	11.045 361	4.959 675 7
123	1 51 29	1 860 867	11.090 536 5	4.973 189 8
124	1 53 76	1 906 624	11.135 528 7	4.986 631
125	1 56 25	1 953 125	11.180 339 9	5
126	1 58 76	2 000 376	11.224 972 2	5.013 297 9
127	1 61 29	2 048 383	11.269 427 7	5.026 525 7
128	1 63 84	2 097 152	11.313 708 5	5.039 684 2
129	1 66 41	2 146 689	11.357 816 7	5.052 774 3
130	1 69 00	2 197 000	11.401 754 3	5.065 797
131	1 71 61	2 248 091	11.445 523 1	5.078 753 1
132	1 74 24	2 299 968	11.489 125 3	5.091 643 4
133	1 76 89	2 352 637	11.532 562 6	5.104 468 7
134	1 79 56	2 406 104	11.575 836 9	5.117 229 9
135	1 82 25	2 460 375	11.618 95	5.129 927 8
136	1 84 96	2 515 456	11.661 903 8	5.142 563 2
137	1 87 69	2 571 353	11.704 699 9	5.155 136 7
138	1 90 44	2 628 072	11.747 340 1	5.167 649 3
139	1 93 21	2 685 619	11.789 826 1	5.180 101 5
140	1 96 00	2 744 000	11.832 159 6	5.192 494 1
141	1 98 81	2 803 221	11.874 342 1	5.204 827 9
142	2 01 64	2 863 288	11.916 375 3	5.217 103 4
143	2 04 49	2 924 207	11.958 260 7	5.229 321 5
144	2 07 36	2 985 984	12	5.241 482 8
145	2 10 25	3 048 625	12.041 594 6	5.253 587 9
146	2 13 16	3 112 136	12.083 046	5.265 637 4
147	2 16 09	3 176 523	12.124 355 7	5.277 632 1
148	2 19 04	3 241 792	12.165 525 1	5.289 572 5
149	2 22 01	3 307 949	12.206 555 6	5.301 459 2
150	2 25 00	3 375 000	12.247 448 7	5.313 292 8
151	2 28 01	3 442 951	12.288 205 7	5.325 074
152	2 31 04	3 511 008	12.328 828	5.336 803 3
153	2 34 09	3 581 577	12.369 316 9	5.348 481 2
154	2 37 16	3 652 264	12.409 673 6	5.360 108 4
155	2 40 25	3 723 875	12.449 899 6	5.371 685 4
156	2 43 36	3 796 416	12.489 996	5.383 212 6
157	2 46 49	3 869 893	12.529 964 1	5.394 690 7
158	2 49 64	3 944 312	12.569 805 1	5.406 120 2
159	2 52 81	4 019 679	12.609 520 2	5.417 501 5
160	2 56 00	4 096 000	12.649 110 6	5.428 835 2
161	2 59 21	4 173 281	12.688 577 5	5.440 121 8
162	2 62 44	4 251 528	12.727 922 1	5.451 361 8
163	2 65 69	4 330 747	12.767 145 3	5.462 555 6
164	2 68 96	4 410 944	12.806 248 5	5.473 703 7
165	2 72 25	4 492 125	12.845 232 6	5.484 806 6
166	2 75 56	4 574 296	12.884 098 7	5.495 864 7

Number.	Square.	Cube.	Square Root.	Cube Root.
167	2 78 89	4 657 463	12.922 848	5.506 878 4
168	2 82 24	4 741 632	12.961 481 4	5.517 848 4
169	2 85 61	4 826 809	13	5.528 774 8
170	2 89 00	4 913 000	13.038 404 8	5.539 658 3
171	2 92 41	5 000 211	13.076 696 8	5.550 499 1
172	2 95 84	5 088 448	13.114 877	5.561 297 8
173	2 99 29	5 177 717	13.152 946 4	5.572 054 6
174	3 02 76	5 268 024	13.190 906	5.582 770 2
175	3 06 25	5 359 375	13.228 756 6	5.593 444 7
176	3 09 76	5 451 776	13.266 499 2	5.604 078 7
177	3 13 29	5 545 233	13.304 134 7	5.614 672 4
178	3 16 84	5 639 752	13.341 664 1	5.625 226 3
179	3 20 41	5 735 339	13.379 088 2	5.635 740 8
180	3 24 00	5 832 000	13.416 407 9	5.646 216 2
181	3 27 61	5 929 741	13.453 624	5.656 652 8
182	3 31 24	6 028 568	13.490 737 6	5.667 051 1
183	3 34 89	6 128 487	13.527 749 3	5.677 411 4
184	3 38 56	6 229 504	13.564 66	5.687 734
185	3 42 25	6 331 625	13.601 470 5	5.698 019 2
186	3 45 96	6 434 856	13.638 181 7	5.708 267 5
187	3 49 69	6 539 203	13.674 794 3	5.718 479 1
188	3 53 44	6 644 672	13.711 309 2	5.728 654 3
189	3 57 21	6 751 269	13.747 727 1	5.738 793 6
190	3 61 00	6 859 000	13.784 048 8	5.748 897 1
191	3 64 81	6 967 871	13.820 275	5.758 965 2
192	3 68 64	7 077 888	13.856 406 5	5.768 998 2
193	3 72 49	7 189 057	13.892 44	5.778 996 6
194	3 76 36	7 301 384	13.928 388 3	5.788 960 4
195	3 80 25	7 414 875	13.964 24	5.798 89
196	3 84 16	7 529 536	14	5.808 785 7
197	3 88 09	7 645 373	14.035 668 8	5.818 647 9
198	3 92 04	7 762 392	14.071 247 3	5.828 476 7
199	3 96 01	7 880 599	14.106 736	5.838 272 5
200	4 00 00	8 000 000	14.142 135 6	5.848 035 5
201	4 04 01	8 120 601	14.177 446 9	5.857 766
202	4 08 04	8 242 408	14.212 670 4	5.867 464 3
203	4 12 09	8 365 427	14.247 806 8	5.877 130 7
204	4 16 16	8 489 664	14.282 856 9	5.886 765 3
205	4 20 25	8 615 125	14.317 821 1	5.896 368 5
206	4 24 36	8 741 816	14.352 700 1	5.905 940 6
207	4 28 49	8 869 743	14.387 494 6	5.915 481 7
208	4 32 64	8 998 912	14.422 205 1	5.924 992 1
209	4 36 81	9 129 329	14.456 832 3	5.934 472 1
210	4 41 00	9 261 000	14.491 376 7	5.943 922
211	4 45 21	9 393 931	14.525 839	5.953 341 8
212	4 49 44	9 528 128	14.560 219 8	5.962 732
213	4 53 69	9 663 597	14.594 519 5	5.972 092 6
214	4 57 96	9 800 344	14.628 738 8	5.981 424
215	4 62 25	9 938 375	14.662 878 3	5.990 726 4
216	4 66 56	10 077 696	14.696 938 5	6
217	4 70 89	10 218 313	14.730 919 9	6.009 245
218	4 75 24	10 360 232	14.764 823 1	6.018 461 7
219	4 79 61	10 503 459	14.798 648 6	6.027 650 2
220	4 84 00	10 648 000	14.832 397	6.036 810 7
221	4 88 41	10 793 861	14.866 068 7	6.045 943 5
222	4 92 84	10 941 048	14.899 664 4	6.055 048 9

Number.	Square.	Cube.	Square Root.	Cube Root.
223	4 97 29	11 089 567	14.933 184 5	6.064 127
224	5 01 76	11 239 424	14.966 629 5	6.073 177 9
225	5 06 25	11 390 625	15	6.082 202
226	5 10 76	11 543 176	15.033 296 4	6.091 199 4
227	5 15 29	11 697 083	15.066 519 2	6.100 170 2
228	5 19 84	11 852 352	15.099 668 9	6.109 114 7
229	5 24 41	12 008 989	15.132 746	6.118 033 2
230	5 29 00	12 167 000	15.165 750 9	6.126 925 7
231	5 33 61	12 326 391	15.198 684 2	6.135 792 4
232	5 38 24	12 487 168	15.231 546 2	6.144 633 7
233	5 42 89	12 649 337	15.264 337 5	6.153 445 5
234	5 47 56	12 812 904	15.297 058 5	6.162 240 1
235	5 52 25	12 977 875	15.329 709 7	6.171 005 8
236	5 56 96	13 144 256	15.362 291 5	6.179 746 6
237	5 61 69	13 312 053	15.394 804 3	6.188 462 8
238	5 66 44	13 481 272	15.427 248 6	6.197 154 4
239	5 71 21	13 651 919	15.459 624 8	6.205 821 8
240	5 76 00	13 824 000	15.491 933 4	6.214 465
241	5 80 81	13 997 521	15.524 174 7	6.223 084 3
242	5 85 64	14 172 488	15.556 349 2	6.231 679 7
243	5 90 49	14 348 907	15.588 457 3	6.240 251 5
244	5 95 36	14 526 784	15.620 499 4	6.248 799 8
245	6 00 25	14 706 125	15.652 475 8	6.257 324 8
246	6 05 16	14 886 936	15.684 387 1	6.265 826 6
247	6 10 09	15 069 223	15.716 233 6	6.274 305 4
248	6 15 04	15 252 992	15.748 015 7	6.282 761 3
249	6 20 01	15 438 249	15.779 733 8	6.291 194 6
250	6 25 00	15 625 000	15.811 388 3	6.299 605 3
251	6 30 01	15 813 251	15.842 979 5	6.307 993 5
252	6 35 04	16 003 008	15.874 507 9	6.316 359 6
253	6 40 09	16 194 277	15.905 973 7	6.324 703 5
254	6 45 16	16 387 064	15.937 377 5	6.333 025 6
255	6 50 25	16 581 375	15.968 719 4	6.341 325 7
256	6 55 36	16 777 216	16	6.349 604 2
257	6 60 49	16 974 593	16.031 219 5	6.357 861 1
258	6 65 64	17 173 512	16.062 378 4	6.366 096 8
259	6 70 81	17 373 979	16.093 476 9	6.374 311 1
260	6 76 00	17 576 000	16.124 515 5	6.382 504 3
261	6 81 21	17 779 581	16.155 494 4	6.390 676 5
262	6 86 44	17 984 728	16.186 414 1	6.398 827 9
263	6 91 69	18 191 447	16.217 274 7	6.406 958 5
264	6 96 96	18 399 744	16.248 076 8	6.415 068 7
265	7 02 25	18 609 625	16.278 820 6	6.423 158 3
266	7 07 56	18 821 096	16.309 506 4	6.431 227 6
267	7 12 89	19 034 163	16.340 134 6	6.439 276 7
268	7 18 24	19 248 832	16.370 705 5	6.447 305 7
269	7 23 61	19 465 109	16.401 219 5	6.455 314 8
270	7 29 00	19 683 000	16.431 676 7	6.463 304 1
271	7 34 41	19 902 511	16.462 077 6	6.471 273 6
272	7 39 84	20 123 648	16.492 422 5	6.479 223 6
273	7 45 29	20 346 417	16.522 711 6	6.487 154 1
274	7 50 76	20 570 824	16.552 945 4	6.495 065 3
275	7 56 25	20 796 875	16.583 124	6.502 957 2
276	7 61 76	21 024 576	16.613 247 7	6.510 83
277	7 67 29	21 253 933	16.643 317	6.518 683 9
278	7 72 84	21 484 952	16.678 332	6.526 518 9

Number.	Square.	Cube.	Square Root.	Cube Root.
279	7 78 41	21 717 639	16.703 293 1	6.534 335 1
280	7 84 00	21 952 000	16.733 200 5	6.542 132 6
281	7 89 61	22 188 041	16.763 054 6	6.549 911 6
282	7 95 24	22 425 768	16.792 855 6	6.557 672 2
283	8 00 89	22 665 187	16.822 603 8	6.565 414 4
284	8 06 56	22 906 304	16.852 299 5	6.573 138 5
285	8 12 25	23 149 125	16.881 943	6.580 844 3
286	8 17 96	23 393 656	16.911 534 5	6.588 532 3
287	8 23 69	23 639 903	16.941 074 3	6.596 202 3
288	8 29 44	23 887 872	16.970 562 7	6.603 854 5
289	8 35 21	24 137 569	17	6.611 489
290	8 41 00	24 389 000	17.029 386 4	6.619 106
291	8 46 81	24 642 171	17.058 722 1	6.626 705 4
292	8 52 64	24 897 088	17.088 007 5	6.634 287 4
293	8 58 49	25 153 757	17.117 242 8	6.641 852 2
294	8 64 36	25 412 184	17.146 428 2	6.649 399 8
295	8 70 25	25 672 375	17.175 564	6 656 930 2
296	8 76 16	25 934 336	17.204 650 5	6.664 443 7
297	8 82 09	26 198 073	17.233 687 9	6.671 940 3
298	8 88 04	26 463 592	17.262 676 5	6.679 42
299	8 94 01	26 730 899	17.291 616 5	6.686 883 1
300	9 00 00	27 000 000	17.320 508 1	6.694 329 5
301	9 06 01	27 270 901	17.349 351 6	6.701 759 3
302	9 12 04	27 543 608	17.378 147 2	6.709 172 9
303	9 18 09	27 818 127	17.406 895 2	6.716 57
304	9 24 16	28 094 464	17.435 595 8	6.723 950 8
305	9 30 25	28 372 625	17.464 249 2	6.731 315 5
306	9 36 36	28 652 616	17.492 855 7	6.738 664 1
307	9 42 49	28 934 443	17.521 415 5	6.745 996 7
308	9 48 64	29 218 112	17.549 928 8	6.753 313 4
309	9 54 81	29 503 629	17.578 395 8	6.760 614 3
310	9 61 00	29 791 000	17.606 816 9	6.767 899 5
311	9 67 21	30 080 231	17.635 192 1	6.775 169
312	9 73 44	30 371 328	17.663 521 7	6.782 422 9
313	9 79 69	30 664 297	17.691 806	6.789 661 3
314	9 85 96	30 959 144	17.720 045 1	6.796 884 4
315	9 92 25	31 255 875	17.748 239 3	6.804 092 1
316	9 98 56	31 554 496	17.776 388 8	6.811 284 7
317	10 04 89	31 855 013	17.804 493 8	6.818 462
318	10 11 24	32 157 432	17.832 554 5	6.825 624 2
319	10 17 61	32 461 759	17.860 571 1	6.832 771 4
320	10 24 00	32 768 000	17.888 543 8	6.839 903 7
321	10 30 41	33 076 161	17.916 472 9	6.847 021 3
322	10 36 84	33 386 248	17.944 358 4	6.854 124
323	10 43 29	33 698 267	17.972 200 8	6.861 212
324	10 49 76	34 012 224	18	6.868 285 5
325	10 56 25	34 328 125	18.027 756 4	6.875 344 3
326	10 62 76	34 645 976	18.055 470 1	6.882 388 8
327	10 69 29	34 965 783	18.083 141 3	6.889 418 8
328	10 75 84	35 287 552	18.110 770 3	6.896 434 5
329	10 82 41	35 611 289	18.138 357 1	6.903 435 9
330	10 89 00	35 937 000	18.165 902 1	6.910 423 2
331	10 95 61	36 264 691	18.193 405 4	6.917 396 4
332	11 02 24	36 594 368	18.220 867 2	6.924 355 6
333	11 08 89	36 926 037	18.248 287 6	6.931 308 8
334	11 15 56	37 259 704	18.275 666 9	6.938 232 1

Number.	Square.	Cube.	Square Root.	Cube Root.
335	11 22 25	37 595 375	18.303 005 2	6.945 149 6
336	11 28 96	37 933 056	18.330 302 8	6.952 053 3
337	11 35 69	38 272 753	18.357 559 8	6.958 943 4
338	11 42 44	38 614 472	18.384 776 3	6.965 819 8
339	11 49 21	38 958 219	18.411 952 6	6.972 682 6
340	11 56 00	39 304 000	18.439 088 9	6.979 532 1
341	11 62 81	39 651 821	18.466 185 3	6.986 368 1
342	11 69 64	40 001 688	18.493 242	6.993 190 6
343	11 76 49	40 353 607	18.520 259 2	7
344	11 83 36	40 707 584	18.547 237	7.006 796 2
345	11 90 25	41 063 625	18.574 175 6	7.013 579 1
346	11 97 16	41 421 736	18.601 075 2	7.020 349
347	12 04 09	41 781 923	18.627 936	7.027 105 8
348	12 11 04	42 144 192	18.654 758 1	7.033 849 7
349	12 18 01	42 508 549	18.681 541 7	7.040 580 6
350	12 25 00	42 875 000	18.708 286 9	7.047 298 7
351	12 32 01	43 243 551	18.734 994	7.054 004 1
352	12 39 04	43 614 208	18.761 663	7.060 696 7
353	12 46 09	43 986 977	18.788 294 2	7.067 376 7
354	12 53 16	44 361 864	18.814 887 7	7.074 044
355	12 60 25	44 738 875	18.841 443 7	7.080 698 8
356	12 67 36	45 118 016	18.867 962 3	7.087 341 1
357	12 74 49	45 499 293	18.894 443 6	7.093 970 9
358	12 81 64	45 882 712	18.920 887 9	7.100 588 5
359	12 88 81	46 268 279	18.947 295 3	7.107 193 7
360	12 96 00	46 656 000	18.973 666	7.113 786 6
361	13 03 21	47 045 831	19	7.120 367 4
362	13 10 44	47 437 928	19.026 297 6	7.126 936
363	13 17 69	47 832 147	19.052 558 9	7.133 492 5
364	13 24 96	48 228 544	19.078 784	7.140 037
365	13 32 25	48 627 125	19.104 973 2	7.146 569 5
366	13 39 56	49 027 896	19.131 126 5	7.153 090 1
367	13 46 89	49 430 863	19.157 244 1	7.159 598 8
368	13 54 24	49 836 032	19.183 326 1	7.166 095 7
369	13 61 61	50 243 409	19.209 372 7	7.172 580 9
370	13 69 00	50 653 000	19.235 384 1	7.179 054 4
371	13 76 41	51 064 811	19.261 360 3	7.185 516 2
372	13 83 84	51 478 848	19.287 301 5	7.191 966 3
373	13 91 29	51 895 117	19.313 207 9	7.198 405
374	13 98 76	52 313 624	19.339 079 6	7.204 832 2
375	14 06 25	52 734 375	19.364 916 7	7.211 247 9
376	14 13 76	53 157 376	19.390 719 4	7.217 652 2
377	14 21 29	53 582 633	19.416 487 8	7.224 045
378	14 28 84	54 010 152	19.442 222 1	7.230 426 8
379	14 36 41	54 439 939	19.467 922 3	7.236 797 2
380	14 44 00	54 872 000	19.493 588 7	7.243 156 5
381	14 51 61	55 306 341	19.519 221 3	7.249 504 5
382	14 59 24	55 742 968	19.544 820 3	7.255 841 5
383	14 66 89	56 181 887	19.570 385 8	7.262 167 5
384	14 74 56	56 623 104	19.595 917 9	7.268 482 4
385	14 82 25	57 066 625	19.621 416 9	7.274 786 4
386	14 89 96	57 512 456	19.646 882 7	7.281 079 4
387	14 97 69	57 960 603	19.672 315 6	7.287 361 7
388	15 05 44	58 411 072	19.697 715 6	7.293 633
389	15 13 21	58 863 869	19.723 082 9	7.299 893 6
390	15 21 00	59 319 000	19.748 417 7	7.306 143 6

Number.	Square.	Cube.	Square Root.	Cube Root.
391	15 28 81	59 776 471	19.773 719 9	7.312 382 8
392	15 36 64	60 236 288	19.798 989 9	7.318 611 4
393	15 44 49	60 698 457	19.824 227 6	7.324 829 5
394	15 52 36	61 162 984	19.849 433 2	7.331 036 9
395	15 60 25	61 629 875	19.874 606 9	7.337 233 9
396	15 68 16	62 099 136	19.899 748 7	7.343 420 5
397	15 76 09	62 570 773	19.924 858 8	7.349 596 6
398	15 84 04	63 044 792	19.949 937 3	7.355 762 4
399	15 92 01	63 521 199	19.974 984 4	7.361 917 8
400	16 00 00	64 000 000	20	7.368 063
401	16 08 01	64 481 201	20.024 984 4	7.374 197 9
402	16 16 04	64 964 808	20.049 937 7	7.380 322 7
403	16 24 09	65 450 827	20.074 859 9	7.386 437 3
404	16 32 16	65 939 264	20.099 751 2	7.392 541 8
405	16 40 25	66 430 125	20.124 611 8	7.398 636 3
406	16 48 36	66 923 416	20.149 441 7	7.404 720 6
407	16 56 49	67 419 143	20.174 241	7.410 795
408	16 64 64	67 917 312	20.199 009 9	7.416 859 5
409	16 72 81	68 417 929	20.223 748 4	7.422 914 2
410	16 81 00	68 921 000	20.248 456 7	7.428 958 9
411	16 89 21	69 426 531	20.273 134 9	7.434 993 8
412	16 97 44	69 934 528	20.297 783 1	7.441 018 9
413	17 05 69	70 444 997	20.322 401 4	7.447 034 2
414	17 13 96	70 957 944	20.346 989 9	7.453 039 9
415	17 22 25	71 473 375	20.371 548 8	7.459 035 9
416	17 30 56	71 991 296	20.396 078 1	7.465 022 3
417	17 38 89	72 511 713	20.420 577 9	7.470 999 1
418	17 47 24	73 034 632	20.445 048 3	7.476 966 4
419	17 55 61	73 560 059	20.469 489 5	7.482 924 2
420	17 64 00	74 088 000	20.493 901 5	7.488 872 4
421	17 72 41	74 618 461	20.518 284 5	7.494 811 3
422	17 80 84	75 151 448	20.542 638 6	7.500 740 6
423	17 89 29	75 686 967	20.566 963 8	7.506 660 7
424	17 97 76	76 225 024	20.591 260 3	7.512 571 5
425	18 06 25	76 765 625	20.615 528 1	7.518 473
426	18 14 76	77 308 776	20.639 767 4	7.524 365 2
427	18 23 29	77 854 483	20.663 978 3	7.530 248 2
428	18 31 84	78 402 752	20.688 160 9	7.536 122 1
429	18 40 41	78 953 589	20.712 315 2	7.541 986 7
430	18 49 00	79 507 000	20.736 441 4	7.547 842 3
431	18 57 61	80 062 991	20.760 539 5	7.553 688 8
432	18 66 24	80 621 568	20.784 609 7	7.559 526 3
433	18 74 89	81 182 737	20.808 652	7.565 354 8
434	18 83 56	81 746 504	20.832 666 7	7.571 174 3
435	18 92 25	82 312 875	20.856 653 6	7.576 984 9
436	19 00 96	82 881 856	20.880 613	7.582 786 5
437	19 09 69	83 453 453	20.904 545	7.588 579 3
438	19 18 44	84 027 672	20.928 449 5	7.594 363 3
439	19 27 21	84 604 519	20.952 326 8	7.600 138 5
440	19 36 00	85 184 000	20.976 177	7.605 904 9
441	19 44 81	85 766 121	21	7.611 662 6
442	19 53 64	86 350 888	21.023 796	7.617 411 6
443	19 62 49	86 938 307	21.047 565 2	7.623 151 9
444	19 71 36	87 528 384	21.071 307 5	7.628 883 7
445	19 80 25	88 121 125	21.095 023 1	7.634 606 7
446	19 89 16	88 716 536	21.118 712 1	7.640 321 3

Number.	Square.	Cube.	Square Root.	Cube Root.
447	19 98 09	89 314 623	21.142 374 5	7.646 027 2
448	20 07 04	89 915 392	21.166 010 5	7.651 724 7
449	20 16 01	90 518 849	21.189 620 1	7.657 413 8
450	20 25 00	91 125 000	21.213 203 4	7.663 094 3
451	20 34 01	91 733 851	21.236 760 6	7.668 766 5
452	20 43 04	92 345 408	21.260 291 6	7.674 430 3
453	20 52 09	92 959 677	21.283 796 7	7.680 085 7
454	20 61 16	93 576 664	21.307 275 8	7.685 732 8
455	20 70 25	94 196 375	21.330 729	7.691 371 7
456	20 79 36	94 818 816	21.354 156 5	7.697 002 3
457	20 88 49	95 443 993	21.377 558 3	7.702 624 6
458	20 97 64	96 071 912	21.400 934 6	7.708 238 8
459	21 06 81	96 702 579	21.424 285 3	7.713 844 8
460	21 16 00	97 336 000	21.447 610 6	7.719 442 6
461	21 25 21	97 972 181	21.470 910 6	7.725 032 5
462	21 34 44	98 611 128	21.494 185 3	7.730 614 1
463	21 43 69	99 252 847	21.517 434 8	7.736 187 7
464	21 52 96	99 897 344	21.540 659 2	7.741 753 2
465	21 62 25	100 544 625	21.563 858 7	7.747 310 9
466	21 71 56	101 194 696	21.587 033 1	7.752 860 6
467	21 80 89	101 847 563	21.610 182 8	7.758 402 3
468	21 90 24	102 503 232	21.633 307 7	7.763 936 1
469	21 99 61	103 161 709	21.656 407 8	7.769 462
470	22 09 00	103 823 000	21.679 483 4	7.774 980 1
471	22 18 41	104 487 111	21.702 534 4	7.780 490 4
472	22 27 84	105 154 048	21.725 561	7.785 992 8
473	22 37 29	105 823 817	21.748 563 2	7.791 487 5
474	22 46 76	106 496 424	21.771 541 1	7.796 974 5
475	22 56 25	107 171 875	21.794 494 7	7.802 453 8
476	22 65 76	107 850 176	21.817 424 2	7.807 925 4
477	22 75 29	108 531 333	21.840 329 7	7.813 389 2
478	22 84 84	109 215 352	21.863 211 1	7.818 845 6
479	22 94 41	109 902 239	21.886 068 6	7.824 294 2
480	23 04 00	110 592 000	21.908 902 3	7.829 735 3
481	23 13 61	111 284 641	21.931 712 2	7.835 168 8
482	23 23 24	111 980 168	21.954 498 4	7.840 594 9
483	23 32 89	112 678 587	21.977 261	7.846 013 4
484	23 42 56	113 379 904	22	7.851 424 4
485	23 52 25	114 084 125	22.022 715 5	7.856 828 1
486	23 61 96	114 791 256	22.045 407 7	7.862 224 2
487	23 71 69	115 501 303	22.068 076 5	7.867 613
488	23 81 44	116 214 272	22.090 722	7.872 994 4
489	23 91 21	116 930 169	22.113 344 4	7.878 368 4
490	24 01 00	117 649 000	22.135 943 6	7.883 735 2
491	24 10 81	118 370 771	22.158 519 8	7.889 094 6
492	24 20 64	119 095 488	22.181 073	7.894 446 8
493	24 30 49	119 823 157	22.203 603 3	7.899 791 7
494	24 40 36	120 553 784	22.226 110 8	7.905 129 4
495	24 50 25	121 287 375	22.248 595 5	7.910 459 9
496	24 60 16	122 023 936	22.271 057 5	7.915 783 2
497	24 70 09	122 763 473	22.293 496 8	7.921 099 4
498	24 80 04	123 505 992	22.315 913 6	7.926 408 5
499	24 90 01	124 251 499	22.338 307 9	7.931 710 4
500	25 00 00	125 000 000	22.360 679 8	7.937 005 3
501	25 10 01	125 751 501	22.383 029 3	7.942 293 1
502	25 20 04	126 506 008	22.405 356 5	7.947 573 9

Number.	Square.	Cube.	Square Root.	Cube Root.
503	25 30 09	127 263 527	22.427 661 5	7.952 847 7
504	25 40 16	128 024 064	22.449 944 3	7.958 114 4
505	25 50 25	128 787 625	22.472 205 1	7.963 374 3
506	25 60 36	129 554 216	22.494 443 8	7.968 627 1
507	25 70 49	130 323 843	22.516 660 5	7.973 873 1
508	25 80 64	131 096 512	22.538 855 3	7.979 112 2
509	25 90 81	131 872 229	22.561 028 3	7.984 344 4
510	26 01 00	132 651 000	22.583 179 6	7.989 569 7
511	26 11 21	133 432 831	22.605 309 1	7.994 788 3
512	26 21 44	134 217 728	22.627 417	8
513	26 31 69	135 005 697	22.649 503 3	8.005 204 9
514	26 41 96	135 796 744	22.671 568 1	8.010 403 2
515	26 52 25	136 590 875	22.693 611 4	8.015 594 6
516	26 62 56	137 388 096	22.715 633 4	8.020 779 4
517	26 72 89	138 188 413	22.737 634	8.025 957 4
518	26 83 24	138 991 832	22.759 613 4	8.031 128 7
519	26 93 61	139 798 359	22.781 571 5	8.036 293 5
520	27 04 00	140 608 000	22.803 508 5	8.041 451 5
521	27 14 41	141 420 761	22.825 424 4	8.046 603
522	27 24 84	142 236 648	22.847 319 3	8.051 747 9
523	27 35 29	143 055 667	22.869 193 3	8.056 886 2
524	27 45 76	143 877 824	22.891 046 3	8.062 018
525	27 56 25	144 703 125	22.912 878 5	8.067 143 2
526	27 66 76	145 531 576	22.934 689 9	8.072 262
527	27 77 29	146 363 183	22.956 480 6	8.077 374 3
528	27 87 84	147 197 952	22.978 250 6	8.082 48
529	27 98 41	148 035 889	23	8.087 579 4
530	28 09 00	148 877 000	23.021 728 9	8.092 672 3
531	28 19 61	149 721 291	23.043 437 2	8.097 758 9
532	28 30 24	150 568 768	23.065 125 2	8.102 839
533	28 40 89	151 419 437	23.086 792 8	8.107 912 8
534	28 51 56	152 273 304	23.108 44	8.112 980 3
535	28 62 25	153 130 375	23.130 067	8.118 041 4
536	28 72 96	153 990 656	23.151 673 8	8.123 096 2
537	28 83 69	154 854 153	23.173 260 5	8.128 144 7
538	28 94 44	155 720 872	23.194 827	8.133 187
539	29 05 21	156 590 819	23.216 373 5	8.138 223
540	29 16 00	157 464 000	23.237 900 1	8.143 252 9
541	29 26 81	158 340 421	23.259 406 7	8.148 276 5
542	29 37 64	159 220 088	23.280 893 5	8.153 293 9
543	29 48 49	160 103 007	23.302 360 4	8.158 305 1
544	29 59 36	160 989 184	23.323 807 6	8.163 310 2
545	29 70 25	161 878 625	23.345 235 1	8.168 309 2
546	29 81 16	162 771 336	23.366 642 9	8.173 302
547	29 92 09	163 667 323	23.388 031 1	8.178 288 8
548	30 03 04	164 566 592	23.409 399 8	8.183 269 5
549	30 14 01	165 469 149	23.430 749	8.188 244 1
550	30 25 00	166 375 000	23.452 078 8	8.193 212 7
551	30 36 01	167 284 151	23.473 389 2	8.198 175 3
552	30 47 04	168 196 608	23.494 680 2	8.203 131 9
553	30 58 09	169 112 377	23.515 952	8.208 082 5
554	30 69 16	170 031 464	23.537 204 6	8.213 027 1
555	30 80 25	170 953 875	23.558 438	8.217 965 7
556	30 91 36	171 879 616	23.579 652 2	8.222 898 5
557	31 02 49	172 808 693	23.600 847 4	8.227 825 4
558	31 13 64	173 741 112	23.622 023 6	8.232 746 3

Number.	Square.	Cube.	Square Root.	Cube Root.
559	31 24 81	174 676 879	23.643 180 8	8.237 661 4
560	31 36 00	175 616 000	23.664 319 1	8.242 570 6
561	31 47 21	176 558 481	23.685 438 6	8.247 474
562	31 58 44	177 504 328	23.706 539 2	8.252 371 5
563	31 69 69	178 453 547	23.727 621	8.257 263 3
564	31 80 96	179 406 144	23.748 684 2	8.262 149 2
565	31 92 25	180 362 125	23.769 728 6	8.267 029 4
566	32 03 56	181 321 496	23.790 754 5	8.271 903 9
567	32 14 89	182 284 263	23.811 761 8	8.276 772 6
568	32 26 24	183 250 432	23.832 750 6	8.281 625 5
569	32 37 61	184 220 009	23.853 720 9	8.286 492 8
570	32 49 00	185 193 000	23.874 672 8	8.291 344 4
571	32 60 41	186 169 411	23.895 606 3	8.296 190 3
572	32 71 84	187 149 248	23.916 521 5	8.301 030 4
573	32 83 29	188 132 517	23.937 418 4	8.305 865 1
574	32 94 76	189 119 224	23.958 297 1	8.310 694 1
575	33 06 25	190 109 375	23.979 157 6	8.315 517 5
576	33 17 76	191 102 976	24	8.320 335 3
577	33 29 29	192 100 033	24.020 824 3	8.325 147 5
578	33 40 84	193 100 552	24.041 630 6	8.329 954 2
579	33 52 41	194 104 539	24.062 418 8	8.334 755 3
580	33 64 00	195 112 000	24.083 189 1	8.339 550 9
581	33 75 61	196 122 941	24.103 941 6	8.344 341
582	33 87 24	197 137 368	24.124 676 2	8.349 125 6
583	33 98 89	198 155 287	24.145 392 9	8.353 904 7
584	34 10 56	199 176 704	24.166 091 9	8.358 678 4
585	34 22 25	200 201 625	24.186 773 2	8.363 446 6
586	34 33 96	201 230 056	24.207 436 9	8.368 209 5
587	34 45 69	202 262 003	24.228 082 9	8.372 966 8
588	34 57 44	203 297 472	24.248 711 3	8.377 718 8
589	34 69 21	204 336 469	24.269 322 2	8.382 465 3
590	34 81 00	205 379 000	24.289 915 6	8.387 206 5
591	34 92 81	206 425 071	24.310 491 6	8.391 942 3
592	35 04 64	207 474 688	24.331 050 1	8.396 672 9
593	35 16 49	208 527 857	24.351 591 3	8.401 398 1
594	35 28 36	209 584 584	24.372 115 2	8.406 118
595	35 40 25	210 644 875	24.392 621 8	8.410 832 6
596	35 52 16	211 708 736	24.413 111 2	8.415 541 9
597	35 64 09	212 776 173	24.433 583 4	8.420 246
598	35 76 04	213 847 192	24.454 038 5	8.424 944 8
599	35 88 01	214 921 799	24.474 476 5	8.429 638 3
600	36 00 00	216 000 000	24.494 897 4	8.434 326 7
601	36 12 01	217 081 801	24.515 301 3	8.439 009 8
602	36 24 04	218 167 208	24.535 688 3	8.443 687 7
603	36 36 09	219 256 227	24.556 058 3	8.448 360 5
604	36 48 16	220 348 864	24.576 411 5	8.453 028 1
605	36 60 25	221 445 125	24.596 747 8	8.457 690 6
606	36 72 36	222 545 016	24.617 067 3	8.462 347 9
607	36 84 49	223 648 543	24.637 37	8.467
608	36 96 64	224 755 712	24.657 656	8.471 647 1
609	37 08 81	225 866 529	24.677 925 4	8.476 289 2
610	37 21 00	226 981 000	24.698 178 1	8.480 926 1
611	37 33 21	228 099 131	24.718 414 2	8.485 557 9
612	37 45 44	229 220 928	24.738 633 8	8.490 184 8
613	37 57 69	230 346 397	24.758 836 8	8.494 806 5
614	37 69 96	231 475 544	24.779 023 4	8.499 423 3

Number.	Square.	Cube.	Square Root.	Cube Root.
615	37 82 25	232 608 375	24.799 193 5	8.504 035
616	37 94 56	233 744 896	24.819 347 3	8.508 641 7
617	38 06 89	234 885 113	24.839 484 7	8.513 243 5
618	38 19 24	236 029 032	24.859 605 8	8.517 840 3
619	38 31 61	237 176 659	24.879 710 6	8.522 432 1
620	38 44 00	238 328 000	24.899 799 2	8.527 018 9
621	38 56 41	239 483 061	24.919 871 6	8.531 600 9
622	38 68 84	240 641 848	24.939 927 8	8.536 178
623	38 81 29	241 804 367	24.959 967 9	8.540 750 1
624	38 93 76	242 970 624	24.979 992	8.545 317 3
625	39 06 25	244 140 625	25	8.549 879 7
626	39 18 76	245 134 376	25.019 992	8.554 437 2
627	39 31 29	246 491 883	25.039 968 1	8.558 989 9
628	39 43 84	247 673 152	25.059 928 2	8.563 537 7
629	39 56 41	248 858 189	25.079 872 4	8.568 080 7
630	39 69 00	250 047 000	25.099 800 8	8.572 618 9
631	39 81 61	251 239 591	25.119 713 4	8.577 152 3
632	39 94 24	252 435 968	25.139 610 2	8.581 680 9
633	40 06 89	253 636 137	25.159 491 3	8.586 204 7
634	40 19 56	254 840 104	25.179 356 6	8.590 723 8
635	40 32 25	256 047 875	25.199 206 3	8.595 238
636	40 44 96	257 259 456	25.219 040 4	8.599 747 6
637	40 57 69	258 474 853	25.238 858 9	8.604 252 5
638	40 70 44	259 694 072	25.258 661 9	8.608 752 6
639	40 83 21	260 917 119	25.278 449 3	8.613 248
640	40 96 00	262 144 000	25.298 221 3	8.617 738 8
641	41 08 81	263 374 721	25.317 977 8	8.622 224 8
642	41 21 64	264 609 288	25.337 718 9	8.626 706 3
643	41 34 49	265 847 707	25.357 444 7	8.631 183
644	41 47 36	267 089 984	25.377 155 1	8.635 655 1
645	41 60 25	268 336 125	25.396 850 2	8.640 122 6
646	41 73 16	269 585 136	25.416 530 1	8.644 585 5
647	41 86 09	270 840 023	25.436 194 7	8.649 043 7
648	41 99 04	272 097 792	25.455 844 1	8.653 497 4
949	42 12 01	273 359 549	25.475 478 4	8.657 946 5
650	42 25 00	274 625 000	25.495 097 6	8.662 391 1
651	42 38 01	275 894 451	25.514 701 6	8.666 831
652	42 51 04	277 167 808	25.534 290 7	8.671 266 5
653	42 64 09	278 445 077	25.553 864 7	8.675 697 4
654	42 77 16	279 726 264	25.573 423 7	8.680 123 7
655	42 90 25	281 011 375	25.592 967 8	8.684 545 6
656	43 03 36	282 300 416	25.612 496 9	8.688 963
657	43 16 49	283 593 393	25.632 011 2	8.693 375 9
658	43 29 64	284 890 312	25.651 510 7	8.697 784 3
659	43 42 81	286 191 179	25.670 995 3	8.702 188 2
660	43 56 00	287 496 000	25.690 465 2	8.706 587 7
661	43 69 21	288 804 781	25.709 920 3	8.710 982 7
662	43 82 44	290 117 528	25.729 360 7	8.715 373 4
663	43 95 69	291 434 247	25.748 786 4	8.719 759 6
664	44 08 96	292 754 944	25.768 197 5	8.724 141 4
665	44 22 25	294 079 625	25.787 593 9	8.728 518 7
666	44 35 56	295 408 296	25.806 975 8	8.732 891 8
667	44 48 89	296 740 963	25.826 343 1	8.737 260 4
668	44 62 24	298 077 632	25.845 696	8.741 624 6
669	44 75 61	299 418 309	25.865 034 3	8.745 984 6
670	44 89 00	300 763 000	25.884 358 2	8.750 340 1

Number.	Square.	Cube.	Square Root.	Cube Root.
671	45 02 41	302 111 711	25.903 667 7	8.754 691 3
672	45 15 84	303 464 448	25.922 962 8	8.759 038 3
673	45 29 29	304 821 217	25.942 243 5	8.763 380 9
674	45 42 76	306 182 024	25.961 51	8.767 719 2
675	45 56 25	307 546 875	25.980 762 1	8.772 053 2
676	45 69 76	308 915 776	26	8.776 383
677	45 83 29	310 288 733	26.019 223 7	8.780 708 4
678	45 96 84	311 665 752	26.038 433 1	8.785 029 6
679	46 10 41	313 046 839	26.057 628 4	8.789 346 6
680	46 24 00	314 432 000	26.076 809 6	8.793 659 3
681	46 37 61	315 821 241	26.095 976 7	8.797 967 9
682	46 51 24	317 214 568	26.115 129 7	8.802 272 1
683	46 64 89	318 611 987	26.134 268 7	8.806 572 2
684	46 78 56	320 013 504	26.153 393 7	8.810 868 1
685	46 92 25	321 419 125	26.172 504 7	8.815 159 8
686	47 05 96	322 828 856	26.191 601 7	8.819 447 4
687	47 19 69	324 242 703	26.210 684 8	8.823 730 7
688	47 33 44	325 660 672	26.229 754 1	8.828 009 9
689	47 47 21	327 082 769	26.248 809 5	8.832 285
690	47 61 00	328 509 000	26.267 851 1	8.836 555 9
691	47 74 81	329 939 371	26.286 878 9	8.840 822 7
692	47 88 64	331 373 888	26.305 892 9	8.845 085 4
693	48 02 49	332 812 557	26.324 893 2	8.849 344
694	48 16 36	334 255 384	26.343 879 7	8.853 598 5
695	48 30 25	335 702 375	26.362 852 7	8.857 848 9
696	48 44 16	337 153 536	26.381 811 9	8.862 095 2
697	48 58 09	338 608 873	26.400 757 6	8.866 337 5
698	48 72 04	340 068 392	26.419 689 6	8.870 575 7
699	48 86 01	341 532 099	26.438 608 1	8.874 809 9
700	49 00 00	343 000 000	26.457 513 1	8.879 04
701	49 14 01	344 472 101	26.476 404 6	8.883 266 1
702	49 28 04	345 948 408	26.495 282 6	8.887 488 2
703	49 42 09	347 428 927	26.514 147 2	8.891 706 3
704	49 56 16	348 913 664	26.532 998 3	8.895 920 4
705	49 70 25	350 402 625	26.551 836 1	8.900 130 4
706	49 84 36	351 895 816	26.570 660 5	8.904 336 6
707	49 98 49	353 393 243	26.589 471 6	8.908 538 7
708	50 12 64	354 894 912	26.608 269 4	8.912 736 9
709	50 26 81	356 400 829	26.627 053 9	8.916 931 1
710	50 41 00	357 911 000	26.645 825 2	8.921 121 4
711	50 55 21	359 425 431	26.664 583 3	8.925 307 8
712	50 69 44	360 944 128	26.683 328 1	8.929 490 2
713	50 83 69	362 467 097	26.702 059 8	8.933 668 7
714	50 97 96	363 994 344	26.720 778 4	8.937 843 3
715	51 12 25	365 525 875	26.739 483 9	8.942 014
716	51 26 56	367 061 696	26.758 176 3	8.946 180 9
717	51 40 89	368 601 813	26.776 855 7	8.950 343 8
718	51 55 24	370 146 232	26.795 522	8.954 502 9
719	51 69 61	371 694 959	26.814 175 4	8.958 658 1
720	51 84 00	373 248 000	26.832 815 7	8.962 809 5
721	51 98 41	374 805 361	26.851 443 2	8.966 957
722	52 12 84	376 367 048	26.870 057 7	8.971 100 7
723	52 27 29	377 933 067	26.888 659 3	8.975 240 6
724	52 41 76	379 503 424	26.907 248 1	8.979 376 6
725	52 56 25	381 078 125	26.925 824	8.983 508 9
726	52 70 76	382 657 176	26.944 387 2	8.987 637 3

Number.	Square.	Cube.	Square Root.	Cube Root.
727	52 85 29	384 240 583	26.962 937 5	8.991 762
728	52 99 84	385 828 352	26.981 475 1	8.995 882 9
729	53 14 41	387 420 489	27	9
730	53 29 00	389 017 000	27.018 512 2	9.004 113 4
731	53 43 61	390 617 891	27.037 011 7	9.008 222 9
732	53 58 24	392 223 168	27.055 498 5	9.012 328 8
733	53 72 89	393 832 837	27.073 972 7	9.016 430 9
734	53 87 56	395 446 904	27.092 434 4	9.020 529 3
735	54 02 25	397 065 375	27.110 883 4	9.024 623 9
736	54 16 96	398 688 256	27.129 319 9	9.028 714 9
737	54 31 69	400 315 553	27.147 743 9	9.032 802 1
738	54 46 44	401 947 272	27.166 155 4	9.036 885 7
739	54 61 21	403 583 419	27.184 554 4	9.040 965 5
740	54 76 00	405 224 000	27.202 941	9.045 041 7
741	54 90 81	406 869 021	27.221 315 2	9.049 114 2
742	55 05 64	408 518 488	27.239 676 9	9.053 183 1
743	55 20 49	410 172 407	27.258 026 3	9.057 248 2
744	55 35 36	411 830 784	27.276 363 4	9.061 309 8
745	55 50 25	413 493 625	27.294 688 1	9.065 367 7
746	55 65 16	415 160 936	27.313 000 6	9.069 422
747	55 80 09	416 832 723	27.331 300 7	9.073 472 6
748	55 95 04	418 508 992	27.349 588 7	9.077 519 7
749	56 10 01	420 189 749	27.367 864 4	9.081 563 1
750	56 25 00	421 875 000	27.386 127 9	9.085 603
751	56 40 01	423 564 751	27.404 379 2	9.089 639 2
752	56 55 04	425 259 008	27.422 618 4	9.093 671 9
753	56 70 09	426 957 777	27.440 845 5	9 097 701
754	56 85 16	428 661 064	27.459 060 4	9.101 726 5
755	57 00 25	430 368 875	27.477 263 3	9.105 748 5
756	57 15 36	432 081 216	27.495 454 2	9.109 766 9
757	57 30 49	433 798 093	27.513 633	9.113 781 8
758	57 45 64	435 519 512	27.531 799 8	9.117 793 1
759	57 60 81	437 245 479	27.549 954 6	9.121 801
760	57 76 00	438 976 000	27.568 097 5	9.125 805 3
761	57 91 21	440 711 081	27.586 228 4	9.129 806 1
762	58 06 44	442 450 728	27.604 347 5	9.133 803 4
763	58 21 69	444 194 947	27.622 454 6	9.137 797 1
764	58 36 96	445 943 744	27.640 549 9	9.141 787 4
765	58 52 25	447 697 125	27.658 633 4	9.145 774 2
766	58 67 56	449 455 096	27.676 705	9.149 757 6
767	58 82 89	451 217 663	27.694 764 8	9.153 737 5
768	58 98 24	452 984 832	27.712 812 9	9.157 713 9
769	59 13 61	454 756 609	27.730 849 2	9.161 686 9
770	59 29 00	456 533 000	27.748 873 9	9.165 656 5
771	59 44 41	458 314 011	27.766 886 8	9.169 622 5
772	59 59 84	460 099 648	27.784 888	9.173 585 2
773	59 75 29	461 889 917	27.802 877 5	9.177 544 5
774	59 90 76	463 684 824	27.820 855 5	9.181 500 3
775	60 06 25	465 484 375	27.838 821 8	9.185 452 7
776	60 21 76	467 288 576	27.856 776 6	9.189 401 8
777	60 37 29	469 097 433	27.874 719 7	9.193 347 4
778	60 52 84	470 910 952	27.892 651 4	9.197 289 7
779	60 68 41	472 729 139	27.910 571 5	9.201 228 6
780	60 84 00	474 552 000	27.928 480 1	9.205 164 1
781	60 99 61	476 379 541	27.946 377 2	9.209 096 2
782	61 15 24	478 211 768	27 964 262 9	9.213 025

Number.	Square.	Cube.	Square Root.	Cube Root.
783	61 30 89	480 048 687	27.982 137 2	9.216 950 5
784	61 46 56	481 890 304	28	9.220 872 6
785	61 62 25	483 736 625	28.017 851 5	9.224 791 4
786	61 77 96	485 587 656	28.035 691 5	9.228 706 8
787	61 93 69	487 443 403	28.053 520 3	9.232 618 9
788	62 09 44	489 303 872	28.071 337 7	9.236 527 7
789	62 25 21	491 169 069	28.089 143 8	9.240 433 3
790	62 41 00	493 039 000	28.106 938 6	9.244 335 5
791	62 56 81	494 913 671	28.124 722 2	9.248 234 4
792	62 72 64	496 793 088	28.142 494 6	9.252 13
793	62 88 49	498 677 257	28.160 255 7	9.256 022 4
794	63 04 36	500 566 184	28.178 005 6	9.259 911 4
795	63 20 25	502 459 875	28.195 744 4	9.263 797 3
796	63 36 16	504 358 336	28.213 472	9.267 679 8
797	63 52 09	506 261 573	28.231 188 4	9.271 559 2
798	63 68 04	508 169 592	28.248 893 8	9.275 435 2
799	63 84 01	510 082 399	28.266 588 1	9.279 308 1
800	64 00 00	512 000 000	28.284 271 2	9.283 177 7
801	64 16 01	513 922 401	28.301 943 4	9.287 044
802	64 32 04	515 849 608	28.319 604 5	9.290 907 2
803	64 48 09	517 781 627	28.337 254 6	9.294 767 1
804	64 64 16	519 718 464	28.354 893 8	9.298 623 9
805	64 80 25	521 660 125	28.372 521 9	9.302 477 5
806	64 96 36	523 606 616	28.390 139 1	9.306 327 8
807	65 12 49	525 557 943	28.407 745 4	9.310 175
808	65 28 64	527 514 112	28.425 340 8	9.314 019
809	65 44 81	529 475 129	28.442 925 3	9.317 859 9
810	65 61 00	531 441 000	28.460 498 9	9.321 697 5
811	65 77 21	533 411 731	28.478 061 7	9.325 532
812	65 93 44	535 387 328	28.495 613 7	9.329 363 4
813	66 09 69	537 367 797	28.513 154 9	9.333 191 6
814	66 25 96	539 353 144	28.530 685 2	9.337 016 7
815	66 42 25	541 343 375	28.548 204 8	9.340 838 6
816	66 58 56	543 338 496	28.565 713 7	9.344 657 5
817	66 74 89	545 338 513	28.583 211 9	9.348 473 1
818	66 91 24	547 343 432	28.600 699 3	9.352 285 7
819	67 07 61	549 353 259	28.618 176	9.356 095 2
820	67 24 00	551 368 000	28.635 642 1	9.359 901 6
821	67 40 41	553 387 661	28.653 097 6	9.363 704 9
822	67 56 84	555 412 248	28.670 542 4	9.367 505 1
823	67 73 29	557 441 767	28.687 976 6	9.371 302 2
824	67 89 76	559 476 224	28.705 400 2	9.375 096 3
825	68 06 25	561 515 625	28.722 813 2	9.378 887 3
826	68 22 76	563 559 976	28.740 215 7	9.382 675 2
827	68 39 29	565 609 283	28.757 607 7	9.386 46
828	68 55 84	567 663 552	28.774 989 1	9.390 241 9
829	68 72 41	569 722 789	28.792 360 1	9.394 020 6
830	68 89 00	571 787 000	28.809 720 6	9.397 796 4
831	69 05 61	573 856 191	28.827 070 6	9.401 569 1
832	69 22 24	575 930 368	28.844 410 2	9.405 338 7
833	69 38 89	578 009 537	28.861 739 4	9.409 105 4
834	69 55 56	580 093 704	28.879 058 2	9.412 869
835	69 72 25	582 182 875	28.896 366 6	9.416 629 7
836	69 88 96	584 277 056	28.913 664 6	9.420 387 3
837	70 05 69	586 376 253	28.930 952 3	9.424 142
838	70 22 44	588 480 472	28.948 229 7	9.427 893 6

Number.	Square.	Cube.	Square Root.	Cube Root.
839	70 39 21	590 589 719	28.965 496 7	9.431 642 3
840	70 56 00	592 704 000	28.982 753 5	9.435 38
841	70 72 81	594 823 321	29	9.439 130 7
842	70 89 64	596 947 688	29.017 236 3	9.442 870 4
843	71 06 49	599 077 107	29.034 462 3	9.446 607 2
844	71 23 36	601 211 584	29.051 678 1	9.450 341
845	71 40 25	603 351 125	29.068 883 7	9.454 071 9
846	71 57 16	605 495 736	29.086 079 1	9.457 799 9
847	71 74 09	607 645 423	29.103 264 4	9.461 524 9
848	71 91 04	609 800 192	29.120 439 6	9.465 247
849	72 08 01	611 960 049	29.137 604 6	9.468 966 1
850	72 25 00	614 125 000	29.154 759 5	9.472 682 4
851	72 42 01	616 295 051	29.171 904 3	9.476 395 7
852	72 59 04	618 470 208	29.189 039	9.480 106 1
853	72 76 09	620 650 477	29.206 163 7	9.483 813 6
854	72 93 16	622 835 864	29.223 278 4	9.487 518 2
855	73 10 25	625 026 375	29.240 383	9.491 22
856	73 27 36	627 222 016	29.257 477 7	9.494 918 8
857	73 44 49	629 422 793	29.274 562 3	9.498 614 7
858	73 61 64	631 628 712	29.291 637	9.502 307 8
859	73 78 81	633 839 779	29.308 701 8	9.505 998
860	73 96 00	636 056 000	29.325 756 6	9.509 685 4
861	74 13 21	638 277 381	29.342 801 5	9.513 369 9
862	74 30 44	640 503 928	29.359 836 5	9.517 051 5
863	74 47 69	642 735 647	29.376 861 6	9.520 730 3
864	74 64 96	644 972 544	29.393 876 9	9.524 406 3
865	74 82 25	647 214 625	29.410 882 3	9.528 079 4
866	74 99 56	649 461 896	29.427 877 9	9 531 749 7
867	75 16 89	651 714 363	29.444 863 7	9 535 417 2
868	75 34 24	653 972 032	29.461 839 7	9.539 081 8
869	75 51 61	656 234 909	29.478 805 9	9.542 743 7
870	75 69 00	658 503 000	29.495 762 4	9.546 402 7
871	75 86 41	660 776 311	29.512 709 1	9.550 058 9
872	76 03 84	663 054 848	29.529 646 1	9.553 712 3
873	76 21 29	665 338 617	29.546 573 4	9.557 363
874	76 38 76	667 627 624	29.563 491	9.561 010 8
875	76 56 25	669 921 875	29.580 398 9	9.564 655 9
876	76 73 76	672 221 376	29.597 297 2	9.568 298 2
877	76 91 29	674 526 133	29.614 185 8	9.571 937 7
878	77 08 84	676 836 152	29.631 064 8	9.575 574 5
879	77 26 41	679 151 439	29.647 934 2	9.579 208 5
880	77 44 00	681 472 000	29.664 793 9	9.582 839 7
881	77 61 61	683 797 841	29.681 644 2	9.586 468 2
882	77 79 24	686 128 968	29.698 484 8	9.590 093 7
883	77 96 89	688 465 387	29.715 315 9	9 593 716 9
884	78 14 56	690 807 104	29.732 137 5	9.597 337 3
885	78 32 25	693 154 125	29.748 949 6	9.600 954 8
886	78 49 96	695 506 456	29.765 752 1	9.604 569 6
887	78 67 69	697 864 103	29.782 545 2	9 608 181 7
888	78 85 44	700 227 072	29.799 328 9	9.611 791 1
889	79 03 21	702 595 369	29.816 103	9.615 397 7
890	79 21 00	704 969 000	29.832 867 8	9.619 001 7
891	79 38 81	707 347 971	29.849 623 1	9.622 603
892	79 56 64	709 732 288	29.866 369	9.626 201 6
893	79 74 49	712 121 957	29.883 105 6	9.629 797 5
894	79 92 36	714 516 984	29.899 832 8	9.633 390 7

Number.	Square.	Cube.	Square Root.	Cube Root.
895	80 10 25	716 917 375	29.916 550 6	9.636 981 2
896	80 28 16	719 323 136	29.933 259 1	9.640 569
897	80 46 09	721 734 273	29.949 958 3	9.644 154 2
898	80 64 04	724 150 792	29.966 648 1	9.647 736 7
899	80 82 01	726 572 699	29.983 328 7	9.651 316 6
900	81 00 00	729 000 000	30	9.654 893 8
901	81 18 01	731 432 701	30.016 662	9.658 468 4
902	81 36 04	733 870 808	30.033 314 8	9.662 040 3
903	81 54 09	736 314 327	30.049 958 4	9.665 609 6
904	81 72 16	738 763 264	30.066 592 8	9.669 176 2
905	81 90 25	741 217 625	30.083 217 9	9.672 740 3
906	82 08 36	743 677 416	30.099 833 9	9.676 301 7
907	82 26 49	746 142 643	30.116 440 7	9.679 860 4
908	82 44 64	748 613 312	30.133 038 3	9.683 416 6
909	82 62 81	751 089 429	30.149 626 9	9.686 970 1
910	82 81 00	753 571 000	30.166 206 3	9.690 521 1
911	82 99 21	756 058 031	30.182 776 5	9.694 069 4
912	83 17 44	758 550 528	30.199 337 7	9.697 615 1
913	83 35 69	761 048 497	30.215 889 9	9.701 158 3
914	83 53 96	763 551 944	30.232 432 9	9.704 698 9
915	83 72 25	766 060 875	30.248 966 9	9.708 236 9
916	83 90 56	768 575 296	30.265 491 9	9.711 772 3
917	84 08 89	771 095 213	30.282 007 9	9.715 305 1
918	84 27 24	773 620 632	30.298 514 8	9.718 835 4
919	84 45 61	776 151 559	30.315 012 8	9.722 363 1
920	84 64 00	778 688 000	30.331 501 8	9.725 888 3
921	84 82 41	781 229 961	30.347 981 8	9.729 410 9
922	85 00 84	783 777 448	30.364 452 9	9.732 930 9
923	85 19 29	786 330 467	30.380 915 1	9.736 448 4
924	85 37 76	788 889 024	30.397 368 3	9.739 963 4
925	85 56 25	791 453 125	30.413 812 7	9.743 475 8
926	85 74 76	794 022 776	30.430 248 1	9.746 985 7
927	85 93 29	796 597 983	30.446 674 7	9.750 493
928	86 11 84	799 178 752	30.463 092 4	9.753 997 9
929	86 30 41	801 765 089	30.479 501 3	9.757 500 2
930	86 49 00	804 357 000	30.495 901 4	9.761 000 1
931	86 67 61	806 954 491	30.512 292 6	9.764 497 4
932	86 86 24	809 557 568	30.528 675	9.767 992 2
933	87 04 89	812 166 237	30.545 048 7	9.771 484 5
934	87 23 56	814 780 504	30.561 413 6	9.774 974 3
935	87 42 25	817 400 375	30.577 769 7	9.778 461 6
936	87 60 96	820 025 856	30.594 117 1	9.781 946 6
937	87 79 69	822 656 953	30.610 455 7	9.785 428 8
938	87 98 44	825 293 672	30.626 785 7	9.788 908 7
939	88 17 21	827 936 019	30.643 106 9	9.792 386 1
940	88 36 00	830 584 000	30.659 419 4	9.795 861 1
941	88 54 81	833 237 621	30.675 723 3	9.799 333 6
942	88 73 64	835 896 888	30.692 018 5	9.802 803 6
943	88 92 49	838 561 807	30.708 305 1	9.806 271 1
944	89 11 36	841 232 384	30.724 583	9.809 736 2
945	89 30 25	843 908 625	30.740 852 3	9.813 198 9
946	89 49 16	846 590 536	30.757 113	9.816 659 1
947	89 68 09	849 278 123	30.773 365 1	9.820 116 9
948	89 87 04	851 971 392	30.789 608 6	9.823 572 3
949	90 06 01	854 670 349	30.805 843 6	9.827 025 2
950	90 25 00	857 375 000	30.822 07	9.830 475 7

Number.	Square.	Cube.	Square Root.	Cube Root.
951	90 44 01	860 085 351	30.838 287 9	9.833 923 8
952	90 63 04	862 801 408	30.854 497 2	9.837 369 5
953	90 82 09	865 523 177	30.870 698 1	9.840 812 7
954	91 01 16	868 250 664	30.886 890 4	9.844 253 6
955	91 20 25	870 983 875	30.903 074 3	9.847 692
956	91 39 36	873 722 816	30.919 247 7	9.851 128
957	91 58 49	876 467 493	30.935 416 6	9.854 561 7
958	91 77 64	879 217 912	30.951 575 1	9.857 992 9
959	91 96 81	881 974 079	30.967 725 1	9.861 421 8
960	92 16 00	884 736 000	30.983 866 8	9.864 848 3
961	92 35 21	887 503 681	31	9.868 272 4
962	92 54 44	890 277 128	31.016 124 8	9.871 694 1
963	92 73 69	893 056 347	31.032 241 3	9.875 113 5
964	92 92 96	895 841 344	31.048 349 4	9.878 530 5
965	93 12 25	898 632 125	31.064 449 1	9.881 945 1
966	93 31 56	901 428 696	31.080 540 5	9.885 357 4
967	93 50 89	904 231 063	31.096 623 6	9.888 767 3
968	93 70 24	907 039 232	31.112 698 4	9.892 174 9
969	93 89 61	909 853 209	31.128 764 8	9.895 580 1
970	94 09 00	912 673 000	31.144 823	9.898 983
971	94 28 41	915 498 611	31.160 872 9	9.902 383 5
972	94 47 84	918 330 048	31.176 914 5	9.905 781 7
973	94 67 29	921 167 317	31.192 947 9	9.909 177 6
974	94 86 76	924 010 424	31.208 973 1	9.912 571 2
975	95 06 25	926 859 375	31.224 99	9.915 962 4
976	95 25 76	929 714 176	31.240 998 7	9.919 351 3
977	95 45 29	932 574 833	31.256 999 2	9.922 737 9
978	95 64 84	935 441 352	31.272 991 5	9.926 122 2
979	95 84 41	938 313 739	31.288 975 7	9.929 504 2
980	96 04 00	941 192 000	31.304 951 7	9.932 883 9
981	96 23 61	944 076 141	31.320 919 5	9.936 261 3
982	96 43 24	946 966 168	31.336 879 2	9.939 636 3
983	96 62 89	949 862 087	31.352 830 8	9.943 009 2
984	96 82 56	952 763 904	31.368 774 3	9.946 379 7
985	97 02 25	955 671 625	31.384 709 7	9.949 747 9
986	97 21 96	958 585 256	31.400 636 9	9.953 113 8
987	97 41 69	961 504 803	31.416 556 1	9.956 477 5
988	97 61 44	964 430 272	31.432 467 3	9.959 838 9
989	97 81 21	967 361 669	31.448 370 4	9.963 198 1
990	98 01 00	970 299 000	31.464 265 4	9.966 554 9
991	98 20 81	973 242 271	31.480 152 5	9.969 909 5
992	98 40 64	976 191 488	31.496 031 5	9.973 261 9
993	98 60 49	979 146 657	31.511 902 5	9.976 612
994	98 80 36	982 107 784	31.527 765 5	9.979 959 9
995	99 00 25	985 074 875	31.543 620 6	9.983 305 5
996	99 20 16	988 047 936	31.559 467 7	9.986 648 8
997	99 40 09	991 026 973	31.575 306 8	9.989 99
998	99 60 04	994 011 992	31.591 138	9.993 328 9
999	99 80 01	997 002 999	31.606 961 3	9.996 665 6
1000	1 00 00 00	1 000 000 000	31.622 776 6	10
1001	1 00 02 01	1 003 003 001	31.638 584	10.003 322 2
1002	1 00 40 04	1 006 012 008	31.654 383 6	10.006 662 2
1003	1 00 60 09	1 009 027 027	31.670 175 2	10.009 989 9
1004	1 00 80 16	1 012 048 064	31.685 959	10.013 315 5
1005	1 01 00 25	1 015 075 125	31.701 734 9	10.016 638 9
1006	1 01 20 36	1 018 108 216	31.717 503	10.019 960 1

Number.	Square.	Cube.	Square Root.	Cube Root.
1007	1 01 40 49	1 021 147 343	31.733 263 3	10.023 279 1
1008	1 01 60 64	1 024 192 512	31.749 015 7	10.026 595 8
1009	1 01 80 81	1 027 243 729	31.764 760 3	10.029 910 4
1010	1 02 01 00	1 030 301 000	31.780 497 2	10.033 222 8
1011	1 02 21 21	1 033 364 331	31.796 226 2	10.036 533
1012	1 02 41 44	1 036 433 728	31.811 947 4	10.039 841
1013	1 02 61 69	1 039 509 197	31.827 660 9	10.043 146 9
1014	1 02 81 96	1 042 590 744	31.843 366 6	10.046 450 6
1015	1 03 02 25	1 045 678 375	31.859 064 6	10.049 752 1
1016	1 03 22 56	1 048 772 096	31.874 754 9	10.053 051 4
1017	1 03 42 89	1 051 871 913	31.890 437 4	10.056 348 5
1018	1 03 63 24	1 054 977 832	31.906 112 3	10.059 643 5
1019	1 03 83 61	1 058 089 859	31.921 779 4	10.062 936 4
1020	1 04 04 00	1 061 208 000	31.937 438 8	10.066 227 1
1021	1 04 24 41	1 064 332 261	31.953 090 6	10.069 515 6
1022	1 04 44 84	1 067 462 648	31.968 734 7	10.072 802
1023	1 04 65 29	1 070 599 167	31.984 371 2	10.076 086 3
1024	1 04 85 76	1 073 741 824	32	10.079 368 4
1025	1 05 06 25	1 076 890 625	32.015 621 2	10.082 648 4
1026	1 05 26 76	1 080 045 576	32.031 234 8	10.085 926 2
1027	1 05 47 29	1 083 206 683	32.046 840 7	10.089 201 9
1028	1 05 67 84	1 086 373 952	32.062 439 1	10.092 475 5
1029	1 05 88 41	1 089 547 389	32.078 029 8	10.095 746 9
1030	1 06 09 00	1 092 727 000	32.093 613 1	10.099 016 3
1031	1 06 29 61	1 095 912 791	32.109 188 7	10.102 283 5
1032	1 06 50 24	1 099 104 768	32.124 756 8	10.105 548 7
1033	1 06 70 89	1 102 302 937	32.140 317 3	10.108 811 7
1034	1 06 91 56	1 105 507 304	32.155 870 4	10.112 072 6
1035	1 07 12 25	1 108 717 875	32.171 415 9	10.115 531 4
1036	1 07 32 96	1 111 934 656	32.186 953 9	10.118 588 2
1037	1 07 53 69	1 115 157 653	32.202 484 4	10.121 842 8
1038	1 07 74 44	1 118 386 872	32.218 007 4	10.126 095 3
1039	1 07 95 21	1 121 622 319	32.233 522 9	10.128 345 7
1040	1 08 16 00	1 124 864 000	32.249 031	10.131 594 1
1041	1 08 36 81	1 128 111 921	32.264 531 6	10.134 840 3
1042	1 08 57 64	1 131 366 088	32.280 024 8	10.138 084 5
1043	1 08 78 49	1 134 626 507	32.295 510 5	10.141 326 6
1044	1 08 99 36	1 137 893 184	32.310 988 8	10.144 566 7
1045	1 09 20 25	1 141 166 125	32.326 459 8	10.147 804 7
1046	1 09 41 16	1 144 445 336	32.341 923 3	10.151 040 6
1047	1 09 62 09	1 147 730 823	32.357 379 4	10.154 274 4
1048	1 09 83 04	1 151 022 592	32.372 828 1	10.157 506 2
1049	1 10 04 01	1 154 320 649	32.388 269 5	10.160 735 9
1050	1 10 25 00	1 157 625 000	32.403 703 5	10.163 963 6
1051	1 10 46 01	1 160 935 651	32.419 130 1	10.167 189 3
1052	1 10 67 04	1 164 252 608	32.434 549 5	10.170 412 9
1053	1 10 88 09	1 167 575 877	32.449 961 5	10.173 634 4
1054	1 11 09 16	1 170 905 464	32.465 366 2	10.176 853 9
1055	1 11 30 25	1 174 241 375	32.480 763 5	10.180 071 4
1056	1 11 51 36	1 177 583 616	32.496 153 6	10.183 286 8
1057	1 11 72 49	1 180 932 193	32.511 536 4	10.186 500 2
1058	1 11 93 64	1 184 287 112	32.526 911 9	10.189 711 6
1059	1 12 14 81	1 187 648 379	32.542 280 2	10.192 920 9
1060	1 12 36 00	1 191 016 000	32.557 641 2	10.196 128 3
1061	1 12 57 21	1 194 389 981	32.572 994 9	10.199 333 6
1062	1 12 78 44	1 197 770 328	32.588 341 5	10.202 536 9

Number.	Square.	Cube.	Square Root.	Cube Root.
1063	1 12 99 69	1 201 157 047	32.603 680 7	10.205 738 2
1064	1 13 20 96	1 204 550 144	32.619 012 9	10.208 937 5
1065	1 13 42 25	1 207 949 625	32.634 337 7	10.212 134 7
1066	1 13 63 56	1 211 355 496	32.649 655 4	10.215 33
1067	1 13 84 89	1 214 767 763	32.664 965 9	10.218 523 3
1068	1 14 06 24	1 218 186 432	32.680 269 3	10.221 714 6
1069	1 14 27 61	1 221 611 509	32.695 565 4	10.224 903 9
1070	1 14 49 00	1 225 043 000	32.710 854 4	10.228 091 2
1071	1 14 70 41	1 228 480 911	32.726 136 3	10.231 276 6
1072	1 14 91 84	1 231 925 248	32.741 411 1	10.234 459 9
1073	1 15 13 29	1 235 376 017	32.756 678 7	10.237 641 3
1074	1 15 34 76	1 238 833 224	32.771 939 2	10.240 820 7
1075	1 15 56 25	1 242 296 875	32.787 192 6	10.243 998 1
1076	1 15 77 76	1 245 766 976	32.802 438 9	10.247 173 5
1077	1 15 99 29	1 249 243 533	32.817 678 2	10.250 347
1078	1 16 20 84	1 252 726 552	32.832 910 3	10.253 518 6
1079	1 16 42 41	1 256 216 039	32.848 135 4	10.256 688 1
1080	1 16 64 00	1 259 712 000	32.863 353 5	10.259 855 7
1081	1 16 85 61	1 263 214 441	32.878 564 4	10.263 021 3
1082	1 17 07 24	1 266 723 368	32.893 768 4	10.266 185
1083	1 17 28 89	1 270 238 787	32.908 965 3	10.269 346 7
1084	1 17 50 56	1 273 760 704	32.924 155 3	10.272 506 5
1085	1 17 72 25	1 277 289 125	32.939 338 2	10.275 664 4
1086	1 17 93 96	1 280 824 056	32.954 514 1	10.278 820 3
1087	1 18 15 69	1 284 365 503	32.969 683	10.281 974 3
1088	1 18 37 44	1 287 913 472	32.984 845	10.285 126 4
1089	1 18 59 21	1 291 467 969	33	10.288 276 5
1090	1 18 81 00	1 295 029 000	33.015 148	10.291 424 7
1091	1 19 02 81	1 298 596 571	33.030 289 1	10.294 570 9
1092	1 19 24 64	1 302 170 688	33.045 423 3	10.297 715 3
1093	1 19 46 49	1 305 751 357	33.060 550 5	10.300 857 7
1094	1 19 68 36	1 309 338 584	33.075 670 8	10.303 998 2
1095	1 19 90 25	1 312 932 375	33.090 784 2	10.307 136 8
1096	1 20 12 16	1 316 532 736	33.105 890 7	10.310 273 5
1097	1 20 34 09	1 320 139 673	33.120 990 3	10.313 408 3
1098	1 20 56 04	1 323 753 192	33.136 083	10.316 541 1
1099	1 20 78 01	1 327 373 299	33.151 168 9	10.319 672 1
1100	1 21 00 00	1 331 000 000	33.166 247 9	10.322 801 2
1101	1 21 22 01	1 334 633 301	33.181 32	10.325 928 4
1102	1 21 44 04	1 338 273 208	33.196 385 3	10.329 053 7
1103	1 21 66 09	1 341 919 727	33.211 443 8	10.332 177
1104	1 21 88 16	1 345 572 864	33.226 695 5	10.335 298 5
1105	1 22 10 25	1 349 232 625	33.241 540 3	10.338 418 1
1106	1 22 32 36	1 352 899 016	33.256 578 3	10.341 535 8
1107	1 22 54 49	1 356 572 043	33.271 609 5	10.344 651 7
1108	1 22 76 64	1 360 251 712	33.286 633 9	10.347 765 7
1109	1 22 98 81	1 363 938 029	33.301 651 6	10.350 877 8
1110	1 23 21 00	1 367 631 000	33.316 662 5	10.353 988
1111	1 23 43 21	1 371 330 631	33.331 666 6	10.357 096 4
1112	1 23 65 44	1 375 036 928	33.346 664	10.360 202 9
1113	1 23 87 69	1 378 749 897	33.361 654 6	10.363 307 6
1114	1 24 09 96	1 382 469 544	33.376 638 5	10.366 410 3
1115	1 24 32 25	1 386 195 875	33.391 615 7	10.369 511 3
1116	1 24 54 56	1 389 928 896	33.406 586 2	10.372 610 3
1117	1 24 76 89	1 393 668 613	33.421 549 9	10.375 707 6
1118	1 24 99 24	1 397 415 032	33.436 507	10.378 803

Number.	Square.	Cube.	Square Root.	Cube Root.
1119	1 25 21 61	1 401 168 159	33.451 457 3	10.381 896 5
1120	1 25 44 00	1 404 928 000	33.466 401 1	10.384 988 2
1121	1 25 66 41	1 408 694 561	33.481 338 1	10.388 078 1
1122	1 25 88 84	1 412 467 848	33.496 268 4	10.391 166 1
1123	1 26 11 29	1 416 247 867	33.511 192 1	10.394 252 3
1124	1 26 33 76	1 420 034 624	33.526 109 2	10.397 336 6
1125	1 26 56 25	1 423 828 125	33.541 019 6	10.400 419 2
1126	1 26 78 76	1 427 628 376	33.555 923 4	10.403 499 9
1127	1 27 01 29	1 431 435 383	33.570 820 6	10.406 578 7
1128	1 27 23 84	1 435 249 152	33.585 711 2	10.409 655 7
1129	1 27 46 41	1 439 069 689	33.600 595 2	10.412 731
1130	1 27 69 00	1 442 897 000	33.615 472 6	10.415 804 4
1131	1 27 91 61	1 446 731 091	33.630 343 4	10.418 876
1132	1 28 14 24	1 450 571 968	33.645 207 7	10.421 945 8
1133	1 28 36 89	1 454 419 637	33.660 065 3	10.425 013 8
1134	1 28 59 56	1 458 274 104	33.674 916 5	10.428 08
1135	1 28 82 25	1 462 135 375	33.689 761	10.431 144 3
1136	1 29 04 96	1 466 003 456	33.704 599 1	10.434 206 9
1137	1 29 27 69	1 469 878 353	33.719 430 6	10.437 267 7
1138	1 29 50 44	1 473 760 072	33.734 255 6	10.440 326 7
1139	1 29 73 21	1 477 648 619	33.749 074 1	10.443 383 9
1140	1 29 96 00	1 481 544 000	33.763 886 0	10.446 439 3
1141	1 30 18 81	1 485 446 221	33.778 691 5	10.449 492 9
1142	1 30 41 64	1 489 355 288	33.793 490 5	10.452 544 8
1143	1 30 64 49	1 493 271 207	33.808 283	10.455 594 8
1144	1 30 87 36	1 497 193 984	33.823 069 1	10.458 643 1
1145	1 31 10 25	1 501 123 625	33.837 848 6	10.461 689 6
1146	1 31 33 16	1 505 060 136	33.852 621 8	10.464 734 3
1147	1 31 56 09	1 509 003 523	33.867 388 4	10.467 777 3
1148	1 31 79 04	1 512 953 792	33.882 148 7	10.470 818 5
1149	1 32 02 01	1 516 910 949	33.896 902 5	10.473 857 9
1150	1 32 25 00	1 520 875 000	33.911 649 9	10.476 895 5
1151	1 32 48 01	1 524 845 951	33.926 390 9	10.479 931 4
1152	1 32 71 04	1 528 823 808	33.941 125 5	10.482 965 6
1153	1 32 94 09	1 532 808 577	33.955 853 7	10.485 998
1154	1 33 17 16	1 536 800 264	33.970 575 5	10.489 028 6
1155	1 33 40 25	1 540 798 875	33.985 291	10.492 057 5
1156	1 33 63 36	1 544 804 416	34	10.495 084 7
1157	1 33 86 49	1 548 816 893	34.014 702 7	10.498 110 1
1158	1 34 09 64	1 552 836 312	34.029 399	10.501 133 7
1159	1 34 32 81	1 556 862 679	34.044 089	10.504 155 6
1160	1 34 56 00	1 560 896 000	34.058 772 7	10.507 175 7
1161	1 34 79 21	1 564 936 281	34.073 450 1	10.510 194 2
1162	1 35 02 44	1 568 983 528	34.088 121 1	10.513 210 9
1163	1 35 25 69	1 573 037 747	34.102 785 8	10.516 225 9
1164	1 35 48 96	1 577 098 944	34.117 444 2	10.519 239 1
1165	1 35 72 25	1 581 167 125	34.132 096 3	10.522 250 6
1166	1 35 95 56	1 585 242 296	34.146 742 2	10.525 260 4
1167	1 36 18 89	1 589 324 463	34.161 381 7	10.528 268 5
1168	1 36 42 24	1 593 413 632	34.176 015	10.531 274 9
1169	1 36 65 61	1 597 509 809	34.190 642	10.534 279 5
1170	1 36 89 00	1 601 613 000	34.205 262 7	10.537 282 5
1171	1 37 12 41	1 605 723 211	34.219 877 3	10.540 283 7
1172	1 37 35 84	1 609 840 448	34.234 485 5	10.543 283 2
1173	1 37 59 29	1 613 964 717	34.249 087 5	10.546 281
1174	1 37 82 76	1 618 096 024	34.263 683 4	10.549 277 1

Number.	Square.	Cube.	Square Root.	Cube Root.
1175	1 38 06 25	1 622 234 375	34.278 273	10.552 271 5
1176	1 38 29 76	1 626 379 776	34.292 856 4	10.555 264 2
1177	1 38 53 29	1 630 532 233	34.307 433 6	10.558 255 2
1178	1 38 76 84	1 634 691 752	34.322 004 6	10.561 244 5
1179	1 39 00 41	1 638 858 339	34.336 569 4	10.564 232 2
1180	1 39 24 00	1 643 032 000	34.351 128 1	10.567 218 1
1181	1 39 47 61	1 647 212 741	34.365 680 5	10.570 202 4
1182	1 39 71 24	1 651 400 568	34.380 226 8	10.573 184 9
1183	1 39 94 89	1 655 595 487	34.394 767	10.576 165 8
1184	1 40 18 56	1 659 797 504	34.409 301 1	10.579 144 9
1185	1 40 42 25	1 664 006 625	34.423 828 9	10.582 122 5
1186	1 40 65 96	1 668 222 856	34.438 350 7	10.585 098 3
1187	1 40 89 69	1 672 446 203	34.452 866 3	10.588 072 5
1188	1 41 13 44	1 676 676 672	34.467 375 9	10.591 045
1189	1 41 37 21	1 680 914 269	34.481 879 3	10.594 015 8
1190	1 41 61 00	1 685 159 000	34.496 376 6	10.596 985
1191	1 41 84 81	1 689 410 871	34.510 867 8	10.599 952 5
1192	1 42 08 64	1 693 669 888	34.525 353	10.602 918 4
1193	1 42 32 49	1 697 936 057	34.539 832 1	10.605 882 6
1194	1 42 56 36	1 702 209 384	34.554 305 1	10.608 845 1
1195	1 42 80 25	1 706 489 875	34.568 772	10.611 806
1196	1 43 04 16	1 710 777 536	34.583 232 9	10.614 765 2
1197	1 43 28 09	1 715 072 373	34.597 687 9	10.617 722 8
1198	1 43 52 04	1 719 374 392	34.612 136 6	10 620 678 8
1199	1 43 76 01	1 723 683 599	34.626 579 4	10.623 633 1
1200	1 44 00 00	1 728 000 000	34.641 016 2	10 626 585 7
1201	1 44 24 01	1 732 323 601	34.655 446 9	10.629 536 7
1202	1 44 48 04	1 736 654 408	34 669 871 6	10 632 486
1203	1 44 72 09	1 740 992 427	34.684 290 4	10.635 433 8
1204	1 44 96 16	1 745 337 664	34.698 703 1	10.638 379 9
1205	1 45 20 25	1 749 690 125	34.713 109 9	10.641 324 4
1206	1 45 44 36	1 754 049 816	34.727 510 7	10.644 267 2
1207	1 45 68 49	1 758 416 743	34.741 905 5	10.647 208 5
1208	1 45 92 64	1 762 790 912	34.756 294 4	10.650 148
1209	1 46 16 81	1 767 172 329	34.770 677 3	10.653 086
1210	1 46 41 00	1 771 561 000	34.785 054 3	10.656 022 3
1211	1 46 65 21	1 775 956 931	34.799 425 3	10.658 957
1212	1 46 89 44	1 780 360 128	34.813 790 4	10.661 890 2
1213	1 47 13 69	1 784 770 597	34.828 149 5	10.664 821 7
1214	1 47 37 96	1 789 188 344	34.842 502 8	10.667 751 6
1215	1 47 62 25	1 793 613 375	34.856 850 1	10.670 679 9
1216	1 47 86 56	1 798 045 696	34.871 191 5	10.673 606 6
1217	1 48 10 89	1 802 485 313	34.885 527 1	10.676 531 7
1218	1 48 35 24	1 806 932 232	34.899 856 7	10.679 455 2
1219	1 48 59 61	1 811 386 459	34.914 180 5	10.682 377 1
1220	1 48 84 00	1 815 848 000	34.928 498 4	10.685 297 3
1221	1 49 08 41	1 820 316 861	34.942 810 4	10.688 216
1222	1 49 32 84	1 824 793 048	34.957 116 6	10.691 133 1
1223	1 49 57 29	1 829 276 567	34.971 416 9	10.694 048 6
1224	1 49 81 76	1 833 767 424	34.985 711 4	10 696 962 5
1225	1 50 06 25	1 838 265 625	35	10.699 874 8
1226	1 50 30 76	1 842 771 176	35.014 282 8	10.702 785 5
1227	1 50 55 29	1 847 284 083	35.028 559 8	10.705 694 7
1228	1 50 79 84	1 851 804 352	35.042 830 9	10.708 602 3
1229	1 51 04 41	1 856 331 989	35.057 096 3	10.711 508 3
1230	1 51 29 00	1 860 867 000	35.071 355 8	10.714 412 7

Number.	Square.	Cube.	Square Root.	Cube Root.
1231	1 51 53 61	1 865 409 391	35.085 609 6	10.717 315 5
1232	1 51 78 24	1 869 959 168	35.099 857 5	10.720 216 8
1233	1 52 02 89	1 874 516 337	35.114 099 7	10.723 116 5
1234	1 52 27 56	1 879 080 904	35.128 336 1	10.726 014 6
1235	1 52 52 25	1 883 652 875	35.142 556 8	10.728 911 2
1236	1 52 76 96	1 888 232 256	35.156 791 7	10.731 806 2
1237	1 53 01 69	1 892 819 053	35.171 010 8	10.734 699 7
1238	1 53 26 44	1 897 413 272	35.185 224 2	10.737 591 6
1239	1 53 51 21	1 902 014 919	35.199 431 8	10.740 481 9
1240	1 53 76 00	1 906 624 000	35.213 633 7	10.743 370 7
1241	1 54 00 81	1 911 240 521	35.227 829 9	10.746 257 9
1242	1 54 25 64	1 915 864 488	35.242 020 4	10.749 143 6
1243	1 54 50 49	1 920 495 907	35.256 205 1	10.752 027 7
1244	1 54 75 36	1 925 134 784	35.270 384 2	10.754 910 3
1245	1 55 00 25	1 929 781 125	35.284 557 5	10.757 791 3
1246	1 55 25 16	1 934 434 936	35.298 725 2	10.760 670 8
1247	1 55 50 09	1 939 096 223	35.312 887 2	10.763 548 8
1248	1 55 75 04	1 943 764 992	35.327 043 5	10.766 425 2
1249	1 56 00 01	1 948 441 249	35.341 194 1	10.769 300 1
1250	1 56 25 00	1 953 125 000	35.355 339 1	10.772 173 5
1251	1 56 50 01	1 957 816 251	35.369 478 4	10.775 045 3
1252	1 56 75 04	1 962 515 008	35.383 612	10.777 915 6
1253	1 57 00 09	1 967 221 277	35.397 74	10.780 784 3
1254	1 57 25 16	1 971 935 064	35.411 862 4	10.783 651 6
1255	1 57 50 25	1 976 656 375	35.425 979 2	10.786 517 3
1256	1 57 75 36	1 981 385 216	35.440 090 3	10.789 381 5
1257	1 58 00 49	1 986 121 593	35.454 195 8	10.792 244 1
1258	1 58 25 64	1 990 865 512	35.468 295 7	10.795 105 3
1259	1 58 50 81	1 995 616 979	35.482 39	10.797 964 9
1260	1 58 76 00	2 000 376 000	35.496 478 7	10.800 823
1261	1 59 01 21	2 005 142 581	35.510 561 8	10.803 679 7
1262	1 59 26 44	2 009 916 728	35.524 639 3	10.806 534 8
1263	1 59 51 69	2 014 698 447	35.538 711 3	10.809 388 4
1264	1 59 76 96	2 019 487 744	35.552 777 7	10.812 240 4
1265	1 60 02 25	2 024 284 625	35.566 838 5	10.815 090 9
1266	1 60 27 56	2 029 089 096	35.580 893 7	10.817 94
1267	1 60 52 89	2 033 901 163	35.594 943 4	10.820 787 6
1268	1 60 78 24	2 038 720 832	35.608 987 6	10.823 633 6
1269	1 61 03 61	2 043 548 109	35.623 026 2	10.826 478 2
1270	1 61 29 00	2 048 383 000	35.637 059 3	10.829 321 3
1271	1 61 54 41	2 053 225 511	35.651 086 9	10.832 162 9
1272	1 61 79 84	2 058 075 648	35.665 109	10.835 003
1273	1 62 05 29	2 062 933 417	35.679 125 5	10.837 841 6
1274	1 62 30 76	2 067 798 824	35.693 136 6	10.840 678 8
1275	1 62 56 25	2 072 671 875	35.707 142 1	10.843 514 4
1276	1 62 81 76	2 077 552 576	35.721 142 2	10.846 348 5
1277	1 63 07 29	2 082 440 933	35.735 136 7	10.849 181 2
1278	1 63 32 84	2 087 336 952	35.749 125 8	10.852 012 5
1279	1 63 58 41	2 092 240 639	35.763 109 5	10.854 842 2
1280	1 63 84 00	2 097 152 000	35.777 087 6	10.857 670 4
1281	1 64 09 61	2 102 071 041	35.791 060 3	10.860 497 2
1282	1 64 35 24	2 106 997 768	35.805 027 6	10.863 322 5
1283	1 64 60 89	2 111 932 187	35.818 989 4	10.866 146 4
1284	1 64 86 56	2 116 874 304	35.832 945 7	10.868 968 7
1285	1 65 12 25	2 121 824 125	35.846 896 6	10.871 789 7
1286	1 65 37 96	2 126 781 656	35.860 842 1	10.874 609 1

Number.	Square.	Cube.	Square Root.	Cube Root.
1287	1 65 63 69	2 131 746 903	35.874 782 2	10.877 427 1
1288	1 65 89 44	2 136 719 872	35.888 716 9	10.880 243 6
1289	1 66 15 21	2 141 700 569	35.902 646 1	10.883 058 7
1290	1 66 41 00	2 146 689 000	35.916 569 9	10.885 872 3
1291	1 66 66 81	2 151 685 171	35.930 488 4	10.888 684 5
1292	1 66 92 64	2 156 689 088	35.944 401 5	10.891 495 2
1293	1 67 18 49	2 161 700 757	35.958 309 2	10.894 304 4
1294	1 67 44 36	2 166 720 184	35.972 211 5	10.897 112 3
1295	1 67 70 25	2 171 747 375	35.986 108 4	10.899 918 6
1296	1 67 96 16	2 176 782 336	36	10.902 723 5
1297	1 68 22 09	2 181 825 073	36.013 886 2	10.905 526 9
1298	1 68 48 04	2 186 875 592	36.027 767 1	10.908 329
1299	1 68 74 01	2 191 933 899	36.041 642 6	10.911 129 6
1300	1 69 00 00	2 197 000 000	36.055 512 8	10.913 928 7
1301	1 69 26 01	2 202 073 901	36.069 377 6	10.916 726 5
1302	1 69 52 04	2 207 155 608	36.083 237 1	10.919 522 8
1303	1 69 78 09	2 212 245 127	36.097 091 3	10.922 317 7
1304	1 70 04 16	2 217 342 464	36.110 940 2	10.925 111 1
1305	1 70 30 25	2 222 447 625	36.124 783 7	10.927 903 1
1306	1 70 56 36	2 227 560 616	36.138 622	10.930 693 7
1307	1 70 82 49	2 232 681 443	36.152 455	10.933 482 9
1308	1 71 08 64	2 237 810 112	36.166 282 6	10.936 270 6
1309	1 71 34 81	2 242 946 629	36.180 105	10.939 056 9
1310	1 71 61 00	2 248 091 000	36.193 922 1	10.941 841 8
1311	1 71 87 21	2 253 243 231	36.207 734	10.944 625 3
1312	1 72 13 44	2 258 403 328	36.221 540 6	10.947 507 4
1313	1 72 39 69	2 263 571 297	36.235 341 9	10.950 188
1314	1 72 65 96	2 268 747 144	36.249 137 9	10.952 967 3
1315	1 72 92 25	2 273 930 875	36.262 928 7	10.955 745 1
1316	1 73 18 56	2 279 122 496	36.276 714 3	10.958 521 5
1317	1 73 44 89	2 284 322 013	36.290 494 6	10.961 296 5
1318	1 73 71 24	2 289 529 432	36.304 269 7	10.964 070 1
1319	1 73 97 61	2 294 744 759	36.318 039 6	10.966 842 3
1320	1 74 24 00	2 299 968 000	36.331 804 2	10.969 613 1
1321	1 74 50 41	2 305 199 161	36.345 563 7	10.972 382 5
1322	1 74 76 84	2 310 438 248	36.359 317 9	10.975 150 5
1323	1 75 03 29	2 315 685 267	36.373 067	10.977 917 1
1324	1 75 29 76	2 320 940 224	36.386 810 8	10.980 682 3
1325	1 75 56 25	2 326 203 125	36.400 549 4	10.983 446 2
1326	1 75 82 76	2 331 473 976	36.414 282 9	10.986 208 6
1327	1 76 09 29	2 336 752 783	36.428 011 2	10.988 969 6
1328	1 76 35 84	2 342 039 552	36.441 734 3	10.991 729 3
1329	1 76 62 41	2 347 334 289	36.455 452 3	10.994 487 6
1330	1 76 89 00	2 352 637 000	36.469 165	10.997 244 5
1331	1 77 15 61	2 357 947 691	36.482 872 7	11
1332	1 77 42 24	2 363 266 368	36.496 575 2	11.002 754 1
1333	1 77 68 89	2 368 593 037	36.510 272 5	11.005 506 9
1334	1 77 95 56	2 373 927 704	36.523 964 7	11.008 258 3
1335	1 78 22 25	2 379 270 375	36.537 651 8	11.011 008 2
1336	1 78 48 96	2 384 621 056	36.551 333 8	11.013 756 9
1337	1 78 75 69	2 389 979 753	36.565 010 6	11.016 504 1
1338	1 79 02 44	2 395 346 472	36.578 682 3	11.019 25
1339	1 79 29 21	2 400 721 219	36.592 348 9	11.021 994 5
1340	1 79 56 00	2 406 104 000	36.606 010 4	11.024 737 7
1341	1 79 82 81	2 411 494 821	36.619 666 8	11.027 479 5
1342	1 80 09 64	2 416 893 688	36.633 318 1	11.030 219 9

Number.	Square.	Cube.	Square Root.	Cube Root.
1343	1 80 36 49	2 422 300 607	36.646 964 4	11.032 959
1344	1 80 63 36	2 427 715 584	36.660 605 6	11.035 696 7
1345	1 80 90 25	2 433 138 625	36.674 241 6	11.038 433
1346	1 81 17 16	2 438 569 736	36.687 872 6	11.041 168
1347	1 81 44 09	2 444 008 923	36.701 498 6	11.043 901 7
1348	1 81 71 04	2 449 456 192	36.715 119 5	11.046 633 9
1349	1 81 98 01	2 454 911 549	36.728 735 3	11.049 364 9
1350	1 82 25 00	2 460 375 000	36.742 346 1	11.052 094 5
1351	1 82 52 01	2 465 846 551	36.755 951 9	11.054 822 7
1352	1 82 79 04	2 471 326 208	36.769 552 6	11.057 549 7
1353	1 83 06 09	2 476 813 977	36.783 148 3	11.060 275 2
1354	1 83 33 16	2 482 309 864	36.796 739	11.062 999 4
1355	1 83 60 25	2 487 813 875	36.810 324 6.	11.065 722 2
1356	1 83 87 36	2 493 326 016	36.823 905 3	11.068 443 7
1357	1 84 14 49	2 498 846 293	36.837 480 9	11.071 163 9
1358	1 84 41 64	2 504 374 712	36.851 051 5	11.073 882 8
1359	1 84 68 81	2 509 911 279	36.864 617 2	11.076 600 3
1360	1 84 96 00	2 515 456 000	36.878 177 8	11.079 316 5
1361	1 85 23 21	2 521 008 881	36.891 733 5	11.082 031 4
1362	1 85 50 44	2 526 569 928	36.905 284 2	11.084 744 9
1363	1 85 77 69	2 532 139 147	36.918 829 9	11.087 457 1
1364	1 86 04 96	2 537 716 544	36.932 370 6	11.090 167 9
1365	1 86 32 25	2 543 302 125	36.945 906 4	11.092 877 5
1366	1 86 59 56	2 548 895 896	36.959 437 2	11.095 585 7
1367	1 86 86 89	2 554 497 863	36.972 963 1	11.098 292 6
1368	1 87 14 24	2 560 108 032	36.986 484	11.100 998 2
1369	1 87 41 61	2 565 726 409	37	11.103 702 5
1370	1 87 69 00	2 571 353 000	37.013 511	11.106 405 4
1371	1 87 96 41	2 576 987 811	37.027 017 2	11.109 107
1372	1 88 23 84	2 582 630 848	37.040 518 4	11.111 807 3
1373	1 88 51 29	2 588 282 117	37.054 014 6	11.114 506 4
1374	1 88 78 76	2 593 941 624	37.067 506	11.117 204 1
1375	1 89 06 25	2 599 609 375	37.080 992 4	11.119 900 4
1376	1 89 33 76	2 605 285 376	37 094 474	11.122 595 5
1377	1 89 61 29	2 610 969 633	37.107 950 6	11.125 289 3
1378	1 89 88 84	2 616 662 152	37.121 422 4	11.127 981 7
1379	1 90 16 41	2 622 362 939	37.134 889 3	11.130 672 9
1380	1 90 44 00	2 628 072 000	37.148 351 2	11.133 362 8
1381	1 90 71 61	2 633 789 341	37.161 808 4	11.136 051 4
1382	1 90 99 24	2 639 514 968	37.175 260 6	11.138 738 6
1383	1 91 26 89	2 645 248 887	37.188 707 9	11.141 424 6
1384	1 91 54 56	2 650 991 104	37.202 150 5	11.144 109 3
1385	1 91 82 25	2 656 741 625	37.215 588 1	11.146 792 6
1386	1 92 09 96	2 662 500 456	37.229 020 9	11.149 474 7
1387	1 92 37 69	2 668 267 603	37.242 448 9	11.152 155 5
1388	1 92 65 44	2 674 043 072	37.255 872	11.154 835
1389	1 92 93 21	2 679 826 869	37.269 290 3	11.157 513 3
1390	1 93 21 00	2 685 619 000	37.282 703 7	11.160 190 3
1391	1 93 48 81	2 691 419 471	37.296 112 4	11.162 865 9
1392	1 93 76 64	2 697 228 288	37.309 516 2	11.165 540 3
1393	1 94 04 49	2 703 045 457	37.322 915 2	11.168 213 4
1394	1 94 32 36	2 708 870 984	37.336 309 4	11.170 885 2
1395	1 94 60 25	2 714 704 875	37.349 698 8	11.173 555 8
1396	1 94 88 16	2 720 547 136	37.363 083 4	11.176 225
1397	1 95 16 09	2 726 397 773	37.376 463 2	11.178 893
1398	1 95 44 04	2 732 256 792	37.389 838 2	11.181 559 8

Number.	Square.	Cube.	Square Root.	Cube Root.
1399	1 95 72 01	2 738 124 199	37.403 208 4	11.184 225 2
1400	1 96 00 00	2 744 000 000	37.416 573 8	11.186 889 4
1401	1 96 28 01	2 749 884 201	37.429 934 5	11.189 552 3
1402	1 96 56 04	2 755 776 808	37.443 290 4	11.192 213 9
1403	1 96 84 09	2 761 677 827	37.456 641 6	11.194 874 3
1404	1 97 12 16	2 767 587 264	37.469 988	11.197 533 4
1405	1 97 40 25	2 773 505 125	37.483 329 6	11.200 191 3
1406	1 97 68 36	2 779 431 416	37.496 666 5	11.202 847 9
1407	1 97 96 49	2 785 366 143	37.509 998 7	11.205 503 2
1408	1 98 24 64	2 791 309 312	37.523 326 1	11.208 157 3
1409	1 98 52 81	2 797 260 929	37.536 648 7	11.210 810 1
1410	1 98 81 00	2 803 221 000	37.549 966 7	11.213 461 7
1411	1 99 09 21	2 809 189 531	37.563 279 9	11.216 112
1412	1 99 37 44	2 815 166 528	37.576 588 5	11.218 761 1
1413	1 99 65 69	2 821 151 997	37.589 892 2	11.221 408 9
1414	1 99 93 96	2 827 145 944	37.603 191 3	11.224 005 4
1415	2 00 22 25	2 833 148 375	37.616 485 7	11.226 700 7
1416	2 00 50 56	2 839 159 296	37.629 775 4	11.229 344 8
1417	2 00 78 89	2 845 178 713	37.643 060 4	11.231 987 6
1418	2 01 07 24	2 851 206 632	37.656 340 7	11.234 629 2
1419	2 01 35 61	2 857 243 059	37.669 616 4	11.237 269 6
1420	2 01 64 00	2 863 288 000	37.682 887 4	11.239 908 7
1421	2 01 92 41	2 869 341 461	37.696 153 6	11.242 546 5
1422	2 02 20 84	2 875 403 448	37.709 415 3	11.245 183 1
1423	2 02 49 29	2 881 473 967	37.722 672 2	11.247 818 5
1424	2 02 77 76	2 887 553 024	37.735 924 5	11.250 452 7
1425	2 03 06 25	2 893 640 625	37.749 172 2	11.253 085 6
1426	2 03 34 76	2 899 736 776	37.762 415 2	11.255 717 3
1427	2 03 63 29	2 905 841 483	37.775 653 5	11.258 347 8
1428	2 03 91 84	2 911 954 752	37.788 887 3	11.260 977
1429	2 04 20 41	2 918 076 589	37.802 116 3	11.263 605
1430	2 04 49 00	2 924 207 000	37.815 340 8	11.266 231 8
1431	2 04 77 61	2 930 345 991	37.828 560 6	11.268 857 3
1432	2 05 06 24	2 936 493 568	37.841 775 9	11.271 481 6
1433	2 05 34 89	2 942 649 737	37.854 986 4	11.274 104 7
1434	2 05 63 56	2 948 814 504	37.868 192 4	11.276 726 6
1435	2 05 92 25	2 954 987 875	37.881 393 8	11.279 347 2
1436	2 06 20 96	2 961 169 856	37.894 590 6	11.281 966 6
1437	2 06 49 69	2 967 360 453	37.907 782 8	11.284 584 9
1438	2 06 78 44	2 973 559 672	37.920 970 4	11.287 201 9
1439	2 07 07 21	2 979 767 519	37.934 153 5	11.289 817 7
1440	2 07 36 00	2 985 984 000	37.947 331 9	11.292 432 3
1441	2 07 64 81	2 992 209 121	37.960 505 8	11.295 045 7
1442	2 07 93 64	2 998 442 888	37.973 675 1	11.297 657 9
1443	2 08 22 49	3 004 685 307	37.986 839 8	11.300 268 8
1444	2 08 51 36	3 010 936 384	38	11.302 878 6
1445	2 08 80 25	3 017 196 125	38.013 155 6	11.305 487 1
1446	2 09 09 16	3 023 464 536	38.026 306 7	11.308 094 5
1447	2 09 38 09	3 029 741 623	38.039 453 2	11.310 700 6
1448	2 09 67 04	3 036 027 392	38.052 595 2	11.313 305 6
1449	2 09 96 01	3 042 321 849	38.065 732 6	11.315 909 4
1450	2 10 25 00	3 048 625 000	38.078 865 5	11.318 511 9
1451	2 10 54 01	3 054 936 851	38.091 993 9	11.321 113 2
1452	2 10 83 04	3 061 257 408	38.105 117 8	11.323 713 4
1453	2 11 12 09	3 067 586 677	38.118 237 1	11.326 312 4
1454	2 11 41 16	3 073 924 664	38.131 351 9	11.328 910 2

Number.	Square.	Cube.	Square Root.	Cube Root.
1455	2 11 70 25	3 080 271 375	38.144 462 2	11.331 506 7
1456	2 11 99 36	3 086 626 816	38.157 568 1	11.334 102 2
1457	2 12 28 49	3 092 990 993	38.170 669 3	11.336 696 4
1458	2 12 57 64	3 099 363 912	38.183 766 2	11.339 289 4
1459	2 12 86 81	3 105 745 579	38.196 858 5	11.341 881 3
1460	2 13 16 00	3 112 136 000	38.209 946 3	11.344 471 9
1461	2 13 45 21	3 118 535 181	38.223 029 7	11.347 061 4
1462	2 13 74 44	3 124 943 128	38.236 108 5	11.349 649 7
1463	2 14 03 69	3 131 359 847	38.249 182 9	11.352 236 8
1464	2 14 32 96	3 137 785 344	38.262 252 9	11.354 822 7
1465	2 14 62 25	3 144 219 625	38.275 318 4	11.357 407 5
1466	2 14 91 56	3 150 662 696	38.288 379 4	11.359 991 1
1467	2 15 20 89	3 157 114 563	38.301 436	11.362 573 5
1468	2 15 50 24	3 163 575 232	38.314 488 1	11.365 154 7
1469	2 15 79 61	3 170 044 709	38.327 535 8	11.367 734 7
1470	2 16 09 00	3 176 523 000	38.340 579	11.370 313 6
1471	2 16 38 41	3 183 010 111	38.353 617 8	11.372 891 4
1472	2 16 67 84	3 189 506 048	38.366 652 2	11.375 467 9
1473	2 16 97 29	3 196 010 817	38.379 682 1	11.378 043 3
1474	2 17 26 76	3 202 524 424	38.392 707 6	11.380 617 5
1475	2 17 56 25	3 209 046 875	38.405 728 7	11.383 190 6
1476	2 17 85 76	3 215 578 176	38.418 745 4	11.385 762 5
1477	2 18 15 29	3 222 118 333	38.431 757 7	11.388 333 2
1478	2 18 44 84	3 228 667 352	38.444 765 6	11.390 902 8
1479	2 18 74 41	3 235 225 239	38.457 769 1	11.393 471 2
1480	2 19 04 00	3 241 792 000	38.470 768 1	11.396 038 4
1481	2 19 33 61	3 248 367 641	38.483 762 7	11.398 604 5
1482	2 19 63 24	3 254 952 168	38.496 753	11.401 169 5
1483	2 19 92 89	3 261 545 587	38.509 739	11.403 733 2
1484	2 20 22 56	3 268 147 904	38.522 720 6	11.406 295 9
1485	2 20 52 25	3 274 759 125	38.535 697 7	11.408 857 4
1486	2 20 81 96	3 281 379 256	38.548 670 5	11.411 417 7
1487	2 21 11 69	3 288 008 303	38.561 638 9	11.413 976 9
1488	2 21 41 44	3 294 646 272	38.574 603	11.416 534 9
1489	2 21 71 21	3 301 293 169	38.587 562 7	11.419 091 8
1490	2 22 01 00	3 307 949 000	38.600 518 1	11.420 647 6
1491	2 22 30 81	3 314 613 771	38.613 469 1	11.424 202 2
1492	2 22 60 64	3 321 287 488	38.626 415 8	11.426 755 6
1493	2 22 90 49	3 327 970 157	38.639 358 2	11.429 307 9
1494	2 23 20 36	3 334 661 784	38.652 296 2	11.431 859 1
1495	2 23 50 25	3 341 362 375	38.665 229 9	11.434 409 2
1496	2 23 80 16	3 348 071 936	38.678 159 3	11.436 958 1
1497	2 24 10 09	3 354 790 473	38.691 084 3	11.439 505 9
1498	2 24 40 04	3 361 517 992	38.704 005	11.442 052 5
1499	2 24 70 01	3 368 254 499	38.716 921 4	11.444 598
1500	2 25 00 00	3 375 000 000	38.729 833 5	11.447 142 4
1501	2 25 30 01	3 381 754 501	38.742 741 2	11.449 685 7
1502	2 25 60 04	3 388 518 008	38.755 644 7	11.452 227 8
1503	2 25 90 09	3 395 290 527	38.768 543 9	11.454 768 8
1504	2 26 20 16	3 402 072 064	38.781 438 9	11.457 308 7
1505	2 26 50 25	3 408 862 625	38.794 329 4	11.459 847 4
1506	2 26 80 36	3 415 662 216	38.807 215 8	11.462 385
1507	2 27 10 49	3 422 470 843	38.820 097 8	11.464 921 5
1508	2 27 40 64	3 429 288 512	38.832 975 7	11.467 456 8
1509	2 27 70 81	3 436 115 229	38.845 849 1	11.469 991 1
1510	2 28 01 00	3 442 951 000	38.858 718 4	11.472 524 2

Number.	Square.	Cube.	Square Root.	Cube Root.
1511	2 28 31 21	3 449 795 831	38.871 583 4	11.475 056 2
1512	2 28 61 44	3 456 649 728	38.884 444 2	11.477 587 1
1513	2 28 91 69	3 463 512 697	38.897 300 6	11.480 116 9
1514	2 29 21 96	3 470 384 744	38.910 152 9	11.482 645 5
1515	2 29 52 25	3 477 265 875	38.923 000 9	11.485 173 1
1516	2 29 82 56	3 484 156 096	38.935 844 7	11.487 699 5
1517	2 30 12 89	3 491 055 413	38.948 684 1	11.490 224 9
1518	2 30 43 24	3 497 963 832	38.961 519 4	11.492 749 1
1519	2 30 73 61	3 504 881 359	38.974 350 5	11.495 272 2
1520	2 31 04 00	3 511 808 000	38.987 177 4	11.497 794 2
1521	2 31 34 41	3 518 743 761	39	11.500 315 1
1522	2 31 64 84	3 525 688 648	39.012 818 4	11.502 834 8
1523	2 31 95 29	3 532 642 667	39.025 632 6	11.505 353 5
1524	2 32 25 76	3 539 605 824	39.038 442 6	11.507 871 1
1525	2 32 56 25	3 546 578 125	39.051 248 3	11.510 387 6
1526	2 32 86 76	3 553 559 576	39.064 049 9	11.512 903
1527	2 33 17 29	3 560 558 183	39.076 847 3	11.515 417 3
1528	2 33 47 84	3 567 549 952	39.089 640 6	11.517 930 5
1529	2 33 78 41	3 574 558 889	39.102 429 6	11.520 442 5
1530	2 34 09 00	3 581 577 000	39.115 214 4	11.522 953 5
1531	2 34 39 61	3 588 604 291	39.127 995 1	11.525 463 4
1532	2 34 70 24	3 595 640 768	39.140 771 6	11.527 972 2
1533	2 35 00 89	3 602 686 437	39.153 543 9	11.530 479 9
1534	2 35 31 56	3 609 741 304	39.166 312	11.532 986 5
1535	2 35 62 25	3 616 805 375	39.179 076	11.535 492
1536	2 35 92 96	3 623 878 656	39.191 835 9	11.537 996 5
1537	2 36 23 69	3 630 961 153	39.204 591 5	11.540 499 8
1538	2 36 54 44	3 638 052 872	39.217 343 1	11.543 002 1
1539	2 36 85 21	3 645 153 819	39.230 090 5	11.545 503 3
1540	2 37 16 00	3 652 264 000	39.242 833 7	11.548 003 4
1541	2 37 46 81	3 659 383 421	39.255 572 8	11.550 502 5
1542	2 37 77 64	3 666 512 088	39.268 307 8	11.553 000 4
1543	2 38 08 49	3 673 650 007	39.281 038 7	11.555 497 3
1544	2 38 39 36	3 680 797 184	39.293 765 4	11.557 993 1
1545	2 38 70 25	3 687 953 625	39.306 488	11.560 487 8
1546	2 39 01 16	3 695 119 336	39.319 206 5	11.562 981 5
1547	2 39 32 09	3 702 294 323	39.331 920 8	11.565 474
1548	2 39 63 04	3 709 478 592	39.344 631 1	11.567 965 5
1549	2 39 94 01	3 716 672 149	39.357 337 3	11.570 455 9
1550	2 40 25 00	3 723 875 000	39.370 039 4	11.572 945 3
1551	2 40 56 01	3 731 087 151	39.382 737 3	11.575 433 6
1552	2 40 87 04	3 738 308 608	39.395 431 2	11.577 920 8
1553	2 41 18 09	3 745 539 377	39.408 121	11.580 406 9
1554	2 41 49 16	3 752 779 464	39.420 806 7	11.582 891 9
1555	2 41 80 25	3 760 028 875	39.433 488 3	11.585 375 9
1556	2 42 11 36	3 767 287 616	39.446 165 8	11.587 858 8
1557	2 42 42 49	3 774 555 693	39.458 839 3	11.590 340 7
1558	2 42 73 64	3 781 833 112	39.471 508 7	11.592 821 5
1559	2 43 04 81	3 789 119 879	39.484 174	11.595 301 3
1560	2 43 36 00	3 796 416 000	39.496 835 3	11.597 779 9
1561	2 43 67 21	3 803 721 481	39.509 492 5	11.600 257 6
1562	2 43 98 44	3 811 036 328	39.522 145 7	11.602 734 2
1563	2 44 29 69	3 818 360 547	39.534 794 8	11.605 209 7
1564	2 44 60 96	3 825 641 144	39.547 439 9	11.607 684 1
1565	2 44 92 25	3 833 037 125	39.560 080 9	11.610 157 5
1566	2 45 23 56	3 840 389 496	39.572 717 9	11.612 629 9

Number.	Square.	Cube.	Square Root.	Cube Root.
1567	2 45 54 89	3 847 751 263	39.585 350 8	11.615 101 2
1568	2 45 86 24	3 855 123 432	39.597 979 7	11.617 571 5
1569	2 46 17 61	3 862 503 009	39.610 604 6	11.620 040 7
1570	2 46 49 00	3 869 893 000	39.623 225 5	11.622 508 8
1571	2 46 80 41	3 877 292 411	39.635 842 4	11.624 975 9
1572	2 47 11 84	3 884 701 248	39.648 455 2	11.627 442
1573	2 47 43 29	3 892 119 517	39.661 064	11.629 907
1574	2 47 74 76	3 899 547 224	39.673 668 8	11.632 371
1575	2 48 06 25	3 906 984 375	39.686 269 6	11.634 833 9
1576	2 48 37 76	3 914 430 976	39.698 866 5	11.637 295 7
1577	2 48 69 29	3 921 887 033	39.711 459 3	11.639 756 6
1578	2 49 00 84	3 929 352 552	39.724 048 1	11.642 216 4
1579	2 49 32 41	3 936 827 539	39.736 632 9	11.644 675 1
1580	2 49 64 00	3 944 312 000	39.749 213 8	11.647 132 9
1581	2 49 95 61	3 951 805 941	39.761 790 7	11.649 589 5
1582	2 50 27 24	3 959 309 368	39.774 363 6	11.652 045 2
1583	2 50 58 89	3 966 822 287	39.786 932 5	11.654 499 8
1584	2 50 90 56	3 974 344 704	39.799 497 5	11.656 953 4
1585	2 51 22 25	3 981 876 625	39.812 058 5	11.659 405 9
1586	2 51 53 96	3 989 418 056	39.824 615 5	11.661 857 4
1587	2 51 85 69	3 996 969 003	39.837 168 6	11.664 307 9
1588	2 52 17 44	4 004 529 472	39.849 717 7	11.666 757 4
1589	2 52 49 21	4 012 099 469	39 862 262 8	11.669 205 8
1590	2 52 81 00	4 019 679 000	39.874 804	11.671 653 2
1591	2 53 12 81	4 027 268 071	39.887 341 3	11.674 099 6
1592	2 53 44 64	4 034 866 688	39.899 874 7	11.676 544 9
1593	2 53 76 49	4 042 474 857	39.912 404 1	11.678 989 2
1594	2 54 08 36	4 050 092 584	39 924 929 5	11.681 432 5
1595	2 54 40 25	4 057 719 875	39 937 451 1	11.683 874 8
1596	2 54 72 16	4 065 356 736	39.949 968 7	11.686 316 1
1597	2 55 04 09	4 073 003 173	39.962 482 4	11.688 756 3
1598	2 55 36 04	4 080 659 192	39.974 992 2	11.691 195 5
1599	2 55 68 01	4 088 324 799	39.987 498	11.693 633 7
1600	2 56 00 00	4 096 000 000	40	11.696 070 9

Uses of preceding table may be greatly extended by aid of following Rules:

To Compute Square or Cube of a higher Number than is contained in Table.

When Number is divisible by a Number without leaving a Remainder.

Rule.—If number exceed by 2, 3, or any other number of times, any number contained in table, multiply square or cube of that number in table by square of 2, 3, etc., and product will give result.

Example.—Required square of 1700.

1700 is 10 *times* 170, and square of 170 is 2 89 00.

Then, $2\,89\,00 \times 10^2 = 2\,89\,00\,00$.

2.—What is cube of 2400?

2400 is twice 1200, and cube of 1200 is 1 728 000 000.

Then $1\,728\,000\,000 \times 2^3 = 13\,824\,000\,000$.

When Number is an Odd Number.

Rule.—Take the two numbers nearest to each other, which, added together, make that sum; then from sum of squares or cubes of these two numbers, multiplied by 2, subtract 1, and remainder will give result.

EXAMPLE.—What is square of 1745?

Two nearest numbers are $\left\{\begin{matrix}873\\872\end{matrix}\right\} = 1745$.

Then, per table, $\left\{\begin{matrix}873^2 = 76\,21\,29\\872^2 = \underline{76\,03\,84}\end{matrix}\right.$

$1\,52\,25\,13 \times 2 = 3\,045\,026 - 1 = 3\,04\,50\,25$.

To Compute Square or Cube Root of a higher Number than is contained in Table.

When Number is divisible by 4 or 8 without leaving a Remainder.

RULE.—Divide number by 4 or 8 respectively, as square or cube root is required; take root of quotient in table, multiply it by 2, and product will give root required.

EXAMPLE.—What are square and cube roots of 3200?

$3200 \div 4 = 800$, and $3200 \div 8 = 400$.

Then, square root for 800, per table, is 28.28 42 71 2, which, being $\times 2 = 56.56\,85\,42\,4$ *root.*

Cube root for 400, per table, is 7.368 063, which, being $\times 2 = 14.736\,126$ *root.*

When the Root (which is taken as Number) does not exceed 1600.

The *Numbers* in table are roots of squares or cubes, which are to be taken as numbers.

ILLUSTRATION.—Square root of 6400 is 80, and cube root of 512 000 is 80.

When a Number has Three or more Ciphers at its right hand.

RULE.—Point off number into periods of two or three figures each, according as square or cube root is required, until remaining figures come within limits of table; then take root for these figures, and remove decimal point one figure for every period pointed off.

EXAMPLE.—What are square or cube roots of 1 500 000?

$1\,5\dot{0}0\,0\dot{0}0 = 150$, remaining figure, square root of which $= 12.247\,45$; hence 1224.745, *square root.*

$1\,50\dot{0}\,00\dot{0} = 1500$, remaining figures, cube root of which $= 11.447\,14$; hence 114.4714, *cube root.*

To Ascertain Cube Root of any Number over 1600.

RULE.—Find by table nearest cube to number given, and term it assumed cube; multiply it and given number respectively by 2; to product of assumed cube add given number, and to product of given number add assumed cube.

Then, as sum of assumed cube is to sum of given number, so is root of assumed cube to root of given number.

EXAMPLE.—What is cube root of 224 809?

By table, nearest cube is 216 000, and its root is 60.

$216\,000 \times 2 + 224\,809 = 656\,809$,
And $224\,809 \times 2 + 216\,000 = 665\,618$.

Then $656\,809 : 665\,618 :: 60 : 60.804+$, *root.*

To Ascertain Square or Cube Root of a Number consisting of Integers and Decimals.

RULE.—Multiply difference between root of integer part and root of next higher integer by decimal, and add product to root of integer given; the sum will give root of number required.

This is correct for Square root to three places of decimals, and for Cube root to seven.

EXAMPLE.—What is square root of 53.75, and cube root of 843.75?

$\sqrt{54} = 7.3484$	$\sqrt[3]{844} = 9.4503$
$\sqrt{53} = 7.2801$	$\sqrt[3]{843} = 9.4466$
$.0683$	$.0037$
$.75$	$.75$
$.051\,225$	$.002\,775$
$\sqrt{53} = 7.2801$	$\sqrt[3]{843} = 9.4466$
$\sqrt{53.75} = 7.331\,325$	$\sqrt[3]{843.75} = 9.449\,375$

When the Square or Cube Root is required for Numbers not exceeding Roots given in Table.

Numbers in table are squares and cubes of roots.

RULE.—Find, by table, in column of numbers that number representing figures of integer and decimals for which root is required, and point it off decimally by places of 2 or 3 figures as square or cube root is required; and opposite to it, in column of roots, take root and point off 1 or 2 additional places of decimals to those in root, as square or cube root is required, and result is root required.

EXAMPLE 1.—What are square roots of .15, 1.50, and 15.00?

In table, 15 has for its root 3.87 298; hence .38 72 98 = *square root for* .15.
150 has for its root 12.24 745; hence 1.22 47 45 = *square root for* 1.50.
1500 has for its root 38.72 98; hence 3.87 29 8 = *square root for* 15.

2.—What are cube roots of .15, 1.50, and 15.00?

Add a cipher to each, to give the numbers three places of figures, as $.15\dot{0}$, $1.50\dot{0}$, and $15.00\dot{0}$.

In table $15\dot{0}$ has for its root 5.3133; hence .531 33 = *cube root of* .15.
$\dot{1}500$ has for its root 11.447; hence 1.1447 = *cube root of* 1.50.
15 has for its root 2.4662; and 15.000, by addition of 3 places of figures, has 24.662; hence 2.4662 = *cube root of* 15.00.

To Ascertain Square or Cube Roots of Decimals alone.

RULE.—Point off number from decimal point into periods of two or three figures each, as square or cube root is required. Ascertain from table or by calculation root of number corresponding to decimal given, the same being read off by removing the decimal point one place to left for every period of 2 figures if square root is required, and one place for every period of 3 figures if cube root is required.

EXAMPLE.—What are square and cube roots of .810, .081, and .0081?

.810, when pointed off $= .8\dot{1}$, and $\sqrt{.8\dot{1}} = .9$.
.081, " " " $= .0\dot{8}1$, " $\sqrt{.0\dot{8}1} = .2846$.
.0081, " " " $= .0\dot{0}8\dot{1}$, " $\sqrt{.0\dot{0}8\dot{1}} = .09$.

.810, when pointed off $= .81\dot{0}$, and $\sqrt[3]{.81\dot{0}} = 932\,17$.
.081, " " " $= .08\dot{1}$, " $\sqrt[3]{.08\dot{1}} = .432\,67$.
.0081, " " " $= .00\dot{8}1$, " $\sqrt[3]{.00\dot{8}1} = .200\,83$.

To Compute 4th Root of a Number.

RULE.—Take square root of its square root.

EXAMPLE.—What is the $\sqrt[4]{}$ of 1600?

$$\sqrt{1600} = 40, \text{ and } \sqrt{40} = 6.32\,45\,55\,3.$$

To Compute 6th Root of a Number.

RULE.—Take cube root of its square root.

EXAMPLE.—What is the $\sqrt[6]{}$ of 441?

$$\sqrt{441} = 21, \text{ and } \sqrt[3]{21} = 2.7\,589\,243.$$

4th and 5th Powers of Numbers.

From 1 to 150.

Number.	4th Power.	5th Power.	Number.	4th Power.	5th Power.
1	1	1	64	16 777 216	1 073 741 824
2	16	32	65	17 850 625	1 160 290 625
3	81	243	66	18 974 736	1 252 332 576
4	256	1 024	67	20 151 121	1 350 125 107
5	625	3 125	68	21 381 376	1 453 933 568
6	1 296	7 776	69	22 667 121	1 564 031 349
7	2 401	16 807	70	24 010 000	1 680 700 000
8	4 096	32 768	71	25 411 681	1 804 229 351
9	6 561	59 049	72	26 873 856	1 934 917 632
10	10 000	100 000	73	28 398 241	2 073 071 593
11	14 641	161 051	74	29 986 576	2 219 006 624
12	20 736	248 832	75	31 640 625	2 373 046 875
13	28 561	371 293	76	33 362 176	2 535 525 376
14	38 416	537 824	77	35 153 041	2 706 784 157
15	50 625	759 375	78	37 015 056	2 887 174 368
16	65 536	1 048 576	79	38 950 081	3 077 056 399
17	83 521	1 419 857	80	40 960 000	3 276 800 000
18	104 976	1 889 568	81	43 046 721	3 486 784 401
19	130 321	2 476 099	82	45 212 176	3 707 398 432
20	160 000	3 200 000	83	47 458 321	3 939 040 643
21	194 481	4 084 101	84	49 787 136	4 182 119 424
22	234 256	5 153 632	85	52 200 625	4 437 053 125
23	279 841	6 436 343	86	54 708 016	4 704 270 176
24	331 776	7 962 624	87	57 289 761	4 984 209 207
25	390 625	9 765 625	88	59 969 536	5 277 319 168
26	456 976	11 881 376	89	62 742 241	5 584 059 449
27	531 441	14 348 907	90	65 610 000	5 904 900 000
28	614 656	17 210 368	91	68 574 961	6 240 321 451
29	707 281	20 511 149	92	71 639 296	6 590 815 232
30	810 000	24 300 000	93	74 805 201	6 956 883 693
31	923 521	28 629 151	94	78 074 896	7 339 040 224
32	1 048 576	33 554 432	95	81 450 625	7 737 809 375
33	1 185 921	39 135 393	96	84 934 656	8 153 726 976
34	1 336 336	45 435 424	97	88 529 281	8 587 340 257
35	1 500 625	52 521 875	98	92 236 816	9 039 207 968
36	1 679 616	60 466 176	99	96 059 601	9 509 900 499
37	1 874 161	69 343 957	100	100 000 000	10 000 000 000
38	2 085 136	79 235 168	101	104 060 401	10 510 100 501
39	2 313 441	90 224 199	102	108 243 216	11 040 808 032
40	2 560 000	102 400 000	103	112 550 881	11 592 740 743
41	2 825 761	115 856 201	104	116 985 856	12 166 529 024
42	3 111 696	130 691 232	105	121 550 625	12 762 815 625
43	3 418 801	147 008 443	106	126 247 696	13 382 255 776
44	3 748 096	164 916 224	107	131 079 601	14 025 517 307
45	4 100 625	184 528 125	108	136 048 896	14 693 280 768
46	4 477 456	205 962 976	109	141 158 161	15 386 239 549
47	4 879 681	229 345 007	110	146 410 000	16 105 100 000
48	5 308 416	254 803 968	111	151 807 041	16 850 581 551
49	5 764 801	282 475 249	112	157 351 936	17 623 416 832
50	6 250 000	312 500 000	113	163 047 361	18 424 351 793
51	6 765 201	345 025 251	114	168 896 016	19 254 145 824
52	7 311 616	380 204 032	115	174 900 625	20 113 581 875
53	7 890 481	418 195 493	116	181 063 936	21 003 416 576
54	8 503 056	459 165 024	117	187 388 721	21 924 480 357
55	9 150 625	503 284 375	118	193 877 776	22 877 577 568
56	9 834 496	550 731 776	119	200 533 921	23 863 536 599
57	10 556 001	601 692 057	120	207 360 000	24 883 200 000
58	11 316 496	656 356 768	121	214 358 881	25 937 424 601
59	12 117 361	714 924 299	122	221 533 456	27 027 081 632
60	12 960 000	777 600 000	123	228 886 641	28 153 056 843
61	13 845 841	844 596 301	124	236 421 376	29 316 250 624
62	14 776 336	916 132 832	125	244 140 625	30 517 578 125
63	15 752 961	992 436 543	126	252 047 376	31 757 969 376

Number.	4th Power.	5th Power.	Number.	4th Power.	5th Power.
127	260 144 641	33 038 369 407	139	373 301 041	51 888 844 699
128	268 435 456	34 359 738 368	140	384 160 000	53 782 400 000
129	276 922 881	35 723 051 649	141	395 254 161	55 730 836 701
130	285 610 000	37 129 300 000	142	406 586 896	57 735 339 232
131	294 499 921	38 579 489 651	143	418 161 601	59 797 108 943
132	303 595 776	40 074 642 432	144	429 981 696	61 917 364 224
133	312 900 721	41 615 795 893	145	442 050 625	64 097 340 625
134	322 417 936	43 204 003 424	146	454 371 856	66 338 290 976
135	332 150 625	44 840 334 375	147	466 948 881	68 641 485 507
136	342 102 016	46 525 874 176	148	479 785 216	71 008 211 968
137	352 275 361	48 261 724 457	149	492 884 401	73 439 775 749
138	362 673 936	50 049 003 168	150	506 250 000	75 937 500 000

To Compute 4th Power of a Number greater than is contained in Table.

RULE.—Ascertain square of number by preceding table or by calculation, and square it; product is power required.

EXAMPLE.—What is 4th power of 1500?

$$1500^2 = 2\,250\,000, \text{ and } 2\,250\,000^2 = 5\,062\,500\,000\,000.$$

To Compute 5th Power of a Number greater than is contained in Table.

RULE.—Ascertain cube of number by preceding table or by calculation, and multiply it by its square; product is power required.

To Compute 4th and 5th Powers by another Method.

RULE.—Reduce number by 2 until it is one contained within table Take power which is required of that number, and multiply it by 16, 16^2, or 16^3 respectively for each division, by 2 for 4th power, and by 32, 32^2, or 32^3 respectively for each division by 2 for 5th power.

EXAMPLE.—What are the 4th and 5th powers of 600?

$$600 \div 2 = 300, \text{ and } 300 \div 2 = 150.$$

The 4th power of 150, per table, = 506 250 000, which $\times\ 16^2$, *multiplier for a second division* 256 = 129 600 000 000, *4th power.*

Again, the 5th power of 150 = 75 937 500 000, which $\times\ 32^2$, *multiplier for a second division* 1024 = 77 760 000 000 000 = *power.*

To Compute 6th Power of a Number.

RULE.—Square its cube.

EXAMPLE.—What is the 6th power of 2?

$$\overline{2^3}^2 = 64.$$

To Compute 4th or 5th Root of a Number per Table.

RULE.—Find in column of 4th and 5th powers number given, and number from which that power is derived will give root required.

EXAMPLE.—What is the 5th root of 3 200 000?

3 200 000 in table is 5th power of 20; hence 20 is root required.

RECIPROCALS.

Reciprocal of a number is quotient arising from dividing 1 by number; thus, reciprocal of 2 is $1 \div 2 = .5$

Product of a number and its reciprocal is always equal to 1; thus, $2 \times .5 = 1$.

Reciprocal of a vulgar fraction is denominator divided by numerator; thus, $\frac{2}{1} = .5$.

LOGARITHMS.

Logarithms of Numbers.

Logarithms are a series of numbers adapted to facilitate the operation of numerical computation,

Addition being substituted for Multiplication, Subtraction for Division, Multiplication for Involution, and Division for Evolution.

The **Logarithm** of a number is the exponent of a power to which 10 must be raised to give that number.

It is not necessary, however, that the *base* should be 10, it may be any other number; but Tables of Logarithms, in common use, are computed with 10 as the base.

Thus, Number 100 Log. $= 2$, as 10^2 base and exponent $= 100$.
" 10000 " $= 4$, " 10^4 " " " $= 10000$.

The *Unit* or *Integral part* of a Logarithm is termed the *Index*, and the *Decimal part* the *Mantissa*; the sum of the index and mantissa is the *Logarithm*.

The Index of the Logarithm of any number, Integral or Mixed, when the base is 10, is equal to the number of digits to the left of the decimal point less 1. From 0 to 9, it is 0; from 10 to 99, it is 1, and from 100 to 999, it is 2, etc.

Thus, logarithm of $3304 = 3.51904$, 3 being the *index* and .51904 the *mantissa*.

The Index of the Logarithm of a Decimal Fraction is a negative number, and is equal to the number of places which the first significant figure of the decimal is removed from the place of units.

Thus, index of logarithm .005 is $\bar{3}$ or -3, the first significant figure, 5, being removed three places from that of units. The bar or minus sign is placed over an index to indicate that this alone is negative, while the decimal part is positive.

The *Difference* is the tabular difference between the two nearest logarithms.

The *Proportional Part* is the difference between the given and the nearest less tabular logarithm.

The *Arithmetical Complement* of a number is the remainder after subtracting it from a number consisting of 1, with as many ciphers annexed as the number has integers. When the index of a logarithm is less than 10, its complement is ascertained by subtracting it from 10.

Illustrations.

Number.	Logarithm.	Number.	Logarithm.
4743	3.676053	.4743	$\bar{1}.676053$
474.3	2.676053	.04743	$\bar{2}.676053$
47.43	1.676053	.004743	$\bar{3}.676053$
4.743	.676053		

Computation of Negative Indices.

To add two Negative Indices. Add them and put the sum negative. As $\bar{5} + \bar{3} = \bar{8}$.

To add a Positive and Negative Index. Subtract the less from the greater, and to remainder give the positive or negative sign, according as the positive or negative *index* is the greater. As $6 + \bar{2} = 4$, and $\bar{6} + 2 = \bar{4}$.

ILLUSTRATION.—Add 6.387 57 and $\bar{2}.924\,59$.

$$\begin{array}{r} 6.387\,57 \\ \bar{2}.924\,59 \\ \hline 5.312\,16 \end{array}$$

Here the excess of 1 from 13 in the first decimal place, being positive, is carried to the positive 6, which makes 7, and $7 - 2 = 5$.

To Subtract a Negative Index. Change its sign to plus or positive, and then add it as in addition. As $\bar{3}$ from $2, = 3 + 2 = 5$. And 5 from $\bar{2}, = \bar{5} + \bar{2} = \bar{3}$; also $\bar{3}$ from $\bar{5}, = 3 + \bar{5} = \bar{2}$.

ILLUSTRATION.—Subtract $\bar{5}.765\,52$ from $\bar{2}.346\,74$.

$$\begin{array}{r} \bar{2}.346\,74 \\ \bar{5}.765\,52 \\ \hline 2.581\,22 \end{array}$$

Here, excess of 1 in the first decimal place used with the .3 in subtracting the .8 from the 1.3 is to be subtracted from the upper number $\bar{2}$, which makes it $\bar{3}$; then $\bar{3} + 5 = 2$.

To Subtract a Positive Index. Change its sign to negative, and then add as in addition. As $\bar{2} - 2 = \bar{2} + \bar{2} = \bar{4}$.

To Multiply a Negative Index. Multiply the fractional parts by the ordinary rule, then multiply the negative *index*, which will give a negative product, and when an excess over 10 is to be carried, subtract the less index from the greater, and the remainder gives the positive or negative *index*, according as the positive or negative index is the greater. As $\bar{2} \times 5 = \overline{10}$, and 1 to be carried $= \bar{9}$.

ILLUSTRATION.—Multiply $\bar{2}.3681$ by 2, and $\bar{3}.7856$ by 6.

$$\begin{array}{r} \bar{2}.3681 \\ 2 \\ \hline \bar{4}.7362 \end{array} \qquad \begin{array}{r} \bar{3}.7856 \\ 6 \\ \hline \overline{14}.7136 \end{array}$$

Here $\bar{2} \times 2 = \bar{4}$, also $\bar{3} \times 6 = \overline{18}$, with a positive excess of $4 = \overline{14}$.

To Divide a Negative Index. If *index* is divisible by divisor, without a remainder, put quotient with a negative sign. If negative exponent is not divisible by divisor, add such a negative number to it as will make it divisible, and prefix an equal positive integer to fractional part of logarithm; then divide increased negative exponent and the other part of logarithm separately by ordinary rules, and former quotient, taken negatively, will be *index* to fractional part of quotient. As $\bar{6} \div 3 = \bar{2}$. $\overline{10} \div 3$ requires $\bar{2}$ to be added or 2 to be subtracted, to make it divisible without a remainder, then $\overline{10} + \bar{2} = \overline{12}$, $\overline{12} \div 3 = \bar{4}$, and 2 (the sum subtracted) $\div 3 = .66$, the quotient therefore is $\bar{4}.66$.

ILLUSTRATION 1.—Divide $\bar{6}.324\,282$ by 3.

$$\bar{6}.324\,282 \div 3 = \bar{2}.108\,094.$$

2.—Divide $\overline{14}.326\,745$ by 9.

$$\overline{14}.326\,745 \div 9 = \overline{18} + 4.326\,745 \div 9 = \bar{2}.480\,749+.$$

Here $\bar{4}$ is added to $\overline{14}$, that the sum $\overline{18}$ may be divided by 9, and as $\bar{4}$ is added, 4 must be prefixed to the fractional part of the logarithm, and thus the value of the logarithm is unchanged, for there is added $\bar{4}$, and $\bar{4} + 4 = 0$, or 4 is subtracted and 4 added.

To Ascertain Logarithm of a Number by Table.

When the Number is less than 101.

Look into first page of table, and opposite to number is its logarithm with its index prefixed.

ILLUSTRATION.—Opposite 7 is .845 098, its logarithm; hence 70 = 1.845 098, .7 = $\bar{1}$.845 098, and .07 = $\bar{2}$.845 098.

When the Number is between 100 *and* 1000.

RULE.—Find the given number in left-hand column of table headed No., and under 0 in next column is decimal part of its logarithm, to which is to be prefixed a whole number for an *index*, of 1 or 2, according as the number consists of 2 or 3 figures.

EXAMPLE.—What is logarithm of 450, and what of .45?

Log. 450 = 2.653 213, and of .45 = $\bar{1}$.653 213.

When the Number is between 1000 *and* 10 000.

RULE.—Find the three left-hand figures of the number in the left-hand column of the table headed No., and under the 4th figure at top of table is the four last figures of the decimal part of logarithm, to which is to be prefixed the proper *index*.

EXAMPLE.—What is logarithm of 4505, and what of .045 05?

Log. 4505 = 3.653 695, and of .045 05 = $\bar{2}$.653 695.

When the Number consists of Five Figures.

RULE.—Find the logarithm of the number composed of the first four figures as preceding, then take the tabular difference from the right-hand column under D and multiply it by the fifth figure; reject the right-hand figure of the product and add the other figures, which are, and are termed, a proportional part to the logarithm found as above, observing that the right-hand figure of the proportional part is to be added to that of the logarithm, and the rest in order.

EXAMPLE.—Required logarithm of 83 407?

NOTE.—When the number consists of less than 4 figures conceive a cipher annexed to make it four.

Log. of 8340 (83 407) =	4.921 166
Tabular difference 52, which × 7 (5th figure) = 364 =	364
	4.921 202 4 *logarithm.*

The difference of the numbers is nearly proportionate to the difference of their logarithms.

Thus, difference between the numbers 8340 and 8341, the next in order, is 1, and the difference between their logarithms or tabular difference is 52.

The log. of this 1 in the 4th place is therefore 52. The correction then, for the 7 of the 5th place, which is .7 of 1 in the 4th place, is ascertained by the proportion 1 : 52 :: .7 : 36.4.

The correction is obtained by multiplying the tabular difference by 7, rejecting the right hand figure of the product, if the log. is to be confined to six decimal places.

When the Number consists of any Number over Four Figures.

RULE.—Proceed as for four figures for the first four, multiplying the tabular difference by the excess of figures over 4 and rejecting one right-hand figure of the product for a number of five figures, and two for one of six, and so on.

EXAMPLE 1.—Required logarithm of 834 079?

Log. of 8340 (834 079) =	5.921 166
Tabular difference 52, which × 79 =	4108
	5.921 207 08 *logarithm.*

2.—Required logarithm of 8 340 794?

Log. of 8340 (8 340 794) =	6.921 166
Tab. diff. 52, which × 794 (5th, 6th, and 7th figures) =	41 288
	6.921 207 288 *logarithm.*

Or, Log. of 8340 =	.921 166
" " 7 (5th figure) × 52 tab. dif. =	364
" " 9 (6th ") × 52 " " =	468
" " 4 (7th ") × 52 " " =	208
Log. with *index* for 7 figures	6.921 207 288

To Ascertain Logarithm of a Mixed Number.

RULE.—Take out logarithm of the number as if it were an integer or whole number, to which prefix the *index* of the integral part of the number.

EXAMPLE.—What is logarithm of 834.0794?

Mantissa of log. of 8 340 794 = 9 212 073; hence log. of 834.0794 = 2.921 207 3.

To Ascertain Logarithm of a Decimal Fraction.

RULE.—Take logarithm from table as if the figures were all integers, and prefix *index* as by previous rules.

EXAMPLE.—Logarithm of .1234 = $\bar{1}.091\,305$.

To Ascertain Logarithm of a Vulgar Fraction.

RULE.—Reduce the fraction to a decimal, and proceed as by preceding rule. Or, subtract logarithm of denominator from that of numerator, and the difference will give logarithm required.

EXAMPLE.—Logarithm of $\frac{3}{16}$?

$\frac{3}{16}$ = .1875. Log. .1875 = $\bar{1}.273\,001$ *logarithm.*

Or, Log. 3 =	.477 121
" 16 =	1.204 12
	$\bar{1}.273\,001$ *logarithm.*

To Ascertain the Number Corresponding to a Given Logarithm.

When the given or exact Logarithm is in the Table.

OPERATION.—Opposite to first two figures of logarithm, neglecting the *index*, in column 0, look for the remaining figures of the log. in that column or in any of the nine at the right thereof; the first three figures of the number will be found at the left in column under No., and the fourth at top directly over the log.

The number is to be made to correspond to *index* of logarithm, by pointing off decimals or prefixing ciphers.

ILLUSTRATION.—What is number corresponding to log. 3.963 977?

Opposite to 963 977, in page 329, is 920, and at top of column is 4; hence, number = 9204.

When the given or exact Logarithm is not in the Table.

OPERATION.—Take the number for the next less logarithm from table, which will give first four figures of required number.

To ascertain the other figures, subtract the logarithm in table from the given logarithm, add ciphers, and divide by the difference in column D opposite the logarithm. Annex quotient to the four figures already ascertained, and place decimal point.

ILLUSTRATION 1.—What is number corresponding to log. 5.921 207?

Given log. =	5.921 207	
Next less in table	5.921 166	834 0
	D = 52) 4100 (78 +	78
	364	834 078
	460	
	416	
Hence, number = 834 078.	44	

2.—What is number corresponding to log. 3.922 853?

Given log. =	3.922 853	
Next less in table	3.922 829	837 2
	D = 52) 2400 (46 +	46
	208	837 246
	320	
	312	
Hence, number = 8372.46.	8	

Multiplication.

RULE.—Add together the logarithms of the numbers and the sum will give the logarithm of the product.

EXAMPLE 1.—Multiply 345.7 by 2.581.

Log. 345.7 = 2.538 699
" 2.581 = .411 788
2.950 487 *log. of product.* Number = 892.251.

2.—Multiply .039 02, 59.71, and .003 147.

Log. .039 02 = $\bar{2}$.591 287
" 59.71 = 1.776 047
" .003 147 = $\bar{3}$.497 897
$\bar{3}$.865 231 *log. of product.* Number = .007 332 15.

Division.

RULE.—From logarithm of dividend subtract that of divisor, and remainder will give logarithm of the quotient.

EXAMPLE.—Divide 371.4 by 52.37.

Log. 371.4 = 2.569 842
" 52.37 = 1.719 083
.850 759 *log. of quotient.* Number = 7.091 85.

Rule of Three, or Proportion.

RULE.—Add together the logarithms of the second and third terms, from their sum subtract logarithm of the first, and the remainder will give logarithm of the fourth term.

Or, instead of subtracting logarithm of first term, add its *Arithmetical Complement*, and subtract 10 from its index.

EXAMPLE 1.—What is fourth proportional to 723.4, .025 19, and 3574?

As	723.4	log. =	2.859 379
Is to	.025 19	" =	$\bar{2}$.401 228
So is	3574	" =	3.553 155
			$\bar{1}$.954 383
First term		"	2.859 379
			$\bar{1}$.095 004 *log. of 4th term.* Number = .124 453.

By Arithmetical Complement.

ILLUSTRATION.—As 723.4 log. = 2.859 379, Ar. com. = $\underline{7}$.140 621

Is to .025 19	" =	2.401 228
So is 3574	" =	3.553 155
		$\bar{1}$.095 004 *log. of 4th term.*

Number = .124 453.

2.—If an engine of 67 HP can raise 57 600 cube feet of water in a given time, what HP is required to raise 8 575 000 cube feet in like time?

$$
\begin{array}{rl}
\text{Log. } 8\,575\,000 = & 6.933\,234 \\
\text{"} \quad 67 = & 1.826\,075 \\
\hline
& 8.759\,309 \\
\text{"} \quad 57\,600 = & 4.760\,422 \\
\hline
& 3.998\,877
\end{array}
$$

log. of 4th term. Number = 9974.4 *cube feet.*

3.—If 14 men in 47 days excavate 5631 cube yards, what time will it require to excavate 47 280 at same rate of excavation? 394.626 *days.*

Involution.

RULE.—Multiply logarithm of given number by exponent of the power to which it is to be raised, and the product will give the logarithm of the required power.

EXAMPLE.—What is cube of 30.71?

$$
\begin{array}{rl}
\text{Log. } 30.71 = & 1.487\,28 \\
& \quad\quad 3 \\
\hline
& 4.461\,84
\end{array}
$$

log. of power. Number = 28 962.73.

Evolution.

RULE.—Divide logarithm of given number by exponent of the root which is to be extracted, and quotient will give logarithm of required root.

EXAMPLE 1.—What is cube root of 1234?

Log. 1234 = 3.091 315

Divide by 3 = 1.030 438 *log. of root.* Number = 10.726 01.

2.—What is 4th root of .007 654?

Log. .007 654 = $\bar{3}$.883 888

Divide by 4 (here $\bar{3} + \bar{1} + 1$) = $\bar{1}$.470 972 *log. of root.* Number = .295 78.

To Ascertain Reciprocal of a Number.

RULE.—Subtract decimal of logarithm of the number from .000 000; add 1 to index of logarithm and change its sign. The result is logarithm of the reciprocal.

EXAMPLE.—Required reciprocal of 230?

$$
\begin{array}{rl}
& .000\,000 \\
\text{Log. } 230 = & 2.361\,728 \\
\hline
& \bar{3}.638\,272
\end{array}
$$

= log. of .004 348 *reciprocal.*

Simple Interest.

Rule.—Add together logarithm of principal, rate per cent., and time in years, from the sum subtract 2, and the remainder will give logarithm of the interest.

Example.—What is interest on \$ 500, @ 6 per cent., for 3 years?

$$
\begin{aligned}
\text{Log. } 500 &= 2.698\,97 \\
6 &= .778\,151 \\
3 &= \underline{.477\,121} \\
& \ 3.954\,242 \\
& \ \underline{2} \\
& \ \bar{1}.954\,242 \ \textit{log. of interest.} \quad \text{Number} = 90 \ \textit{dollars.}
\end{aligned}
$$

Compound Interest.

Rule.—Compute amount of \$ 1 or £ 1, etc., at the given rate of interest for one year for the first term, which is termed the *ratio*.

Multiply logarithm of *ratio* by the time, add to product logarithm of the principal, and sum is logarithm of the amount.

Logarithms of Ratios at given Rates Per Cent.

Rate.	Log. of Ratio.	Rate.	Log. of Ratio.	Rate.	Log. of Ratio.	Rate.	Log. of Ratio.
1	.004 321 4	3.25	.013 890 1	5.5	.023 252 5	7.75	.032 417 3
1.25	.005 395	3.5	.014 940 3	5.75	.024 280 4	8	.033 423 8
1.5	.006 466	3.75	.015 988 1	6	.025 305 9	8.25	.034 427 9
1.75	.007 534 4	4	.017 033 3	6.25	.026 328 9	8.5	.035 429 7
2	.008 600 2	4.25	.018 076 1	6.5	.027 349 6	8.75	.036 429 3
2.25	.009 663 3	4.5	.019 116 3	6.75	.028 363 9	9	.037 426 5
2.5	.010 723 9	4.75	.020 154	7	.029 383 8	9.25	.038 421 4
2.75	.011 781 8	5	.021 189 3	7.25	.030 397 3	9.5	.039 414 1
3	.012 837 2	5.25	.022 222 1	7.5	.031 408 5	9.75	.040 404 5

Example.—What will \$364, at 6 per cent. per annum, compounded yearly, amount to in 23 years?

$$
\begin{aligned}
&\text{Log. of ratio from above table} && .025\,305\,9 \\
& && \underline{23} \\
& && .542\,035\,7 \\
&\text{“ “ } 364 && \underline{2.561\,101} \\
& && 3.103\,136\,7 \ \textit{log. of amount.} \quad \text{Number} = 1268.05 \ \textit{doll.}
\end{aligned}
$$

Miscellaneous Illustrations.

1. What is area and circumference of a circle of 21.72 feet in diameter?

$$
\begin{aligned}
& && 1.336\,860 \\
& && \underline{2} \\
&\text{Log. of } 21.72^2 &&= 2.673\,720 \\
&\text{“ “ } .7854 &&= \underline{\bar{1}.895\,091} \\
&\text{“ “} && \ 2.568\,811 = 370.54 \ \textit{feet area.} \\
&\text{Log. of } 21.72 &&= 2.336\,86 \\
&\text{“ “ } 3.1416 &&= \underline{.497\,15} \\
&\text{“ “} && \ 1.839\,71 = 68.236 \ \textit{feet circum.}
\end{aligned}
$$

2. Sides of a triangle are 564, 373, and 747 feet; what is its area?

$$
\begin{aligned}
&\text{Log. of sides } \frac{564 + 373 + 747}{2} &&= 2.925\,312 \\
&\text{“ “ } .5 \text{ side} - a = 842 - 564 &&= 2.444\,045 \\
&\text{“ “ } .5 \text{ side} - b = 842 - 373 &&= 2.671\,173 \\
&\text{“ “ } .5 \text{ side} - c = 842 - 747 &&= \underline{1.977\,724} \\
& && 2)\overline{10.018\,254} \\
&\text{Area} = \text{Number of} && \ 5.009\,127 = 1021.24 \ \textit{feet.}
\end{aligned}
$$

3.—What is logarithm of $8^{3.6}$?

$$\text{Log. } \frac{8 \times 36}{10} = \frac{36}{10} \times \text{log. } 8 = 3.6 \times .903\,09 = 3.251\,124. \quad \text{Number} = 1782.89.$$

Logarithms of Numbers.

From 1 *to* 10 000.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
1	.0	26	1.414 973	51	1.707 57	76	1.880 814
2	.301 03	27	1.431 364	52	1.716 003	77	1.886 491
3	.477 121	28	1.447 158	53	1.724 276	78	1.892 095
4	.602 06	29	1.462 398	54	1.732 394	79	1.897 627
5	.698 97	30	1.477 121	55	1.740 363	80	1.903 09
6	.778 151	31	1.491 362	56	1.748 188	81	1.908 485
7	.845 098	32	1.505 15	57	1.755 875	82	1.913 814
8	.903 09	33	1.518 514	58	1.763 428	83	1.919 078
9	.954 243	34	1.531 479	59	1.770 852	84	1.924 279
10	1	35	1.544 068	60	1.778 151	85	1.929 419
11	1.041 393	36	1.556 303	61	1.785 33	86	1.934 498
12	1.079 181	37	1.568 202	62	1.792 392	87	1.939 519
13	1.113 943	38	1.579 784	63	1.799 341	88	1.944 483
14	1.146 128	39	1.591 065	64	1.806 18	89	1.949 39
15	1.176 091	40	1.602 06	65	1.812 913	90	1.954 243
16	1.204 12	41	1.612 784	66	1.819 544	91	1.959 041
17	1.230 449	42	1.623 249	67	1.826 075	92	1.963 788
18	1.255 273	43	1.633 468	68	1.832 509	93	1.968 483
19	1.278 754	44	1.643 453	69	1.838 849	94	1.973 128
20	1.301 03	45	1.653 213	70	1.845 098	95	1.977 724
21	1.322 219	46	1.662 758	71	1.851 258	96	1.982 271
22	1.342 423	47	1.672 098	72	1.857 332	97	1.986 772
23	1.361 728	48	1.681 241	73	1.863 323	98	1.991 226
24	1.380 211	49	1.690 196	74	1.869 232	99	1.995 635
25	1.397 94	50	1.698 97	75	1.875 061	100	2

No.	0	1	2	3	4	5	6	7	8	9	D
100	00- 0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	00- 4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	00- 86	9026	9451	9876	—	—	—	—	—	—	425
102	01- —	—	—	—	03	0724	1147	157	1993	2415	424
103	01- 2837	3259	368	41	4521	494	536	5779	6197	6616	420
104	01- 7033	7451	7868	8284	87	9116	9532	9947	—	—	417
104	02- —	—	—	—	—	—	—	—	0361	0775	416
105	02- 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	02- 5306	5715	6125	6533	6942	735	7757	8164	8571	8978	408
107	02- 9384	9789	—	—	—	—	—	—	—	—	405
107	03- —	—	0195	06	1004	1408	1812	2216	2619	3021	404
108	03- 3424	3826	4227	4628	5029	543	583	623	6629	7028	400
109	03- 7426	7825	8223	862	9017	9414	9811	—	—	—	398
109	04- —	—	—	—	—	—	—	0207	0602	0998	397
110	04- 1393	1787	2182	2576	2969	3362	3755	4148	454	4932	393
111	04- 5323	5714	6105	6495	6885	7275	7664	8053	8442	883	389
112	04- 9218	9606	9993	—	—	—	—	—	—	—	388
112	05- —	—	—	038	0766	1153	1538	1924	2309	2694	386
113	05- 3078	3463	3846	423	4613	4996	5378	576	6142	6524	383
114	05- 6905	7286	7666	8046	8426	8805	9185	9563	9942	—	383
114	06- —	—	—	—	—	—	—	—	—	032	379
No.	0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
115	06-	0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	06-	4458	4832	5206	558	5953	6326	6699	7071	7443	7815	373
117	06-	8186	8557	8928	9298	9668	—	—	—	—	—	380
117	07-	—	—	—	—	—	0038	0407	0776	1145	1514	370
118	07-	1882	225	2617	2985	3352	3718	4085	4451	4816	5182	366
119	07-	5547	5912	6276	664	7004	7368	7731	8094	8457	8819	363
120	07-	9181	9543	9904	—	—	—	—	—	—	—	362
120	08-	—	—	—	0266	0626	0987	1347	1707	2067	2426	360
121	08-	2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	08-	636	6716	7071	7426	7781	8136	849	8845	9198	9552	355
123	08-	9905	—	—	—	—	—	—	—	—	—	355
123	09-	—	0258	0611	0963	1315	1667	2018	237	2721	3071	353
124	09-	3422	3772	4122	4471	482	5169	5518	5866	6215	6562	349
125	09-	691	7257	7604	7951	8298	8644	899	9335	9681	—	348
125	10-	—	—	—	—	—	—	—	—	—	0026	346
126	10-	0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	10-	3804	4146	4487	4828	5169	551	5851	6191	6531	6871	341
128	10-	721	7549	7888	8227	8565	8903	9241	9579	9916	—	338
128	11-	—	—	—	—	—	—	—	—	—	0253	337
129	11-	059	0926	1263	1599	1934	227	2605	294	3275	3609	335
130	11-	3943	4277	4611	4944	5278	5611	5943	6276	6608	694	333
131	11-	7271	7603	7934	8265	8595	8926	9256	9586	9915	—	331
131	12-	—	—	—	—	—	—	—	—	—	0245	330
132	12-	0574	0903	1231	156	1888	2216	2544	2871	3198	3525	328
133	12-	3852	4178	4504	483	5156	5481	5806	6131	6456	6781	325
134	12-	7105	7429	7753	8076	8399	8722	9045	9368	969	—	323
134	13-	—	—	—	—	—	—	—	—	—	0012	323
135	13-	0334	0655	0977	1298	1619	1939	226	258	29	3219	321
136	13-	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	13-	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	13-	9879	—	—	—	—	—	—	—	—	—	315
138	14-	—	0194	0508	0822	1136	145	1763	2076	2389	2702	314
139	14-	3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	14-	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	14-	9219	9527	9835	—	—	—	—	—	—	—	308
141	15-	—	—	—	0142	0449	0756	1063	137	1676	1982	307
142	15-	2288	2594	29	3205	351	3815	412	4424	4728	5032	305
143	15-	5336	564	5943	6246	6549	6852	7154	7457	7759	8061	303
144	15-	8362	8664	8965	9266	9567	9868	—	—	—	—	302
144	16-	—	—	—	—	—	—	0168	0469	0769	1068	301
145	16-	1368	1667	1967	2266	2564	2863	3161	346	3758	4055	299
146	16-	4353	465	4947	5244	5541	5838	6134	643	6726	7022	297
147	16-	7317	7613	7908	8203	8497	8792	9086	938	9674	9968	295
148	17-	0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	17-	3186	3478	3769	406	4351	4641	4932	5222	5512	5802	291
150	17-	6091	6381	667	6959	7248	7536	7825	8113	8401	8689	289
151	17-	8977	9264	9552	9839	—	—	—	—	—	—	287
151	18-	—	—	—	—	0126	0413	0699	0986	1272	1558	287
152	18-	1844	2129	2415	27	2985	327	3555	3839	4123	4407	285
153	18-	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	18-	7521	7803	8084	8366	8647	8928	9209	949	9771	—	281
154	19-	—	—	—	—	—	—	—	—	—	0051	281
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
155	19-	0332	0612	0892	1171	1451	173	201	2289	2567	2846	279
156	19-	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	19-	59	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	19-	8657	8932	9206	9481	9755	—	—	—	—	—	275
158	20-	—	—	—	—	—	0029	0303	0577	085	1124	274
159	20-	1397	167	1943	2216	2488	2761	3033	3305	3577	3848	272
160	20-	412	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	20-	6826	7096	7365	7634	7904	8173	8441	871	8979	9247	269
162	20-	9515	9783	—	—	—	—	—	—	—	—	268
162	21-	—	—	0051	0319	0586	0853	1121	1388	1654	1921	267
163	21-	2188	2454	272	2986	3252	3518	3783	4049	4314	4579	266
164	21-	4844	5109	5373	5638	5902	6166	643	6694	6957	7221	264
165	21-	7484	7747	801	8273	8536	8798	906	9323	9585	9846	262
166	22-	0108	037	0631	0892	1153	1414	1675	1936	2196	2456	261
167	22-	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	22-	5309	5568	5826	6084	6342	66	6858	7115	7372	763	258
169	22-	7887	8144	84	8657	8913	917	9426	9682	9938	—	257
169	23-	—	—	—	—	—	—	—	—	—	0193	256
170	23-	0449	0704	096	1215	147	1724	1979	2234	2488	2742	255
171	23-	2996	325	3504	3757	4011	4264	4517	477	5023	5276	253
172	23-	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	23-	8046	8297	8548	8799	9049	9299	955	98	—	—	251
173	24-	—	—	—	—	—	—	—	—	005	03	250
174	24-	0549	0799	1048	1297	1546	1795	2044	2293	2541	279	249
175	24-	3038	3286	3534	3782	403	4277	4525	4772	5019	5266	248
176	24-	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	24-	7973	8219	8464	8709	8954	9198	9443	9687	9932	—	246
177	25-	—	—	—	—	—	—	—	—	—	0176	245
178	25-	042	0664	0908	1151	1395	1638	1881	2125	2368	261	243
179	25-	2853	3096	3338	358	3822	4064	4306	4548	479	5031	242
180	25-	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	25-	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26-	0071	031	0548	0787	1025	1263	1501	1739	1976	2214	238
183	26-	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	26-	4818	5054	529	5525	5761	5996	6232	6467	6702	6937	235
185	26-	7172	7406	7641	7875	811	8344	8578	8812	9046	9279	234
186	26-	9513	9746	998	—	—	—	—	—	—	—	234
186	27-	—	—	—	0213	0446	0679	0912	1144	1377	1609	233
187	27-	1842	2074	2306	2538	277	3001	3233	3464	3696	3927	232
188	27-	4158	4389	462	485	5081	5311	5542	5772	6002	6232	230
189	27-	6462	6692	6921	7151	738	7609	7838	8067	8296	8525	229
190	27-	8754	8982	9211	9439	9667	9895	—	—	—	—	228
190	28-	—	—	—	—	—	—	0123	0351	0578	0806	228
191	28-	1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	28-	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	28-	5557	5782	6007	6232	6456	6681	6905	713	7354	7578	225
194	28-	7802	8026	8249	8473	8696	892	9143	9366	9589	9812	223
195	29-	0035	0257	048	0702	0925	1147	1369	1591	1813	2034	222
196	29-	2256	2478	2699	292	3141	3363	3584	3804	4025	4246	221
197	29-	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	29-	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	29-	8853	9071	9289	9507	9725	9943	—	—	—	—	218
199	30-	—	—	—	—	—	—	0161	0378	0595	0813	218
No.		0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
200	30- 103	1247	1464	1681	1898	2114	2331	2547	2764	298	217
201	30- 3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	30- 5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	30- 7496	771	7924	8137	8351	8564	8778	8991	9204	9417	213
204	30- 963	9843	—	—	—	—	—	—	—	—	213
204	31- —	—	0056	0268	0481	0693	0906	1118	133	1542	212
205	31- 1754	1966	2177	2389	26	2812	3023	3234	3445	3656	211
206	31- 3867	4078	4289	4499	471	492	513	534	5551	576	210
207	31- 597	618	639	6599	6809	7018	7227	7436	7646	7854	209
208	31- 8063	8272	8481	8689	8898	9106	9314	9522	973	9938	208
209	32- 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	32- 2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	32- 4282	4488	4694	4899	5105	531	5516	5721	5926	6131	205
212	32- 6336	6541	6745	695	7155	7359	7563	7767	7972	8176	204
213	32- 838	8583	8787	8991	9194	9398	9601	9805	—	—	204
213	33- —	—	—	—	—	—	—	—	0008	0211	203
214	33- 0414	0617	0819	1022	1225	1427	163	1832	2034	2236	202
215	33- 2438	264	2842	3044	3246	3447	3649	385	4051	4253	202
216	33- 4454	4655	4856	5057	5257	5458	5658	5859	6059	626	201
217	33- 646	666	686	706	726	7459	7659	7858	8058	8257	200
218	33- 8456	8656	8855	9054	9253	9451	965	9849	—	—	200
218	34- —	—	—	—	—	—	—	—	0047	0246	199
219	34- 0444	0642	0841	1039	1237	1435	1632	183	2028	2225	198
220	34- 2423	262	2817	3014	3212	3409	3606	3802	3999	4196	197
221	34- 4392	4589	4785	4981	5178	5374	557	5766	5962	6157	196
222	34- 6353	6549	6744	6939	7135	733	7525	772	7915	811	195
223	34- 8305	85	8694	888[illegible]	9083	9278	9472	9666	986	—	194
223	35- —	—	—	—	—	—	—	—	—	0054	194
224	35- 0248	0442	0636	0829	1023	1216	141	1603	1796	1989	193
225	35- 2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	35- 4108	4301	4493	4685	4876	5068	526	5452	5643	5834	192
227	35- 6026	6217	6408	6599	679	6981	7172	7363	7554	7744	191
228	35- 7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	35- 9835	—	—	—	—	—	—	—	—	—	189
229	36- —	0025	0215	0404	0593	0783	0972	1161	135	1539	189
230	36- 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	36- 3612	38	3988	4176	4363	4551	4739	4926	5113	5301	188
232	36- 5488	5675	5862	6049	6236	6423	661	6796	6983	7169	187
233	36- 7356	7542	7729	7915	8101	8287	8473	8659	8845	903	186
234	36- 9216	9401	9587	9772	9958	—	—	—	—	—	186
235	37- —	—	—	—	—	0143	0328	0513	0698	0883	185
235	37- 1068	1253	1437	1622	1806	1991	2175	236	2544	2728	184
236	37- 2912	3096	328	3464	3647	3831	4015	4198	4382	4565	184
237	37- 4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	37- 6577	6759	6942	7124	7306	7488	767	7852	8034	8216	182
239	37- 8398	858	8761	8943	9124	9306	9487	9668	9849	—	182
239	38- —	—	—	—	—	—	—	—	—	003	181
240	38- 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	38- 2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	38- 3815	3995	4174	4353	4533	4712	4891	507	5249	5428	179
243	38- 5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	38- 739	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
No.	0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
245	38- 9166	9343	952	9698	9875	—	—	—	—	—	177
245	39- —	—	—	—	—	0051	0228	0405	0582	0759	177
246	39- 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	39- 2697	2873	3048	3224	34	3575	3751	3926	4101	4277	176
248	39- 4452	4627	4802	4977	5152	5326	5501	5676	585	6025	175
249	39- 6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	39- 794	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	39- 9674	9847	—	—	—	—	—	—	—	—	173
251	40- —	—	002	0192	0365	0538	0711	0883	1056	1228	173
252	40- 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	40- 3121	3292	3464	3635	3807	3978	4149	432	4492	4663	171
254	40- 4834	5005	5176	5346	5517	5688	5858	6029	6199	637	171
255	40- 654	671	6881	7051	7221	7391	7561	7731	7901	807	170
256	40- 824	841	8579	8749	8918	9087	9257	9426	9595	9764	169
257	40- 9933	—	—	—	—	—	—	—	—	—	169
257	41- —	0102	0271	044	0609	0777	0946	1114	1283	1451	169
258	41- 162	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	41- 33	3467	3635	3803	397	4137	4305	4472	4639	4806	167
260	41- 4973	514	5307	5474	5641	5808	5974	6141	6308	6474	167
261	41- 6641	6807	6973	7139	7306	7472	7638	7804	797	8135	166
262	41- 8301	8467	8633	8798	8964	9129	9295	946	9625	9791	165
263	41- 9956	—	—	—	—	—	—	—	—	—	165
263	42- —	0121	0286	0451	0616	0781	0945	111	1275	1439	165
264	42- 1604	1768	1933	2097	2261	2426	259	2754	2918	3082	164
265	42- 3246	341	3574	3737	3901	4065	4228	4392	4555	4718	164
266	42- 4882	5045	5208	5371	5534	5697	586	6023	6186	6349	163
267	42- 6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	42- 8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	42- 9752	9914	—	—	—	—	—	—	—	—	162
269	43- —	—	0075	0236	0398	0559	072	0881	1042	1203	161
270	43- 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	43- 2969	313	329	345	361	377	393	409	4249	4409	160
272	43- 4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	43- 6163	6322	6481	664	6799	6957	7116	7275	7433	7592	159
274	43- 7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	43- 9333	9491	9648	9806	9964	—	—	—	—	—	158
275	44- —	—	—	—	—	0122	0279	0437	0594	0752	158
276	44- 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	44- 248	2637	2793	295	3106	3263	3419	3576	3732	3889	157
278	44- 4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	44- 5604	576	5915	6071	6226	6382	6537	6692	6848	7003	155
280	44- 7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	44- 8706	8861	9015	917	9324	9478	9633	9787	9941	—	154
281	45- —	—	—	—	—	—	—	—	—	0095	154
282	45- 0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	45- 1786	194	2093	2247	24	2553	2706	2859	3012	3165	153
284	45- 3318	3471	3624	3777	393	4082	4235	4387	454	4692	153
285	45- 4845	4997	515	5302	5454	5606	5758	591	6062	6214	152
286	45- 6366	6518	667	6821	6973	7125	7276	7428	7579	7731	152
287	45- 7882	8033	8184	8336	8487	8638	8789	894	9091	9242	151
288	45- 9392	9543	9694	9845	9995	—	—	—	—	—	151
288	46- —	—	—	—	—	0146	0296	0447	0597	0748	151
289	46- 0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
No.	0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
290	46-	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	46-	3893	4042	4191	434	449	4639	4788	4936	5085	5234	149
292	46-	5383	5532	568	5829	5977	6126	6274	6423	6571	6719	149
293	46-	6868	7016	7164	7312	746	7608	7756	7904	8052	82	148
294	46-	8347	8495	8643	879	8938	9085	9233	938	9527	9675	148
295	46-	9822	9969	—	—	—	—	—	—	—	—	147
295	47-	—	—	0116	0263	041	0557	0704	0851	0998	1145	147
296	47-	1292	1438	1585	1732	1878	2025	2171	2318	2464	261	146
297	47-	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	47-	4216	4362	4508	4653	4799	4944	509	5235	5381	5526	146
299	47-	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	47-	7121	7266	7411	7555	77	7844	7989	8133	8278	8422	145
301	47-	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	48-	0007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	48-	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	48-	2874	3016	3159	3302	3445	3587	373	3872	4015	4157	143
305	48-	43	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	48-	5721	5863	6005	6147	6289	643	6572	6714	6855	6997	142
307	48-	7138	728	7421	7563	7704	7845	7986	8127	8269	841	141
308	48-	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	48-	9958	—	—	—	—	—	—	—	—	—	140
309	49-	—	0099	0239	038	052	0661	0801	0941	1081	1222	140
310	49-	1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	49-	276	29	304	3179	3319	3458	3597	3737	3876	4015	139
312	49-	4155	4294	4433	4572	4711	485	4989	5128	5267	5406	139
313	49-	5544	5683	5822	596	6099	6238	6376	6515	6653	6791	139
314	49-	693	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	49-	8311	8448	8586	8724	8862	8999	9137	9275	9412	955	138
316	49-	9687	9824	9962	—	—	—	—	—	—	—	137
316	50-	—	—	—	0099	0236	0374	0511	0648	0785	0922	137
317	50-	1059	1196	1333	147	1607	1744	188	2017	2154	2291	137
318	50-	2427	2564	27	2837	2973	3109	3246	3382	3518	3655	136
319	50-	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	50-	515	5286	5421	5557	5693	5828	5964	6099	6234	637	136
321	50-	6505	664	6776	6911	7046	7181	7316	7451	7586	7721	135
322	50-	7856	7991	8126	826	8395	853	8664	8799	8934	9068	135
323	50-	9203	9337	9471	9606	974	9874	—	—	—	—	134
323	51-	—	—	—	—	—	—	0009	0143	0277	0411	134
324	51-	0545	0679	0813	0947	1081	1215	1349	1482	1616	175	134
325	51-	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	51-	3218	3351	3484	3617	375	3883	4016	4149	4282	4415	133
327	51-	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	51-	5874	6006	6139	6271	6403	6535	6668	68	6932	7064	132
329	51-	7196	7328	746	7592	7724	7855	7987	8119	8251	8382	132
330	51-	8514	8646	8777	8909	904	9171	9303	9434	9566	9697	131
331	51-	9828	9959	—	—	—	—	—	—	—	—	131
331	52-	—	—	009	0221	0353	0484	0615	0745	0876	1007	131
332	52-	1138	1269	14	153	1661	1792	1922	2053	2183	2314	131
333	52-	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	52-	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
No.		0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
335	52- 5045	5174	5304	5434	5563	5693	5822	5951	6081	621	129
336	52- 6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	52- 763	7759	7888	8016	8145	8274	8402	8531	866	8788	129
338	52- 8917	9045	9174	9302	943	9559	9687	9815	9943	—	128
338	53- —	—	—	—	—	—	—	—	—	0072	128
339	53- 02	0328	0456	0584	0712	084	0968	1096	1223	1351	128
340	53- 1479	1607	1734	1862	199	2117	2245	2372	25	2627	128
341	53- 2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	53- 4026	4153	428	4407	4534	4661	4787	4914	5041	5167	127
343	53- 5294	5421	5547	5674	58	5927	6053	618	6306	6432	126
344	53- 6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	53- 7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	53- 9076	9202	9327	9452	9578	9703	9829	9954	—	—	126
346	54- —	—	—	—	—	—	—	—	0079	0204	125
347	54- 0329	0455	058	0705	083	0955	108	1205	133	1454	125
348	54- 1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	54- 2825	295	3074	3199	3323	3447	3571	3696	382	3944	124
350	54- 4068	4192	4316	444	4564	4688	4812	4936	506	5183	124
351	54- 5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	54- 6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	54- 7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	54- 9003	9126	9249	9371	9494	9616	9739	9861	9984	—	123
354	55- —	—	—	—	—	—	—	—	—	0106	123
355	55- 0228	0351	0473	0595	0717	084	0962	1084	1206	1328	122
356	55- 145	1572	1694	1816	1938	206	2181	2303	2425	2547	122
357	55- 2668	279	2911	3033	3155	3276	3398	3519	364	3762	121
358	55- 3883	4004	4126	4247	4368	4489	461	4731	4852	4973	121
359	55- 5094	5215	5336	5457	5578	5699	582	594	6061	6182	121
360	55- 6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	55- 7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	55- 8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	55- 9907	—	—	—	—	—	—	—	—	—	120
363	56- —	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	56- 1101	1221	134	1459	1578	1698	1817	1936	2055	2174	119
365	56- 2293	2412	2531	265	2769	2887	3006	3125	3244	3362	119
366	56- 3481	36	3718	3837	3955	4074	4192	4311	4429	4548	119
367	56- 4666	4784	4903	5021	5139	5257	5376	5494	5612	573	118
368	56- 5848	5966	6084	6202	632	6437	6555	6673	6791	6909	118
369	56- 7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	56- 8202	8319	8436	8554	8671	8788	8905	9023	914	9257	117
371	56- 9374	9491	9608	9725	9842	9959	—	—	—	—	117
371	57- —	—	—	—	—	—	0076	0193	0309	0426	117
372	57- 0543	066	0776	0893	101	1126	1243	1359	1476	1592	117
373	57- 1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	57- 2872	2988	3104	322	3336	3452	3568	3684	38	3915	116
375	57- 4031	4147	4263	4379	4494	461	4726	4841	4957	5072	116
376	57- 5188	5303	5419	5534	565	5765	588	5996	6111	6226	115
377	57- 6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	57- 7492	7607	7722	7836	7951	8066	8181	8295	841	8525	115
379	57- 8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

No. 0 1 2 3 4 5 6 7 8 9 D

No.		0	1	2	3	4	5	6	7	8	9	D
380	57-	9784	9898	—	—	—	—	—	—	—	—	114
380	58-	—	—	0012	0126	0241	0355	0469	0583	0697	0811	114
381	58-	0925	1039	1153	1267	1381	1495	1608	1722	1836	195	114
382	58-	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	58-	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	58-	4331	4444	4557	467	4783	4896	5009	5122	5235	5348	113
385	58-	5461	5574	5686	5799	5912	6024	6137	625	6362	6475	113
386	58-	6587	67	6812	6925	7037	7149	7262	7374	7486	7599	112
387	58-	7711	7823	7935	8047	816	8272	8384	8496	8608	872	112
388	58-	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	58-	995	—	—	—	—	—	—	—	—	—	112
389	59-	—	0061	0173	0284	0396	0507	0619	073	0842	0953	112
390	59-	1065	1176	1287	1399	151	1621	1732	1843	1955	2066	111
391	59-	2177	2288	2399	251	2621	2732	2843	2954	3064	3175	111
392	59-	3286	3397	3508	3618	3729	384	395	4061	4171	4282	111
393	59-	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	59-	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	59-	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	59-	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	59-	8791	89	9009	9119	9228	9337	9446	9556	9665	9774	109
398	59-	9883	9992	—	—	—	—	—	—	—	—	109
398	60-	—	—	0101	021	0319	0428	0537	0646	0755	0864	109
399	60-	0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	60-	206	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	60-	3144	3253	3361	3469	3577	3686	3794	3902	401	4118	108
402	60-	4226	4334	4442	455	4658	4766	4874	4982	5089	5197	108
403	60-	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	60-	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	60-	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	60-	8526	8633	874	8847	8954	9061	9167	9274	9381	9488	107
407	60-	9594	9701	9808	9914	—	—	—	—	—	—	107
407	61-	—	—	—	—	0021	0128	0234	0341	0447	0554	107
408	61-	066	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	61-	1723	1829	1936	2042	2148	2254	236	2466	2572	2678	106
410	61-	2784	289	2996	3102	3207	3313	3419	3525	363	3736	106
411	61-	3842	3947	4053	4159	4264	437	4475	4581	4686	4792	106
412	61-	4897	5003	5108	5213	5319	5424	5529	5634	574	5845	105
413	61-	595	6055	616	6265	637	6476	6581	6686	679	6895	105
414	61-	7	7105	721	7315	742	7525	7629	7734	7839	7943	105
415	61-	8048	8153	8257	8362	8466	8571	8676	878	8884	8989	105
416	61-	9093	9198	9302	9406	9511	9615	9719	9824	9928	—	105
416	62-	—	—	—	—	—	—	—	—	—	0032	104
417	62-	0136	024	0344	0448	0552	0656	076	0864	0968	1072	104
418	62-	1176	128	1384	1488	1592	1695	1799	1903	2007	211	104
419	62-	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	62-	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	62-	4282	4385	4488	4591	4695	4798	4901	5004	5107	521	103
422	62-	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	62-	634	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	62-	7366	7468	7571	7673	7775	7878	798	8082	8185	8287	102
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
425	62-	8389	8491	8593	8695	8797	89	9002	9104	9206	9308	102
426	62-	941	9512	9613	9715	9817	9919	—	—	—	—	102
426	63-	—	—	—	—	—	—	0021	0123	0224	0326	102
427	63-	0428	053	0631	0733	0835	0936	1038	1139	1241	1342	102
428	63-	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	63-	2457	2559	266	2761	2862	2963	3064	3165	3266	3367	101
430	63-	3468	3569	367	3771	3872	3973	4074	4175	4276	4376	101
431	63-	4477	4578	4679	4779	488	4981	5081	5182	5283	5383	101
432	63-	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	63-	6488	6588	6688	6789	6889	6989	7089	7189	729	739	100
434	63-	749	759	769	779	789	799	809	819	829	8389	100
435	63-	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	63-	9486	9586	9686	9785	9885	9984	—	—	—	—	100
436	64-	—	—	—	—	—	—	0084	0183	0283	0382	99
437	64-	0481	0581	068	0779	0879	0978	1077	1177	1276	1375	99
438	64-	1474	1573	1672	1771	1871	197	2069	2168	2267	2366	99
439	64-	2465	2563	2662	2761	286	2959	3058	3156	3255	3354	99
440	64-	3453	3551	365	3749	3847	3946	4044	4143	4242	434	99
441	64-	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	64-	5422	5521	5619	5717	5815	5913	6011	611	6208	6306	98
443	64-	6404	6502	66	6698	6796	6894	6992	7089	7187	7285	98
444	64-	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	64-	836	8458	8555	8653	875	8848	8945	9043	914	9237	97
446	64-	9335	9432	953	9627	9724	9821	9919	—	—	—	97
446	65-	—	—	—	—	—	—	—	0016	0113	021	97
447	65-	0308	0405	0502	0599	0696	0793	089	0987	1084	1181	97
448	65-	1278	1375	1472	1569	1666	1762	1859	1956	2053	215	97
449	65-	2246	2343	244	2536	2633	273	2826	2923	3019	3116	97
450	65-	3213	3309	3405	3502	3598	3695	3791	3888	3984	408	96
451	65-	4177	4273	4369	4465	4562	4658	4754	485	4946	5042	96
452	65-	5138	5235	5331	5427	5523	5619	5715	581	5906	6002	96
453	65-	6098	6194	629	6386	6482	6577	6673	6769	6864	696	96
454	65-	7056	7152	7247	7343	7438	7534	7629	7725	782	7916	96
455	65-	8011	8107	8202	8298	8393	8488	8584	8679	8774	887	95
456	65-	8965	906	9155	925	9346	9441	9536	9631	9726	9821	95
457	65-	9916	—	—	—	—	—	—	—	—	—	95
457	66-	—	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
458	66-	0865	096	1055	115	1245	1339	1434	1529	1623	1718	95
459	66-	1813	1907	2002	2096	2191	2286	238	2475	2569	2663	95
460	66-	2758	2852	2947	3041	3135	323	3324	3418	3512	3607	94
461	66-	3701	3795	3889	3983	4078	4172	4266	436	4454	4548	94
462	66-	4642	4736	483	4924	5018	5112	5206	5299	5393	5487	94
463	66-	5581	5675	5769	5862	5956	605	6143	6237	6331	6424	94
464	66-	6518	6612	6705	6799	6892	6986	7079	7173	7266	736	94
465	66-	7453	7546	764	7733	7826	792	8013	8106	8199	8293	93
466	66-	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	66-	9317	941	9503	9596	9689	9782	9875	9967	—	—	93
467	67-	—	—	—	—	—	—	—	—	006	0153	93
468	67-	0246	0339	0431	0524	0617	071	0802	0895	0988	108	93
469	67-	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
470	67-	2098	219	2283	2375	2467	256	2652	2744	2836	2929	92
471	67-	3021	3113	3205	3297	339	3482	3574	3666	3758	385	92
472	67-	3942	4034	4126	4218	431	4402	4494	4586	4677	4769	92
473	67-	4861	4953	5045	5137	5228	532	5412	5503	5595	5687	92
474	67-	5778	587	5962	6053	6145	6236	6328	6419	6511	6602	92
475	67-	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	67-	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	67-	8518	8609	87	8791	8882	8973	9064	9155	9246	9337	91
478	67-	9428	9519	961	97	9791	9882	9973	—	—	—	91
478	68-	—	—	—	—	—	—	—	0063	0154	0245	91
479	68-	0336	0426	0517	0607	0698	0789	0879	097	106	1151	91
480	68-	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	68-	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	68-	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	68-	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	68-	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	68-	5742	5831	5921	601	61	6189	6279	6368	6458	6547	89
486	68-	6636	6726	6815	6904	6994	7083	7172	7261	7351	744	89
487	68-	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	68-	842	8509	8598	8687	8776	8865	8953	9042	9131	922	89
489	68-	9309	9398	9486	9575	9664	9753	9841	993	—	—	89
489	69-	—	—	—	—	—	—	—	—	0019	0107	89
490	69-	0196	0285	0373	0462	055	0639	0728	0816	0905	0993	89
491	69-	1081	117	1258	1347	1435	1524	1612	17	1789	1877	88
492	69-	1965	2053	2142	223	2318	2406	2494	2583	2671	2759	88
493	69-	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	69-	3727	3815	3903	3991	4078	4166	4254	4342	443	4517	88
495	69-	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	69-	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	69-	6356	6444	6531	6618	6706	6793	688	6968	7055	7142	87
498	69-	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	69-	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	69-	897	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	69-	9838	9924	—	—	—	—	—	—	—	—	87
501	70-	—	—	0011	0098	0184	0271	0358	0444	0531	0617	87
502	70-	0704	079	0877	0963	105	1136	1222	1309	1395	1482	86
503	70-	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	70-	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	70-	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	70-	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	70-	5008	5094	5179	5265	535	5436	5522	5607	5693	5778	86
508	70-	5864	5949	6035	612	6206	6291	6376	6462	6547	6632	85
509	70-	6718	6803	6888	6974	7059	7144	7229	7315	74	7485	85
510	70-	757	7655	774	7826	7911	7996	8081	8166	8251	8336	85
511	70-	8421	8506	8591	8676	8761	8846	8931	9015	91	9185	85
512	70-	927	9355	944	9524	9609	9694	9779	9863	9948	—	85
512	71-	—	—	—	—	—	—	—	—	—	0033	85
513	71-	0117	0202	0287	0371	0456	054	0625	071	0794	0879	85
514	71-	0963	1048	1132	1217	1301	1385	147	1554	1639	1723	84
No.		0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
515	71- 1807	1892	1976	206	2144	2229	2313	2397	2481	2566	84
516	71- 265	2734	2818	2902	2986	307	3154	3238	3323	3407	84
517	71- 3491	3575	3659	3742	3826	391	3994	4078	4162	4246	84
518	71- 433	4414	4497	4581	4665	4749	4833	4916	5	5084	84
519	71- 5167	5251	5335	5418	5502	5586	5669	5753	5836	592	84
520	71- 6003	6087	617	6254	6337	6421	6504	6588	6671	6754	83
521	71- 6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	71- 7671	7754	7837	792	8003	8086	8169	8253	8336	8419	83
523	71- 8502	8585	8668	8751	8834	8917	9	9083	9165	9248	83
524	71- 9331	9414	9497	958	9663	9745	9828	9911	9994	—	83
524	72- —	—	—	—	—	—	—	—	—	0077	83
525	72- 0159	0242	0325	0407	049	0573	0655	0738	0821	0903	83
526	72- 0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	72- 1811	1893	1975	2058	214	2222	2305	2387	2469	2552	82
528	72- 2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	72- 3456	3538	362	3702	3784	3866	3948	403	4112	4194	82
530	72- 4276	4358	444	4522	4604	4685	4767	4849	4931	5013	82
531	72- 5095	5176	5258	534	5422	5503	5585	5667	5748	583	82
532	72- 5912	5993	6075	6156	6238	632	6401	6483	6564	6646	82
533	72- 6727	6809	689	6972	7053	7134	7216	7297	7379	746	81
534	72- 7541	7623	7704	7785	7866	7948	8029	811	8191	8273	81
535	72- 8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	72- 9165	9246	9327	9408	9489	957	9651	9732	9813	9893	81
537	72- 9974	—	—	—	—	—	—	—	—	—	81
537	73- —	0055	0136	0217	0298	0378	0459	054	0621	0702	81
538	73- 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	73- 1589	1669	175	183	1911	1991	2072	2152	2233	2313	81
540	73- 2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	73- 3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	73- 3999	4079	416	424	432	44	448	456	464	472	80
543	73- 48	488	496	504	512	52	5279	5359	5439	5519	80
544	73- 5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	73- 6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	73- 7193	7272	7352	7431	7511	759	767	7749	7829	7908	79
547	73- 7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	73- 8781	886	8939	9018	9097	9177	9256	9335	9414	9493	79
549	73- 9572	9651	9731	981	9889	9968	—	—	—	—	79
549	74- —	—	—	—	—	—	0047	0126	0205	0284	79
550	74- 0363	0442	0521	06	0678	0757	0836	0915	0994	1073	79
551	74- 1152	123	1309	1388	1467	1546	1624	1703	1782	186	79
552	74- 1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	74- 2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	74- 351	3588	3667	3745	3823	3902	398	4058	4136	4215	78
555	74- 4293	4371	4449	4528	4606	4684	4762	484	4919	4997	78
556	74- 5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	74- 5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	74- 6634	6712	679	6868	6945	7023	7101	7179	7256	7334	78
559	74- 7412	7489	7567	7645	7722	78	7878	7955	8033	811	78
No.	0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
560	74-	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	74-	8963	904	9118	9195	9272	935	9427	9504	9582	9659	77
562	74-	9736	9814	9891	9968	—	—	—	—	—	—	77
562	75-	—	—	—	—	0045	0123	02	0277	0354	0431	77
563	75-	0508	0586	0663	074	0817	0894	0971	1048	1125	1202	77
564	75-	1279	1356	1433	151	1587	1664	1741	1818	1895	1972	77
565	75-	2048	2125	2202	2279	2356	2433	2509	2586	2663	274	77
566	75-	2816	2893	297	3047	3123	32	3277	3353	343	3506	77
567	75-	3583	366	3736	3813	3889	3966	4042	4119	4195	4272	77
568	75-	4348	4425	4501	4578	4654	473	4807	4883	496	5036	76
569	75-	5112	5189	5265	5341	5417	5494	557	5646	5722	5799	76
570	75-	5875	5951	6027	6103	618	6256	6332	6408	6484	656	76
571	75-	6636	6712	6788	6864	694	7016	7092	7168	7244	732	76
572	75-	7396	7472	7548	7624	77	7775	7851	7927	8003	8079	76
573	75-	8155	823	8306	8382	8458	8533	8609	8685	8761	8836	76
574	75-	8912	8988	9063	9139	9214	929	9366	9441	9517	9592	76
575	75-	9668	9743	9819	9894	997	—	—	—	—	—	76
575	76-	—	—	—	—	—	0045	0121	0196	0272	0347	75
576	76-	0422	0498	0573	0649	0724	0799	0875	095	1025	1101	75
577	76-	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	76-	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	76-	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	76-	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	76-	4176	4251	4326	44	4475	455	4624	4699	4774	4848	75
582	76-	4923	4998	5072	5147	5221	5296	537	5445	552	5594	75
583	76-	5669	5743	5818	5892	5966	6041	6115	619	6264	6338	74
584	76-	6413	6487	6562	6636	671	6785	6859	6933	7007	7082	74
585	76-	7156	723	7304	7379	7453	7527	7601	7675	7749	7823	74
586	76-	7898	7972	8046	812	8194	8268	8342	8416	849	8564	74
587	76-	8638	8712	8786	886	8934	9008	9082	9156	923	9303	74
588	76-	9377	9451	9525	9599	9673	9746	982	9894	9968	—	74
588	77-	—	—	—	—	—	—	—	—	—	0042	74
589	77-	0115	0189	0263	0336	041	0484	0557	0631	0705	0778	74
590	77-	0852	0926	0999	1073	1146	122	1293	1367	144	1514	74
591	77-	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	77-	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	77-	3055	3128	3201	3274	3348	3421	3494	3567	364	3713	73
594	77-	3786	386	3933	4006	4079	4152	4225	4298	4371	4444	73
595	77-	4517	459	4663	4736	4809	4882	4955	5028	51	5173	73
596	77-	5246	5319	5392	5465	5538	561	5683	5756	5829	5902	73
597	77-	5974	6047	612	6193	6265	6338	6411	6483	6556	6629	73
598	77-	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	77-	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	77-	8151	8224	8296	8368	8441	8513	8585	8658	873	8802	72
601	77-	8874	8947	9019	9091	9163	9236	9308	938	9452	9524	72
602	77-	9596	9669	9741	9813	9885	9957	—	—	—	—	72
602	78-	—	—	—	—	—	—	0029	0101	0173	0245	72
603	78-	0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	78-	1037	1109	1181	1253	1324	1396	1468	154	1612	1684	72
No.		0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
605	78- 1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	78- 2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	78- 3189	326	3332	3403	3475	3546	3618	3689	3761	3832	71
608	78- 3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	78- 4617	4689	476	4831	4902	4974	5045	5116	5187	5259	71
610	78- 533	5401	5472	5543	5615	5686	5757	5828	5899	597	71
611	78- 6041	6112	6183	6254	6325	6396	6467	6538	6609	668	71
612	78- 6751	6822	6893	6964	7035	7106	7177	7248	7319	739	71
613	78- 746	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	78- 8168	8239	831	8381	8451	8522	8593	8663	8734	8804	71
615	78- 8875	8946	9016	9087	9157	9228	9299	9369	944	951	71
616	78- 9581	9651	9722	9792	9863	9933	—	—	—	—	70
616	79- —	—	—	—	—	—	0004	0074	0144	0215	70
617	79- 0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	79- 0988	1059	1129	1199	1269	134	141	148	155	162	70
619	79- 1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	79- 2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	79- 3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	79- 379	386	393	4	407	4139	4209	4279	4349	4418	70
623	79- 4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	79- 5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	79- 588	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	79- 6574	6644	6713	6782	6852	6921	699	706	7129	7198	69
627	79- 7268	7337	7406	7475	7545	7614	7683	7752	7821	789	69
628	79- 796	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	79- 8651	872	8789	8858	8927	8996	9065	9134	9203	9272	69
630	79- 9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80- 0029	0098	0167	0236	0305	0373	0442	0511	058	0648	69
632	80- 0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	80- 1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	80- 2089	2158	2226	2295	2363	2432	25	2568	2637	2705	69
635	80- 2774	2842	291	2979	3047	3116	3184	3252	3321	3389	68
636	80- 3457	3525	3594	3662	373	3798	3867	3935	4003	4071	68
637	80- 4139	4208	4276	4344	4412	448	4548	4616	4685	4753	68
638	80- 4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	80- 5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	80- 618	6248	6316	6384	6451	6519	6587	6655	6723	679	68
641	80- 6858	6926	6994	7061	7129	7197	7264	7332	74	7467	68
642	80- 7535	7603	767	7738	7806	7873	7941	8008	8076	8143	68
643	80- 8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	80- 8886	8953	9021	9088	9156	9223	929	9358	9425	9492	67
645	80- 956	9627	9694	9762	9829	9896	9964	—	—	—	67
645	81- —	—	—	—	—	—	—	0031	0098	0165	67
646	81- 0233	03	0367	0434	0501	0569	0636	0703	077	0837	67
647	81- 0904	0971	1039	1106	1173	124	1307	1374	1441	1508	67
648	81- 1575	1642	1709	1776	1843	191	1977	2044	2111	2178	67
649	81- 2245	2312	2379	2445	2512	2579	2646	2713	278	2847	67
650	81- 2913	298	3047	3114	3181	3247	3314	3381	3448	3514	67
651	81- 3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	81- 4248	4314	4381	4447	4514	4581	4647	4714	478	4847	67
653	81- 4913	498	5046	5113	5179	5246	5312	5378	5445	5511	66
654	81- 5578	5644	5711	5777	5843	591	5976	6042	6109	6175	66
No.	0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
655	81- 6241	6308	6374	644	6506	6573	6639	6705	6771	6838	66
656	81- 6904	697	7036	7102	7169	7235	7301	7367	7433	7499	66
657	81- 7565	7631	7698	7764	783	7896	7962	8028	8094	816	66
658	81- 8226	8292	8358	8424	849	8556	8622	8688	8754	882	66
659	81- 8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	81- 9544	961	9676	9741	9807	9873	9939	—	—	—	66
660	82- —	—	—	—	—	—	—	0004	007	0136	66
661	82- 0201	0267	0333	0399	0464	053	0595	0661	0727	0792	66
662	82- 0858	0924	0989	1055	112	1186	1251	1317	1382	1448	66
663	82- 1514	1579	1645	171	1775	1841	1906	1972	2037	2103	66
664	82- 2168	2233	2299	2364	243	2495	256	2626	2691	2756	65
665	82- 2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	82- 3474	3539	3605	367	3735	38	3865	393	3996	4061	65
667	82- 4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	82- 4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	82- 5426	5491	5556	5621	5686	5751	5815	588	5945	601	65
670	82- 6075	614	6204	6269	6334	6399	6464	6528	6593	6658	65
671	82- 6723	6787	6852	6917	6981	7046	7111	7175	724	7305	65
672	82- 7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	82- 8015	808	8144	8209	8273	8338	8402	8467	8531	8595	65
674	82- 866	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	82- 9304	9368	9432	9497	9561	9625	969	9754	9818	9882	64
676	82- 9947	—	—	—	—	—	—	—	—	—	64
676	83- —	0011	0075	0139	0204	0268	0332	0396	046	0525	64
677	83- 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	83- 123	1294	1358	1422	1486	155	1614	1678	1742	1806	64
679	83- 187	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	83- 2509	2573	2637	27	2764	2828	2892	2956	302	3083	64
681	83- 3147	3211	3275	3338	3402	3466	353	3593	3657	3721	64
682	83- 3784	3848	3912	3975	4039	4103	4166	423	4294	4357	64
683	83- 4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	83- 5056	512	5183	5247	531	5373	5437	55	5564	5627	64
685	83- 5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	83- 6324	6387	6451	6514	6577	6641	6704	6767	683	6894	63
687	83- 6957	702	7083	7146	721	7273	7336	7399	7462	7525	63
688	83- 7588	7652	7715	7778	7841	7904	7967	803	8093	8156	63
689	83- 8219	8282	8345	8408	8471	8534	8597	866	8723	8786	63
690	83- 8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	83- 9478	9541	9604	9667	9729	9792	9855	9918	9981	—	63
691	84- —	—	—	—	—	—	—	—	—	0043	63
692	84 0106	0169	0232	0294	0357	042	0482	0545	0608	0671	63
693	84- 0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	84- 1359	1422	1485	1547	161	1672	1735	1797	186	1922	63
695	84- 1985	2047	211	2172	2235	2297	236	2422	2484	2547	63
696	84- 2609	2672	2734	2796	2859	2921	2983	3046	3108	317	62
697	84- 3233	3295	3357	342	3482	3544	3606	3669	3731	3793	62
698	84- 3855	3918	398	4042	4104	4166	4229	4291	4353	4415	62
699	84- 4477	4539	4601	4664	4726	4788	485	4912	4974	5036	62
700	84- 5098	516	5222	5284	5346	5408	547	5532	5594	5656	62
701	84- 5718	578	5842	5904	5966	6028	609	6151	6213	6275	62
702	84- 6337	6399	6461	6523	6585	6646	6708	677	6832	6894	62
703	84- 6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	84- 7573	7634	7696	7758	7819	7881	7943	8004	8066	8127	62
No.	0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
705	84- 8189	8251	8312	8374	8435	8497	8559	862	8682	8743	62
706	84- 8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	84- 9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85- 0033	0095	0156	0217	0279	034	0401	0462	0524	0585	61
709	85- 0646	0707	0769	083	0891	0952	1014	1075	1136	1197	61
710	85- 1258	132	1381	1442	1503	1564	1625	1686	1747	1809	61
711	85- 187	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	85- 248	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	85- 309	315	3211	3272	3333	3394	3455	3516	3577	3637	61
714	85- 3698	3759	382	3881	3941	4002	4063	4124	4185	4245	61
715	85- 4306	4367	4428	4488	4549	461	467	4731	4792	4852	61
716	85- 4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	85- 5519	558	564	5701	5761	5822	5882	5943	6003	6064	61
718	85- 6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	85- 6729	6789	685	691	697	7031	7091	7152	7212	7272	60
720	85- 7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	85- 7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	85- 8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	85- 9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	85- 9739	9799	9859	9918	9978	—	—	—	—	—	60
724	86- —	—	—	—	—	0038	0098	0158	0218	0278	60
725	86- 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	86- 0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	86- 1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	86- 2131	2191	2251	231	237	243	2489	2549	2608	2668	60
729	86- 2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	86- 3323	3382	3442	3501	3561	362	368	3739	3799	3858	59
731	86- 3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	86- 4511	457	463	4689	4748	4808	4867	4926	4985	5045	59
733	86- 5104	5163	5222	5282	5341	54	5459	5519	5578	5637	59
734	86- 5696	5755	5814	5874	5933	5992	6051	611	6169	6228	59
735	86- 6287	6346	6405	6465	6524	6583	6642	6701	676	6819	59
736	86- 6878	6937	6996	7055	7114	7173	7232	7291	735	7409	59
737	86- 7467	7526	7585	7644	7703	7762	7821	788	7939	7998	59
738	86- 8056	8115	8174	8233	8292	835	8409	8468	8527	8586	59
739	86- 8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	86- 9232	929	9349	9408	9466	9525	9584	9642	9701	976	59
741	86- 9818	9877	9935	9994	—	—	—	—	—	—	59
741	87- —	—	—	—	0053	0111	017	0228	0287	0345	59
742	87- 0404	0462	0521	0579	0638	0696	0755	0813	0872	093	58
743	87- 0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	87- 1573	1631	169	1748	1806	1865	1923	1981	204	2098	58
745	87- 2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	87- 2739	2797	2855	2913	2972	303	3088	3146	3204	3262	58
747	87- 3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	87- 3902	396	4018	4076	4134	4192	425	4308	4366	4424	58
749	87- 4482	454	4598	4656	4714	4772	483	4888	4945	5003	58
750	87- 5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	87- 564	5698	5756	5813	5871	5929	5987	6045	6102	616	58
752	87- 6218	6276	6333	6391	6449	6507	6564	6622	668	6737	58
753	87- 6795	6853	691	6968	7026	7083	7141	7199	7256	7314	58
754	87- 7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
No.	0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
755	87- 7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	87- 8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	87- 9096	9153	9211	9268	9325	9383	944	9497	9555	9612	57
758	87- 9669	9726	9784	9841	9898	9956	—	—	—	—	57
758	88- —	—	—	—	—	—	0013	007	0127	0185	57
759	88- 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
760	88- 0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	88- 1385	1442	1499	1556	1613	167	1727	1784	1841	1898	57
762	88- 1955	2012	2069	2126	2183	224	2297	2354	2411	2468	57
763	88- 2525	2581	2638	2695	2752	2809	2866	2923	298	3037	57
764	88- 3093	315	3207	3264	3321	3377	3434	3491	3548	3605	57
765	88- 3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	88- 4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	88- 4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	88- 5361	5418	5474	5531	5587	5644	57	5757	5813	587	57
769	88- 5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	88- 6491	6547	6604	666	6716	6773	6829	6885	6942	6998	56
771	88- 7054	7111	7167	7223	728	7336	7392	7449	7505	7561	56
772	88- 7617	7674	773	7786	7842	7898	7955	8011	8067	8123	56
773	88- 8179	8236	8292	8348	8404	846	8516	8573	8629	8685	56
774	88- 8741	8797	8853	8909	8965	9021	9077	9134	919	9246	56
775	88- 9302	9358	9414	947	9526	9582	9638	9694	975	9806	56
776	88- 9862	9918	9974	—	—	—	—	—	—	—	56
776	89- —	—	—	003	0086	0141	0197	0253	0309	0365	56
777	89- 0421	0477	0533	0589	0645	07	0756	0812	0868	0924	56
778	89- 098	1035	1091	1147	1203	1259	1314	137	1426	1482	56
779	89- 1537	1593	1649	1705	176	1816	1872	1928	1983	2039	56
780	89- 2095	215	2206	2262	2317	2373	2429	2484	254	2595	56
781	89- 2651	2707	2762	2818	2873	2929	2985	304	3096	3151	56
782	89- 3207	3262	3318	3373	3429	3484	354	3595	3651	3706	56
783	89- 3762	3817	3873	3928	3984	4039	4094	415	4205	4261	55
784	89- 4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	89- 487	4925	498	5036	5091	5146	5201	5257	5312	5367	55
786	89- 5423	5478	5533	5588	5644	5699	5754	5809	5864	592	55
787	89- 5975	603	6085	614	6195	6251	6306	6361	6416	6471	55
788	89- 6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	89- 7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	89- 7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	89- 8176	8231	8286	8341	8396	8451	8506	8561	8615	867	55
792	89- 8725	878	8835	889	8944	8999	9054	9109	9164	9218	55
793	89- 9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	89- 9821	9875	993	9985	—	—	—	—	—	—	55
794	90- —	—	—	—	0039	0094	0149	0203	0258	0312	55
795	90- 0367	0422	0476	0531	0586	064	0695	0749	0804	0859	55
796	90- 0913	0968	1022	1077	1131	1186	124	1295	1349	1404	55
797	90- 1458	1513	1567	1622	1676	1731	1785	184	1894	1948	55
798	90- 2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	90- 2547	2601	2655	271	2764	2818	2873	2927	2981	3036	54
800	90- 309	3144	3199	3253	3307	3361	3416	347	3524	3578	54
801	90- 3633	3687	3741	3795	3849	3904	3958	4012	4066	412	54
802	90- 4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	90- 4716	477	4824	4878	4932	4986	504	5094	5148	5202	54
804	90- 5256	531	5364	5418	5472	5526	558	5634	5688	5742	54
No.	0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
805	90-	5796	585	5904	5958	6012	6066	6119	6173	6227	6281	54
806	90-	6335	6389	6443	6497	6551	6604	6658	6712	6766	682	54
807	90-	6874	6927	6981	7035	7089	7143	7196	725	7304	7358	54
808	90-	7411	7465	7519	7573	7626	768	7734	7787	7841	7895	54
809	90-	7949	8002	8056	811	8163	8217	827	8324	8378	8431	54
810	90-	8485	8539	8592	8646	8699	8753	8807	886	8914	8967	54
811	90-	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	90-	9556	9609	9663	9716	977	9823	9877	993	9984	—	54
812	91-	—	—	—	—	—	—	—	—	—	0037	53
813	91-	0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	91-	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	91-	1158	1211	1264	1317	1371	1424	1477	153	1584	1637	53
816	91-	169	1743	1797	185	1903	1956	2009	2063	2116	2169	53
817	91-	2222	2275	2328	2381	2435	2488	2541	2594	2647	27	53
818	91-	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	91-	3284	3337	339	3443	3496	3549	3602	3655	3708	3761	53
820	91-	3814	3867	392	3973	4026	4079	4132	4184	4237	429	53
821	91-	4343	4396	4449	4502	4555	4608	466	4713	4766	4819	53
822	91-	4872	4925	4977	503	5083	5136	5189	5241	5294	5347	53
823	91-	54	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	91-	5927	598	6033	6085	6138	6191	6243	6296	6349	6401	53
825	91-	6454	6507	6559	6612	6664	6717	677	6822	6875	6927	53
826	91-	698	7033	7085	7138	719	7243	7295	7348	74	7453	53
827	91-	7506	7558	7611	7663	7716	7768	782	7873	7925	7978	52
828	91-	803	8083	8135	8188	824	8293	8345	8397	845	8502	52
829	91-	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	91-	9078	913	9183	9235	9287	934	9392	9444	9496	9549	52
831	91-	9601	9653	9706	9758	981	9862	9914	9967	—	—	52
831	92-	—	—	—	—	—	—	—	—	0019	0071	52
832	92-	0123	0176	0228	028	0332	0384	0436	0489	0541	0593	52
833	92-	0645	0697	0749	0801	0853	0906	0958	101	1062	1114	52
834	92-	1166	1218	127	1322	1374	1426	1478	153	1582	1634	52
835	92-	1686	1738	179	1842	1894	1946	1998	205	2102	2154	52
836	92-	2206	2258	231	2362	2414	2466	2518	257	2622	2674	52
837	92-	2725	2777	2829	2881	2933	2985	3037	3089	314	3192	52
838	92-	3244	3296	3348	3399	3451	3503	3555	3607	3658	371	52
839	92-	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	92-	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	92-	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	92-	5312	5364	5415	5467	5518	557	5621	5673	5725	5776	52
843	92-	5828	5879	5931	5982	6034	6085	6137	6188	624	6291	51
844	92-	6342	6394	6445	6497	6548	66	6651	6702	6754	6805	51
845	92-	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	92-	737	7422	7473	7524	7576	7627	7678	773	7781	7832	51
847	92-	7883	7935	7986	8037	8088	814	8191	8242	8293	8345	51
848	92-	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	92-	8908	8959	901	9061	9112	9163	9215	9266	9317	9368	51
850	92-	9419	947	9521	9572	9623	9674	9725	9776	9827	9879	51
851	92-	993	9981	—	—	—	—	—	—	—	—	51
851	93-	—	—	0032	0083	0134	0185	0236	0287	0338	0389	51
852	93-	044	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	93-	0949	1	1051	1102	1153	1203	1254	1305	1356	1407	51
854	93-	1458	1509	156	161	1661	1712	1763	1814	1865	1915	51
No.		0	1	2	3	4	5	6	7	8	9	D

No.	0	1	2	3	4	5	6	7	8	9	D
855	93- 1966	2017	2068	2118	2169	222	2271	2322	2372	2423	51
856	93- 2474	2524	2575	2626	2677	2727	2778	2829	2879	293	51
857	93- 2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	93- 3487	3538	3589	3639	369	374	3791	3841	3892	3943	51
859	93- 3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	93- 4498	4549	4599	465	47	4751	4801	4852	4902	4953	50
861	93- 5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	93- 5507	5558	5608	5658	5709	5759	5809	586	591	596	50
863	93- 6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	93- 6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	93- 7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	93- 7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	93- 8019	8069	8119	8169	8219	8269	8319	837	842	847	50
868	93- 852	857	862	867	872	877	882	887	892	897	50
869	93- 902	907	912	917	922	927	932	9369	9419	9469	50
870	93- 9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94- 0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	94- 0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	94- 1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	94- 1511	1561	1611	166	171	176	1809	1859	1909	1958	50
875	94- 2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	94- 2504	2554	2603	2653	2702	2752	2801	2851	2901	295	50
877	94- 3	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	94- 3495	3544	3593	3643	3692	3742	3791	3841	389	3939	49
879	94- 3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	94- 4483	4532	4581	4631	468	4729	4779	4828	4877	4927	49
881	94- 4976	5025	5074	5124	5173	5222	5272	5321	537	5419	49
882	94- 5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	94- 5961	601	6059	6108	6157	6207	6256	6305	6354	6403	49
884	94- 6452	6501	6551	66	6649	6698	6747	6796	6845	6894	49
885	94- 6943	6992	7041	709	714	7189	7238	7287	7336	7385	49
886	94- 7434	7483	7532	7581	763	7679	7728	7777	7826	7875	49
887	94- 7924	7973	8022	807	8119	8168	8217	8266	8315	8364	49
888	94- 8413	8462	8511	856	8609	8657	8706	8755	8804	8853	49
889	94- 8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	94- 939	9439	9488	9536	9585	9634	9683	9731	978	9829	49
891	94- 9878	9926	9975	—	—	—	—	—	—	—	49
891	95- —	—	—	0024	0073	0121	017	0219	0267	0316	49
892	95- 0365	0414	0462	0511	056	0608	0657	0706	0754	0803	49
893	95- 0851	09	0949	0997	1046	1095	1143	1192	124	1289	49
894	95- 1338	1386	1435	1483	1532	158	1629	1677	1726	1775	49
895	95- 1823	1872	192	1969	2017	2066	2114	2163	2211	226	48
896	95- 2308	2356	2405	2453	2502	255	2599	2647	2696	2744	48
897	95- 2792	2841	2889	2938	2986	3034	3083	3131	318	3228	48
898	95- 3276	3325	3373	3421	347	3518	3566	3615	3663	3711	48
899	95- 376	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	95- 4243	4291	4339	4387	4435	4484	4532	458	4628	4677	48
901	95- 4725	4773	4821	4869	4918	4966	5014	5062	511	5158	48
902	95- 5207	5255	5303	5351	5399	5447	5495	5543	5592	564	48
903	95- 5688	5736	5784	5832	588	5928	5976	6024	6072	612	48
904	95- 6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
No.	0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
905	95-	6649	6697	6745	6793	684	6888	6936	6984	7032	708	48
906	95-	7128	7176	7224	7272	732	7368	7416	7464	7512	7559	48
907	95-	7607	7655	7703	7751	7799	7847	7894	7942	799	8038	48
908	95-	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	95-	8564	8612	8659	8707	8755	8803	885	8898	8946	8994	48
910	95-	9041	9089	9137	9185	9232	928	9328	9375	9423	9471	48
911	95-	9518	9566	9614	9661	9709	9757	9804	9852	99	9947	48
912	95-	9995	—	—	—	—	—	—	—	—	—	48
912	96-	—	0042	009	0138	0185	0233	028	0328	0376	0423	48
913	96-	0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	96-	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	96-	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	96-	1895	1943	199	2038	2085	2132	218	2227	2275	2322	47
917	96-	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	96-	2843	289	2937	2985	3032	3079	3126	3174	3221	3268	47
919	96-	3316	3363	341	3457	3504	3552	3599	3646	3693	3741	47
920	96-	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	96-	426	4307	4354	4401	4448	4495	4542	459	4637	4684	47
922	96-	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	96-	5202	5249	5296	5343	539	5437	5484	5531	5578	5625	47
924	96-	5672	5719	5766	5813	586	5907	5954	6001	6048	6095	47
925	96-	6142	6189	6236	6283	6329	6376	6423	647	6517	6564	47
926	96-	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	96-	708	7127	7173	722	7267	7314	7361	7408	7454	7501	47
928	96-	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	96-	8016	8062	8109	8156	8203	8249	8296	8343	839	8436	47
930	96-	8483	853	8576	8623	867	8716	8763	881	8856	8903	47
931	96-	895	8996	9043	909	9136	9183	9229	9276	9323	9369	47
932	96-	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	96-	9882	9928	9975	—	—	—	—	—	—	—	47
933	97-	—	—	—	0021	0068	0114	0161	0207	0254	03	47
934	97-	0347	0393	044	0486	0533	0579	0626	0672	0719	0765	46
935	97-	0812	0858	0904	0951	0997	1044	109	1137	1183	1229	46
936	97-	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	97-	174	1786	1832	1879	1925	1971	2018	2064	211	2157	46
938	97-	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	97-	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	97-	3128	3174	322	3266	3313	3359	3405	3451	3497	3543	46
941	97-	359	3636	3682	3728	3774	382	3866	3913	3959	4005	46
942	97-	4051	4097	4143	4189	4235	4281	4327	4374	442	4466	46
943	97-	4512	4558	4604	465	4696	4742	4788	4834	488	4926	46
944	97-	4972	5018	5064	511	5156	5202	5248	5294	534	5386	46
945	97-	5432	5478	5524	557	5616	5662	5707	5753	5799	5845	46
946	97-	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	97-	635	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	97-	6808	6854	69	6946	6992	7037	7083	7129	7175	722	46
949	97-	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97-	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	97-	8181	8226	8272	8317	8363	8409	8454	85	8546	8591	46
952	97-	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	97-	9093	9138	9184	923	9275	9321	9366	9412	9457	9503	46
954	97-	9548	9594	9639	9685	973	9776	9821	9867	9912	9958	46
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
955	98-	0003	0049	0094	014	0185	0231	0276	0322	0367	0412	45
956	98-	0458	0503	0549	0594	064	0685	073	0776	0821	0867	45
957	98-	0912	0957	1003	1048	1093	1139	1184	1229	1275	132	45
958	98-	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	98-	1819	1864	1909	1954	2	2045	209	2135	2181	2226	45
960	98-	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	98-	2723	2769	2814	2859	2904	2949	2994	304	3085	313	45
962	98-	3175	322	3265	331	3356	3401	3446	3491	3536	3581	45
963	98-	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	98-	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	98-	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	98-	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	98-	5426	5471	5516	5561	5606	5651	5696	5741	5786	583	45
968	98-	5875	592	5965	601	6055	61	6144	6189	6234	6279	45
969	98-	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	98-	6772	6817	6861	6906	6951	6996	704	7085	713	7175	45
971	98-	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	98-	7666	7711	7756	78	7845	789	7934	7979	8024	8068	45
973	98-	8113	8157	8202	8247	8291	8336	8381	8425	847	8514	45
974	98-	8559	8604	8648	8693	8737	8782	8826	8871	8916	896	45
975	98-	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	98-	945	9494	9539	9583	9628	9672	9717	9761	9806	985	44
977	98-	9895	9939	9983	—	—	—	—	—	—	—	44
977	99-	—	—	—	0028	0072	0117	0161	0206	025	0294	44
978	99-	0339	0383	0428	0472	0516	0561	0605	065	0694	0738	44
979	99-	0783	0827	0871	0916	096	1004	1049	1093	1137	1182	44
980	99-	1226	127	1315	1359	1403	1448	1492	1536	158	1625	44
981	99-	1669	1713	1758	1802	1846	189	1935	1979	2023	2067	44
982	99-	2111	2156	22	2244	2288	2333	2377	2421	2465	2509	44
983	99-	2554	2598	2642	2686	273	2774	2819	2863	2907	2951	44
984	99-	2995	3039	3083	3127	3172	3216	326	3304	3348	3392	44
985	99-	3436	348	3524	3568	3613	3657	3701	3745	3789	3833	44
986	99-	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	99-	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	99-	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	99-	5196	524	5284	5328	5372	5416	546	5504	5547	5591	44
990	99-	5635	5679	5723	5767	5811	5854	5898	5942	5986	603	44
991	99-	6074	6117	6161	6205	6249	6293	6337	638	6424	6468	44
992	99-	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	99-	6949	6993	7037	708	7124	7168	7212	7255	7299	7343	44
994	99-	7386	743	7474	7517	7561	7605	7648	7692	7736	7779	44
995	99-	7823	7867	791	7954	7998	8041	8085	8129	8172	8216	44
996	99-	8259	8303	8347	839	8434	8477	8521	8564	8608	8652	44
997	99-	8695	8739	8782	8826	8869	8913	8956	9	9043	9087	44
998	99-	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	99-	9565	9609	9652	9696	9739	9783	9826	987	9913	9957	43
No.		0	1	2	3	4	5	6	7	8	9	D

Hyperbolic Logarithms of Numbers.

From 1.01 *to* 30.

In following table, the numbers range from 1.01 to 30, advancing by .01, up to the whole number 10; and thence by larger intervals up to 30. The hyperbolic logarithms of numbers, or Neperian logarithms, as they are sometimes termed, are computed by multiplying the common logarithms of numbers by the constant multiplier, 2.302 585.

The hyperbolic logarithms of numbers intermediate between those which are given in the table may be readily obtained by interpolating proportional differences.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1.01	.0099	1.41	.3436	1.81	.5933	2.21	.793	2.61	.9594
1.02	.0198	1.42	.3507	1.82	.5988	2.22	.7975	2.62	.9632
1.03	.0296	1.43	.3577	1.83	.6043	2.23	.802	2.63	.967
1.04	.0392	1.44	.3646	1.84	.6098	2.24	.8065	2.64	.9708
1.05	.0488	1.45	.3716	1.85	.6152	2.25	.8109	2.65	.9746
1.06	.0583	1.46	.3784	1.86	.6206	2.26	.8154	2.66	.9783
1.07	.0677	1.47	.3853	1.87	.6259	2.27	.8198	2.67	.9821
1.08	.077	1.48	.392	1.88	.6313	2.28	.8242	2.68	.9858
1.09	.0862	1.49	.3988	1.89	.6366	2.29	.8286	2.69	.9895
1.1	.0953	1.5	.4055	1.9	.6419	2.3	.8329	2.7	.9933
1.11	.1044	1.51	.4121	1.91	.6471	2.31	.8372	2.71	.9969
1.12	.1133	1.52	.4187	1.92	.6523	2.32	.8416	2.72	1.0006
1.13	.1222	1.53	.4253	1.93	.6575	2.33	.8458	2.73	1.0043
1.14	.131	1.54	.4318	1.94	.6627	2.34	.8502	2.74	1.008
1.15	.1398	1.55	.4383	1.95	.6678	2.35	.8544	2.75	1.0116
1.16	.1484	1.56	.4447	1.96	.6729	2.36	.8587	2.76	1.0152
1.17	.157	1.57	.4511	1.97	.678	2.37	.8629	2.77	1.0188
1.18	.1655	1.58	.4574	1.98	.6831	2.38	.8671	2.78	1.0225
1.19	.174	1.59	.4637	1.99	.6881	2.39	.8713	2.79	1.026
1.2	.1823	1.6	.47	2	.6931	2.4	.8755	2.8	1.0296
1.21	.1906	1.61	.4762	2.01	.6981	2.41	.8796	2.81	1.0332
1.22	.1988	1.62	.4824	2.02	.7031	2.42	.8838	2.82	1.0367
1.23	.207	1.63	.4886	2.03	.708	2.43	.8879	2.83	1.0403
1.24	.2151	1.64	.4947	2.04	.7129	2.44	.892	2.84	1.0438
1.25	.2231	1.65	.5008	2.05	.7178	2.45	.8961	2.85	1.0473
1.26	.2311	1.66	.5068	2.06	.7227	2.46	.9002	2.86	1.0508
1.27	.239	1.67	.5128	2.07	.7275	2.47	.9042	2.87	1.0543
1.28	.2469	1.68	.5188	2.08	.7324	2.48	.9083	2.88	1.0578
1.29	.2546	1.69	.5247	2.09	.7372	2.49	.9123	2.89	1.0613
1.3	.2624	1.7	.5306	2.1	.7419	2.5	.9163	2.9	1.0647
1.31	.27	1.71	.5365	2.11	.7467	2.51	.9203	2.91	1.0682
1.32	.2776	1.72	.5423	2.12	.7514	2.52	.9243	2.92	1.0716
1.33	.2852	1.73	.5481	2.13	.7561	2.53	.9282	2.93	1.075
1.34	.2927	1.74	.5539	2.14	.7608	2.54	.9322	2.94	1.0784
1.35	.3001	1.75	.5596	2.15	.7655	2.55	.9361	2.95	1.0818
1.36	.3075	1.76	.5653	2.16	.7701	2.56	.94	2.96	1.0852
1.37	.3148	1.77	.571	2.17	.7747	2.57	.9439	2.97	1.0886
1.38	.3221	1.78	.5766	2.18	.7793	2.58	.9478	2.98	1.0919
1.39	.3293	1.79	.5822	2.19	.7839	2.59	.9517	2.99	1.0953
1.4	.3365	1.8	.5878	2.2	.7885	2.6	.9555	3	1.0986

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
3.01	1.1019	3.51	1.2556	4.01	1.3888	4.51	1.5063	5.01	1.6114
3.02	1.1053	3.52	1.2585	4.02	1.3913	4.52	1.5085	5.02	1.6134
3.03	1.1086	3.53	1.2613	4.03	1.3938	4.53	1.5107	5.03	1.6154
3.04	1.1119	3.54	1.2641	4.04	1.3962	4.54	1.5129	5.04	1.6174
3.05	1.1151	3.55	1.2669	4.05	1.3987	4.55	1.5151	5.05	1.6194
3.06	1.1184	3.56	1.2698	4.06	1.4012	4.56	1.5173	5.06	1.6214
3.07	1.1217	3.57	1.2726	4.07	1.4036	4.57	1.5195	5.07	1.6233
3.08	1.1249	3.58	1.2754	4.08	1.4061	4.58	1.5217	5.08	1.6253
3.09	1.1282	3.59	1.2782	4.09	1.4085	4.59	1.5239	5.09	1.6273
3.1	1.1314	3.6	1.2809	4.1	1.411	4.6	1.5261	5.1	1.6292
3.11	1.1346	3.61	1.2837	4.11	1.4134	4.61	1.5282	5.11	1.6312
3.12	1.1378	3.62	1.2865	4.12	1.4159	4.62	1.5304	5.12	1.6332
3.13	1.141	3.63	1.2892	4.13	1.4183	4.63	1.5326	5.13	1.6351
3.14	1.1442	3.64	1.292	4.14	1.4207	4.64	1.5347	5.14	1.6371
3.15	1.1474	3.65	1.2947	4.15	1.4231	4.65	1.5369	5.15	1.639
3.16	1.1506	3.66	1.2975	4.16	1.4255	4.66	1.539	5.16	1.6409
3.17	1.1537	3.67	1.3002	4.17	1.4279	4.67	1.5412	5.17	1.6429
3.18	1.1569	3.68	1.3029	4.18	1.4303	4.68	1.5433	5.18	1.6448
3.19	1.16	3.69	1.3056	4.19	1.4327	4.69	1.5454	5.19	1.6467
3.2	1.1632	3.7	1.3083	4.2	1.4351	4.7	1.5476	5.2	1.6487
3.21	1.1663	3.71	1.311	4.21	1.4375	4.71	1.5497	5.21	1.6506
3.22	1.1694	3.72	1.3137	4.22	1.4398	4.72	1.5518	5.22	1.6525
3.23	1.1725	3.73	1.3164	4.23	1.4422	4.73	1.5539	5.23	1.6514
3.24	1.1756	3.74	1.3191	4.24	1.4446	4.74	1.556	5.24	1.6563
3.25	1.1787	3.75	1.3218	4.25	1.4469	4.75	1.5581	5.25	1.6582
3.26	1.1817	3.76	1.3244	4.26	1.4493	4.76	1.5602	5.26	1.6601
3.27	1.1848	3.77	1.3271	4.27	1.4516	4.77	1.5623	5.27	1.662
3.28	1.1878	3.78	1.3297	4.28	1.454	4.78	1.5644	5.28	1.6639
3.29	1.1909	3.79	1.3324	4.29	1.4563	4.79	1.5665	5.29	1.6658
3.3	1.1939	3.8	1.335	4.3	1.4586	4.8	1.5686	5.3	1.6677
3.31	1.1969	3.81	1.3376	4.31	1.4609	4.81	1.5707	5.31	1.6696
3.32	1.1999	3.82	1.3403	4.32	1.4633	4.82	1.5728	5.32	1.6715
3.33	1.203	3.83	1.3429	4.33	1.4656	4.83	1.5748	5.33	1.6734
3.34	1.206	3.84	1.3455	4.34	1.4679	4.84	1.5769	5.34	1.6752
3.35	1.209	3.85	1.3481	4.35	1.4702	4.85	1.579	5.35	1.6771
3.36	1.2119	3.86	1.3507	4.36	1.4725	4.86	1.581	5.36	1.679
3.37	1.2149	3.87	1.3533	4.37	1.4748	4.87	1.5831	5.37	1.6808
3.38	1.2179	3.88	1.3558	4.38	1.477	4.88	1.5851	5.38	1.6827
3.39	1.2208	3.89	1.3584	4.39	1.4793	4.89	1.5872	5.39	1.6845
3.4	1.2238	3.9	1.361	4.4	1.4816	4.9	1.5892	5.4	1.6864
3.41	1.2267	3.91	1.3635	4.41	1.4839	4.91	1.5913	5.41	1.6882
3.42	1.2296	3.92	1.3661	4.42	1.4861	4.92	1.5933	5.42	1.6901
3.43	1.2326	3.93	1.3686	4.43	1.4884	4.93	1.5953	5.43	1.6919
3.44	1.2355	3.94	1.3712	4.44	1.4907	4.94	1.5974	5.44	1.6938
3.45	1.2384	3.95	1.3737	4.45	1.4929	4.95	1.5994	5.45	1.6956
3.46	1.2413	3.96	1.3762	4.46	1.4951	4.96	1.6014	5.46	1.6974
3.47	1.2442	3.97	1.3788	4.47	1.4974	4.97	1.6034	5.47	1.6993
3.48	1.247	3.98	1.3813	4.48	1.4996	4.98	1.6054	5.48	1.7011
3.49	1.2499	3.99	1.3838	4.49	1.5019	4.99	1.6074	5.49	1.7029
3.5	1.2528	4	1.3863	4.5	1.5041	5	1.6094	5.5	1.7047

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
5.51	1.7066	6.01	1.7934	6.51	1.8733	7.01	1.9473	7.51	2.0162
5.52	1.7084	6.02	1.7951	6.52	1.8749	7.02	1.9488	7.52	2.0176
5.53	1.7102	6.03	1.7967	6.53	1.8764	7.03	1.9502	7.53	2.0189
5.54	1.712	6.04	1.7984	6.54	1.8779	7.04	1.9516	7.54	2.0202
5.55	1.7138	6.05	1.8001	6.55	1.8795	7.05	1.953	7.55	2.0215
5.56	1.7156	6.06	1.8017	6.56	1.881	7.06	1.9544	7.56	2.0229
5.57	1.7174	6.07	1.8034	6.57	1.8825	7.07	1.9559	7.57	2.0242
5.58	1.7192	6.08	1.805	6.58	1.884	7.08	1.9573	7.58	2.0255
5.59	1.721	6.09	1.8066	6.59	1.8856	7.09	1.9587	7.59	2.0268
5.6	1.7228	6.1	1.8083	6.6	1.8871	7.1	1.9601	7.6	2.0281
5.61	1.7246	6.11	1.8099	6.61	1.8886	7.11	1.9615	7.61	2.0295
5.62	1.7263	6.12	1.8116	6.62	1.8901	7.12	1.9629	7.62	2.0308
5.63	1.7281	6.13	1.8132	6.63	1.8916	7.13	1.9643	7.63	2.0321
5.64	1.7299	6.14	1.8148	6.64	1.8931	7.14	1.9657	7.64	2.0334
5.65	1.7317	6.15	1.8165	6.65	1.8946	7.15	1.9671	7.65	2.0347
5.66	1.7334	6.16	1.8181	6.66	1.8961	7.16	1.9685	7.66	2.036
5.67	1.7352	6.17	1.8197	6.67	1.8976	7.17	1.9699	7.67	2.0373
5.68	1.737	6.18	1.8213	6.68	1.8991	7.18	1.9713	7.68	2.0386
5.69	1.7387	6.19	1.8229	6.69	1.9006	7.19	1.9727	7.69	2.0399
5.7	1.7405	6.2	1.8245	6.7	1.9021	7.2	1.9741	7.7	2.0412
5.71	1.7422	6.21	1.8262	6.71	1.9036	7.21	1.9755	7.71	2.0425
5.72	1.744	6.22	1.8278	6.72	1.9051	7.22	1.9769	7.72	2.0438
5.73	1.7457	6.23	1.8294	6.73	1.9066	7.23	1.9782	7.73	2.0451
5.74	1.7475	6.24	1.831	6.74	1.9081	7.24	1.9796	7.74	2.0464
5.75	1.7492	6.25	1.8326	6.75	1.9095	7.25	1.981	7.75	2.0477
5.76	1.7509	6.26	1.8342	6.76	1.911	7.26	1.9824	7.76	2.049
5.77	1.7527	6.27	1.8358	6.77	1.9125	7.27	1.9838	7.77	2.0503
5.78	1.7544	6.28	1.8374	6.78	1.914	7.28	1.9851	7.78	2.0516
5.79	1.7561	6.29	1.839	6.79	1.9155	7.29	1.9865	7.79	2.0528
5.8	1.7579	6.3	1.8405	6.8	1.9169	7.3	1.9879	7.8	2.0541
5.81	1.7596	6.31	1.8421	6.81	1.9184	7.31	1.9892	7.81	2.0554
5.82	1.7613	6.32	1.8437	6.82	1.9199	7.32	1.9906	7.82	2.0567
5.83	1.763	6.33	1.8453	6.83	1.9213	7.33	1.992	7.83	2.058
5.84	1.7647	6.34	1.8469	6.84	1.9228	7.34	1.9933	7.84	2.0592
5.85	1.7664	6.35	1.8485	6.85	1.9242	7.35	1.9947	7.85	2.0605
5.86	1.7681	6.36	1.85	6.86	1.9257	7.36	1.9961	7.86	2.0618
5.87	1.7699	6.37	1.8516	6.87	1.9272	7.37	1.9974	7.87	2.0631
5.88	1.7716	6.38	1.8532	6.88	1.9286	7.38	1.9988	7.88	2.0643
5.89	1.7733	6.39	1.8547	6.89	1.9301	7.39	2.0001	7.89	2.0656
5.9	1.775	6.4	1.8563	6.9	1.9315	7.4	2.0015	7.9	2.0669
5.91	1.7766	6.41	1.8579	6.91	1.933	7.41	2.0028	7.91	2.0681
5.92	1.7783	6.42	1.8594	6.92	1.9344	7.42	2.0042	7.92	2.0694
5.93	1.78	6.43	1.861	6.93	1.9359	7.43	2.0055	7.93	2.0707
5.94	1.7817	6.44	1.8625	6.94	1.9373	7.44	2.0069	7.94	2.0719
5.95	1.7834	6.45	1.8641	6.95	1.9387	7.45	2.0082	7.95	2.0732
5.96	1.7851	6.46	1.8656	6.96	1.9402	7.46	2.0096	7.96	2.0744
5.97	1.7867	6.47	1.8672	6.97	1.9416	7.47	2.0109	7.97	2.0757
5.98	1.7884	6.48	1.8687	6.98	1.943	7.48	2.0122	7.98	2.0769
5.99	1.7901	6.49	1.8703	6.99	1.9445	7.49	2.0136	7.99	2.0782
6	1.7918	6.5	1.8718	7	1.9459	7.5	2.0149	8	2.0794

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
8.01	2.0807	8.41	2.1294	8.81	2.1759	9.21	2.2203	9.61	2.2628
8.02	2.0819	8.42	2.1306	8.82	2.177	9.22	2.2214	9.62	2.2638
8.03	2.0832	8.43	2.1318	8.83	2.1782	9.23	2.2225	9.63	2.2649
8.04	2.0844	8.44	2.133	8.84	2.1793	9.24	2.2235	9.64	2.2659
8.05	2.0857	8.45	2.1342	8.85	2.1804	9.25	2.2246	9.65	2.267
8.06	2.0869	8.46	2.1353	8.86	2.1815	9.26	2.2257	9.66	2.268
8.07	2.0882	8.47	2.1365	8.87	2.1827	9.27	2.2268	9.67	2.269
8.08	2.0894	8.48	2.1377	8.88	2.1838	9.28	2.2279	9.68	2.2701
8.09	2.0906	8.49	2.1389	8.89	2.1849	9.29	2.2289	9.69	2.2711
8.1	2.0919	8.5	2.1401	8.9	2.1861	9.3	2.23	9.7	2.2721
8.11	2.0931	8.51	2.1412	8.91	2.1872	9.31	2.2311	9.71	2.2732
8.12	2.0943	8.52	2.1424	8.92	2.1883	9.32	2.2322	9.72	2.2742
8.13	2.0956	8.53	2.1436	8.93	2.1894	9.33	2.2332	9.73	2.2752
8.14	2.0968	8.54	2.1448	8.94	2.1905	9.34	2.2343	9.74	2.2762
8.15	2.098	8.55	2.1459	8.95	2.1917	9.35	2.2354	9.75	2.2773
8.16	2.0992	8.56	2.1471	8.96	2.1928	9.36	2.2364	9.76	2.2783
8.17	2.1005	8.57	2.1483	8.97	2.1939	9.37	2.2375	9.77	2.2793
8.18	2.1017	8.58	2.1494	8.98	2.195	9.38	2.2386	9.78	2.2803
8.19	2.1029	8.59	2.1506	8.99	2.1961	9.39	2.2396	9.79	2.2814
8.2	2.1041	8.6	2.1518	9	2.1972	9.4	2.2407	9.8	2.2824
8.21	2.1054	8.61	2.1529	9.01	2.1983	9.41	2.2418	9.81	2.2834
8.22	2.1066	8.62	2.1541	9.02	2.1994	9.42	2.2428	9.82	2.2844
8.23	2.1078	8.63	2.1552	9.03	2.2006	9.43	2.2439	9.83	2.2854
8.24	2.109	8.64	2.1564	9.04	2.2017	9.44	2.245	9.84	2.2865
8.25	2.1102	8.65	2.1576	9.05	2.2028	9.45	2.246	9.85	2.2875
8.26	2.1114	8.66	2.1587	9.06	2.2039	9.46	2.2471	9.86	2.2885
8.27	2.1126	8.67	2.1599	9.07	2.205	9.47	2.2481	9.87	2.2895
8.28	2.1138	8.68	2.161	9.08	2.2061	9.48	2.2492	9.88	2.2905
8.29	2.115	8.69	2.1622	9.09	2.2072	9.49	2.2502	9.89	2.2915
8.3	2.1163	8.7	2.1633	9.1	2.2083	9.5	2.2513	9.9	2.2925
8.31	2.1175	8.71	2.1645	9.11	2.2094	9.51	2.2523	9.91	2.2935
8.32	2.1187	8.72	2.1656	9.12	2.2105	9.52	2.2534	9.92	2.2946
8.33	2.1199	8.73	2.1668	9.13	2.2116	9.53	2.2544	9.93	2.2956
8.34	2.1211	8.74	2.1679	9.14	2.2127	9.54	2.2555	9.94	2.2966
8.35	2.1223	8.75	2.1691	9.15	2.2138	9.55	2.2565	9.95	2.2976
8.36	2.1235	8.76	2.1702	9.16	2.2148	9.56	2.2576	9.96	2.2986
8.37	2.1247	8.77	2.1713	9.17	2.2159	9.57	2.2586	9.97	2.2996
8.38	2.1258	8.78	2.1725	9.18	2.217	9.58	2.2597	9.98	2.3006
8.39	2.127	8.79	2.1736	9.19	2.2181	9.59	2.2607	9.99	2.3016
8.4	2.1282	8.8	2.1748	9.2	2.2192	9.6	2.2618	10	2.3026
10.25	2.3279	12.25	2.5052	14.25	2.6567	17.5	2.8621	23	3.1355
10.5	2.3513	12.5	2.5262	14.5	2.674	18	2.8904	24	3.1781
10.75	2.3749	12.75	2.5455	14.75	2.6913	18.5	2.9173	25	3.2189
11	2.3979	13	2.5649	15	2.7081	19	2.9444	26	3.2581
11.25	2.4201	13.25	2.584	15.5	2.7408	19.5	2.9703	27	3.2958
11.5	2.443	13.5	2.6027	16	2.7726	2	2.9957	28	3.3322
11.75	2.4636	13.75	2.6211	16.5	2.8034	21	3.0445	29	3.3673
12	2.4849	14	2.6391	17	2.8332	22	3.0911	3	3.4012

MENSURATION OF AREAS, LINES, AND SURFACES.

Parallelograms.

Definition.—Quadrilaterals, having their opposite sides parallel.

To Compute Area of a Square, Rectangle, Rhombus, or Rhomboid.—Figs. 1, 2, 3, and 4.

Rule.—Multiply length by breadth or height.

Or, $l \times b =$ *area, l representing length, and b breadth.*

Fig. 1.

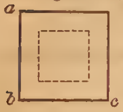


Fig. 2.

Fig. 3.

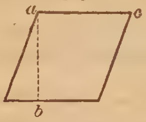
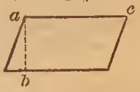


Fig. 4.



Example.—Sides $a\,b$, $b\,c$, Fig. 1, are 5 feet 6 ins.; what is area?

$5.5 \times 5.5 = 30.25$ *square feet.*

Note 1.—Opposite angles of a Rhombus and a Rhomboid are equal.

2.—In any parallelogram the four angles equal 360°.

3.—Side of a square multiplied by 1.52 is equal to side of an equilateral triangle of equal area.

Gnomon.

Definition.—Space included between the lines forming two similar parallelograms, of which smaller is inscribed within larger, so that one angle in each is common to both, as shown by dotted lines, Fig. 1.

To Compute Area of a Gnomon.—Fig. 1.

Rule.—Ascertain areas of the two parallelograms, and subtract less from greater.

Or, $a - a' =$ *area, a and a′ representing areas.*

Example.—Sides of a gnomon are 10 by 10 and 6 by 6 ins.; what is its area?

$10 \times 10 = 100$, and $6 \times 6 = 36$. Then $100 - 36 = 64$ *square ins.*

Triangles.

Definition.—Plain superficies having three sides and angles.

To Compute Area of a Triangle.—Figs. 5, 6, and 7.

Rule.—Multiply base by height, and divide product by 2.

Or, $\frac{ab \times cd}{2}$. Or, $\frac{b\,h}{2} =$ *area, b representing base, and h height.*

Note 1.—*Hypothenuse* of a right angle is side opposite to right angle.

2.—Perpendicular height of a triangle = twice its area divided by its base.

3.—Perpendicular height of an equilateral triangle = a side × .866.

4.—Side of an equilateral triangle × .658 255 = side of a square of equal area, Or ÷ 1.3468 = diameter of a circle of equal area.

Fig. 5.

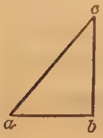


Fig. 6.

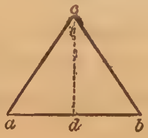
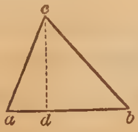


Fig. 7.



Example.—Base $a\,b$, Fig. 5, is 4 feet, and height $c\,b$, 6; what is area?

$4 \times 6 = 24$, and $24 \div 2 = 12$ *square feet.*

To Compute Area of a Triangle by Length of its Sides.—Figs. 6 and 7.

RULE.—From half sum of the three sides subtract each side separately: then multiply half sum and the three remainders continually together, and take square root of product.

Or, $\sqrt{(s-a) \times (s-b) \times (s-c)\,S} = area$, *a, b, c representing sides, and* S *half sum of the three sides.*

EXAMPLE.—Sides of a triangle, Figs. 6 and 7, are 30, 40, and 50 feet; what is area?

$\frac{30+40+50}{2} = \frac{120}{2} = 60$, or half sum of sides. $\left.\begin{array}{l}60-30=30\\60-40=20\\60-50=10\end{array}\right\}$ *remainders.*

Whence, $30 \times 20 \times 10 \times 60 = 360\,000$, and $\sqrt{360\,000} = 600$ *square feet.*

When all Sides are Equal. RULE.—Square length of a side, and multiply product by .433.

Or, $S^2 \times .433 = area$, S *representing length of a side.*

To Compute Length of One Side of a Right-Angled Triangle.

When Length of the other Two Sides are given.

To Ascertain Hypothenuse.—Fig. 5.

RULE.—Add together squares of the two legs, and take square root of sum.

Or, $\sqrt{a\,b^2 + b\,c^2} = hypothenuse$. Or, $\sqrt{b^2 + h^2}$.

EXAMPLE.—Base, $a\,b$, Fig. 5, is 30 ins., and height, $b\,c$, 40; what is length of hypothenuse?

$30^2 + 40^2 = 2500$, and $\sqrt{2500} = 50$ *ins.*

To Ascertain other Leg.

When Hypothenuse and One of the Legs are given.—Fig. 5. RULE.—Subtract square of given leg from square of hypothenuse, and take square root of remainder.

Or, $\sqrt{hyp.^2 - \begin{cases} b^2 = h. \\ h^2 = b. \end{cases}}$ Or, $\sqrt{a\,c^2 - \begin{cases} a\,b^2 = b\,c. \\ b\,c^2 = a\,b. \end{cases}}$

EXAMPLE.—Base of a triangle, $a\,b$, Fig. 5, is 30 feet, and hypothenuse, $a\,c$, 50; what is height of it?

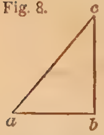
$50^2 - 30^2 = 1600$, and $\sqrt{1600} = 40$ *feet.*

To Compute Length of a Side.

When Hypothenuse of a Right-angled Triangle of Equal Sides alone is given.—Fig. 8. RULE.—Divide hypothenuse by 1.414 213.

Or, $\frac{hyp.}{1.414\,213} = length\ of\ a\ side.$

Fig. 8.



EXAMPLE.—Hypothenuse $a\,c$ of a right-angled triangle, Fig. 8, is 300 feet; what is length of its sides?

$300 \div 1.414\,213 = 212.1321$ *feet.*

To Compute Perpendicular or Height of a Triangle.

When Base and Area alone are given.—Fig. 9. RULE.—Divide twice area by its base. Or, $2a \div b = h$.

EXAMPLE.—Area of a triangle, Fig. 9, is 10 feet, and length of its base, $a\,b$, 5; what is its perpendicular, $c\,d$?

$10 \times 2 = 20$, and $20 \div 5 = 4$ *feet.*

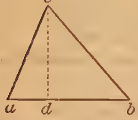
To Compute Perpendicular or Height of a Triangle.

When Base and Two Sides are given. RULE.—As base is to sum of the sides, so is difference of sides to difference of divisions of base. Half this difference being added to or subtracted from half base will give the two divisions thereof. Hence, as the sides and their opposite division of base constitute a right-angled triangle, the perpendicular thereof is readily ascertained by preceding rules.

$$\text{Or, } \frac{bc+ca\times\overline{bc\backsim ca}}{ba}=bd\backsim da.$$

$$\text{Or, } \frac{ac^2+ab^2-bc^2}{2ab}=ad;\ \text{whence } \sqrt{ac^2-ad^2}=dc.$$

Fig. 9.



EXAMPLE.—Three sides of a triangle, abc, Fig. 9, are 9.928, 8, and 5 feet; what is length of perpendicular on longest side?

As $9.928 : 8+5 :: 8\backsim 5 : 3.928 =$ *difference of divisions of the base.*

Then $3.928 \div 2 = 1.964$, which, added to $\frac{9.928}{2} = 4.964 +$ $1.964 = 6.928 =$ *length of longest division of base.*

Hence, there is a right-angled triangle with its base 6.928, and its hypothenuse 8; consequently, its remaining side or perpendicular is $\sqrt{(8^2-6.928^2)} = 4$ *feet.*

When any Two of the Dimensions of a Triangle and One of the corresponding Dimensions of a similar Figure are given, and it is required to ascertain the other corresponding Dimensions of the last Figure.

Fig. 10.

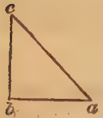
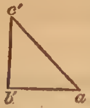


Fig. 11.



Let abc, $a'b'c'$, be two similar triangles, Figs. 10 and 11.

Then $ab : bc :: a'b' : b'c'$, or $a'b' : b'c' :: ab : bc$.

NOTE.—Same proportion holds with respect to the similar lineal parts of any other similar figures, whether plane or solid.

EXAMPLE.—Shadow of a vertical stake 4 feet in length was 5 feet; at same time, shadow of a tree, both on level ground, was 83 feet; what was height of tree?

$$5\ a'b' : 4\ b'c' :: 83\ ab : 66.4\ \textit{feet}.$$

To Compute Acreage.

Divide area into convenient triangles, and multiply base of each triangle in links by half perpendicular in links; cut off 5 figures at the *right*, remaining figures will give acres; multiply the 5 figures so cut off by 4, and again cut off 5, and remainder will give roods; multiply the 5 by 40, and again cut off 5 for perches.

Trapezium.

DEFINITION.—A Quadrilateral having unequal sides of which no two are parallel.

To Compute Area of a Trapezium.—Fig. 12.

RULE.—Multiply diagonal by sum of the two perpendiculars falling upon it from the opposite angles, and divide product by 2.

$$\text{Or, } \frac{db\times\overline{a+c}}{2}=\textit{area}.$$

Fig. 12.

EXAMPLE.—Diagonal db, Fig. 12, is 125 feet, and perpendiculars a and c 50 and 37; what is area?

$125\times\overline{50+37} = 10875$, and $10875 \div 2 = 5437.5$ *square feet.*

When the Two opposite Angles are Supplements to each other, that is, when a Trapezium can be inscribed in a Circle, the Sum of its opposite Angles being equal to Two Right Angles, or 180°. RULE.—From half sum of the four sides, subtract each side severally; then multiply the four remainders continually together, and take square root of product.

EXAMPLE.—In a trapezium the sides are 15, 13, 14, and 12 feet; its opposite angles being supplements to each other, required its area.

$$15+13+14+12=54, \text{ and } \frac{54}{2}=27.$$

27	27	27	27
15	13	14	12

$12 \times 14 \times 13 \times 15 = 32\,760$, and $\sqrt{32\,760} = 180.997$ *square feet.*

Trapezoid.

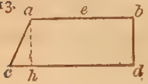
DEFINITION.—A Quadrilateral with only one pair of opposite sides parallel.

To Compute Area of a Trapezoid.—Fig. 13.

RULE.—Multiply sum of the parallel sides by perpendicular distance between them, and divide product.

Or, $\frac{ab+dc \times ah}{2}$. Or, $\frac{s+s' \times h}{2}$ = *area, s and s' representing sides.*

Fig. 13.



EXAMPLE.—Parallel sides *a b*, *c d*, Fig. 13, are 100 and 132 feet, and distance between them 62.5 feet; what is area?

$100+132 \times 62.5 = 14\,500$, and $14\,500 \div 2 = 7250$ *square feet.*

Polygons.

DEFINITION.—Plane figures having three or more sides, and are either regular or irregular, according as their sides or angles are equal or unequal, and they are named from the number of their sides and angles.

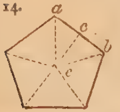
Regular Polygons.

To Compute Area of a Regular Polygon.—Fig. 14.

RULE.—Multiply length of a side by perpendicular distance to centre; multiply product by number of sides, and divide it by 2.

Or, $\frac{ab \times ce \times n}{2}$ = *area, n representing number of sides.*

Fig. 14.



EXAMPLE.—What is area of a pentagon, side *a b*, Fig. 14, being 5 feet, and distance *c e* 4.25 feet?

$5 \times 4.25 \times 5\,(n) = 106.25$ = *product of length of a side, distance to centre, and number of sides.*

Then, $106.25 \div 2 = 53.125$ *square feet.*

To Compute Radius of a Circle that contains a Given Polygon.

When Length of a Perpendicular from Centre alone is given. RULE.—Multiply distance from centre to a side of the polygon, by unit in column A of following Table.

EXAMPLE.—What is radius of a circle that contains a hexagon, distance to centre being 4.33 inches?

$4.33 \times 1.156 = 5$ *ins.*

To Compute Length of a Side of a Polygon that is contained in a Given Circle.

When Radius of Circle is given. RULE.—Multiply radius of circle, by unit in column B of following Table.

EXAMPLE.—What is length of side of a pentagon contained in a circle 8.5 feet in diameter?

$8.5 \div 2 = 4.25$ radius, and $4.25 \times 1.1756 = 5$ *feet.*

To Compute Radius of a Circumscribing Circle.

When Length of a Side is given. RULE.—Multiply length of a side of the polygon, by unit in column C of following Table.

EXAMPLE.—What is radius of a circle that will contain a hexagon, a side being 5 inches?

$$5 \times 1 = 5 \text{ ins.}$$

To Compute Radius of a Circle that can be Inscribed in a Given Polygon.

When Length of a Side is given. RULE.—Multiply length of a side of polygon, by unit in column D of following Table.

EXAMPLE.—What is radius of the circle that is bounded by a hexagon, its sides being 5 inches?

$$5 \times .866 = 4.33 \text{ ins.}$$

To Compute Area of a Regular Polygon.

When Length of a Side only is given. RULE.—Multiply square of side, by multiplier opposite to term of polygon in following Table:

No. of Sides.	POLYGON.	AREA.	A. Radius of Circumscribed Circle.	B. Length of a Side.	C. Radius of Circumscribing Circle.	D. Radius of Inscribed Circle.
3	Trigon	.433 01	2	1.732	.5773	.2887
4	Tetragon	1	1.414	1.4142	.7071	.5
5	Pentagon	1.720 48	1.238	1.1756	.8506	.6882
6	Hexagon	2.598 08	1.156	1	1	.866
7	Heptagon	3.633 91	1.11	.8677	1.1524	1.0383
8	Octagon	4.828 43	1.083	.7653	1.3066	1.2071
9	Nonagon	6.181 82	1.064	.684	1.4619	1.3737
10	Decagon	7.694 21	1.051	.618	1.618	1.5388
11	Undecagon	9.365 64	1.042	.5634	1.7747	1.7028
12	Dodecagon	11.196 15	1.037	.5176	1.9319	1.866

EXAMPLE. — What is area of a square (tetragon) when length of its sides is 7.071 067 8 inches?

$$7.071\,067\,8^2 = 50, \text{ and } 50 \times 1 = 50 \text{ square ins.}$$

To Compute Length of a Side and Radii of a Regular Polygon.

When Area alone is given. RULE.—Multiply square root of area of polygon by multiplier in column E of the following table for length of side; by multiplier in column G for radius of circumscribing circle, and by multiplier in column H for radius of inscribed circle or perpendicular.

No. of Sides.	POLYGON.	E. Length of Side.	G. Radius of Circumscribing Circle.	H. Radius of Inscribed Circle.	Angle.	Angle of Polygon.	Tangent.
3	Trigon	1.5197	.8774	.4387	120°	60°	.5774
4	Tetragon	1	.7071	.5	90	90	1
5	Pentagon	.7624	.6485	.5247	72	108	1.3764
6	Hexagon	.6204	.6204	.5373	60	120	1.7321
7	Heptagon	.5246	.6045	.5446	51 25.71′	128 34.29′	2.0765
8	Octagon	.4551	.5946	.5493	45	135	2.4142
9	Nonagon	.4022	.588	.5525	40	140	2.7475
10	Decagon	.3605	.5833	.5548	36	144	3.0777
11	Undecagon	.3268	.5799	.5564	32 43.64′	147 16.36′	3.4057
12	Dodecagon	.2989	.5774	.5577	30	150	3.7321

EXAMPLE 1.—Area of a square (tetragon) is 16 inches; what is length of its side?

$$\sqrt{16} = 4, \text{ and } 4 \times 1 = 4 \text{ ins.}$$

2.—Area of an octagon is 70.698 yards; what is diameter of its circumscribing circle?

$$\sqrt{70.698} \times .5946 = 5, \text{ and } 5 \times 2 = 10 \text{ yards.}$$

Additional Uses of foregoing Table.—6th and 7th columns of table facilitate construction of these figures with aid of a sector. Thus, if it is required to describe an octagon, opposite to it in column 6th, is 45; then, with chord of 60 on sector as radius, describe a circle, taking length 45 on same line of sector; mark this distance off on the circumference, which, being repeated around the circle, will give points of the sides.

7th column gives angle which any two adjoining sides of the respective figures make with each other; and 8th gives tangent of the angle in column 6th.

To Compute Radius of Inscribed or Circumscribed Circles.

When Radius of Circumscribing Circle is given. RULE.—Multiply radius given by unit in column E, in following Table, opposite to term of polygon for which radius is required.

When Radius of Inscribed Circle is given. RULE.—Multiply radius given by unit in column F, in following Table, opposite to term of polygon for which radius is required.

To Compute Area.

When Radii of Inscribed or Circumscribing Circles are given. RULE.—Square radius given, and multiply it by unit in columns G or H, in following Table, and opposite to term of polygon for which area is required.

When Length of a Side is given. RULE.—Square length of side and multiply it by unit in column I, in following Table, opposite to term of polygon for which area is required.

To Compute Length of a Side.

When Radius of Inscribed Circle is given. RULE.—Multiply radius given by unit in column K, in following Table, and opposite to term of polygon for which length is required.

No. of Sides.	POLYGON.	E. Radius of Inscribed by Circumscribing Circle.	F. Radius of Circumscribing by Inscribing Circle.	G. Area. By Radius of Inscribed Circle.	H. Area. By Radius of Circumscribing Circle.	I. Area. By Length of Side.	K. Length of Side. By Radius of Inscribed Circle.
3	Trigon	.5	2	5.196 15	1.299 04	.433 01	3.464 1
4	Tetragon	.707 11	1.414 21	4	2	1	2
5	Pentagon	.809 02	1.236 07	3.632 72	2.377 64	1.720 48	1.453 08
6	Hexagon	.866 02	1.154 7	3.464 1	2.598 08	2.598 08	1.154 7
7	Heptagon	.900 97	1.109 92	3.371 02	2.736 41	3.633 91	.963 15
8	Octagon	.923 88	1.082 39	3.313 71	2.828 42	4.818 43	.828 43
9	Nonagon	.939 69	1.064 18	3.275 73	2.892 54	6.182 82	.727 94
10	Decagon	.951 06	1.051 46	3.249 2	2.938 93	7.694 21	.649 84
11	Undecagon	.959 49	1.042 22	3.229 89	2.973 53	9.365 64	.587 25
12	Dodecagon	.965 93	1.035 28	3.215 39	3	11.196 15	.535 9

Regular Bodies.

To Compute Surface or Linear Edge of Regular Body.

RULE.—Multiply square of linear edge, or radius of circumscribed or inscribed sphere, by units in following table, under head of dimension used:

No. of Sides.	BODY.	Surface by Linear Edge.	Radius of Circumscribed Sphere.	Radius of Inscribed Sphere.	Linear Edge by Surface.
4	Tetrahedron	1.732 05	1.632 99	4.898 98	.759 84
6	Hexahedron	6	1.154 7	2	.408 25
8	Octahedron	3.464 1	1.414 21	2.449 49	.537 29
12	Dodecahedron	20.645 78	.713 64	.898 06	.220 08
20	Icosahedron	8.660 25	1.051 46	1.323 17	.339 81

EXAMPLE.—What is surface of a hexahedron or cube, having sides of 5 inches?

$5^2 \times 6 = 25 \times 6 = 150$ *square ins.*

To Compute Linear Edge.

When Surface alone is given. RULE.—Multiply square root of surface, by multiplier opposite to term of body under head of Linear Edge by Surface in preceding Table.

EXAMPLE.—What is linear edge of a hexahedron, surface being 6 inches?

$$\sqrt{6} \times .40825 = 1 \textit{ inch.}$$

When Radius of an Inscribed or Circumscribed Sphere is given. RULE.—Multiply radius given, by multiplier opposite to term of body in preceding Table, under head of the Radius given.

EXAMPLE.—Radius of circumscribing sphere of a hexahedron is 10 inches; what is its linear edge?

$$10 \times 1.1547 = 11.547 \textit{ ins.}$$

To Compute Surface.

When Linear Edge is given. RULE.—Multiply square of edge, by multiplier opposite to term of body in preceding Table, under head of Surface.

EXAMPLE.—Linear edge of a hexahedron is 1 inch; what is its surface?

$$1^2 \times 6 = 6 \textit{ square ins.}$$

Irregular Polygons.

DEFINITION.—Figures with unequal sides.

To Compute Area of an Irregular Polygon.—Figs. 15 and 16.

Fig. 15.

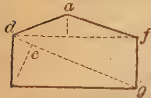
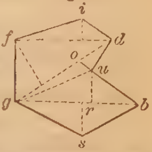


Fig. 16.



RULE.—Draw diagonals and perpendiculars, as df, dg, a, and c, Fig. 15, and fd, gd, gb, gu, and i, o, r, and s, Fig. 16, to divide the figures into triangles and quadrilaterals: ascertain areas of these separately, and take their sum.

NOTE.—To ascertain area of mixed or compound figures, or such as are composed of rectilineal and curvilineal figures together, compute areas of the several figures of which the whole is composed, then add them together, and the sum will give area of compound figure. In this manner any irregular surface or field of land may be measured by dividing it into trapeziums and triangles, and computing area of each separately.

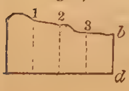
When any Part of a Figure is bounded by a Curve the Area may be ascertained as follows:

Erect any number of perpendiculars upon base, at equal distances, and ascertain their lengths.

Add lengths of the perpendiculars thus ascertained together, and their sum, divided by their number, will give mean breadth; then multiply mean breadth by length of base.

To Compute Area of a Long, Irregular Figure.—Fig. 17.

Fig. 17.



RULE.—Take mean breadths at several places, at equal distances apart, as 1, 2, 3, $b\ d$, etc.; add them together, divide their sum by number of breadths for total mean breadth, and multiply quotient by length of figure.

$$\text{Or, } \frac{b + b' + b'', \text{ etc.}}{a} = \textit{area.}$$

To Compute an Area bounded by a Curve.—Fig. 18.

(Simpson's Rule.)

Fig. 18.

OPERATION.—Divide line $a\,b$ into any number of *equal* parts, by perpendiculars from base, as 1, 2, 3, etc., which will give an *odd* number of points of division. Measure lengths of these perpendiculars or ordinates, and proceed as follows:

To sum of lengths of first and last ordinates, add four times sum of lengths of all even numbered ordinates and twice sum of odd; multiply their sum by one third of distance between ordinates, and product will give area required.

ILLUSTRATION.—Water-line of a vessel has a length of 90 feet, and ordinates 0, 1, 1.2, 1.5, 2, 1.9, 1.5, 1.1, and 0, each 10 feet apart; what is its area?

Ordinates.

Even.	Odd.	Sums.
1	1.2	first 0
1.5	2	last 0
1.9	1.5	even 22
1.1		odd 9.4
$5.5 \times 4 = 22.$	$4.7 \times 2 = 9.4$	$31.4 \times 10 = 314$, which $\div 3 = 104.66$ *square feet.*

Circle.

Diameter is a right line drawn through its centre, bounded by its periphery.

Radius is a right line drawn from its centre to its circumference.

Circumference is assumed to be divided into 360 equal parts, termed *degrees*; each degree is divided into 60 parts, termed *minutes*; each minute into 60 parts, termed *seconds*; and each second into 60 parts, termed *thirds*, and so on.

To Compute Circumference of a Circle.

RULE.—Multiply diameter by 3.1416.

Or, as 7 is to 22, so is diameter to circumference.
Or, as 113 is to 355, so is diameter to circumference.

EXAMPLE.—Diameter of a circle is 1.25 inches; what is its circumference?

$1.25 \times 3.1416 = 3.927$ *ins.*

To Compute Diameter of a Circle.

RULE.—Divide circumference by 3.1416.

Or, as 22 is to 7, so is circumference to diameter.

NOTE.—Divide area by .7854, and square root of quotient will give diameter of circle.

To Compute Area of a Circle.

RULE.—Multiply square of diameter by .7854.

Or, multiply square of circumference by .079 58.
Or, multiply half circumference by half diameter.
Or, multiply square of radius by 3.1416.
Or, $p\,r^2 = area$, *r representing radius.*

EXAMPLE.—The diameter of a circle is 8 inches; what is the area of it?

$8^2 = 64$, and $64 \times .7854 = 50.2656$ *ins.*

Proportions of a Circle, its Equal, Inscribed, and Circumscribed Squares.

CIRCLE.

1. Diameter × .8862 } = Side of an Equal Square.
2. Circumference × .2821 } = Side of an Equal Square.
3. Diameter × .7071 } = Side of Inscribed Square.
4. Circumference × .2251 } = Side of Inscribed Square.
5. Area × .9003 ÷ diam. } = Side of Inscribed Square.
6. Diameter × 1.3468 = Side of an Equilateral Triangle.

SQUARE.

7. A Side × 1.1442 = Diameter of its Circumscribing Circle.
8. " × 4.443 = Circumference of its Circumscribing Circle.
9. " × 1.128 = Diameter } of an Equal Circle.
10. " × 3.545 = Circumference } of an Equal Circle.
11. Square inches × 1.273 = Circle inches } of an Equal Circle.

NOTE.—Square described within a circle is one half area of one described without it.

To Compute Side of Greatest Square that can be Inscribed in a Circle.

RULE.—Multiply diameter by .7071, or take twice square of radius.

Useful Factors.

In which p or π represents Circumference of a Circle.

Diameter = 1.

$p = 3.141\,592\,653\,589+$	$\frac{4}{3}p = 4.188\,79+$	$\sqrt{p} = 1.772\,453$
$2p = 6.283\,185\,307\,179+$	$\frac{1}{6}p = .523\,598+$	$\sqrt{\frac{2}{p}} = .797\,884$
$4p = 12.566\,370\,614\,359+$	$\frac{1}{8}p = .392\,699+$	Log. $p = .497\,149\,87$
$\frac{1}{2}p = 1.570\,796\,326\,794+$	$\frac{1}{12}p = .261\,799+$	$\frac{1}{2}\sqrt{p} = .886\,226+$
$\frac{1}{4}p = .785\,398\,163\,397+$	$\frac{1}{360}p = .008\,726+$	$36\,p = 113.097\,335+$

Diameter = 10.

1.	Chord of arc of semicircle	= 10
2.	Chord of half arc of semicircle	= 7.071 067
3.	Versed sine of arc of semicircle.	= 5
4.	Versed sine of half arc of semicircle	= 1.464 466
5.	Chord of half arc, of half of arc of semicircle	= 3.826 83
6.	Half chord, of chord of half arc	= 3.535 533
7.	Length of arc of semicircle	= 15.707 963
8.	Length of half arc of semicircle	= 7.853 981
9.	Square of chord, of half arc of semicircle (2)	= 50
10.	Square root of versed sine of half arc (4)	= 1.210 151
11.	Square of versed sine of half arc (4)	= 2.144 664
12.	Square of chord of half arc, of half arc of semicircle (5)	= 14.644 67
13.	Square of half chord, of chord of half arc (6)	= 12.5

NOTE.—In all computations p is taken at 3.1416, $\frac{1}{4}p$ at .7854, $\frac{1}{6}p$ at .5236; and whenever the decimal figure next to the one last taken exceeds 5, one is added. Thus, 3.141 59 for four places of decimals is taken as 3.1416.

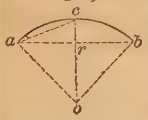
To Compute Length of an Arc of a Circle.—Fig. 19.

When Number of Degrees and Radius are given. RULE 1.—Multiply number of degrees in the arc by 3.1416 times the radius, and divide by 180.

2.—Multiply radius of circle by .017 453 29, and product by degrees in the arc.

If length is required for minutes, multiply radius by .000 290 889; if for seconds, by .000 004 848.

Fig. 19.



EXAMPLE 1.—Number of degrees in an arc, $o\,a\,b$, Fig. 19, are 90, and radius, $o\,b$, 5 inches; what is length of arc?

90 × (3.1416 × 5) = 1413.72, which ÷ 180 = 7.854 *ins.*

2.—Radius of an arc is 10, and measure of its angle 44° 30′ 30″; what is length of arc?

10 × .017 453 29 = .174 532 9, which × 44 = 7.679 447 6, *length for* 44°.

10 × .000 290 889 = .002 908 89, which × 30 = .087 266 7, *length for* 30′.
10 × .000 004 848 = .000 048 48, which × 30 = .001 454 4, *length for* 30″.

$$\text{Then } \left.\begin{matrix} 7.679\,447\,6 \\ .087\,266\,7 \\ .001\,454\,4 \end{matrix}\right\} = 7.768\,168\,7 \textit{ ins.}$$

Or, reduce minutes and seconds to decimal of a degree, and multiply by it.

See Rule, page 93. 30′ 30″ = .5083, and .174 532 9 from above × 44.5083 = 7.768 163 *ins.*

When Chord of Half Arc and Chord of Arc are given. RULE.—From eight times chord of half arc subtract chord of arc, and one third of remainder will give length nearly.

Or, $\frac{8\,c' - c}{3}$, *c' representing chord of half arc, and c chord of arc.*

EXAMPLE.—Chord of half arc, *a c*, Fig. 19, is 30 inches, and chord of arc, *a b*, 48; what is length of arc?

$30 \times 8 = 240 = 8$ *times chord of half arc;* $240 - 48 = 192$, and $192 \div 3 = 64$ *ins.*

When Chord of Arc and Versed Sine of Arc are given. RULE.—Multiply square root of sum of square of chord, and four times square of the versed sine (equal to twice chord of half arc), by ten times square of versed sine; divide this product by sum of fifteen times square of chord and thirty-three times square of versed sine; then add this quotient to twice chord of half arc,* and sum will give length of arc very nearly.

Or, $\frac{\sqrt{c^2 + 4\ v.\ sin.^2} \times 10\ v.\ sin.^2}{15\ c^2 + 33\ v.\ sin.^2} + 2\ c'$, *v. sin. representing versed sine.*

EXAMPLE.—Chord of an arc is 80, and its versed sine, *c r*, 30; what is length of arc?

$80^2 = 6400$ = *square of chord;* $30^2 = 900$ = *square of versed sine.*

$\sqrt{(6400 + 900 \times 4)} = 100$ = *square root of square of chord and four times square of versed sine = twice chord of half arc.*

Then $100 \times 30^2 \times 10 = 900\,000$ = *product of* 10 *times square of versed sine and root above obtained.*

And $80^2 \times 15 = 96\,000 = 15$ *times square of chord.*
$30^2 \times 33 = 29\,700 = 33$ *times square of versed sine.*
$125\,700$

Hence $\frac{100 \times 900\,000}{125\,700} = 7.1599$, and $7.1599 + 100$, *or twice chord of half arc* = 107.1599 *length.*

When Diameter and Versed Sine are given. RULE.—Multiply twice chord of half the arc by 10 times versed sine; divide product by 27 times versed sine subtracted from 60 times diameter, add quotient to twice chord of half arc, and the sum will give length of arc very nearly.

Or, $\frac{2c' \times 10\ v.\ sin.}{60d - 27\ v.\ sin.} + 2\ c' = c.$

EXAMPLE.—Diameter of a circle is 100 feet, and versed sine, *c r*, of arc 25; what is length of arc?

$\sqrt{25 \times 100} = 50$ = *chord of half arc.* See Rule, page 345.

$50 \times 2 \times 25 \times 10 = 25\,000$ = *twice chord of half arc by* 10 *times versed sine.*
$100 \times 60 - 25 \times 27 = 5325 = 27$ *times versed sine from* 60 *times diameter.*

Then $\frac{25\,000}{5325} = 4.6948$, and $4.6948 + 50 \times 2 = 104.6948$ *feet.*

To Compute Chord of an Arc.

When Chord of Half the Arc and Versed Sine are given. RULE.—From square of chord of half arc subtract square of versed sine, and take twice square root of remainder.

Or, $\sqrt{(c'^2 - v.\ sin.^2)} \times 2 = c.$

EXAMPLE.—Chord of half arc, *a c*, is 60, and versed sine, *c r*, 36; what is length of chord of arc?

$60^2 - 36^2 = 2304$, and $\sqrt{2304} \times 2 = 96.$

* Square root of sum of square of chord and four times square of the versed sine is equal to twice chord of half arc.

When Diameter and Versed Sine are given. Multiply versed sine by 2, and subtract product from diameter; subtract square of remainder from square of diameter, and take square root of that remainder.

$$\text{Or, } \sqrt{d^2 - (d - v.\ sin. \times 2)^2} = c.$$

EXAMPLE.—Diameter of a circle is 100, and versed sine of half arc is 36; what is length of chord of arc?

$$(36 \times 2 - 100)^2 - 100^2 = 9216, \text{ and } \sqrt{9216} = 96.$$

To Compute Chord of Half an Arc.

When Chord of the Arc and Versed Sine are given. RULE 1.—Divide square root of sum of square of chord of the arc and four times square of versed sine by two.

2.—Take square root of sum of squares of half chord of arc and versed sine.

$$\text{Or, } \frac{\sqrt{c^2 + 4\ v.\ sin.^2}}{2} = c'. \quad \text{Or, } \sqrt{\left(\frac{c}{2}\right)^2 + v.\ sin.^2} = c'.$$

When Diameter and Versed Sine are given. RULE.—Multiply diameter by versed sine, and take square root of their product.

$$\text{Or, } \sqrt{d \times v.\ sin.} = c'.$$

To Compute Diameter.

RULE 1.—Divide square of chord of half arc by versed sine.

$$\text{Or, } c'^2 \div v.\ sin. = diameter.$$

2.—Add square of half chord of arc to the square of versed sine, and divide this sum by versed sine.

$$\text{Or, } \frac{(c \div 2)^2 + v.\ sin.^2}{v.\ sin.} = d.$$

To Compute Versed Sine.

RULE.—Divide square of chord of half arc by diameter.

$$\text{Or, } \frac{c'^2}{d} = v.\ sin.$$

When Chord of the Arc and Diameter are given. RULE.—From square of diameter subtract square of chord, and extract square root of remainder; subtract this root from diameter, and divide remainder by 2.

$$\text{Or, } \frac{d - \sqrt{d^2 - c^2}}{2} = v.\ sin.$$

When it is greater than a Semidiameter. RULE.—Proceed as before, but add square root of remainder (of squares of diameter and chord) to diameter, and halve the sum.

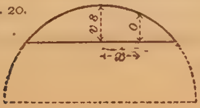
$$\text{Or, } \frac{d + \sqrt{d^2 - c^2}}{2} = v.\ sin.$$

EXAMPLE.—Diameter of a circle is 100, and chord of arc 97.9796; what is its versed sine?

$$\frac{100 + \sqrt{100^2 - 97.9796^2}}{2} = \frac{100 + 20}{2} = 60.$$

To Compute Ordinate of a Circular Curve.—Fig. 20.

Fig. 20.



$$\sqrt{r^2 - x^2} - (r - v) = ordinate.$$

ILLUSTRATION.—Radius of circle 5 ins., versed sine 2, and distance x 2; what is length of ordinate o?

$$\sqrt{5^2 - 2^2} - (5 - 2) = 4.58 - 3 = 1.58 \text{ ins.}$$

Sector of a Circle.

DEFINITION.—A part of a circle bounded by an arc and two radii.

To Compute Area of a Sector of a Circle.

When Degrees in the Arc are given.—Fig. 21. RULE.—As 360 is to number of degrees in a sector, so is area of circle of which sector is a part to area of sector.

Fig. 21.

Or, $\frac{d\,a}{306}$ = *area, d representing degrees in arc, and a area of circle.*

EXAMPLE. —Radius of a circle, *o a*, Fig. 21, is 5 ins., and number of degrees of sector, *a b o*, is 22° 30′; what is area?

Area of a circle of 5 ins. radius = 78.54 ins.

Then, as 360°: 22° 30′ :: 78.54 : 4.908 75 *ins.*

When Length of the Arc, etc., are given. RULE.—Multiply length of arc by half length of radius, and product is area.

Or, $b \times \overline{r \div 2}$ = *area, b representing arc, and r radius.*

Segment of a Circle.

DEFINITION.—A part of a circle bounded by an arc and a chord.

To Compute Area of a Segment of a Circle.

When Chord and Versed Sine of Arc, and Radius or Diameter of Circle are given.

When Segment is less than a Semicircle, as a b c, Fig. 21. RULE.—Ascertain area of sector having same arc as segment; then ascertain area of triangle formed by chord of segment and radii of sector, and take difference of these areas.

NOTE. —Subtract versed sine from radius; multiply remainder by one half of chord of arc, and product will give area of triangle.

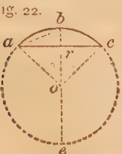
Or, $a - a'$ = *area, a and a′ representing areas of sector and triangle.*

When Segment is greater than a Semicircle. RULE.—Ascertain, by preceding rule, area of lesser portion of circle; subtract it from area of whole circle, and remainder will give area.

Or, $a - a'$ = *area, a and a′ representing areas of circle and lesser portion.*

See Table of Areas of Segments, page 267.

Fig. 22.



EXAMPLE. —Chord, *a c*, Fig. 22, is 14.142; diameter, *b e*, is 20 ins.; and versed sine, *b r*, is 2.929; what is area of segment?

$14.142 \div 2 = 7.071$ = *half chord of arc.*

$\sqrt{7.071^2 + 2.929^2} = 7.654$ = *square root of sum of squares of half chord of arc and versed sine, which is chord a b of half arc a b c.*

By Rule, page 346,

$7.654 \times 2 \times 2.929 \times 10 = 448.371$ = *twice chord of half arc by 10 times versed sine.*

$20 \times 60 - 2.929 \times 27 = 1120.917$ = 60 *times diameter subtracted from 27 times versed sine.*

Then $448.371 \div 1120.917 = .4$, and .4 added to $\overline{7.654 \times 2}$ (twice chord of half arc) = 15.708 *inches, length of arc.*

By Rule above, $15.708 \times \frac{10}{2} = 78.54$ = *the arc multiplied by half length of radius,* = *area of sector.*

$10 - 2.929 = 7.071$ = *versed sine subtracted from a radius, which is height of triangle a o c,* and $7.071 \times \frac{14.142}{2} = 50$ = *area of triangle.*

Consequently, $78.54 - 50 = 28.54$.

When the Chords of Arc, and of half of Arc, and Versed Sine are given. RULE.—To chord of whole arc add chord of half arc and one third of it more; multiply this sum by versed sine, and this product, multiplied by .404 26, will give area nearly.

$$\text{Or, } c + c' + \frac{c'}{3}\ v.\ sin. \times .404\,26 = \textit{area nearly.}$$

EXAMPLE.—Chord of a segment, $a\ c$, Fig. 22, is 28 feet; chord of half arc, $a\ b$, is 15; and versed sine, $b\ r$, 6; what is area of segment?

$28 + 15 + \frac{15}{3}$ = *chord of arc added to chord of half arc and one third of it more.*

$48 \times 6 = 288$ = *product of above sum and versed sine.* Hence $288 \times .404\,26 = 116.427$ *square feet.*

When the Chord of Arc and Versed Sine only are given. RULE.—Ascertain chord of half arc, and proceed as before.

To Compute Chord and Height of a Segment of a Circle.

When Area is given. RULE.—Divide area by square of diameter of circle, take tab. height for area from table of Areas of Segments of a Circle, p. 267, multiply it by diameter, and product will give required height.

From diameter subtract height, multiply remainder by height, take square root of product and multiply it by 2 for required chord.

$$\text{Or, } \frac{a}{d^2} = (\textit{tab. area for height}) \times d = h, \text{ and } \sqrt{d - h \times h} \times 2 = c.$$

Circular Measure. (See Rule, page 113.)

Sphere.

DEFINITION.—A figure, surface of which is at a uniform distance from centre.

To Compute Convex Surface of a Sphere.—Fig. 23.

Fig. 23.

RULE.—Multiply diameter by circumference, and product will give surface.

Or, $4\,p\,r^2$ = *surface.** Or, $p\,d^2$ = *surface.*

EXAMPLE.—What is convex surface of a sphere, Fig. 23, having a diameter, $a\ b$, of 10 ins?

$10 \times 31.416 = 314.16$ *square ins.*

Segment of a Sphere.

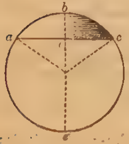
DEFINITION.—A section of a sphere.

To Compute Surface of a Segment of a Sphere.—Fig. 24.

RULE.—Multiply height by the circumference of sphere, and add product to the area of base.

Or, $2\,p\,r\,h$ = *convex surface alone.*

Fig. 24.



EXAMPLE.—Height, $b\ o$, of a segment, $a\ b\ c$, Fig. 24, is 36 ins., and diameter, $b\ e$, of sphere 100; what is convex surface, and what whole surface?

$36 \times 100 \times 3.1416 = 11\,309.76$ = *height of segment multiplied by circumference of sphere.*

To ascertain area of base; diameter and versed sine being given, diameter of base of segment, being equal to chord of arc, is, by Rule, page 347,

$$100 - \overline{36 \times 2} = 28;\ \sqrt{100^2 - 28^2} = 96.$$

$96^2 \times .7854 = 7238.2464$ = *convex surface*, and $7238.2464 + 11\,309.76 = 18\,548.0064$ = *convex surface added to area of base* = *square ins.*

NOTE.—When convex surface of a figure alone is required, area or areas of base or ends must be omitted.

* p or π represents in this, and in all cases where it is used, ratio of circumference of a circle to its diameter, or 3.1416.

When the Diameter of Base of Segment and Height of it are alone given. RULE.—Add square of half diameter of base to the square of height; divide this sum by height, and result will give diameter of sphere.

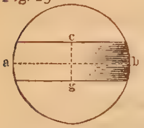
$$\text{Or, } \overline{d \div 2}^2 + h^2 \div h = \textit{diameter}.$$

Spherical Zone (or Frustum of a Sphere).

DEFINITION.—The part of a sphere included between two parallel chords.

To Compute Surface of a Spherical Zone.—Fig. 25.

Fig. 25.



RULE.—Multiply height by the circumference of sphere, and add product to area of the two ends.

$$\text{Or, } h\,c + a + a' = \textit{surface}.$$
$$\text{Or, } 2\,p\,r\,h = \textit{convex surface alone}.$$

EXAMPLE.—Diameter of a sphere, $a\,b$, Fig. 25, from which a zone, $c\,g$, is cut, is 25 inches, and height, $c\,g$, is 8; what is convex surface?

$$25 \times 3.1416 \times 8 = 628.32 = \textit{height} \times \textit{circumference of sphere} = \textit{square ins.}$$

When the Diameter of Sphere is not given. RULE.—Multiply mean length of the two chords by half their difference; divide this product by breadth of zone, and to quotient add breadth. To square of this sum add square of lesser chord, and square root of their sum will give diameter of sphere.

$$\text{Or, } \sqrt{\left(\overline{\frac{l+l'}{2} \times \frac{l \sim l'}{2} \div b + b}^2 + l'^2\right)} = d.$$

Spheroids or Ellipsoids.

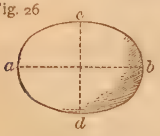
DEFINITION.—Figures generated by the revolution of a semi-ellipse about one of its diameters.

When revolution is about Transverse diameter they are Prolate, and when it is about Conjugate they are Oblate.

To Compute Surface of a Spheroid.—Fig. 26.

When Spheroid is Prolate. RULE.—Square diameters, and multiply square root of half their sum by 3.1416, and this product by conjugate diameter.

Fig. 26



Or, $\sqrt{\dfrac{d^2+d'^2}{2}} \times 3.1416 \times d = \textit{surface}$, d and d' *representing conjugate and transverse diameters.*

EXAMPLE.—A prolate spheroid, Fig. 26, has diameters, $c\,d$ and $a\,b$, of 10 and 14 inches; what is its surface?

$$10^2 + 14^2 = 296 = \textit{sum of squares of diameters.}$$

$296 \div 2 = 148$, and $\sqrt{148} = 12.1655 = $ *square root of half sum of squares of diameters.*

$12.1655 \times 3.1416 \times 10 = 382.191$ *ins.* = *product of root above obtained* $\times$ 3.1416, *and by conjugate diameter.*

When Spheroid is Oblate. RULE.—Square diameters, and multiply square root of half their sum by 3.1416, and this product by transverse diameter.

$$\text{Or, } \sqrt{\frac{d^2+d'^2}{2}} \times 3.1416 \times d' = \textit{surface}.$$

EXAMPLE.—An oblate spheroid has diameters of 14 and 10 inches; what is its surface?

$$12^2 + 10^2 = 296 = \textit{sum of squares of diameters.}$$

$296 \div 2 = 148$, and $\sqrt{148} = 12.1655 = $ *square root of half sum of squares of diameter.*

$12.1655 \times 3.1416 \times 14 = 535.0679$ *ins.* = *product of root above obtained* $\times$ 3.1416, *and by transverse diameter.*

To Compute Convex Surface of a Segment of a Spheroid.—Figs. 27 and 28.

RULE.—Square diameters, and take square root of half their sum; then, as diameter from which the segment is cut is to this root, so is the height of segment to proportionate height required. Multiply product of other diameter and 3.1416 by proportionate height of segment, and this last product will give surface.

$$\text{Or, } \frac{\sqrt{d^2 + d'^2 \div 2}}{d \text{ or } d'} \times h \times d' \text{ or } d \times 3.1416 = \textit{surface.}$$

Fig. 27.

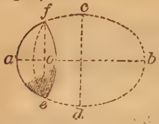
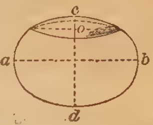


Fig. 28.



EXAMPLE.—Height, *a o*, of a segment, *e f*, of a prolate spheroid, Fig. 27, is 4 inches, diameters being 10 and 14; what is convex surface of it?

Square root of half sum of squares of diameters, 12.1655.

Then 14 : 12.1655 :: 4 : 3.4758 = *height of segment, proportionate to mean of diameters, and* 10 × 3.1416 × 3.4758 = 109.1957 *ins.*

2.—Height, *c o*, of a segment of an oblate spheroid, Fig. 28, is 4 inches, the diameters being 14 and 10; what is convex surface of it? 214.0272 *square ins.*

To Compute Convex Surface of a Frustum or Zone of a Spheroid.—Figs. 29 and 30.

RULE.—Proceed as by previous rule for surface of a segment, and obtain proportionate height of frustum; then multiply product of diameter parallel to base of frustum and 3.1416 by proportionate height of frustum, and it will give surface.

Fig. 29.

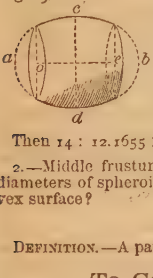
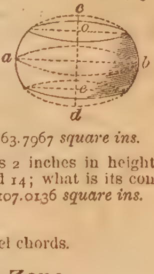


Fig. 30.



EXAMPLE.—Middle frustum, *o e*, of a prolate spheroid, Fig. 29, is 6 inches, diameters of spheroid being 10 and 14; what is its convex surface?

Mean diameter, as per preceding example, is 12.1655.

Diameter parallel to base of frustum is 10.

Then 14 : 12.1655 :: 6 : 5.2138, and 10 × 3.1416 × 5.2138 = 163.7967 *square ins.*

2.—Middle frustum of an oblate spheroid, as *o c*, Fig. 30, is 2 inches in height, diameters of spheroid, as in preceding examples, being 10 and 14; what is its convex surface? 107.0136 *square ins.*

Circular Zone.

DEFINITION.—A part of a circle included between two parallel chords.

To Compute Area of a Circular Zone.

RULE.—From area of circle subtract areas of segments.

Or, see Table of Areas of Zones, page 269.

When Diameter of Circle is not given.—Multiply mean length of the two chords by half their difference; divide this product by breadth of zone, and to quotient add the breadth.

To square of this sum add square of lesser chord, and square root of their sum will give diameter of circle.

EXAMPLE.—Greater chord, *h g*, is 90 inches; lesser, *a c*, is 80; and breadth of zone, *a o*, is 72.526; what is its diameter?

$$\frac{80 + 90}{2} \times \frac{90 \sim 80}{2} = 85 \times 5 = 425, \text{ and } \frac{425}{72.526} + 72.526 = 78.385.$$

Then $\sqrt{78.385^2 + 80^2} = \sqrt{12\,544.2} = 112 = \textit{diameter.}$

Cylinder.

DEFINITION.—A figure formed by revolution of a right-angled parallelogram around one of its sides.

To Compute Surface of a Cylinder.—Fig. 31.

RULE.—Multiply length by circumference, and add product to area of the two ends.

Fig. 31.

Or, $l\,c + 2\,a = s$, *a representing area of end.*

NOTE.—When internal or convex surface alone is wanted, areas of ends are omitted.

EXAMPLE.—Diameter of a cylinder, *b c*, Fig. 31, is 30 inches, and its length, *a b*, 50; what is its surface?

$30 \times 3.1416 = 94.248$, and $94.248 \times 50 = 4712.4$.

Then $30^2 \times .7854 = 706.86 =$ *area of one end;* $706.86 \times 2 = 1413.72 =$ *area of both ends*, and $4712.4 + 1413.72 = 6125.12$ *square ins.*

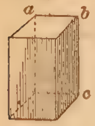
Prisms.

DEFINITION. — Figures, sides of which are parallelograms, and ends equal and parallel.

NOTE.—When ends are triangles, they are termed *triangular prisms;* when they are square, *square* or *right prisms;* and when they are a pentagon, *pentagonal prisms*, etc.

To Compute Surface of a Right Prism.—Figs. 32 and 33.

Fig. 32.



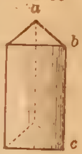
RULE.—Ascertain areas of ends and sides, and add them together.

Or, $2\,a + n\,a' = s$, *a representing area of ends, a′ area of sides, and n their number.*

EXAMPLE. —Side, *a b*, Fig. 32, of a square prism is 12 inches, and length, *b c*, 30; what is surface?

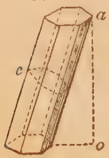
$12 \times 12 = 144 =$ *area of one end;* $144 \times 2 = 288 =$ *area of both ends;* $12 \times 30 = 360 =$ *area of one side;* $360 \times 4 = 1440 =$ *area of four sides*, and $288 + 1440 = 1728$ *sq. ins.*

Fig. 33.



To Compute Surface of an Oblique or Irregular Prism.—Fig. 34.

Fig. 34.



RULE.—Multiply circumference of one end, by perpendicular height, *a o*. Or, multiply circumference, *c*, at a right angle to sides by actual length of figure, and add area of ends.

EXAMPLE.—Sides, *a c*, of an oblique hexagonal prism, Fig. 34, are 10 inches, and perpendicular height, *a o*, is 5 feet; what is its surface?

$10 \times 6 = 60$ *ins. = length of sides.*

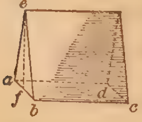
$60 \times 5 \times 12 = 3600$ *square ins. = area of sides*, and by table, page 342, $10^2 \times 2.598\,08 \times 2 = 519.616$ *square ins., which added to* $3600 = 4119.616$ *square ins.*

Wedge.

DEFINITION.—A wedge is a prolate triangular prism, and its surface is computed by rule for that of a right prism.

To Compute Surface of a Wedge.—Fig. 35.

Fig. 35.



EXAMPLE.—Back of a wedge, *a b c d*, Fig. 35, is 20 by 2 inches, and its end, *e f*, 20 by 2; what is its surface?

$20^2 + \overline{2 \div 1}^2 = 401 =$ *sum of squares of half base, a f, and height, e f, of triangle, e f a.*

$\sqrt{401} = 20.025 =$ *square root of above sum = length of e a.*

Then $20.025 \times 20 \times 2 = 801 =$ *area of sides.*

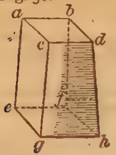
And $20 \times 2 = 40 =$ *area of back;* and $20 \times \overline{2 \div 2} \times 2 = 40 =$ *area of ends.* Hence $801 + 40 + 40 = 881$ *square ins.*

Prismoids.

DEFINITION.—Figures alike to a prism, having only one pair of sides parallel.

To Compute Surface of a Prismoid.—Fig. 36.

Fig. 36.



RULE.—Ascertain area of sides and ends as by rules for squares, triangles, etc., and add them together.

EXAMPLE.—Ends of a prismoid, $e f g h$ and $a b c d$, Fig. 36, are 10 and 8 inches square, and its slant height, $d h$, 25; what is its surface?

$10 \times 10 = 100 =$ *area of base*; $8 \times 8 = 64 =$ *area of top*.

$\frac{10+8}{2} \times 25 = 225$, and $225 \times 4 = 900 =$ *area of sides*.

Then $100 + 64 + 900 = 1064 =$ *square ins.*

To Compute Surface of an Oblique or Irregular Prismoid.

Proceed as directed for an Oblique or Irregular Prism, page 350.

Ungulas.

DEFINITION.—Cylindrical ungulas are the parts (including all or part of the base) left by a plane cutting a cylinder through any portion and at any angle.

To Compute Curved Surface of an Ungula.—Figs. 37, 38, 39, and 40.

When Section is parallel to Axis of the Cylinder, Fig. 37. RULE 1.—Multiply length of arc of one end by height.

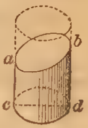
Fig. 37.

EXAMPLE.—Diameter of a cylinder, $a c$, from which an ungula, Fig. 37, is cut, is 10 inches, its length, $b d$, 50, and versed sine or depth of ungula is 5 inches; what is curved surface?

$10 \div 2 = 5 =$ *radius of cylinder.*

Hence radius and versed sine are equal; the arc, therefore, of ungula is one half circumference of the cylinder, which is $31.416 \div 2 = 15.708$, and $15.708 \times 50 = 785.4$ *square ins.*

Fig. 38.



When Section passes obliquely through opposite Sides of Cylinder, Fig. 38. RULE 2.—Multiply circumference of base of cylinder by half sum of greatest and least heights of ungula.

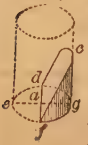
EXAMPLE.—Diameter, $c d$, of a cylindrical ungula, Fig. 38, is 10 inches, and greater and less heights, $b d$ and $a c$, are 25 and 15 inches; what is its curved surface?

10 *diameter* $= 31.416$ *circumference*; $25 + 15 = 40$, and $40 \div 2 = 20$. Hence $31.416 \times 20 = 628.32$ *square ins.*

When Section passes through Base of Cylinder and one of its Sides, and Versed Sine does not exceed Sine, or Base is equal to or less than a Semicircle, Fig. 39. RULE 3.—Multiply sine, $a d$, of half arc, $d g$, of base, $d g f$, by diameter, $e g$, of cylinder, and from this product subtract product * of arc and cosine, $a o$. Multiply difference thus found by quotient of height, $g c$, divided by versed sine, $a g$.

NOTE.—The *sine of base* is half of the longest chord that can be drawn in base.

Fig. 39.



EXAMPLE.—Sine, $a d$, of half arc of base of an ungula, Fig. 39, is 5, diameter of cylinder, $e g$, is 10, and height, $c g$, of ungula 10 inches; what is curved surface?

$5 \times 10 = 50 =$ *sine of half arc by diameter.*

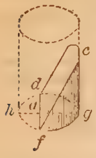
Length of arc, versed sine and radius being equal, under Rule, page 346 $= 15.708$, and as versed sine and radius are equal, cosine is 0.

Hence, when cosine is 0, product is 0. Therefore $50 - 0 = 50 =$ *difference of product before obtained and product of arc and cosine*, and $50 \times \overline{10 \div 5} = 50 \times 2 = 100$ *square ins.*

* When the cosine is 0, this product is 0.

When Section passes through Base of Cylinder, and Versed Sine, a g, exceeds Sine, or when Base exceeds a Semicircle, Fig. 40. RULE 4.—Multiply sine of half the arc of base by diameter of cylinder, and to this product add product of arc and the excess of versed sine over the sine of base. Multiply sum thus found by quotient of height divided by versed sine.

Fig. 40.



EXAMPLE.—Sine, *a d*, of half arc of an ungula, Fig. 40, is 12 inches; versed sine, *a g*, is 16; height, *c g*, 16; and diameter of cylinder, *h g*, 25 inches; what is curved surface?

$12 \times 25 = 300 =$ *sine of half arc by diameter of cylinder*, and length of arc of base, Rule, page 344 = *arc of d h f — circumference of base* = 46.392.

Then $46.392 \times \overline{16 - 12.5} = 162.372$, and $300 + 162\ 272 = 462.372$; $16 \div 16 = 1$, and $462.372 \times 1 = 462.372$ *square ins.*

NOTE.—When sine of an arc is 0, the versed sine is equal to diameter.

Fig. 41.

When Section passes obliquely through both Ends of Cylinder, Fig. 41. RULE 5.—Conceive section to be continued to *m*, till it meets side of cylinder produced; then, as difference of versed sines, *a e* and *d o*, of arcs of two ends of ungula is to versed sine, *a e*, of arc of the less end, so is height of cylinder, *a d*, to the part of side produced.

Ascertain surface of each of ungulas thus found by Rules 3 and 4, and their difference will give curved surface.

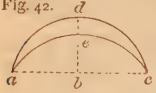
Lune.

DEFINITION.—Space between intersecting arcs of two eccentric circles.

To Compute Area of a Lune.—Fig. 42.

RULE.—Ascertain areas of the two segments from which lune is formed, and their difference will give area.

Fig. 42.



EXAMPLE.—Length of chord *a c*, Fig. 42, is 20 inches, height *e d* is 3, and *e b* 2; what is area of lune?

By Rule 2, page 345, diameters of circles of which lune is formed are thus ascertained:

For *a d c*, $\dfrac{10^2 + (3+2)^2}{5} = 25$. For *a e c*, $\dfrac{10^2 + 2^2}{2} = 52$.

Then, by Rule for Areas of Segments of a Circle, page 267,

Area of *a d c* is	70.5577	*sq. ins.*
" *a e c* "	27.1638	"
Their difference	43.3939	*sq. ins.*

NOTE.—If semicircles be described on the three sides of a right-angled triangle as diameters, two lunes will be formed, and their united areas will be equal to that of triangle.

Cycloid.

DEFINITION.—A curve generated by revolution of a circle on a plane.

To Compute Area of a Cycloid.—Fig. 43.

Fig. 43.

RULE.—Multiply area of generating circle by 3.

EXAMPLE.—Generating circle of a cycloid, *a b c*, Fig. 43, has an area of 115.45 sq. inches; what is area of cycloid?

$115.45 \times 3 = 346.35$ *square ins.*

To Compute Length of a Cycloidal Curve.

RULE.—Multiply diameter of generating circle by 4.

EXAMPLE.—Diameter of generating circle of a cycloid, Fig. 43, is 8 inches; what is length of curve *d s c*?

$8 \times 4 = 32 =$ *product of diameter and* 4 = *ins.*

NOTE.—The curve of a cycloid is line of swiftest descent; that is, a body will fall through arc of this curve, from one point to another, in less time than through any other path.

Circular Rings.

DEFINITION.—Space between two concentric circles.

To Compute Sectional Area of a Circular Ring.—Fig. 44.

RULE.—From area of greater circle subtract that of less.

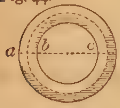
Cylindrical Rings.

DEFINITION.—A ring formed by curvature of a cylinder.

To Compute Surface of a Cylindrical Ring.—Fig. 44.

RULE.—To diameter of body of the ring add inner diameter of the ring; multiply this sum by diameter of the body, and product by 9.8696.

Fig. 44.



Or, $c \times l =$ *surface.*

EXAMPLE.—Diameter of body of a cylindrical ring, *a b*, Fig. 44, is 2 inches, and inner diameter, *b c*, is 18; what is surface of it?

$2 + 18 = 20 =$ *thickness of ring added to inner diameter.*

$20 \times 2 \times 9.8696 =$ *sum above obtained* $\times$ *thickness of ring, and that product by* $9.8696 = 394.784$ *ins.*

Link.

DEFINITION.—An elongated ring.

To Compute Surface of a Link.—Figs. 45 and 46.

RULE.—Multiply length of axis of link by circumference of a section of body, *a b*.

Or, $l \times c =$ *surface.*

To Compute Length of Axis and Circumference.

When Ring is Elongated. RULE.—To less diameter add the diameter of the body of the link, and multiply sum by 3.1416; subtract less diameter from greater, multiply remainder by 2, and sum of these products is length of axis.

Fig. 45.

Fig. 46.

EXAMPLE.—Link of a chain, Fig. 45, is 1 inch in diameter of body, *a b*, and its inner diameters, *b c* and *e f*, are 12.5 and 2.5 inches; what is its circumference?

$2.5 + 1 \times 3.1416 = 10.9956 =$ *length of axis of ends.*

$12.5 - 2.5 \times 2 = 20 =$ *length of sides of body.*

Then $10.9956 + 20 = 30.9956 =$ *length of axis of link*, and 30.9956×3.1416 (cir. of 1 inch) $= 97.3758$ *square ins.*

When Ring is Elliptical, Fig. 46. RULE.—Square diameters of axes of ring, multiply square root of half their sum by 3.1416, and product is length of axis.

Cones.

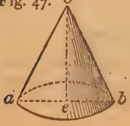
DEFINITION. — A figure described by revolution of a right-angled triangle about one of its legs.

For Sections of a Cone, see Conic Sections, page 379.

To Compute Surface of a Cone.—Fig. 47.

RULE.—Multiply perimeter or circumference of base by slant height, or side of cone; divide product by 2, and add the quotient to area of the base.

Fig. 47.



Or, $c \times h \div 2 + a' =$ *surface, c representing perimeter.*

EXAMPLE.—Diameter, *a b*, Fig. 47, of base of a cone is 3 feet, and slant height, *a c*, 15; what is surface of cone?

Circum. of 3 feet $= 9.4248$, and $\frac{9.4248 \times 15}{2} = 70.686 =$ *surface of side; area of base* $3 = 7.068$, and $70.686 + 7.068 = 77.754$ *square feet.*

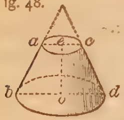
To Compute Surface of the Frustum of a Cone.—Fig. 48.

RULE.—Multiply sum of perimeters of two ends by slant height of frustum; divide product by 2, and add it to areas of two ends.

$$\text{Or, } \frac{c+c'\times h}{2}+a+a'=\textit{surface.}$$

EXAMPLE.—Frustum, $a\,b\,c\,d$, Fig. 48, has a slant height, $c\,d$, of 26 inches, and circumferences of its ends are 15.71 and 22 inches respectively; what is its surface?

Fig. 48.



$$\frac{15.71+22\times 26}{2}=490.23=\textit{surface of sides};\ \left(\frac{15.71}{3.1416}\right)^2\times .7854$$
$$+\left(\frac{22}{3.1416}\right)^2\times .7854=58.119=\textit{areas of ends.}$$ Then $490.23+58.119=548.349$ *square ins.*

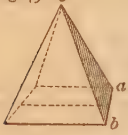
Pyramids.

DEFINITION.—A figure, base of which has three or more sides, and sides of which are plane triangles.

To Compute Surface of a Pyramid.—Figs. 49 and 50.

RULE.—Multiply perimeter of base by slant height; divide product by 2, and add it to area of base.

Fig. 49.



$$\text{Or, } \frac{c\,h}{2}+a=\textit{surface.}$$

Fig. 50.

EXAMPLE.—Side of a quadrangular pyramid, $a\,b$, Fig. 49, is 12 inches, and its slant height, $a\,c$, 40; what is its surface?

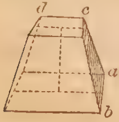
$12\times 4=48=$ *perimeter of base.* $\frac{48\times 40}{2}=960=$ *area of sides*, and $12\times 12+960=1104$ *square ins.*

To Compute Surface of Frustum of a Pyramid.—Fig. 51.

RULE.—Multiply sum of perimeters of two ends by slant height; divide product by 2, and add it to areas of ends.

$$\text{Or, } \frac{c+c'\times h}{2}+a+a'=\textit{surface.}$$

Fig. 51.



EXAMPLE.—Sides $a\,b$, $c\,d$, Fig. 51, of frustum of a quadrangular pyramid are 10 and 9 inches, and its slant height, $a\,c$, 20; what is its surface?

$10\times 4=40$, and $9\times 4=36$; $40+36=76=$ *sum of perimeters.*

$76\times 20=1520$, and $\frac{1520}{2}=760=$ *area of sides*; $10\times 10=100$, and $9\times 9=81$. Then $100+81+760=941=$ *square ins.*

When Pyramid is Irregular sided or Oblique. RULE.—The surfaces of each of the sides and ends must be computed and added together.

Helix (Screw).

DEFINITION.—A line generated by progressive rotation of a point around an axis and equidistant from its centre.

To Compute Length of a Helix.—Fig. 52.

RULE.—To square of circumference described by generating point, add square of distance advanced in one revolution, extract square root of their sum, and multiply it by number of revolutions of generating point.

Fig. 52.

Or, $\sqrt{(p^2 + l^2)}\, n =$ *length, n representing number of revolutions.*

EXAMPLE.—What is length of a helical line, Fig. 52, running 3.5 times around a cylinder of 22 inches in circumference, and advancing 16 inches in each revolution?

$22^2 + 16^2 = 740 =$ *sum of squares of circumference and of distance advanced.** Then $\sqrt{740} \times 3.5 = 95.21$ *ins.*

To Compute Length of a Revolution of Thread of a Screw.

RULE.—Proceed as above for length and omit number of revolutions.

Spirals.

DEFINITION.—Lines generated by the progressive rotation of a point around a fixed axis.

A *Plane Spiral* is when the point rotates around a central point.

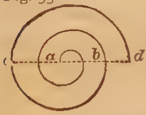
A *Conical Spiral* is when the point rotates around an axis at a progressing distance from its centre, as around a cone.

To Compute Length of a Plane Spiral Line.—Fig. 53.

RULE.—Add together greater and less diameters; divide their sum by 2; multiply quotient by 3.1416, and again by number of revolutions.

Or, when circumferences are given, take their mean length, and multiply it by number of revolutions.

Fig. 53.



Or, $d + d' \div 2 \times 3.1416\, n =$ *length of line;* $P \times n =$ *radius*, and $p\, r^2 \div l =$ *pitch.* P *representing the pitch.*

EXAMPLE.—Less and greater diameters of a plane spiral spring, as *a b*, *c d*, Fig. 53, are 2 and 20 inches, and number of revolutions 10; what is length of it?

$\overline{2 + 20} \div 2 = 11 =$ *sum of diameters* $\div 2$; $11 \times 3.1416 = 34.5576$ and 34.5576×3.1416.

Then $34.5576 \times 10 = 345.576$ *inches.*

NOTE.—Above rule is applicable to winding engines, see page 862, where it is required to ascertain length of a rope, its thickness, number of revolutions, diameter of drum, etc.

To Compute Length of a Conical Spiral Line.—Fig. 54.

RULE.—Add together greater and less diameters; divide their sum by 2, and multiply quotient by 3.1416.

To square of product of this circumference and number of revolutions of spiral, add square of height of its axis, and take square root of the sum.

Fig. 54.

Or, $\sqrt{(\overline{d + d' \div 2 \times 3.1416\, n}^2 + h^2)} =$ *length of line.*

EXAMPLE.—Greater and less diameters of a conical spiral, Fig. 54, are 20 and 2 inches; its height, *c d*, 10; and number of revolutions 10; what is length of it?

$20 + 2 \div 2 = 11 \times 3.1416 = 34.5576 =$ *sum of diameters* $\div 2$, and $\times$ 3.1416; $35.5576 \times 10 = 345.576$.

Then $\sqrt{345.576^2 + 10^2} = 345.72$ *inches.*

Spindles.

DEFINITION.—Figures generated by revolution of a plane area, when the curve is revolved about a chord perpendicular to its axis, or about its double ordinate, and they are designated by the name of the arc or curve from which they are generated, as Circular, Elliptic, Parabolic, etc.

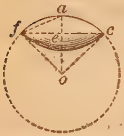
* When the spiral is other than a line, measure diameters of it from middle of body composing it.

Circular Spindle.

To Compute Convex Surface of a Circular Spindle, Zone, or Segment of it.—Figs. 55, 56, and 57.

RULE.—Multiply length by radius of revolving arc; multiply this arc by central distance, or distance between centre of spindle and centre of revolving arc; subtract this product from former, double remainder, and multiply it by 3.1416.

Fig. 55.



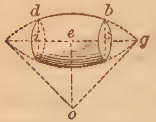
Or, $l\,r - \left(a\sqrt{r^2 - \left(\frac{c}{2}\right)^2}\right)2\,p$ = *surface, a representing length of arc, and c the spindle chord.*

EXAMPLE.—What is surface of a circular spindle, Fig. 55, length of it, *f c*, being 14.142 inches, radius of its arc, *o c*, 10, and central distance, *o e*, 7.071?

$14.142 \times 10 = 141.42 =$ *length* $\times$ *radius. Length of arc, f a c,* by Rules, page 344 $= 15.708$.

$15.708 \times 7.071 = 111.0713 =$ *length of arc* $\times$ *central distance;* $141.42 - 111.0713 = 30.3487 =$ *difference of products.* Then $30.3487 \times 2 \times 3.1416 = 190.687$ *square ins.*

Fig. 56.



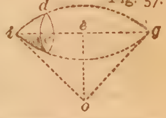
Zone.

EXAMPLE.—What is convex surface of zone of a circular spindle, Fig. 56, length of it, *i c*, being 7.653 inches, radius of its arc, *o g*, 10, central distance, *o e*, 7.071, and length of its side or arc, *d b*, 7.854 inches?

$7.653 \times 10 = 76.53 =$ *length* $\times$ *radius;* $7.854 \times 7.071 = 55.5356 =$ *length of arc* $\times$ *central distance;* $76.53 - 55.5356 = 20.9944 =$ *difference of products.*

Then $20.9944 \times 2 \times 3.1416 = 131.912$ *square ins.*

Fig. 57.



Segment.

EXAMPLE.—What is convex surface of a segment of a circular spindle, Fig. 57, length of it, *i c*, being 3.2495 inches, radius of its arc, *o g*, 10, central distance, *o e*, 7.071, and length of its side, *i d*, 3.927 inches?

$3.2495 \times 10 = 32.495 =$ *length* $\times$ *radius;* $3.927 \times 7.071 = 27.7678 =$ *length of arc* $\times$ *central distance;* $32.495 - 27.7678 = 4.7272 =$ *difference of products.*

Then $4.7272 \times 2 \times 3.1416 = 29.702$ *square ins.*

GENERAL FORMULA.—$S = 2\,(l\,r - a\,c)\,p$ = *surface, l representing length of spindle, segment, or zone, a length of its revolving arc, r radius of generating circle, and c central distance.*

ILLUSTRATION.—Length of a circular spindle is 14.142 inches, length of its revolving arc is 15.708, radius of its generating circle is 10, and distance of its centre from centre of the circle from which it is generated is 7.071; what is its surface?

$$2 \times (14.142 \times 10 - \overline{15.708 \times 7.071}) \times 3.1416 = 190.687 \text{ square inches.}$$

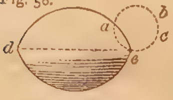
NOTE.—Surface of a frustum of a spindle may be obtained by division of the surface of a zone.

Cycloidal Spindle.

To Compute Convex Surface of a Cycloidal Spindle.—Fig. 58.

RULE.—Multiply area of generating circle by 64, and divide it by 3.

Fig. 58.



Or, $\frac{a \times 64}{3}$ = *surface.*

EXAMPLE.—Area of generating circle, *a b c*, of a cycloidal spindle, *d e*, is 32 inches; what is surface of spindle?

$32 \times 64 = 2048 =$ *area of circle* $\times 64$, and $2048 \div 3 = 682.667$ *square ins.*

NOTE.—Area of greatest or centre section of a cycloidal spindle is twice area of the cycloid.

Ellipsoid, Paraboloid, or Hyperboloid of Revolution.

DEFINITION.—Figures alike to a cone, generated by revolution of a conic section around its axis.

NOTE.—These figures are usually known as Conoids.

When they are generated by revolution of an ellipse, they are termed Ellipsoids, and when by a parabola, Paraboloids, etc.

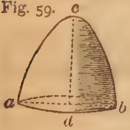
Revolution of an arc of a conic section around the axis of the curve will give a segment of a conoid.

Ellipsoid.

To Compute Convex Surface of an Ellipsoid.—Fig. 59.

RULE.—Add together square of base and four times square of height; multiply square root of half their sum by 3.1416, and this product by radius of the base.

Fig. 59.



Or, $\sqrt{\frac{b^2+4h^2}{2}}\,3.1416\,r = surface.$

EXAMPLE.—Base, $a\,b$, of an ellipsoid, Fig. 59, is 10 inches, and vertical height, $c\,d$, 7; what is its surface?

$10^2+\overline{7^2\times 4}=296=$ *sum of square of base and 4 times square of height*; $296\div 2=148$, and $\sqrt{148}=12.1655=$ *square root of half above sum.* Then $12.1655\times 3.1416\times\frac{10}{2}=191.0957$ *square ins.*

To Compute Convex Surface of a Segment, Frustum, or Zone of an Ellipsoid.—Fig. 59.

See Rules for Convex Surface of a Segment, Frustum, or Zone of a Spheroid or Ellipsoid, pages 348-9.

$$d \text{ or } d'\times 3.1416\times h = surface,$$

$$\text{and } \frac{mean.\ diam.\times h}{d \text{ or } d'}=h\text{; then } d\times 3.1416\times h = surface.$$

Paraboloid.

To Compute Convex Surface of a Paraboloid.—Fig. 60.

RULE.—From cube of square root of sum of four times square of height, and square of radius of base, subtract cube of radius of base; multiply remainder by quotient of 3.1416 times radius of base divided by six times square of height.

Fig. 60.

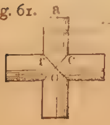
Or, $(\sqrt{4h^2+r^2})^3-r^3\times\frac{r\times p}{6\times h^2}=surface.$

EXAMPLE.—Axis, $b\,d$, of a paraboloid, Fig. 60, is 40 inches; radius, $a\,d$, of its base is 18 inches; what is its convex surface?

$40^2\times 4=6400=4$ *times square of height*; $6400+18^2=6724=$ *sum of above product and square of radius of base*; $(\sqrt{6724})^3-18^3$ $545\,536=$ *remainder of cube of radius of base subtracted from cube of square root of preceding sum*; $3.1416\times 18\div(6\times 40^2)=.005\,890\,5$ $=$ *quotient of* 3.1416 *times radius of base* $\div$ 6 *times square of height.*

Then $545\,536\times .005\,890\,5=3213.48$ *square ins.*

Fig. 61.



Cylinder Sections.

To Compute Surface of a Cylinder Section.—Fig. 61.

RULE.—From entire surface of cylinder $a\,o$ subtract surface of the two ungulas, $r\,o$, $o\,c$, as per rule, page 351, and multiply result by 4.

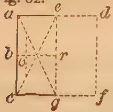
Any Figure of Revolution.

To Ascertain Convex Surface of any Figure of Revolution.—Figs. 62, 63, and 64.

RULE.—Multiply length of generating line by circumference described by its centre of gravity.

Or, $l\,2\,r\,p$ = *surface, r representing radius of centre of gravity.*

Fig. 62.



EXAMPLE 1.—If generating line, $a\,c$, of cylinder, $a\,c\,d\,f$, 10 inches in diameter, Fig. 62, is 10, then centre of gravity of it will be in b, radius of which is $b\,r = 5$.

Hence $10 \times \overline{5 \times 2} \times 3.1416 = 314.16$ *ins.*

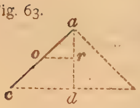
Again, if generating line is $e\,a\,c\,g$, and it is ($e\,a = 5$, $a\,c = 10$, and $c\,g = 5$) = 20, then centre of gravity, o, will be in middle of line joining centres of gravity of triangles $e\,a\,c$ and $a\,c\,g = 3.75$ from r.

Hence $20 \times \overline{3.75 \times 2} \times 3.1416 = 471.24$ *square ins. = entire surface.*

VERIFICATION. { Convex surface as above........................314.16
{ Area of each end, $10^2 \times .7854 \times 2$ = ...157.08

471.24 *inches.*

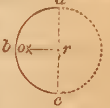
Fig. 63.



2.—If generating elements of a cone, Fig. 63, are $a\,d = 10$, $d\,c = 10$, and $a\,c$, generating line, = 14.142, centre of gravity of which is in o, and $o\,r = 5$,

Then $14.142 \times \overline{5 \times 2} \times 3.1416 = 444.285$, *convex surface*, and $\overline{10 \times 2}^2 \times .7854 = 314.16$, *area of base.*

Fig. 64.



Hence $444.285 + 314.16 = 758.445$, *entire surface.*

3.—If generating elements of a sphere, Fig. 64, are $a\,c = 10$, $a\,b\,c$ will be 15.708, centre of gravity of which is in o, and by Rule, page 606, $o\,r = 3.183$.

Hence $15.708 \times \overline{3.183 \times 2} \times 3.1416 = 314.16$ *square ins.*

Capillary Tube.

To Compute Diameter of a Capillary Tube.

RULE.—Weigh tube when empty, and again when filled with mercury; subtract one weight from the other; reduce difference to grains, and divide it by length of tube in inches. Extract square root of this quotient, multiply it by .019 224 5, and product will give diameter of tube in inches.

Or, $\sqrt{\frac{w}{l}} \times .019\,224\,5$ = *diameter, w representing difference in weights in grains and l length of tube.*

EXAMPLE.—Difference in weights of a capillary tube when empty and when filled with mercury is 90 grains, and length of tube is 10 inches; what is diameter of it?

$90 \div 10 = 9$ = *weight of mercury ÷ length of tube*; $\sqrt{9} = 3$, and $3 \times .019\,224\,5 = .057\,673\,5$ = *square root of above quotient* × .019 224 5 *inches = diameter of tube.*

PROOF.—Weight of a cube inch of mercury is 3442.75 grains, and diameter of a circular inch of equal area to a square inch is 1.128 (page 342).

If, then, 3442.75 grains occupy 1 cube inch, 90 grains will require .026 141 9 cube inch, which, ÷ 10 for height of tube = .002 614 19 inch for area of section of tube.

Then $\sqrt{.002\,614\,19} = .051\,129$ = *side of square of a column of mercury of this area.*

Hence $.051\,129 \times 1.128$ (which is ratio between side of a square and diameter of a circle of equal area) = .057 673 5 *ins.*

To Ascertain Area of an Irregular Figure.

RULE.—Take a uniform piece of board or pasteboard, weigh it, cut out figure of which area is required, and weigh it; then, as weight of board or pasteboard is to entire surface, so is weight of figure as cut out to its surface.

Or, see rule page 341, or Simpson's rule, page 342.

To Ascertain Area of any Plane Figure.

RULE. — Divide surfaces into squares, triangles, prisms, etc.; ascertain their areas and add them together.

Reduction of an Ascending or Descending Line to Horizontal Measurement.

In Link and Foot.

Degrees.	Link.	Foot.	Degrees.	Link.	Foot.	Degrees.	Link.	Foot.
1	.000 099	.000 15	7	.004 917	.007 45	13	.016 915	.025 63
2	.000 403	.000 61	8	.006 421	.009 73	14	.019 602	.029 7
3	.000 904	.001 37	9	.008 125	.012 31	15	.022 486	.034 07
4	.001 61	.002 44	10	.010 025	.015 19	16	.025 569	.038 74
5	.002 515	.003 81	11	.012 124	.018 37	17	.028 925	.043 7
6	.003 617	.005 48	12	.014 421	.022 85	18	.032 3	.048 94

ILLUSTRATION 1.—In an ascending grade of 14°, what is reduction in 500 feet?

$14^\circ = 500 \times .0297 = 14.85$ *feet* $= 14$ *feet* 10.2 *ins.*

2.—What is reduction in 500 links?

$14^\circ = 500 \times .019\,602 = 9.801$ *feet* $= 9$ *feet* 9.6 *ins.*

Reduction of Grade of an Ascending or Descending Line to Degrees.

Per 100 Links, Feet, etc.

Grade.	Degrees.	Grade.	Degrees.	Grade.	Degrees.	Grade.	Degrees.
	′ ″		° ′ ″		° ′ ″		° ′ ″
.25	8 35.2	1.75	1 0 10.3	4.5	2 34 45.5	10	5 44 20.7
.5	17 10.3	2	1 8 45.5	5	2 51 57.6	11	6 18 55.8
.75	25 47.6	2.5	1 25 57.6	6	3 26 22.7	12	6 53 31
1	34 22.7	3	1 43 8.3	7	4 0 49.6	13	7 28 10.3
1.25	42 57.9	3.5	2 0 20.7	8	4 35 18.6	14	8 2 51.7
1.5	51 35.2	4	2 17 33.1	9	5 9 49.6	15	8 37 37.2

To Plot Angles without a Protractor.

On a given line prick off 100 with any convenient scale, and from the point so pricked off lay off at right angle with the same scale the natural tangent due to the angle (see table of Natural Tangents and Sines); or strike out a portion of a circle with radius 100 and lay off a chord = 2 sin. of half the angle required.

To Compute Chord of an Angle.

Double sine of half angle.

ILLUSTRATION.—What is the chord of $21^\circ\ 30'$?

Sine of $\frac{21^\circ\ 30'}{2} = 10^\circ\ 45'$, and sine of $10^\circ\ 45' = .186\,52$, which, $\times\ 2 = .373\,04$ *chord.*

To Ascertain Value of a Power of a Quantity.

RULE.—Multiply logarithm of quantity by fractional exponent, and product is logarithm of required number.

EXAMPLE.—What is the value of $16^{\frac{3}{4}}$?

$\frac{3}{4} \times \log. 16 = \frac{3}{4} \times 1.204\,12 = .903\,09$. Number for which = 8.

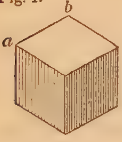
MENSURATION OF VOLUMES.

Cubes and Parallelopipedons.

Cube.

DEFINITION.—A volume contained by six equal square sides.

Fig. 1.



To Compute Volume of a Cube.—Fig. 1.

RULE.—Multiply a side of cube by itself, and that product again by a side.

Or, $s^3 = V$, *s representing length of a side, and V volume.*

EXAMPLE.—Side, *a b*, Fig. 1, is 12 inches; what is volume of it?

$12 \times 12 \times 12 = 1728$ *cube ins.*

Parallelopipedon.

DEFINITION.—A volume contained by six quadrilateral sides, every opposite two of which are equal and parallel.

Fig. 2.

To Compute Volume of a Parallelopipedon. —Fig. 2.

RULE.—Multiply length by breadth, and that product again by depth.

Or, $l\,b\,d = V$.

Prisms, Prismoids, and Wedges.

Prisms.

DEFINITION.—Volumes, ends of which are equal, similar, and parallel planes, and sides of which are parallelograms.

NOTE.—When ends of a prism or prismoid are triangles, it is termed a *triangular prism* or *prismoid*; when rhomboids, a *rhomboidal prism*, and when squares, a *square prism*, etc.

Fig. 3.

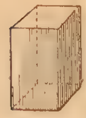


Fig. 4.

To Compute Volume of a Prism.—Figs. 3 and 4.

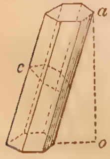
RULE.—Multiply area of base by height.

Or, $a\,h = V$.

EXAMPLE.—A triangular prism, *a b c*, Fig. 4, has sides of 2.5 feet, and a length, *c b*, of 10; what is its volume?

By Rule, page 339, $2.5^2 \times .433 = 2.70625 =$ *area of end a b*, and $2.70625 \times 10 = 27.0625$ *cube feet.*

Fig. 5.



When a Prism is Oblique or Irregular.

RULE.—Multiply area of an end by height, as *a o*; or, multiply area taken at a right angle to sides, as at *c*, by actual length.

To Compute Volume of any Frustum of a Prism, whether Right or Oblique.—Figs. 6 and 7.

Fig. 6.

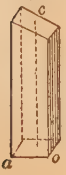
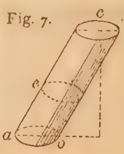


Fig. 7.



RULE.—Multiply area of base by perpendicular distances between it and centre of gravity of upper or other end.

Or, area at right angle to side as at *e* by actual length.

EXAMPLE.—Area of base, *a o*, of frustum of a rectangular or cylindrical prism, Fig. 6, is 15 inches, and height to centre of gravity, *c*, is 12; what is its volume?

$10 \times 12 = 120$ *cube ins.*

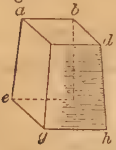
Prismoids.*

To Compute Volume of a Prismoid.—Fig. 8.

RULE.—To sum of areas of the two ends add four times area of middle section, parallel to them, and multiply this sum by one sixth of perpendicular height.

NOTE.—This is the general rule, and known as the *Prismoidal Formula*, and it applies equally to all figures of proportionate or dissimilar ends.

Fig. 8.



Or, $a + a' + 4\,m \times \overline{h \div 6} = V$, *a and a′ representing areas of ends, and m area of middle section.*

EXAMPLE.—What is volume of a rectangular prismoid, Fig. 8, lengths and breadths, $e\,g$ and $g\,h$, $a\,b$ and $b\,d$, of two ends being 7×6 and 3×2 inches, and height 15 feet?

$7 \times 6 + \overline{3 \times 2} = 42 + 6 = 48 =$ *sum of areas of two ends;* $7 + 3 \div 2 = 5 =$ *length of middle section;* $6 + 2 \div 2 = 4 =$ *breadth of middle section;* $5 \times 4 \times 4 = 80 =$ *four times area of middle section.*

Then $48 + 80 \times \frac{15 \times 12}{6} = 128 \times 30 = 3840$ *cube ins.*

NOTE 1.—Length and breadth of middle section are respectively equal to half sum of lengths and breadths of the two ends.

2.—Prismoids, alike to prisms, derive their designation from figure of their ends as triangular, square, rectangular, pentagonal, etc.

When it is Irregular or Oblique and their ends are united by plane or curved surfaces, through which and every point of them, a right line may be drawn from one of the ends or parallel faces to the other.—Figs. 9, 10, and 11.

Fig. 9.

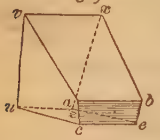


Fig. 10.

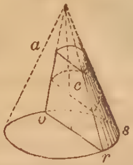
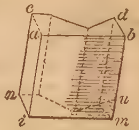


Fig. 11.



EXAMPLE.—Areas of ends, $a\,c$ and $o\,r\,s$, Fig. 10, $a\,b\,c\,d$, and $i\,m\,n\,u$, Fig. 11, and $a\,b\,c\,e$ and $v\,x\,w\,z$, Fig. 9, are each 10 and 30 inches, that of their middle section 20, and their perpendicular heights 18; what is their volume?

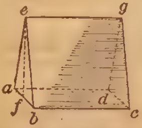
$10 + 30 + \overline{20 \times 4} = 120 =$ *sum of areas of ends* $+ 4$ *times middle section.* And $120 \times \frac{18}{6} = 360$ *cube ins.*

Wedge.

To Compute Volume of a Wedge.—Fig. 12.

RULE.—To length of edge add twice length of back; multiply this sum by perpendicular height, and then by breadth of back, and take one sixth of product.

Fig. 12.



Or, $(l + \overline{l' \times 2} \times h\,b) \div 6 = V$.

EXAMPLE.—Length of edge of a wedge, $e\,g$, is 20 inches, back, $a\,b\,c\,d$, is 20 by 2, and its height, $e\,f$, 20; what is its volume?

$20 + \overline{20 \times 2} = 60 =$ *length of edge added to twice length of back;* $60 \times 20 \times 2 = 2400 =$ *above sum multiplied by height, and that product by breadth of back.*

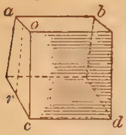
Then $2400 \div 6 = 400$ *cube ins.*

NOTE.—When a wedge is a true prism, as represented by Fig. 12, volume of it is equal to area of an end multiplied by its length.

* An excavation or embankment of a road, when terminated by parallel cross sections, is a rectangular prismoid.

To Compute Frustum of a Wedge.—Fig. 13.

Fig. 13.



RULE.—To sum of areas of both ends, add 4 times area of section parallel to and equally distant from both ends, and multiply sum by one sixth of length.

$$\text{Or, } A+a+4a'\times\frac{l}{6}=V.$$

EXAMPLE.—Lengths of edge and back of a frustum of a wedge $a\,b$ and $c\,d$ are 20×1 and 20×2 ins., and height $o\,r$ is 20 ins.; what is its volume?

$$\frac{20\times\overline{2+1}}{2}\times 2+4\times\left(20\times\frac{2+1}{2}\right)\times\frac{20}{6}=60+120\times\frac{20}{6}=600 \textit{ cube ins.}$$

NOTE.—When frustum is a true prism, as represented Fig. 13, volume of it is equal to mean area of ends multiplied by its length.

Regular Bodies (Polyhedrons).

DEFINITION.—A regular body is a solid contained under a certain number of similar and equal plane faces,* all of which are equal regular polygons.

NOTE 1.—Whole number of regular bodies which can possibly be formed is five.

2.—A sphere may always be inscribed within, and may always be circumscribed about a regular body or polyhedron, which will have a common centre.

Fig. 14.

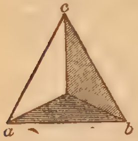


Fig. 15.

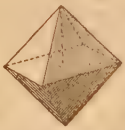


Fig. 16.

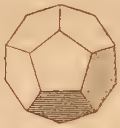
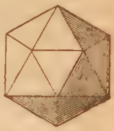


Fig. 17.



1. Tetrahedron, or Pyramid, Fig. 14, which has four triangular faces.
2. Hexahedron, or Cube, Fig. 1, which has six square faces.
3. Octahedron, Fig. 15, which has eight triangular faces.
4. Dodecahedron, Fig. 16, which has twelve pentagonal faces.
5. Icosahedron, Fig. 17, which has twenty triangular faces.

To Compute Elements of any Regular Body.—Figs. 14, 15, 16, and 17.

To Compute Radius of a Sphere that will Circumscribe a given Regular Body, or that may be Inscribed within it.

When Linear Edge is given. RULE.—Multiply it by multiplier opposite to body in columns A and B in following Table, under head of element required.

EXAMPLE.—Linear edge of a hexahedron or cube, Fig. 1, is 2 inches; what are radii of circumscribing and inscribed spheres?

$2\times .86602=1.73204$ *inches = radius of circumscribing sphere;* $2\times .5=1$ *inch = radius of inscribed sphere.*

When Surface is given. RULE.—Multiply square root of it by multiplier opposite to body in columns C and D in following Table, under head of element required.

When Volume is given. RULE.—Multiply cube root of it by multiplier opposite to body in columns E and F in following Table, under head of element required.

* Angle of adjacent faces of a polygon is termed diedral angle.

When one of the Radii of Circumscribing or Inscribed Sphere alone is required, the other being given. RULE.—Multiply given radius by multiplier opposite to body in columns G and H in Table, page 364, under head of other radius.

To Compute Linear Edge.

When Radius of Circumscribing or Inscribed Sphere is given. RULE.—Multiply radius given by multiplier opposite to body in columns I and K in Table, page 364.

When Surface is given. RULE.—Multiply square root of it by multiplier opposite to body in column L in Table, page 364.

When Volume is given. RULE.—Multiply cube root of it by multiplier opposite to body in column M in Table, page 364.

To Compute Surface.

When Radius of Circumscribing Sphere is given. RULE.—Multiply square of radius by multiplier opposite to body in column N in Table, page 364.

When Radius of Inscribed Sphere is given. RULE.—Multiply square of radius by multiplier opposite to body in column O in Table, page 364.

When Linear Edge is given. RULE.—Multiply square of edge by multiplier opposite to body in column P in Table, page 364.

When Volume is given. RULE.—Extract cube root of volume, and multiply square of root by multiplier opposite to body in column Q in Table, page 364.

To Compute Volume.

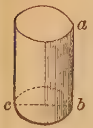
When Linear Edge is given. RULE.—Cube linear edge, and multiply it by multiplier opposite to body in column R in Table, page 364.

When Radius of Circumscribing Sphere is given. RULE.—Multiply cube of radius given by multiplier opposite to body in column S in Table, page 364.

When Radius of Inscribed Sphere is given. RULE.—Multiply cube of radius given by multiplier opposite to body in column T in Table, page 364.

When Surface is given. RULE.—Cube surface given, extract square root, and multiply the root by multiplier opposite to body in column U in Table, page 364.

Fig. 18.



Cylinder.

To Compute Volume of a Solid Cylinder.—Fig. 18.

RULE.—Multiply area of base by height.

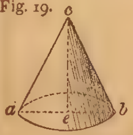
EXAMPLE.—Diameter of a cylinder, $b\,c$, is 3 feet, and its length, $a\,b$, 7 feet; what is its volume?

Area of 3 feet $= 7.068$. Then $7.068 \times 7 = 49.176$ *cube feet.*

To Compute Volume of a Hollow Cylinder.

RULE.—Subtract volume of internal cylinder from that of cylinder.

Fig. 19.



Cone.

To Compute Volume of a Cone.—Fig. 19.

RULE.—Multiply area of base by perpendicular height, and take one third of product.

EXAMPLE.—Diameter, $a\,b$, of base of a cone is 15 inches, and height, $c\,e$, 32.5 inches; what is its volume?

Area of 15 inches $= 176.7146$. Then $\frac{176.715 \times 32.5}{3} = 1914.4125$ *cube ins.*

Units for Elements of the Regular Bodies.

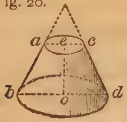
	Figure.	A. Radius of Circumscribing Sphere. By Linear Edge.	B. Radius of Inscribed Sphere. By Linear Edge.	C. Radius of Circumscribing Sphere. By Surface.	D. Radius of Inscribed Sphere. By Surface.	E. Radius of Circumscribing Sphere. By Volume.	F. Radius of Inscribed Sphere. By Volume.	G. Radius of Inscribed Sphere. By Circumscribing Sphere.	H. Radius of Circumscribing Sphere. By Inscribed Sphere.	I. Linear Edge. By Radius of Circumscribing Sphere.	K. Linear Edge. By Radius of Inscribed Sphere.
4	Tetrahedron....	.612 37	.204 12	.465 3	.155 1	1.248 96	.416 34	.333 33	3	1.632 99	4.898 98
6	Hexahedron ...	.866 02	.5	.353 55	.204 12	.866 02	.5	.577 35	1.732 05	1.154 7	2
8	Octahedron	.707 11	.408 25	.379 92	.219 35	.908 06	.524 56	.577 35	1.732 05	1.414 21	2.449 49
12	Dodecahedron ..	1.401 26	1.113 52	.308 39	.245 07	.710 75	.564 8	.794 65	1.258 41	.713 64	.898 06
20	Icosahedron	.951 06	.755 76	.323 18	.256 81	.733 29	.582 71	.794 65	1.258 41	1.051 46	1.323 17

	Figure.	L. Linear Edge. By Surface.	M. Linear Edge. By Volume.	N. Surface. By Radius of Circumscribing Sphere.	O. Surface. By Radius of Inscribed Sphere.	P. Surface. By Linear Edge.	Q. Surface. By Volume.	R. Volume. By Linear Edge.	S. Volume. By Radius of Circumscribing Sphere.	T. Volume. By Radius of Inscribed Sphere.	U. Volume. By Surface.
4	Tetrahedron ...	.759 84	2.039 55	4.618 8	41.569 22	1.732 05	7.205 62	.117 85	.513 2	13.856 41	.051 7
6	Hexahedron ...	.408 25	1	8	24	6	6	1	1.539 6	8	.068 04
8	Octahedron	.537 29	1.284 9	6.928 2	20.784 61	3.464 1	5.719 1	.471 4	1.333 33	6.928 2	.073 11
12	Dodecahedron ..	.220 08	.507 22	10.514 62	16.650 87	20.645 73	5.311 61	7.663 12	2.785 17	5.550 29	.081 69
20	Icosahedron	.339 81	.771 02	9.574 54	15.162 17	8.660 25	5.148 35	2.181 7	2.536 15	5.054 06	.085 6

To Compute Volume of Frustum of a Cone.—Fig. 20.

RULE.—Add together squares of the diameters or circumferences of greater and lesser ends and product of the two diameters or circumferences; multiply their sum respectively by .7854 and .079 58, and this product by height; then divide this last product by 3.

Fig. 20.



$$\text{Or, } d^2 + d'^2 + \overline{d \times d'} \times .7854\, h \div 3 = V.$$

$$\text{Or, } c^2 + c'^2 + \overline{c \times c'} \times .079\,58\, h \div 3 = V.$$

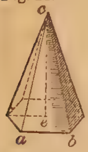
EXAMPLE.—What is volume of frustum of a cone, diameters of greater and lesser ends, *b d*, *a c*, being 5 and 3 feet, and height, *e o*, 9?

$5^2 + 3^2 + 5 \times 3 = 49$; and $49 \times .7854 = 38.4846 =$ *above sum by* .7854; and $\frac{38.4846 \times 9}{3} = 115.4538$ *cube feet.*

Pyramid.

NOTE.—Volume of a pyramid is equal to one third of that of a prism having equal bases and altitude.

Fig. 21.



To Compute Volume of a Pyramid.—Fig. 21.

RULE.—Multiply area of base by perpendicular height, and take one third of product.

EXAMPLE.—What is the volume of a hexagonal pyramid, Fig. 21, a side, *a b*, being 40 feet, and its height, *e c*, 60?

$40^2 \times 2.5981$ (tabular multiplier, page 341) $= 4156.96 =$ *area of base.*

$$\frac{4156.96 \times 60}{3} = 83\,139.2 \textit{ cube feet.}$$

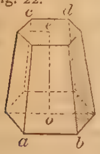
To Compute Volume of Frustum of a Pyramid.—Fig. 22.

RULE.—Add together squares of sides of greater and lesser ends, and product of these two sides; multiply sum by tabular multiplier for areas in Table, page 341, and this product by height; then divide last product by 3.

$$\text{Or, } s^2 + s'^2 + \overline{s \times s'} \times \textit{tab. mult.} \times h \div 3 = V.$$

When Areas of Ends are known, or can be obtained without reference to a tabular multiplier, use following.

Fig. 22.



$$\text{Or, } a + a' + \sqrt{\overline{a \times a'}} \times h \div 3 = V.$$

EXAMPLE.—What is the volume of the frustum of a hexagonal pyramid, Fig. 22, the lengths of the sides of the greater and lesser ends, *a b*, *c d*, being respectively 3.75 and 2.5 feet, and its perpendicular height, *e o*, 7.5?

$3.75^2 + 2.5^2 = 20.3125 =$ *sum of squares of sides of greater and lesser ends;* $20.3125 + \overline{3.75 \times 2.5} = 29.6875 =$ *above sum added to product of the two sides;* $29.6875 \times 2.5981 \times 7.5 = 578.48 \times$ *tab. mult., and again by the height,* which, $\div 3 = 192.83$ *cube feet.*

When Ends of a Pyramid are not those of a Regular Polygon, or when Areas of Ends are given

RULE.—Add together areas of the two ends and square root of their product; multiply sum by height, and take one third of product.

$$\text{Or, } a + a' + \sqrt{a\,a'} \times h \div 3 = V.$$

EXAMPLE.—What is the volume of an irregular-sided frustum of a pyramid, the areas of the two ends being 22 and 88 inches, and the length 20?

$22 + 88 = 110 =$ *sum of areas of ends;* $22 \times 88 = 1936$, and $\sqrt{1936} = 44 =$ *square root of product of areas.* Then $\frac{\overline{110 + 44} \times 20}{3} = 1026.66$ *cube ins.*

Spherical Pyramid.

A Spherical Pyramid is that part of a sphere included within three or more adjoining plane surfaces meeting at centre of sphere. The spherical polygon defined by these plane surfaces of pyramid is termed the base, and the lateral faces are sectors of circles.

NOTE.—To compute the Elements of Spherical Pyramids, see Docharty and Hackley's Geometry.

Cylindrical Ungulas.

DEFINITION.—Cylindrical Ungulas are frusta of cylinders. Conical Ungulas are frusta of cones.

To Compute Volume of a Cylindrical Ungula.—Fig. 23.

1. *When Section is parallel to Axis of Cylinder.* RULE.—Multiply area of base by height of the cylinder.

Fig. 23.

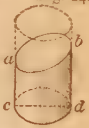
$$\text{Or, } a\,h = V.$$

EXAMPLE.—Area of base, *d e f*, Fig. 23, of a cylindrical ungula is 15.5 inches, and its height, *a e*, 20; what is its volume?

$$15.5 \times 20 = 310 \textit{ cube ins.}$$

2. *When Section passes Obliquely through opposite sides of Cylinder*, Fig. 24. RULE.—Multiply area of base of cylinder by half sum of greatest and least lengths of ungula.

Fig. 24.



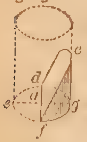
$$\text{Or, } a \times \overline{l + l'} \div 2 = V.$$

EXAMPLE.—Area of base, *c d*, of a cylindrical ungula, Fig. 24, is 25 inches, and the greater and less heights of it, *a c*, *b d*, are 15 and 17; what is its volume?

$$25 \times \frac{15 + 17}{2} = 400 \textit{ cube ins.}$$

3. *When Section passes through Base of Cylinder and one of its Sides, and Versed Sine does not exceed Sine, or the Base is equal to or less than a Semicircle*, Fig. 25. RULE.—From two thirds of cube of sine of half arc of base subtract product of area of base and cosine * of half arc. Multiply difference thus found by quotient arising from height divided by versed sine.

Fig. 25.



$$\text{Or, } \frac{2\,sin.^3}{3} - \overline{a\,c} \times \frac{h}{v.\ sin.} = V, \ \textit{v. sin. representing versed sine.}$$

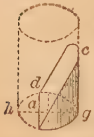
EXAMPLE.—Sine, *a d*, of half arc, *d e f*, of base of an ungula, Fig. 25, is 5 inches, diameter of cylinder is 10, and height, *c g*, of ungula 10; what is its volume?

Two thirds of $5^3 = 83.333$ = *two thirds of cube of sine.* As versed sine and radius of base are equal, cosine is 0. Hence, area of base × cosine = 0, and $83.333 - 0 \times 10 \div 5 = 166.666$ *cube ins.*

4. *When Section passes through Base of Cylinder, and Versed Sine exceeds Radius, or when the Base exceeds a Semicircle*, Fig. 26. RULE.—To two thirds of cube of sine of half arc of base add product of area of base and cosine. Multiply sum thus found by quotient arising from height, divided by versed sine.

Fig. 26.

$$\text{Or, } \frac{2\,sin.^3}{3} + \overline{a\,c} \times \frac{h}{v.\ sin.} = V.$$



EXAMPLE.—Sine, *a d*, of half arc of an ungula, Fig. 26, is 12 inches, versed sine, *a g*, is 16, height, *g c*, 10, and diameter of cylinder 25; what is its volume?

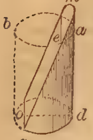
Two thirds of $12^3 = 1152$ = *two thirds of cube of sine of half arc of base.* Area of base = 331.78; $1152 + 331.78 \times \overline{16 - 12.5} = 2313.23$ = *sum of two thirds of cube of sine of half the arc of base, and product of area of base and cosine.* Then $2313.23 \times \overline{20 \div 16} = 2891.5375$ *cube ins.*

* When the cosine is 0, the product is 0.

5. *When Section passes Obliquely through both Ends of Cylinder*, Fig. 27.
RULE.—Conceive section to be continued till it meets side of cylinder produced; then, as the difference of versed sines of the arcs of the two ends of ungula is to the versed sine of arc of less end, so is the height of cylinder to the part of side produced.

Ascertain volume of each of the ungulas by Rules 3 and 4, and take their difference.

Fig. 27.



Or, $\frac{v.\ sin.'\ h}{v.\ sin. - v.\ sin.'} = h'$, *v. sin. and v. sin.' representing versed sines of arcs of the two ends, h height of cylinder, and h' height of part produced.*

EXAMPLE.—Versed sines, *a e*, *d o*, and sines, *e* and *o*, of arcs of two ends of an ungula, Fig. 27, are assumed to be respectively 8.5 and 25, and 11.5 and 0 inches, length of ungula, *b o*, within cylinder, cut from one having 25 inches diameter, *d o*, is 20 inches; what is height of ungula produced beyond cylinder, and what is volume of it?

$25 \backsim 8.5 : 8.5 :: 20 : 10.303 =$ *height of ungula produced beyond cylinder.*

Greater ungula, sine *o* being 0, versed sine = *the diameter*. Base of ungula being a circle of 25 inches diameter, area = 490.875. Versed sine and diameter of base being equal (25), sine = 0. $490.875 \times 25 \backsim \frac{25}{2} = 6135.9375 =$ *product of area of base and cosine, or excess of versed sine over sine of base.* $30.303 \div 25 = 1.212\,12 =$ *quotient of height ÷ versed sine.*

Then $6135.9375 \times 1.212\,12 = 7437.4926$ *cube inches;* and by Rules 3 and 4, volumes of less and greater ungulas = 515.444, and 6922.0486 = 7437.4926 *cube inches.*

Sphere.

DEFINITION.—A solid, surface of which is at a uniform distance from the centre.

Fig. 28.

To Compute Volume of a Sphere.—Fig. 28.

RULE.—Multiply cube of diameter by .5236.

Or, $d^3 \times .5236 = V$, *d representing diameter.*

EXAMPLE.—What is volume of a sphere, Fig. 28, its diameter, *a b*, being 10 inches?

$10^3 = 1000$, and $1000 \times .5236 = 523.6$ *cube ins.*

To Compute Volume of a Hollow Sphere.

RULE.—Subtract volume of internal space from that of sphere.

Or, $V - v =$ *volume.*

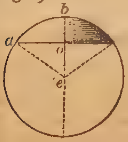
Segment of a Sphere.

DEFINITION.—A section of a sphere.

To Compute Volume of a Segment of a Sphere.—Fig. 29.

RULE 1.—To three times square of radius of its base add square of its height; multiply this sum by height, and product by .5236.

Fig. 29.



Or, $\overline{3\,r^2 + h^2}\,h \times .5236 = V.$

2.—From three times diameter of sphere subtract twice height of segment; multiply this remainder by square of height, and product by .5236.

Or, $\overline{3\,d - 2\,h}\,h^2 \times .5236 = V.$

EXAMPLE.—Segment of a sphere, Fig. 29, has a radius, *a e*, of 7 inches for its base, and a height, *b o*, of 4; what is its volume?

$7^2 \times 3 + 4^2 = 163 =$ *the sum of three times square of radius and square of height;* $163 \times 4 \times .5236 = 331.3872$ *cube ins.*

Spherical Zone (or Frustum of a Sphere).

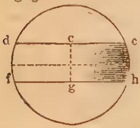
DEFINITION.—Part of a sphere included between two parallel chords.

To Compute Volume of a Spherical Zone.—Fig. 30.

DEFINITION.—Part of a sphere included between two parallel planes.

RULE.—To sum of squares of the radii of the two ends add one third of square of height of zone; multiply this sum by height, and again by 1.5708.

Fig. 30.



Or, $r^2 + r'^2 + \overline{h^2 \div 3}\, h \times 1.5708 = V$.

EXAMPLE.—What is the volume of a spherical zone, Fig. 30, greater and less diameters, *f h* and *d e*, being 20 and 15 inches, and distance between them, or height of zone, *c g*, being 10 ins.?

$10^2 + 7.5^2 = 156.25 =$ *sum of squares of radii of the two ends;*

$156.25 + \overline{10^2 \div 3} = 189.58 =$ *above sum added to one third of square of the height.*

Then $189.58 \times 10 \times 1.5708 = 2977.9226$ *cube ins.*

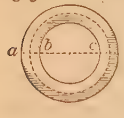
Cylindrical Ring.

DEFINITION.—A ring formed by the curvature of a cylinder.

To Compute Volume of a Cylindrical Ring.—Fig. 31.

RULE.—To diameter of body of ring add inner diameter of ring; multiply sum by square of diameter of body, and product by 2.4674.

Fig. 31.



Or, $\overline{d + d'}\, d^2\; 2.4674 = V$.

Or, $a\,l = V$, *a representing area of section of body, and l length of axis of body..*

EXAMPLE.—What is volume of an anchor ring, Fig 31, diameter of metal, *a b*, being 3 inches, and inner diameter of ring, *b c*, 8?

$3 + 8 \times 3^2 = 99 =$ *product of sum of diameters and square of diameter of body of ring.*

Then $99 \times 2.4674 = 244.2726$ *cube ins.*

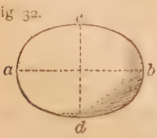
Spheroids (Ellipsoids).

DEFINITION.—Solids generated by the revolution of a semi-ellipse about one of its diameters. When the revolution is about the transverse diameter they are termed Prolate, and when about the conjugate they are Oblate.

To Compute Volume of a Spheroid.—Fig. 32.

RULE.—Multiply square of revolving axis by fixed axis, and this product by .5236.

Fig 32.



Or, $a^2\, a' \times .5236 = V$, *a and a′ representing revolving and fixed axes.*

Or, $4 \div 3 \times 3.1416\, r^2\, r' = V$, *r and r′ representing semi-axes.*

EXAMPLE.—In a prolate spheroid, Fig. 32, fixed axis, *a b*, is 14 inches, and revolving axis, *c d*, 10; what is its volume?

$10^2 \times 14 = 1400 =$ *product of square of revolving axis and fixed axis.* Then $1400 \times .5236 = 733.04$ *cube ins.*

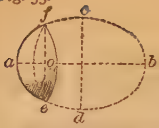
NOTE.—Volume of a spheroid is equal to ⅔ of a cylinder that will circumscribe it.

Segments of Spheroids.

To Compute Volume of Segment of a Spheroid.—Fig. 33.

When Base, e f, is Circular, or parallel to revolving Axis, as c d, Fig. 33, *or as e f to Axis a b,* Fig. 34. RULE.—Multiply fixed axis by 3, height of segment by 2, and subtract one product from the other; multiply remainder by square of height of segment, and product by .5236. Then, as square of fixed axis is to square of revolving axis, so is last product to volume of segment.

Fig. 33.



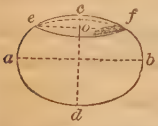
Or, $\frac{3\,a - \overline{2\,h}\,h^2 \times .5236 \times a'^2}{a^2} = V.$

EXAMPLE.—In a prolate spheroid, Fig. 33, fixed or transverse axis, $a\,b$, is 100 inches, revolving or conjugate, $c\,d$, 60, and height of segment, $a\,o$, 10; what is its volume?

$100 \times 3 - \overline{10 \times 2} = 280$ = *twice the height of segment subtracted from three times fixed axis;* $280 \times 10^2 \times .5236 =$ 14 660.8 *inches = product of above remainder, square of height, and* .5236. Then $100^2 : 60^2 :: 14\,669.8 : 5277.888$ *cube ins.*

When Base, e f, is Elliptical, or perpendicular to revolving Axis, a b, Fig. 33, *or as e f to Axis c d,* Fig. 34. RULE.—Multiply fixed axis by 3, and height of segment by 2, and subtract one from the other; multiply remainder by square of height of segment, and product by .5236. Then, as fixed axis is to revolving axis, so is last product to volume of segment.

Fig. 34.



Or, $\frac{3\,a' - \overline{2\,h}\,h^2 \times .5236 \times a}{a'} = V.$

EXAMPLE.—Diameters of an oblate spheroid, Fig. 34, are 100 and 60 inches, and height of a segment thereof is 12; what is its volume?

$100 \times 3 - \overline{12 \times 2} = 276$ = *twice the height of the segment subtracted from three times the revolving axis;* $276 \times 12^2 \times .5236$ $= 20\,809.9584$ = *product of above remainder, the square of height, and* .5236.

Then $100 : 60 :: 20\,809.9584 : 12\,485.975$ *cube ins.*

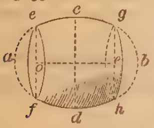
Frusta of Spheroids.

To Compute Volume of Middle Frustum of a Spheroid.—Fig. 35.

When Ends, e f and g h, are Circular, or parallel to revolving Axis, as c d, Fig. 35, *or a b,* Fig. 36. RULE.—To twice square of revolving axis add square of diameter of either end; multiply this sum by length of frustum, and product by .2618.

Or, $2\,a'^2 + d^2 \times l\,.2618 = V.$

Fig. 35.

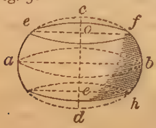


EXAMPLE.—Middle frustum of a prolate spheroid, $i\,o$, Fig. 35, is 36 inches in length, diameter of it being, in middle, $c\,d$, 50, and at its ends, $e\,f$ and $g\,h$, 40; what is its volume?

$50^2 \times 2 + 40^2 = 6600$ = *sum of twice square of middle diameter added to square of diameter of ends.* Then $6600 \times 36 \times 2618 = 62\,203.68$ *cube ins.*

When Ends, e f and g h, are Elliptical, or perpendicular to revolving Axis, a b, Fig. 35, *or e f and g h to Axis, c d,* Fig. 36. RULE.—To twice product of transverse and conjugate diameters of middle section, add product of transverse and conjugate of either end; multiply this sum by length of frustum, and product by .2618.

Fig. 36.



Or, $d\,d' \times 2 + \overline{d\,d'}\,l \times .2618 = V.$

EXAMPLE.—In middle frustum of a prolate spheroid, Fig. 36, diameters of its middle section are 50 and 30 inches, its ends 40 and 24, and its length, $o\,i$, 18; what is its volume?

$50 \times 30 \times 2 = 3000$ = *twice product of transverse and conjugate diameters;* $3000 + \overline{40 \times 24} = 3960$ = *sum of above product and product of transverse and conjugate diameters of ends.*

Then $3960 \times 18 \times .2618 = 18\,661.104$ *cube ins.*

Links.

Definition.—Elongated or Elliptical rings.

Elongated or Elliptical Links.

To Compute Volume of an Elongated or Elliptical Link. —Figs. 37 and 38.

Rule.—Multiply area of a section of the body of link by its length, or circumference of its axis.

Or, $a\,l$ or $c = V$.

Note.—By Rule, page 353, Circumference or length of axis of an Elongated link = the sum of 3.1416 times sum of less diameter added to thickness of ring, and product of twice remainder of less diameter subtracted from greater.

Also, Circumference or length of axis of an Elliptical ring = square root of half sum of diameters added to thickness of ring or axes squared × 3.1416.

Fig. 37.

Example.—Elongated link of a chain, Fig. 37, is 1 inch in diameter of body, $a\,b$, and its inner diameters, $b\,c$ and $e\,f$, are 10 and 2.5 inches; what is its volume?

Area of 1 inch = .7854; $2.5 + 1 \times 3.1416 = 10.9956$ = 3.1416 *times sum of less diameter and thickness of ring = length of axis of ends;* $10 - 2.5 \times 2 = 15$ = *twice remainder of the less diameter subtracted from greater = length of sides of body.*

Then $10.9956 + 15 = 25.9956$ = *length of axis of length.*

Hence $.7854 \times 25.9956 = 20.417$ *cube ins.*

Fig. 38.

2.—Elliptical link of a chain, Fig. 38, is of the same dimensions as preceding; what is its volume?

$\overline{2.5+1}^2 + \overline{10+1}^2 = 133.25$ = *diameter of axes squared;* $\sqrt{\frac{133.25}{2}}\ 3.1416$ = 25.643 = *square root of half sum of diameters squared* × 3.1416 = *circumference of axis of ring.* Area of 1 inch = .7854.

Then $25.643 \times .7854 = 20.14$ *cube ins.*

Spherical Sector.

Definition.—A figure generated by the revolution of a sector of a circle about a straight line through the vertex of the sector as an axis.

Note.—Arc of sector generates surface of a zone, termed base of sector of a sphere, and the radii generate surfaces of two cones, having a vertex in common with the sector at the centre of the sphere.

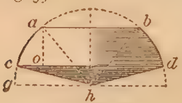
To Compute Volume of a Spherical Sector.—Fig. 39.

Rule.—Multiply external surface of zone, which is base of sector, by one third of the radius of sphere.

Or, $a\,r \div 3 = V$, *a representing area of base.*

Note.—Surface of a spherical sector = sum of surface of zone and surfaces of the two cones.

Fig. 39.



Example.—What is volume of a spherical sector, Fig. 39, generated by sector, $c\,a\,h$, height of zone, $a\,b\,c\,d$, being $a\,o$, 12 inches, and radius, $g\,h$, of sphere 15?

$12 \times 94.248 = 1130.976$ = *height of zone × circumference of sphere = external surface of zone* (see page 350).

$1130.976 \times \overline{15 \div 3}$ = *surface × one third of radius* = 5654.88 *cube ins.*

Spindles.

Definition.—Figures generated by revolution of a plane area bounded by a curve, when the curve is revolved about a chord perpendicular to its axis or about its double ordinate, and they are designated by the name of arc from which they are generated, as Circular, Elliptic, Parabolic, etc.

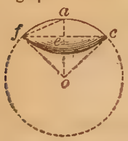
Circular Spindle.

To Compute Volume of a Circular Spindle.—Fig. 40.

RULE.—Multiply central distance by half area of revolving segment; subtract product from one third of cube of half length, and multiply remainder by 12.5664.

Or, $\frac{(l \div 2)^3}{3} - \left(c \times \frac{a}{2}\right) \times 12.5664 = V$, *a representing area of revolving segment.*

Fig. 40.



EXAMPLE.—What is volume of a circular spindle, Fig. 40, when central distance, *o e*, is 7.071 067 inches, length, *f c*, 14.142 13, and radius, *o c*, 10?

NOTE.—Area of revolving segment; *f e*, being = side of square that can be inscribed in a circle of 20, is $\overline{20^2 \times .7854 - 14.142\,13^2} \div 4 = 28.54$ *area.*

$7.071\,067 \times \overline{28.54 \div 2} = 100.9041$ = *central distance × half area of revolving segment;* $\frac{7.071\,67^3}{3} - 100.9041 = 16.947$ = *remainder of above product and one third of cube of half length.*

Then $16.497 \times 12.5664 = 212.9628$ *cube ins.*

Frustum or Zone of a Circular Spindle.*

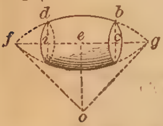
To Compute Volume of a Frustum or Zone of a Circular Spindle.—Fig. 41.

RULE.—From square of half length of whole spindle take one third of square of half length of frustum, and multiply remainder by said half length of frustum; multiply central distance by revolving area which generates the frustum; subtract this product from former, and multiply remainder by 6.2832.

Or, $\overline{l \div 2}^2 - \frac{\overline{l' \div 2}^2}{3} \times \frac{l'}{2} - (c \times a) \times 6.2832 = V$, *l and l′ representing lengths of spindle and of frustum, and a area of revolving section of frustum.*

NOTE.—Revolving area of frustum can be obtained by dividing its plane into a segment of a circle and a parallelogram.

Fig. 41.



EXAMPLE.—Length of middle frustum of a circular spindle, *i c*, Fig. 41, is 6 inches; length of spindle, *f g*, is 8; central distance, *o e*, is 3; and area of revolving or generating segment is 10; what is volume of frustum?

$(8 \div 2)^2 - \frac{(6 \div 2)^2}{3} = 13$, and $13 \times 3 = 39$ = *product of* $\frac{1}{2}$ *length of frustum, and remainder of one third square of half length of frustum subtracted from square of half length of spindle;* $39 - \overline{3 \times 10} = 9$ = *product of central distance and area of segment subtracted from preceding product.*

Then $9 \times 6.2832 = 56.5488$ *cube ins.*

Segment of a Circular Spindle.

To Compute Volume of a Segment of a Circular Spindle.—Fig. 42.

RULE.—Subtract length of segment from half length of spindle; double remainder, and ascertain volume of a middle frustum of this length. Subtract result from volume of whole spindle, and halve remainder.†

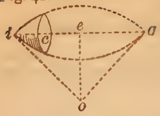
Or, $C - c \div 2 = V$, *C and c representing volume of spindle and middle frustum.*

---

* Middle frustum of a Circular Spindle is one of the various forms of casks.

† This rule is applicable to segment of any Spindle or any Conoid, volume of the figure and frustum being first obtained.

Fig. 42.



EXAMPLE.—Length of a circular spindle, *i a*, Fig. 42, is 14.142 13 inches; central distance, *o e*, is 7.071 07; radius of arc, *o a*, is 10; and length of segment, *i c*, is 3.535 53; what is its volume?

$\frac{14.142\,13}{2} - 3.535\,53 \times 2 = 7.071\,07 =$ *double remainder of length of segment subtracted from half length of spindle = length of middle frustum.*

NOTE.—Area of revolving or generating segment of whole spindle is 28.54 inches, and that of middle frustum is 19.25.

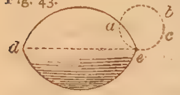
The volume of whole spindle is........ 212.9628 *cube ins.*
" " middle frustum is...... 162.8982 " "
Hence.............. 50.0646 ÷ 2 = 25.0323 *cube ins.*

Cycloidal Spindle.*

To Compute Volume of a Cycloidal Spindle.—Fig. 43.

RULE.—Multiply product of square of twice diameter of generating circle and 3.927 by its circumference, and divide this product by 8.

Fig. 43.



Or, $\frac{\overline{2\,d}^{2} \times 3.927 \times d \times 3.1416}{8} = V$, *d representing diameter of circle, or half width of spindle.*

EXAMPLE.—Diameter of generating circle, *a b c*, of a cycloid, Fig. 43, is 10 inches; what is volume of spindle, *d e*?

$\overline{10 \times 2}^{2} \times 3.927 = 1570.8 =$ *product of twice diameter squared and* 3.927.

Then $1570.8 \times 10 \times 3.1416 \div 8 = 6168.5316$ *cube ins.*

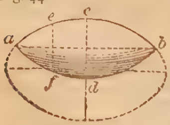
Elliptic Spindle.

To Compute Volume of an Elliptic Spindle.—Fig. 44.

RULE.—To square of its diameter add square of twice diameter at one fourth of its length; multiply sum by length, and product by .1309.†

Or, $d^2 + \overline{2\,d'}^{2}\ l.1309 = V$, *d and d′ representing diameters as above.*

Fig. 44.



EXAMPLE.—Length of an elliptic spindle, *a b*, Fig. 44, is 75 inches, its diameter, *c d*, 35, and diameter, *e f*, at .25 of its length, 25; what is its volume?

$35^2 + \overline{25 \times 2}^{2} = 3725 =$ *sum of squares of diameter of spindle and of twice its diameter at one fourth of its length;*
$3725 \times 75 \quad 279\,375 =$ *above sum × length of spindle.*

Then $279\,375 \times .1309 = 36\,570.1875$ *cube ins.*

NOTE.—For all such solid bodies this rule is exact when body is formed by a conic section, or a part of it, revolving about axis of section, and will always be very near when figure revolves about another line.

To Compute Volume of Middle Frustum or Zone of an Elliptic Spindle.—Fig. 45.

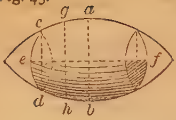
RULE.—Add together squares of greatest and least diameters, and square of double diameter in middle between the two; multiply the sum by length, and product by .1309.‡

Or, $d^2 + d'^2 + \overline{2\,d''}^{2}\ l.1309 = V$, *d, d′, and d″ representing different diameters.*

---

* Volume of a Cycloidal Spindle is equal to .625 of its circumscribing cylinder.
† See preceding Note. ‡ See Note above.

Fig. 45.



EXAMPLE.—Greatest and least diameters, *a b* and *c d*, of the frustum of an elliptic spindle, Fig. 45, are 68 and 50 inches, its middle diameter, *g h*, 60, and its length, *e f*, 75; what is its volume?

$68^2 + 50^2 + \overline{60 \times 2}^2 = 21\,524$ = *sum of squares of greatest and least diameters and of double middle diameter.*

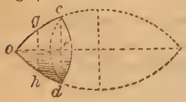
Then $21\,524 \times 75 \times .1309 = 211\,311.87$ *cube ins.*

To Compute Volume of a Segment of an Elliptic Spindle.—Fig. 46.

RULE.—Add together square of diameter of base of segment and square of double diameter in middle between base and vertex; multiply sum by length of segment, and product by .1309.*

Or, $d^2 + \overline{2\,d''}^2\, l \times .1309 = V$, *d and d″ representing diameters.*

Fig. 46.



EXAMPLE.—Diameters, *c d* and *g h*, of the segment of an elliptic spindle, Fig. 46, are 20 and 12 inches, and length, *o e*, is 16; what is its volume?

$20^2 + \overline{12 \times 2}^2 = 976$ = *sum of squares of diameter at base and in middle.*

Then $976 \times 116 \times .1309 = 2044.134$ *cube ins.*

Parabolic Spindle.

To Compute Volume of a Parabolic Spindle.—Fig. 47.

RULE 1.—Multiply square of diameter by length, and the product by .418 88.†

Or, $d^2\, l \times .418\,88 = V$.

RULE 2.—To square of its diameter add square of twice diameter at one fourth of its length; multiply sum by length, and product by .1309.‡

Or, $d^2 + \overline{2\,d'}^2\, l \times .1309 = V$.

Fig. 47.

EXAMPLE.—Diameter of a parabolic spindle, *a b*, Fig. 47, is 40 ins., and its length, *c d*, 10; what is its volume?

$40^2 \times 10 = 16\,000$ = *square of diameter × length.*

Then $16\,000 \times .418\,88 = 6702.08$ *cube ins.*

Again, If middle diam. at .25 of its length is 30, Then,

by Rule 2, $40^2 + \overline{30 \times 2}^2 \times 40 \times .1309 = 6806.8$ *cube ins.*

To Compute Volume of Middle Frustum of a Parabolic Spindle.—Fig. 48.

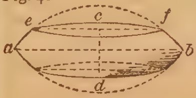
RULE 1.—Add together 8 times square of greatest diameter, 3 times square of least diameter, and 4 times product of these two diameters; multiply sum by length, and product by .052 36.

Or, $d^2\, 8 + \overline{d'^2\, 3} + \overline{d\, d' \times 4}\, l \times .052\,36 = V$.

RULE 2.—Add together squares of greatest and least diameters and square of double diameter in middle between the two; multiply the sum by length, and product by .1309.

Or, $d^2 + d'^2 + 2\,d''^2\, l \times .1309 = V$, *d″ representing diameter between the two.*

Fig. 48.



EXAMPLE.—Middle frustum of a parabolic spindle, Fig. 48, has diameters, *a b* and *e f*, of 40 and 30 inches, and its length, *c d*, is 10; what is its volume?

$40^2 \times 8 + 30^2 \times 3 + \overline{40 \times 30} \times 4 = 20\,300$ = *sum of 8 times square of greatest diameter, 3 times square of least diameter, and 4 times product of these.*

Then $20\,300 \times 10 \times .052\,36 = 10\,629.08$ *cube ins.*

---

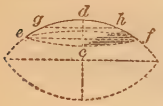
* See Note, page 372. † 8–15 of .7854. ‡ See Note, page 372.

To Compute Volume of a Segment of a Parabolic Spindle.—Fig. 49.

RULE.—Add together square of diameter of base of segment and square of double diameter in middle between base and vertex; multiply sum by height of segment, and product by .1309.

Or, $d^2 + d''^2\, l \times .1309 = V$.

Fig. 49.



EXAMPLE.—Segment of a parabolic spindle, Fig. 49, has diameters, *e f* and *g h*, of 15 and 8.75 inches, and height, *c d*, is 2.5; what is its volume?

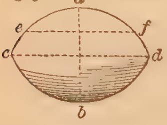
$15^2 + \overline{8.75 \times 2}^2 = 531.25$ = *sum of square of base and of double diameter in middle of segment.* Then $531.25 \times 2.5 \times .1309 = 173.852$ *cube ins.*

Hyperbolic Spindle.

To Compute Volume of a Hyperbolic Spindle.—Fig. 50.

RULE.—To square of diameter add square of double diameter at one fourth of its length; multiply sum by length, and product by .1309.*

Fig. 50.



Or, $d^2 + \overline{2\,d'}^2\, l \times .1309 = V$.

EXAMPLE.—Length, *a b*, Fig. 50, of a hyperbolic spindle is 100 inches, and its diameters, *c d* and *e f*, are 150 and 110; what is its volume?

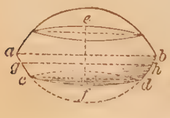
$150^2 + \overline{110 \times 2}^2 \times 109 = 7\,090\,000$ = *product of sum of squares of greatest diameter and of twice diameter at one fourth of length of spindle and length.* Then $7\,090\,000 \times .1309 = 928\,081$ *cube inches.*

To Compute Volume of Middle Frustum of a Hyperbolic Spindle.—Fig. 51.

RULE.—Add together squares of greatest and least diameters and square of double diameter in middle between the two; multiply this sum by length, and product by .1309.†

Or, $d^2 + d'^2 + (2\,d'')^2\, l \times .1309 = V$.

Fig. 51.



EXAMPLE.—Diameters, *a b* and *c d*, of middle frustum of a hyperbolic spindle, Fig. 51, are 150 and 110 inches; diameter, *g h*, 140; and length, *e f*, 50; what is its volume?

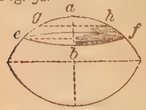
$150^2 + 110^2 + \overline{140 \times 2}^2 = 113\,000$ = *sum of squares of greatest and least diameters and of double middle diameter.* Then $113\,000 \times 50 \times .1309 = 739\,585$ *cube ins.*

To Compute Volume of a Segment of a Hyperbolic Spindle.—Fig. 52.

RULE.—Add together square of diameter of base of segment and square of double diameter in middle between base and vertex; multiply sum by length of segment, and product by .1309.

Or, $d^2 + d''^2\, l \times .1309 = V$.

Fig. 52.



EXAMPLE.—Segment of a hyperbolic spindle, Fig. 52, has diameters, *e f* and *g h*, of 110 and 65 inches, and its length, *a b*, 25; what is its volume?

$110^2 + \overline{65 \times 2}^2 = 29\,000$ = *sum of squares of diameter of base and of double middle diameter.*

Then $29\,000 \times 25 \times .1309 = 94\,902.5$ *cube ins.*

---

* See Note, page 372. † Ibid.

Ellipsoid, Paraboloid, and Hyperboloid of Revolution* (Conoids).

Definition.—Figures like to a cone, described by revolution of a conic section around and at a right angle to plane of their fixed axes.

Ellipsoid of Revolution (Spheroid).

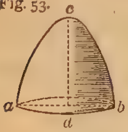
Definition.—An ellipsoid of revolution is a semi-spheroid. (See page 368.)

Paraboloid of Revolution.†

To Compute Volume of a Paraboloid of Revolution.—Fig. 53.

Rule.—Multiply area of base by half height.

Fig. 53.



Or, $a\,h \div 2 = V$.

Note.—This rule will hold for any segment of paraboloid, whether base be perpendicular or oblique to axis of solid.

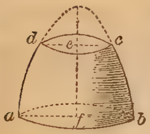
Example.—Diameter, $a\,b$, of base of a paraboloid of revolution, Fig. 53, is 20 inches, and its height, $d\,c$, 20; what is its volume?

Area of 20 inches diameter of base $= 314.16$. Then $314.16 \times 20 \div 2 = 3141.6$ *cube ins.*

Frustum of a Paraboloid of Revolution.

To Compute Volume of a Frustum of a Paraboloid of Revolution.—Fig. 54.

Fig. 54.



Rule.—Multiply sum of squares of diameters by height of frustum, and this product by .3927.

Or, $\overline{d^2 + d'^2}\, h \times .3927 = V$.

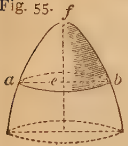
Example.—Diameters, $a\,b$ and $d\,c$, of the base and vertex of frustum of a paraboloid of revolution, Fig. 54, are 20 and 11.5 inches, and its height, $e\,f$, 12.6; what is its volume?

$20^2 + 11.5^2 = 532.25 =$ *sum of squares of diameters.* Then $532.25 \times 12.6 \times .3927 = 2633.5837$ *cube ins.*

Segment of a Paraboloid of Revolution.

To Compute Volume of Segment of a Paraboloid of Revolution.—Fig. 55.

Fig. 55.



Rule.—Multiply area of base by half height.

Or, $a \times \overline{h \div 2} = V$.

Note.—This rule will hold for any segment of paraboloid, whether base be perpendicular or oblique to axis of solid.

Example.—Diameter, $a\,b$, of the base of a segment of a paraboloid of revolution, Fig. 55, is 11.5 inches, and its height, $e\,f$, is 7.4; what is its volume?

Area of 11.5 inches diameter of base $= 103.869$. Then $103.869 \times 7.4 \div 2 = 384.315$ *cube ins.*

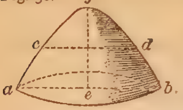
Hyperboloid of Revolution.

To Compute Volume of a Hyperboloid of Revolution.—Fig. 56.

Rule.—To square of radius of base add square of middle diameter; multiply this sum by height, and product by .5236.

* These figures have been known as Conoids. For the definition of a Conoid, see *Haswell's Mensuration*, page 233.
† Volume of a Paraboloid of Revolution is = .5 of its circumference.

Fig. 56.



Or, $r^2+d^2\,h\times.5236=V$, *d representing middle diameter.*

EXAMPLE.—Base, $a\,b$, of a hyperboloid of revolution, Fig. 56, is 80 inches; middle diameter, $c\,d$, 66; and height, $e\,f$, 60; what is its volume?

$\overline{80\div 2}^2+66^2=5956=$ *sum of square of radius of base and middle diam.* Then $5956\times 60\times.5236=87\,113.7$ *cube ins.*

Segment of a Hyperboloid of Revolution.

To Compute Volume of Segment of a Hyperboloid of Revolution, as Fig. 56.

RULE.—To square of radius of base add square of middle diameter: multiply this sum by height, and product by .5236.

Or, $r^2+d''^2\,h\times.5236=V$, *r representing radius of base.*

EXAMPLE.—Radius, $a\,e$, of base of a segment of a hyperboloid of revolution, as Fig. 56, is 21 inches; its middle diameter, $c\,d$, is 30; and its height, $e\,f$, 15; what is its volume?

$21^2+30^2\times 15=20\,115=$ *product of sum of squares of radius of base and middle diameter multiplied by height.* Then $20\,115\times.5236=10\,532.214$ *cube ins.*

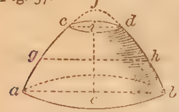
Frustum of a Hyperboloid of Revolution.

To Compute Volume of Frustum of a Hyperboloid of Revolution.—Fig. 57.

RULE.—Add together squares of greatest and least semi-diameters and square of diameter in middle of the two; multiply this sum by height, and product by .5236.

Or, $\left(\frac{d}{2}\right)^2+\left(\frac{d'}{2}\right)^2+d''^2\,h\times.5236=V$, *d, d', and d'' representing several diameters.*

Fig. 57.



EXAMPLE.—Frustum of a hyperboloid of revolution, Fig. 57, is in height, $e\,i$, 50 inches; diameters of greater and lesser ends, $a\,b$ and $c\,d$, are 110 and 42; and that of middle diameter, $g\,h$, is 80; what is volume?

$110\div 2=55$, and $42\div 9=21$. Hence $55^2+21^2+80^2=9866=$ *sum of squares of semi-diameters of ends and of middle diam.* Then $9866\times 50\times.5236=258\,291.88$ *cube ins.*

Any Figure of Revolution.

To Compute Volume of any Figure of Revolution.—Fig. 58.

RULE.—Multiply area of generating surface by circumference described by its centre of gravity.

Or, $a\,2r\,p=V$, *r representing radius of centre of gravity.*

Fig. 58.

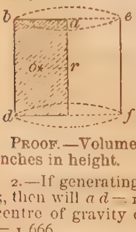
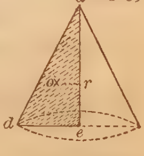


ILLUSTRATION 1.—If generating surface, $a\,b\,c\,d$, of cylinder, $b\,e\,d\,f$, Fig. 58, is 5 inches in width and 10 in height, then will $a\,b=5$ and $b\,d=10$, and centre of gravity will be in o, the radius of which is $r\,o=5\div 2=2.5$. Hence $10\times 5=50=$ *area of generating surface.*

Then $50\times 2.5\times 2\times 3.1416=785.4=$ *area of generating surface* $\times$ *circumference of its centre of gravity = volume of cylinder.*

Fig. 59.

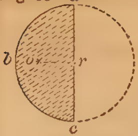


PROOF.—Volume of a cylinder 10 inches in diameter and 10 inches in height. $10^2\times.7854=78.54$, and $78.54\times 10=785.4$.

2.—If generating surface of a cone, Fig. 59, is $a\,e=10$, $d\,e=5$, then will $a\,d=11.18$, and area of triangle $=10\times 5\div 2=25$, centre of gravity of which is in o, and $o\,r$, by Rule, page 607, $=1.666$.

Hence, $25\times 1.666\times 2\times 3.1416=261.8=$ *area of generating surface* $\times$ *circumference of its centre of gravity = volume of cone.*

Fig. 60.



3.—If generating surface of a sphere, Fig. 60, is $a\,b\,c$, and $a\,c = 10$, $a\,b\,c$ will be $\left(\frac{10^2 \times .7854}{2}\right) = 39.27$, centre of gravity of which is in o, and by Rule, page 607, $o\,r = 2.122$.

Hence, $39.27 \times 2.122 \times 2 \times 3.1416 = 523.6 =$ *area of generating surface* $\times$ *circumference of its centre of gravity* $=$ *volume of sphere.*

Irregular Bodies.

To Compute Volume of an Irregular Body.

Rule.—Weigh it both in and out of fresh water, and note difference in lbs.; then, as 62.5* is to this difference, so is 1728† to number of cube inches in body.

Or, divide difference in lbs. by 62.5, and quotient will give volume in cube feet.

Note.—If salt water is to be used, ascertained weight of a cube foot of it, or 64, is to be used for 62.5.

Example.—An irregular-shaped body weighs 15 lbs. in water, and 30 out; what is its volume in cube inches?

$30 - 15 = 15 =$ *difference of weights in and out of water.*
$62.5 : 15 :: 1728 : 414.72 =$ *volume in cube ins.*
Or, $15 \div 62.5 = .24$, and $.24 \times 1728 = 414.72 =$ *volume in cube ins.*

CASK GAUGING.

Varieties of Casks.

To Compute Volume of a Cask.

1st Variety. Ordinary form of middle frustum of a Prolate Spheroid.

This class comprises all casks having a spherical outline of staves, as Rum puncheons, Whiskey barrels, etc.

Rule.—To twice square of bung diameter add square of head diameter; multiply this sum by length of the cask, and product by .2618, and it will give volume in cube inches, which, being divided by 231, will give result in gallons.

2d Variety. Middle frustum of a Parabolic Spindle.

This class comprises all casks in which curve of staves quickens at the chime, as Brandy casks and Provision barrels.

Rule.—To square of a head diameter add double square of bung diameter, and from sum subtract .4 of square of difference of diameters; multiply remainder by length, and product by .2618, which, being divided by 231, will give volume in gallons.

3d Variety. Middle frustum of a Paraboloid.

This class comprises all casks in which curve of staves quickens slightly at bilge, as Wine casks.

Rule.—To square of bung diameter add square of head diameter; multiply sum by length, and product by .3927, which, being divided by 231, will give volume in gallons.

4th Variety. Two equal frustums of Cones.

This class comprises all casks in which curve of staves quickens sharply at bilge, as Gin pipes.

Rule.—Add square of difference of diameters to three times square of their sum; multiply sum by length, and product by .065 66, and it will give volume in cube inches, which, being divided by 231, will give result in gallons.

---

* Weight of a cube foot of fresh water. † Number of inches in a cube foot.

EXAMPLE.—Bung and head diameters of a cask are 24 and 16 inches, and length 36; what is its volume in gallons?

$\overline{24-16}^2 + \overline{(24+16)^2 \times 3} = 4864$, which $\times\ 36 = 175\,104$, and $175\,104 \times .065\,66 = 11\,497.329$, which $\div\ 231 = 49.77$ *gallons.*

Generally.

$\overline{\overline{D\,d} + M^2}\ .001\,692\ L =$ *U. S. gallons*, and $.001\,416\,2 =$ *Imperial gallons.*

D, *d*, and M *representing interior, head and bung diameters. and* L *length of cask in inches.*

To Ascertain Mean Diameter of a Cask.

RULE.—Subtract head diameter from bung diameter in inches, and multiply difference by following units for the four varieties; add product to head diameter, and sum will give mean diameter of varieties required.

1st Variety	.7	3d Variety	.56
2d Variety	.68	4th Variety	.52

EXAMPLE.—Bung and head diameters of a cask of 1st variety are 24 and 20 inches; what is its mean diameter?

$24 - 20 = 4$, and $4 \times .7 = 2.8$, which, added to 20, $= 22.8$ *ins.*

ULLAGE CASKS.

To Compute Volume of Ullage Casks.

When a cask is only partly filled, it is termed an *ullage cask*, and is considered in two positions, viz., as lying on its side, when it is termed a *Segment Lying*, or as standing on its end, when it is termed a *Segment Standing.*

To Ullage a Lying Cask.

RULE.—Divide wet inches (depth of liquid) by bung diameter; find quotient in column of versed sines in table of circular segments, page 267, and take its corresponding segment; multiply this segment by capacity of cask in gallons, and product by 1.25 for ullage required.

EXAMPLE.—Capacity of a cask is 90 gallons, bung diameter being 32 inches; what is its volume at 8 inches depth?

$8 \div 32 = .25$, *tab. seg.* of which is $.153\,55$, which $\times\ 90 = 13.8195$, and again $\times\ 1.25 = 17.2744$ *gallons.*

To Ullage a Standing Cask.

RULE.—Add together square of diameter at surface of liquor, square of head diameter, and square of double diameter taken in middle between the two; multiply sum by wet inches, and product by .1309, and divide by 231 for result in gallons.

To Compute Volume of a Cask by Four Dimensions.

RULE.—Add together squares of bung and head diameters, and square of double diameter taken in middle between bung and head; multiply the sum by length of cask, and product by .1309, and divide this product by 231 for result in gallons.

To Compute Volume of any Cask from Three Dimensions only.

RULE.—Add into one sum 39 times square of bung diameter, 25 times square of head diameter, and 26 times product of the two diameters; multiply sum by length, and product by .008 726; and divide quotient by 231 for result in gallons.

For Rules in Gauging in all its conditions and for description and use of instruments, see *Haswell's Mensuration*, pages 307-23.

CONIC SECTIONS.

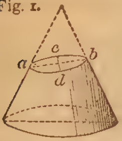
A *Cone* is a figure described by revolution of a right-angled triangle about one of its legs, or it is a solid having a circle for its base, and terminated in a vertex.

Conic Sections are figures made by a plane cutting a cone.

If a cone is cut by a plane through vertex and base, section will be a triangle, and if cut by a plane parallel to its base, section will be a circle.

Axis is line about which triangle revolves. *Base* is circle which is described by revolving base of triangle.

Fig. 1.

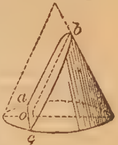


An *Ellipse* is a figure generated by an oblique plane cutting a cone above its base.

Transverse axis or diameter is longest right line that can be drawn in it, as *a b*, Fig. 1.

Conjugate axis or diameter is a line drawn through centre of ellipse perpendicular to transverse axis, as *c d*.

Fig. 2.

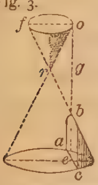


A *Parabola* is a figure generated by a plane cutting a cone parallel to its side, as *a b c*, Fig. 2.

Axis is a right line drawn from vertex to middle of base, as *b o*.

NOTE.—A parabola has not a conjugate diameter.

Fig. 3.



A *Hyperbola* is a figure generated by a plane cutting a cone at any angle with base greater than that of side of cone, as *a b c*, Fig. 3.

Transverse axis or diameter, *o b*, is that part of axis, *e b*, which, if continued, as at *o*, would join an opposite cone, *o f r*.

Conjugate axis or diameter is a right line drawn through centre, *g*, of transverse axis, and perpendicular to it.

Straight line through *foci* is indefinite transverse axis; that part of it between vertices of curves, as *o b*, is definite transverse axis. Its middle point, *g*, is centre of curve.

Eccentricity of a hyperbola is ratio obtained by dividing distance from centre to either *focus* by semi-transverse axis.

Parameter is cord of curve drawn through *focus* at right angles to axis.

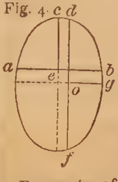
Asymptotes of a hyperbola are two right lines to which the curve continually approaches, touches at an infinite distance but does not pass; they are prolongations of diagonals of rectangle constructed on extremes of the axes.

Two hyperbolas are *conjugate* when transverse axis of one is conjugate of the other, and contrariwise.

General Definitions.

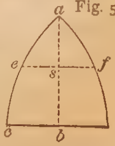
An *Ordinate* is a right line from any point of a curve to either of diameters, as *a e* and *d o*, Fig. 4, and *a b* and *d f*, are double ordinates; *c b*, Fig. 5, is an ordinate, and *a b* an abscissa.

Fig. 4.



An *Abscissa* is that part of diameter which is contained between vertex and an ordinate, as *c e*, *g o*, Fig. 4, and *a b*, Fig. 5.

Fig. 5.



Parameter of any diameter is equal to four times distance from *focus* to vertex of curve; parameter of axis is least possible, and is termed parameter of curve.

Parameter of curve of a conic section is equal to chord of curve drawn through *focus* perpendicular to axis.

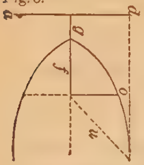
Parameter of transverse axis is least, and is termed parameter of curve.

Parameter of a conic section and *foci* are sufficient elements for construction of curve.

A *Focus* is a point on principal axis where double ordinate to axis, through point, is equal to parameter, as *e f*, Fig. 5.

It may be determined arithmetically thus: Divide square of ordinate by four times abscissa, and quotient will give focal distances, *a s* and *s*, in preceding figures.

Fig. 6.



Directrix of a conic section is a right line at right angles to major axis, and it is in such a position that

$$f : g :: u : o.$$

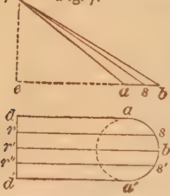
Here *a d*, Fig. 6, is directrix, and *o* is *offset* to directrix.

Latus Rectum, or principal parameter, passes through a focus; it is a double ordinate, which is a third proportion to the axis.

Or, A : *a* :: *a* : L.

A *and a representing major and minor axes.* (See *Haswell's Mensuration*, page 232.)

Fig. 7.



A *Conoid* is a warped surface generated by a right line being moved in such a manner that it will touch a straight line and curve, and continue parallel to a given plane. Straight line and curve are called *directrices*, plane a plane *directrix*, and moving line the *generatrix*.

Thus, let *a b a'*, Fig. 7, be a circle in a horizontal plane, and *d d'* projection of right lines perpendicular to a vertical plane, *r' b e*; if right lines, *d a*, *r s*, *r' b*, *r'' s*, and *d' a*, be moved so as to touch circle and right line *d d'* and be constantly parallel to plane *r' b e*, it will generate conoid *d a b a' d'*.

Radii vectores are lines drawn from the foci to any point in the curve; hence a *radius vector* is one of these lines.

Traced angle is angle formed by the radii vectores and the transverse diameter.

Ellipsoid, *Paraboloid*, and *Hyperboloid of Revolution*—Figures generated by the revolution of an ellipse, parabola, etc., around their axes. (See *Mensuration of Surfaces and Solids*, pages 357-75.)

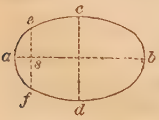
NOTE 1.—All figures which can possibly be formed by cutting of a cone are mentioned in these definitions, and are five following—viz., a *Triangle*, a *Circle*, an *Ellipse*, a *Parabola*, and a *Hyperbola*; but last three only are termed *Conic Sections*.

2.—In *Parabola* parameter of any diameter is a third proportional to abscissa and ordinate of any point of curve, abscissa and ordinate being referred to that diameter and tangent at its vertex.

3.—In *Ellipse* and *Hyperbola* parameter of any diameter is a third proportional to diameter and its conjugate.

To Determine Parameter of an Ellipse or Hyperbola.

Fig. 8.



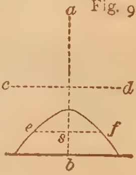
RULE. — Divide product of conjugate diameter, multiplied by itself, by transverse, and quotient is equal to parameter.

In annexed Figs. 8 and 9, of an *Ellipse* and *Hyperbola*, transverse and conjugate diameters, *a b*, *c d*, are each 30 and 20.

Then 30 : 20 :: 20 : 13.333 = *parameter*.

Parameter of curve = *e f*, a double ordinate passing through focus, *s*.

Fig. 9.



Ellipse.

To Describe Ellipses. (See Geometry, page 226.)

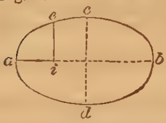
To Compute Terms of an Ellipse.

When any three of four Terms of an Ellipse are given, viz., Transverse and Conjugate Diameters, an Ordinate, and its Abscissa, to ascertain remaining Terms.

To Compute Ordinate.

Transverse and Conjugate Diameters and Abscissa being given. RULE.—As transverse diameter is to conjugate, so is square root of product of abscissæ to ordinate which divides them.

Fig. 10.



EXAMPLE.—Transverse diameter, $a\,b$, of an ellipse, Fig. 10, is 25; conjugate, $c\,d$, 16; and abscissa, $a\,i$, 7; what is length of ordinate, $i\,e$?

$$25 - 7 = 18 \text{ less abscissa; } \sqrt{7 \times 18} = 11.225.$$

$$\text{Hence } 25 : 16 :: 11.225 : 7.184 \text{ ordinate.}$$

Or, $\sqrt{c^2 - \left(\frac{c\,x}{t}\right)^2} =$ *any ordinate, c and t representing semi-conjugate and transverse diameters, and x distance of ordinate from centre of figure.*

To Compute Abscissæ.

Transverse and Conjugate Diameters and Ordinate being given. RULE.—As conjugate diameter is to transverse, so is square root of difference of squares of ordinate and semi-conjugate to distance between ordinate and centre; and this distance being added to, or subtracted from, semi-transverse, will give abscissæ required.

EXAMPLE.—Transverse diameter, $a\,b$, of an ellipse, Fig. 10, is 25; conjugate, $c\,d$, 16; and ordinate, $i\,e$, 7.184; what is abscissa, $i\,b$?

$$\sqrt{8^2 - 7.184^2} = 3.519943. \quad \text{Hence, as } 16 : 25 :: 3.52 : 5.5.$$

$$\text{Then } \left.\begin{array}{l} 25 \div 2 = 12.5, \text{ and } 12.5 + 5.5 = 18 = b\,i, \\ 25 \div 2 = 12.5, \text{ and } 12.5 - 5.5 = 7 = a\,i, \end{array}\right\} \text{abscissæ.}$$

To Compute Transverse Diameter.

Conjugate, Ordinate, and Abscissa being given. RULE.—To or from semi-conjugate, according as great or less abscissa is used, add or subtract square root of difference of squares of ordinate and semi-conjugate. Then, as this sum or difference is to abscissa, so is conjugate to transverse.

EXAMPLE.—Conjugate diameter, $c\,d$, of an ellipse, Fig. 10, is 16; ordinate, $i\,e$, 7.184; and abscissæ, $b\,i$, $i\,a$, 18 and 7; what is length of transverse diameter?

$$\sqrt{(16 \div 2)^2 - 7.184^2} = 3.52.$$

$$16 \div 2 + 3.52 : 18 :: 16 : 25; \quad 16 \div 2 - 3.52 : 7 :: 16 : 25 \text{ transverse diameter.}$$

To Compute Conjugate Diameter.

Transverse, Ordinate, and Abscissa being given. RULE.—As square root of product of abscissæ is to ordinate, so is transverse diameter to conjugate.

EXAMPLE.—Transverse diameter, $a\,b$, of an ellipse, Fig. 10, is 25; ordinate, $i\,e$, 7.184; and abscissæ, $b\,i$ and $i\,a$, 18 and 7; what is length of conjugate diameter?

$$\sqrt{18 \times 7} = 11.225. \quad \text{Hence } 11.225 : 7.184 :: 25 : 16 \text{ conjugate diameter.}$$

To Compute Circumference of an Ellipse.

RULE.—Multiply square root of half sum of the squares of two diameters by 3.1416.

EXAMPLE.—Transverse and conjugate diameters, $a\,b$ and $c\,d$, of an ellipse, Fig. 10, are 24 and 20; what is its circumference?

$$\frac{24^2 + 20^2}{2} = 488, \text{ and } \sqrt{488} = 22.09. \quad \text{Hence } 22.09 \times 3.1416 = 69.398 \text{ circumference.}$$

To Compute Area of an Ellipse.

RULE.—Multiply the diameters together, and the product by .7854. Or, multiply one diameter by .7854, and the product by the other.

EXAMPLE.—The transverse diameter of an ellipse, $a\,b$, Fig. 10, is 12, and its conjugate, $c\,d$, 9; what is its area?

$$12 \times 9 \times .7854 = 84.8232 \text{ area.}$$

NOTE.—Area of an ellipse is a mean proportional between areas of two circles, diameter of one being major axis and of the other minor axis.

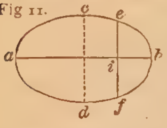
ILLUSTRATION.—Area of circle of $40 = 1256.64$; area of ellipse $40 \times 20 = 628.32$; area of circle of $20 = 314.16$, mean proportional of the two circles $1256.64 + 314.16 = 785.4$. Therefore the conjugate diameter of an ellipse of an area of 785.4 sq. ins., its transverse being 40, is 25 *feet*, as $40 \times 25 \times .7854 = 785.4$ *sq. ins.*

Segment of an Ellipse.

To Compute Area of a Segment of an Ellipse.

When its Base is parallel to either Axis, as e i f. RULE.—Divide height of segment, $b\,i$, by diameter or axis, $a\,b$, of which it is a part, and find in Table of Areas of Segments of a Circle, page 267, a segment having same versed sine as this quotient; then multiply area of segment thus found and the axes of ellipse together.

Fig. 11.



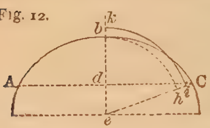
EXAMPLE.—Height, $b\,i$, Fig. 11, is 5, and axes of ellipse are 30 and 20; what is area of segment?

$5 \div 30 = .1666$ *tabular versed sine, the area of which* (page 267) *is* .085 54.

Hence $.085\,54 \times 30 \times 20 = 51.324$ *area.*

To Ascertain Length of an Elliptic Curve which is less than half of entire Figure.

Fig. 12.



Let curve of which length is required be A b C, Fig. 12.

Extend versed sine $b\,d$ to meet centre of curve in e.

Draw line e C, and from e, with distance $e\,b$, describe $b\,h$; bisect h C in i, and from e, with radius $e\,i$, describe $k\,i$, and it is equal to half arc A b C.

To Ascertain Length when Curve is greater than half entire Figure.

Ascertain by above problem curve of less portion of figure; subtract it from circumference of ellipse, and remainder will be length of curve required.

Parabola.

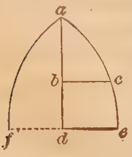
To Describe a Parabola. (See Geometry, page 229.)

To Compute either Ordinate or Abscissa of a Parabola.

When the other Ordinate and Abscissæ, or other Abscissa and Ordinates are given. RULE.—As either abscissa is to square of its ordinate, so is other abscissa to square of its ordinate.

Or, as square of any ordinate is to its abscissa, so is square of other ordinate to its abscissa.

Fig. 13.



EXAMPLE 1.—Abscissa, $a\,b$, of parabola, Fig. 13, is 9; its ordinate, $b\,c$, 6; what is ordinate, $d\,e$, abscissa of which, $a\,d$, is 16?

Hence $9 : 6^2 :: 16 : 64$, and $\sqrt{64} = 8$ *length.*

2.—Abscissæ of a parabola are 9 and 16, and their corresponding ordinates 6 and 8; any three of these being taken, it is required to compute the fourth.

1. $\frac{6^2 \times 16}{9} = 8$ *ordinate.* 2. $\sqrt{\frac{8^2 \times 9}{16}} = 6$ *ordinate.*

3. $\frac{16 \times 6^2}{8^2} = 9$ *less abscissa.* 4. $\frac{9 \times 8^2}{6^2} = 16$ *abscissa.*

Parabolic Curve.

To Compute Length of Curve of a Parabola cut off by a Double Ordinate.—Fig. 13.

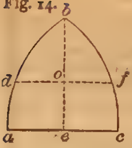
RULE.—To square of ordinate add $\frac{4}{3}$ of square of abscissa, and square root of this sum, multiplied by two, will give length of curve nearly.

EXAMPLE.—Ordinate, $d\,e$, Fig. 13, is 8, and its abscissa, $a\,d$, 16; what is length of curve, $f\,a\,e$?

$$8^2 + \frac{4 \times 16^2}{3} = 405.333, \text{ and } \sqrt{405.333} \times 2 = 40.267 \text{ length.}$$

To Compute Area of a Parabola.

Fig. 14.



RULE.—Multiply base by height, and take two thirds of product.

Corollary.—A parabola is two thirds of its circumscribing parallelogram.

EXAMPLE.—What is area of parabola, *a b c*, Fig. 14, height, *b e*, being 16, and base, or double ordinate, *a c*, 16?

$$16 \times 16 = 256, \textit{ and } \frac{2}{3} \textit{ of } 256 = 170.667 \textit{ area.}$$

To Compute Area of a Segment of a Parabola.

RULE.—Multiply difference of cubes of two ends of segment, *a c*, *d f*, by twice its height, *e o*, and divide product by three times difference of squares of ends.

EXAMPLE.—Ends of a segment of a parabola, *a c* and *d f*, Fig. 14, are 10 and 6, and height, *e o*, is 10; what is its area?

$$10^3 \sim 6^3 \times \overline{10 \times 2} = 15\,680, \textit{ and } \div \overline{10^2 \sim 6^2 \times 3} = 81.667 \textit{ area.}$$

NOTE.—Any parabolic segment is equal to a parabola of the same height, the base of which is equal to base of segment, increased by a third proportional to sum of the two ends and lesser end.

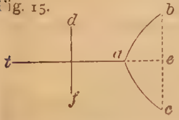
Hyperbola.

To Describe a Hyperbola. (See Geometry, page 230.)

To Compute Ordinate of a Hyperbola,

Transverse and Conjugate Diameters and Abscissæ being given. RULE.—As transverse diameter is to conjugate, so is square root of product of abscissæ to ordinate required.

Fig. 15.



EXAMPLE.—Hyperbola, *a b c*, Fig. 15, has a transverse diameter, *a t*, of 120; a conjugate, *d f*, of 72; and abscissa, *a e*, 40; what is the length of ordinate, *e c*?

$40 + 120 = 160$ *greater abscissa*, and
$120 : 72 :: \sqrt{(40 \times 160)} : 48$ *ordinate.*

NOTE 1.—In hyperbolas lesser abscissa, added to axis (the transverse diameter), gives greater.

2.—Difference of two lines drawn from *foci* of any hyperbola to any point in curve is equal to its transverse diameter.

To Compute Abscissæ,

Transverse and Conjugate Diameters and Ordinate being given. RULE.—As conjugate diameter is to transverse, so is square root of sum of squares of ordinate and semi conjugate to distance between ordinate and centre, or half sum of abscissæ. Then the sum of this distance and semi-transverse will give greater abscissa, and their difference the lesser abscissa.

EXAMPLE.—Transverse diameter, *a t*, of a hyperbola, Fig. 15, is 120; conjugate, *d f*, 72; and ordinate, *e c*, 48; what are lengths of abscissæ, *t e* and *a e*?

$72 : 120 :: \sqrt{48^2 + (72 \div 2)^2} = 60 : 100$ *half sum of abscissæ*, and $100 + (120 \div 2) = 160$ *greater abscissa*, and $100 - (120 \div 2) = 40$ *lesser abscissa.*

To Compute Conjugate Diameter,

Transverse Diameter, Abscissæ, and Ordinate being given. RULE.—As square root of product of abscissæ is to ordinate, so is transverse diameter to conjugate.

EXAMPLE.—Transverse diameter, *a t*, of a hyperbola, Fig. 15, is 120; ordinate, *e c*, 48; and abscissæ, *t e* and *a e*, 160 and 40; what is length of conjugate, *d f*?

$$\sqrt{40 \times 160} = 80 : 48 :: 120 : 72 \textit{ conjugate.}$$

To Compute Transverse Diameter,

Conjugate, Ordinate, and an Abscissa being given. RULE.—Add square of ordinate to square of semi-conjugate, and extract square root of their sum.

Take sum or difference of semi-conjugate and this root, according as greater or lesser abscissa is used. Then, as square of ordinate is to product of abscissa and conjugate, so is sum or difference above ascertained to transverse diameter required.

NOTE.—When the greater abscissa is used, the difference is taken, and contrariwise.

EXAMPLE.—Conjugate diameter, *d f*, of a hyperbola, Fig. 15, is 72; ordinate, *e c*, 48; and lesser abscissa, *a e*, 40; what is length of transverse diameter, *a t*?

$\sqrt{48^2 + (72 \div 2)^2} = 60$, and $60 + \overline{72 \div 2} = 96$ *lesser abscissa*, and $40 \times 72 = 2880$.
Hence, $48^2 : 2880 :: 96 : 120$ *transverse diameter.*

To Compute Length of any Arc of a Hyperbola, commencing at Vertex.

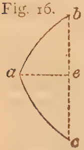
RULE.—To 19 times transverse diameter add 21 times parameter of axis.

To 9 times transverse diameter add 21 times parameter, and multiply each of these sums respectively by quotient of lesser abscissa divided by transverse diameter.

To each of products thus ascertained add 15 times parameter, and divide former by latter; then this quotient, multiplied by ordinate, will give length of arc, nearly.

NOTE.—*To Compute Parameter*, divide square of conjugate by transverse diameter.

Fig. 16.



EXAMPLE.—In hyperbola, *a b c*, Fig. 16, transverse diameter is 120, conjugate, 72, ordinate, *e c*, 48, and lesser abscissa, *a e*, 40; what is length of arc, *a b*?

$\frac{72^2}{120} = 43.2$ *parameter.* $120 \times 19 + \overline{43.2 \times 21} \times \frac{40}{120} = 1062.4$.

$\overline{120 \times 9 + \overline{43.2 \times 21} \times \frac{40}{120}} = 662.4$. Then $1062.4 + \overline{43.2 \times 15} \div \overline{662.4 + 43.2 \times 15} = 1.305$, which $\times 48 = 62.64$ *length.*

NOTE.—As transverse diameter is to conjugate, so is conjugate to parameter. (See Rule, page 380.)

To Compute Area of a Hyperbola,

Transverse, Conjugate, and Lesser Abscissa being given. RULE.—To product of transverse diameter and lesser abscissa add five sevenths of square of this abscissa, and multiply square root of sum by 21.

Add 4 times square root of product of transverse diameter and lesser abscissa to product last ascertained, and divide sum by 75.

Divide 4 times product of conjugate diameter and lesser abscissa by transverse diameter, and this last quotient, multiplied by former, will give area, nearly.

EXAMPLE.—Transverse diameter of a hyperbola, Fig. 16, is 60, conjugate 36, and lesser abscissa or height, *a e*, 20; what is area of figure?

$60 \times 20 + \frac{5}{7}$ of $20^2 = 1485.7143$, and $\sqrt{1485.7143} \times 21 = 809.43$, and $\sqrt{60 \times 20} \times 4 + 809.43 = 901.02$, which $\div 75 = 12.0136$ and $\frac{36 \times 20 \times 4}{60} \times 12.0136 = 576.653$ *area.*

NOTE.—For ordinates of a parabola in divisions of eighths and tenths, see page 229.

Delta Metal.

Delta Metal is an improved composition of Aluminium and its alloys; it is non-corrosive, capable of being cast, forged, and hot rolled.

Tensile Strength per Sq. Inch.

Cast in green sand	48 380 lbs.	Rolled, annealed	60 920 lbs.
Rolled, hard	75 260 "	Wire, No. 22 W G	140 000 "

PLANE TRIGONOMETRY.

By *Plane Trigonometry* is ascertained how to compute or determine four of the seven elements of a plane or rectilinear triangle from the other three, for when any three of them are given, one of which being a side or the area, the remaining elements may be determined; and this operation is termed *Solving the Triangle.*

The determination of the mutual relation of the Sines, Tangents, Secants, etc., of the sums, differences, multiples, etc., of arcs or angles is also classed under this head.

For Diagram and Explanation of Terms, see Geometry, pp. 219–21.

Right-angled Triangles.

For Solution by Lines and Areas, see Mensuration of Areas, Lines, and Surfaces, pp. 335–39.

To Compute a Side.

When a Side and its Opposite Angle is given. RULE.—As sine of angle opposite given side is to sine of angle opposite required side, so is given side to required side.

To Compute an Angle.

RULE.—As side opposite to given angle is to side opposite to required angle, so is sine of given angle to sine of required angle.

To Compute Base or Perpendicular in a Right-angled Triangle.

When Angles and One Side next Right Angle are given. RULE.—As radius is to tangent of angle adjacent to given side, so is this side to other side.

To Compute the other Side.

When Two Sides and Included Angle are given. RULE.—As sum of two given sides is to their difference, so is tangent of half sum of their opposite angles to tangent of half their difference; add this half difference to half sum, to ascertain greater angle; and subtract half difference from half sum, to ascertain less angle. The other side may then be ascertained by Rule above.

To Compute Angles.

When Sides are given. RULE.—As one side is to other side, so is radius to tangent of angle adjacent to first side.

To Compute an Angle.

When Three Sides are given. RULE 1.—Subtract sum of logarithms of sides which contain required angle, from 20; to remainder add logarithm of half sum of three sides, and that of difference between this half sum and side opposite to required angle. Half the sum of these three logarithms is logarithmic cosine of half required angle. The other angles may be ascertained by Rule above.

2.—Subtract sum of logarithms of two sides which contain required angle, from 20, and to remainder add logarithms of differences between these two sides and half sum of the three sides. Half result is logarithmic sine of half required angle.

NOTE.—In all ordinary cases either of these rules will give sufficiently accurate results. Rule 1 should be used when required angle exceeds 90°; and Rule 2 when it is less than 90°.

EXAMPLE.—The sides of a triangle are 3, 4, and 5; what are the angles of the hypothenuse?

20 — (Log. 4 = .602 06 + Log. 5 = .698 97) = 18.698 97; Log. 3 + 4 + 5 ÷ 2 — 4 = .301 03; and Log. 3 + 4 + 5 ÷ 2 — 5 = 0.

Then 18.698 97 + .301 03 = 19, which ÷ 2 = 9.5 = log. sin. of half angle = 18° 26′, which × 2 = 36° 52′ *angle.*

Hence 90° — 36° 52′ = 53° 8′ *remaining angle.*

In following figures, 1 and 2:

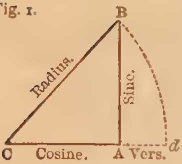
A = 90°, B = 45°, C = 45°, Radius = 1, Secant = 1.4142, Cosine = .7071, Sin. 45° = .7071, Tangent = 1, Area = .25.

By Sin., Tan., Sec., etc., A B, etc., is expressed Sine, Tangent, Secant, etc., of angles, A, B, etc.

To Compute Sides A C and B C.—Figs. 1 and 2.

When Hyp., Side B A, *and Angles* B *and* C *are given.*

Fig. 1.



$$\frac{\text{Sin. B} \times \text{B A}}{\text{Sin. C}} = \text{A C.}$$

$$\text{B A} \times \text{Cot. C} = \text{A C.}$$

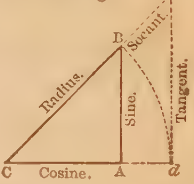
$$\text{Hyp.} \times \text{Cos. C} = \text{A C.}$$

$$\text{Hyp.} \times \text{Sin. B} = \text{A C.}$$

$$\frac{\text{B A}}{\text{Sin. C}} = \text{B C.}$$

$$\frac{\text{A C}}{\text{Sin. B}} = \text{B C.}$$

Fig. 2.



To Compute Side A C and Angles.

When Hyp. and Side B A *are given.*—Fig. 1 and 2.

$$\frac{\text{A C}}{\text{Hyp.}} = \text{Sin. B.} \qquad \frac{\text{B A}}{\text{Hyp.}} = \text{Sin. C.} \qquad \frac{\text{B A} \times \text{Sin. B}}{\text{Sin. C}} = \text{A C.} \qquad \text{B C} \times \text{Sin. B} = \text{A C.}$$

To Compute Side B C and Hyp. or Angles.

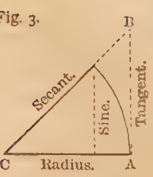
When both Sides are given.—Fig. 2.

$$\frac{\text{A C}}{\text{B A}} = \text{Tan. B.} \qquad \frac{\text{B A}}{\text{Sin. C}} = \text{B C.} \qquad \sqrt{\text{A C}^2 + \text{B A}^2} = \text{B C.} \qquad \frac{\text{B A}}{\text{A C}} = \text{Tan. C.}$$

$$\frac{\text{B A}}{\text{B C}} = \text{Sin. C.} \qquad \frac{\text{A C}}{\text{B C}} = \text{Sin. B.}$$

To Compute Sides.—Figs. 3 and 4.

Fig. 3.



When a Side and an Angle are given.

$$\text{B C} \times \text{Cos. B} = \text{B A.}$$

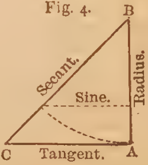
$$\text{B C} \times \text{Sin. B} = \text{A C.}$$

$$\text{A B} \times \text{Sec. B} = \text{B C.}$$

$$\frac{\text{A C} \times \text{Tan. C}}{\text{Rad.}} = \text{B A.} \qquad \frac{\text{A C} \times \text{Sin. C}}{\text{Sin. B}} = \text{B A.}$$

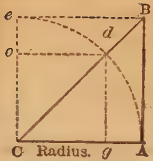
$$\frac{\text{A C} \times \text{Sec. C}}{\text{Rad.}} = \text{B C.} \qquad \frac{\text{A C} \times \text{Rad.}}{\text{Sin. B}} = \text{B C.}$$

Fig. 4.



In B A C, Fig. 5, a right-angled triangle, C A, is assumed to be radius; B A tangent of C, and B C secant to that radius; Or, dividing each of these by base, there is obtained the tangent and secant of C respectively to radius 1.

Fig. 5.



Radius	$CA = 1$	Sine	$dg = .7071$
Secant	$CB = 1.4142$	Cosine	Cg or $od = .7071$
Tangent	$AB = 1$	Versed sine	$gA = .2929$
Co-secant	$CB = 1.4142$	Co-versed sine	$oe = .2929$
Co-tangent	$eB = 1$	Angle	$CAB = 90°$

$\sqrt{AC^2 + BA^2} = \text{hyp. } BC.$ $\quad BA \div \text{Sin. } C = \text{hyp. } BC.$

$AC \div \text{Cos. } C = \text{hyp. } BC.$ $\quad 1 \div \text{Tan. } C = \text{Cot. } C.$

$$\sqrt{\frac{2 \text{ Area}}{\text{Tan. } C}} = \text{Rad.} \quad \frac{\text{Cos. } C}{\text{Sin. } C} = \text{Cot. } C. \quad \frac{BC^2 \times \text{Sin. } 2C}{4} = \textit{Area.}$$

$BA \times \text{Sec. } B = BC.$

$BC \times \text{Cos. } C = \text{Rad.}$	$BA \times \text{Cot. } C = \text{Rad.}$	$BC \times \text{Sin. } B = \text{Rad.}$
$BA \times \text{Tan. } B = \text{Rad.}$	$BC \times \text{Sin. } C = BA.$	$AC \times \text{Tan. } C = BA.$
$BC \div BA = \text{Sec. } B.$	$1 \div \text{Sin. } C = \text{Cosec. } C.$	$1 - \text{Sin. } C = \text{Co-ver. sin.}$
$BC \times \text{Cos. } B = BA.$	$\text{Cos. } C \div \text{Sin. } C = \text{Cot. } C.$	$CB \times \text{Sin. } B = AC.$

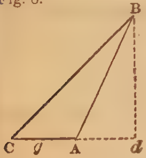
Trigonometrical Equivalents.

Perp. ÷ hyp. = Sin. C.	Hyp. ÷ base = Sec. C.	Perp. ÷ base = Tan. C.
Base ÷ hyp. = Cos. C.	Base ÷ perp. = Tan. B.	Hyp. ÷ perp. = Sec. B.
Base ÷ hyp. = Sin. B.	Perp. ÷ hyp. = Cos. B.	Hyp. ÷ perp. = Cosec. C.
Base ÷ perp. = Cotan. C.	Hyp. — Base = Versin.	Hyp. — Perp. = Co-ver. sin. C.

$\sqrt{(1 - \text{sin.}^2)} = \text{Cos.}$	Tan. ÷ sin. = Sec.	1 ÷ cos. = Sec.
Sin. ÷ tan. = Cos.	Tan. ÷ sec. = Sin.	1 ÷ cosec. = Sin.
Sin. × cot. = Cos.	Tan. × cot. = Rad.	1 ÷ sec. = Cos.
Sin. ÷ cos. = Tan.	$\sqrt{(1 - \text{cos.}^2)} = \text{Sin.}$	1 — cos. = Versin.
Cos. ÷ cot. = Sin.	1 ÷ cot. = Tan.	1 — sin. = Co-ver. sin.
Cos. ÷ sin. = Cot.	1 ÷ sin. = Cosec.	1 ÷ tan. = Cotan.

ILLUSTRATIONS.—Assume side A B of a right-angled triangle is 100, and angle C 53° 8′; what are its elements?

Fig. 6.



Oblique-angled Triangles.

To Compute Sides B A and B C.

When Side A C *and Angles are given.*—Fig. 6.

$$\frac{\text{Sin. } C \times AC}{\text{Sin. } B} = BA. \qquad \frac{\text{Sin. } C \times BC}{\text{Sin. } A} = BA.$$

$$\frac{\text{Sin. } A \times AC}{\text{Sin } B} = BC.$$

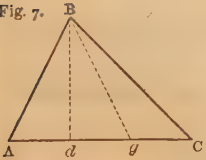
To Compute Angles and Side A C.

When Sides A B, B C, *and one of the Angles are given.*—Fig. 6.

$$\frac{BC \times \text{Sin. } B}{AC} = \text{Sin. } A. \qquad \frac{\text{Sin. } C \times AC}{BA} = \text{Sin. } B. \qquad \frac{AB \times \text{Sin. } B}{AC} = \text{Sin. } C.$$

$$\frac{\text{Sin. } B \times BC}{\text{Sin. } A} = AC.$$

Fig. 7.



To Compute Sides B A and B C.

When Side A C *and Angles are given.*—Fig. 7.

$$\frac{\text{Sin. } C \times BC}{\text{Sin. } A} = BA. \qquad \frac{\text{Sin. } A \times AC}{\text{Sin. } B} = BC.$$

When Side B C *and Angles are given.*—Fig. 7.

$$\frac{BC \times \text{Sin. } C}{\text{Sin. } (C + B)} = BA. \qquad \frac{\text{Sin. } C \times AC}{\text{Sin. } B} = BA.$$

NOTE.—*Sine and Cosine of an arc are each equal to sine and cosine of their supplements.*

Spherical Triangles, Right-angled and Oblique. For full formulas see Molesworth, Lond., 1878, pp. 435-6.

To Compute Angles and Side A C.

When Sides A B, B C, *and Angle* B *are given.*—Fig. 7.

$$\frac{\text{Sin. B} \times \text{B C}}{\text{Sin. A}} = \text{A C.} \quad \frac{\text{B C} \times \text{Sin. B}}{\text{A C}} = \text{Sin. A.} \quad \frac{\text{B A} \times \text{Sin. B}}{\text{A C}} = \text{Sin. C.}$$

$$\frac{\text{A C} \times \text{Sin. A}}{\text{B C}} = \text{Sin. B.} \quad \frac{\text{B A} \times \text{Sin. A}}{\text{B C}} = \text{Sin. C.} \quad \frac{\text{B C} \times \text{Sin. C}}{\text{A B}} = \text{Sin. A.}$$

To Compute all the Angles.

When all the Sides are given, Figs. 6 and 7. RULE.—Let fall a perpendicular, B d, opposite to required angle. Then, as A C : sum of A B, B C :: their difference : twice $d\,g$, the distance of perpendicular, B d, from middle of the base.

Hence A d, C g are known, and triangle, A B C, is divided into two right-angled triangles, B C d, B A d; then, by rules for right-angled triangles, ascertain angle A or C.

OPERATION.—A C, Fig. 6, .5014 : A B + B C, $1.1174 + 1.4142 = 2.5316$ :: A B ∽ B C, $1.4142 - 1.1174 = .2968$: $2 \times d\,g = 1.4986$.

Hence $A\,d = \overline{d\,g - A\,C} \div 2 = \frac{1.4986}{2} - \frac{.5014}{2} = .4986$, and $C\,d = A\,d + A\,C = 1$.

Consequently, triangle B d C, Fig. 6, is divided into two triangles, B A C and B d A.

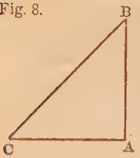
To Compute Side A B and Angles.

When Two Sides and One Angle, or One Side and Two Angles, are given.—Fig. 6.

$$\frac{\text{A C} \times \text{Sin. C}}{\text{Sin. B}} = \text{A B.} \quad \frac{\text{B C} \times \text{Sin. B}}{\text{A C}} = \text{Sin. A.} \quad \frac{\text{A C} \times \text{Sin. A}}{\text{A B} - (\text{A C} \times \text{Cos. A})} = \text{Tan. B.}$$

$$\frac{\text{A C} \times \text{Sin. C}}{\text{A B}} = \text{Sin. B.} \quad \frac{\text{A B} \times \text{Sin. B}}{\text{A C}} = \text{Sin. C.} \quad \frac{\text{A C} \times \text{Sin. C}}{\text{B C} - (\text{A C} \times \text{Cos. C})} = \text{Tan. B.}$$

Fig. 8.



To Compute Area of a Triangle.—Fig. 8.

$$\frac{\text{B A} \times \text{B C} \times \text{Sin. B}}{2}, \quad \frac{\text{A C} \times \text{B C} \times \text{Sin. C}}{2}, \quad \frac{\text{B A} \times \text{A C} \times \text{Sin. A}}{2},$$

$$\frac{\text{Sin. 2 C, B C}^2}{4}, \quad \frac{\text{A C}^2\text{, Tan. C}}{2}, \text{ and } \frac{\text{B A}^2\text{, Cot. C}}{2} = \textit{Area.}$$

NOTE.—For other rules, see Mensuration of Areas, Lines, and Surfaces, page 335.

To Compute Sides.

When Areas and Angles are given.—Figs. 6 and 7.

$$\frac{2\text{ Area}}{\text{B C, Sin. C}} = \text{A C.} \quad \frac{2\text{ Area}}{\text{A C, Sin. A}} = \text{B A.} \quad \sqrt{\frac{2\text{ Area, Sin. A}}{\text{Sin. C, Sin. (A + C)}}} = \text{B C.}$$

To Ascertain Distance of Inaccessible Objects on a Level Plane.—Figs. 9 and 10.

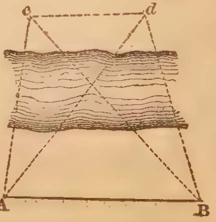
Fig. 9.

OPERATION.—Lay off perpendiculars to line A B, Fig. 9, as B c, $d\,e$, on line A d, terminating on line e A.

Then $e\,d - c\,B : c\,B :: B\,d : B\,A$.

When there are Two Inaccessible Objects, as Fig. 10.

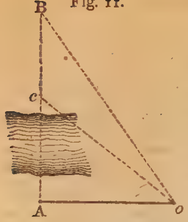
Fig. 10.



OPERATION. — Measure a base line, A B, Fig. 10, and angles c A B, d B A, A $c\,d$, B $d\,c$, etc. Then proceed by formulas, page 387, to deduce $c\,d$.

NOTE.—If course of $c\,d$ is required, take difference of angles $d\,c$ A and $c\,d$ B from course A B.

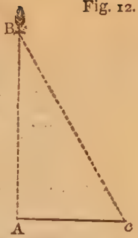
Fig. 11.



When the Objects can be aligned.—Fig. 11.

OPERATION.—Align c B, Fig. 11, at A, measure a base line at any angle thereto, as A o, and angles o A c, B A c, and B o A. Then proceed as per formula, page 386, to deduce c B.

Fig. 12.

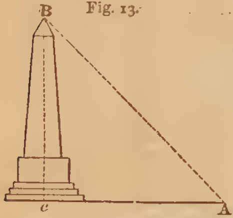


To Compute Distance from a Given Point to an Inaccessible Object.—Fig. 12.

OPERATION.—Measure a level line, A c, Fig. 12, and ascertain angles, B A c, c A B. Hence, having side, A c, and two angles, proceed as per formula, page 386, to determine A B.

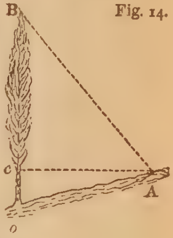
To Compute Height of an Elevated Point.—Fig. 13.

Fig. 13.



OPERATION. — Measure distance on a horizontal line, A c, Fig. 13; ascertain Angle B A c. Then proceed as per formulas, pp. 386–8, to ascertain B c.

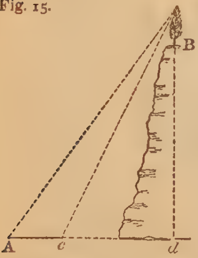
Fig. 14.



When a Horizontal Base is not Attainable.—Fig. 14.

OPERATION.—Measure or compute distance A c, Fig. 14; ascertain angle of depression A o c and of elevation B A o. Then proceed as per formula, page 386, to ascertain B c.

Fig. 15.

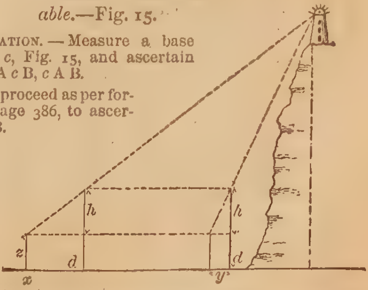


When a Full Base Line is not Attainable.—Fig. 15.

OPERATION. — Measure a base line, A c, Fig. 15, and ascertain angles A c B, c A B.

Then proceed as per formula, page 386, to ascertain d B.

Fig. 16.



Without Use of an Instrument.—Fig. 16.

OPERATION.—Lay off any suitable and level distance, d d, set up a staff at each extremity at like elevation from base line d d, and note distances y and x, at which the lines of sight of object range with tops of the staffs; deduct height of eye from length of staffs, and ascertain heights h.

Then $\frac{\mathrm{D}\,h}{x-y}+h+s=$ *height.* *s representing height of line of sight from base d d, and* D *length of line d d.*

Natural Sines and Cosines.

Prop. parts.	′	0°		1°		2°		3°			Prop. parts.
29	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		2
0	0	.00000	1	.01745	.99985	.0349	.99939	.05234	.99863	60	2
0	1	.00029	1	.01774	.99984	.03519	.99938	.05263	.99861	59	2
1	2	.00058	1	.01803	.99984	.03548	.99937	.05292	.9986	58	2
1	3	.00087	1	.01832	.99983	.03577	.99936	.05321	.99858	57	2
2	4	.00116	1	.01862	.99983	.03606	.99935	.0535	.99857	56	2
2	5	.00145	1	.01891	.99982	.03635	.99934	.05379	.99855	55	2
3	6	.00175	1	.0192	.99982	.03664	.99933	.05408	.99854	54	2
3	7	.00204	1	.01949	.99981	.03693	.99932	.05437	.99852	53	2
4	8	.00233	1	.01978	.9998	.03723	.99931	.05466	.99851	52	2
4	9	.00262	1	.02007	.9998	.03752	.9993	.05495	.99849	51	2
5	10	.00291	1	.02036	.99979	.03781	.99929	.05524	.99847	50	2
5	11	.0032	.99999	.02065	.99979	.0381	.99927	.05553	.99846	49	2
6	12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	48	2
6	13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	47	2
7	14	.00407	.99999	.02152	.99977	.03897	.99924	.0564	.99841	46	2
7	15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	45	2
8	16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	44	1
8	17	.00495	.99999	.0224	.99975	.03984	.99921	.05727	.99836	43	1
9	18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	42	1
9	19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	41	1
10	20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	40	1
10	21	.00611	.99998	.02356	.99972	.041	.99916	.05844	.99829	39	1
11	22	.0064	.99998	.02385	.99972	.04129	.99915	.05873	.99827	38	1
11	23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	37	1
12	24	.00698	.99998	.02443	.9997	.04188	.99912	.05931	.99824	36	1
12	25	.00727	.99997	.02472	.99969	.04217	.99911	.0596	.99822	35	1
13	26	.00756	.99997	.02501	.99969	.04246	.9991	.05989	.99821	34	1
13	27	.00785	.99997	.0253	.99968	.04275	.99909	.06018	.99819	33	1
14	28	.00814	.99997	.0256	.99967	.04304	.99907	.06047	.99817	32	1
14	29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	31	1
15	30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	30	1
15	31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	29	1
15	32	.00931	.99996	.02676	.99964	.0442	.99902	.06163	.9981	28	1
16	33	.0096	.99995	.02705	.99963	.04449	.99901	.06192	.99808	27	1
16	34	.00989	.99995	.02734	.99963	.04478	.999	.06221	.99806	26	1
17	35	.01018	.99995	.02763	.99962	.04507	.99898	.0625	.99804	25	1
17	36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	24	1
18	37	.01076	.99994	.02821	.9996	.04565	.99896	.06308	.99801	23	1
18	38	.01105	.99994	.0285	.99959	.04594	.99894	.06337	.99799	22	1
19	39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	21	1
19	40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	20	1
20	41	.01193	.99993	.02938	.99957	.04682	.9989	.06424	.99793	19	1
20	42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	18	1
21	43	.01251	.99992	.02996	.99955	.0474	.99888	.06482	.9979	17	1
21	44	.0128	.99992	.03025	.99954	.04769	.99886	.06511	.99788	16	1
22	45	.01309	.99991	.03054	.99953	.04798	.99885	.0654	.99786	15	1
22	46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	14	0
23	47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	13	0
23	48	.01396	.9999	.03141	.99951	.04885	.99881	.06627	.9978	12	0
24	49	.01425	.9999	.0317	.9995	.04914	.99879	.06656	.99778	11	0
24	50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	10	0
25	51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	9	0
25	52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	8	0
26	53	.01542	.99988	.03286	.99946	.0503	.99873	.06773	.9977	7	0
26	54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	6	0
27	55	.016	.99987	.03345	.99944	.05088	.9987	.06831	.99766	5	0
27	56	.01629	.99987	.03374	.99943	.05117	.99869	.0686	.99764	4	0
28	57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	3	0
28	58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.9976	2	0
29	59	.01716	.99985	.03461	.9994	.05205	.99864	.06947	.99758	1	0
29	60	.01745	.99985	.0349	.99939	.05234	.99863	.06976	.99756	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		89°		88°		87°		86°			

Prop. parts.	′	4°		5°		6°		7°			Prop. parts.
29		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		4
0	0	.06976	.99756	.08716	.99619	.10453	.99452	.12187	.99255	60	4
0	1	.07005	.99754	.08745	.99617	.10482	.99449	.12216	.99251	59	4
1	2	.07034	.99752	.08774	.99614	.10511	.99446	.12245	.99248	58	4
1	3	.07063	.9975	.08803	.99612	.1054	.99443	.12274	.99244	57	4
2	4	.07092	.99748	.08831	.99609	.10569	.9944	.12302	.9924	56	4
2	5	.07121	.99746	.0886	.99607	.10597	.99437	.12331	.99237	55	4
3	6	.0715	.99744	.08889	.99604	.10626	.99434	.1236	.99233	54	4
3	7	.07179	.99742	.08918	.99602	.10655	.99431	.12389	.9923	53	4
4	8	.07208	.9974	.08947	.99599	.10684	.99428	.12418	.99226	52	3
4	9	.07237	.99738	.08976	.99596	.10713	.99424	.12447	.99222	51	3
5	10	.07266	.99736	.09005	.99594	.10742	.99421	.12476	.99219	50	3
5	11	.07295	.99734	.09034	.99591	.10771	.99418	.12504	.99215	49	3
6	12	.07324	.99731	.09063	.99588	.108	.99415	.12533	.99211	48	3
6	13	.07353	.99729	.09092	.99586	.10829	.99412	.12562	.99208	47	3
7	14	.07382	.99727	.09121	.99583	.10858	.99409	.12591	.99204	46	3
7	15	.07411	.99725	.0915	.9958	.10887	.99406	.1262	.992	45	3
8	16	.0744	.99723	.09179	.99578	.10916	.99402	.12649	.99197	44	3
8	17	.07469	.99721	.09208	.99575	.10945	.99399	.12678	.99193	43	3
9	18	.07498	.99719	.09237	.99572	.10973	.99396	.12706	.99189	42	3
9	19	.07527	.99716	.09266	.9957	.11002	.99393	.12735	.99186	41	3
10	20	.07556	.99714	.09295	.99567	.11031	.9939	.12764	.99182	40	3
10	21	.07585	.99712	.09324	.99564	.1106	.99386	.12793	.99178	39	3
11	22	.07614	.9971	.09353	.99562	.11089	.99383	.12822	.99175	38	3
11	23	.07643	.99708	.09382	.99559	.11118	.9938	.12851	.99171	37	2
12	24	.07672	.99705	.09411	.99556	.11147	.99377	.1288	.99167	36	2
12	25	.07701	.99703	0944	.99553	.11176	.99374	.12908	.99163	35	2
13	26	.0773	.99701	.09469	.99551	.11205	.9937	.12937	.9916	34	2
13	27	.07759	.99699	.09498	.99548	.11234	.99367	.12966	.99156	33	2
14	28	.07788	.99696	.09527	.99545	.11263	.99364	.12995	.99152	32	2
14	29	.07817	.99694	.09556	.99542	.11291	.9936	.13024	.99148	31	2
15	30	.07846	.99692	.09585	.9954	.1132	.99357	.13053	.99144	30	2
15	31	.07875	.99689	.09614	.99537	.11349	.99354	.13081	.99141	29	2
15	32	.07904	.99687	.09642	.99534	.11378	.99351	.1311	.99137	28	2
16	33	.07933	.99685	.09671	.99531	.11407	.99347	.13139	.99133	27	2
16	34	.07962	.99683	.097	.99528	.11436	.99344	.13168	.99129	26	2
17	35	.07991	.9968	.09729	.99526	.11465	.99341	.13197	.99125	25	2
17	36	.0802	.99678	.09758	.99523	.11494	.99337	.13226	.99122	24	2
18	37	.08049	.99676	.09787	.9952	.11523	.99334	.13254	.99118	23	2
18	38	.08078	.99673	.09816	.99517	.11552	.99331	.13283	.99114	22	1
19	39	.08107	.99671	.09845	.99514	.1158	.99327	.13312	.9911	21	1
19	40	.08136	.99668	.09874	.99511	.11609	.99324	.13341	.99106	20	1
20	41	.08165	.99666	.09903	.99508	.11638	.9932	.1337	.99102	19	1
20	42	.08194	.99664	.09932	.99506	.11667	.99317	.13399	.99098	18	1
21	43	.08223	.99661	.09961	.99503	.11696	.99314	.13427	.99094	17	1
21	44	.08252	.99659	.0999	.995	.11725	.9931	.13456	.99091	16	1
22	45	.08281	.99657	.10019	.99497	.11754	.99307	.13485	.99087	15	1
22	46	.0831	.99654	.10048	.99494	.11783	.99303	.13514	.99083	14	1
23	47	.08339	.99652	.10077	.99491	.11812	.993	.13543	99079	13	1
23	48	.08368	.99649	.10106	.99488	.1184	.99297	.13572	.99075	12	1
24	49	.08397	.99647	.10135	.99485	.11869	.99293	.136	.99071	11	1
24	50	.08426	.99644	.10164	.99482	.11898	.9929	.13629	.99067	10	1
25	51	.08455	.99642	.10192	.99479	.11927	.99286	.13658	.99063	9	1
25	52	.08484	.99639	.10221	.99476	.11956	.99283	.13687	.99059	8	1
26	53	.08513	.99637	.1025	.99473	.11985	.99279	.13716	.99055	7	0
26	54	.08542	.99635	.10279	.9947	.12014	.99276	.13744	.99051	6	0
27	55	.08571	.99632	.10308	.99467	.12043	.99272	.13773	.99047	5	0
27	56	.086	.9963	.10337	.99464	.12071	.99269	.13802	.99043	4	0
28	57	.08629	.99627	.10366	.99461	.121	.99265	.13831	.99039	3	0
28	58	.08658	.99625	.10395	.99458	.12129	.99262	.1386	.99035	2	0
29	59	.08687	.99622	.10424	.99455	.12158	.99258	.13889	.99031	1	0
29	60	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		85°		84°		83°		82°			

Prop. parts. 28	′	8° N. sine.	8° N. cos.	9° N. sine.	9° N. cos.	10° N. sine.	10° N. cos.	11° N. sine.	11° N. cos.		Prop. parts. 6
0	0	.13917	.99027	.15643	.98769	.17365	.98481	.19081	.98163	60	6
0	1	.13946	.99023	.15672	.98764	.17393	.98476	.19109	.98157	59	6
1	2	.13975	.99019	.15701	.9876	.17422	.98471	.19138	.98152	58	6
1	3	.14004	.99015	.1573	.98755	.17451	.98466	.19167	.98146	57	6
2	4	.14033	.99011	.15758	.98751	.17479	.98461	.19195	.9814	56	6
2	5	.14061	.99006	.15787	.98746	.17508	.98455	.19224	.98135	55	6
3	6	.1409	.99002	.15816	.98741	.17537	.9845	.19252	.98129	54	5
3	7	.14119	.98998	.15845	.98737	.17565	.98445	.19281	.98124	53	5
4	8	.14148	.98994	.15873	.98732	.17594	.9844	.19309	.98118	52	5
4	9	.14177	.9899	.15902	.98728	.17623	.98435	.19338	.98112	51	5
5	10	.14205	.98986	.15931	.98723	.17651	.9843	.19366	.98107	50	5
5	11	.14234	.98982	.15959	.98718	.1768	.98425	.19395	.98101	49	5
6	12	.14263	.98978	.15988	.98714	.17708	.9842	.19423	.98096	48	5
6	13	.14292	.98973	.16017	.98709	.17737	.98414	.19452	.9809	47	5
7	14	.1432	.98969	.16046	.98704	.17766	.98409	.19481	.98084	46	5
7	15	.14349	.98965	.16074	.987	.17794	.98404	.19509	.98079	45	5
7	16	.14378	.98961	.16103	.98695	.17823	.98399	.19538	.98073	44	4
8	17	.14407	.98957	.16132	.9869	.17852	.98394	.19566	.98067	43	4
8	18	.14436	.98953	.1616	.98686	.1788	.98389	.19595	.98061	42	4
9	19	.14464	.98948	.16189	.98681	.17909	.98383	.19623	.98056	41	4
9	20	.14493	.98944	.16218	.98676	.17937	.98378	.19652	.9805	40	4
10	21	.14522	.9894	.16246	.98671	.17966	.98373	.1968	.98044	39	4
10	22	.14551	.98936	.16275	.98667	.17995	.98368	.19709	.98039	38	4
11	23	.1458	.98931	.16304	.98662	.18023	.98362	.19737	.98033	37	4
11	24	.14608	.98927	.16333	.98657	.18052	.98357	.19766	.98027	36	4
12	25	.14637	.98923	.16361	.98652	.18081	.98352	.19794	.98021	35	4
12	26	.14666	.98919	.1639	.98648	.18109	.98347	.19823	.98016	34	3
13	27	.14695	.98914	.16419	.98643	.18138	.98341	.19851	.9801	33	3
13	28	.14723	.9891	.16447	.98638	.18166	.98336	.1988	.98004	32	3
14	29	.14752	.98906	.16476	.98633	.18195	.98331	.19908	.97988	31	3
14	30	.14781	.98902	.16505	.98629	.18224	.98325	.19937	.97992	30	3
14	31	.1481	.98897	.16533	.98624	.18252	.9832	.19965	.97987	29	3
15	32	.14838	.98893	.16562	.98619	.18281	.98315	.19994	.97981	28	3
15	33	.14867	.98889	.16591	.98614	.18309	.9831	.20022	.97975	27	3
16	34	.14896	.98884	.1662	.98609	.18338	.98304	.20051	.97969	26	3
16	35	.14925	.9888	.16648	.98604	.18367	.98299	.20079	.97963	25	3
17	36	.14954	.98876	.16677	.986	.18395	.98294	.20108	.97958	24	2
17	37	.14982	.98871	.16706	.98595	.18424	.98288	.20136	.97952	23	2
18	38	.15011	.98867	.16734	.9859	.18452	.98283	.20165	.97946	22	2
18	39	.1504	.98863	.16763	.98585	.18481	.98277	.20193	.9794	21	2
19	40	.15069	.98858	.16792	.9858	.18509	.98272	.20222	.97934	20	2
19	41	.15097	.98854	.1682	.98575	.18538	.98267	.2025	.97928	19	2
20	42	.15126	.98849	.16849	.9857	.18567	.98261	.20279	.97922	18	2
20	43	.15155	.98845	.16878	.98565	.18595	.98256	.20307	.97916	17	2
21	44	.15184	.98841	.16906	.98561	.18624	.9825	.20336	.9791	16	2
21	45	.15212	.98836	.16935	.98556	.18652	.98245	.20364	.97905	15	2
21	46	.15241	.98832	.16964	.98551	.18681	.9824	.20393	.97899	14	1
22	47	.1527	.98827	.16992	.98546	.1871	.98234	.20421	.97893	13	1
22	48	.15299	.98823	.17021	.98541	.18738	.98229	.2045	.97887	12	1
23	49	.15327	.98818	.1705	.98536	.18767	.98223	.20478	.97881	11	1
23	50	.15356	.98814	.17078	.98531	.18795	.98218	.20507	.97875	10	1
24	51	.15385	.98809	.17107	.98526	.18824	.98212	.20535	.97869	9	1
24	52	.15414	.98805	.17136	.98521	.18852	.98207	.20563	.97863	8	1
25	53	.15442	.988	.17164	.98516	.18881	.98201	.20592	.97857	7	1
25	54	.15471	.98796	.17193	.98511	.1891	.98196	.2062	.97851	6	1
26	55	.155	.98791	.17222	.98506	.18938	.9819	.20649	.97845	5	1
26	56	.15529	.98787	.1725	.98501	.18967	.98185	.20677	.97839	4	0
27	57	.15557	.98782	.17279	.98496	.18995	.98179	.20706	.97833	3	0
27	58	.15586	.98778	.17308	.98491	.19024	.98174	.20734	.97827	2	0
28	59	.15615	.98773	.17336	.98486	.19052	.98168	.20763	.97821	1	0
28	60	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		81°		80°		79°		78°			

Prop. parts. 27	′	12° N. sine.	12° N. cos.	13° N. sine.	13° N. cos.	14° N. sine.	14° N. cos.	15° N. sine.	15° N. cos.		Prop. parts. 9
0	0	.20791	.97815	.22495	.97437	.24192	.9703	.25882	.96593	60	9
0	1	.2082	.97809	.22523	.9743	.2422	.97023	.2591	.96585	59	9
1	2	.20848	.97803	.22552	.97424	.24249	.97015	.25938	.96578	58	9
1	3	.20877	.97797	.2258	.97417	.24277	.97008	.25966	.9657	57	9
2	4	.20905	.97791	.22608	.97411	.24305	.97001	.25994	.96562	56	8
2	5	.20933	.97784	.22637	.97404	.24333	.96994	.26022	.96555	55	8
3	6	.20962	.97778	.22665	.97398	.24362	.96987	.2605	.96547	54	8
3	7	.2099	.97772	.22693	.97391	.2439	.9698	.26079	.9654	53	8
4	8	.21019	.97766	.22722	.97384	.24418	.96973	.26107	.96532	52	8
4	9	.21047	.9776	.2275	.97378	.24446	.96966	.26135	.96524	51	8
5	10	.21076	.97754	.22778	.97371	.24474	.96959	.26163	.96517	50	8
5	11	.21104	.97748	.22807	.97365	.24503	.96952	.26191	.96509	49	7
5	12	.21132	.97742	.22835	.97358	.24531	.96945	.26219	.96502	48	7
6	13	.21161	.97735	.22863	.97351	.24559	.96937	.26247	.96494	47	7
6	14	.21189	.97729	.22892	.97345	.24587	.9693	.26275	.96486	46	7
7	15	.21218	.97723	.2292	.97338	.24615	.96923	.26303	.96479	45	7
7	16	.21246	.97717	.22948	.97331	.24644	.96916	.26331	.96471	44	7
8	17	.21275	.97711	.22977	.97325	.24672	.96909	.26359	.96463	43	6
8	18	.21303	.97705	.23005	.97318	.247	.96902	.26387	.96456	42	6
9	19	.21331	.97698	.23033	.97311	.24728	.96894	.26415	.96448	41	6
9	20	.2136	.97692	.23062	.97304	.24756	.96887	.26443	.9644	40	6
9	21	.21388	.97686	.2309	.97298	.24784	.9688	.26471	.96433	39	6
10	22	.21417	.9768	.23118	.97291	.24813	.96873	.265	.96425	38	6
10	23	.21445	.97673	.23146	.97284	.24841	.96866	.26528	.96417	37	6
11	24	.21474	.97667	.23175	.97278	.24869	.96858	.26556	.9641	36	5
11	25	.21502	.97661	.23203	.97271	.24897	.96851	.26584	.96402	35	5
12	26	.2153	.97655	.23231	.97264	.24925	.96844	.26612	.96394	34	5
12	27	.21559	.97648	.2326	.97257	.24954	.96837	.2664	.96386	33	5
13	28	.21587	.97642	.23288	.97251	.24982	.96829	.26668	.96379	32	5
13	29	.21616	.97636	.23316	.97244	.2501	.96822	.26696	.96371	31	5
14	30	.21644	.9763	.23345	.97237	.25038	.96815	.26724	.96363	30	5
14	31	.21672	.97623	.23373	.9723	.25066	.96807	.26752	.96355	29	4
14	32	.21701	.97617	.23401	.97223	.25094	.968	.2678	.96347	28	4
15	33	.21729	.97611	.23429	.97217	.25122	.96793	.26808	.9634	27	4
15	34	.21758	.97604	.23458	.9721	.25151	.96786	.26836	.96332	26	4
16	35	.21786	.97598	.23486	.97203	.25179	.96778	.26864	.96324	25	4
16	36	.21814	.97592	.23514	.97196	.25207	.96771	.26892	.96316	24	4
17	37	.21843	.97585	.23542	.97189	.25235	.96764	.2692	.96308	23	3
17	38	.21871	.97579	.23571	.97182	.25263	.96756	.26948	.96301	22	3
18	39	.21899	.97573	.23599	.97176	.25291	.96749	.26976	.96293	21	3
18	40	.21928	.97566	.23627	.97169	.2532	.96742	.27004	.96285	20	3
18	41	.21956	.9756	.23656	.97162	.25348	.96734	.27032	.96277	19	3
19	42	.21985	.97553	.23684	.97155	.25376	.96727	.2706	.96269	18	3
19	43	.22013	.97547	.23712	.97148	.25404	.96719	.27088	.96261	17	3
20	44	.22041	.97541	.2374	.97141	.25432	.96712	.27116	.96253	16	2
20	45	.2207	.97534	.23769	.97134	.2546	.96705	.27144	.96246	15	2
21	46	.22098	.97528	.23797	.97127	.25488	.96697	.27172	.96238	14	2
21	47	.22126	.97521	.23825	.9712	.25516	.9669	.272	.9623	13	2
22	48	.22155	.97515	.23853	.97113	.25545	.96682	.27228	.96222	12	2
22	49	.22183	.97508	.23882	.97106	.25573	.96675	.27256	.96214	11	2
23	50	.22212	.97502	.2391	.971	.25601	.96667	.27284	.96206	10	2
23	51	.2224	.97496	.23938	.97093	.25629	.9666	.27312	.96198	9	1
23	52	.22268	.97489	.23966	.97086	.25657	.96653	.2734	.9619	8	1
24	53	.22297	.97483	.23995	.97079	.25685	.96645	.27368	.96182	7	1
24	54	.22325	.97476	.24023	.97072	.25713	.96638	.27396	.96174	6	1
25	55	.22353	.9747	.24051	.97065	.25741	.9663	.27424	.96166	5	1
25	56	.22382	.97463	.24079	.97058	.25769	.96623	.27452	.96158	4	1
26	57	.2241	.97457	.24108	.97051	.25798	.96615	.2748	.9615	3	0
26	58	.22438	.9745	.24136	.97044	.25826	.96608	.27508	.96142	2	0
27	59	.22467	.97444	.24164	.97037	.25854	.966	.27536	.96134	1	0
27	60	.22495	.97437	.24192	.9703	.25882	.96593	.27564	.96126	0	
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		77°		76°		75°		74°			

Prop. parts.		16°		17°		18°		19°			Prop. parts.
27	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		9
0	0	.27564	.96126	.29237	.9563	.30902	.95106	.32557	.94552	60	9
0	1	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59	9
1	2	.2762	.9611	.29293	.95613	.30957	.95088	.32612	.94533	58	9
1	3	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57	9
2	4	.27676	.96094	.29348	.95596	.31012	.9507	.32667	94514	56	8
2	5	.27704	.96086	.29376	.95588	.3104	.95061	.32694	.94504	55	8
3	6	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54	8
3	7	.27759	.9607	.29432	.95571	.31095	.95043	.32749	.94485	53	8
4	8	.27787	.96062	.2946	.95562	.31123	.95033	.32777	.94476	52	8
4	9	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51	8
5	10	.27843	96046	.29515	.95545	.31178	.95015	.32832	.94457	50	8
5	11	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49	7
5	12	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48	7
6	•13	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47	7
6	14	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46	7
7	15	.27983	.96005	.29654	.95502	.31316	.9497	.32969	.94409	45	7
7	16	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44	7
8	17	.28039	.95989	.2971	.95485	.31372	.94952	.33024	.9439	43	6
8	18	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.9438	42	6
9	19	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.9437	41	6
9	20	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40	6
9	21	.2815	.95956	.29821	.9545	.31482	.94915	.33134	94351	39	6
10	22	.28178	.95948	.29849	.95441	.3151	.94906	.33161	.94342	38	6
10	23	.28206	.9594	.29876	.95433	.31537	.94897	.33189	.94332	37	6
11	24	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36	5
11	25	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35	5
12	26	.2829	.95915	.2996	.95407	.3162	.94869	.33271	.94303	34	5
12	27	.28318	.95907	.29987	.95398	.31648	.9486	.33298	.94293	33	5
13	28	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32	5
13	29	.28374	.9589	.30043	.9538	.31703	.94842	.33353	.94274	31	5
14	30	.28402	.95882	.30071	.95372	.3173	.94832	.33381	.94264	30	5
14	31	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29	4
14	32	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28	4
15	33	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27	4
15	34	.28513	.95849	.30182	.95337	31841	.94795	.3349	.94225	26	4
16	35	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25	4
16	36	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24	4
17	37	.28597	.95824	.30265	.9531	.31923	.94768	.33573	.94196	23	3
17	38	.28625	95816	.30292	.95301	.31951	.94758	.336	.94186	22	3
18	39	.28652	.95807	.3032	.95293	.31979	.94749	.33627	.94176	21	3
18	40	.2868	.95799	.30348	.95284	.32006	.9474	.33655	.94167	20	3
18	41	.28708	.95791	.30376	.95275	.32034	.9473	.33682	.94157	19	3
19	42	.28736	.95782	.30403	.95266	.32061	.94721	.3371	.94147	18	3
19	43	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17	3
20	44	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16	2
20	45	.2882	.95757	.30486	.9524	.32144	.94693	.33792	.94118	15	2
21	46	.28847	.95749	.30514	.95231	.32171	94684	.33819	.94108	14	2
21	47	.28875	.9574	.30542	.95222	.32199	.94674	.33846	.94098	13	2
22	48	.28903	.95732	.3057	.95213	.32227	.94665	.33874	.94088	12	2
22	49	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11	2
23	50	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10	2
23	51	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9	1
23	52	.29015	.95698	.3068	.95177	.32337	.94627	.33983	.94049	8	1
24	53	.29042	.9569	.30708	.95168	.32364	.94618	.34011	.94039	7	1
24	54	.2907	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6	1
25	55	.29098	.95673	.30763	.9515	.32419	.94599	.34065	.94019	5	1
25	56	.29126	.95664	.30791	.95142	.32447	.9459	.34093	.94009	4	1
26	57	.29154	.95656	.30819	.95133	.32474	.9458	.3412	.93999	3	0
26	58	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2	0
27	59	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1	0
27	60	.29237	.9563	.30902	.95106	.32557	.94552	.34202	.93969	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		73°		72°		71°		70°			

Prop. parts.	′	20°		21°		22°		23°			Prop. parts.
27		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		11
0	0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.9205	60	11
0	1	.34229	.93959	.35864	.93348	.37488	.92707	.391	.92039	59	11
1	2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	58	11
1	3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	57	10
2	4	.34311	.93929	.35945	.93316	.37569	.92675	.3918	.92005	56	10
2	5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	55	10
3	6	.34366	.93909	.36	.93295	.37622	.92653	.39234	.91982	54	10
3	7	.34393	.93899	.36027	.93285	.37649	.92642	.3926	.91971	53	10
4	8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	52	10
4	9	.34448	.93879	.36081	.93264	.37703	.9262	.39314	.91948	51	9
5	10	.34475	.93869	.36108	.93253	.3773	.92609	.39341	.91936	50	9
5	11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	49	9
5	12	.3453	.93849	.36162	.93232	.37784	.92587	.39394	.91914	48	9
6	13	.34557	.93839	.3619	.93222	.37811	.92576	.39421	.91902	47	9
6	14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	46	8
7	15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	45	8
7	16	.34639	.93809	.36271	.9319	.37892	.92543	.39501	.91868	44	8
8	17	.34666	.93799	.36298	.9318	.37919	.92532	.39528	.91856	43	8
8	18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	42	8
9	19	.34721	.93779	.36352	.93159	.37973	.9251	.39581	.91833	41	8
9	20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	40	7
9	21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.9181	39	7
10	22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	38	7
10	23	.3483	.93738	.36461	.93116	.3808	.92466	.39688	.91787	37	7
11	24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	36	7
11	25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	35	6
12	26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	34	6
12	27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	33	6
13	28	.34966	.93688	.36596	.93063	.38215	.9241	.39822	.91729	32	6
13	29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	31	6
14	30	.35021	.93667	.3665	.93042	.38268	.92388	.39875	.91706	30	6
14	31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	29	5
14	32	.35075	.93647	.36704	.9302	.38322	.92366	.39928	.91683	28	5
15	33	.35102	.93637	.36731	.9301	.38349	.92355	.39955	.91671	27	5
15	34	.3513	.93626	.36758	.92999	.38376	.92343	.39982	.9166	26	5
16	35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	25	5
16	36	.35184	.93606	.36812	.92978	.3843	.92321	.40035	.91636	24	4
17	37	.35211	.93596	.36839	.92967	.38456	.9231	.40062	.91625	23	4
17	38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	22	4
18	39	.35266	.93575	.36894	.92945	.3851	.92287	.40115	.91601	21	4
18	40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.9159	20	4
18	41	.3532	.93555	.36948	.92924	.38564	.92265	.40168	.91578	19	3
19	42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	18	3
19	43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	17	3
20	44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	16	3
20	45	.35429	.93514	.37056	.92881	.38671	.9222	.40275	.91531	15	3
21	46	.35456	.93503	.37083	.9287	.38698	.92209	.40301	.91519	14	3
21	47	.35484	.93493	.3711	.92859	.38725	.92198	.40328	.91508	13	2
22	48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	12	2
22	49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	11	2
23	50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	10	2
23	51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	9	2
23	52	.35619	.93441	.37245	92805	.38859	.92141	.40461	.91449	8	1
24	53	.35647	.93431	.37272	.92794	.38886	.9213	.40488	.91437	7	1
24	54	.35674	.9342	.37299	.92784	.38912	.92119	.40514	.91425	6	1
25	55	.35701	.9341	.37326	.92773	.38939	.92107	.40541	.91414	5	1
25	56	.35728	.934	.37353	.92762	.38966	.92096	.40567	.91402	4	1
26	57	.35755	.93389	.3738	.92751	.38993	.92085	.40594	.9139	3	1
26	58	.35782	.93379	.37407	.9274	.3902	.92073	.40621	.91378	2	0
27	59	.3581	.93368	.37434	.92729	.39046	.92062	.40647	.91366	1	0
27	60	.35837	.93358	.37461	.92718	.39073	.9205	.40674	.91355	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		69°		68°		67°		66°			

Prop. parts.		24°		25°		26°		27°			Prop. parts.
26	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		14
0	0	.40674	.91355	.42262	.90631	.43837	.89879	.45399	.89101	60	14
0	1	.407	.91343	.42288	.90618	.43863	.89867	.45425	.89087	59	14
1	2	.40727	.91331	.42315	.90606	.43889	.89854	.45451	.89074	58	14
1	3	.40753	.91319	.42341	.90594	.43916	.89841	.45477	.89061	57	13
2	4	.4078	.91307	.42367	.90582	.43942	.89828	.45503	.89048	56	13
2	5	.40806	.91295	.42394	.90569	.43968	.89816	.45529	.89035	55	13
3	6	.40833	.91283	.4242	.90557	.43994	.89803	.45554	.89021	54	13
3	7	.4086	.91272	.42446	.90545	.4402	.8979	.4558	.89008	53	12
3	8	.40886	.9126	.42473	.90532	.44046	.89777	.45606	.88995	52	12
4	9	.40913	.91248	.42499	.9052	.44072	.89764	.45632	.88981	51	12
4	10	.40939	.91236	.42525	.90507	.44098	.89752	.45658	.88968	50	12
5	11	.40966	.91224	.42552	.90495	.44124	.89739	.45684	.88955	49	11
5	12	.40992	.91212	.42578	.90483	.44151	.89726	.4571	.88942	48	11
6	13	.41019	.912	.42604	.9047	.44177	.89713	.45736	.88928	47	11
6	14	.41045	.91188	.42631	.90458	.44203	.897	.45762	.88915	46	11
7	15	.41072	.91176	.42657	.90446	.44229	.89687	.45787	.88902	45	11
7	16	.41098	.91164	.42683	.90433	.44255	.89674	.45813	.88888	44	10
7	17	.41125	.91152	.42709	.90421	.44281	.89662	.45839	.88875	43	10
8	18	.41151	.9114	.42736	.90408	.44307	.89649	.45865	.88862	42	10
8	19	.41178	.91128	.42762	.90396	.44333	.89636	.45891	.88848	41	10
9	20	.41204	.91116	.42788	.90383	.44359	.89623	.45917	.88835	40	9
9	21	.41231	.91104	.42815	.90371	.44385	.8961	.45942	.88822	39	9
10	22	.41257	.91092	.42841	.90358	.44411	.89597	.45968	.88808	38	9
10	23	.41284	.9108	.42867	.90346	.44437	.89584	.45994	.88795	37	9
10	24	.4131	.91068	.42894	.90334	.44464	.89571	.4602	.88782	36	8
11	25	.41337	.91056	.4292	.90321	.4449	.89558	.46046	.88768	35	8
11	26	.41363	.91044	.42946	.90309	.44516	.89545	.46072	.88755	34	8
12	27	.4139	.91032	.42972	.90296	.44542	.89532	.46097	.88741	33	8
12	28	.41416	.9102	.42999	.90284	.44568	.89519	.46123	.88728	32	7
13	29	.41443	.91008	.43025	.90271	.44594	.89506	.46149	.88715	31	7
13	30	.41469	.90996	.43051	.90259	.4462	.89493	.46175	.88701	30	7
13	31	.41496	.90984	.43077	.90246	.44646	.8948	.46201	.88688	29	7
14	32	.41522	.90972	.43104	.90233	.44672	.89467	.46226	.88674	28	7
14	33	.41549	.9096	.4313	.90221	.44698	.89454	.46252	.88661	27	6
15	34	.41575	.90948	.43156	.90208	.44724	.89441	.46278	.88647	26	6
15	35	.41602	.90936	.43182	.90196	.4475	.89428	.46304	.88634	25	6
16	36	.41628	.90924	.43209	.90183	.44776	.89415	.4633	.8862	24	6
16	37	.41655	.90911	.43235	.90171	.44802	.89402	.46355	.88607	23	5
16	38	.41681	.90899	.43261	.90158	.44828	.89389	.46381	.88593	22	5
17	39	.41707	.90887	.43287	.90146	.44854	.89376	.46407	.8858	21	5
17	40	.41734	.90875	.43313	.90133	.4488	.89363	.46433	.88566	20	5
18	41	.4176	.90863	.4334	.9012	.44906	.8935	.46458	.88553	19	4
18	42	.41787	.90851	.43366	.90108	.44932	.89337	.46484	.88539	18	4
19	43	.41813	.90839	.43392	.90095	.44958	.89324	.4651	.88526	17	4
19	44	.4184	.90826	.43418	.90082	.44984	.89311	.46536	.88512	16	4
20	45	.41866	.90814	.43445	.9007	.4501	.89298	.46561	.88499	15	4
20	46	.41892	.90802	.43471	.90057	.45036	.89285	.46587	.88485	14	3
20	47	.41919	.9079	.43497	.90045	.45062	.89272	.46613	.88472	13	3
21	48	.41945	.90778	.43523	.90032	.45088	.89259	.46639	.88458	12	3
21	49	.41972	.90766	.43549	.90019	.45114	.89245	.46664	.88445	11	3
22	50	.41998	.90753	.43575	.90007	.4514	.89232	.4669	.88431	10	2
22	51	.42024	.90741	.43602	.89994	.45166	.89219	.46716	.88417	9	2
23	52	.42051	.90729	.43628	.89981	.45192	.89206	.46742	.88404	8	2
23	53	.42077	.90717	.43654	.89968	.45218	.89193	.46767	.8839	7	2
23	54	.42104	.90704	.4368	.89956	.45243	.8918	.46793	.88377	6	1
24	55	.4213	.90692	.43706	.89943	.45269	.89167	.46819	.88363	5	1
24	56	.42156	.9068	.43733	.8993	.45295	.89153	.46844	.88349	4	1
25	57	.42183	.90668	.43759	.89918	.45321	.8914	.4687	.88336	3	1
25	58	.42209	.90655	.43785	.89905	.45347	.89127	.46896	.88322	2	0
26	59	.42235	.90643	.43811	.89892	.45373	.89114	.46921	.88308	1	0
26	60	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		65°		64°		63°		62°			

Prop. parts.	′	28° N. sine.	28° N. cos.	29° N. sine.	29° N. cos.	30° N. sine.	30° N. cos.	31° N. sine.	31° N. cos.		Prop. parts.
25											14
0	0	.46947	.88295	.48481	.87462	.5	.86603	.51504	.85717	60	14
0	1	.46973	.88281	.48506	.87448	.50025	.86588	.51529	.85702	59	14
1	2	.46999	.88267	.48532	.87434	.5005	.86573	.51554	.85687	58	14
1	3	.47024	.88254	.48557	.8742	.50076	.86559	.51579	.85672	57	13
2	4	.4705	.8824	.48583	.87406	.50101	.86544	.51604	.85657	56	13
2	5	.47076	.88226	.48608	.87391	.50126	.8653	.51628	.85642	55	13
3	6	.47101	.88213	.48634	.87377	.50151	.86515	.51653	.85627	54	13
3	7	.47127	.88199	.48659	.87363	.50176	.86501	.51678	.85612	53	12
3	8	.47153	.88185	.48684	.87349	.50201	.86486	.51703	.85597	52	12
4	9	.47178	.88172	.4871	.87335	.50227	.86471	.51728	.85582	51	12
4	10	.47204	.88158	.48735	.87321	.50252	.86457	.51753	.85567	50	12
5	11	.47229	.88144	.48761	.87306	.50277	.86442	.51778	.85551	49	11
5	12	.47255	.8813	.48786	.87292	.50302	.86427	.51803	.85536	48	11
5	13	.47281	.88117	.48811	.87278	.50327	.86413	.51828	.85521	47	11
6	14	47306	.88103	.48837	.87264	.50352	.86398	.51852	.85506	46	11
6	15	.47332	.88089	.48862	.8725	.50377	.86384	.51877	.85491	45	11
7	16	.47358	.88075	.48888	.87235	.50403	.86369	.51902	.85476	44	10
7	17	.47383	.88062	.48913	.87221	.50428	.86354	.51927	.85461	43	10
8	18	.47409	.88048	.48938	.87207	.50453	.8634	.51952	.85446	42	10
8	19	.47434	.88034	.48964	.87193	.50478	.86325	.51977	.85431	41	10
8	20	.4746	.8802	.48989	.87178	.50503	.8631	.52002	.85416	40	9
9	21	.47486	.88006	.49014	.87164	.50528	.86295	.52026	.85401	39	9
9	22	.47511	.87993	.4904	.8715	.50553	.86281	.52051	.85385	38	9
10	23	.47537	.87979	.49065	.87136	.50578	.86266	.52076	.8537	37	9
10	24	.47562	.87965	.4909	.87121	.50603	.86251	.52101	.85355	36	8
10	25	.47588	.87951	.49116	.87107	.50628	.86237	.52126	.8534	35	8
11	26	.47614	.87937	.49141	87093	.50654	.86222	.52151	.85325	34	8
11	27	.47639	.87923	.49166	.87079	.50679	.86207	.52175	.8531	33	8
12	28	.47665	.87909	.49192	.87064	.50704	.86192	.522	.85294	32	7
12	29	.4769	.87896	.49217	.8705	.50729	.86178	.52225	.85279	31	7
13	30	.47716	.87882	.49242	.87036	.50754	.86163	.5225	.85264	30	7
13	31	.47741	.87868	.49268	.87021	.50779	.86148	.52275	.85249	29	7
13	32	.47767	.87854	.49293	.87007	.50804	.86133	.52299	.85234	28	7
14	33	.47793	.8784	.49318	.86993	.50829	.86119	.52324	.85218	27	6
14	34	.47818	.87826	.49344	.86978	.50854	.86104	.52349	.85203	26	6
15	35	.47844	.87812	.49369	.86964	.50879	.86089	.52374	.85188	25	6
15	36	.47869	.87798	.49394	.86949	.50904	.86074	.52399	.85173	24	6
15	37	.47895	.87784	.49419	.86935	.50929	.86059	.52423	.85157	23	5
16	38	.4792	.8777	.49445	.86921	.50954	.86045	.52448	.85142	22	5
16	39	.47946	.87756	.4947	.86906	.50979	.8603	.52473	.85127	21	5
17	40	.47971	.87743	.49495	.86892	.51004	.86015	.52498	.85112	20	5
17	41	.47997	.87729	.49521	.86878	.51029	.86	.52522	.85096	19	4
18	42	.48022	.87715	.49546	.86863	.51054	.85985	.52547	.85081	18	4
18	43	.48048	.87701	.49571	.86849	.51079	.8597	.52572	.85066	17	4
18	44	.48073	.87687	.49596	.86834	.51104	.85956	.52597	.85051	16	4
19	45	.48099	.87673	.49622	.8682	.51129	.85941	.52621	.85035	15	4
19	46	.48124	.87659	.49647	.86805	.51154	.85926	.52646	.8502	14	3
20	47	.4815	.87645	.49672	.86791	.51179	.85911	.52671	.85005	13	3
20	48	.48175	.87631	.49697	.86777	.51204	.85896	.52696	.84989	12	3
20	49	.48201	.87617	.49723	.86762	.51229	.85881	.5272	.84974	11	3
21	50	.48226	.87603	.49748	.86748	.51254	.85866	.52745	.84959	10	2
21	51	.48252	.87589	.49773	.86733	.51279	.85851	.5277	.84943	9	2
22	52	.48277	.87575	.49798	.86719	.51304	.85836	.52794	.84928	8	2
22	53	.48303	.87561	.49824	.86704	.51329	.85821	.52819	.84913	7	2
23	54	.48328	.87546	.49849	.8669	.51354	.85806	.52844	.84897	6	1
23	55	.48354	.87532	.49874	.86675	.51379	.85792	.52869	.84882	5	1
23	56	.48379	.87518	.49899	.86661	.51404	.85777	.52893	.84866	4	1
24	57	.48405	.87504	.49924	.86646	.51429	.85762	.52918	.84851	3	1
24	58	.4843	.8749	.4995	.86632	.51454	.85747	.52943	.84836	2	0
25	59	.48456	.87476	.49975	.86617	.51479	.85732	.52967	.8482	1	0
25	60	.48481	.87462	.5	.86603	.51504	.85717	.52992	.84805	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		61°		60°		59°		58°			

Prop. parts.		32°		33°		34°		35°			Prop. parts.
23	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		11
0	0	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	60	16
0	1	.53017	.84789	.54488	.83851	.55943	.82887	.57381	.81899	59	16
1	2	.53041	.84774	.54513	.83835	.55968	.82871	.57405	.81882	58	15
1	3	.53066	.84759	.54537	.83819	.55992	.82855	.57429	.81865	57	15
2	4	.53091	.84743	.54561	.83804	.56016	.82839	.57453	.81848	56	15
2	5	.53115	.84728	.54586	.83788	.5604	.82822	.57477	.81832	55	15
2	6	.5314	.84712	.5461	.83772	.56064	.82806	.57501	.81815	54	14
3	7	.53164	.84697	.54635	.83756	.56088	.8279	.57524	.81798	53	14
3	8	.53189	.84681	.54659	.8374	.56112	.82773	.57548	.81782	52	14
3	9	.53214	.84666	.54683	.83724	.56136	.82757	.57572	.81765	51	14
4	10	.53238	.8465	.54708	.83708	.5616	.82741	.57596	.81748	50	13
4	11	.53263	.84635	.54732	.83692	.56184	.82724	.57619	.81731	49	13
5	12	.53288	.84619	.54756	.83676	.56208	.82708	.57643	.81714	48	13
5	13	.53312	.84604	.54781	.8366	.56232	.82692	.57667	.81698	47	13
5	14	.53337	.84588	.54805	.83645	.56256	.82675	.57691	.81681	46	12
6	15	.53361	.84573	.54829	.83629	.5628	.82659	.57715	.81664	45	12
6	16	.53386	.84557	.54854	.83613	.56305	.82643	.57738	.81647	44	12
7	17	.53411	.84542	.54878	.83597	.56329	.82626	.57762	.81631	43	11
7	18	.53435	.84526	.54902	.83581	.56353	.8261	.57786	.81614	42	11
7	19	.5346	.84511	.54927	.83565	.56377	.82593	.5781	.81597	41	11
8	20	.53484	.84495	.54951	.83549	.56401	.82577	.57833	.8158	40	11
8	21	.53509	.8448	.54975	.83533	.56425	.82561	.57857	.81563	39	10
8	22	.53534	.84464	.54999	.83517	.56449	.82544	.57881	.81546	38	10
9	23	.53558	.84448	.55024	.83501	.56473	.82528	.57904	.8153	37	10
9	24	.53583	.84433	.55048	.83485	.56497	.82511	.57928	.81513	36	10
10	25	.53607	.84417	.55072	.83469	.56521	.82495	.57952	.81496	35	9
10	26	.53632	.84402	.55097	.83453	.56545	.82478	.57976	.81479	34	9
10	27	.53656	.84386	.55121	.83437	.56569	.82462	.57999	.81462	33	9
11	28	.53681	.8437	.55145	.83421	.56593	.82446	.58023	.81445	32	9
11	29	.53705	.84355	.55169	.83405	.56617	.82429	.58047	.81428	31	8
12	30	.5373	.84339	.55194	.83389	.56641	.82413	.5807	.81412	30	8
12	31	.53754	.84324	.55218	.83373	.56665	.82396	.58094	.81395	29	8
12	32	.53779	.84308	.55242	.83356	.56689	.8238	.58118	.81378	28	7
13	33	.53804	.84292	.55266	.8334	.56713	.82363	.58141	.81361	27	7
13	34	.53828	.84277	.55291	.83324	.56736	.82347	.58165	.81344	26	7
13	35	.53853	.84261	.55315	.83308	.5676	.8233	.58189	.81327	25	7
14	36	.53877	.84245	.55339	.83292	.56784	.82314	.58212	.8131	24	6
14	37	.53902	.8423	.55363	.83276	.56808	.82297	.58236	.81293	23	6
15	38	.53926	.84214	.55388	.8326	.56832	.82281	.5826	.81276	22	6
15	39	.53951	.84198	.55412	.83244	.56856	.82264	.58283	.81259	21	6
15	40	.53975	.84182	.55436	.83228	.5688	.82248	.58307	.81242	20	5
16	41	.54	.84167	.5546	.83212	.56904	.82231	.5833	.81225	19	5
16	42	.54024	.84151	.55484	.83195	.56928	.82214	.58354	.81208	18	5
16	43	.54049	.84135	.55509	.83179	.56952	.82198	.58378	.81191	17	5
17	44	.54073	.8412	.55533	.83163	.56976	.82181	.58401	.81174	16	4
17	45	.54097	.84104	.55557	.83147	.57	.82165	.58425	.81157	15	4
18	46	.54122	.84088	.55581	.83131	.57024	.82148	.58449	.8114	14	4
18	47	.54146	.84072	.55605	.83115	.57047	.82132	.58472	.81123	13	3
18	48	.54171	.84057	.5563	.83098	.57071	.82115	.58496	.81106	12	3
19	49	.54195	.84041	.55654	.83082	.57095	.82098	.58519	.81089	11	3
19	50	.5422	.84025	.55678	.83066	.57119	.82082	.58543	.81072	10	3
20	51	.54244	.84009	.55702	.8305	.57143	.82065	.58567	.81055	9	2
20	52	.54269	.83994	.55726	.83034	.57167	.82048	.5859	.81038	8	2
20	53	.54293	.83978	.5575	.83017	.57191	.82032	.58614	.81021	7	2
21	54	.54317	.83962	.55775	.83001	.57215	.82015	.58637	.81004	6	2
21	55	.54342	.83946	.55799	.82985	.57238	.81999	.58661	.80987	5	1
21	56	.54366	.8393	.55823	.82969	.57262	.81982	.58684	.8097	4	1
22	57	.54391	.83915	.55847	.82953	.57286	.81965	.58708	.80953	3	1
22	58	.54415	.83899	.55871	.82936	.5731	.81949	.58731	.80936	2	1
23	59	.5444	.83883	.55895	.8292	.57334	.81932	.58755	.80919	1	0
23	60	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		57°		56°		55°		54°			

Prop. parts. 23	′	36° N. sine.	36° N. cos.	37° N. sine.	37° N. cos.	38° N. sine.	38° N. cos.	39° N. sine.	39° N. cos.		Prop. parts. 18
0	0	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60	18
0	1	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59	18
1	2	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58	17
1	3	.58849	.8085	.60251	.79811	.61635	.78747	.63	.7766	57	17
2	4	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56	17
2	5	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55	17
2	6	.5892	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54	16
3	7	.58943	.80782	.60344	.79741	.61726	.78676	.6309	.77586	53	16
3	8	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52	16
3	9	.5899	.80748	.6039	.79706	.61772	.7864	.63135	.7755	51	15
4	10	.59014	.8073	.60414	.79688	.61795	.78622	.63158	.77531	50	15
4	11	.59037	.80713	.60437	.79671	.61818	.78604	.6318	.77513	49	15
5	12	.59061	.80696	.6046	.79653	.61841	78586	.63203	.77494	48	14
5	13	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47	14
5	14	.59108	.80662	.60506	.79618	.61887	.7855	.63248	.77458	46	14
6	15	.59131	.80644	.60529	.796	.61909	.78532	.63271	.77439	45	14
6	16	.59154	.80627	.60553	79583	61932	.78514	.63293	.77421	44	13
7	17	.59178	.8061	.60576	.79565	.61955	.78496	.63316	.77402	43	13
7	18	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42	13
7	19	.59225	.80576	.60622	.7953	.62001	.7846	.63361	.77366	41	12
8	20	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40	12
8	21	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39	12
8	22	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.7731	38	11
9	23	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37	11
9	24	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36	11
10	25	.59365	.80472	.60761	.79424	.62138	.78351	63496	.77255	35	11
10	26	.59389	.80455	.60784	.79406	.6216	.78333	.63518	.77236	34	10
10	27	.59412	.80438	.60807	.79388	.62183	.78315	.6354	.77218	33	10
11	28	.59436	.8042	.6083	.79371	.62206	.78297	.63563	.77199	32	10
11	29	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31	9
12	30	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30	9
12	31	.59506	.80368	.60899	.79318	.62274	.78243	.6363	.77144	29	9
12	32	.59529	.80351	.60922	.793	.62297	.78225	.63653	.77125	28	8
13	33	.59552	.80334	.60945	.79282	.6232	.78206	.63675	.77107	27	8
13	34	.59576	.80316	.60968	.79264	62342	.78188	.63698	.77088	26	8
13	35	.59599	.80299	.60991	.79247	.62365	.7817	.6372	.7707	25	8
14	36	.59622	.80282	.61015	.79229	.62388	78152	.63742	.77051	24	7
14	37	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23	7
15	38	.59669	.80247	61061	.79193	.62433	78116	.63787	.77014	22	7
15	39	.59693	.8023	.61084	.79176	.62456	.78098	.6381	.76996	21	6
15	40	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20	6
16	41	.59739	.80195	.6113	.7914	.62502	.78061	.63854	.76959	19	6
16	42	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.7694	18	5
16	43	.59786	.8016	.61176	.79105	.62547	.78025	.63899	.76921	17	5
17	44	.59809	.80143	.61199	.79087	.6257	.78007	.63922	.76903	16	5
17	45	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15	5
18	46	.59856	.80108	.61245	.79051	.62615	.7797	.63966	.76866	14	4
18	47	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13	4
18	48	.59902	.80073	.61291	.79016	.6266	.77934	.64011	.76828	12	4
19	49	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.7681	11	3
19	50	.59949	.80038	.61337	.7898	.62706	.77897	.64056	76791	10	3
20	51	.59972	.80021	.6136	.78962	.62728	.77879	.64078	.76772	9	3
20	52	.59995	.80003	.61383	.78944	.62751	.77861	.641	.76754	8	2
20	53	.60019	.79986	61406	.78926	.62774	.77843	.64123	.76735	7	2
21	54	.60042	.79968	.61429	.78908	.62796	.77824	64145	.76717	6	2
21	55	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5	2
21	56	.60089	.79934	.61474	.78873	62842	.77788	.6419	.76679	4	1
22	57	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3	1
22	58	.60135	.79899	.6152	.78837	.62887	.77751	.64234	.76642	2	1
23	59	.60158	.79881	.61543	78819	.62909	.77733	.64256	.76623	1	0
23	60	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		53°		52°		51°		50°			

Prop. parts.		40°		41°		42°		43°			Prop. parts.
22	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		19
0	0	.64279	.76604	.65606	.75471	.66913	.74314	.682	.73135	60	19
0	1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	59	19
1	2	.64323	.76567	.6565	.75433	.66956	.74276	.68242	.73096	58	18
1	3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	57	18
1	4	.64368	.7653	.65694	.75395	.66999	.74237	.68285	.73056	56	18
2	5	.6439	.76511	.65716	.75375	.67021	.74217	.68306	.73036	55	17
2	6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	54	17
3	7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	53	17
3	8	.64457	.76455	.65781	.75318	.67086	.74159	.6837	.72976	52	16
3	9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	51	16
4	10	.64501	.76417	.65825	.7528	.67129	.7412	.68412	.72937	50	16
4	11	.64524	.76398	.65847	.75261	.67151	.741	.68434	.72917	49	16
4	12	.64546	.7638	.65869	.75241	.67172	.7408	.68455	.72897	48	15
5	13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	47	15
5	14	.6459	.76342	.65913	.75203	.67215	.74041	.68497	.72857	46	15
6	15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	45	14
6	16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	44	14
6	17	.64657	.76286	.65978	.75146	.6728	.73983	.68561	.72797	43	14
7	18	.64679	.76267	.66	.75126	.67301	.73963	.68582	.72777	42	13
7	19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	41	13
7	20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	40	13
8	21	.64746	.7621	.66066	.75069	.67366	.73904	.68645	.72717	39	12
8	22	.64768	.76192	.66088	.7505	.67387	.73885	.68666	.72697	38	12
8	23	.6479	.76173	.66109	.7503	.67409	.73865	.68688	.72677	37	12
9	24	.64812	.76154	.66131	.75011	.6743	.73846	.68709	.72657	36	11
9	25	.64834	.76135	.66153	.74992	.67452	.73826	.6873	.71637	35	11
10	26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	34	11
10	27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	33	10
10	28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	32	10
11	29	.64923	.76059	.6624	.74915	.67538	.73747	.68814	.72557	31	10
11	30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	30	10
11	31	.64967	.76022	.66284	.74876	.6758	.73708	.68857	.72517	29	9
12	32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	28	9
12	33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	27	9
12	34	.65033	.75965	.66349	.74818	.67645	.73649	.6892	.72457	26	8
13	35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	25	8
13	36	.65077	.75927	.66393	.7478	.67688	.7361	.68962	.72417	24	8
14	37	.651	.75908	.66414	.7476	.67709	.7359	.68983	.72397	23	7
14	38	.65122	.75889	.66436	.74741	.6773	.7357	.69004	.72377	22	7
14	39	.65144	.7587	.66458	.74722	.67752	.73551	.69025	.72357	21	7
15	40	.65166	.75851	.6648	.74703	.67773	.73531	.69046	.72337	20	6
15	41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	19	6
15	42	.6521	.75813	.66523	.74664	.67816	.73491	.69088	.72297	18	6
16	43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	17	5
16	44	.65254	.75775	.66566	.74625	.67859	.73452	.6913	.72257	16	5
17	45	.65276	.75756	.66588	.74606	.6788	.73432	.69151	.72236	15	5
17	46	.65298	.75738	.6661	.74586	.67901	.73413	.69172	.72216	14	4
17	47	.6532	.75719	.66632	.74567	.67923	.73393	.69193	.72196	13	4
18	48	.65342	.757	.66653	.74548	.67944	.73373	.69214	.72176	12	4
18	49	.65364	.7568	.66675	.74528	.67965	.73353	.69235	.72156	11	3
18	50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	10	3
19	51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	9	3
19	52	.6543	.75623	.6674	.7447	.68029	.73294	.69298	.72095	8	3
19	53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	7	2
20	54	.65474	.75585	.66783	.74431	.68072	.73254	.6934	.72055	6	2
20	55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	5	2
21	56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	4	1
21	57	.6554	.75528	.66848	.74373	.68136	.73195	.69403	.71995	3	1
21	58	.65562	.75509	.6687	.74353	.68157	.73175	.69424	.71974	2	1
22	59	.65584	.7549	.66891	.74334	.68179	.73155	.69445	.71954	1	0
22	60	.65606	.75471	.66913	.74314	.682	.73135	.69466	.71934	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		49°		48°		47°		46°			

Prop. parts. 11	′	44° N. sine.	N. cos.		Prop. parts. 19	Prop. parts. 22	′	44° N. sine.	N. cos.		Prop. parts. 9
0	0	.69466	.71934	60	19	11	31	.70112	.71305	29	9
0	1	.69487	.71914	59	19	12	32	.70132	.71284	28	9
1	2	.69508	.71894	58	18	12	33	.70153	.71264	27	9
1	3	.69529	.71873	57	18	12	34	.70174	.71243	26	8
1	4	.69549	.71853	56	18	13	35	.70195	.71223	25	8
2	5	.6957	.71833	55	17	13	36	.70215	.71203	24	8
2	6	.69591	.71813	54	17	14	37	.70236	.71182	23	7
3	7	.69612	.71792	53	17	14	38	.70257	.71162	22	7
3	8	.69633	.71772	52	16	14	39	.70277	.71141	21	7
3	9	.69654	.71752	51	16	15	40	.70298	.71121	20	6
4	10	.69675	.71732	50	16	15	41	.70319	.711	19	6
4	11	.69696	.71711	49	16	15	42	.70339	.7108	18	6
4	12	.69717	.71691	48	15	16	43	.7036	.71059	17	5
5	13	.69737	.71671	47	15	16	44	.70381	.71039	16	5
5	14	.69758	.7165	46	15	17	45	.70401	.71019	15	5
6	15	.69779	.7163	45	14	17	46	.70422	.70998	14	4
6	16	.698	.7161	44	14	17	47	.70443	.70978	13	4
6	17	.69821	.7159	43	14	18	48	.70463	.70957	12	4
7	18	.69842	.71569	42	13	18	49	.70484	.70937	11	3
7	19	.69862	.71549	41	13	18	50	.70505	.70916	10	3
7	20	.69883	.71529	40	13	19	51	.70525	.70896	9	3
8	21	.69904	.71508	39	12	19	52	.70546	.70875	8	3
8	22	.69925	.71488	38	12	19	53	.70567	.70855	7	2
8	23	.69946	.71468	37	12	20	54	.70587	.70834	6	2
9	24	.69966	.71447	36	11	20	55	.70608	.70813	5	2
9	25	.69987	.71427	35	11	21	56	.70628	.70793	4	1
10	26	.70008	.71407	34	11	21	57	.70649	.70772	3	1
10	27	.70029	.71386	33	10	21	58	.7067	.70752	2	1
10	28	.70049	.71366	32	10	22	59	.7069	.70731	1	0
11	29	.7007	.71345	31	10	22	60	.70711	.70711	0	0
11	30	.70091	.71325	30	10						
		N. cos. 45°	N. sine.	′				N. cos. 45°	N. sine.	′	

Preceding Table contains Natural Sine and Cosine for every minute of the Quadrant to Radius 1.

If Degrees are taken at head of columns, Minutes, Sine, and Cosine must be taken from head also; and if they are taken at foot of column, Minutes, etc., must be taken from foot also.

ILLUSTRATION.—.3173 is sine of 18° 30′, and cosine of 71° 30′.

To Compute Sine or Cosine for Seconds.

When Angle is less than 45°. RULE.—Ascertain sine or cosine of angle for degrees and minutes from Table; take difference between it and sine or cosine of angle next below it. Look for this difference or remainder,* if *Sine* is required, at head of column of *Proportional Parts*, on left side; and if *Cosine* is required, at head of column on right side; and in these respective columns, opposite to number of seconds of angle in column, is number or correction in seconds to be added to Sine, or subtracted from Cosine of angle.

ILLUSTRATION 1.—What is sine of 8° 9′ 10″?

$$\left.\begin{array}{l}\text{Sine of } 8^\circ\ 9', \text{ per Table} = .141\,77;\\ \text{Sine of } 8^\circ\ 10', \quad \text{“} \quad = .142\,05;\end{array}\right\}\ .000\,28 \textit{ difference.}$$

In left side column of proportional parts, under 28, and opposite to 10′, is 5, correction for 10′, which, being added to .141 77 = .141 82 *Sine*.

* The table in some instances will give a unit too much, but this, in general, is of little importance.

2.—What is cosine of 8° 9′ 10″?

Cosine of 8° 9′, per Table = .989 90; } .000 04 *difference.*
Cosine of 8° 10′, " = .999 86; }

In right side column of *proportional parts*, under 4, and opposite to 10′, is 1, the correction for 10′, which, being subtracted from .989 90 = .989 89 *cosine.*

When Angle exceeds 45°. RULE.—Ascertain sine or cosine for angle in degrees and minutes from Table, taking degrees at the foot of it; then take difference between it and sine or cosine of angle next above it. Look for remainder, if *Sine* is required, at head of column of *Proportional Parts*, on right side; and if *Cosine* is required, at head of column on left side; and in these respective columns, opposite to seconds of angle, is number or correction in seconds to be added to Sine, or subtracted from Cosine of angle.

ILLUSTRATION.—What is the Sine and Cosine of 81° 50′ 50″?

Sine of 81° 50′, per Table = .989 86; } .000 04 *difference.*
Sine of 81° 51′, " = .989 9; }

In right-side column of *proportional parts*, and opposite to 50′, is 3, which, added to .989 86 = .989 89 *Sine.*

Cosine of 81° 50′, per Table = .142 05; } .000 25 *difference.*
Cosine of 81° 51′, " = .141 77; }

In left-side column of *proportional parts*, and opposite to 50′, is 24, which, subtracted from .142 05 = .141 81 *Cosine.*

To Ascertain or Compute Number of Degrees, Minutes, and Seconds of a given Sine or Cosine.

When Sine is given. RULE.—If given *sine* is in Table, the degrees of it will be at top or bottom of page, and minutes in marginal column, at left or right side, according as sine corresponds to an angle less or greater than 45°.

If given sine is not in Table, take sine in Table which is next *less* than the one for which degrees, etc., are required, and note degrees, etc., for it. Subtract this sine from next *greater* tabular sine, and also from given sine.

Then, as tabular difference is to difference between given sine and tabular sine, so is 60 seconds to seconds for sine given.

EXAMPLE.—What are the degrees, minutes, and seconds for sine of .75?

Next less sine is .749 92, *arc for which is* 48° 35′. Next greater sine is .750 11, *difference between which and next less is* .750 11 — .749 92 = .000 19. *Difference between less tabular sine and one given is* .75 — .749 92 = 8.

Then 19 : 8 :: 60″ : 25+, which, added to 48° 35′ = 48° 35′ 25″.

When Cosine is given. RULE.—If given *cosine* is found in Table, degrees of it will be found as in manner specified when *sine* is given.

If given cosine is not in Table, take cosine in Table which is next *greater* than one for which degrees, etc., are required, and note degrees, etc., for it. Subtract this cosine from next *less* tabular cosine, and also from given cosine.

Then, as tabular difference is to difference between given cosine and tabular cosine, so is 60 seconds to seconds for cosine given.

EXAMPLE.—What are the degrees, minutes, and seconds for cosine of .75?

Next greater cosine is .750 11, *arc for which is* 41° 24′. Next less cosine is .749 92, *difference between which and next greater is* .750 11 — .749 92 = .000 19. *Difference between greater tabular cosine and one given is* .750 11 — .750 00 = 11.

Then 19 : 11 :: 60 : 35 —, which, added to 41° 24′ = 41° 24′ 35″.

To Compute Versed Sine of an Angle.

Subtract cosine of angle from 1.

ILLUSTRATION.—What is the versed sine of 21° 30′?

Cosine of 21° 30′ is .930 42, which, — 1 = .069 58 *versed sine.*

To Compute Co-versed Sine of an Angle.

Subtract sine of angle from 1.

ILLUSTRATION.—What is the co-versed sine of 21° 30′?

The sine of 21° 30′ is .3665, which, — 1 = .6335 *co-versed sine.*

Natural Secants and Co-secants.

′	0° Secant.	0° Co-secant.	1° Secant.	1° Co-sec't.	2° Secant.	2° Co-sec't.	3° Secant.	3° Co-sec't.	′
0	1	Infinite.	1.0001	57.299	1.0006	28.654	1.0014	19.107	60
1	1	3437.7	.0001	6.359	.0006	8.417	.0014	9.002	59
2	1	1718.9	.0002	5.45	.0006	8.184	.0014	8.897	58
3	1	145.9	.0002	4.57	.0006	7.955	.0014	8.794	57
4	1	859.44	.0002	3.713	.0006	7.73	.0014	8.692	56
5	1	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	55
6	1	572.96	.0002	2.09	.0007	7.29	.0015	8.491	54
7	1	491.11	.0002	1.313	.0007	7.075	.0015	8.393	53
8	1	29.72	.0002	0.558	.0007	6.864	.0015	8.295	52
9	1	381.97	.0002	49.826	.0007	6.655	.0015	8.198	51
10	1	343.77	1.0002	49.114	1.0007	26.45	1.0015	18.103	50
11	1	12.52	.0002	8.422	.0007	6.249	.0015	8.008	49
12	1	286.48	.0002	7.75	.0007	6.05	.0016	7.914	48
13	1	64.44	.0002	7.096	.0007	5.854	.0016	7.821	47
14	1	45.55	.0002	6.46	.0008	5.661	.0016	7.73	46
15	1	229.18	1.0002	45.84	1.0008	25.471	1.0016	17.639	45
16	1	14.86	.0002	5.237	.0008	5.284	.0016	7.549	44
17	1	02.22	.0002	4.65	.0008	5.1	.0016	7.46	43
18	1	190.99	.0002	4.077	.0008	4.918	.0017	7.372	42
19	1	80.73	.0003	3.52	.0008	4.739	.0017	7.285	41
20	1	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	40
21	1	63.7	.0003	2.445	.0008	4.358	.0017	7.113	39
22	1	56.26	.0003	1.928	.0008	4.216	.0017	7.028	38
23	1	49.47	.0003	1.423	.0009	4.047	.0017	6.944	37
24	1	43.24	.0003	40.93	.0009	3.88	.0018	6.861	36
25	1	137.51	1.0003	40.448	1.0009	23.716	1.0018	16.779	35
26	1	32.22	.0003	39.978	.0009	3.553	.0018	6.698	34
27	1	27.32	.0003	9.518	.0009	3.393	.0018	6.617	33
28	1	22.78	.0003	9.069	.0009	3.235	.0018	6.538	32
29	1	18.54	.0003	8.631	.0009	3.079	.0018	6.459	31
30	1	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.38	30
31	1	10.9	.0003	7.782	.001	2.774	.0019	6.303	29
32	1	07.43	.0003	7.371	.001	2.624	.0019	6.226	28
33	1	04.17	.0004	6.969	.001	2.476	.0019	6.15	27
34	1	01.11	.0004	6.576	.001	2.33	.0019	6.075	26
35	1	98.223	1.0004	36.191	1.001	22.186	1.0019	16	25
36	1	5.495	.0004	5.814	.001	2.044	.002	5.926	24
37	1	2.914	.0004	5.445	.001	1.904	.002	5.853	23
38	1.0001	2.469	.0004	5.084	.001	1.765	.002	5.78	22
39	.0001	88.149	.0004	4.729	.0011	1.629	.002	5.708	21
40	1.0001	85.946	1.0004	34.382	1.0011	21.494	1.002	15.637	20
41	.0001	3.849	.0004	4.042	.0011	1.36	.0021	5.566	19
42	.0001	1.853	.0004	3.708	.0011	1.228	.0021	5.496	18
43	.0001	79.95	.0004	3.381	.0011	1.098	.0021	5.427	17
44	.0001	8.133	.0004	3.06	.0011	20.97	.0021	5.358	16
45	1.0001	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.29	15
46	.0001	4.736	.0005	2.437	.0012	0.717	.0022	5.222	14
47	.0001	3.146	.0005	2.134	.0012	0.593	.0022	5.155	13
48	.0001	1.622	.0005	1.836	.0012	0.471	.0022	5.089	12
49	.0001	1.16	.0005	1.544	.0012	0.35	.0022	5.023	11
50	1.0001	68.757	1.0005	31.257	1.0012	20.23	1.0022	14.958	10
51	.0001	7.409	.0005	30.976	.0012	0.112	.0023	4.893	9
52	.0001	6.113	.0005	0.699	.0012	19.995	.0023	4.829	8
53	.0001	4.866	.0005	0.428	.0013	9.88	.0023	4.765	7
54	.0001	3.664	.0005	0.161	.0013	9.766	.0023	4.702	6
55	1.0001	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.64	5
56	.0001	1.391	.0006	9.641	.0013	9.541	.0024	4.578	4
57	.0001	1.314	.0006	9.388	.0013	9.431	.0024	4.517	3
58	.0001	59.274	.0006	9.139	.0013	9.322	.0024	4.456	2
59	.0001	8.27	.0006	8.894	.0013	9.214	.0024	4.395	1
60	1.0001	57.299	1.0006	28.654	1.0014	19.107	1.0024	14.335	0
′	89° Co-sec't.	89° Secant.	88° Co-sec't.	88° Secant.	87° Co-sec't.	87° Secant.	86° Co-sec't.	86° Secant.	′

	4°		5°		6°		7°		
′	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	′
0	1.0024	14.335	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	60
1	.0025	4.276	.0038	1.436	.0055	.5404	.0075	.1861	59
2	.0025	4.217	.0039	1.398	.0056	.5141	.0076	.1668	58
3	.0025	4.159	.0039	1.36	.0056	.488	.0076	.1476	57
4	.0025	4.101	.0039	1.323	.0056	.462	.0076	.1285	56
5	1.0025	14.043	1.0039	11.286	1.0057	9.4362	1.0077	8.1094	55
6	.0026	3.986	.004	1.249	.0057	.4105	.0077	.0905	54
7	.0026	3.93	.004	1.213	.0057	.385	.0078	.0717	53
8	.0026	3.874	.004	1.176	.0057	.3596	.0078	.0529	52
9	.0026	3.818	.004	1.14	.0058	.3343	.0078	.0342	51
10	1.0026	13.763	1.0041	11.104	1.0058	9.3092	1.0079	8.0156	50
11	.0027	3.708	.0041	1.069	.0058	.2842	.0079	7.9971	49
12	.0027	3.654	.0041	1.033	.0059	.2593	.0079	.9787	48
13	.0027	3.6	.0041	0.988	.0059	.2346	.008	.9604	47
14	.0027	3.547	.0042	0.963	.0059	.21	.008	.9421	46
15	1.0027	13.494	1.0042	10.929	1.006	9.1855	1.008	7.924	45
16	.0028	3.441	.0042	0.894	.006	.1612	.0081	.9059	44
17	.0028	3.389	.0043	0.86	.006	.137	.0081	.8879	43
18	.0028	3.337	.0043	0.826	.0061	.1129	.0082	.87	42
19	.0028	3.286	.0043	0.792	.0061	.089	.0082	.8522	41
20	1.0029	13.235	1.0043	10.758	1.0061	9.0651	1.0082	7.8344	40
21	.0029	3.184	.0044	0.725	.0062	.0414	.0083	.8168	39
22	.0029	3.134	.0044	0.692	.0062	.0179	.0083	.7992	38
23	.0029	3.084	.0044	0.659	.0062	8.9944	.0084	.7817	37
24	.0029	3.034	.0044	0.626	.0063	.9711	.0084	.7642	36
25	1.003	12.985	1.0045	10.593	1.0063	8.9479	1.0084	7.7469	35
26	.003	2.937	.0045	0.561	.0063	.9248	.0085	.7296	34
27	.003	2.888	.0045	0.529	.0064	.9018	.0085	.7124	33
28	.003	2.84	.0046	0.497	.0064	.879	.0085	.6953	32
29	.0031	2.793	.0046	0.465	.0064	.8563	.0086	.6783	31
30	1.0031	12.745	1.0046	10.433	1.0065	8.8337	1.0086	7.6613	30
31	.0031	2.698	.0046	0.402	.0065	.8112	.0087	.6444	29
32	.0031	2.652	.0047	0.371	.0065	.7888	.0087	.6276	28
33	.0032	2.606	.0047	0.34	.0066	.7665	.0087	.6108	27
34	.0032	2.56	.0047	0.309	.0066	.7444	.0088	.5942	26
35	1.0032	12.514	1.0048	10.278	1.0066	8.7223	1.0088	7.5776	25
36	.0032	2.469	.0048	0.248	.0067	.7004	.0089	.5611	24
37	.0032	2.424	.0048	0.217	.0067	.6786	.0089	.5446	23
38	.0033	2.379	.0048	0.187	.0067	.6569	.0089	.5282	22
39	.0033	2.335	.0049	0.157	.0068	.6353	.009	.5119	21
40	1.0033	12.291	1.0049	10.127	1.0068	8.6138	1.009	7.4957	20
41	.0033	2.248	.0049	0.098	.0068	.5924	.009	.4795	19
42	.0034	2.204	.005	0.068	.0069	.5711	.0091	.4634	18
43	.0034	2.161	.005	0.039	.0069	.5499	.0091	.4474	17
44	.0034	2.118	.005	0.01	.0069	.5289	.0092	.4315	16
45	1.0034	12.076	1.005	9.9812	1.007	8.5079	1.0092	7.4156	15
46	.0035	2.034	.0051	.9525	.007	.4871	.0092	.3998	14
47	.0035	1.992	.0051	.9239	.007	.4663	.0093	.384	13
48	.0035	1.95	.0051	.8955	.0071	.4457	.0093	.3683	12
49	.0035	1.909	.0052	.8672	.0071	.4251	.0094	.3527	11
50	1.0036	11.868	1.0052	9.8391	1.0071	8.4046	1.0094	7.3372	10
51	.0036	1.828	.0052	.8112	.0072	.3843	.0094	.3217	9
52	.0036	1.787	.0053	.7834	.0072	.3640	.0095	.3063	8
53	.0036	1.747	.0053	.7558	.0073	.3439	.0095	.2909	7
54	.0037	1.707	.0053	.7283	.0073	.3238	.0096	.2757	6
55	1.0037	11.668	1.0053	9.701	1.0073	8.3039	1.0096	7.2604	5
56	.0037	1.628	.0054	.6739	.0074	.2840	.0097	.2453	4
57	.0037	1.589	.0054	.6469	.0074	.2642	.0097	.2302	3
58	.0038	1.55	.0054	.62	.0074	.2446	.0097	.2152	2
59	.0038	1.512	.0055	.5933	.0075	.225	.0098	.2002	1
60	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	.0098	7.1853	0
′	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	′
	85°		84°		83°		82°		

′	8° Secant.	8° Co-sec't.	9° Secant.	9° Co-sec't.	10° Secant.	10° Co-sec't.	11° Secant.	11° Co-sec't.	′
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	60
1	.0099	.1704	.0125	.3807	.0155	.7493	.0188	.233	59
2	.0099	.1557	.0125	.369	.0155	.7398	.0188	.2252	58
3	.0099	.1409	.0126	.3574	.0156	.7304	.0189	.2174	57
4	.01	.1263	.0126	.3458	.0156	.721	.0189	.2097	56
5	1.01	7.1117	1.0127	6.3343	1.0157	5.7117	1.019	5.2019	55
6	.0101	.0972	.0127	.3228	.0157	.7023	.0191	.1942	54
7	.0101	.0827	.0128	.3113	.0158	.693	.0191	.1865	53
8	.0102	.0683	.0128	.2999	.0158	.6838	.0192	.1788	52
9	.0102	.0539	.0129	.2885	.0159	.6745	.0192	.1712	51
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	50
11	.0103	.0254	.013	.2659	.016	.6561	.0193	.156	49
12	.0103	.0112	.013	.2546	.016	.647	.0194	.1484	48
13	.0104	6.9971	.0131	.2434	.0161	.6379	.0195	.1409	47
14	.0104	.983	.0131	.2322	.0162	.6288	.0195	.1333	46
15	1.0104	6.969	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	45
16	.0105	.955	.0132	.21	.0163	.6107	.0196	.1183	44
17	.0105	.9411	.0133	.199	.0163	.6017	.0197	.1109	43
18	.0106	.9273	.0133	.188	.0164	.5928	.0198	.1034	42
19	.0106	.9135	.0134	.177	.0164	.5838	.0198	.096	41
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	40
21	.0107	.8861	.0135	.1552	.0165	.566	.0199	.0812	39
22	.0107	.8725	.0135	.1443	.0166	.5572	.02	.0739	38
23	.0108	.8589	.0136	.1335	.0166	.5484	.0201	.0666	37
24	.0108	.8454	.0136	.1227	.0167	.5396	.0201	.0593	36
25	1.0109	6.832	1.0136	6.112	1.0167	5.5308	1.0202	5.052	35
26	.0109	.8185	.0137	.1013	.0168	.5221	.0202	.0447	34
27	.011	.8052	.0137	.0906	.0169	.5134	.0203	.0375	33
28	.011	.7919	.0138	.08	.0169	.5047	.0204	.0302	32
29	.0111	.7787	.0138	.0694	.017	.496	.0204	.023	31
30	1.0111	6.7655	1.0139	6.0588	1.017	5.4874	1.0205	5.0158	30
31	.0111	.7523	.0139	.0483	.0171	.4788	.0205	.0087	29
32	.0112	.7392	.014	.0379	.0171	.4702	.0206	.0015	28
33	.0112	.7262	.014	.0274	.0172	.4617	.0207	4.9944	27
34	.0113	.7132	.0141	.017	.0172	.4532	.0207	.9873	26
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	25
36	.0114	.6874	.0142	5.9963	.0174	.4362	.0208	.9732	24
37	.0114	.6745	.0142	.986	.0174	.4278	.0209	.9661	23
38	.0115	.6617	.0143	.9758	.0175	.4194	.021	.9591	22
39	.0115	.649	.0143	.9655	.0175	.411	.021	.9521	21
40	1.0115	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	20
41	.0116	.6237	.0144	.9452	.0176	.3943	.0211	.9382	19
42	.0116	.6111	.0145	.9351	.0177	.386	.0212	.9313	18
43	.0117	.5985	.0145	.925	.0177	.3777	.0213	.9243	17
44	.0117	.586	.0146	.915	.0178	.3695	.0213	.9175	16
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	15
46	.0118	.5612	.0147	.895	.0179	.353	.0215	.9037	14
47	.0119	.5488	.0147	.885	.018	.3449	.0215	.8969	13
48	.0119	.5365	.0148	.8751	.018	.3367	.0216	.8901	12
49	.0119	.5243	.0148	.8652	.0181	.3286	.0216	.8833	11
50	1.012	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	10
51	.012	.4999	.015	.8456	.0182	.3124	.0218	.8697	9
52	.0121	.4878	.015	.8358	.0182	.3044	.0218	.863	8
53	.0121	.4757	.0151	.8261	.0183	.2963	.0219	.8563	7
54	.0122	.4637	.0151	.8163	.0184	.2883	.022	.8496	6
55	1 0122	6.4517	1.0152	5.8067	1.0184	5.2803	1.022	4.8429	5
56	.0123	.4398	.0152	.797	.0185	.2724	.0221	.8362	4
57	.0123	.4279	.0153	.7874	.0185	.2645	.0221	.8296	3
58	.0124	.416	.0153	.7778	.0186	.2566	.0222	.8229	2
59	.0124	.4042	.0154	.7683	.0186	.2487	.0223	.8163	1
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	0
′	Co-sec't. 81°	Secant. 81°	Co-sec't. 80°	Secant. 80°	Co-sec't. 79°	Secant. 79°	Co-sec't. 78°	Secant. 78°	′

	12°		13°		14°		15°		
′	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	′
0	1.0223	4.8097	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	60
1	.0224	.8032	.0264	.4398	.0307	.1287	.0353	.8595	59
2	.0225	.7966	.0264	.4342	.0308	.1239	.0354	.8553	58
3	.0225	.7901	.0265	.4287	.0308	.1191	.0355	.8512	57
4	.0226	.7835	.0266	.4231	.0309	.1144	.0356	.847	56
5	1.0226	4.777	1.0266	4.4176	1.031	4.1096	1.0357	3.8428	55
6	.0227	.7706	.0267	.4121	.0311	.1048	.0358	.8387	54
7	.0228	.7641	.0268	.4065	.0311	.1001	.0358	.8346	53
8	.0228	.7576	.0268	.4011	.0312	.0953	.0359	.8304	52
9	.0229	.7512	.0269	.3956	.0313	.0906	.036	.8263	51
10	1.023	4.7448	1.027	4.3901	1.0314	4.0859	1.0361	3.8222	50
11	.023	.7384	.0271	.3847	.0314	.0812	.0362	.8181	49
12	.0231	.732	.0271	.3792	.0315	.0765	.0362	.814	48
13	.0232	.7257	.0272	.3738	.0316	.0718	.0363	.81	47
14	.0232	.7193	.0273	.3684	.0317	.0672	.0364	.8059	46
15	1.0233	4.713	1.0273	4.363	1.0317	4.0625	1.0365	3.8018	45
16	.0234	.7067	.0274	.3576	.0318	.0579	.0366	.7978	44
17	.0234	.7004	.0275	.3522	.0319	.0532	.0367	.7937	43
18	.0235	.6942	.0276	.3469	.032	.0486	.0367	.7897	42
19	.0235	.6879	.0276	.3415	.032	.044	.0368	.7857	41
20	1.0236	4.6817	1.0277	4.3362	1.0321	4.0394	1.0369	3.7816	40
21	.0237	.6754	.0278	.3309	.0322	.0348	.037	.7776	39
22	.0237	.6692	.0278	.3256	.0323	.0302	.0371	.7736	38
23	.0238	.6631	.0279	.3203	.0323	.0256	.0371	.7697	37
24	.0239	.6569	.028	.315	.0324	.0211	.0372	.7657	36
25	1.0239	4.6507	1.028	4.3098	1.0325	4.0165	1.0373	3.7617	35
26	.024	.6446	.0281	.3045	.0326	.012	.0374	.7577	34
27	.0241	.6385	.0282	.2993	.0327	.0074	.0375	.7538	33
28	.0241	.6324	.0283	.2941	.0327	.0029	.0375	.7498	32
29	.0242	.6263	.0283	.2888	.0328	3.9984	.0376	.7459	31
30	1.0243	4.6202	1.0284	4.2836	1.0329	3.9939	1.0377	3.742	30
31	.0243	6142	0285	.2785	.033	.9894	.0378	.738	29
32	.0244	6081	.0285	.2733	.033	.985	.0379	.7341	28
33	.0245	.6021	.0286	.2681	.0331	.9805	.038	.7302	27
34	.0245	.5961	.0287	.263	.0332	.976	.0381	.7263	26
35	1.0246	4.5901	1.0288	4.2579	1.0333	3.9716	1.0382	3.7224	25
36	.0247	.5841	.0288	.2527	.0334	.9672	.0382	.7186	24
37	.0247	.5782	.0289	.2476	.0334	.9627	.0383	.7147	23
38	.0248	.5722	.029	.2425	.0335	.9583	.0384	.7108	22
39	.0249	.5663	.0291	.2375	.0336	.9539	.0385	.707	21
40	1.0249	4.5604	1.0291	4.2324	1.0337	3.9495	1.0386	3.7031	20
41	.025	.5545	.0292	.2273	.0338	.9451	.0387	.6993	19
42	.0251	.5486	.0293	.2223	.0338	.9408	.0387	.6955	18
43	.0251	.5428	.0293	.2173	.0339	.9364	.0388	.6917	17
44	.0252	.5369	.0294	.2122	.034	.932	.0389	.6878	16
45	1.0253	4.5311	1.0295	4.2072	1.0341	3.9277	1.039	3.684	15
46	.0253	.5253	.0296	.2022	.0341	.9234	.0391	.6802	14
47	.0254	.5195	.0296	.1972	.0342	.919	.0392	.6765	13
48	.0255	.5137	.0297	.1923	.0343	.9147	.0393	.6727	12
49	.0255	.5079	.0298	.1873	.0344	.9104	.0393	.6689	11
50	1.0256	4.5021	1.0299	4.1824	1.0345	3.9061	1.0394	3.6651	10
51	.0257	.4964	.0299	.1774	.0345	.9018	.0395	.6614	9
52	.0257	.4907	.03	.1725	.0346	.8976	.0396	.6576	8
53	.0258	.485	.0301	.1676	.0347	.8933	.0397	.6539	7
54	.0259	.4793	.0302	.1627	.0348	.899	.0398	.6502	6
55	1.026	4.4736	1.0302	4.1578	1.0349	3.8848	1.0399	3.6464	5
56	.026	.4679	.0303	.1529	.0349	.8805	.0399	.6427	4
57	.0261	.4623	.0304	.1481	.035	.8763	.04	.639	3
58	.0262	.4566	.0305	.1432	.0351	.8721	.0401	.6353	2
59	.0262	.451	.0305	.1384	.0352	.8679	.0402	.6316	1
60	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	1.0403	3.6279	0
′	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	′
	77°		76°		75°		74°		

′	16° Secant.	16° Co-sec't.	17° Secant.	17° Co-sec't.	18° Secant.	18° Co-sec't.	19° Secant.	19° Co-sec't.	′
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	60
1	.0404	.6243	.0458	.417	.0516	.2332	.0577	.069	59
2	.0405	.6206	.0459	.4138	.0517	.2303	.0578	.0664	58
3	.0406	.6169	.046	.4106	.0518	.2274	.0579	.0638	57
4	.0406	.6133	.0461	.4073	.0519	.2245	.058	.0612	56
5	1.0407	3.6096	1.0461	3.4041	1.052	3.2216	1.0581	3.0586	55
6	.0408	.606	.0462	.4009	.0521	.2188	.0582	.0561	54
7	.0409	.6024	.0463	.3977	.0522	.2159	.0584	.0535	53
8	.041	.5987	.0464	.3945	.0523	.2131	.0585	.0509	52
9	.0411	.5951	.0465	.3913	.0524	.2102	.0586	.0484	51
10	1.0412	3.5915	1.0466	3.3881	1.0525	3.2074	1.0587	3.0458	50
11	.0413	.5879	.0467	.3849	.0526	.2045	.0588	.0433	49
12	.0413	.5843	.0468	.3817	.0527	.2017	.0589	.0407	48
13	.0414	.5807	.0469	.3785	.0528	.1989	.059	.0382	47
14	.0415	.5772	.047	.3754	.0529	.196	.0591	.0357	46
15	1.0416	3.5736	1.0471	3.3722	1.053	3.1932	1.0592	3.0331	45
16	.0417	.57	.0472	.369	.0531	.1904	.0593	.0306	44
17	.0418	.5665	.0473	.3659	.0532	.1876	.0594	.0281	43
18	.0419	.5629	.0474	.3627	.0533	.1848	.0595	.0256	42
19	.042	.5594	.0475	.3596	.0534	.182	.0596	.0231	41
20	1.042	3.5559	1.0476	3.3565	1.0535	3.1792	1.0598	3.0206	40
21	.0421	.5523	.0477	.3534	.0536	.1764	.0599	.0181	39
22	.0422	.5488	.0478	.3502	.0537	.1736	.06	.0156	38
23	.0423	.5453	.0478	.3471	.0538	.1708	.0601	.0131	37
24	.0424	.5418	.0479	.344	.0539	.1681	.0602	.0106	36
25	1.0425	3.5383	1.048	3.3409	1.054	3.1653	1.0603	3.0081	35
26	.0426	.5348	.0481	.3378	.0541	.1625	.0604	.0056	34
27	.0427	.5313	.0482	.3347	.0542	.1598	.0605	0031	33
28	.0428	.5279	.0483	.3316	.0543	.157	.0606	.0007	32
29	.0428	.5244	.0484	.3286	.0544	.1543	.0607	2.9982	31
30	1.0429	3.5209	1.0485	3.3255	1.0545	3.1515	1.0608	2.9957	30
31	.043	.5175	.0486	.3224	.0546	.1488	.0609	.9933	29
32	.0431	.514	.0487	.3194	.0547	.1461	.0611	.9908	28
33	.0432	.5106	.0488	.3163	.0548	.1433	.0612	.9884	27
34	.0433	.5072	.0489	.3133	.0549	.1406	.0613	.9859	26
35	1.0434	3.5037	1.049	3.3102	1.055	3.1379	1.0614	2.9835	25
36	.0435	.5003	.0491	.3072	.0551	.1352	.0615	.981	24
37	.0436	.4969	.0492	.3042	.0552	.1325	.0616	.9786	23
38	.0437	.4935	.0493	.3011	.0553	.1298	.0617	.9762	22
39	.0438	.4901	.0494	.2981	.0554	.1271	.0618	.9738	21
40	1.0438	3.4867	1.0495	3.2951	1.0555	3.1244	1.0619	2.9713	20
41	.0439	.4833	.0496	.2921	.0556	.1217	.062	.9689	19
42	.044	.4799	.0497	.2891	.0557	.119	.0622	.9665	18
43	.0441	.4766	.0498	.2861	.0558	.1163	.0623	.9641	17
44	.0442	.4732	.0499	.2831	.0559	.1137	.0624	.9617	16
45	1.0443	3.4698	1.05	3.2801	1.056	3.111	1.0625	2.9593	15
46	.0444	.4665	.0501	.2772	.0561	.1083	.0626	.9569	14
47	.0445	.4632	.0502	.2742	.0562	.1057	.0627	.9545	13
48	.0446	.4598	.0503	.2712	.0363	.103	.0628	.9521	12
49	.0447	.4565	.0504	.2683	.0565	.1004	.0629	.9497	11
50	1.0448	3.4532	1.0505	3.2653	1.0566	3.0977	1.063	2.9474	10
51	.0448	.4498	.0506	.2624	.0567	.0951	.0632	.945	9
52	.0449	.4465	.0507	.2594	.0568	.0925	.0633	.9426	8
53	.045	.4432	.0508	.2565	.0569	.0898	.0634	.9402	7
54	.0451	.4399	.0509	.2535	.057	.0872	.0635	.9379	6
55	1.0452	3.4366	1.051	3.2506	1.0571	3.0846	1.0636	2.9355	5
56	.0453	.4334	.0511	.2477	.0572	.082	.0637	.9332	4
57	.0454	.4301	.0512	.2448	.0573	.0793	.0638	.9308	3
58	.0455	.4268	.0513	.2419	.0574	.0767	.0639	.9285	2
59	.0456	.4236	.0514	.239	.0575	.0741	.0641	.9261	1
60	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	0
′	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	′
	73°		72°		71°		70°		

′	20° Secant.	20° Co-sec't	21° Secant.	21° Co-sec't.	22° Secant.	22° Co-sec't.	23° Secant.	23° Co-sec't.	′
0	1.0642	2.9238	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	60
1	.0643	.9215	.0713	.7883	.0787	.6675	.0865	.5575	59
2	.0644	.9191	.0714	.7862	.0788	.6656	.0866	.5558	58
3	.0645	.9168	.0715	.7841	.0789	.6637	.0868	.554	57
4	.0646	.9145	.0716	.782	.079	.6618	.0869	.5523	56
5	1.0647	2.9122	1.0717	2.7799	1.0792	2.6599	1.087	2.5506	55
6	.0648	.9098	.0719	.7778	.0793	.658	.0872	.5488	54
7	.065	.9075	.072	.7757	.0794	.6561	.0873	.5471	53
8	.0651	.9052	.0721	.7736	.0795	.6542	.0874	.5453	52
9	.0652	.9029	.0722	.7715	.0797	.6523	.0876	.5436	51
10	1.0653	2.9006	1.0723	2.7694	1.0798	2.6504	1.0877	2.5419	50
11	.0654	.8983	.0725	.7674	.0799	.6485	.0878	.5402	49
12	.0655	.896	.0726	.7653	.0801	.6466	.088	.5384	48
13	.0656	.8937	.0727	.7632	.0802	.6447	.0881	.5367	47
14	.0658	.8915	.0728	.7611	.0803	.6428	.0882	.535	46
15	1.0659	2.8892	1.0729	2.7591	1.0804	2.641	1.0884	2.5333	45
16	.066	.8869	.0731	.757	.0806	.6391	.0885	.5316	44
17	.0661	.8846	.0732	.755	.0807	.6372	.0886	.5299	43
18	.0662	.8824	.0733	.7529	.0808	.6353	.0888	.5281	42
19	.0663	.8801	.0734	.7509	.081	.6335	.0889	.5264	41
20	1.0664	2.8778	1.0736	2.7488	1.0811	2.6316	1.0891	2.5247	40
21	.0666	.8756	.0737	.7468	.0812	.6297	.0892	.523	39
22	.0667	.8733	.0738	.7447	.0813	.6279	.0893	.5213	38
23	.0668	.8711	.0739	.7427	.0815	.626	.0895	.5196	37
24	.0669	.8688	.074	.7406	.0816	.6242	.0896	.5179	36
25	1.067	2.8666	1.0742	2.7386	1.0817	2.6223	1.0897	2.5163	35
26	.0671	.8644	.0743	.7366	.0819	.6205	.0899	.5146	34
27	.0673	.8621	.0744	.7346	.082	.6186	.09	.5129	33
28	.0674	.8599	.0745	.7325	.0821	.6168	.0902	.5112	32
29	.0675	.8577	.0747	.7305	.0823	.615	.0903	.5095	31
30	1.0676	2.8554	1.0748	2.7285	1.0824	2.6131	1.0904	2.5078	30
31	.0677	.8532	.0749	.7265	.0825	.6113	.0906	.5062	29
32	.0678	.851	.075	.7245	.0826	.6095	.0907	.5045	28
33	.0679	.8488	.0751	.7225	.0828	.6076	.0908	.5028	27
34	.0681	.8466	.0753	.7205	.0829	.6058	.091	.5011	26
35	1.0682	2.8444	1.0754	2.7185	1.083	2.604	1.0911	2.4995	25
36	.0683	.8422	.0755	.7165	.0832	.6022	.0913	.4978	24
37	.0684	.84	.0756	.7145	.0833	.6003	.0914	.4961	23
38	.0685	.8378	.0758	.7125	.0834	.5985	.0915	.4945	22
39	.0686	.8356	.0759	.7105	.0836	.5967	.0917	.4928	21
40	1.0688	2.8334	1.076	2.7085	1.0837	2.5949	1.0918	2.4912	20
41	.0689	.8312	.0761	.7065	.0838	.5931	.092	.4895	19
42	.069	.829	.0763	.7045	.084	.5913	.0921	.4879	18
43	.0691	.8269	.0764	.7026	.0841	.5895	.0922	.4862	17
44	.0692	.8247	.0765	.7006	.0842	.5877	.0924	.4846	16
45	1.0694	2.8225	1.0766	2.6986	1.0844	2.5859	1.0925	2.4829	15
46	.0695	.8204	.0768	.6967	.0845	.5841	.0927	.4813	14
47	.0696	.8182	.0769	.6947	.0846	.5823	.0928	.4797	13
48	.0697	.816	.077	.6927	.0847	.5805	.0929	.478	12
49	.0698	.8139	.0771	.6908	.0849	.5787	.0931	.4764	11
50	1.0699	2.8117	1.0773	2.6888	1.085	2.577	1.0932	2.4748	10
51	.0701	.8096	.0774	.6869	.0851	.5752	.0934	.4731	9
52	.0702	.8074	.0775	.6849	.0853	.5734	.0935	.4715	8
53	.0703	.8053	.0776	.683	.0854	.5716	.0936	.4699	7
54	.0704	.8032	.0778	.681	.0855	.5699	.0938	.4683	6
55	1.0705	2.801	1.0779	2.6791	1.0857	2.5681	1.0939	2.4666	5
56	.0707	.7989	.078	.6772	.0858	.5663	.0941	.465	4
57	.0708	.7968	.0781	.6752	.0859	.5646	.0942	.4634	3
58	.0709	.7947	.0783	.6733	.0861	.5628	.0943	.4618	2
59	.071	.7925	.0784	.6714	.0862	.561	.0945	.4602	1
60	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	.0946	2.4586	0
′	Co-sec't. 69°	Secant. 69°	Co-sec't. 68°	Secant. 68°	Co-sec't. 67°	Secant. 67°	Co-sec't. 66°	Secant. 66°	′

	24°		25°		26°		27°		
′	SECANT.	CO-SEC'T.	SECANT.	CO-SEC'T.	SECANT.	CO-SEC'T.	SECANT.	CO-SEC'T.	′
0	1.0946	2.4586	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	60
1	.0948	.457	.1035	.3647	.1127	.2798	.1225	.2014	59
2	.0949	.4554	.1037	.3632	.1129	.2784	.1226	.2002	58
3	.0951	.4538	.1038	.3618	.1131	.2771	.1228	.1989	57
4	.0952	.4522	.104	.3603	.1132	.2757	.123	.1977	56
5	1.0953	2.4506	1.1041	2.3588	1.1134	2.2744	1.1231	2.1964	55
6	.0955	.449	.1043	.3574	.1135	.273	.1233	.1952	54
7	.0956	.4474	.1044	.3559	.1137	.2717	.1235	.1939	53
8	.0958	.4458	.1046	.3544	.1139	.2703	.1237	.1927	52
9	.0959	.4442	.1047	.353	.114	.269	.1238	.1914	51
10	1.0961	2.4426	1.1049	2.3515	1.1142	2.2676	1.124	2.1902	50
11	.0962	.4411	.105	.3501	.1143	.2663	.1242	.1889	49
12	.0963	.4395	.1052	.3486	.1145	.265	.1243	.1877	48
13	.0965	.4379	.1053	.3472	.1147	.2636	.1245	.1865	47
14	.0966	.4363	.1055	.3457	.1148	.2623	.1247	.1852	46
15	1.0968	2.4347	1.1056	2.3443	1.115	2.261	1.1248	2.184	45
16	.0969	.4332	.1058	.3428	.1151	.2596	.125	.1828	44
17	.0971	.4316	.1059	.3414	.1153	.2583	.1252	.1815	43
18	.0972	.43	.1061	.3399	.1155	.257	.1253	.1803	42
19	.0973	.4285	.1062	.3385	.1156	.2556	.1255	.1791	41
20	1.0975	2.4269	1.1064	2.3371	1.1158	2.2543	1.1257	2.1778	40
21	.0976	.4254	.1065	.3356	.1159	.253	.1258	.1766	39
22	.0978	.4238	.1067	.3342	.1161	.2517	.126	.1754	38
23	.0979	.4222	.1068	.3328	.1163	.2503	.1262	.1742	37
24	.0981	.4207	.107	.3313	.1164	.249	.1264	.173	36
25	1.0982	2.4191	1.1072	2.3299	1.1166	2.2477	1.1265	2.1717	35
26	.0984	.4176	.1073	.3285	.1167	.2464	.1267	.1705	34
27	.0985	.416	.1075	.3271	.1169	.2451	.1269	.1693	33
28	.0986	.4145	.1076	.3256	.1171	.2438	.127	.1681	32
29	.0988	.413	.1078	.3242	.1172	.2425	.1272	.1669	31
30	1.0989	2.4114	1.1079	2.3228	1.1174	2.2411	1.1274	2.1657	30
31	.0991	.4099	.1081	.3214	.1176	.2398	.1275	.1645	29
32	.0992	.4083	.1082	.32	.1177	.2385	.1277	.1633	28
33	.0994	.4068	.1084	.3186	.1179	.2372	.1279	.162	27
34	.0995	.4053	.1085	.3172	.118	.2359	.1281	.1608	26
35	1.0997	2.4037	1.1087	2.3158	1.1182	2.2346	1.1282	2.1596	25
36	.0998	.4022	.1088	.3143	.1184	.2333	.1284	.1584	24
37	.1	.4007	.109	.3129	.1185	.232	.1286	.1572	23
38	.1001	.3992	.1092	.3115	.1187	.2307	.1287	.156	22
39	.1003	.3976	.1093	.3101	.1189	.2294	.1289	.1548	21
40	1.1004	2.3961	1.1095	2.3087	1.119	2.2282	1.1291	2.1536	20
41	.1005	.3946	.1096	.3073	.1192	.2269	.1293	.1525	19
42	.1007	.3931	.1098	.3059	.1193	.2256	.1294	.1513	18
43	.1008	.3916	.1099	.3046	.1195	.2243	.1296	.1501	17
44	.101	.3901	.1101	.3032	.1197	.223	.1298	.1489	16
45	1.1011	2.3886	1.1102	2.3018	1.1198	2.2217	1.1299	2.1477	15
46	.1013	.3871	.1104	.3004	.12	.2204	.1301	.1465	14
47	.1014	.3856	.1106	.299	.1202	.2192	.1303	.1453	13
48	.1016	.3841	.1107	.2976	.1203	.2179	.1305	.1441	12
49	.1017	.3826	.1109	.2962	.1205	.2166	.1306	.143	11
50	1.1019	2.3811	1.111	2.2949	1.1207	2.2153	1.1308	2.1418	10
51	.102	.3796	.1112	.2935	.1208	.2141	.131	.1406	9
52	.1022	.3781	.1113	.2921	.121	.2128	.1312	.1394	8
53	.1023	.3766	.1115	.2907	.1212	.2115	.1313	.1382	7
54	.1025	.3751	.1116	.2894	.1213	.2103	.1315	.1371	6
55	1.1026	2.3736	1.1118	2.288	1.1215	2.209	1.1317	2.1359	5
56	.1028	.3721	.112	.2866	.1217	.2077	.1319	.1347	4
57	.1029	.3706	.1121	.2853	.1218	.2065	.132	.1335	3
58	.1031	.3691	.1123	.2839	.122	.2052	.1322	.1324	2
59	.1032	.3677	.1124	.2825	.1222	.2039	.1324	.1312	1
60	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	1.1326	2.13	0
′	CO-SEC'T.	SECANT.	CO-SEC'T.	SECANT.	CO-SEC'T.	SECANT.	CO-SEC'T.	SECANT.	′
	65°		64°		63°		62°		

′	28° Secant.	28° Co-sec't.	29° Secant.	29° Co-sec't.	30° Secant.	30° Co-sec't.	31° Secant.	31° Co-sec't.	′
0	1.1326	2.13	1.1433	2.0627	1.1547	2	1.1666	1.9416	60
1	.1327	.1289	.1435	.0616	.1549	1.999	.1668	9407	59
2	.1329	.1277	.1437	.0605	.1551	.998	.167	.9397	58
3	.1331	.1266	.1439	.0594	.1553	.997	.1672	.9388	57
4	.1333	.1254	.1441	.0583	.1555	.996	.1674	.9378	56
5	1.1334	2.1242	1.1443	2.0573	1.1557	1.995	1.1676	1.9369	55
6	.1336	.1231	.1445	.0562	.1559	.994	.1678	.936	54
7	.1338	.1219	.1446	.0551	.1561	.993	.1681	.935	53
8	.134	.1208	.1448	.054	.1562	.992	.1683	.9341	52
9	.1341	.1196	.145	.053	.1564	.991	.1685	.9332	51
10	1.1343	2.1185	1.1452	2.0519	1.1566	1.99	1.1687	1.9322	50
11	.1345	.1173	.1454	.0508	.1568	.989	.1689	.9313	49
12	.1347	.1162	.1456	.0498	.157	.988	1691	.9304	48
13	.1349	.115	.1458	.0487	.1572	.987	.1693	.9295	47
14	.135	.1139	.1459	.0476	.1574	.986	.1695	.9285	46
15	1.1352	2.1127	1.1461	2.0466	1.1576	1.985	1.1697	1.9276	45
16	.1354	.1116	.1463	.0455	.1578	.984	1699	.9267	44
17	.1356	.1104	.1465	.0444	.158	.983	.1701	.9258	43
18	.1357	.1093	.1467	.0434	.1582	.982	.1703	.9248	42
19	.1359	.1082	.1469	.0423	.1584	.9811	.1705	.9239	41
20	1.1361	2.107	1.1471	2.0413	1.1586	1.9801	1.1707	1.923	40
21	.1363	.1059	.1473	.0402	.1588	.9791	.1709	.9221	39
22	.1365	.1048	.1474	.0392	.159	.9781	.1712	.9212	38
23	.1366	.1036	.1476	.0381	.1592	.9771	.1714	.9203	37
24	.1368	1025	.1478	.037	.1594	.9761	.1716	.9193	36
25	1.137	2.1014	1.148	2.036	1.1596	1.9752	1.1718	1.9184	35
26	.1372	.1002	.1482	.0349	.1598	.9742	.172	.9175	34
27	.1373	.0991	.1484	.0339	.16	.9732	.1722	.9166	33
28	.1375	.098	.1486	.0329	.1602	.9722	.1724	.9157	32
29	.1377	.0969	.1488	.0318	.1604	.9713	.1726	.9148	31
30	1.1379	2.0957	1.1489	2.0308	1.1606	1.9703	1.1728	1.9139	30
31	.1381	.0946	.1491	.0297	.1608	.9693	.173	.913	29
32	.1382	.0935	.1493	.0287	.161	.9683	.1732	.9121	28
33	.1384	.0924	.1495	.0276	.1612	.9674	.1734	.9112	27
34	.1386	.0912	.1497	.0266	.1614	.9664	.1737	.9102	26
35	1.1388	2.0901	1.1499	2.0256	1.1616	1.9654	1.1739	1.9093	25
36	.139	.089	.1501	.0245	.1618	.9645	.1741	.9084	24
37	.1391	.0879	.1503	.0235	.162	.9635	.1743	.9075	23
38	.1393	.0868	.1505	.0224	.1622	.9625	.1745	.9066	22
39	.1395	.0857	.1507	.0214	.1624	.9616	.1747	.9057	21
40	1.1397	2.0846	1.1508	2.0204	1.1626	1.9606	1.1749	1.9048	20
41	.1399	.0835	.151	.0194	.1628	.9596	.1751	.9039	19
42	.1401	.0824	.1512	.0183	.163	.9587	.1753	.903	18
43	.1402	.0812	.1514	.0173	.1632	.9577	.1756	.9021	17
44	.1404	.0801	.1516	.0163	.1634	.9568	.1758	.9013	16
45	1.1406	2.079	1.1518	2.0152	1.1636	1.9558	1.176	1.9004	15
46	.1408	.0779	.152	.0142	.1638	.9549	.1762	.8995	14
47	.141	.0768	.1522	.0132	.164	.9539	.1764	.8986	13
48	.1411	.0757	.1524	.0122	.1642	.953	.1766	.8977	12
49	.1413	.0746	.1526	.0111	.1644	.952	.1768	.8968	11
50	1.1415	2.0735	1.1528	2.0101	1.1646	1.951	1.177	1.8959	10
51	.1417	.0725	.153	.0091	.1648	.9501	.1772	.895	9
52	.1419	.0714	.1531	.0081	.165	.9491	.1775	.8941	8
53	.1421	.0703	.1533	.0071	.1652	.9482	.1777	.8932	7
54	.1422	.0692	.1535	.0061	.1654	.9473	.1779	.8924	6
55	1.1424	2.0681	1.1537	2.005	1.1656	1.9463	1.1781	1.8915	5
56	.1426	.067	.1539	.004	.1658	.9454	.1783	.8906	4
57	.1428	.0659	.1541	.003	.166	.9444	.1785	.8897	3
58	.143	.0648	.1543	.002	.1662	.9435	.1787	.8888	2
59	.1432	.0637	.1545	.001	.1664	.9425	.179	.8879	1
60	1.1433	2.0627	1.1547	2	1.1666	1.9416	1.1792	1.8871	0
′	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	′
	61°		60°		59°		58°		

′	32° Secant.	32° Co-sec't.	33° Secant.	33° Co-sec't.	34° Secant.	34° Co-sec't.	35° Secant.	35° Co-sec't.	′
0	1.1792	1.8871	1.1924	1.8361	1.2062	1.7883	1.2208	1.7434	60
1	.1794	.8862	.1926	.8352	.2064	.7875	.221	.7427	59
2	.1796	.8853	.1928	.8344	.2067	.7867	.2213	.742	58
3	.1798	.8844	.193	.8336	.2069	.786	.2215	.7413	57
4	.18	.8836	.1933	.8328	.2072	.7852	.2218	.7405	56
5	1.1802	1.8827	1.1935	1.832	1.2074	1.7844	1.222	1.7398	55
6	.1805	.8818	.1937	.8311	.2076	.7837	.2223	.7391	54
7	.1807	.8809	.1939	.8303	.2079	.7829	.2225	.7384	53
8	.1809	.8801	.1942	.8295	.2081	.7821	.2228	.7377	52
9	.1811	.8792	.1944	.8287	.2083	.7814	.223	.7369	51
10	1.1813	1.8783	1.1946	1.8279	1.2086	1.7806	1.2233	1.7362	50
11	.1815	.8785	.1948	.8271	.2088	.7798	.2235	.7355	49
12	.1818	.8766	.1951	.8263	.2091	.7791	.2238	.7348	48
13	.182	.8757	.1953	.8255	.2093	.7783	.224	.7341	47
14	.1822	.8749	.1955	.8246	.2095	.7776	.2243	.7334	46
15	1.1824	1.874	1.1958	1.8238	1.2098	1.7768	1.2245	1.7327	45
16	.1826	.8731	.196	.823	.21	.776	.2248	.7319	44
17	.1828	.8723	.1962	.8222	.2103	.7753	.225	.7312	43
18	.1831	.8714	.1964	.8214	.2105	.7745	.2253	.7305	42
19	.1833	.8706	.1967	.8206	.2107	.7738	.2255	.7298	41
20	1.1835	1.8697	1.1969	1.8198	1.211	1.773	1.2258	1.7291	40
21	.1837	.8688	.1971	.819	.2112	.7723	.226	.7284	39
22	.1839	.868	.1974	.8182	.2115	.7715	.2263	.7277	38
23	.1841	.8671	.1976	.8174	.2117	.7708	.2265	.727	37
24	.1844	.8663	.1978	.8166	.2119	.77	.2268	.7263	36
25	1.1846	1.8654	1.198	1.8158	1.2122	1.7693	1.227	1.7256	35
26	.1848	.8646	.1983	.815	.2124	.7685	.2273	.7249	34
27	.185	.8637	.1985	.8142	.2127	.7678	.2276	.7242	33
28	.1852	.8629	.1987	.8134	.2129	.767	.2278	.7234	32
29	.1855	.862	.199	.8126	.2132	.7663	.2281	.7227	31
30	1.1857	1.8611	1.1992	1.8118	1.2134	1.7655	1.2283	1.722	30
31	.1859	.8603	.1994	.811	.2136	.7648	.2286	.7213	29
32	.1861	.8595	.1997	.8102	.2139	.764	.2288	.7206	28
33	.1863	.8586	.1999	.8094	.2141	.7633	.2291	.7199	27
34	.1866	.8578	.2001	.8086	.2144	.7625	.2293	.7192	26
35	1.1868	1.8569	1.2004	1.8078	1.2146	1.7618	1.2296	1.7185	25
36	.187	.8561	.2006	.807	.2149	.761	.2298	.7178	24
37	.1872	.8552	.2008	.8062	.2151	.7603	.2301	.7171	23
38	.1874	.8544	.201	.8054	.2153	.7596	.2304	.7164	22
39	.1877	.8535	.2013	.8047	.2156	.7588	.2306	.7157	21
40	1.1879	1.8527	1.2015	1.8039	1.2158	1.7581	1.2309	1.7151	20
41	.1881	.8519	.2017	.8031	.2161	.7573	.2311	.7144	19
42	.1883	.851	.202	.8023	.2163	.7566	.2314	.7137	18
43	.1886	.8502	.2022	.8015	.2166	.7559	.2316	.713	17
44	.1888	.8493	.2024	.8007	.2168	.7551	.2319	.7123	16
45	1.189	1.8485	1.2027	1.7999	1.2171	1.7544	1.2322	1.7116	15
46	.1892	.8477	.2029	.7992	.2173	.7537	.2324	.7109	14
47	.1894	.8468	.2031	.7984	.2175	.7529	.2327	.7102	13
48	.1897	.846	.2034	.7976	.2178	.7522	.2329	.7095	12
49	.1899	.8452	.2036	.7968	.218	.7514	.2332	.7088	11
50	1.1901	1.8443	1.2039	1.796	1.2183	1.7507	1.2335	1.7081	10
51	.1903	.8435	.2041	.7953	.2185	.75	.2337	.7075	9
52	.1906	.8427	.2043	.7945	.2188	.7493	.234	.7068	8
53	.1908	.8418	.2046	.7937	.219	.7485	.2342	.7061	7
54	.191	.841	.2048	.7929	.2193	.7478	.2345	.7054	6
55	1.1912	1.8402	1.205	1.7921	1.2195	1.7471	1.2348	1.7047	5
56	.1915	.8394	.2053	.7914	.2198	.7463	.235	.704	4
57	.1917	.8385	.2055	.7906	.22	.7456	.2353	.7033	3
58	.1919	.8377	.2057	.7898	.2203	.7449	.2355	.7027	2
59	.1921	.8369	.206	.7891	.2205	.7442	.2358	.702	1
60	1.1922	1.8361	1.2062	1.7883	1.2208	1.7434	1.2361	1.7013	0
′	57° Co-sec't.	57° Secant.	56° Co-sec't.	56° Secant.	55° Co-sec't.	55° Secant.	54° Co-sec't.	54° Secant.	′

′	36° Secant.	36° Co-sec't.	37° Secant.	37° Co-sec't.	38° Secant.	38° Co-sec't.	39° Secant.	39° Co-sec't.	′
0	1.2361	1.7013	1.2521	1.6616	1.269	1.6243	1.2867	1.589	60
1	.2363	.7006	.2524	.661	.2693	.6237	.2871	.5884	59
2	.2366	.6999	.2527	.6603	.2696	.6231	.2874	.5879	58
3	.2368	.6993	.253	.6597	.2699	.6224	.2877	.5873	57
4	.2371	.6986	.2532	.6591	.2702	.6218	.288	.5867	56
5	1.2374	1.6979	1.2535	1.6584	1.2705	1.6212	1.2883	1.5862	55
6	.2376	.6972	.2538	.6578	.2707	.6206	.2886	.5856	54
7	.2379	.6965	.2541	.6572	.271	.62	.2889	.585	53
8	.2382	.6959	.2543	.6565	.2713	.6194	.2892	.5845	52
9	.2384	.6952	.2546	.6559	.2716	.6188	.2895	.5839	51
10	1.2387	1.6945	1.2549	1.6552	1.2719	1.6182	1.2898	1.5833	50
11	.2389	.6938	.2552	.6546	.2722	.6176	.2901	.5828	49
12	.2392	.6932	.2554	.654	.2725	.617	.2904	.5822	48
13	.2395	.6925	.2557	.6533	.2728	.6164	.2907	.5816	47
14	.2397	.6918	.256	.6527	.2731	.6159	.291	.5811	46
15	1.24	1.6912	1.2563	1.6521	1.2734	1.6153	1.2913	1.5805	45
16	.2403	.6905	.2565	.6514	.2737	.6147	.2916	.5799	44
17	.2405	.6898	.2568	.6508	.2739	.6141	.2919	.5794	43
18	.2408	.6891	.2571	.6502	.2742	.6135	.2922	.5788	42
19	.2411	.6885	.2574	.6496	.2745	.6129	.2926	.5783	41
20	1.2413	1.6878	1.2577	1.6489	1.2748	1.6123	1.2929	1.5777	40
21	.2416	.6871	.2579	.6483	.2751	.6117	.2932	.5771	39
22	.2419	.6865	.2582	.6477	.2754	.6111	.2935	.5766	38
23	.2421	.6858	.2585	.647	.2757	.6105	.2938	.576	37
24	.2424	.6851	.2588	.6464	.276	.6099	.2941	.5755	36
25	1.2427	1.6845	1.2591	1.6458	1.2763	1.6093	1.2944	1.5749	35
26	.2429	.6838	.2593	.6452	.2766	.6087	.2947	.5743	34
27	.2432	.6831	.2596	.6445	.2769	.6081	.295	.5738	33
28	.2435	.6825	.2599	.6439	.2772	.6077	.2953	.5732	32
29	.2437	.6818	.2602	.6433	.2775	.607	.2956	.5727	31
30	1.244	1.6812	1.2605	1.6427	1.2778	1.6064	1.296	1.5721	30
31	.2443	.6805	.2607	.642	.2781	.6058	.2963	.5716	29
32	.2445	.6798	.261	.6414	.2784	.6052	.2966	.571	28
33	.2448	.6792	.2613	.6408	.2787	.6046	.2969	.5705	27
34	.2451	.6785	.2616	.6402	.279	.604	.2972	.5699	26
35	1.2453	1.6779	1.2619	1.6396	1.2793	1.6034	1.2975	1.5694	25
36	.2456	.6772	.2622	.6389	.2795	.6029	.2978	.5688	24
37	.2459	.6766	.2624	.6383	.2798	.6023	.2981	.5683	23
38	.2461	.6759	.2627	.6377	.2801	.6017	.2985	.5677	22
39	.2464	.6752	.263	.6371	.2804	.6011	.2988	.5672	21
40	1.2467	1.6746	1.2633	1.6365	1.2807	1.6005	1.2991	1.5666	20
41	.247	.6739	.2636	.6359	.281	.6	.2994	.5661	19
42	.2472	.6733	.2639	.6352	.2813	.5994	.2997	.5655	18
43	.2475	.6726	.2641	.6346	.2816	.5988	.3	.565	17
44	.2478	.672	.2644	.634	.2819	.5982	.3003	.5644	16
45	1.248	1.6713	1.2647	1.6334	1.2822	1.5976	1.3006	1.5639	15
46	.2483	.6707	.265	.6328	.2825	.5971	.301	.5633	14
47	.2486	.67	.2653	.6322	.2828	.5965	.3013	.5628	13
48	.2488	.6694	.2656	.6316	.2831	.5959	.3016	.5622	12
49	.249	.6687	.2659	.6309	.2834	.5953	.3019	.5617	11
50	1.2494	1.6681	1.2661	1.6303	1.2837	1.5947	1.3022	1.5611	10
51	.2497	.6674	.2664	.6297	.284	.5942	.3025	.5606	9
52	.2499	.6668	.2667	.6291	.2843	.5936	.3029	.56	8
53	.2502	.6661	.267	.6285	.2846	.593	.3032	.5595	7
54	.2505	.6655	.2673	.6279	.2849	.5924	.3035	.559	6
55	1.2508	1.6648	1.2676	1.6273	1.2852	1.5919	1.3038	1.5584	5
56	.251	.6642	.2679	.6267	.2855	.5913	.3041	.5579	4
57	.2513	.6636	.2681	.6261	.2858	.5907	.3044	.5573	3
58	.2516	.6629	.2684	.6255	.2861	.5901	.3048	.5568	2
59	.2519	.6623	.2687	.6249	.2864	.5896	.3051	.5563	1
60	1.2521	1.6616	1.269	1.6243	1.2867	1.589	1.3054	1.5557	0
′	53° Co-sec't.	53° Secant.	52° Co-sec't.	52° Secant.	51° Co-sec't.	51° Secant.	50° Co-sec't.	50° Secant.	′

	40°		41°		42°		43°		
′	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	′
0	1.3054	1.5557	1.325	1.5242	1.3456	1.4945	1.3673	1.4663	60
1	.3057	.5552	.3253	.5237	.346	.494	.3677	.4658	59
2	.306	.5546	.3257	.5232	.3463	.4935	.3681	.4654	58
3	.3064	.5541	.326	.5227	.3467	.493	.3684	.4649	57
4	.3067	.5536	.3263	.5222	.347	.4925	.3688	.4644	56
5	1.307	1.553	1.3267	1.5217	1.3474	1.4921	1.3692	1.464	55
6	.3073	.5525	.327	.5212	.3477	.4916	.3695	.4635	54
7	.3076	.552	.3274	.5207	.3481	.4911	.3699	.4631	53
8	.308	.5514	.3277	.5202	.3485	.4906	.3703	.4626	52
9	.3083	.5509	.328	.5197	.3488	.4901	.3707	.4622	51
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.371	1.4617	50
11	.3089	.5498	.3287	.5187	.3495	.4892	.3714	.4613	49
12	.3092	.5493	.329	.5182	.3499	.4887	.3718	.4608	48
13	.3096	.5487	.3294	.5177	.3502	.4882	.3722	.4604	47
14	.3099	.5482	.3297	.5171	.3506	.4877	.3725	.4599	46
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	45
16	.3105	.5471	.3304	.5161	.3513	.4868	.3733	.459	44
17	.3109	.5466	.3307	.5156	.3517	.4863	.3737	.4586	43
18	.3112	.5461	.3311	.5151	.352	.4858	.374	.4581	42
19	.3115	.5456	.3314	.5146	.3524	.4854	.3744	.4577	41
20	1.3118	1.545	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	40
21	.3121	.5445	.3321	.5136	.3531	.4844	.3752	.4568	39
22	.3125	.544	.3324	.5131	.3534	.4839	.3756	.4563	38
23	.3128	.5434	.3328	.5126	.3538	.4835	.3759	.4559	37
24	.3131	.5429	.3331	.5121	.3542	.483	.3763	.4554	36
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.455	35
26	.3138	.5419	.3338	.5111	.3549	.4821	.3771	.4545	34
27	.3141	.5413	.3342	.5106	.3552	.4816	.3774	.4541	33
28	.3144	.5408	.3345	.5101	.3556	.4811	.3778	.4536	32
29	.3148	.5403	.3348	.5096	.356	.4806	.3782	.4532	31
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	30
31	.3154	.5392	.3355	.5087	.3567	.4797	.379	.4523	29
32	.3157	.5387	.3359	.5082	.3571	.4792	.3794	.4518	28
33	.3161	.5382	.3362	.5077	.3574	.4788	.3797	.4514	27
34	.3164	.5377	.3366	.5072	.3578	.4783	.3801	.451	26
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	25
36	.317	.5366	.3372	.5062	.3585	.4774	.3809	.4501	24
37	.3174	.5361	.3376	.5057	.3589	.4769	.3813	.4496	23
38	.3177	.5356	.3379	.5052	.3592	.4764	.3816	.4492	22
39	.318	.5351	.3383	.5047	.3596	.476	.382	.4487	21
40	1.3184	1.5345	1.3386	1.5042	1.36	1.4755	1.3824	1.4483	20
41	.3187	.534	.339	.5037	.3603	.475	.3828	.4479	19
42	.319	.5335	.3393	.5032	.3607	.4746	.3832	.4474	18
43	.3193	.533	.3397	.5027	.3611	.4741	.3836	.447	17
44	.3197	.5325	.34	.5022	.3614	.4736	.3839	.4465	16
45	1.32	1.5319	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	15
46	.3203	.5314	.3407	.5013	.3622	.4727	.3847	.4457	14
47	.3207	.5309	.3411	.5008	.3625	.4723	.3851	.4452	13
48	.321	.5304	.3414	.5003	.3629	.4718	.3855	.4448	12
49	.3213	.5299	.3418	.4998	.3633	.4713	.3859	.4443	11
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	10
51	.322	.5289	.3425	.4988	.364	.4704	.3867	.4435	9
52	.3223	.5283	.3428	.4983	.3644	.4699	.387	.443	8
53	.3227	.5278	.3432	.4979	.3647	.4695	.3874	.4426	7
54	.323	.5273	.3435	.4974	.3651	.469	.3878	.4422	6
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	5
56	.3237	.5263	.3442	.4964	.3658	.4681	.3886	.4413	4
57	.324	.5258	.3446	.4959	.3662	.4676	.389	.4408	3
58	.3243	.5253	.3449	.4954	.3666	.4672	.3894	.4404	2
59	.3247	.5248	.3453	.4949	.3669	.3667	.3898	.44	1
60	1.325	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	0
′	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	Co-sec't.	Secant.	′
	49°		48°		47°		46°		

	44°				44°				44°		
′	Secant.	Co-sec't.	′	′	Secant.	Co-sec't.	′	′	Secant.	Co-sec't	′
0	1.3902	1.4395	60	21	1.3984	1.4305	39	41	1.4065	1.4221	19
1	.3905	.4391	59	22	.3988	.4301	38	42	.4069	.4217	18
2	.3909	.4387	58	23	.3992	.4297	37	43	.4073	.4212	17
3	.3913	.4382	57	24	.3996	.4292	36	44	.4077	.4208	16
4	.3917	.4378	56	25	1.4	1.4288	35	45	1.4081	1.4204	15
5	1.3921	1.4374	55	26	.4004	.4284	34	46	.4085	.42	14
6	.3925	.437	54	27	.4008	.428	33	47	.4089	.4196	13
7	3929	.4365	53	28	.4012	.4276	32	48	.4093	.4192	12
8	.3933	.4361	52	29	.4016	.4271	31	49	.4097	.4188	11
9	.3937	.4357	51	30	1.402	1.4267	30	50	1.4101	1.4183	10
10	1.3941	1.4352	50	31	.4024	.4263	29	51	.4105	.4179	9
11	.3945	.4348	49	32	.4028	.4259	28	52	.4109	.4175	8
12	.3949	.4344	48	33	.4032	.4254	27	53	.4113	.4171	7
13	.3953	.4339	47	34	.4036	.425	26	54	.4117	.4167	6
14	.3957	.4335	46	35	1.404	1.4246	25	55	1.4122	1.4163	5
15	1.396	1.4331	45	36	.4044	.4242	24	56	.4126	.4159	4
16	.3964	.4327	44	37	.4048	.4238	23	57	.413	.4154	3
17	.3968	.4322	43	38	.4052	.4333	22	58	.4134	.415	2
18	.3972	.4318	42	39	.4056	.4229	21	59	.4138	.4146	1
19	.3976	.4314	41	40	1.406	1.4225	20	60	1.4142	1.4142	0
20	1.398	1.431	40								
′	Co-sec't.	Secant.	′	′	Co-sec't.	Secant.	′	′	Co-sec't.	Secant.	′
	45°				45°				45°		

Preceding Table contains Natural Secants and Co-secants for every minute of the Quadrant to Radius 1.

If Degrees are taken at head of column, Minutes, Secant, and Co-secant must be taken from head also; and if they are taken at foot of column, Minutes, etc., must be taken from foot also.

Illustration.—1.05 is secant of 17° 45′ and co-secant of 72° 15′.

To Compute Secant or Co-secant of any Angle.

Rule.—Divide 1 by Cosine of angle for Secant, and by Sine for Co-secant.

Example 1.—What is secant of 25° 25′?

Cosine of angle = .903 21. Then 1 ÷ .903 21 = 1.1072, *Secant.*

2.—What is co-secant of 64° 35′?

Sine of angle = .903 21. Then 1 ÷ .903 21 = 1.1072, *Co-secant.*

To Compute Degrees, Minutes, and Seconds of a Secant or Co-secant.

When Secant is given,

Proceed as by Rule, page 402, for Sines, substituting Secants for Sines.

Example.—What is secant for 1.1607?

The next less secant is 1.1606, *arc for which* = 30° 30′.

Next greater secant is 1.1608, *difference between which and next less is* 1.1608 − 1.1606 = .0002.

Difference between less tab. secant and one given is 1.1607 − 1.1606 = .0001.

Then .0002 : .0001 :: 60 : 30, which, added to 30° 30′ = 30° 30′ 30″.

When Co-secant is given,

Proceed as by Rule, page 402, substituting Co-secants for Cosines.

Natural Tangents and Co-tangents.

′	0° Tang.	0° Co-tang.	1° Tang.	1° Co-tang.	2° Tang.	2° Co-tang.	3° Tang.	3° Co-tang.	′
0	.000 00	Infinite.	.017 46	57.29	.034 92	28.6363	.052 41	19.0811	60
1	.000 29	3437.75	.017 75	6.3506	.035 21	8.3994	.052 7	8.9755	59
2	.000 58	1718.87	.018 04	5.4415	.035 5	8.1664	.052 99	8.8711	58
3	.000 87	145.92	.018 33	4.5613	.035 79	7.9372	.053 28	8.7678	57
4	.001 16	859.436	.018 62	3.7086	.036 09	7.7117	.053 57	8.6656	56
5	.001 45	687.549	.018 91	52.8821	.036 38	27.4899	.053 87	18.5645	55
6	.001 75	572.957	.019 2	2.0807	.036 67	7.2715	.054 16	8.4645	54
7	.002 04	491.106	.019 49	1.3032	.036 96	7.0566	.054 45	8.3655	53
8	.002 33	29.718	.019 78	0.5485	.037 25	6.845	.054 74	8.2677	52
9	.002 62	381.971	.020 07	49.8157	.037 54	6.6367	.055 03	8.1708	51
10	.002 91	343.774	.020 36	49.1039	.037 83	26.4316	.055 33	18.075	50
11	.003 2	12.521	.020 66	8.4121	.038 12	6.2296	.055 62	7.9802	49
12	.003 49	286.478	.020 95	7.7395	.038 42	6.0307	.055 91	7.8863	48
13	.003 78	64.441	.021 24	7.0853	.038 71	5.8348	.056 2	7.7934	47
14	.004 07	45.552	.021 53	6.4489	.039	5.6418	.056 49	7.7015	46
15	.004 36	229.182	.021 82	45.8294	.039 29	25.4517	.056 78	17.6106	45
16	.004 65	14.858	.022 11	5.2261	.039 58	5.2644	.057 08	7.5205	44
17	.004 95	92.219	.022 4	4.6386	.039 87	5.0798	.057 37	7.4314	43
18	.005 24	190.984	.022 69	4.0661	.040 16	4.8978	.057 66	7.3432	42
19	.005 53	80.932	.022 98	3.5081	.040 46	4.7185	.057 95	7.2558	41
20	.005 82	171.885	.023 28	42.9641	.040 75	24.5418	.058 24	17.1693	40
21	.006 11	63.7	.023 57	2.4335	.041 04	4.3675	.058 54	7.0837	39
22	.006 4	56.259	.023 86	1.9158	.041 33	4.1957	.058 83	6.999	38
23	.006 69	49.465	.024 15	1.4106	.041 62	4.0263	.059 12	6.915	37
24	.006 98	43.237	.024 44	0.9174	.041 91	3.8593	.059 41	6.8319	36
25	.007 27	137.507	.024 73	40.4358	.042 2	23.6945	.059 7	16.7496	35
26	.007 56	32.219	.025 02	39.9655	.042 5	3.5321	.059 99	6.6681	34
27	.007 85	27.321	.025 31	9.5059	.042 79	3.3718	.060 29	6.5874	33
28	.008 14	22.774	.025 6	9.0568	.043 08	3.2137	.060 58	6.5075	32
29	.008 44	18.54	.025 89	8.6177	.043 37	3.0577	.060 87	6.4283	31
30	.008 73	114.589	.026 19	38.1885	.043 66	22.9038	.061 16	16.3499	30
31	.009 02	10.892	.026 48	7.7686	.043 95	2.7519	.061 45	6.2722	29
32	.009 31	07.426	.026 77	7.3579	.044 24	2.602	.061 75	6.1952	28
33	.009 6	04.171	.027 06	6.956	.044 54	2.4541	.062 04	6.119	27
34	.009 89	01.107	.027 35	6.5627	.044 83	2.3081	.062 33	6.0435	26
35	.010 18	98.2179	.027 64	36.1776	.045 12	22.164	.062 62	15.9687	25
36	.010 47	5.4895	.027 93	5.8006	.045 41	2.0217	.062 91	5.8945	24
37	.010 76	2.9085	.028 22	5.4313	.045 7	1.8813	.063 21	5.8211	23
38	.011 05	0.4633	.028 51	5.0695	.045 99	1.7426	.063 5	5.7483	22
39	.011 35	88.1436	.028 81	4.7151	.046 28	1.6056	.063 79	5.6762	21
40	.011 64	85.9398	.029 1	34.3678	.046 58	21.4704	.064 08	15.6048	20
41	.011 93	3.8435	.029 39	4.0273	.046 87	1.3369	.064 37	5.534	19
42	.012 22	1.847	.029 68	3.6935	.047 16	1.2049	.064 67	5.4638	18
43	.012 51	79.9434	.029 97	3.3662	.047 45	1.0747	.064 96	5.3943	17
44	.012 8	8.1263	.030 26	3.0452	.047 74	0.946	.065 25	5.3254	16
45	.013 09	76.39	.030 55	32.7303	.048 03	20.8188	.065 54	15.2571	15
46	.013 38	4.7292	.030 84	2.4213	.048 32	0.6932	.065 84	5.1893	14
47	.013 67	3.139	.031 14	2.1181	.048 62	0.5691	.066 13	5.1222	13
48	.013 96	1.6151	.031 43	1.8205	.048 91	0.4465	.066 42	5.0557	12
49	.014 25	0.1533	.031 72	1.5284	.049 2	0.3253	.066 71	4.9898	11
50	.014 55	68.7501	.032 01	31.2416	.049 49	20.2056	.067	14.9244	10
51	.014 84	7.4019	.032 3	0.9599	.049 78	0.0872	.067 3	4.8596	9
52	.015 13	6.1055	.032 59	0.6833	.050 07	19.9702	.067 59	4.7954	8
53	.015 42	4.858	.032 88	0.4116	.050 37	9.8546	.067 88	4.7317	7
54	.015 71	3.6567	.033 17	0.1446	.050 66	9.7403	.068 17	4.6685	6
55	.016	62.4992	.033 46	29.8823	.050 95	19.6273	.068 47	14.6059	5
56	.016 29	1.3829	.033 76	9.6245	.051 24	9.5156	.068 76	4.5438	4
57	.016 58	0.3058	.034 05	9.3711	.051 53	9.4051	.069 05	4.4823	3
58	.016 87	59.2659	.034 34	9.122	.051 82	9.2959	.069 34	4.4212	2
59	.017 16	8.2612	.034 63	8.8771	.052 12	9.1879	.069 63	4.3607	1
60	.017 46	57.29	.034 92	28.6363	.052 41	19.0811	.069 93	14.3007	0
′	Co-tang. 89°	Tang. 89°	Co-tang. 88°	Tang. 88°	Co-tang. 87°	Tang. 87°	Co-tang. 86°	Tang. 86°	′

	4°		5°		6°		7°		
′	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	′
0	.069 93	14.3007	.087 49	11.430 1	.105 1	9.514 36	.122 78	8.144 35	60
1	.070 22	4.2411	.087 78	1.391 9	.105 4	.487 81	.123 08	.124 81	59
2	.070 51	4.1821	.088 07	1.354	.105 69	.461 41	.123 38	.105 36	58
3	.070 8	4.1235	.088 37	1.316 3	.105 99	.435 15	.123 67	.086	57
4	.071 1	4.0655	.088 66	1.278 9	.106 28	.409 04	.123 97	.066 74	56
5	.071 39	14.0079	.088 95	11.241 7	.106 57	9.383 07	.124 26	8.047 56	55
6	.071 68	3.9507	.089 25	1.204 8	.106 87	.357 24	.124 56	.028 48	54
7	.071 97	3.894	.089 54	1.168 1	.107 16	.331 54	.124 85	8.009 48	53
8	.072 27	3.8378	.089 83	1.131 6	.107 46	.305 99	.125 15	7.990 58	52
9	.072 56	3.7821	.090 13	1.095 4	.107 75	.280 58	.125 44	.971 76	51
10	.072 85	13.7267	.090 42	11.059 4	.108 05	9.255 3	.125 74	7.953 02	50
11	.073 14	3.6719	.090 71	1.023 7	.108 34	.230 16	.126 03	.934 38	49
12	.073 44	3.6174	.091 01	0.988 2	.108 63	.205 16	.126 33	.915 82	48
13	.073 73	3.5634	.091 3	0.952 9	.108 93	.180 28	.126 62	.897 34	47
14	.074 02	3.5098	.091 59	0.917 8	.109 22	.155 54	.126 92	.878 95	46
15	.074 31	13.4566	.091 89	10.882 9	.109 52	9.130 93	.127 22	7.860 64	45
16	.074 61	3.4039	.092 18	0.848 3	.109 81	.106 46	.127 51	.842 42	44
17	.074 9	3.3515	.092 47	0.813 9	.110 11	.082 11	.127 81	.824 28	43
18	.075 19	3.2996	.092 77	0.779 7	.110 4	.057 89	.128 1	.806 22	42
19	.075 48	3.248	.093 06	0.745 7	.110 7	.033 79	.128 4	.788 25	41
20	.075 78	13.1969	.093 35	10.711 9	.110 99	9.009 83	.128 69	7.770 35	40
21	.076 07	3.1461	.093 65	0.678 3	.111 28	8.985 98	.128 99	.752 54	39
22	.076 36	3.0958	.093 94	0.645	.111 58	.962 27	.129 29	.734 8	38
23	.076 65	3.0458	.094 23	0.611 8	.111 87	.938 67	.129 58	.717 15	37
24	.076 95	2.9962	.094 53	0.578 9	.112 17	.915 2	.129 88	.699 57	36
25	.077 24	12.9469	.094 82	10.546 2	.112 46	8.891 85	.130 17	7.682 08	35
26	.077 53	2.8981	.095 11	0.513 6	.112 76	.868 62	.130 47	.664 66	34
27	.077 82	2.8496	.095 41	0.481 3	.113 05	.845 51	.130 76	.647 32	33
28	.078 12	2.8014	.095 7	0.449 1	.113 35	.822 52	.131 06	.630 05	32
29	.078 41	2.7536	.096	0.417 2	.113 64	.799 64	.131 36	.612 87	31
30	.078 7	12.7062	.096 29	10.385 4	.113 94	8.776 89	.131 65	7.595 75	30
31	.078 99	2.6591	.096 58	0.353 8	.114 23	.754 25	.131 95	.578 72	29
32	.079 29	2.6124	.096 88	0.322 4	.114 52	.731 72	.132 24	.561 76	28
33	.079 58	2.566	.097 17	0.291 3	.114 82	.709 31	.132 54	.544 87	27
34	.079 87	2.5199	.097 46	0.260 2	.115 11	.687 01	.132 84	.528 06	26
35	.080 17	12.4742	.097 76	10.229 4	.115 41	8.664 82	.133 13	7.511 32	25
36	.080 46	2.4288	.098 05	0.198 8	.115 7	.642 75	.133 43	.494 65	24
37	.080 75	2.3838	.098 34	0.168 3	.116	.620 78	.133 72	.478 06	23
38	.081 04	2.339	.098 64	0.138 1	.116 29	.598 93	.134 02	.461 54	22
39	.081 34	2.2946	.098 93	0.108	.116 59	.577 18	.134 32	.445 09	21
40	.081 63	12.2505	.099 23	10.078	.116 88	8.555 55	.134 61	7.428 71	20
41	.081 92	2.2067	.099 52	0.048 3	.117 18	.534 02	.134 91	.412 4	19
42	.082 21	2.1632	.099 81	0.018 7	.117 47	.512 59	.135 21	.396 16	18
43	.082 51	2.1201	.100 11	9.989 3	.117 77	.491 28	.135 5	.379 99	17
44	.082 8	2.0772	.100 4	.960 07	.118 06	.470 07	.135 8	.363 89	16
45	.083 09	12.0346	.100 69	9.931 01	.118 36	8.448 96	.136 09	7.347 86	15
46	.083 39	1.9923	.100 99	.902 11	.118 65	.427 95	.136 39	.331 9	14
47	.083 68	1.9504	.101 28	.873 38	.118 95	.407 05	.136 69	.316	13
48	.083 97	1.9087	.101 58	.844 82	.119 24	.386 25	.136 98	.300 18	12
49	.084 27	1.8673	.101 87	.816 41	.119 54	.365 55	.137 28	.284 42	11
50	.084 56	11.8262	.102 16	9.788 17	.119 83	8.344 96	.137 58	7.268 73	10
51	.084 85	1.7853	.102 46	.760 09	.120 13	.324 46	.137 87	.253 1	9
52	.085 14	1.7448	.102 75	.732 17	.120 42	.304 06	.138 17	.237 54	8
53	.085 44	1.7045	.103 05	.704 41	.120 72	.283 76	.138 46	.222 04	7
54	.085 73	1.6645	.103 34	.676 8	.121 01	.263 55	.138 76	.206 61	6
55	.086 02	11.6248	.103 63	9.649 35	.121 31	8.243 45	.139 06	7.191 25	5
56	.086 32	1.5853	.103 93	.622 05	.121 6	.223 44	.139 35	.175 94	4
57	.086 61	1.5461	.104 22	.594 9	.121 9	.203 52	.139 65	.160 71	3
58	.086 9	1.5072	.104 52	.567 91	.122 19	.183 7	.139 95	.145 53	2
59	.087 2	1.4685	.104 81	.541 06	.122 49	.163 98	.140 24	.130 42	1
60	.087 49	11.4301	.105 1	9.514 36	.122 78	8.144 35	.140 54	7.115 37	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	85°		84°		83°		82°		

′	8° Tang.	8° Co-tang.	9° Tang.	9° Co-tang.	10° Tang.	10° Co-tang.	11° Tang.	11° Co-tang.	′
0	.140 54	7.115 37	.158 38	6.313 75	.176 33	5.671 28	.194 38	5.144 55	60
1	.140 84	.100 38	.158 68	.301 89	.176 63	.661 65	.194 68	.136 58	59
2	.141 13	.085 46	.158 98	.290 07	.176 93	.652 05	.194 98	.128 62	58
3	.141 43	.070 59	.159 28	.278 29	.177 23	.642 48	.195 29	.120 69	57
4	.141 73	.055 79	.159 58	.266 55	.177 53	.632 95	.195 59	.112 79	56
5	.142 02	7.041 05	.159 88	6.254 86	.177 83	5.623 44	.195 89	5.104 9	55
6	.142 32	.026 37	.160 17	.243 21	.178 13	.613 97	.196 19	.097 04	54
7	.142 62	.011 74	.160 47	.231 6	.178 43	.604 52	.196 49	.089 21	53
8	.142 91	6.997 18	.160 77	.220 03	.178 73	.595 11	.196 8	.081 39	52
9	.143 21	.982 68	.161 07	.208 51	.179 03	.585 73	.197 1	.073 6	51
10	143 51	6.968 23	.161 37	6.197 03	.179 33	5.576 38	.197 4	5.065 84	50
11	.143 81	.953 85	.161 67	.185 59	.179 63	.567 06	.197 7	.058 09	49
12	.144 1	.939 52	.161 96	.174 19	.179 93	.557 77	.198 01	.050 37	48
13	.144 4	.925 25	.162 26	.162 83	.180 23	.548 51	.198 31	.042 67	47
14	144 7	.911 04	.162 56	.151 51	.180 53	.539 27	.198 61	.034 99	46
15	.144 99	6.896 88	.162 86	6.140 23	.180 83	5.530 07	.198 91	5.027 34	45
16	.145 29	.882 78	.163 16	.128 99	.181 13	.520 9	.199 21	.019 71	44
17	.145 59	.868 74	.163 46	.117 79	.181 43	.511 76	.199 52	.012 1	43
18	.145 88	.854 75	.163 76	.106 64	.181 73	.502 64	.199 82	.004 51	42
19	.146 18	.840 82	.164 05	.095 52	.182 03	.493 56	.200 12	4.996 95	41
20	.146 48	6.826 94	.164 35	6.084 44	.182 33	5.484 51	.200 42	4.989 4	40
21	.146 78	.813 12	.164 65	.073 4	.182 63	.475 48	.200 73	.981 88	39
22	.147 07	.799 36	.164 95	.062 4	.182 93	.466 48	.201 03	.974 38	38
23	.147 37	.785 64	.165 25	.051 43	.183 23	.457 51	.201 33	.966 9	37
24	.147 67	.771 99	.165 55	.040 51	.183 53	.448 57	.201 64	.959 45	36
25	.147 96	6.758 38	.165 85	6.029 62	.183 83	5.439 66	.201 94	4.952 01	35
26	.148 26	.744 83	.166 15	.018 78	.184 14	.430 77	.202 24	.944 6	34
27	.148 56	.731 33	.166 45	.007 97	.184 44	.421 92	.202 54	.937 21	33
28	.148 86	.717 89	.166 74	5.997 2	.184 74	.413 09	.202 85	.929 84	32
29	.149 15	.704 5	.167 04	.986 46	.185 04	.404 29	.203 15	.922 49	31
30	.149 45	6.691 16	.167 34	5.975 76	.185 34	5.395 52	.203 45	4.915 16	30
31	.149 75	.677 87	.167 64	.965 1	.185 64	.386 77	.203 76	.907 85	29
32	.150 05	.664 63	.167 94	.954 48	.185 94	.378 05	.204 06	.900 56	28
33	.150 34	.651 44	.168 24	.943 9	.186 24	.369 36	.204 36	.893 3	27
34	.150 64	.638 31	.168 54	.933 35	.186 54	.360 7	.204 66	.886 05	26
35	.150 94	6.625 23	.168 84	5.922 83	.186 84	5.352 06	.204 97	4.878 82	25
36	.151 24	.612 19	.169 14	.912 35	.187 14	.343 45	.205 27	.871 62	24
37	.151 53	.599 21	.169 44	.901 91	.187 45	.334 87	.205 57	.864 44	23
38	.151 83	.586 27	.169 74	.891 51	.187 75	.326 31	.205 88	.857 27	22
39	.152 13	.573 39	.170 04	.881 14	.188 05	.317 78	.206 18	.850 13	21
40	.152 43	6.560 55	.170 33	5.870 8	.188 35	5.309 28	.206 48	4.843	20
41	.152 72	.547 77	.170 63	.860 51	.188 65	.300 8	.206 79	.835 9	19
42	.153 02	.535 03	.170 93	.850 24	.188 95	.292 35	.207 09	.828 82	18
43	.153 32	.522 34	.171 23	.840 01	.189 25	.283 93	.207 39	.821 75	17
44	.153 62	.509 7	.171 53	.829 82	.189 55	.275 53	.207 7	.814 71	16
45	.153 91	6.497 1	.171 83	5.819 66	.189 86	5.267 15	.208	4.807 69	15
46	.154 21	.484 56	.172 13	.809 53	.190 16	.258 8	.208 3	.800 68	14
47	.154 51	.472 06	.172 43	.799 44	.190 46	.250 48	.208 61	.793 7	13
48	.154 81	.459 61	.172 73	.789 38	.190 76	.242 18	.208 91	.786 73	12
49	.155 11	.447 2	.173 03	.779 36	.191 06	.233 91	.209 21	.779 78	11
50	.155 4	6.434 84	.173 33	5.769 37	.191 36	5.225 66	.209 52	4.772 86	10
51	.155 7	.422 53	.173 63	.759 41	.191 66	.217 44	.209 82	.765 95	9
52	.156	.410 26	.173 93	.749 49	.191 97	.209 25	.210 13	.759 06	8
53	.156 3	.398 04	.174 23	.739 6	.192 27	.201 07	.210 43	.752 19	7
54	.156 6	.385 87	.174 53	.729 74	.192 57	.192 93	.210 73	.745 34	6
55	.156 89	6.373 74	.174 83	5.719 92	.192 87	5.184 8	.211 04	4.738 51	5
56	.157 19	.361 65	.175 13	.710 13	.193 17	.176 71	.211 34	.731 7	4
57	.157 49	.349 61	.175 43	.700 37	.193 47	.168 63	.211 64	.724 9	3
58	.157 79	.337 61	.175 73	.690 64	.193 78	.160 58	.211 95	.718 13	2
59	.158 09	.325 66	.176 03	.680 94	.194 08	.152 56	212 25	.711 37	1
60	.158 38	6.313 75	.176 33	5.671 28	.194 38	5.144 55	.212 56	4.704 63	0
′	81° Co-tang.	81° Tang.	80° Co-tang.	80° Tang.	79° Co-tang.	79° Tang.	78° Co-tang.	78° Tang.	′

′	12° Tang.	12° Co-tang.	13° Tang.	13° Co-tang.	14° Tang.	14° Co-tang.	15° Tang.	15° Co-tang.	′
0	.212 56	4.704 63	.230 87	4.331 48	.249 33	4.010 78	.267 95	3.732 05	60
1	.212 86	.697 91	.231 17	.325 73	.249 64	.005 82	.268 26	.727 71	59
2	.213 16	.691 21	.231 48	.320 01	.249 95	.000 86	.268 57	.723 38	58
3	.213 47	.684 52	.231 79	.314 3	.250 26	3.995 92	.268 88	.719 07	57
4	.213 77	.677 86	.232 09	.308 6	.250 56	.990 99	.269 2	.714 76	56
5	.214 08	4.671 21	.232 4	4.302 91	.250 87	3.986 07	.269 51	3.710 46	55
6	.214 38	.664 58	.232 71	.297 24	.251 18	.981 17	.269 82	.706 16	54
7	.214 69	.657 97	.233 01	.291 59	.251 49	.976 27	.270 13	.701 88	53
8	.214 99	.651 38	.233 32	.285 95	.251 8	.971 39	.270 44	.697 61	52
9	.215 29	.644 8	.233 63	.280 32	.252 11	.966 51	.270 76	.693 35	51
10	.215 6	4.638 25	.233 93	4.274 71	.252 42	3.961 65	.271 07	3.689 09	50
11	.215 9	.631 71	.234 24	.269 11	.252 73	.956 8	.271 38	.684 85	49
12	.216 21	.625 18	.234 55	.263 52	.253 04	.951 96	.271 69	.680 61	48
13	.216 51	.618 68	.234 85	.257 95	.253 35	.947 13	.272 01	.676 38	47
14	.216 82	.612 19	.235 16	.252 39	.253 66	.942 32	.272 32	.672 17	46
15	.217 12	4.605 72	.235 47	4.246 85	.253 97	3.937 51	.272 63	3.667 96	45
16	.217 43	.599 27	.235 78	.241 32	.254 28	.932 71	.272 94	.663 76	44
17	.217 73	.592 83	.236 08	.235 8	.254 59	.927 93	.273 26	.659 57	43
18	.218 04	.586 41	.236 39	.230 3	.254 9	.923 16	.273 57	.655 38	42
19	.218 34	.580 01	.236 7	.224 81	.255 21	.918 39	.273 88	.651 21	41
20	.218 64	4.573 63	.237	4.219 33	.255 52	3.913 64	.274 19	3.647 05	40
21	.218 95	.567 26	.237 31	.213 87	.255 83	.908 9	.274 51	.642 89	39
22	.219 25	.560 91	.237 62	.208 42	.256 14	.904 17	.274 82	.638 74	38
23	.219 56	.554 58	.237 93	.202 98	.256 45	.899 45	.275 13	.634 61	37
24	.219 86	.548 26	.238 23	.197 56	.256 76	.894 74	.275 45	.630 48	36
25	.220 17	4.541 96	.238 54	4.192 15	.257 07	3.890 04	.275 76	3.626 36	35
26	.220 47	.535 68	.238 85	.186 75	.257 38	.885 36	.276 07	.622 24	34
27	.220 78	.529 41	.239 16	.181 37	.257 69	.880 68	.276 38	.618 14	33
28	.221 08	.523 16	.239 46	.176	.258	.876 01	.276 7	.614 05	32
29	.221 39	.516 93	.239 77	.170 64	.258 31	.871 36	.277 01	.609 96	31
30	.221 69	4.510 71	.240 08	4.165 3	.258 62	3.866 71	.277 32	3.605 88	30
31	.222	.504 51	.240 39	.159 97	.258 93	.862 08	.277 64	.601 81	29
32	.222 31	.498 32	.240 69	.154 65	.259 24	.857 45	.277 95	.597 75	28
33	.222 61	.492 15	.241	.149 34	.259 55	.852 84	.278 26	.593 7	27
34	.222 92	.486	.241 31	.144 05	.259 86	.848 24	.278 58	.589 66	26
35	.223 22	4.479 86	.241 62	4.138 77	.260 17	3.843 64	.278 89	3.585 62	25
36	.223 53	.473 74	.241 93	.133 5	.260 48	.839 06	.279 2	.581 6	24
37	.223 83	.467 64	.242 23	.128 25	.260 79	.834 49	.279 52	.577 58	23
38	.224 14	.461 55	.242 54	.123 01	.261 1	.829 92	.279 83	.573 57	22
39	.224 44	.455 48	.242 85	.117 78	.261 41	.825 37	.280 15	.569 57	21
40	.224 75	4.449 42	.243 16	4.112 56	.261 72	3.820 83	.280 46	3.565 57	20
41	.225 05	.443 38	.243 47	.107 36	.262 03	.816 3	.280 77	.561 59	19
42	.225 36	.437 35	.243 77	.102 16	.262 35	.811 77	.281 09	.557 61	18
43	.225 67	.431 34	.244 08	.096 99	.262 66	.807 26	.281 4	.553 64	17
44	.225 97	.425 34	.244 39	.091 82	.262 97	.802 76	.281 72	.549 68	16
45	.226 28	4.419 36	.244 7	4.086 66	.263 28	3.798 27	.282 03	3.545 73	15
46	.226 58	.413 4	.245 01	.081 52	.263 59	.793 78	.282 34	.541 79	14
47	.226 89	.407 45	.245 32	.076 39	.263 9	.789 31	.282 66	.537 85	13
48	.227 19	.401 52	.245 62	.071 27	.264 21	.784 85	.282 97	.533 93	12
49	.227 5	.395 6	.245 93	.066 16	.264 52	.780 4	.283 29	.530 01	11
50	.227 81	4.389 69	.246 24	4.061 07	.264 83	3.775 95	.283 6	3.526 09	10
51	.228 11	.383 81	.246 55	.055 99	.265 15	.771 52	.283 91	.522 19	9
52	.228 42	.377 93	.246 86	.050 92	.265 46	.767 09	.284 23	.518 29	8
53	.228 72	.372 07	.247 17	.045 86	.265 77	.762 68	.284 54	.514 41	7
54	.229 03	.366 23	.247 47	.040 81	.266 08	.758 28	.284 86	.510 53	6
55	.229 34	4.360 4	.247 78	4.035 78	.266 39	3.753 88	.285 17	3.506 66	5
56	.229 64	.354 59	.248 09	.030 75	.266 7	.749 5	.285 49	.502 79	4
57	.229 95	.348 79	.248 4	.025 74	.267 01	.745 12	.285 8	.498 94	3
58	.230 26	.343	.248 71	.020 74	.267 33	.740 75	.286 12	.495 09	2
59	.230 56	.337 23	.249 02	.015 76	.267 64	.736 4	.286 43	.491 25	1
60	.230 87	4.331 48	.249 33	4.010 78	.267 95	3.732 05	.286 75	3.487 41	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	77°		76°		75°		74°		

′	16° Tang.	16° Co-tang.	17° Tang.	17° Co-tang.	18° Tang.	18° Co-tang.	19° Tang.	19° Co-tang.	′
0	.286 75	3.487 41	.305 73	3.270 85	.324 92	3.077 68	.344 33	2.904 21	60
1	.287 06	.483 59	.306 05	.267 45	.325 24	.074 64	.344 65	.901 47	59
2	.287 38	.479 77	.306 37	.264 06	.325 56	.071 6	.344 98	.898 73	58
3	.287 69	.475 96	.306 69	.260 67	.325 88	.068 57	.345 3	.896	57
4	.288	.472 16	.307	.257 29	.326 21	.065 54	.345 63	.893 27	56
5	.288 32	3.468 37	.307 32	3.253 92	.326 53	3.062 52	.345 96	2.890 55	55
6	.288 64	.464 58	.307 64	.250 55	.326 85	.059 5	.346 28	.887 83	54
7	.288 95	.460 8	.307 96	.247 19	.327 17	.056 49	.346 61	.885 11	53
8	.289 27	.457 03	.308 28	.243 83	.327 49	.053 49	.346 93	.882 4	52
9	.289 58	.453 27	.308 6	.240 49	.327 82	.050 49	.347 26	.879 7	51
10	.289 9	3.449 51	.308 91	3.237 14	.328 14	3.047 49	.347 58	2.877	50
11	.290 21	.445 76	.309 23	.233 81	.328 46	.044 5	.347 91	.874 3	49
12	.290 53	.442 02	.309 55	.230 48	.328 78	.041 52	.348 24	.871 61	48
13	.290 84	.438 29	.309 87	.227 15	.329 11	.038 54	.348 56	.868 92	47
14	.291 16	.434 56	.310 19	.223 84	.329 43	.035 56	.348 89	.866 24	46
15	.291 47	3.430 84	.310 51	3.220 53	.329 75	3.032 6	.349 22	2.863 56	45
16	.291 79	.427 13	.310 83	.217 22	.330 07	.029 63	.349 54	.860 89	44
17	.292 1	.423 43	.311 15	.213 92	.330 4	.026 67	.349 87	.858 22	43
18	.292 42	.419 73	.311 47	.210 63	.330 72	.023 72	.350 19	.855 55	42
19	.292 74	.416 04	.311 78	.207 34	.331 04	.020 77	.350 52	.852 89	41
20	.293 05	3.412 36	.312 1	3.204 06	.331 36	3.017 83	.350 85	2.850 23	40
21	.293 37	.408 69	.312 42	.200 79	.331 69	.014 89	.351 17	.847 58	39
22	.293 68	.405 02	.312 74	.197 52	.332 01	.011 96	.351 5	.844 94	38
23	.294	.401 36	.313 06	.194 26	.332 33	.009 03	.351 83	.842 29	37
24	.294 32	.397 71	.313 38	.191	.332 66	.006 11	.352 16	.839 65	36
25	.294 63	3.394 06	.313 7	3.187 75	.332 98	3.003 19	.352 48	2.837 02	35
26	.294 95	.390 42	.314 02	.184 51	.333 3	.000 28	.352 81	.834 39	34
27	.295 26	.386 79	.314 34	.181 27	.333 63	2.997 38	.353 14	.831 76	33
28	.295 58	.383 17	.314 66	.178 04	.333 95	.994 47	.353 46	.829 14	32
29	.295 9	.379 55	.314 98	.174 81	.334 27	.991 58	.353 79	.826 53	31
30	.296 21	3.375 94	.315 3	3.171 59	.334 6	2.988 68	.354 12	2.823 91	30
31	.296 53	.372 34	.315 62	.168 38	.334 92	.985 8	.354 45	.821 3	29
32	.296 85	.368 75	.315 94	.165 17	.335 24	.982 92	.354 77	.818 7	28
33	.297 16	.365 16	.316 26	.161 97	.335 57	.980 04	.355 1	.816 1	27
34	.297 48	.361 58	.316 58	.158 77	.335 89	.977 17	.355 43	.813 5	26
35	.297 8	3.358	.316 9	3.155 58	.336 21	2.974 3	.355 76	2.810 91	25
36	.298 11	.354 43	.317 22	.152 4	.336 54	.971 44	.356 08	.808 33	24
37	.298 43	.350 87	.317 54	.149 22	.336 86	.968 58	.356 41	.805 74	23
38	.298 75	.347 32	.317 86	.146 05	.337 18	.965 73	.356 74	.803 16	22
39	.299 06	.343 77	.318 18	.142 88	.337 51	.962 88	.357 07	.800 59	21
40	.299 38	3.340 23	.318 5	3.139 72	.337 83	2.960 04	.357 4	2.798 02	20
41	.299 7	.336 7	.318 82	.136 56	.338 16	.957 21	.357 72	.795 45	19
42	.300 01	.333 17	.319 14	.133 41	.338 48	.954 37	.358 05	.792 89	18
43	.300 33	.329 65	.319 46	.130 27	.338 81	.951 55	.358 38	.790 33	17
44	.300 65	.326 14	.319 78	.127 13	.339 13	.948 72	.358 71	.787 78	16
45	.300 97	3.322 64	.320 1	3.124	.339 45	2.945 9	.359 04	2.785 23	15
46	.301 28	.319 14	.320 42	.120 87	.339 78	.943 09	.359 37	.782 69	14
47	.301 6	.315 65	.320 74	.117 75	.340 1	.940 28	.359 69	.780 14	13
48	.301 92	.312 16	.321 06	.114 64	.340 43	.937 48	.360 02	.777 61	12
49	.302 24	.308 68	.321 39	.111 53	.340 75	.934 68	.360 35	.775 07	11
50	.302 55	3.305 21	.321 71	3.108 42	.341 08	2.931 89	.360 68	2.772 54	10
51	.302 87	.301 74	.322 03	.105 32	.341 4	.929 1	.361 01	.770 02	9
52	.303 19	.298 29	.322 35	.102 23	.341 73	.926 32	.361 34	.767 5	8
53	.303 51	.294 83	.322 67	.099 14	.342 05	.923 54	.361 67	.764 98	7
54	.303 82	.291 39	.322 99	.096 06	.342 38	.920 76	.361 99	.762 47	6
55	.304 14	3.287 95	.323 31	3.092 98	.342 7	2.917 99	.362 32	2.759 96	5
56	.304 46	.284 52	.323 63	.089 91	.343 03	.915 23	.362 65	.757 46	4
57	.304 78	.281 09	.323 96	.086 85	.343 35	.912 46	.362 98	.754 96	3
58	.305 09	.277 67	.324 28	.083 79	.343 68	.909 71	.363 31	.752 46	2
59	.305 41	.274 26	.324 6	.080 73	.344	.906 96	.363 64	.749 97	1
60	.305 73	3.270 85	.324 92	3.077 68	.344 33	2.904 21	.363 97	2.747 48	0
′	Co-tang. 73°	Tang. 73°	Co-tang. 72°	Tang. 72°	Co-tang. 71°	Tang. 71°	Co-tang. 70°	Tang. 70°	′

′	20° Tang.	20° Co-tang.	21° Tang.	21° Co-tang.	22° Tang.	22° Co-tang.	23° Tang.	23° Co-tang.	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.3643	.74499	.3842	.60283	.40436	.47302	.42482	.35395	59
2	.36463	.74251	.38453	.60057	.4047	.47095	.42516	.35205	58
3	.36496	.74004	.38487	.59831	.40504	.46888	.42551	.35015	57
4	.36529	.73756	.3852	.59606	.40538	.46682	.42585	.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	.73263	.38587	.59156	.40606	.4627	.42654	.34447	54
7	.36628	.73017	.3862	.58932	.4064	.46065	.42688	.34258	53
8	.36661	.72771	.38654	.58708	.40674	.4586	.42722	.34069	52
9	.36694	.72526	.38687	.58484	.40707	.45655	.42757	.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.3676	.72036	.38754	.58038	.40775	.45246	.42826	.33505	49
12	.36793	.71792	.38787	.57815	.40809	.45043	.4286	.33317	48
13	.36826	.71548	.38821	.57593	.40843	.44839	.42894	.3313	47
14	.36859	.71305	.38854	.57371	.40877	.44636	.42929	.32943	46
15	.36892	2.71062	.38888	2.5715	.40911	2.44433	.42963	2.32756	45
16	.36925	.70819	.38921	.56928	.40945	.4423	.42998	.3257	44
17	.36958	.70577	.38955	.56707	.40979	.44027	.43032	.32383	43
18	.36991	.70335	.38988	.56487	.41013	.43825	.43067	.32197	42
19	.37024	.70094	.39022	.56266	.41047	.43623	.43101	.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.3709	.69612	.39089	.55827	.41115	.4322	.4317	.31641	39
22	.37124	.69371	.39122	.55608	.41149	.43019	.43205	.31456	38
23	.37157	.69131	.39156	.55389	.41183	.42819	.43239	.31271	37
24	.3719	.68892	.3919	.5517	.41217	.42618	.43274	.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	.68414	.39257	.54734	.41285	.42218	.43343	.30718	34
27	.37289	.68175	.3929	.54516	.41319	.42019	.43378	.30534	33
28	.37322	.67937	.39324	.54299	.41353	.41819	.43412	.30351	32
29	.37355	.677	.39357	.54082	.41387	.4162	.43447	.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2 41421	.43481	2.29984	30
31	.37422	.67225	.39425	.53648	.41455	.41223	.43516	.29801	29
32	.37455	.66989	.39458	.53432	.4149	.41025	.4355	.29619	28
33	.37488	.66752	.39492	.53217	.41524	.40827	.43585	.29437	27
34	.37521	.66516	.39526	.53001	.41558	.40629	.4362	.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	.66046	.39593	.52571	.41626	.40235	.43689	.28891	24
37	.37621	.65811	.39626	.52357	.4166	.40038	.43724	.2871	23
38	.37654	.65576	.3966	.52142	.41694	.39841	.43758	.28528	22
39	.37687	.65342	.39694	.51929	.41728	.39645	.43793	.28348	21
40	.3772	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	.64875	.39761	.51502	.41797	.39253	.43862	.27987	19
42	.37787	.64642	.39795	.51289	.41831	.39058	.43897	.27806	18
43	.3782	.6441	.39829	.51076	.41865	.38862	.43932	.27626	17
44	.37853	.64177	.39862	.50864	.41899	.38668	.43966	.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.3792	.63714	.3993	.5044	.41968	.38279	.44036	.27088	14
47	.37953	.63483	.39963	.50229	.42002	.38084	.44071	.26909	13
48	.37986	.63252	.39997	.50018	.42036	.37891	.44105	.2673	12
49	.3802	.63021	.40031	.49807	.4207	.37697	.4414	.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	.62561	.40098	.49386	.42139	.37311	.4421	.26196	9
52	.3812	.62332	.40132	.49177	.42173	.37118	.44244	.26018	8
53	.38153	.62103	.40166	.48967	.42207	.36925	.44279	.2584	7
54	.38186	.61874	.402	.48758	.42242	.36733	.44314	.25663	6
55	.3822	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	.61418	.40267	.4834	.4231	.36349	.44384	.25309	4
57	.38286	.6119	.40301	.48132	.42345	.36158	.44418	.25132	3
58	.3832	.60963	.40335	.47924	.42379	.35967	.44453	.24956	2
59	.38353	.60736	.40369	.47716	.42413	.35776	.44488	.2478	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Co-tang. 69°	Tang. 69°	Co-tang. 68°	Tang. 68°	Co-tang. 67°	Tang. 67°	Co-tang. 66°	Tang. 66°	′

	24°		25°		26°		27°		
′	TANG.	CO-TANG.	TANG.	CO-TANG.	TANG.	CO-TANG.	TANG.	CO-TANG.	′
0	.445 23	2.246 04	.466 31	2.144 51	.487 73	2.050 3	.509 53	1.962 61	60
1	.445 58	.244 28	.466 66	.142 88	.488 09	.048 79	.509 89	.961 2	59
2	.445 93	.242 52	.467 02	.141 25	.488 45	.047 28	.510 26	.959 79	58
3	.446 27	.240 77	.467 37	.139 63	.488 81	.045 77	.510 63	.958 38	57
4	.446 62	.239 02	.467 72	.138 01	.489 17	.044 26	.510 99	.956 98	56
5	.446 97	2.237 27	.468 08	2.136 39	.489 53	2.042 76	.511 36	1.955 57	55
6	.447 32	.235 53	.468 43	.134 77	.489 89	.041 25	.511 73	.954 17	54
7	.447 67	.233 78	.468 79	.133 16	.490 26	.039 75	.512 09	.952 77	53
8	.448 02	.232 04	.469 14	.131 54	.490 62	.038 25	.512 46	.951 37	52
9	.448 37	.230 3	.469 5	.129 93	.490 98	.036 75	.512 83	.949 97	51
10	.448 72	2.228 57	.469 85	2.128 32	.491 34	2.035 26	.513 19	1.948 58	50
11	.449 07	.226 83	.470 21	.126 71	.491 7	.033 76	.513 56	.947 18	49
12	.449 42	.225 1	.470 56	.125 11	.492 06	.032 27	.513 93	.945 79	48
13	.449 77	.223 37	.470 92	.123 5	.492 42	.030 78	.514 3	.944 4	47
14	.450 12	.221 64	.471 28	.121 9	.492 78	.029 29	.514 67	.943 01	46
15	.450 47	2.219 92	.471 63	2.120 3	.493 15	2.027 8	.515 03	1.941 62	45
16	.450 82	.218 19	.471 99	.118 71	.493 51	.026 31	.515 4	.940 23	44
17	.451 17	.216 47	.472 34	.117 11	.493 87	.024 83	.515 77	.938 85	43
18	.451 52	.214 75	.472 7	.115 52	.494 23	.023 35	.516 14	.937 46	42
19	.451 87	.213 04	.473 05	.113 92	.494 59	.021 87	.516 51	.936 08	41
20	.452 22	2.211 32	.473 41	2.112 33	.494 95	2.020 39	.516 88	1.934 7	40
21	.452 57	.209 61	.473 77	.110 75	.495 32	.018 91	.517 24	.933 32	39
22	.452 92	.207 9	.474 12	.109 16	.495 68	.017 43	.517 61	.931 95	38
23	.453 27	.206 19	.474 48	.107 58	.496 04	.015 96	.517 98	.930 57	37
24	.453 62	.204 49	.474 83	.106	.496 4	.014 49	.518 35	.929 2	36
25	.453 97	2.202 78	.475 19	2.104 42	.496 77	2.013 02	.518 72	1.927 82	35
26	.454 32	.201 08	.475 55	.102 84	.497 13	.011 55	.519 09	.926 45	34
27	.454 67	.199 38	.475 9	.101 26	.497 49	.010 08	.519 46	.925 08	33
28	.455 02	.197 69	.476 26	.099 69	.497 86	.008 62	.519 83	.923 71	32
29	.455 37	.195 99	.476 62	.098 11	.498 22	.007 15	.520 2	.922 35	31
30	.455 73	2.194 3	.476 98	2.096 54	.498 58	2.005 69	.520 57	1.920 98	30
31	.456 08	.192 61	.477 33	.094 98	.498 94	.004 23	.520 94	.919 62	29
32	.456 43	.190 92	.477 69	.093 41	.499 31	.002 77	.521 31	.918 26	28
33	.456 78	.189 23	.478 05	.091 84	.499 67	.001 31	.521 68	.916 9	27
34	.457 13	.187 55	.478 4	.090 28	.500 04	1.999 86	.522 05	.915 54	26
35	.457 48	2.185 87	.478 76	2.088 72	.500 4	1.998 41	.522 42	1.914 18	25
36	.457 84	.184 19	.479 12	.087 16	.500 76	.996 95	.522 79	.912 82	24
37	.458 19	.182 51	.479 48	.085 6	.501 13	.995 5	.523 16	.911 47	23
38	.458 54	.180 84	.479 84	.084 05	.501 49	.994 06	.523 53	.910 12	22
39	.458 89	.179 16	.480 19	.082 5	.501 85	.992 61	.523 9	.908 76	21
40	.459 24	2.177 49	.480 55	2.080 94	.502 22	1.991 16	.524 27	1.907 41	20
41	.459 6	.175 82	.480 91	.079 39	.502 58	.989 72	.524 64	.906 07	19
42	.459 95	.174 16	.481 27	.077 85	.502 95	.988 28	.525 01	.904 72	18
43	.460 3	.172 49	.481 63	.076 3	.503 31	.986 84	.525 38	.903 37	17
44	.460 65	.170 83	.481 98	.074 76	.503 68	.985 4	.525 75	.902 03	16
45	.461 01	2.169 17	.482 34	2.073 21	.504 04	1.983 96	.526 13	1.900 69	15
46	.461 36	.167 51	.482 7	.071 67	.504 41	.982 53	.526 5	.899 35	14
47	.461 71	.165 85	.483 06	.070 14	.504 77	.981 1	.526 87	.898 01	13
48	.462 06	.164 2	.483 42	.068 6	.505 14	.979 66	.527 24	.896 67	12
49	.462 42	.162 55	.483 78	.067 06	.505 5	.978 23	.527 61	.895 33	11
50	.462 77	2.160 9	.484 14	2.065 53	.505 87	1.976 8	.527 98	1.894	10
51	.463 12	.159 25	.484 5	.064	.506 23	.975 38	.528 36	.892 66	9
52	.463 48	.157 6	.484 86	.062 47	.506 6	.973 95	.528 73	.891 33	8
53	.463 83	.155 96	.485 21	.060 94	.506 96	.972 53	.529 1	.89	7
54	.464 18	.154 32	.485 57	.059 42	.507 33	.971 11	.529 47	.888 67	6
55	.464 54	2.152 68	.485 93	2.057 9	.507 69	1.969 69	.529 84	1.887 34	5
56	.464 89	.151 04	.486 29	.056 37	.508 06	.968 27	.530 22	.886 02	4
57	.465 25	.149 4	.486 65	.054 85	.508 43	.966 85	.530 59	.884 69	3
58	.465 6	.147 77	.487 01	.053 33	.508 79	.965 44	.530 96	.883 37	2
59	.465 95	.146 14	.487 37	.051 82	.509 16	.964 02	.531 34	.882 05	1
60	.466 31	2.144 51	.487 73	2.050 3	.509 53	1.962 61	.531 71	1.880 73	0
′	CO-TANG.	TANG.	CO-TANG.	TANG.	CO-TANG.	TANG.	CO-TANG.	TANG.	′
	65°		64°		63°		62°		

′	28° Tang.	28° Co-tang.	29° Tang.	29° Co-tang.	30° Tang.	30° Co-tang.	31° Tang.	31° Co-tang.	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	.87941	.55469	.80281	.57774	.73089	.60126	.66318	59
2	.53246	.87809	.55507	.80158	.57813	.72973	.60165	.66209	58
3	.53283	.87677	.55545	.80034	.57851	.72857	.60205	.66099	57
4	.5332	.87546	.55583	.79911	.5789	.72741	.60245	.6599	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	.87283	.55659	.79665	.57968	.72509	.60324	.65772	54
7	.53432	.87152	.55697	.79542	.58007	.72393	.60364	.65663	53
8	.5347	.87021	.55736	.79419	.58046	.72278	.60403	.65554	52
9	.53507	.86891	.55774	.79296	.58085	.72163	.60443	.65445	51
10	.53545	1.8676	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	.8663	.5585	.79051	.58162	.71932	.60522	.65228	49
12	.5362	.86499	.55888	.78929	.58201	.71817	.60562	.6512	48
13	.53657	.86369	.55926	.78807	.5824	.71702	.60602	.65011	47
14	.53694	.86239	.55964	.78685	.58279	.71588	.60642	.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	.85979	.56041	.78441	.58357	.71358	.60721	.64687	44
17	.53807	.8585	.56079	.78319	.58396	.71244	.60761	.64579	43
18	.53844	.8572	.56117	.78198	.58435	.71129	.60801	.64471	42
19	.53882	.85591	.56156	.78077	.58474	.71015	.60841	.64363	41
20	.5392	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	.85333	.56232	.77834	.58552	.70787	.60921	.64148	39
22	.53995	.85204	.5627	.77713	.58591	.70673	.6096	.64041	38
23	.54032	.85075	.56309	.77592	.58631	.7056	.61	.63934	37
24	.5407	.84946	.56347	.77471	.5867	.70446	.6104	.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.6108	1.63719	35
26	.54145	.84689	.56424	.7723	.58748	.70219	.6112	.63612	34
27	.54183	.84561	.56462	.7711	.58787	.70106	.6116	.63505	33
28	.5422	.84433	.565	.7699	.58826	.69992	.612	.63398	32
29	.54258	.84305	.56539	.76869	.58865	.69879	.6124	.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.6128	1.63185	30
31	.54333	.84049	.56616	.7663	.58944	.69653	.6132	.63079	29
32	.54371	.83922	.56654	.7651	.58983	.69541	.6136	.62972	28
33	.54409	.83794	.56693	.7639	.59022	.69428	.614	.62866	27
34	.54446	.83667	.56731	.76271	.59061	.69316	.6144	.6276	26
35	.54484	1.8354	.56769	1.76151	.59101	1.69203	.6148	1.62654	25
36	.54522	.83413	.56808	.76032	.5914	.69091	.6152	.62548	24
37	.5456	.83286	.56846	.75913	.59179	.68979	.61561	.62442	23
38	.54597	.83159	.56885	.75794	.59218	.68866	.61601	.62336	22
39	.54635	.83033	.56923	.75675	.59258	.68754	.61641	.6223	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	.8278	.57	.75437	.59336	.68531	.61721	.62019	19
42	.54748	.82654	.57039	.75319	.59376	.68419	.61761	.61914	18
43	.54786	.82528	.57078	.752	.59415	.68308	.61801	.61808	17
44	.54824	.82402	.57116	.75082	.59454	.68196	.61842	.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.549	.8215	.57193	.74846	.59533	.67974	.61922	.61493	14
47	.54938	.82025	.57232	.74728	.59573	.67863	.61962	.61388	13
48	.54975	.81899	.57271	.7461	.59612	.67752	.62003	.61283	12
49	.55013	.81774	.57309	.74492	.59651	.67641	.62043	.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.6753	.62083	1.61074	10
51	.55089	.81524	.57386	.74257	.5973	.67419	.62124	.6097	9
52	.55127	.81399	.57425	.7414	.5977	.67309	.62164	.60865	8
53	.55165	.81274	.57464	.74022	.59809	.67198	.62204	.60761	7
54	.55203	.8115	.57503	.73905	.59849	.67088	.62245	.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	.80901	.5758	.73671	.59928	.66867	.62325	.60449	4
57	.55317	.80777	.57619	.73555	.59967	.66757	.62366	.60345	3
58	.55355	.80653	.57657	.73438	.60007	.66647	.62406	.60241	2
59	.55393	.80529	.57696	.73321	.60046	.66538	.62446	.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.624 87	1.600 33	.649 41	1.539 86	.674 51	1.482 56	.700 21	1.428 15	60
1	.625 27	.599 3	.649 82	.538 88	.674 93	.481 63	.700 64	.427 26	59
2	.625 68	.598 26	.650 23	.537 91	.675 36	.480 7	.701 07	.426 38	58
3	.626 08	.597 23	.650 65	.536 93	.675 78	.479 77	.701 51	.425 5	57
4	.626 49	.596 2	.651 06	.535 95	.676 2	.478 85	.701 94	.424 62	56
5	.626 89	1.595 17	.651 48	1.534 97	.676 63	1.477 92	.702 38	1.423 74	55
6	.627 3	.594 14	.651 89	.534	.677 05	.476 99	.702 81	.422 86	54
7	.627 7	.593 11	.652 31	.533 02	.677 48	.476 07	.703 25	.421 98	53
8	.628 11	.592 08	.652 72	.532 05	.677 9	.475 14	.703 68	.421 1	52
9	.628 52	.591 05	.653 14	.531 07	.678 32	.474 22	.704 12	.420 22	51
10	.628 92	1.590 02	.653 55	1.530 1	.678 75	1.473 3	.704 55	1.419 34	50
11	.629 33	.589	.653 97	.529 13	.679 17	.472 38	.704 99	.418 47	49
12	.629 73	.587 97	.654 38	.528 16	.679 6	.471 46	.705 42	.417 59	48
13	.630 14	.586 95	.654 8	.527 19	.680 02	.470 53	.705 86	.416 72	47
14	.630 55	.585 93	.655 21	.526 22	.680 45	.469 62	.706 29	.415 84	46
15	.630 95	1.584 9	.655 63	1.525 25	.680 88	1.468 7	.706 73	1.414 97	45
16	.631 36	.583 88	.656 04	.524 29	.681 3	.467 78	.707 17	.414 09	44
17	.631 77	.582 86	.656 46	.523 32	.681 73	.466 86	.707 6	.413 22	43
18	.632 17	.581 84	.656 88	.522 35	.682 15	.465 95	.708 04	.412 35	42
19	.632 58	.580 83	.657 29	.521 39	.682 58	.465 03	.708 48	.411 48	41
20	.632 99	1.579 81	.657 71	1.520 43	.683 01	1.464 11	.708 91	1.410 61	40
21	.633 4	.578 79	.658 13	.519 46	.683 43	.463 2	.709 35	.409 74	39
22	.633 8	.577 78	.658 54	.518 5	.683 86	.462 29	.709 79	.408 87	38
23	.634 21	.576 76	.658 96	.517 54	.684 29	.461 37	.710 23	.408	37
24	.634 62	.575 75	.659 38	.516 58	.684 71	.460 46	.710 66	.407 14	36
25	.635 03	1.574 74	.659 8	1.515 62	.685 14	1.459 55	.711 1	1.406 27	35
26	.635 44	.573 72	.660 21	.514 66	.685 57	.458 64	.711 54	.405 4	34
27	.635 84	.572 71	.660 63	.513 7	.686	.457 73	.711 98	.404 54	33
28	.636 25	.571 7	.661 05	.512 75	.686 42	.456 82	.712 42	.403 67	32
29	.636 66	.570 69	.661 47	.511 79	.686 85	.455 92	.712 85	.402 81	31
30	.637 07	1.569 69	.661 89	1.510 84	.687 28	1.455 01	.713 29	1.401 95	30
31	.637 48	.568 68	.662 3	.509 88	.687 71	.454 1	.713 73	.401 09	29
32	.637 89	.567 67	.662 72	.508 93	.688 14	.453 2	.714 17	.400 22	28
33	.638 3	.566 67	.663 14	.507 97	.688 57	.452 29	.714 61	.399 36	27
34	.638 71	.565 66	.663 56	.507 02	.689	.451 39	.715 05	.398 5	26
35	.639 12	1.564 66	.663 98	1.506 07	.689 42	1.450 49	.715 49	1.397 64	25
36	.639 53	.563 66	.664 4	.505 12	.689 85	.449 58	.715 93	.396 79	24
37	.639 94	.562 65	.664 82	.504 17	.690 28	.448 68	.716 37	.395 93	23
38	.640 35	.561 65	.665 24	.503 22	.690 71	.447 78	.716 81	.395 07	22
39	.640 76	.560 65	.665 66	.502 28	.691 14	.446 88	.717 25	.394 21	21
40	.641 17	1.559 66	.666 08	1.501 33	.691 57	1.445 98	.717 69	1.393 36	20
41	.641 58	.558 66	.666 5	.500 38	.692	.445 08	.718 13	.392 5	19
42	.641 99	.557 66	.666 92	.499 44	.692 43	.444 18	.718 57	.391 65	18
43	.642 4	.556 66	.667 34	.498 49	.692 86	.443 29	.719 01	.390 79	17
44	.642 81	.555 67	.667 76	.497 55	.693 29	.442 39	.719 46	.389 94	16
45	.643 22	1.554 67	.668 18	1.496 61	.693 72	1.441 49	.719 9	1.389 09	15
46	.643 63	.553 68	.668 6	.495 66	.694 16	.440 6	.720 34	.388 24	14
47	.644 04	.552 69	.669 02	.494 72	.694 59	.439 7	.720 78	.387 38	13
48	.644 46	.551 7	.669 44	.493 78	.695 02	.438 81	.721 22	.386 53	12
49	.644 87	.550 71	.669 86	.492 84	.695 45	.437 92	.721 66	.385 68	11
50	.645 28	1.549 72	.670 28	1.491 9	.695 88	1.437 03	.722 11	1.384 84	10
51	.645 69	.548 73	.670 71	.490 97	.696 31	.436 14	.722 55	.383 99	9
52	.646 1	.547 74	.671 13	.490 03	.696 75	.435 25	.722 99	.383 14	8
53	.646 52	.546 75	.671 55	.489 09	.697 18	.434 36	.723 44	.382 29	7
54	.646 93	.545 76	.671 97	.488 16	.697 61	.433 47	.723 88	.381 45	6
55	.647 34	1.544 78	.672 39	1.487 22	.698 04	1.432 58	.724 32	1.380 6	5
56	.647 75	.543 79	.672 82	.486 29	.698 47	.431 69	.724 77	.379 76	4
57	.648 17	.542 81	.673 24	.485 36	.698 91	.430 8	.725 21	.378 91	3
58	.648 58	.541 83	.673 66	.484 42	.699 34	.429 92	.725 65	.378 07	2
59	.648 99	.540 85	.674 09	.483 49	.699 77	.429 03	.726 1	.377 22	1
60	.649 41	1.539 86	.674 51	1.482 56	.700 21	1.428 15	.726 54	1.376 38	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	57°		56°		55°		54°		

′	36°		37°		38°		39°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.2349	60
1	.72699	.37554	.75401	.32624	.78175	.27917	.81027	.23416	59
2	.72743	.3747	.75447	.32544	.78222	.27841	.81075	.23343	58
3	.72788	.37386	.75492	.32464	.78269	.27764	.81123	.2327	57
4	.72832	.37302	.75538	.32384	.78316	.27688	.81171	.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.8122	1.23123	55
6	.72921	.37134	.75629	.32224	.7841	.27535	.81268	.2305	54
7	.72966	.3705	.75675	.32144	.78457	.27458	.81316	.22977	53
8	.7301	.36967	.75721	.32064	.78504	.27382	.81364	.22904	52
9	.73055	.36883	.75767	.31984	.78551	.27306	.81413	.22831	51
10	.731	1.368	.75812	1.31904	.78598	1.2723	.81461	1.22758	50
11	.73144	.36716	.75858	.31825	.78645	.27153	.8151	.22685	49
12	.73189	.36633	.75904	.31745	.78692	.27077	.81558	.22612	48
13	.73234	.36549	.7595	.31666	.78739	.27001	.81606	.22539	47
14	.73278	1.36466	.75996	.31586	.78786	.26925	.81655	.22467	46
15	.73323	.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	.363	.76088	.31427	.78881	.26774	.81752	.22321	44
17	.73413	.36217	.76134	.31348	.78928	.26698	.818	.22249	43
18	.73457	.36133	.7618	.31269	.78975	.26622	.81849	.22176	42
19	.73502	.36051	.76226	.3119	.79022	.26546	.81898	.22104	41
20	.73547	1.35968	.76272	1.3111	.7907	1.26471	.81946	1.22031	40
21	.73592	.35885	.76318	.31031	.79117	.26395	.81995	.21959	39
22	.73637	.35802	.76364	.30952	.79164	.26319	.82044	.21886	38
23	.73681	.35719	.7641	.30873	.79212	.26244	.82092	.21814	37
24	.73726	.35637	.76456	.30795	.79259	.26169	.82141	.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.8219	1.2167	35
26	.73816	.35472	.76548	.30637	.79354	.26018	.82238	.21598	34
27	.73861	.35389	.76594	.30558	.79401	.25943	.82287	.21526	33
28	.73906	.35307	.7664	.3048	.79449	.25867	.82336	.21454	32
29	.73951	.35224	.76686	.30401	.79496	.25792	.82385	.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.2131	30
31	.74041	.3506	.76779	.30244	.79591	.25642	.82483	.21238	29
32	.74086	.34978	.76825	.30166	.79639	.25567	.82531	.21166	28
33	.74131	.34896	.76871	.30087	.79686	.25492	.8258	.21094	27
34	.74176	.34814	.76918	.30009	.79734	.25417	.82629	.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	.3465	.7701	.29853	.79829	.25268	.82727	.20879	24
37	.74312	.34568	.77057	.29775	.79877	.25193	.82776	.20808	23
38	.74357	.34487	.77103	.29696	.79924	.25118	.82825	.20736	22
39	.74402	.34405	.77149	.29618	.79972	.25044	.82874	.20665	21
40	.74447	1.34323	.77196	1.2954	.8002	1.24969	.82923	1.20593	20
41	.74492	.34242	.77242	.29463	.80067	.24895	.82972	.20522	19
42	.74538	.3416	.77289	.29385	.80115	.2482	.83022	.20451	18
43	.74583	.34079	.77335	.29307	.80163	.24746	.83071	.20379	17
44	.74628	.33998	.77382	.29229	.80211	.24672	.8312	.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	.33835	.77475	.29074	.80306	.24523	.83218	.20166	14
47	.74764	.33754	.77521	.28997	.80354	.24449	.83268	.20095	13
48	.7481	.33673	.77568	.28919	.80402	.24375	.83317	.20024	12
49	.74855	.33592	.77615	.28842	.8045	.24301	.83366	.19953	11
50	.749	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	.3343	.77708	.28687	.80546	.24153	.83465	.19811	9
52	.74991	.33349	.77754	.2861	.80594	.24079	.83514	.1974	8
53	.75037	.33268	.77801	.28533	.80642	.24005	.83564	.19669	7
54	.75082	.33187	.77848	.28456	.8069	.23931	.83613	.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	.33026	.77941	.28302	.80786	.23784	.83712	.19457	4
57	.75219	.32946	.77988	.28225	.80834	.2371	.83761	.19387	3
58	.75264	.32865	.78035	.28148	.80882	.23637	.83811	.19316	2
59	.7531	.32785	.78082	.28071	.8093	.23563	.8386	.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.2349	.8391	1.19175	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	53°		52°		51°		50°		

′	40°		41°		42°		43°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.839 1	1.191 75	.869 29	1.150 37	.900 4	1.110 61	.932 52	1.072 37	60
1	.839 6	.191 05	.869 8	.149 69	.900 93	.109 96	.933 06	.071 74	59
2	.840 09	.190 35	.870 31	.149 02	.901 46	.109 31	.933 6	.071 12	58
3	.840 59	.189 64	.870 82	.148 34	.901 99	.108 67	.934 15	.070 49	57
4	.841 08	.188 94	.871 33	.147 67	.902 51	.108 02	.934 69	.069 87	56
5	.841 58	1.188 24	.871 84	1.146 99	.903 04	1.107 37	.935 24	1.069 25	55
6	.842 08	.187 54	.872 36	.146 32	.903 57	.106 72	.935 78	.068 62	54
7	.842 58	.186 84	.872 87	.145 65	.904 1	.106 07	.936 33	.068	53
8	.843 07	.186 14	.873 38	.144 98	.904 63	.105 43	.936 88	.067 38	52
9	.843 57	.185 44	.873 89	.144 3	.905 16	.104 78	.937 42	.066 76	51
10	.844 07	1.184 74	.874 41	1.143 63	.905 69	1.104 14	.937 97	1.066 13	50
11	.844 57	.184 04	.874 92	.142 96	.906 21	.103 49	.938 52	.065 51	49
12	.845 07	.183 34	.875 43	.142 29	.906 74	.102 85	.939 06	.064 89	48
13	.845 56	.182 64	.875 95	.141 62	.907 27	.102 2	.939 61	.064 27	47
14	.846 06	.181 94	.876 46	.140 95	.907 81	.101 56	.940 16	.063 65	46
15	.846 56	1.181 25	.876 98	1.140 28	.908 34	1.100 91	940 71	1.063 03	45
16	.847 06	.180 55	.877 49	.139 61	.908 87	.100 27	.941 25	.062 41	44
17	.847 56	.179 86	.878 01	.138 94	.909 4	.099 63	.941 8	.061 79	43
18	.848 06	.179 16	.878 52	.138 28	.909 93	.098 99	.942 35	.061 17	42
19	.848 56	.178 46	.879 04	.137 61	.910 46	.098 34	.942 9	.060 56	41
20	.849 06	1.177 77	.879 55	1.136 94	.910 99	1.097 7	.943 45	1.059 94	40
21	.849 56	.177 08	.880 07	.136 27	.911 53	.097 06	.944	.059 32	39
22	.850 06	.176 38	.880 59	.135 61	.912 06	.096 42	.944 55	.058 7	38
23	.850 57	.175 69	.881 1	.134 94	.912 59	.095 78	.945 1	.058 09	37
24	.851 07	.175	.881 62	.134 28	.913 13	.095 14	.945 65	.057 47	36
25	.851 57	1.174 3	.882 14	1.133 61	.913 66	1.094 5	.946 2	1.056 85	35
26	.852 07	.173 61	.882 65	.132 95	.914 19	.093 86	.946 76	.056 24	34
27	.852 57	.172 92	.883 17	.132 28	.914 73	.093 22	.947 31	.055 62	33
28	.853 07	.172 23	.883 69	.131 62	.915 26	.092 58	.947 86	.055 01	32
29	.853 58	.171 54	.884 21	.130 96	.915 8	.091 95	.948 41	.054 39	31
30	.854 08	1.170 85	.884 73	1.130 29	.916 33	1.091 31	.948 96	1.053 78	30
31	.854 53	.170 16	.885 24	.129 63	.916 87	.090 67	.949 52	.053 17	29
32	.855 09	.169 47	.885 76	.128 97	.917 4	.090 03	.950 07	.052 55	28
33	.855 59	.168 78	.886 28	.128 31	.917 94	.089 4	.950 62	.051 94	27
34	.856 09	.168 09	.886 8	.127 65	.918 47	.088 76	.951 18	.051 33	26
35	.856 6	1.167 41	.887 32	1.126 99	.919 01	1.088 13	.951 73	1.050 72	25
36	.857 1	.166 72	.887 84	.126 33	.919 55	.087 49	.952 29	.050 1	24
37	.857 61	.166 03	.888 36	.125 67	.920 08	.086 86	.952 84	.049 49	23
38	.858 11	.165 35	.888 88	.125 01	.920 62	.086 22	.953 4	.048 88	22
39	.858 62	.164 66	.889 4	.124 35	.921 16	.085 59	.953 95	.048 27	21
40	.859 12	1.163 98	.889 92	1.123 69	.921 7	1.084 96	.954 51	1.047 66	20
41	.859 63	.163 29	.890 45	.123 03	.922 23	.084 32	.955 06	.047 05	19
42	.860 14	.162 61	.890 97	.122 38	.922 77	.083 69	.955 62	.046 44	18
43	.860 64	.161 92	.891 49	.121 72	.923 31	.083 06	.956 18	.045 83	17
44	.861 15	.161 24	.892 01	.121 06	.923 85	.082 43	.956 73	.045 22	16
45	.861 66	1.160 56	.892 53	1.120 41	.924 39	1.081 79	.957 29	1.044 61	15
46	.862 16	.159 87	.893 06	.119 75	.924 93	.081 16	.957 85	.044 01	14
47	.862 67	.159 19	.893 58	.119 09	.925 47	.080 53	.958 41	.043 4	13
48	.863 18	.158 51	.894 1	.118 44	.926 01	.079 9	.958 97	.042 79	12
49	.863 68	.157 83	.894 63	.117 78	.926 55	.079 27	.959 52	.042 18	11
50	.864 19	1.157 15	.895 15	1.117 13	.927 09	1.078 64	.960 08	1.041 58	10
51	.864 7	.156 47	.895 67	.116 48	.927 63	.078 01	.960 64	.040 97	9
52	.865 21	.155 79	.896 2	.115 82	.928 17	.077 38	.961 2	.040 36	8
53	.865 72	.155 11	.896 72	.115 17	.928 72	.076 76	.961 76	.039 76	7
54	.866 23	.154 43	.897 25	.114 52	.929 26	.076 13	.962 32	.039 15	6
55	.866 74	1.153 75	.897 77	1.113 87	.929 8	1.075 5	.962 88	1.038 55	5
56	.867 25	.153 08	.898 3	.113 21	.930 34	.074 87	.963 44	.037 94	4
57	.867 76	.152 4	.898 83	.112 56	.930 88	.074 25	.964	.037 34	3
58	.868 27	.151 72	.899 35	.111 91	.931 43	.073 62	.964 57	.036 74	2
59	.868 78	.151 04	.899 88	.111 26	.931 97	.072 99	.965 13	.036 13	1
60	.869 29	1.150 37	.900 4	1.110 61	.932 52	1.072 37	.965 69	1.035 53	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	49°		48°		47°		46°		

	44°				44°				44°		
′	Tang.	Co-tang.	′	′	Tang.	Co-tang.	′	′	Tang.	Co-tang.	′
0	.965 69	1.035 53	60	21	.977 56	1.022 95	39	41	.989 01	1.011 12	19
1	.966 25	1.034 93	59	22	.978 13	1.022 36	38	42	.989 59	1.010 53	18
2	.966 81	1.034 33	58	23	.978 7	1.021 76	37	43	.990 16	1.009 94	17
3	.967 38	1.033 72	57	24	.979 27	1.021 17	36	44	.990 73	1.009 35	16
4	.967 94	1.033 12	56	25	.979 84	1.020 57	35	45	.991 31	1.008 76	15
5	.968 5	1.032 52	55	26	.980 41	1.019 98	34	46	.991 89	1.008 18	14
6	.969 07	1.031 92	54	27	.980 98	1.019 39	33	47	.992 47	1.007 59	13
7	.969 63	1.031 32	53	28	.981 55	1.018 79	32	48	.993 04	1.007 01	12
8	.970 2	1.030 72	52	29	.982 13	1.018 2	31	49	.993 62	1.006 42	11
9	.970 76	1.030 12	51	30	.982 7	1.017 61	30	50	.994 2	1.005 83	10
10	.971 33	1.029 52	50	31	.983 27	1.017 02	29	51	.994 78	1.005 25	9
11	.971 89	1.028 92	49	32	.983 84	1.016 42	28	52	.995 36	1.004 67	8
12	.972 46	1.028 32	48	33	.984 41	1.015 83	27	53	.995 94	1.004 08	7
13	.973 02	1.027 72	47	34	.984 99	1.015 24	26	54	.996 52	1.003 5	6
14	.973 59	1.027 13	46	35	.985 56	1.014 65	25	55	.997 1	1.002 91	5
15	.974 16	1.026 53	45	36	.986 13	1.014 06	24	56	.997 68	1.002 33	4
16	.974 72	1.025 93	44	37	.986 71	1.013 47	23	57	.998 26	1.001 75	3
17	.975 29	1.025 33	43	38	.987 28	1.012 88	22	58	.998 84	1.001 16	2
18	.975 86	1.024 74	42	39	.987 86	1.012 29	21	59	.999 42	1.000 58	1
19	.976 43	1.024 14	41	40	.988 43	1.011 7	20	60	1	1	0
20	.977	1.023 55	40								
′	Co-tang.	Tang.	′	′	Co-tang.	Tang.	′	′	Co-tang.	Tang.	′
	45°				45°				45°		

Preceding Table contains Natural Tangents and Co-tangents for every minute of the quadrant, to the radius of 1.

If Degrees are taken at head of columns, Minutes, Tangents, and Co-tangents must be taken from head also; and if they are taken at foot of columns, Minutes, etc., must be taken from foot also.

ILLUSTRATION.—.1974 is tangent for 11° 10′, and co-tangent for 78° 50′.

To Compute Tangents and Co-tangents for Seconds.

Ascertain tangent or co-tangent of angle for degrees and minutes from Table; take difference between it and tangent or co-tangent next below it.

Then as 60 seconds is to difference, so are seconds given to result required, which is to be added to tangent and subtracted from co-tangent.

ILLUSTRATION.—What is the tangent and co-tangent of 54° 40′ 40″?

Tangent of 50° 40′, per Table = 1.410 61 }
Tangent of 54° 41′, " = 1.411 48 } .000 87 *difference.*

Then 60 : .000 87 :: 40 : .000 58, which, added to 1.410 61 = 1.411 19 *tangent.*

Co-tangent of 54° 40′, per Table = .708 91 }
Co-tangent of 54° 41′, " = .708 48 } .000 43 *difference.*

Then 60° : .000 43 :: 40 : 29, which, subtracted from .708 91 = .708 62 *co-tangent.*

To Compute Tangent or Co-tangent of any Angle in Degrees, Minutes, and Seconds.

Divide Sine by Cosine for Tangent, and Cosine by Sine for Co-tangent.

EXAMPLE.—What is tangent of 25° 18′?

$$\text{Sine} = .427\,36;\ \text{cosine} = .904\,08 \quad \text{Then } \frac{.427\,36}{.904\,08} = .4727 \ tangent.$$

To Compute Number of Degrees, Minutes, and Seconds of a given Tangent or Co-tangent.

When Tangent is given.—Proceed as by Rule, page 402, for Sines, substituting Tangents for Sines.

EXAMPLE.—What is tangent for 1.411 19?

Next less tangent is 1.410 61, *arc for which is* 54° 40′ Next greatest tangent is 1.411 48, *difference between which and next less is* .000 87.

Difference between less tabular tangent and one given is 1.410 61 — 1.411 19 = .000 58.
Then .000 87 : .000 58 :: 60 : 40, which, added to 54° 40′ = 54° 40′ 40″.

When Co-tangent is given.—Proceed as by Rule, page 402, for Cosines, substituting Co-tangents for Cosines.

AEROSTATICS.

Atmospheric Air consists, by volume, of Oxygen 21, and Nitrogen 79 parts; and in 10 000 parts there are 4.9 parts of Carbonic acid gas. By weight, it consists of 77 parts of Oxygen, and 23 of Nitrogen.

One cube foot of Atmospheric Air at surface of Earth, when barometer is at 30 ins., and at a temperature of 32°, weighs 565.0964 grains = .080 728 lbs. avoirdupois, being 773.19 times lighter than water.

Specific gravity compared with water, at 62.418 = .001 293 345.

Mean weight of a column of air a foot square, and of an altitude equal to height of atmosphere (barometer 30 ins.), is 2124.6875 lbs. = 14.7548 lbs. per sq. inch = support of 34.0393 feet of water.

Standard pound is computed with a mercurial barometer at 30 ins.; hence, as a cube inch of mercury at 60° weighs .490 776 9 lbs., pressure of atmosphere at 60° = 14.723 307 lbs. per square inch.

12.3873 cube feet of air weigh a pound, and its weight varies about 1 gr. for each degree of heat.

Extreme height of barometer in latitude 30° to 35° N. = 30.21 ins.

Rate of expansion of Air, *and all other Elastic Fluids* for all temperatures, is essentially uniform. From 32° to 212° they expand from 1000 to 1376 volumes = .002 088 or $\frac{1}{479}$th part of their bulk for every degree of heat. From 212° to 680° they expand from 1376 to 2322 = .002 021 for each degree of heat.

Thus, if volume of air at 132° is required. $132° - 32° = 100$, and $1000 + \overline{100 \times .002\,088} = 1209$ *volumes*.

Height, at Equator is estimated at 300 feet greater than at Poles, its mean height at 45° latitude.

In like latitudes, air loses about 1° for every foot in height above level of sea.

Below surface of Earth, temperature increases.

Elasticity of air is inversely as space it occupies, and directly as its density.

When altitude of air is taken in arithmetical proportion, its *Rarity* will be in geometric proportion. Thus, at 7 miles above surface of Earth, air is 4 times rarer or lighter than at Earth's surface; at 14 miles, 16 times; at 21 miles, 64 times, and so on.

Density of an aeriform fluid mass at 32° and at t° will be to each other as $1 + .002\,088\,(t° - 32°)$ is to 1.

For Volume, Pressure, and Density of Air, see Heat, page 521.

Altitude of Atmosphere at ordinary density is = a column of mercury 30 ins. in height, divided by specific gravity of air compared with mercury.

Hence 30 ins. = 2.5 feet, which, divided by .000 094 987, specific gravity of air compared with mercury, = 26 319 *feet* = 4.985 *miles*.

Gay Lussac, Humboldt, and Boussingault estimated it at a minimum of 30 miles, Sir John Herschell 83, Bravais 66 to 100, Dalton 102, and Liais at 180 or 204 miles.

The aqueous vapor always existing in air, in a greater or less quantity, being lighter than air, diminishes its weight in mixing with it; and as, other things equal, its quantity is greater the higher the temperature of the air, its effect is to be considered by increasing the multiplier of t by raising it to .002 22.

Glaisher and Coxwell, in 1862, ascended in a balloon to a height of 37 000 feet.

At temperature of 32°, mean velocity of sound is 1089 feet per second. It is increased or diminished about one foot for each degree of temperature above or below 32°.

Velocity of sound in water is estimated at 4750 feet per second.

Velocity of Sound at Various Temperatures.

°	Per Second.	°	Per Second.	°	Per Second.	°	Per Second.
	Feet.		Feet.		Feet.		Feet.
5	1056	32	1089	68	1122	95	1152
14	1070	50	1102	77	1132	104	1161
23	1079	59	1112	86	1142	113	1171

Motions of air and all gases, by force of gravity, are precisely alike to those of fluids.

Sensation of hearing, or sound, cannot exist in an absolute vacuum. The human voice can be heard a distance of 3300 feet.

Echo.—At a less distance than 100 feet there is not a sufficient interval between the delivery of a sound and its reflection to render one perceptible.

To Compute Velocity of Sound through Air.

$1089 \times 13\sqrt{1 + [.002\,088\,(t - 32)]} = v$ *in feet per second, t representing temperature of air.*

Illustration.—Flash of a cannon from a vessel was observed 13 seconds before report was heard; temperature of air 60°; what was distance to vessel?

$1089 \times 13\sqrt{1 + [.002\,088\,(60° - 32)]} = 1089 \times 13 \times 1.029 = 14\,567.55$ *feet* = 2.76 *miles.*

Theoretical velocity with which air will flow into a vacuum, if wholly unobstructed, is $\sqrt{2gh} = 1347.4$ *feet per second.* In operation, however, it is $1347.4 \times .707 = 952.61$ *feet.*

To Compute Velocity of Air Flowing into a Vacuum.

$\sqrt{2gh} \times c = v$ *in feet per second, c representing coefficient of efflux.*

Coefficients for openings are as follows:

Circular aperture in a thin plate.................................. .65 to .7
Cylindrical adjutage.......... .92 | Conical adjutage.......... .93

Velocity of Sound in Several Solids.

Velocity in Air = 1.

Lead......	3.9	Zinc......	9.8	Pine......	12.5	Glass....	11.9	Steel.....	14.3
Gold......	5.6	Oak.......	9.9	Copper...	11.2	Pine.....	12.5	Iron.....	15.1

To Compute Elevations by a Barometer.

*Approximately** 60 000 (log. B — log. *b*) C = *height in feet; B and b representing heights of barometer at lower and upper stations,* and C *correction due to* T + *t or temperatures of lower and upper stations.*

Values of C or T + *t*.

°	C	°	C	°	C	°	C	°	C	°	C	°	C
40	.973	60	.996	80	1.018	100	1.04	120	1.062	140	1.084	160	1.106
42	.976	62	.998	82	1.02	102	1.042	122	1.064	142	1.087	162	1.108
44	.978	64	1	84	1.022	104	1.044	124	1.067	144	1.089	164	1.111
46	.98	66	1.002	86	1.024	106	1.047	126	1.069	146	1.091	166	1.113
48	.982	68	1.004	88	1.027	108	1.049	128	1.071	148	1.093	168	1.115
50	.984	70	1.007	90	1.029	110	1.051	130	1.073	150	1.096	170	1.117
52	.987	72	1.009	92	1.031	112	1.053	132	1.076	152	1.098	172	1.12
54	.989	74	1.011	94	1.033	114	1.056	134	1.078	154	1.1	174	1.122
56	.991	76	1.013	96	1.036	116	1.058	136	1.08	156	1.102	176	1.124
58	.993	78	1.016	98	1.038	118	1.06	138	1.082	158	1.104	178	1.126

* For more exact formulas, see Tables and Formulas, by Capt. T. S. Lee, U. S. Top. Eng., 1853.

Their *values* vary approximately .0011 per degree.

	Upper Station.	Lower Station.
ILLUSTRATION.—Thermometer	70.4	77.6
Barometer	23.66	30.05

$C = 77.6 + 70.4 = 1.093$, log. $B = 1.4778$, log. $b = 1.374$.

Then $60\,000 \times (1.4778 - 1.374) \times 1.093 = 6807.2$ *feet.*

To Compute Elevations by a Thermometer.

$520\ B + B^2 \times C =$ *height in feet.* B *representing temperature of water boiling at elevated station deducted from* 212°.

Correction for temperatures of air at lower and upper stations, or $T + t$, to be taken from table, page 428, as before.

ILLUSTRATION.—Temperature of water boiling at upper station 192°; temperature of air 50° and 32°. $C = 1.02$.

Then $520 \times \overline{212 - 192} + \overline{212 - 192}^2 \times 1.02 = 11\,010$ *feet.*

To Compute Capacity of a Balloon, etc., see page 218.

Barometer.

Elevations by Barometer Readings. (*Astronomer Royal.*)

Mean Temperature of Air 50°.

For correction for temperature, see note at foot.

Height.	Barom.	Height.	Barom.	Height.	Barom.	Height.	Barom.	Height.	Barom.
Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.
0	31	600	30.325	1500	29.34	4000	26.769	7 000	23.979
50	30.943	650	30.269	1600	29.233	4250	26.524	7 500	23.543
100	30.886	700	30.214	1750	29.072	4500	26.282	8 000	23.115
150	30.83	750	30.159	1800	29.019	4750	26 042	8 500	22.695
200	30.773	800	30.103	2000	28.807	5000	25.804	9 000	22.282
250	30.717	850	30.048	2250	28.544	5250	25.569	9 500	21.877
300	30.661	900	29.993	2500	28.283	5500	25.335	10 000	21.479
350	30.604	1000	29.883	2750	28.025	5750	25.104	10 500	21.089
400	30.548	1100	29 774	3000	27.769	6000	24.875	11 000	20.706
450	30.492	1200	29.665	3250	27.515	6250	24.648	11 500	20.329
500	30.436	1300	29.556	3500	27.264	6500	24.423	12 000	19.959
550	30.381	1400	29.448	3750	27.015	6750	24.2	12 500	19.952

Barometer.

Correction for Capillary Attraction to be added in Inches.

Diameter of tube........	.6	.55	.5	.45	.4	.55	.3	.25	.2	.1
Correction, unboiled.....	.004	005	.007	.01	.014	.02	.025	04	059	.087
Correction, boiled	.002	.003	004	005	.007	01	.014	.02	029	.044

To Compute Height.

RULE.—Subtract reading at lower station from reading at upper station, difference is height in feet.

Table assumes mean temperature of atmosphere to be 50° F. or 10° C. For other temperatures following correction must be applied.

Add together temperatures at upper and lower station. If this sum, in degrees in F., is greater than 100°, *increase* height by $\frac{1}{1000}$ part for every degree of excess above 100°; if sum is less than 100°, *diminish* height by $\frac{1}{1000}$ part for every degree of defect from 100°. Or if sum in C° is greater than 20°, *increase* height by $\frac{1}{550}$ part for every degree of excess above 20°; if sum is less than 20°, *diminish* height by $\frac{1}{550}$ part for every degree of defect from 20°.

Barometer Indications.

Increasing storm.—If mercury falls during a high wind from S. W., S. S. W., W., or S.

Violent but short.—If fall be rapid.

Less violent but of longer continuance.—If fall be slow.

Snow.—If mercury falls when thermometer is low.

Improved weather.—When a gradual continuous rise of mercury occurs with a falling thermometer.

Heavy gales from N.—*Soon* after *first* rise of mercury from a very low point.
Unsettled weather.—With a *rapid* rise of mercury.
Settled weather.—With a *slow* rise of mercury.
Very fine weather.—With a continued *steadiness* of mercury with dry air.
Stormy weather with rain (or snow).—With a rapid and considerable fall of mercury.
Threatening, unsettled weather.—With an alternate rising and falling of mercury.
Lightning *only*.—When mercury is low, storm being beyond horizon.
Fine weather.—With a rosy sky at sunset.
Wind and rain.—When sky has a sickly greenish hue.
Rain.—When clouds are of a dark Indian red.
Foul weather or much wind.—When sky is red in morning.

Weather Glasses.

Explanatory Card. *Vice-Admiral Fitzroy, F. R. S.*

Barometer Rises for Northerly wind (including from N. W. by N. to E.), for dry, or less wet weather, for less wind, or for more than one of these changes—
Except on a few occasions when rain, hail, or snow comes from N. with *strong* wind.

Barometer Falls for Southerly wind (including from S. E. by S. to W.), for wet weather, for stronger wind, or for more than one of these changes—
Except on a few occasions when *moderate* wind with rain (or snow) comes from N.

For change of wind toward Northerly directions, *a Thermometer falls.*

For change of wind toward Southerly directions, *a Thermometer rises.*

Moisture or dampness in air (shown by a Hygrometer) increases *before* rain, fog, or dew.

Add one tenth of an inch to observed height for each hundred feet Barometer is above half-tide level.

Average height of Barometer, in England, at sea-level, is about 29 94 inches; and *average* temperature of air is nearly 50 degrees (latitude of London).

Thermometer falls about one degree for each 300 feet of elevation from ground, but varies with wind.

"When the wind shifts against the sun,
Trust it not, for back it will run."

First rise after very low
Indicates a stronger blow.

Long foretold—long last,
Short notice—soon past.

Rarefaction of Air.

In consequence of rarefaction of air, gas loses of its illuminating power 1 cube inch for each 2.69 feet of elevation above the sea. (*M. Bremond.*)

Clouds.

Classification.—1. *Cirrus*—Like to a feather, commonly termed *Mare's tails.* 2. *Cirro-cumulus*—Small round clouds, termed *mackerel sky.* 3. *Cirro-stratus*—Concave or undulated stratus. 4. *Cumulus*—Conical, round clusters, termed *wool-packs* and *cotton balls.* 5. *Cumulo-stratus*—Two latter mixed. 6. *Nimbus*—A cumulus spreading out in arms, and precipitating rain beneath it. 7. *Stratus*—A level sheet.

NOTE.—*Cirrus* is most elevated.

Height.—Clouds have been seen at a greater height than 37 000 feet.

Velocity.—At an apparent moderate speed, they attain a velocity of 80 miles per hour.

Lightning.

Classification.—1 *Striped* or *Zigzag*—Developed with great rapidity. 2. *Sheet*—Covering a large surface. 3. *Globular*—When the electric fluid appears condensed, and it is developed at a comparatively lower velocity. 4. *Phosphoric*—When the flash appears to rest upon the edges of the clouds.

WEATHER INDICATIONS.

Weather.	*Clouds.*	*Sky.*
Fine and Fair.	Soft or delicate-looking and indefinite outlines.	Gray in morning and light, delicate tints and low dawn.
Wind.	Hard-edged, oily-looking, and tawny or copper-colored, and the more hard, "greasy," and ragged, the more wind.	High dawn, and sunset of a bright yellow.
Wind only.	Light scud alone.	
Rain.	Small and inky.	Sunset of a pale yellow.
Wind and Rain.	Light scud driving across heavy masses.	Orange or copper color.
Rain and Wind.	Hard defined outlines.	Gaudy unusual hues.
Change of Wind.	High upper, cross lower in a direction different to their course or that of wind.	

General.

Fair.—When sea-birds fly early and far out, when dew is deposited, and when a leech, confined in a bottle of water, will curl up at the bottom.

Rain.—Clear atmosphere near to horizon and light atmospheric pressure, or a good "hearing day," as it is termed.

Storm.—When sea-birds remain near to shore or fly inland.

Rain, Snow, or Wind.—When a leech, confined in a bottle of water, will rise excitedly to the surface.

Thunder.—When a leech, confined as above, will be much excited and leave the water.

Value of Indications of Fair Weather, in Days, Compared to one of Rain.

From an extended series of observations. (*Lowe.*)

Profuse Dew	4.5	Mock Sun or Moon	3.3
White Stratus in a valley	7.2	Stars falling abundant	3.2
Colored Clouds at sunset	2.9	Stars bright	3.4
Solar Halo	1.9	Stars dim	1.5
Sun red and rayless	10.3	Stars scintillated	6
Sun pale and sparkling	1	Aurora borealis	1.8
White Frost	4.2	Toads in evening	2.4
Lunar Halo	1	Landrails noisy	13
Lunar burr, or rough-edged	2.8	Ducks and Geese noisy	2.3
Moon dim	2	Fish rising	1.5
Moon rising red	7	Smoke rising vertically	5

For weather-foretelling plants, see page 185.

ATMOSPHERIC AIR.

Very pure air contains Oxygen 20.96, Nitrogen 79, and Carbonic Acid .04.

Air respired by a human being in one hour is about 15 cube feet, producing 500 grains of carbonic acid, corresponding to 137 grains carbon, and during this time about 200 grains of water will be exhaled by the lungs.

During this period there would be consumed about 415 grains of oxygen. In one hour, then, there would be vitiated 73 cube feet pure air.

A man, weighing 150 lbs., requires 930 cube feet of air per hour, in order that the air he breathes may not contain more than 1 per 1000 of carbonic acid (at which proportion its impurity becomes sensible to the nose); he ought, therefore, to have 800 cube feet of well ventilated space.

An adult human being consumes in food from 145 to 165 grains of carbon per hour, and gives off from 12 to 16 cube feet of carbonic acid gas.

An assemblage of 1000 persons will give off in two hours, in vapor, 8.5 gallons water, and nearly as much carbon as there is in 56 lbs. of bituminous coal.

Proportion of Oxygen and Carbonic Acid at following Locations.

Pure Air represented by Oxygen 20.96.

Street in Glasgow	20.895	Metropolitan Railway (underground)	20.6
Regent Street, London	20.865	Pit of a Theatre	20.74
Centre Hyde Park	21.005	Gallery of a Theatre	20.63

Carbonic Acid .04 Per cent.

Open field, Manchester	.0383	Top of Monument, London	.0358
Churchyard	.0323	Hyde Park	.0334
Market, Smithfield	.0446	Metropolitan Railway (underground)	.338
Factory mills	.283	Lake of Geneva	.046
School-rooms	.097	Boys' school	.31*
Pitt of theatre, 11 P. M.	.32	Girls' "	.723†
Boxes " 12 "	.218	Horse stable	.7
Gallery " 10 "	.101	Convict prison	.045

* Roscoe. † Peltenhoffer.

Consumption of Atmospheric Air. (*Coathupe.*)

One wax candle (three in a lb.) destroys, during its combustion, as much oxygen per hour as respiration of one adult.

A lighted taper, when confined within a given volume of atmospheric air, will become extinguished as soon as it has converted 3 per cent. of given volume of air into carbonic acid.

Carbonic Acid Exhaled per Minute by a Man. (*Dr. Smith.*)

During sleep 4.99 per cent., lying down 5.91, walking at rate of 2 miles per hour 18.1, at 3 miles 25.83, hard labor 44.97.

ANIMAL POWER.

Work.

Work is measured by product of the resistance and distance through which its point of application is moved. In performance of work by means of mechanism, work done upon weight is equal to work done by power.

Unit of Work is the *moment* or effect of 1 pound through a distance of 1 foot, and it is termed a *foot-pound*.

In France a kilogrammetre is the expression, or the pressure of a kilogramme through a distance of 1 meter = 7.233 foot-pounds.

Result of observation upon animal power furnishes the following as maximum daily effect:

1. When effect produced varied from .2 to .33 of that which could be produced without velocity during a brief interval.

2. When the velocity varied from .16 to .25 for a man, and from .08 to .066 for a horse, of the velocity which they were capable for a brief interval, and not involving any effort.

3. When duration of the daily work varied from .33 to .5 for a brief interval, during which the work could be constantly sustained without prejudice to health of man or animal; the time not extending beyond 18 hours per day, however limited may be the daily task, so long as it involved a constant attendance.

Men.

Mean effect of power of men working to best practicable advantage, is raising of 70 lbs. 1 foot high in a second, for 10 hours per day = 4200 *foot-pounds* per minute.

Windlass.—Two men, working at a windlass at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

Labor.—A man of ordinary strength can exert a force of 30 lbs. for 10 hours in a day, with a velocity of 2.5 feet in a second = 4500 lbs. raised one foot in a minute = .2 of work of a horse.

A man can travel, without a load, on level ground, during 8.5 hours a day, at rate of 3.7 miles an hour, or 31.25 miles a day. He can carry 111 lbs. 11 miles in a day. Daily allowance of water, 1 gallon for all purposes; and he requires from 220 to 240 cube feet of fresh air per hour.

A porter going short distances, and returning unloaded, can carry 135 lbs. 7 miles a day, or he can transport, in a wheelbarrow, 150 lbs. 10 miles in a day.

Crane.—The maximum power of a man at a crane, as determined by Mr. Field, for constant operation, is 15 lbs., exclusive of frictional resistance. which, at a velocity of 220 feet per minute = 3300 foot-pounds, and when exerted for a period of 2.5 minutes was 17.329 foot-pounds per minute.

Pile-driving.—G. B. Bruce states that, in average work at a pile-driver, a laborer, for 10 hours, exerts a force of 16 lbs., plus resistance of gearing, and at a velocity of 270 feet per minute, making one blow every four minutes.

Rowing.—A man rowing a boat 1 mile in 7 minutes, performs the labor of 6 fully-worked laborers at ordinary occupations of 10 hours per day.

Drawing or Pushing.—A man drawing a boat in a canal can transport 110 000 lbs. for a distance of 7 miles, and produce 156 times the effect of a man weighing 154 lbs., and walking 31.25 miles in a day; and he can push on a horizontal plane 20 lbs. with a velocity of 2 feet per second for 10 hours per day.

Tread-mill.—A man either inside or outside of a tread-mill can raise 30 lbs. at a velocity of 1.3 feet per second for 10 hours, = 1 404 000 foot-pounds.

Pulley.—A man can raise by a single pulley 36 lbs., with a velocity of .8 of a foot per second, for 10 hours.

Walking.—A man can pass over 12.5 times the space horizontally that he can vertically, and, according to J. Robison, by walking in alternate directions upon a platform supported on a fulcrum in its centre, he can, weighing 165 lbs., produce an effect of 3 984 000 foot-pounds. for 10 hours per day.

Pump, Crank, Bell, and Rowing.—Mr. Buchanan ascertained that, in working a pump, turning a crank, ringing a bell, and rowing a boat, the effective power of a man is as the numbers 100, 167, 227, and 248.

Pumping.—A practised laborer can raise, during 10 hours, 1 000 000 lbs. water 1 foot in height, with a properly designed and constructed pump.

Crank.—A man can exert on the handle of a screw-jack of 11 inches radius for a short period a force of 25 lbs., and continuously 15 lbs., a net power of 20 lbs. Mr. J. Field's tests gave 11.5 lbs. as easily attained, 17.3 as difficult, and 27.6 with great difficulty.

Mowing.—A man can mow an acre of grass in 1 day.

Reaping.—A man can reap an acre of wheat in 2 days.

Ploughing.—A man and horse .8 of an acre per day.

Day's Work. (*D. K. Clark.*)

Laborer.—Carrying bricks or tiles, net load 106 lbs. = 600 lbs. 1 mile.

Carrying coal in a mine, net load 95 to 115 lbs. = 342 lbs. 1 mile.

Loading coke into a wagon, net load 100 lbs. = 270 lbs. 1 mile.

Loading a boat with coal, net load 190 lbs. = 1230 lbs. 1 mile, or 20 cube yards of earth in a wagon.

Digging stubble land .055 of an acre per day, or 2000 cube feet of superficial earth.

Breaking 1.5 cube yards hard stone into 2 inch cubes.

Quarrying.—A man can quarry from 5 to 8 tons of rock per day.

A foot-soldier travels in 1 minute, in common time, 90 steps = 70 yards.

He occupies in ranks a front of 20 inches, and a depth of 13, without a knapsack; interval between the ranks is 13 inches.

Average weight of men, 150 lbs. each, and five men can stand in a space of 1 square yard.

Effective Power of Men for a Short Period.

Manner of Application.	Force.	Manner of Application.	Force.
	Lbs.		Lbs.
Bench-vice or Chisel............	72	Screw-driver, one-hand	84
Drawing-knife or Auger.........	100	Small screw driver	14
Hand-plane	50	Thumb and fingers..............	14
Hand-saw.......................	36	Windlass or Pincers	60

The muscles of the human jaw exert a force of 534 lbs.

Mr. Smeaton estimated power of an ordinary laborer at ordinary work was equivalent to 3762 foot-pounds per minute. But, according to a particular case made by him in the pumping of water 4 feet high, by good English laborers, their power was equivalent to 3904 foot-pounds per minute; and this he assigned as twice that of ordinary persons promiscuously operated with.

Mr. J. Walker deduced from experiments that the power of an ordinary laborer, in turning a crank, was 13 lbs., at a velocity of 320 feet per minute for 8 hours per day.

Amount of Labor produced by a Man. (*Morin.*)

For 10 hours per day.

MANNER OF APPLICATION.	Power.	Velocity per Second.	Weight raised. Feet per Minute.	IP for Period given.
	Lbs.	Feet.	Lbs.	No.
Throwing earth with a shovel, a height of 5 feet..	6	1.33	480	8.7
Wheeling a loaded barrow up an inclined plane, 1 to 12.......................................	132	.625	4950	90
Raising and pitching earth in a shovel 13 feet horizontally	6	2.25	810	14.7
Pushing and drawing alternately in a vertical direction	13	2.5	1950	35.5
Transporting weight upon a barrow, and returning unloaded..................................	132	1	7920	144
FOR 8 HOURS PER DAY.				
Ascending a slight elevation, unloaded...........	143	.5	4290	62
Walking, and pushing or drawing in a horizontal direction	26	2	3120	45.2
Turning a crank	18	2.5	2790	39
Upon a tread-mill*..............................	140	.5	4200	61.1
Rowing..	26	5	7800	113
FOR 7 HOURS PER DAY.				
Walking with a load upon his back..............	88	2.5	13200	160.5
FOR 6 HOURS PER DAY.				
Transporting a weight upon his back, and returning unloaded..............................	140	1.75	14700	160.5
Transporting a weight upon his back up a slight elevation, and returning unloaded.............	140	.2	1680	19
Raising a weight by his hands	44	.5	1320	14.4

* Morin gives *amount* of labor of a man upon tread-mill, in an individual case, at 140 lbs., at a velocity of .5 feet per second for 8 hours per day = 70 lbs. at 1 foot per second; hence 70 ÷ 1.3 feet as

To Compute Number of Men to Perform Work upon a Tread-mill or Pile-driver.

RULE.—To product of weight to be raised and radius of crank, add friction of wheel, and divide sum by product of power and radius of wheel.

EXAMPLE.—How many men are required upon a tread-mill, 20 feet in diameter, to raise a weight of 9233.33 lbs., crank 9 inches in length, weight of wheel and its load estimated at 5000 lbs., and friction at .015.

Weight of a man assumed at 25 lbs. Radius of crank .75 feet.

Effect of a man on a tread-mill, page 433, 30 lbs. at a velocity of 1.3 feet per second, $= 1.3 \times 60 = 78$ *feet per minute.*

$9233.33 \times .75 + \overline{5000 \times .015} = 7000$ *lbs. resistance of load and wheel*, and $7000 \div \frac{78}{20 \times 3.1416} \times 10 \times 30 = 7000 =$ *load and weight* $\div$ *product of power increased by its velocity over load, radius of wheel and power* $= 7000 \div \overline{1.241 \times 10 \times 30} = 18.8$ *men.*

Horse.

Amount of Labor produced by a Horse under different Circumstances. (*Morin.*)

For 10 hours per day.

MANNER OF APPLICATION.	Power.	Velocity per Second.	Weight drawn. Feet per Minute.	HP for Period given.
	Lbs.	Feet.	Lbs.	No.
Drawing a 4-wheeled carriage at a walk	154	3	27 720	504
With load upon his back at a walk	264	3.75	59 400	1080
Transporting a loaded wagon, and returning unloaded at a walk	1540	2	184 800	3360
Drawing a loaded wagon at a walk	1540	3.75	346 500	6300
FOR 8 HOURS PER DAY.				
Upon a revolving platform at a walk	100	3	18 000	260.8
FOR 4.5 HOURS PER DAY.				
Upon a revolving platform at a trot	66	6.75	26 730	218.7
Drawing an unloaded 4-wheeled carriage at a trot.	97	7.25	43 195	353.5
Drawing a loaded 4-wheeled carriage at a trot	770	7.25	334 950	2741

If traction power of a horse, when continuously at a walk, is equal to 120 lbs., and grade of road 1 in 30, resistance on a level being one thirtieth of load, he can draw a load of $120 \times 30 \div 2 = 1500$ lbs.

Street Rails or Tramways. (*Henry Hughes.*)

Cars, 26 lbs. per ton, or 1 to 86 as a mean.

Performance of Horses in France. (*M. Charié-Marsaines.*)

SEASON.	Road.	Weight per Horse.	Speed per Hour.	Work per Hour, drawn One Mile.	Ratio of Pavement to Macadam.
		Tons.	Miles.	Ton-miles.	
Winter	Pavement	1.306	2.05	2.677	1.644 to 1
	Macadam	.851	1.91	1.625	
Summer	Pavement	1.395	2.17	3.027	1.229 to 1
	Macadam	1.141	2.16	2.464	

Average daily work of a Flemish horse in North of France, where country is flat and loads heavy, is, on same authority, as follows:

Winter, 21.82 ton-miles per day.
Summer, 27.82 " " } Mean for the year, 25.

given in example = 53.8 lbs., from which a deduction is to be made for excess of amount of labor that can be performed in 8 hours over 10. Or, as 10 : 8 :: 53.8 : 43.04 lbs., which does not essentially differ from effect of 30 lbs. for that of an average performance.

Greatest mechanical effect of an ordinary horse is produced in operating a gin or drawing a load on a railroad, when travelling at rate of 2.5 miles per hour, where he can exert a tractive force of 150 lbs. for 8 hours per day.

Horse upon Turnpike Road.

At a speed of 10 miles per hour, a horse will perform 13 miles per day for 3 years. In ordinary staging, a horse will perform 15 miles per day.

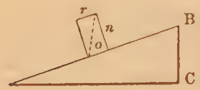
To Compute Tractive Power of a Horse Team, see Traction, page 848.

Assuming maximum load that a horse can draw on a gravel road as a standard, he can draw,

On best-broken stone road	2 to 3 times.
On a well-made stone pavement	3 to 5 "
On a stone trackway	7 to 8 "
On plank road	4 to 12 "
On a railway	18 to 20 "

NOTE.—Track of an iron railway compared with a plank-road is as 27 to 10.

To Compute Power of Draught of a Horse at Different Elevations.



Let A B C represent an inclined plane, *o* weight of a horse which, being resolved into two component forces, one of which, *n*, is perpendicular to plane of inclination, and other, *r*, is parallel to it.

Hence, *r* represents force which horse must overcome to move his own weight.

Then, by similar triangles, A C or l : B C or h :: o : r. Or, $\frac{h\,o}{l} = r$.

If t represents tractive power of horse, upon a level, of 100 lbs., t' tractive power upon a plane of inclination, and r that part of force exerted by horse which is expended upon his own body, then $t' = t - r$, or $t - \frac{h\,o}{l} = t'$ in *lbs.*

ILLUSTRATION.—If inclination is 1 in 50.
Assume $t = 100$, weight of horse 900 lbs., and $l = 50.01$.

$$\text{Then, } 100 - \frac{1 \times 900}{50.01} = 100 - 17.99 = 82.01 \textit{ lbs.}$$

Assuming load that a horse can draw on a level at 100, he can draw upon inclinations as follows:

1 in 100	91	1 in 75	88	1 in 50	82	1 in 35	74	1 in 20	55
1 " 90	90	1 " 70	87	1 " 45	80	1 " 30	70	1 " 15	40
1 " 80	89	1 " 60	85	1 " 40	77	1 " 25	64	1 " 10	10

On his back a horse can carry from 220 to 390 lbs., or about 27.5 per cent. of his weight.

Labor.—The work of a horse as assigned by Boulton & Watt, Tredgold, Rennie, Beardmore, and others, ranges from 20 000 to 39 320 foot-pounds per minute for 8 hours, a mean of 27 750 lbs.

A horse can travel, at a walk, 400 yards in 4.5 minutes; at a trot, in 2 minutes; and at a gallop, in 1 minute. He occupies in ranks, a front of 40 ins., and a depth of 10 feet; in a stall, from 3.5 to 4.5 feet front; and at a picket, 3 feet by 9; and his average weight = 1000 lbs.

Carrying a soldier and his equipments (225 lbs.) he can travel 25 miles in a day of 8 hours.

A draught-horse can draw 1600 lbs. 23 miles a day, weight of carriage included.

Ordinary work of a horse may be stated at 22 500 lbs., raised 1 foot in a minute, for 8 hours per day.

In a mill, he moves at rate of 3 feet in a second. Diameter of track should not be less than 25 feet.

Rennie ascertained that a horse weighing 1232 lbs. could draw a canal-boat at a speed of 2.5 miles per hour, with a power of 108 lbs., 20 miles per day. This is equivalent to a work of 23 760 foot-lbs. per minute, He estimated that the average work of horses, strong and weak, is at the rate of 22 000 foot-lbs. per minute.

From results of trials upon strength and endurance of horses at Bedford, Eng., it was determined that average work of a horse = 20 000 foot-lbs. per minute. A good horse can draw 1 ton at rate of 2.5 miles per hour, from 10 to 12 hours per day.

Expense of conveying goods at 3 miles per hour, per horse teams being 1, expense at 4.33 miles will be 1.33, and so on, expense being doubled when speed is 5.125 miles per hour.

Strength of a horse is equivalent to that of 5 men, and his daily allowance of water should be 4 gallons.

Amount of Labor a Horse of average Strength is capable of performing, at different Velocities, on Canal, Railroad, and Turnpike.

Traction estimated at 83.3 lbs.

Velocity per Hour.	Duration of Work.	Useful Effect, drawn 1 Mile. On a Canal.	On a Railroad.	On a Turnpike.	Velocity per Hour.	Duration of Work.	Useful Effect, drawn 1 Mile. On a Canal.	On a Railroad.	On a Turnpike.
Miles.	Hours.	Tons.	Tons.	Tons.	Miles.	Hours.	Tons.	Tons.	Tons.
2.5	11.5	520	115	14	6	2	30	48	6
3	8	243	92	12	7	1.5	19	41	5.1
4	4.5	102	72	9	8	1.125	12.8	36	4.5
5	2.9	52	57	7.2	10	.75	6.6	28.8	3.6

Actual labor performed by horses is greater, but they are injured by it.

Tractive Power of a horse decreases as his speed is increased, and within limits of low speed, or up to 4 miles per hour, it decreases nearly in an inverse ratio.

For 10 Hours per Day.

Miles.	Traction.	Miles.	Traction.	Miles.	Traction.	Miles.	Traction.
Per Hour.	Lbs.	Per Hour.	Lbs.	Per Hour.	Lbs.	Per Hour.	Lbs.
.75	330	1.5	165	2.25	110	3	82
1	250	1.75	140	2.5	100	3.5	70
1.25	200	2	125	2.75	90	4	62

For Ordinary or Short Periods. (Molesworth.)

Miles per hour..........	2	3	3.5	4	4.5	5
Power in lbs............	166	125	104	83	62	41

Mule. (*D. K. Clark.*)

Load on back, 170 to 220 lbs. day's work = 6400 lbs. 1 mile; 400 lbs. at 2.9 miles per hour = 5300 lbs. 1 mile, and 330 lbs. at 2 miles per hour = 5000 lbs. 1 mile.

Upon a revolving platform, at a velocity of 3 feet per second, = 11 880 lbs. raised one foot per minute, or 172.2 HP for 8 hours per day

Ass.

Load on back, 176 lbs. carried 19 miles day's work = 3300 lbs. 1 mile.

In Syria an ass carries 450 to 550 lbs. grain.

Upon a revolving platform, at a velocity of 2.75 feet per second, = 5280 lbs. raised one foot per minute, or 76.5 HP for 8 hours per day.

Ox.

An Ox, walking at a velocity of 2 feet in a second (1.36 miles per hour), exerts a power of 154 lbs., = 18 480 lbs. raised one foot per minute, or 268.8 HP for 8 hours per day.

A pair of well-conditioned bullocks in India have performed work = 8000 foot-lbs. per minute.

Camel.

Load on back, 550 lbs. carried 30 miles per day for 4 days, 4 days' work 16 500 lbs. 1 mile, for 5 days 13 000 lbs. 1 mile = 44 HP for 10 hours per day.

Load of a Dromedary, 770 lbs.

Llama.

Load on back, 110 lbs., day's work 2000 to 3000 lbs. 1 mile = .5 to .75 HP for 10 hours per day.

Birds and Insects.

Area of their wing surface is in an inverse ratio to their weight.

Assuming weight of each of the following Birds to be one pound, and each Insect one ounce, the relative area of their wing surface proportionate to that of their actual weight would be as follows (*M. De Lucy*):

	Sq. ft.		Sq. ft.		Sq. ft.		Sq. ft.
Swallow	4.85	Pigeon	1.27	Gnat	3.05	Cockchafer	.32
Sparrow	2.7	Vulture	.82	Dragon-fly, sm'll,	1.83	Bee	.33
Turtle-dove	2.13	Crane, Australia,	.41	Lady-bird	1.66	Meat-fly	.35

Crocodile and Dog.

The direct power of their jaws is estimated at 120 lbs. for the former and 44 for the latter, which, with the leverage, will give respectively 6000 and 1500 lbs.

PERFORMANCES OF MEN, HORSES, ETC.

Following are designed to furnish an authentic summary of the fastest or most successful *recorded* performances in each of the feats, etc., given.

MAN. Walking.

1874, *Wm. Perkins*, London, Eng., .5 mile, in 2 *min.* 56 *sec.*; 1, in 6 *min.* 23 *sec.*; 2, in 13 *min.* 30 *sec.*; 1876, 8, in 59 *min.* 5 *sec.*; 1877, 20, in 2 *hours* 39 *min.* 57 *sec.*

1880, *T. Smith*, London, Eng., 12 miles, in 1 *hour* 31 *min.* 42.4 *sec.*

1881, *C. A. Harriman*, Chicago, Ill., 530 miles, in 5 *days* 20 *hours* 47 *min.*

1851, *J. Smith*, London, Eng., 25 miles, in 3 *hours* 42 *min.* 16 *sec.*

1878, *W. Howes*, London, Eng., 50 miles, in 7 *hours* 57 *min.* 44 *sec.*; 1880, 75 miles, in 13 *hours* 7 *min.* 27 *sec.*, and 100, in 18 *hours* 8 *min.* 15 *sec.*

1880, *John Dobler*, Buffalo, N. Y., 150 miles 850 yards, in 24 *hours*.

1801, *Capt. R. Barclay*, Eng., country road, 90 miles, in 20 *hours* 22 *min.* 4 *sec.*, including rests; 1803, .25 mile, in 56 *sec.*, and Charing Cross to Newmarket, 64, in 10 *hours*, including rests; 1806, 100, in 19 *hours*, including 1 *hour* 30 *min.* in rests; 1809, 1000, in 1000 consecutive *hours*, walking a mile only at commencement of each hour.

1877, *D. O'Leary*, London, Eng., 200 miles, in 45 *hours* 21 *min.* 33 *sec.*

1818, *Jos. Eaton*, Stowmarket, Eng., 4032 quarter miles, in 4032 consecutive quarter *hours*.

1877, *Wm. Gale*, London, Eng., 1500 miles, in 1000 consecutive *hours*, 1.5 miles each *hour*; and 4000 quarter miles, in 4000 consecutive periods of 10 *minutes*.

1882, *Chas. Rowell*, New York, N. Y., 89 miles 1640 yards, in 12 hours.

1882, *Geo. Hazael*, New York, N. Y., 600 miles 220 yards, in 6 *days*.

Running.

1710, *Levi Whitehead*, Branham Moor, Eng., 4 miles, in 19 *min.*

1844, *Geo. Seward*, of U. S., Manchester, Eng., 100 yards, in 9.25 *sec.*

1869, *Geo. Forbes*, Providence, R. I., 150 yards, in 15 *sec.*

1851, *Chas. Westhall*, Manchester, Eng., 150 yards, in 15 *sec.*, and 200, in 19.5 *sec.*

1864, *Jas. Nuttall*, Manchester, Eng., 600 yards, in 1 *min.* 13 *sec.*

1881, *L. E. Myers*, New York, N. Y., 1000 yards, in 2 *min.* 13 *sec.*

1863, *Wm. Lang*, Newmarket, Eng., 1 mile, in 4 *min.* 2 *sec.*, descending ground; Manchester, 2, in 9 *min.* 11.5 *sec.*; 1865, 11 miles 1660 yards, in 1 *hour* 2 *min.* 2.5 *sec.*

1852. *Wm. Howitt*, "American Deer," London, Eng., 10 miles, in 51 *min.* 34 *sec.*, walking last 200 yards, time, if run, 51 *min.* 20 *sec.*; and 15, in 1 *hour* 22 *min.*

1863, *L. Bennett*, "Deerfoot," Hackney Wick, Eng., 12 m., in 1 *hour* 2 *min.* 2.5 *sec.*

1879, *Patrick Byrnes*, Halifax, N. S., 20 miles, in 1 *hour* 54 *sec.*

1880, *D. Donovan*, Providence, R. I., 40 miles, in 4 *hours* 48 *min.* 22 *sec.*

1879, *G. Hazael*, London, Eng., 50 miles, in 6 *hours* 15 *min.* 57 *sec.*

17—, *A Courier*, East Indies, 102 miles, in 24 *hours*.

Jumping, Leaping, etc.

1848, *P. M'Neely*, Petersburg, Ky., 10 jumps, standing, 110 *feet* 4 *ins.*

1854, *J. Howard*, Chester, Eng., 1 jump, board raised 4 ins. in front, running start, with dumb-bells, 5 lbs., 29 *feet* 7 *ins.*

1868, *Geo. M. Kelley*, Corinth, Miss., running, and from a spring board, leaped over 17 horses standing side by side.

1874, *J. Lane*, Dublin, Ireland, running start, 1 jump, without aid, 23 *feet* 1.5 *ins.*

1878, *E. W. Johnson*, Baltimore, Md., standing leap, 5 *feet* 3 *ins.*

1879, *G. W. Hamilton*, Romeo, Mich., dumb-bells, 22 lbs., standing jump, 14 *feet* 5.5 *ins.*; and 1880, dumb-bells, 12 lbs., 3 standing jumps, 39 *feet* 1 *inch.*

1880, *P. Davin*, Dublin, Ireland, running leap, 6 *feet* 2.75 *ins.*

Lifting.

1825, *Thomas Gardner*, of New Brunswick, N. S., a barrel of pork, 320 *lbs.*, under each arm; also transported across a pier an anchor, 1200 *lbs.*

1868, *Wm. B. Curtis*, New York, N. Y., 3239 *lbs.*, in harness.

1881, *D. L. Dowd*, Springfield, Mass., by hands, 1317 *lbs.*

Throwing Weights.

1870, *D. Dinnie*, New York, N. Y., *light stone*, 18 lbs., 43 *feet*; *heavy stone*, 24 lbs., 34 *feet* 6 *ins.*; *heavy hammer*, 24 lbs., 83 *feet* 8 *ins.*; 1872, Aberdeen, Scotland, *light hammer*, 138 *feet*; run, 16 lbs., 162 *feet.*

1877, *M. Davin*, Dublin, Ireland, run, 56 lb. weight, 30 *feet* 2 *ins.*

Swimming.

1835, *S. Bruck*, 15 miles, in a rough sea, in 7 *hours* 30 *min.*

1846, *A Native*, off Sandwich Islands, 7 miles at sea, with a live pig under one arm.

1878, *E. T. Jones*, London, Eng., 100 yards, in 1 *min.* 8.5 *sec.*

1870, *Pauline Rohn*, Milwaukee, Wis., 650 feet, still water, in 2 *min.* 43 *sec.*

1881, *Wm. Beckwith*, London, Eng., 1000 yards, in 15 *min.* 8.5 *sec.*

1872, *J. B. Johnson*, Hendon, Eng., open water, 1 mile, in 28 *min.* 24.6 *sec.*; Agricultural Hall, London, Eng., remained under water, 3 *min.* 35 *sec.*

1875, *Capt. M. Webb*, Dover, Eng., to Calais, France, 23 miles, crossing two full and two half tides = 35 miles, in 21 *hours* 45 *min.*

1880, *J. Strickland*, Melbourne, Australia, plunged 73 *feet* 1 *inch.*

Skating.

1854, *Wm. Clark*, Madison, Wis., 1 mile, in 1 *min.* 56 *sec.*

1868, *John Conyers*, Lake Simcoe, Can., 8 miles, in 18 *min.* 40.5 *sec.*

1876, *E. St. Clair Milliard*, Chicago, Ill., 50 miles, in 4 *hours* 57 *min.* 3 *sec.*

1877, *John Ennis*, Chicago, Ill., 100 yards, calm, in 11.75 *sec.*; 9 laps to a mile, 100 miles, in 11 *hours* 37 *min.* 45 *sec.*; and 145 inside of 19 *hours*.

NOTE.—The *Sporting Magazine*, London, vol. ix., page 135, reports a man in 1767 to have skated a mile upon the Serpentine, Hyde Park, London, in 57 seconds.

HORSE. Trotting.

1878, "Controller," San Francisco, Cal., 10 miles, harness, in 27 *min*, 27.25 *sec.*, and 20 miles, wagon, in 58 *min.* 57 *sec.*

1875, "Steel Grey," Yorkshire, Eng., 10 miles, saddle, in 27 *min.* 56.5 *sec.*

1867, "John Stewart," Boston, Mass., half-mile track, 20 miles, harness, in 58 *min.* 5.75 *sec.*, and 20.5 miles in 59 *min.* 31 *sec.*

1830, "Top Gallant," Philadelphia, Penn., 12 miles, harness, in 38 *min.*

1829, "Tom Thumb," Sunbury Common, Eng., 16.5 miles, harness, 248 lbs., in 56 *min.* 45 *sec.*; and 100 miles, in 10 *hours* 7 *min.*, including 37 *min.* in rests.

1869, "Morning Star," Doncaster, Eng., 18 miles, harness (sulky 100 lbs.), in 57 *min.* 27 *sec.*

1835, "Black Joke," Providence, R. I., 50 miles, saddle, 175 lbs., in 3 *hours* 57 *min.*

1855, "Spangle," Long Island, N. Y., 50 miles, wagon and driver 400 lbs., in 3 *hours* 59 *min.* 4 *sec.*

1837, "Mischief," Jersey City, N. J., to Philadelphia, Penn., 84.25 miles, harness, very hot day and sandy road, in 8 *hours* 30 *min.*

1853, "Conqueror," Long Island, N. Y., 100 miles, harness, in 8 *hours* 55 *min.* 53 *sec.*, including 15 short rests.

1873, *M. Delaney's* mare, St. Paul's, Minn., 200 miles, race track, harness, in 44 *hours* 20 *min.*, including 15 *hours* 49 *min.* in rests.

1834, "Master Burke" and "Robin," Long Island, N. Y., 100 miles, wagon, in 10 *hours*, 17 *min.* 22 *sec.*, including 28 *min.* 34 *sec.* in rests.

Stage-coaching.

1750, *By the Duke of Queensberry*, Newmarket, Eng., 19 miles, in 53 *min.* 24 *sec.*

1830, *London to Birmingham, Eng.*, "Tally-ho," 109 miles, in 7 *hours* 50 *min.*, including stop for breakfast of passengers.

Leaping.*

1821, A horse of Mr. Mane, at Loughborough, Leicestershire, Eng., 173 lbs., over a hedge 6 *feet* in height, 35 *feet.*

1821, A horse of Lieut. Green, Third Dragoon Guards, at Inchinnan, Eng., ridden by a heavy dragoon, over a wall 6 *feet* in height and 1 *foot* in width at top.

1839, "Lottery," Liverpool, Eng., over a wall, 33 *feet.*

1847, "Chandler," Warwick, Eng., over water, 37 *feet.*

NOTE.—The maximum stride of a horse is estimated to be 28 *feet* 9 *ins.*; "Eclipse" has covered 25 *feet.* The maximum stride of an elk is 34 *feet*, and of an elephant 14 *feet.*

Running.

1701, *Mr. Sinclair*, on the Swift at Carlisle, a gelding, 1000 miles, in 1000 consecutive *hours.*

1731, *Geo. Osbaldeston*, Newmarket, 156 lbs., 100 miles, by 16 horses, in 4 *hours* 19 *min.* 40 *sec.*, and 200, by 28 horses, in 8 *hours* 39 *min.*, including 1 *hour* 2 *min.* 56 *sec.* in rests; 1 horse, "Tranby," 16 miles, in 33 *min.* 15 *sec.*

1752, *Spedding's* mare, 100 miles, in 12 *hours* 30 *min.*, for 2 consecutive days.

1754, A Galloway mare of Daniel Corker's, Newmarket, 300 miles, by one rider, 67 lbs., in 64 *hours* 20 *min.*

1761, *John Woodcock*, Newmarket, 100 miles per day, by 14 horses, one each day, for 29 consecutive days.

1814, *An Officer of 14th Dragoons*, Blackwater, 12 miles, 1 horse, in 25 *min.* 11 *sec.*

1868, *N. H. Mowry*, San Francisco, Cal., race track, 160 lbs., 300 miles, by 30 horses (Mexican), in 14 *hours* 9 *min.*, including 40 minutes for rests; the first 200, in 8 *hours* 2 *min.* 48 *sec.*, and the fastest mile in 2 *min.* 8 *sec.*

1869, *Nell Coker*, San Pedro, Texas, 61 miles, in 2 *hours* 55 *min.* 15 *sec.*, including rests.

1870, *John Faylor*, Carson City, Nevada, 50 miles, by 18 horses, in 1 *hour* 58 *min.* 33 *sec.*; and Omaha, Neb., 56 miles, in 2 *hours* 26 *min.*, including rests.

1876, *John Murphy*, New York, N. Y., 155 miles, by 20 horses, in 6 *hours* 45 *min.* 7 *sec.*

1878, *Capt. Salvi*, Bergamo to Naples, Italy, 580 miles, in 10 *days.*

1880, "Mr. Brown," Rancocas, N. J., *aged*, 160 lbs., 10 miles, in 26 *min.* 18 *sec.*

1828, "Chapeau de Paille" (*Arabian*), India, 1.5 miles, 115 lbs., in 2 *min.* 53 *sec.*

183–, *Capt. Horne* (*Arabians*), Madras to Bungalore, India, 200 miles, in less than 10 *hours.*

DOGS. Coursing and Chasing.

A Greyhound and Hare ran 12 miles in 30 *min.*

1794, A Fox, at Brende, Eng., ran 50 miles in 6.5 *hours.*

A Greyhound, at Bushy Park, Eng., leaped over a brook 30 *feet* 6 *ins.*

BIRDS. Flying.

Vulture, 150 miles; *Wild Goose and Swallow*, 90 miles; *Crow*, 25 miles per hour.

1870, *Carrier Pigeons*, Pesth to Cologne, Germany, 600 miles, in 8 *hours.*

1875, *Carrier Pigeon*, Dundee Lake to Paterson, N. J., 3 miles, in 3 *min.* 24 *sec.*

---

* A Salmon can leap a dam 14 *feet* in height.—*Sporting Magazine*, London, vol. xii., page 79.

HORSE-POWER.

Horse-power.—HP is the principal measure of rate at which work is performed. One horse-power is computed to be equivalent to raising of 33 000 lbs. one foot high per minute, or 550 lbs. per second. Or, 33 000 foot-lbs. of work, and it is designated as being Nominal, Indicated, or Actual.

A HP in work is estimated at 33 000 lbs., raised 1 foot in a minute; but as a horse can exert that force for only 6 hours per day, one work HP is equivalent to that of 4.5 horses.

Cheval-vapeur of France is computed to be equivalent to 75 kilogrammeters of work per second, or 7.233 foot-lbs., or $75 \times 7.233 = 542.5$ foot-lbs., which is 1.37 per cent. less than American or English value.

BELTS AND BELTING.

Capacity of belts to transmit power is determined by extent of their adhesion to surface of pulley, and it is very limited in comparison with tensile strength of belt.

Resistance of a belt to slipping depends essentially upon character of surface of pulley, its degree of tension, and width, and as adhesion is in proportion to pressure on surface of pulley, long belts, by having greater weight, give greater adhesion.

Tensile strength of Belting per square inch of section ranges as follows:

Tanned Leather, .186 inch thick, from 2846 to 5000 lbs., or from 530 to 930 lbs. per inch of width; when *spliced* 385 lbs., and when *laced* 210 lbs.

Taking .3 as a factor of safety, 70 and 128 lbs. represent resistance per sq. inch that belts in operation may be subjected to, and they have been run successfully at these tensions.

Raw hide has a tensile strength of 1.5 times that of tanned.

By Experiments of H. R. Towne and Mr. Kirkaldy. (*England.*)

Tensile strength of Single leather belting per square inch of section.

Laced, 960 *lbs.* Riveted, 1740 *lbs.* Solid, 3080 *lbs.*

Norris & Co.—Double, 2 ins., 2942 *lbs.*; 6 ins., 5603 *lbs.*; 12 ins., 14 861 *lbs.* Single, 3.5 ins., 3007 *lbs.*; 5 ins., 4060 *lbs.*; 10 ins., 8846 *lbs.*

Spill's belting, from flax, saturated with an endurable substance, gave tensile strength per inch of width as follows:

No. 1, 5 ins. wide, 1254 *lbs.* No. 2, 5 ins. wide, 1489 *lbs.* No. 3, 10 ins. wide, 1663 *lbs.*

At a velocity of 1000 feet per minute, a width of leather belt of 1 inch will transmit power of 1 horse, and at a velocity of 1800 feet, .56 of an inch will transmit a like power, pulley being fully three feet in diameter, equal to a stress of 33 lbs. per inch of width of belt of ordinary thickness.

To Compute Width of a Leather Belt.

Assuming a well-defined case (where limit of adhesion was ascertained), a belt of ordinary construction (laced), and 9 inches in width, transmitted the power of 15 horses over a pulley 4 feet in diameter, at a velocity of 1800 feet per minute, with an arc of adhesion of 210°, or of .6 or 7.54 feet of circumference, and with an area of 95 square feet of belt per HP.

Hence, $\frac{4400 \text{ to } 5000 \text{ HP}}{d\,v} = w$; *w representing width of belt in inches, d diameter of pulley in feet, and v velocity of belt in feet per minute.*

NOTE.—Thickness of belt should be added to diameter of pulley. Applying these elements to the formulas of 13 different authors, the result varies from 7.85 to 13.5 ins., mean of which is 10.675. For double belting width $= .6\,w$.

ILLUSTRATION.—If HP 25, diameter of pulley 4 feet, and velocity 2250 feet; what should be width of belt?

$$\frac{4500 \times 25}{4 \times 2250} = 12.5 \textit{ ins. for ordinary thickness of } .1875 \textit{ in.}$$

To Compute Elements of Belting.

$$\frac{v\,w}{1000} = \text{HP}; \quad \frac{\text{HP}\;33\,000}{v\,w} = \text{P}; \quad \frac{33\,000\;\text{HP}}{v} = \text{W}; \quad \frac{\text{W}}{t} = \text{S}; \quad \frac{\text{A}-a}{l\,t} = l.$$

P *representing power or stress transmitted,* W *weight or stress on belt,* t *thickness of belt,* S *stress on belt per inch of width,* A *and* a *areas of coil and eye, and* l *length in feet.*

NOTE.—70 square feet of good belting are capable of transmitting an indicated HP.

India Rubber Belting. (*Vulcanized.*)

Results of Experiments upon Adhesion of India Rubber and Leather Belting.—(J. H. Cheever).

Rubber.	Lbs.	*Leather.*	Lbs.
Rubber belt slipped on iron pulley at	90	Leather belt slipped on iron pulley at	48
" " leather "	128	" " leather "	64
" " rubber "	183	" " rubber "	128

Hence it appears that a Rubber Belt for equal resistances with a Leather Belt may be reduced respectively 46, 50, and 30 per cent.

Iron Wire.—A wire rope .375 inch in diameter, over a pulley 4 feet in diameter, and running at a velocity of 1250 feet per minute, will transmit 4.5 HP.

Diameter of pulley should not be less than 140 times diameter of rope, in order to avoid undue bending of wires.

A sheet-iron belt 7 inches in width proved more effective than one of leather of like width.

General Notes.

Leather Belts—Are best when oak tanned, should be frequently oiled,* and when run with hair side over pulley will give greatest adhesion.

Ordinary thickness .1875 inch, and weight 60 lbs. per cube foot.

Relative effect of different pulleys and belts:

Pulleys.—Leather surface	1.	Turned iron	.64
Rough iron	.41	Turned wood	.7

Tensile strength of calf and sheep skins is about one half that of beeve and horse.

Morin assigns 50 lbs. as a proper stress per inch of width of good belting.

Presence of small holes in a belt will prevent its slipping or squealing.

Rubber Belts.—Best vulcanized rubber is stronger than leather, and its resistance is from 50 to 85 per cent. greater.

To increase adhesion, coat driving surface with boiled oil or cold tallow, and then apply powdered chalk.

When new, cut them .1875 inch short for each foot in length required, to admit of the stretch that occurs in their early operation.

They should be kept free from contact with an animal oil.

Three ply, .1875 inch thick, has a tensile resistance of 600 lbs. per inch of width.

Relative slipping of a vulcanized belt, over smooth or turned leather or rubber-faced iron pulleys is as .5, .7, and 1.

Rubber, Gutta percha, and Canvas belts will stretch continuously.

Memoranda.

Belts should be set as near horizontal as practicable, in order that the sag may increase adhesion on pulley, and hence power should be communicated through under side.

The "creeping" or lost speed by belts is about 2 per cent., hence, to maintain a uniform or required speed, driver must be increased in diameter *pro rata* with slip.

* See *Cements, etc.*, page 871, for compositions, etc.

A belt, 11 ins. in width, over a driver 4 feet in diameter, running from 1200 to 2250 feet per minute, will transmit the power from two steam cylinders, 6 ins. in diameter and 11 ins. stroke, averaging 125 revolutions per minute, with a pressure of 60 lbs. per sq. inch.

A double belt, 75 ins. in width and 153.5 feet in length, transmitted 650 IHP.

Pulleys should have a slight convexity of surface. Authorities differ, from .5 inch per foot of breadth to .1 of breadth. Belts run at a high speed are less liable to slip than at low speed.

The best speeds for economy are from 1200 to 1500 feet per minute, and the best for result not to exceed 1800.

Belts.—Leather, hair-side.... 1	Leather, flesh-side... .74	Rubber...... .51
Gutta percha .... .44	Canvas .... .35	

Coefficient of Friction of a Belt in operation is assumed to be .423.

Smooth surface belts are most endurable and soft most adherent.

Round belts .25 and .5 inch in diameter are fully equal in operation to flat of 1 and 3 ins., and grooves in their pulleys should be angular or V shaped.

The neutral point of a rope belt is at .33 of diameter from inside surface.

Friction of driving and pulley bearings is about .025.

A fan-blower No. 6*, driven by a belt 3.875 ins. in width and .18 in thickness, at a velocity of 2820 revolutions per minute, requires power of 9.7 horses.

Area of belts per IP varies essentially, ranging from 25 to 100 square feet; the mean is 75.

BLASTING.

In Blasting, rock requires from .25 to 1.5 lbs. gunpowder per cube yard, according to its degree of hardness and position. In small blasts 2 cube yards have been rent and loosened, and in very large blasts 2 to 4 cube yards have been rent and loosened, by 1 lb. of powder.

Tunnels and shafts require 1.5 to 2 lbs. per cube yard of rock.

Gunpowder has an explosive force varying from 40000 to 90000 lbs. per sq. inch. That used for blasting is much inferior to that used for projectiles, the proportion being fully one third less.

Nitro-glycerine is an unctuous liquid, which explodes by concussion, an extreme pressure (2000 lbs. per sq. inch), or a temperature exceeding 600° if quickly applied to it; it will inflame, however, and burn gradually.

At a temperature below 40° it solidifies in crystals.

Its explosion is so instantaneous that in rock-blasting tamping is not necessary; its explosive power by weight is from 4 to 5 times that of gunpowder.

Dynamite is nitro-glycerine 75 parts, absorbed in 25 parts of a siliceous earth termed kieselguhr; it also explodes so instantaneously as to render tamping in blasting quite unnecessary.

It is insoluble in water, and may be used in wet holes; it congeals at 40°, is rendered ineffective at 212°, and has an explosive force by weight of 3 times that of gunpowder, and by bulk 4.25 times.

Gun-cotton is insoluble in water, and has an explosive force by weight of from 2.75 to 3 times that of gunpowder, and by bulk 2.5 times. It may be detonated in a wet state with a small quantity of dry material.

Tonite is nitrated gun-cotton, and is known also as *cotton powder*. It is produced in a granulated form.

Litho-fracteur is a nitro-glycerine compound in which a portion of the base or absorbent material is made explosive by the admixture therein of nitrate of baryta and charcoal.

---

* For a table of Belts for Fan-blowers, etc., see J. H. Cooper, in "Jour. Franklin Inst.," vol. 66, p. 409.

Cellulose Dynamite is when gun-cotton is used as the absorbent for nitro-glycerine; it will explode frozen dynamite, and is more sensitive to percussion than it.

To Compute Charge of Gunpowder for Rock Blasting.

Rule.—Divide cube of line of least resistance by 25, as for limestone, to 32 for granite, and remainder will give charge of powder in lbs.

$$\text{Or, } L^3 \div 32 = lbs.$$

Example.—When line of least resistance is 6 feet, what is charge required?

$$6^3 \div 32 = 6.75\ lbs.$$

Line of least resistance should not exceed .5 depth of hole.

Tamping.—*Dried clay* is the most effective of all materials for tamping; *Broken Brick* the next, and *Loose Sand* the least.

Relative Costs of a Tunnel and Shaft in England. (*Sir John Burgoyne.*)

Iron and steel	8.98	Powder	29.04
Smiths and coal	6	Labor	48.8
Fuses	7.18		100

Weight of Explosive Materials in Holes of Different Diameters.
Per Inch of Length.

Diam.	Powder or Gun-cotton.	Dynamite.	Diam.	Powder or Gun-cotton.	Dynamite.	Diam.	Powder or Gun-cotton.	Dynamite.
Ins.	Oz.	Oz.	Ins.	Oz.	Oz.	Ins.	Oz.	Oz.
1	.419	.67	1.75	1.283	2.053	2.5	2.618	4.189
1.25	.654	1.046	2	1.675	2.68	2.75	3.166	5.066
1.5	.942	1.507	2.25	2.12	3.392	3	3.769	6.03

Boring Holes in Granite.

Diam. of Jumper.	Depth of Hole.	Men.	Depth bored per Day.	Hammer.	Diam. of Jumper.	Depth of Hole.	Men.	Depth bored per Day.	Hammer.
Ins.	Ins.	No.	Feet.	Lbs.	Ins.	Ins.	No.	Feet.	Lbs.
1	1 to 2	1	8	6	2.25	5 to 10	3	6	16
1.75	2.5 to 6	3	12	14	2.5	9 to 12	3	5	16
2	4 to 7	3	8	14	3	9 to 15	3	4	18

Drill.—Width of bit compared to stock .625.

Charges of Powder.

Usual practice of charging to one third depth of hole is erroneous, inasmuch as volume of charge increases as square of diameter of hole. Hence holes of 1.5 and 2 inches, although of equal depths, would require charges in proportion of 2.25 and 4.

Line of least resistance.	Powder.	Line of least resistance.	Powder.		Line of least resistance.	Powder.		Line of least resistance.	Powder.	
Feet.	Oz.	Feet.	Lbs.	Oz.	Feet.	Lbs.	Oz.	Feet.	Lbs.	Oz.
1	.75	3		13.5	5	3	14.5	7	10	11.5
2	4	4	2		6	6	12	8	16	

Effects.

Gunpowder.—From its gradual combustion, rends and projects rather than shatters.

A hole 5.5 ins. in diameter and 19 feet 7 ins. in depth, filled to 8 feet 10 ins. with 75 lbs. powder, has removed and rent 1200 cube yards, equal to 2400 tons. The labor expended was that of 3 men for 14 days.

Temperature of gases of explosion 4000°.

Gun-cotton.—From the rapidity of its combustion, shatters.

Dynamite.—From the greater rapidity of its combustion over gun-cotton, is more shattering in its explosion.

Drilling.

Churn-drilling.—A churn-driller will drill, in ordinary hard rock, from 8 to 12 feet, 2 inch holes of 2.5 feet depth, per day, and at a cost of from 12 to 18 cents per foot, on a basis of ordinary labor at $1 per day. Drillers receiving $2.50.

One man can bore, with a bit 1 inch in diameter, from 50 to 100 inches per day of 10 hours in granite, or 300 to 400 inches per day in limestone.

Tamping.—Two strikers and a holder can bore, with a bit 2 inches in diameter, 10 feet in a day in rock of medium hardness.

Composition for waterproof charger or fuse consists by weight of Pitch, 8 parts; Beeswax and Tallow each 1 part.

Mining. (*Lefroy's Handbook.*)

In demolition of walls line of least resistance L = half thickness, and C is a coefficient depending on structure.
Charge in lbs. $= C \times L^3$.
In a wall without counterforts, where interval between the charge is 2 L, C = .15.
In a wall with counterforts the charge to be placed in centre of each counterfort at junction with wall, C = .2.
Where the charge is placed under a foundation, having equal support on both sides, C = .4.

A leather bag, containing 50 to 60 lbs. powder, hung or supported against a gate or like barrier, will demolish it.

For ordinary mines in average rock charge in ounces $= L^3 \div 160$.

BLOWING ENGINES.

For Smelting.

Volume of oxygen in air is different at different temperatures. Thus, dry air at 85° contains 10 per cent. less oxygen than when it is at temperature of 32°; and when it is saturated with vapor, it contains 12 per cent. less. If an average supply of 1500 cube feet per minute is required in winter, 1650 feet will be required in summer.

Smelting of Iron Ore.

Coke or Anthracite Coal.—18 to 20 tons of air are required for each ton of Pig Iron, and with *Charcoal* 17 to 18 tons are required.

(1 ton of air at 34° = 29 751, and at 60° = 31 366 *cube feet.*)

Pressure.—Pressure ordinarily required for smelting purposes is equal to a column of mercury from 3 to 10 inches, or a pressure of 1.5 to 5 lbs. per square inch.

Reservoir.—Capacity of it, if dry, should be 15 to 20 times that of cylinder if single acting, and 10 times if double acting.

Pipes.—Their area, leading to reservoir, should be .2 that of blast cylinder, and velocity of the air should not exceed 35 feet per second.

A smith's forge requires 150 cube feet of air per minute. Pressure of blast .25 to 2 lbs. per square inch. A ton of iron melted per hour in a cupola requires 3500 cube feet of air per minute. A finery forge requires 100 000 cube feet of air for each ton of iron refined. A blast furnace requires 20 cube feet per minute for each cube yard capacity of furnace.

A Ton of Pig Iron requires for its reduction from the ore 310 000 cube feet of air, or 5.3 cube feet of air for each pound of carbon consumed. Pressure, .7 lb. per square inch.

To Compute Power Required to Drive a Blowing Engine.

$$\frac{.000\,0509}{c} V^3 \left(\frac{L}{d^5} + \frac{42}{d^4}\right) 60 \div 33\,000 = \text{IP}.$$

$d' = \sqrt{\frac{V}{.93 \times .7854 \times v}}$. *v representing velocity of air in feet per second, d and d' diameters of pipe and of nozzle in feet.* $= \sqrt{\frac{35}{.93 \times .7854 \times 500}}$ $= .309$.

Illustration.—What should be power of a steam-engine to drive 35 cube feet of air at a velocity of 500 feet per second, through a pipe 1 foot in diameter and 300 feet in length?

c = ratio between power employed and effect produced by it = in a well-constructed engine .5, *and* C = .93. *d* = .2974, assumed at .3.

$$\frac{.000\,0509}{.5} \times 35^3 \left(\frac{300}{1^5} + \frac{42}{.3^4}\right) 60 \div 33\,000 = 22\,631.625 \times 60 \div 33\,000 = 41.15 \text{ IP}.$$

To Compute Required Power of a Blowing Engine.

$\frac{P + f \times a\,v}{33\,000} = \text{IP}$. P *representing pressure of blast in lbs. per sq. inch; a area of cylinder in sq. ins.; v velocity of piston in feet per minute; f friction of piston and from curvatures, etc., estimated at* 1.25 *per sq. inch of piston.*

Note.—If cylinder is single acting, divide result by 2.

Illustration.—Assume area of blast cylinder 5600 sq. ins., pressure of blast 2.25 lbs. per sq. inch, and velocity of piston 96 feet per second.

$\frac{2.25 + 1.25 \times 5600 \times 96}{33\,000} = \frac{1\,881\,600}{33\,000} = 57$ *horses*, the exact power developed in this case.

To Compute Dimensions of a Driving Engine.

Rule 1.—Divide power in lbs. by product of mean effective pressure upon piston of steam cylinder in lbs. per sq. inch, and velocity of piston in feet per minute, and quotient will give area of cylinder in sq. ins.

2.—Divide velocity of piston by twice number of revolutions, and quotient will give stroke of piston in feet.

Volume of air at atmospheric density delivered into reservoir, in consequence of escape through valves, and partial vacuum necessary to produce a current, will be about .2 less than capacity of cylinder.

Example.—Assume elements of preceding case, with a pressure of 50 lbs. steam, cut off at .375, and with 12 revolutions of engine per minute, what should be area of cylinder of a non-condensing engine?

Mean effective pressure of steam with 5 per cent. clearance = 50 lbs., and $50 - f^* + 14.7 = 50 - 2.5 + 3.33 + 14.7 = 29.47$ *lbs.*, and velocity of piston = 192 *feet.*

$\frac{5600 \times 2.25 + 1.25 \times 96}{29.47 \times 192} = \frac{1\,881\,600}{5658} = 332.5$ *sq. ins.*, and $\frac{192}{12 \times 2} = 8$ *feet stroke.*

Area of cylinder in this case was 324 *sq. ins.*

For Volume, Pressure, and Density of Air, see Heat, page 521.

* See formula and note for power of non-condensing engine, page 733.

To Compute Elements of a Blowing Engine.

Single Stroke.

$$\frac{V\,\overline{P+f}}{230} \text{ or } \frac{A\,s\,n\,\overline{P+f}}{33\,000} = \text{IP}; \quad \frac{\sqrt{V+10\,L}}{3} = d; \quad \frac{D^2\,s\,n}{40\,a} = v;$$

$$\frac{D^2\,s\,n}{40\,v} = a; \quad \frac{230\,\text{IP}}{P+f} = V; \quad \frac{D^2\,s\,n}{92} = V; \text{ and } 34\,P + 32 = t.$$

V representing volume of air in cube feet per minute, P pressure of air and f frictional resistance in lbs. per sq. inch, A area of cylinder and a area of its valves in sq. ins., s stroke of piston in feet, n number of single strokes of piston per minute, L length of air-pipe from reservoir to discharge in feet, d diameter of air or blast pipe and D diameter of cylinder in ins., v velocity of blast in feet per second, and t temperature of blast consequent upon compression in degrees.

ILLUSTRATIONS. —Assume blowing cylinder 50 ins. in diam., stroke of piston 10 feet, number of single strokes 10 per minute, pressure by mercurial manometer 6.12 ins., frictional resistance .4 lb., length of pipe 25.25 feet, and area of valves 95 sq. ins.

$V = 1363.54$ *cube feet,* $P = 3$ *lbs.,* $A = 1963.5$ *sq. ins.*

Then $\frac{1363.54 \times \overline{3+.4}}{230} = 20.16$ IP, and $\frac{1963.5 \times 10 \times 10 \times \overline{3+.4}}{33\,000} = 20.23$ IP.

$$\frac{\sqrt{1363.54+10\times 25.25}}{3} = 13.4 \textit{ ins.} \quad \frac{50^2\times 10\times 10}{40\times 95} = 65.8 \textit{ feet.} \quad \frac{50^2\times 10\times 10}{40\times 65.8} = 95 \textit{ sq. ins.}$$

To Compute Volume of Air transmitted by an Engine.

When Pressure, Temperature, etc., are given.

$$34.5\sqrt{h\left(\frac{1+.004\,t}{h+H}\right)}\,C = v. \quad \text{Then } a\,v \times 60 = V \textit{ in cube feet per minute.}$$

H and h representing height of barometer and pressure of blast in ins. of mercury; t temperature of blast; and v velocity in feet per second.

ILLUSTRATION.—A furnace having 2 tuyeres of 5 ins. diameter, pressure and temperature of blast 3 ins. and 350°, and barometer 30 ins.; what is volume of air transmitted per minute?

C for a conical opening = .94.

$$34.5\sqrt{3\left(\frac{1+.004\times 350^\circ}{3+30}\right)}\times .94 = 34.5\sqrt{3\left(\frac{2.4}{33}\right)} = 34.5\times .467\times .94 = 15.14$$

feet velocity per second.

Then, area 5 ins. = 19.635, which × 2 = 39.27 ins., and 39.27 × 15.14 × 60 ÷ 144 = 247.73 *cube feet.*

To Compute Pressure of Blast from Water or Mercurial Gauge.

RULE.—Divide Water and Mercurial Gauge in ins. by 27.67 and 2.04 respectively, and quotient will give pressure in lbs. per sq. inch.

Fan-blowers.

Proportions of Parts. Blades.—Their width and length should be at least equal to .4 or .5 radius of fan.

Openings.—Inlet should be equal to radius of fan; and outlet, or discharge, should be in depth not less than .125 diameter, its width being equal to width of fan.

Eccentricity.—.1 of diameter of fan. Journals, 4 diameters of shaft.

By the experiments of Mr. Buckle, he deduced

1. That velocity of periphery of blades should be .9 that of their *theoretical* velocity; that is, velocity a body would acquire in falling height of a homogeneous column of air equivalent to required density.

2. That a diminution of inlet from proportions here given involved a greater expenditure of power to produce same density.

3. That greater the depth of blade, greater the density of air produced with same number of revolutions.

To Compute Elements of a Fan-blower.

$$\left(\frac{v}{8.02}\right)^2 \div 939.45 = d; \quad 244\sqrt{d} = v; \quad \frac{a\,v\,60}{160} = V; \quad \text{and} \quad \frac{d\,a\,v}{400} = \text{IHP}.$$

v representing velocity of periphery of fan in feet per second, d inches of mercury, V *volume of air in cube feet, and a area of discharge in sq. ins.*

ILLUSTRATION.—Assume velocity of periphery of fan 123 feet per second, density of blast .25 inch, volume of air 1845 cube feet, and area of discharge 40 sq. ins.

$$\overline{123 \div 8.02}^{\,2} \div 939.454 = .25\,\textit{inch}. \quad 244\sqrt{.25} = 122\,\textit{feet}. \quad \frac{40 \times 123 \times 60}{160} = 1845\,\textit{cub. ft.}$$

$$\frac{.242 \times 40 \times 123}{400} = 2.97\ \text{IHP}, \textit{ independent of friction of blast in pipes and tuyeres.}$$

To Compute Power of a Centrifugal Fan.

$V^2 \div 97\,300 = P$. V *representing velocity of tips of fan in feet per second.*

Memoranda.

Operation of a blower requires about 2.5 per cent. of power of attached boiler.

An increase in number of blades renders operation of fan smoother, but does not increase its capacity.

Pressure or density of a blast is usually measured in ins. of mercury, a pressure of 1 lb. per sq. inch at 60° = 2.0376 ins.

When water is used, a pressure of 1 lb. = 27.671 ins.

Cupola blast .8 lbs., and *Smith's forge* .25 to .3 lbs. per sq. inch.

An ordinary Eccentric Fan, 4 feet in diameter, with 5 blades 10 ins. wide and 14 in length, set 1.5 ins. eccentric, with an inlet opening of 17.5 ins. in diameter, and an outlet of 12 ins. square, making 870 revolutions per minute, will supply air to 40 tuyeres, each of 1.625 ins. in diameter, and at a pressure per sq. inch of .5 inch of mercury.

An ordinary eccentric fan blower, 50 ins. in diameter, running at 1000 revolutions per minute, will give a pressure of 15 ins. of water, and require for its operation a power of 12 horses. Area of tuyere discharge 500 sq. ins.

A non-condensing engine, diameter of cylinder 8 ins., stroke of piston 1 foot, pressure of steam 18 lbs. (mercurial gauge), and making 100 revolutions per minute, will drive a fan, 4 feet by 2, opening 2 feet by 2, 500 revolutions per minute.

Such a blower was applied as an exhausting draught to smoke-pipe of steamer *Keystone State*, cylinder 80 ins. by 8 feet, and evaporation was doubled over that of when wind was calm.

In French blowing engines, volume of air discharged 75 per cent. that of volume of piston space in cylinder, stroke equal diameter of cylinder, and velocity of piston from 100 to 200 feet per minute.

Area of admission valves from .066 to .083 of that of cylinder for speeds of 100 to 150 feet per minute, and from .1 to .111 for higher speeds. Area of exit valves from .066 to .05 of cylinder. (*M. Claudel.*)

By some experiments lately concluded in England with boilers of two steamers, to determine relative effects of natural and forced draught furnaces, the results were as follows (*R. J. Butler*):

Per Sq. Foot of Grate Surface.—Natural Draught, 10 to 10.87 IHP; *Steam Blast*, 12.5 to 13; *Forced or Blast Draught*, 15 to 16.

Heating Surface per IHP.—*Natural Draught*, 2.44 to 2.61; *Steam Blast*, 1.71 to 2.86; *Forced or Blast Draught*, 1.56 to 2.5.

Tube Surface per IHP *in Sq. Feet.—Natural Draught*, 2.03 to 2.18; *Steam Blast*, 2.02 to 2.08; *Forced or Blast Draught*, 1.3 to 2.8.

IHP *per Sq. Foot of Grate in these Trials.—Natural Draught*, 10.15 to 10.87; *Steam Blast*, 12.76 to 13.1; *Forced or Blast Draught*, 10.6 to 16.9.

Root's Rotary Blower—Is constructed from .125 to 14 nominal HP, supplying from 150 to 10 800 cube feet of air per minute. Delivery pipe 2.5 to 19 ins. in diameter. Efficiency 65 to 80 per cent. of power.

For *Ventilation of Mines*—From 40 to 280 revolutions per minute, equal to discharge of 12 500 to 200 000 cube feet of air per minute. 15.5 to 189 HP. Steam cylinder from 14 × 18 ins. to 28 × 48 ins.

For other details of Blowing Engines see page 898.

CENTRAL FORCES.

All bodies moving around a centre or fixed point have a tendency to fly off in a straight line: this is termed *Centrifugal Force;* it is opposed to a *Centripetal Force*, or that power which maintains a body in its curvilineal path.

Centrifugal Force of a body, moving with different velocities in same circle, is proportional to square of velocity. Thus, centrifugal force of a body making 10 revolutions in a minute is 4 times as great as centrifugal force of same body making 5 revolutions in a minute. Hence, in equal circles, the forces are inversely as squares of times of revolution.

If times are equal, velocities and forces are as radii of circle of revolution.

The squares of times are as cubes of distances of centrifugal force from axis of revolution.

Centrifugal forces of two unequal bodies, having same velocity, and at same distance from central body, are to one another as the respective quantities of matter in the two bodies.

Centrifugal forces of two bodies, which perform their revolutions in same time, the quantities of matter of which are inversely as their distances from centre, are equal to one another.

Centrifugal forces of two equal bodies, moving with equal velocities at different distances from centre, are inversely as their distances from centre.

Centrifugal forces of two unequal bodies, moving with equal velocities at different distances from centre, are to one another as their quantities of matter, multiplied by their respective distances from centre.

Centrifugal forces of two unequal bodies, having unequal velocities, and at different distances from their axes are in compound ratio of their quantities of matter, squares of their velocities, and their distances from centre.

Centrifugal force is to weight of body, as double height due to velocity is to radius of rotation.

A *Radius Vector* is a line drawn from centre of force to moving body.

To Compute Centrifugal Force of any Body.

Rule 1.—Divide its velocity in feet per second by 4.01, also square of quotient by diameter of circle; this quotient is centrifugal force, assuming the weight of body as 1. Then this quotient, multiplied by weight of body, will give centrifugal force required.

Example.—What is the centrifugal force of the rim of a fly-wheel having a diameter of 10 feet, and running with a velocity of 30 feet per second?

$30 \div 4.01 = 7.48$, and $7.48^2 \div 10 = 5.59$, or *times weight of rim.*

Or, $\frac{W\,n^2\sqrt{R^2 + r^2}}{4100} = C$. *r representing radius of inner diameter of ring*

Note.—Diameter of a fly-wheel should be measured from centres of gravity of rim.

When great accuracy is required, ascertain centre of gyration of body, and take twice distance of it from axis for diameter.

Rule 2.—Multiply square of number of revolutions in a minute by diameter of circle of centre of gyration in feet, and divide product by constant number 5217; quotient is centrifugal force when weight of body is 1. Then, as in previous Rule, this quotient, multiplied by weight of body, is centrifugal force required.

Or, $\frac{n^2\,d}{5217} = W$. *n representing number of revolutions per minute, d diameter of circle of gyration in feet, and W weight of revolving body in lbs.*

Example.—What is centrifugal force of a grindstone weighing 1200 lbs., 42 inches in diameter, and turning with a velocity of 400 revolutions in a minute?

Centre of gyration = rad. $(42 \div 2) \times .7071 = 14.85$ *ins.*, which $\div 12$ and $\times 2 =$ 2.475 *feet = diameter of circle of gyration.* Then $\frac{400^2 \times 2.475}{5217} \times 1200 = 91\,080$ *lbs.*

Formulas to Determine Various Elements.

$$C^* = \frac{W\,v^2}{32.166\,R};\ = \frac{W\,R\,n^2}{2930};\ = W\,R\,v'\,1.225;\quad W = \frac{C\,32.166\,R}{v^2};$$

$$R = \frac{2930\,C}{W\,n^2};\ = \frac{W\,v^2}{32.166\,C};\quad n = \sqrt{\frac{2930\,C}{W\,R}};\quad v = \sqrt{\frac{C\,R\,32.166}{W}};\ = 6.28\,v'\,R.$$

C representing centrifugal force, W mass or weight of revolving body, both in lbs., R radius of circle of revolving body in feet, n number of revolutions per minute, and v and v′ linear or circumferential and angular velocities of body in feet per second.

Illustration.—What is centrifugal force of a sphere weighing 30 lbs., revolving around a centre at a distance of 5 feet, at 30 revolutions per second?

$v = \frac{5 \times 2 \times 3.1416 \times 30}{60} = 15.71$ *feet.* Then $C\ \frac{30 \times 15.71^2}{32.166 \times 5} = 46.04$ *lbs.*

Centrifugal forces of two bodies are as radii of circles of revolution directly, and as squares of times inversely.

Illustration.—If a fly-wheel, 12 feet in diameter and 3 tons in weight, revolves in 8 seconds, and another of like weight revolves in 6, what should be the diameter of the second when their centrifugal forces are equal?

Then $3 : 3 :: \frac{12}{8^2} : \frac{x}{6^2}$; or $x = \frac{12 \times 6^2}{8^2} = 6.75$ *feet, x = unknown element.*

Centrifugal forces of two bodies, when weights are unequal, are directly as squares of times.

Illustration.—What should be the ratio of the weights of the wheels in the preceding case, their forces being equal?

Then $3 : x :: 6^2 : 8^2$, or $x = \frac{3 \times 8^2}{6^2} = \frac{3 \times 64}{36} = 5.333$ *tons.*

Molesworth gives $.000\,34\,W\,R\,n^2 = C$.

---

* This is termed the *Vis Viva*, or living force.

FLY-WHEEL.

A Fly-wheel by its inertia becomes a reservoir as well as a regulator of force, and to be effective should have high velocity, and its diameter should be from 3 to 4 times that of stroke of driving engine.

Co-efficient of fluctuation of energy in a machine ranges from .015 to .035.

Weight of a fly-wheel in engines that are subjected to irregular motion, as in a cotton-press, rolling-mill, etc., must be greater than in others where so sudden a check is not experienced, and its diameter should range from 3.5 to 5 times length of the crank.

A single acting engine requires a weight of wheel about 2.5 times greater than that for a double acting, and 5 times for double engines of double action.

To Compute Weight of Rim of a Fly-wheel.

Rule.—Multiply mean effective pressure upon piston in lbs. by its stroke in feet, and divide product by product of square of number of revolutions, diameter of wheel, and .000 23.

Note.—If a light wheel is required, multiply by .0003; and if a heavy one, by .000 16.

Example 1.—A non-condensing engine (double acting), having a diameter of cylinder of 14 ins., and a stroke of piston of 4 feet, working full stroke, at a pressure of 65 lbs. mercurial gauge, and making 40 revolutions per minute, develops about 65 IP; what should be the weight of its fly-wheel, when adapted to ordinary work?

Area of cylinder 154 *sq. ins. Mean pressure assumed* 50 *lbs. per sq. inch. Diameter of wheel* 4 *feet stroke* $\times$ 3.5 = 14 *feet.*

$$50 \times 154 \times 4 = 30\,800, \text{ which } \div \overline{40^2 \times 14 \times .000\,23} = 5978 \textit{ lbs.}$$

2.—If a fly-wheel, 16 feet in diameter and 4 tons in weight, is sufficient to regulate an engine (double acting) when it revolves in 4 seconds, what should be the weight of a wheel, 12 feet in diameter, revolving in 2 seconds, so that it may have like centrifugal force?

Note.—The centrifugal forces of two bodies are as the radii of the circles of revolution directly, and as squares of times inversely.

Then $\frac{4 \times 16}{4^2} = \frac{x \times 12}{2^2}$. Or, $x = \frac{4 \times 16 \times 2^2}{12 \times 4^2} = \frac{4 \times 16 \times 4}{12 \times 16} = 1.333$ *tons.*

Assume elements of example 1.

$$5978 \times .1 \div 13.25 = 45.12 \textit{ square ins.}$$

To Compute Dimensions of Rim.

Rule.—Multiply weight of wheel in lbs. by .1, and divide product by mean diameter of rim in feet; quotient will give sectional area of rim in square inches of cast iron.

Or, $\frac{P\,S}{.4\,D} = W$, and $\frac{W}{10\,D} = A$. P *representing pressure on piston and* W *weight of wheel in lbs.,* S *stroke of piston and* D *mean diameter of wheel, both in feet, and* A *area of section of rim in sq. ins.*

Or, $\frac{1.16\,n\,P\,S\,C}{60\,D} = W$. C *representing coefficient varying from* 3 *to* 4 *ordinarily, and increasing to* 6 *when great regularity of speed is required, and n number of revolutions per minute.*

Note.—Maximum safe velocity for cast iron is assumed at 80 feet per second.

For engines at high expansion of steam, or with irregular loads, as with a rolling-mill, multiply W by 1.5, or put W 100 lbs. for each IHP. (*Molesworth.*)

In corn or like mills, the velocity of periphery of fly-wheel should exceed that of the stones.

GOVERNORS.

A GOVERNOR or CONICAL PENDULUM in its operation depends upon the principles of Central Forces.

When in a *Ball Governor* the balls diverge, the ring on vertical shaft raises, and in proportion to the increase of velocity of the balls squared, or the square roots of distances of ring from fixed point of arms, corresponding to two velocities, will be as these velocities.

Thus, if a governor makes 6 revolutions in a second when ring is 16 ins. from fixed point or top, the distance of ring will be 5.76 ins. when speed is increased to 10 revolutions in same time.

For $10 : 6 :: \sqrt{16} : 2.4$, which, squared $= 5.76$ *ins., distance of ring from top.* Or, $6^2 : 10^2 :: 5.76 : 16$ *ins.*

A governor performs in one minute half as many revolutions as a pendulum vibrates, the length of which is perpendicular distance between plane in which the balls move and the fixed point or centre of suspension.

To Compute Number of Revolutions of a Ball Governor per Minute to maintain Balls at any given Height.

$188 \div \sqrt{H} =$ *revolutions.* H *representing vertical height between plane of balls and points of their suspension in ins.*

ILLUSTRATION.—If the rise of the balls of a centrifugal governor is 22 ins., what are the number of revolutions per minute?

$$188 \div \sqrt{22} = 40.09 \textit{ revolutions.}$$

To Compute Vertical Height between Plane of Balls and their Points of Suspension.

$(188 \div r)^2 =$ *vertical height in ins.* r *representing number of revolutions per minute.*

ILLUSTRATION.—If number of revolutions of a centrifugal governor is 100, what will be rise of balls?

$$\overline{188 \div 100}^2 = 1.88^2 = 3.53 \textit{ ins.}$$

To Compute Angle of Arms or Plane of Balls with Centre Shaft.

$r \div l = \sin. \angle.$ r *representing distance of balls from plane of centre shaft, and* l *distance between balls and point of suspension measured in plane of shaft.*

ILLUSTRATION.—Distance of balls from plane of centre shaft is 10 inches, and their distance from point of suspension is 25; what is the angle?

$$10 \div 25 = .4, \text{ and } \sin. .4 = 23^\circ 35'.$$

When Number of Revolutions are given. $\frac{(54.16 \div n)^2}{l} = \cos. \angle.$

ILLUSTRATION.—Revolutions of a governor per minute are 50, and length of its arms 2 feet; what is their angle with plane of shaft?

$$\frac{(54.16 \div 50)^2}{2} = \frac{1.173}{2} = .5865 = \cos. 54^\circ 6'.$$

PENDULUMS.

Pendulums are *Simple* or *Compound*, the former being a material point, or single weight suspended from a fixed point, about which it oscillates, or vibrates, by a connection void of weight; and the latter, a like body or number of bodies suspended by a rod or connection. Any such body will have as many centres of oscillation as there are given points of suspension to it, and when any one of these centres are determined the others are readily ascertained.

Thus, $s\,o \times s\,g =$ a *constant product*, and $s\,r = \sqrt{s\,o \times s\,g}$, *s g o and r representing points of suspension, gravity, oscillation, and gyration.*

Or, any body, as a cone, a cylinder, or of any form, regular or irregular, so suspended as to be capable of vibrating, is a compound pendulum, and distance of its centre of oscillation from any assumed point of suspension is considered as the length of an equivalent simple pendulum.

The *Amplitude* of a simple pendulum is the distance through which it passes from its lowest position to its farthest on either side.

Complete Period of a pendulum in motion is the time it occupies in making two vibrations.

All vibrations of same pendulum, whether great or small, are performed very nearly in same time.

Number of Oscillations of two different pendulums in same time and at same place are in inverse ratio of square roots of their lengths.

Length of a Pendulum vibrating seconds is in a constant ratio to force of gravity.

Time of Vibration is half of a complete period, and it is proportional to square root of length of pendulum. Consequently, lengths of pendulums for different vibrations are—

Latitude of Washington.

39.0958 ins. for one second.	4.344 for third of a second.
9.774 ins. for half a second.	2.4435 for quarter of a second.

Lengths of Pendulums vibrating Seconds at Level of the Sea in several Places.

	Ins.		Ins.		Ins.
Equator	39.0152	New York	39.1017	Paris	39.1284
Washington	39.0958	Lat. 45°	39.127	London	39.1393

To Compute Length of a Simple Pendulum for a given Latitude.

$39.127 - .099\,82 \cos. 2\,L = l$. *L representing latitude.*

ILLUSTRATION.—Required the length of a simple pendulum vibrating seconds in the latitude of 50° 31′.

$L = 50°\,31' \cos. 2\,L = 2 \times 50°\,31' = \cos. 180° - \overline{50°\,31' \times 2} = \cos. 78°\,58' = .191\,38$
$- 39.127 + .191\,38 \times .099\,82$ (*two* — or *negative* = an *affirmative* or +) $= 39.1461$ *ins.*

To Compute Length of a Simple Pendulum for a given Number of Vibrations.

$L\,t^2 = l$. *L representing length for latitude, t time in seconds, and l length of pendulum in ins.*

ILLUSTRATION.—Required vibrations of a pendulum in a minute at New York, are 60; what should be its length?

$39.1017 \times 1^2 = 39.1017$. Or, $\frac{L}{n^2} = l$. *n representing number of vibrations per second.*

To Compute Number of Vibrations of a Simple Pendulum in a given Time.

$\frac{\sqrt{L}\,t}{\sqrt{l}} = n$, $\frac{t}{n}$ *representing time of one vibration in seconds.*

To Compute Centre of Gravity of a Compound Pendulum of Two Weights connected in a Right Line.

When Weights are both on one Side of Point of Suspension.

$\frac{l\,W + l'\,w}{W + w} = o =$ *distance of centre of gravity from point of suspension.*

When Weights are on Opposite Sides of Point of Suspension.

$\frac{l\,W - l'\,w}{W + w} = o =$ *distance of centre of gravity of greater weight from point of suspension.*

Note.—To obtain strictly isochronous vibrations, the circular arc must be substituted for the cycloid curve, which possesses the property of having an inclination, the sine of which is simply proportional to distance measured on the curve from its lowest point.

For construction of a Cycloidal pendulum, see Deschanel's *Physics*, Part I., pp. 71–2.

To Compute Length of a Simple Pendulum, Vibrations of which will be same in Number as Inches in its Length.

$$\sqrt[3]{(60\sqrt{L})^2} = l \text{ in inches.}$$

Illustration.—What will be length of a pendulum in New York, vibrations of which will be same number as the ins. in its length?

$$\sqrt[3]{(\sqrt{39.1017} \times 60)^2} = 7.211^2 = 52 \text{ ins.}$$

To Compute Time of Vibration of a Simple Pendulum, Length being given.

$$\sqrt{l \div L} = t \text{ in seconds.}$$

Illustration.—Length of a pendulum is 156.4 ins.; what is the time of its vibration in New York?

$$\sqrt{\frac{156.4}{39.1017}} = 2 \text{ seconds.}$$

Or, $\sqrt{\frac{l}{g}} \times 3.1416 = t$. *l representing length of a pendulum vibrating seconds in ins., g measure of force of gravity, and t time of one oscillation.*

Illustration.—Length of a simple pendulum vibrating seconds, and measure of force of gravity at Washington, are 39.0958 ins., and 32.155 feet.

$$3.1416\sqrt{\frac{39.0958}{32.155 \times 12}} = 3.1416 \times \sqrt{1.013} = 3.1416 \times .3183 = 1 \text{ second.}$$

To Compute Number of Vibrations of a Simple Pendulum in a given Time.

$$\sqrt{\frac{L}{l}} \times t = n. \quad n \text{ representing number of vibrations.}$$

Illustration.—The length of a pendulum in New York is 156.4 ins., and time of its vibration is 2 seconds; what are number of its vibrations?

$\sqrt{\frac{39.1017}{156.4}} \times 2 = \sqrt{\frac{6.253}{12.506}} \times 2 = .5 \times 2 = 1$ *vibration.* Hence, $1 \times \frac{60}{2} = 30$ *vibrations per minute.*

To Compute Measure of Gravity, Length of Pendulum and Number of its Vibrations being given.

$$\frac{.82246\,l\,n^2}{t^2} = g. \quad g \text{ representing measure of gravity in feet.}$$

To Compute Number of Revolutions of a Conical Pendulum per Minute.

$\sqrt{\frac{2933.5}{h}} = n$. *h representing distance between point of suspension and plane of revolutions in ins.*

Note.—Number of revolutions per minute are constant for any given height, and the time of a revolution is directly as square root of height.

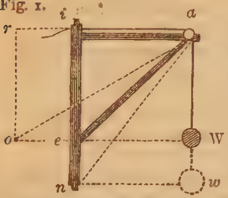
CRANES.

Usual form of a Crane is that of a right-angled triangle, the sides being post or jib, and stay or strut, which is hypothenuse of triangle.

When jib and post are equal in length, and stay is diagonal of a square, this form is theoretically strongest, as the whole stress or weight is borne by stay, tending to compress it in direction of its length; stress upon it, compared to weight supported, being as diagonal to side of square, or as 1.4142 to 1. Consequently, if weight borne by crane is 1000 lbs., thrust or compression upon stay will be 1414.2 lbs., or as *a e* to *e* W, Fig. 1.

When Post is Supported at both Head and Foot, as Fig. 1.

Fig. 1.



Weight W is sustained by a rope or chain, and tension is equal upon both parts of it; that is, on two sides of square, *i a* and *e* W. Consequently jib, *i a*, has no stress upon it, and serves merely to retain stay, *a e*.

If foot of stay is set at *n*, thrust upon it, as compared with weight, will be as *a n* to *a w*; and if chain or rope from *i* to *a* is removed, and weight is suspended from *a*, tension on jib will be as *i a* to *a* W.

If foot of stay is raised to *o*, thrust, as compared with weight, will be as line *a o* is to *a* W, and tension on jib will be as line *a r*.

By dividing line representing weight, as *a* W or *a w*, into equal parts, to represent tons or pounds, and using it as a scale, stress upon any other part may be measured upon described parallelogram.

Thus, as length of *a* W, compared to *a e*, is as 1 to 1.4142: if *a* W is divided into 10 parts representing tons, *a e* would measure 14.142 parts or tons.

When Post is Supported at Foot only.

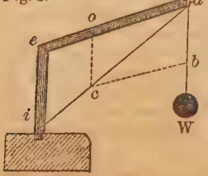
If post is wholly unsupported at head, and its foot is secured up to line *o* W, then W, acting with leverage, *e* W, will tend to rupture post at *e*, with same effect as if twice that weight was laid upon middle of a beam equal to twice length of *e* W, *e* being at middle of beam, which is assumed to be supported at both ends, and of like dimensions to those of post.

Or, force exerted to rupture post will be represented by stress, W, multiplied by 4 times length of lever, *e* W, divided by depth of post in line of stress, squared, and multiplied by breadth of it and *Value** of its material.

Post of such a crane is in condition of half a beam supported at one end, weight suspended from other; consequently, it must be estimated as a beam of twice the length supported at both ends, stress applied in middle.

To Compute Stress on Jib, and on Stay or Strut.—Fig. 2.

Fig. 2.



On diagram of crane, Fig. 2, mark off on line of chain, *a* W, a distance, *a b*, representing weight on chain; from point *b* draw a line, *b c*, parallel to jib, *a e*, and where this intersects stay or strut, draw a vertical line, *c o*, extending to jib, and distances from *a* to points *b c* and *o c*, measured upon a scale of equal parts, will represent proportional strain.

Illustration.—In figure, weight being 10 tons, stress on stay or strut compressing, *a c*, will be 31 tons, and on jib or tension-rods, *a o*, 26 tons.

* For *Value* of Materials, see page 779.

To Compute Dimensions of Post of a Crane.

When Post is Supported at Feet only. RULE.—Multiply weight or stress to be borne in lbs. by length of jib in feet measured upon a horizontal plane; divide product by *Value* of material to be used, and product, divided by breadth in ins., will give square of depth, also in ins.

EXAMPLE.—Stress upon a crane is to be 22 400 lbs., and distance of it from centre of post 20 feet; what should be dimension of post if of American white oak?

Value of American white oak 50. Assumed breadth 12 ins.

$$\frac{22\,400 \times 20}{50} = 8960, \text{ and } \frac{8960}{12} = 746.67. \text{ Then } \sqrt{746.67} = 27.32 \textit{ ins.}$$

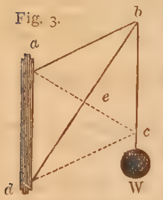
When Post is Supported at both Ends. RULE.—Multiply weight or stress to be borne in lbs. by twice length of jib in feet measured upon a horizontal plane; divide product by *Value* of material to be used, and product, divided by four times breadth in ins., will give square of depth, also in ins.

EXAMPLE.—Take same elements as in preceding case. Assumed breadth 10 ins.

$$\text{Then } \frac{22\,400 \times 20 \times 2}{50} = 17\,920, \ \frac{17\,920}{4 \times 10} = 448, \text{ and } \sqrt{448} = 21.166 \textit{ ins.}$$

In Fig. 3, angle *a b e* and *e b c* being equal, chain or rope is represented by *a b c*, and weight by W; stress upon stay *b d*, as compared with weight, is as *b d* to *a b* or *b c*.

Fig. 3.



In practice, however, it is not prudent to consider chain as supporting stay; but it is proper to disregard chain or rope as forming part of system, and crane should be designed to support load independent of it. It is also proper that angles on each side of diagonal stay, in this case, should not be equal. If side *a b* is formed of tension-rods of wrought iron, point *a* should be depressed, so as to lengthen that side, and decrease angle *a b e*; but if it is of timber, point *a* should be raised, and angle *a b e* increased.

Fig. 4.

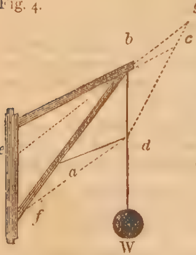


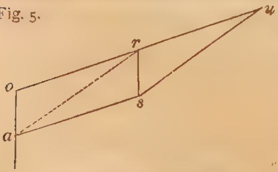
Fig. 4 shows a form of crane very generally used; angles are same as in Fig. 3, and weight suspended from it, being attached to point *d*, is represented by line *b d*. The tension, which is equal to weight, is shown by length of line *b c*, and thrust by length of line *b a*, measured by a scale of equal parts, into which line *b d*, representing weight, is supposed to be divided.

But if *b e* be direction of jib, then *b g* will show tension, and *b f* the thrust (*d f* being taken parallel to *b e*), both of them being now greater than before; line *b d* representing weight, and being same in both cases.

To Ascertain Stress on Jib, on Strut of a Crane.—Fig. 5.

Through *a* draw *a s*, parallel to jib or tension-rod *o r*, and also *s u* parallel to strut *a r*; then *r s* is a diagonal of parallelogram, sides of which are equal to *r a* and *r u*.

Fig. 5.



If then *r s* represents a stress of 20 lbs., the two forces into which it is decomposed are shown by *r u* and *r a*; *o r* is equal to *r u*, as each of them is equal to *a s*, and *r s* is equal to *o a*. Hence, 20 represented by *a o*, stress on jib will be represented by *o r*, and that on strut by *r a*.

Assuming then *o r* 3 feet, *a r* 3.5, and *o a* 1, stress on jib will be 60 lbs., and on strut 70.

Thus, in all cases, stress on jib or tension-rod and on strut can be determined by relative proportions of sides of triangle formed.

To Compute Stress upon Strut of a Crane.

RULE.—Multiply length of strut in feet by weight to be borne in lbs.; divide product by height of jib from point of bearing of strut in feet, and quotient will give stress or thrust in lbs.

EXAMPLE.—Length of strut of a crane is 28.284 feet, height of post is 26.457 feet, and weight to be borne is 22 400 lbs.; what is stress?

$$\frac{28.284 \times 22\,400}{26.457} = \frac{633\,561.6}{26.457} = 23\,947 \textit{ lbs.}$$

Chains and Ropes.

Chains for Cranes should be made of short oval links, and should not exceed 1 inch in diameter.

Short-linked Crane Chains and Ropes showing Dimensions and Weight of each, and Proof of Chain in Tons.

Diam. of Chains.	Weight per Fathom.	Proof Strain.	Circumf. of Rope.	Weight of Rope per Fath.	Diam. of Chains.	Weight per Fathom.	Proof Strain.	Circumf. of Rope.	Weight of Rope per Fath.
Ins.	Lbs.	Tons.	Ins.	Lbs.	Ins.	Lbs.	Tons.	Ins.	Lbs.
.3125	6	.75	2.5	1.5	.6875	28	6.5	7	10.5
.375	8.5	1.5	3.25	2.5	.75	32	7.75	7.5	12
.4375	11	2.5	4	3.75	.8125	36	9.25	8.25	15
.5	14	3.5	4.75	5	.875	44	10.75	9	17.5
.5625	18	4.5	5.5	7	.9375	50	12.5	9.5	19.5
.625	24	5.25	6.25	8.7	1	56	14	10	22

Ropes of circumferences given are considered to be of equal strength with the chains, which, being short-linked, are made without studs.

A crane chain will stretch, under a proof of 15 tons, half an inch per fathom.

Machinery of Cranes.

To attain greater effect of application of power to a crane, the wheel-work must be properly designed and executed.

If manual labor is employed, it should be exerted at a speed of 220 feet per minute.

Proportions.—Capacity of Crane, 5 tons.

Radius of winch or handle 15 to 18 ins. Height of axle from floor 36 to 39.

1st pinion, 11 teeth, 1.25 ins. pitch.	2d pinion, 12 teeth, 1.5 ins. pitch.
1st wheel, 89 " 1.25 " "	2d wheel, 96 " " " "

$$\frac{\text{Barrel } 8 \textit{ ins.} \times 11 \textit{ teeth} \times 12 \textit{ teeth} \times 11\,200 \textit{ lbs.} = 30\,800}{\text{Winch } 17 \textit{ ins.} \times 89 \textit{ teeth} \times 96 \textit{ teeth} \times 4 \text{ men} = 1513} = 20.35 \textit{ lbs.}$$ = statical resistance to each of the 4 men at winches.

An experiment upon capacity of a crane, geared 1 to 105, developed that a strong man for a period of 2.5 minutes exerted a power of 27 562 foot-pounds per minute, which, when friction of crane is considered, is fully equal to the power of a horse for one minute.

In practice an ordinary man can develop a power of 15 lbs. upon a crane, handle moved at a velocity of 220 feet per minute, which is equivalent to 3300 foot-pounds.

For Treatise on Cranes, see Weales' Series, No. 33.

COMBUSTION.

Combustion is one of the many sources of heat, and denotes combination of a body with any of the substances termed Supporters of Combustion; with reference to generation of steam, we are restricted to but one of these combinations, and that is Oxygen.

All bodies, when intensely heated, become luminous. When this heat is produced by combination with oxygen, they are said to be ignited; and when the body heated is in a gaseous state, it forms what is termed Flame.

Carbon exists in nearly a pure state in charcoal and in soot. It combines with no more than 2.66 of its weight of oxygen. In its combustion, 1 lb. of it produces sufficient heat to increase temperature of 14 500 lbs. of water 1°.

Hydrogen exists in a gaseous state, and combines with 8 times its weight of oxygen, and 1 lb. of it, in burning, raises heat of 50 000 lbs. of water 1°.*

An increase in the rapidity of combustion is accompanied by a diminution in the evaporative efficiency of the combustible.

Mr. D. K. Clark furnishes the following: When coal is exposed to heat in a furnace, the carbon and hydrogen, associated in various chemical unions, as hydrocarbons, are volatilized and pass off. At lowest temperature, naphthaline, resins, and fluids with high boiling-points are disengaged; at a higher temperature, volatile fluids are disengaged; and still higher, olefiant gas, followed by light carburetted hydrogen, which continues to be given off after the coal has reached a low red heat. As temperature rises, pure hydrogen is also given off, until finally, in the fifth or highest stage of temperature for distillation, hydrogen alone is discharged. What remains after distillatory process is over, is coke, which is the fixed or solid carbon of coal, with earthy matter or ash of the coal.

The hydrocarbons, especially those which are given off at lowest temperatures, being richest in carbon, constitute the flame-making and smoke-making part of the coal. When subjected to heat much above the temperatures required to vaporize them, they become decomposed, and pass successively into more and more permanent forms by precipitating portions of their carbon. At temperature of low redness none of them are to be found, and the olefiant gas is the densest type that remains, mixed with carburetted and free hydrogen. It is during these transformations that the great volume of smoke is made, consisting of precipitated carbon passing off uncombined. Even olefiant gas, at a bright red heat, deposits half its carbon, changing into carburetted hydrogen; and this gas, in its turn, may deposit the last remaining equivalent of carbon at highest furnace heats, and be converted into pure hydrogen.

Throughout all this distillation and transformation, the element of hydrogen maintains a prior claim to the oxygen present above the fuel; and until it is satisfied, the precipitated carbon remains unburned.

Summary of Products of Decomposition in the Furnace.

Reverting to statement of average composition of coal, page 485, it appears that the fixed carbon or coke remaining in a furnace after volatile portions of coal are driven off, averages 61 per cent. of gross weight of the coal. Taking it at 60 per cent., proportion of carbon volatilized in combination with hydrogen will be 20 per cent., making total of 80 per cent. of constituent carbon in average coal.

Of the 5 per cent. of constituent hydrogen, 1 part is united to the 8 per cent. of oxygen, in the combining proportions to form water, and remaining 4 parts of hydrogen are found partly united to the volatilized carbon, and partly free.

* Mean effect.

These particulars are embodied in following summary of condition of elements of 100 lbs. of average coal, after having been decomposed, and prior to entering into combustion—

100 Lbs. of Average Coal in a Furnace.

Composition	Lbs.		Lbs.	Decomposition.
Carbon { Fixed	60	forming	60	fixed carbon.
Carbon { Volatilized	20		24	hydrocarbons and free hydrogen.
Hydrogen	5		1.25	sulphur.
Sulphur	1.25		85.25	
Oxygen	8		9	water or steam.
Nitrogen	1.2		1.2	nitrogen.
Ash, etc.	4.55		4.55	ash, etc.
	100		100	

showing a total useful combustible of 85.25 per cent., of which 25.25 per cent. is volatilized. While the decomposition proceeds, combustion proceeds, and the 25.25 per cent. of volatilized portions, and the 60 per cent. of fixed carbon, successively, are burned.

It may be added that the sulphur and a portion of the nitrogen are disengaged in combination with hydrogen, as sulphuretted hydrogen and ammonia. But these compounds are small in quantity, and, for the sake of simplicity, they have not been indicated in the synopsis.

Volume of Air chemically consumed in complete Combustion of Coal.

Assume 100 lbs. of average coal. Then, by following

$$80 + 3\left(5 - \frac{8}{8}\right) + \overline{.4 \times 1.25} \times 152 = 14\,060 \textit{ cube feet of air at } 62° \textit{ for } 100 \textit{ lbs. coal.}$$

For volatilized portion, Hydrogen (H),	4	lbs. × 457 =	1 828	cube feet.
Carbon (C),	20	" × 152 =	3 040	" "
Sulphur (S),	1.25	" × 57 =	71	" "
			4 939	" "
For fixed portion, Carbon,	60	lbs. × 152 =	9 120	" "
Total useful combustible,	85.25	"	14 059	" " *for com-*

plete combustion of 100 *lbs. coal of average composition at* 62°.

To Compute Volume of Air at 62°, under One Atmosphere, chemically consumed in Complete Combustion of 1 Lb. of a given Fuel.

RULE.—Express constituent carbon, hydrogen, oxygen, and sulphur, as percentages of whole weight of fuel; divide oxygen by 8, deduct quotient from hydrogen, and multiply remainder by 3; multiply sulphur by .4; add products to the carbon, and multiply sum by 1.52. Final product is volume of air in cube feet.

To compute weight of air chemically consumed.—Divide volume thus found by 13.14; quotient is weight of air in lbs.

$$\text{Or, } 1.52\left(C + 3\left(H - \frac{O}{8}\right) + .4\,S\right) = \textit{Air.} \quad O \textit{ Oxygen.}$$

NOTE.—In ordinary or approximate computations, sulphur may be neglected.

EXAMPLE.—Assume 1 lb. Newcastle coal. $C = 82.24$, $H = 5.42$, $O = 6.44$, and $S = 1.35$.

$\frac{6.44}{8} = .805$, $5.42 - .805 = 4.615 \times 3 = 13.845$, $1.35 \times .4 = .54$, $13.845 + .54 + 82.24 = 96.625$, and $96.625 \times 1.52 = 146.87$ *cube feet.*

Then $146.87 \div 13.14 = 11.18$ *lbs.*

To Compute Total Weight of Gaseous Products of Complete Combustion of 1 Lb. of a given Fuel.

RULE.—Express the elements as per-centages of fuel; multiply carbon by .126, hydrogen by .358, sulphur by .053, and nitrogen by .01, and add products together. Sum is total weight of gases in lbs.

Or, $.126\ C + .358\ H + .053\ S + .01\ N = \mathit{Weight}$.

EXAMPLE.—Assume as preceding case. $N = 1.61$.

$$82.24 \times 1.26 + 5.42 \times .358 + 1.35 \times 053 + 1.61 \times .01 = 12.39 \ \mathit{lbs.}$$

To Compute Total Volume, at 62°, of Gaseous Products of Complete Combustion of 1 Lb. of given Fuel.

RULE.—Express elements as per-centages; multiply carbon by 1.52, hydrogen by 5.52, sulphur by .567, and nitrogen by .135, and add products together. Sum is total volume, at 62° F., of gases, in cube feet.

Or, $1.52\ C + 5.52\ H + .567\ S + .135\ N = \mathit{Volume}$.

To Compute Volume of the several Gases separately from their Respective Quantities.

RULE.—Multiply weight of each gaseous product by volume of 1 lb. in cube feet at 62°, as below.

Volume of 1 Lb. of Gases at 62° under a Pressure of 14.7 Lbs.

	Cube feet.		Cube feet.		Cube feet.
Aqueous Vapor or Gaseous Steam	21.125	Oxygen	11.887	Nitrogen	13.501
		Hydrogen	190	Carbonic Acid	8.594

Air................ 13.141 cube feet.

For a lb. of oxygen in combustion, 4.35 lbs. air are consumed; or, by volume, for a cube foot of oxygen 4.76 cube feet of air are consumed.

1 lb. Hydrogen consumes	34.8 lbs.,	or 457	cube feet. at 62°.
1 " Carbon, completely burned, consumes	11.6 "	" 152	" " " "
1 " " partially " "	5.8 "	" 76	" " " "
1 " Sulphur consumes	4.35 "	" 57	" " " "

Composition and Equivalents of Gases, combined in Combustion of Fuel.

GASES.	Elements.	By Weight.	GASES.	Elements.	By Weight.
ELEMENTS.	Equivalents.		COMPOUNDS.	Equivalents.	
Oxygen	O. 1	8	Light Carburetted Hydrogen	C. 2 H. 4	12 4
Hydrogen	H. 1	1	Carbonic Oxide	O. 1 C. 1	8 6
Carbon	C. 1	6	Carbonic Acid	O. 2 C. 1	16 6
Sulphur	S. 1	16	Olefiant Gas (Bi-carburetted Hyd.)	C. 4 H. 4	24 4
Nitrogen	N. 1	14	Sulphurous Acid	O. 2 S. 1	16 16
COMPOUNDS.					
*Atmospheric Air (mech. mixture)	O. 23 N. 77	8 26.8			
Aqueous Vapor or Water	O. 1 H. 1	8 1			

Weights of products in combustion of 1 lb. of given fuel, are—

$C = .0366$. $H = .09$. $S = .02$. $N = .0893\ C + .268\ H + .0335\ S + .01\ N$.

Cube Feet.		Cube Feet.	
$.0366 \times 8.59 = .315$	volume carbonic acid.	$.02 \times 5.85 = .117$	volume sulph. acid.
$.09 \times 190 = 17.1$	" steam.	$.0893 + .268 + .0335 + .01 \times 13.501 = 5.409$	volume nitrogen.

Volume of Air or Gases at higher temperatures than here given (62°) is ascertained by, $V \dfrac{t' + 461}{t + 461} = V'$. *V representing volume of air or gas at temperature t, and V′ at temperature t′.*

* By Volume 1 Oxygen, 3.762 Nitrogen.

Chemical Composition of some Compound Combustibles.

Combustible.	Combining equivalents. Car.	Hyd.	Oxy.	In 100 parts by weight. Car.	Hyd.	Oxy.
				Per Cent.	Per Cent.	Per Cent.
Carbonic oxide	1	—	1	42.9	—	57.1
Light carburetted hydrogen	2	4	—	75	25	—
Olefiant gas, Bicarburetted hyd.	4	4	—	85.7	14.3	—
Sulphuric ether	4	5	1	64.8	13.5	21.7
Alcohol	4	6	2	52.2	13	34.8
Turpentine	20	16	—	88.2	11.8	—
Wax	—	—	—	81.6	13.9	4.5
Olive oil	—	—	—	77.2	13.4	9.4
Tallow	—	—	—	79	11.7	9.3

Heating powers of compound bodies are approximately equal to sum of heating powers of their elements.

Thus, carburetted hydrogen, which consists of two equivalents of carbon and four of hydrogen, weighing respectively $2 \times 6 = 12$ and $1 \times 4 = 4$, in proportion of 3 to 1, or .75 lb. of carbon and .25 lb. of hydrogen in one lb. of gas. Elements of heat of combustion of one lb. are, then—

	Units of heat.
For carbon	$14\,544 \times .75 = 10\,908$
For hydrogen	$62\,032 \times .25 = 15\,508$
Total heat of combustion, as computed	26 416
Total heat, by direct trial	23 513

Heating Powers of Combustibles.

(MM. Favre and Silbermann, D. K. Clark and others.)

1 Lb. of Combustible.	Oxygen consumed per lb. of Combustible.	Weight and Volume of Air consumed per lb. of Combustible.		Total Heat of Combustion of 1 lb. of Combustible.	Equivalent evaporative Power of 1 lb. of Combustible, under one Atmosphere.	
	Lbs.	Lbs.	Cube Feet at 62°.	Units.	Lbs. of water at 62°.	Lbs. of water at 212°.
Hydrogen	8	34.8	457	62 032	55.6	64.2
Carbon, making carbonic oxide	1.33	5.8	76	4 452	4	4.61
Carbon, making carbonic acid	2.66	11.6	152	14 500	13	15
Carbonic oxide	.57	2.48	33	4 325	3.88	4.48
Light carburetted hydrogen	4	17.4	229	23 513	21.07	24.34
Olefiant gas	3.43	15	196	21 343	19.12	22.09
Sulphuric ether	2.6	11.3	149	16 249	14.56	16.82
Alcohol	2.78	12.1	159	12 929	11.76	13.38
Turpentine	3.29	14.3	188	19 534	17.5	20.22
Sulphur	1	4.35	57	4 032	3.61	4.17
Tallow	2.95	12.83	169	18 028	16.15	18.66
Petroleum	4.12	17.93	235	27 531	—	28.5
Coal (average)	2.46	10.7	141	14 133	12.67	14.62
Coke, desiccated	2.5	10.9	143	13 550	12.14	14.02
Wood, desiccated	1.4	6.1	80	7 792	6.98	8.07
Wood-charcoal, desiccated	2.25	9.8	129	13 309	11.92	13.13
Peat, desiccated	1.75	7.6	100	9 951	8.91	10.3
Peat-charcoal, desiccated	2.28	9.9	129	12 325	11.04	12.76
Lignite	2.03	8.85	116	11 678	—	12.1
Asphalt	2.73	11.87	156	16 655	—	17.24

When carbon is not completely burned, and becomes carbonic oxide, it produces less than a third of heat yielded when it is completely burned. For heating power of carbon an average of 14 500 units is adopted.

To Compute Heating Power of 1 Lb. of a given Combustible.

When proportions of Carbon, Hydrogen, Oxygen, and Sulphur are given. RULE.—Ascertain difference between hydrogen and .125 of oxygen; multiply remainder by 4.28; multiply sulphur by .28, add products to the carbon, multiply sum by 14 500, divide by 100, and product is total heating power in units of heat.

$$\text{Or, } 145\ (C + 4.28\ \overline{H - O \times .125} + .28\ S) = \textit{heat.}$$

ILLUSTRATION.—Assume as preceding case.

$$5.42 \sim \overline{82.28} \times .125 \times 4.28 + \overline{1.35 \times .28} + 82.28 \times 14\,500 \div 100 = 15\,005.$$

To Compute Evaporative Power of 1 Lb. of a Given Combustible.

When Proportions of Carbon, Hydrogen, Oxygen, and Sulphur are given. RULE.—Ascertain difference between hydrogen and .125 of oxygen, multiply remainder by 4.28; multiply sulphur by .28, add products to the carbon, and multiply sum by .13, when water is supplied at 62°, and .15 when at 212°; product is evaporative power in lbs. of water at 212°.

Or, When total heating power is known, divide it by 1116 when water is at 62°, or 996 when at 212°.

ILLUSTRATION.—By table, heating power of Tallow is 18 028 units.

Hence, 18 028 ÷ 1116 = 16.15 *lbs. water evaporated at* 62°.

Temperature of Combustion.

Temperature of combustion is determined by product of volumes and specific heats of products of combustion.

ILLUSTRATION.—1 lb. carbon, when completely burned, yields 3.66 lbs. carbonic acid and 8.94 of nitrogen. Specific heats .2164 and .244.

3.66 × .2164	=	.792	units of heat for 1°.
8.94 × .244	=	2.181	" " " 1°.
12.6		2.973	" " " 1°.

Consequently, products of combustion of 1 lb. carbon absorbs 2.973 units of heat in producing 1° temperature.

Weight and Specific Heat of Products of Combustion, and Temperature of Combustion. (*D. K. Clark.*)

1 LB. OF COMBUSTIBLE.	Gaseous Products for 1 Lb. of Combustible. Weight.	Mean specific Heat.	Heat to raise the Temperature 1°.	Temperature of Combustion.	
	Lbs.	Water = 1.	Units.	°	Ratio.
Hydrogen	35.8	.302	10.814	5744	100
Sulphuric ether	11.97	.256	3.063	5305	92
Olefiant gas (Bi-carburetted hyd.)	15.9	.257	4.089	5219	91
Tallow	13.84	.256	3.54	5093	88.7
Coal (average)	11.94	.246	2.935	4879	85
Carbon, or pure coke	12.6	.236	2.973	4877	85
Wax	15.21	.257	3.914	4826	84
Alcohol	10.09	.27	2.68	4825	84
Light carburetted hydrogen	18.4	.268	4.933	4766	83
Sulphur	5.35	.211	1.128	3575	62
Turpentine	12.18	.257	3.127	3470	60
Coal, with double supply of air	22.64	.242	5.478	2614	45

Whence it appears, that mean specific heat of products of combustion, omitting hydrogen .302 and sulphur .211, is about .25.

Hence, *To Ascertain Temperature of Combustion.*—Divide total heat of combustion in units by units of heat for 1°, and quotient will give temperature.

ILLUSTRATION.—What is temperature of combustion of coal of average composition?

Gaseous products as per preceding table 11.94, which $\times$ .246 specific heat $=$ 2.935 *units of heat at* 1°.

Hence, 14 133 units of combustion (from table, page 461) $\div$ 2.935 $=$ 4812° *temperature of combustion of average coal.*

If surplus air is mixed with products of combustion equal to volume of air chemically combined, total weight of gases for one lb. of this coal is increased to 22.64. See following table, having a mean specific heat of .242.

Then 22.64 $\times$ .242 $=$ 5.478 *units for* 1°.

Hence, 14 133 total heat of combustion $\div$ 5.478 $=$ 2614° temperature of combustion, or a little more than half that of undiluted products.

Taking averages, it is seen that the evaporative efficiency of coal varies directly with volume of constituent carbon, and inversely with volume of constituent oxygen; and that it varies, not so much because there is more or less carbon, as, chiefly, because there is less or more oxygen. The per-centages of constituent hydrogen, nitrogen, sulphur, and ash, taking averages, are nearly constant, though there are individual exceptions, and their united effect, as a whole, appears to be nearly constant also.

Heat of Combustion.

Or, number of times in combustion of a substance, its equivalent weight of water would be raised 1°, *by heat evolved in combustion of substance.*

Alcohol	12 930	Ether	16 246	Olefiant gas	21 340
Charcoal	14 545	Olive oil	17 750	Hydrogen	62 030

Combustion of Fuel.

Constituents of coal are *Carbon*, *Hydrogen*, *Azote*, and *Oxygen*.

Volatile products of combustion of coal are hydrogen and carbon, the unions of which (relating to combustion in a furnace) are *Carburetted hydrogen* and *Bi-carburetted hydrogen* or *Olefiant gas*, which, upon combining with atmospheric air, becomes *Carbonic acid* or *Carbonic oxide*, *Steam*, and uncombined *Nitrogen*.

Carbonic oxide is result of imperfect combustion, and Carbonic acid that of perfect combustion.

Perfect combustion of carbon evolves heat as 15 to 4.55 compared with imperfect combustion of it, as when carbonic oxide is produced.

1 lb. carbon combines with 2.66 lbs. of oxygen, and produces 3.66 lbs. of carbonic acid.

Smoke is the combustible and incombustible products evolved in combustion of fuel, which pass off by flues of a furnace, and it is composed of such portions of hydrogen and carbon of the fuel gas as have not been supplied or combined with oxygen, and consequently have not been converted either into steam or carbonic acid; the hydrogen so passing away is invisible, but the carbon, upon being separated from the hydrogen, loses its gaseous character, and returns to its elementary state of a black pulverulent body, and as such it becomes visible.

Bituminous portion of coal is converted into gaseous state alone, carbonaceous portion only into solid state. It is partly combustible and partly incombustible.

To effect combustion of 1 cube foot of coal gas, 2 cube feet of oxygen are required; and, as 10 cube feet of atmospheric air are necessary to supply this volume of oxygen, 1 cube foot of gas requires oxygen of 10 cube feet of air.

In furnaces with a natural draught, volume of air required exceeds that when the draught is produced artificially.

An insufficient supply of air causes imperfect combustion; an excessive supply, a waste of heat.

Volume of atmospheric air that is chemically required for combustion of 1 lb. of bituminous coal is 150.35 cube feet. Of this, 44.64* cube feet combine with the gases evolved from the coal, and remaining 105.71 cube feet combine with the carbon of the coal.

Combination of gases evolved by combustion gives a resulting volume proportionate to volume of atmospheric air required to furnish the oxygen, as 11 to 10. Hence the 44.64 cube feet must be increased in this proportion, and it becomes $44.64 + 4.46 = 49.1$.

Gases resulting from combustion of the carbon of coal and oxygen of the atmosphere, are of same bulk as that of atmospheric air required to furnish the oxygen, viz., 105.71 cube feet. Total volume, then, of the atmospheric air and gases at bridge wall, flues, or tubes, becomes $105.71 + 49.1 = 154.81$ cube feet, assuming temperature to be that of the external air. Consequently, augmentation of volume due to increase of temperature of a furnace is to be considered and added to this volume, in the consideration of the capacity of flue or *calorimeter* of a furnace.

There is required, then, to be admitted through the grates of a furnace for combustion of 1 lb. of bituminous coal as follows:

Coal containing 80 per cent. of carbon, or .7047 per cent. of coke.

1 lb. coal × 44.64 cube feet of gas........ =	44.64
.7047 lb. carbon × 150 cube feet of air ... =	105.71
	150.35 *cube feet.*

For anthracite, by observations of W. R. Johnston, an increase of 30 per cent. over that for bituminous coal is required = 195.45 *cube feet.*

Coke does not require as much air as coal, usually not to exceed 108 cube feet, depending upon its purity.

Heat of an ordinary furnace may be safely considered at 1000°; hence air entering ash-pit and gases evolved in furnace under general law of expansion of permanently elastic fluids of $\frac{1}{479}$ths of its volume (or .002 087) for each degree of heat imparted to it, the 154.81 is increased in volume from 100° (assumed ordinary temperature of air at ash-pit) to $1000° = 900°$; then $900 \times .002\,087 = 1.8783$ times, or $154.81 + \overline{154.81 \times 1.8783} = 445.59$ *cube feet.*

If the combustion of the gases evolved from coal and air was complete, there would be required to give passage to volume of but 445.59 cube feet over bridge wall or through flues of a furnace; but by experiments it appears that about one half of the oxygen admitted beneath grates of a furnace passes off uncombined; the area of the bridge wall, or flues or tubes, must consequently be increased in this proportion, hence the 445.59 becomes 891.18.

Velocity of the gases passing from furnace of a proper-proportioned boiler may be estimated at from 30 to 36 feet per second. Then $\frac{891.18}{60' \times 60'' \times 36} =$ .006 87 sq. feet, or .99 sq. ins., of area at bridge wall for each lb. of coal consumed per hour.

A limit, then, is here obtained for area at the bridge wall, or of flues or tubes immediately behind it, below which it must not be decreased, or combustion will be imperfect. In ordinary practice it will be found advantageous to make this area .014 sq. feet, or 2 sq. ins. for every lb. of bituminous coal consumed per sq. foot of grate per hour, and so on in proportion for any other quantity.

Volumes of heat evolved are very nearly same for same substance, whatever temperature of combustible.

* By experiment, 4.464 cube feet of gas are evolved from 1 lb. of bituminous coal, requiring 44.64 cube feet of air.

Relative Volumes of Air required for Combustion of Fuels.

	Lbs.		Lbs.		Lbs.
Warlich's patent....	13.1	Anthracite Coal....	12.13	Bitum. Coal, lowest..	5.92
Charcoal...........	11.16	Bituminous "	10.98	Peat, dry...........	7.08
Coke..............	11.28	Bitum. Coal, average	10.7	Wood, dry..........	6

Perfect combustion of 1 lb. of carbon requires 11.18 lbs. air at 62°, and total weight = 12.39 *lbs.* Total heat of combustion of 1 lb. carbon or charcoal is 14 500 thermal units; mean specific heat of products of combustion is .25, which, multiplied by 12.39 as above = 3.0975, and 14 500* ÷ 3.0975 = 4681° temperature of a furnace, assuming every atom of oxygen that was ignited in it entered into combination.

If, however, as in ordinary furnaces, twice volume of air enters, then products of combustion of 1 lb. of coal will be 12.39 + 11.18 = 23.57, which, multiplied by its specific heat of .25 as before, and if divided into 14 500, quotient will be 2641°, which is temperature of an ordinary furnace.

Ratio of Combustion.—Quantity of fuel burned per hour per sq. foot of grate varies very much in different classes of boilers. In Cornish boilers it is 3.5 lbs. per sq. foot; in ordinary Land boilers, 10 to 20 lbs.; (English) 13 to 14 lbs.; in Marine boilers (natural draught), 10 to 24 lbs.; (blast) 30 to 60 lbs.; and in Locomotive boilers, 80 to 120 lbs.

Volumes of air and smoke for each cube foot of water converted into steam, is for coal and coke 2000 cube feet, for wood 4000 cube feet; and for each lb. of fuel as follows:

Coal.......... 207 | Cannel coal... 315 | Coke......... 216 | Wood......... 173

Calorific power of 1 lb. good coal = 14 000 × 772 = 10 808 000 *lbs.*

Relative Evaporation of Several Combustibles in Lbs. of Water, Heated 1° by 1 Lb. of Material.

Combustible.	Composition.	Water.	Combustible.	Composition.	Water.
		Lbs.			Lbs.
Alcohol..... .812	Hyd. .12, Carb. .45	8 120	Olive oil..........	Hyd. .13, Carb. .77	14 560
Bituminous coal...	Hyd. .04, Carb. .75	9 830	Peat, moist.......	Hyd. .04, Carb. .43	3 481
Carbon..........	—	14 220	" dry.........	Hyd. .06, Carb. .58	3 900
Coke.............	Carb. .84	9 028			
Hydrogen (mean)..	—	50 854	Pine wood, dry....	Hyd. .06, Carb. .7	3 618
Oak wood, dry....	Hyd. .06, Carb. .53	6 018	Sulphuric ether, .7	Hyd. .13, Carb. .6	8 680
" " green...	Hyd. .08, Carb. .37	5 662	Tallow............	—	14 560

1 lb. Hydrogen will evaporate 62.6 lbs. water from 212° = 60.509 lbs. heated 1°.
1 lb. Carbon " 14.6 lbs. " 212°, or raise 12 lbs. water at 60° to steam at 120 lbs. pressure.

1 lb. of Oxygen will generate same quantity of heat whether in combustion with hydrogen, carbon, alcohol, or other combustible.

Relative Volumes of Gases or Products of Combustion per Lb. of Fuel.

Temp. Air.	Supply of Air per lb. of Fuel. 12 lbs. Volume per lb.	18 lbs. Volume per lb.	24 lbs. Volume per lb.	Temp. Air.	Supply of Air per lb. of Fuel. 12 lbs. Volume per lb.	18 lbs. Volume per lb.	24 lbs. Volume per lb.
°	Cube Feet.	Cube Feet.	Cube Feet.	°	Cube Feet.	Cube Feet.	Cube Feet.
32	150	225	300	572	314	471	628
68	161	241	322	752	369	553	738
104	172	258	344	1112	479	718	957
212	205	307	409	1472	588	882	1176
392	259	389	519	2500	906	1359	1812

* Mean of all experiments 13 964.

To Compute Consumption of Fuel to Heat Air.

RULE.—Divide volume of air to be heated by volume of 1 lb. of it, at its temperature of supply; multiply result by number of heat-units necessary to raise 1 lb. air through the range of temperature to which it is to be heated, and product, divided by number of heat-units of fuel used, will give result in lbs. per hour.

EXAMPLE.—What is required consumption per hour of coal of an average composition to heat 776 400 cube feet of air at 54° to 114°?

Coal of an average composition (Table, page 461) = 14 133 *heat-units*. Volume of 1 lb. air at 54° (see formula, page 522) $= \frac{461+54}{39.8} = 12.94$ *cube feet*. $1 \times \overline{114-54} \times .2377$ (specific heat of air) = 14.262 *heat-units*.

$$\frac{776\,400}{12.94} \times 14.262 \div 14\,133 = 60.55 \text{ lbs.}$$

Loss of heat by conduction of it to walls of apartment is to be added to this.

EXCAVATION AND EMBANKMENT.

Labor and Work upon Excavation and Embankment.

Elements of Estimate of Work and Cost.

Per Day of 10 Hours.

Cart.—One horse. Distance or *lead* assumed at 100 feet, or 200 *feet for a trip*, at a speed of 200 *feet per minute*.

Earths.—Of gravelly, loam, and sandy, a laborer will load per day into a cart respectively 10, 12, and 14 cube yards as measured in embankment, and if measured in excavation, .11 more is to be added, in consequence of the greater density of earth when placed in embankment than in excavation.

NOTE.—Earth, when first loosened, increases in volume about .2, but when settled in embankment it has less volume than when in bank or excavation.

Carting.—Descending, load .33 cube yard, Level, .28, and Ascending .25, measured in embankment; and number of cart-loads in a cube yard of embankment are, Gravelly earth 3, Loam 3.5, and Sandy earth 4.

Loosening.—Loam, a three-horsed plough will loosen from 250 to 800 cube yards per day.

Trimming.—Cost of trimming and superintendence 1 to 2 cents per cube yard.

Scooping.—A scoop load measures about .1 cube yard in excavation; time lost in loading, unloading, and turning, 1.125 minutes per load; in double scooping it is 1 minute. Time occupied for every 100 feet of distance from excavation to embankment, 1.43 minutes.

Time.—Time occupied in loading, unloading, awaiting, etc., 4 minutes per load.

To Compute Number of Loads or Trips in Cube Yards per Cart per Day.

$\left(\frac{60}{E \div 100 + 4}\right) h \div y = n$. *E representing average distance of carting from embankment in stations of 100 feet each, y number of cart-loads to cube yard of excavation, and n number of cube yards in embankment, hauled by a cart per day to distance* E.

ILLUSTRATION. — What is number of cube yards of loam that can be removed by one cart from an embankment on level ground for an average distance of 250 feet?

$$E = 250 \div 100 = 2.5, \text{ and } y = 3.5.$$

$$\frac{60}{2.5+4} \times 10 \div 3.5 = \frac{60}{6.5} \times 10 \div 3.5 = 26.37 \textit{ cube yards.}$$

Substituting for 3, 3.5, and 4 number of cart-loads in a cube yard of embankment, 20, 17.14, and 15, = 60 minutes, divided respectively by these numbers.

$\frac{h \times 20}{E+4} = n$, *in descending carting;* $\frac{17.14 \times h}{E+4} = n$, *in level, and* $\frac{15 \times h}{E+4} = n$, *in ascending. h representing number of hours actually at work.*

To Compute Cost of Excavating and Embanking per Cube Yard.

$\frac{L}{v} + \frac{c}{n} + l + s = V$. *L representing pay of laborers, v value or result of loading in different earths, as 10, 12, and 14, c of one cart and driver per day, l cost of loosening material per cube yard, and s cost of trimming and superintendence, both per cube yard, and all in cents.*

ILLUSTRATION.—Volume of excavation in loam 30 000 cube yards. Level carting 650 feet = 6.5 trips or courses. Loosening by plough 1.7 cents per cube yard, laborers 106 cents per day, carts 160, and trimming and superintendence 1.5 cents per cube yard.

$v = 12$, and $\frac{17.14 \times 10}{6.5+4} = 16.33$, *number of loads per day by preceding formula.*

Then $\frac{106}{12} + \frac{160}{16.33} + 1.7 + 1.5 = 8.833 + 9.797 + 1.7 + 1.5 = 21.83$ *cents per cube yard.*

Earthwork.

By Carts.—A laborer can load a cart with one third of a cube yard of sandy earth in 5 minutes, of loam in 6, and of heavy soil in 7. This will give a result, for a day of 10 hours, of 24, 20, and 17.2 cube yards of the respective earths, after deducting the necessary and indispensable losses of time, which is estimated at .4.

It is not customary to alter the volume of a cart-load in consequence of any difference in density of the earths, or to modify it in consequence of a slight inclination in the grade of the lead.

In a lead of ordinary length one driver can operate 4 carts. With labor at $1 per day, the expense of a horse and cart, including harness, repairs, etc., is $1.25 per day.

A laborer will spread from 50 to 100 cube yards of earth per day.

The removal of stones requires more time than earth.

The cost of maintaining the lead in good order, the wear of tools, superintendence, trimming, etc., is fully 2.5 cents per cube yard.

By Wheel-barrows.—A laborer in wheeling travels at the rate of 200 feet per minute, and the time occupied in loading, emptying, etc., is about 1.25 minutes, without including lead. The actual time of a man in wheeling in a day of 10 hours is .9 or 2.25 minutes per lead of 100 feet. Hence,

To Compute Number of Barrow-Loads removed by a Laborer per Day.

$\frac{10 \times 60 \times .9}{1.25 + n'} = n$. *n′ representing number of leads of 100 feet.*

A barrow-load is about .04 of a cube yard.

Rock.

By Carts.—Quarried rock will weigh upon an average 4250 lbs. per cube yard, and a load may be estimated at .2 cube yard, and weighing a very little more than a load of average earth.

Hence, the comparative cost of carting earth and rock is to be computed on the basis of a cube yard of earth averaging 3.5 loads and one of rock 5 loads, with the addition of an increase in time of loading, and wear of cart.

Labor.

For labor of a man, see Animal Power, pp. 433–34.

By Wheel-barrow.—A barrow-load may be assumed at 175 lbs. = 2 cube feet of space.

Blasting.—When labor is $1 per day, hard rock in ordinary position may be blasted and loaded for 45 cents per cube yard.

The cost, however, in consequence of condition, position, etc., may vary from 20 cents to $1.

See Blasting, page 443.

17 cube yards of hard rock may be carted per day over a lead of 100 feet, at a cost of 7.29 cents per yard.

The preceding elements are essentially deduced from notes furnished by Ellwood Morris, C.E., and the valuable treatise of John C. Trautwine, C.E., Phila., 1872.

Stone.

Hauling Stone.—A cart drawn by horses over an ordinary road will travel 1.15 miles per hour of trip = 2.3 miles per hour.

A four-horse team will haul from 25 to 36 cube feet of stone at each load.

Time expended in loading, unloading, etc., including delays, averages 35 minutes per trip. Cost of loading and unloading a cart, using a horse-crane at the quarry, and unloading by hand, when labor is $1.25 per day, and a horse 75 cents, is 25 cents per perch = 24.75 cube feet = 1 cent per cube foot.

Work done by an animal is greatest when velocity with which he moves is .125 of greatest with which he can move when not impeded, and force then exerted .45 of utmost force the animal can exert at a dead pull.

Earthwork. (*Molesworth.*)

Proportion of Getters, Fillers, and Wheelers in different soils, Wheelers being calculated at 50 yards run.

	Gett's.	Fill's.	Wheel's.		Gett's.	Fill's.	Wheel's.
In loose earth, sand, etc.	1	1	1	In Hard clay	1	1.25	1.25
" Compact	1	2	2	" Compact gravel	1	1	1
" Marl	1	2	2	" Rock, from	3	1	1

Average Weight of Earths, Rocks, etc.

Per cube yard.

	Lbs.		Lbs.		Lbs.		Lbs.
Sand	3360	Marl	2912	Sandstone	4368	Granite	4700
Gravel	3360	Clay	3472	Shale	4480	Trap	4700
Mud	2800	Chalk	4032	Quartz	4492	Slate	4710

Bulk of Rock Earthwork, etc., original Excavation assumed at 1.

When in Embankment.

Rock, large	1.5	Sand and gravel	1.07
Medium	1.25 to 1.3	Clay and earth after subsidence	1.08
Metal	1.2	" " before "	1.2

FRICTION.

Friction is the force that resists the bearing or movement of one surface over another, and it is termed *Sliding* when one surface moves over another, as on a slide or over a pin; and *Rolling* when a body rotates upon the surface of some other, as a wheel upon a plane, so that new parts of both surfaces are continually being brought in contact with each other.

The force necessary to abrade the fibres or particles of a body is termed *Measure of friction;* this is determined by ascertaining what portion of the weight of a moving body must be exerted to overcome the resistance arising from this cause.

Coefficient of Friction expresses ratio between pressure and resistance of one surface over or upon another, or of surfaces upon each other.

Angle of Repose is the greatest angle of obliquity of pressure between two planes, consistent with stability, the tangent of which is the coefficient of friction.

Experiments and *Investigations* have adduced the following observations and results:

1. Amount of friction in surfaces of like material is very nearly proportioned to pressure perpendicularly exerted on such surfaces.

2. With equal pressure and similar surfaces, friction increases as dimensions of surfaces are increased.

3. A regular velocity has no considerable influence on friction; if velocity is increased friction may be greater, but this depends on secondary or incidental causes, as generation of heat and resistance of the air.

M. Morin's experiments afford the principal available data for use. Though constancy of friction holds good for velocities not exceeding 15 or 16 feet per second, yet, for greater velocities, resistance of friction appears, from experiments of M. Poirée, in 1851, to be diminished in same proportion as velocity is increased.

4. Similar substances excite a greater degree of friction than dissimilar. If pressures are light, the hardest bodies excite least friction.

5. In the choice of unguents, those of a viscous nature are best adapted for rough or porous surfaces, as tar and tallow are suitable for surfaces of woods, and oils best adapted for surfaces of metals.

6. A rolling motion produces much less friction than a sliding one.

7. Hard metals and woods have less friction than soft.

8. Without unguents or lubrication, and within the limits of 33 lbs. pressure per sq. inch, the friction of hard metals upon each other may be estimated generally at about one sixth the pressure.

9. Within limits of abrasion friction of metals is nearly alike.

10. With greatly increased pressures friction increases in a very sensible ratio, being greatest with steel or cast iron, and least with brass or wrought iron.

11. With woods and metals, without lubrication, velocity has very little influence in augmenting friction, except under peculiar circumstances.

12. When no unguent is interposed, the amount of the friction is, in every case, independent of extent of surfaces of contact; so that, the force with which two surfaces are pressed together being the same, their friction is the same, whatever may be the extent of their surfaces of contact.

13. Friction of a body sliding upon another will be the same, whether the body moves upon its face or upon its edge.

14. When fibres of materials cross each other, friction is less than when they run in the same direction.

15. Friction is greater between surfaces of the same character than between those of different characters.

16. With hard substances, and within limits of abrasion, friction is as pressure, without regard to surfaces, time, or velocity.

17. The influence of duration of contact (friction of rest) varies with the nature of substances; thus, with hard bodies resting upon each other, the effect reaches a maximum very quickly; with soft bodies, very slowly; with wood upon wood, the limit is attained in a few minutes; and with metal on wood, the greatest effect is not attained for some days.

Coefficient of Friction of Journals.

Diameters from 2 to 4 ins. Speeds varied as 1 to 4. Pressure up to 2 tons.

(*From data of M. Morin.*)

Surfaces of Contact. (Journals. / Bearings.)	Lubrication.	Coefficient* pressure = 1. Ordinary Lubrication.
Cast iron on cast iron	Olive oil, or tallow	.07 to .08
	Unctuous and wet	.14
Cast iron on gun metal	Olive oil, or tallow	.07 to .08
	Unctuous and wet	.16
Cast iron on lignum-vitæ	Slightly unctuous	.18
	Oil, or lard	—
	Lard and plumbago	.14
Wrought iron on cast iron	Olive oil, or tallow	.07 to .08
Wrought iron on gun metal	Olive oil, or tallow	.07 to .08
	Unctuous and wet	.19
	Slightly unctuous	.25
Wrought iron on lignum-vitæ	Oil	.11
	Unctuous	.19
Gun metal on gun metal	Oil	.1
Lignum-vitæ on cast iron	Unctuous	.15

* Continuous lubrication reduces the coefficients fully one half.

Surfaces of Contact.	Disposition of Fibres and Lubrication.	Coefficient pressure = 1.
Oak on oak	Parallel and soaped	.16
Wrought iron on oak	" " wet	.26
" " "	" " soaped	.21
Cast iron on oak	" " wet	.22
" "	" " soaped	.19
Leather on oak	" " wet	.29
Leather belt on oak (flat)	" " dry	.27
" " on oak pulley	Perpendicular " "	.47

Leather belts over wood drums .47 of pressure, and over turned cast-iron pulleys .28 of pressure.

Coefficients of Friction of Motion.

Substances.		Dry.	Water.	Olive-oil.	Lard.	Tallow.	Dry Soap.	Greasy and wet.
Hemp cords, etc.	On wood	.45	.33	—	—	—	—	—
	On iron	—	—	.15	—	.19	—	—
Metal upon wood	Mean	.18	.31	.07	.09	.09	.2	.13
Sole-leather, smooth, upon wood or metal	Raw	.54	.36	.16	—	.2	—	—
	Dry	.34	.31	.14	—	.14	—	—
Wood upon metal	Mean	.42	.24	.06	.07	.08	.2	.14
Wood upon wood		.36	.25	—	.07	.07	.15	.12

(Condition of Surfaces and Unguents: Dry, Water, Olive-oil, Lard, Tallow, Dry Soap, Greasy and wet.)

Relative Value of Unguents to Reduce Friction.

Unguents.	Wood upon Wood.	Wood upon Metals.	Metals upon Metals.	Unguents.	Wood upon Wood.	Wood upon Metals.	Metals upon Metals.
Dry soap	.4	.32	.27	Olive oil	—	1	1
Lard	.82	.85	.7	Tallow	1	.93	.8
Lard and plumbago.	—	.67	.96	Water	.22	.24	.18

To Determine Coefficient of Friction of Bodies.

Place them upon a horizontal plane, attach a cord to them, and lead it in a direction parallel to the plane over a pulley, and suspend from it a scale in which weights are to be placed until body moves.

Then weight that moves the body is numerator, and weight of body moved is denominator of a fraction, which represents coefficient required.

Illustration.—If, by a pressure of 320 lbs. friction amounts to 80 lbs., its coefficient of friction in this case would be $80 \div 320 = .25$.

Hence, if coefficient of friction of a wagon over a gravel road was .25, and the load 8400 lbs., the power required to draw it would be $8400 \times .25 = 2100$ *lbs.*

Coefficients of Axle Friction. (*M. Morin.*)

Substances.	Condition of Surfaces and Unguents. Dry and a little Greasy.	Greasy and wet with Water.	Oil, Tallow, or Lard. In usual way.	Oil, Tallow, or Lard. Continuously.	Very soft and purified Carriage Grease.
Bell metal upon bell metal	....	....	.097	....	....
Cast iron upon bell metal	.194	.161	.075	.054	.065
Cast iron upon cast iron	....	.079	.075	.054	....
Cast iron upon lignum-vitæ	.185	....	.1	.092	.109
Wrought iron upon bell metal	.251	.189	.075	.054	.09
Wrought iron upon cast iron	....	....	.075	.054	....
Wrought iron upon lignum-vitæ	.188	....	.125	....	....

Friction of a journal of an axle which presses on one side only, as in a worn bearing, is less than when it presses at all points, the difference being about .005.

Friction of Axles.—With axles, friction of motion has alone been experimented upon. When weight upon axle and radius of its journal is given, *mechanical effect* of friction may be readily determined.

The mechanical effect absorbed by, or of friction, increases with pressure or weight upon journal of axle and number of revolutions.

Friction of an axle is greater the deeper it lies in its bearing.

If journal of an axle lies in a prismatic bearing, as in a triangle, etc., friction is greater, as there is more pressure on, and consequently greater friction in contact: in a triangular bearing it is about double that of a cylindrical bearing.

To Compute Mechanical Effect of Friction on Journal of an Axle.

$\frac{p\,n\,f\,W\,r}{30} = F.$ *n representing number of revolutions, and r radius of journal in feet.*

Illustration.—Weight of a wheel, with its axle or shaft resting on its journals, is 360 lbs.; diameter of journals 2 ins.; and number of revolutions 30; what is mechanical effect of the friction, the coefficient of it being .16?

$$\frac{3.1416 \times 30 \times .16 \times 360 \times 1 \div 12}{30} = \frac{452.4}{30} = 15.08 \text{ } lbs.$$

By application of friction-wheels (rollers) friction is much reduced, and mechanical effect then becomes, when weights of friction-wheels are disregarded,

$\frac{p\,n\,f\,W\,r}{30} \times \frac{r'}{a' \cos. \overline{a \div 2}} = F.$ *r′ representing radii of axles of friction-wheels, a′ radii of friction-wheels, and a angle of lines of direction between axis of roller and axis of friction-wheels.*

When a single friction-wheel is used, $\frac{2\,p\,r\,n}{60} \times f\,W = F$. and $\frac{F}{r' \div a'} = F'$. *F′ representing mechanical effect.*

ILLUSTRATION.—A wheel and its shaft, making 5 revolutions per minute, weighs 30 000 lbs.; its diameter and that of its journals are 32 feet and 10 ins. The journals rest upon a friction-wheel, the radius of which is 5 times greater than its axle.

1. What is the power at circumference of wheel necessary to overcome friction? 2. What is mechanical effect of the friction? 3. What is reduction of friction by use of the friction-wheel?

1. $\frac{\overline{32 \div 2 \times 12}}{10 \div 2} = 38.4$, *circum. of wheel = 38.4 times that of axle.*

Coefficient of friction assumed at .075. Hence $\frac{30\,000 \times .075}{38.4} = 58.59$ *lbs. = power at circum. to overcome friction at axle.* 2. $\frac{10 \times 3.1416}{12} = 2.618$ *feet = distance passed by friction.*

Consequently, $\frac{2.618 \times 5}{60} = .2181$ *feet = distance passed by friction in one second.*

Hence, $.2181 \times 2250\ (30\,000 \times .075) = 490.725$. 3. $1 \div 5 = .2$ = *radius of friction-axle ÷ by radius of friction-wheel,* and $38.4 \times .2 = 7.68$ = *friction referred to circum. of wheel, and* $\frac{490.725}{5} = 98.145$ = *mechanical effect by application of friction-wheel = a reduction of four fifths.*

Friction of Pivots.

Friction on Pivots is independent of their velocity, increases in a greater degree than their pressures, and approximates very near to that of sliding and axle friction.

Friction on Conical Bearings is greater than with like elements on plane surfaces.

Figure of point of a pivot, as to its acuteness, affects friction: with great pressure the most advantageous angle for the figure ranges from 30° to 45°; with less pressure it may be reduced to 10° and 12°.

Relative Value of Angles of Pivots.

6°	1	15°	.66	45°	.39

Relative Values of different Materials for use as Pivots.

Agate	.83	Granite	1	Tempered steel	.44
Glass	.55	Rock crystal	.76		

Friction and Rigidity of Cordage.

Experiments by Amonton and Coulomb, with an apparatus of Amonton's, furnish the following deductions:

1. That resistance caused by stiffness of cords about the same or like pulleys varies directly as the suspended weight.

2. That resistance caused by stiffness of cords increases not only in direct proportion of suspended weights, but also in direct proportion of diameter of the cords.

Consequently, that resistance to motion over the same or like pulleys, arising from stiffness of cords, is in direct compound proportion of suspended weight and diameter of cords.

3. That resistance to bending varied inversely as diameter of sheave or drum.

4. That complete resistance is represented by expression $\frac{S+C\,T}{d}$. S *representing constant for each rope and sheave, expressing stiffness of rope;* T *tension of rope which is being bent, expressed by* C T; C *constant for each rope and sheave; and* d *diameter of sheave, including diameter of rope.*

5. That stiffness of tarred ropes is sensibly greater than that of white ropes.

Extending results obtained by Coulomb, Morin furnishes following *formulas:*

For White Ropes: $12\,n \div d\,(.002\,15 + .001\,77\,n + .0012\,W) = R$. *For Tarred Ropes:* $12\,n \div d\,(.010\,54 + .0025\,n + .0014\,W) = R$. R *representing rigidity in lbs.,* n *number of yarns,* d *diameter of sheave in ins. and rope combined, and* W *weight in lbs.*

Illustration.—What is value of stiffness or resistance of a dry white rope having a diameter of 60 yarns, which runs over a sheave 6 ins. in diameter in the groove, with an attached weight of 1000 lbs.?

Assume diameter for 60 yarns to be 1.2 *ins.* Then $\frac{12 \times 60}{7.2}(.002\,15 + .001\,77 \times 60 + .0012 \times 1000) = 100 \times 1.308\,35 = 130.835$ *lbs.*

Value of natural stiffness of ropes increases as the square of number of threads nearly, and value of stiffness proportional to tension is directly as number of threads, being a constant number. Hence, having the rigidity for any number of threads, the rigidity for a greater or lesser number is readily ascertained.

Wire Ropes.

Weisbach deduced from his experiments on wire ropes that their rigidity for diameters capable of supporting equal strains with hemp ropes is considerably less.

Wire ropes, newly tarred or greased, have about 40 per cent. less rigidity than untarred ropes.

Rolling Friction.

Rolling Friction increases with pressure, and is inversely as diameter of rolling body.

For rolling upon compressed wood, $f = .019$ to $.031$.

When a Body is moved upon Rollers and Power applied at the Base of the Body, $(f+f')\frac{W}{r} = F$. f *and* f' *representing coefficients of friction of two surfaces upon which rollers act.*

When Power is applied at Circumference of Roller, $f\,W \div r = F$.

When Power is applied at Axis of Roller, $f\,W \div \overline{r \div 2} = F$.

Bearings for Propeller Shaft. (*Mr. John Penn.*)

Bearings.	Pressure per Sq. Inch.	Time of Operation.	Bearings.	Pressure per Sq. Inch.	Time of Operation.
	Lbs.	Min.		Lbs.	Min.
Babbit's metal on iron*	1600	8	Brass on iron†	675	60
Box on brass	4480	5	Brass on iron‡	4480	—
Box on iron	448	30	Lignum-vitæ on brass	4000	5
Brass on brass	448	30	Snake-wood on brass	4000	5
Brass on iron	448	30	Lignum-vitæ on iron	1250	2160

* Rolled out. † Abraded. ‡ Set fast.

Result of Experiments upon Friction of Several Instruments. (*R. S. Ball.*)

Instrument.	Friction. F L	Velocity ratio.	Mechanical efficiency.	Useful effect. Per Cent.
Pulley, single	2.21 + .5453	2	1.8	90
" 3 sheaves	2.36 + .238	6	4	64
" differential	3.87 + .151	16	6.1	38
Screw	.0 + .014	193	70	36
Inclined plane, angle 17° 2′	.09 + .55	3.4	1.72	51
Screw Jack	.66 + .007	414	116	28
Wheel and Axle	.204 + .043	31	22	70
" " Barrel	.5 + .169	5.95	5.55	93
" " Pinion	2.46 + .21	8	4.1	51
Crane	.0 + .056	23	18	78
"	.185 + .008	137	87	63

F *representing friction, and* L *load.*

ILLUSTRATION 1.—If it is required to ascertain power necessary to raise 200 lbs. 2 feet, by a single movable pulley, $200 \times .5453 + 2.21 = 111.27$ *lbs.*, which must be applied as power to raise 200 lbs. 2 feet. $111.27 \times 2 = 222.54$ *lbs.* Hence, for application of 222.54 lbs., 200 or 89.87 *per cent. are usefully or effectively employed.*

2.—If it is required to raise 100 lbs. by a three-sheave pulley, then $100 \times .238 + 2.36 = 26.16$ *lbs*, which must be applied as power to raise 100 lbs. 6 feet ($3 \times 2 = 6$). $26.16 \times 6 = 156.96$ *lbs.* Hence, for application of 156.96 lbs., 100 or 63.71 *per cent. are effectively employed.*

3.—The velocity ratio of a crane being 137, and its mechanical efficiency 87, a man applying 26 lbs. to it can raise $87 \times 26 = 2262$ *lbs.*

Application of preceding Results.

ILLUSTRATION.—If a vessel, including cradle, weighing 1000 tons, is to be drawn upon an inclined plane having a rise of 10 feet in 100 of its length, what will be the resistance to be overcome, the cradle being supported on wrought-iron axles in cast-iron rollers, running on cast-iron rails?

$\frac{1000 \times 10}{100} = 100$ *tons = power required to draw vessel independent of friction.*

Ratio of friction to pressure of wrought iron on cast, in an axle and its bearing, .075. Ratio of ditto of cast iron upon cast, say .005.

Hence $.075 + .005 = .08$ of 1000 tons = 80 *tons*, which, added to 100 tons before deducted, gives 180 *tons, or resistance to be overcome.*

Power or effect lost by friction in axles and their bearing may be expressed by formula

$\frac{W f d r}{230} = P$. *f representing coefficient of friction, d diameter of axle in ins., and r number of revolutions per minute.*

ILLUSTRATION.—Pressure on piston of a steam-engine is 20000 lbs., number of revolutions 20, and diameter of driving shaft of wrought iron in a brass journal is 8 ins.; what is the effect of friction?

$$\frac{20\,000 \times .07 \times 8 \times 20}{230} = 973.91 \textit{ lbs.}$$

Hence $P\,v \div 33\,000 =$ IP. *v representing circumference of shaft in feet × by revolutions per minute.*

The power or effect lost by friction in guides or slides may be expressed by following formula:

$\frac{W f s r}{60 \times \sqrt{(5\,l^2 - s^2)}} = P$. *s representing stroke of cross-head, and l length of connecting rod in feet.*

Frictional Resistances.

Friction of Steam-engines.

Friction of Condensing Engines in Lbs. per Sq. Inch of Piston.

Diameter of Cylinder.	Oscillating and Trunk.	Beam and Geared.	Direct-acting and Vertical.	Diameter of Cylinder.	Oscillating and Trunk.	Beam and Geared.	Direct-acting and Vertical.
10	5	6	7	50	2.5	2.7	3.3
15	4	5	6	60	2.4	2.6	3
20	3.5	4	5	70	2.3	2.5	2.7
25	3	3.6	4.5	80	2	2.3	2.6
30	3	3.5	4	100	1.6	2.2	2.5
35	2.6	3	3.5	110	1.5	2	2.1

Experiments upon different steam-engines have determined that friction, when pressure on piston is about 12 lbs. per sq. inch, does not exceed 1.5 lbs., or about one tenth of power exerted.

Friction of double cylinder (50-inch diam.) direct-acting condensing propeller engine is 1.25 lbs. per sq. inch of piston = 10.3 per cent. of total power developed; friction of load is .9 lbs. per sq. inch of piston = 7.5 per cent. of total pressure; and friction of propeller is 1.3 lbs. per sq. inch of piston = 10.8 per cent. of total power = 28.6 per cent.

Friction of double cylinder (70-inch diam.) inclined condensing water-wheel engine with its load is 15 per cent. of total power developed.

In general, when engines are in good order, their efficiency ranges from 80 per cent. for small engines to 93 per cent. for large.

Power required to work air-pumps is 5 per cent., and to work feed-pumps 1 per cent.

Results of Experiments upon Friction of Machinery.

(*Davison.*)

Steam-engine, vertical beam, one tenth its power; 190 feet horizontal, and 180 feet vertical shafting, with 34 bearings, having an area of 3300 sq. ins., with 11 pair of spur and bevel wheels; 7.65 IP.

Set of three-throw Pumps, 6 ins. in diam., delivering 5000 gallons per hour at an elevation of 165 feet; 4.7 IP, or about 13 per cent.

Two pair iron Rollers and an elevator, grinding and raising 320 bushels malt per hour; 8.5 IP.

Ale-mashing Machine, 800 bushels malt at a time; 5.68 IP.

Archimedes Screw (ninety-five feet), 15 ins. in diameter, and an elevator conveying 320 bushels malt per hour to a height of 65 feet; 3.13 IP.

Friction Clutch.—Driven by a leather belt 14 ins. in width; face of clutch 5 ins. deep; broke a cast-iron shaft 6.5 ins. in diameter.

Flax Mill (*M. Cornut*, 1872).—Two condensing engines, cylinders, 12.9 ins. × 44.3 ins. stroke, and 22 ins. × 59.8 ins. stroke. Pressure of steam, 50 lbs. per sq. inch; revolutions, 25 per minute. Friction of entire machinery, 20 per cent.

With vegetable oil and hand oiling a steam pressure of 62 lbs. per sq. inch was required, and with mineral oil and continuous oiling a pressure of 50 lbs. only was required.

By continuous oiling, a saving of 44 per cent. was effected over hand oiling.

Flax Mill.

Power required to Drive Engine, Shafting, and entire Machinery. (*M. Cornut.*)

Parts.	Total.	Indicated Horse-power. One Machine at work.	empty.	Effect of Machines.
Engines, shafting, and belts..........	20.41	—	—	—
4 cards..............................	8.42	2.105	1.423	32
14 drawing frames (29 heads or 156 slivers)...........................	7.19	.0934	.0794	15
4 combing machines..................	2.22	.555	.151	78
6 roving frames (330 spindles)........	7.78	.02627*	2.434	7.3
20 spinning frames.				
Dry (1480 spindles)................	47.5	.0321*	2.515	21.6
Wet (2080 ")................	46.59	.0224*	1.613	19

Total 150.11 HP.

* Per 100 spindles.

Estimate of Horse's Power.—2080 spindles, wet, 34.4 per HP, long fibre.
640 " dry, 20.1 " " " "
840 " " 14.5 " " tow.
3560 " average, 23.7 " "

The HP per 100 spindles varies inversely as sq. root of their number.

Winding Engine (*G. H. Daglish*).

Shafts 738 *to* 1740 *feet in depth; cylinder* 65 × 84 *ins. stroke; pressure of steam* 19 *lbs. per sq. inch; revolutions* 12.5 *per minute; mean diameter of drum*, 26 *feet.* HP 313.4; effect 235 = 75 per cent.

Tools. (*Dr. Hartig*).

Single shearing, $.1 + \frac{n\,t^2}{20.7}$ = HP *to drive tool. n representing number of cuts per minute, t thickness of plate*, and $\frac{a\,F}{1\,980\,000}$ = HP *to shear. a representing area of surface cut or punched per hour in sq. ins., and* F $(1166 + 1691\,t)$ *a factor expressing work required to cut or shear a surface of* 1 *inch square.*

Illustration.—A shearing machine cutting 4648 sq. ins. of surface per hour, in plates .4 inch thick, required .68 HP to run and 4.3 to operate it, equal to 5 horses.

Iron Plate-bending. $\frac{85\,000\,b\,t^2\,l}{r}$ = P *for cold plates*, and $\frac{11\,300\,b\,t^2\,l}{r}$ = P *for red-hot plates. b, t, and l representing breadth, thickness, and length of plate, r radius of curvature, all in ins., and* P *net power of bending.*

Power for large rolls when running only .5 to .6 HP.

Ordinary Cutting Tools, in Metal.

Materials of a brittle nature, as cast iron, are reduced most economically in power consumed, by heavy cuts; while materials which yield tough curling shavings are more economically reduced by thinner cuttings. Following formulas apply to light cutting work:

Power required to plane cast iron is—

Planing Cast iron, $W\left(.0155 + \frac{1}{11\,000\,s}\right)$ = HP. W *representing weight of cast iron removed per hour, in lbs.*, and *s average sectional area of shavings, in sq. ins.*

Steel, Wrought iron, and Gun-metal, with cuts of an average character—

Steel .... .112 W = HP | Wrought iron, .052 W = HP | Gun-metal, .0127 W = HP

Planing and Molding.—Run without cutting. $\frac{N}{2000}$ = HP. N *representing sum of revolutions of all the shafts per minute.*

Molding. — Pine, $.0566 + \frac{.02268}{h}$, and Red Beech, $.08895 + \frac{.00731}{h} =$ IP. *h representing depth of wood cut down to form molding.*

Turning. — Steel, $.047\ W =$ IP; Wrought iron, $.0327\ W =$ IP; Cast iron, $.0314\ W =$ IP.

For turning off metals, power required is less than for planing, and it is ascertained that greater power is required for small diameters than large.

Light Lathes, $.05 + .0005\ n =$ IP; 1 or 2 shafts, $.05 + .0012\ n =$ IP; 3 or 4 shafts, $.05 + .05\ n =$ IP. *Heavy Lathes*, $.025 + .0031\ n$; $.025 + .053\ n$; $.025 + .18\ n$.

n representing number of revolutions of spindle per minute.

Drilling.—Power required to remove a given weight of metal is greater than in planing. Volume being taken in place of weight.

Holes from .4 to 2 ins. in diameter.

Cast iron, dry. $V\left(.0168 + \frac{.00067}{d}\right) =$ IP. Wrought iron, oil. $V\left(.0168 + \frac{.0269}{d}\right) =$ IP.

V representing volume removed in cube ins. per hour, and d diameter of hole.

Without gearing, $.0006\ n + .0005\ n'$; with gearing, $.0006\ n + .001\ n'$; radial drills without gearing, $.0006\ n + .004\ n'$; radial drills with gearing, $.04 + .0006\ n + .004\ n'$. *n representing number of revolutions per minute of gearing shaft, and n′ of drill.*

Slotting.—Stroke 8 ins. $.045 + \frac{n\,s}{4000} =$ IP. *n representing number of strokes per minute, and s stroke in ins.*

Wood-sawing, Circular.—A cube foot of soft wood and half a cube foot of hard, reduced to sawdust, requires 1 IP.

Hard wood, $\frac{A\,c}{6} =$ IP′. Soft wood, $\frac{A\,c}{12} =$ IP′. *A representing area in sq. feet and IP′ horse-power per sq foot, both cut per hour, and c width of cut in ins.*

From .4 to 4 ins. in diameter.—Pine. $V\left(.000125 + \frac{.00656}{d}\right) =$ IP.

Dry pine timber. $.00428 + .0065\ \frac{S\,c}{f} =$ IP′. *S representing stroke of saw in feet, and f feed per cut in ins.*

$\frac{n\,d}{32000} =$ IP *for horse-power to run only without cutting. d representing diameter of saw in ins., and n number of revolutions per minute.*

Net power required to cut with a circular saw is proportional to volume of material removed. For a saw cutting hot iron, at a circumferential speed of 7875 feet per minute, and making a cut .14 inch wide, power is expressed by formulas—

$.702\ A =$ IP, for red-hot iron. $1.013\ A =$ IP, for red-hot steel.

A representing sectional area of surface cut through, in sq. feet.

Vertical Saw. $.00428 + .0065\ \frac{S\,c}{f} =$ IP *in dry pine timber per sq. foot per hour. S representing stroke of saw in feet, c width of cut in ins., and f feed of cut in ins.*

Band Saw. $.0034 + \frac{.758\ c\ v}{10000\ f} =$ IP′ *in Pine.* $.00483 + \frac{.957\ c\ v}{10000\ f}$ IP′ *in Oak.* $.00576 + \frac{1.127\ c\ v}{10000\ f} =$ IP′ *in Beech. v representing velocity of saw, and f rate of feed, in feet per minute.*

Screw Cutting. Screws, $\frac{5\,l\,d^3}{64} =$ IP. Taps, $\frac{l\,d^3}{29} =$ IP. *d representing diameter in ins., and l length cut in feet per hour.*

Machine of medium dimensions, .2 IP.

Grindstones. $\frac{p\,C\,v}{33\,000} = HP$. *p representing pressure upon stone, v circumferential velocity of stone in feet per minute, and C coefficient of friction.*

Coefficients of Friction between Grindstones and Metals.

Cast iron, .22 at high speed, .72 at low speed; Wrought iron, .44 at high speed, 1 at low; Steel, .29 at high speed, .94 at low.

Power required to run them alone.

Large	$.000\,040\,9\,d\,v = HP$	Small......	$.16 + .000\,089\,5\,d\,v = HP$
or	$.000\,128\,d^2\,n = HP$	or......	$.16 + .000\,28\,d^2\,n = HP$

Grain Conveyers.

Conveyers of Grain horizontally by Screws and Bands.—A 12-inch screw, having 4 ins. pitch, turning in a trough, with a clearance of .25 inch, revolving with a speed of maximum effect, 60 turns per minute, will discharge 6.75 tons of grain per hour, expending .04 HP per foot run. Sectional area of body of grain moved 49 per cent. of that of screw. At speeds above 60 turns per minute, the grain will not advance, but will revolve with screw.

Screw Steamer. (*Vice-admiral C. R. Moorsom, R. N.*)

Moving friction of hull.............	.07	Slip of screw	.171
Moving friction of load.............	.063	Resistance of hull.................	.606
Moving friction of rotation of blades of screw...........	.09		
			1

Side Lever Steam-engine. (*J. V. Merrick.*)

In Pressure of Steam.

Friction to work air-pump...	.585	to	.7	lb.
Friction of weight of parts...	.5	"	.5	"
Friction of cylinder packing ...	.15	"	.3	"
Friction of air-pump packing ...	.046	"	.092	"
Friction of valves, parallel motion, resistance to air, etc.	.169	"	.258	"
	1.45		1.85	*lbs.*

Hence $\frac{1.45 + 1.85}{2} = 1.65$ lbs. per sq. inch. If journals are kept constantly lubricated, as with automatic lubrications, friction of weight will be reduced to .33, and pressure will be reduced from 1.65 — .33 to 1.32 lbs. per sq. inch of piston to work engine without load. Friction of load, according as journals are lubricated, ends keyed up, etc., will range from 2 to 5 per cent.

Locomotives and Railway Trains. See Railways, page 682.

Friction developed in Launching of Vessels.

Experiments made by a committee of Franklin Institute on friction of launching vessels gave, when pressure or weight was from 2280 to 3560 per sq. foot, a coefficient of .0335.

Marine Railway.—To draw 3000 tons upon greased slides a power of 250 tons was necessary to move it, but when started 150 tons would draw it.

Woollen Machinery. (*Dr. Hartig.*) When running empty 8.15 IHP, and at work 32.97.

The efficiency of the various machines averaging 60.5 per cent.

Friction of a Non-condensing Steam-engine.

Friction of an Engine. Diameter of cylinder 20 ins. by 40 ins. stroke of piston. Revolutions, 15 to 70 per minute.

Engine, unloaded, 2 lbs. per sq. inch	= 1.86 to 8.69 HP.
Shafting, unloaded, 2.5 to 45 lbs. per sq. inch......	= 2.36 to 19.61 "
Total 4.5 to 6.5 lbs. per sq. inch	= 4.22 to 28.3 "

FUEL.

With equal weights, where each kind is exposed under like advantageous circumstances, that which contains most hydrogen ought, in its combustion, to produce greatest volume of flame. Thus, pine wood is preferable to hard, and bituminous to anthracite coal.

When wood is used as a fuel, it should be as dry as practicable. To produce greatest quantity of heat, it should be dried by direct application of heat; usually it has about 25 per cent. of water combined with it, heat necessary for evaporation of which is lost.

Different fuels require different volumes of oxygen; for different kinds of coal it varies from 1.87 to 3 lbs. for each lb. of coal. 60 cube feet of air is necessary to furnish 1 lb. of oxygen; and, making a due allowance for loss, nearly 90 cube feet of air are required in furnace of a boiler for each lb. of oxygen applied to combustion.

Classification of Coal.	Semi-bituminous....	Cherry. Splint.	Hydrogenous or Gas coal.....	Cannel. Shaly. Asphalt.
	Bituminous....	Caking. Cherry. Splint.	Anthracites..........	Hard. Semi or gaseous.

Bituminous Coal.

Lignite. Brown Coal or Bituminous Wood.—Presents a distinct woody structure; is brittle, and burns readily, leaving a white ash, and contains and absorbs moisture in some cases fully 40 per cent.

Caking.—Fractures uneven, and when heated breaks into small pieces, which afterwards agglomerate and form a compact body. When the proportion of bitumen is great, it fuses into a pasty mass. This coal is unsuited where great heat is required, as the draught of a furnace is impeded by its caking. It is applicable for production of gas and coke.

Splint or Hard.—Color black or brown-black, lustre resinous and glistening. It kindles less readily than caking coal, but when ignited produces a clear and hot fire.

Cherry or Soft.—Alike to splint coal in fracture, but its lustre is more splendent. Does not fuse when heated, is very brittle, ignites readily, and produces a bright fire with a yellow flame, but consumes rapidly.

Cannel.—Color jet, or gray or brown-black, compact and even texture, a shining, resinous lustre. Fractures smooth or flat, conchoidal in every direction, and polishes readily.

Experiments upon practical burning of this description of coal in furnace of a steam-boiler give an evaporation of from 6 to 10 lbs. of fresh water, under a pressure of 30 lbs. per sq. inch per lb. of coal; Cumberland (Md., U. S.) coal being most effective, and Scotch least.

Limit of evaporation from 212° for 1 lb. of best coal, assuming all of heat evolved from it to be absorbed, would be 14.9 lbs.

Coals that contain sulphur, and are in progress of decay, are liable to spontaneous combustion.

There are very great variations in the chemical composition and properties of coals.

American.	*British.*
Carbon, from 75 to 80 *per cent.*	Carbon, from 70 to 91 *per cent.*
Hydrogen, from 5 to 6.	Hydrogen, from 3.5 to nearly 7.
Oxygen, from 4 to 10.	Oxygen, from about .5 to 20.
Nitrogen, from 1 to 2.	Nitrogen, from a mere trace to 2.2.
Sulphur, from .4 to 3.	Sulphur, from 0 to 5.
Ash, from 3 to 10.	Ash, from .2 to 15.
Coke, from 48.5 to 79.5.	Coke, from 49 to 93.

For Volume of Air, etc., see Combustion, page 465.

Coal.

Anthracite.

Anthracite or Glance Coal, or Culm—Is hard, compact, lustrous, and sometimes iridescent, most perfect being entirely free from bitumen; it ignites with difficulty, and breaks into fragments when heated.

Evaporative power, in furnace of a steam-boiler and under pressure, is from 7.5 to 9.5 lbs. of fresh water per lb. of coal.

Coal from one pit will sometimes vary 6 per cent. in evaporative value.

Elements of Various American Coals.

	Specific Gravity.	Fixed Carbon.	Volatile Matter.	Water.	Moisture	Ash.	Earthy Matter.
		Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.
Illinois, Warren Co.	1.23	51.7	43.1	—	—	—	5.2
Bureau "	1.32	57.6	28.8	—	11.2	2.4	—
Mercer "	1.26	54.8	31.2	—	8.4	5.6	—
Indiana, Clay "	1.28	56.5	32.5	8.5	—	2.5	—
Coopriders	1.28	50.5	42.5	3	—	4	—
Pennsylvania, Connellsville	1.28	65	24	4.5	—	6.5	—
Pennsylvania, Youghiogheny	1.3	58.4	35	—	1	5.6	—
Fayette Co.	1.29	58	34	3	—	5	—
Kentucky, Sardric	1.32	51	42.5	2	—	4.5	—
Mud River	1.28	57	37	3.5	—	2.5	—
Ohio, Nelsonville	1.27	58.4	33.05	6.65	—	1.9	—
Colorado, Carbon City	1.21	56.8	34.2	4.5	—	4.5	—
Washington Territory	1.32	58.25	31.75	7	—	3	—

Coke.

Coke.—Coking in a close oven will give an increase of yield of 40 per cent. over coking in heaps, gain in bulk being 22 per cent. Coals when coked in heaps will lose in bulk.

Cannel and Welsh (Cardiff) coals when coked in retorts will gain from 10 to 30 per cent. in bulk and lose 36.5 per cent. in weight.

Relative costs of coal and coke for like results, as developed by an experiment in a locomotive boiler, are as 1 to 2.4.

Evaporative power in furnace of a steam-boiler and under pressure, is from 7.5 to 8.5 lbs. of fresh water per lb.

Bituminous coal will yield from 60 to 80 per cent. of coke. Averaging 66 per cent. It is capable of absorbing 15 to 20 per cent. of moisture.

Heat of combustion lost in coking of bituminous coal 40 per cent.

Charcoal.

Charcoal, properly termed, is not made below a temperature of 536°. The best quality is made from Oak, Maple, Beech, and Chestnut.

Wood will furnish, when properly burned, about 23 per cent. of coal.

Charcoal absorbs, upon an average of the various kinds, from .8 per cent. of water for Beech, to 16.3 for Black Poplar, Oak absorbing about 4.28, and Pine 8.9.

Evaporative power, in furnace of a boiler and under pressure, is 5.5 lbs. of fresh water per lb. of coal.

Volume of air chemically required for combustion of 1 lb. of charcoal is, when it consists of 79 carbon, 129 cube feet at 62°.

138 bushels charcoal and 432 lbs. limestone, with 2612 lbs. of ore, will produce 1 ton of pig iron.

Produce of Charcoal from Various Woods dried at 300° and Carbonized at 572°. (*M. Violette.*)

Wood.	Weight.	Wood.	Weight.	Wood.	Weight.
	Per Cent.		Per Cent.		Per Cent.
Cork	62.8	Larch	40.31	Maple	33.75
Oak	46.09	Chestnut	36.06	Willow	33.74
Beech	44.25	Apple	34.69	Black elder	33.61
Pine	41.48	Elm	34.59	Ash	33.28
Poplar roots	40.9	Birch	34.17	Pear	31.88

Poplar .... 31.12 per cent.

In a Green or Ordinary State. (*Weight per cent.*)

Apple	23.8	Birch	24.1	Oak	22.85	Red Pine	23
Ash	26.7	Elm	25.1	" young	33.3	White Pine	23.5
Beech	21.1	Maple	22.9	Poplar	20.5	Willow	18.6

It appears from this that cork, the lightest of woods, yields largest per-centage of charcoal, about 63 per cent.; and that poplar yields lowest, about 31 per cent. There does not appear to be any definite relation between density of wood and volume of yield.

Produce by a slow process of charring is very nearly 50 per cent. greater than by a quick process.

Lignite.

Lignite is an imperfect mineral coal. It is distinguished from coal by its large proportion of oxygen, being from 13 to 29 per cent. Its specific gravity ranges from 1.12 to 1.35.

Elements of Various American Lignites. (*W. M. Barr.*)

Location.	Spec. Grav.	Fixed Carbon.	Volatile Matter.	Water.	Ash.	Total Volatile.	Coke.
		Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.
Kentucky	1.2	40	23	30	7	53	47
" Blandville	1.17	31	48	11.5	9.5	59.5	40.5
Washington Terr'y	—	58.25	31.75	7	3	38.75	61.25
Vancouver's Island	—	62	31	4	3	35	65
Colorado, Carbon City	1.27	41.25	46	3.5	9.25	49.5	50.5
Canon City	1.28	56.8	34.2	4.5	4.5	38.7	61.3
Arkansas	—	34.5	28.5	32	5	60.5	39.5
Texas, Robertson Co.	1.23	45	39.5	11	4.5	50.5	49.5

Asphalt.

Asphalt, alike to Lignite, contains a large proportion of oxygen.

Wood.

Wood, as a combustible, is divided into two classes, the hard, as Oak, Ash, Elm, Beech, Maple, and Hickory, and soft, as Pine, Cotton, Birch, Sycamore, and Chestnut.

Green wood subjected to a temperature ranging from 340° to 440° will lose 30 to 45 per cent. of its weight.

At a temperature of 300°, Oak, Ash, Elm, and Walnut, in a comparatively seasoned state, lost from 16 to 18 per cent.

Woods contain an average of 56 per cent. of combustible matter.

From an analysis of M. Violette it appears that composition of wood is about same throughout the tree, and that of the bark also; that wood and bark have about same proportion of carbon (49 per cent.), but that bark has more ash than wood. Leaves and small roots have less carbon than wood (45 per cent.), and more ash, 5 and 7 per cent.

Leaves when dried at 212° lost 60 per cent. of water, and branches 45 per cent.

Evaporative power of 1 cube foot of pine wood is equal to that of 1 cube foot of fresh water; or, in the furnace of a steam-boiler and under pressure, it is 4.75 lbs. fresh water for 1 lb. of wood.

Northern Wood.—One cord of *hard* wood and one cord of *soft* wood, such as is used upon Lakes Ontario and Erie, is equal in evaporative effects to 2000 lbs. of anthracite coal.

Western Wood.—One cord of the description used by the river steamboats is equal in evaporative qualities to 12 bushels (960 lbs.) of Pittsburgh coal. 9 cords cotton, ash, and cypress wood are equal to 7 cords of yellow pine.

Solid portion (*lignin*) of all woods, wherever and under whatever circumstances of growth, are nearly similar, specific gravity being as 1.46 to 1.53.

Densest woods give greatest heat, as charcoal produces greater heat than flame.

For every 14 parts of an ordinary pile of wood there are 11 parts of space; or a cord of wood in pile has 71.68 feet of solid wood and 56.32 feet of voids.

Trees in the early part of April contain 20 per cent. more water than they do in the end of January.

Ash.

Proportion of Ash in 100 Lbs. of several Woods.

Woods.	Wood.	Leaves.	Woods.	Wood.	Leaves.
	Per Cent.	Per Cent.		Per Cent.	Per Cent.
Ash	.5	—	Elm	1.88	11.8
Beech	.35	5.4	Oak	.21	4
Birch	.34	5	Pitch Pine	.25	3.15

Peat.

Peat is the organic matter, or soil, of bogs, swamps, and marshes—decayed moss, sedge, coarse grass, etc.—in beds varying from 1 to 40 feet in depth. That near the surface, and less advanced in transformation, is light, spongy, and fibrous, of reddish-brown color; lower down, it is more compact, of a darker brown color; and, in lowest strata, it is of a blackish brown, or almost black, of a pitchy or unctuous surface, the fibrous texture nearly or altogether transformed.

Peat, in its natural condition, contains from 75 to 80 per cent. of water. Occasionally its constituent water amounts to 85 or 90 per cent., in which case peat is of the consistency of mire. It shrinks very much in drying; and its specific gravity varies from .22 to 1.06, surface peat being lightest, and deepest peat densest.

When peat is milled, so that its fibre is broken up, its contraction in drying is much increased, and in this condition it is termed *condensed.*

When ordinarily air dried, it will contain 20 to 30 per cent. of moisture, and when effectively dried at least 15 per cent.

Products of Distillation of Peat.

Water 31.4. Tar 2.8. Gas 36.6. Charcoal 29.2.

The distillation of the tar will yield paraffine, oil, gas, water, and charcoal, and the water acetic acid, wood spirit, and chloride of ammonia.

Evaporative power, in furnace of a steam-boiler and under pressure, is from 3.5 to 5 lbs. of fresh water per lb. of fuel.

Tan.

Tan, oak or hemlock bark, after having been used in the process of tanning, is combustible as a fuel. It consists of the fibre of the bark, and, according to M. Peclet, 5 parts of bark produce 4 parts of dry tan; and heating power of it when perfectly dry, or containing but 15 per cent. of ash, is 6100 units; while that of tan in an ordinary state of dryness, containing 30 per cent. of water, is 4284. Weight of water evaporated at 212° by 1 lb., equivalent to these units, is 6.31 lbs. for dry, and 4.44 for moist.

Relative Values of different Fuels.

Description.	Lbs. of Steam from Water at 212° by 1 lb. of Fuel.	Relative Evaporative Power for equal Weights.	Relative Evaporative Power for equal Volumes.	Relative Rapidities of Ignition.	Relative Freedom from Waste.	Relative Completeness of Combustion.	Relative Weights.
Anthracites.							
Peach Mountain, Pa.	10.7	1	1	.505	.633	.725	.945
Beaver Meadow	9.88	.923	.982	.207	.748	.6	1
Bituminous.							
Newcastle	8.66	.809	.776	.595	.887	.346	.904
Pictou	8.48	.792	.738	.588	.418	1	.876
Liverpool	7.84	.733	.663	.581	1	.333	.852
Cannelton, Ind.	7.34	.686	.616	1	.984	.578	.848
Scotch	6.95	.649	.625	.521	.499	.649	.909
Pine wood, dry	4.69	.436	.175	—	16.417	—	—

Weights, Evaporative Powers per Weight and Bulk, etc., of different Fuels. (*W. R. Johnson and others.*)

Fuel.	Specific Gravity.	Weight per Cube Foot.	Steam from Water at 212° by 1 lb. of Fuel.	Clinker from 100 lbs.	Cube Feet in a Ton.
Bituminous.		Lbs.	Lbs.	Lbs.	No.
Cumberland, *maximum*	1.313	52.92	10.7	2.13	42.3
" *minimum*	1.337	54.29	9.44	4.53	41.2
Duffryn	1.326	53.22	10.14	—	42.09
Cannel, Wigan	1.23	48.3	7.7	—	46.37
Blossburgh	1.324	53.05	9.72	3.4	42.2
Midlothian, *screened*	1.283	45.72	8.94	3.33	49
" *average*	1.294	54.04	8.39	8.82	41.4
Newcastle, Hartley	1.257	50.82	8.76	3.14	44
Pictou	1.318	49.25	8.41	6.13	45
Pittsburgh	1.252	46.81	8.2	.94	47.8
Sydney	1.338	47.44	7.99	2.25	47.2
Carr's Hartley	1.262	47.88	7.84	1.86	46.7
Clover Hill, Va.	1.285	45.49	7.67	3.86	49.2
Cannelton, Ind.	1.273	47.65	7.34	1.64	47
Scotch, Dalkeith	1.519	51.09	7.08	5.63	43.8
Chili	—	—	5.72	—	—
Japan	1.231	48.3	—	—	—
Anthracite.					
Peach Mountain	1.464	53.79	10.11	3.03	41.6
Forest Improvement	1.477	53.66	10.06	.81	41.7
Beaver Meadow	1.554	56.19	9.88	.6	39.8
Lackawanna	1.421	48.89	9.79	1.24	45.8
Beaver Meadow, No. 3	1.61	54.93	9.21	1.01	40.7
Lehigh	1.59	55.32	8.93	1.08	40.5
Coke.					
Natural Virginia	1.323	46.64	8.47	5.31	48.3
Midlothian	—	32.7	8.63	10.51	68.5
Cumberland	—	31.6	8.99	3.55	70.9
Miscellaneous.				Ash.	
Charcoal, Oak	1.5	24	5.5	3.06	104
Peat	.53	30	5	—	75
Warlich's fuel	1.15	69.1	10.4	2.91	32.44
Wylam's "	—	65	8.9	—	—
Pine wood, dry	—	21	4.7	.31	106.6

Weights and Comparative Values of different Woods.

Woods.	Cord.	Value.	Woods.	Cord.	Value.
	Lbs.			Lbs.	
Shell-bark Hickory...	4469	1	New Jersey Pine.....	2137	.54
Red-heart Hickory ...	3705	.81	Yellow Pine.........	1904	.43
White Oak...........	3821	.81	White Pine..........	1868	.42
Red Oak.............	3254	.69	Beech...............	—	.7
Virginia Pine........	2689	.61	Spruce..............	—	.52
Southern Pine........	3375	.73	Hemlock.............	—	.44
Hard Maple..........	2878	.6	Cottonwood..........	—	.33

Liquid Fuels.

Petroleum.

Petroleum is a hydro-carbon liquid which is found in America and Europe. According to analysis of M. Sainte-Claire Deville, composition of 15 petroleums from different sources was found to be practically constant. Average specific gravity was .87. Extreme and average elementary composition was as follows:

Carbon..............	82 to 87.1 per cent.	Average,	84.7 per cent.
Hydrogen...........	11.2 to 14.8 "	"	13.1 "
Oxygen.............	.5 to 5.7 "	"	2.2 "
			100

Its heat of combustion is 20240, and its evaporative power at 212° 20.33.

Petroleum Oils—Are obtained by distillation from petroleum, and are compounds of carbon and hydrogen, in average proportion of 72.6 and 27.4.

Boiling-point ranges from 86° to 495°.

Schist Oil—Consists of carbon 80.3 parts, hydrogen 11.5, and oxygen 8.2.

Pine Wood Oil—Consists of carbon 87.1 per cent., hydrogen 10.4, and oxygen 2.5.

Coal-gas.

Coal Gas—As furnished by Chartered Gas Co. of London is composed as follows:

	Carbon.	Hydrogen.		Oxygen.	Hydrogen.	Nitrogen.
Olefiant Gas, Bi-carb. hyd.	3.096	.434	Hydrogen......	—	51.8	—
Marsh gas, Carb. hyd.	26.445	8.815	Oxygen........	.08	—	—
Carbonic oxide....	3.84	5.11	Nitrogen.......	—	—	.38
			Total........ 100 parts.			

Heat of combustion at 212° 52961 units, and evaporative power 47.51 lbs.

Coal-gas. (*V. Harcourt.*)

	Carb.	Hyd.	Oxy.	Nit.		Carb.	Hyd.	Oxy.	Nit.
	Per ct.	Per ct.	Per ct.	Per ct.		Per ct.	Per ct.	Per ct.	Per ct.
Olefiant gas.....	10.5	1.7	—	—	Hydrogen......	—	8.1	—	—
Marsh gas......	39.7	13.2	—	—	Nitrogen.......	—	—	—	5.8
Carbonic oxide..	5.9	—	7.9	—	Oxygen........	—	—	.3	—
Carbonic dioxide	1.9	—	5	—	Total.......	58	23	13.2	5.8

One lb. of this gas had a volume of 30 cube feet at 62°; heat of combustion 22684 units; and of one cube foot 756 units, which is equivalent to evaporation of .68 lb. of water from 62°, or of .78 lb. from 212° per cube foot.

Average Composition of Fuels.

	Specific Gravity.	Carbon.	Hydrogen.	Nitrogen.	Oxygen.	Sulphur.	Ash.
BITUMINOUS COALS.		Per ct.	Per ct.	Per ct.	Per ct.	Per ct.	Per ct.
Australian	1.31	—	—	—	—	.5	8.38
Borneo	1.28	64.52	4.74	.8	20.75	1.45	7.74
British, lowest	—	68.72	4.76	—	18.63	1.35	—
Boghead, dry, average	1.18	63.94	8.86	.96	4.7	.32	21.22
Chili, Conception Bay	1.29	70.55	5.76	.95	13.24	1.98	7.52
" Chiriqui	—	38.98	4.01	.58	13.38	6.14	36.91
Cannel, Wigan	1.23	79.23	6.08	1.18	7.24	1.43	4.84
Cumberland, Md.	1.31	93.81	1.82	—	2.77	—	1.6
Coke, Garesfield	—	97.6	—	—	—	.85	1.55
" Durham	—	89.5	—	—	—	1.25	9.25
" Average	—	93.44	—	—	—	1.22	5.34
Duffryn	1.33	88.26	4.66	1.45	.6	1.77	3.26
Formosa Island	1.24	78.26	5.7	.64	10.95	.49	3.96
French, hard	1.32	88.56	4.88	(4.38)		—	2.19
" caking	1.29	87.73	5.08	(5.65)		—	1.54
" long flame	1.3	82.94	5.35	(8.63)		—	3.08
" average*	1.31	85	4.5	(7)		—	3.5
Indian, average	—	47.3	—	—	—	—	22.9
" Kotbec	—	90	—	—	—	—	4
Patagonia	—	62.25	5.05	.63	17.54	1.13	13.4
Russian, Miouchi†	—	91.45	4.5	(4.05)		—	—
Sydney, S. W.	—	82.39	5.32	1.27	8.32	.07	2.04
Splint, Wylam	—	74.82	6.18	(5.09)		—	13.91
" Glasgow	—	82.92	5.49	(10.46)		—	1.13
" Cannel, Lancashire	—	83.75	5.66	(8.04)		—	2.55
" " Edinburgh	—	67.6	5.4	(12.43)		—	14.57
" Cherry, Newcastle	—	84.85	5.05	(8.43)		—	1.67
" Caking, Garesfleld	—	87.95	5.24	(5.42)		—	1.39
" Ebbro Vale, Welsh	—	89.78	5.15	2.16	.39	1.02	1.5
" Llangenneck "	—	84.97	4.26	1.45	3.5	.42	5.4
Vancouver's Island	—	66.93	5.32	1.02	8.7	2.2	15.83
ANTHRACITES.							
Anthracite	1.5	88.54	—	—	—	.52	8.67
French	1.5	86.17	2.67	(2.85)		—	8.56
Russian	—	96.66	1.35	(1.99)		—	—
WOODS.							
Beech	—	50.17	6.12	1.05	40.38	—	1.77
Birch	—	48.12	6.37	1.15	43.95	—	.48
Oak	—	48.13	5.25	.82	44.5	—	1.3
White Pine	—	49.95	6.41	—	43.65	—	.31
Woods, average	—	49.7	6.06	1.05	41.3	—	1.8
CHARCOAL.							
Oak	—	87.68	2.83	—	6.43	—	3.06
Pine	—	71.36	5.95	—	22.19‡	—	.4
Maple	—	70.07	4.61	—	24.89‡	—	.43
MISCELLANEOUS.							
Asphalt	1.06	79.18	9.3	(8.72)		—	2.8
Lignite, perfect	1.29	69.02	5.05	(20.12)		—	5.82
" imperfect	1.25	60.18	5.29	(29.03)		—	5.57
" bituminous	1.18	74.82	7.36	(13.38)		—	4.45
" Colorado	1.28	56.8	—	—	—	—	4.5
" Kentucky	1.2	40	—	—	—	—	7
" Arkansas	—	34.5	—	—	—	—	5
Peat, dense	—	61.02	5.77	.81	32.4	—	—
" Irish, average	.528	58.18	5.96	1.23	31.21	—	3.43
Patent, Warlich	1.15	90.02	5.56	—	—	1.62	2.91§
" Wylam's	1.1	79.91	5.69	1.68	6.63	1.25	4.84

* Heat of Combustion of 1 Lb. 14 723.
‡ Including Nitrogen.
† Heat of Combustion of 1 Lb. 15 651.
§ Including Oxygen.

Average Composition of Coals and Fuels, Heat of Combustion, and Evaporative Power.

Deduced from analysis and experiments of Messrs. De La Bèche, Playfair, and Peclet.

Coals and Fuels.	Specific Gravity.	Composition. Carbon.	Hydro-gen.	Nitro-gen.	Sul-phur.	Oxy-gen.	Ash.	Heat of Combustion of 1 lb.	Evaporation from water at 212°.
		Per ct.	Per ct.	Per ct.	Per ct.	Per ct.	Per ct.	Units.	Lbs.
Derbyshire and Yorkshire	1.29	79.68	4.94	1.41	1.01	10.28	2.65	13 860	14.34
Lancashire	1.27	77.9	5.32	1.3	1.44	9.53	4.88	13 918	14.56
Newcastle	1.26	82.12	5.31	1.35	1.24	5.69	3.77	14 820	15.32
Scotch	1.26	78.53	5.61	1	1.11	9.69	4.03	14 164	14.77
Welsh	1.32	83.78	4.79	.98	1.43	4.15	4.91	14 858	15.52
Average of British	1.28	80.4	5.19	1.21	1.25	7.87	4.05	14 320	14.82
Patent fuels	1.17	83.4	4.97	1.08	1.26	2.79	5.93	15 000	15.66
Van Diemen's Land	—	65.8	3.5	1.3	1.1	5.58	22.71	11 320	11.83
Chili	—	63.56	5.43	.82	2.5	14.84	13.31	11 030	11.68
Lignite, Trinidad	—	65.2	4.25	1.33	.69	21.69	6.84	10 438	10.87
" French Alps	1.28	70.02	5.2	—	—	—	3.01	11 790	12.1
" Bitum., Cuba	1.2	75.85	7.25	—	—	—	3.94	14 562	14.96
" Wash. Ter.*	—	67	4.55	—	1	—	3.1	12 538	12.91
Asphalt	1.06	79.18	9.3	—	—	—	2.8	16 655	17.24
Petroleum	.87	84.7	13.1	—	—	2.2	—	20 240	20.33
" oils	.75	—	—	—	—	—	—	27 530	28.5
Oak bark Tan, dry	—	—	—	—	—	—	15	6 100	6.31
" " moist	—	—	—	—	—	—	15	4 284	4.24
Charcoal at 302°	1.5	47.51	6.12	(O and N 46.29)			.8	8 130	8.4
" " 572°	1.4	73.24	4.25	(O and N 21.96)			.57	11 861	12.27
" " 810°	1.71	81.64	4.96	(O and N 15.24)			1.61	14 916	15.43
Peat, dry, average	.53	58.18	5.96	1.23	—	31.21	3.43	9 951	10.3
" moist,† "	—	43.1	4.3	(O and N 21.4)			3.3	8 917	9.22
Coal-gas	.42	33.38	66.16	.38	—	.08	—	52 961	47.51

* Water 7. Oxygen and Nitrogen 17.36. † Moisture 27.8. Sulphur .2.

Elements of Fuels not included in Preceding Tables.

Fuel.	Heat of Combustion of 1 lb.	Evaporative Power of 1 lb. at 212°.	Coke produced.	Weight of 1 Cub. Foot.	Volume of 1 Ton.
Bituminous Coal.	Units.	Lbs.	Per cent.	Lbs.	Cube Feet.
Welsh	14 858	9.05	73	82	42.7
Newcastle	14 820	8.01	61	78.3	45.3
Lancashire	13 918	7.94	58	79.4	45.2
Scotch	14 164	7.7	54	78.6	42
Boghead	14 478	7.87	30.94	—	—
British, average	14 133	8.13	61	79.8	44.52
Irish, lowest	—	9.85	90	99.6	35.7
Cumberland, Md.	—	—	83.7	84.93	42.4
American, average	—	—	82.5	87.54	43.49
French, average	14 723	—	64.2	—	40
Australian	—	—	68.27	—	—
Anthracite.					
American	—	—	94.82	93.78	42.35
French	14 038	—	88.83	—	—
Miscellaneous.					
Warlich's fuel	16 495	—	—	73.5	34.5
Coke......Mickley	15 600	—	—	—	80
Virginia, average	13 550	14.02	—	45	69.8
Charcoal	12 325	—	—	—	12.76
Lignite, perfect	11 678	12.1	47	—	—
" imperfect	9 834	10.18	37.5	—	—
" Russian	15 837	—	—	—	—
Asphalt	16 555	17.24	9	—	—
Woods, dry, average	7 792	8.07	—	—	114

Miscellaneous.

Experiments undertaken by Baltimore and Ohio R. R. Co. determined evaporating effect of 1 ton of Cumberland coal equal to 1.25 tons of anthracite, and 1 ton of anthracite to be equal to 1.75 cords of pine wood; also that 2000 lbs. of Lackawanna coal were equal to 4500 lbs. best pine wood.

One lb. of anthracite coal in a cupola furnace will melt from 5 to 10 lbs. of cast iron; 8 bushels bituminous coal in an air furnace will melt 1 ton of cast iron.

Small coal produces about .75 effect of large coal of same description.

Experiments by Messrs. Stevens, at Bordentown, N. J., gave following results:

Under a pressure of 30 *lbs.*, 1 lb. pine wood evaporated 3.5 to 4.75 lbs. of water. 1 lb. Lehigh coal, 7.25 to 8.75 lbs.

Bituminous coal is 13 per cent. more effective than coke for equal weights; and in England effects are alike for equal costs.

Radiation from Fuel.—Proportion which heat radiated from incandescent fuel bears to total heat of combustion is,

From Wood	.29	From Charcoal and Peat	.5

Least consumption of coal yet attained is 1.5 lbs. per IHP. It usually varies in different engines from 2 to 8 lbs.

Volume of pine wood is about 5.5 times as great as its equivalent of bituminous coal.

GRAVITATION.

Gravity is an attraction common to all material substances, and they are affected by it directly, in exact proportion to their mass, and inversely, as square of their distance apart.

This attraction is termed *terrestrial gravity*, and force with which a body is drawn toward centre of Earth is termed the *weight* of that body.

Force of gravity differs a little at different latitudes: the law of variation, however, is not accurately ascertained; but following theorems represent it very nearly:

$$\left.\begin{array}{l} g'\ (1 - .002\,837 \cos.\ 2\ \text{lat.}) \\ g'\ (1 + .002\,837),\ \text{at the poles} \\ g'\ (1 - .002\,837),\ \text{at the equator} \end{array}\right\} = g.$$

g′ representing force of gravity at latitude 45°, *and g force at other places.*

Or, $32.171\ (\text{lat. } 45^\circ)\ (1 + \overline{.005\,133 \sin.\ L})\left(1 - \frac{2\,H}{R}\right) = g.$ L *representing latitude*, H *height of elevation above level of sea, and* R *radius of Earth, both in feet.*

Note.—If 2 L exceeds 90°, put cos. 180 — 2 L, and R at Equator = 20 926 062, at Poles 20 853 429, and mean 20 889 746.

Illustration.—What is force of gravity at latitude 45°, at an elevation of 209 feet, and radius = 20 900 000 feet?

$$32.171\ (1 + .005\,133 \sin.\ 45^\circ)\left(1 - \frac{2\,H}{20\,900\,000}\right) = 32.171 \times 1.003\,63 \times .999\,98 = 32.287.$$

Gravity at Various Locations at Level of Sea.

Equator	32.088	New York	32.161	London	32.189
Washington	32.155	Lat. 45°	32.171	Poles	32.253

In bodies descending freely by their own weight, their *velocities* are as *times* of their descent, and *spaces* passed through as square of the times.

Times, then, being 1, 2, 3, 4, etc., *Velocities* will be 1, 2, 3, 4, etc.

Spaces passed through will be as square of the velocities acquired at end of those times, as 1, 4, 9, 16, etc.; and *spaces* for each time as 1, 3, 5, 7, 9, etc.

A body falling freely will descend through 16.0833 feet in first second of time, and will then have acquired a velocity which will carry it through 32.166 feet in next second.

If a body descends in a curved line, it suffers no loss of velocity, and the curve of a cycloid is that of quickest descent.

Motion of a falling body being uniformly accelerated by gravity, motion of a body projected vertically upwards is uniformly retarded in same manner.

A body projected perpendicularly upwards with a velocity equal to that which it would have acquired by falling from any height, will ascend to the same height before it loses its velocity. Hence, a body projected upwards is ascending for one half of time it is in motion, and descending the other half.

Various Formulas here given are for Bodies Projected Upwards or Falling Freely, in Vacuo.

When, however, weight of a body is great compared with its volume, and velocity of it is low, deductions given are sufficiently accurate for ordinary purposes.

In considering action of gravitation on bodies not far distant from surface of the Earth, it is assumed, without sensible error, that the directions in which it acts are parallel, or perpendicular to the horizontal plane.

A distance of one mile only produces a deviation from parallelism less than one minute, or the 60th part of a degree.

Relation of Time, Space, and Velocities.

Time from Beginning of Descent.	Velocity acquired at End of that Time.	Squares of Time.	Space fallen through in that Time.	Spaces for this Time.	Space fallen through in last Second of Fall.
Seconds.	Feet.	Seconds.	Feet.	No.	Feet.
1	32.166	1	16.083	1	16.08
2	64.333	4	64.333	3	48.25
3	96.5	9	144.75	5	80.41
4	128.665	16	257.33	7	112.58
5	160.832	25	402.08	9	144.75
6	193	36	579	11	176.91
7	225.166	49	788.08	13	209.08
8	257.333	64	1029.33	15	241.25
9	289.5	81	1302.75	17	273.42
10	321.666	100	1608.33	19	305.58

and in same manner this Table may be continued to any extent.

Velocity acquired due to given Height of Fall and Height due to given Velocity.

$$8.04\sqrt{h} = v; \qquad 32.2\,t = v; \qquad \frac{v^2}{64.4} = h; \quad \text{and} \quad 16.083\,t^2 = h.$$

h representing height of fall in feet, v velocity acquired in feet per second, and t time of fall in seconds.

To Compute Action of Gravity.

Time.

When Space is given. RULE.—Divide space by 16.083, and square root of quotient will give time.

EXAMPLE.—How long will a body be in falling through 402.08 feet?

$$\sqrt{402.08 \div 16.083} = 5 \text{ seconds.}$$

When Velocity is given. RULE.—Divide given velocity by 32.166, and quotient will give time.

EXAMPLE.—How long must a body be in falling to acquire a velocity of 800 feet per second? $800 \div 32.166 = 24.87$ *seconds.*

Velocity.

When Space is given. RULE.—Multiply space in feet by 64.333, and square root of product will give velocity.

EXAMPLE.—Required velocity a body acquires in descending through 579 feet.

$$\sqrt{579 \times 64.333} = 193 \textit{ feet.}$$

Velocity acquired at any period is equal to twice the mean velocity during that period.

ILLUSTRATION.—If a ball fall through 2316 feet in 12 seconds, with what velocity will it strike?

$$2316 \div 12 = 193, \textit{ mean velocity, which } \times 2 = 386 \textit{ feet} = \textit{velocity.}$$

When Time is given. RULE.—Multiply time in seconds by 32.166, and product will give velocity.

EXAMPLE.—What is velocity acquired by a falling body in 6 seconds?

$$32.166 \times 6 = 192.996 \textit{ feet.}$$

Space.

When Velocity is given. RULE.—Divide velocity by 8.04, and square of quotient will give distance fallen through to acquire that velocity.

Or, Divide square of velocity by 64.33.

EXAMPLE.—If the velocity of a cannon-ball is 579 feet per second, from what height must a body fall to acquire the same velocity?

$$579 \div 8.04 = 72.014, \text{ and } 72.014^2 = 5186.02 \textit{ feet.}$$

When Time is given. RULE.—Multiply square of time in seconds by 16.083, and it will give space in feet.

EXAMPLE.—Required space fallen through in 5 seconds.

$$5^2 = 25, \text{ and } 25 \times 16.083 = 402.08 \textit{ feet.}$$

Distance fallen through in feet is very nearly equal to square of time in fourths of a second.

ILLUSTRATION 1.—A bullet dropped from the spire of a church was 4 seconds in reaching the ground; what was height of the spire?

$$4 \times 4 = 16, \text{ and } 16^2 = 256 \textit{ feet.}$$

By Rule, $4 \times 4 \times 16.0833 = 257.33$ *feet.*

2.—A bullet dropped into a well was 2 seconds in reaching bottom; what is the depth of the well?

$$\text{Then } 2 \times 4 = 8, \text{ and } 8^2 = 64 \textit{ feet.}$$

By Rule, $2 \times 2 \times 16.0833 = 64.33$ *feet.*

By Inversion.—In what time will a bullet fall through 256 feet?

$$\sqrt{256} = 16, \text{ and } 16 \div 4 = 4 \textit{ seconds.}$$

Space fallen through in last Second of Fall.

When Time is given. RULE.—Subtract half of a second from time, and multiply remainder by 32.166.

EXAMPLE.—What is space fallen through in last second of time, of a body falling for 10 seconds?

$$10 - .5 \times 32.166 = 305.58 \textit{ feet.}$$

Promiscuous Examples.

1. If a ball is 1 minute in falling, how far will it fall in last second?

Space fallen through = square of time, and 1 *minute* = 60 *seconds.*

$$\begin{array}{rl} 60^2 \times 16.083 = 57\,898 & \textit{feet} \text{ for } 60 \textit{ seconds.} \\ 59^2 \times 16.083 = \underline{55\,984} & \text{“} \quad \text{“} \quad \underline{59} \quad \text{“} \\ 1\,914 & \qquad\quad 1 \end{array}$$

2. Compute time of generating a velocity of 193 feet per second, and whole space descended.

$$193 \div 32.166 = 6 \textit{ seconds; } 6^2 \times 16.083 = 579 \textit{ feet.}$$

3. If a body was to fall 579 feet, what time would it be in falling, and how far would it fall in the last second?

$$\sqrt{\frac{579 \times 2}{32.166}} = \sqrt{36} = 6 \textit{ seconds}, \text{ and } 6 - .5 \times 32.166 = 5.5 \times 32.166 = 176.91 \textit{ feet.}$$

Formulas to determine the various Elements.

$$\text{1. } T = \sqrt{\frac{S}{.5\,g}}; \; = \frac{V}{g}; \; = \frac{2\,S}{V}; \; = \sqrt{\frac{2\,S}{g}}; \; = \frac{h}{g} + .5.$$

$$\text{2. } S = \left(\frac{V}{.25\,g}\right)^2; \; = \frac{V^2}{2\,g}; \; = \frac{V\,T}{2}; \; = \frac{g\,T^2}{2}; \; = T^2\,.5\,g. \qquad \text{3. } h = T - .5\,g.$$

$$\text{4. } V = \sqrt{S \times 2\,g}; \; = T\,g; \; = 2\sqrt{.5\,g\,S}; \; = \frac{2\,S}{T}.$$

T *representing time of falling in seconds,* V *velocity acquired in feet per second,* S *space or vertical height in feet,* h *space fallen through in last second,* g 32.166 *and* .5 g *and* .25 g *representing* 16.083 *and* 8.04.

Retarded Motion.

A body projected vertically upward is affected inversely to its motion when falling freely and directly downward, inasmuch as a like cause retards it in one case and accelerates it in the other.

In air a ball will not return with same velocity with which it started. In *vacuo* it would. Effect of the air is to lessen its velocity both ascending and descending. Difference of velocities will depend upon relative specific gravity of ball and density of medium through which it passes. Thus, greater weight of ball, greater its velocity.

To Compute Action of Gravity by a Body projected Upward or Downward with a given Velocity.

Space.

When projected Upward. Rule.—From the product of the given velocity and the time in seconds subtract the product of 32.166, and half the square of the time, and the remainder will give the space in feet.

Or, Square velocity, divide result by 64.33, and quotient will give space in feet.

Example.—If a body is projected upward with a velocity of 96.5 feet per second, through what space will it ascend before it stops?

$$96.5 \div 32.166 = 3 \textit{ seconds} = \textit{time to acquire this velocity.}$$

$$\text{Then, } 96.5 \times 3 - \left(32.166 \times \frac{3^2}{2}\right) = 289.5 - 144.75 = 144.75 \textit{ feet.}$$

Time.

Rule.—Divide velocity in feet by 32.166, and quotient will give time in seconds.

Example.—Velocity as in preceding example.

$$96.5 \div 32.166 = 3 \textit{ seconds.}$$

Velocity.

Rule.—Multiply time in seconds by 32.166, and product will give velocity in feet per second.

Example.—Time as in preceding example.

$$3 \times 32.166 = 96.5 \textit{ feet velocity.}$$

Space fallen through in last Second.

Rule.—Subtract .5 from time, multiply remainder by 32.166, and product will give space in feet per second.

Example.—Time as in preceding example.

$$3 - .5 \times 32.166 = 2.5 \times 32.166 = 80.416 \textit{ feet.}$$

When projected Downward.

Space.

RULE.—Proceed as for projection upwards and take sum of products.

EXAMPLE I.—If a body is projected downward with a velocity of 96.5 feet per second, through what space will it fall in 3 seconds?

$$96.5 \times 3 + \left(32.166 \times \frac{3^2}{2}\right) = 289.5 + 144.75 = 434.25 \textit{ feet.}$$

$$\text{Or, } t^2 \times 16.083 + \overline{v \times t} = s.$$

2.—If a body is projected downward with a velocity of 96.5 feet per second, through what space must it descend to acquire a velocity of 193 feet per second?

$96.5 \div 32.166 = 3$ *seconds, time to acquire this velocity.*
$193 \div 32.166 = 6$ *seconds, time to acquire this velocity.*

Hence $6 - 3 = 3$ *seconds, time of body falling.*

Then $96.5 \times 3 = 289.5 =$ *product of velocity of projection and time.*

$16.083 \times 3^2 = 144.75 =$ *product of* 32.166, *and half square of time.*

Therefore $289.5 + 144.75 = 434.25$ *feet.*

Time.

RULE.—Subtract space for velocity of projection from space given, and remainder, divided by velocity of projection, will give time.

EXAMPLE.—In what time will a body fall through 434.25 feet of space, when projected with a velocity of 96.5 feet?

Space for velocity of $96.5 = 144.75$ *feet.*

Then, $434.25 - 144.75 \div 96.5 = 289.5 - 96.5 = 3$ *seconds.*

Velocity.

RULE.—Divide twice space fallen through in feet by time in seconds.

EXAMPLE.—Elements as in preceding example.

Space fallen through when projected at velocity of 96.5 *feet* = 144.75 *feet, and* 434.25 *feet = space fallen through in* 3 *seconds.*

Then, $144.75 + 434.25 = 579$ *feet* space fallen through, and $\sqrt{579 \div 16.083} = 6$ *seconds.*

Hence, $579 \times 2 \div 6 = 1158 \div 6 = 193$ *feet.*

Space Fallen through in last Second.

RULE.—Subtract .5 from time, multiply remainder by 32.166, and product will give space in feet per second.

EXAMPLE.—Elements as in preceding example.

$$6 - .5 \times 32.166 = 5.5 \times 32.166 = 176.91 \textit{ feet.}$$

Ascending bodies, as before stated, are retarded in same ratio that descending bodies are accelerated. Hence, a body projected upward is ascending for one half of the time it is in motion, and descending the other half.

ILLUSTRATION I.—If a body projected vertically upwards return to earth in 12 seconds, how high did it ascend?

The body is half time in ascending. $12 \div 2 = 6$.

Hence, by Rule, p. 489, $6^2 \times 16.083 = 579$ *feet = product of square of time and* 16.083.

2.—If a body is projected upward with a velocity of 96.5 feet per second, it is required to ascertain point of body at end of 10 seconds.

$96.5 \div 32.166 = 3$ *seconds, time to acquire this velocity,* and $3^2 \times 16.083 = 144.75$ *feet, height body reached with its initial velocity.*

Then $10 - 3 = 7$ *seconds left for body to fall in.*

Hence, by Rule, as in preceding example, $7^2 \times 16.083 = 788.07$, and $788.07 - 144.75 = 643.32$ *feet = distance below point of projection.*

Or, $10^2 \times 16.083 = 1608.3$ *feet, space fallen through under the effect of gravity,* and $96.5 \times 10 = 965$ *feet, space if gravity did not act.* Hence $1608.3 - 965 = 643.3$ *feet.*

3.—A body is projected vertically with a velocity of 135 feet; what velocity will it have at 60 feet?

$135^2 \div 64.33 = 283.3$ *feet space projected at that velocity*, $135 \div 32.16 = 4.197$ *seconds = time of projection*, and $283.3 - 60 = 223.3 =$ *space to be passed through after attainment of* 60 *feet*. Hence, $\sqrt{223.3 \times 64.33} = 119.85$ *feet velocity*, and $223.3 + 60 = 283.3$ *feet*.

By Inversion.—Velocity 119.85. Hence, $\frac{119.85^2}{64.33} = 223.3$ *feet space*, and $283.3 - 223.3 = 60$ *feet*.

Formulas to Determine Elements of Retarded Motion.

1. $v = V - \overline{gt}$. 2. $v = \frac{s}{t} - \frac{gt}{2}$. 3. $V = v + \overline{gt}$.

4. $V = \frac{s}{t} + \frac{gt}{2}$. 5. $s = Vt - \frac{gt^2}{2}$. 6. $s = tv + \frac{gt^2}{2}$.

7. $t = \frac{V - v}{g}$. 8. $t = \frac{V}{g} - \sqrt{\frac{V^2}{g^2} - \frac{2s}{g}}$. 9. $h = T - \overline{t - t'} - .5\,g$.

v representing velocity at expiration of time, t any less time than T. *t′ less time than t, s space through which a body ascends in time t*, V, T, S, *and h as in previous formulas*, page 490.

Illustration.—A body projected upwards with a velocity of 193 feet per second, was arrested in 5 seconds.

$T = 6,\ t' = 1.$

1. What was its velocity when arrested? (1.)
2. What was the time of its passing through 562.92 feet of space? (7.)
3. What space had it passed through? (5.)
4. What was the time of its projection, when it had a velocity of 96.5 feet? (4.)
5. What was the height it was projected in the last second of time? (8.)

1. $193 - \overline{32.166 \times 5} = 32.17$ *feet*.
3. $32.17 + \overline{32.166 \times 5} = 193$ *velocity*.
2. $\frac{562.92}{5} + \frac{32.166 \times 5}{2} = 193$ *velocity*.
4. $\frac{193 - 96.5}{32.166} = 3$ *seconds*.
5. $193 \times 5 - \frac{32.166 \times 5^2}{2} = 562.92$ *feet*.
6. $\frac{193 - 32.17}{32.166} = 5$ *seconds*.
7. $\frac{193}{32.166} - \sqrt{\frac{193^2}{32.166^2} - \frac{2 \times 562.92}{32.166}} = 6 - \sqrt{36 - 35} = 5$ *seconds*.
8. $6 - \overline{5 - 1} - .5 \times 32.166 = 48.25$ *feet*.

Gravity and Motion at an Inclination.

If a body freely descend at an inclination, as upon an inclined plane, by force of gravity alone, the velocity acquired by it when it arrives at termination of inclination is that which it would acquire by falling freely through vertical height thereof. Or, velocity is that due to height of inclination of the plane.

Time occupied in making descent is greater than that due to height, in ratio of length of its inclination, or distance passed, to its height.

Consequently, times of descending different inclinations or planes of like heights are to one another as lengths of the inclinations or planes.

Space which a body descends upon an inclination, *when descending by gravity*, is to space it would freely fall in same time as height of inclination is to its length; and spaces being same, times will be inversely in this proportion.

If a body descend in a curve, it suffers no loss of velocity.

If two bodies begin to descend from rest, from same point, one upon an inclined plane, and the other falling freely, their velocities at all equal heights below point of starting will be equal.

ILLUSTRATION.—What distance will a body roll down an inclined plane 300 feet long and 25 feet high in one second, by force of gravity alone?

As 300 : 25 :: 16.083 : 1.34025 *feet.*

Hence, if proportion of height to length of above plane is reduced from 25 to 300 to 25 to 600, the time required for body to fall 1.34025 feet would be determined as follows:

As 25 : 600 :: 1.34025 : 32.166, and $32.166 = 16.083 \times 2 =$ *twice time or space in which it would fall freely required for one half proportion of height to length.*

$$\text{Or, as } \frac{300}{25} : \frac{600}{25} :: 1.34025 : 32.166, \textit{ as above.}$$

Impelling or accelerating force by gravitation acting in a direction parallel to an inclination, is less than weight of body, in ratio of height of inclination to its length. It is, therefore, inversely in proportion to length of inclination, when height is the same.

Time of descent, under this condition, is inversely in proportion to accelerating force.

If, for instance, length of inclination is five times height, time of making freely descent at inclination by gravitation is five times that in which a body would freely fall vertically through height; and impelling force down inclination is .2 of weight of body.

When bodies move down inclined planes, the accelerating force is expressed by $h \div l$, *quotient of height ÷ length of plane;* or, what is equivalent thereto, sine of inclination of plane, *i. e.*, sin. *a.*

ILLUSTRATION.—An inclined plane having a height of one half its length, the space fallen through in any time would be *one half of that which it would fall freely.*

Velocity which a body rolling down such a plane would acquire in 5 seconds is 80.416 feet.

Thus, $32.166 \times 5 = 160.833$ feet, and an inclined plane, having a height one half of its length, has an angle or sine of 30°. Hence, sin. $30° = .5$, and $160.833 \times .5 = 80.416$ *feet.*

Formulas to Determine various Elements of Gravitation on an Inclined Plane.

1. $S = .5\, g\, T^2 \sin. a; \;= \dfrac{V^2}{2\, g \sin. a}; \;= .5\, T\, V.$ 4. $V = v \mp \overline{g\, T \sin. a}.$

2. $V = g\, T \sin. a; \;= \sqrt{(2\, g\, S \sin. a)}; \;= \dfrac{2\, S}{T}.$ 6. $H = \dfrac{l^2}{.5\, g\, T^2}.$

3. $T = \sqrt{\left(\dfrac{2\, S}{g \sin. a}\right)}; \;= \dfrac{2\, S}{V}; \;= .25\sqrt{\dfrac{l^2}{H}}; \;= \dfrac{l}{4\sqrt{H}}.$ 7. $l = 4\, T\sqrt{H}.$

5. $S = V\, T \mp \overline{.5\, g\, T^2 \sin. a}.$ Or, $\dfrac{V^2}{2\, g \sin. a}.$

v representing velocity of projection in feet per second, S *space or vertical height of velocity and projection, a angle of inclination of plane, l length, and* H *height of plane.*

ILLUSTRATION.—Assume elements of preceding illustration. $V = 80.416$, $T = 5$, and $H = 201.04$.

1. $.5 \times 32.166 \times 5^2 \times .5 = 201.04$ *feet.* 2. $32.166 \times 5 \times .5 = 80.416$ *feet.*

3. $\sqrt{\left(\dfrac{2 \times 201.04}{32.166 \times .5}\right)} = \sqrt{\left(\dfrac{402.08}{16.083}\right)}\; \sqrt{25} = 5$ *seconds.*

6. $\dfrac{283.42^2}{.5 \times 16.083 \times 5^2} = 201.04$ *feet.* 7. $4 \times 5 \times \sqrt{201.04} = 283.42$ *feet.*

If projected downward with an initial velocity of 16.083 feet per second. $V + g$.

4. $16.083 + \overline{32.166 \times 5 \times .5} = 96.5$ *feet.*

5. $80.416 + 16.083 \times 5 - \overline{.5 \times 32.166 \times 5^2} \times .5 = 281.46$ *feet.*

ILLUSTRATION.—What time will it take for a ball to roll 38 feet down an inclined plane, the angle $a = 12° 20'$, and what velocity will it attain at 38 feet from its starting-point?

$T = \sqrt{\frac{2 S}{g \sin. a}} = \sqrt{\frac{2 \times 38}{32.166 \times .2136}} = 3.33$ *seconds.* $V = g T \sin. a = 32.166 \times 3.33 \times .2136 = 22.88$ *feet per second.*

When a body is projected upward it is retarded in the same ratio that a descending body is accelerated.

ILLUSTRATION.—If a body is projected up an inclined plane having a length of twice its height, at a velocity of 96.5 feet per second,

Then, $T = 96.5 \div 32.166 = 3$ *seconds.* $S = .5 \; 32.166 \times 3^2 \times .5 = 72.375$ *feet.* $v = 32.166 \times 3 \times .5 = 48.25$ *feet.*

Inclined Plane.

Problems on descent of bodies on inclined planes are soluble by formulas 1 to 9, page 495, for relations of accelerating forces. As a preliminary step, however, accelerating force is to be determined by multiplying weight of descending body by height of plane, and dividing product by length of plane.

ILLUSTRATION.—If a body of 15 lbs. weight gravitate freely down an inclined plane, length of which is five times height, accelerating force is $15 \div 5 = 3$ lbs. If length of plane is 100 feet and height 20, velocity acquired in falling freely from top to bottom of plane would be

$$v = 8\sqrt{\frac{3 \times 100}{15}} = 8\sqrt{20} = 35.776 \textit{ feet.}$$

Time occupied in making descent,

$$t = .25\sqrt{\frac{15 \times 100}{3}} = .25\sqrt{500} = 5.59 \textit{ seconds.}$$

Whereas, for a free vertical fall through height of 20 feet, time would be,

$$t = \frac{35.776}{32.166} = 1.118 \textit{ seconds,}$$

which is .2 of time of making descent on inclined plane.

Velocities acquired by bodies in falling down planes of like height will all be equal when arriving at base of plane.

When Length of an Inclined Plane and Time of Free Descent are given.

RULE.—Divide square of length by square of time in seconds and by 16; the quotient is height of inclined plane.

EXAMPLE.—Length of plane is 100 feet, and time of descent is 5.59 seconds; then vertical height of descent is

$$\frac{100^2}{5.59^2 \times 16.08} = 20 \textit{ feet.}$$

Accelerated and Retarded Motion.

If an Accelerating or Retarding force is greater than gravity, that is, weight of the body, the constant, g, or 32.166, is to be varied in proportion thereto, and to do this it is to be multiplied by the accelerating force, and product divided by weight of body.

Thus, *Let f represent accelerating force, and w weight of body.*

Then, $\frac{64.333 f}{w}$, or $\frac{32.166 f}{w}$, or $\frac{16.083 f}{w}$ become the constants.

The same rules and formulas that have been given for action of gravity alone are applicable to the action of any other uniformly accelerating or retarding force, the numerical constants above given being adapted to the force.

Average Velocity of a Moving Body uniformly Accelerated or Retarded.

Average velocity of a moving body uniformly accelerated or retarded, during a given time or in a given space, is equal to half sum of initial and final velocities; and if body begin from a state of rest or arrive at a state of rest, its average speed is half the final or initial velocity, as the case may be.

Thus, in example of a ball rolling, initial speed or velocity is, in either case, 60 feet per second, and terminal speed is nothing; average speed is therefore $\frac{60+0}{2}$, namely, one half of that, or 30 feet per second.

When a cannon-ball is projected at an angle to horizon, there are two forces acting on it at same time—viz., force of charge, which propels it uniformly in a right line, and force of gravity, which causes it to fall from a right line with an accelerated motion; these two motions (uniform and accelerated) cause the ball to move in the curved line of a Parabola.

Formulas for Flight of a Cannon-ball.

$$V = 2800\sqrt{\frac{P}{w}}; \quad P = \frac{w\,V^2}{7\,840\,000};$$

$$b = \frac{V^2 \sin. a, \cos. a}{g}; \quad t = \frac{V \sin. a}{g}; \quad h = \frac{V^2 \sin.^2 a}{2g}.$$

w representing weight of ball and P *of powder in lbs.; t time of flight in seconds; b horizontal range, and h vertical height of range of projection of ball in feet.*

ILLUSTRATION.—A cannon loaded to give a ball a velocity of 900 feet per second, the angle $a = 45°$; what is horizontal range, the time t and height of range h?

$$b = \frac{900^2 \times \sin. 45° \times \cos. 45°}{32.166} = \frac{900^2 \times .5}{32.166} = 12\,590 \textit{ feet.}$$

$$t = \frac{900 \times .7071}{32.166} = 19.78 \textit{ seconds}; \quad h = \frac{900^2 \times .7071^2}{2 \times 32.166} = 6295 \textit{ feet.}$$

NOTE.—As distance b will be greatest when angle $a = 45°$, product of *sine* and *cosine* is greatest for that angle. Sin. $45° \times$ cos. $45° = .5$.

24 lb. ball with a velocity of 2000 feet per second at 45° range 7300 feet.

General Formulas for Accelerating and Retarding Forces.

1. $V = \frac{g\,f\,t}{w}$. 2. $S = \frac{.5\,g\,f\,t^2}{w}$. 3. $t = \frac{w\,V}{g\,f}$. 4. $S = \frac{w\,V^2}{2\,g\,f}$.

5. $t = .25\sqrt{\frac{w\,S}{f}}$. 6. $V = 8\sqrt{\frac{f\,S}{w}}$.

7. $f = \frac{w\,V^2}{2\,g\,S}$. 8. $f = \frac{w\,V}{t\,32.2}$. 9. $w = \frac{g\,f\,t}{V}$.

NOTE 1.—When accelerating or retarding force bears a simple ratio to weight of body, the ratio may, for facility of calculation, be substituted in the quantities representing modified constants, for force and weight. Thus, if accelerating force is a tenth part of weight, then ratio is 1 to 10, and $\frac{32.166}{10} = 3.2166$; or, $\frac{16.083}{10} = 1.6083$, and $\frac{64.333}{10} = 6.4333$; and these quotients may be substituted for 16.083, 32.166, and 64.333 respectively, in formulas for action of gravity 1 to 9, to fit them for computation in an accelerating or retarding force one-tenth of gravity.

2.—Table, page 488, giving relations of velocity and height of falling bodies, may be employed in solving questions of accelerating force general.

EXAMPLE.—A ball weighing 10 lbs. is projected with an initial velocity of 60 feet per second on a level plane, and frictional resistance to its motion is 1 lb. What distance will it traverse before it comes to a state of rest? By formula 4:

$$\frac{10 \text{ lbs.} \times 60^2}{64.333 \times 1 \text{ lb.}} = 559.59 \textit{ feet.}$$

Again, same result may be arrived at, according to Note 1, by multiplying constant 64.333, in Rule, page 494, for gravity, by ratio of force and weight, which in this case is $\frac{1}{10}$, and $64.333 \times \frac{1}{10} = 6.4333$. Substituting 6.4333 for 64.333 in that rule, formula becomes

$$S = \frac{V^2}{6.4333} = \frac{60^2}{6.4333} = 559.59 \textit{ feet.}$$

The question may be answered more directly by aid of table for falling bodies, page 488. Height due to a velocity of 60 feet per second, is 55.9 feet; which is to be multiplied by inverse ratio of accelerating force and weight of body, or $\frac{10}{1}$, or 10; that is,

$$55.9 \times 10 = 559 \textit{ feet.}$$

If the question is put otherwise—What space will a weight move over before it comes to a state of rest, with an initial velocity of 60 feet per second, allowing friction to be one tenth weight? The answer is that friction, which is retarding force, being one tenth of weight, or of gravity, space described will be 10 times as great as is necessary for gravity, supposing the weight to be projected vertically upwards to bring it to a state of rest. The height due to velocity being 55.9 feet; then

$$55.9 \times 10 = 559 \textit{ feet.}$$

Average velocity of a moving body, uniformly accelerated or retarded during a given period or space, is equal to half sum of initial and final velocities.

To Compute Velocity of a Falling Stream of Water per Second at End of any given Time.

When Perpendicular Distance is given.

EXAMPLE.—What is the distance a stream of water will descend on an inclined plane 10 feet high, and 100 feet long at base, in 5 seconds?

$5^2 \times 16.083 = 402.08$ *feet = space a body will freely fall in this time.*

Then, as 100 : 10 :: 402.08 : 40.21 *feet = proportionate velocity on a plane of these dimensions to velocity when falling freely.*

Miscellaneous Illustrations.

1.—What is the space descended vertically by a falling body in 7 seconds.

$$S = .5\, g \times t^2. \quad \text{Then } 16.083 \times 7^2 = 788.067 \textit{ feet.}$$

2.—What is the time of a falling body descending 400 feet, and velocity acquired at end of that time?

$$t = \frac{v}{g}. \quad \text{Then } \frac{160.4}{32.166} = 4.98 \textit{ sec.} \qquad v = \sqrt{2\, g \times S}. \quad \text{Then } \sqrt{64.333 \times 400} = 160.4 \textit{ feet.}$$

3.—If a drop of rain fall through 176 feet in last second of its fall, how high was the cloud from which it fell?

$$S = \frac{h^2}{2\, g}. \quad \text{Then } \frac{176^2}{64.166} = 482.75 \textit{ feet.}$$

4.—If two weights, one of 5 lbs. and one of 3, hanging freely over a sheave, are set free, how far will heavier one descend or lighter one rise in 4 seconds.

$$\frac{5-3}{5+3} \times 16.083 \times 4^2 = \frac{2}{8} \times 257.328 = 64.33 \textit{ feet.}$$

5.—If length of an inclined plane is 100 feet, and time of descent of a body is 6 seconds, what is vertical height of plane or space fallen through?

$$\frac{100^2}{6^2 \times .5\, g} = \frac{10\,000}{579} = 17.27 \textit{ feet.}$$

6.—If a bullet is projected vertically with a velocity of 135 feet per second, what velocity will it have at 60 feet?

Formula 7, page 492. $\frac{135}{32.166} - \sqrt{\frac{135^2}{32.166^2} - \frac{2 \times 60}{32.166}} = .41$ *seconds.*

GUNNERY.

A heavy body impelled by a force of projection describes in its flight or track a parabola, *parameter* of which is four times height due to velocity of projection.

Velocity of a shot projected from a gun varies as square root of charge directly, and as square root of weight of shot reciprocally.

To Compute Velocity of a Shot or Shell.

RULE.—Multiply square root of triple weight of powder in lbs. by 1600; divide product by square root of weight of shot; and quotient will give velocity in feet per second.

EXAMPLE.—What is velocity of a shot of 196 lbs., projected with a charge of 9 lbs. of powder?

$$\sqrt{9 \times 3} \times 1600 \div \sqrt{196} = 8320 \div 14 = 594.3 \text{ } lbs.$$

To Compute Range for a Charge, or Charge for a Range.

When Range for a Charge is given.—Ranges have same proportion as charges of powder; that is, as one range is to its charge, so is any other range to its charge, elevation of gun being same in both cases. *Consequently,*

To Compute Range.

RULE.—Multiply range determined by charge in lbs. for range required, divide product by given charge, and quotient will give range required.

EXAMPLE.—If, with a charge of 9 lbs. of powder, a shot ranges 4000 feet, how far will a charge of 6.75 lbs. project same shot at same elevation?

$$4000 \times 6.75 \div 9 = 3000 \text{ } feet.$$

To Compute Charge.

RULE.—Multiply given range by charge in lbs. for range determined, divide product by range determined, and quotient will give charge required.

EXAMPLE.—If required range of a shot is 3000 feet, and charge for a range of 4000 feet has been determined to be 9 lbs. of powder, what is charge required to project same shot at same elevation?

$$3000 \times 9 \div 4000 = 6.75 \text{ } lbs.$$

To Compute Range at one Elevation, when Range for another is given.

RULE.—As sine of double first elevation in degrees is to its range, so is sine of double another elevation to its range.

EXAMPLE.—If a shot range 1000 yards when projected at an elevation of 45°, how far will it range when elevation is 30° 16′, charge of powder being same?

Sine of $45° \times 2 = 100\,000$; sine of $30° \, 16' \times 2 = 87\,064$.

Then, as $100\,000 : 1000 :: 87\,064 : 870.64$ *feet.*

To Compute Elevation at one Range, when Elevation for another is given.

RULE.—As range for first elevation is to sine of double its elevation, so is range for elevation required to sine for double its elevation.

EXAMPLE.—If range of a shell at 45° elevation is 3750 feet, at what elevation must a gun be set for a shell to range 2810 feet with a like charge of powder?

Sine of $45° \times 2 = 100\,000$.

Then, as $3750 : 100\,000 :: 2810 : 74\,933 =$ *sine for double elevation* $= 24° \, 16'$.

Approximate Rule for Time of Flight.

Under 4000 yards, velocity of projectile 900 feet in one second; under 6000 yards, velocity 800 feet; and over 6000 yards, velocity 700 feet.

Guns and Howitzers take their denomination from weights of their solid shot in round numbers, up to the 42-pounder; larger pieces, rifled guns, and mortars, from diameter of their bore.

Initial Velocity and Ranges of Shot and Shells.

The *Range* of a shot or shell is the distance of its first graze upon a horizontal plane, the piece mounted upon its proper carriage.

Arms and Ordnance.	Projectile. Description.	Projectile. Weight.	Powder.	Initial Velocity.	Time of Flight.	Elevation.	Range.
		Grains.	Grains.	Feet.	Seconds.	° ′	Yards.
Rifle Musket.........	Elongated.	510	60	963	—	—	—
Musket, 1841..........	Round.	412	110	1500	—	—	—
		Lbs.	Lbs.				
6-Pounder..........	"	6.15	1.25	—	—	5	1523
12 "	"	12.3	2.5	1826	1.75	1	575
24 "	"	24.25	6	1870	—	2	1147
32 "	"	32.3	8	1640	—	1	713
42 "	"	42.5	10.5	—	—	5	1955
8-inch Columbiad...	"	65	10	—	14.19	15	3224
10 " " ...	"	127.5	15	—	14.32	15	3281
10 " Mortar.......	Shell.	98	10	—	36	45	4250
13 " "	"	200	20	—	—	45	4325
15 " Columbiad...	"	302	40	—	—	7	1948
15 " " ...	"	315	50	—	23.29	25	4680
RIFLED.							
10-pounder Parrott..	"	9.75	1	—	21	20	5000
20 " " ..	"	19	2	—	17.25	15	4400
30 " " ..	"	29	3.25	—	27	25	6700
100 " " ..	Elongated.	100	10	—	29	25	6910
100 " " ..	Shell.	101	10	1250	28	25	6820
200 " " ..	"	150	16	—	—	4	2200
12-inch Rodman.....	"	—	50	1154	5.5	40	—
Hall's Rockets.......	3-inch.	16	—	—	—	47	1720

Penetration of Shot and Shell.

Experiments at Fort Monroe, 1839, and at West Point, 1853.

Ordnance.	Charge.	Distance.	Mean Penetration. White Oak.	Mean Penetration. Hard Brick.	Mean Penetration. Granite.	Ordnance.	Charge.	Distance.	Mean Penetration. White Oak.	Mean Penetration. Hard Brick.	Mean Penetration. Granite.
	Lbs.	Yds.	Ins.	Ins.	Ins.		Lbs.	Yds.	Ins.	Ins.	Ins.
32 Lbs. Shot.	8	880	—	15.25	3.5	8-inch Howitz.*	6	880	—	8.5	1
32 " "	11	100	60	—	—	8 " Columb.[1]†	12	200	—	—	—
42 " "	10.5	100	54.75	18	4	10 " " †	18	114	63.5	44	7.75
42 " Shell.	7	100	40.75	—	—	10 " " *	18	100	56.75	—	—

[1] 24 ins. of Concrete. * Shell. † Shot.

Solid shot broke against granite, but not against freestone or brick, and general effect is less upon brick than upon granite.

Shells broke into small fragments against each of the three materials.

Penetration in earth of shell from a 10-inch Columbiad was 33 ins.

Experiments—England. (*Holley.*)

Ordnance.	Charge.	Projectile.	Weight.	Velocity.	Range.	Target and Effects.
	Lbs.		Lbs.	Feet.	Yards.	
11-inch U. S. Navy.	30	Shot.	169	1400	50	Iron plates, 14 ins.—loosened.
15-inch Rodman...	60	"	400	1480	50	Iron plates, 6 ins.—destroyed.
RIFLED.						
7-inch Whitworth..	25	Shot.	150	1241	200	Inglis'st—destr'd.
10.5-inch Armstrong	45	"	307	1228	200	" "
13-inch "	90	* "	344.5	1760	200	Solid plates, 11 ins. thick—destr'd.

* Steel. † 8-inch vertical and 5-inch horizontal slabs, and 7-inch vertical and 5-in. horizontal slabs, 9 × 5 ins. ribs and 3-inch ribs.

Elements of Report of Board of Engineers for Fortifications, U. S. A. Professional Papers No. 25. (Brev. Maj.-Gen. Z. B. Tower.)

Experimental firings for penetration during the past twenty years have determined that wrought iron and cast iron, unless chilled, are unsuitable for projectiles to be used against iron armor; that the best material for that purpose is hammered steel or Whitworth's compressed steel.

2. That cast-iron and cast-steel armor-plates will break up under the impact of the heaviest projectiles now in service, unless made so thick as to exclude their use in ship-protection.

3. That wrought-iron plates have been so perfected that they do not break up, but are penetrated by displacement or crowding aside of the material in the path of the shot, the rate of penetration bearing an approximately determined ratio to the striking energy of the projectile, measured per inch of shot's circumference, as expressed by the following formula:

$$\sqrt[2.035]{\frac{V^2 P}{2g \times 2r\pi \times 2240 \times .86}} = \textit{penetration in ins.}$$ *V representing velocity in feet per second,* P *weight of shot in lbs., and r radius of shot in ins.*

That such plates can therefore be safely used in ship construction, their thickness being determined by the limit of flotation and the protection needed.

4. That, though experiments with wrought-iron plates, faced with steel, have not been sufficiently extended to determine the best combination of these two materials, we may nevertheless assume that they give a resistance of about one fourth greater than those of homogenous iron.

5. That hammered steel in the late Spezzia trials proved superior to any other material hitherto tested for armor-plates. The 19-inch plate resisted penetration, and was only partially broken up by 4 shots, three of which had a striking energy of between 33 000 and 34 000 foot-tons each. Not one shot penetrated the plate. Those of chilled iron were broken up, and the steel projectile, though of excellent quality, was set up to about two thirds of its length.

Velocity and Ranges of Shot. *(Krupp's Ballistic Tables.)*

Penetration in Wrought Iron.

$$\sqrt{\frac{V^2 P}{2g \times 2r\pi \times 2240 \times C}} = \textit{penetration in ins.} \quad C = 2.53.$$

Gun.	Caliber.	Powder.	Shot.	Velocity at Muzzle per Sec.	Velocity Range 3000	Velocity Range 6000	Penetration at Muzzle	Penetration Range 600	Penetration Range 3000	Penetration Range 6000
Tons.	Ins.	Lbs.	Lbs.	Feet.	Yds.	Yds.	Ins.	Ins.	Ins.	Ins.
Armstrong, 100..	17.75	550	2022	1715	1424	1191	34.76	33.2	27.55	22.04
" " ..	17.75	776	2000	1832	1518	1259	37.52	35.81	29.66	23.47
Woolwich, 81..	16	445	1760	1657	1393	1181	32.6	31.23	26.24	21.35
Krupp, 71..	15.75	485	1715	1703	1434	1211	33.52	32.12	27.04	21.89
" 18..	9.45	165	474	1688	1351	1113	20.42	19.31	15.46	12.14
U. S.* 8-inch	8	35	180	1450	1036	840	10.23	9.22	6.72	5.17

* Unchambered.

Target.—For 100-ton gun, steel plate 22 ins. thick, backed with 28.8 ins. of wood, 2 wrought-iron plates 1.5 ins. thick, and the frame of a vessel.

Effect.—Total destruction of steel plate, and backing entered to a depth of 22 ins., but not perforated.

Summary of Record of Practice in Europe with Heavy Armstrong, Woolwich, and Krupp Guns.

Board of Engineers for Fortifications, U. S. A., Professional Papers No. 25.

Gun.	Powder.	Projectile.	Charge of Powder.	Weight of Projectile.	Initial Velocity per Second. V.	Energy: Initial $\frac{PV^2}{2g \cdot 2240}$	Energy: per inch of circumference of shot. $\frac{PV^2}{2g \cdot 2\pi r \cdot 2240}$
			Lbs.	Lbs.	Feet.	Ft.-tons.	Foot-tons.
ARMSTRONG, 100 Tons, caliber 17 ins., bore 30.5 feet.	1.5-inch cubes..	Shot....	330	2000	1446	28 990	544.05
	Waltham Abbey	"	375	2000	1543	33 000	623
	Fossano.......	"	400	2000	1502	31 282	585.74
	"	"	776	2000	1832	46 580	835.32
WOOLWICH, 81 Tons, caliber 14.5 ins., bore 24 feet.	.75-inch cubes.	"	170	1258	1393	16 922	371.5
	1.5 " "	"	220	1450	1440	20 842	457.57
	2 " "	"	250	1260	1523	20 259	444.78
caliber 16 ins.....	1.5 " "	"	310	1466	1553	24 508	520.4
38 Tons, caliber 12.5 ins., bore 16.5 feet.	1.5 " "	Pall. shell	130	800	1451	11 668	297.64
	1.5 " "	"	200	800	1421	11 210	285.4
	1.5 " "	"	180	800	1504	12 545	319.4
KRUPP, 71 Tons, caliber 15.75 ins., bore 28.58 feet.	Prism A.......	Plain ...	298	1707	1184	16 602	335.42
	" H.......	Shrapnel	485	1725	1703	34 503	697.91
	" 2 inch...	Shell....	441	1419	1761	30 484	616.14
18 Tons, caliber 9.45 ins., bore 17.5 feet.	" 1 hole...	Plain ...	132	300	1873	7 298	246.03
	" 2 inch...	Shrapnel	145	474	1688	9 367	315.66
	"	Shell....	165	300	1991	8 244	277.69

Penetration in Ball Cartridge Paper, No. 1.

Musket, with 134 grains, at 13.3 yards.................... 653 sheets.
Common rifle, 92 grains, at 13.3 yards.................... 500 sheets.

Penetration of Lead Balls in Small Arms.

Experiments at Washington Arsenal in 1839, and at West Point in 1837.

Arm.	Diameter of Ball.	Charge Powder.	Distance.	Weight of Ball.	Penetration. White Oak.	Penetration. White Pine.
	Inch.	Grains.	Yards.	Grains.	Ins.	Ins.
Musket...............	.64	134	9	397.5	1.6	—
	.64	144	5	397.5	3	—
Common Rifle........	—	100	5	219	2.05	—
	—	92	9	—	1.8	—
Hall's rifle...........	—	100	5	219	2	—
	—	70	9	219	.6	—
Hall's carbine, musket caliber.............	.5775	70	5	—	1.7	—
		80	5	219	.8	—
		90*	5	—	1.1	—
		100*	5	—	1.2	—
Pistol................	—	51	5	219	.725	—
Rifle musket..........	.5775	—	200	500	—	11
Altered musket.......	.685	60	200	730	—	10.5
Rifle, Harper's Ferry..	.5775	70	200	500	—	9.33
Pistol carbine.........	.5775	40	200	450	—	5.75
Sharpe's carbine......	.55	60	30	463	—	7.17
Burnside's "	.55	55	30	350	—	6.15

* Charges too great for service.

Musket discharged at 9 yards distance, with a charge of 134 grains, 1 ball and 3 buckshot, gave for ball a penetration of 1.15 ins., buckshot, .41 inch.

Loss of Force by Windage.

A comparison of results shows that 4 lbs. of powder give to a ball without windage nearly as great a velocity as is given by 6 lbs. having .14 inch windage, which is true windage of a 24-lb. ball; or, in other words, this windage causes a loss of nearly *one third* of force of charge.

Vents.—Experiments show that loss of force by escape of gas from vent of a gun is altogether inconsiderable when compared with whole force of charge.

Diameter of *Vent* in U. S. Ordnance is in all cases .2 inch.

Effect of different Waddings with a Charge of 77 Grains of Powder.

Wad.	Velocity of Ball per Second.
	Feet.
Ball wrapped in cartridge paper and crumpled	1377
1 felt wad upon powder and 1 upon ball	1346
2 felt wads upon powder and 1 upon ball	1482
1 elastic wad upon powder and 1 upon ball	1132
2 pasteboard wads upon powder	1200
2 elastic wads upon powder	1100

Felt wads cut from body of a hat, weight 3 grains.

Pasteboard wads .1 of an inch thick, weight 8 grains.

Cartridge paper 3 × 4.5 ins., weight 12.82 grains.

Elastic wads, "Baldwin's indented," a little more than .1 of an inch thick, weight 5.127 grains.

Most advantageous wads are those made of thick pasteboard, or of ordinary cartridge paper.

In service of *cannon*, heavy wads over ball are in all respects injurious.

For purpose of retaining the ball in its place, light *grommets* should be used.

On the other hand, it is of great importance, and especially so in use of small arms, that there should be a good wad over powder for developing full force of charge, unless, as in the rifle, the ball has but very little windage. (*Capt. Mordecai.*)

Weight and Dimensions of Lead Balls.

Number of Balls in a Lb., from 1.3125 to .237 of an Inch Diameter.

Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.
Ins.		Inch.		Inch.		Inch.		Inch.		Inch.	
1.67	1	.75	11	.57	25	.388	80	.301	170	.259	270
1.326	2	.73	12	.537	30	.375	88	.295	180	.256	280
1.157	3	.71	13	.51	35	.372	90	.29	190	.252	290
1.051	4	.693	14	.505	36	.359	100	.285	200	.249	300
.977	5	.677	15	.488	40	.348	110	.281	210	.247	310
.919	6	.662	16	.469	45	.338	120	.276	220	.244	320
.873	7	.65	17	.453	50	.329	130	.272	230	.242	330
.835	8	.637	18	.426	60	.321	140	.268	240	.239	340
.802	9	.625	19	.405	70	.314	150	.265	250	.237	350
.775	10	.615	20	.395	75	.307	160	.262	260		

Heated shot do not return to their original dimensions upon cooling, but retain a permanent enlargement of about .02 per cent. in volume.

Number of Pellets in an Ounce of Lead Shot of the different Sizes.

A A	40	B	75	No. 3	135	No. 6	280	No. 9	984
A	50	No. 1	82	4	177	7	341	10	1726
B B	58	2	112	5	218	8	600	12	2140

No. 14 3150

Proportion of Powder to Shot for following Numbers of Shot.

No.	Shot.	Powder.	No.	Shot.	Powder.	No.	Shot.	Powder.
	Oz.	Drams.		Oz.	Drams.		Oz.	Drams.
2	2	1.5	4	1.5	1.875	6	1.25	2.375
3	1.75	1.625	5	1.375	2.125	7	1.125	2.625

NOTE.—2 oz. of No. 2 shot, with 1.5 drams of powder, produced greatest effect.

Increase of powder for greater number of pellets is in consequence of increased friction of their projection.

Numbers of Percussion Caps corresponding with Birmingham Numbers.

Eley's	5	6	7	8	9	24	10	11	18	12	13	14
Birmingham	43	44	46	48	49	50	51 and 52	53 and 54	55 and 56	57	58	58

Where there are two numbers of Birmingham sizes corresponding with only one of Eley's, it is in consequence of two numbers being of *same size*, varying only in length of caps.

Comparison of Force of a Charge in various Arms.

Arm.	Lock.	Powder, A 5.	Windage.	Weight of Ball.	Velocity.
		Grains.	Inch.	Grains.	Feet.
Ordinary rifle	Percussion.	100	.015	219	2018
" "	"	70	.015	219	1755
Hall's rifle	Flint.	70	.0	219	1490
Hall's carbine	Percussion.	70	.0	219	1240
Jenks's carbine	"	70	.0	219	1687
Cadet's musket	Flint.	70	.045	219	1690
Pistol	Percussion.	35	.015	218.5	947

Ranges for Small Arms.

Musket.—With a ball of 17 to pound, and a charge of 110 grains of powder, etc., an elevation of 36′ is required for a range of 200 yards; and for a range of 500 yards, an elevation of 3° 30′ is necessary, and at this distance a ball will pass through a pine board 1 inch in thickness.

Rifle.—With a charge of 70 grains, an effective range of from 300 to 350 yards is obtained; but as 75 grains can be used without *stripping* the ball, it is deemed better to use it, to allow for accidental loss, deterioration of powder, etc.

Pistol.—With a charge of 30 grains, the ball is projected through a pine board 1 inch in thickness at a distance of 80 yards.

Gunpowder.

Gunpowder is distinguished as *Musket*, *Mortar*, *Cannon*, *Mammoth*, and *Sporting* powder; it is all made in same manner, of same proportions of materials, and differs only in size of its grain.

Bursting or Explosive Energy.—By the experiments of Captain Rodman, U. S. Ordnance Corps, a pressure of 45 000 lbs. per square inch was obtained with 10 lbs. of powder, and a ball of 43 lbs.

Also, a pressure of 185 000 lbs. per sq. inch was obtained *when the powder was burned in its own volume*, in a cast-iron shell having diameters of 3.85 and 12 ins.

Proof of Powder. (*U. S. Ordnance Manual.*)

Powder in magazines that does not range over 180 yards is held to be unserviceable.

Good powder averages from 280 to 300 yards; *small grain*, from 300 to 320 yards.

Restoring Unserviceable Powder.—When powder has been damaged by being stored in damp places, it loses its strength, and requires to be worked over. If quantity of moisture absorbed does not exceed 7 per cent., it is sufficient to dry it to restore it for service. This is done by exposing it to the sun.

When powder has absorbed more than 7 per cent. of water it should be sent to a powder mill to be worked over.

Properties and Results of Gunpowder, determined by Experiments. (*Captain A. Mordecai, U. S. A.*)

24-Pounder Gun.		Musket Pendulum.	
Weight of ball and wad....	24.25 lbs.	Weight of ball............	397.5 grains.
" " powder.........	6 "	" " powder.........	120 "
Windage of ball...........	.135 inch.	Windage of ball...........	.09 inch.

Grain.	Composition. Salt-petre.	Char-coal.	Sul-phur.	Manufacture. Where from.	Number of Grains in 10 Troy Grains.	Relative Quickness of Burning.	Water absorbed by exposure to Air. Per c't.	Relative Force.
Cannon, large...	76	14	12	* Dupont's Mills, Del.	77	275	2.77	.677
" small...					569	314	3.35	.72
Musket.........					1 134	214	—	.808
Rifle............					6 174	142	—	.907
Rifle............					5 344	282	3.55	.728
Musket.........					1 642	—	—	.834
Rifle					13 152	—	—	.943
Cannon, uneven.	75	12.5	12.5	† Dupont's Mills, Del.	166	183	2.09	.788
" large ...					103	182	1.91	.756
Sporting........	77	13	10	* Dupont's Mills, Del.	72 808	100	4.42	1
Blasting, uneven	70	15	15		295	212	—	—
Rifle............	76	15	9	Loomis, Hazard, & Co., Conn.*	2 378	204	—	.82
Sporting........					—	—	—	.888
Rifle............	75	15	10	Waltham Abbey, England.*	11 600	—	—	.865

* Glazed. † Rough.

Manufacture of Powder.—Powder of greatest force, whether for cannon or small arms, is produced by incorporation in the "cylinder mills."

Effect of Size of Grain.—Within limits of difference in size of grain, which occurs in ordinary cannon powder, the granulation appears to exercise but little influence upon force of it, unless grain be exceedingly dense and hard.

Effect of Glazing.—Glazing is favorable to production of greatest force, and to quick combustion of grains, by affording a rapid transmission of flame through mass of the powder.

Effect of using Percussion Primers.—Increase of force by use of primers, *which nearly closes vent*, is constant and appreciable in amount, yet not of sufficient value to authorize a reduction of charge.

*Ratio of Relative Strength of different Powders for use under water differ but little from the reciprocal of the ratio between the sizes of the grains, showing that the strength is nearly inversely proportional thereto.**

Mammoth, .08; Oliver, .09; Cannon, .18; Mortar, 1; Musket, 1.57; Sporting 2.61, and Safety Compound 30.62.

Dualin is nitro-glycerine absorbed by Schultze's powder.

For other powders and explosive materials see Gunnery, page 443.

Heat and Explosive Power. (*Capt. Noble and F. A. Abel.*)

One gram of fired powder evolves a mean temperature of 730°. Temperature of explosion 3970°. Volume of permanent gas (which is in an inverse ratio to units of heat evolved) at 32° = 250°.

The explosive power of powder, as tested in Ordnance, ranges, for volumes of expansion of 1.5 to 50 times, from 36 to 170 *foot-tons* per lb. burned.

A charge of 70 lbs. gave to an 180 lbs. shot a velocity of 1694 feet per second, equal to a total energy of 3637 *foot-tons*, and a charge of 100 lbs. gave a velocity of 2182 *feet*, and an energy of 5940 *foot-tons.*

* Report of Experiments and Investigations to develop a system of submarine mines. Professional Papers, U. S. E., No. 23.

HEAT.

Heat, alike to gravity, is a universal force, and is referred to both as cause and effect.

Caloric is usually treated of as a material substance, though its claims to this distinction are not decided; the strongest argument in favor of this position is that of its power of radiation. Upon touching a body having a higher temperature than our own, caloric passes from it, and excites the feeling of warmth; and when we touch a body having a lower temperature than our own, caloric passes from our body to it, and thus arises the sensation of cold.

To avoid any ambiguity that may arise from use of the same expression, it is usual and proper to employ the word *Caloric* to signify the principle or cause of sensation of heat.

Heat Unit.—For purpose of expressing and comparing quantities of heat, it is convenient and customary to adopt a *Unit of heat* or *Thermal unit*, being that quantity of heat which is raised or lost in a defined period of temperature in a defined weight of a particular substance.

Thus, a Thermal unit, *Is quantity of heat which corresponds to an interval of 1° in temperature of 1 lb. of pure liquid water, at and near its temperature of greatest density.*

Thermal unit in France, termed *Caloric*, *Is quantity of heat which corresponds to an interval of 1° C. in temperature of 1 kilogramme of pure liquid water, at and near its temperature of greatest density.*

Thermal unit to Caloric, 3.968 32; Caloric to Thermal unit, .251 996.

One Thermal unit or 1° in 1 lb. of water, 772 foot-lbs.
One Caloric or 1° C. in 1 kilogramme of water, 423 55 kilogrammetres.
1° C. in 1 lb. water, 1389.6 foot-lbs.

Ratio of Fahrenheit to Centigrade, 1.8; of Centigrade to Fahrenheit, .555.

Absolute Temperature, Is a temperature assigned by deduction, as an opportunity of observing it cannot occur, it being the temperature corresponding to entire absence of gaseous elasticity, or when pressure and volume = 0. By Fahrenheit it is—461.2°, by Reaumur—229.2°, and by Centigrade—274°.

Heat is termed *Sensible* when it diffuses itself to all surrounding bodies; hence it is free and uncombined, passing from one substance to another, affecting the senses in its passage, determining the height of the thermometer, etc.

Temperature of a body, is the quantity of sensible heat in it, present at any moment.

Heat is developed by water when it is violently agitated.

Heat is developed by percussion of a metal, and it is greatest at the first blow.

Quantities of heat evolved are nearly the same for same substance, without reference to temperature of its combustion.

Mechanical power may be expended in production of heat either by friction or compression, and quantity of heat produced bears the same proportion to quantity of mechanical power expended, being 1 unit for power necessary to raise 1 lb. 772 feet in height. This number of 772 is termed the *mechanical equivalent of heat* (Joules).

Specific Heat.

Specific Heat of a body signifies its capacity for heat, or quantity required to raise temperature of a body 1°, or it is that which is absorbed by different bodies of equal weights or volumes when their temperature is equal, based upon the law, *That similar quantities of different bodies require unequal quantities of heat at any given temperature.* It is also the *quantity of heat* requisite to change the temperature of a body any stated number of degrees compared with that which would produce same effect upon water at 32°.

Quantity of heat, therefore, is the quantity necessary to change the temperature of a body by any given amount (as 1°), divided by quantity of heat necessary to change an equal weight or volume of water 32° by same amount.

NOTE.—Water has greater specific heat than any known body.

Every substance has a specific heat peculiar to itself, whence a change of composition will be attended by a change of its capacity for heat.

Specific heat of a body varies with its form. A solid has a less capacity for heat than same substance when in state of a liquid; specific heat of water, for instance, being .5 in solid state (ice), .622 in gaseous (steam), and 1 in liquid.

Specific heat of equal weights of same gas increases as density decreases; exact rate of increase is not known, but ratio is less rapid than diminution in density.

Change of capacity for heat always occasions a change of temperature. Increase in former is attended by diminution of latter, and contrariwise.

Specific heat multiplied by atomic weight of a substance will give the constant 37.5 as an average, which shows that the atoms of all substances have equal capacity for heat. This is a result for which as yet no reason has been assigned.

Thus: atomic weights of lead and copper are respectively 1294.5 and 395.7, and their specific heats are .031 and .095. Hence $1294.5 \times .031 = 40.129$, and $395.7 \times .095 = 37.591$.

It is important to know the relative Specific Heat of bodies. The most convenient method of discovering it is by mixing different substances together at different temperatures, and noting temperature of mixture; and by experiments it appears that the same quantity of heat imparts twice as high a temperature to mercury as to an equal quantity of water; thus, when water at 100° and mercury at 40° are mixed together, the mixture will be at 80°, the 20° lost by the water causing a rise of 40° in the mercury; and when weights are substituted for measures, the fact is strikingly illustrated; for instance, on mixing a pound of mercury at 40° with a pound of water at 160°, a thermometer placed in it will fall to 155°. Thus it appears that same quantity of heat imparts twice as high a temperature to mercury as to an equal volume of water, and that the heat which gives 5° to water will raise an equal weight of mercury 115°, being the ratio of 1 to 23. Hence, if equal quantities of heat be added to equal weights of water and mercury, their temperatures will be expressed in relation to each other by numbers 1 and 23; or, in order to increase the temperature of equal weights of those substances to the same extent, the water will require 23 times as much heat as the mercury.

Capacity for Heat is relative power of a body in receiving and retaining heat in being raised to any given temperature; while *Specific* applies to actual quantity of heat so received and retained.

Specific Heat of Air and other Gases.

Specific heat, or capacity for heat, of permanent gases is sensibly constant for all temperatures, and for all densities. Capacity for heat of each gas is same for each degree of temperature. M. Regnault proved that capacity for heat for air was uniform for temperatures varying from —22° to +437°; consequently, specific heat for equal weights of air, at constant pressure, averaged .2377.

Specific Heat. *Water at* 32° = 1.

Metals from 32° *to* 212°.							
Antimony	.0508	Silver	.056	*Woods.*		Sulphur	.2026
Bismuth	.0308	Steel	.1165	Oak	.57	*Liquids.*	
Brass	.0939	Tin	.0562	Pear	.5	Alcohol	.6588
Copper	.092	Wrought iron	.1138	Pine	.65	Ether	.4554
Cast iron	.1298	Zinc	.0955	*Min'l Substances.*		Linseed oil	.31
Gold	.0324	*Stones.*		Charcoal	.2415	Olive oil	.3096
Lead	.0314	Chalk	2149	Coal	.2411	Steam	.365
Mercury	.0333	Limestone	.2174	Coke	.203	Turpentine	.416
Nickel	.1086	Masonry	.2	Glass	.1977	Vinegar	.92
Platinum	.0324	Marble, gray	.2694	Gypsum	.1966	*Solid.*	
		" white	.2158	Phosphorus	2503	Ice	.504

Gases.

Air	.2377	Hydrogen	.2356
Oxygen	.2412	Carbonic Acid	.3308

For Equal Weights.

Air	.1688	Hydrogen	2.4096
Oxygen	.1559	Carbonic Acid	.1714

Metals have least, ranging from Bismuth .0308 to Cast Iron .1298. Stones and Mineral Substances have .2 that of water, and Woods about .5. Liquids, with exception of Bromine, are less than water, Olive oil being lowest and Vinegar highest.

Illustration.—If 1 lb. of coal will heat 1 lb. of water to 100°, $\frac{1}{.033} = \frac{1}{30.3}$ of a lb. will heat 1 lb. of mercury to 100°.

To Compute Temperature of a Mixture of like Substances.

$\frac{W\,T + w\,t}{W + w} = t'$; $\frac{w\,(t' - t)}{T - t'} = W$; $\frac{w\,(t' - t)}{W} + t' = T$. W *representing weight or volume of a substance of temperature* T, *w weight or volume of a like substance of temperature t, and t′ temperature of mixture* W + *w*.

Illustration 1. — When 5 cube feet of water (W) at a temperature of 150° (T) is mixed with 7.5 cube feet (*w*) at 50° (*t*), what is the resultant temperature of the mixture?

$$\frac{5 \times 150° + 7.5 \times 50°}{5 + 7.5} = \frac{1125}{12.5} = 90°.$$

2.—How much water at (T) 100° should be mixed with 30 gallons (*w*) at 60°, for a required temperature of 80°?

$$\frac{30\,(80° - 60°)}{100° - 80°} = \frac{600}{20} = 30 \textit{ gallons}.$$

To Compute Temperature of a Mixture of Unlike Substances.

$\frac{W\,S\,T + w\,s\,t}{W\,S + w\,s} = t'$; $\frac{w\,s\,(t - t')}{S\,(T - t)} = W$; $\frac{t'\,(W\,S + w\,s) \backsim w\,s\,t}{W\,S} = T$. W *and w representing weights, and* S *and s specific heat of substances.*

Illustration.—To what temperature should 20 lbs. cast iron (W) be heated to raise 150 lbs. (*w*) of water to a temperature (*t*) of 50° to 60°?

$s = 1$, and $S = .1298$. $\frac{60°\,(20 \times .1298 + 150 \times 1) \backsim 150 \times 1 \times 50°}{20 \times .1298} = \frac{1655.76}{2.596} = 638°.$

To Compute Specific Heat at Constant Volume.

When Specific Heat at Constant Pressure is known. $\frac{S\,p}{H} = s$. S *representing specific heat at constant pressure, p proportion of heat absorbed at constant volume,* H *total heat absorbed at constant pressure, and s specific heat at constant volume.*

Or, $\frac{S\,(t' - t) - 2.742\,(V - v)}{t' - t} = s$. *t and t′ representing initial and final temperature of the gas and that to which it is raised, and* V *and v initial and final volumes of the gas under* 14.7 *lbs. per sq. inch, and of it heated under constant pressure in cube feet.*

Illustration.—Assume 1 lb. air at atmospheric pressure and at 32°, doubled in volume by heat. S = .2377*, $t - t' = 32° \sim 525° = 493°$ and $V - v = 12.387$* *cube feet.*

$$\frac{.2377 \times 493 - (2.742 \times 12.387)}{493} = .1688 \text{ specific heat.}$$

For comparative volumes of other gases, see Table, page 506.

To Compute Specific Heat for Equal Volume of Gas and Air.

Rule.—Multiply specific heat of the gas for equal weights of gas and air by specific gravity of gas, and product is specific heat for equal volume.

Example.—What is specific heat of air at equal volume with hydrogen?

Specific heat of hydrogen for equal weights at constant volume, 2.4096, and specific gravity of the gas, .0692. (See Table, page 506.)

Then, $2.4096 \times .0692 = .1667$ *specific heat for equal volumes at constant volume.*

Specific heat of steam, air at unity = 1.281.

Capacity for Heat.

When a body has its density increased, its capacity for heat is diminished. The rapid reduction of air to .2 of its volume evolves heat sufficient to inflame tinder, which requires 550°.

Relative Capacity for Heat of Various Bodies. (*Water at* 32° = 1.)

Bodies.	Equal Weights.	Equal Volumes.	Bodies.	Equal Weights.	Equal Volumes.	Bodies.	Equal Weights.	Equal Volumes.
Water..	1	1	Gold....	.05	.966	Mercury	.036	—
Brass...	.116	.971	Ice.....	.9	—	Silver..	.082	.833
Copper..	.114	1.027	Iron....	.126	.993	Tin.....	.06	—
Glass...	.187	.448	Lead....	.043	.487	Zinc....	.102	—

To Ascertain Relative Capacities of Different Bodies, combined with experiment.

Rule.—Multiply weight of each body by number of degrees of heat lost or gained by mixture, and capacities of bodies will be inversely as products.

Or, if bodies be mingled in unequal quantities, capacities of the bodies will be reciprocally as quantities of matter, multiplied into their respective changes of temperature.

Illustration.—If 1 lb. of water at 156° is mixed with 1 lb. of mercury at 40°, resultant temperature is 152°.

Thus, $1 \times 156° - 152° = 4°$, and $1 \times 40° \sim 152° = 112°$. Hence capacity of water for heat is to capacity of mercury as 112° to 4°, or as 28 to 1.

Sensible Heat.

Sensible heat or temperature to raise water from 32° to 212° = 180.9°, or heat units.

* See Tables, pages 506 and 520–21.

Latent Heat.

Latent Heat is that which is insensible to the touch of our bodies, and is incapable of being detected by a thermometer.

When a solid body is exposed to heat, and ultimately passes into the liquid state under its influence, its temperature rises until it attains the point of fusion, or melting point. The temperature of the body at this point remains stationary until the whole of it is melted; and the heat meantime absorbed, without affecting the temperature or being sensible to the touch or to the indications of a thermometer, is said to become *latent*. It is, in fact, *the latent heat of fusion*, or *the latent heat of liquidity*, and its function is to separate the particles of the body, hitherto solid, and change their condition into that of a liquid. When, on the contrary, a liquid is solidified, the latent heat is disengaged.

If to a pound of newly-fallen snow were added a pound of water at 172°, the snow would be melted, and 32° would be resulting temperature.

When a body is *fusing*, no rise in its temperature occurs, however great the additional quantity of heat may be imparted to it, as the increased heat is absorbed in the operation of fusion. The quantity of heat thus made latent varies in different bodies.

A pound of water, in passing from a liquid at 212° to steam at 212°, receives as much heat as would be sufficient to raise it through 966.6 thermometric degrees, if that heat, instead of becoming *latent*, had been *sensible*.

If 5.5 lbs. of water, at temperature of 32°, be placed in a vessel, communicating with another one (in which water is kept constantly boiling at temperature of 212°), until former reaches temperature of latter quantity, then let it be weighed, and it will be found to weigh 6.5 lbs., showing that one lb. of water has been received in form of steam through communication, and reconverted into water by lower temperature in vessel. Now this pound of water, received in the form of steam, had, when in that form, a temperature of 212°. It is now converted into liquid form, and still retains same temperature of 212°; but it has caused 5.5 lbs. of water to rise from the temperature of 32° to 212°, and this without losing any temperature of itself. Now this heat was combined with the steam, but as it is not sensible to a thermometer, it is termed *Latent*.

Quantity of heat necessary to enable ice to resume the fluid state is equal to that which would raise temperature of same weight of water 140°; and an equal quantity of heat is set free from water when it assumes the solid form.

Sum of Sensible and Latent Heats.

From Water at 32°.

Pressure.	Latent.	Sum.	Pressure.	Latent.	Sum.	Pressure.	Latent.	Sum.	Pressure.	Latent.	Sum.
Lbs.	°	°	Lbs.	°	°	Lbs.	°	°	Lbs.	°	°
14.7	964.3	1146.1	26	943.7	1155.3	55	912	1169	120	873.7	1185.4
16	962.1	1147.4	27	942.2	1155.8	60	908	1170.7	130	869.4	1187.3
17	959.8	1148.3	28	940.8	1156.4	65	904.2	1172.3	140	865.4	1189
18	957.7	1149.2	29	939.4	1157.1	70	900.8	1173.8	150	861.5	1190.7
19	955.7	1150.1	30	937.9	1157.8	75	897.5	1175.2	160	857.9	1192.2
20	952.8	1150.9	32	935.3	1158.9	80	894.3	1176.5	170	854.5	1193.7
21	951.3	1151.7	35	931.6	1160.5	85	891.4	1177.9	180	851.3	1195.1
22	949.9	1152.5	37	929.3	1161.5	90	888.5	1179.1	190	848	1196.5
23	948.5	1153.2	40	926	1162.9	95	885.8	1180.3	200	845	1197.8
24	946.9	1153.9	45	920.9	1164.6	100	883.1	1181.4	220	829.2	1200.3
25	945.3	1154.6	50	916.3	1167.1	110	878.3	1183.5	250	831.2	1203.7

Latent Heat of Vaporization, or Number of Degrees of Heat required to convert following Substances from their respective Solidities to Vapor at Pressure of Atmosphere.

Alcohol	364°	Ice	142.6°	Water	966.6°
Ammonia	860°	Mercury	157°	Zinc	493°
Ether (Sulph.)	163°	Carbonic Acid	298°	Oil of Turpentine	124°

Latent Heat of Fusion of Solids. (*Person.*)

Substances.	Melting Point.	Specific Heat. Liquid.	Specific Heat. Solid.	In Heat-units of 1 lb.
	°	°	°	
Tin	442	.0637	.0562	25.6
Bismuth	507	.0363	.0308	22.7
Lead	617	.0402	.0314	9.86
Zinc	773	—	.0956	50.6
Silver	1873	—	.057	37.9
Mercury	39	.0333	.0319	5
Cast iron	3400	—	.129	233
Ice	32	1	.504	142.85
Phosphorus	112	.2045	.1788	9
Spermaceti	120	—	—	148
Wax	142	—	—	175
Sulphur	239	.234	.2026	17
Nitrate of soda	591	.413	.2782	113
Nit. of potassia	642	.3319	.2388	85

To Compute Latent Heat of Fusion of a Non-metallic Substance.

$C \backsim c\,(t + 256^\circ) = L$. *C and c representing specific heats of substance in solid and liquid state, t temperature of fusion, and L latent heat.*

ILLUSTRATION.—What is latent heat of fusion of ice?

$C = .504$; $c = 1$; and $t = 32^\circ$.

$$.504 \backsim 1 \times \overline{32 + 256} = 142.85^\circ \textit{ units.}$$

NOTE.—For Latent Heat of Fusion of some substances, see Deschanel's, New York, 1872, Heat, part 2.

Radiation of Heat.

Radiation of Heat is diffusion of heat by projection of it in diverging right lines into space, from a body having a higher temperature than space surrounding it, or body or bodies enveloping it.

Radiation is affected by nature of surface of body; thus, black and rough surfaces radiate and absorb more heat than light and polished surfaces. Bodies which radiate heat best absorb it best.

Radiant heat passes through moderate thicknesses of air and gas without suffering any appreciable loss or heating them. When a polished surface receives a ray of heat, it absorbs a portion of it and reflects the rest. The quantity of heat absorbed by the body from its surface is the measure of its *absorbing power*, and the heat reflected, that of its *reflecting power.*

When temperature of a body remains constant it is in consequence of quantity of heat emitted being equal to quantity of heat absorbed by body. Reflecting power of a body is complement of its absorbing power; or, sum of absorbing and reflecting powers of all bodies is the same.

Thus, if quantity of heat which strikes a body = 100, and radiating and reflecting powers each 90, the absorbent would be 10.

Radiating or Absorbent and Reflecting Powers of Substances.

SUBSTANCES.	Radiating or Absorbing.	Reflecting.
Lamp Black	100	—
Water	100	—
Carbonate of Lead	100	—
Lead, white	100	—
Writing Paper	98	2
Ivory, Jet, Marble	93 to 98	7 to 2
Resin	96	4
Glass	90	10
India Ink	85	15
Ice	85	15
Shellac	72	28
Lead	45	55
Cast Iron, bright polished	25	75
Platinum, a little polish'd	24	76
Mercury	23	77
Wrought Iron, polished	23	77
Lead, polished	19	81
Zinc, polished	19	81
Steel, polished	17	83
Platinum, in sheet	17	83
Tin	15	85
Copper, varnished	14	86
Brass, dead polished	11	89
" bright polished	7	93
Copper, ham'ered or cast	7	93
" deposited on iron	7	93
Gold, plated	5	95
" polished	3	97
Silver, polished	3	97
" cast, polished	3	97

Radiating and Absorbing Power of various Bodies, in Units of Heat per Sq. Foot per Hour for a Difference of 1°. (*Peclet.*)

	Unit.		Unit.		Unit.
Silver, polished	.0266	Iron, ordinary	.5662	Woollen stuff	.7522
Copper	.0327	Glass	.5948	Oil paint	.7583
Tin	.0439	Iron, cast	.648	Paper	.7706
Brass, polished	.0491	Wood sawdust	.7225	Lamp-black	.8196
Iron, sheet	.092	Stone, Brick, etc.	.7358	Water	1.0853

To Compute Loss of Heat by Radiation per Sq. Foot.

$\frac{1.7\, l\,(T-t)}{d\, v} = R.$ *T representing temperature of pipe, which is assumed to be .05 less than that of steam; t temperature of air; l length of pipe, and v velocity of heat in feet per second; d diameter in ins., and R radiation in degrees per second.*

Illustration.—Assume temperatures of a steam-pipe, steam, 212°, 200°, and air 60°, length of pipe 20 feet, velocity of heat (steam) 15 feet per second, and diameter of pipe 16 ins.; what will be loss of heat by radiation?

$$\frac{1.7 \times 20\,(200-60)}{16 \times 15} = 15.66°.$$

Reflection.

Reflection of Heat is passage of heat from surface of one substance to another or into space, and it is the converse of radiation.

Heat is reflected from surface upon which its rays fall in same manner as light, angle of reflection being opposite and equal to that of incidence. Metals are the strongest reflectors.

Reflecting Power of various Substances.

Silver	.97	Specular metal	.86	Zinc	.81
Gold	.95	Tin	.85	Iron	.77
Brass	.93	Steel	.83	Lead	.6

Communication and Transmission of Heat.

Communication of Heat is passage of heat through different bodies with different degrees of velocity. This has led to division of bodies into *Conductors* and *Non-conductors*; former includes such as metals, which allow caloric to pass freely through their substance, and latter comprise those that do not give an easy passage to it, such as stones, glass, wood, charcoal, etc.

Velocity of cooling, other things being equal, increases with extent of surface compared with volume of substance; and of two bodies of same material, temperature, and form, but differing in volume.

Transmission of Heat is passage of heat through different bodies with different degrees of intensity. Gaseous bodies and a vacuum are highest in order of transmittents.

Relative Power of various Substances to Transmit Heat.

All bodies capable of transmitting heat are more or less translucent, though their powers of transmitting heat and light are not in same relative proportions.

Air	1	Flint-glass	.67	Nitric acid	.15	Sulphuric acid	.17
Alcohol	.15	Gypsum	.2	Rock-crystal	.62	Turpentine	.31
Crown-glass	.49	Ice	.06	Rape-seed oil	.3	Water	.11

Heat which passes through one plate of glass is less subject to absorption in passing through a second and a third plate. Of 1000 rays, 451 were intercepted by 4 plates as follows:

1st. 381. 2d. 43. 3d. 18. 4th. 9.

Average Results of Heating and Evaporating Water by Steam in Copper Pipes and Boilers. (*D. K. Clark.*)

	Steam condensed per sq. foot for 1° difference per hour. Heating.	Evaporating.	Heat transmitted per sq. foot for 1° difference per hour. Heating.	Evaporating.
	Lbs.	Lbs.	Units.	Units.
Cast-iron-plate surface	.077	.105	82	100
Copper-plate surface	.248	.483	276	534
Copper-pipe surface	.291	1.07	312	1034

Whence.—Efficiency of copper-plate surface for evaporation of water is double its efficiency for heating; for copper-pipe surface efficiency is more than three times as much; and for cast-iron-plate surface, a fourth more.

Efficiency of pipe surface is a fifth more than that of plate surface for heating, and more than twice as much for evaporation.

Generally, copper-plate surface condenses .5 lb. of steam, copper-pipe 1 lb., and cast-iron-plate surface .1 lb. per sq. foot per 1° of temperature per hour, for evaporation.

Quantity of heat transmitted is at rate of about 1000 units per lb. of steam condensed.

Transmission of Heat through Glass of different Colors.

Direct = 100.

Plate	65.5	Blue, deep	19	Yellow	40
Window	52	" light	42	Orange	44
Violet, deep	53	Green	26	Red	53

M. Peclet defines law of transmission of heat as: The flow of heat which traverses an element of a body in a unit of time is proportional to its surface, and to difference of temperature of the two faces perpendicular to direction of flow, and is in inverse of thickness of element.

Or, $(t - t')\frac{C}{T} = H$. *t and t′ representing temperatures of surfaces,* C *constant for material* 1 *inch thick, or quantity of heat transmitted per hour for* 1° *difference of temperature through* 1 *unit of thickness,* T *thickness, and* H *quantity of heat in units passed through plate per sq. foot per hour.*

Quantities of Heat transmitted from Water to Water through Plates or Beds of Metals and other Solid Bodies, 1 Inch thick, per Sq. Foot.

For 1° *Difference of Temperature between the two Faces per Hour.*

Selected from M. Peclet's tables. (*D. K. Clark.*)

Substance.	C or Quantity of Heat.	Substance.	C or Quantity of Heat.	Substance.	C or Quantity of Heat.
	Units.		Units.		Units.
Gold	620	Iron	225	Marble	24
Platinum	604	Zinc	225	Plaster	2.6
Silver	596	Tin	177	Glass	6.56
Copper	555	Lead	112	Sand	2.16

The conditions are, that the surfaces of conducting material must be perfectly clean, that they be in contact with water at both faces of different temperatures, and that the water in contact with surfaces be thoroughly and constantly changed. M. Peclet found that when metallic surfaces became dull, rate of transmission of heat through all metals became very nearly the same.

To Compute Units of Heat Transmitted.

Illustration 1.—If 2000 lbs. beet root juice at 40° are contained in a copper boiler with a double bottom, and heated to 212°, with a heating surface of 25 sq. feet, and subjected to steam at a temperature of 275°, for a period of 15 minutes, what will be the total heat, and heat per degree of difference transmitted per sq. foot per hour?

$212^\circ - 40^\circ \times 60 \div 15 = 688^\circ$ *per hour*, and $2000 \times 688 \div 25 = 55\,040$ *units per sq. foot per hour.*

$(212^\circ + 40^\circ) \div 2 = 126^\circ$ *mean temperature of juice*, and $275^\circ - 126^\circ = 149^\circ$ *mean difference of temperature.*

Hence, $55\,040 \div 149 = 369.4$ *units per sq. foot per degree of difference per hour.*

2.—If 48.2 sq. feet of iron pipe 1.36 ins. in diameter, is supplied with steam at 275°, and it raises temperature of 882 lbs. water from 46° to 212° in 4 minutes, what will be total heat per sq. foot per hour, total heat per sq. foot per degree, and quantity condensed per sq. foot per degree per hour?

$212^\circ - 46^\circ \times 60 \div 4 = 2490^\circ$ *in an hour;* $46^\circ + 212^\circ \div 2 = 129^\circ$ *mean temperature*, and $275^\circ - 129^\circ = 146^\circ$ *difference of temperature.*

$\frac{2490^\circ \times 882}{48.2} = 45\,563$ *units per sq. foot per hour*, $45\,563 \div 146 = 312.1$ *units per sq. foot per degree*, and total heat of steam above $129^\circ = 1068^\circ$.

Hence $\frac{312.1}{1068} = .292$ *lbs. steam condensed per sq. foot per degree per hour.*

Evaporation.

Evaporation or *Vaporization* is conversion of a fluid into vapor, and it produces cold in consequence of heat being absorbed to form vapor.

It proceeds only from surface of fluids, and therefore, *other things equal*, must depend upon extent of surface exposed.

When a liquid is covered by a stratum of dry air, evaporation is rapid, even when temperature is low.

As a large quantity of heat passes from a sensible to a latent state during formation of vapor, it follows that cold is generated by evaporation.

Fluids evaporate in vacuo at from 120° to 125° below their boiling-point.

Heat required to Evaporate 1 lb. Water at Temperatures below 212° from a Vessel in open air at 32°.

(*Thomas Box.*)

Temperature.	Water evapor'd per sq. foot of surface p'r hour.	HEAT lost by radiation from surface.	HEAT lost in air.	HEAT to evaporate 1 lb. of water.	HEAT Total lost per hour.	Temperature.	Water evapor'd per sq. foot of surface p'r hour.	HEAT lost by radiation from surface.	HEAT lost in air.	HEAT to evaporate 1 lb. of water.	HEAT Total lost per hour.
°	Lbs.	Units.	Units.	Units.	Units.	°	Lbs.	Units.	Units.	Units.	Units.
32	.027	—	—	1091	29	132	.706	182	202	1506	1068
42	.04	270	424	1788	71	142	.916	158	162	1445	1326
52	.058	375	581	2052	119	152	1.178	137	127	1392	1637
62	.083	405	605	2110	174	162	1.505	118	97	1346	2039
72	.117	386	566	2055	239	172	1.895	106	72	1312	2475
82	.162	358	504	1968	319	182	2.373	92	50	1279	3045
92	.223	319	434	1862	415	192	2.947	81	32	1253	3685
102	.303	280	366	1758	533	202	3.633	71	14	1228	4465
112	.406	245	304	1664	671	212	4.471	63	—	1209	5397
122	.528	211	250	1580	849	—	—	—	—	—	—

To Compute Surface of a Refrigerator.

Illustration of Table.—If it is required to cool 20 barrels, of 42 gallons each, of beer, from 202° to 82° in an hour.

Result to be attained is to dissipate 42×8.33 (lbs. U. S. gallons) $\times 20 \times \overline{202 - 82} = 840\,000$ *units of heat per hour.*

At 202°, 4465 units are lost, and at 82°, 319, hence, average loss for each temperature between extremes $= 1850$ *units per sq. foot per hour.*

$$\text{Then } \frac{840\,000}{1850} = 454 \textit{ sq. feet in a still air.}$$

The volume of air required per hour in this case would be about 100 000 cube feet.

To Compute Area of Grate and Consumption of Fuel for Evaporation.

Illustration of Table.—If it is required to evaporate 6 Beer gallons (282 cube ins.) of liquid per hour, at a temperature not exceeding 152°.

6 gallons = 50 lbs. At 152°, water evaporated as per table = 1.178 lbs. per hour.

$\frac{50}{1.178} = 42$ *sq. feet.* Heat required to effect this = $1392 \times 50 = 69\,600$ *units.*

Assuming 6000 units as average economic value of coals, then $\frac{69\,600}{6000} = 11.6$ *lbs. coal, on a grate of 1 sq. foot.*

When it is practicable to evaporate at a high temperature, as at or above 212°, it is most economical.

Thus, water requires only 1209 units per lb. if surface is exposed, but if enclosed, heat is reduced $(1209 - 63)$ to 1146 units.

Evaporative Powers of Different Tubes per Degree of Heat, per Sq. Foot of Surface.—In Units.

Vertical tube, 230; Double-bottomed vessel, 330; Horizontal tube or Worm, 430.

To Compute Volume of Water Evaporated in a given Time.

ILLUSTRATION.—What is volume evaporated at 212°, in 15 minutes per sq. foot of surface, in a double-bottomed vessel having an area of heating surface of 17 feet, and subjected to steam at a pressure of 25 lbs. ?

Temperature of steam at $25 + 14.7$ lbs. = 269°. $269° - 212° = 57°$, and latent heat = 927.

$$\text{Then } \frac{330 \times 57 \times 17 \times 15}{927 \times 60} = 86.2 \textit{ lbs. water.}$$

When Water is at a Lower Temperature than 212°.

If 120 gallons or 1000 lbs. of water were to be evaporated from 42° in an hour, from same vessel and under like pressure as preceding:

There would be required $1000 \times (212° - 42°)$ 170 000 *units of heat.* Mean temperature of water while being heated $= \frac{42° + 212°}{2} = 127°$.

Difference between temperature of steam and water $= 267° - 127° = 140°$.

Then, $\frac{170\,000}{330 \times 140 \times 17} = .216$ *hour = time to raise water to* 212°; hence $1 - .216 =$ .784 *hour* left for evaporation, and quantity evaporated $= \frac{330 \times 57 \times 17 \times .784}{927} =$ 270.4 *lbs.*, or 32.44 *gallons.*

Dessiccation.

Dessiccation, or the drying of a substance, is best effected in a drying chamber, and it is imperative that to attain greatest effect the hot air should be admitted at highest point of exposed substance and discharged at its lowest.

Wood, submitted to an average temperature of 300° in an enclosed space for a period of 2.5 days, will lose its moisture at a consumption of 1 lb. of wood for 10.5 lbs. of wood dried, and evaporating 4 lbs. of water, equal to 2.66 lbs. of water per lb. of undried wood.

Limit of temperature for drying of wood is 340°.

Evaporation of Water per Sq. Foot of Surface per Hour.

(Dr. Dalton.)

Temperature of Water.	Evaporation. Calm.	Light Air.	Brisk Wind.	Temperature of Water.	Evaporation. Calm.	Light Air.	Brisk Wind.
°	Lbs.	Lbs.	Lbs.	°	Lbs.	Lbs.	Lbs.
32	.0349	.0448	.055	100	.3248	.4169	.5116
40	.0459	.0589	.0723	125	.6619	.8494	1.043
50	.0655	.0841	.1032	150	1.296	1.663	2.043
60	.0917	.1175	.1441	175	2.378	3.053	3.746
70	.1257	.1616	.1983	200	4.128	5.298	6.502
80	.1746	.2241	.2751	212	5.239	6.724	8.252

The rates of evaporation for these conditions of the air when perfectly dry are as 1, 1.28, and 1.57.

To Compute Quantity of Water exposed to Air that would be evaporated as above.—Subtract tabulated weight of water corresponding to dew-point from weight of water corresponding to temperature of dry air, and remainder is weight of water that would be evaporated per sq. foot of surface per hour.

Distillation.

Distillation is depriving vapor of its latent heat, and, though it may be effected in a vacuum with very little heat, no advantage in regard to a saving of fuel is gained, as latent heat of vapor is increased proportionately to diminution of sensible heat.

A temperature of 70° is sufficient for distillation of water in a vessel exhausted of air.

Conduction or Convection of Heat.

Air and gases are very imperfect conductors. Heat appears to be transmitted through them almost entirely by conveyance, the heated portions of air becoming lighter, and diffusing the heat through the mass in their ascent. Hence, in heating a room with air, the hot air should be introduced at lowest part. The advantage of double windows for retention of heat depends, in a great measure, upon sheet of air confined between them, through which heat is very slowly transmitted.

Convection of heat refers to transfer and diffusion of heat in a fluid mass, by means of the motion of the particles of the mass.

Relative Internal Conducting Powers of Various Substances.

Metals.

Brass	.76	Gold	1	Porcelain	.012	Tin	.3
Cast Iron	.517	Lead	.18	Silver	.97	Wrought Iron	.44
Copper	.89	Platinum	.98	Terra Cotta	.011	Zinc	.36

Minerals.

Cement	.21	Coal, anth'cite	1.92	Fire brick	.61	Gypsum	.2
Chalk	.6	" bitumin.	1.68	Fire clay	.76	Lime	.24
Charcoal	.07	Coke	1.98	Glass	.96	Marble	1.22
Slate	1			Wood ash	.08		

Woods with Birch = .41 with Silver.

Apple	.68	Birch	1	Ebony	.5	Oak	.73
Ash	.73	Chestnut	.7	Elm	.73	Pine	.73

Hair and Fur with Air = 1.

Cotton	.55	Flannel	2.44	Hair	2	Silk	.43
Eider down	.44	Hemp Canvas	.28	Hare's fur	.43	Wool	.5

Liquids with Water.

Alcohol	.93	Proof spirit	.85	Turpentine	3.1
Mercury	2.8	Sulphuric acid	1.7	Water	1

Practical Deductions from preceding Results.

Asphalt is best composition for resisting moisture, and, being a slow conductor of heat, it is best adapted for economy of heat and dryness.

Slate is a very dry material, but, from its quick conducting power, it is not adapted for retention of heat.

Cements.—Plaster of Paris and Woods are well adapted for lining of rooms, having low conductive powers, while *Hair and Lime*, being a quick conductor, is one of the coldest compositions.

Fire-brick absorbs much heat, and is well adapted for lining of fire-places, etc.; while *Iron*, being a high conductor of heat, is one of the worst of substances for this purpose. *Common brick* is not a very slow conductor of heat.

Communication.

Communication of Heat is passage of heat through different bodies with different degrees of velocity. This has led to the division of bodies into *Conductors* and *Non-conductors* of caloric; the former includes such as metals, which allow caloric to pass freely through their substance, and the latter comprise those that do not give an easy passage to it, such as stones, glass, wood, charcoal, etc.

The velocity of cooling, other things being equal, increases with the extent of surface compared with volume of substance; and of two bodies of same material, temperature, and form, but differing in volume.

Condensation.

Tredgold ascertained by experiment that steam at pressure (absolute) of 17.5 lbs. per sq. inch, 221°, produced 1 cube foot of water per hour by condensation in 182 sq. feet of cast-iron pipe, at a uniform and quiescent temperature of 60°. Hence, condensation .352 lb. water per hour, or .0022 lbs. per degree of difference of temperature (221 − 60).

From experiments of Mr. B. G. Nichol in England, 1875, it was deduced:

That rates of transmission of heat, between temperature of steam and that of water of condensation at its exit, at the rate of 150 feet per minute, may be taken as 380 units for vertical tubes and 520 for horizontal.

Condensation of Steam in Cast-iron Pipes. (*M. Burnat.*)

Average Press. per Sq. Inch.	Temperature.			Condensation per sq. foot of external surface of pipe per hour.				
	Steam.	Air.	Difference.	Bare.	Straw.	Pipe.	Waste.	Plaster.
Lbs.	°	°	°	Lb.	Lb.	Lb.	Lb.	Lb.
22	233	36.5	196.5	.581	.2	.229	.286	.324

From these data, following constants are deduced for an absolute pressure of 22 lbs. per sq. inch of steam condensed, and heat passed off per sq. foot of external surface of pipe per hour of 1° difference of temperature.

Surface of Pipe.	Steam condensed per Sq. Foot.	Heat passed off.	Surface of Pipe.	Steam condensed per Sq. Foot.	Heat passed off.
	Lb.	Units.		Lb.	Units.
Bare pipe	.003	2.812	Cotton waste 1 inch	.001 46	1.384
Straw coat	.001 02	.968	Earth and hair	.001 65	1.568
Cased with clay pipe	.001 15	1.108	White paint	.001 56	1.486

Pipes were 4.72 ins. diameter, .25 inch thick, and had area of 58.5 sq. feet. *Bare*—rough surface as cast. *Straw coat*—laid lengthwise .6 inch thick and bound. *Pipe*—laid in clay pipe with an air space between them, the whole covered with loam and straw. *Waste cotton*—1 inch thick and bound with twine. *Plaster*—laid in clay and hair 2.36 ins. thick.

A wrought-iron pipe 3.75 ins. in external diameter, .25 inch thick, and lagged with felt and spun yarn .5 inch thick, condensed steam at 245° at rate of .262 lb. per sq. foot per hour, in an external temperature of 60°.

Steam Condensed per Sq. Foot and per Degree per Hour.

Mean Results of several Experiments with bare Cast-iron Pipes, with Steam at Absolute Pressure of 20 lbs. per Sq. Inch.

.4 lb. per sq. foot, and .002 39 lb. per degree.

Hence, to ascertain quantity of heat lost by condensation of .002 39 lb. $= \frac{1}{420}$ of a lb.

Difference of total and sensible heats of 1 lb. steam at 20 lbs. absolute pressure $= 1151° + 32° - 228° = 955$ units, and $955 \div 420 = 2.274$ units $=$ *heat condensed.*

The loss of heat from a naked boiler in air at 62°, under an absolute pressure of 50 lbs. per sq. inch, was 5.8 units.

Congelation and Liquefaction.

Freezing water gives out 140° of heat. All solids absorb heat when becoming fluid.

Particular quantity of heat which renders a substance fluid is termed its caloric of fluidity, or latent heat.

Temperature of Solidification of Several Gases. (Faraday.)

Cyanogen	31°	Ammonia	103°	Sulphuretted Hydrogen,	123°
Carbonic Acid	72°	Sulphurous Acid	105°	Protoxide of Nitrogen	148°

Frigorific Mixtures.

Mixtures.	Parts.	Fall of Temperature. °	Mixtures.	Parts.	Fall of Temperature. °
Sea salt Nitrate of ammonia Snow, or pounded ice	5 5 12	−18 to −25	Nitrate of ammonia. Water	1 1	+50 to +4
Muriate of ammonia Nitrate of potash Snow, or pounded ice	5 1	−5 to −18	Snow Dilute sulphuric acid	1 1	−10 to −60
Phosphate of soda Nitrate of ammonia Dilute mixed acids	3 2 4	−34 to −50	Sulphate of soda Diluted nitric acid	3 2	+50 to −3
Snow Crystallized muriate of lime	1 3	−40 to −73	Nitrate of ammonia. Carbonate of soda Water	1 1 1	+50 to −7
Snow Dilute sulphuric acid	8 10	−68 to −91	Sulphate of soda Muriate of ammonia. Nitrate of potash Dilute nitric acid	6 4 2 4	+50 to −10
Phosphate of soda Nitrate of ammonia Dilute nitric acid	5 3 4	0 to −34	Phosphate of soda Dilute nitric acid	9 4	+50 to −12
Snow Dilute nitric acid	3 2	0 to −46	Snow Muriate of lime	3 4	+20 to −48
			Potash Snow	4 3	+32 to −51

A Mixture of Solid Carbonic Acid and Sulphuric Ether, under receiver of an air-pump, under pressures of .6 lbs. to 14 lbs., exhibited a temperature ranging from −107° to −166°, which is the most intense cold as yet known. (*Faraday.*)

Melting-points.

Metals.	°
Aluminium at red heat	—
Antimony	810
Arsenic	365
Bismuth	476
Bronze	1692
Calcium at red heat	—
Copper	1996
Gold, pure	2282 2590
" standard	2156
Iron, cast	2000 2250 3479*
" 2d melting	2200 2450 3700*
" Wrought	2700 2912 3509*
" malleable forge	—
Lead	608
Lithium	356
Mercury	—39
Platinum	3080
Nickel, highest forge heat	—
Potassium	136
Silver	1250 1873
Sodium	194
Steel	2500
Tin	446
Zinc	680

Alloys.	°
Lead 2, Tin 3, Bismuth 5	212
" 1, " 3, " 5	210
Lead 1, Tin 4, Bismuth 5	240
" 2, " 3	334
" 3, " 2, Bismuth 5	199
" 3, " 1	552
" 2, " 1 (solder)	475
" 1, " 2 (soft solder)	360
" 1, " 1	368
" 1, " 1, Bism. 4, Cadm. 1	155
Tin 1, Bismuth 1	286
" 2, " 1	336
" 8, " 1	392
Zinc 1, Tin 1	399

Fusible Plugs.

Lead 2, Tin 2	372
" 6, " 2	383
" 7, " 2	388
" 8, " 2	410

Various Substances.

Ambergris	145
Beeswax	151
Carbonic acid	—108
Glass	2377
Ice	32
Lard	95
Nitro-Glycerine	45
Phosphorus	112
Pitch	91
Saltpetre	606
Spermaceti	112
Stearine	114
Sulphur	239
Tallow	92
Wax, white	142

* Rankine.

Volume of Water, Antimony, and Cast iron, in the solid state, exceeds that of the liquid, as evidenced by the floating of ice on water, and of cold iron on iron in a liquid state.

Boiling-points. *(Under One Atmosphere.)*

Liquids.	°
Alcohol, s. g. 813	173
Ammonia	140
Benzine	173
Chloroform	146
Ether	100
Linseed oil	597
Mercury	648
Milk	213
Nitric acid, s. g. 1.42	248
" " " 1.5	210
Oil of Turpentine	315
Petroleum, rectified	316
Phosphorus	554
Sea water, average	213.2
Sulphur	570
Sulphuric acid, s. g. 1.848	590
" " " 1.3	240
" ether	100
Turpentine	315
Water	212
" in vacuo	98
Whale oil	630

Saturated Solutions.

Acetate of Soda	255.8
" " Potash	336
Brine	226
Carbonate of Soda	220.3
" " Potash	275
Nitrate of Soda	250
" " Potash	240.6
Salt, common	227.2

Various Substances.

Coal Tar	325
Naphtha	186

Boiling-points of Saturated Vapors under Various Pressures. (*Regnault.*)

Temperature.	Water.	Alcohol.	Ether.	Chloroform.	Temperature.	Water.	Alcohol.	Ether.	Chloroform.
°	Lbs.	Lbs.	Lbs.	Lbs.	°	Lbs.	Lbs.	Lbs.	Lbs.
32	.089	.246	3.53	—	212	14.7	32.6	95.17	45.54
50	.178	.466	5.54	2.52	230	20.8	45.5	120.9	58.42
68	.337	.851	8.6	3.68	240.8	25.37	—	137	Turp'tine.
86	.609	1.52	12.32	5.34	248	29.88	62.05	—	4.97
104	1.06	2.59	17.67	7.04	266	39.27	83.8	—	6.71
122	1.78	4.26	24.53	10.14	276.8	46.87	—	—	—
140	2.88	6.77	33.47	14.27	284	52.56	109.1	—	8.94
158	4.51	10.43	44.67	18.88	302	69.27	140.4	—	11.7
176	6.86	15.72	57.01	26.46	305.6	73.07	147.3	—	—
194	10.16	23.02	75.41	35.03	320	89.97	—	—	13.1

Boiling-points of Water corresponding to Altitudes of Barometer between 62 and 31 Ins.

Barom.	Boiling-point.	Barom.	Boiling-point.	Barom.	Boiling-point.	Barom.	Boiling-point.
	°		°		°		°
26	204.91	27.5	207.55	29	210.19	30.5	212.88
26.5	205.79	28	208.43	29.5	211.07	32	213.76
27	206.67	28.5	209.31	30	212		

Boiling-point of Salt water, 213.2°. Water may be heated in a Digester to 400° without boiling.

Fluids boil in a vacuum with less heat than under pressure of atmosphere. On Mont Blanc water boils at 187°; and in a vacuum water boils at 98° to 100°, according as it is more or less perfect.

Water may be reduced to 5° if confined in tubes of from .003 to .005 inch in diameter: this is in consequence of adhesion of water to surface of tube, interfering with a change in its state. It may also be reduced in its temperature below 32° if it is kept perfectly quiescent.

Effect upon Various Bodies by Heat.

Wedgewood's zero is 1077° (Fahrenheit), and each degree = 130°.

In designation of degrees of temperature, symbol + is omitted when temperature is above 0; but when below it, symbol — must be prefixed.

	Degrees.
Acetification ends....	88
Acetous fermentation begins.. ...	78
Air Furnace.........	3300
Ammonia (liq.) freezes	—46
Blood (hum.), heat of.	98
" freezes.	25
Brandy freezes.......	—7
Charcoal burns.......	800
Cold, greatest artific.	—166
" " natural	—56
Common fire.........	790
Fire brick....	4000 to 5000
Gutta-percha softens..	145
Heat, cherry red	1500
" " (Daniell)	1141
" bright red......	1860
" red, visible by day........	1077
" white..........	2900
Highest natural temperature, Egypt..	117
India-rubber and Gutta-percha vulcanize...........	293
Iron, bright red in the dark.........	752
Iron, red hot in twilight............	884
Iron, wrought, welds..	2700
Ignition of bodies....	750
Combustion of do...	800
Mercury volatilizes...	680
Milk freezes.........	30
Nitric Acid (sp. grav. 1.424) freezes....	—45
Nitrous Oxide freezes	—150
Olive-oil freezes......	36
Petroleum boils......	306
Proof Spirit freezes...	—7
Sea-water freezes....	28
Snow and Salt, equal parts............	0
Spirits Turpen. freezes	14
Steel, faint yellow....	430
" full "	470
" purple.........	530
" blue...........	550
" full blue.......	560
" dark "	600
" polished, blue ..	580
" " straw color	460
Strong Wines freeze..	20
Sulph. Acid (sp. grav. 1.641) freezes....	—45
Sulph. Ether freezes..	—46
Vinegar freezes......	28
Vinous ferment. ..	60 to 77
Zinc boils............	1872
Wood, dried..........	340

Volume of Several Liquids at their Boiling-point.

	Steam.		Steam.		Steam.		Steam.
1 Water......	1700	1 Alcohol......	528	1 Ether........	298	1 Turpentine..	193

Height corresponding to Boiling-point of Pure Water.

Boiling-point at Level of Sea = 212°.

Degree.	Feet.	Degree.	Feet.	Degree.	Feet.	Degree.	Feet.	Degree.	Feet.
211	521	207	2625	203	4761	199	6929	195	9129
210	1044	206	3156	202	5300	198	7476	194	9684
209	1569	205	3689	201	5841	197	8025	193	10241
208	2096	204	4224	200	6384	196	8576	192	10800

Correction for temperature of air same as given at page 428 for Elevation by a Barometer by multiplying by C.

ILLUSTRATION.—If water boils at a temperature of 200° and C = 136°,

Then 6384 × 1.08 = 6894.72 *feet.*

Underground Temperature.

Mean increase of underground temperature per foot, from observations in 36 mines in various and extended localities, is .015 65° = 1° in 64 feet.

Linear Expansion or Dilatation of a Bar or Prism by Heat.

For 1° in a Length of 100 Feet.

METALS, MINERALS, ETC.

	Inch.
Antimony	.007 22
Bismuth	.009 28
Brass	.012 5
" yellow	.012 6
Brick	.001 44
Cast Iron	.007 4
Cement	.009 56
Copper from 0° to 212°	.011 5
" from 32° to 572°	.004 18
Fire brick	.003 3
Glass	.005 74
" flint	.005 41
" tube	.061 2
Gold—Paris standard annealed	.010 1
" " " unannealed	.010 3
Granite	.005 25
Gun Metal—16 copper + 1 tin	.012 7
" " 8 copper + 1 tin	.012 1
Ice	.033 3
Iron, forged	.008 14
" from 0° to 212°	.007 88
Iron, from 32° to 572°	.003 26
Iron wire	.008 23
Lead	.019
Marble	.005 66
Palladium	.006 67
Platinum	.005 71
" from 32° to 572°	.002 04
Sandstone	.013
"	.008 14
Silver	.012 7
Speculum metal	.013
Steel, rod	.007 63
" cast	.007 2
" tempered	.008 26
" not tempered	.007 19
Tin	.014 5
Water	.000 222 9
White Solder—tin 1 + 2 lead	.016 7
Zinc, forged	.020 7
" sheet	.019 6
" 8 + 1 tin	.017 9

Superficial expansion is twice linear, and cubical, three times linear.

To Compute Linear Expansion of a Substance.

Divide 1 by decimal given in above Table, and quotient will give proportion.

ILLUSTRATION 1.—A rod of copper 100 feet in length will expand between temperatures of 32° and 212°. 212 − 32 = 180 × .0115 = 2.07 *ins.*

2.—A cube of cast iron of 1 foot will expand in volume between temperatures of 62° and 212°. 212 − 62 = 150, and 150 × .0074 = 1.11, which ÷ 100 for 1 foot = .0111 *inch*, and 12 + .0111 × 3 = 12.0333 *ins.*

Some solids, as ice, cast iron, etc., have more volume when near to their melting-point than when melted. This is illustrated in floating of solid metal in the liquid.

Expansion of Water.

Water expands from temperature of maximum density (see page 520), 39.1°, to 46°, at which degree it regains its initial volume of 32°, and from thence it expands under one atmosphere to 212°; and its cubical expansion is .0466, that is, its volume is dilated from 1 at 32° to 1.0466 at 212°.

Its expansion increases in a greater ratio than that of temperature.

To Compute Density of Water at a given Temperature.

$$\frac{62.5 \times 2}{\frac{t+461}{500}+\frac{500}{t+461}} = \textit{approximate density, } t \textit{ representing temperature of water.}$$

ILLUSTRATION.—What is density of pure water at 298°?

$$\frac{62.5 \times 2}{\frac{298+461}{500}+\frac{500}{298+461}} = 57.42 \textit{ lbs. or weight of 1 cube foot.}$$

Expansion of Water. (*Dalton.*)

Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.
°		°		°		°	
22	1.0009	52	1.00021	112	1.0088	172	1.02575
32	1	72	1.0018	132	1.01367	192	1.03265
*46	1	92	1.00477	152	1.01934	212	1.0466

* Greatest density 39.1°.

Hence, at 72°, water expands $\frac{1}{.0018}$ = 555.55th part of its original bulk.

Expansion of Liquids from 32° to 212°. *Volume at* 32° = 1.

Liquids.	Volume at 212°.	Liquids.	Volume at 212°.
Alcohol	1.11	Olive oil	1.08
Linseed oil	1.08	Sulphuric acid	1.06
Mercury	1.0154	" ether	1.07
" 212° to 392°	1.0184331	Turpentine	1.07
" 392° to 572°	1.0188679	Water	1.0466
Nitric acid	1.11	Water sat. with salt	1.05

Expansion of Gases from 32° to 212°. *Volume at* 32° = 1.

Gases.	Volume at 212°.	Gases.	Volume at 212°.
Air .... 1 Atmosphere	1.36706	Nitrous oxide ... 1 Atmosphere	1.3179
3.45 "	1.36964	Sulphurous acid, 1 "	1.3903
Hydrogen .... 1 "	1.36613	1.16 "	1.398
3.35 "	1.36016	Carbonic oxide .. 1 "	1.3669
Carbonic acid, 1 "	1.37099	Cyanogen .... 1 "	1.3877
3.32 "	1.38455		

Expansion of Gases is uniform for all temperatures.

Volume of One Pound of Various Gases at 32° under one Atmosphere.

	Cube feet.		Cube feet.		Cube feet.
Air	12.387	Hydrogen	178.83	Oxygen	12.205
Carbonic acid	8.101	Nitrogen	12.753	Mercury	1.776
Ether, vapor	4.777	Olefiant	12.58	Steam	19.913

Expansion of Air. (*Dalton.*)

Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.	Temp.	Expansion.
°		°		°		°		°		°	
32	1	40	1.021	60	1.066	80	1.110	100	1.152	392	1.739
33	1.002	45	1.032	65	1.077	85	1.121	200	1.354	482	1.912
34	1.004	50	1.043	70	1.089	90	1.132	212	1.376	680	2.028
35	1.007	55	1.055	75	1.099	95	1.142	302	1.558	772	2.312

To Compute Volume of a Constant Weight of Air or Permanent Gas for any Pressure.

When volume at a given pressure is known, temperature remaining constant. RULE.—Multiply given volume by given pressure and divide by new pressure.

EXAMPLE.—Pressure at 212° = 18.92 lbs. per sq. inch, and volume 16.91 cube feet; what is volume at pressure of 13.86 lbs.

$16.91 \times 13.86 \div 18.92 = 12.39$ *cube feet.*

Relative Densities of some Vapors.

Water 1. Alcohol 2.59. Ether 4.16. Spirits of Turpentine 8.06. Sulphur 3.59.

Volume, Pressure, and Density of Air at Various Temperatures.

Volume and Atmospheric Pressure at 62° = 1.

Temperature.	Volume of 1 lb. of air at atmospheric pressure of 14.7 lbs.	Pressure of a given weight of air.	Density, or weight of one cube foot of air at 14.7 lbs.	Temperature.	Volume of 1 lb. of air at atmospheric pressure of 14.7 lbs.	Pressure of a given weight of air.	Density, or weight of one cube foot of air at 14.7 lbs.
°	Cube feet.	Lbs. per Sq. Inch.	Lbs.	°	Cube feet.	Lbs. per Sq. Inch.	Lbs.
0	11.583	12.96	.086 331	360	20.63	23.08	.048 476
32	12.387	13.86	.080 728	380	21.131	23.64	.047 323
40	12.586	14.08	.079 439	400	21.634	24.2	.046 223
50	12.84	14.36	.077 884	425	22.262	24.9	.044 92
62	13.141	14.7	.076 097	450	22.89	25.61	.043 686
70	13.342	14.92	.074 95	475	23.518	26.31	.042 52
80	13.593	15.21	.073 565	500	24.146	27.01	.041 414
90	13.845	15.49	.072 23	525	24.775	27.71	.040 364
100	14.096	15.77	.070 942	550	25.403	28.42	.039 365
120	14.592	16.33	.068 5	575	26.031	29.12	.038 415
140	15.1	16.89	.066 221	600	26.659	29.82	.037 51
160	15.603	17.5	.064 088	650	27.915	31.23	.035 822
180	16.106	18.02	.062 09	700	29.171	32.635	.034 28
200	16.606	18.58	.060 21	750	30.428	34.04	.032 865
210	16.86	18.86	.059 313	800	31.684	35.445	.031 561
212	16.91	18.92	.059 135	850	32.941	36.85	.030 358
220	17.111	19.14	.058 442	900	34.197	38.255	.029 242
240	17.612	19.7	.056 774	950	35.454	39.66	.028 206
260	18.116	20.27	.055 2	1000	36.811	41.065	.027 241
280	18.621	20.83	.053 71	1500	49.375	55.115	.020 295
300	19.121	21.39	.052 297	2000	61.94	69.165	.016 172
320	19.624	21.95	.050 959	2500	74.565	83.215	.013 441
340	20.126	22.51	.049 686	3000	87.13	97.265	.011 499

To Compute Volume of a Constant Weight of Air or other Permanent Gas for any other Pressure and Temperature.

When volume is known at a given pressure and temperature. RULE.—Multiply given volume by given pressure, and by new absolute temperature, and divide by new pressure, and by given absolute temperature.

EXAMPLE.—Given volume 16.91 cube feet, pressure 13.86 lbs., and temperature 32°; what is volume at this temperature?

Temperature for volume 16.91 = 212°.

$$16.91 \times 13.86 \times \overline{32 + 461} \div 13.86 \times \overline{212 + 461} = 12.39 \textit{ cube feet.}$$

To Compute Pressure of a Constant Weight of Air or other Permanent Gas for any other Volume and Temperature.

When pressure is known for a given volume and temperature. RULE.—Multiply given pressure by new absolute temperature, and divide by given absolute temperature.

NOTE.—Absolute temperature is found by adding 461° to temperature.

EXAMPLE.—Given pressure 13.86 lbs., and temperature at this volume 32°; what is pressure at temperature of 212°?

$$13.86 \times \overline{212 + 461} \div \overline{32 + 461} = 18.92 \textit{ lbs.}$$

To Compute Volume of a Constant Weight of Air or other Permanent Gas at any Temperature.

When volume at a given temperature is known, pressure being constant. RULE.—Multiply given volume by new absolute temperature, and divide by given absolute temperature.

Absolute zero-point by different thermometrical scales is: Fahrenheit —461.2°; Reaumur —219.2°; Centigrade —274°.

EXAMPLE.—Volume of 1 lb. air at 32° = 12.387 cube feet; what is its volume at 212°?

$$12.387 \times \overline{212 + 461} \div \overline{32 + 461} = 16.91 \textit{ cube feet.}$$

To Compute Increased Volume of a Constant Weight of Air.

When initial volume at 62° = 1 under 1 atmosphere. RULE.—To given temperature add 461, and divide sum by 523 (32 + 461).

EXAMPLE.—Assume elements of preceding case.

$$212^\circ + 461 \div 523 = 1.287 \textit{ comparative volume to } 1.$$

To Compute Pressure of a Constant Weight of Air or other Gas at 62°, and at 14.7 lbs. Pressure per Sq. In., with Constant Volume, for a given Temperature.

RULE.—Add 461 to given temperature, and divide sum by 35.58.

EXAMPLE.—Temperature is 212°; what is pressure?

$$212 + 461 \div 35.58 = 18.92 \textit{ lbs.}$$

To Compute Volume, Pressure, Temperature, and Density of Air.

$\frac{t+461}{p\;2.71} = V$; $\frac{t+461}{39.8} = V$; $\frac{t+461}{V\;2.71} = p$; $V\;2.7074\;p - 461 = t$; and $2.71\;\frac{p}{t+461} = D$. *t representing temperature, p pressure in lbs. per sq. inch, V volume in cube feet, and D weight of 1 cube foot at 14.7 lbs. per sq. inch.*

Product of volume and pressure of a constant weight of air, or any other permanent gas, is equal to product of absolute temperature and a coefficient, determined for each gas by its density.

$$\text{Or, } V\,p = C\,\overline{t + 461}.$$

Coefficients, as determined by volumes and consequent densities.*

Air	2.71	Hydrogen	.1875	Oxygen	2.99
Carbonic acid	4.14	Nitrogen	2.63	Mercury	18.88
Ether, vapor	7.02	Olefiant	2.67	Steam	1.68

* See D. K. Clark, London, 1877, page 349.

Decrease of Temperature by Altitudes.

	In clear sky.	*With cloudy sky.*
From 1 to 1 000 feet	1° in 139 feet	1° in 222 feet.
1 " 10 000 "	1° " 288 "	1° " 331 "
1 " 20 000 "	1° " 365 "	1° " 468 "

To Compute Temperature to which a Substance of a given Length or Dimension must be Submitted or Reduced, to give it a Greater or Less Length or Volume by Expansion or Contraction.

Lineal.—When Length is to be increased. $\frac{L-l}{C} + t = T$. L *and* l *representing lengths of increased and primitive substance in like denominations,* T *and* t *temperatures of* L *and* l, *and* C *expansion of substance for each degree of heat.*

ILLUSTRATION.—A copper rod at 32° is 100 feet in length; to what temperature must it be subjected to increase its length 1.1633 ins.?

Expansion for a length of 100 feet of copper for $1° = .0115$.

$$\frac{100 \times 12 + 1.1633 - \overline{100 \times 12}}{.0115} + 32 = \frac{1.1633}{.0115} + 32 = 133.16°.$$

When Length is to be reduced. $\frac{L - l}{C} - T = t.$

ILLUSTRATION.—Take elements of preceding case.

$$\frac{1201.1633 - 1200}{.0115} - 133.16° = 101.16 - 133.16 = 32°.$$

To Reduce Degrees of Fahrenheit to Reaumur and Centigrade, and Contrariwise.

Fahrenheit to Reaumur. *If above zero.*—Multiply difference between number of degrees and 32 by 4, and divide product by 9.

Thus, $212° - 32° = 180°$, and $180° \times 4 \div 9 = 80°$.

If below zero.—Add 32 to number of degrees; multiply sum by 4, and divide product by 9.

Thus, $-40° + 32° = 72°$, and $72° \times 4 \div 9 = -32°$.

Reaumur to Fahrenheit. *If above freezing-point.*—Multiply number of degrees by 9, divide by 4, and add 32 to quotient.

Thus, $80° \times 9 \div 4 = 180°$, and $180° + 32 = 212°$.

If below freezing-point.—Multiply number of degrees by 9, divide by 4, and subtract 32 from product.

Thus, $-32° \times 9 \div 4 = 72°$, and $72° - 32 = -40°$.

Fahrenheit to Centigrade. *If above zero.*—Multiply difference between number of degrees and 32 by 5, and divide product by 9.

Thus, $212° - 32° \times 5 \div 9 = 180° \times 5 \div 9 = 100°$.

If below zero.—Add 32 to number of degrees, multiply sum by 5, and divide product by 9.

Thus, $-40° + 32° \times 5 \div 9 = 72° \times 5 \div 9 = -40°$.

Centigrade to Fahrenheit. *If above freezing-point.*—Multiply number of degrees by 9, divide product by 5, and add 32 to quotient.

Thus, $100° \times 9 \div 5 = 180°$, and $180° + 32 = 212°$.

If below freezing-point.—Multiply number of degrees by 9, divide product by 5, and take difference between 32 and quotient.

Thus, $-10° \times 9 \div 5 = 18°$, and $18° \backsim 32 = 14°$.

Reaumur to Centigrade.—Divide by 4, and add product.

Thus, $80° \div 4 = 20°$, and $20° + 80° = 100°$.

Centigrade to Reaumur.—Divide by 5, and subtract product.

Thus, $100° \div 5 = 20°$, and $100° - 20 = 80°$.

Corresponding Degrees upon the Three Scales.

Fahr.	Cent.	Reaum.	Fahr.	Cent.	Reaum.	Fahr.	Cent.	Reaum.
212	100	80	32	0	0	—40	—40	—32

To Compute Expansion of Fluids in Volume.

RULE.—Proceed by preceding formulas for computing length of a substance. Substitute V and v for volume, instead of L and l, the lengths.

ILLUSTRATION.—A closed vessel contains 6 cube feet of water at a temperature of 40°; to what height will a column of it rise in a pipe 1.152 ins. in area, when it is exposed to a temperature of 130°?

1.152 ins. $=$ .008 *sq. foot.* C for water $=$ .000 222 9.

$$6\left(1+\overline{.000\,222\,9\,(130-40)}\right)=6.125\,95, \text{ and } \frac{6.125\,95-6}{.008}=15.744 \textit{ lineal feet.}$$

Temperature by Agitation.

Results of Experiments with Water enclosed in a Vessel and violently Agitated.

Temperature of Air, 60.5°; of Water, 59.5°.

Duration of Agitation.	Increase of Temperature.	Duration of Agitation.	Increase of Temperature.	Duration of Agitation.	Increase of Temperature.
Hour.	°	Hours.	°	Hours.	°
.5	10	2	19.5	5	39.5
1	14.5	3	29.5	6	42.5

VENTILATION.

Buildings, Apartments, etc.

In Ventilation of Apartments.—From 3.5 to 5 cube feet of air are required per minute in winter, and 5 to 10 feet in summer for each occupant. In *Hospitals*, this rate must be materially increased.

Ventilation is attained by both natural draught and artificial means. In first case the ascensional force is measured by difference in weight of two columns of air of same height, the height being determined by total difference of level between entrance for warm air and its escape into the atmosphere. The difference of weight is ascertained from difference of temperatures of ascending warm air and the external atmosphere, as by Table, page 521, or by formula, page 522.

Volumes of Air Discharged through a Ventilator One Foot Square of Opening, at Various Heights and Temperatures.

Height of Ventilator from Base-line.	Excess of Temperature of Apartment above that of External Air. 5°	10°	15°	20°	25°	30°	Height of Ventilator from Base-line.	Excess of Temperature of Apartment above that of External Air. 5°	10°	15°	20°	25°	30°
Feet.	C. ft.	C. ft.	C. ft.	C. ft.	C. ft.	C. ft.	Feet.	C. ft.	C. ft.	C. ft.	C. ft.	C. ft.	C. ft.
10	116	164	200	235	260	284	35	218	306	376	436	486	531
15	142	202	245	284	318	348	40	235	329	403	465	518	570
20	164	232	285	330	368	404	45	248	348	427	493	551	605
25	184	260	318	368	410	450	50	260	367	450	518	579	635
30	201	284	347	403	450	493	55	270	385	472	541	605	663

Velocity of draft having been ascertained for any particular case, together with volume of air to be supplied per minute, sectional area of both air passages may be computed from these data.

Heating by Hot Water.

One sq. foot of plate or pipe surface at 200° will heat from 40 to 100 cube feet of enclosed space to 70° where extreme depression of temperature is −10°.

The range from 40 to 100 is to meet conditions of exposed or corner buildings, of buildings less exposed, as intermediate ones of a cluster or block, and of rooms intermediate between the front and rear.

When the air is in constant course of change, as required for ventilation or occupation of space, these proportions are to be very materially increased as per following rules.

In determining length of pipe for any given space it is proper to include in the computation the character and occupancy of the space. Thus, a church, during hours of service, or a dwelling-room, will require less service of plate or length of pipe than a hallway or a public building.

Reduction of Heat by Surfaces of Glass or Metal.—In addition to the volume of air to be heated per minute for each occupant, 1.25 cube feet for each sq. foot of glass or metal the space is enclosed with must be added. The communicating power of the glass and metal being directly proportionate to difference of external and internal temperature of the air. Thus, 80 feet of glass will reduce 100 feet of air per minute.

When Pipes are laid in Trenches in the Earth.—The loss of heat is estimated by Mr. Hood at from 5 to 7 per cent.

Circulation of Water in Pipes.—In consequence of the complex forms of heating-pipes and the roughness of their internal surface, it is impracticable to apply a rule to determine the velocity of circulation, as consequent upon difference of weights of ascending and descending columns of the water.

For a difference of temperature in the two columns of 30° (190° — 160°) and a height of 20 feet, the velocity due to the height would be 3.74 feet. In practice not .3, and in some cases but .1, would be attained.

In Churches and Large Public Rooms, with ordinary area of doors and windows and moderate ventilation, a large amount of heat is generated by the respiration of the persons assembled therein.

In these cases it is not necessary to heat the air above 55°, and a rule that will meet the ordinary ranges of temperature from 10° is to divide volume in cube feet by 200, and quotient will give area of plate in sq. feet or length of 4-inch pipe in lineal feet.

Volume of Air required per Hour for each Occupant in an Enclosed Space.
(*General Morin.*)

	Cube Feet.		Cube Feet.		Cube Feet.
Hospitals....	2100 to 3700	Lecture-rooms	1000 to 2100	Prisons.......	1800
Workshops ..	2100 " 3500	Theatres......	1400 " 1800	Schools.......	424 to 1060

To Compute Length of Iron Pipe required to Heat Air in an Enclosed Space.

By Hot Water.

Rule.—Multiply volume of air to be heated per minute in cube feet by difference of temperatures in space and external air, divide product by difference of temperatures of surface of pipe and space, multiply result by following coefficients, and product will give length of pipe in feet.

For diameter of 4 ins. multiply by .5 to .55, for 3 ins. by .7 to .75, and for 2 ins. by 1 to 1.1.

A pipe 4 ins. in diameter, .375 inch thick, and 1 foot in length has an area of internal surface of 1.05 *sq. feet.*

Example.—Volume of a room of a protected dwelling is 4000 cube feet; what length of 4 ins. pipe, at 200°, is necessary to maintain a temperature of 70°, when external air is at 0°?

$$\frac{4000 \times \overline{70 - 0}}{200 - 70} \times .4 = 862 \textit{ feet.}$$

In computing length of pipe or surface of plate it is to be borne in mind that the coefficients here given and computation in following table are based upon a ventilation or change of air ordinarily of 3.5 to 5 cube feet per person. and from 5 to 10 cube feet in summer per minute. Hence, when the ventilation is restricted the coefficient may be correspondingly increased.

Lengths of Four-Inch Pipe to Heat 1000 Cube Feet of Air per Minute. (*Chas. Hood.*)

Temperature of Pipe 200°.

Temperature of External Air.	Temperature of Building. 45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
°	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
10	126	150	174	200	229	259	292	328	367	409
16	105	127	151	176	204	223	265	300	337	378
20	91	112	135	160	187	216	247	281	318	358
26	69	90	112	136	162	190	220	253	288	327
30	54	75	97	120	145	173	202	234	269	307
36	32	52	73	96	120	147	175	206	239	276
40	18	37	58	80	104	129	157	187	220	255
50	—	—	19	40	62	86	112	140	171	204

Proper Temperatures of Enclosed Spaces.

Spaces.	Temperature required.	Spaces.	Temperature required.
	°		°
Work-rooms, manufactories, etc.	55	Dwelling-rooms	70
Churches and like spaces	55	Graperies	70
Greenhouses	55	Hot-houses	80
Schools, lecture-rooms	58	Drying-rooms, when filled	80
Halls, shops, waiting-rooms, etc.	60	" " for curing paper	70
Dwelling-rooms	65	" "	120

Boiler.

Boiler for steam-heating should be capable of evaporating as much water as the pipes or surfaces will condense in equal times. Mr. Hood recommends that 6 sq. feet of direct heating-surface of boiler should be provided to evaporate a cube foot per hour. Adopt mean weight of steam of 5 lbs. above pressure of atmosphere, or 20 lbs. absolute pressure, condensed per sq. foot of pipe per degree of difference of temperature per hour, viz., .002 35 lb. (as given by D. K. Clark), the quantity of pipe or plate surface that would form a cube foot of condensed water per hour, weight of like volume of water 62.4 lbs., would be, per 1° difference of temperature,

$62.4 \div .00235 = 26550$ *sq. feet*, and for differences of 168°, 158°, 148°, and 108°, required surface would be respectively ($26550 \div 168 = 158$) 158, 168, 179, and 246 *sq. feet*.

Hence, assuming, as previously stated, that 4 sq. feet of direct and effective heating boiler-surface, or its equivalent flue or tube surface, will evaporate 1 cube foot of water per hour, 158 sq. feet of steam-pipe or plate will require 4 sq. feet of direct surface, etc., for a temperature of 60°, and correspondingly for other temperatures.

Boiler-power.—One sq. foot of boiler-surface exposed to direct action of fire, or 3 sq. feet of flue-surface, will suffice, with good coal, for heating 50 sq. feet of 4-inch, 66 of 3-inch, and 100 of 2-inch pipe. Mr. Hood assigns the proportion at 40 feet of 4-inch pipe for all purposes. Usual rate of combustion of coal is 10 or 11 lbs. per sq. foot of grate-surface, and at this rate, 20 sq. ins. of grate suffice for heating 40 feet of 4-inch pipe.

Four sq. feet of direct heating boiler-surface, or equivalent flue or tube surface, exposed to direct action of a good fire, are capable of evaporating 1 cube foot of water per hour.

According to M. Grouvelle, 1 sq. meter of pipe-surface (10.76 sq. feet), heated to 60° an ordinary room alike to a library or office, of from 90 to 100 cube meters (3178 to 3531 cube feet).

If a workshop to be heated to a high temperature, 1 sq. meter (10.76 sq. feet) of surface is assigned to 70 cube meters (2472 cube feet) = 4.35 sq. feet or 5.11 lineal feet of 4-inch pipe per 1000 cube feet.

For heating workshops, having a transverse section of 260 sq. feet, with a window-surface of one sixth total surface, it is customary in France to assign 1.33 sq. feet of iron pipe surface per lineal foot of shop = 5.2 sq. feet per 1000 cube feet.

Illustrations of extensive Heating by Steam. (*R. Briggs, M. I. C. E.*)

1. Total number of rooms, including halls and vaults............	286
" Area of floor surface	137 370 sq. feet.
" Volume of rooms..	1 923 500 cube feet.
" Number of occupants	650
Maximum average of occupants at any time.................	1300
Volume per occupant, excluding vaults	1443 cube feet.

Boilers.—8 with 173 sq. feet of grate surface and 8000 sq. feet of heating surface. Furnishing steam in addition to the above, to operate the elevators and electric dynamos, elevating water, and supplying steam to heat a distant building, requiring one third of their capacity.

By Steam.

To Compute Length of Iron Pipe required to Heat Air in an Enclosed Space, with Steam at 5 lbs. per Sq. Inch above Pressure of Atmosphere.

Rule.—Multiply volume of air in cube feet to be heated per minute, by difference of temperature in space and external air, divide product by coefficients in preceding table, and quotient will give length of 4-inch pipe in lineal feet, or area of plate-surface in sq. feet.

Temperature of steam at 5 lbs. + pressure = 228°. Hence, if temperature of space required is 60°, 70°, 80°, or 120°, the differences will be 168°, 158°, 148°, and 108°, which for a coefficient of .5, as given in rule for hot water, would be 336, 316, 296, and 216, for a pipe 4 ins. in diameter, and for

	60°	70°	80°	120°
3-inch pipe........	252	237	222	162
2 " "	168	158	148	108
1 " "	84	79	74	54

Illustration.—Volume of combined spaces of a factory is 50 000 cube feet; what surface of wrought-iron plate at 200° is necessary to maintain a temperature of 50° when external air is at 0°?

$$\frac{50\,000 \times \overline{50-0}}{200-50} \times .4 = 6666 \textit{ square feet.}$$

Coal Consumed per Hour to Heat 100 Feet of Pipe.

(*Chas. Hood.*)

Diam. of Pipe.	Difference of Temperature of Pipe and Air in Space, in Degrees.														
	150	145	140	135	130	125	120	115	110	105	100	95	90	85	80
Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1	1.1	1.1	1.1	1	1	.9	.9	.9	.8	.8	.7	.7	.7	.6	.6
2	2.3	2.2	2.2	2.1	2	1.9	1.8	1.8	1.7	1.6	1.5	1.4	1.4	1.3	1.2
3	3.5	3.4	3.3	3.1	3	2.9	2.8	2.7	2.5	2.4	2.3	2.2	2.1	2	1.8
4	4.7	4.5	4.4	4.2	4.1	3.9	3.7	3.6	3.4	3.2	3.1	2.9	2.8	2.6	2.5

To warm a factory, according to M. Claudel, 43 feet in width by 10.5 high, a single line of hot-water pipe 6.25 ins. in diameter per foot of length of room, appears to be sufficient, temperature in pipe being from 170° to 180°. Also, water being at 180°, and air at 60°, making a difference of 120°, it is convenient to estimate from 1.5 to 1.75 sq. feet of water-heated surface as equivalent to one sq. foot of steam-heated surface, and to allow from 8 to 9 sq. feet of hot-water pipe-surface per 1000 cube feet of room.

M. Grouvelle states that 4 sq. feet of cast-iron pipe-surface, whether heated by steam or by water at 176° to 194°, will warm 1000 cube feet of workshop, maintaining a temperature of 60°. Steam is condensed at rate of .328 lb. per sq. foot per hour.

2. (*R. L. Greene.*) Length of fronts of buildings................ 2000 lineal feet.
Total volume of rooms.................... 2 574 084 cube feet.
Radiating surfaces, direct, 10 804.......... } 34 100 sq. feet.
indirect, 23 296.......... }
Boilers.—Grate-surface.......................... 180 "
Heating surface................... 5 863 "

Volume of Air Heated by Radiators; Consumption of Coal; Areas of Grate and Heating-surface of Boiler.

(*Rob't Briggs.*)

Per 100 Sq. Feet of Warming-surface of Radiator.

Pressure of steam per sq. inch + atmosphere in *lbs*.	—	3	10	30	60
Heat from radiators per minute in *units*	456	486	537	642	741
Volume of air heated 1° per minute in *cube feet*	25 110	26 772	29 570	35 352	40 803
Efficiency of radiators in *ratio*	1	1.066	1.178	1.408	1.625
Coal consumed per hour in *lbs*.	3.64	3.24	3.58	4.28	4.94
Area of grate consuming 8 lbs. coal per hour in *sq. feet*	.38	.405	.448	—	—
do. 12 lbs.	—	—	.298	.357	.412
Heating surface of boiler; coal consumed per hour × 2.8 in *sq. feet*	8.512	9.072	10.02	11.98	13.83
8 lbs. × 2.8	22.4	22.4	22.4	—	—
12 lbs. × 2.8	—	—	33.6	33.6	33.6

By Hot-Air Furnaces or Stoves.

A square foot of heating surface in a hot-air furnace or stove is held to be equivalent to 7 sq. feet of hot water pipe.

M. Peclet deduced that when the flue-pipe of a stove radiated its heat directly to air of a space, the heat radiated per sq. foot per hour, for 1° difference of temperature, were, for: Cast iron, 3.65 units; Wrought iron, 1.45 units, and Terra cotta .4 inch thick, 1.42 units.

In ordinary practice, 1 sq. foot of cast iron is assigned to 328 cube feet of space.

Open Fires.

According to M. Claudel, the quantity of heat radiated into an apartment from an ordinary fireplace is .25 of total heat radiated by combustible.

For wood the heat utilized is but from 6 to 7 per cent., and for coal 13 per cent.

In combustion of wood, chimney of an ordinary open fireplace draws from 1000 to 1600 cube feet of air per pound of fuel, and a sectional area of from 50 to 60 sq. ins. is sufficient for an ordinary apartment.

Proportions of fuel required to heat an apartment are: For ordinary fireplaces, 100; metal stoves, 63; and open fires, 13 to 16.

Furnaces.

By D. K. Clark, from investigations of Mr. J. Lothian Bell.

Cupola.—M. Peclet estimates that in melting pig-iron by combustion of 30 per cent. of its weight of coke, 14 per cent. only of the heat of combustion is utilized.

Metallurgical.—According to Dr. Siemens, 1 ton of coal is consumed in heating 1.66 tons of wrought iron to welding-point of 2700°, and a ton of coal is capable of heating up 39 tons of iron; from which it appears that only 4.5 per cent. of whole heat is appropriated by the iron. Similarly, he estimates 1.5 per cent. of whole heat generated is utilized in melting pot steel in ordinary furnaces, whilst, in his regenerative furnace, 1 ton of steel is melted by combustion of 1344 lbs. of small coal, showing that 6 per cent. of the heat is utilized.

Blast-furnace.—Mr. Bell has formed detailed estimates of appropriation of the heat of Durham coke in a blast-furnace; from which is deduced following abstract:

Durham coke consists of 92.5 per cent. of carbon, 2.5 of water, and 5 of ash and sulphur. To produce 1 ton of pig-iron, there are required 1232 lbs. of limestone, and 5388 lbs. of calcined iron-stone; the iron-stone consists of 2083 lbs. of iron, 1008 lbs. of oxygen, and 2509 lbs. of earths. There is formed 813 lbs. of slag, of which 123 lbs. is formed with ash of the coke, and 690 lbs. with the limestone. There are 2397 lbs. of earths from the iron-stone, less 93 lbs. of bases taken up by the pig-iron and dissipated in fume, say 2314 lbs. Total of slag and earths, 3127 lbs.

Mr. Bell assumes that 30.4 per cent. of the carbon of the fuel, which escapes in a gaseous form, is carbonic acid; and that, therefore, only 51.27 per cent. of heating power of fuel is developed, and remaining 48.73 per cent. leaves tunnel-head undeveloped. He adopts, as a unit of heat, the heat required to raise the temperature of 112 lbs. of water 33.8°.

HYDRAULICS.

Descending Fluids are actuated by same laws as *Falling Bodies.*

A Fluid will fall through 1 foot in .25 of a second, 4 feet in .5 of a second, and through 9 feet in .75 of a second, and so on.

Velocity of a fluid, flowing through an aperture in side of a vessel, reservoir, or bulkhead, is same that a heavy body would acquire by falling freely from a height equal to that between surface of fluid and middle of aperture.

Velocity of a fluid flowing out of an aperture is as square root of height of head of fluid. *Theoretical* velocity, therefore, in feet per second, is as square root of product of space fallen through in feet and $64.333 = \sqrt{2gh}$; consequently, for one foot it is $\sqrt{64.333} = 8.02$ *feet*. *Mean* velocity, however, of a number of experiments gives 5.4 feet, or .673.

In short ajutages accurately rounded, and of form of contracted vein, (*vena contracta*), coefficient of discharge = .974 of theoretical.

Fluids subside to a natural level, or curve similar to Earth's convexity; apparent level, or level taken by any instrument for that purpose, is only a tangent to Earth's circumference; hence, in leveling for canals, etc., difference caused by Earth's curvature must be deducted from apparent level, to obtain true level.

Deductions from Experiments on Discharge of Fluids from Reservoirs.

1. That volumes of a fluid discharged in equal times by same apertures from same head are nearly as areas of apertures.

2. That volumes of a fluid discharged in equal times by same apertures, under different heads, are nearly as square roots of corresponding heights of fluid above surface of apertures.

3. That, on account of friction, small-lipped or thin orifices discharge proportionally more fluid than those which are larger and of similar figure, under same height of fluid.

4. That in consequence of a slight augmentation which contraction of the fluid vein undergoes, in proportion as the height of a fluid increases, the flow is a little diminished.

5. That if a cylindrical horizontal tube is of greater length than its diameter, discharge of a fluid is much increased, and may be increased with advantage, up to a length of tube of four times diameter of aperture.

6. That discharge of a fluid by a vertical pipe is augmented, on the principle of gravitation of falling bodies; consequently, greater the length of a pipe, greater the discharge of the fluid.

7. That discharge of a fluid is inversely as square root of its density.

8. That velocity of a fluid line passing from a reservoir at any point is equal to ordinate of a parabola, of which twice the action of gravity ($2\,g$) is parameter, the distance of this point below surface of reservoir being the abscissa.* Or, velocity of a jet being ascertained, its curve is a parabola, parameter of which $= 4\,h$, due to velocity of projection.†

9. Volume of water discharged through an aperture from a prismatic vessel which empties itself, is only half of what it would have been during the time of emptying, if flow had taken place constantly under same head and corresponding velocity as at commencement of discharge: consequently, the time in which such a vessel empties itself is double the time in which all its fluid would have run out if the head had remained uniform.

10. Mean velocity of a fluid flowing from a rectangular slit in side of a reservoir is two thirds of that due to velocity at sill or lowest point, or it is that due to a point four ninths of whole height from surface of reservoir.

11. When a fluid issues through a short tube, the vein is less contracted than in preceding case, in proportion of 16 to 13; and if it issues through an aperture which is alike to frustum of a cone, base of which is the aperture, the height of frustum half diameter of aperture, and area of small end to area of large end as 10 to 16, there will be no contraction of the vein. Hence this form of aperture will give greatest attainable discharge of a fluid.

12. Velocity of efflux increases as square root of pressure on surface of a fluid.

13. In efflux under water, difference of levels between the surfaces must be taken as head of the flowing water.

14. To attain greatest mechanical effect, or *vis viva*, of water flowing through an opening, it should flow through a circular aperture in a thin plate, as it has less frictional surface.

From Conduits or Pipes. (*Bossut.*)

1. Less diameter of pipe, the less is proportional discharge of fluid.

2. Discharges made in equal times by horizontal pipes of different lengths, but of same diameter, and under same altitude of fluid, are to one another in inverse ratio of sq. roots of their lengths.

3. In order to have a perceptible and continuous discharge of fluid, the altitude of it in a reservoir, above plane of conduit pipe, must not be less than .082 ins. for every 100 feet of length of pipe.

4. In construction of hydraulic machines, it is not enough that elbows and contractions be avoided, but also any intermediate enlargements, the injurious effects of which are proportionate, as in following Table, for like volumes of fluid, under like heads in pipes, having a different number of enlarged parts.

No. of Parts.	Velocity.	No. of Parts.	Velocity.	No. of Parts.	Velocity.	No. of Parts.	Velocity.
0	1	1	.741	3	.569	5	.454

* See D'Aubuisson, page 66.

† Humber, page 57.

Friction.

Flowing of liquids through pipes or in natural channels is materially affected by friction.

If equal volumes of water were to be discharged through pipes of equal diameters and lengths, but of following figures:

Fig. 1.

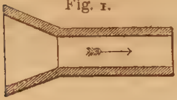


Fig. 2.

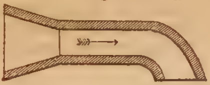
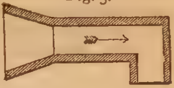


Fig. 3.



	Figs. 1.	2.	3.
The *times* would be as	1,	1.11, and	1.55.
And *velocities* as	1,	.72, and	.64.

Discharges from Compound or Divided Reservoirs.

Velocity in each may be considered as generated by difference of heights in contiguous reservoirs; consequently, square root of difference will represent velocities, which, if there are several apertures, must be inversely as their respective areas.

NOTE.—When water flows into a vacuum, 32.166 feet must be added to height of it; and when into a rarefied space only, height due to difference of external and internal pressure must be added.

VELOCITY OF WATER OR OF FLUIDS.

Coefficients of Discharge.

Coefficient of Discharge or *Efflux* is product of coefficients of *Contraction* and *Velocity*.

It is ascertained in practice that water issuing from a *Circular Aperture* in a thin plate contracts its section at a distance of .5 its diameter from aperture to very nearly .8 diameter of aperture, so as to reduce its area from 1 to about .61.* Velocity at this point is also ascertained to be about .974 times theoretical velocity due to a body falling from a height equal to head of water. Mean velocity *in aperture* is therefore .974, which, $\times$ $.61 = .594$, theoretical discharge; and in this case .594 becomes *coefficient of discharge*, which, if expressed generally by C, will give for discharge itself

$a\sqrt{2\,g\,h} \times C = V$. *a representing area of aperture, and* V *volume discharged per second.* Or, $4.95\,a\sqrt{h} = V$. Or, $3.91\,d^2\sqrt{h} = V$. *d representing diameter in feet.*

Hence, for cube feet per second, $4.95\,a\sqrt{h}$, or $3.91\,d^2\sqrt{h}$.

ILLUSTRATION.—Assume head of water 10 feet, diameter of opening 1.127 feet, area 1 sq. foot, and $C = .62$.

Then $1\sqrt{2\,g\,10} \times .62 = 15.72$ *cube feet.* $4.97 \times 1 \times \sqrt{10} = 15.72$ *cube feet*, and $3.91 \times 1.127^2 \times \sqrt{10} = 15.7$ *cube feet.*

For square aperture it is .615, and for rectangular .621.

Volume of water or a fluid discharged in a given time from an aperture of a given area depends on head, form of aperture, and nature of approaches.

$\overline{64.333\ h} = v^2$, and $\dfrac{v^2}{64.333} = h$. *h representing height to centre of opening in feet.*

NOTE. — *Head*, or *height*, *h*, may be measured from surface of water to centre of aperture without practical error, for it has been proved by Mr. Neville that for circular apertures, having their centre at the depth of their radius below the surface, and therefore circumference touching the surface, the error cannot exceed 4 per cent. in excess of the true theoretical discharge, and that for depths exceeding three

* Bayer, .61. Observed discharges of water coincide nearer to unit of Bayer than that of all others.

times the diameter, the error is practically immaterial. For rectangular apertures it is also shown that, when their upper side is at surface of the water, as in notches, the extreme error cannot exceed 4.17 per cent. in excess; and when the upper is three times depth of aperture below the surface, the excess is inappreciable.

For notches, weirs, slits, etc., however, it is usual to take full depth for head, when .666 only of above equation must be taken to ascertain the discharge.

Experiments show that *coefficient* for similar apertures in thin plates, for small apertures and low velocities, is greater than for large apertures and high velocities, and that for elongated and small apertures it is greater than for apertures which have a regular form, and which approximate to the circle.

When Discharge of a Fluid is under the Surface of another body of a like Fluid.—The difference of levels between the two surfaces must be taken as the head of the fluid.

$$\text{Or, } \sqrt{2\,g\,(h-h')}=v.$$

When Outer Side of opening of a discharging Vessel is pressed by a Force.—The difference of height of head of fluid and quotient of pressures on two sides of vessel, divided by density of fluid, must be taken as heads of fluid.

$$\text{Or, } \sqrt{2\,g\left(h-\frac{p-p'\times 144}{S}\right)}=v. \quad S \text{ representing density of fluid.}$$

Illustration.—Assume head of water in open reservoir is 12 feet above water-line in boiler, and pressures of atmosphere and steam are 14.7 and 19.7 lbs.

$$\text{Then } \sqrt{2\,g\left(12-\frac{19.7-14.7\times 144}{62.5}\right)}=\sqrt{64.333\times\left(12-\frac{5\times 144}{62.5}\right)}=5.56 \textit{ feet.}$$

When Water flows into a rarefied Space, as into Condenser of a Steam-engine, and is either pressed upon or open to Atmosphere.—The height due to mean pressure of atmosphere within condenser, added to height of water above internal surface of it, must be taken as head of the water.

$$\text{Or, } \sqrt{2\,g\,(h+h')}=v.$$

Illustration.—Assume head of water external to condenser of a steam-engine to be 3 feet, vacuum gauge to indicate a column of mercury of 26.467 ins. (= 13 lbs.), and a column of water of 13 lbs. = 29.9 *feet.*

$$\text{Then } \sqrt{2\,g\,(3+29.9)}=\sqrt{64.333\times 32.9}=\sqrt{2116.57}=46 \textit{ feet.}$$

Relative Velocity of Discharge of Water through different Apertures and under like Heads.

Velocity that would result from direct, unretarded action of the column of water which produces it, being a constant, or	1
Through a cylindrical aperture in a thin plate	.625
A tube from 2 to 3 diameters in length, projecting outward	.8125
A tube of the same length, projecting inward	.6812
A conical tube of form of contracted vein	.974
Wide opening, bottom of which is on a level with that of reservoir; sluice with walls in a line with orifice; or bridge with pointed piers	.96
Narrow opening, bottom of which is on a level with that of reservoir; abrupt projections and square piers of bridges	.86
Sluice without side walls	.63

Discharge or Efflux of Water for various Openings and Apertures.

Rectangular Weir.

Weirs are designated *Perfect* when their sill is above surface of natural stream, and *Imperfect*, *Submerged*, or *Drowned* when it is below that surface.

Height measured from Surface of Water to Sill. (*Jas. B. Francis.*)

Mean Head.	Length of Opening.	Mean Discharge per Second.	Mean Coefficient.
.62 to 1.55 feet.	10 feet.	32.9 cube feet.	.623

Principal causes for variation in coefficients derived from most experiments giving discharge of water over weirs arises from,

1. Depth being taken from only one part of surface, for it has been proved that heads *on, at, and above a weir* should be taken in order to determine true discharge.

2. Nature of the approaches, including ratio of the water-way in channel above, to water-way on weir.

When a weir extends from side to side of a channel, the contraction is less than when it forms a notch, or Poncelet weir, and coefficient sometimes rises as high as .667.

When weir or notch extends only one fourth, or a less portion of width, coefficient has been found to vary from .584 to .6.

When wing-boards are added at an angle of about 64°, coefficient is greater than even when head is less.

Computation of Volume of Discharge.

Mean velocity of a fluid issuing through a rectangular opening in side of a vessel is two thirds of that due to velocity at sill or lower edge of opening, or it is that due to a point four ninths of whole height from surface of fluid.

Height measured from Surface of Head of Water to Sill of Opening.

RULE.—Multiply square root of product of 64.333 and height or whole depth of the fluid in feet, by area in feet, and by *coefficient* for opening, and two thirds of product will give volume in cube feet per second.

Or, $\frac{2}{3}\, b\, h \sqrt{2\, gh}\, C = V$; $\quad \dfrac{V}{\frac{2}{3}\, b\, h \sqrt{2\, gh}\, C} = t$; and $\left(\dfrac{V}{\frac{2}{3}\, C\, b\, h}\right)^2 \div 2\, g = h$.

t representing time in seconds and V *volume in cube feet.*

EXAMPLE.—Sill of a weir is 1 foot below surface of water, and its breadth is 10 feet; what volume of water will it discharge in one second?

$C = .623, \quad \sqrt{64.33 \times 1} \times 10 \times 1 = 80.2$, and $\frac{2}{3}\ 80.2 \times .623 = 33.32$ *cube feet.*

NOTE.—Mean coefficient of discharge of weirs, breadth of which is no more than third part of breadth of stream, is two thirds of $.6 = .4$; and for weirs which extend whole width of stream it is two thirds of $.666 = .444$.

Or, $214 \sqrt{h^3} = V$ *in cube feet per minute.* *When h is in ins., put* 5.15 *for* 214.

Or, $C\, b\, h \sqrt{2\, g\, h} = V$. C for a depth .1 of length $= .417$, and for .33 of length $= .4$.

Or, by formula of Jas. B. Francis: $3.33\, (L - .1\, n\, H)\, H^{\frac{3}{2}} = V$.

L *representing length of weir and* H *depth of water in canal, sufficiently far from weir to be unaffected by depression caused by the current, both in feet, and n number of end contractions.*

NOTE.—When contraction exists at each end of weir, $n = 2$; and when weir is of width of canal or conduit, end contraction does not exist, and $n = 0$.

This formula is applicable only to rectangular and horizontal weirs in side of a dam, vertical on water-side, with sharp edges to current; for if bevelled or rounded off in any perceptible degree, a material effect will be produced in the discharge; it is essential also that the stream, after passing the edges, should in nowise be restricted in its flow and descent.

In cases in which depth exceeds one third of length of weir, this formula is not applicable. In the observations from which it was deduced, the depth varied from 7 to nearly 19 ins.

With end contraction, a distance from side of canal to weir equal to depth on weir is least admissible, in order that formula may apply correctly.

Depth of water in canal should not be less than three times that on weir for accurate computation of flow.

ILLUSTRATION.—If an overfull weir has a length of 7.94 feet and a depth of .986 (as determined by a hook gauge), what volume will it discharge in 24 hours?

$3.33\ (7.94 - .2 \times .986)\ .986^{\frac{3}{2}} = 3.33 \times \overline{7.94 - .1972} \times .97907 = 3.33 \times 7.7428 \times .97907 = 25.243\,875$, which $\times 60 \times 60 \times 24 = 2\,181\,061$ *cube feet.*

By Logarithms.—Log. $3.33 = .522\,444$

$7.7428 = .888\,898$

$.986^{\frac{3}{2}} = \bar{1}.993\,877$

$\times 3$

$2)\ \bar{1}.981\,631$

$\bar{1}.990\,815 = \bar{1}.990\,815$

$1.402\,157$

Log. 24 hours = 86 400 *seconds.* $4.936\,514$

$6.338\,671$

Log. $6.338\,67 = 2\,181\,073$ *cube feet.* C in this case = .615.

Or. $214\sqrt{H^3}$ and $5.15\sqrt{h^3} = V$, if stream above the sill is not in motion. H *representing height of surface of water above sill in feet, h in inches;* and $214\sqrt{H^3 + .035\, v^2\, H^3} = V$, if in motion. *v representing velocity of approach of water in feet per second, and V volume in cube feet discharged over each lineal foot of sill per minute.*

In gauging, waste-board must have a thin edge. Height measured to level of surface not affected by the current of overfall. (*Molesworth.*)

To Compute Depth of Flow over a Sill that will Discharge a given Volume of Water.

$\left(\frac{3\,V}{2\,C\,b\sqrt{2\,g}} + k^{\frac{3}{2}}\right)^{\frac{2}{3}} - k = d.$ $k = \frac{v^2}{2\,g}$ *representing height due to velocity* (v) *as it flows to the weir.*

NOTE.—When back-water is raised considerably, say 2 feet, velocity of water approaching weir (k) may be neglected.

Rectangular Notches, or Vertical Apertures or Slits.

A *Notch* is an opening, either vertical or oblique, in side of a vessel, reservoir, etc., alike to a narrow and deep weir.

Vertical Apertures or *Slits* are narrow notches or weirs, running to or near to bottom of vessel or reservoir.

Coefficient for opening, 8 ins. by 5, mean .606 (*Poncelet and Lesbros*).

Coefficient increases as depth decreases, or as ratio of length of notch to its depth increases.

When sides and under edge of a notch increase in thickness, so as to be converted into a short open channel, coefficients reduce considerably, and to an extent beyond what increased resistance from friction, particularly for small depths, indicates.

Poncelet and Lesbros found, for apertures 8 × 8 ins., that addition of a horizontal shoot 21 ins. long reduced coefficient from .604 to .601, with a head of about 4 feet; but for a head of 4.5 ins. coefficient fell from .572 to .483.

For Rule and Formulas, see preceding page.

Rectangular Openings or Sluices, or Horizontal Slits.

Height measured from Surface of Head of Water to Upper Side and to Sill of Opening.

Coefficient for { Opening, 1 inch by 1 inch. Head, 7 to 23 feet. $= .621$.
" 3 " " 3 " " 7 " 23 " $= .614$.
" 2 feet " 1 foot. " 1 " 2 " $= .641$. }

Poncelet and Lesbros deduced that coefficient of discharge increases with small and very oblong apertures as they approach the surface, and decreases with large and square apertures under like circumstances.

Coefficients ranged, in square apertures of 8 by 8 ins., under a head of 6 ins. to rectangular apertures, 8 by 4 ins.; under a head of 10 feet, from .572 to .745.

In a Thin Plate, C = .616 (*Bossut*); C = .61 (*Michelotti*).

To Compute Discharge.

Rule.—Multiply square root of 64.333 and breadth of opening in feet, by coefficient for opening, and by difference of products of heights of water and their square roots, and two thirds of whole product will give discharge in cube feet per second.

Or, $\frac{2}{3}\, b\sqrt{2\,g}\,(h\sqrt{h}-h'\sqrt{h'})\,C = V$; $\frac{V}{\frac{2}{3}\, b\sqrt{2\,g}\,(h\sqrt{h}-h'\sqrt{h'})\,C} = t$; and $\frac{V}{b\,(h-h')} = v$. *h and h' representing depth to sill and opening in feet, and v velocity in feet per second.*

Example.—Sill of a rectangular sluice, 6 feet in width by 5 feet in depth, is 9 feet below surface of water; what is discharge in cube feet per second?

$C = .625$, $9-5=4$. and $\frac{2}{3}\sqrt{2g}\times 6\times .625\times(9\sqrt{9}-4\times\sqrt{4}) = 380.95$ *cube feet.*

Or, $\sqrt{2\,g\,d}\,a\,C = V$. *d representing depth to centre of opening in feet.*

$d = 9-2.5 = 6.5$, $a = 6\times 5 = 30$, and $\sqrt{64.33\times 6.5}\times 30\times .625 = 383.44$ *cube ft.*

Sluice Weirs or Sluices.

Discharge of water by Sluices occurs under three forms—viz., *Unimpeded*, *Impeded*, or *Partly Unimpeded.*

To Compute Discharge when Unimpeded.

$C\,d\,b\sqrt{2\,g\,h} = V$. *$d$ representing depth of opening and h taken from centre of opening to surface of water.*

If velocity, k, with which water flows to sluice is considered,

$\frac{1}{2\,g}\left(\frac{V}{C\,d\,b}\right)^2 - k = h$; $\frac{V}{C\,b\sqrt{2\,g\,h}} = d$; and $\frac{V}{C\,b\sqrt{2\,g\left(h'-\frac{d}{2}\right)}} = d$.

h' representing height to which water is raised by dam above sill.

Illustration.—How high must the gate of a sluice weir be raised, to discharge 250 cube feet of water per second, its breadth being 24 feet and height, h', 5 feet?

C by experiment $= .6$. d approximately $= 1$.

$$\frac{250}{.6\times 24\sqrt{64.33\left(5-\frac{1}{2}\right)}} = \frac{250}{14.4\times 17.014} = 1.0204 \textit{ feet.}$$

To Compute Discharge when Impeded.

$C\,d\,b\sqrt{2\,g\,h} = V$, and $\frac{V}{C\,b\sqrt{2\,g\,h}} = d$.

h representing difference of level between supply and back-water.

To Compute Discharge when partly Impeded.

$C\,b\sqrt{2\,g}\left(d\sqrt{h-\frac{d}{2}}+d'\sqrt{h}\right)=V.$ *d′ representing depth of back-water above upper edge of sill.*

Illustration.—Dimensions of a sluice are 18 feet in breadth by .5 in depth; height of opening above surface of water .7 feet, and difference between levels of supply and surface water is 2 feet; what is discharge per second?

$$.6\times 18\times 8.02\left(.7\sqrt{2-\frac{.7}{2}}+.5\sqrt{2}\right)=86.62\times\overline{.896+.707}=138.85 \textit{ cube feet.}$$

Coefficients of Circular Openings or Sluices.

Height measured from Surface of Head of Water to Centre of Opening.

Contraction of section from 1 to .633, and reduction of velocity to .974; hence $.633\times .974=.617$ (*Neville*).

In a Thin Plate, C = .666 (*Bossut*); .631 (*Venturi*); .64 (*Eytelwein*).

Cylindrical Ajutages, or Additional Tubes, give a greater discharge than apertures in a thin side, head and area of opening being the same; but it is necessary that the flowing water should entirely fill mouth of ajutage.

Mean coefficient, as deduced by *Castel, Bossut*, and *Eytelwein*, is .82.

Short Tubes, Mouth-pieces, and Cylindrical Prolongations or Ajutages.

Fig. 4.

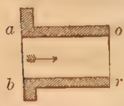
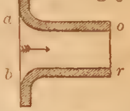


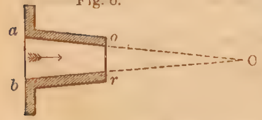
Fig. 5.



If an aperture be placed in side of a vessel of from 1.5 to 2.5 diameters in thickness, it is converted thereby into a short tube, and coefficient, instead of being reduced by increased friction, is increased from mean value up to about .815, when opening is cylindrical, as in Fig. 4; and when junction is rounded, as in Fig. 5, to form of contracted vein, coefficient increases to .958, .959, and .975 for heads of 1, 10, and 15 feet.

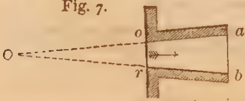
Conically Convergent and Divergent Tubes.

Fig. 6.



In conically divergent tube, Fig. 6, coefficient of discharge is greater than for same tube placed convergent, fluid filling in both cases, and the smaller diameters, or those at same distance from centres, O O, being used in the computations.

Fig. 7.



A tube, angle of convergence, O, of which is 5° nearly, with a head of from 1 to 10 feet, axial length of which is 3.5 ins., small diameter 1 inch, and large diameter 1.3 ins., gives, when placed as at Fig. 6, .921 for coefficient; but when placed as at Fig. 7, coefficient increases up to .948. Coefficient of velocity is, however, larger for Fig. 6 than for Fig. 7, and discharging jet has greater amplitude in falling. If a prismatic tube project beyond sides into a vessel, coefficient will be reduced to .715 nearly.

Form of tube which gives greatest discharge is that of a truncated cone, lesser base being fitted to reservoir, Fig. 7. Venturi concluded from his experiments that tube of greatest discharge has a length 9 times diameter of lesser opening base, and a diverging angle of 5° 6′—discharge being 2.5 greater than that through a thin plate, 1.9 times greater than through a short cylindrical tube, and 1.46 greater than theoretic discharge.

Compound Mouth-pieces and Ajutages.

Fig. 8.

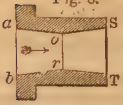


Fig. 9.

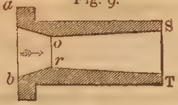
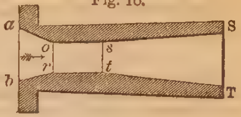


Fig. 10.



Coefficients for Mouth-pieces, Short Tubes, and Cylindrical Prolongations.

Computed and reduced by Mr. Neville, from Venturi's Experiments.

Description of Aperture, Mouth-piece, or Tube.	C. for Diam. $a\,b$.	C. for Diam. $o\,r$.
1. An aperture 1.5 ins. diameter, in a thin plate	.622	.974
2. Tube 1.5 ins. diameter, and 4.5 ins. long, Fig. 4	.823	.823
3. Tube, Fig. 5, having junction rounded to form of contracted vein	.611	.956
4. Short conical convergent mouth-piece, Fig. 6	.607	.934
5. Like tube divergent, with smaller diameter at junction with reservoir; length 3.5 ins., $o\,r = 1$ in., and $a\,b = 1.3$ ins.	.561	.948
6. Double conical tube, $a\,o$, S T, $r\,b$, Fig. 9, when $a\,b = $ S T $= 1.5$ ins., $o\,r = 1.21$ ins., $a\,o = .92$ in., and o S $= 4.1$ ins.	.928	1.428
7. Like tube when, as in Fig. 8, $a\,o\,r\,b = o$ S T r, and $a\,o$ S $=$ 1.84 ins.	.823	1.266
8. Like tube when S T $= 1.46$ ins., and o S $= 2.17$ ins.	.823	1.266
9. Like tube when S T $= 3$ ins., and o S $= 9.5$ ins.	.911	1.4
10. Like tube when o S $= 6.5$ ins., and S T $= 1.92$ ins.	1.02	1.569
11. Like tube when S T $= 2.25$ ins., and o S $= 12.125$ ins.	1.215	1.855
12. A tube, Fig. 10, when $o\,s = r\,t = 3$ ins., $o\,r \mp s\,t = 1.21$ ins., and tube o S T r, as in No. 6, S T $= 1.5$ ins., and s S $= 4.1$ ins.	.895	1.377

Mean of various experiments with tubes of .5 to 3 ins. in diameter, and with a head of fluid of from 3 to 20 feet, gave a coefficient of .813; and as mean for circular apertures in a thin plate is .63, it follows that under similar circumstances, $.813 \div .63 = 1.29$ times as much fluid flows through a tube as through a like aperture in a thin plate.

Preceding Table gives coefficients of discharge for figures given, and it will be found of great value, as coefficients are calculated for large as well as small diameters, and the necessity for taking into consideration form of junction of a pipe with a reservoir will be understood from the results.

Circular Sluices, etc.

To Compute Discharge.

Height measured from Surface of Head of Water to Centre of Opening.

Rule.—Multiply square root of product of 64.333 and depth of centre of opening from surface of water, by area of opening in square feet, and this product by coefficient for the opening, and whole product will give discharge in cube feet per second.

Or, $\sqrt{2\,g\,d}\,a\,C = V$. *a representing area in sq. feet, and d depth of surface of fluid from centre of opening in feet.*

EXAMPLE.—Diameter of a circular sluice is 1 foot, and its centre is 1.5 feet below surface of the water; what is discharge in cube feet per second?

Area of 1 foot $= .7854$; $C = .64$, and $\sqrt{64.333 \times 1.5} \times .7854 \times .64 = 4.938$ *cube feet.*

When Circumference reaches Surface of Water. $\sqrt{2\,g\,r}$, $.9604\,a\,C = V$. *r representing radius of circle in feet.*

ILLUSTRATION.—In what time will 800 cube feet of water be discharged through a circular opening of .025 sq foot, centre of which is 8 feet below surface of water?

$$C = .63. \quad \frac{800}{\sqrt{2\,g\,d} \times .025 \times .63} = \frac{800}{22.68 \times .025 \times .63} = 2239.58 = 37\ min.\ 19.6\ sec.$$

NOTE.—For circular orifices, the formula $\sqrt{2\,g\,d}\,a\,C = V$ is sufficiently exact for all depths exceeding 3 times diameter; the finish of openings being of more effect than extreme accuracy in coefficient.

Semicircular Sluices.

When Diameter is either Upward or Downward. $\sqrt{2\,g\,d}\,a\,C = V$. *d representing depth of centre of gravity of figure from surface.*

When Diameter as above is at Depth d, below Surface. $\sqrt{2\,g\,d}\ 1.188\,a\,C = V$.

Circular, Semicircular, Triangular, Trapezoidal, Prismatic Wedges, Sluices, Slits, etc.

See Neville, London, 1860, *pp.* 51–63, *and Weisbach, vol.* i. *p.* 456.

For greater number of apertures at any depth below surface of water, product of area, and velocity of depth of centre, or centre of gravity, if practicable to obtain it, will give discharge with sufficient accuracy.

Discharge from Vessels not Receiving any Supply.

For prismatic vessels the general law applies, that twice as much would be discharged from like apertures if the vessels were kept full during the time which is required for emptying them.

To Compute Time. $\dfrac{2\,A\sqrt{h}}{C\,a\sqrt{2\,g}} = \dfrac{2\,A\,h}{V} = t.$

ILLUSTRATION.—A rectangular cistern has a transverse horizontal section of 14 feet, a depth of 4 feet, and a circular opening in its bottom of 2 ins. in diameter; in what time will it discharge its volume of water, when supply to it is cut off and cistern allowed to be emptied of its contents?

$h = 4$ *feet,* $a = 2^2 \times .7854 \div 144 = .0218$, $C = .613$, and $\sqrt{2\,g\,h} \times a \times C = .2143$ *cube foot per second.* Then $\dfrac{2 \times 14 \times 4}{.2143} = 522.6$ *seconds.*

To Compute Time and Fall.

Depression or subsidence of surface of water in a vessel, corresponding to a given time of efflux, is $h - h'$. *h' representing lesser depth.*

$$\frac{2\,A}{C\,a\sqrt{2\,g}}(\sqrt{h} - \sqrt{h'}) = t. \quad \text{Inversely,} \left(\sqrt{h} - \frac{C\,a\sqrt{2\,g}}{2\,A}\,t\right)^2 = h'.$$

ILLUSTRATION.—In what time will the water in cistern, as given in preceding case, subside 1.6 feet, and how much will it subside in that time?

$$A = 14, \quad C = .6, \quad a = .0218, \quad \sqrt{2\,g} = 8.02, \quad h = 4, \quad h' = 4 - 1.6 = 2.4.$$

$$\frac{2 \times 14}{.6 \times .0218 \times 8.02} \times (\sqrt{4} - \sqrt{2.4}) = \frac{28}{.1049} \times (2 - 1.55) = 120.1\ seconds.$$

$$\left(\sqrt{4} - \frac{.6 \times .0218 \times 8.02}{2 \times 14} \times 120.1\right)^2 = \overline{2 - .45} = 2.4\ feet;\ \text{hence, } 4 - 2.4 = 1.6\ feet.$$

When Supply is maintained.—Divide result obtained as preceding by 2.

Discharge, when Form and Dimensions of Vessel of Efflux are not known.

Volume discharged may be estimated by observing heads of the water at equal intervals of time; and at end of half time of discharge, head of water will be .25 of whole height from surface to delivery.

When $t =$ *such interval. For openings in bottom or side,* $C\,a\,t\sqrt{2g}\left(\frac{\sqrt{h}+\sqrt{h_1}}{2}\right) = V$, *for* 1 *depth;* $C\,a\,t\sqrt{2g}\left(\frac{\sqrt{h}+4\sqrt{h_1}+\sqrt{h_2}}{3}\right) = V$ *for* 2 *depths;* and $C\,a\,t\sqrt{2g}\left(\frac{\sqrt{h}+4\sqrt{h_1}+2\sqrt{h_2}+4\sqrt{h_3}+\sqrt{h_4}}{3}\right) = V$ *for* 4 *depths.*

NOTE.—At end of half time of discharge, head of water will be .25 of whole height from surface to delivery.

Weirs or Notches.

$\frac{2}{9}\,C\,b\,t\sqrt{2g}\,(\sqrt{h^3}+4\sqrt{h^3_1}+\sqrt{h^3_2}) = V$. *b representing breadth in feet.*

ILLUSTRATION.—A prismatic reservoir 9 feet in depth is discharged through a notch 2.222 feet wide, surface subsiding 6.75 feet in 935 seconds; what is volume discharged?

$C = .6$, $h_1 = 9 - 6.75 = 2.25$ *feet,* and $\frac{2}{9}\;6 \times 2.222 \times 935 \times 8.02\;(\sqrt{9^3}+4\sqrt{2.25^3}+\sqrt{0^3}) = 2221.6 \times 40.5 = 89\,974.8$ *cube feet.*

When there is an Influx and Efflux.

If a reservoir during an efflux from it has an influx into it, determination of time in which surface of water rises or falls a certain height becomes so complicated that an approximate determination is here alone essayed.

A state of permanency or constant height occurs whenever head of water is increased or decreased by $\frac{1}{2g}\left(\frac{I}{C\,a}\right) = k$. I *representing influx in cube feet per second.*

Time (t) in which variable head (x) increases by volume (v) $= \frac{A_1\,v}{I - C\,a\sqrt{2gx}}$; and time in which it sinks height, k, by $\frac{A_1\,v}{C\,a\sqrt{2gx} - I}$. Time of efflux, in which subsiding surface falls from A to A_1, etc., and head of water from h to h_1, when k is represented by $\frac{I}{C\,a\sqrt{2g}} = \sqrt{k}$, is

$$\frac{h - h_4}{12\,C\,a\sqrt{2g}}\left(\frac{A}{\sqrt{h}-\sqrt{k}}+\frac{4\,A_1}{\sqrt{h_1}-\sqrt{k}}+\frac{2\,A_2}{\sqrt{h_2}-\sqrt{k}}+\frac{4\,A_3}{\sqrt{h_3}-\sqrt{k}}+\frac{A_4}{\sqrt{h_4}-\sqrt{k}}\right) = t.$$

ILLUSTRATION.—In what time will surface of water in a pond, as in a previous example, fall 6 feet, if there is an influx into it of 3.0444 cube feet per second?

$$\sqrt{k} = \frac{3.044}{.537 \times .8836 \times 8.02} = .8. \quad C = .537 \text{ and } a = .8836.$$

$$\frac{20 - 14}{12 \times .537 \times .8836 \times 8.02} \times \left(\frac{600\,000}{4.472 - .8}+\frac{4 \times 495\,000}{4.301 - .8}+\frac{2 \times 410\,000}{4.123 - .8}+\frac{4 \times 325\,000}{3.937 - .8}+\frac{265\,000}{3.742 - .8}\right) = \frac{6}{45.665} \times 1\,480\,201 = 194\,486 \textit{ seconds} = 54\ h.,\ 1\ min.,\ 26\ sec.$$

Prismatic Vessels.

If vessel has a uniform transverse section, A.

Then $\frac{2\,A}{C\,a\sqrt{2g}}\left[\sqrt{h}-\sqrt{h_1}+\sqrt{k} \times \text{hyp. log.}\left(\frac{\sqrt{h}-\sqrt{k}}{\sqrt{h_1}-\sqrt{k}}\right)\right] = t =$ *time in which head of water flows from* h *to* h_1.

ILLUSTRATION.—A reservoir has a surface of 500000 sq. feet, a depth of 20 feet; it is fed by a stream affording a supply of 3.0444 cube feet per second, and outlet has an area of .8836 sq. foot; in what time will it subside 6 feet?

$\sqrt{k}$, as before, $= .8$, $C = .537$, and $\frac{2 \times 500000}{C\,a\sqrt{2g}} \times \Big[\sqrt{20} - \sqrt{14} + .8 \times$ hyp. log. $\left(\frac{\sqrt{20}-.8}{\sqrt{14}-.8}\right) \times 2.303\Big] = 238414$ *seconds* $= 66$ *h.* 13 *min.* 34 *sec.*

To Compute Fall in a given Time.

This is determining head h_1 at end of that time, and it should be subtracted from head h at commencement of discharge. Put into preceding equation several values of h_1, until one is found to meet the condition.

ILLUSTRATION.—Take a prismatic pond having a surface of 38750 sq. feet, a depth to centre of opening of sluice of 10.5 feet, a supply of 33.6 cube feet, and a discharge of 40 cube feet per second.

$$\sqrt{k} = .84.$$

Putting these numerical values into the equation, and assuming different values for h_1, a value which nearly satisfies the equation is 4. Consequently, $10.5 - 4 = 6.5$ *feet, fall.*

$$\frac{A\,k}{3\,I}\left[\text{hyp. log}\ \frac{h_1 + \sqrt{h_1 k} + k}{(\sqrt{h_1} - \sqrt{k})^2} + \sqrt{12}\ \text{arc}\left(\text{tang.} = \frac{-\sqrt{3\,h_1}}{2\sqrt{k} + \sqrt{h_1}}\right)\right] = t;$$

$\left(\frac{I}{\frac{2}{3}\,C\,b\sqrt{2g}}\right)^{\frac{2}{3}} = k$; arc (tang. $= y$, *arc tangent of which* $= y$, *and* I *as preceding.*

According as k is $\lessgtr h$, and influx of water, $I \gtrless \frac{2}{3}\,C\,l\sqrt{2\,g\,h^3}$, there is a rise or fall of fluid surface, the condition of permanency occurring when $h_1 = k$, and time corresponding becomes ∞.

ILLUSTRATION.—In what time will water in a rectangular tank, 12 feet in length by 6 feet in breadth, rise from sill of a weir or notch, 6 inches broad, to 2 feet above it, when 5 cube feet of water flow into the tank per second?

$$h_1 = 2, \quad h = 0, \quad A = 12 \times 6 = 72, \quad I = 5, \quad b = .5, \quad C = .6.$$

$$k = \left(\frac{5}{\frac{2}{3}\,.6 \times .5 \times 8.02}\right)^{\frac{2}{3}} = \sqrt[3]{3.117^2} = 2.1338.$$

Then $\frac{72 \times 2.1338}{3 \times 5}\left[\text{hyp. logarithm}\ \frac{2 + \sqrt{2 \times 2.1338} + 2.1338}{(\sqrt{2} - \sqrt{2.1338})^2} + \sqrt{12}\ \text{arc}\left(\text{tang.} = \frac{-\sqrt{3 \times 2}}{2\sqrt{2.1338} + \sqrt{2}}\right)\right] = 10.2423 \times \text{hyp. log.}\ \frac{6.1996}{.002162} - 3.4641 \times \text{arc}\left(\text{tang.}\ \frac{\sqrt{6}}{4.3356}\right) =$ $10.2423 \times [7.961 - (3.461 \times$ arc, tangent of which $= .56497$, or $29°\ 28' = 29.466$, length of which $= .5143) = 1.781] = 10.2423 - 7.961 - 1.781 = 10.2423 \times 6.18 = 63.297$ *seconds.*

Discharge of Water under Variable Pressures.

To Compute Time, Rise and Fall, and Volume.

$\frac{a}{A}\sqrt{2\,g\,x} = v$. *x representing variable head,* A *and a areas of transverse horizontal section of vessel and discharge, and v theoretical velocity of efflux.*

To Compute Volume.

$\frac{A\,y}{h - h'} = V$. *y representing extent of fall, and* V *volume of water discharged,* as

ILLUSTRATION.—Assume elements of preceding case.

$$A = 14. \qquad y = 4\ \textit{feet.} \qquad \text{Then } 56 \times 4 = 224\ \textit{cube feet.}$$

Discharge from Vessels of Communication.

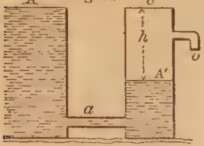
When Reservoir of Supply is maintained at a uniform Height.—Fig. 11.

To Compute Time. $\frac{2\,A\sqrt{h}}{C\,a\sqrt{2\,g}}=t.$

ILLUSTRATION 1.—In what time will level of water in a receiving vessel having a section of 14 sq. feet attain height of that in supply, through a pipe 2 ins. in diameter, placed 4 feet below level of supply?

$$C=.613.\quad \frac{2\times14\times\sqrt{4}}{.613\times.0218\times8.02}=\frac{56}{.1072}=522.3\ \textit{seconds.}$$

Fig. 11.



2.—Assume C, vessel, Fig. 11, to be a cylinder 18 ins. in diameter, head of water in A = 4 feet, at A′ 1 foot, and 2 feet below outlet o; in what time will water in vessel run out and over at o through a pipe, a, 1.5 ins. diameter?

$$h-h'=4-1-2=1\ \textit{foot.}\qquad C=.8.$$

$$\frac{A}{a}=\left(\frac{18}{1.5}\right)^2=144.$$

Then $\frac{2\times144}{.8\times8.03}(\sqrt{3}-\sqrt{1})=\frac{288}{6.424}\times\overline{1.73-1}=32.73$ *seconds.*

When Vessel of Supply has no Influx, and is not indefinitely great compared with Receiving Vessel.

$\frac{2\,A\,A'\sqrt{h}}{C\,a\,(A+A')\sqrt{2\,g}}=t.$ *A′ representing section of receiving vessel, t time in which the two surfaces of water attain same level; and* $\frac{2\,A\,A'(\sqrt{h}-\sqrt{h'})}{C\,a\,(A+A')\sqrt{2\,g}}=t$, *time within which level falls from h to h′.*

ILLUSTRATION.—Section of a cistern from which water is to be drawn is 10 sq. feet, and section of receiving cistern is 4 sq. feet; initial difference of level is 3 feet, and diameter of communicating pipe is 1 inch; in what time will surfaces of water in both vessels attain like levels?

$$C=.82.\qquad 1''=.7854.\qquad \frac{2\times10\times4\sqrt{3}}{.82\times.7854\times\frac{14}{144}\times8.02}=\frac{138.56}{.502}=276\ \textit{seconds.}$$

Discharge from a Notch* in Side of a Vessel.

When it has no Influx. $\frac{3\,A}{C\,b\times\sqrt{2\,g}}\left(\frac{1}{\sqrt{h'}}-\frac{1}{\sqrt{h}}\right)=t.$ *b breadth of notch in feet.*

ILLUSTRATION.—If a reservoir of water, 110 feet in length by 40 in breadth, has a notch in end of 9 ins. in width; in what time will head of water of 15 ins. fall to 6?

$$C=.6.\qquad 9''=.75\ \textit{foot.}\qquad h'=.5.\qquad h=1.25.$$

$$\frac{3\times110\times40}{.6\times.75\times8.02}\times\left(\frac{1}{\sqrt{.5}}-\frac{1}{\sqrt{1.25}}\right)=\frac{13\,200}{3.61}\times\overline{1.414-.894}=1901\ \textit{seconds.}$$

NOTE.—For discharge of vessels in motion, see Weisbach, vol. 1, pp. 394–396.

Reservoirs or Cisterns.

To Compute Time of Filling and of Emptying a Reservoir under Operation of both Supply and Discharge.

$\frac{V}{S-D}=T$, and $\frac{V}{D-S}=t.$ *V representing volume of vessel, S supply of water, and D discharge of water, both per minute, and in cube feet. T time of filling vessel, and t time of discharging it, both in minutes.*

---

* When the notch extends to the bottom of the reservoir, etc., the time for the water to run out is indefinite, as $h'=0$.

Irregular-Shaped Vessels, as a Pond, Lake, etc.

To Compute Time and Volume Discharged.

Operation.—Divide whole mass of water into four or six strata of equal depths.

Then, *for* 4 *Strata*, $\frac{h-h^4}{12\,C\,a\sqrt{2g}} \times \left(\frac{a}{\sqrt{h}} + \frac{4\,a^1}{\sqrt{h^1}} + \frac{2\,a^2}{\sqrt{h^2}} + \frac{4\,a^3}{\sqrt{h^3}} + \frac{a^4}{\sqrt{h^4}}\right) = t$; h, h', etc., *representing depths of strata at* a, a^1, etc., *commencing at surface*; a^1, a^2, etc., *being areas of first, second*, etc., *transverse sections of pond*, etc.; and $\frac{h-h^4}{12} \times a + 4\,a^1 + 2\,a^2 + 4\,a^3 + a^4 = V$.

Fig. 12.

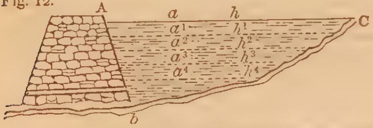


Illustration.—In what time will depth of water in a lake, A b C, Fig. 12, subside 6 feet, surfaces of its strata having following areas, outline of sluice being a semicircle, 18 ins. wide, 9 deep, and 60 feet in length?

a	at	20	feet	(h)	depth of water	=	area of	600 000	sq. feet.
a^1	"	18.5	"	(h^1)	" "	=	"	495 000	"
a^2	"	17	"	(h^2)	" "	=	"	410 000	"
a^3	"	15.5	"	(h^3)	" "	=	"	325 000	"
a^4	"	14	"	(h^4)	" "	=	"	265 000	"

a = area of $18 \div 2 = .8836$ *sq. feet*; C = .537.

Then $\frac{20-14}{12 \times .537 \times .8836 \times 8.02} \times \left(\frac{600\,000}{4.472} + \frac{4 \times 405\,000}{4.301} + \frac{2 \times 410\,000}{4.123} + \frac{4 \times 325\,000}{3.937} + \frac{265\,000}{3.742}\right) = \frac{6}{45.665} \times 1\,194\,431 = 156\,938$ *sec.* = 43 *h.*, 35 *min.* 38 *sec.*

And discharge $= \frac{6}{12} \times (600\,000 + 4 \times 495\,000 + 2 \times 410\,000 + 4 \times 325\,000 + 265\,000) = .5 \times 4\,965\,000 = 2\,482\,500$ *cube feet.*

For 6 *Strata*, put $2\,a^4$, instead of a^4, and $4\,a^5$ and a^6 additional, and divide by 18 instead of 12.

Flow of Water in Beds.

Flow of water in beds is either *Uniform* or *Variable*. It is uniform when mean velocity at all transverse sections is the same, and consequently when areas of sections are equal; it is variable when mean velocities, and therefore areas of sections, vary.

To Compute Fall of Flow.

$C\,\frac{l\,p}{a} \times \frac{v^2}{2g} = h$. C *representing coefficient of friction*, l *length of flow*, p *perimeter of sides and bottom of bed, and* h *fall in feet.*

Illustration.—A canal 2600 feet in length has breadths of 3 and 7 feet, a depth of 3 feet, with a flow of 40 cube feet per second; what is its fall?

C = as per table below .007 565; $p = \sqrt{3^2 + 2^2} \times 2 + 3 = 10.2$; $a = 15$; and $v = 40 \div 15 = 2.66$. Hence $.007\,565 \times \frac{2600 \times 10.2}{15} \times \frac{2.66^2}{64.33} = 1.47$ *feet.*

To Compute Velocity of Flow. $\sqrt{\frac{a}{C \times l\,p}\,2\,g\,h} = v$.

Illustration.—A canal 5800 feet in length has breadths of 4 and 12 feet, a depth of 5, and a fall of 3; what is velocity and volume of flow?

$$p = \sqrt{5^2 + 4^2} \times 2 + 4 = 16.8, \text{ and } a = 40.$$

Then $\sqrt{\frac{40}{.007\,565 \times 5800 \times 16.8} \times 64.33 \times 3} = \sqrt{.0542 \times 193} = 3.23$ *feet.* Hence volume $= 40 \times 3.23 = 129.2$ *cube feet.*

Coefficients of Friction of Flow of Water in Beds, as in Rivers, Canals, Streams, etc.

In Feet per Second.

Velocity.	C.	Velocity.	C.	Velocity.	C.	Velocity.	C.
.3	.008 15	.7	.007 73	1.5	.007 59	5	.007 45
.4	.007 97	.8	.007 69	2	.007 52	8	.007 44
.5	.007 85	.9	.007 66	2.5	.007 51	10	.007 43
.6	.007 78	1	.007 63	3	.007 49	12	.007 42

Forms of Transverse Sections of Canals, etc.

Resistance or friction which bed of a stream, etc., opposes to flow of water, in consequence of its adhesion or viscosity, increases with surface of contact between bed and water, and therefore with the perimeter of water profile, or of that portion of transverse section which comprises the bed.

Friction of flow of water in a bed is inversely as area of it.

Of all regular figures, that which has greatest number of sides has for same area least perimeter; hence, for enclosed conduits, nearer its transverse profile approaches to a regular figure, less the coefficient of its friction; consequently, a circle has the profile which presents minimum of friction.

When a canal is cut in earth or sand and not walled up, the slope of its sides should not exceed 45°.

Variable Motion.

Variable motion of water in beds of rivers or streams may be reduced to rules of uniform motion when resistance of friction for an observed length of river can be taken as constant.

To Compute Volume of Water flowing in a River.

$$\frac{\sqrt{2gh}}{\sqrt{\frac{1}{A_1^2}-\frac{1}{A^2}+C\frac{lp}{A_1+A}\left(\frac{1}{A_1^2}+\frac{1}{A^2}\right)}}=V.$$

A and A_1 representing areas of upper and lower transverse sections of flow.

Illustration.—A stream having a mean perimeter of water profile of 40 feet for a length of 300 feet has a fall of 9.6 ins.; area of its upper section is 70 sq. feet, and of its lower 60; what is volume of its discharge?

To obtain C for velocity due to this case, $92.35\sqrt{\frac{70+60\times\frac{9.6}{12}}{40\times300}}=8.59$ *feet*, coefficient for which, see Table above, $=.007\,44$.

$$\frac{\sqrt{64.33\times(9.6\div12)}}{\sqrt{\frac{1}{70^2}-\frac{1}{60^2}+.007\,44\,\frac{300\times40}{70+60}\left(\frac{1}{70^2}+\frac{1}{60^2}\right)}}=\frac{7.174}{\sqrt{.000\,330\,89}}=394.6 \text{ cube feet;}$$

and mean velocity $=\frac{394.6\times2}{70+60}=6.07$ *feet*, C for which is .007 45.

FRICTION IN PIPES AND SEWERS.

Friction in flow of water through pipes, etc., of a uniform diameter is independent of pressure, and increases directly as length, very nearly as square of velocity of flow, and inversely as diameter of pipe.

With wooden pipes friction is 1.75 times greater than in metallic.

Time occupied in flowing of an equal quantity of water through Pipes or Sewers of equal lengths, and with equal heads, is proportionally as follows:

In a Right Line as 90, in a True Curve as 100, and in a Right Angle as 140.

To Compute Head necessary to overcome Friction of Pipe. (*Weisbach.*)

$\left(.0144 + \frac{.01746}{\sqrt{v}}\right) \times \frac{l}{d} \times \frac{v^2}{5.4} = h'$. *h′ representing head to overcome friction of flow in pipe, l length of pipe, and v velocity of water per second, both in feet, and d internal diameter of pipe in ins.*

Illustration.—Length of a conduit-pipe is 1000 feet, its diameter 3 ins., and the required velocity of its discharge 4 feet per second; what is required head of water to overcome friction of flow in pipe?

$$\left(.0144 + \frac{.01746}{\sqrt{4}}\right) \times \frac{1000}{3} \times \frac{16}{5.4} = .02313 \times 333.333 \times 2.963 = 22.845 \textit{ feet.}$$

Head here deduced is height necessary to overcome friction of water in pipe alone.

Whole or entire head or fall includes, in addition to above, height between surface of supply and centre of opening of pipe at its upper end. Consequently, it is whole height or vertical distance between supply and centre of outlet.

To Compute whole Head, or Height from Surface of Supply to Centre of Discharge.

$$\left(C \times \frac{l}{d} + 1.5\right) \times \frac{v^2}{2g} = h.$$

1.5 is taken as a mean, and is coefficient of friction for interior orifice, or that of upper portion of pipe.

To obtain C *or coefficient.* $\left(.0144 + \frac{.01746}{\sqrt{v}}\right) = C.$

For facilitating computation, following Table of coefficients of resistance is introduced, being a reduction of preceding formula:

Coefficients of Friction of Water.

In Pipes at Different Velocities.

V. Ft. Ins.	C.	V. Ft. Ins.	C.	V. Ft. Ins.	C.	V. Ft. Ins.	C.	V. Ft. Ins.	C
4	.0443	2 8	.025	5	.0221	7 4	.0208	11 6	.0195
8	.0356	3	.0244	5 4	.0219	7 8	.0206	12	.0194
1	.0317	3 4	.0239	5 8	.0217	8	.0205	12 6	.0193
1 4	.0294	3 8	.0234	6	.0215	8 6	.0204	13	.0191
1 8	.0278	4	.0231	6 4	.0213	9	.0202	14	.0189
2	.0266	4 4	.0227	6 8	.0211	10	.0199	15	.0188
2 4	.0257	4 8	.0224	7	.0209	11	.0196	16	.0187

Illustration 1.—Coefficient due to a velocity of 4 feet per second is .0231.

2.—Take elements of preceding case.

$$\left(.0231 \times \frac{1000 \times 12}{3} + 1.5\right) \times \frac{4^2}{64.33} = 93.9 \times \frac{16}{64.33} = 23.35 \textit{ feet.}$$

Note.—In preceding formula l was taken in feet, as the multiplier of 12 for ins. was cancelled by taking 5.4 for $2g$, but in above formula it is necessary to restore this multiplier.

Radii of Curvatures.

When Pipes branch off from Mains, or when they are deflected at right angles, radius of curvature should be proportionate to their diameter. Thus,

	Ins.	Ins.	Ins.	Ins.	Ins.
Diameter	2 to 3	3 to 4	6	8	10
Radius..................	18	20	30	42	60

Curves and Bends.

Resistance or loss of head due to curves and bends, alike to that of friction, increases as square of velocity; when, however, curves have a long radius and bends are obtuse, the loss is small.

Curved Circular Pipe. (*Weisbach*). $\frac{a}{180} \times \left[.131 + 1.847 \left(\frac{d}{2\,r}\right)^{\frac{7}{2}}\right] \times \frac{v^2}{2\,g} = h.$

a representing angle of curve, d diameter of pipe, r radius of curve, and h height due to friction or resistance of curve, all in feet.

For facility of computations, following values of $.131 + 1.847 \left(\frac{d}{2\,r}\right)^{\frac{7}{2}}$ are introduced.

Coefficients of Resistance.

In Curved Pipes with Section of a Circle.

$\frac{d}{2\,r}$											
.1	.131	.25	.145	.4	.206	.6	.44	.75	.806	.9	1.408
.15	.133	.3	.158	.45	.244	.65	.54	.8	.977	.95	1.674
.2	.138	.35	.178	.5	.294	.7	.661	.85	1.177	1	1.978

ILLUSTRATION.—If in a pipe 18 ins. in diameter and 1 mile in length there is a right angled curve of 5 feet radius, what additional head of flow should be given to attain velocity due to a head of 20 feet?

$a = 90°$, v for such a pipe and head $= 4$ feet per second; $18 = 1.5$ and $\frac{1.5}{2 \times 5} = .15$, and .15 by table $= .133$.

$$\text{Hence, } \frac{90}{180} \times .133 \times \frac{4^2}{64.33} = .5 \times .133 \times \frac{16}{64.33} = .016\,53 \textit{ foot.}$$

NOTE.—If angle is greater than 90°, head should be proportionately increased.

Bent or Angular Circular Pipes.

Coefficient for angle of bend $= .9457 \sin.^2 x + 2.047 \sin.^4 x$. Hence,

x	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
C	.046	.139	.364	.74	.984	1.26	1.556	1.861	2.158	2.431

and $\frac{v^2}{2\,g} \times C = h$. *x representing half angle of bend.*

ILLUSTRATION.—Assume $v = 4$ *feet*, and angle $= 90°$; $x = \frac{90°}{2} = 45°$.

Then $\frac{4^2}{64.33} \times .984 = .2447$ *foot additional head required.*

In Valve Gates or Slide Valves.

In Rectangular Pipes.

r	1	.9	.8	.7	.6	.5	.4	.3	.2	.1
C	.0	.09	.39	.95	2.08	4.02	8.12	17.8	44.5	193

r = ratio of cross section.

In Cylindrical Pipes.

h	0	.125	.25	.375	.5	.625	.75	.875
r	1	.948	.856	.74	.609	.466	.315	.159
C	.0	.07	.26	.81	2.06	5.52	17	97.8

h = relative height of opening.

In a Throttle Valve. *In Cylindrical Pipes.*

A	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°	70°
r	.913	.826	.741	.658	.577	.5	.426	.357	.293	.234	.134	.06
C	.24	.52	.9	1.54	2.51	3.91	6.22	10.8	18.7	32.6	118	751

A = angle of position.

In a Clack or Trap Valve.

Angle of opening.....	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°
C	90	62	42	30	20	14	9.5	6.6	4.6	3.2	2.3	1.7

In a Cock. *In Cylindrical Pipes.*

A	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
r	.926	.85	.772	.692	.613	.535	.458	.385	.315	.25	.19	.137	.091
C	.05	.29	.75	1.56	3.1	5.47	9.68	17.3	31.2	52.6	106	206	486

In a Conical Valve. $\left(1.645\frac{a}{a'} - 1\right)^2 = C$. *$a$ and a' = areas of pipe and opening.*

In Imperfect Contractions. $\left(\frac{a}{c\,a'} - 1\right)^2 = C$. *$c$ = a factor, ranging from .624 for $\frac{a}{a'} = .1$ to 1 for $\frac{a}{a'} = 1$, being greater the greater the ratio.*

Illustration.—If a slide valve is set in a cylindrical pipe 3 ins. in diameter and 500 feet in length, is opened to .375 of diameter of pipe (hence .625 diameter closed), what volume of water will it discharge under a head of 100 feet, coefficient of entrance of pipe assumed at .5?

C, *by table, p. 545, pipe being .625 closed* = 5.52. $\qquad \frac{\sqrt{2g}\,\sqrt{h}}{\sqrt{\left(1.5 + C + c\frac{h \times n}{a}\right)}} = v.$

C = *from table, p. 544, for an assumed velocity of 11 feet 6 ins.* = .0195.

$$\text{Then}\quad \frac{\sqrt{64.33} \times \sqrt{100}}{\sqrt{\left(1.5 + 5.52 + .0195\frac{500 \times 12}{3}\right)}} = \frac{8.03 \times 10}{\sqrt{(7.02 + 39)}} = \frac{80.3}{6.78} = 11.85 \textit{ feet.}$$

Hence, area of 3 ins. = 7.07, and 7.07 × 12 × 11.85 = 1005.4 *cube feet per second.*

Valves. (*Conical, Spherical, or Flap.*)

Conical or Spherical Valve Puppet.

Height due to resistance or loss of head of water $= 11\frac{v^2}{2g}$. *v representing velocity of water in full diameter of pipe or vessel.*

$\left(\frac{A}{A'\,C'} - 1\right)^2 = C$. A and A′ *representing transverse areas of vessel and of valve opening, and* $\left(1.645\frac{A}{A'} - 1\right)^2 = C$ *of contraction in general.*

Illustration.—If A′ = .5 of vessel, $C = \left(1.645 \times \frac{1}{.5} - 1\right)^2 = 2.29^2 = 5.24$.

Clack or Trap Valve.—C decreases with diameter of vessel.

Illustration.—If a single acting force pump, 6 ins. in diameter, delivers at each stroke 5 cube feet of water in 4 seconds, diameter of valve seat 3.5 ins., and of valve 4.5; what resistance has water in its passage, and what is loss of mechanical effect?

$a = .196$. $\left(\frac{3.5}{6}\right)^2 = .34$ *ratio of transverse area of opening.* $1 - \left(\frac{4.5}{6}\right)^2 = .44$ *ratio of annular contraction to transverse area of vessel.*

Hence, $\frac{.34 + .44}{2} = .39$ *mean ratio, and coefficient of resistance corresponding thereto* $= \left(\frac{1.645}{.39} - 1\right)^2 = 3.22^2 = 10.37$. $\qquad \frac{5}{4 \times .196} = 6.37$ *velocity per second.*

$\frac{6.37^2}{64.33} = .63$ *height due to velocity.* Consequently, $10.37 \times .63 = 6.53$ *height due to resistance of valve, and* $\frac{5}{4} \times 62.5 \times 6.53 = 510.15$ *lbs. mechanical effect lost.*

Discharge of Water in Pipes.

For any Length and Head, and for Diameters from 1 Inch to 10 Feet. *In Cube Feet per Minute. (Beardmore.)*

Diam.	Tab. No.	Diam.	Tab. No.	Diam.	Tab. No.	Diam.	Tab. No.	Diam.	Tab. No.
Ins.		Ft. Ins.		Ft. Ins.		Ft. Ins.		Ft. Ins.	
1	4.71	9	1 147.6	1 11	11 983	3 1	39 329	4 9	115 854
1.25	8.48	10	1 493.5	2	13 328	3 2	42 040	5	131 703
1.5	13.02	11	1 894.9	2 1	14 758	3 3	44 863	5 3	148 791
1.75	19.15	1	2 356	2 2	16 278	3 4	47 794	5 6	167 139
2	26.69	1 1	2 876.7	2 3	17 889	3 5	50 835	5 9	186 786
2.5	46.67	1 2	3 463.3	2 4	19 592	3 6	53 995	6	207 754
3	73.5	1 3	4 115.9	2 5	21 390	3 7	57 265	6 6	253 781
3.5	108.14	1 4	4 836.9	2 6	23 282	3 8	60 648	7	305 437
4	151.02	1 5	5 628.5	2 7	25 270	3 9	64 156	7 6	362 935
4.5	194.84	1 6	6 493.1	2 8	27 358	3 10	67 782	8	426 481
5	263.87	1 7	7 433	2 9	29 547	3 11	71 526	8 6	496 275
6	416.54	1 8	8 449	2 10	31 834	4	75 392	9	572 508
7	612.32	1 9	9 544	2 11	34 228	4 3	87 730	9 6	655 369
8	854.99	1 10	10 722	3	36 725	4 6	101 207	10	745 038

This Table is applicable to Sewers and Drains by taking same proportion of tabular numbers that area of cross-section of water in sewer or drain bears to whole area of sewer or drain.

Formula upon which the table is constructed is, $2356\sqrt{\frac{h}{l} \times d^5} = V$ *in cube feet per minute,* and $39.27\sqrt{\frac{h}{l} \times d^5} = V$ *in cube feet per second.* *h representing height of fall of water and d diameter of pipe and l length, all in feet.*

To Compute Discharge.

(Eytelwein.) $\sqrt{\frac{d^5 h}{l}}\, 4.71 = V$, and $\sqrt[5]{\frac{l V^2}{h}}\, .538 = d$. *d = diameter of pipe in ins., l length of pipe and h head of water, both in feet.*

(Hawksley.) $\sqrt[5]{\frac{G^2 l}{h}}\, \frac{1}{15} = d$, and $\sqrt{\frac{(15\, d)^5\, h}{l}} = G$. G = *number of Imperial gallons per hour, and l length of pipes in yards.*

(Neville.) $140\sqrt{r\,s} - 11\sqrt[3]{r\,s} = v$ *in feet per second, r = hydraulic mean depth in feet, and s sine of the inclination or total fall divided by total length.*

$v\, 47.124\, d^2 = V$, and $v\, 293.7286\, d^2 =$ *Imperial gallons per minute, d = diameter of pipe in feet.*

To Compute Volume discharged.

When Length of Pipe, Height or Fall, and Diameter are given. Rule. — Divide tabular number, opposite to diameter of tube, by square root of rate of inclination, and quotient will give volume required in cube feet per minute.

Example.—A pipe has a diameter of 9 ins., and a length of 4750 feet; what is its discharge per minute under a head of 17.5 feet?

Tab. No. 9 ins. $= 1147.6$, and $\frac{1147.61}{\sqrt{\frac{4750}{17.5}}} = \frac{1147.61}{16.47} = 69.67$ *cube feet.*

To Compute Diameter.

When Length, Head, and Volume are given. RULE.—Multiply discharge per minute by square root of ratio of inclination; take nearest corresponding number in Table, and opposite to it is diameter required.

EXAMPLE.—Take elements of preceding case.

$$69.67 \times \sqrt{\frac{4750}{17.5}} = 1147.61, \text{ and opposite to this is } 9 \textit{ ins.}$$

Or, $\sqrt[5]{\frac{v\ l}{1542\ h}} = d$ *in feet.* *v representing velocity in feet per second and l length in feet.*

To Compute Head.

When Length, Discharge, and Diameter are given. RULE. — Divide tabular number for diameter by discharge per minute, square quotient, and divide length of pipe by it; quotient will give head necessary to force given volume of water through pipe in one minute.

EXAMPLE.—Take elements of preceding cases.

$$\frac{1147.61}{69.67} = 16.47;\ 16.47^2 = 271.3;\ 4750 \div 271.2 = 17.5 \textit{ feet.}$$

To Compute whole Head necessary to furnish requisite Discharge.

See Formula and Illustration, page 544.

To Compute Velocity.

When Volume and Diameter alone are given. RULE. — Divide volume when in feet per minute by area in feet, and quotient, divided by 60, will give velocity in feet per second.

EXAMPLE.—Take elements of preceding case.

$$\frac{69.67}{.75^2 \times .7854} \div 60 = 2.63 \textit{ feet.}$$

When Volume is not given. RULE. — Multiply square root of product of height of pipe by diameter in feet, divided by length in feet, by 50, and product will give velocity in feet per second. (*Beardmore.*)

To Compute Inclination of a Pipe.

When Volume, Diameter, and Length are given. $\left(\frac{V}{2356}\right)^2 \frac{1}{d^5} = \frac{h}{l}$.

ILLUSTRATION.—Take elements of preceding case.

$\left(\frac{69.67}{2356}\right)^2 \times \frac{1}{.75^5} = .000874 \times 4.214 = .00368$, and $\frac{17.5}{4750} = .00368$, or $4750 \times .00368 = 17.49$ *feet head.*

To Compute Elements of Long Pipes.

$$\frac{V}{A} = \frac{4\ V}{3.1416 \times d^2} = 1.2732 \frac{V}{d^2} = v; \quad \left(1 + c + C\frac{l}{d}\right)\frac{v^2}{2\ g} = h; \quad \frac{\sqrt{2\ g\ h}}{\sqrt{1 + c + C\frac{l}{d}}} = v;$$

and $.4787 \sqrt[5]{(1.505 \times d + c\ l)\frac{V^2}{h}} = d$ *in ins.*

This latter formula will only give an approximate dimension in consequence of unknown element d, and also of C, as $v = \frac{4\ V}{3.1416 \times d^2}$.

For Illustration, see Miscellaneous Illustration, page 556.

To Compute Vertical Height of a Stream projected from Pipe of a Fire-engine or Pump.

RULE.—Ascertain velocity of stream by computing volume of water running or forced through opening in a second; then, by Rule in Gravitation, page 488, ascertain height to which stream would be elevated if wholly unobstructed, which multiply by a coefficient for particular case.

In great heights and with small apertures, coefficients should be reduced. In consequence of the varying elements and conditions of operation of fire-engines, it is difficult to assign a coefficient for them. Difference between actual discharge and that as computed by capacity and stroke of cylinder, as ascertained by Mr. Larned, 1859, was 18 per cent. = a coefficient of .82.

A steam fire-engine of the Portland Company, discharging a stream 1.125 ins. in diameter, through 100 feet 2.5 inch hose, gave a theoretical head, computed from actual discharge, of 225 feet, and stream vertically projected was 200 feet; hence coefficient in this case was .88.

EXAMPLE.—If a fire-engine discharges 14 cube feet of water vertically through a pipe .75 inch in diameter in one minute, how high will the water be projected?

$14 \times 1728 \div .4417$ area of pipe, $\div 12$ ins. in a foot, $\div 60$ seconds $= 76.07$ *feet* velocity; and as coefficient of such a stream = at .85, then $114.1 \times .85 = 96.98$ *feet.*

Or, $H - \frac{.0022\ H^2}{d} = h$. H *representing head at nozzle, and d height of jet, both in feet, and d diameter of nozzle in ins.* (*R. F. Hartford.*)

ILLUSTRATION.—Assume head of 110 feet and diameter of nozzle .75 inch.

$$110 - \frac{.0022 \times 110^2}{.75} = 110 - 35.5 = 74.5 \textit{ feet.}$$

NOTE.—The loss of head is greater with ring than with smooth nozzles. E. B. Weston, Am. Soc. C. E., puts the difference at $.000\,171\ v^2$.

The loss of head increases with the absolute height of the jet, and is less with an increase of its diameter. This loss increases nearly in ratio of square of height of jet, and varies nearly in inverse ratio to its diameter.

Cylindrical Ajutage.

Mean coefficient as determined by Mariotte and Bossut = .003 066 square of effective head for cylindrical ajutages; hence, for conical, alike to that of an engine pipe, coefficient ranges from .72 to .9, or a mean of .81.

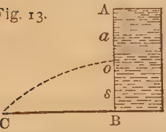
By formula of D'Aubuisson, $.003\,047\ h^2 = h'$.

Effective head, or h, in preceding example = 114.1. Then $114.1 - .003\,047 \times 114.1^2 = 114.1 - 39.67 = 74.43$ *feet height of jet.*

Hence, for a conical or engine pipe, $74.43 \times .81 = 60.29$ *feet*, or a coefficient of .535.

To Compute Distance a Jet of Water will be projected from a Vessel through an Opening in its Side.

Fig. 13.



B C, Fig. 13, is equal to twice square root of A $o \times o$ B.

If s is 4 times as deep below A as a is, s will discharge twice volume of water that will flow from a in same time, as 2 is $\sqrt{}$ of A s and 1 is $\sqrt{}$ of A a.

NOTE.—Water will spout farthest when o is equidistant from A and B; and if vessel is raised above a plane, B must be taken upon plane.

Volumes of water passing through equal apertures in same time are as square roots of their depths from surface.

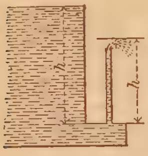
RULE.—Multiply square root of product of distance of opening from surface of water, and its height from plane upon which water flows, in feet by 2, and product will give distance in feet.

EXAMPLE.—A vessel 20 feet deep is raised 5 feet above a plane; how far will a jet reach that is 5 feet from bottom of vessel?

$$20 - 5 \times \overline{5 + 5} = 150, \text{ and } \sqrt{150} \times 2 = 24.495 \textit{ feet.}$$

Velocity of a jet of water flowing from a cylindrical tube is determined to be .974 to .98 of actual to theoretic velocity, or $= .82$ of that due to height of reservoir. Hence volume of discharge through a cylindrical opening $= .82\, a \sqrt{2\, g\, h}$.

Fig. 14.



Jets d'Eau. (Fig. 14.)

That a jet may ascend to greatest practicable height, communication with supply should be perfectly free.

Short tubes shaped alike to contracted fluid vein, and conically convergent pipes, are those which give greatest velocities of efflux. Hence, to attain greatest effect, as in fire-engines, long and slightly conically convergent tubes or pipes should be applied.

In order to diminish resistance of descending water, a jet must be directed with a slight inclination from vertical.

Effect of combined causes which diminish height of a jet from that due to elevation of its supply can only be determined by experiments. Great jets rise higher than small ones.

With cylindrical tubes, velocity being reduced in ratio of 1 to .82, and as heights of jets are as squares of these coefficients or ratios, or as 1 to .67, height of a jet through a cylindrical tube is two thirds that of head of water from which it flows.

$H\,C = h$. H *representing head of water*, C *coefficient, and* h *height of jet.* (*Molesworth.*)

When $d = H \div 300$, $C = .96$.	When $d = H \div 1500$, $C = .8$.
" " $=$ " $\div$ 450, " $= .93$.	" " $=$ " $\div$ 1800, " $= .7$.
" " $=$ " $\div$ 600, " $= .9$.	" " $=$ " $\div$ 2800, " $= .6$.
" " $=$ " $\div$ 800, " $= .87$.	" " $=$ " $\div$ 3500, " $= .5$.
" " $=$ " $\div$ 1000, " $= .85$.	" " $=$ " $\div$ 4500, " $= .25$.

FLOW OF WATER IN RIVERS, CANALS, AND STREAMS.

Running Water.—Water flows either in a natural or artificial bed or course. In first case it forms Streams, Brooks, and Rivers; in second, Drains, Cuts, and Canals.

Bed of a water-course is formed of a *Bottom* and two *Banks* or *Shores.*

Transverse Section is a vertical plane at right angles to course of the flowing water; *Perimeter* is length of this section in its bed.

Longitudinal Section or *Profile* is a vertical plane in the course or *thread of current* of flowing water.

Slope or *Declivity* is the mean angle of inclination of surface of the water to the horizon.

Fall is vertical distance of the two extreme points of a defined length of the flowing course, measured upon a horizontal plane, and this fall assigns angle for defined length of the course.

Line or *Thread of Current* is the point where flowing water attains its maximum velocity.

Mid-channel is deepest point of the bed in thread of current. *Velocity* is greatest at surface and in middle of current; and surface of flowing water is highest in current, and lowest at banks or shore.

A River, Canal, etc., is in a state of *permanency* when an equal quantity of water flows through each of its transverse sections in an equal time, or when V, *product of area of section, and mean velocity through whole extent of the stream, is a constant number.*

To Compute Mean Depth of Flowing Water.

RULE.—Set off breadth of the stream, etc., into any convenient number of divisions; ascertain mean depths of these divisions; then divide their sum by number of divisions, and quotient is the mean depth.

To Compute Mean Area of Flowing Water.

RULE 1.—Multiply breadth or breadths of the stream, etc., by the mean depth or depths, and product is the area.

2.—Divide the volume flowing in cube feet per second by mean velocity in feet per second, and quotient is area in sq. feet.

To Compute Volume of Flowing Water.

RULE.—Multiply area of the stream, etc., in sq. feet, by the mean velocity of its flow in feet, and product is volume in cube feet.

To Compute Mean Velocity of Flowing Water.

RULE.—Divide surface velocity of flow in feet per second by area of the stream, etc., and quotient, multiplied by coefficient of velocity, will give mean velocity in feet.

Mean velocity at half depth of a stream has been ascertained to be as .915 to 1, and at bottom of it as .83 to 1, compared with velocity at surface. Again, the velocity diminishes from line of current toward banks, and, to obtain mean superficial velocity, $\frac{v_1+v_2+v_3}{n}=.915\,v$; hence,

To Compute Mean Velocity in whole Profile of a Navigable River, etc.,

$\overline{V+1}-2\sqrt{V}=$ *velocity at bottom*, and $\overline{V+.5}-\sqrt{V}=$ *mean velocity*.

In rivers of low velocities multiply mean velocity by .8.

Obstruction in Rivers. (*Molesworth.*)

$\frac{v^2}{58.6}+.05\times\left(\frac{A}{a}\right)^2-1=R$. *v representing velocity in ins. per second previous to obstruction*, A *and* a *areas of river unobstructed and at obstruction in sq. feet, and* R *rise in feet.*

ILLUSTRATION.—Velocity of obstructed flow of a river is 6 feet per second, and areas of section before and after obstruction are 100 and 90 sq. feet; what would be rise in feet?

$$\frac{6^2}{58.6}+.05\times\left(\frac{100}{90}\right)^2-1=.664\times.232=.154\ \textit{feet}.$$

Flow of Water in Lined Channels. (*Bazin.*)

$\sqrt{\frac{C\,D}{F}}=V$; $\frac{1}{x\left(y+\frac{1}{D}\right)}=C$. D *representing mean hydraulic depth in feet*, F *fall, or length of channel to fall of* 1, x *and* y *as per table, and* C *as per table p.* 543.

	x	y		x	y
Plastered.........	.000 004 5	10.16	Rubble Masonry......	.000 06	1.219
Cut Stone........	.000 013	4.354	Earth...............	.000 35	.214

For Sections of Uniform Area, as Canals, Sewers, etc. $\sqrt{\frac{A}{P}\,2\,D}=v$. A = *area of flow in sq. feet*, P *wet perimeter of section, and* D *fall of stream per mile in feet.*

ILLUSTRATION.—Area of transverse section of a sewer is 50 sq. feet, its wet perimeter 20 feet, and its fall 5 feet per mile.

$\sqrt{\left(\frac{50}{20}\times2\times5\right)}=\sqrt{25}=5$ *feet*. *For Sections of Rivers.* $12\sqrt{D\,\frac{A}{P}}=v$.

ILLUSTRATION.—Assume area 500 sq. feet, wet perimeter 200, and fall 5 feet per mile.

$$12\sqrt{5\times\frac{500}{200}}=12\sqrt{12.5}=42.4\ \textit{feet}.$$

Hydraulic Radius or Mean Depth is obtained by dividing area of transverse section by wet perimeter, both in feet.

To Compute Fall per Mile for a required Mean Velocity.

$\left(\frac{v \times 12}{12}\right)^2 \div 2\,r = D.$ *r representing hydraulic radius in ins.*

Upper surface of flowing water is not exactly horizontal, as water at its surface flows with different velocities with respect to each other, and consequently exert on each other different pressures.

If v and v_1 are velocities at line of current and bank of a stream, the difference of the two levels is $\frac{v^2 - v_1^2}{2\,g} = h$.

Illustration.—If $v = 5$ *feet*, and v_1 .9 v; then $\frac{5^2 - \overline{.9 \times 5}^2}{2\,g} = \frac{4.75}{64.33} = .0738$ *foot.*

A velocity of 7 to 8 ins. per second is necessary to prevent deposit of slime and growth of grass, and 15 ins. is necessary to prevent deposit of sand.

Maximum velocity of water in a canal should depend on character of bed of the channel.

Thus, *Mean Velocity* should not exceed per second over

Fine clay..........	6 ins.	River sand..........	1 ft.	Broken stones......	4 ft.
A slimy bed.......	8 "	Small gravel........	1 "	Stones..............	6 "
Common clay......	6 "	Large shingle........	3 "	Loose rocks.........	10 "

To Compute Velocity of Flow or Discharge of Water in Streams, Pipes, Canals, etc.

1. *When Volume discharged per Minute is given in Cube Feet, and Area of Canal, etc., in Sq. Feet.* Rule.—Divide volume by area, and quotient, divided by 60, will give velocity in feet per second.

2. *When Volume is given in Cube Feet, and Area in Sq. Ins.* Rule.—Divide volume by area; multiply quotient by 144, and divide product by 60.

3. *When Volume is given in Cube Ins., and Area in Sq. Ins.* Rule.—Divide volume by area, and again by 12 and by 60.

To Compute Flow or Volume of Discharge.

1. *When Area is given in Sq. Feet.* Rule.—Multiply area of flow by its velocity in feet per second, and product, multiplied by 60, will give volume in cube feet per minute.

2. *When Area is given in Sq. Ins.* Rule.—Multiply area by its velocity, and again by 60, and divide product by 144.

Note 1.—Velocities and discharges here deduced are theoretical, actual results depending upon coefficient of efflux used. Mean velocity, however, as before given, page 529, may be taken at $\sqrt{2\,g}\ .673 = 5.4$ feet, instead of 8.02 feet.

2.—As a rule, with large bodies, as vessels, etc., their floating velocity is somewhat greater than that of flow of water, not only because in floating they descend an inclined plane, formed by surface of the water, but because they are but slightly affected by the irregular intimate motion of water: the variation for small bodies is so slight that it may be neglected.

To Compute Height of Head of Flowing Water.

When Volume and Area of Flow are given in Feet. Rule.—Divide volume in feet per second by product of area, and $\frac{5}{8}$ coefficient for opening, and square of quotient, divided by 64.33, will give height in feet.

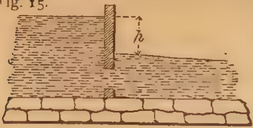
Example.—Assume volume 266.48 cube feet, area 40 sq. feet, and C = .623.

Then $\left(\frac{266.48}{40 \times \frac{5}{8} .623}\right)^2 \div 64.33 = \frac{257.28}{64.33} = 4$ *feet.*

Submerged or Drowned Orifices and Weirs.

When wholly submerged (Fig. 15).—Available pressure at any point in depth of orifice is equal to difference of pressure on each side.

Fig. 15.



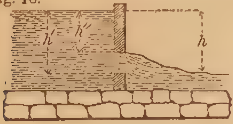
Whence, $C\sqrt{2gh}=v$, and $Ca\sqrt{2gh}=V$.
a representing area of sluice in sq. feet.

ILLUSTRATION.—Assume opening 3 feet by 5, $h=4$ feet, and $C=.5$.

Then, $.5\times3\times5\sqrt{64.33\times4}=7.5\times16.04=$ 120.3 *cube feet per second.*

When partly submerged (Fig. 16). $h'-h=d=$ submerged depth, and $h-h''=d'=$ remaining portion of depth; whence $d'+d=$ entire depth, and

Fig. 16.

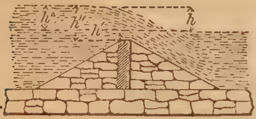


$$C\,l\sqrt{2g}\,(d\sqrt{h}+\tfrac{2}{3}h\sqrt{h-h''\sqrt{h''}})=V.$$

ILLUSTRATION.—Assume opening as above, $h=4$ *feet*, $h'=6$, $h''=3$, and $C=.5$. Then $d=6-4=2$ *feet.*

Then $.5\times5\times8.02\,(2\sqrt{4}+\frac{2}{3}\times4\sqrt{4-3\sqrt{3}})$ $=20.05\times5.869=117.67$ *cube feet per second.*

Fig. 17.



When drowned (Fig. 17).

$$C\,l\sqrt{2gh}\,(d+\tfrac{2}{3}h)=V.$$

ILLUSTRATION.—Assume opening as above, $h=4$ *feet*, $d=2$, and $C=.52$.

Then, $.52\times5\times\sqrt{64.33\times4}\times(2+\frac{2}{3}4)=2.6\times16.04\times4.66=194.34$ *cube feet per second.*

CANAL LOCKS.

Single Locks.

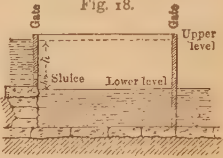
When a fluid passes from one level or reservoir to another, through an aperture covered by the fluid in the latter, effective head on each point of aperture, and consequently head due to velocity of efflux at each instant, is the difference of levels of the two reservoirs at that instant.

Hence $C\,a\sqrt{2gh'}=V$ per second. *h' representing difference of levels.*

To Compute Time of Filling and Discharging a Single Lock.—Fig. 18.

When Sluice in Upper Gate is entirely under Water, and above Lower Level.

Fig. 18.



$\dfrac{A\,h'}{C\,a\sqrt{2gh}}=$ *time of filling up to centre of sluice.*

h representing height of centre of sluice in upper gate from surface of canal or reservoir, and h' height of centre of sluice in upper gate from lower surface, or water in the lock or river, all in feet; and

$\dfrac{2\,A\,h}{C\,a\sqrt{2gh}}=$ *time of filling the remaining space, where a gradual diminution of head of water occurs.*

Consequently, $\dfrac{(h'+2h)\,A}{C\,a\sqrt{2gh}}=t$ *time of filling a single lock.*

When Aperture or Sluice in Lower Gate is entirely under Water, and above Lower Level. $\dfrac{2\,A\sqrt{h+h'}}{C\,a'\sqrt{2g}}=$ *time of emptying or discharging it. a' representing area of lower sluice.*

Illustration.—Mean dimensions of a lock, Fig. 18, are 200 feet in length by 24 in breadth; height of centre of aperture of sluice from upper and lower surfaces is 5 feet; breadth of both upper and lower sluices is 2.5 feet; height of upper is 4 feet, and of lower—entirely under water—5 feet; required the times of filling and discharging.

$h=5$, $h'=5$, $A=200\times24=4800$, $C=.545$, $a=4\times2.5=10$, $a'=5\times2.5=12.5$.

$\frac{4800\times5}{.545\times10\times\sqrt{2gh}}=\frac{24\,000}{97.72}=245.59$ *seconds = time of filling lock up to centre of sluice;* and $\frac{2\times4800\times5}{.545\times10\times\sqrt{2gh}}=\frac{48\,000}{97.72}=491.18$ *seconds = time of filling remaining space, or lock above centre of sluice,* and $245.59+491.18=736.77$ *seconds, whole time.*

Or, $\frac{(5+\overline{2\times5})\times4800}{.545\times10\times\sqrt{2gh}}=\frac{72\,000}{97.72}=736.77$ *sec. = time of filling.* $\frac{2\times4800\sqrt{5+5}}{.545\times12.5\times\sqrt{2g}}=\frac{30\,358.08}{54.7}=554.9$ *seconds = time of discharging.*

When Aperture or Sluice in Upper Gate is entirely under Water and below Lower Level. $\frac{2A\sqrt{h-h'}}{Ca\sqrt{2g}}$ = *time of filling lock.*

When Sluice in the Lower Gate is in part above Surface of Lower Level and in part below it. $\frac{2A(h+h')}{Cb\sqrt{2g}\left(d\sqrt{h+h'-\frac{d}{2}}+d'\sqrt{h+h'}\right)}$ = *time of discharging. d and d′ representing distances of part of aperture above and of below surface of lower water, b breadth of aperture, and h and h′ as before.*

Illustration.—Assume sluice in preceding example to be 1 foot above lower level of water, or that of lower canal; what is time of discharge of lock, distance of part of aperture 1 foot and of that below surface of water 4 feet?

$$\frac{2\times4800(5+5)}{.545\times2.5\times8.02[1\times\sqrt{5+5-(1\div2)}+4\times\sqrt{5+5}]}=\frac{96\,000}{10.93\times(3.082+12.65)}=\frac{96\,000}{171.95}=558.3 \textit{ seconds.}$$

Double Lock. *(J. D. Van Buren, Jr.)*

A double lock is not a duplication of a single lock in its operation, for in lower chamber supply of water is from upper one, having no influx, instead of a uniform supply flowing directly from surface level of canal or feeder.

Operation, therefore, of a double lock is complex, addition to formula for a single lock being that of discharging of water in upper lock to fill lower, the head of water gradually decreasing in the chamber, which is closed from upper reach during discharge into lower.

Fig. 19.

To Compute Time required for Water to Fall from Upper to Uniform Water Level.

1. $\frac{A}{Ca\sqrt{g}}\left(\sqrt{f}+\sqrt{2h}-\sqrt{2h-2d}\right)=t$. *A representing horizontal area of lock, and a area of sluice opening, both in sq. feet, C coefficient of discharge = .545 for openings with square arrises, g acceleration of gravity, f depth of centre of sluice*

below uniform level, h depth of centre sluice opening below upper water level, and d height of centre of sluice above lower water level, all in feet, and t time for water to fall from upper to uniform water level, in seconds.

ILLUSTRATION.—$A = 2000$ *sq. feet;* $C = .545$; $a = 5$; $f = 6$; $h = 14$; and $d = 2$ *feet.* (Fig. 19.)

Then, $\frac{2000}{.545 \times 5 \times 5.67} = \frac{2000}{15.45} \times \overline{7.74 - 4.9} = 367.6$ *seconds.*

2. If $d = 0$; $\frac{A\sqrt{f}}{C\,a\sqrt{g}} = t$; $= \frac{2000 \times \sqrt{8}}{.545 \times 5 \times 5.67} = \frac{5660}{15.45} = 366.34$ *seconds.*

NOTE.—f is never greater than l (lift in feet); it is equal to l when $d = 0$; f_2 is equal to l when $f_1 = 0$, never greater. In each case it is the unbalanced head above sluice, however far below the lowest water level the sluice is.

To Fill Upper Lock or Empty Lower.

To fill upper lock or empty lower, when the sluice is below the lowest water-line, in either case, takes the same time; for the head diminishes at the same rate, one from the upper surface, the other from the bottom.

3. $\frac{A\sqrt{2f}}{C\,a\sqrt{g}} = t.$ Here, *f being below lowest water level of lock* $= 8$ *feet*, as $d = 0$, *and* $f =$ *whole lift* $= \frac{2000\sqrt{2 \times 8}}{.545 \times 5 \times 5.67} = \frac{8000}{15.45} = 517.8$ *seconds.*

To Discharge a like Volume under a Constant Head.

4. $\frac{A\sqrt{f}}{C\,a\sqrt{2g}} = \frac{A}{C\,a}\sqrt{\frac{f}{2g}} = t. \quad = \frac{2000}{.545 \times 5}\sqrt{\frac{8}{64.33}} = 258.9$ *seconds,*

Or, one half the time given by preceding case.

The times deduced by preceding formulas are in the following proportions in order, as $1 : \sqrt{2} : \frac{\sqrt{2}}{2}$, or $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$.

If sluice of upper lock, through which it is filled, is above lowest water level, then, by combining formulas 3 and 4, the time is thus deduced.

To fill from Lowest Water Level of said Lock to Level of Centre of Sluice.

5. $\frac{A\sqrt{f'}}{C\,a\sqrt{2g}} = t'.$ *f' representing height of centre of sluice above said lowest water level.*

To fill remaining Portion of Lock above Sluice.

6. $\frac{2A\sqrt{f''}}{C\,a\sqrt{2g}} = t''.$ *f'' representing depth below upper water level of centre of sluice or remaining portion of lift.* Hence, $t' + t'' = \frac{A}{C\,a\sqrt{2g}}(\sqrt{f'} + 2\sqrt{f''}) = t.$

To fill Lower Lock under Constant Head from Upper Canal Level.

7. $$\frac{A\sqrt{h}}{C.a\sqrt{2g}}\left(2 + \frac{d}{h} - \frac{2\sqrt{h-f}}{\sqrt{h}}\right) = t.$$

8. If both lifts are the same, $h - f = l$, and $\frac{A\sqrt{h}}{C\,a\sqrt{2g}} \cdot \left(2 + \frac{d}{h} - 2\sqrt{\frac{l}{12}}\right) = t.$

If lower lock is filled from upper one under a constant head, when latter is drawn down to lowest level, formula 7 will apply by making $h = f$, and

$\frac{A}{C\,a\sqrt{2g}}\left(2\sqrt{f} + \frac{d}{\sqrt{f}}\right)$, which is identical with 7, for $f = f_2$ and $d = f$, the cases being the same.

MISCELLANEOUS ILLUSTRATIONS.

1. If external height of fresh water, at 60° above injection opening in condenser of a steam-engine, is 3 feet, and the indicated vacuum at 23 ins., velocity of water flowing into condenser is thus determined. *(Formula page 532.)*

$v = \sqrt{2\,g\,(h + h')}$. *h′ representing height of a column of water equivalent to pressure of atmosphere within condenser.*

Assuming mean pressure of atmosphere = 14.7 *lbs. per sq. inch*, height of a column of fresh water equivalent thereto = 33.95 *feet.*

Then, if 1 inch = .4912 *lbs.*, 23 ins. = 11.3 *lbs.;* and if 14.7 *lbs.* = 33.95 *feet*, 11.3 *lbs.* = 26.1 *feet.*

Hence $v = \sqrt{2\,g\,(3 + 26.1)} = 43.27$ *feet*, less retardation due to coefficient of both influx and efflux.

2. What breadth must be given to a rectangular weir, to admit of a flow of 6 cube feet of water, under a head of 8 ins.? *(Formula page 533.)*

$$\frac{6}{\frac{2}{3} \times .625\sqrt{2g\ .66}} = \frac{6}{.417 \times 6.55} = 2.21 \textit{ feet.}$$

3. It being required to ascertain volume of water flowing in a stream, a temporary dam is raised across it, with a notch in it 2 feet in breadth by 1 in depth, which so arrests flow that it raises to a head of 1.75 feet above sill of notch; what is volume of flow per second? *(Formula page 533.)*

$$C = .635. \quad \frac{2}{3} \times .635 \times 2 \times 1.75\,\sqrt{2\,g \times 1.75} = 1.481 \times 10.6 = 15.7 \textit{ cube feet.}$$

4. A rectangular sluice 6 feet in breadth by 5 in depth, has a depth of 9 feet of water over its sill, and discharges, as per example page 535, 380.95 cube feet per second; what is velocity of flow? *(Formula page 535.)*

$$\frac{380.95}{6 \times (9 - 4)} = \frac{380.95}{30} = 12.7 \textit{ feet.}$$

If volume was not given: $\frac{2}{3}\,C\sqrt{2\,g} \times \frac{\sqrt{h^3} - \sqrt{h'^3}}{h - h'} = v.\quad C = .625.$

$$\text{Then } \frac{2}{3} \times .625 \times 8.02 \times \frac{\sqrt{729} - \sqrt{64}}{9 - 4} = 3.341 \times 3.8 = 12.7 \textit{ feet.}$$

5. If a river has an inclination of 1.5 feet per mile, is 40 feet in breadth with nearly vertical banks, and 3 feet depth; what is volume of its discharge? *(Formula p. 542.)*

Perimeter $40 + 2 \times 3 = 46$ *feet;* *hydraulic mean depth* $\frac{120}{46} = 2.61$ *feet;*

$a = 120$ *feet;* C *per table*, page 543, *for assumed velocity of* 2.5 *feet* = .0075.

$$\text{Then } \sqrt{\frac{120}{.0075 \times 5280 \times 46} \times 64.33 \times 1.5} = \sqrt{.0659 \times 96.5} = 2.52 \textit{ feet velocity.}$$

Hence $120 \times 2.52 = 302.4$ *cube feet.*

6. What is head of water necessary to give a discharge of 25 cube feet of water per minute, through a pipe 5 ins. in diam. and 150 feet in length? *(Formula p. 548.)*

Tabular number for diameter 5 *ins., page* 547, = 263.87.

Then $\overline{263.87 \div 25}^{\,2} = 111.3$, and $150 \div 111.3 = 1.35$ *feet.*

If this pipe had 2 rectangular knees or bends, what then would be head of water required? *(Formula page 545.)*

C, *page* 545, *for* $\frac{90°}{2} = .984$, *area of* 5 *ins.* = .136 *feet, and* $\frac{25}{.136} \div 60 = 3.06$ *feet velocity.* Then $\frac{3.06^2}{64.33} \times .984 \times 2 = .2863$, *which, added to* $1.35 = 1.64$ *feet.*

By formulas foot of page 548, C = .024, and c .505 *velocity* = 3.06 *feet; head* = 1.49 *feet*, and *volume* 26.38 *cube feet.*

7. If a stream of water has a mean velocity of 2.25 feet per second at a breadth of 560 feet, and a mean depth of 9 feet, what will be its mean velocity when it has a breadth of 320 feet, and a mean depth of 7.5 feet? *(Rule page 548.)*

$$\frac{560 \times 9 \times 2.25}{320 \times 7.5} = \frac{11\,340}{2400} = 4.725 \textit{ feet.}$$

8. What volume will a pipe 48 feet in length and 2 ins. in diameter, under a head of 5 feet, deliver per second? (*Formula page* 547.)

Tabular number for diameter 2 *ins., page* 547, = 26.69.

$$\sqrt{\frac{48}{5}} = 3.1. \quad \text{Then } \frac{26.69}{3.1} = 8.61, \text{ which} \div 60 = .143 \textit{ cube feet.}$$

If this pipe had 5 curves of 90°, with radii $\frac{d}{2\,r} = \frac{2}{4} = .5$; what would be its discharge per second?

$V = .143$; $a = 2 \div 144 = .0139$; *C per table* $= \frac{d}{2\,r} = .294$; $v = \frac{.143}{.0139} = 10.29$ *feet.*

Then $.294 \times \frac{90^{\circ}}{180^{\circ}} \times \frac{10.29^2}{64.33} = .147 \times 1.64 = .241$, *which* $\times$ 5 *for* 5 *curves* = 1.2 = *height due to resistance of curves.* $h = 5 - 1.2 = 3.8$.

Hence, if $\sqrt{2\,g\;5} = .143$; $\sqrt{2\,g\;3.8} = .125$ *cube feet.*

9. If a slide stop valve, set in a cylindrical conduit 500 feet in length and 3 ins. in diameter, is raised so as to close .625 of conduit; what volume will it discharge under a head of 4 feet? (*Formula page* 546.)

C *for conduit* = .5, *for friction* .025, *and for slide valve* .375 *open, table, page* 545, 5.52, $d = .25$, and $a = 7.07$ *sq. ins.*

$$\text{Then } \frac{2\,g\,h}{\sqrt{\left(1 + .5 + 5.52 + .025\,\frac{500}{.25}\right)}} = \frac{16.06}{\sqrt{(7.02 + 50)}} = 2.13 \textit{ feet velocity, and}$$

$2.13 \times 12 \times 7.07 = 180.71$ *cube ins.*

10. If a single lock chamber is 200 feet in length by 24 in breadth, with a depth of 10 feet, centre of upper gate, which is 4 feet in depth by 2.5 in breadth, is at middle of depth of chamber, lower gate, 5 feet in depth by 2.5 in breadth and wholly immersed; what is time required for filling and discharging it? (*Formula p.* 553.)

$C = .615$, $h = 5$, $h' = 5$, $A = 200 \times 24 = 4800$, $a = 4 \times 2.5 = 10$, and $a' = 5 \times 2.5 = 12.5$

$$\frac{(2 \times 5 + 5)\,4800}{.615 \times 10\sqrt{64.33 \times 5}} = \frac{72\,000}{110.27} = 652.8 \textit{ seconds time of filling.}$$

$$\frac{2 \times 4800 \times \sqrt{5 + 5}}{.615 \times 12.5\,\sqrt{2\,g}} = \frac{30\,336}{61.73} = 491.4 \textit{ seconds time of emptying.}$$

11. In a moderately direct and uniform course of a river, the depths and velocities are as follows; what is the volume of its flow and what its mean velocity? (*p.* 551.)

	Feet.	Feet.	Feet.	Feet.	Feet.
Distances..........	5	12	20	15	7
Depths	3	6	11	8	4
Mean velocity......	1.9	2.3	2.8	2.4	2.1

Area of profiles $= 5 \times 3 + 12 \times 6 + 20 \times 11 + 15 \times 8 + 7 \times 4 = 455$ *sq. feet.*

$15 \times 1.9 + 72 \times 2.3 + 220 \times 2.8 + 120 \times 2.4 + 28 \times 2.1 = 1156.9$ *cube feet volume,* and $\frac{1156.9}{455} = 2.54$ *feet velocity.*

Miner's Inch.

A "Miner's inch" is a measure for flow of water, and is an opening one inch square through a plank two inches in thickness, under a head of six inches of water to upper edge of opening.

It will discharge 11.625 U. S. gallons water in one minute.

Theoretical HP under different Heads.

Heads in feet.	100	90	80	70	60	50	40	30	20	15	10	5	3	1
Ins. per HP...	3.25	3.61	4.06	4.64	5.41	6.5	8.12	10.8	16.2	21.6	32.5	65	108	325

Water Inch (*Pouce d'eau*).—Circular opening of 1 inch in a thin plate is equal to a discharge of 19.1953 cube meters per 24 hours.

HYDRODYNAMICS.

Hydrodynamics treats of the force of action of Liquids or Inelastic Fluids, and it embraces *Hydraulics* and *Hydrostatics:* the former of which treats of liquids in motion, as flow of water in pipes, etc., and latter of pressure, weight, and equilibrium of liquids in a state of rest.

Fluids are of two kinds, aeriform and liquid, or elastic and inelastic, and they press equally in all directions, and any pressure communicated to a fluid at rest is equally transmitted throughout the whole fluid.

Pressure of a fluid at any depth is as depth or vertical height, and pressure upon bottom of a containing vessel is as base and perpendicular height, whatever may be the figure of vessel. Pressure, therefore, of a fluid, *upon any surface*, whether *Vertical*, *Oblique*, or *Horizontal*, is equal to weight of a column of the fluid, base of which is equal to surface pressed, and height equal to distance of centre of gravity of surface pressed, below surface of the fluid.

Side of any vessel sustains a pressure equal to its area, multiplied by half depth of fluid, and whole pressure upon bottom and against sides of a vessel is equal to three times weight of fluid.

Pressure upon a number of surfaces is ascertained by multiplying sum of surfaces into depth of their common centre of gravity, below surface of fluid.

When a body is partly or wholly immersed in a fluid, vertical pressure of the fluid tends to raise the body with a force equal to weight of fluid displaced; hence weight of any quantity of a fluid displaced by a buoyant body equals weight of that body.

Centre of Pressure is that point of a surface against which any fluid presses, to which, if a force equal to whole pressure were applied, it would keep surface at rest. Hence distance of centre of pressure of any given surface from surface of fluid is same as *Centre of Percussion.*

Centres of Pressure.

Parallelogram, Side, Base, Tangent, or Vertex of Figure at Surface of Fluid, is at .66 of line (measuring downward) that joins centres of two horizontal sides.

Triangle, Base uppermost, is at centre of a line raised vertically from lower apex, and joining it with centre of base; and *Vertex uppermost*, it is at .75 of a line let fall perpendicularly from vertex, and joining it with centre of base.

Right angled Triangle, Base uppermost, is at intersection of a line extended from centre of base to extremity of triangle by a line running horizontally from centre of side of triangle. *Vertex or Extremity uppermost*, is at intersection of a line extended from the centre of the base to the vertex, by a line running horizontally from .375 of side of triangle, measured from base.

Trapezoid, either of parallel Sides at Surface, $\frac{b + 3\,b'}{2\,b + 4\,b'} \times a = d$. *b and b' representing breadths of figure, d distance from surface of fluid, and a length of line joining opposite sides.*

Circle, at 1.25 of its radius, measured from upper edge.

Semicircle, Diameter at Surface of Fluid, $\frac{3\,p\,r}{16} = d$. *r representing radius of circle and* $p = 3.1416$. *Diam. downward,* $\frac{15\,p\,r - 32\,r}{12\,p - 16} = d$.

Side, Base, or Tangent of Figure below Surface of Fluid.

Rectangle or Parallelog'm. $\frac{2}{3} \times \frac{h'^3 - h^3}{h'^2 - h^2} = d$; or, $\frac{3\,m\,o + m^2}{3\,o} = d$; and $\frac{m^2}{3\,o} = d''$.
h and h' representing depths of upper and lower surfaces of figure and d depth, both from surface of fluid, m half depth of figure, o depth of centre of gravity of figure from surface of fluid, d' distance from upper side of figure, and d'' distance from centre of gravity.

Triangle.—Vertex Uppermost. $\frac{l^2 + 18\,o^2}{18\,o} = d$; $\frac{l^2}{18\,o} = d'$. *Base Uppermost.* $\frac{l^2 + 18\,o^2}{18\,o} = d$. *l representing depth of figure, d distance from surface of fluid upon a line from vertex to centre of base, and d' distance from centre of gravity of figure.*

Circle. $\frac{4\,o^2 + r^2}{4\,o} = d$, or $\frac{r^2}{4\,o}$ = *distance from centre of circle.*

Semicircle.—Diam. Horizontal and Upward or Downward. $\frac{l^2}{4\,o} - \frac{16\,l^2}{9\,p\,o} + o = d$; $\frac{3\,p\,l - 4\,l}{3\,p} = d'$; $\frac{4\,l}{3\,p} = d''$, and $\frac{l^2}{4\,o} - \frac{16\,l^2}{9\,p\,o} = c$. *d representing distance from surface of fluid, d' distance of centre of gravity from centre of arc, d'' distance of centre of gravity from diameter when it is uppermost, and c centre of pressure.*

Pressure.

To Compute Pressure of a Fluid upon Bottom of its Containing Vessel.

Rule.—Multiply area of base by height of fluid in feet, and product by weight of a cube foot of fluid.

To Compute Pressure of a Fluid upon a Vertical, Inclined, Curved, or any Surface.

Rule.—Multiply area of surface by height of centre of gravity of fluid in feet, and product by weight of a cube foot of fluid.

Example 1.—What is pressure upon a sloping side of a pond of fresh water 10 feet square and 8 feet in depth?

Centre of gravity, $8 \div 2 = 4$ *feet from surface.* Then $10^2 \times 4 \times 62.5 = 25\,000$ *lbs.*

2.—What is pressure upon staves of a cylindrical reservoir when filled with fresh water, depth being 6 feet, and diameter of base 5 feet?

$5 \times 3.1416 = 15.708$ *feet curved surface of reservoir,* which is considered as a plane.

$$15.708 \times 6 \times \overline{6 \div 2} = 282.744, \text{ which } \times 62.5 = 17\,671.5 \textit{ lbs.}$$

3.—A rectangular flood-gate in fresh water is 25 feet in length by 12 feet deep; what is pressure upon it?

$$25 \times 12 \times \overline{12 \div 2} = 1800, \text{ which } \times 62.5 = 112\,500 \textit{ lbs.}$$

When water presses against both sides of a plane surface, there arises from resultant forces, corresponding to the two sides, a new resultant, which is obtained by subtraction of former, as they are opposed to each other.

Illustration.—Depth of water in a canal is 7 feet; in its adjoining lock it is 4 feet, and breadth of gates is 15 feet; what mean pressure have they to sustain, and what is depth of point of its application below surface?

$7 \times 15 = 105$, and $4 \times 15 = 60$ *sq. feet.* $(105 \times \frac{7}{2} - \overline{60 \times 2}) \times 62.5 = 1546.875$ *lbs., mean pressure.*

Then $1546.875 \div 62.5 = 247.5$ = *cube feet pressing upon gates upon high side,* and $247.5 \div 15 \times 7 = 2.35$ *feet = depth of centre of gravity of mean pressure.*

To Compute Pressure on a Sluice.

$A\,w\,d = P$, and $C\,P = P'$. *A representing area of sluice in sq. feet, w weight of water per cube foot, d mean depth of sluice below surface, in feet, P pressure on sluice, and P' power required to operate it, both in lbs.*

C = .68 when sluice is of wood, and .31 when of iron.

EXAMPLE.—What is pressure on a sluice-gate 3 feet square, its centre of gravity being 30 feet below surface of a pond of fresh water?

$3 \times 3 \times 30 = 270$, which $\times 62.5 = 16875$ *lbs.*

To Compute Pressure of a Column of a Fluid per Sq. Inch.

RULE.—Multiply height of column in feet by weight of a cube foot of fluid, and divide product by 144; quotient will give weight or pressure per sq. inch in lbs.

NOTE.—When height is given in ins., omit division by 144.

PIPES.

To Compute required Thickness of a Pipe.

RULE.—Multiply pressure in lbs. per sq. inch by diameter of pipe in ins., and divide product by twice assumed tensile resistance or *value* of a sq. inch of material of which pipe is constructed.

By experiment, it has been found that a cast iron pipe 15 ins. in diameter, and .75 of an inch thick, will support a head of water of 600 feet; and that one of oak, of same diameter, and 2 ins. thick, will support a head of 180 feet?

EXAMPLE 1.—Pressure upon a cast-iron pipe 15 ins. in diameter is 300 lbs. per sq. inch; what is required thickness of metal?

$300 \times 15 = 4500$, which $\div \overline{3000 \times 2} = .75$ *inch.*

NOTE.—Here 3000 is taken as *value* of tensile strength of cast iron in ordinary small water-pipes. This is in consequence of liability of such castings to be imperfect from honey-combs, springing of core, etc.

2.—Pressure upon a lead pipe 1 inch in diameter is 150 lbs. per sq. inch; what is required thickness of metal?

Here 500 is taken as *value* of tensile strength.

$150 \times 1 = 150$, which $\div \overline{500 \times 2} = .15$ *inch.*

Cast-iron Pipes.

To Compute Thickness, etc., of Flanged Pipes.

For 75 lbs. Pressure.		For 100 lbs. Pressure.	
$.025\,D + .25$	$= T$	$.03\,D + .3$	$= T$
$.03\,D + .3$	$= t$	$.035\,D + .45$	$= t$
$.05\,D + 1.15$	$= l$	$.05\,D + 1.15$	$= l$
$.03\,D + .35$	$= f$	$.04\,D + .6$	$= f$
$1.05\,D + \overline{4.25\,d} + 1.25$	$= o$	$1.1\,D + \overline{5 \times d} + 1.5$	$= o$
$1.05\,D + 2 \times d + 1$	$= o'$	$1.1\,D + 2.5 \times d + 1.4$	$= o'$

$$.7\,D + 2.2 = n; \quad \frac{A \times p \div n}{4000} = a, \text{ and } \sqrt{\frac{a}{.7854}} + C = d.$$

D *representing diam. of pipe*, T *thickness of metal*, *t thickness and l length of boss*, *f thickness of flange*, *o diam. of flange*, *o′ diam. of centres at bolt holes, and d diam. of bolts, all in ins.;* A *area of pipe and a area of bolt at base of its thread, in sq. ins.*, *p pressure in lbs. per sq. inch, and C a coefficient due to diam. of bolt.*

Thus, diam. $.125 + .032$, $.25 + .064$, $.5 + .107$, $1 + .16$, $1.5 + .214$, and $2 + .285$.

ILLUSTRATION.—What should be dimensions of a flanged pipe, 10 ins. in diameter, for a pressure of 100 lbs. per sq. inch?

$.7 \times 10 + 2.2 = 9.2 = 10$ *number of bolts*, and *diam.* 10 *ins.* $= 78.54$ *ins. area* $=$ A.

$\frac{78.54 \times 100 \div 10}{4000} = .19635$, and $\sqrt{\frac{.19635}{.7854}} + C = \sqrt{.25} = .5$; hence, $.5 + .107 = .607 = .625$ *lbs. diameter of bolts;* $.03 \times 10 + .3 = .6 =$ *thickness of metal;* $.035 \times 10 + .45 = .8 =$ *thickness of flange;* $.05 \times 10 + 1.15 = 1.65 =$ *length of boss;* $.04 \times 10 + .6 = 1 =$ *thickness of flange;* $1.1 \times 10 + \overline{5 \times .625} + 1.5 = 15.625 =$ *diameter of flange;* and $1.1 \times 10 + 2.5 \times .625 + 1.4 = 13.9625 =$ *diameter of bolt holes.*

For Tables of Cast-iron Pipes, see page 132.

To Compute Elements of Water-pipes.

$.000\,124\,5\ P\,d + C = t$; or, $.000\,054\ H\,d + C = t$; $.4336\ H = P$; and $D^2 - d^2 \times 2.45 = W$. P *representing pressure of water in lbs. per sq. inch*, D *and* d *external and internal diameters of pipe, and* t *thickness of metal, all in ins.*, C *coefficient for diameter of pipe, and* H *head of water in feet.*

C = .37 for pipes less than 12 ins. in diameter, .5 from 12 to 30, and .6 from 30 to 50.

To Compute Weight of Pipes.

To Diameter add thickness of metal, multiply sum by 10 times thickness, and product will give weight in lbs. per foot of length.

Weight of Faucet end is equal to 8 ins. of length of pipe.

Hydrostatic Press.

To Compute Elements of a Hydrostatic Press.

$\frac{P\,l\,A}{l'\,a} = W$; $\frac{W\,l'\,a}{P\,l} = A$; $\frac{W\,l'\,a}{l\,A} = P$; $\frac{P\,A\,l}{W\,l'} = a$. P *representing power or pressure applied*, W *weight or resistance in lbs.*, l *and* l′ *lengths of lever and fulcrum in ins. or feet, and* A *and* a *areas of ram and piston in sq. ins.*

ILLUSTRATION.—Areas of a ram and piston are 86.6 and 1 sq. ins., lengths of lever and fulcrum 4 feet and 9 ins., and power applied 20 lbs.; what is weight that may be sustained?

$$\frac{20 \times \overline{4 \times 12} \times 86.6}{9 \times 1} = \frac{83\,136}{9} = 9237.3\ lbs.$$

To Compute Thickness of Metal to Resist a given Pressure.

RULE.—Multiply pressure per sq. inch in lbs. by diameter of cylinder in ins., and divide product by twice estimated tensile resistance or *value* of metal in lbs. per sq. inch, and quotient will give thickness of metal required.

EXAMPLE.—Pressure required is 9000 lbs. per sq. inch, and diameter of cylinder is 5.3 ins.; what is required thickness of metal of cast iron?

Value of metal is taken at 6000. $\frac{9000 \times 5.3}{6000 \times 2} = \frac{47\,700}{12\,000} = 3.975\ ins.$

Values of Different Metals in Tons. (*Molesworth.*)

Cast iron...... .41 | Gun metal..... .22 | Wrought iron.. .14 | Steel......... .06

Hydraulic Ram.

Useful effect of an Hydraulic Ram, as determined by Eytelwein, varied from .9 to .18 of power expended. When height to which water is raised compared to fall is low, effect is greater than with any other machine; but it diminishes as height increases.

Length of supply pipe should not be less than .75 of height to which water is to be raised, or 5 times height of supply; it may be much longer.

To Compute Elements.

$.001\,13\ V\,h = \text{IP}$; $\frac{881\ \text{IP}}{h} = V$; $1.45\sqrt{V} = D$; $.75\sqrt{V} = d$; and $\frac{5}{6} \times \frac{v\,h'}{V\,h} =$ *efficiency.* V *and* v *representing volumes expended and raised, in cube feet per minute,* h *and* h′ *heights from which water is drawn and elevated in feet,* D *and* d *diameters of supply and discharging pipes in ins., and* IP *effective horse-power.*

ILLUSTRATION.—Heights of a fall and of elevation are 10 and 26.3 feet, and volumes expended and raised per minute are 1.71 and .543 cube feet.

$.001\,13 \times 1.71 \times 10 = .0193\ \text{IP}$; $\frac{881 \times .0193}{10} = 1.71$ *cube feet;* $1.45\sqrt{1.71} = 1.89$ *ins.;* $.75\sqrt{1.71} = .975$ *ins.;* and $\frac{5}{6} \times \frac{.543 \times 26.3}{1.71 \times 10} = .696$ *efficiency.*

Results of Operations of Hydraulic Rams.

Strokes per M.	Fall.	Elevation.	Water Expen'd.	Water Raised.	Useful Effect.	Strokes per M.	Fall.	Elevation.	Water Expen'd	Water Raised.	Useful Effect.
No.	Feet.	Feet.	C. Ft.	C. Ft.		No.	Feet.	Feet.	C. Ft.	C. Ft.	
66	10.06	26.3	1.71	.543	.9	15	3.22	38.6	1.98	.058	.35
50	9.93	38.6	1.93	.421	.85	10	1.97	38.6	1.58	.014	.18
36	6.05	38.6	1.43	.169	.75	—	22.8	196.8	.38	.029	.67
31	5.06	38.6	1.29	.113	.67	—	8.5	52.7	2	.186	.57

NOTE.—Volume of air vessel = volume of delivery pipe. One seventh of water may be raised to about 4 times head of fall, or one fourteenth 8 times, or one twenty-eighth 16 times.

WATER POWER.

Water acts as a moving power, either by its *weight* or by its *vis viva*, and in latter case it acts either by *Pressure* or by *Impact*.

Natural Effect or *Power* of a fall of water is equal to weight of its volume and vertical height of its fall.

If water is made to impinge upon a machine, the velocity with which it impinges may be estimated in the effect of the machine. Result or effect, however, is in nowise altered; for in first case $P = V\,w\,h$, and in latter $= \frac{v^2}{2\,g} V\,w$. V *representing volume in cube feet, w weight in lbs., and v velocity of flow in feet per second.*

$62.5\,V\,h = P$, and $3.2^*\,a\sqrt{h} = V$. P *representing pressure in lbs., a area of opening in sq. feet, and h height of flow in feet per second.*

To Compute Power of a Fall of Water.

RULE.—Multiply volume of flowing water in cube feet per minute by 62.5, and this product by vertical height of fall in feet.

NOTE.—*When Flow is over a Weir or Notch*, height is measured from surface of tail-race to a point four ninths of height of weir, or to centre of velocity or pressure of opening of flow.

When Flow is through a Sluice or Horizontal Slit, height is measured from surface of tail-race to centre of pressure of opening.

EXAMPLE.—What is power of a stream of water when flowing over a weir 5 feet in breadth by 1 in depth, and having a fall of 20 feet from centre of pressure of flow?

By Rule, page 533, $\frac{2}{3}\,5 \times 1\sqrt{2\,g\,1} \times .625 = 16.68$ *cube feet per second.*

$16.68 \times 60 \times 62.5 \times 20 = 1\,251\,000$ *lbs.*, which $\div\, 33\,000 = 37.91$ *horses' power.*

Or, $.1135\,V\,h =$ *theoretical* IP. *h representing height from race in feet.*

ILLUSTRATION.—If flow of a stream is 17.9 cube feet per second, to what height and area of flow of 1 foot in depth should it be dammed to attain a power of 10 horses.

$\frac{33\,000 \times 10}{60} = 5500$ *lbs. per second, and* $\frac{5500}{62.5} = 88$ *cube feet per second.* $\frac{88}{17.9} = 4.92$ *feet height.* Hence, $\frac{2}{3}\,.6\sqrt{2\,g \times 1} = 3.2$, and $17.9 \div 3.2 = 5.59$ *sq. feet.*

Water sometimes acts by its weight and *vis viva* simultaneously, by combining effect of an acquired velocity with fall through which it flows upon wheel or instrument.

In this case $\left(h + \frac{v^2}{2\,g}\right) V \times 62.5 =$ *mechanical effect.*

* As determined by $\frac{2}{3}$ C.

WATER-WHEELS.

WATER-WHEELS are divided into two classes, Vertical and Horizontal. Vertical comprises *Overshot*, *Breast*, and *Undershot*; and Horizontal, *Turbine*, *Impact*, or *Reaction* wheels.

Vertical wheels are limited by construction to falls of less than 60 feet. Turbines are applicable to falls of any height from 1 foot upward.

Vertical wheels applied to a fall of from 20 to 40 feet give a greater effect than a Turbine, and for very low falls Turbines give a greater effect.

Sluices.—Methods of admitting water to an Overshot or Breast Wheel are various, consisting of *Overfall*, *Guide-bucket*, and *Penstock*.

An *Overfall Sluice* is a saddle-beam with a curved surface, so as to direct the current of water tangentially to buckets; a *Guide-bucket* is an apron by which water is guided in a course tangential to buckets; and a *Penstock* is sluice-board or gate, placed as close to wheel as practicable, and of such thickness at its lower edge as to avoid a contraction of current. Bottom surface of penstock is formed with a parabolic lip.

Shrouding of a wheel consists of plates at its periphery, which form the sides of the bucket.

Height of *fall* of a water-wheel is measured between surfaces of water in *penstock* and in *tail-race*, and, ordinarily, two thirds of height between level of reservoir and point at which water strikes a wheel is lost for all effective operation.

Velocity of a wheel at centre of percussion of fluid should be from .5 to .6 that of flow of the water.

Total effect in a fall of water is expressed by product of its weight and height of its fall.

Ratio of Effective Power of Water Motors.

Overshot and high breast	from	.68 to .6 to 1	Undershot, Poncelet's,	from	.6 to .4 to 1
Turbine	"	.6 to .8 to 1	Undershot	"	.27 to .45 to 1
Breast	"	.45 to .65 to 1	Impact and Reaction	"	.3 to .5 to 1
Hydraulic Ram	"	.6 to 1	Water-pressure engine	"	.8 to 1

Overshot-wheel.

OVERSHOT-WHEEL.—The flow of water acts in some degree by impact, but chiefly by its weight.

Lower the speed of wheel at its circumference, the greater will be mechanical effect of the water, in some cases rising to 80 per cent.; with velocities of from 3 to 6.5 feet, efficiency ranges from 70 to 75 per cent. Proper velocity is about 5 feet per second.

Number of buckets should be as great, and should retain water as long, as practicable. Maximum effect is attained when the buckets are so numerous and close that water surface in the bucket commencing to be emptied should come in contact with the under side of the bucket next above it. Molesworth gives 12 ins. apart.

Curved buckets give greatest effect, and Radial give but .78 of effect of Elbow buckets. Wheel 40 feet in diameter should have 152 buckets.

Small wheels give a less effect than large, in consequence of their greater centrifugal action, and discharging water from the buckets at an earlier period than with larger wheels, or when their velocity is lower.

When head of water bears to fall or height of wheel a proportion as great as 1 to 4 or 5, ratio of effect to power is reduced. The general law therefore is, that ratio of effect to power decreases as proportion of *head* to *total head and fall* increases.

Wheel with shallow *Shrouding* acts more efficiently than one where it is deep, and depth is usually made 10 or 12 ins., but in some cases it has been increased to 15.

Breadth of a wheel depends upon capacity necessary to give the buckets to receive required volume of water.

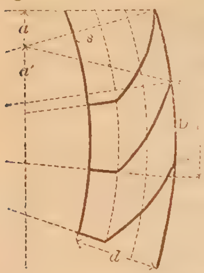
Form of Buckets.—Radial buckets—that is, when the bottom is a right line—involve so great a loss of mechanical effect as to render their use incompatible with economy; and when a bucket is formed of two pieces, lower or inner piece is termed *bottom* or *floor*, and outer piece *arm* or *wrist*. Former is usually placed in a line with radius of wheel.

Line of a circle passing through *elbow*, made by junction of *floor* and *arm*, is termed division circle, or *bucket pitch*, and it is usual to put this at one half depth of shrouding.

When *arm* of a bucket is included in division angle of buckets, that is, $\frac{360^\circ}{n}$, *n representing number of buckets*, the cells are not sufficiently covered, except for very shallow shrouding; hence it is best to extend arm of a bucket over 1.2 of division angle, so as to cover or overlap *elbow* of bucket next in advance of it.

Construction of Buckets (Fig. 1).—Capacity of bucket should be 3 times volume of water.

Fig. 1.



Fairbairn gives area of opening of a bucket in a wheel of great diameter, compared to the volume of it, as 5 to 24.

Buckets having a bottom of two planes, that is, with two bottoms, and two division circles or bucket pitches and an arm, give a greater effect than with one bottom.

When an opening is made in base of buckets, so as to afford an escape of air contained within, without a loss of water admitted, the buckets are termed *ventilated*, and effective power of wheel is much greater than with closed buckets.

D = distance apart at periphery = *d*, *d* depth of shrouding, *s* length of radial start = .33 *d*, *l* length of bucket curve = 1.25 *d* in large wheels, and 1 in wheels under 25 feet, *a* angle of radius of curve of bucket, with radial line of wheel at points of bucket = 15°. (*Molesworth.*)

To Compute Radius and Revolutions of an Overshot-wheel, and Height of Fall of Water.

When whole Fall and Velocity of Flow, etc., are given. $\frac{h - h'}{1 + \cos. a} = r$, $\frac{h\,c}{3.1416\,r} = n$, $\frac{v^2}{2\,g}\,1.1 = h'$, and $\frac{3.1416\,n\,r}{30} = c$. *h representing height of whole fall, h′ height between the centre of gravity of discharge and half depth of bucket upon which water flows, v velocity of flow in feet per second, a angle which point of entrance of water into a bucket makes with summit of wheel, n number of revolutions per minute, c velocity of wheel at its circumference per second, and r its radius.*

Note.—Height of whole fall is distance between surface of water in flume and point at which lower buckets are emptied of water, and as a proportion of velocity of flow is lost, it is proper to assume height *h′* as above given.

Illustration.—A fall of water is 30 feet, velocity of its flow is 16 feet per second, angle of its impact upon buckets is 12°, and required velocity of wheel is 8 feet per second; what is required radius, number of revolutions, and height of fall upon wheel?

$h' = \frac{16^2}{2\,g} \times 1.1 = 4.38$ *feet;* $\cos.\ 12^\circ = .978$; $\frac{30 - 4.38}{1 + .978} = \frac{25.62}{1.978} = 12.95$ *feet radius;*
$\frac{30 \times 8}{3.1416 \times 12.95} = \frac{240}{40.68} = 5.9$, *revolutions.*

When Number of Revolutions and Ratio between Velocities of Flow and at Circumference of Wheel are given.

$$\frac{\sqrt{.000\,772\,(x\,n)^2\,h + (1+\cos.\,a)^2} - 1 + \cos.\,a)}{.000\,386\,(x\,n)^2} = r,\ x = \frac{v}{c},\ \text{and}\ \frac{3.1416\,n\,r}{30} = c.$$

ILLUSTRATION.—If number of revolutions are 5, $x = 2$, and fall, etc., as in previous case; what is radius of wheel, velocity of flow, and height of fall?

$$\frac{\sqrt{.000\,772\ (2 \times 5)^2 \times 30 + (1.978)^2} - 1.978}{.000\,386\ (2 \times 5)^2} = \frac{.518}{.0386} = 13.41\ \textit{feet.}$$

$\frac{3.1416 \times 5 \times 13.41}{30} = 7.03$ *feet.* Hence $7.03 \times 2 = 14.06$ *velocity of flow*, and $\frac{14.06^2}{64.33} \times 1.1 = 3.37$ *feet.*

To Compute Width of an Overshot-wheel.

$\frac{C\,V}{s\,c'} = w$. *C representing a coefficient* = 3, *when buckets are filled to an excess, and* 5 *when they are deficiently filled,* V *volume of water in cube feet per second, s depth of shrouding, w width of buckets, both in feet, and c′ velocity of wheel at centre of shrouding, in feet per second.*

ILLUSTRATION.—A wheel is to be 31 feet in diameter, with a depth of shrouding of 1 foot, and is required to make 5 revolutions per minute under a discharge of 10 cube feet of water per second; what should be width of buckets?

Assume C = 4, and $c' = \frac{31 - 1 \times 3\ 1416 \times 5}{60} = 7.854$. Then $\frac{4 \times 10}{1 \times 7.854}$ 5.09 *feet.*

To Compute Number of Buckets.

$7\left(1 + \frac{s}{.83}\right) \div 12 = d$, and $\frac{D\,p\,s}{d} = n$. D *representing diameter of wheel, d distance between centres of buckets, in feet, and n number of buckets.*

ILLUSTRATION.—Take elements of preceding case.

Then $7\left(1 + \frac{1}{.83}\right) = 7 \times 2.2 \div 12 = 1.283$, and $\frac{31 - 1 \times 3.1416 \times 1}{1\ 283} = 73.4$, say 72 *buckets;* hence $\frac{360°}{72} = 5°$, *angle of subdivision of buckets.*

To Compute Effect of an Overshot-wheel.

$\frac{V\,h'\,w - \left(\frac{v'^2}{2\,g}\,V\,w + f\right)}{V\,h\,w} = P$. *w representing weight of cube foot of water in lbs., v′ velocity of it discharged at tail of wheel, in feet per second,* V *volume of flow in cube feet, and f friction of wheel in lbs.*

ILLUSTRATION.—A volume of 12 cube feet per second has a fall of 10 feet, wheel using but 8.5 feet of it, and velocity of water discharged is 9 feet per second; what is effect of fall?

Friction of wheel is assumed to be 750 *lbs.*

$\frac{12 \times 8.5 \times 62.5 - \left(\frac{9^2}{64.33} \times 12 \times 62.5 + 750\right)}{12 \times 10 \times 62.5} = \frac{6375 - (1.26 \times 750 + 750}{7500} = \frac{4680}{7500} =$ .624 = *ratio of effect to power;* and 4680 × 60 *seconds* ÷ 33 000 = 8.51 HP.

To Compute Power of an Overshot-wheel.

RULE.—Multiply weight of water in lbs. discharged upon wheel in one minute by height or distance in feet from centre of opening in gate to surface of tail-race; divide product by 33 000, and multiply quotient by assumed or determined ratio of effect to power. Or, for general purposes, divide product by 50 000, and quotient is HP.

Or, $.0852\,V\,h$ = HP, and $\frac{11.7\ \text{HP}}{h}$ = V *per second;* or, $\frac{771\ \text{HP}}{h}$ = V *per minute.*

Mechanical Effect of water is product of its weight into height from which it falls.

EXAMPLE.—Volume of water discharged upon an overshot-wheel is 640 cube feet per minute, and effective height of fall is 22 feet; what is HP?

$$\frac{640 \times 62.5 \times 22}{33\,000} = 26.67, \text{ which, } \times .75 = \text{assumed ratio of effect to power} = 20 \text{ HP}.$$

Useful Effect of an Overshot-wheel.

With a large wheel running in most advantageous manner, .84 of power may be taken for effect.

Velocity of a wheel bears a constant ratio, for maximum effects, to that of the flowing water, and this ratio is at a mean .55.

Ratio of effect to power with radial-buckets is .78 that of elbow-buckets. Ratio of effect decreases as proportion of head to total head and fall increases. Thus, a wheel 10 feet in diameter gave, with heads of water above gate, ranging from .25 to 3.75 feet, a ratio of effect decreasing from .82 to .67 of power.

Higher an overshot-wheel is, in proportion to whole descent of water, greater will be its effect. Effect is as product of volume of water and its perpendicular height.

Weight of arch of loaded buckets in lbs. is ascertained by multiplying .444 of their number by number of cube feet in each, and that product by 40.

Undershot-wheel.

UNDERSHOT-WHEEL is usually set in a curb, with as little clearance for escape of water as practicable; hence a curb concentric to this wheel is more effective than one set straight or tangential to it.

Computations for an undershot-wheel and rules for construction are nearly identical with those for a breast-wheel.

Buckets are usually set radially, but they may be inclined upward, so as to be more effectively relieved of water upon their return side, and they are usually filled from .5 to .6 of their volume. Depth of shrouding should be from 15 to 18 ins., in order to prevent overflow of water within the wheel, which would retard it.

Velocity of periphery should equal theoretical velocity due to head of water $\times$ .57.

NOTE.—When constructed without shrouding, as in a current-wheel, etc., buckets become *blades*.

Sluice-gate should be set at an inclination to plane of curb, or tangential to wheel, in order that its aperture may be as close to wheel as practicable; and in order to prevent partial contraction of flow of water, lower edge of sluice should be rounded.

Effect of an undershot-wheel is less than that of a breast-wheel, as the fall available as weight is less than with latter.

To Compute Power of an Undershot-wheel.

Proceed as per rule for an overshot-wheel, using 93 750 for 50 000, and .4 for .75.

Or, $V\,h\,.000\,66 = \text{HP}$; or, $\frac{1515 \text{ HP}}{h} = V$. V *representing volume of water in cube feet per minute, and h head of water in feet.*

Poncelet's Wheel.

PONCELET'S WHEEL.—Buckets are curved, so that flow of water is in course of their concave side, pressing upon them without impact; and effect is greater than when water impinges at nearly right angles to a plane surface or blade.

This wheel is advantageous for application to falls under 6 feet, as its effect is greater than that of other undershot wheels with a curb, and for falls from 3 to 6 feet its effect is equal to that of a Turbine.

For falls of 4 feet and less, efficiency is 65 per cent., for 4.25 to 5 feet, 60 per cent., and from 6 to 6.5 feet, 55 to 50 per cent.

In its arrangement, aperture of sluice should be brought close to face of wheel. First part of course should be inclined from 4° to 6°; remainder of course, which should cover or embrace at least three buckets, should be carried concentric to wheel, and at end of it a quick fall of 6 ins. made, to guard against effect of back-water. Sluice should not be opened over 1 foot in any case, and 6 ins. is a suitable height for falls of 5 and 6 feet.

Distance between two buckets should not exceed 8 or 10 ins., and radius of wheel should not be less than 40 ins., or more than 8 feet.

Plane of stream or head of water should meet periphery of wheel at an angle of from 24° to 30°. Space between wheel and its *curb* should not exceed .4 of an inch.

Depth of shrouding should be at least .25 depth of head of water, or such as to prevent water from flowing through it and over the buckets, and width of wheel should be equal to that of stream of impinging water.

Effect of this wheel increases with depth of water flow, and, therefore, other elements being equal, as filling of buckets, to obtain maximum effect, water should flow to buckets without impact, and velocity of wheel should be only a little less than half that of velocity of water flowing upon wheel.

To Compute Proportions of a Poncelet Wheel.

NOTE.—As it is impracticable to arrive at the results by a direct formula, they must be obtained by gradual approximation.

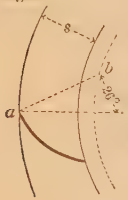
EXAMPLE.—Height of fall is 4.5 feet; volume of water 40 cube feet per second; radius of wheel $= 2\,h$, or 9 feet; depth of the stream $= .75$ *feet;* and C assumed at .9.

V *representing volume of water in cube feet per second,* h *height of fall,* d *depth of shrouding* $= \frac{1}{4} \cdot \frac{v^2}{2\,g} + d'$; d' *opening of and* e *width of sluice,* r *radius of curvature of buckets* $= \frac{d}{\cos. z}$, *and* a *of wheel, all in feet;* n *number of revolutions* $= \frac{30\,c}{p\,a}$ *per minute;* c *velocity of circumference of wheel and* v *velocity of water, both in feet per second;* C *coefficient of resistance of flow of water;* x *angle between plane of flowing water and that of circumference of wheel at point of contact,* sin. *of* $\frac{x}{2} = \sqrt{\cos. z}$; z *angle made by circumference of wheel with end of buckets* $= 2$ tang. y; *and* y *angle of direction of water from circumference of wheel* $= \frac{p\,c}{2\,a}\sqrt{\frac{d}{g+\frac{c^2}{a}}}$.

Then $v = .9\sqrt{2\,g\left(h - \frac{d'}{2}\right)} = .9 \times 16.29 = 14.66$ *feet* ∴ *velocity of wheel,* being less than half velocity of water; $c = \frac{14.66 - .66}{2} = 7$ *feet*; $d = \frac{1}{4} \times \frac{14.66^2}{2g} + .75 = 1.58$ *feet*; $y = \frac{3.1416 \times 7}{2 \times 9} \times \sqrt{\frac{1.58}{32.166 + \frac{7^2}{9}}} = 1.222 \times \sqrt{.0403} = .25$, angle corresponding to which $= 14^\circ\ 30'$; $n = \frac{30 \times 7}{3.1416 \times 9} = 7.43$ *revolutions*;

$z = 2$ tang. $y = 2 \times .25862 = .51724 \therefore z = 27^\circ\ 20'$; $c = \frac{40}{.75 \times 14.66} = 3.63$ *feet*; $r = \frac{1.58}{\cos. 27^\circ\ 20'} = \frac{1.58}{.88835} = 1.78$ *feet*; $x = \sin. \frac{x}{2} = \sqrt{\cos. z} = \sqrt{\cos. 27^\circ\ 20'} = .943$ $= \sin.$ of $70^\circ\ 34' \therefore x = 141^\circ\ 8'$. Effect is a maximum when $c = .5\ v \cos. y$.

Fig. 2.



Construction of Buckets (Fig. 2). (*Molesworth.*)

From point of bucket, *a*, draw a line, *a b*, at an angle of 26° with radial line, point *b*, where this line cuts an imaginary circle, drawn at a distance of $s \times 1.17$ from periphery of wheel, is centre from which bucket is struck with radius, *b a*. Radius of wheel should not be less than 7, or more than 16 feet.

Curb should fit wheel accurately for 18 or 20 ins., measured back from perpendicular line which passes through axis of wheel, the breast should then incline 1 in 10, or 1 in 15 towards sluice.

After passing axis of wheel in tail-race, curb should make a sudden dip of 6 ins.

To Compute Power of a Poncelet Wheel.

$V\,h\,.00113 = \text{IP}$, and $\frac{880\ \text{IP}}{h} = V$. $V =$ *velocity of theoretical periphery* $= .55.$*

Number of buckets $1.6\ D + 1.6$, $D =$ diameter of wheel in feet. Shrouding .33 to .5 depth of head of water, and $D = 2\,h$, and not less than 7 or more than 16 feet.

Breast-wheel.

Breast-wheel is designed for falls of water varying from 5 to 15 feet, and for flows of from 5 to 80 cube feet per second. It is constructed with either ordinary buckets or with blades confined by a *Curb*.

Enclosure within which water flows to a breast-wheel as it leaves the sluice is termed a *Curb* or *Mantle*.

When blades are enclosed in a *curb*, they are not required to hold water; hence they may be set *radial*, and they should be numerous, as the loss of water escaping between the wheel and the curb is less the greater their number; and that they may not lift or carry up water with them from tail-race, it is proper to give them such a plane that it may leave the water as nearly vertical as may be practicable.

Distance between two buckets or blades should be from 1.3 to 1.5 times head over gate for low velocity of wheel and more for a high velocity, or equal to depth of shrouding, or at from 10 to 15 ins.

It is essential that there should be air-holes in floor of buckets, to prevent air from impeding flow of water into them, as the water admitted is nearly as deep as the interval between them; and velocity of wheel should be such that buckets should be filled to .5 or .625 of their volume.

When wheels are constructed of iron, and are accurately set in masonry, a clearance of .5 of an inch is sufficient.

---

* $\sqrt{2\,g\,h}$ *in feet per second.*

High Breast-wheel is used when level of water in *tail-race* and *penstock* or *forebay* are subject to variation of heights, as wheel revolves in direction in which water flows from blades, and *back-water* is therefore less disadvantageous, added to which, penstocks can be so constructed as to admit of an adjustable point of opening for the water to flow upon the wheel.

Effect of this wheel is equal to that of the overshot, and in some instances, from the advantageous manner in which water is admitted to it, it is greater when both wheels have same general proportions.

Under circumstances of a variable supply of water, *Breast-wheel* is better designed for effective duty than *Overshot*, as it can be made of a greater diameter; whereby it affords an increased facility for reception of water into its buckets, also for its discharge at bottom; and further, its buckets more easily overcome retardation of back-water, enabling it to be worked for a longer period in back-water consequent upon a flood.

In a well-constructed wheel an efficiency of 93 per cent. was observed by M. Morin, and Sir Wm. Fairbairn gives, at a velocity of circumference of wheel of 5 feet, an efficiency of 75 per cent. Velocity usually adopted by him was from 4 to 6 feet per second, both for high and low falls; a minimum of 3.5 feet for a fall of 40 and a maximum of 7 feet for a fall of 5 to 6 feet.

When water flows at from 10° to 12° above horizontal centre of wheel, Fairbairn gives area of opening of buckets, compared with their volume, as 8 to 24.

The capacity between two buckets or blades should be very nearly double that of volume of water expended.

To Compute Proportions and Effect of a Breast-wheel.

Illustration.—Flow of water is 15 cube feet per second; height of fall, measured from centre of pressure of opening to tail race, is 8.5 feet; velocity of circumference of wheel 5 feet per second; and depth of buckets or blades 1 foot, filled to .5 of their volume.

Width of wheel $= \frac{V}{s\,d}$, *d representing depth, and v velocity of buckets;* $\frac{15}{1 \times 5} = 3$; and as buckets are but .5 filled, $3 \div .5 = 6$ *feet.* Assume water is to flow with double velocity of circumference of wheel; $v = 5 \times 2 = 10$ *feet;* and fall required to generate this velocity $= \frac{v^2}{2g} \times 1.1 = h' = \frac{100}{64.33} \times 1.1 = 1.71$ *feet.*

Deducting this height from total fall, there remains for height of curb or shrouding, or fall during which weight of water alone acts, $h - h' = 8.5 - 1.71 = 6.79$ *feet.*

Making radius of wheel 12 feet, and radius of bucket circle 11 feet, whole mechanical effect of flow of water $= 15 \times 62.5 \times 8.5 = 7968.75$ *lbs.*, from which is to be deducted from 10 to 15 per cent. for loss of water by escape.

Theoretical effect, as determined by M. Morin, velocity of circumference about .5 of that of water, and within velocities of 1.66 to 6 feet.

$\left(\frac{(v \cos. a - v)\,v}{g} + h''\right) V\,62.5.$ *a representing angle of direction of velocity with which water flows to wheel at centre of thread of flow and direction of velocity of wheel at this line, and h″ h — h′ in feet.*

a is here assumed at 20°. See Weisbach, London, 1848, vol. ii. page 197, and for the necessarily small value of *a*, its cosine may be taken at 1. Cos. $20° = .94$.

Then $\left(\frac{(10 \times .94 - 5)\,5}{32.16} + 6.79\right) \times 15 \times 62.5 = 7.474 \times 15 \times 62.5 = 7006.9$ *lbs.*, which is to be reduced by a coefficient of .77 for a penstock sluice, and .8 for an overfall sluice.

Theoretical effect, as determined by Weisbach, 7273 *lbs.*, from which are to be deducted losses, which he computes as follows:

Loss by escape of water between wheel and curb............	=	916
Loss by escape at sides of wheel and curb....................	=	180
Friction and resistance of water = 2.5 per cent...............	=	160
		1256 *lbs.*

Friction of wheel as per formula, page 571, $= W\,r\,n\,C\,.0086$; $a = .048\sqrt{\frac{W}{2}} = .048\sqrt{\frac{16\,500}{2}} = 4.36$ *ins.*; and $n = \frac{5 \times 60}{12 \times 2 \times 3.1416} = 4$ *revolutions*. $C = .08$. $r = 4.36 \div 2 = 2.18$. Then $16\,500 \times 2.18 \times 4 \times .08 \times .0086 = 98.99$ *lbs.*

Whence, $\frac{7006.9 - \overline{1256 + 9.9}}{7968.75} = .72$ *efficiency*, upon assumption of losses as computed by Weisbach.

To Compute Power of a Breast-wheel.

RULE.—Proceed as per rule for an overshot-wheel, using 55 000 and .65 with a high breast, and 62 500 and .6 for a low breast.

Or, High breast, $.0612\,V\,h = \text{HP}$, and $\frac{13.3\,\text{HP}}{h} = V$; and Low breast $.0546\,V\,h = \text{HP}$, and $\frac{14.5\,\text{HP}}{h} = V$.

ILLUSTRATION.—Assume elements of preceding case. Then $\frac{15 \times 62.5 \times 8.5 \times 60}{33\,000} = 14.49$, which $\times .7 = 10.14$ *horses*.

$$\text{Or, } \frac{7006.9 - \overline{1256 + 102.6} \times 60}{33\,000} = 10.27 \textit{ horses.}$$

Openings of Buckets or Blades.—High Breast, .33 sq. foot, and *Low Breast*, .2 sq. foot for each cube foot of their volume, or generally 6 to 8 in opening in a high breast and 9 to 12 in a low breast.

Forms of Buckets.—Two Part. $d = D$, $s = .5\,d$, l $1.25\,d$ in large wheels, and $= d$ in wheels less than 25 feet in diameter.

Three Part Buckets.—d divided into 3 equal parts; $l = .25\,d$, $d = D$, $s = .33\,d$, $l = d$ in large wheels, and $.75\,d$ in wheels less than 25 feet in diameter.

Ventilating Buckets (*Fairbairn's*). Spaces are about 1 inch in width.

NOTES.—A Committee of the Franklin Institute ascertained that, with a high breast wheel 20 feet in diameter, water admitted under a head of 9 ins., and at 17 feet above bottom of wheel, elbow-buckets gave a ratio of effect to power of .731 at a maximum, and radial blades .653. With water admitted at a height of 33 feet 8 ins., elbow-buckets gave .658, and radial blades .628.

At 10.96 feet above bottom of wheel, with a head of 4.29 feet, elbow-buckets gave .544, and blades .329.

At 7 feet above bottom of wheel, and a head of 2 feet, a *low breast* gave for elbow-buckets .62, and for blades .531.

At 3 feet 8 ins. above bottom of wheel, and a head of 1 foot, elbow-buckets gave .555, and blades .533.

Current-wheel.

CURRENT-WHEEL.—D. K. Clark assigns the most suitable ratio of velocity of blades to that of current as 40 per cent.

Depth of blades should be from .25 to .2 of radius; it should not be less than 12 or 14 ins. Diameter is usually from 13 to 16.5 feet, with 12 blades; but it is thought that there might be an advantage in applying 18 or even 24. The blades should be completely submerged at lower side, but not more than 2 ins. under water, and not less than 2 at one time.

$\frac{a\,s}{150}(v - s)^2 = \text{HP}$. *a representing area of vertical section of immersed blades in sq. feet, s velocity of wheel at circumference, and v of stream, both in feet per second.*

Or, $.38\,\frac{v^2}{2g}\,V\,62.5 =$ *useful effect.* Hence, efficiency $= .38$.

Flutter-wheel.

Flutter or Saw-mill Wheel—Is a small, low breast-wheel operating under a high head of water; the design of its construction, water being plenty, is the attainment of a simple application to high-speed connections, as a gang or circular saw. In effect it is from .6 to .7 that of an overshot-wheel of like head of fall.

$$\frac{V\,s}{150}(v-s)=\text{IP}. \quad \textit{v and s as preceding.}$$

Friction of Journals or Gudgeons.

A very considerable portion of mechanical effect of a wheel is lost in effect absorbed by friction of its gudgeons.

To Compute Friction of Journals or Gudgeons of a Water-wheel.

$W\,r\,n\,C\,.0086=f$. *W representing weight of wheel in lbs., r radius of gudgeon in ins., and n number of revolutions of wheel per minute.*

For well turned surfaces and good bearings, $C=.075$ with oil or tallow; when best of oil is well supplied $=.054$; and, as in ordinary circumstances, when a black-lead unguent is alone applied $=.11$.

ILLUSTRATION.—A wheel weighing 25 000 lbs. has gudgeons 6 ins. in diameter, and makes 6 revolutions per minute; what is loss of effect?

$$\text{Assume } C=.08. \quad \text{Then } 25\,000\times\frac{6}{2}\times 6\times .08\times .0086=309.6 \textit{ lbs.}$$

Weights.—*Iron wheels* of 18 to 20 feet in diameter will weigh from 800 to 1000 lbs. per IP

Wood wheels of 30 feet in diameter, 2000 to 2500 lbs. per IP.

To Compute Diameter and Journals of a Shaft, Stress laid uniformly along its Length.

Cast Iron, $\frac{\sqrt[3]{W\,l}}{9.6}=d$. *Wood*, $6.12\sqrt[3]{\frac{\text{IP}}{4}}=d$. *W representing weight or load in lbs., l length of shaft between journals in feet, and d diameter of shaft in its body in ins.*

Journals or *Gudgeons.—Cast Iron*, $.048\sqrt{\frac{W}{2}}=d$.

When Shaft has to resist both Lateral and Torsional Stress.—Ascertain the diameter for each stress, and cube root of sum of their cubes will give diameter.

To Compute Dimensions of Arms.

Cast Iron, $\frac{1.7\,d}{\sqrt[3]{n}}=w$. *d representing diameter of shaft, and w width of arm, both in ins., n number of arms,* $\frac{w}{5}=t$, *and t thickness of arm.*

When Arm is of Oak, *w* should be 1.4 times that of iron, and thickness .7 that of width.

Memoranda.

A volume of water of 17.5 cube feet per second, with a fall of 25 feet, applied to an undershot wheel, will drive a hammer of 1500 lbs. in weight from 100 to 120 blows per minute, with a lift of from 1 to 1.5 feet.*

A volume of water of 21.5 cube feet per second, with a fall of 12.5 feet, applied to a wheel having a great height of water above its summit, being 7.75 feet in diameter, will drive a hammer of 500 lbs. in weight 100 blows per minute, with a lift of 2 feet 10 ins. Estimate of power 31.5 horses.

* Volume of water required for a hammer increases in a much greater ratio than velocity to be given to it, it being nearly as cube of velocity.

A Stream and Overshot Wheel of following dimensions—viz., height of head to centre of opening, 24.875 ins.; opening, 1.75 by 80 ins; wheel, 22 feet in diameter by 8 feet face; 52 buckets, each 1 foot in depth, making 3.5 revolutions per minute—drove 3 run of 4.5 feet stones 130 revolutions per minute, with all attendent machinery, and ground and dressed 25 bushels of wheat per hour.

4.5 bushels Southern and 5 bushels Northern wheat are required to make 1 barrel of flour.

A Breast-wheel and Stream of following dimensions—viz., head, 20 feet; height of water upon wheel, 16 feet; opening, 18 feet by 2 ins.; diameter of wheel, 26 feet 4 ins.; face of wheel, 20 feet 9 ins.; depth of buckets, 15.75 ins.; number of buckets, 70; revolutions, 4.5 per minute—drove 6144 self-acting mule spindles; 160 looms, weaving printing cloths 27 ins. wide of No. 33 yarn (33 hanks to a lb.), and producing 24000 hanks in a day of 11 hours.

Horizontal Wheels.

In horizontal water-wheels, water produces its effect either by *Impact*, *Pressure*, or *Reaction*, but never directly by its weight.

These wheels are therefore classed as Impact, Pressure, and Reaction, but are now designated by the generic term of Turbine.

Turbines.

TURBINES, being operated at a higher number of revolutions than Vertical Wheels, are more generally applicable to mechanical purposes; but in operations requiring low velocities, Vertical Wheel is preferred.

For variable resistances, as rolling-mills, etc., Vertical Wheel is far preferable, as its mass serves to regulate motion better than a small wheel.

In economy of construction there is no essential difference between a Vertical Wheel and a Turbine. When, however, fall of water and volume of it are great, the Turbine is least expensive. Variations in supply of water affect vertical wheels less than Turbines.

Durability of a Turbine is less than that of a Vertical Wheel; and it is indispensable to its operation that the water should be free from sand, silt, branches, leaves, etc.

With Overshot and Breast Wheels, when only a small quantity of water is available, or when it is required or becomes necessary to produce only a portion of the power of the *fall*, their efficiency is relatively increased, from the blades being but proportionately filled; but with Turbines the effect is contrary, as when the sluice is lowered or supply decreased water enters the wheel under circumstances involving greater loss of effect. To produce maximum effect of a stream of water upon a wheel, it must flow without impact upon it, and leave it without velocity; and distance between point at which the water flows upon a wheel and level of water in reservoir should be as short as practicable.

Small wheels give less effect than large, in consequence of their making a greater number of revolutions and having a smaller water arc.

In *High-pressure Turbines* reservoir (of wheel) is enclosed at top, and water is admitted through a pipe at its side. In *Low-pressure*, water flows into reservoir, which is open.

In Turbines working under water, height is measured from surface of water in supply to surface of discharged water or race; and when they work in air, height is measured from surface in supply to centre of wheel.

In order to obtain maximum effect from water, velocity of it, when leaving a Turbine, should be the least practicable.

Efficiency is greater when sluice or supply is wide open, and it is less affected by head than by variations in supply of water. It varies but little with velocity, as it was ascertained by experiment that when 35 revolutions gave an effect of .64, 55 gave but .66.

When Turbines operate under water, the flow is always full through them; hence they become *Reaction-wheels*, which are the most efficient.

Experiments of Morin gave efficiency of Turbines as high as .75 of power.

Angle of plane of water entering a Turbine, with inner periphery of it, should be greater than 90°, and angle which plane of water leaving reservoir makes with inner circumference of Turbine should be less than 90°.

When Turbines are constructed without a *guide curve**, angle of plane of flowing water and inner circumference of wheel = 90°.

Great curvature involves greater resistance to efflux of water; and hence it is advisable to make angle of plane of entering water rather obtuse than acute, say 100°; angle of plane of water leaving, then, should be 50°, if internal pressure is to balance the external; and if wheel operates free of water, it may be reduced to 25° and 30°.

If blades are given increased length, and formed to such a hollow curve that the water leaves wheel in nearly a horizontal direction, water then both impinges on blades and exerts a pressure upon them; therefore effect is greater than with an impact-wheel alone.

Turbines are of three descriptions: Outward, Downward, and Inward flow.

Outward-flow Turbines.

FOURNEYRON TURBINE, as recently constructed, may be considered as one of the most perfect of horizontal wheels; it operates both in and out of back-water, is applicable to high or low falls, and is either a high or low pressure turbine.

In high-pressure, the reservoir is closed at top and the water is led to it through a pipe. In low-pressure, the water flows directly into an open reservoir. Pressure upon the step is confined to weight of wheel alone.

Fourneyron makes angle of plane of water entering = 90°, and angle of plane of water leaving = 30°.

Efficiency is reduced in proportion as sluice is lowered, for action of water on wheel is less favorably exerted. M. Morin tested a Fourneyron turbine 6.56 feet in diameter, and he found that efficiency varied from a minimum of 24, to 79 per cent., when supply of water was reduced to .25 of full supply. In practice, radial length of blades of wheel is .25 of radius, for falls not exceeding 6.5 feet, .3 for falls of from 6.5 to 19 feet, and .66 for higher falls.

To Compute Elements and Results.

High Pressure, $6.6\sqrt{h} = v$; $\frac{V}{v} = A$; $\frac{\sqrt{1.77\,V}}{\sqrt{h}} = D\,†$; $12.6\,\frac{IP}{h} = V$; and $.079\,V\,h = IP$. *h representing head of water, v velocity of turbine at periphery per minute, and D internal diameter of turbine, all in feet, V volume of water in cube feet per second, A sum of area of orifices in sq. feet, and IP effective horse-power.*

1.2 D = external diameter of turbine in feet, when it is more than 6 feet, and 1.4 when it is less than 6 feet. Number of guides = number of blades ‡ when less than 24, and number ÷ 3 when greater than 24. Area of section of supply pipe = .4 V.

For construction of blades and guides, see Molesworth, London, 1882, page 540.

---

* Guide curves are plates upon centre body of a Turbine, which give direction to flowing water, or to blades of wheel which surround them.

† In extreme cases of very high falls diameter given by this formula may be increased.

‡ Fourneyron's rule for the number of blades is constant number 36, irrespective of size of turbine.

Operation of High-Pressure Turbines.

h	30	40	50	60	70	80	90	100	120	140	160	180	200
V	4.2	3.1	2.5	2.1	1.8	1.6	1.4	1.27	1.05	.9	.8	.7	.63
v	36	42	47	51	55	59	63	66	73	78	84	89	94

h = head of water in feet, V *volume of water in cube feet required for each* 10 IP, *and v velocity of periphery of turbine in feet per second.*

Boyden Turbine.—Mr. —— Boyden, of Massachusetts, designed an outward-flow turbine of 75 IP, which realized an efficiency of 88 per cent. Peculiar features, as compared with a Fourneyron turbine, are, 1st, and most important, the conduction of the water to turbine through a vertical truncated cone, concentric with the shaft. The water, as it descends, acquires a gradually increasing velocity, together with a spiral movement in direction of motion of wheel. The spiral movement is, in fact, a continuation of the motion of the water as it enters cone.—2d. Guide-plates at base are inclined, so as to meet tangentially the approaching water.—3d. A "diffuser," or annular chamber surrounding wheel, into which water from wheel is discharged. This chamber expands outwardly, and, thus escaping velocity of water, is eased off and reduced to a fourth when outside of diffuser is reached. Effect of diffuser is to accelerate velocity of water through machine; and gain of efficiency is 3 per cent. Diffuser must be entirely submerged. (*D. K. Clark.*)

Poncelet Turbine.—This wheel is alike to one of his undershot-wheels set horizontally, and it is the most simple of all horizontal wheels.

To Compute Elements of General Proportion and Results. (*Lt. F. A. Mahan, U. S. A.*)

$$.0425\,D^2 h\sqrt{h} = \text{IP};\quad 4.85\sqrt{\frac{P}{h\sqrt{h}}} = D;\quad .5\,D^2\sqrt{h} = V;\quad .1\,D = H;\quad 4.49\sqrt{h} = v;$$

$$3\,(D + 10) = N;\quad \frac{D}{N} = w;\quad \frac{4\,D}{N} = W;\quad D - \frac{8\,D}{N} = d;\quad .5\,N \text{ to } .75\,N = n;\quad \frac{d}{n} = w';$$

and C coefficient for V′ in terms of $V = \frac{V'}{V}$. D *and* d *representing exterior and interior diameters of wheel,* H *and* h *heights of orifices of discharge at outer circumference and of fall acting on wheel,* w *and* w′ *shortest distances between two adjacent blades and two adjacent guides, all in feet,* V, V′, *and* v *velocities due to fall of water passing through narrowest section of wheel, and of interior circumference of wheel, all in feet per second,* N *and* n *numbers of blades and guides, and* IP *actual horsepower.*

For falls of from 5 feet to 40, and diameters not less than 2 feet, $n\,w$ should be equal to diameter of wheel. H equal to .1 D, $n\,w' = d$, and $4\,w$ = width of crown. For falls exceeding this, H should be smaller, in proportion to diameter of wheel.

Downward-flow Turbines.

In turbines with downward flow, wheel is placed below an annular series of guide-blades, by which water is conducted to wheel. The water strikes curved blades, and falls vertically, or nearly so, into tail-race; consequently, centrifugal action is avoided, and downward flow is more compact.

Fontaine Turbine yields an efficiency of 70 per cent., when fully charged. When supply of water is shut off to .75, by sluice, efficiency is 57 per cent. Best velocity at mean circumference of wheel is equal to 55 per cent. of that due to height of fall. It may vary .25 of this either way, without materially affecting efficiency.

In operation the water in race is in immediate contact with wheel, and its efficiency is greatest when sluice is fully opened. Its efficiency, also, is less affected by variations of head of flow than in volume of water supplied; hence they are adapted for *Tide-mills.*

Jonval Turbine.—This wheel is essentially alike in its principal proportions to Fontaine's, and in principle of operation it is the same. Water in race must be at a certain depth below wheel.

For convenience, it is placed at some height above level of tail-race, within an air-tight cylinder, or "draft-tube," so that a partial vacuum or reduction of pressure is induced under wheel, and effect of wheel is by so much increased. Resulting efficiency is same as if wheel was placed at level of tail-race; and thus, while it may be placed at any level, advantage is taken of whole height of fall, and its efficiency decreases as volume of water is diminished or as sluice is contracted.

To Compute Elements and Results.

Low Pressure.—For falls of 30 feet and less.

$$6\sqrt{h} = v; \quad = \frac{V}{v} = A; \quad \frac{\sqrt{1.77\,V}}{\sqrt{h}} = D^*; \quad 12.7\frac{HP}{h} = V; \quad \text{and } .079\,V\,h = HP.$$

h representing head of water, v velocity of turbine at periphery per minute, and D *internal diameter of turbine, all in feet,* V *volume of water in cube feet per second,* A *sum of area of orifices in sq. feet, and* HP *effective horse-power.*

1.2 D = external diameter of turbine in feet, when it is more than 6 feet, and 1.4 when it is less than 6 feet. Number of guides = number of blades† when less than 24, and number ÷ 3 when greater than 24. Area of section of supply-pipe = .4 V.

For construction of blades and guides, see Molesworth, London, 1882, page 540.

Low-Pressure Turbines. *(Molesworth.)*

Head.	v	5 HP		10 HP		15 HP		20 HP		30 HP		40 HP		50 HP	
		V	R	V	R	V	R	V	R	V	R	V	R	V	R
2.5	9.48	25	34	50	24	75	20	100	17	—	—	—	-	—	—
5	13.38	12.5	81	25	57	38	47	50	41	75	33	100	28	126	26
7.5	16.38	8.5	136	17	97	25	79	33	68	51	56	68	48	85	43
10	18.96	6.3	180	12.6	128	19	105	25	90	38	75	50	64	63	58
15	23.22	4.2	319	8.4	226	12.6	185	17	160	25	131	33	113	42	100
20	26.82	—	—	6.3	329	9.3	273	12.6	232	18.9	194	25	164	31	148
25	30	—	—	—	—	7.5	358	10	310	15	253	20	220	25	196
30	32.88	—	—	—	—	—	—	8.4	380	12.6	310	17	268	21	240

v representing velocity of centre of blades in feet and V *volume of water, in cube feet, both per second,* R *revolutions per minute, and* HP *effective horse-power.*

Vertical Shaft. $\sqrt[3]{\frac{230\,HP}{R}}$ = *diameter of shaft in ins.*

Inward-flow Turbine.

Inward-flow Turbine. — Inward-flow or vortex wheel is made with radiating blades, and is surrounded by an annular case, closed externally, and open internally to wheel, having its inner circumference fitted with four curved guide-passages. The water is admitted by one or more pipes to the case, and it issues centripetally through the guide-passages upon circumference of wheel. The water acting against the curved blades, wheel is driven at a velocity dependent on height of fall, and water having expended its force, passes out at centre. This wheel has realized an efficiency as high as 77.5 per cent. It was originally designed by Prof. James Thomson.

Swain Turbine.—Combines an inward and a downward discharge. Receiving edges of buckets of wheel are vertical opposite guide-blades, and lower portions of the edges are bent into form of a quadrant. Each bucket thus forms, with the surface of adjoining bucket, an outlet which combines an inward and a downward discharge. One, 72 ins. in diameter, was tested

* In extreme cases of very high falls diameter given by this formula may be increased.

† Fourneyron's rule for the number of blades is constant number 36, irrespective of size of turbine.

by Mr. J. B. Francis, for several heights of gate or sluice, from 2 to 13.08 ins., and circumferential velocities of wheel ranging from 60 to 80 per cent. of respective velocities due to heads acting on wheel.

For a velocity of 60 per cent., and for heights of gate varying within limits already stated, efficiency ranged from 47.5 to 76.5 per cent., and for a velocity of 80 per cent. it ranged from 37.5 to 83 per cent. Maximum efficiency attained was 84 per cent., with a 12-inch gate and a velocity-ratio of 76 per cent.; but from 9-inch to 13-inch gate, or from .66 gate to full gate, maximum efficiency varied within very narrow limits—from 83 to 84 per cent.,—velocity-ratios being 72 per cent for 9-inch gate, and 76.5 per cent. for full gate. At half gate, maximum efficiency was 78 per cent., when velocity-ratio was 68 per cent. At quarter-gate, maximum efficiency was 61 per cent., and velocity-ratio 66 per cent.

Tremont Turbine, as observed by Mr. Francis, in his experiments at Lowell, Mass., gave a ratio of effect to power as .793 to 1.

Victor Turbine is alleged to have given an effect of .88 per cent. under a head of 18.34 feet, with a discharge of 977 cube feet of water per minute, and with 343.5 revolutions.

Tangential Wheel.

Wheels to which water is applied at a portion only of the circumference are termed tangential. They are suited for very high falls, where diameter and high tangential velocity may be combined with moderate revolutions. The *Girard* turbine belongs to this class. It is employed at Goeschenen station for St. Gothard tunnel, it operates under a head of 279 feet. The wheels are 7 feet 10.5 ins. in diam., having 80 blades, and their speed is 160 revolutions per minute, with a maximum charge of water of 67 gallons per second. An efficiency of 87 per cent. is claimed for them at the Paris water-works; ordinarily it is from 75 to 80 per cent. (*D. K. Clark.*)

Impact and Reaction Wheel.

Impact-wheel.—Impact Turbine is most simple but least efficient form of impact-wheel. It consists of a series of rectangular buckets or blades, set upon a wheel at an angle of 50° to 70° to horizon; the water flows to blades through a pyramidal trough set at an angle of 20° to 40°, so that the water impinges nearly at right angles to blades. Effect is .5 entire mechanical effect, which is increased by enclosing blades in a border or frame.

If buckets are given increased length, and formed to such a hollow curve that the water leaves wheel in nearly a horizontal direction, the water then impinges on buckets and exerts a pressure upon them; effect therefore is greater than with the force of impact alone.

By deductions of Weisbach it appears that effect of impact is only half available effect under most favorable circumstances.

Reaction-wheel.—Reaction of water issuing from an orifice of less capacity than section of vessel of supply, is *equal to weight of a column of water, basis of which is area of orifice or of stream, and height of which is twice height due to velocity of water discharged.*

Hence, the expression is $2 \cdot \frac{v^2}{2g} a w = R$. *w representing weight of a cube foot of water in lbs., and a area of opening in sq. feet.*

Whitelaw's is a modification of Barker's; the arms taper from centre towards circumference and are curved in such a manner as to enable the water to pass from central openings to orifices in a line nearly right and radial, when instrument is operating at a proper velocity; in order that very little centrifugal force may be imparted to the water by the revolution of the arms, and consequently a minimum of frictional resistance is opposed to course of the water.

A Turbine 9.55 feet in diameter, with orifices 4.944 ins. in diameter, operated by a fall of 25 feet, gave an efficiency of 75 per cent., including friction of gearing of an inclined plane.

When a reaction wheel is loaded, so that height due to velocity, corresponding to velocity of rotation v, is equal to fall, or $\frac{v^2}{2\,g}=h$, or $v=\sqrt{2\,g\,h}$, there is a loss of 17 per cent. of available effect; and when $\frac{v^2}{2\,g}=2\,h$, there is a loss of but 10 per cent.; and when $\frac{v^2}{2\,g}=4\,h$, there is a loss of but 6 per cent. Consequently, for moderate falls, and when a velocity of rotation exceeding velocity due to height of fall may be adopted, this wheel works very effectively.

Efficiency of wheel is but one half that of an undershot-wheel.

When sluice is lowered, so that only a portion of wheel is opened, efficiency of a Reaction-wheel is less than that of a Pressure Turbine.

Ratio of Effect to Power of several Turbines is as follows:

Poncelet	.65 to .75 to 1	Jonval	.6 to .7 to 1
Fourneyron	.6 to .75 to 1	Fontaine	.6 to .7 to 1

Barker's Mill.—Effect of this mill is considerably greater than that which same quantity of water would produce if applied to an undershot-wheel, but less than that which it would produce if properly applied to an overshot-wheel.

For a description of it, see *Grier's Mechanics' Calculator*, page 234; and for its formulas, see *London Artisan*, 1845, page 229.

IMPULSE AND RESISTANCE OF FLUIDS.

Impulse and Resistance of Water.—Water or any other fluid, when flowing against a body, imparts a force to it by which its condition of motion is altered. Resistance which a fluid opposes to motion of a body does not essentially differ from Impulse.

Impulse of one and same mass of fluid under otherwise similar circumstances is proportional to relative velocities $c \mp v$ of fluid.

For an equal transverse section of a stream, the impulse against a surface at rest increases as square of velocity of water.

Impulse against Plane Surfaces.—The impulse of a stream of water depends principally upon angle under which, after impulse, it leaves the water; it is nothing if the angle is 0, and a maximum if it is deflected back in a line parallel to that of its flow, or 180°, $2\,\frac{c \mp v}{g}\,V\,w = P$*.

When Surface of Resistance is a Plane, and $=90°$, then $\frac{c \mp v}{g}\,V\,w = P$, and *for a surface at rest,* $2\,a\,h\,w = P$. *a representing area of opening in sq. feet.*

$P = 2\,A\,h\,w$; *c and v representing velocities of water and of surface upon which it impinges in feet per second, w weight of fluid per cube foot in lbs.,* A *transverse section of stream in sq. ins., and* $c \mp v$ *relative motions of water and surface.*

Normal impulse of water against a plane surface is equivalent to weight of a column which has for its base transverse section of stream, and for altitude twice height due to its velocity, $2\,h = 2\,\frac{c^2}{2\,g}$.

Resistance of a fluid to a body in motion is same as impulse of a fluid moving with same velocity against a body at rest.

* Weisbach, New York, 1870, vol. i. page 1008.

Maximum Effect of Impulse.—Effect of impulse depends principally on velocity v of impinged surface. It is, for example, 0, both when $v=c$ and $v=0$; hence there is a velocity for which effect of impulse is a maximum $=(c-v)\,v$; that is, $v=\frac{c}{2}$, and maximum effect of impulse of water is obtained when surface impinged moves from it with half velocity of water.

ILLUSTRATION.—A stream of water having a transverse section of 40 sq. ins., discharges 5 cube feet per second against a plane surface, and flows off with a velocity of 12 feet per second; effect of its impulse, then, is $\frac{c \times v}{g}$ V $w=$ P; $c=\frac{5 \times 144}{40}=18$;

$$g=32.16;\quad w=62.5;\quad \frac{18-12}{32.16} \times 5 \times 62.5=58.28 \textit{ lbs.}$$

Hence mechanical effect upon surface $=$ P $v=58.28 \times 12=699.36$ *lbs.*

Maximum effect would be $v=\frac{c}{2}=\frac{1}{2} \times \frac{5 \times 144}{40}=9$ *feet, and* $\frac{1}{2} \times \frac{18^2}{2\,g} \times 5 \times 62.5$ $=\frac{1}{2} \times 5.036 \times 312.5=786.87$ *lbs.; and hydraulic pressure* $=\frac{786.87}{9}=87.44$ *lbs.*

When Surface is a Plane and at an Angle, then $(1-\cos. a)\,\frac{c}{g}$ V $w=$ P.

ILLUSTRATION.—A stream of water, having a transverse section of 64 sq. ins., discharges 17.778 cube feet per second against a fixed cone, having an angle of convergence from flow of stream of 50°, hydraulic pressure in direction of stream; then $c=\frac{17.778}{64 \div 144}=40$; cos. $50°=.64279$. $(1-.64279)\;\frac{40}{32.16} \times 17.778 \times 62.5=$ $.35721 \times 1382.2=494.26$ *lbs.*

When Surface of Resistance is a Plane at 90°, *and has Borders added to its Perimeter*, effect will be greater, depending upon height of border and ratio of transverse section between stream and part confined.

Oblique Impulse.—In oblique impulse against a plane, the stream may flow in one, two, or in all directions over plane.

When Stream is confined at Three Sides, $(1 \cos. a)\;\frac{c-v}{g}$ V $w=$ P.

When Stream is confined at Two Sides, $\frac{c-v}{g}$ sin. a^2 V $w=$ P.

Normal impulse of a stream increases as sine of angle of incidence; parallel impulse as square of sine of angle; and lateral impulse as double the angle.

When an Inclined Surface is not Bordered, then stream can spread over it in all directions, and impulse is greater, because of all the angles by which the water is deflected, a is least; hence each particle that does not move in normal plane exerts a greater pressure than particle in that plane, and $\frac{2 \sin. a^2}{1+\sin. a^2} \times \frac{c-v}{g}$ V $w=$ P.

Impulse and Resistance against Surfaces.

Coefficient of resistance, C, or number with which height due to velocity is to be multiplied, to obtain height of a column of water measuring this hydraulic pressure, varies for bodies of different figures, and only for surfaces which are at right angles to direction of motion is it nearly a definite quantity.

According to experiments of Du Buat and Thibault, C $=1.85$ for impulse of air or water against a plane surface at rest, and for resistance of air or water against a surface in motion, C $=1.4$. In each case about .66 of effect is expended upon front surface, and .34 upon rear.

Comparison between Turbines and other Water-wheels.

Turbines are applicable to falls of water at any height, from 1 to 500 feet.

Their efficiency for very high falls is less than for smaller, in consequence of the hydraulic resistances involved, and which increase as the square of the velocity of the water. They can only be operated in clear water.

With Fourneyron's, the stress and pressure on the step is that of the wheel in motion; with Fontaine's, the whole weight of the water is added to that of the wheel; they are well adapted, however, for tide-mills. Experiments on Jouval's gave equal results with Fontaine's.

Vertical Water-wheels are limited in their application to falls under 60 feet in height.

For falls of from 40 to 20 feet they give a greater effect than any turbine; for falls of from 20 to 10 feet, they are equal to them; and for very low falls, they have much less efficiency.

Variations in the supply of water effect them less than turbines.

Water-pressure Engine.

By experiments of M. Jordan, he ascertained that a mean useful effect of .84 was attainable.

Weisbach, London, 1848, vol. ii. page 349.

PERCUSSION OF FLUIDS.

When a stream strikes a plane perpendicular to its action, force with which it strikes is estimated by product of area of plane, density of fluid, and square of its velocity.

Or, $A\,d\,v^2 = P$. A *representing area in sq. feet, d weight of fluid in lbs., and v velocity in feet per second.*

If plane is itself in motion, then force becomes $A\,d\,(v - v')^2 = P$. v' *representing velocity of plane.*

If C represent a coefficient to be determined by experiment, and h height due to velocity v, then $v^2 = 2\,g\,h$, and expression for force becomes $A\,C\,2\,g\,h = P$.

CENTRIFUGAL PUMPS. (*D. K. Clark.*)

Appold Pump, made with curved receding blades, is the form of centrifugal pump most widely known and accepted. M. Morin tested three kinds of centrifugal or revolving pumps:

1st, on model of Appold pump; 2d, one having straight receding blades inclined at an angle of 45° with the radius, and 3d, one having radial blades. They were 12 ins. in diameter and 3.125 ins. in length, and had central openings of 6 ins. Their efficiencies were as follows:

1. Curved blades.. 48 to 68 per cent. | 2. Inclined blades.. 40 to 43 per cent.
3. Radial blades........ 24 per cent.

Height to which water ascends in a pipe, by action of a centrifugal pump, would, if there were no other resistances, be that due to velocity of circumference of revolving wheel, or to $\frac{v^2}{2\,g}$. Results of experiments made by the author on two pumps, in 1862, yielded following data, showing height to which water was raised, without any discharge:

	Gwynne's Pump (blades partly radial, curved at ends).	Appold Pump (blades, curved).
Diameter of pump-wheel...............	4 feet.	4 feet 7 ins.
Revolutions per minute................	177	95.4
Velocity of circumference per second...	37.05 feet.	22.9 feet.
Head due to the velocity...............	21.45 "	8.194 "
Actual head..........................	18.21 "	5.833 "
Do. do. in parts of head due to velocity,	85 per cent.	71.2 per cent.

Mr. David Thomson made similar experiments with Appold pumps of from 1.25 to 1.71 feet in diameter, the results of which showed that the actual head was about 90 per cent. of the head due to the velocity.

M. Tresca, in 1861, tested two centrifugal pumps, 18 ins. in diameter, with a central opening of 9 ins. at each side. The blades were six in number, of which three sprung from centre, where they were .5 inch thick; the alternate three only sprung at a distance equal to radius of opening from centre. They were radial, except at ends, where they were curved backward, to a radius of about 2.25 ins.; and they joined the circumference nearly at a tangent. Width of blades was taper, and they were 5.75 ins. wide at nave, and only 2.625 ins. at ends; so designed that section of outflowing water should be nearly constant.

M. Tresca deduced from his experiments that, in making from 630 to 7[illegible]0 revolutions per minute, efficiency of the pump, or actual duty in raising water, through a height of 31.16 feet, amounted to from 34 to 54 per cent. of work applied to shaft; or that, in the conditions of the experiment, the pump could raise upward of 16 200 cube feet of water per hour, through a height of 33 feet, with about 30 IP applied to shaft, and an efficiency of 45 per cent.

According to Mr. Thomson, maximum duty of a centrifugal pump worked by a steam-engine varies from 55 per cent for smaller pumps to 70 per cent for larger pumps. They may be most effectively used for low or for moderately high lifts, of from 15 to 20 feet; and, in such conditions, they are as efficient as any pumps that can be made. For lifts of 4 or 5 feet they are even more efficient than others.

At same time, larger the pump higher lift it may work against. Thus, an 18 inch pump works well at 20-feet lift, and a 3-feet pump at 30-feet lift. A 21-inch wheel at 40 feet lift has not given good results; high lifts demand very high velocities.

Efficiency is influenced by form of casing of pump. Hon. R. C. Parsons made experiments with two 14-inch wheels on Appold's and on Rankine's forms. In Rankine's wheel blades are curved backwards, like those of Appold's, for half their length; and curved forwards, reversely, for outer half of their length. Deducing results of performance arrived at, following are the several amounts of work done per lb. of water evaporated from boiler:

		Work done per lb. of water evaporated. Foot-lbs.	Ratio.
Appold wheel,	in concentric circular casing..........	11 385	1.06
" "	in spiral casing	15 996	1.5
Rankine wheel,	in concentric circular casing..........	10 748	1
" "	in spiral casing	12 954	1.2

These data prove:—1st, that spiral casing was better than concentric casing; 2d, that Appold's wheel was more efficient than Rankine's wheel.

IMPACT OR COLLISION.

IMPACT is *Direct* or *Oblique*. Bodies are Elastic or Inelastic. The division of them into *hard* and *elastic* is wholly at variance with these properties; as, for instance, glass and steel, which are among hardest of bodies, are most elastic of all.

Product of mass and velocity of a body is the *Momentum* of the body.

Principle upon which motions of bodies from percussion or collision are determined belongs both to elastic and inelastic bodies; thus there exists in bodies the same momentum or quantity of motion, estimated in any one and same direction, both before collision and after it.

Action and reaction are always equal and contrary. If a body impinge obliquely upon a plane, force of blow is as the sine of angle of incidence.

When a body impinges upon a plane surface, it rebounds at an angle equal to that at which it impinged the plane, that is, angle of reflection is equal to that of incidence.

Effect of a blow of an elastic body upon a plane is double that of an inelastic one, velocity and mass being equal in each; for the force of blow from inelastic body is as its mass and velocity, which is only destroyed by resistance of the plane; but in an elastic body that force is not only destroyed, being sustained by plane, but another, also equal to it, is sustained by plane, in consequence of the restoring force, and by which the body is repelled with an equal velocity; hence intensity of the blow is doubled.

If two perfectly elastic bodies impinge on one another, their relative velocities will be same, both before and after impact; that is, they will recede from each other with same velocity with which they approached and met.

If two bodies are imperfectly elastic, sum of their moments will be same, both before and after collision, but velocities after will be less than in case of perfect elasticity, in ratio of imperfection.

Effect of collision of two bodies, as B and b, velocities of which are different, as v and v', is given in following formulas, in which B is assumed to have greatest momentum before impact.

If bodies move in same direction before and after impact, *sum of their moments before impact will be equal to their sum after.*

If bodies move in same direction before, and in opposite direction after impact, *sum of their moments before impact will be equal to difference of their sums after.*

If bodies move in opposite directions before, and in same direction after impact, *difference of their moments before impact will be equal to their sum after.*

If bodies move in opposite directions before, and in opposite directions after impact, *difference of their moments before impact will be equal to their difference after.*

To Compute Velocities of Inelastic Bodies after Impact.

When Impelled in Same Direction. $\frac{BV + \overline{b\,v}}{B+b} = r$. *B and b representing weights of the two bodies, V and v their velocities before impact, and r velocity of bodies after impact, all in feet.*

Consequently, $\frac{V-v}{B+b} \times b =$ *velocity lost by* B, and $\frac{V-v}{B+b} \times B =$ *velocity gained by b.*

NOTE.—In these formulas it is assumed that $V > v$. If $V < v$ the result will be negative, but may be read as positive if *lost* and *gained* are reversed in places.

ILLUSTRATION.—An inelastic body, b, weighing 30 lbs., having a velocity of 3 feet, is struck by another body, B, of 50 lbs., having a velocity of 7 feet; the velocity of b after impact will be

$$\frac{50 \times 7 + \overline{30 \times 3}}{50 + 30} = \frac{440}{80} = 5.5 \textit{ feet.}$$

When Impelled in Opposite Directions. $\frac{BV - \overline{b\,v}}{B+b} = r$.

ILLUSTRATION.—Assume elements of preceding case.

$$\frac{50 \times 7 - \overline{30 \times 3}}{50 + 30} = \frac{260}{80} = 3.25 \textit{ feet.}$$

When One Body is at Rest. $\frac{BV}{B+b} = r$.

ILLUSTRATION.—Assume elements as preceding.

$$\frac{50 \times 7}{50 + 30} = \frac{350}{80} = 4.75 \textit{ feet.}$$

When Bodies are inelastic, their velocities after impact will be alike.

To Compute Velocities of Elastic Bodies after Impact.

When Impelled in One Direction. $\frac{\overline{B-b}\,V+2\,b\,v}{B+b}=R$. and $\frac{2\,B\,V-\overline{B-b}\,v}{B+b}=r$.

ILLUSTRATION.—Assume elements as preceding.

$\frac{\overline{50-30}\times 7+2\times 30\times 3}{50+30}=\frac{320}{80}=4$ *feet*, and $\frac{2\times 50\times 7-\overline{50-30}\times 3}{50+30}=\frac{640}{80}=8$ *feet.*

Or, $V-\frac{2\,b}{B+b}\,\overline{V-v}=$ velocity of R, and $v+\frac{2\,B}{B+b}\,\overline{V-v}=$ velocity of r.

When Impelled in Opposite Directions.

$$\frac{\overline{B-b}\,V\sim 2\,b\,v}{B+b}=R,\text{ and }\frac{2\,B\,V-\overline{B-b}\,v}{B+b}=r.$$

ILLUSTRATION.—Assume elements as preceding.

$\frac{\overline{50-30}\times 7\sim 2\times 30\times 3}{50+30}=\frac{140\sim 180}{80}=-.5$ *feet*, and $\frac{2\times 50\times 7+\overline{50-30}\times 3}{50+30}=$ $\frac{700+60}{80}=9.5$ *feet.* Or, $\frac{2\,b\,(V+v)}{B+b}=$ *velocity lost by* B. As $\frac{2\times 30\times\overline{7+3}}{50+30}=\frac{600}{80}$ $=7.5$ *feet.*

When One Body is at Rest. $\frac{V\,\overline{B-b}}{B+b}=R$, and $\frac{2\,B\,V}{B+b}=r$.

ILLUSTRATION.—Assume elements as preceding.

$$\frac{7\times\overline{50-30}}{50+30}=\frac{140}{80}=1.75\textit{ feet, and }\frac{2\times 50\times 7}{50+30}=\frac{700}{80}=8.75\textit{ feet.}$$

To Compute Velocities of Imperfect Elastic Bodies after Impact.

Effect of Collision is increased over that of perfectly inelastic bodies, but not doubled, as in case of perfectly elastic bodies; it must be multiplied by $1+\frac{n}{m}$ or $\frac{m+n}{m}$, when $\frac{n}{m}$ represents degree of elasticity relative to both perfect inelasticity and elasticity.

Moving in same Direction. $V-\frac{m+n}{m}\times\frac{B}{B+b}\,(V-v)=R$; and $v+\frac{m+n}{m}\times\frac{B}{B+b}\,(V-v)=r$. *m and n representing ratio of perfect to imperfect elasticity.*

ILLUSTRATION.—Assume elements as preceding. m and $n=2$ and 1.

$7-\frac{2+1}{2}\times\frac{30}{50+30}\times\overline{7-3}=7-1.5\times\frac{30}{80}\times 4=7-2.25=4.75$ *feet*, and $3+\frac{2+1}{2}\times\frac{50}{50+30}\times\overline{7-3}=3+3.75=6.75$ *feet.*

When Moving in Opposite Directions.

$$V-\frac{m+n}{m}\times\frac{b\,(V+v)}{B+b}=R,\text{ and }\frac{m+n}{m}\times\frac{B}{B+b}\times(V+v)-v=r.$$

When One Body is at Rest. $\frac{V\left(B-\frac{n}{m}\,b\right)}{B+b}=R$, and $\frac{B\,V\left(1+\frac{n}{m}\right)}{B+b}=r$.

ILLUSTRATION.—Assume elements of preceding case.

$\frac{7\times\left(50-\frac{1}{2}\times 30\right)}{50+30}=\frac{7\times\overline{50-15}}{80}=3.0625$ *feet*, and $\frac{50\times 7\times\left(1+\frac{1}{2}\right)}{50+30}=$ $\frac{350\times 1.5}{80}=6.5625$ *feet.*

LIGHT.

LIGHT is similar to Heat in many of its qualities, being emitted in form of rays, and subject to same laws of reflection.

It is of two kinds, *Natural* and *Artificial ;* one proceeding from Sun and Stars, the other from heated bodies.

Solids shine in dark only at a temperature from 600° to 700°, and in daylight at 1000°.

Intensity of Light is inversely as square of distance from luminous body.

Velocity of Light of Sun is 185 000 miles per second.

Standard of Intensity or of comparison of light between different methods of Illumination is a Sperm Candle "short 6," burning 120 grains per hour.

Candles.

A Spermaceti candle .85 of a inch in diameter consumes an inch in length in 1 hour.

Decomposition of Light.

COLORS.	Maximum Ray.	Contrasts. Primary.	Contrasts. Second'y.	Contrasts. Tertiary.	Combinations. Primary.	Combinations. Secondary.	Combinations. Tertiary.
Violet....	Chemical.	—	—	—	Blue...	Green..	—
Indigo....	—	—	—	Brown.	Yellow.		Dark.
Blue	Electrical.	Blue.	—	—	Blue...	Purple.	Green.
Green	—	—	Green.	Green.	Red....	Orange.	Gray.
Yellow ...	Light.	Yellow.	—	—	—	Green..	
Orange ...	—	—	Orange.	Broken.	Yellow.	Purple.	Brown.
Red......	Heat.	Red.	Purple.	Green.	Red....	Orange.	

All colors of spectrum, when combined, are white.

Consumption and Comparative Intensity of Light of Candles.

CANDLE.	No. in a Lb.	Diameter.	Length.	Consumption per Hour.	Light comp'd with Carcel.
		Inch.	Ins.	Grains.	
Wax........................	3	1	12	135	.09
"	3	.875	15		
Spermaceti	3	.9	15	156	.09
"	4	.8	13.5		
"	6	.84	8.5		
Tallow.....................	3	1	12.5	204	.07
"	3	.9	15		
"	4	.8	13.75		

Compared with 1000 *Cube Feet of Gas.*

CANDLE.	Gas=1.	Con-sump-tion.	Light.	Con-sumption for equal Light.	CANDLE.	Gas=1.	Con-sump-tion.	Light.	Con-sumption for equal Light.
		Lbs.	Lbs.				Lbs.	Lbs.	
Paraffine.	.098	3.5	35.5	103	Adamantine.	.108	5.1	47.2	137
Sperm...	.095	3.9	41.1	120	Tallow......	.074	5.1	53.8	155

In combustion of oil in an ordinary lamp, a straight or horizontally cut wick gives great economy over one irregularly cut.

Relative Intensity, Consumption, Illumination, and Cost of various Modes of Illumination.

Oil at 11 cents, Tallow at 14 cents, Wax at 52 cents, and Stearine at 32 cents per lb. 100 cube feet coal gas at 14 cents, and 100 cube feet of oil gas at 52 cents.

Illuminator.	Illumination. Carcel Lamp = 100.	Actual Cost per Hour.	Cost for equal Intensity.	Illuminator.	Illumination. Carcel Lamp = 100.	Actual Cost per Hour.	Cost for equal Intensity.
		Cents.	Per H'r.			Cents.	Per H'r.
Carcel Lamp.....	100	.87	.87	Stearine Candle 5 to lb	66.6	.59	4.13
Lamp with inverted reserv'r.	57.8	.89	.99	Tallow " 6 "	54	.25	2.34
				Sperm " 6 "	67.5	.89	5.7
Astral Lamp......	48.7	.56	1.78	Coal Gas.............	—	1.22	.96
Wax Candle 6 to lb.	61.6	.92	6.31	Oil Gas..............	—	1.25	.98

1000 cube feet of 13-candle coal gas is equal to 7.5 gallons sperm oil, 52.9 lbs. mold, and 44.6 lbs. sperm candles.

Candles, Lamps, Fluids, and Gas.

Comparison of several Varieties of Candles, Lamps, and Fluids, with Coal Gas, deduced from Reports of Com. of Franklin Institute, and of A. Frye, M.D., etc.*

Candle.	Intensity of Light.†	Light at Equal Costs.	Cost compared with Gas for Equal Light.	Candle.	Intensity of Light.†	Light at Equal Costs.	Cost compared with Gas for Equal Light.
Diaphane...........	.7	.5	15.1	Tallow, short 6's, double wick..	1	1	7.1
Spermaceti, short 6's.	.8	.54	16.2	Wax, short 6's....	.8	.61	14.4
Tallow, short 6's, single wick...	.58	.85	7.5	Palm oil.........	7	.77	10.5

* City of Philadelphia. † Compared with a fish-tail jet of Edinburgh gas, containing 12 per cent. of condensable matter and consuming 1 cube foot per hour.

Lamp and Fluid.	Intensity of Light.	Light at Equal Cost.	Time of Burning 1 Pint of Oil.	Lamp and Fluid.	Intensity of Light.	Light at Equal Cost.	Time of Burning 1 Pint of Oil.
Carcel.			Hours.				Hours.
Sperm oil, *max'm*	2.15	1.8	6.32	Gas.................	1	1	—
" *mean.*	1.22	1.35	9.87	Semi-solar, Sperm oil	1.15	.93	6.75
" *min'm*	.69	1.2	14.6	Solar, Sperm oil.....	1.76	1.55	8.42
Lard oil.........	.77	.97	11.3	Camphene..........	1.75	3.08	9.31

Loss of Light by Use of Glass Globes.

Clear Glass, 12 per cent. | Half ground, 35 per cent. | Full ground, 40 per cent.

Refraction.

Relative Index of Refraction—Is, Ratio of sine of angle of incidence to sine of angle of refraction, when a ray of light passes from one medium into another.

Absolute Index or Index of Refraction—Is, When a ray passes from a vacuum into any medium, the ratio is greater than unity.

Relative index of refraction from any medium, as A, into another, as B, is always equal to absolute index of B, divided by absolute index of A.

Absolute index of air is so small, that it may be neglected when compared with liquids or solids; strictly, however, relative index for a ray passing from air into a given substance must be multiplied by absolute index for air, in order to obtain like index of refraction for the substance.

Mean Indices of Refraction.

Air at 32°...........	1	Glass, fluid.........	1.58 / 1.64	Humors of eye....	1.34
Alcohol.............	1.37			Salt, rock.........	1.55
Canada balsam.....	1.54	" crown.......	1.53 / 1.56	Water, fresh......	1.34
Crystalline lens.....	1.34			" sea........	1.34

Gas.

Retort.—A retort produces about 600 cube feet of gas in 5 hours with a charge of about 1.5 cwt. of coal, or 2800 cube feet in 24 hours.

In estimating number of retorts required, one fourth should be added for being under repairs, etc.

Pressure with which gas is forced through pipes should seldom exceed 2.5 ins. of water at the Works, or leakage will exceed advantages to be obtained from increased pressure.

The average mean pressure in street mains is equal to that of 1 inch of water.

When pipes are laid at an inclination either above or below horizon, a correction will have to be made in estimating supply, by adding or deducting .01 inch from initial pressure for every foot of rise or fall in the length of pipe.

It is customary to locate a governor at each change of level of 30 feet.

Illuminating power of coal-gas varies from 1.6 to 4.4 times that of a tallow candle 6 to a lb.; consumption being from 1.5 to 2.3 cube feet per hour, and specific gravity from .42 to .58.

Higher the flame from a burner greater the intensity of the light, the most effective height being 5 ins.

Standard of gas burning is a 15-hole Argand lamp, internal diameter .44 inch, chimney 7 ins. in height, and consumption 5 cube feet per hour, giving a light from ordinary coal-gas of from 10 to 12 candles, with Cannel coal from 20 to 24 candles, and with rich coals of Virginia and Pennsylvania of from 14 to 16 candles.

In Philadelphia, with a fish-tail burner, consuming 4.26 cube feet per hour, illuminating power was equal to 17.9 candles, and with an Argand burner, consuming 5.28 cube feet per hour, illuminating power was 20.4 candles.

Gas, which at level of sea would have a *Value* of 100, would have but 60 in city of Mexico.

Internal lights require 4 cube feet, and external lights about 5 per hour. When large or Argand burners are used, from 6 to 10 are required.

An ordinary single-jet house burner consumes 5 to 6 cube feet per hour.

Street-lamps in city of New York consume 3 cube feet per hour. In some cities 4 and 5 cube feet are consumed. Fish-tail burners for ordinary coal gas consume from 4 to 5 cube feet of gas per hour.

A cube foot of good gas, from a jet .033 inch in diameter and height of flame of 4 ins., will burn for 65 minutes.

Resin Gas.—Jet .033, flame 5 ins., 1.25 cube feet per hour.

Purifiers.—Wet purifiers require 1 bushel of lime mixed with 48 bushels of water for 10 000 cube feet of gas.

Dry purifiers require 1 bushel of lime to 10 000 cube feet of gas, and 1 superficial foot for every 400 cube feet of gas.

Intensity of Light with Equal Volumes of Gas from different Burners.

Equal to Spermaceti Candle burning 120 Grains per Hour.

Burners.	Expenditure in Cube Feet per Hour.				Burners.	Expenditure in Cube Feet per Hour.			
	1	2	3	4		1	2	3	4
Single-jet, 1 foot.....	2.6	—	—	—	Argand, 16 holes....	.32	1.9	3.3	3.8
Fish-tail No. 3......	3.5	4	4.2	—	Argand, 24 holes....	.33	2.2	3.4	5.3
Bat's-wing..........	3	4.1	4.3	4.5	Argand, 28 holes....	.34	2.3	3.5	5.8

Volume of Gas obtained from a Ton of Coal, Resin, etc.

Material.	Cube Feet.	Material.	Cube Feet.	Material.	Cube Feet.
Boghead Cannel...	13 334	Cumberland........	9 800	Pittsburgh	9 520
Wigan Cannel.....	15 426	English, mean......	11 000	Resin............	15 600
Cannel..........	8 960 / 15 000	Newcastle........	9 500 / 10 000	Scotch.........	10 300 / 15 000
Cape Breton, "Cow Bay," etc.........	9 500	Oil and Grease	23 000	Virginia..........	8 960
		Pictou and Sidney..	8 000	" West'n..	9 500
		Pine wood	11 800	Walls-end........	12 000

1 Chaldron Newcastle coal, 3136 lbs., will furnish 8600 cube feet of gas at a specific gravity of .4, 1454 lbs. coke, 14.1 gallons tar, and 15 gallons ammoniacal liquor.

Australian coal is superior to Welsh in producing of gas.

Wigan Cannel, 1 ton, has produced coke, 1326 lbs.; gas, 338 lbs.; tar, 250 lbs.; loss, 326 lbs.

Peat, 1 lb. will produce gas for a light of one hour.

Fuel, required for a retort 18 lbs. per 100 lbs. of coal.

In distilling 56 lbs. of coal, volume of gas produced in cube feet when distillation was effected in 3 hours was 41.3, in 7, 37.5, in 20, 33.5, and in 25, 31.7.

Flow of Gas in Pipes.

Flow of Gas is determined by same rules as govern that of flow of water. Pressure applied is indicated and estimated in inches of water, usually from .5 to 1 inch.

Volumes of gases of like specific gravities discharged in equal times by a horizontal pipe, under same pressure and for different lengths, are inversely as square roots of lengths.

Velocity of gases of different specific gravities, under like pressure, are inversely as square roots of their gravities.

By experiment, 30 000 cube feet of gas, specific gravity of .42, were discharged in an hour through a main 6 ins. in diameter and 22.5 feet in length.

Loss of volume of discharge by friction, in a pipe 6 ins. in diameter and 1 mile in length, is estimated at 95 per cent.

Diameter and Length of Gas-pipes to transmit given Volumes of Gas to Branch-pipes. (*Dr. Ure.*)

Volume per Hour.	Diameter.	Length.	Volume per Hour.	Diameter.	Length.	Volume per Hour.	Diameter.	Length.
Cube Feet.	Ins.	Feet.	Cube Feet.	Ins.	Feet.	Cube Feet.	Ins.	Feet.
50	.4	100	1000	3.16	1000	2000	7	6000
250	1	200	1500	3.87	1000	6000	7.75	1000
500	1.97	600	2000	5.32	2000	6000	9.21	2000
700	2.65	1000	2000	6.33	4000	8000	8.95	1000

Regulation of Diameter and Extreme Length of Tubing, and Number of Burners permitted.

Diameter of Tubing.	Length.	Capacity of Meters.	Burners.	Diameter of Tubing.	Length.	Capacity of Meters.	Burners.
Ins.	Feet.	Light.	No.	Ins.	Feet.	Light.	No.
.25	6	3	9	.75	50	30	90
.375	20	5	15	1	70	45	135
.5	30	10	30	1.25	100	60	180
.625	40	20	60	1.5	150	100	300

Temperature of Gases.—Combustion of a cube foot of common gas will heat 650 lbs. of water 1°.

Services for Lamps.

Lamps.	Length from Main.	Diameter of Pipe.	Lamps.	Length from Main.	Diameter of Pipe.	Lamps.	Length from Main.	Diameter of Pipe.
No.	Feet.	Ins.	No.	Feet.	Ins.	No.	Feet.	Ins.
2	40	.375	10	100	.75	25	180	1.5
4	40	.5	15	130	1	30	200	1.75
6	50	.625	20	150	1.25			

Volumes of Gas Discharged per Hour under a Pressure of Half an Inch of Water.

Specific Gravity .42.

Diam. of Opening.	Volume.	Diam. of Opening.	Volume.	Diam. of Opening.	Volume.	Diam. of Opening.	Volume.
Ins.	Cube Feet.	Ins.	Cube Feet.	Ins.	Cube Feet.	Ins.	Cube Feet.
.25	80	.75	723	1.125	1625	1.5	2885
.5	321	1	1287	1.25	2010	5	46150

To Compute Volume of Gas Discharged through a Pipe.

$1000\sqrt{\frac{d^5\,h}{G\,l}} = V$, and $.063\sqrt[5]{\frac{V_2\,G\,l}{h}} = d$. *d representing diameter of pipe, and h height of water in ins., denoting pressure upon gas, l length of pipe in yards,* G *specific gravity of gas, and* V *volume in cube feet per hour.*

G may be assumed for ordinary computation at .42, and h .5 to 1 inch.

ILLUSTRATION.—Assume diameter of pipe 1 inch, pressure 1.68 ins., and length of pipe 1 yard.

$$1000 \times \sqrt{\frac{1 \times 1.68}{.42 \times 1}} = 1000 \times \sqrt{\frac{1.68}{.42}} = 2000 \textit{ cube feet},$$

$$\text{and } .063 \times \sqrt[5]{\frac{4\,000\,000 \times .42 \times 1}{1.68}} = \sqrt[5]{\frac{168\,000\,000}{1.68}} = 1.05 \textit{ ins.}$$

NOTE.—For tables deduced by above formulas see Molesworth, 1878, page 226.

Dimensions of Mains, with Weight of One Length.

Diameter in ins.	4	6	8	9	10	14	18	20
Length in feet......	9	9	9	9	9	9	9	9
Thickness in ins. ...	.375	.375	.5	.5	.5	.625	.75	.75
Weight in lbs.	288	224	400	454	489	868	1316	1484

GAS ENGINES.

In the Lenoir engine, the best proportions of air and gas are, for common gas, 8 volumes of air to 1 of gas, and for cannel gas, 11 of air to 1 of gas.

The time of explosion is about the 27th part of a second.

An engine, having a cylinder 4.625 ins. in diameter and 8.75 ins. stroke of piston, making 185 revolutions per minute, develops a half horse-power.

Distribution of Heat Generated in the Cylinder. (*M. Tresca.*)

	Per cent.		Per cent.
Dissipated by the water and products of combustion...............	69	Losses............................	27
Converted into work................	4		100

Hence efficiency as determined by the brake = 4 per cent.

Atmospheric Gas Engine.

A single-acting cylinder 6 ins. in diameter, making 81 strokes per minute, developed .456 IP, and the gas consumed per minute for cylinder 20 cube feet and for inflaming 2 cube feet. (*M. Tresca.*)

LIMES, CEMENTS, MORTARS, AND CONCRETES.

*Essentially from a Treatise by Brig.-Gen'l Q. A. Gillmore, U.S.A.**

Lime.

Calcination of marble or any pure limestone produces *lime* (*quick-lime*). Pure limestones burn white, and give richest limes.

Finest calcareous minerals are rhombohedral prisms of calcareous spar, the transparent double-reflecting Iceland spar, and white or statuary marble.

Property of hardening under water, or when excluded from air, conferred upon a paste of lime, is effected by presence of foreign substances—as silicum, alumina, iron, etc.—when their aggregate presence amounts to .1 of whole.

Limes are classed: 1. Common or Fat limes, which do not set in water. 2. Poor or Meagre, mixed with sand, which does not alter its condition. 3. Hydraulic Lime, containing 8 to 12 per cent. of silica, alumina, iron, etc., set slowly in water. 4. Hydraulic, containing 12 to 20 per cent. of similar ingredients, sets in water in 6 or 8 days. 5. Eminently Hydraulic, containing 20 to 30 per cent. of similar ingredients, sets in water in 2 to 4 days. 6. Hydraulic Cement, containing 30 to 50 per cent. of argil, sets in a few minutes, and attains the hardness of stone in a few months. 7. Natural Pozzuolanas, including pozzuolana properly so called, Trass or Terras, Arènes, Ochreous earths, Basaltic sands, and a variety of similar substances.

Indications of Limestones. They dissolve wholly or partly in weak acids with brisk effervescence, and are nearly insoluble in water.

Rich Limes are fully dissolved in water frequently renewed, and they remain a long time without hardening; they also increase greatly in volume, from 2 to 3.5 times their original bulks, and will not harden without the action of air. They are rendered *Hydraulic* by admixture of pozzuolana or trass.

Rich, fat, or *common Limes* usually contain less than 10 per cent. of impurities.

Hydraulic Limestones are those which contain iron and clay, so as to enable them to produce cements which become solid when under water.

Poor Limes have all the defects of rich limes, and increase but slightly in bulk, the poorer limes are invariably basis of the most rapidly-setting and most durable cements and mortars, and they are also the only limes which have the property, when in combination with silica, etc., of indurating under water, and are therefore applicable for admixture of hydraulic cements or mortars. Alike to rich limes, they will not harden if in a state of paste under water or in wet soil, or if excluded from contact with the atmosphere or carbonic acid gas. They should be employed for mortar only when it is impracticable to procure common or hydraulic lime or cement, in which case it is recommended to reduce them to powder by grinding.

Hydraulic Limes are those which readily harden under water. The most valuable or *eminently hydraulic set* from the 2d to the 4th day after immersion; at end of a month they become hard and insoluble, and at end of six months they are capable of being worked like the hard, natural limestones. They absorb less water than pure limes, and only increase in bulk from 1.75 to 2.5 times their original volume.

---

* See also his Treatises on Limes, Hydraulic Cements, and Mortars, in Papers on Practical Engineering, Engineer Department, U. S. A.

Inferior grades, or *moderately hydraulic*, require a period of from 15 to 20 days' immersion, and continue to harden for a period of 6 months.

Resistance of hydraulic limes increase if sand is mixed in proportion of 50 to 180 per cent. of the part in volume; from thence it decreases.

M. Vicat declares that lime is rendered hydraulic by admixture with it of from 33 to 40 per cent. of clay and silica, and that a lime is obtained which does not slake, and which quickly sets under water.

Artificial Hydraulic Limes do not attain, even under favorable circumstances, the same degree of hardness and power of resistance to compression as natural limes of same class.

Close-grained and densest limestones furnish best limes.

Hydraulic limes lose or depreciate in value by exposure to the air.

Pastes of fat limes shrink, in hardening, to such a degree that they cannot be used as mortar without a large proportion of sand.

Arènes is a species of ochreous sand. It is found in France. On account of the large proportion of clay it contains, sometimes as great as .7, it can be made into a paste with water without any addition of lime; hence it is sometimes used in that state for walls constructed *en pisé*, as well as for mortar. Mixed with rich lime it gives excellent mortar, which attains great hardness under water, and possesses great hydraulic energy.

Pozzuolana is of volcanic origin. It comprises Trass or Terras, the Arènes, some of the ochreous earths, and the sand of certain graywackes, granites, schists, and basalts; their principal elements are silica and alumina, the former preponderating. None contain more than 10 per cent. of lime.

When finely pulverized, without previous calcination, and combined with paste of fat lime in proportions suitable to supply its deficiency in that element, it possesses hydraulic energy to a valuable degree. It is used in combination with rich lime, and may be made by slightly calcining clay and driving off the water of combination at a temperature of 1200°.

Brick or *Tile Dust* combined with rich lime possesses hydraulic energy.

Trass or *Terras* is a blue-black trap, and is also of volcanic origin. It requires to be pulverized and combined with rich lime to render it fit for use, and to develop any of its hydraulic properties.

General Gillmore designates the varieties of hydraulic limes as follows: If, after being slaked, they harden under water in periods varying from 15 to 20 days after immersion, *slightly hydraulic;* if from 6 to 8 days, *hydraulic;* and if from 1 to 4 days, *eminently hydraulic.*

Pulverized silica burned with rich lime produces hydraulic lime of excellent quality. Hydraulic limes are injured by air-slaking in a ratio varying directly with their hydraulicity, and they deteriorate by age.

For foundations in a damp soil or exposure, hydraulic limes must be exclusively employed.

Hydraulic Lime of Teil is a silicious hydraulic lime; it is slow in setting, requiring a period of from 18 to 24 hours.

Cements.

Hydraulic Cements contain a larger proportion of silica, alumina, magnesia, etc., than any of preceding varieties of lime; they do not slake after calcination, and are superior to the very best of hydraulic limes, as some of them set under water at a moderate temperature (65°) in from 3 to 4 minutes; others require as many hours. They do not shrink in hardening, and make an excellent mortar without any admixture of sand.

When exposed to air, they absorb moisture and carbonic acid gas, and are rapidly deteriorated thereby.

Roman Cement is made from a lime of a peculiar character, found in England and France, derived from argillo-calcareous kidney-shaped stones termed *Septaria*.

It is about .33 strength of Portland, and is not adapted for use with sand.

Rosendale Cement is from Rosendale, New York.

Portland Cement is made in England and France. It requires less water (cement 1, water .29) than Roman cement, sets slowly, and *can be remixed with additional water after an interval of* 12 *or even* 24 *hours from its first mixture.*

Property of setting slow may be an obstacle to use of some designations of this cement, as the Boulogne, when required for local ties having to contend against immediate causes of destruction, as in sea constructions, having to be executed under water and between tides. On the other hand, a quick setting cement is always difficult of use; it requires special workmen and an active supervision. A slow-setting cement, however, like natural Portland, possesses the advantage of being managed by ordinary workmen, and *it can also be remixed with additional water after an interval of* 12 *or even* 24 *hours from its first mixing.*

Conclusions derived from Mr. Grant's Experiments.

1. Portland cement improves by age, if kept from moisture.
2. Longer it is in setting, stronger it will be.
3. At end of a year, 1 of cement to 1 sand is about .75 strength of neat cement; 1 to 2, .5 strength; 1 to 3, .33; 1 to 4, .25; 1 to 5, .16.
4. Cleaner and sharper the sand, greater the strength.
5. Strong cement is heavy; blue gray, slow setting. Quick setting has generally too much clay in its composition—is brownish and weak.
6. Less water used in mixing cement the better.
7. Bricks, stones, etc., used with cement should be well wetted before use.
8. Cement setting under *still* water will be stronger than if kept dry.
9. Bricks of neat Portland cement in a few months are equal to Blue bricks, Bramley-Fall stone, or Yorkshire landings.
10. Bricks of 1 cement to 4 or 5 of sand are equal to picked stock bricks.
11. When concrete is being used, a current of water will wash away the cement.

Artificial Cement is made by a combination of slaked lime with unburned clay in suitable proportions.

Artificial Pozzuolana is made by subjecting clay to a slight calcination.

Salt water has a tendency to decompose cements of all kinds, and their strength is considerably impaired by their mixture with it.

Mortar.

Lime or Cement paste is the cementing substance in mortar, and its proportion should be determined by the rule that *Volume of cementing substance should be somewhat in excess of volume of voids or spaces in sand or coarse material to be united,* the excess being added to meet imperfect manipulation of the mass.

Hydraulic Mortar, if re-pulverized and formed into a paste after having once *set*, immediately loses a great portion of its hydraulicity, and descends to the level of moderate hydraulic limes.

The retarding influence of sea-water upon initial hydraulic induration is not very great, if the cement is mixed with fresh water. The strength of mortars, however, is considerably impaired by being mixed with sea-water.

Pointing Mortar is composed of a paste of finely-ground cement and clean sharp siliceous sand, in such proportions that the volume of cement paste is slightly in excess of the volume of voids or spaces in the sand. The volume of sand varies from 2.5 to 2.75 that of the cement paste, or by weight, 1 of cement powder to 3 to 3.33 of sand. The mixture should be made under shelter, and in small quantities.

All mortars are much improved by being worked or manipulated; and as rich limes gain somewhat by exposure to the air, it is advisable to work mortar in large quantities, and then render it fit for use by a second manipulation.

White lime will take a larger proportion of sand than brown lime.

Use of salt-water in the composition of mortar injures adhesion of it.

When a small quantity of water is mixed with slaked lime, a stiff paste is made, which, upon becoming dry or hard, has but very little tenacity, but, by being mixed with sand or like substance, it acquires the properties of a cement or mortar.

Proportion of sand that can be incorporated with mortar depends partly upon the degree of fineness of the sand itself, and partly upon character of the lime. For rich limes, the resistance is increased if the sand is in proportions varying from 50 to 240 per cent. of the paste in volume; beyond this proportion the resistance decreases.

Lime, 1, clean sharp sand, 2.5. An excess of water in slaking the lime swells the mortar, which remains light and porous, or shrinks in drying; an excess of sand destroys the cohesive properties of the mass.

It is indispensable that the sand should be sharp and clean.

Stone Mortar.—8 parts cement, 3 parts lime, and 31 parts of sand; or 1 cask cement, 325 lbs., .5 cask of lime, 120 lbs., and 14.7 cube feet of sand= 18.5 cube feet of mortar.

Brick Mortar.—8 parts cement, 3 parts lime, and 27 parts of sand; or 1 cask cement, 325 lbs., .5 cask of lime, 120 lbs., and 12 cube feet of sand= 16 cube feet of mortar.

Brown Mortar.—Lime 1 part, sand 2 parts, and a small quantity of hair.

Lime and sand, and cement and sand, lessen about .33 in volume when mixed together.

Calcareous Mortar, being composed of one or more of the varieties of lime or cement, natural or artificial, mixed with sand, will vary in its properties with quality of the lime or cement used, the nature and quality of sand, and method of manipulation.

Turkish Plaster, or Hydraulic Cement.

100 lbs. fresh lime reduced to powder, 10 quarts linseed-oil, and 1 to 2 ounces cotton. Manipulate the lime, gradually mixing the oil and cotton, in a wooden vessel, until mixture becomes of the consistency of bread-dough.

Dry, and when required for use, mix with linseed-oil to the consistency of paste, and then lay on in coats. Water-pipes of clay or metal, joined or coated with it, resist the effect of humidity for very long periods.

Stucco.

Stucco or Exterior Plaster is term given to a certain mortar designed for exterior plastering; it is sometimes manipulated to resemble variegated marble, and consists of 1 volume of cement powder to 2 volumes of dry sand.

In India, to water for mixing the plaster is added 1 lb. of sugar or molasses to 8 Imperial gallons of water, for the first coat; and for second or finishing, 1 lb. sugar to 2 gallons of water.

Powdered slaked lime and Smith's forge scales, mixed with blood in suitable proportions, make a moderate hydraulic mortar, which adheres well to masonry previously coated with boiled oil.

Plaster should be applied in two coats laid on in one operation, first coat being thinner than second. Second coat is applied upon first while latter is yet soft.

The two coats should form one of about 1.5 inches in thickness, and when finished it should be kept moist for several days.

When the cement is of too dark a color for desired shade, it may be mixed with white sand in whole or in part, or lime paste may be added until its volume equals that of the cement paste.

Khorassar, or Turkish Mortar,

Used for the construction of buildings requiring great solidity, .33 powdered brick and tiles, .66 fine sifted lime. Mix with water to required consistency, and lay between the courses of brick or stones.

Mortars.

Mortars used for inside plastering are termed Coarse, Fine, Gauge or hard finish, and Stucco.

Plastering.—1 bushel, or 1.25 cube feet of cement, mortar, etc., will cover 1.5 square yards .75 inch thick. 75 volumes are required upon brick work for 70 upon laths.

When full time for hardening cannot be allowed, substitute from 15 to 20 per cent. of the lime by an equal proportion of hydraulic cement.

For the second or *brown coat* the proportion of hair may be slightly diminished.

Coarse Stuff.—Common lime mortar, as made for brick masonry, with a small quantity of hair; or by volumes, lime paste (30 lbs. lime) 1 part, sand 2 to 2.25 parts, hair .16 part.

Fine Stuff (lime putty).—Lump lime slaked to a paste with a moderate volume of water, and afterwards diluted to consistency of cream, and then to harden by evaporation to required consistency for working.

In this state it is used for a *slipped coat*, and when mixed with sand or plaster of Paris, it is used for *finishing coat.*

Gauge, or Hard Finish, is composed of from 3 to 4 volumes fine stuff and 1 volume plaster of Paris, in proportions regulated by rapidity required in hardening; for cornices, etc., proportions are equal volumes of each, fine stuff and plaster.

Scratch Coat.—First of three coats when laid upon laths, and is from .25 to .375 of an inch in thickness.

One-coat Work.—Plastering in one coat without finish, either on masonry or laths—that is, *rendered* or *laid.*

Two-coat Work.—Plastering in two coats is done either in a *laid coat and set*, or in a *screed coat and set.*

Screed coat is also termed a *Floated coat. Laid* first coat in two-coat work is resorted to in common work instead of *screeding*, when finished surface is not required to be exact to a straight-edge. It is laid in a coat of about .5 inch in thickness.

Laid coat, except for very common work, should be *hand-floated.*

Firmness and tenacity of plastering is very much increased by hand-floating.

Screeds are strips of mortar 6 to 8 inches in width, and of required thickness of first coat, applied to the angles of a room, or edge of a wall and parallelly, at intervals of 3 to 5 feet over surface to be covered. When these have become sufficiently hard to withstand pressure of a straight-edge, the interspaces between the screeds are filled out flush with them.

Slipped Coat is the smoothing off of a brown coat with a small quantity of lime putty, mixed with 3 per cent. of white sand, so as to make a comparatively even surface.

This finish answers when the surface is to be finished in distemper, or paper.

Concrete or Beton

Is a mixture of mortar (generally hydraulic) with coarse materials, as gravel, pebbles, stones, shells, broken bricks, etc. Two or more of these materials, or all of them, may be used together. As lime or cement paste is the cementing substance in mortar, so is mortar the cementing substance in concrete or beton. The original distinction between cement and beton was, that latter possessed hydraulic energy, while former did not.

Hydraulic.—1.5 parts unslaked hydraulic lime, 1.5 parts sand, 1 part gravel, and 2 parts of a hard broken limestone.

This mass contracts one fifth in volume. Fat lime may be mixed with concrete, without serious prejudice to its hydraulic energy.

Various Compositions of Concrete.

Hydraulic.—308 lbs. cement = 3.65 to 3.7 cube feet of stiff paste. 12 cube feet of loose sand = 9.75 cube feet of dense.

For Superstructure.—11.75 cube feet of mortar as above, and 16 cube feet of stone fragments.

Sea Wall.—Boston Harbor.—Hydraulic.—308 lbs. cement, 8 cube feet of sand, and 30 cube feet of gravel. Whole producing 32.3 cube feet.

Superstructure.—308 lbs. cement, 80 lbs. lime, and 14.6 cube feet dense sands. Whole producing 12.825 cube feet.

Pisé is made of clay or earth rammed in layers of from 3 to 4 ins. in depth. In moist climates, it is necessary to protect the external surface of a wall constructed in this manner with a coat of mortar.

Asphalt Composition.

1. Mineral pitch 1 part, bitumen 11, powdered stone, or wood ashes, 7 parts.

2. Ashes 2 parts, clay 3 parts, and sand 1 part, mixed with a little oil, makes a very fine and durable cement, suitable for external use.

Flooring.—8 lbs. of composition will cover 1 sup. foot, .75 inch thick.

Asphaltum 55 lbs. and gravel 28.7 lbs. will cover an area of 10.75 sq. feet.

Mastic.—Pulverized burnt clay 93 parts, litharge, ground very fine, 7 parts, mixed with a sufficient quantity of pure linseed oil.

3. Siliceous sand 14, pulverized calcareous stone 14, litharge 2, and linseed oil 4 parts by weight.

The powders to be well dried in an oven, and the surface upon which it is to be applied must be saturated with oil.

4. *For Roads.*—Bitumen 16.875 parts, asphaltum 225 parts, oil of resin 6.25 parts, and sand 135 parts. Thickness, from 1.25 to 1.375 ins.

Artificial Mastic.—Composition of 1 square yard .9 inch thick:

Mineral tar	205 cube ins.	Gravel	275 cube ins.
Pitch	165 " "	Slaked lime	55 " "
Sand	549 " "		1249 cube ins.

Mural Efflorescence.—White alkaline efflorescence upon the surface of brick walls laid in mortar, of which natural hydraulic lime or cement is the basis. Mortar mixed with animal fat in the proportion of .025 of its weight will prevent its formation.

Crystallization of these salts within the pores of bricks, into which they have been absorbed from the mortar, causes disintegration.

Distemper is term for all coloring mixed with water and size.

Grouting.—Mortar composed of lime and fine sand, in a semi-fluid state, poured into the upper beds and internal joints of masonry.

Laitance is the pulpy and gelatinous fluid, of a milky hue, that is washed from cement upon its being deposited in water. It is produced more abundantly in sea water than in fresh; it sets very imperfectly, and has a tendency to lessen the strength of the concrete.

Slaking.

Slaked Lime is a hydrate of lime, and it absorbs a mean of 2.5 times its volume, and 2.25 times its weight of water.

Lime (*quicklime*) must be slaked before it can be used as a matrix for mortar.

Ordinary method of slaking is by submitting the lime to its full proportion of water (previously known or attained by trial) in order to reduce it to the consistency of a thick pulp. The volume of water required for this purpose will vary with different limes, and will range from 2.5 to 3 volumes that of the lime, and it is imperative that it should all be poured upon it so nearly at one time as to be in advance of the elevation of the temperature consequent upon its reduction.

This process, when the water used is in an excessive quantity, is termed "drowning," and when the volume of lime has increased by the absorption of water it is termed its "growth."

If too much water is used, the binding qualities of the lime is injured by its semi-fluidity; and if too little, it is injurious to add after the reduction of the lime has commenced, as it reduces its temperature and renders it granular and lumpy.

While lime is in progress of slaking it should be covered with a tarpaulin or canvas (a layer of sand will suffice), in order to concentrate its evolved heat.

The essential point in slaking is to attain the complete reduction of the lime, and the greater the hydraulic energy of a lime, the more difficult it becomes to effect it.

Whitewash or Grouting.—When lime is required for a whitewash or for grouting, it should be thoroughly "drowned," and then run off into tight vessels and closed.

Slaking by Immersion is the method of suspending lime in a suitable vessel in water for a very brief period, and withdrawing it before reduction commences. The lime is then transferred to casks or like suitable receptacles, and tightly enclosed, until it is reduced to a fine powder, in which condition, if secured from absorption of air, it may be preserved for several months without essential deterioration.

Spontaneous or Air Slaking.—When lime is not wholly secured from exposure to the air, it absorbs moisture therefrom, slakes, and falls into a powder.

Limes and Cements.—A Cask of Lime = 240 lbs., will make from 7.8 to 8.15 cube feet of stiff paste.

A Cask of Cement = 300* lbs., will make from 3.7 to 3.75 cube feet of stiff paste.

A Cask of Portland Cement = 4 bushels or 5 cube feet = 420 lbs.

A Cask of Roman Cement = 3 bushels or 3.75 cube feet = 364 lbs.

	.5 inch.	.75 inch.	1 inch.
A Bushel of cement will cover.......	2.25 yards	1.5 yards	1.14 yards.

From experiments of General Totten, it appeared that

1	volume of lime	slaked	with .33	its volume of water	gave	2.27	volumes	of powder.
1	"	"	" .66	"	"	1.74	"	"
1	"	"	" 1	"	"	2.06	"	"

One cube foot of dry cement, mixed with .33 cube foot of water, will make .63 to 635 cube foot of stiff paste.

Lime should be slaked at least one day before it is incorporated with the sand, and when they are thoroughly mixed, the mortar should be heaped into one volume or mass, for use as required.

---

* 300 lbs. net is standard; it usually overruns 8 lbs.

Mortar, Cement, &c. (*Molesworth.*)

Mortar.—1 of lime to 2 to 3 of sharp river sand.

Or, 1 of lime to 2 sand and 1 blacksmith's ashes, or coarsely ground coke.

Coarse Mortar.—1 of lime to 4 of coarse gravelly sand.

Concrete.—1 of lime to 4 of gravel and 2 of sand.

Hydraulic Mortar.—1 of blue lias lime to 2.5 of burnt clay, ground together.

Or, 1 of blue lias lime to 6 of sharp sand, 1 of pozzuolana and 1 of calcined ironstone.

Beton.—1 of hydraulic mortar to 1.5 of angular stones.

Cement.—1 of sand to 1 of cement.—If great tenacity is required, the cement should be used without sand.

Portland Cement

Is composed of clayey mud and chalk ground together, and afterwards calcined at a high temperature—after calcining it is ground to a fine powder.

Strength of Mortars, Cements, and Concretes.

Deduced from Experiments of Vicat, Paisley, Treussart, and Voisin.

Tensile

Weight or Power required to Tear asunder One Sq. Inch.

Cement Mortar. (42 *days old.*)

	Proportion of Sand to 1 of Cement.										
	0	1	2	3	4	5	6	7	8	9	10
Roman.......	284	284	199	166	142	128	116	106	99	92	95 lbs.
Portland......	142	142	113	92	79	67	57	42	35	25	— "

Brick, Stone, and Granite Masonry. (320 *days old.*)

Experiments of General Gillmore, U. S. A.

Cement on Bricks.

	Lbs.
Pure, average	30.8
Sand 1, Cement 1	15.7
Sand 1, Cement 2	12.3
Sand 1, Cement 3	6.8

Cement on Granite.

	Lbs.		Lbs.
Pure.................	27.5	Sand 1, Cement 4	7.9
Sand 1, Cement 1	20.8	Water 1, Cement 2	20.5
Sand 1, Cement 2	12.6	Water .42, Cement 1	37.25
Sand 1, Cement 3	9.2	Water .33, Cement 1	29.15

Delafield and Baxter.	Lbs.
Pure cement........	68
Cement 4, Sand 1	68
Cement 8, Siftings 1	80
Cement 1, Siftings 1	82
Cement 1, Siftings 2	74
Lawrence Cement Co.	
Pure cement	87
" "	54

James River.	Lbs.
Pure cement........	87
Cement 4, Sand 1	62
Newark Lime and Cement Co.	
Pure cement........	93
Cement 1, Sand 2	40
Newark and Rosendale.	
Pure cement........	75
Cement 1, Sand 1	16

Newark and Rosendale.	Lbs.
Cement 1, Sand 3	7
Pure, without mortar, mean	45

Mortar.

Lime paste 1, sand 2.5,	6
" " 1, " 2	4
" " 1, " 3	6
" " 1, " 3, cement paste 5......	11

Pure Cement.

	Lbs.		Lbs.
Boulogne 100, water 50	112	Portland, in sea-water, 45 days	266
Portland, natural, 1 year	675	" English, 6 months	424
" artificial, Eng., 1 year	462	Roman "Septaria," 1 year	191
" English, 320 days	1152	" masonry, 5 months	77
" " 1 month	393	Rosendale, 9 months	700
Newark and Rosendale	339	Lawrence Cement Co.	1210

Transverse.

Reduced to a uniform Measure of One Inch Square and One Foot in Length. Supported at Both Ends.

Experiments of General Gillmore.

Formed in molds under a pressure of 32 lbs. per sq. inch, applied until mortar had set. Exposed to moisture for 24 hours, and then immersed in sea-water.

Prisms 2 by 2 by 8 ins. between supports.

Reduced by Formula $\frac{2}{3}\frac{l\,W}{4\,b\,d^2} - \frac{a}{2} = C$. *C coefficient of rupture, and a weight of portion of prism l.*

Cement. | *Mortar.*

Material.	Age.	Pure.	Material.	Age.	Cement 1. Sand 1.	Cement 1. Sand 2.
	Days.	Lbs.		Days.	Lbs.	Lbs.
James River.			Portland, Eng., stiff paste	320	13	10
Thick cream	59	3.9	Roman, " " "	20	2.5	—
Thin paste	320	5.8	" " " "	100	6	—
Stiff paste	59	6.9	Cumberland, Md.	320	12 8	7.8
Rosendale "Hoffman."			Akron, N. Y.	320	8.8	8.4
Thin paste	320	9	James River, Va.	320	8.6	8.8
Stiff paste	320	8.9	Pulverized and re-mixed after set	3	3.6	—
"Delafield and Baxter."		..				
Thin paste	320	8.5	Fresh	320	9	—
Stiff paste	320	12	Kingston and Rosendale	320	7.6	6.6
English.			High Falls, Ulster Co., N.Y.	95	—	3.2
Portland, pure	320	16				
Stiff paste	320	13	Fresh water to a stiff paste	95	—	4.4
Cumberland, Md., pure	320	13.2				
High Falls, Ulster Co., N. Y.	95	8.4	Sea-water to a stiff paste	95	—	2.6
			Lawrence Cement Co.			
Complete calcination	95	4.2	Fresh	320	10.2	—

Crushing.

Cements, Stones, etc. (*Crystal Palace, London.*)

Reduced to a uniform Measure of One Sq. Inch.

Material.	Destructive Pressure.	Material.	Destructive Pressure.
	Lbs.		Lbs.
Portl'd cem't, area 1, height 1.	1680	Portland cement 1, " sand 4	1244
" cement, " sand	1244	" cement 1, " sand 7	692
" stone	1144	Roman cement, pure	342

General Deductions.

1. Particles of unground cement exceeding .0125 of an inch in diameter may be allowed in cement paste without sand, to extent of 50 per cent. of whole, without detriment to its properties, while a corresponding proportion of sand injures the strength of mortar about 40 per cent.

2. When these unground particles exist in cement paste to extent of 66 per cent. of whole, adhesive strength is diminished about 28 per cent. For a corresponding proportion of sand the diminution is 68 per cent.

3. Addition of siftings exercises a less injurious effect upon the cohesive than upon the adhesive property of cement. The converse is true when sand, instead of siftings, is used.

4. In all mixtures with siftings, even when the latter amounted to 66 per cent. of whole, cohesive strength of mortars exceeded their adhesion to bricks. Same results appear to exist when siftings are replaced by sand, until volume of the latter exceeds 20 per cent. of whole, after which adhesion exceeds cohesion.

5. At age of 320 days (and perhaps considerably within that period) cohesive strength of pure cement mortar exceeds that of Croton front bricks. The converse is true when the mortar contains 50 per cent. or more of sand.

6. When cement is to be used without sand, as may be the case when *grouting* is resorted to, or when old walls are to be repaired by injections of thin paste, there is no advantage in having it ground to an impalpable powder.

7. For economy it is customary to add lime to cement mortars, and this may be done to a considerable extent when in positions where hydraulic activity and strength are not required in an eminent degree.

8. Ramming of concrete under water is held to be injurious.

9. Mortars of common lime, when suitably made, *set* in a very few days, and with such rapidity that there is no need of awaiting its hardening in the prosecution of work.

Fire Clay.—The fusibility of clay arises from the presence of impurities, such as lime, iron, and manganese. These may be removed by steeping the clay in hot muriatic acid, then washing it with water. Crucibles from common clay may be made in this manner.

Notes by General Gillmore, U. S. A.—Recent experiments have developed that most American cements will sustain, without any great loss of strength, a dose of lime paste equal to that of the cement paste, while a dose equal to .5 to .75 the volume of cement paste may be safely added to any Rosendale cement without producing any essential deterioration of the quality of the mortar. Neither is the hydraulic activity of the mortars so far impaired by this limited addition of lime paste as to render them unsuited for concrete under water, or other submarine masonry. By the use of lime is secured the double advantages of slow setting and economy.

Notes by General Totten, U. S. A.—240 lbs. lime = 1 cask, will make from 7.8 to 8.15 cube feet of stiff paste.

1 cube foot of dry cement powder, measured when loose, will measure .78 to .8 cube foot when packed, as at a manufactory.

For composition of Concretes, at Toulon, Marseilles, Cherbourg, Dover, Alderney, etc., see Treatise of General Gillmore, pp. 253–256.

MASONRY.

Brickwork.

Bond is an arrangement of bricks or stones, laid aside of and above each other, so that the vertical joint between any two bricks or stones does not coincide with that between any other two.

This is termed "breaking joints."

Header is a brick or stone laid with an end to face of wall.

Stretcher is a brick or stone laid parallel to face of wall.

Header Course or *Bond* is a course or courses of headers alone.

Stretcher Course or *Bond* is a course or courses of stretchers alone.

Closers are pieces of bricks inserted in alternate courses, in order to obtain a bond by preventing two headers from being exactly over a stretcher.

English Bond is laying of headers and stretchers in alternates courses.

Flemish Bond is laying of headers and stretchers alternately in each course.

Gauged Work.—Bricks cut and rubbed to exact shape required.

String Course is a horizontal and projecting course around a building.

Corbelling is projection of some courses of a wall beyond its face, in order to support wall-plates or floor-beams, etc.

Wood Bricks, *Pallets*, *Plugs*, or *Slips* are pieces of wood laid in a wall in order the better to secure any woodwork that it may be necessary to fasten to it.

Reveals are portions of sides of an opening in a wall in front of the recesses for a door or window frame.

Brick Ashlar.—Walls with ashlar-facing backed with brick.

Grouting is pouring liquid mortar over last course for the purpose of filling all vacuities.

Larrying is filling in of interior of thick walls or piers, after exterior faces are laid, with a bed of soft mortar and floating bricks or spawls in it.

Rendering (Eng.) is application of first coat on masonry, *Laying* if one or two coats on laths, and "Pricking up" if three-coat work on laths.

Bricks should be well wetted before use. *Sea sand* should not be used in the composition of mortar, as it contains salt and its grains are round, being worn by attrition, and consequently having less tenacity than sharp-edged grains.

A common burned brick will absorb 1 pint or about one sixth of its weight of water to saturate it. The volume of water a brick will absorb is inversely a test of its quality.

A good brick should not absorb to exceed .067 of its weight of water.

The courses of brick walls should be of same height in front and rear, whether front is laid with stretchers and thin joints or not.

In ashlar facing the stones should have a width or depth of bed at least equal to height of stone.

Hard bricks set in cement and 3 months set will sustain a pressure of 40 tons per sq. foot.

The compression to which a stone should be subjected should not exceed .1 of its crushing resistance.

The extreme stress upon any part of the masonry of St. Peter's at Rome is computed at 15.5 tons per sq. foot; of St. Paul's, London, 14 tons; and of piers of New York and Brooklyn Bridge, 5.5 tons.

The absorption of water in 24 hours by granites, sandstones, and limestones of a durable description is 1, 8, and 12 per cent. of volume of the stone.

Color of Bricks depends upon composition of the clay, the molding sand, temperature of burning, and volume of air admitted to kiln.

Pure clay free of iron will burn *white*, and mixing of chalk with the clay will produce a like effect.

Presence of iron produces a tint ranging from *red* and *orange* to *light yellow*, according to proportion of iron.

A large proportion of oxide of iron, mixed with a pure clay, will produce a *bright red*, and when there is from 8 to 10 per cent., and the brick is exposed to an intense heat, the oxide fuses and produces a *dark blue* or *purple*, and with a small volume of manganese and an increased proportion of the oxide the color is darkened, even to a *black*.

Small volume of lime and iron produces a *cream color*, an increase of iron produces *red*, and an increase of lime *brown*.

Magnesia in presence of iron produces *yellow*.

Clay containing alkalies and burned at a high temperature produces a *bluish green*.

For other notes on materials of masonry, their manipulation, etc., see "Limes, Cements, Mortars, and Concretes," pp. 588–597.

Pointing.—Before pointing, the joints should be reamed, and in close masonry they must be open to .2 of an inch, then thoroughly saturated with water, and maintained in a condition that they will neither absorb water from the mortar or impart any to it. Masonry should not be allowed to dry rapidly after pointing, but it should be well driven in by the aid of a calking iron and hammer.

In pointing of rubble masonry the same general directions are to be observed.

Sand is *Argillaceous, Siliceous*, or *Calcareous*, according to its composition. Its use is to prevent excessive shrinking, and to save cost of lime or cement. Ordinarily it is not acted upon by lime, its presence in mortar being mechanical, and with hydraulic limes and cements it weakens the mortar. Rich lime adheres better to the surface of sand than to its own particles; hence the sand strengthens the mortar.

It is imperative that sand should be perfectly clean, freed from all impurities, and of a sharp or angular structure. Within moderate limits size of grain does not affect the strength of mortar; preference, however, should be given to coarse.

Calcareous sand is preferable to siliceous.

Sea and River sand are suitable for plastering, but are deficient in the sharpness required for mortar, from the attrition they are exposed to.

Clean sand will not soil the hands when rubbed upon them, and the presence of salt can be detected by its taste.

Scoriæ, Slag, Clinker, and Cinder, when properly crushed and used, make good substitutes for sand.

Concrete.—In the mixing of concrete, slake lime first, mix with cement, and then with the chips, etc., deposit in layers of 6 ins., and hammer down.

Bricks.

Variations in dimensions by various manufacturers, and different degrees of intensity of their burning, render a table of exact dimensions of different manufactures and classes of bricks altogether impracticable.

As an exponent, however, of the ranges of their dimensions, following averages are given:

Description.	Ins.	Description.	Ins.
Baltimore front	8.25 × 4.125 × 2.375	Maine...........	7.5 × 3.375 × 2.375
Philadelphia "	8.25 × 4.125 × 2.375	Milwaukee......	8.5 × 4.125 × 2.375
Wilmington "	8.25 × 4.125 × 2.375	North River.....	8 × 3.5 × 2.25
Croton "	8.5 × 4 × 2.25	Ordinary........	7.75 × 3.625 × 2.25
Colabaugh.......	8.25 × 3.625 × 2.375	Ordinary........	8 × 4.125 × 2.5
Eng. ordinary...	9 × 4.5 × 2.5	Stourbridge fire-brick....	9.125 × 4.625 × 2.375
" Lond. stock	8.75 × 4.25 × 2.5		
Dutch Clinker...	6.25 × 3 × 1.5	Amer. do., N. Y..	8.875 × 4.5 × 2.625

In consequence of the variations in dimensions of bricks, and thickness of the layer of mortar or cement in which they may be laid, it is also impracticable to give any rule of general application for volume of laid brick-work. It becomes necessary, therefore, when it is required to ascertain the volume of bricks in masonry, to proceed as follows:

To Compute Volume of Bricks, and Number in a Cube Foot of Masonry.

Rule.—To face dimensions of particular bricks used, add one half thickness of the mortar or cement in which they are laid, and compute the area; divide width of wall by number of bricks of which it is composed; multiply this area by quotient thus obtained, and product will give volume of the mass of a brick and its mortar in ins.

Divide 1728 by this volume, and quotient will give number of bricks in a cube foot.

Example.—Width of a wall is to be 12.75 ins., and front of it laid with Philadelphia bricks in courses .25 of an inch in depth; how many bricks will there be in face and backing in a cube foot?

Philadelphia front brick, 8.25 × 2.375 ins. face.

$8.25 + \overline{.25 \times 2 \div 2} = 8.25 + .25 = 8.5$ = *length of brick and joint;*

$2.375 + \overline{.25 \times 2 \div 2} = 2.375 + .25 = 2.625$ = *width of brick and joint.*

Then $8.5 \times 3.625 = 22.3125$ *ins.* = *area of face;* $12.75 \div 3$ (*number of bricks in width of wall*) $= 4.25$ *ins.*

Hence $22.3125 \times 4.25 = 94.83$ *cube ins.;* and $1728 \div 94.83 = 18.22$ *bricks.*

One rod of brick masonry (Eng.) = 11.33 cube yards and weighs 15 tons, or 272 superficial feet by 13.5 thick, averaging 4300 bricks, requiring 3 cube yards mortar and 120 gallons water.

Bricklayers' hod will contain 16 bricks or .7 cube feet mortar.

Fire-bricks.

Fire-clay contains Silica, Alumina, Oxide of Iron, and a small proportion of Lime, Magnesia, Potash, and Soda. Its fire-resisting properties depending upon the relative proportions of these constituents and character of its grain.

A good clay should be of a uniform structure, a coarse open grain, greasy to the hand, and free from any alkaline earths.

The Stourbridge clay is black and is composed as follows:

Silica..... 63.3 | Alumina..... 23.3 | Lime..... .73 | Protoxide of iron.... 1.8
Water and organic matter........ 10.3

Newcastle clay is very similar.

Stone Masonry.

Masonry is classed as *Ashlar* or *Rubble.*

Ashlar is composed of blocks of stone dressed square and laid with close joints.

Coursed Ashlar consists of blocks of same height throughout each course.

Fig. 1.

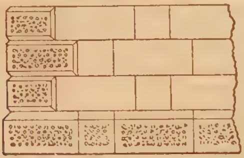


Fig. 1.—*Coursed*, with chamfered and rusticated quoins and plinth.

Fig. 2.

Fig. 2.—*Regular Coursed.*

Fig. 3.

Fig. 3.—*Irregular Coursed.*

Fig. 4.

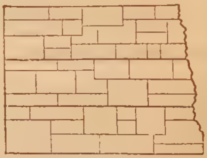


Fig. 4.—*Random Coursed.*

Fig. 5.

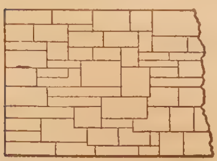


Fig. 5.—*Ranged Random*, level, and broken courses.

Fig. 6.

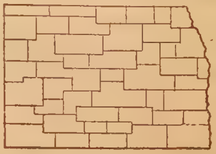


Fig. 6.—*Random*, level, and broken.

Rubble Ashlar

Is ashlar faced stone with rubble backing.

Rubble Masonry

Is composed of small stones irregular in form, and rough.

Fig. 7.

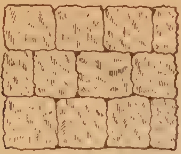


Fig. 7. *Block Coursed.*—Large blocks in courses (regular or irregular), Beds and Joints roughly dressed.

Fig. 8.

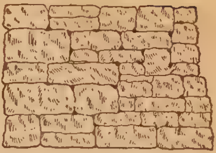


Fig. 8.—*Coursed and Ranged Random.*

Fig. 9.

Fig. 9. *Ranged Random.*—Squared rubble laid in level and broken courses.

Fig. 10.

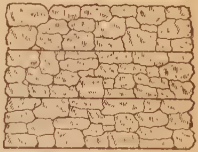


Fig. 10. *Coursed Random.*—Stones laid in courses at intervals of from 12 to 18 ins. in height.

Fig. 11.

Fig. 11. *Uncoursed or Random.*—Beds and Joints undressed, projections knocked off, and laid at random. Interstices filled with spalls and mortar.

Dry Rubble

Is a wall laid without cement or mortar.

Fig. 12.

Fig. 12. *Dry Rubble.*—Without mortar or cement.

Fig. 13.

Fig. 13. *Laced Coursed.*—Horizontal bands of stone or bricks, interposed to give stability.

Fig. 14.

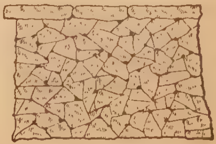


Fig. 14. *Rustic or Rag.*—Stones of irregular form, and dressed to make close joints.

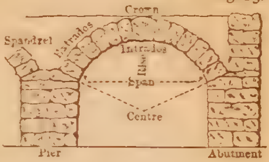
Note.—Rustic or Rag work is frequently laid in mortar.

Terra Cotta.

Terra Cotta in blocks should not exceed 4 cube feet in volume. When properly burned, it is unaffected by the atmosphere or by fumes of any acid.

Arches and Walls.

Fig. 15.



Springing.—Point *s*, Fig. 15, on each side, from which arch springs.

Crown.—Highest point of arch.

Haunches.—Sides of arch, from springing half-way up to crown.

Spandrel.—Space between extrados, a horizontal line drawn through crown and a vertical line through upper end of skewback.

Skewback is upper surface of an abutment or pier from which an arch springs, and its face is on a line radiating from centre of arch.

Abutment is outer body that supports arch and from which it springs.

Pier is the intermediate support for two or more arches.

Jambs are sides of abutments or piers.

Voussoirs are the blocks forming an arch.

Key-stone is centre voussoir at crown.

Span is horizontal distance from springing to springing of arch.

Rise.—Height from springing line to under side of arch at key-stone.

Length is that of springing line or span.

Ring-course of a wall or arch is parallel to face of it, and in direction of its span.

String and Collar courses are projecting ashlar dressed broad stones at right angles to face of a wall or arch, and in direction of its length.

Camber is a slight rise of an arch as .125 to .25 of an inch per foot of span.

Quoin is the external angle or course of a wall.

Plinth is a projecting base to a wall.

Footing is projecting course at bottom of a wall, in order to distribute its weight over an increased area. Its width should be double that of base of wall, diminishing in regular offsets .5 width of their height.

Blocking Course.—A course placed on top of a cornice.

Parapet is a low wall, over edge of a roof or terrace.

Extrados.—Back or upper and outer surface of an arch.

Intrados or *Soffit* is underside of lower surface of arch or an opening.

Groined is when arches intersect one another.

Invert.—An inverted arch, an arch with its intrados below axis or springing line.

Ashlar masonry requires .125 of its volume of mortar. *Rubble*, 1.2 cube yards stone and .25 cube yard mortar for each cube yard.

Rubble masonry in cement, 160 feet in height, will stand and bear 20 000 lbs. per sq. inch.

Stones should be laid with their strata horizontal.

When "through" or "thorough bonds" are not introduced, headers should overlap one another from opposite sides, known as *dogs' tooth bond.*

Aggregate surface of ends of bond stones should be from .125 to .25 of area of each face of wall.

Weak stones, as sandstone and granular limestone, should not have a length over 3 times their depth. Strong or hard stones may have a length from 4 to 5 times their depth.

Gallets are small and sharp pieces of stone stuck into mortar joints, in which case the work is termed *galleted.*

Snapped work is when stones are split and roughly squared.

Quarry or Rock-faced.—Quarried stones with their faces undressed.

Pitch-faced.—Stones on which the arris or angles of their face, with their sides and ends, is defined by a chisel, in order to show a right-lined edge.

Drafted or Drafted Margin is a narrow border chiselled around edges of faces of a block of rough stone.

Diamond-faced is when planes are either sunk or raised from each edge and meet in the centre.

Squared Stones.—Stones roughly squared and dressed.

Rubble.—Unsquared stones, as taken from a quarry or elsewhere, in their natural form, or their extreme projections removed.

Cut Stones.—Stones squared and with dressed sides and ends.

Dressed Stones.

The following are the modes of dressing the faces of ashlar in engineering:

Rough Pointed.—Rough dressing with a pick or heavy point.

Fine Pointed.—Rough dressing, followed by dressing with a fine point.

Crandalled.—Fine pointing in right lines with a hammer, the face of which is close serried with sharp edges.

Cross Crandalled.—When the operation of crandalling is right angled.

Hammered.—The surface of stone may be finished or smooth dressed by being *Axed* or *Bushed;* the former is a finish by a heavy hammer alike to a crandall, the latter is a final finish by a heavy hammer with a face serried with sharp points at right angles.

Thickness of Brick Walls for Warehouses. (*Molesworth.*)

Length.	Height.	Thickness.	Length.	Height.	Thickness.	Length.	Height.	Thickness.
Feet.	Feet.	Ins.	Feet.	Feet.	Ins.	Feet.	Feet.	Ins.
Unlimited.	25	13	Unlimit'd.	100	34	45	30	13
do.	30	17.5	60	40	17.5	30	40	13
do.	40	21.5	70	50	21.5	40	50	17.5
do.	50	26	50	60	21.5	35	60	17.5
do.	60	26	45	70	21.5	30	70	17.5
do.	70	26	60	80	26	45	80	21.5
do.	80	30	70	90	30	60	90	26
do.	90	34	70	100	30	55	100	26

For drawings and a description of stone-dressing tools, see a paper by J. R. Cross, W. E. Merrill, and E. B. Van Winkle, "A. S. Civil Engineer Transactions," Nov. 1877.

Walls not exceeding 30 feet in height, upper story walls may be 8.5 ins. thick.

From 16 feet below top of wall to base of it, it should not be less than the space defined by two right lines drawn from each side of wall at its base to 16 feet from top.

Thickness not to be less in any case than one fourteenth of height of story.

Laths.

Laths are 1.25 to 1.5 ins. by 4 feet in length, are usually set .25 of an inch apart, and a bundle contains 100.

Plastering.

Volumes required for Various Thickness.

Material.	Square Yards. .5	.75	1	Material.	Square Yards. .5	.75	1
Cube Feet.	Ins.	Ins.	Ins.	Cube Feet.	Ins.	Ins.	Ins.
Cement 1	2.25	1.5	1.15	Lime 1, sand 2, hair 3.75......	75 yards sup'l rendered and set on brick or 70 on lath.		
Cement 1, sand 1...	4.5	3	2.25				
Cement 1, sand 2...	6.75	4.5	3.33				

Estimate of Materials and Labor for 100 Sq. Yards of Lath and Plaster.

Materials and Labor.	Three Coats Hard Finish.	Two Coats Slipped.	Materials and Labor.	Three Coats Hard Finish.	Two Coats Slipped.
Lime...........	4 casks.	3.5 casks.	White sand....	2.5 bushels.	
Lump lime	.66 "		Nails	13 lbs.	13 lbs.
Plaster of Paris..	.5 "		Masons	4 days.	3.5 days.
Laths...........	2000.	2000.	Laborer........	3 "	2 "
Hair............	4 bushels.	3 bushels.	Cartage.......	1 "	.75 "
Sand	7 loads.	6 loads.			

Rough Cast is washed gravel mixed with hot hydraulic lime and water and applied in a semi-fluid condition.

Arches and Abutments.

To Compute Depth of Keystone of Circular or Elliptic Arch.

$\frac{\sqrt{R+s\div 2}}{4}+.25=d.$ *R representing radius, s span, and d depth, all in feet.*

This is for a rise of about .25 of span; when it is reduced, as to .125, add .5 instead of .25.

Illustration.—Arch of Washington aqueduct at "Cabin John" has a span of 220 feet, a rise of 57.25, and a radius of 134.25; what should be depth of its keystone?

$$\frac{\sqrt{134.25+220\div 2}}{4}+.25=\frac{15.63}{4}+.25=4.16\textit{ feet.}$$ Depth is 4.16 feet.

Viaducts of several arches increase results as determined above by adding .125 to .15 to depth.

For arches of 2d class materials and work, and for spans exceeding 10 feet, add .125 to depth of keystone, and for good rubble or brick-work add .25.

Note.—It is customary to make the keystones of elliptic arches of greater depth than that obtained by above formula. Trautwine, however, who is high authority in this case, declares it is unnecessary.

To Compute Radius of an Arch, Circular or Ellipse.

$$\left(\frac{s}{2}\right)^2+r^2\div 2\,r=R.$$ *r representing rise.*

Railway Arches.

For Spans between 25 and 70 feet. Rise .2 of span. Depth of arch .055 of span. Thickness of abutments .2 to .25 of span, and of pier .14 to .16 of span.

Abutments.

When height does not exceed 1.5 *times base.* $R\div 5+.1\,r+2=$ *thickness at spring of arch in feet.* (*Trautwine.*)

Batter.—From .5 to 1.5 ins. per foot of height of wall.

To Compute Depth of Arch. (*Hurst.*)

$c\sqrt{R} = D$. c = Stone (block) .3. Brick = .4. Rubble = .45.

When there are a series of arches, put .3 = .35, .4 = .45, and .45 = .5.

Minimum Thickness of Abutments for Bridge and similar Arches of 120°. (*Hurst.*)

When depth of crown does not exceed 3 feet. Computed from formula.

$\sqrt{6\,R + \left(\frac{3\,R}{2\,H}\right)^2} - \frac{3\,R}{2\,H} = T$. H *representing height of abutment to springing in feet.*

Radius of Arch.	Height of Abutment to Springing. 5	7.5	10	20	30	Radius of Arch.	Height of Abutment to Springing. 5	7.5	10	20	30
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
4	3.7	4.2	4.3	4.6	4.7	12	5.6	6.4	6.9	7.6	7.9
4.5	3.9	4.4	4.6	4.9	5	15	6	7	7.5	8.4	8.8
5	4.2	4.6	4.8	5.1	5.2	20	6.5	7.7	8.4	9.6	10
6	4.5	4.7	5.2	5.6	5.7	25	6.9	8.2	9.1	10.5	11.1
7	4.7	5.2	5.5	6	6.1	30	7.2	8.7	9.7	11.4	12
8	4.9	5.5	5.8	6.4	6.5	35	7.4	9.1	10.2	11.8	12.9
9	5.1	5.8	6.1	6.7	6.9	40	7.6	9.4	10.6	12.8	13.6
10	5.3	6	6.4	7.1	7.3	45	7.8	9.7	11	13.4	14.3
11	5.5	6.2	6.6	7.3	7.6	50	7.9	10	11.4	14	15

NOTE.—Abutments in Table are assumed to be without counterforts or wing-walls. A sufficient margin of safety must be allowed beyond dimensions here given.

Culverts for a road having double tracks are not necessarily twice the length for a single track.

For other and full notes, tables, etc., see Trautwine's Pocket Book, pp. 341–356.

MECHANICAL CENTRES.

There are four Mechanical centres of force in bodies, namely, Centre of Gravity, Centre of Gyration, Centre of Oscillation, and Centre of Percussion.

Centre of Gravity.

CENTRE OF GRAVITY of a body, or any system of bodies rigidly connected together, is point about which, if suspended, all parts will be in equilibrium.

A body or system of bodies, suspended at a point *out* of centre of gravity, will rest with its centre of gravity vertical under point of suspension.

A body or system of bodies, suspended at a point *out* of centre of gravity, and successively suspended at two or more such points, the vertical lines through these points of suspension will intersect each other at centre of gravity of body or bodies.

Centre of gravity of a body is not always within the body itself.

If centres of gravity of two bodies, as B C, be connected by a line, distances of B and C from their common centre of gravity, c, *is as the weights of the bodies.* Thus, B : C :: C c : c B.

To Ascertain Centre of Gravity of any Plane Figure Mechanically.

Suspend the figure by any point near its edge, and mark on it direction of a plumb-line hung from that point; then suspend it from some other point, and again mark direction of plumb-line. Then centre of gravity of surface will be at point of intersection of the two marks of plumb-line.

Centre of gravity of parallel-sided objects may readily be found in this way. For instance, to ascertain centre of gravity of an arch of a bridge, draw elevation upon paper to a scale, cut out figure, and proceed with it as above directed, in order to find position of centre of gravity in elevation of the model. In actual arch, centre of gravity will have same relative position as in paper model.

In regular figures or solids, centre of gravity is same as their geometrical centres.

Line.

Circular Arc. $\frac{r\,c}{l}$ = *distance from centre, r representing radius, c chord, and l length of arc.*

Surfaces.

Square, Rectangle, Rhombus, Rhomboid, Gnomon, Cube, Regular Polygon, Circle, Sphere, Spheroid or Ellipsoid, Spheroidal Zone, Cylinder, Circular Ring, Cylindrical Ring, Link, Helix, Plain Spiral, Spindle, all Regular Figures, and Middle Frusta of all Spheroids, Spindles, etc.

The centre of gravity of the surfaces of these figures is *in their geometrical centre.*

Triangle.—On a line drawn from any angle to the middle of opposite side, at two thirds of the distance from angle.

Trapezium.—Draw two diagonals, and ascertain centres of gravity of each of four triangles thus formed; join each opposite pair of these centres, *and it is at intersection of the lines.*

Trapezoid. $\left(\frac{B+2\,b}{B+b}\right) \times \frac{m}{3}$ = *distance from* B *on a line joining middle of two parallel sides* B *b, m representing middle line.*

Circular Arc. $\frac{c\,r}{l}$ = *distance from centre of circle.*

Sector of a Circle. $.4244\,r$ = *distance from centre of circle.*

Semicircle. $.4244\,r$ = *distance from centre.*

Semi-semicircle. $.4244\,r$ = *distance from both base and height and at their intersection.*

Segment of a Circle. $\frac{c^3}{12\,a}$ = *distance from centre, a representing area of segment.*

Sector of a Circular Ring. $\frac{4}{3} \times \frac{\sin .5 \angle}{\text{arc} \angle} \times \frac{r^3 - r'^3}{r^2 - r'^2}$ = *distance from centre of arcs, r and r′ representing the radii.*

Illustration.—Radii of surfaces of a dome are 5 and 3.5 feet, and angle ($\angle$) at centre = 130°.

$$\frac{4}{3} \times \frac{\sin 65°}{\text{arc } 130°} \times \frac{125 - 42.875}{25 - 12.25} = \frac{4}{3} \times \frac{.9063}{2.2689} \times \frac{82.125}{6.8067 \times 12.75} = 3.43 \textit{ feet.}$$

Hemisphere, Spherical Segment, and Spherical Zone, At centre of their heights.

Circular Zone.—Ascertain centres of gravity of trapezoid and segments comprising zone; draw a line (equally dividing zone) perpendicular to chords; connect centres of segments by a line cutting perpendicular to chords.

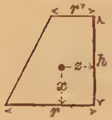
Then centre of gravity of figure will be on perpendicular, toward lesser chord, at such proportionate distance of difference between centres of gravity of trapezoid and line connecting centres of segments, as area of segments bears to area of trapezoid.

Prism and Wedge.—When end is a Parallelogram, *in their geometrical centres;* when the end is a Triangle, Trapezium, etc., it is *in middle of its length, at same distance from base, as that of triangle or trapezoid of which it is a section.*

Parabola in its axis = *.6 distance from vertex.*

Prismoid.—At same distance from its base as that of the trapezoid or trapezium, which is a section of it.

Lune.—On a line connecting centres of gravity of arcs at a proportionate point to respective areas of arcs.



Co-ordinates. $\left(r + r' - \frac{r\,r'}{r + r'}\right)\frac{1}{3} = z,$

and $\left(\frac{2\,r' + r}{r + r'}\right)\frac{h}{3} = x.$

Solids.

Cube, Parallelopipedon, Hexahedron, Octahedron, Dodecahedron, Icosahedron, Cylinder, Sphere, Right Spherical Zone, Spheroid or Ellipsoid, Cylindrical Ring, Link, Spindle, all Regular Bodies, and Middle Frusta of all Spheroids and Spindles, etc. Centre of gravity of these figures *is in their geometrical centre.*

Tetrahedron.—In common centre of centres of gravity of the triangles made by a section through centre of each side of the figures.

Cone and Pyramid. .25 *of line joining vertex and centre of gravity of base* = *distance from base.*

Frustum of a Cone or Pyramid. $\frac{(r + r')^2 + 2\,r^2}{(r + r')^2 - r\,r'} \times \frac{1}{4}\,h$ = *distance from centre of lesser end, r and r', in a cone representing radii, and in a pyramid sides, and h height.*

Cone, Frustum of a Cone, Pyramid, Frustum of a Pyramid, and Ungula.—At same distance from base as in that of triangle, parallelogram, or semicircle, which is a right section of them.

Hemisphere. .375 r = *distance from centre.*

Spherical Segment. $3.1416\,vs^2\left(r - \frac{vs}{2}\right)^2 \div v$ = *distance from centre, vs representing versed sine, and v volume of segment.* $\left(\frac{8\,r - 3\,h}{12\,r - 4\,h}\right) \times h$ = *distance from vertex.*

Spherical Sector. $.75\ (r - .5\,h)$ = *distance from centre.* $\frac{2\,r + 3\,h}{8}$ = *distance from vertex.*

Spirals.—Plane, *in its geometrical centre.* Conical, *at a distance from the base,* .25 *of line joining vertex and centre of gravity of base.*

Frustum of a Circular Spindle. $\frac{r^2 - r'^2}{2\,(h - D.z)}$ = *distance from centre of spindle, h representing distance between two bases,* D *distance of centre of spindle from centre of circle, and z generating arc, expressed in units of radius.*

Segment of a Circular Spindle. $\frac{r^2}{2\,(h - D.z)}$ = *distance from centre of spindle.*

Semi-spheroids.—Prolate. .375 a.—*Oblate.* .375 a = *distance from centre.*

Semi-spheroid or Ellipsoid and its Segment.—See *Haswell's Mensuration*, pages 281 and 282.

Frusta of Spheroids or Ellipsoids. Prolate. $.75\ \frac{h\,(2\,a^2 - h^2)}{3\,a^2 - h^2}$ = *distance from centre of spheroid, a representing semi-transverse diameter in a prolate frustum, and semi-conjugate in an oblate frustum.*

Segments of Spheroids.—Prolate. $.75\,\frac{(a+d)^2}{2\,a+d}$.—*Oblate.* $.75\,\frac{(a+d')^2}{2\,a+d'}$ = *distance from centre of spheroid, d and d′ representing distances of base of segments from centre of spheroid.*

Any Frustum. $.75\,\frac{(d+d')\times(2\,a^2-d'^2+d^2)}{3\,a^2-d'^2+d'\,d+d^2}$ = *distance from centre of spheroid, d and d′ representing distances of base and end of segments from centre of the spheroid.*

Segment of an Elliptic Spindle at two thirds of height from vertex.

Paraboloid of Revolution, at two thirds of height from vertex.

Segment of a Hyperbolic Spindle, at 75 *of height from vertex.*

Frustum of Paraboloid of Revolution. $\frac{2\,r^2+r'}{r^2+r'}\times\frac{h}{3}$ = *distance from base, r and r′ representing radii of base and vertex.*

Segment of Paraboloid of Revolution, at two thirds of height from vertex.

Segments of a Circular and a Parabolic Spindle.—See Haswell's Mensuration, pages 192 and 199.

Parabola. .4 *of height* = *distance from base.*

Hyperboloid of Revolution. $\frac{4\,b+3\,h}{6\,b+4\,h}\times h$ = *distance from vertex, b representing diameter of base.*

Frustum of Hyperboloid of Revolution. $.75\,\frac{(d+d')\,(2\,a^2-d'^2+d^2)}{3\,a^2-d'^2+d'\,d+d^2}$ = *distance from centre of base, a representing semi-transverse axis, or distance from centre of curve to vertex of figure; d and d′ distances from centre of curve to centre of lesser and greater diameter of frustum.*

Segment of Hyperboloid of Revolution. $\frac{4\,b+3\,h}{6\,b+4\,h}\times h$ = *distance from vertex.*

Of Two Bodies. $\frac{d\,v}{V+v}$ = *distance from V or volume or area of larger body, d representing distance between centres of gravity of bodies, and v volume or area of less body.*

Cycloid. — .833 *of radius of generating circle* = *distance from centre of chord of curve.*

Any Plane Figure.—Divide it into triangles, and ascertain centre of gravity of each; connect two centres together, and ascertain their common centre; then connect this common centre and centre of a third, and ascertain the common centre, and so on, connecting the last-ascertained common centre to another centre till whole are included, and last common centre will give centre required.

Of an Irregular Body of Rotation.

Divide figure into four or six equidistant divisions; ascertain volume of each, their moments with reference to first horizontal plane or base, and then connect them thus:

$(A+4\,A_1+2\,A_2+4\,A_3+A_4)\,\frac{h}{12}=V$, $A\,A_1$, etc., *representing volume of divisions, and h height of body from base;*

and $\frac{(0\,A+1\times4\,A_1+2\times2\,A_2+3\times4\,A_3+4\,A_4)}{A+4\,A_1+2\,A_2+4\,A_3+A_4}\times\frac{h}{4}$ = *distance of centre of gravity from base.*

Centre of Gyration.

Centre of Gyration is that point in any revolving body or system of bodies in which, if the whole quantity of matter were collected, the *Angular velocity* would be the same; that is, the *Momentum* of the body or system of bodies is centred at this point, and the position of it is a mean proportional between the centres of Oscillation and Gravity.

If a straight bar of uniform dimensions was struck at this point, the stroke would communicate the same *angular velocity* to the bar as if the whole bar was collected at that point.

The *Angular velocity* of a body or system of bodies is the motion of a line connecting any point and the centre or axis of motion: it is the same in all parts of the same revolving body.

In different unconnected bodies, each oscillating about a common centre, their *angular velocity* is as the velocity directly, and as the distance from the centre inversely. Hence, if their velocities are as their radii, or distances from the axis of motion, their angular velocities will be equal.

When a body revolves on an axis, and a force is impressed upon it sufficient to cause it to revolve on another, it will revolve on neither, but on a line in the plane of the axes, dividing the angle which they contain; so that the sine of each part will be in the inverse ratio of the angular velocities with which the bodies would have revolved about these axes separately.

Weight of revolving body, multiplied into height due to the velocity with which centre of gyration moves in its circle, is energy of body, or mechanical power, which must be communicated to it to give it that motion.

Distance of centre of gyration from axis of motion is termed the *Radius of gyration;* and the moment of inertia is equal to product of square of radius of gyration and mass or weight of body.

The moment of inertia of a revolving body is ascertained exactly by ascertaining the moments of inertia of every particle separately, and adding them together; or, approximately, by adding together the moments of the small parts arrived at by a subdivision of the body.

To Compute Moment of Inertia of a Revolving Body.

Rule.—Divide body into small parts of regular figure. Multiply mass or weight of each part by square of distance of its centre of gravity from axis of revolution. The sum of products is moment of inertia of body.

Note.—The value of moment of inertia obtained by this process will be more exact, the smaller and more numerous the parts into which body is divided.

To Compute Radius of Gyration of a Revolving Body about its Axis of Revolution.

Rule.—Divide moment of inertia of body by its mass, or its weight, and square root of quotient is length of radius of gyration.

Note.—When the parts into which body is divided are equal, radius of gyration may be determined by taking mean of all squares of distances of parts from axis of revolution, and taking square root of their sum.

Or, $\sqrt{R^2 + r^2 \div 2} = G$. R *and* r *representing radii.*

Example.—A straight rod of uniform diameter and 4 feet in length, weighs 4 lbs.; what is its inertia, and where is its radius or centre of gyration?

Each foot of length weighs 1 lb., and if divided into 4 parts, centre of gyration of each is respectively .5, 1.5, 2.5, and 3.5 feet. Hence,

$$\left.\begin{array}{r} 1 \times .5^2 = .25 \\ 1 \times 1.5^2 = 2.25 \\ 1 \times 2.5^2 = 6.25 \\ 1 \times 3.5^2 = 12.25 \end{array}\right\} 21 = \textit{inertia}, \text{ which} \div 4 = 5.25, \text{ and } \sqrt{5.25} = 2.291 \textit{ feet radius.}$$

Following are distances of centres of gyration from centre of motion in various revolving bodies:

Straight, uniform Rod or *Cylinder* or *thin Rectangular Plate* revolving about one end; *length* $\times$ .5773, and revolving about their centre; *length* $\times$ .2886.

The general expression is, when revolving at any point of its length,

$$\sqrt{\left(\frac{l^3 + l'^3}{3\,(l + l')}\right)}.$$ *l and l′ representing length of the two points.*

Circular Plane, revolving on its centre; *radius of circle* $\times$ .7071; *Circle Plane, as a Wheel or Disc of uniform Thickness*, revolving about one of its diameters as an axis; *radius* $\times$ .5.

Solid Cylinder, revolving about its axis; *radius* $\times$ .7071.

Solid Sphere, revolving about its diameter as an axis; *radius* $\times$ .6325.

Thin, hollow Sphere, revolving about one of its diameters as an axis; *radius* $\times$ .8164. *Surface of sphere* .8615 *r*.

Sphere and Solid Cylinder (vertical), at a distance from axis of revolution $= \sqrt{l^2 + .4\,r^2}$ *for sphere*, and $\sqrt{l^2 + .5\,r^2}$ *for cylinder, l representing length of connection to centre of sphere and cylinder.*

Cone, revolving about its axis; *radius of base* $\times$ .5447; revolving about its vertex $= \sqrt{12\,h^2 + 3\,r^2 \div 20}$, *h representing height, and r radius of base;* revolving about its base $= \sqrt{2\,h^2 + 3\,r^2 \div 20}$.

Circular Ring, as *Rim of a Fly wheel or Hollow Cylinder*, revolving about its diameter $= \sqrt{R^2 + r^2 \div 2}$, R *representing radius of periphery, and r of inner circle of ring.*

Fly-wheel $= \sqrt{\dfrac{6\,W\,(R^2 + r^2) + w\,(4\,r^2 + l^2)}{12\,(W + w)}}$, *W and w representing weights of rim and of arms and hub, and l length of arms from axis of wheel.*

Section of Rim. $\sqrt{\dfrac{4\,d^2 + c^2}{12} + r^2 + r\,d}$. *d representing depth and c periphery of rim.*

Parallelopiped, revolving about one end, distance from end $= \sqrt{\dfrac{4\,l^2 + b^2}{12}}$, *b representing breadth.*

Illustration.—In a solid sphere revolving about its diameter, diameter being 2 feet, distance of centre of gyration is $12 \times .6325 = 7.59$ *ins.*

To Compute Elements of Gyration.

$$\frac{G\,W\,v}{r\,t\,g} = P; \quad \frac{P\,r\,t\,g}{W\,v} = G; \quad \frac{G\,W\,v}{P\,t\,g} = r; \quad \frac{P\,r\,t\,g}{G\,v} = W; \quad \frac{G\,W\,v}{P\,r\,g} = t;$$

$$\frac{P\,r\,t\,g}{G\,W} = v.$$ G *representing distance of centre of gyration from axis of rotation,* W *weight of body, t time power acts in seconds, v velocity in feet per second acquired by revolving body in that time, and r distance of point of application of power from axis of body, as length of crank, etc.*

Illustration 1.—What is distance of centre of gyration in a fly-wheel, power 224 lbs., length of crank 7 feet, time of rotation 10 seconds, weight of wheel 5600 lbs., and velocity of it 8 feet per second?

$$\frac{224 \times 7 \times 10 \times 32.166}{5600 \times 8} = \frac{504\,373}{42\,800} = 11.78\ \textit{feet}.$$

2.—What should be weight of a fly wheel making 12 revolutions per minute, its diameter 8 feet, power applied at 2 feet from its axis 84 lbs., time of rotation 6 seconds, and distance of centre of gyration of wheel 3.5 feet?

$$\frac{8 \times 3.1416 \times 12}{60} = 5.0265\ \textit{feet} = \textit{velocity}. \quad \text{Then}\ \frac{84 \times 2 \times 6 \times 32.166}{3.5 \times 5.0265} = 1843.2\ \textit{lbs.}$$

When the Body is a Compound one. RULE.—Multiply weight of several particles or bodies by squares of their distances in feet from centre of motion or rotation, and divide sum of their products by weight of entire mass; the square root of quotient will give distance of centre of gyration from centre of motion or rotation.

EXAMPLE.—If two weights, of 3 and 4 lbs. respectively, be laid upon a lever (which is here assumed to be without weight) at the respective distances of 1 and 2 feet, what is distance of centre of gyration from centre of motion (the fulcrum)?

$$3 \times 1^2 = 3; \quad 4 \times 2^2 = 16; \quad \frac{3+16}{3+4} = \frac{19}{7} = 2.71, \text{ and } \sqrt{2.71} = 1.64 \textit{ feet.}$$

That is, a single weight of 7 lbs., placed at 1.64 feet from centre of motion, and revolving in same time, would have same *momentum* as the two weights in their respective places.

When Centre of Gravity is given. RULE.—Multiply distance of centre of oscillation from centre or point of suspension, by distance of centre of gravity from same point, and square root of product will give distance of centre of gyration.

EXAMPLE.—Centre of oscillation of a body is 9 feet, and that of its gravity 4 feet from centre of rotation or point of suspension; at what distance from this point is centre of gyration?

$$9 \times 4 = 36, \text{ and } \sqrt{36} = 6 \textit{ feet.}$$

To Compute Centre of Gyration of a Water-wheel.

RULE.—Multiply severally twice weight of rim, as composed of buckets, shrouding, etc., and twice that of arms and that of water in the buckets (when wheel is in operation) by square of radius of wheel in feet; divide sum by twice sum of these several weights, and square root of quotient will give distance in feet.

EXAMPLE.—In a wheel 20 feet in diameter, weight of rim is 3 tons, weight of arms 2 tons, and weight of water in buckets 1 ton; what is distance of centre of gyration from centre of wheel?

$$\begin{aligned} \text{Rim} &= 3 \text{ tons} \times 10^2 \times 2 = 600 \\ \text{Buckets} &= 2 \text{ tons} \times 10^2 \times 2 = 400 \\ \text{Water} &= 1 \text{ ton} \times 10^2 = \underline{100} \\ & \phantom{= 1 \text{ ton} \times 10^2 \times 2 =} 1100 \end{aligned}$$

$$3 + 2 + 1 \times 2 = 12 \textit{ sum of weights.}$$

$$\text{Hence } \sqrt{\frac{1100}{12}} = \sqrt{91.67} = 9.57 \textit{ feet.}$$

GENERAL FORMULAS.—P *representing power*, H *horses' power*, F *force applied to rotate body in lbs.*, M *mass of revolving body in lbs.*, r *radius upon which* F *acts in feet*, d *distance from axis of motion to centre of gyration in feet*, t *time force is applied in seconds*, n *number of revolutions in time* t, x *angular velocity, or number of revolutions per minute at end of time* t, *and* $G = \frac{32.166\, F\, r^2}{M\, d^2}$.

$$\sqrt{\frac{4\,p\,r\,n}{G}} = t; \quad \frac{2\,p\,r^2\,x}{60\,G} = t; \quad \frac{M\,x\,d^2}{153.5\,t\,r} = F; \quad \frac{M\,n\,d^2}{2.56\,t^2\,F} = r; \quad \frac{2.56\,t^2\,F\,r}{M\,d^2} = n;$$

$$\frac{153.5\,t\,F\,r}{M\,d^2} = x; \quad \frac{244\,t\,P}{x^2\,d^2} = M; \quad \frac{x^2\,M\,d^2}{244\,t} = P; \quad \frac{x^2\,M\,d^2}{134\,100\,t} = H.$$

ILLUSTRATION.—Rim of a fly-wheel weighing 7000 lbs. has radii of 6.5 and 5.75 feet; what is its centre of gyration, and what force must be applied to it 2 feet from axis of motion to give it an angular velocity of 130 revolutions per minute in 40 seconds? how many revolutions will it make in 40 seconds? and what is its power?

$$\frac{130^2 \times 7000 \times 6.14^2}{134\,100 \times 40} = \frac{4\,459\,862\,680}{5\,364\,000} = 829.7 \textit{ horses.}$$

$$\text{Centre of gyration} = \sqrt{\frac{6.5^2 + 5.75^2}{2}} = 6.14 \textit{ feet.} \quad \text{Then } F = \frac{130 \times 7000 \times 6.14^2}{153.5 \times 40 \times 2} =$$

$$\frac{34\,306\,636}{12\,280} = 2793.7 \textit{ lbs.}, \text{ and } \frac{2.56 \times 40^2 \times 2793.7 \times 2}{7000 \times 6.14^2} = 86.67 \textit{ revolutions.}$$

Centres of Oscillation and Percussion.

CENTRE OF OSCILLATION of a body, or a system of bodies, is that point in axis of vibration of a vibrating body in which, if, as an equivalent condition, the whole matter of vibrating body was concentrated, it would continue to vibrate in same time. It is resultant point of whole vibrating energy, or of action of gravity in producing oscillation.

As particles of a body further from centre of its suspension have greater velocity of vibration than those nearer to it, it is apparent that centre of oscillation is further from its centre than centre of gravity is from axis of suspension, but it is situated in centre of a line drawn from axis of a body through its centre of gravity. It further differs from centre of gyration in this, that while motion of oscillation is produced by gravity of a body, that of gyration is caused by some other force acting at one place only.

Radius of oscillation, or distance of centre of oscillation from axis of suspension, is a third proportional, to distance of centre of gravity from axis of suspension and radius of gyration.

CENTRE OF PERCUSSION of a body, or a system of bodies, revolving about a point or axis, is that point at which, if resisted by an immovable obstacle, all the motion of the body, or system of bodies, would be destroyed, and without impulse on the point of suspension. It is also that point which would strike any obstacle with greatest effect, and from this property it has been termed percussion.

Centres of Oscillation and Percussion are in same point.—If a blow is struck by a body oscillating or revolving about a fixed centre, percussive action is same as if its entire mass was concentrated at centre of oscillation. That is, centre of percussion is identical with centre of oscillation, and its position is ascertained by same rules as for centre of oscillation. If an external body is struck so that the mean line of its resistance passes through centre of percussion, then entire force of percussion is transmitted directly to the external body; on the contrary, if a revolving body is struck at its centre of percussion, its motion will be absolutely destroyed, so that the body will not incline either way.

As in bodies at rest, the entire weight may be considered as collected in centre of gravity: so in bodies in vibration, the entire force may be considered as concentrated in centre of oscillation; and in bodies in motion, the whole force may be considered as concentrated in centre of percussion.

If centre of oscillation is made point of suspension, point of suspension will become centre of oscillation.

Angle of Oscillation or *Percussion* is determined by angle delineated by vertical plane of body in vibration, in plane of motion of body.

Velocity of a Body in Oscillation or Percussion through its vertical plane is equal to that acquired by a body freely falling through a vertical line equal in height to versed sine of the arc.

To Compute Centre of Oscillation or Percussion of a Body of Uniform Density and Figure.

RULE.—Multiply weight of body by distance of its centre of gravity from point of suspension; multiply also weight of body by square of its length, and divide product by 3.

Divide this last quotient by product of weight of body and distance of its centre of gravity, and quotient is distance of centres from point of suspension.

Or, $\frac{W\,l^2}{3} \div W \times g =$ *distance from axis.* Or, square radius of gyration of body and divide by distance of centre of gravity from axis of suspension.

EXAMPLE.—Where is centre of oscillation in a rod 9 feet in length from its point of suspension, and weighing 9 lbs.?

$9 \times \frac{9}{2} = 40.5 =$ *product of weight and its centre of gravity;* $\frac{9 \times 9^2}{3} = 243 =$ *quotient of product of weight of body and square of its length* $\div 3$; $\frac{243}{40.5} = 6$ *feet.*

When Point of Suspension is not at End of Rod. RULE.—To cube of distance of point of suspension from top of rod or bar, add cube of its distance from lower end, and multiply sum by 2.

Divide product by three times difference of squares of these distances, and quotient is distance of point of oscillation from point of suspension.

EXAMPLE.—A homogeneous rod of uniform dimensions, 6 feet in length, is suspended 1.5 feet from its upper end; what is distance of point of oscillation from that of suspension?

$$6 - 1.5 = 4.5. \qquad \frac{2\,(4.5^3 + 1.5^3)}{3\,(4.5^2 - 1.5^2)} = \frac{189}{54} = 3.5 \text{ feet.}$$

Centres of Oscillation and Percussion in Bodies of Various Figures.

When Axis of Motion is in Vertex of Figure, and when Oscillation or Motion is Facewise.

Right Line, or any figure of uniform shape and density $= .66\,l$.
Isosceles Triangle $= .75\,h$. *Circle* $= 1.25\,r$.
Parabola $= .714\,h$. *Cone* $= .8\,h$.
When Axis of Motion is in Centre of Body. *Wheel* $= .75$ *radius.*

When Oscillation or Motion is Sidewise. Right Line, or any figure of uniform shape and density $= .66\,l$. *Rectangle*, suspended at one angle $= .66$ *of diagonal.*

Parabola, if suspended by its vertex $= .714$ of axis $+ .33$ parameter; if suspended by middle of its base $= .57$ of axis $+ .5$ parameter.

Sector of a Circle $= \frac{3 \text{ arc} \cdot r}{4\,c}$, *c representing chord of arc, and r radius of base.*

Circle $= .75\,d$. *Cone* $= \frac{4}{5}$ axis $+ \frac{r^2}{5 \text{ axis}}$.

Sphere $= \frac{2}{5}\frac{r^2}{(c+r)} + r + c$, *c representing length of cord by which it is suspended.*

To Ascertain Centres of Oscillation and Percussion experimentally.

Suspend body very freely from a fixed point, and make it vibrate in small arcs, noting number of vibrations it makes in a minute, and let number made in a minute be represented by n; then will distance of centre of oscillation from point of suspension be $= \frac{140\,850}{n^2} =$ *ins.*

For length of a pendulum vibrating seconds, or 60 times in a minute, being 39.125 ins., and lengths of pendulums being reciprocally as the squares of number of vibrations made in same time, therefore $n^2 : 60^2 :: 39.125 : \frac{60^2 \times 39.125}{n^2} = \frac{140\,850}{n^2}$, *being length of pendulum which vibrates n times in a minute, or distance of centre of oscillation below axis of motion.*

To Compute Centres of Oscillation or Percussion of a System of Particles or Bodies.

Rule.—Multiply weight of each particle or body by square of its distance from point of suspension, and divide sum of their products by sum of weights, multiplied by distance of centre of gravity from point of suspension, and quotient will give centre required, measured from point of suspension.

$$\text{Or, } \frac{W\,d^2 + W'\,d'^2}{W\,g + W'\,g'} = \textit{distance of centre.}$$

Example 1.—Length of a suspended rod being 20 feet, and weight of a foot in length of it equal 100 oz., has a ball attached at under end weighing 100 oz.; at what point of rod from point of suspension is centre of percussion?

$100 \times 20 = 2000 =$ *weight of rod;* $2000 \times \frac{20}{2} = 20\,000 =$ *momentum of rod, or product of its weight, and distance of its centre of gravity;* $\frac{2000 \times 20^2}{3} = 266\,666.66 =$ *force of rod;* $1000 \times 20^2 = 400\,000 =$ *force of ball.*

Then $\frac{266\,666.66 + 400\,000}{20\,000 + 20\,000} = 16.66$ *feet.*

2.—Assume a rod 12 feet in length, and weighing 2 lbs. for each foot of its length, with 2 balls of 3 lbs. each—one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$12 \times 2 \times \frac{12}{2} = 144 =$ *momentum of rod.*
$3 \times 6 = 18 =$ " *of 1st ball.*
$3 \times 12 = 36 =$ " *of 2d ball.*
198 *sum of moments.*

$\frac{24 \times 12^2}{3} = \frac{3456}{3} = 1152 =$ *force of rod.*
$3 \times 6^2 = 3 \times 36 = 108 =$ " *of 1st ball.*
$3 \times 12^2 = 3 \times 144 = 432 =$ " *of 2d ball.*
1692 *sum of forces.*

Then $1692 \div 198 = 8.545$ *feet.*

MECHANICS.

Mechanics is the science which treats of and investigates effects of forces, motion and resistance of material bodies, and of equilibrium: it is divided into two parts—Statics and Dynamics.

Statics treats of equilibrium of forces or bodies at rest. Dynamics of forces that produce motion, or bodies in motion.

These bodies are further divided into *Mechanics of Solid, Fluid,* and *Aeriform bodies;* hence the following combinations:

1. *Statics of Solid Bodies, or Geostatics.*
2. *Dynamics of Solid Bodies, or Geodynamics.*
3. *Statics of Fluids, or Hydrostatics.*
4. *Dynamics of Fluids, or Hydrodynamics.*
5. *Statics of Aeriform Bodies, or Aerostatics.*
6. *Dynamics of Aeriform Bodies, Pneumatics or Aerodynamics.*

Forces are various, and are divided into moving forces or resistances; as

Gravity,	*Heat or Caloric,*	*Inertia,*
Muscular,	*Magnetism,*	*Cohesion,*
Elasticity and Contractility,	*Percussion,*	*Adhesion,*
Central,	*Expansion,*	*and Explosion.*

Couple.—Two forces of equal magnitude applied to or operating upon same body in parallel and opposite directions, but not in same line of action, constitute a *couple,* and its force is sum or magnitude of the two equal forces.

Moment.—Quantity of motion in a moving body, which is always equal to product of quantity of matter and its velocity.

When velocities of two moving bodies are inversely as their quantities of matter, their *momenta* are equal.

STATICS.

Composition and Resolution of Forces.

When two forces act upon a body in same or in an opposite direction, effect is same as if only one force acted upon it, being sum or difference of the forces. Hence, when a body is drawn or projected in directions immediately opposite, by two or more unequal forces, it is affected as if it were drawn or projected by a single force equal to difference between the two or more forces, and acting in direction of greater force.

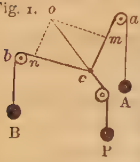
This single force, derived from the combined action of two or more forces, is their *Resultant.*

The process by which the *resultant* of two or more forces, or a single force equidistant in its effect to two or more forces, is determined, is termed the *Composition of Forces*, and the inverse operation; or, when combined effects of two or more forces are equivalent to that of a single given force, the process by which they are determined is termed the *Decomposition or Resolution of Forces.* Two or more forces which are equivalent to a single force are termed *Components.*

When two forces act on same point their intensities are represented by sides of a parallelogram, and their combined effect will be equivalent to that of a single force acting on point in direction of diagonal of parallelogram, the intensity of which is proportional to diagonal.

Illustration.—Attach three cords to a fixed point, c, Fig. 1; let $c\,a$ and $c\,b$ pass over fixed rollers, and suspend weights A and B therefrom.

Fig. 1.



Point c will be drawn by the forces A and B in directions $a\,c$ and $b\,c$. Now, in order to ascertain which single force, P, would produce the same effect upon it, set off the distances $c\,m$ and $c\,n$ on the cords in the same proportion of length as weights of A and B; that is, so that $c\,m : c\,n :: A : B$; then draw parallelogram $c\,m\,o\,n$ and diagonal $o\,c$, and it will represent a single force, P, acting in its direction, and having same ratio to weights A or B as it has to sides $c\,m$ or $c\,n$ of parallelogram. Consequently, it will produce same effect on point c as combined actions of A and B.

A parallelogram, constructed from lateral forces, and diagonal of which is mean force, is termed a *Parallelogram of Forces.*

Fig. 2.

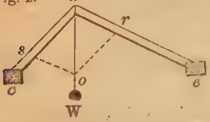


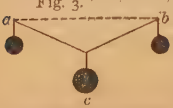
Illustration.—Assume a weight, W, Fig. 2, to be suspended from a; then, if any distance, $a\,o$, is set off in numerical value upon the vertical line, a W, and the parallelogram, $o\,r\,a\,s$, is completed, $a\,s$ and $a\,r$, measured upon the scale, $a\,o$, will represent strain upon $a\,c$ and $a\,e$ in same proportion that $a\,o$ bears to weight W.

If several forces act upon same point, and their intensities taken in order are represented by sides of a polygon, except one, a single force proportioned to and acting in direction of that one side will be their resultant.

To Resolve a Single Force into a Pair of Forces.—Figs. 3 and 4.

The ends of a cord, Fig. 3, are led over two points, a and b, and in centre of cord at c a weight of 4 lbs. is suspended. If distances $a\,c$, $b\,c$, are each 1 foot, distance $a\,b$ should be 18 ins.

Fig. 3.



When cord is in this position, weight at c draws upon $c\,a$ and $c\,b$ with a force of 3 lbs.; hence c of 4 lbs. is equal to two forces of 3 lbs. each in direction of $a\,c$ and $b\,c$.

Fig. 4.

Apply ends of cord to $e\,f$, Fig. 4, distance being 22 ins., then the strain on $c\,e$, $c\,f$, are each 5 lbs.; hence one force of 4 lbs. is equal to two of 5 lbs. each.

Equilibrium of Forces.

Two bodies which act directly against each other in same line are in equilibrium when their quantities of motion are equal: that is, when product of mass of one, into velocity with which it moves or tends to move, is equal to product of mass of other, into its actual or virtual * velocity.

When the velocities with which bodies are moved are same, their forces are proportional to their masses or quantities of matter. Hence, when equal masses are in motion, their forces are proportional to their velocities.

Relative magnitudes and directions of any two forces may be represented by two right lines, which shall bear to each other the relations of the forces, and which shall be inclined to each other in an angle equal to that made by direction of the forces.

Fig. 4.

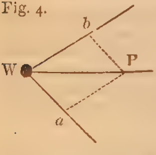


ILLUSTRATION.—Assume a body, W, to weigh 150 lbs., and resting upon a smooth surface, to be drawn by two forces, a and b, Fig. 5, = 24 and 30 lbs., which make with each other an angle; a W b = 105°, in which direction and with what acceleration will motion occur?

Cos. a W b = 105°, and cos. 180° — 105° = cos. 75°, *mean force.*

$$P = \sqrt{30^2 + 24^2 - 2 \times 30 \times 24 \cos. 75^\circ} = \sqrt{900 + 576 - 1440 \cos. 75^\circ}$$
$$= \sqrt{1476 - (1440 \times .258\,82)} = \sqrt{1103.3} = 33.21 \textit{ lbs.}$$

The acceleration is $\frac{P\,g}{W} = \frac{33.21 \times 32.166}{150} = 7.1215$ *feet.*

Angle of Repose is greatest inclination of a plane to horizon at which a body will remain in equilibrium upon it.

Hence greatest angle of obliquity of pressure between two planes, consistent with stability, is the angle tangent of which is equal to coefficient of friction of the two planes.

Inertia is resistance which a body at rest offers to an external power to be put in motion or to change its velocity or direction when in motion.

To Compute Inertia of a Revolving Body.

Divide it into small parts of a regular figure, multiply weight of each part by square of its distance of its centre of gravity from axis of revolution, and sum of products will give moment of inertia of body.

DYNAMICS.

DYNAMICS is the investigation of the laws of *Motion of Solid Bodies*, or *of Matter, Force, Velocity, Space*, and *Time*.

Mass of a body is the quantity of matter of which it is composed.

Force is divided into Motive, Accelerative, or Retardative.

Motive Force, or *Momentum*, of a body, is the product of its mass and its velocity, and is its quantity of motion. This force can, therefore, be ascertained and compared in any number of bodies when these two quantities are known.†

Accelerative or *Retardative Force* is that which respects velocity of motion only, accelerating or retarding it; and it is denoted by quotient of motive force, divided by mass or weight of body. Thus, if a body

---

* Virtual velocity is the velocity which a body in equilibrium would acquire were the equilibrium to be disturbed.

† It is compared, because it is not referable to any standard, as a ton, pound, etc. Thus, suppose a cannon-ball weighing 15 lbs., projected with a velocity of 1500 feet per second, strike a resisting body, its momentum, according to the above rule, would be 15 × 1500 = 22 500; not pounds, for weight is a pressure with which it cannot be compared.

of 5 lbs. is impelled by a force of 40 lbs., accelerating force is 8 lbs.; but if a force of 40 lbs. act upon a body of 10 lbs., accelerating force is only 4 lbs., or half former, and will produce only half velocity.

With equal masses, velocities are proportional to their forces.

With equal forces, velocities are inversely as the masses.

With equal velocities, forces are proportional to the masses.

Work is product of force, velocity, and time.

Motion.—The succession of positions which a body in its motion progressively occupies forms a line which is termed the trajectory, or path of the moving body.

A motion is *Uniform* when equal spaces are described by it in equal times, and *Variable* when this equality does not occur. When spaces described in equal times increase continuously with the time, a variable motion is termed *accelerated*, when spaces decrease, *retarded*, and when equal spaces are described within certain intervals only, the motion is termed *periodic*, and intervals periods. Uniform motion is illustrated in progressive motion of hands of a watch; variable in progressive velocity of falling and upwardly projected bodies; and periodic by oscillation of a pendulum or strokes of a piston of a steam-engine.

Uniform Motion.

FORMULAS. $f\,v$, $\frac{f\,s}{t}$, $H\,550$, and $\frac{W}{t} = P$; $\frac{P}{v}$, $\frac{W}{s}$, and $\frac{H\,550}{v} = f$; $\frac{s}{t}$, $\frac{P}{f}$, $\frac{H\,550}{f}$, and $\frac{W}{f\,t} = v$; $v\,t$, $\frac{P\,t}{f}$, $\frac{W}{f}$, and $\frac{H\,550\,t}{f} = s$; $\frac{s\,f}{P}$, $\frac{s}{v}$, $\frac{W}{f\,v}$, and $\frac{f\,s}{H\,550} = t$; $f\,s$, $H\,550\,t$, $P\,t$, and $f\,v\,t = W$; $\frac{P}{550}$, $\frac{f\,v}{550}$, $\frac{f\,s}{550\,t}$, and $\frac{W}{550\,t} = H$. P *representing power in effect, body, or momentum, f force in lbs., v and s velocity and space in feet per second, t time in seconds,* H *horse-power, and* W *work in foot-lbs.*

If two or more bodies, etc., are compared, two or more corresponding letters, as P, *p*, *p′*, V, *v*, *v′*, *etc., are employed.*

ILLUSTRATION 1.—Two bodies, one of 20, the other of 10 lbs., are impelled by same momentum, say 60. They move uniformly, first for 8 seconds, second for 6; what are the spaces described by both?

$$60 \div 20 = 3 = V, \text{ and } 60 \div 10 = 6 = v.$$

Then $T\,V = 3 \times 8 = 24 = S$, and $t\,v = 6 \times 6 = 36 = s$, *spaces respectively.*

2.—If a power of 12 800 effects has a velocity of 10 feet per second, what is its force?

$$12\,800 \div 10 = 1280 \textit{ lbs.}$$

Uniform Variable Motion.

Space described by a body having uniform variable motion is represented by sum or difference of velocity, and product of acceleration and time, according as the motion is accelerated or retarded.

ILLUSTRATION 1.—A sphere rolling down an inclined plane with an initial velocity of 25 feet, acquires in its course an additional velocity at each second of time of 5 feet; what will be its velocity after 3 seconds?

$$25 + \overline{5 \times 3} = 40 \textit{ feet.}$$

2.—A locomotive having an initial velocity of 30 feet per second is so retarded that in each second it loses 4 feet; what is its velocity after 6 seconds?

$$30 - \overline{4 \times 6} = 6 \textit{ feet.}$$

Uniform Motion Accelerated.

In this motion, velocity acquired at end of any time whatever is equal to product of accelerating force into time, and space described is equal to product of half accelerating force into square of time, or half product of velocity and time of acquiring the velocity.

Spaces described in successive seconds of time are as the odd numbers, 1, 3, 5, 7, 9, etc.

Gravity is a constant force, and its effect upon a body falling freely in a vertical line is represented by g, and the motion of such body is uniformly accelerated.

The following theorems are applicable to all cases of motion uniformly accelerated by any constant force, F:

$$.5\,t\,v = .5\,g\,F\,t^2 = \frac{v^2}{2\,g\,F} = s. \qquad \frac{2\,s}{v} = \frac{v}{g\,F} = \sqrt{\frac{s}{.5\,g\,F}} = t.$$

$$\frac{2\,s}{t} = g\,F\,t = \sqrt{2\,g\,F\,s} = v. \qquad \frac{v}{g\,t} = \frac{2\,s}{g\,t^2} = \frac{v^2}{2\,g\,s} = F.$$

When gravity acts alone, as when a body falls in a vertical line, F *is omitted. Thus,*

$$.5\,g\,t^2 = \frac{v^2}{2\,g} = s. \qquad g\,t = \sqrt{2\,g\,s} = v. \qquad \frac{v}{g} = \sqrt{\frac{2\,s}{g}} = t. \qquad \frac{v}{t} = \frac{2\,s}{t^2} = \frac{v^2}{2\,s} = g.$$

t representing time in seconds, and s velocity in feet per second.

If, instead of a heavy body falling freely, it be projected vertically upward or downward with a given velocity, v, then $s = t\,v \mp .5\,g\,t^2$; an expression in which $-$ must be taken when the projection is upward, and $+$ when it is downward.

ILLUSTRATION 1. — If a body in 10 seconds has acquired a velocity by uniformly accelerated motion of 26 feet, what is accelerating force, and what space described, in that time?

$$26 \div 10 = 2.6 = \textit{accelerating force}; \quad \frac{2.6}{2} \times 10^2 = 130 \textit{ feet} = \textit{space described.}$$

2.—A body moving with an acceleration of 15.625 feet describes in 1.5 seconds a space $= \dfrac{15.625 \times (1.5)^2}{2} = 17.578$ *feet.*

3.—A body propelled with an initial velocity of 3 feet, and with an acceleration of 5 feet, describes in 7 seconds a space $= 3 \times 7 + 5 \times \dfrac{7^2}{2} = 143.5$ *feet.*

4.—A body which in 180 seconds changes its velocity from 2.5 to 7.5 feet, traverses in that time a distance of $\dfrac{2.5 + 7.5}{2} \times 180 = 900$ *feet.*

5.—A body which rolls up an inclined plane with an initial velocity of 40 feet per second, by which it suffers a retardation of 8 feet, ascends only $\dfrac{40}{8} = 5$ *seconds*, and $40^2 \div \overline{2 \times 8} = 100$ *feet* in height, then rolls back, and returns, after 10 seconds, with a velocity of 40 feet, to its initial point; and after 12 seconds arrives at a distance of $40 \times 12 - \overline{4 \times 12^2} = 96$ *feet below point, assuming plane to be extended backward.*

Circular Motion.

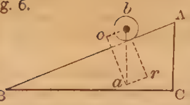
$$\frac{2\,p\,r\,n}{60} = \frac{2\,p\,r\,n'}{t} = v; \qquad \frac{5500\ \text{HP}}{r\,n} = \frac{W}{2\,p\,r\,n'} = f; \qquad \frac{f\,r\,n}{5500} = \frac{f\,2\,p\,r\,n}{550 \times 60} = \text{HP};$$

$f\,2\,p\,r\,n' = \dfrac{f\,t\,2\,p\,r\,n}{60} = W.$ *r representing radius in feet, n number of revolutions of circle per minute, n' total revolutions, f force in lbs., t time in seconds, and* HP *horse-power.*

Motion on an Inclined Plane.

To Ascertain Conditions of Motion by Gravity.

Fig. 6.



Assume A B, Fig. 6, an inclined plane, B C its base, A C its height, and b a body descending the plane; from dot, centre of gravity of body, draw $b\ a$ perpendicular to B C, representing pressure of b by gravity; draw $b\ o$ parallel and $b\ r$ perpendicular to A B, and complete parallelogram; then force $b\ a$ is equal to both $b\ o$, $b\ r$, of which $b\ r$ is sustained by reaction of plane, and force $b\ o$ is wholly effective in accelerating motion of body.

Let this force be represented by f, and $b\ a$, by g or force of gravity, then by similar triangle, $f : g :: b\ o : b\ a : \text{A C} : \text{A B}$. Hence, $\frac{\text{A C} \times g}{\text{A B}} = f$.

Put A B $= l$, A C $= h$ and $\angle$ A B C $= a$, then force which produces motion on the plane on f becomes $g\ \frac{h}{l}$, and g sin. a.

Therefore, accelerating force on an inclined plane is constant, and equations of motion will be obtained by substituting its value of f for g in equations 1, 2, and 3, page 618.

$$\frac{g h t^2}{2 l}, \quad \frac{l v^2}{2 g h}, \quad .5\, t\, v, \quad .5\, g\, t^2 \sin. a, \quad \text{and} \quad \frac{v^2}{2 g \sin. a} = s.$$

$$\frac{2 s}{t}, \quad \frac{g h t}{l}, \quad \sqrt{\frac{2 g h s}{l}}, \quad g\, t \sin. a, \quad \text{and} \quad \sqrt{2 g s \sin. a} = v.$$

$$\frac{2 s}{v}, \quad \frac{l v}{g h}, \quad \sqrt{\frac{2 l s}{g h}}, \quad \frac{v}{g \sin. a}, \quad \text{and} \quad \sqrt{\frac{2 s}{g \sin. a}} = t.$$ *a representing* $\angle$ A B C.

When a Body is projected down or up an Inclined Plane, with a given Velocity.—The distance which it will be from point of projection in a given time will be

$$t\, v \pm \frac{g h t^2}{2 l}, \quad \text{and} \quad \frac{t}{2 l}(2 l v \pm g h t) = s.$$

Illustration 1.—Length of an inclined plane is 100 feet, and its angle of inclination 60°; what is time of a body rolling down it, and velocity acquired?

$$\sin. 60^\circ = .866.$$

$$\sqrt{\frac{2 \times 100}{32.16 \times .866}} = \sqrt{7.18} = 2.68 \textit{ seconds}, \text{ and } 32.16 \times 2.68 \times .866 = 74.64 \textit{ feet}.$$

2.—If a body is projected up an inclined plane, which rises 1 in 6, with a velocity of 50 feet per second, what will be its place and velocity at end of 6 seconds?

$$6 \times 50 - \frac{32.16 \times 1 \times 6^2}{2 \times 6} = 203.52 \textit{ feet from bottom}, \text{ and } 50 - \left(32.16 \times 6 \times \frac{1}{6}\right) =$$
$50 - 32.16 = 17.84$ *feet.*

To effect an ascent up an inclined plane in least time, its length, to its height, must be as twice weight to power.

Work Accumulated in Moving Bodies.

Quantity of work stored in a body in motion is same as that which would be accumulated in it by gravity if it fell from the height due to the velocity. Accumulated work expressed in foot-lbs. is equal to product of height so found in feet, and weight of body in lbs. Height due to velocity is equal to square of velocity divided by 64.4, and work and velocity may be deduced directly from each other by following rules:

To Compute Accumulated Work.

Rule.—Multiply weight in lbs. by square of velocity in feet per second, and divide by 64.4, and quotient is accumulated work in foot-lbs.

Or, $W = \frac{v^2 \times w}{64.4}$, or, $= w \times h$. *W representing work, w weight in lbs., and h height due to velocity in feet per second.*

Work by Percussive Force.

If a wedge is driven by strokes of a hammer or other heavy mass, effect of percussive force is measured by quantity of work accumulated in stricken body. This work is computed by preceding rules, from weight of body and velocity with which a stroke is delivered, or directly from height of fall, if gravity be percussive power.

Useful work done through a wedge is equal to work expended upon it, assuming that there is no elastic or vibrating reaction from the stroke, as if the work had been exerted by a constant pressure equal to weight of striking body, exerted through a space equal to height of fall, or height due to its final velocity.

If elastic action intervenes, a portion of work exerted is absorbed in an elastic stress to resisting body; and the elastic action may be, in some cases, so great as to absorb the work expended.

The principle of action of a blow on a wedge is alike applicable to action of the stroke of a monkey of a pile-driver upon a pile.

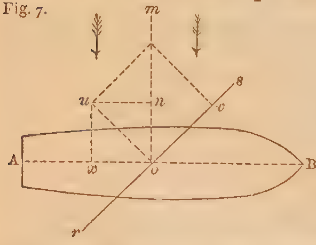
If there be no elastic action, the work expended being product of weight of monkey by height of its fall, is equal to work performed in driving the pile: that is, to product of resistance to its descent by depth through which it is driven by each blow of monkey.

ILLUSTRATION.—If a horse draws 200 lbs. out of a mine, at a speed of 2 miles per hour, how many units of work does he perform in a minute, coefficient of friction .05?

$\frac{2 \times 5280}{60} = 176$ *feet per minute.* Hence. $176 \times \overline{200 + .05 \times 200} = 35\,210$ *units.*

Decomposition of Force.

Fig. 7.



By parallelogram of force it is illustrated how a vessel is enabled to be sailed with a free wind and against one.

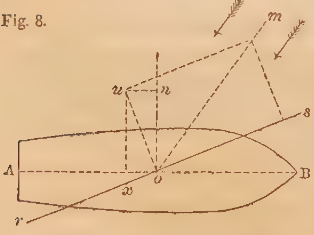
Assume wind to be free or in direction of arrows, Fig. 7, and perpendicular to line A B, the course of vessel.

Let line *m o* represent direction and force of wind, and *r s* plane of sail; from *o* draw *o u* perpendicular to *r s*, and from *m* perpendicular, *m v* on *r s*, and *m u* on *o u*.

By principle of parallelogram of forces, force *m o* may be decomposed into *o v* and *o u*, since they are the sides of parallelogram of which *m o*, representing force of wind, is diagonal. Force of wind, therefore, is measured by *o u*, both in magnitude and direction, and represents actual pressure on sail.

Draw *u n* and *u x* parallel to *o* A and *o m*, thus forming parallelogram *u n o x*. Hence force *o u* is equal to the two, *o n* and *o x*. Force *o n* acts in a direction perpendicular to vessel's course and that of *o x* is to drive vessel onward.

Fig. 8.



It can thus be shown that when direction of sail bisects angle *m o* B, the effect of *o x* is greater than when sail is in any other position.

Assume wind to be ahead as in direction of arrows, Fig. 8. Let *o m* represent direction and force of wind, and *r s* direction of sail; from *o* draw *o u*, and proceed as before, and *o u* represents the effective force that acts upon the sail, *o n* that which drives her to leeward, and *o x* that which drives her on her course.

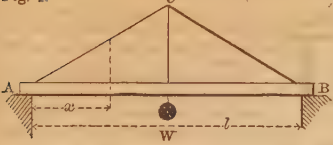
For full treatises on this subject, see John C. Trautwine's Engineer's Pocket-book, 1872; Bull's Experimental Mechanics, London, 1871; and Dynamics, Construction of Machinery, etc., by G. Finden Warr, London, 1851.

MOMENTS OF STRESS.

To Describe and Compute Moments of Stress in Girders or Beams.

Beam Supported at Both Ends.

Fig. 1.



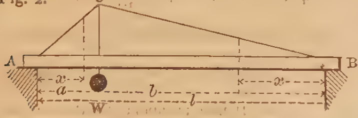
Loaded in Middle, Fig. 1.—Assume A B beam. At middle erect $W\,c = \frac{W\,l}{4}$. Connect A c and c B, and vertical distances between them and A B will give moment required.

Thus, $\frac{W\,x}{2} = M$ at any point. W *representing weight or load, l length of span, x horizontal distance from nearest support at which* M *is required, and* M *moment of stress.*

Illustration.—Assume $l = 10$ feet, $W = 10$ lbs., and $x = 3$ feet.

Then, $W\,c = \frac{10 \times 10}{4} = 25$ *lbs. at centre of span*; $\frac{10 \times 3}{2} = 15$ *lbs. at x.*

Fig. 2.



Loaded at Any Point, Fig 2.—Proceed as for previous figure.

$\frac{W\,a\,b}{l}$ or $W\,c =$ *maximum load.*

$\frac{W\,x\,b}{l} = M$ *between* A *and* W.

$\frac{W\,x\,a}{l} = M$ *between* W *and* B.

a representing least distance of W *to support, and b greatest distance.*

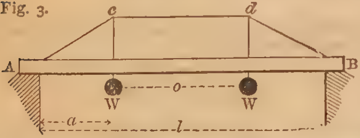
Illustration.—Take elements as before with $a = 3$ feet, and $x = 1.5$ and 3.5 feet.

Then, $W\,c = \frac{10 \times 3 \times 7}{10} = 21$ *lbs. at point of stress*; $\frac{10 \times 1.5 \times 7}{10} = 10.5$ *lbs. at x between* A *and* W, *and* $\frac{10 \times 3.5 \times 3}{10} = 10.5$ *lbs. at x between* W *and* B.

Note.—x must be taken from the pier which is on the same side of W as x is.

Loaded with Two Equal Weights at Equal Distances from Ends, alike to a Transverse Girder as for a Single Line of Railway.—Fig. 3.

Fig. 3.

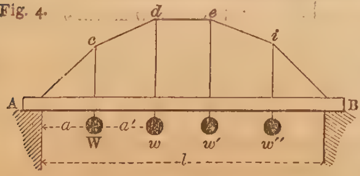


At point of stress of weights erect W c and W d, each = W a. Connect A c d and B, and vertical distances between A B, as defined by c d, will give moments.

$\frac{W\,(l - o)}{2} = W\,a = W\,b = M$ *at any point between weights.*

Loaded with Four Equal Weights, symmetrically bearing from Centre, alike to a Transverse Girder as for a Double Line of Railway.—Fig. 4.

Fig. 4.



At W and w'' erect W c, and $w''\,i = 2\,W\,a$, and at w and w' erect $w\,d$, $w'\,e$, each $= W\,(2\,a + a')$.

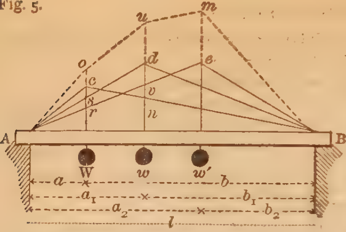
Connect A c d e i and B, and ordinates to A B will give moments.

$W\,(2\,a + a') = M$ at w and w'; $2\,W\,a = M$ at W and w''.

Illustration.—Assume W each 10 lbs. 2 feet apart, and l 10 feet.

Then, $10\,(2 \times 2 + 2) = 60$ at w or w', and $2 \times 10 \times 2 = 40$ at W or w''.

Fig. 5.



Loaded at Different Points.—Fig. 5.

Locate three weights, W, w, and w', as at $a\,b$, $a_1\,b_1$, $a_2\,b_2$.

Draw $A\,c\,B$, $A\,d\,B$, and $A\,e\,B$, for three separate cases, as by formula, $\frac{W\,a\,b}{l}$, Fig. 2.

Produce $W\,c$ until $W\,o = W\,r$, $W\,s$, and $W\,c$; $W\,d$ until $w\,u = w\,n$, $w\,v$ and $w\,d$, and $w'\,e$ to $w'\,m$ in like manner.

Connect $A\,o\,u\,m$ and B, and an ordinate therefrom, to A B will give moment or stress at the point taken.

Illustration.—Take $a = 2$ feet, $a_1 = 4$, $a_2 = 6$, $b = 8$, $b_1 = 6$, $b_2 = 4$, $x = 2$, W, w, and w' each 10 lbs., and $l = 10$ feet.

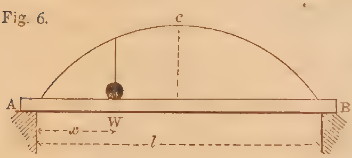
Then $\frac{1}{l}(W\,a\,x + w\,a_1\,x + w'\,b_2\,x) = M$ at x.

Take $x = 2$. Then $\frac{1}{10}(10 \times 2 \times 2 + \overline{10 \times 4 \times 2} + \overline{10 \times 8 \times 2}) = \frac{280}{10} = 28$ *lbs.*

$x = 4$. Then $\frac{1}{10}(10 \times 2 \times 2 + \overline{10 \times 4 \times 2} + \overline{10 \times 8 \times 4}) = \frac{440}{10} = 44$ *lbs.*

Take $x = 5$. Then $\frac{1}{10}(10 \times 2 \times 5 + \overline{10 \times 4 \times 5} + \overline{10 \times 4 \times 5}) = \frac{500}{10} = 50$ *lbs.*

Fig. 6.



Loaded with a Rolling Weight.—Fig. 6.

Define parabola A c B as determined by $\frac{W\,l}{4}$ = the ordinate at c, and vertical distances between A B will give moments.

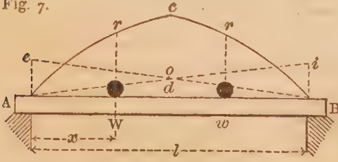
$$\frac{W\,x\,(l - x)}{l} = M \textit{ at any point.}$$

Loaded Uniformly its Entire Length.—Define parabola as at Fig. 6, ordinate of which at $c = \frac{w\,l^2}{8}$. L *representing stationary or dead load per unit of length.*

$$\frac{L\,x}{2}\,(l - x) = M \textit{ at any point, and } \frac{w\,l^2}{8} = M \textit{ at centre.}$$

Loaded with Two Connected Weights, moving in either Direction, alike to a Locomotive or Car on a Railway.—Fig. 7.

Fig. 7.



Define parabola A c B as determined by $\frac{(W + w)\,l}{4} = c$.

At A and B erect A e, B $i = w\,d$, connect A i and B e, and vertical distances between A o B and A c B will give moments.

$$\frac{x}{l}\left[(W + w)\,(l - x) - \overline{w\,d}\right] = M$$

at any point.

Position of W *at greatest moment, when* $x = \frac{l}{2} \pm \frac{w\,d}{2\,(W + w)}$. Or if W and w are equal, when $x = \frac{l}{2} \pm \frac{d}{4}$.

Illustration.—Assume $x = 3$, $d = 4$, and W w each 10 lbs., and l 10 feet.

Then $\frac{3}{10}\left(\overline{10 + 10} \times \overline{10 - 3} - \overline{10 \times 4}\right) = M$ *at any point*, as at W r, $w\,r$.

Shearing Stress.

To Determine Shearing Stress at any Part of a Girder or Beam and under any Distribution of Load.

Fig. 8.

Required to determine stress of a beam at any point as c, Fig. 8.

Assume W = load between A and c, and w that between B and c.

Then $S x$ at $c = P - W$, or $P' - w$.

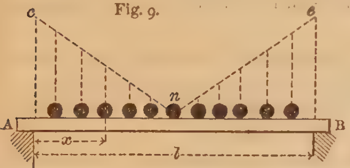
The greater of the two values to be taken.

S x representing shearing stress at any point x, P and P′ the reaction on supports due to total load on beam between supports, W and w loads or stress concentrated at any point.

To Describe and Ascertain Shearing Stress in a Girder or Beam.

Supported or Fixed at Both Ends.

Fig. 9.



Loaded Uniformly. Fig. 9.

At A and B, erect A c, B e, each equal to $\frac{W\,l}{2}$. Connect c and e at middle of span as at n, and vertical distances between A B and $c\,n\,e$ will give shearing stresses as determined by the ordinates to $c\,n\,e$.

$L\left(\frac{l}{2} - x\right) = S$. Sign of result to be disregarded. *L representing distributed load per unit of length.*

ILLUSTRATION.—Assume W = 10 lbs. per foot, $l = 10$, and $x = 2.5$ feet.

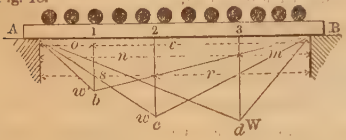
$$\text{Then } 10\left(\frac{10}{2} - 2.5\right) = 25 \textit{ lbs.}$$

NOTE.—The moment of rupture at any point, produced by several loads acting simultaneously on a beam, is equal to the sum of the moments produced by the several loads acting separately.

For other Formulas and Diagrams see Strains in Girders, by William Humber, A. L. C. E, London, 1872.

Operation deduced by Graphic Delineation of Greatest Stress, with a Uniformly Distributed Load of 4000 *Lbs.*—Fig. 10.

Fig. 10.



Determine moment of weights by formulas $\frac{W\,m\,n}{l}$, $\frac{w\,r\,s}{l}$, and $\frac{w'\,o\,v}{l}$.

Assume W = 7000 lbs., $w = 4000$, and $w' = 3000$, $m = 7$ feet, $n = 13$, $r = 13$, $s = 7$, $o = 3$, $v = 17$, and $l = 20$.

Then $W = \frac{7000 \times 13 \times 7}{20} = 31\,850$, $w = \frac{4000 \times 13 \times 7}{20} = 18\,200$, and $w' = \frac{3000 \times 3 \times 17}{20} = 7650$, and let fall perpendiculars thereto, as 3 d, 2 c, and 1 b.

Connect d, c, and b with A B, and sum of distances of intersections of these lines upon perpendiculars, from 3, 2, and 1 respectively, will give stress upon A B at these points.

To determine Greatest Stress at Greatest Load.

Stress at 3 d = 31 850
" " 2 c = 13 : 18 200 : 7 = 9 800
Stress at 1 b = 17 : 7650 : 3 = 1 350
43 000

$43\,000 + \frac{7 \times 13 \times 4000 \times .5}{20} = 52\,100$ *lbs.*, concentrated load at W, and proportion of uniformly distributed load of 4000 lbs.

MECHANICAL POWERS.

MECHANICAL POWER is a compound of *Weight*, or *Force* and *Velocity:* it cannot be increased by mechanical means.

The Powers are three in number—viz., LEVER, INCLINED PLANE, and PULLEY.

NOTE.—A Wheel and Axle is a *continuous* or *revolving lever*, a Wedge a *double* inclined plane, and a Screw a *revolving* inclined plane.

LEVER.

Levers are straight, bent, curved, single, or compound.

To Compute Length of a Lever.

When Weight and Power are given. RULE.—Divide weight by power, and quotient is leverage, or distance from fulcrum at which power supports weight.

Or, $\frac{W}{P} = p$. W *representing weight*, P *power, and* p *distance of power from fulcrum.*

EXAMPLE.—A weight of 1600 lbs. is to be raised by a power or force of 80; required length of longest arm of lever, shortest being 1 foot.

$$1600 \div 80 = 20 \textit{ feet.}$$

To Compute Weight that can be raised by a Lever.

When its Length, Power, and Position of its Fulcrum are given. RULE.—Multiply power by its distance from fulcrum, and divide product by distance of weight from fulcrum.

Or, $\frac{P\,p}{w} = W$. *w representing distance of weight from fulcrum.*

EXAMPLE.—What weight can be raised by 375 lbs. suspended from end of a lever 8 feet from fulcrum, distance of weight from fulcrum being 2 feet?

$$375 \times 8 \div 2 = 1500 \textit{ lbs.}$$

To Compute Position of Fulcrum.

When Weight and Power and Length of Lever are given, and when Fulcrum is between Weight and Power. RULE.—Divide weight by power, add 1 to quotient, and divide length by sum thus obtained.

Or, $L \div \left(\frac{W}{P} + 1\right) = w$. L *representing entire length of lever.*

EXAMPLE.—A weight of 2460 lbs. is to be raised with a lever 7 feet long and a power of 300; at what part of lever must fulcrum be placed?

$$2460 \div 300 = 8.2, \text{ and } 8.2 + 1 = 9.2. \quad \text{Then } (7 \times 12)\ 84 \div 9.2 = 9.13 \textit{ ins.}$$

When Weight is between Fulcrum and Power. RULE.—Divide length by quotient of weight, divided by power.

$$\text{Or, } L \div \frac{W}{P} = w.$$

To Compute Length of Arm of Lever to which Weight is attached.

When Weight, Power, and Length of Arm of Lever to which Power is applied are given. RULE.—Multiply power by length of arm to which it is applied, and divide product by weight.

$$\text{Or, } \frac{P\,p}{W} = w.$$

EXAMPLE.—A weight of 1600 lbs., suspended from a lever, is supported by a power of 80, applied at other end of arm, 20 feet in length; what is length of arm?

$80 \times 20 \div 1600 = 1$ *foot.*

NOTE.—These rules apply equally *When fulcrum (or support) of lever is between weight and power;* when fulcrum is at one extremity of lever, and power, or weight, at the other;† and when arms of lever are equally or unequally bent or curved.*

To Compute Power Required to Raise a given Weight.

When Length of Lever and Position of Fulcrum are given. RULE.—Multiply weight to be raised by its distance from fulcrum, and divide product by distance of power from fulcrum.

$$\text{Or, } \frac{W\,w}{p} = P.$$

EXAMPLE.—Length of a lever is 10 feet, weight to be raised is 3000 lbs., and its distance from fulcrum is 2 feet; what is power required?

$$\frac{3000 \times 2}{10 - 2} = \frac{6000}{8} = 750 \text{ lbs.}$$

To Compute Length of Arm of Lever to which Power is applied.

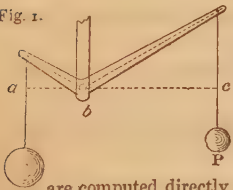
When Weight, Power, and Distance of Fulcrum are given. RULE.—Multiply weight by its distance from fulcrum, and divide product by power.

$$\text{Or, } \frac{W\,w}{P} = p.$$

EXAMPLE.—A weight of 400 lbs., suspended 15 ins. from fulcrum, is supported by a power of 50, applied at other; what is length of the arm?

$400 \times 15 \div 50 = 120$ *ins.*

Fig. 1.



When Arms of a Lever are bent or curved, Distances taken from perpendiculars, drawn from lines of direction of weight and power, must be measured on a line running horizontally through fulcrum, as *a b c*, Figs. 1 and 2.

When Arms of a Lever are at Right Angles, and Power and Weight are applied at a Right Angle to each other, Fig. 3, The moments are computed directly as *a b* to *b c*.

Fig. 2.

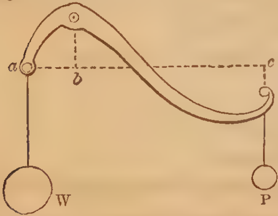
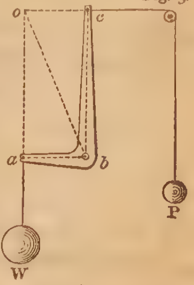


Fig. 3.

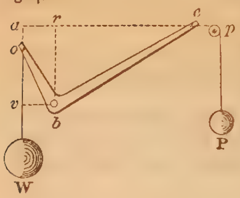


Thrust, or pressure on fulcrum, is in this case less than sum of power and weight; and it may be determined by drawing a parallelogram upon the two arms of lever, arms representing inversely their respective forces. That is, *a b* represents magnitude and direction of weight W, and *b c* of power P. Diagonal *o b* of parallelogram represents magnitude and direction of third force, or thrust upon fulcrum.

---

* Pressure upon fulcrum is equal to sum of weight and power.
† Pressure upon fulcrum is equal to difference of weight and power.

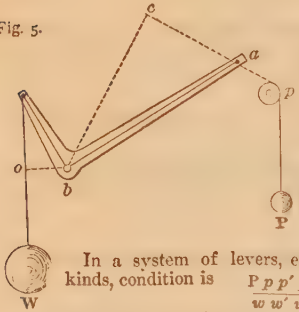
Fig. 4.



When same Lever is borne into an Oblique Position, Power continuing to act Horizontally, Fig. 4, Draw vertical $a\,v$ through end o of lever, and produce the power line $p\,c$ to meet it at a. Complete parallelogram $a\,v\,b\,r$; then sides $r\,b$ and $b\,v$ are perpendiculars to directions to power and weight, on which moments are computed.

Consequently, moment $P \times r\,b =$ moment $W \times a\,v$, and a diagonal, $b\,a$, is resultant thrust at fulcrum.

Fig. 5.



When Power does not act Horizontally, Fig. 5, but in some other direction, $a\,p$, produce the power-line $p\,a$ and draw $b\,c$ perpendicular to it; draw $b\,o$, then moments are computed on perpendiculars $b\,c$, $b\,o$, and $P \times c\,b = W \times b\,o$.

If several weights or powers act upon one or both ends of a lever, condition of equilibrium is

$P\,p + P'\,p' + P''\,p''$, etc., $= W\,w + W'\,w'$, etc.

In a system of levers, either of similar, compound, or mixed kinds, condition is $\dfrac{P\,p\,p'\,p''}{w\,w'\,w''} = W$.

Illustration.—Let $P = 1$ lb., p and p' each 10 feet, p'' 1 foot; and if w and w' be each 1 foot, and w'' 1 inch, then

$\dfrac{1 \times 120 \times 120 \times 12}{12 \times 12 \times 1} = \dfrac{172\,800}{144} = 1200$; that is, 1 lb. will support 1200, with levers of the lengths above given.

Note.—Weights of levers in above formulas are not considered, centre of gravity being assumed to be over fulcrums.

General Rule, therefore, for ascertaining relation of Power to Weight in a lever, whether straight or curved, is. *Power multiplied by its distance from fulcrum is equal to weight multiplied by its distance from fulcrum.*

Or, $P : W :: w : p$, or $P\,p = W\,w$; and

1. $\dfrac{W\,w}{p} = P$. 2. $\dfrac{P\,p}{w} = W$. 3. $\dfrac{W\,w}{P} = p$. 4. $\dfrac{P\,p}{W} = w$.

WHEEL AND AXLE.

A Wheel and Axle is a revolving lever.

Power, multiplied by radius of wheel, is equal to weight, multiplied by radius of axle.

As radius of wheel is to radius of axle, so is effect to power.

Or, $P\,R = W\,r$. Or, $P\,V = W\,v$. Or, $R : r :: W : P$. Or, $P\dfrac{R}{r} = W$; $\dfrac{R\,P}{W} = r$; $\dfrac{W\,r}{P} = R$. *R and r representing radii, and V and v velocities of wheel and axle.*

When a series of wheels and axles act upon each other, either by belts or teeth, weight or velocity will be to power or unity as product of radii, or circumferences of wheels, to product of radii, or circumferences of axles.

ILLUSTRATION.—If radii of a series of wheels are 9, 6, 9, 10, and 12, and their pinions have each a radius of 6 ins., and power applied is 10 lbs., what weight will they raise?

$$\frac{10 \times 9 \times 6 \times 9 \times 10 \times 12}{6 \times 6 \times 6 \times 6 \times 6} = \frac{583\,200}{7776} = 75 \textit{ lbs.}$$

Or, if 1st wheel make 10 revolutions, last will make 75 in same time.

To Compute Power of a Combination of Wheels and an Axle or Axles, as in Cranes, etc.

RULE.—Divide product of driven teeth by product of drivers, and quotient is their relative velocity; which, multiplied by length of lever or arm and power applied to it in pounds, and divided by radius of barrel, will give weight that can be raised.

Or, $\frac{v\,l\,P}{r} = W$; Or, $W\,r = v\,l\,P$; Or, $\frac{W\,r}{v\,l} = P$. *l representing length of lever or arm, r radius of barrel, P power, v velocity, and W weight.*

EXAMPLE 1.—A power of 18 lbs. is applied to lever or winch of a crane, length of it being 8 ins., pinion having 6 teeth, driving-wheel 72, and barrel 6 ins. diameter.

$\frac{72}{6} = 12$, and $12 \times 8 \times 18 = 1728$, which, $\div 3$, *radius of barrel*, $= 576$ *lbs.*

2.—A weight of 94 tons is to be raised 360 feet in 15 minutes, by a power, velocity of which is 220 feet per minute; what is power required?

$360 \div 15 = 24$ *feet per minute.* Hence $\frac{24 \times 94}{220} = 10.2545$ *tons.*

Compound Axle, or Chinese Windlass.

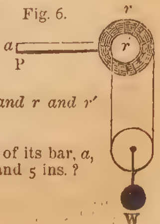
Axle or drum of windlass consists of two parts, diameter of one being less than that of the other.

The operation is thus: At a revolution of axle or drum, a portion of sustaining rope or chain equal to circumference of larger axle is wound up, and at same time a portion equal to circumference of lesser axle is unwound. Effect, therefore, is to wind up or shorten rope or chain, by which a weight or stress is borne, by a length equal to difference between circumferences of the two axles. Consequently, half that portion of the rope or chain will be shortened by half difference between circumferences.

To Compute Elements of a Wheel and Compound Axle, or Chinese Windlass.—Fig. 6.

RULE.—Multiply power by radius of wheel, arm, or bar to which it is applied, and divide product by half difference of radii of axle, and quotient is weight that can be sustained.

Fig. 6.



Or, $\frac{P\,R}{.5\,(r - r')} = W$. *R representing radius of wheel, etc., and r and r' radii of axle at its greatest and least diameters.*

EXAMPLE.—What weight can be raised by a capstan, radius of its bar, *a*, 5 feet, power applied 50 lbs., and radii, *r r'*, of axle or drum 6 and 5 ins.?

$$\frac{50 \times 5 \times 12}{.5\,(6 - 5)} = \frac{3000}{.5} = 6000 \textit{ lbs.}$$

Wheel and Pinion Combinations, or Complex Wheel-work.

Power, multiplied by product of radii or circumferences, or number of teeth of wheels, is equal to weight, multiplied by product of radii or circumferences, or number of teeth or leaves of pinions.

$$\text{Or, } P\,R\,R'\,R'', \text{ etc.}, = W\,r\,r'\,r'', \text{ etc.}$$

NOTE.—Cogs on face of wheel are termed *teeth*, and those on surface of axle are termed *leaves;* the axle itself in this case is termed a *pinion*.

Rack and Pinion.

To Compute Power of a Rack and Pinion.

RULE.—Multiply weight to be sustained by quotient of radius of pinion, divided by radius of crank, and product is power required.

$$\text{Or, } W\,\frac{r}{R} = P.$$

When Pinion on Crank Axle communicates with a Wheel and Pinion. RULE.—Multiply weight to be sustained by quotient of product of radii of pinions, divided by radii of crank and wheel, and product is power required.

$$\text{Or, } W\,\frac{r\,r'}{R\,R'} = P.$$

EXAMPLE.—If radii of pinions of a jack-screw are each one inch; of crank and wheel 10 and 5 ins.; what power will sustain a weight of 750 lbs.?

$$750 \times \frac{1 \times 1}{10 \times 5} = \frac{750}{50} = 15 \textit{ lbs.}$$

INCLINED PLANE.

To Compute Length of Base, Height, or Length.

When any Two of them are given, and when Line of Direction of Power or Traction is Parallel to Face of Plane.—Proceed as in Mensuration or Trigonometry to determine side of a right-angled triangle, any two of three being given.

To Compute Power necessary to Support a Weight on an Inclined Plane.

When Height and Length are given. RULE.—Multiply weight by height of plane, and divide product by length.

$$\text{Or, } \frac{W\,h}{l} = P. \quad \textit{h and l representing height and length of plane.}$$

EXAMPLE.—What is power necessary to support 1000 lbs. on an inclined plane 4 feet in height and 6 feet in length?

$$1000 \times 4 \div 6 = 666.67 \textit{ lbs.}$$

To Compute Weight that may be Sustained by a given Power on an Inclined Plane.

When Height and Length of Plane are given. RULE.—Multiply power by length of plane, and divide product by height.

$$\text{Or, } \frac{P\,l}{h} = W.$$

EXAMPLE.—What is weight that can be sustained on an inclined plane 5 feet in height and 7 feet in length by a power of 700 lbs.?

$$700 \times 7 \div 5 = 980 \textit{ lbs.}$$

NOTE.—In estimating power required to overcome resistance of a body being drawn up or supported upon an inclined plane, and contrariwise, if body is descending; weight of body, in proportion of power of plane (*i. e.*, as its length to its height), must be added to *resistance*, if being drawn up or supported, or to the *moment* if descending.

To Compute Height or Length of an Inclined Plane.

When Weight and Power and one of required Elements are given, and when Height is required. RULE.—Multiply power by length, and divide product by weight.

When Length is required. RULE.—Multiply weight by height, and divide product by power.

$$\text{Or, } \frac{P\,l}{W} = h, \text{ and } \frac{W\,h}{P} = l$$

To Compute Pressure on an Inclined Plane.

RULE.—Multiply weight by length of base of plane, and divide product by length of face.

Or, $\frac{W\,b}{l}$ = *pressure.* *b representing length of base of plane.*

EXAMPLE.—Weight on an inclined plane is 100 lbs., base of plane is 4 feet, and length of it 5; required pressure on plane.

$100 \times 4 \div 5 = 80$ *lbs.*

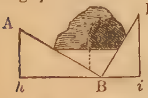
When Two Bodies on Two Inclined Planes sustain each other, as by Connection of a Cord over a Pulley, their Weights are directly as Lengths of Planes.

ILLUSTRATION.—If a weight of 50 lbs. upon an inclined plane, of 10 feet rise in 100 of an inclination, is sustained by a weight on another plane of 10 feet rise in 90, what is the weight of the latter?

100 : 90 :: 50 : 45 = *weight that on shortest plane would sustain that on largest.*

When a Body is Supported by Two Planes, as Fig. 7, pressure upon them will be reciprocally as sines of inclinations of planes.

Fig. 7.



Thus, weight is as sin. A B D.
Pressure on A B as sin. D B i
Pressure on B D as sin. A B h.

Assume angle A B D to be 90°, and D B i, 60°; then angle A B h will be 30°; and as sines of 90°, 60°, and 30° are respectively .1, .866, and .5, if weight = 100 lbs., then pressures on A B and B D will be 86.6 and 50 lbs., *centre of gravity of weight assumed to be in its centre.*

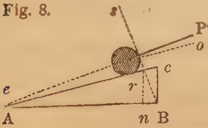
When Line of Direction of Power is parallel to Base of Plane, power is to weight as height of plane to length of its base.

$$\text{Or, } P : W :: h : b.$$

$$\text{Hence, } P = \frac{W\,h}{b}; \quad W = \frac{P\,b}{h}; \quad h = \frac{P\,b}{W}; \quad b = \frac{W\,h}{P}.$$

When Line of Direction of Power is neither parallel to Face of Plane nor to its Base, but in some other Direction, as P′, Fig. 8, power is to weight as sine of angle of plane's elevation to cosine of angle which line of power or traction describes with face of plane.

Fig. 8.



Thus, P′ : W :: sin. A : cos. P′ e c.
Sin. A : cos. P′ e c :: P′ : W.
Cos. P′ e c : sin. A :: W : P′.

ILLUSTRATION.—A weight of 500 lbs. is required to be sustained on a plane, angle of elevation of which, c A B, is 10°; line of direction of power or traction, P′ e c, is 5°; what is sustaining power required?

Cos. P′ e c (5°) = .996 19 : sin. A (10°) = .173 65 :: 500 : 87.16 *lbs.*

Or, draw a line, B s, perpendicular to direction of power's action from end of base line (at back of plane), and intersection of this line on length, A c, will determine length and height (n r) of the plane.

ILLUSTRATION.—By Trigonometry (page 385), A B, assumed to be 1, A r and $n\,r$ are $=$ .985 and .171.

Hence $\frac{500 \times .171}{.985} = 86.8$ *lbs.* $=$ *product of weight* $\times$ *height of plane* $\div$ *length of it.*

NOTE.—When line of direction of power is parallel to plane, power is least.

Wedge.

A WEDGE is a double inclined plane.

To Compute Power.

1. *When One Body is to be Forced or Sustained.* RULE.—Multiply weight or resistance to be sustained by depth of back of wedge, and divide product by length of its base.

EXAMPLE.—What power, applied to the back of a wedge 6 ins. deep, will raise a weight of 15 000 lbs., the wedge being 100 ins. long on its base?

$$\frac{15\,000 \times 6}{100} = \frac{90\,000}{100} = 900 \text{ lbs.}$$

2. *When Two Bodies or Two Parts of a Body are Forced or Sustained in a Direction Parallel to Back of Wedge.* RULE.—Multiply weight or resistance to be sustained by half depth of back of wedge, and divide product by length of wedge.

Or, $\frac{W\,\overline{d \div 2}}{l} = P$. *d representing depth of back, and l length.*

NOTE.—The length of a single wedge is measured on its base, and of a double wedge, from centre of its head to its point.

EXAMPLE.—The depth of the back of a double-faced wedge is 6 ins., and the length of it through the middle 10; what power applied to it is necessary to sustain or overcome a resistance of 150 lbs.?

$$\frac{150 \times \overline{6 \div 2}}{10} = \frac{450}{10} = 45 \text{ lbs.}$$

To Compute Elements of a Wedge.

$$\frac{W\,d}{P} = l. \qquad \frac{P\,l}{W} = d. \qquad \frac{P\,l}{d} = W. \qquad \frac{W\,d}{l} = P.$$

$$\frac{P\,l}{d \div 2} = W. \qquad \frac{W\,\overline{d \div 2}}{P} = l. \qquad \frac{P\,l}{W} = \overline{d \div 2}. \qquad \frac{W\,d}{l} \div 2 = P.$$

NOTE.—As power of wedge in practice depends upon split or rift in wood to be cleft, or in rise of body to be raised, the above rules as regards length of wedge are only theoretical when a rift or rise exists.

SCREW.

A SCREW is a revolving inclined plane.

To Compute Length and Height of Plane of a Screw.

As a screw is an inclined plane wound around a cylinder, length of plane is ascertained by adding square of circumference of screw to square of distance between threads, and taking square root of sum.

The *Pitch* or height of a screw is distance between its consecutive threads.

To Compute Power.

RULE.—Multiply weight or resistance, to be sustained by pitch of threads, and divide product by circumference described by power.

Or, $\frac{W\,p}{c} = P$. *p representing pitch, and c circumference.*

EXAMPLE.—What is power requisite to raise a weight of 8000 lbs. by a screw of 12 ins. circumference and 1 inch pitch?

$8000 \times 1 \div 12 = 666.66$ *lbs.*

To Compute Weight.

RULE.—Multiply power by circumference described by it, and divide product by pitch of threads.

$$\text{Or, } \frac{P\,c}{p} = W.$$

To Compute Pitch.

RULE.—Multiply power by circumference described by it, and divide product by weight.

$$\text{Or, } \frac{P\,c}{W} = p.$$

To Compute Circumference.

RULE.—Multiply weight by pitch, and divide product by power.

$$\text{Or, } \frac{W\,p}{P} = c. \quad \text{Or, } \frac{W\,p}{6.28\,P} = r. \quad r \text{ representing radius.}$$

When Power is applied by a Lever or Wheel, substitute radius of power for circumference.

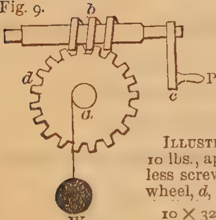
ILLUSTRATION.—If a lever 30 ins. in length was added to circumference of screw in preceding example,

Then, $12 \div 3.416 = 3.819$, and $\frac{3.819}{2} + 30 = 31.9095 =$ *radius of power.*

$$\text{Hence } \frac{8000 \times 1}{r \times 6.28} = 39.92 \text{ lbs.}$$

Compound Screw.

Fig. 9.



When a Lever and Endless Screw or a Series of Wheels are applied to a Screw, as Fig. 9. RULE.—Ascertain result of each application, and take their continued product.

NOTE.—*If there is more than one thread to a screw, pitch must be increased as many times as there are threads.*

ILLUSTRATION.—What weight can be raised with a power of 10 lbs., applied to a crank, c, Fig 9, 32 ins. long, turning an endless screw, b, of 3.5 ins. diameter and 1 inch pitch, applied to a wheel, d, of 20 ins. diameter, upon an axle, a, of 5 ins.?

$\frac{10 \times 32 \times 6.28}{1} = 2009.6 =$ *quotient of product of power and circumference described by it, and pitch, and* $\frac{2009.6 \times 20}{5} = 8038.4$ *lbs. = quotient of power applied to wheel, divided by its axle.*

When a Series of Wheels and Axles are in Connection with each other, Weight is to power, as continued product of radii of wheels is to continued product of radii of axles.

$$W : P :: R\,n : r\,n.$$

Or, $r\,n : R\,n :: P : W$. *n representing continued product of number of wheels or axles.*

ILLUSTRATION.—If a power of 150 lbs. is applied to a crank of 20 ins. radius, turning an endless screw with a pitch of half an inch, geared to a wheel, pinion of which is geared to another wheel, and pinion of second wheel is geared to a third wheel, to axle or barrel of which is suspended a weight; it is required to know what weight can be sustained in that position, diameter of wheels being 18, and pinions and axle 2 ins.

$\frac{150 \times 20 \times 2 \times 3.1416}{.5} = 37\,699.2$ *lbs. = power applied to face of first wheel.*

Diameters of wheels and pinions being 18 and 2, *their radii are 9 and 1.*

$$\text{Hence, } 1 \times 1 \times 1 : 9 \times 9 \times 9 :: 37\,699.2 : 27\,482\,716.8 \text{ lbs.}$$

Differential Screw.

When a hollow screw revolves upon one of less diameter and pitch (as designed by Mr. Hunter), effect is same as that of a single screw, in which the distance between threads is equal to difference of distances between threads of the two screws.

Therefore power, to effect or weight sustained, is as difference between distances of threads of the two screws to circumference described by power.

ILLUSTRATION.—If external screw has 20 threads, and internal one 21 threads in pitch of 1 inch, and power applied describes a circumference of 35 ins., the result or power is as $\frac{1}{21} \backsim \frac{1}{20} = \frac{1}{420}$, or .002 38. Hence $\frac{35}{.00\,238} = 14\,706$.

PULLEY.

PULLEYS are designated as *Fixed* and *Movable*, according as cord is passed over a fixed or a movable pulley. A *movable* pulley is when cord passes through a second pulley or block in suspension; a single movable pulley is termed a *runner;* and a combination of pulleys is termed a *system of pulleys.*

A *Whip* is a single cord over a fixed pulley.

To Compute Power Required to Raise a given Weight.

When Number of Parts of Cord supporting Lower Block are given, and when only one Cord or Rope is used. RULE.—Divide weight to be raised by number of parts of cord supporting lower or movable block.

Or, $W \div n = P$. Or, $n\,P = W$. *n representing number of parts of cord sustaining lower block.*

EXAMPLE.—What power is required to raise 600 lbs. when lower block contains six sheaves?

When Cord is attached to Upper or Fixed Block.

$\frac{600}{6 \times 2} = 50$ *lbs. = weight ÷ number of parts of rope sustaining lower block.*

When Cord is attached to Lower or Movable Block.

$\frac{600}{6 \times 2 + 1} = 46.15$ *lbs. = weight ÷ number of parts of rope sustaining lower block.*

To Compute Weight a given Power will Raise.

When Number of Parts of Cord supporting Lower Block are given. RULE.—Multiply power by number of parts of cord supporting lower block.

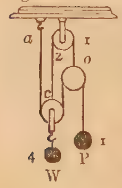
Or, $P\,n = W$.

To Compute Number of Cords necessary to Sustain Lower Block.

When Weight and Power are given. RULE.—Divide weight by power.

Or, $W \div P = n$.

Fig. 10.

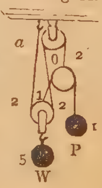


When more than one Cord is used.

In a Spanish Burton, Fig. 10, where ends of one cord, *a* P, are fastened to support and power, and ends of the other, *c o*, to lower and upper blocks, weight is to power as 4 to 1.

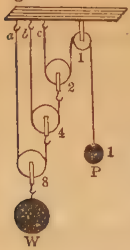
In another, Fig. 11, where there are two cords, *a* and *o*, two movable pulleys, and one fixed pulley, with ends of one rope fastened to support and upper movable pulley, and ends of other fastened to lower block and power, weight is to power as 5 to 1.

Fig. 11.



Compound or Fast and Loose Pulleys.

Fig. 12.



When Cord is attached to Fixed Block, Fig. 12. RULE.—Multiply power by the power of 2, of which the index is number of movable pulleys.

Or, $P\,2^n = W$.

Or, Multiply power successively by 2 for each pulley.

EXAMPLE 1.—What weight will one pound support in a system of three movable pulleys, the cords being connected to a fixed block on Fig. 12. $1 \times 2^3 = 8$ *lbs.*

EXAMPLE 2.—What would a like power support, fixed block being made movable and cord attached thereto?

$1 \times 2^4 - 1 = 15$ *lbs.*

If fixed pulleys were substituted for hooks *a b c*, Fig. 12, power would be increased threefold; hence $1 \times 3^3 = 27$.

In a System of Pulleys, Figs. 13 and 14, *with any Number of Cords, o o, e e, Ends being fastened to Support.*

Fig. 13.

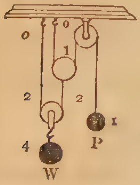
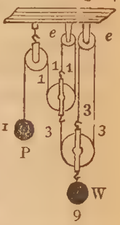


Fig. 14.



$W \div 2^n = P$; $2^n \times P = W$; $\frac{W}{P} = 2^n$. *n representing number of distinct cords.*

ILLUSTRATION.—What weight will a power of 1 lb. sustain in a system of two movable pulleys and two cords?

$1 \times 2 \times 2 = 4$ *lbs.*

When fixed Pulleys, e e, are used in Place of Hooks, to Attach Ends of Rope to Support.—Fig. 14.

$W \div 3^n = P$; $3^n \times P = W$; $W \div P = 3^n$.

ILLUSTRATION.—What weight will a power of 5 lbs. sustain with two movable and three fixed pulleys, and two cords? $5 \times 3 \times 3 = 45$ *lbs.*

When Ends of Cord or Fixed Pulleys are fastened to Weight, as by an Inversion of the last Figures, putting Supports for Weights, and contrariwise.—Figs. 13 and 14.

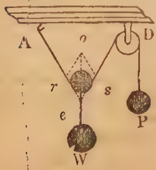
Fig. 13. $\frac{W}{(2^n - 1)} = P$; $(2^n - 1)\,P = W$; $\frac{W}{P} = (2^n - 1)$.

Fig. 14. $\frac{W}{(3^n - 1)} = P$; $(3^n - 1)\,P = W$; $\frac{W}{P} = (3^n - 1)$.

ILLUSTRATION.—What weight will a power of 1 lb. sustain in a system of two movable pulleys and two cords, and one of two movable and two fixed pulleys and two cords? $1 \times 2 \times 2 - 1 = 3$ *lbs.* $1 \times 3 \times 3 - 1 = 8$ *lbs.*

When Cords sustaining Pulleys are not in a Vertical Direction.—Fig. 15.

Fig. 15.



e o, Fig. 15, is vertical line through which weight bears, and from *o* draw *o r*, *o s* parallel to D *e* and A *e*.

Forces acting at *e* are represented by lines *e s*, *e r*, and *e o*; and as tension of every part of cord is same, and equal to power P, sides *o s* and *o r* of parallelogram must be equal, and therefore diagonal *e o* divides the angle *r o s* into two equal portions. Hence the weight will always fall into the position in which the two parts of cord A *e* and *e* D will be equally inclined to vertical line, and it will bear to power same ratio as *e o* to *e s*.

Therefore W : P :: 2 cos. .5 *e* : 1. *e representing angle* A *e* D.

Or, $2\,P \times \cos .5\,e = W$. That is, twice power, multiplied by cosine of half angle of cord, at point of suspension of weight, is equal to weight.

ILLUSTRATION.—What weight will be sustained by a power of 5 lbs , with an oblique movable pulley, Fig. 15, having an angle, A *e* D, of 30°?

$$5 \times 2 \times .96593 = 9.6593 \textit{ lbs.} = \textit{twice power} \times \textit{cos.}\ 15^\circ.$$

When Direction of Cord is Irregular, Weight not resting in Centre of it.

$\frac{P}{W} = \frac{\sin. a}{\sin. (a+b)}$; $\frac{P \sin. (a+b)}{\sin. a} = W$; $\frac{W \sin. a}{\sin. (a+b)} = P$. *a and b representing greater and lesser angles of cord at e.*

METALS.

ALLOYS AND COMPOSITIONS.

Alloy is the proportion of a baser metal mixed with a finer or purer, as copper is mixed with gold, etc.

Amalgam is a compound of Mercury and a metal—a soft alloy.

Compositions of copper contract in admixture, and all *Amalgams* expand.

In manufacture of Alloys and Compositions, the less fusible metals should be melted first.

In Compositions of Brass, as proportion of Zinc is increased, so is malleability decreased.

Tenacity of Brass is impaired by addition of Lead or Tin.

Steel alloyed with one five-hundredth part of Platinum, or Silver, is rendered harder, more malleable, and better adapted for cutting instruments.

Specific gravity of alloys* does not follow the ratios of those of their components; it is sometimes greater and sometimes less than the mean.

Composition for Welding Cast Steel.

Borax, 91 parts; Sal ammoniac, 9 parts. Grind or pound them roughly together; fuse them in a metal-pot over a clear fire, continuing heat until all spume has disappeared from surface. When liquid is clear, pour composition out to cool and concrete, and grind to a fine powder; then it is ready for use.

To use this composition, the steel to be welded should be raised to a bright yellow heat; then dip it in the welding powder, and again raise it to a like heat as before; it is then ready to be submitted to the hammer.

Fusible Compounds.

COMPOUNDS.	Zinc.	Tin.	Lead.	Bismuth.	Cadmium.
Rose's, fusing at 200°	—	25	25	50	—
Fusing at less than 200°	33.3	—	33.3	33.4	—
Newton's, fusing at less than 212°.	—	19	31	50	—
Fusing at 150° to 160°	—	12	25	50	13

Solders.

Solder is an alloy used to make joints between metals, and it mu... more fusible than the metals it is designed to unite, and it is distingui... as *hard* and *soft*, according to the temperature of its fusing.

The addition of a small portion of Bismuth increases its fusibility.

---

* For a table of Alloys, having densities different from a mean of their components, see D. K. (Manual, London, 1877, page 201.

Alloys and Compositions.

	Copper.	Zinc.	Tin.	Nickel.	Lead.	Antimony.	Bismuth.	Aluminum.
Argentan	55	24	—	21	—	—	—	—
Aluminum, brown	95	—	—	—	—	—	—	5
Babbitt's metal*	3.7	—	89	—	—	7.3	—	—
Brass, common	84.3	5.2	10.5	—	—	—	—	—
" "	75	25	—	—	—	—	—	—
" " hard	79.3	6.4	14.3	—	—	—	—	—
" instruments	92.2	—	7.8	—	—	—	—	—
" locomot. bearings	90	1	9	—	—	—	—	—
" Pinchbeck	80	20	—	—	—	—	—	—
" red Tombac	88.8	11.2	—	—	—	—	—	—
" rolled	74.3	22.3	3.4	—	—	—	—	—
" Tutenag	50	31	—	19	—	—	—	—
" very tenacious	88.9	2.8	8.3	—	—	—	—	—
" wheels, valves	90	—	10	—	—	—	—	—
" white	10	80	10	—	—	—	—	—
" "	3	90	—	—	—	7	—	—
" "	7	—	—	—	46	47	—	—
" wire	67	33	—	—	—	—	—	—
" yellow, fine	66	34	—	—	—	—	—	—
Britannia metal	—	—	25	—	—	25	—	—
When fused add	—	—	—	—	—	25	25	—
Bronze, red	87	13	—	—	—	—	—	—
" "	86	11.1	2.9	—	—	—	—	—
" yellow	67.2	31.2	1.6	—	—	—	—	—
" Gun metal, large	90	—	10	—	—	—	—	—
" " small	93	—	7	—	—	—	—	—
" " soft	95	—	5	—	—	—	—	—
" Cymbals	80	—	20	—	—	—	—	—
" Medals	93	—	7	—	—	—	—	—
" Statuary	91.4	5.5	1.4	—	1.7	—	—	—
Chinese silver	58.1	17.2	—	11.6	—	—	Silver. 2	Cobalt of Iron. 11.1
" white copper	40.4	25.4	2.6	31.6	—	—	—	—
Church bells	80	5.6	10.1	—	4.3	—	—	—
" "	69	—	31	—	—	—	—	—
Clocks, Musical bells	87.5	—	12.5	—	—	—	—	—
Clock bells	72	—	26.5	—	—	—	—	Iron. 1.5
German silver	33.3	33.4	—	33.3	—	—	—	—
" "	40.4	25.4	—	31.6	—	—	—	2.6
" " fine	49.5	24	—	24	—	—	—	2.5
Gongs	81.6	—	18.4	—	—	—	—	—
House bells	77	—	23	—	—	—	—	—
Lathe bushes	80	—	20	—	—	—	—	—
Machinery bearings	87.5	—	12.5	—	—	—	—	—
" " hard	77.4	7	15.6	—	—	—	—	—
Metal that expands in cooling	—	—	—	—	75	16.7	Bismuth. 8.3	—
Muntz metal, 10 oz. lead	60	40	—	—	—	—	—	—
Pewter, best	—	—	86	—	—	14	—	—
"	—	—	80	—	20	—	—	—
Sheathing metal	56	45	—	—	—	—	—	—
Speculum "	66	—	22	—	—	—	—	Arsenic. 12
" "	50	21	29	—	—	—	—	—
Telescopic mirrors	66.6	—	33.4	—	—	—	—	—
Temper†	33.4	—	66.6	—	—	—	—	—
Type metal and stereo-	—	—	—	—	75	25	—	—
type plates	—	—	—	—	87.5	12.5	—	—
White metal	7.4	7.4	28.4	—	—	56.8	—	—
" " hard	69.8	25.8	4.4	—	—	—	—	—
Oreide	73	12.3	Magnesia 4.4; Sal-ammoniac 2.5			Cream of tartar 6.5; Quicklime 1.3		

* See page 636 for directions.

† For adding small quantities of copper.

Solders.

	Copper.	Tin.	Lead.	Zinc.	Silver.	Bismuth.	Gold.	Cadmium.	Antimony.
Tin	—	25	75	—	—	—	—	—	—
"	—	58	16	—	—	16	—	—	10
" coarse, melts at 500°	—	33	67	—	—	—	—	—	—
" ordi'y, melts at 360°	—	67	33	—	—	—	—	—	—
Spelter, soft	50	—	—	50	—	—	—	—	—
" hard	65	—	—	35	—	—	—	—	—
Lead	—	33	67	—	—	—	—	—	—
Steel	13	—	—	5	82	—	—	—	—
Brass or Copper	50	—	—	50	—	—	—	—	—
Fine brass	47	—	—	47	6	—	—	—	—
Pewterers' or Soft	—	33	45	—	—	22	—	—	—
" "	—	50	25	—	—	25	—	—	—
Plumbers' pot-metal	—	33	67	—	—	—	—	—	—
" coarse	—	25	75	—	—	—	—	—	—
" fine	—	67	33	—	—	—	—	—	—
" fusible	—	50	50	—	—	—	—	—	—
" very "	—	25	25	—	—	50	—	—	—
Gold	4	—	—	—	7	—	89	—	—
" hard	66	—	—	34	—	—	—	—	—
" soft	—	66	34	—	—	—	—	—	—
Silver, hard	20	—	—	—	80	—	—	—	—
" soft	12	—	—	—	67	—	—	21	—
Pewter	—	40	20	—	—	40	—	—	—
Iron	66	—	—	33	—	—	—	—	1
Copper	53	47	—	—	—	—	—	—	—

A Plastic Metallic Alloy.—See Journal of Franklin Institute, vol. xxxix., page 55, for its composition and manufacture.

Soldering Fluid for use with Soft Solder.

To 2 fluid oz. of Muriatic acid add small pieces of Zinc until bubbles cease to rise. Add .5 a teaspoonful of Sal-ammoniac and two fluid oz. of Water.

By the application of this to Iron or Steel, they may be soldered without their surfaces being previously tinned.

Fluxes for Soldering or Welding.

Iron	Borax.	Zinc	Chloride of zinc.
Tinned iron	Resin.	Lead	Tallow or resin.
Copper and Brass	Sal-ammoniac.	Lead and tin	Resin and sweet oil.

Babbitt's Anti-attrition Metal.

Melt 4 lbs. Copper; add by degrees 12 lbs. best Banca tin, 8 lbs. Regulus of antimony, and 12 lbs. more of Tin. After 4 or 5 lbs. Tin have been added, reduce heat to a dull red, then add remainder of metal as above.

This composition is termed *hardening*; for *lining*, take 1 lb. of this *hardening*, melt with it 2 lbs. Banca tin, which produces the lining metal for use. Hence, the proportions for lining metal are 4 lbs. of copper, 8 of regulus of antimony, and 96 of tin.

Brass.

Brass is an alloy of copper and zinc, in proportions varying with purpose of metal required, its color depending upon the proportions.

It is rendered brittle by continued impacts, more malleable than copper when cold, but is impracticable of being forged, as its zinc melts at a low temperature.

Its fusibility is governed by its proportion of zinc; a small quantity of phosphorus gives it fluidity.

Bronze.

Bronze is an alloy of copper and tin; it is harder, more fusible, and stronger than copper. It is usually known as *Gun-metal.*

Aluminum Bronze contains 90 to 95 per cent. of copper, and 5 to 10 per cent. aluminum.

Phosphor Bronze contains copper and tin and a small proportion of phosphorus. It wears better than bronze.

IRON.

Foreign substances which iron contains modify its essential properties. *Carbon* adds to its hardness, but destroys some of its qualities, and produces Cast Iron or Steel, according to proportion it contains. Thus, .25 per cent. renders it malleable, .5 steel, 1.75 is limit of welding steel, and 2 is lowest limit of cast iron. *Sulphur* renders it fusible, difficult to weld, and brittle when heated, or "*hot short.*" *Phosphorus* renders it "*cold short,*" but may be present in proportion of .002 to .003, without affecting injuriously its tenacity. *Antimony*, *Arsenic*, and *Copper* have same effect as sulphur, the last in a greater degree. *Silicon* renders it hard and brittle. *Manganese*, in proportion of .02, renders it "*cold short,*" and *Vanadium* adds to its ductility.

Cast Iron.

Process of making Cast Iron depends much upon description of fuel used; whether charcoal, coke, bituminous, or anthracite coals. A larger yield from same furnace, and a great economy in fuel, are effected by use of a *hot blast.* The greater heat thus produced causes the iron to combine with a larger percentage of foreign substances.

Cast Iron for purposes requiring great strength should be smelted with a *cold blast.* *Pig-iron*, according to proportion of carbon which it contains, is divided into *Foundry Iron* and *Forge Iron*, latter adapted only to conversion into malleable iron; while former, containing largest proportion of carbon, can be used either for castings or bars.

High temperature in melting injures gun-metal.

There are many varieties of Cast Iron, differing by almost insensible shades; the two principal divisions are *gray* and *white*, so termed from color of their fracture. Their properties are very different.

Gray Iron is softer and less brittle than white; it is in a slight degree malleable and flexible, and is insonorous; it can easily be drilled or turned, and does not resist the file. It has a brilliant fracture, of a gray, or sometimes a bluish-gray, color; color is lighter as grain becomes closer, and its hardness increases. It melts at a lower heat than white, and preserves its fluidity longer. Color of the fluid metal is red, and deeper in proportion as the heat is lower; it does not adhere to the ladle; it fills molds well, contracts less, and contains fewer cavities than white; edges of its castings are sharp, and surfaces smooth and convex. It is used for machinery and ordnance where the pieces are to be bored or fitted. Its tenacity and specific gravity are *diminished* by annealing.

White Iron is very brittle and sonorous; it resists file and chisel, and is susceptible of high polish; surface of its castings is concave; fracture presents a silvery appearance, generally fine grained and compact, sometimes radiating or lamellar. When melted it is white, throws off a great number of sparks, and its qualities are the reverse of those of gray iron; it is therefore unsuitable for machinery purposes. Its tenacity is *increased*, and its specific gravity *diminished*, by annealing.

Mottled Iron is a mixture of white and gray; it has a spotted appearance; flows well, and with few sparks; its castings have a plane surface, with edges slightly rounded. It is suitable for shot, shells, etc. A fine mottled is only kind suitable for castings which require great strength. The kind of mottle will depend much upon volume of the casting. A medium-sized grain, bright gray color, fracture sharp to touch, and a close, compact texture, indicate a good quality of iron. A grain either very large or very small, a dull, earthy aspect, loose texture, dissimilar crystals mixed together, indicate an inferior quality.

Besides these general divisions, the different varieties of pig-iron are more particularly distinguished by numbers, according to their relative hardness.

No. 1.—Fracture dark gray, crystals large and highly lustrous, alike to new surface of lead. It is the softest iron, possessing in highest degree the qualities belonging to gray iron; it has not much strength, but on account of its fluidity when melted, and of its mixing advantageously with scrap iron and with the harder kinds of cast iron, it is of great use to a foundry.

No. 2 is harder, closer grained, and stronger than No. 1; it has a gray color and considerable lustre. It is most suitable for shot and shells.

No. 3 is harder than No. 2. Fracture white, crystals larger and brighter at centre than at the sides; color gray, but inclining to white; has considerable strength, but is principally used for mixing with other kinds of iron and for large castings.

No. 4 or *Bright.*—Fracture light gray, with small crystals and little lustre, and not being sufficiently fusible for castings it is used for conversion to wrought iron.

No. 5. *Mottled.*—Fracture dull white, with gray specks, and a line of white around edge or sides of fracture.

No. 6. *White.*—Fracture white, with little lustre, granulated with radiating crystalline surface. It is hardest and most brittle of all descriptions, and is unfit for use unless mixed with other grades, or for being converted to an inferior wrought iron.

Qualities of these descriptions depend upon proportion of carbon, and upon state in which it exists in the metal; in darker kinds of iron, where proportion is sometimes 7 per cent., it exists partly in state of graphite or plumbago, which makes the iron soft. In white iron the carbon is thoroughly combined with the metal, as in steel.

Cast iron frequently retains a portion of foreign ingredients from the ore, such as earths or oxides of other metals, and sometimes sulphur and phosphorus, which are all injurious to its quality.

Foreign substances, and also a portion of the carbon, are separated by melting iron in contact with air, and soft iron is thus rendered harder and stronger. Effect of remelting varies with nature of the iron and character of ore from which it has been extracted; that from hard ores, such as magnetic oxides, undergoes less alteration than that from hematites, the latter being sometimes changed from No. 1 to *white* by a single remelting in an air furnace.

Color and texture of cast iron depend greatly upon volume of casting and rapidity of its cooling; a small casting, which cools quickly, is almost always *white*, and surface of large castings partakes more of the qualities of white metal than the interior.

All cast iron expands at moment of becoming liquid, and contracts in cooling; gray iron expands more and contracts less than other iron.

Remelting iron improves its tenacity; thus, a mean of 14 cases for two fusions gave, for 1st fusion, a tenacity of 29 284 lbs.; for 2d fusion, 33 790 lbs. For two cases—for first fusion, 15 129 lbs.; for 2d fusion, 35 786 lbs.

Malleable Castings.

Malleable cast iron is made by subjecting a casting to a process of annealing, by enclosing it in a box with hematite iron ore or black oxide of iron, and maintaining it in an equable heat for a period depending upon form and volume of casting.

Wrought Iron.

Wrought iron is made from pig-iron in a *Bloomery Fire* or in a *Puddling Furnace*—generally in latter. Process consists in melting and keeping it exposed to a great heat, constantly stirring the mass, bringing every part of it under action of the flame until it loses its remaining carbon, when it becomes malleable iron. When, however, it is desired to obtain iron of best quality, pig-iron should be *refined*.

Refining.—This operation deprives iron of a considerable portion of its carbon; it is effected in a *Blast Furnace*, where iron is melted by means of charcoal or coke, and exposed for some time to action of a great heat; the metal is then run into a cast-iron mold, by which it is formed into a large broad plate. As soon as surface of plate is chilled, cold water is poured on to render it brittle.

A *Bloomery* resembles a large forge fire, where charcoal and a strong blast are used; and the *refined* metal or pig-iron, after being broken into pieces of proper size, is placed before the blast, directly in contact with charcoal; as the metal fuses, it falls into a cavity left for that purpose below the blast, where the "bloomer" works it into the shape of a *ball*, which he places again before the blast, with fresh charcoal; this operation is generally again repeated, when ball is ready for the "shingler."

Shingling is performed in a strong *squeezer* or under a trip-hammer. Its object is to press out as perfectly as practicable the liquid cinder which a ball contains; it also forms a ball into shape for the puddle rolls. A heavy hammer, weighing from 6 to 7 tons, effects this object most thoroughly, but not so cheaply as the squeezer. A ball receives from 15 to 20 blows of a hammer, being turned from time to time as required: it is now termed a *Bloom*, and is ready to be rolled or hammered; or a ball is passed once through the squeezer, and is still hot enough to be passed through the puddle rolls.

A *Puddling Furnace* is a reverberatory furnace, where flame of bituminous coal is brought to act directly upon the melted metal. The "puddler" then stirs it, exposing each portion in turn to action of flame, and continues this as long as he is able to work it. When it has lost its fluidity, he forms it into balls, weighing from 80 to 100 lbs., which are then passed to the "shingler."

Puddle Rolls. — By passing through different grooves in these rolls, a bloom is reduced to a *rough bar* from 3 to 4 feet in length, its term conveying an idea of its condition, which is rough and imperfect.

Piling.—To prepare rough bars for this operation, they are cut, by a pair of *shears*, into such lengths as are best adapted to the volume of finished bar required; the sheared bars are then piled one over the other, according to volume required, when pile is ready for *balling*.

Balling.—This operation is performed in balling furnace, which is similar to puddling furnace, except that its bottom or hearth is made up, from time to time, with sand; it is used to give a welding heat to piles to prepare them for rolling.

Finishing Rolls.—The *balls* are passed successively between rollers of various forms and dimensions, according to shape of finished bar required.

Quality of iron depends upon description of pig-iron used, skill of the "puddler," and absence of deleterious substances in the furnace.

Strongest cast irons do not produce strongest malleable iron.

For many purposes, such as sheets for tinning, best boiler-plates, and bars for converting into steel, *charcoal iron* is used exclusively; and, generally, this kind of iron is to be relied upon, for strength and toughness, with greater confidence than any other, though iron of a superior quality is made from pigs made with other fuel, and with a hot blast. Iron for gun-barrels has been lately made from anthracite hot-blast pigs.

Iron is improved in quality by judicious working, reheating, hammering, or rolling: other things being equal, best iron is that which has been wrought the most.

Best quality of iron has greatest elasticity.

Tests.—It will not blacken if exposed to nitric acid. Long silky fibres in a fracture denote a soft and strong metal: short black fibres denote a badly refined metal, and a fine grain denotes hardness and condition known as "cold short." Coarse grain with bright and crystallized fracture, with discolored spots, also denotes "cold short" and brittle metal, working easily and welding well. Cracks upon edges of a bar, etc., indicate "hot short." Good iron heats readily, is worked easily, and throws off but few sparks.

A high breaking strain may not be conclusive as to quality, as it may be due to a hard, elastic metal, or a low one may be due to great softness.

When iron is fractured suddenly, a crystalline surface is produced, and when gradually, a fibrous one. Breaking strain of iron is increased by heating it and suddenly cooling it in water. Iron exposed to a welding or white heat and not reduced by hammering or rolling is weakened.

Specific gravity of iron is a good indication of its quality, as it indicates very correctly its relative degree of strength.

LEAD.

Sheet Lead is either *Cast* or *Milled*, the former in sheets 16 to 18 feet in length and 6 feet in width; the latter is rolled, is thinner than the former, is more uniform in its thickness, and is made into sheets 25 to 35 feet in length, and from 6 to 7.5 feet in width.

Soft or Rain Water, when aerated, Silt of rivers, Vegetable matter, Acids, Mortar, and Vitiated Air will oxidize lead. The waters which act with greatest effect on it are the purest and most highly oxygenated, also nitrites, nitrates, and chlorides, and those which act with least effect are such as contain carbonate and phosphate of lime.

Coating of Pipes, except with substances insoluble in water, as Bitumen and Sulphide of lead, is objectionable.

Lead-encased Pipes.—An inner pipe of tin is encased in one of lead.

STEEL.

Steel is a compound of Iron and Carbon, in which proportion of latter is from 1 to 5 per cent., and even less in some descriptions. It is distinguished from iron by its fine grain, and by action of diluted nitric acid, which leaves a black spot upon it.

There are many varieties of steel, principal of which are:

Natural Steel, obtained by reducing rich and pure descriptions of iron ore with charcoal, and refining cast iron, so as to deprive it of a sufficient portion of carbon to bring it to a malleable state. It is used for files and other tools.

Indian Steel, termed *Wootz*, is said to be a natural steel, containing a small portion of other metals.

Blistered Steel, or *Steel of Cementation*, is prepared by direct combination of iron and carbon. For this purpose, iron in bars is put in layers, alternating with powdered charcoal, in a close furnace, and exposed for 7 or 8 days to a high temperature, and then put to cool for a like period. The bars, on being taken out, are covered with blisters, have acquired a brittle quality, and exhibit in fracture a uniform crystalline appearance. The degree of carbonization is varied according to purposes for which the steel is intended, and the very best qualities of iron are used for the finest kinds of steel.

Tilted Steel is made from blistered steel moderately heated, and subjected to action of a tilt hammer, by which means its tenacity and density are increased.

Shear Steel is made from blistered or natural steel, refined by piling thin bars into fagots, which are brought to a welding heat in a reverberatory furnace, and hammered or rolled again into bars; this operation is repeated several times to produce finest kinds of shear steel, which are distinguished by the terms of *Half shear*, *Single shear*, and *Double shear*, or steel of 1, 2, or 3 *marks*, etc., according to number of times it has been piled.

Spring Steel is blister steel heated to an orange red color and rolled or hammered.

Cast or *Crucible Steel* is made by breaking blistered steel into small pieces and melting it in close crucibles, from which it is poured into iron molds; *ingot* is then reduced to a bar by hammering or rolling. Cast steel is best kind of steel, and best adapted for most purposes; it is known by a very fine, even, and close grain, and a silvery, homogeneous fracture; it is very brittle, and acquires extreme hardness, but is difficult to weld without use of a flux. Other kinds of steel have a similar appearance to cast steel, but grain is coarser and less homogeneous; they are softer and less brittle, and weld more readily. A fibrous or lamellar appearance in fracture indicates an imperfect steel. A material of great toughness and elasticity, as well as hardness, is made by forging together steel and iron, forming the celebrated *Damasked Steel*, which is used for sword-blades, springs, etc.; damask appearance of which is produced by a diluted acid, which gives a black tint to the steel, while the iron remains white.

With cast steel, breaking strength is greater across fibres of rolling than with them.

Heath's Process is an improvement on this method, and consists in adding to molten metal a small quantity of carburet of manganese.

Heaton's Process consists in adding nitrate of soda to molten pig-iron, in order to remove carbon and silica.

Mushet's Process.—Malleable iron is melted in crucibles with oxide of manganese and charcoal.

Puddled Steel is produced by arresting the puddling in the manufacture of the wrought iron before all the carbon has been removed, the small amount of carbon remaining, .3 to 1 per cent., being sufficient to make an inferior steel.

Mild Steel contains from .2 to .5 per cent. of carbon; when more is present it is termed *Hard Steel.*

Bessemer Steel is made direct from pig-iron. The carbon is first removed, in order to obtain pure wrought iron, and to this is added the exact quantity of carbon required for the steel. The pig should be free from sulphur and phosphorus. It is melted in a blast or cupola, and run into a *converter* (a pear-shaped iron vessel suspended on hollow trunnions and lined with fire-brick or clay), where it is subjected to an air blast for a period of 20 minutes, in order to dispel the carbon, after which from 5 to 10 per cent. of spiegeleisen is added.

The blast is then resumed for a short period, to incorporate the two metals, when the steel is run off into molds. The moment at which all the carbon has been removed is indicated by color of the flame at mouth of converter. The ingots, when thus produced, contain air holes, and it becomes necessary to heat them and render them solid under a hammer.

Siemen's Process.—Pig-iron is fused upon open hearth of a regenerative furnace, and when raised to a steel-melting temperature, rich and pure ore and limestone are added gradually, whereby a reaction is established between the oxygen of the ferrous oxide and the carbon and silicon in the metal. The silicon is thus converted into silicic acid, which with the lime forms a fusible slag, and the carbon, combining with oxygen, escapes as carbonic acid, and induces a powerful ebullition.

Modification of this process.—The ore is treated in a separate rotatory furnace with carbonaceous material, and converted into balls of malleable iron, which are transferred from the rotatory to the bath of the steel-melting furnace.

This process is adapted to the production of steel of a very high quality, because the sulphur and phosphorus of the ore are separated from the metal in the rotatory furnace.

Siemen's-Martin Process.—Scrap-iron or steel is gradually added in a highly heated condition to a bath of about .25 its weight, of highly heated pig, and melted. Samples are occasionally taken from the bath, in order to ascertain the percentage of carbon remaining in the metal, and ore is added in small quantities, in order to reduce the carbon to about .1 per cent.

At this stage of the process, siliceous iron, spiegeleisen, or ferro-manganese is added in such proportions as are necessary to produce steel of the required degree of hardness. The metal is then tapped into a ladle.

Landore-Siemen's Steel is a variety of steel made by the *Modification of Siemen's Process.* Its great value is due to its extreme ductility, and its having nearly like strength in both directions of its plates.

Whitworth's Compressed Steel is molten steel subjected to a pressure of about 6 tons per square inch, by which all its cavities are dispelled, and it is compressed to about .875 of its original volume, its density and strength being proportionately increased.

Chrome and Tungsten Steel are made by adding a small percentage of Chromium or Tungsten to crucible steel, the result producing a steel of great hardness and tenacity, suitable for tools, such as drills, etc.

Homogeneous Steel is a variety of cast steel containing .25 per cent. of carbon.

Remarks on Manufacture of Steel, and Mode of Working it.

(*D. Chernoff*, 1868).

Steel, when cast and allowed to cool quietly, assumes a crystalline structure. Higher temperature to which it is heated, softer it becomes, and greater is liberty its particles possess to group themselves into crystals.

Steel, however hard it may be, will not harden if heated to a temperature lower than what may be distinguished as dark cherry-red, *a*, however quickly it is cooled; on contrary, it will become sensibly softer, and more easily worked with a file.

Steel, heated to a temperature lower than red, but not sparkling, *b*, does not change its structure whether cooled quickly or slowly. When temperature has reached *b*, substance of steel quickly passes from granular or crystalline condition to amorphous, or wax-like structure, which it retains up to its melting-point, *c*.

Points *a*, *b*, and *c* have no permanent place in scale of temperature, but their positions vary with quality of steel; in pure steel, they depend directly on quantity of constituent carbon. Harder the steel, lower the temperatures. Tints above specified have reference only to hard and medium qualities of steel; in very soft kinds of steel, nearly approaching to wrought iron, points *a* and *b* range very high, and in wrought iron point *b* rises to a white heat.

Assumption of the crystalline structure takes place entirely in cooling, between temperatures *c* and *b*; when temperature sinks below *b* there is no change of structure. For successful forging, therefore, heated ingot, after it is taken out of furnace, must be forged as quickly as practicable, so as not to leave any spot untouched by hammer, where the steel might crystallize quietly, as formation of crystals should be hindered, and the steel should be kept in an amorphous condition until temperature sinks below point *b*.

Below this temperature, if piece is cooled in quiet, mass will no longer be disposed to crystallize, but will possess great tenacity and homogeneousness of structure.

When steel is forged at temperatures lower than *b*, its crystals or grains, being driven against each other, change their shapes, becoming elongated in one direction, and contracted in another; while density and tensile strength are considerably increased. But available hammer-power is only sufficient for treatment of small steel forgings; and object of preventing coarse crystalline structure in large forgings is more easily and more certainly effected, if, after having given forging desired shape, its structure be altered to an homogeneous amorphous condition by heating it to a temperature somewhat higher than *b*, and the condition be fixed by rapid cooling to a temperature lower than *b*, the piece should then be allowed to finish cooling gradually, so as to prevent, as far as practicable, internal strains due to sudden and unequal contraction.

Alloys of steel with *Silver*, *Platinum*, *Rhodium*, and *Aluminum* have been made with a view to imitating Damascus steel, Wootz, etc., and improving fabrication of some finer kinds of surgical and other instruments.

Properties of Steel.—After being tempered it is not easily broken; it welds readily; does not crack or split; bears a very high heat, and preserves the capability of hardening after repeated working.

Hardening and Tempering.—Upon these operations the quality of manufactured steel in a great measure depends.

Hardening is effected by heating steel to a cherry-red, or until scales of oxide are loosened on surface, and plunging it into a cooling liquid; degree of hardness depends upon heat and rapidity of cooling. Steel is thus rendered so hard as to resist files, and it becomes at same time extremely brittle. Degree of heat, and temperature and nature of cooling medium, must be chosen with reference to quality of steel and purpose for which it is intended. Cold water gives a greater hardness than oils or like substances, sand, wet-iron scales, or cinders, but an inferior degree of hardness to that given by acids. Oil, tallow, etc., prevent cracks caused by too rapid cooling. Lower the heat at which steel becomes hard, the better.

Tempering.—Steel in its hardest state being too brittle for most purposes, the requisite strength and elasticity are obtained by tempering—or "*letting down the temper*"—which is performed by heating hardened steel to a certain degree and cooling it quickly. Requisite heat is usually ascertained by color which surface of the steel assumes from film of oxide thus formed. Degrees of heat to which these several colors correspond are as follows:

At 430°, very faint yellow.. At 450°, pale straw color....	Suitable for hard instruments; as hammer-faces, drills, lancets, razors, etc.
At 470°, full yellow........ At 490°, brown color.......	For instruments requiring hard edges without elasticity; as shears, scissors, turning tools, penknives, etc.
At 510°, brown, with purple spots.................... At 538°, purple............	For tools for cutting wood and soft metals; such as plane-irons, saws, knives, etc.
At 550°, dark blue......... At 560°, full blue...........	For tools requiring strong edges without extreme hardness; as cold chisels, axes, cutlery, etc.
At 600°, grayish blue, verging on black.............	For spring temper, which will bend before breaking; as saws, sword-blades, etc.

If steel is heated to a higher temperature than this, effect of the hardening process is destroyed.

A high breaking strain may not be conclusive as to quality, as it may be due to a hard, elastic metal, or a low one may be due to great softness.

Case-hardening.

This operation consists in converting surface of wrought iron into steel, by cementation, for purpose of adapting it to receive a polish or to bear friction, etc.; it is effected by heating iron to a cherry-red, in a close vessel, in contact with carbonaceous materials, and then plunging it into cold water. Bones, leather, hoofs, and horns of animals are generally used for this purpose, after having been burned or roasted so that they can be pulverized. Soot is also frequently used.

The operation reduces strength of the iron.

TIN.

Tin is more readily fused than any other metal, and oxidizes very slowly.

Its purity is tested by its extreme brittleness at high temperature.

Tin plate is iron plate coated with tin.

Block Tin is tin plate with an additional coating of tin.

ZINC.

Zinc, if pure, is malleable at 220°: at higher temperatures, such as 400°, it becomes brittle. It is readily acted upon by moist air, and when a film of oxide is formed, it protects the surface from further action. When, however, the air is acid, as from the sea or large towns, it is readily oxidized to destruction.

Iron, Copper, Lead, and Soot are very destructive of it, in consequence of the voltaic action generated, and it should not be in contact with calcareous water or acid woods.

The best quality, as that known as "Vielle Montagne," is composed of zinc .995, iron .004, and lead .001. Its expansion and contraction by differences of temperature is in excess of that of any other metal.

STRENGTH OF MODELS.

The forces to which Models are subjected are,

1. To draw them asunder by *tensile* stress. 2. To break them by *transverse* stress. 3. To crush them by *compression*.

The stress upon side of a model is to corresponding side of a structure as cube of its corresponding magnitude. Thus, if a structure is six times greater than its model, the stress upon it is as 6^3 to $1 = 216$ to 1: but resistance of rupture increases only as squares of the corresponding magnitudes, or as 6^2 to $1 = 36$ to 1. A structure, therefore, will bear as much less resistance than its model as its side is greater.

To Compute Dimensions of a Beam, etc., which a Structure can bear.

Rule.—Divide greatest weight which the beam, etc. (including its weight), in the model can bear, by the greatest weight which the structure is required to bear (including its weight), and quotient, multiplied by length of beam, etc., in model, will give length of beam, etc., in structure.

Example.—A beam in a model 7 inches in length is capable of bearing a weight of 26 lbs., but it is required to sustain only a weight or stress of 4 lbs.; what is the greatest length that a corresponding beam can be made in the structure?

$26 \div 4 = 6.5$, and $6.5 \times 7 = 45.5$ *ins.*

Resistance in a model to crushing increases directly as its dimensions; but as stress increases as cubes of dimensions, a model is stronger than the structure, inversely as the squares of their comparative magnitudes.

Hence, greatest magnitude of a structure is ascertained by taking square root of quotient, as obtained by preceding rule, instead of quotient itself.

Example.—If greatest weight which a column in a model can sustain is 26 lbs., and it is required to bear only 4 lbs.; height of column being 18 ins., what should be height of it in structure?

$\sqrt{\left(\frac{26}{4}\right)} = \sqrt{6.5} = 2.55$, and $2.55 \times 18 = 45.9$ *ins., height of column in structure.*

If, when length or height and breadth are retained, and it is required to give to the beam, etc., such a thickness or depth that it will not break in consequence of its increased dimensions,

Then $\sqrt{\left(\frac{26}{4}\right)} = \sqrt{6.5} = 2.55$, which, $\times$ square of relative size of model $=$ *thickness required.*

To Compute Resistance of a Bridge from a Model.

$$n^2 W - \left[\frac{n^2}{2}(n-1)\,w\right] = \textit{load bridge will bear in its centre.}$$

Example.—If length of the platform of a model between centres of its repose upon the piers is 12 feet, its weight 30 lbs., and the weight it will just sustain at its centre 350 lbs., the comparative magnitudes of model and bridge as 20, and actual length of bridge 240 feet; what weight will bridge sustain?

$$20^2 \times 350 - \left[\frac{400}{2} \times (20-1) \times 30\right] = 140\,000 - 3800 \times 30 = 26\,000 \textit{ lbs.}$$

MOTION OF BODIES IN FLUIDS.

If a body move through a fluid at rest, or fluid move against body at rest, resistance of fluid against body is as square of velocity and density of fluid; that is, $R = d\,v^2$. For resistance is as quantity of matter or particles struck, and velocity with which they are struck. But quantity or number of particles struck in any time are as velocity and density of fluid; therefore, resistance of a fluid is as density and square of velocity.

$\frac{v^2}{2\,g} = h$, and $\frac{a\,d\,v^2}{2\,g} = R$. *h representing height due to velocity, d density of fluid, and* R *resistance or motive force.*

Resistance to a plane is as plane is greater or less, and therefore resistance to a plane is as its area, density of medium, and square of velocity; that is, $R = a\,d\,v^2$.

Motion is not perpendicular, but oblique, to plane or to face of body in any angle, sine of which is s to radius 1; then resistance to plane, or force of fluid against plane, in direction of motion, will be diminished in triplicate ratio of radius to sine of angle of inclination, or in ratio of 1 to s^3.

Hence, $\frac{a\,d\,v^2\,s^3}{2\,g} = R$, and $\frac{a\,d\,v^2\,s^3}{2\,g\,w} = F$. *w representing weight of body, and* F *retarding force.*

Progression of a solid floating body, as a boat in a channel of still water, gives rise to a displacement of water surface, which advances with an undulation in direction of body, and this undulation is termed *Wave of Displacement.*

Resistance of a fluid to progression of a floating body increases as velocity of body attains velocity of wave of displacement, and it is greatest when the two velocities are equal.

In the motion of elastic fluids, it appears from experiments that oblique action produces nearly same effect as in motion of water, in the passage of curvatures, apertures, etc.

Resistance to an Area of One Sq. Foot moving through Water, or Contrariwise.

Angle of Surface with Plane of Current.	*Pressure per Sq. Foot for following Velocities per Foot per Minute.* 120	240	480	900	Angle of Surface with Plane of Current.	*Pressure per Sq. Foot for following Velocities per Foot per Minute.* 120	240	480	900
°	Lbs.	Lbs.	Lbs.	Lbs.	°	Lbs.	Lbs.	Lbs.	Lbs.
6	.09	.359	1.435	5.046	45	2.66	10.639	42.557	149.614
8	.133	.53	2.122	7.459	50	2.995	11.981	47.923	168.48
9	.156	.624	2.496	8.775	55	3.249	12.975	51.979	182.739
10	.179	.718	2.87	10.091	60	3.455	13.822	55.286	194.366
15	.355	1.42	5.678	19.963	65	3.607	14.43	57.72	202.922
20	.608	2.434	9.734	34.222	70	3.728	14.914	59.654	209.722
25	.94	3.76	15.038	52.869	75	3.81	15.241	60.065	214.329
30	1.353	5.413	21.653	76.123	80	3.857	15.428	61.714	216.926
35	1.798	7.192	28.766	101.132	85	3.892	15.569	62.275	218.936
40	2.258	9.032	36.13	127.018	90	3.9	15.6	62.4	219.375

Resistance to a plane, from a fluid acting in a direction perpendicular to its face, is equal to weight of a column of fluid, base of which is plane and altitude equal to that which is due to velocity of the motion, or through which a heavy body must fall to acquire that velocity.

Resistance to a plane running through a fluid is same as force of fluid in motion with same velocity on plane at rest. But force of fluid in motion is equal to weight or pressure which generates that motion, and this is equal to weight or pressure of a column of fluid, base of which is area of the plane, and its altitude that which is due to velocity.

Illustration.—If a plane 1 foot square be moved through water at rate of 32.166 feet per second, then $\frac{32.166^2}{64.333} = 16.083$, space a body would require to fall to acquire a velocity of 32.166 feet per second; therefore 1×62.5 (weight of a cube foot of water) $\times \frac{32.166^2}{64.333} = 1005$ *lbs.* = *resistance of plane.*

Resistance of different Figures at different Velocities in Air.

Velocity per Second.	Cone. Vertex.	Cone. Base.	Sphere.	Cylinder.	Hemisphere. Round.	Velocity per Second.	Cone. Vertex.	Cone. Base.	Sphere.	Cylinder.	Hemisphere. Round.
Feet.	Oz.	Oz.	Oz.	Oz.	Oz.	Feet.	Oz.	Oz.	Oz.	Oz.	Oz.
3	.028	.064	.027	.05	.02	12	.376	.85	.37	.826	.347
4	.048	.109	.047	.09	.039	14	.512	1.166	.505	1.145	.478
5	.071	.162	.068	.143	.063	15	.589	1.346	.581	1.327	.552
8	.168	.382	.162	.36	.16	16	.673	1.546	.663	1.526	.634
9	.211	.478	.205	.456	.199	18	.858	2.002	.848	1.986	.818
10	.26	.587	.255	.565	.242	20	1.069	2.54	1.057	2.528	1.033

Diameter of all the figures was 6.375 ins., and altitude of the cone 6.625 ins. Angle of side of cone and its axis is, consequently, 25° 42′ nearly.

From the above, several practical inferences may be drawn.

1. That resistance is nearly as surface, increasing but a very little above that proportion in greater surfaces.

2. Resistance to same surface is nearly as square of velocity, but gradually increasing more and more above that proportion as velocity increases.

3. When after parts of bodies are of different forms, resistances are different, though fore parts be alike.

4. The resistance on base* of a cone is to that on vertex nearly as 2.3 to 1. And in same ratio is radius to sine of angle of inclination of side of cone to its path or axis. So that, in this instance, resistance is directly as sine of angle of incidence, transverse section being same, instead of square of sine.

Resistance on base of a hemisphere is to that on convex side nearly as 2.4 to 1, instead of 2 to 1, as theory assigns the proportion.

Sphere.—Resistance to a sphere moving through a fluid is but half resistance to its great circle, or to end of a cylinder of same diameter, moving with an equal velocity, being half of that of a cylinder of same diameter.

$\sqrt{2\,g \times \frac{4}{3}\,d \times \frac{N-n}{n}} = V.$ *d representing diameter of sphere, and* N *and* n *specific gravities of sphere and resisting fluid.*

$\frac{N}{n} \times \frac{4}{3}\,d = S.$ S *representing space through which a sphere passes while acquiring its maximum velocity, in falling through a resisting fluid.*

ILLUSTRATION.—If a ball of lead 1 inch in diameter, specific gravity 11.33, be set free in water, specific gravity 1, what is greatest velocity it will attain in descending, and what space will it describe in attaining this velocity?

$$g = 32.166, \quad d = \frac{1}{12}\text{ foot}, \quad N = 11.33, \text{ and } n = 1.$$

Then $\sqrt{2 \times 32.166 \times \frac{4}{3} \text{ of } \frac{1}{12} \times \frac{11.33-1}{1}} = \sqrt{7.148 \times 10.33} = 8.593$ *feet per sec.*

Hence, $\frac{11.33}{1} \times \frac{4}{3}$ of $\frac{1}{12} = 1.259$ *feet.* $\frac{3\,n\,v^2}{8\,g\,N\,d} = f =$ *retardive force* $= \frac{v^2}{2\,g\,s}$.

Cylinder. $\frac{n\,a\,v^2}{2\,g} = R$, and $\frac{n\,a\,v^2}{2\,g\,w} = f$. *a representing area or* $p\,r^2$, *and* w *weight of body.*

ILLUSTRATION.—Assume $a = 32$ sq. feet, $v = 10$ feet per second, and $n = .0012$.

Then $\frac{.0012 \times 32 \times 10^2}{64.33} = .06$ *of a cube foot of water* $= .06$ of $62.5 = 3.75$ *lbs.*

Conical Surface. $\frac{n\,a\,v^2\,s^3}{2\,g} = R$, also $\frac{n\,p\,d^2\,v^2\,s^2}{8\,g} = R$, and $\frac{n\,p\,d^2\,v^2\,s^2}{8\,g\,w} = f$. *s representing sine of inclination, and a convex surface of cone.*

Curved End as a Sphere or Hemispherical End. $\frac{p\,n\,v^2\,d^2}{16\,g} = R$, and **Circle** .5 *of spherical end.*

In general, when n is to water as a standard, result is in cube feet of water, if a is in sq. feet; and in cube ins. of water, if a is in sq. ins., v in ins., and g in ins.

If n is given in lbs. in a cube foot, a is in sq. feet, v and g are in feet, result is in lbs.

To Compute Altitude of a Column of Air, Pressure of which shall be equal to Resistance of a Body moving through it, with any Velocity.

$\frac{5}{6} \times \frac{r}{a} = x =$ *altitude in feet.* $a\,x =$ *volume of column in feet, and* $\frac{6}{5}\,a\,x =$ *weight in ounces.* *a representing area of section of body, similar to any in table, perpendicular to direction of motion, r resistance to velocity in table, and x altitude sought of a column of air, base of which is a, and pressure r.*

---

* This is a refutation of the popular assertion that a taper spar can be towed in water easiest when the base is foremost.

When $a=\frac{2}{9}$ of a foot, as in all figures in table, x becomes $\frac{15}{4}$ r when $r=$ *resistance in table to similar body.*

Illustration.—Assume convex face of hemisphere resistance = .634 oz. at a velocity of 16 feet per second.

Then $r=.634$, and $x=\frac{15}{4}r=2.3775$ *feet = altitude of column of air, pressure of which = resistance to a spherical surface at a velocity of* 16 *feet.*

To Compute when Pressure of Air in rear of a Projectile is Inferior to Pressure due to its Velocity.

Assume height of barometer = 2.5 feet, and weight of atmosphere = 14.7 lbs.

Weight of cube inch of mercury $=\frac{14.7}{30}=.49$ lbs., and weight of cube inch of air = .000 043 57 lbs.; hence, $.49 \div .000\,043\,57 = 11\,246$, which $\times$ 2.5 feet = 28 115 feet.

Then $\sqrt{16.08} : \sqrt{28\,115} :: 32.16 : x$, and $x=\frac{32.16\times\sqrt{28\,115}}{\sqrt{16}}=1341.6$ *feet.*

To Compute Velocity with which a Plane Surface must be projected to generate a Resistance just equal to Pressure of Atmosphere upon it.

By table, resistance on a circle with an area of .222 sq. foot $(2\div 9)=.051$ oz., at a velocity of 3 feet per second. Hence $3^2 : 1^2 :: .051 : .0056$ *oz. at a velocity of* 1 *foot,* and $1\times144\times14.7\times16\times2\div9=7526.4$ *oz.* Hence, $\sqrt{.0056} : \sqrt{7526.4} :: 1 : 1160$ *feet.*

To Compute Velocity lost by a Projectile.

If a body is projected with any velocity in a medium of same density with itself, and it describes a space = 3 of its diameters,

$$\text{Then } x=3\,d, \text{ and } b=\frac{3\,n}{8\,\mathrm{N}\,d}=\frac{3}{8\,d}.$$

Hence, $b\,x=\frac{9}{8}$, and $\frac{c^{b\,x}-1}{c^{b\,x}}=\frac{2.08}{3.08}=$ *velocity lost nearly .66 of projectile velocity.*

$c=$ base of Nap. system of log.; hence $c^{b\,x}=$ number corresponding to Nap. log. $b\,x$. Hence, if $b\,x\times.4343$, result = com. log. of $c^{b\,x}$.

$b\,x=\frac{9}{8}=1.125$, which $\times .4343=.488\,587\,5$, and number to this com. log. = 3.0803.

Hence, velocity lost $=\frac{3.0803-1}{3.0803}=\frac{2.08}{3.08}$.

Illustration.—If an iron ball 2 ins. diam. were projected with a velocity of 1200 feet per second, what would be velocity lost after moving through 500 feet of space?

$$d=\frac{2}{12}=\frac{1}{6},\quad x=500,\quad \mathrm{N}=7\tfrac{1}{8},\quad \text{and } n=.0012.$$

Hence, $b\,x=\frac{3\,n\,x}{8\,\mathrm{N}\,d}=\frac{3\times12\times500\times3\times6}{8\times21\times10\,000}=\frac{81}{440}$, and $v=\frac{1200}{c^{\frac{81}{440}}}=998$ *feet per second,* having lost 202 feet, or nearly $\frac{1}{6}$ of its initial velocity.

$\frac{12}{10\,000}=.0012$, $\frac{3}{22}$ and $\frac{6}{\cdot}=\frac{22}{3}$ and $\frac{1}{6}$ inverted, because N and n are in denominator.

To Compute Time and Velocity.

$$\frac{1}{b}\left(\frac{1}{v}-\frac{1}{a}\right)=time,\quad \frac{3\,n}{8\,\mathrm{N}\,d}=b,\ \text{and}\ \frac{a}{c^{b\,x}}=v.$$

Illustration.—If an iron ball 2 ins. in diameter were projected in air with a velocity of 1200 feet per second, in what time would it pass over 1500 feet, and what its velocity at end of that time?

$b=\frac{3\times12\times3\times6}{8\times22\times10\,000}=\frac{1}{2716}$, and $b\,x=\frac{1500}{2716}$; hence $\frac{1}{b}=\frac{2716}{1}$; $\frac{1}{a}=\frac{1}{1200}$, and $\frac{1}{v}=\frac{c^{b\,x}}{a}=\frac{1.7372}{1200}=\frac{1}{690}$ *nearly.* $\therefore v=690$ and $t=2716\times\left(\frac{1}{690}-\frac{1}{1200}\right)=1.67$ *sec.*

NAVAL ARCHITECTURE.

Results of Experiments upon Form of Vessels.

(*Wm. Bland.*)

Cubical Models. *Head Resistance.*—Increases directly with area of its surface. *Weight Resistance.*—Increases directly as weight.

Vessels' Models. *Lateral Resistance.*—About one twelfth of length of body immersed, varying with speed.

Order of Superiority of Amidship Section.—Rectangle, Semicircular, Ellipse, and Triangle.

Centre of lateral resistance moves forward as model progresses.

Centre of gravity has no influence upon centre of lateral resistance.

Relative Speeds.

Length.—Increased length gives increased speed or less resistance.

Depth of Flotation.—Less depth of immersion of a vessel, less the resistance.

Amidship Section.—Curved sections give higher speed than angled.

Sides.—Slight horizontal curves present less resistance than right lines. Curved sides with one fourth more beam give equal speeds with straight sides of less beam. *Keel.*—Length of keel has greater effect than depth. *Stern.*—Parallel-sided after bodies give greater speed than taper-sided.

Form of Bow.	Order of Speed.
Isosceles triangle, sides slightly convex	1
" " " right lines	2
" " " slightly concave at entrance and running out convex	3

Spherical equilateral triangle compared to *Equilateral triangle*, speed is as 11 to 12. *Equilateral triangle*, with its isosceles sides bevelled off at an angle of 45°, compared to bow with vertical sides, is as 5 to 4.

When bow has an angle of 14° with plane of keel, compared with one of 7°, its speed is greater.

Bodies Inclined Upwards from Amidship Section.

1. Model with bow inclined from ⊠, has less resistance than model without any inclination.

2. Model with stern inclined from ⊠, has less resistance than model without any inclination.

Model 1 had less resistance than model 2. Model with both bow and stern inclined from ⊠, has less resistance than either 1 or 2.

Stability.

Results of Experiments upon Stability of Rectangular Blocks of Wood of Uniform Length and Depth, but of Different Breadths. (*Wm. Bland.*)

Length 15, *Depth* 2, *and Depression* 1 *inch.*

Width.	Weight.	Ratio of Stability. As Observed.	With like Weights.	By Squares of Breadth.	By Cubes of Breadth.
Ins.	Oz.				
3	24	1	1	1	1
4.5	35	2.5	2.4	2.25	3.375
6	45	7	3.7	4	8
7	55	11	4.8	6.25	15.625

Hence it appears that rectangular and homogeneous bodies of a uniform length, depth, weight, and immersion in a fluid, but of different breadths, have stability for uniform depressions at their sides (heeling) nearly *as squares of their breadth*; and that, when weights are directly as their breadths, their stability under like circumstances is nearly *as cubes of their breadth.*

With equal lengths, ratio of stability is at its limit of rapid increase when width is one third of length, being nearly in cube ratio; afterwards it approaches to arithmetic ratio.

Results of Experiments upon Stability and Speed of Models having Amidship Sections of different Forms, but Uniform Length, Breadth, and Weights. (*W. Bland.*)

Immersion different, depending upon Form of Section.

Form of Immersed Section.	Stability.	Speed.
Half-depth triangle, other half rectangle	12	4
Rectangle	14	3
Right-angled triangle *	7	3
Semicircle	9	2

* Draught of water or immersion double that of rectangle.

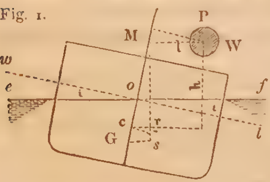
Statical Stability is moment of force which a body in flotation exerts to attain its normal position or that of equilibrium, it having been deflected from it, and it is equal to product of weight of fluid displaced and horizontal distances between the two centres of gravity of body and of displacement, or it is product of weight of displacement, height of *Meta-centre*, and *Sine* of angle of inclination.

Dynamical Stability is amount of mechanical work necessary to deflect a body in flotation from its normal position or that of equilibrium, and it is equal to product of sum of vertical distances through which centre of gravity of body ascends and centre of buoyancy descends, in moving from vertical to inclined position by weight of body or displacement.

To Determine Measure of Stability of Hull of a Vessel or Floating Body.—Fig. 1.

Measure of stability of a floating body depends essentially upon horizontal distance, G *s*, of *meta-centre* of body from centre of gravity of body; and it is product of force of the water, or resistance to displacement of it, acting upward, and distance of G *s*, or P × G *s*. If distance, *c* M, represented by *r*, and angle of rolling, *c* M *r*, by M°, measure of stability, or S is determined by P *r*, sin. M° = S; and this is therefore greater, the greater the weight of body, the greater distance of *meta-centre* from centre of gravity of body, and the greater the angle of inclination of this or of *c* M *r*.

Fig. 1.



Assume figure to represent transverse section of hull of a vessel, G centre of gravity of hull, *w l* water-line, and *c* centre of buoyancy or of displacement of immersed hull in position of equilibrium. Conceive vessel to be heeled or inclined over, so that *e f* becomes water-line, and *s* centre of buoyancy; produce *s* M, and point M is *meta-centre* of hull of vessel.

Transverse *meta-centre* depends upon position of centre of buoyancy, for it is that point where a vertical line drawn from centre intersects a line passing through centre of gravity of hull of vessel perpendicular to plane of keel.

Point of *meta-centre* may be the same, or it may differ slightly for different angles of heeling. Angle of direction adopted to ascertain position of *meta-centre* should be greatest which, under ordinary circumstances, is of probable occurrence; in different vessels this angle ranges from 20° to 60°.

If *meta-centre* is above centre of gravity, equilibrium is Stable; if it coincides with it, equilibrium is Indifferent; and if it is below it, equilibrium is Unstable.

Comparative Stability of different hulls of vessels is proportionate to the distance of G M for same angles of heeling, or of distance G s. Oscillations of hull of a vessel may be resolved into a rolling about its longitudinal axis, pitching about its transverse axis, and vertical pitching, consisting in rising and sinking below and above position of equilibrium.

If transverse section of hull of a vessel is such that, when vessel heels, level of centre of gravity is not altered, then its rolling will be about a permanent longitudinal axis traversing its centre of gravity, and it will not be accompanied by any vertical oscillations or pitchings, and moment of its *inertia* will be constant while it rolls. But if, when hull heels, level of its centre of gravity is altered, then axis about which it rolls becomes an instantaneous one, and moment of its *inertia* will vary as it rolls; and rolling must then necessarily be accompanied by vertical oscillations.

Such oscillations tend to strain a vessel and her spars, and it is desirable, therefore, that transverse section of hull should be such that centre of its gravity should not alter as it rolls, a condition which is always secured if all water-lines, as $w\,l$ and $e\,f$, are tangents to a common sphere described about G; or, in other words, if point of their intersections, o, with vertical plane of keel, is always equidistant from centre of gravity of hull.

To Compute Statical Stability.

D c M sin. M = S. D *representing displacement*, M *angle of inclination, and* S *stability.*

Illustration 1.—Assume a ship weighing 6000 tons is heeled to an angle of 9°, distance c M = 3 feet,

Sin. 9° = .1564. Then 6000 × 3 × .1564 = 2815.2 *foot-tons.*

2.—Weight of a floating body is 5515 lbs., distance between its centre of gravity and *meta-centre* is 11.32 feet, and angle M = 20°.

Sin. M = .342 02. Hence 5515 × 11.32 × .342 02 = 21 352.24 *foot-lbs.*

Statical Surface Stability.

Moment of Statical surface stability at any angle is $c\,z$ D. Assuming centre of gravity of vessel coincided with c; coefficient of a vessel's stability at any angle of heel is expressed when the displacement is multiplied by vertical height of the *meta-centre* for given angle of heel above centre of gravity, or D c M.

Approximately. Rule.—Divide moment of inertia of plane of flotation for upright position, relatively to middle line by volume of displacement; and quotient multiplied by sine of angle of heel will give result.

Per Foot of Length of Vessel, $\frac{2}{3}$ (B^3 sin. M). B *representing half breadth.*

Dynamical Surface Stability.

Moment of Dynamical surface stability is expressed by product of weight of vessel or displacement and depression of centre of buoyancy during the inclination, that is, for angle M.

To Compute Dynamical Stability of a Vessel.

Approximately. Rule.—Multiply displacement by height of *meta-centre* above centre of gravity, and product by versed sine of angle of heel.

Or multiply statical stability for given angle by tangent of .5 angle of heel.

To Compute Elements of Stability of a Floating Body.

$\frac{A'}{A}\,a = s$, $\frac{c}{\sin. M} = r$, $\frac{s}{\sin. M} = g$, and sin. M $r = c$. A *representing area of immersed section;* A′ *section immersed by careening of body, as* $f\,o\,l$; s *horizontal distance,* $c\,r$, *between centres of buoyancy;* a *horizontal distance between centres of gravity,* $i\,i$, *of areas immersed and emerged by careening;* g *distance,* c M, *between centre of buoyancy or of water displaced and meta-centre;* r *distance,* G M, *between centre of gravity and meta-centre;* c *horizontal distance,* G s, *between centre of gravity and of line of displacement of it when careened;* e *vertical distance between centres of gravity and buoyancy, all in feet; and* M *angle of careening.*

NOTE.—When centre of gravity, G, is below that of displacement, c, then e is $+$; when it is above c it is $-$; and when it coincides with c it is o; or e is $-$ when $\frac{S}{P} < s$; and a body will roll over when e sin. $M =$ or $> s$.

Assumed elements of figure illustrated are $A = 86$, $A' = 21.5$, $b = 21.5$, and $e = .5$. The deduced are $s = 3.7$, $c = 3.87$, $g = 10.82$, $a = 14.9$, and $r = 11.32$. *b representing breadth at water-line or beam in feet, and P weight or displacement in lbs. or tons.*

Then $s = \frac{21.5}{86} \times 14.9 = 3.7$ *feet*, $r = \frac{3.87}{.34202} = 11.32$ *feet*, $e = r - g$, $g = \frac{3.7}{.34202} = 10.82$ *feet*, $c = .34202 \times 11.32 = 3.87$ *feet.*

Of Hull of a Vessel. $\left(\frac{b^3}{10.7 \text{ to } 13^* A} \pm e\right)$ P, sin. $M = S$; $\quad d \cos. .5\, M = d'$, $\frac{b^3}{10.7 \text{ to } 13\,(11.93)\, A} = g$, $\quad \frac{1}{\sin. M}\left(\frac{S}{P} - s\right) = \pm e$; $\quad P\left(\frac{b\,a}{A} + \overline{e \sin. M}\right) = S$; and $P\,(s \pm \overline{e \sin. M}) = S$. *d representing depth of centre of gravity of displacement under water in equilibrium, and d′ depth when out of equilibrium, both in feet.*

ILLUSTRATION 1.—Displacement of a vessel is 10 000 000 lbs.; breadth of beam, 50 feet; area of immersed section, 800 sq. feet; vertical distance from centre of gravity of hull up to centre of buoyancy or displacement, 1.9 feet, and horizontal distance a between centres of gravity of areas immersed and emerged, when careened to an angle of $9° 10' = 33.4$ feet, immersed area being 50 sq. feet.

Sin. $9° 10' = .1593$. Then $s = \frac{50}{800} \times 33.4 = 2.0875$ *feet.* $800 \times 2.0875 = 50 \times 33.4$. $r = \frac{2.39}{.1593} = 15$ *feet.* $\quad g = \frac{50^3}{11.93 \times 800} = 13.1$ *feet*, $\quad S = \left(\frac{50^3}{11.93 \times 800}\right) + 1.9 \times 10\,000\,000 \times .1593 = 23\,905\,396$ *lbs.*, and $e = \frac{1}{.1593}\left(\frac{23\,905\,396}{10\,000\,000} - 2.0875\right) = 1.9$ *feet.*

2.—Assume a ship having a displacement of 5000 tons, and a height of *meta-centre* of 3.25 feet, to be careened to $6° 12'$. What is her statical stability?

Sin. $6° 12' = .1079$. Then $5000 \times 3.25 \times .1079 = 1753.37$ *foot-tons.*

3.—Assume a weight, W, of 50 tons to be placed upon her spar deck, having a common centre of gravity of 15 feet above her load-line,

Then $5000 \times 3.25 - \overline{50 + 15} \times .1079 = 1745.29$ *foot-tons.*

4.—Assume 100 tons of water ballast to be admitted to her tanks at a common centre of gravity of 15 feet below her load-line,

Then $5000 \times 3.25 + \overline{100 \times 15} \times .1079 = 1915.22$ *foot-tons.*

5.—Assume her masts, weighing 6 tons, to be cut down 20 feet,

Then $\frac{10 \times 20}{5000} = \frac{2}{50}$ foot = *fall of centre of gravity*, and $5000 \times \left(3.25 + \frac{2}{50}\right) \times .1079 = 1774.95$ *tons.*

To Compute Elements of Power, etc., required to Careen a Body or Vessel.

Sin. $M\,(h - \overline{n \sin. M}) + \overline{n \sec. M} - s = l$. $\quad \frac{b^3}{10.7 \text{ to } 13^* A}\sqrt[3]{\frac{P}{64.125\, L\, A}} = m$.

$W\,l\,r = P\,c$, and $W\,l = S$. *W representing weight or power exerted and l distance at which weight or power acts to careen body, taken from centre of gravity of displacement perpendicular to careening force, h vertical height from centre of gravity of displacement to centre of weight or power to careen body when it is in equilibrium, n horizontal distance from centre of vessel to centre of weight or power, L length of vessel, m meta-centre, and S as in preceding case, all in feet.*

---

* Unit for section of a parallelogram is 10.7; of a semicircle 12, and of a triangle 12.8.

ILLUSTRATION.—A weight is placed upon deck of a vessel at a mean height of 3.87 feet from centre line of hull; height at which it is placed is 11.32, and other elements as in first case given.

Sec. $20^\circ = .342$. Then $h = 11.32$, $n = 3.87$, and $l = .342\ (11.3 - \overline{3.87 \times .342}) + 3.87 \times 1.0642 - 3.7 = .342 \times 10 + 4.12 - 3.7 = 3.84$ *feet.*

Assume W = 5515. Then $5515 \times 3.84 = 21\,187.6$ *foot-lbs.*

Or P (w cos. M + h sin. M) = S. *w representing distance of weight from centre of vessel, and h height of w above water-line, both in feet.*

ILLUSTRATION.—If a weight of 30 tons placed at 20 feet from centre of hull or deck, 10 feet above water-line, careens it to an angle of $2^\circ\ 9'$, what is its stability?

$$\cos.\ 2^\circ\ 9' = .9993;\ \sin.\ 2^\circ\ 9' = .0375.$$

$$30\ (20 \times .9993 + 10 \times .0375) = 30 \times 20.361 = 610.83\ \textit{foot-tons.}$$

Bottom and Immersed Surface of Hull of Vessels.

To Compute Bottom and Side Surface of Hull.

Bottom and Side. RULE.—Multiply length of curve of amidship section, taken from top of tonnage or main deck beams upon one side to same point upon other (omitting width of keel), by mean of lengths of keel and between perpendiculars in feet, multiply product by .85 or .9 (according to the capacity of vessel), and product will give surface required in sq. feet.

EXAMPLE.—Lengths of a steamer are as follows: keel 201 feet, and between perpendiculars 210 feet, curved surface of amidship section 76 feet; what is surface?

Coefficient .87. $\overline{210 + 201} \div 2 = 205.5$, and $76 \times 205.5 \times .87 = 13\,587$ *sq. feet.*

NOTE.—Exact surface as measured was 13 650 sq. feet.

Bottom Surface. RULE.—Multiply length of hull at load-line by its breadth, and this product by depth of immersion (omitting the depth of keel) in feet; and this product multiplied by from .07 to .08 (according to capacity of vessel) will give surface required in sq. feet.

EXAMPLE.—Length upon load-line of a vessel is 310 feet, beam 40 feet, depth of keel 1 foot, and draught of water 20 feet; what is bottom or wet surface?

Coefficient assumed .073. $310 \times 40 \times \overline{20 - 1} \times .073 = 17\,199$ *sq. feet.*

To Compute Resistance to Wet Surface of Hull.

$C\,a\,v^2 = R$. *C representing a coefficient of resistance, a area of wet surface in sq. feet, and v velocity of hull in feet per second.*

Values of C, $\left\{\begin{array}{l}.007,\ \text{clean copper.}\\ .01,\ \text{smooth paint.}\end{array}\right.$ | .014, iron plate. .019, iron plate, moderately foul.

Power required to propel one sq. foot of immersed amidship section at $\otimes$ is .073 that of smooth wet surface.

To Compute Elements of a Vessel.

Displacement and its Centre of Gravity.

Displacement of a vessel is volume of her body below water-line.

Centre of Gravity, or Centre of Buoyancy of Displacement, is centre of gravity of water displaced by hull of vessel.

For Displacement. RULE.—Divide vessel, on half breadth plan, into a number of equidistant sections, as one, two, or more frames, commencing at $\otimes$ and running each side of it. Add together lengths of these lines in both fore and aft bodies, except first and last, by Simpson's rule for areas (see page 344); multiply sum of products by one third distance between sections, and product will give area of water-line between fore and aft sections.

Then compute areas contained in sections forward and aft of sections taken, including stern and rudder-post, rudder and stem, and add sum to area of body-sections already ascertained.*

---

* *To Compute Area of a Water-line,* see Mensuration of Surfaces, page 344.

Compute area of remaining water-lines in like manner. Tabulate results, and multiply them by Simpson's rule in like manner as for a water-line, and again by consecutive number of water-lines, and sum of products between water line and product will give volume between load and lower water-line.

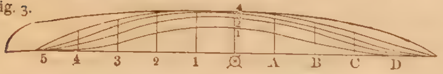
Add area of lower water line to area of upper surface of keel; multiply half sum by distance between them, and product will give volume; then compute areas contained in sections forward and aft of sections taken as before directed.

If keel is not parallel to lower water-line, take average of distance between them.

Compute volume of keel, rudder post and rudder below water line; add to volume already ascertained; multiply product by two, for full breadth, and product will give volume required in cube feet, all dimensions being taken in feet.

Fig. 2.

Fig. 3.



EXAMPLE.—Assume a vessel 100 feet in length by 20 feet in extreme breadth, on load-line of 8 feet 9 inches immersion. Figs. 2 and 3.

Distance between sections, for purpose of simplifying this example, is taken at 10 feet; usually frames are 18 to 30 ins. apart, and two or more included in a section. Water-lines 2 feet apart.

1st Water-line.

4	5	=	5
3	7.7 × 4	=	30.8
2	9.5 × 2	=	19
1	9.9 × 4	=	39.6
0	10 × 2	=	20
A	9.6 × 4	=	38.4
B	7.8 × 2	=	15.6
C	6.8 × 4	=	27.2
D	4	=	4
			199.6
	10 ÷ 3	=	3⅓
			665.3
	Abaft section 4, rudder and post		25
	Forward section D and stem		20.7
			711

2d Water-line.

4	2.7	=	2.7
3	6.9 × 4	=	27.6
2	8.7 × 2	=	17.4
1	9.5 × 4	=	38
0	9.6 × 2	=	19.2
A	9 × 4	=	36
B	7 × 2	=	14
C	5 × 4	=	20
D	2	=	2
			176.9
	10 ÷ 3	=	3⅓
			589.7
	Abaft section 4, rudder and post		13.2
	Forward section D and stem		9.1
			612

3d Water-line.

4	1.5	=	1.5
3	5 × 4	=	20
2	6.6 × 2	=	13.2
1	8.7 × 4	=	34.8
0	8.9 × 2	=	17.8
A	7.6 × 4	=	30.4
B	7 × 2	=	14
C	3 × 4	=	12
D	1.2	=	1.2
			144.9
	10 ÷ 3	=	3⅓
			483
	Abaft section 4, rudder and post		7
	Forward section D and stern........		5.4
			495.4

4th Water-line.

4	.7	=	.7
3	2 × 4	=	8
2	4.3 × 2	=	8.6
1	6.5 × 4	=	26
0	6.8 × 2	=	13.6
A	5 × 4	=	20
B	3.6 × 2	=	7.2
C	.9 × 4	=	3.6
D	.3	=	.3
			88
	10 ÷ 3	=	3⅓
			293.3
	Abaft section 4, rudder and post		3.2
	Forward section D and stem.........		.8
			297.3

Keel.

Half breadth = .25 × length of 98 feet =	24.5
Rudder-post and rudder................	.3
	24.8

Results.

1st water-line	711		711	
2d "	612	× 4 =	2448	× 1 = 2448
3d "	495.4	× 2 =	990.8	× 2 = 1981.6
4th "	297.3	× 4 =	1189.2	× 3 = 3567.6
Keel	24.8		24.8	× 4 = 99.2
			5363.8	8096.4
			2	
			3)10727.6	

Displacement, 3575.9 × 2 = 7151.8 *cube ft.*

To Compute Centre of Gravity of Displacement.

RULE.—Divide sum of products obtained as above, by consecutive water-lines, by sum of products obtained in column of products by Simpson's multipliers, and quotient, multiplied by distance between water-lines, will give depth of centre below load water-line.

ILLUSTRATION I. 8096.4, from above, $\div$ 5363.8 = 1.5, which $\times$ 2 = 3 feet.

Or, $\dfrac{n}{2\left(2-\dfrac{D}{a\,n}\right)} = d.$ *n representing draught of water exclusive of any drag of keel, a area of immersed surface of hull in sq. feet, and* D *displacement in cube feet.*

2.—Assume draught of water 8 feet, displacement 7152 cube feet, and area of immersed surface of hull 1100 sq. feet.

$$\text{Then } \frac{8}{2\left(2-\dfrac{7152}{1100 \times 8}\right)} = \frac{8}{2 \times 1.187} = 3.37 \textit{ feet.}$$

To Compute Displacement Approximately.

Coefficient of Displacement of a vessel is ratio that volume of displacement bears to parallelopipedon circumscribing immersed body.

$\dfrac{V}{L\,B\,D} = C.$ V *representing volume of displacement in cube feet,* L *length at immersed water-line,* B *extreme breadth, and* D *draught in depth of immersion, both in feet.*

Coefficient of Area of Amidship Section in Plane of a Water-line is ratio which their areas bear to that of circumscribing rectangle.

L *representing length of water-line, and* D *distance between water-lines, both in feet.*

Coefficients. (By S. M. Pook, Constructor U. S. Navy.)

RULE.—Multiply length of vessel at load-line by breadth, and product by depth (from load-line to under side of garboard-strake) in feet, and this product by coefficient for vessel as follows: divide by 35 for salt water, 36 for fresh water, and quotient will give displacement in tons.

Amidship sections range from .7 to .9 of their circumscribing square, and mean of horizontal lines from .55 to .75 of their respective parallelograms. Hence, ranges for vessels of least capacity to greatest are $7 \times .55 = .385$, and $.9 \times .75 = .675$.

Merchant ship, very full	.6 to .7	Merchant steamer, medium	.52 to .54
" " medium	.58 to .62	Clipper	.5 to .54
River steamer, stern-wheel	.6 to .65	Schooner, medium	.48 to .52
Ship of the line	.5 to 6	River steamer, tug boat, sharp	.45 to .5
Naval steamer, first class	.5 to .6	" " medium	.45 to .5
" "	.52 to .58	" " sharp	.42 to .45
Merchant steamer, sharp	.54 to .58	Schooner, sharp	46 to .5
Half clipper	.52 to .56	Yachts, sharp	.4 to .45
Brigs, barks, etc	.52 to .56	" very sharp	.3 to .4
River steamer, tug-boat, med'm	.52 to .56	River steamers, very sharp	.36 to .42

In steam launch *Miranda*, when making 16.2 knots per hour, with a displacement of 58 tons, her coefficient was 3.

To Compute Change of Trim.

$\dfrac{W\,d}{D} \times \dfrac{L}{m} = d'.$ D *representing displacement at line of draught in tons,* L *length at same line in feet, and m longitudinal meta-centre.*

ILLUSTRATION.—"*Warrior*," at draught of 25.5 feet, has L = 380 feet, $m = 475$ feet, and D = 8625 tons. If, then, a weight of 20 tons was shifted fore and aft 100 feet,

$$\frac{20 \times 100}{8625} \times \frac{380}{475} = .1856 \textit{ feet} = 2.22 \textit{ ins.}$$

Illustration.—*Vertical Plane at ⊠ and Horizontal at Load-line.*

Elements of a Steam Frigate.	Weight.	Horizontal. Distances. Forward.	Horizontal. Distances. Abaft.	Horizontal. Moments. Forward.	Horizontal. Moments. Abaft.	Vertical. Distances. Above.	Vertical. Distances. Below.	Vertical. Moments. Above.	Vertical. Moments. Below.
	Tons.	Feet.	Feet.			Feet.	Feet.		
Hull, bunkers, and cement in bottom	1075	1.6	—	1 720	—	—	1	—	1075
Engines, boilers, water, and stores	470	—	29	—	13 630	—	6.3	—	3011
Coal	252	16	—	4 032	—	—	4	—	1008
Battery and ammunition	131.5	62	—	8 153	—	2	—	263	—
Masts, spars, sails, and rigging	24	27	—	648	—	31	—	744	—
Anchors and cables	25	40	—	1 000	—	—	6	—	150
Boats	3.25	—	48	—	156	16	—	52	—
Water and ship's stores	22	40	—	880	—	—	3	—	66
Provisions and galley	30	1.2	—	360	—	5	—	150	—
Crew and effects	30	17	—	510	—	7	—	210	—
Officers' and mess stores	7.25	—	40	—	290	—	8	—	58
Total	2070			17 303	14 076			1419	5368

Moments forward ⊠, 17 303 — moments abaft, 14 076 = 3227 ÷ 2070 tons (*weight*) = 1.56 *feet* = *distance of centre forward of* ⊠.

Moments above load-line, 5368 — moments below, 1419 = 3949 ÷ 2070 tons (*weight*) = 1.91 *feet* = *distance of centre below load-line.*

Note.—Rule, in Strength of Materials, to compute common centre of gravity, page 819, would apply in this case.

To Compute Centre of Gravity of Bottom Plating of a Vessel.

Longitudinal.

Rule.—Measure half girths of plating at equidistant sections, as at two or more frames. Multiply these in accordance with Simpson's rule for areas and add products together.

Multiply each of these products in their order, by number representing number of intervals of section forward and abaft of ⊠.

Divide difference of these moments by sum of products of half girths, previously obtained.

Multiply product by common distance between sections, and result will give distance of centre of gravity from ⊠ in a horizontal plane.

To Compute Depth of Centre of Gravity or Buoyancy Below Meta-Centre.

$\frac{S}{D \sin. M} = d.$ S *representing statical stability,* D *displacement in tons, and* sin. M *sine of angle of heel.*

Illustration.—Elements of Fig. 2, page 654, are, statical stability at angle of 5.44°, 90 tons, and displacement 204.33 tons.

$$\text{Sin. } 5.44° = .0999. \text{ Then } \frac{90}{204.33 \times .0999} = 4.41 \textit{ feet.}$$

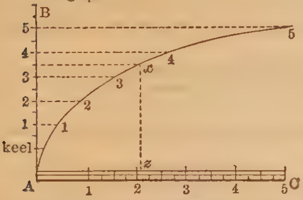
To Compute Centre of Gravity or Buoyancy Approximately.

$\frac{2}{5}$ to $\frac{9}{20}$ of *mean draught of hull*, using larger coefficient for full-bodied vessels.

To Delineate Curve of Displacement.

This curve is for purpose of ascertaining volume of water or tons weight, displaced by immersed hull of a vessel at any given or required draught; or weight required to depress a hull to any given or required draught. From results of computation for displacement of vessel, proceed as follows, Fig. 4:

Fig. 4.



On a vertical scale of feet and ins., as A B, set off depths of keel and water-lines, draw ordinates thereto representing displacement of keel, and at each water-line, in tons.

Through points 1, 2, 3, 4, and 5 delineate curve A 5, which will represent displacement at any given or required draught.

Draw a horizontal scale corresponding to weight due to displacement at load-line, as A C, and subdivide it into tons and decimals thereof, and a vertical line let fall from any point, as x, at a given draught, will indicate weight of displacement at depth, on scale A C, and, contrariwise, a line raised from any point, as z, on A C will give draught at that weight.

Illustration.—Displacement of hull (page 654) at load line = 7151.8 cube feet, which ÷ 35 for salt water = 204.3 tons, hence A C represents tons, and is to be subdivided accordingly.

Assume launching draught to have been 4 feet, then a vertical let fall from 4 will indicate weight of hull in tons on A C.

Coefficients. (*By C. Mackrow, M. I. N. A.*)

Description of Vessel.	Length.	Breadth.	Mean Draught	Coefficient. Displacement.	Coefficient. Amidship Section.	Coefficient. Water-lines.
Iron-Clads	225	45	15	.715	.932	.755
	325	59	24.75	.64	.81	.71
Mail Steamers	350	35	21	.687	.85	.84
	385	42	22	.659	.88	.8
	368.27	42.5	18.71	.516	.812	.635
Merchant, small	220	27	8	.702	.912	.742
	90	15	4	.637	.914	.704
Gunboats	125	23	8	.536	.87	.616
	160	31.3	12	.466	.745	.603
Troop Ships	350	49.12	23.5	.47	.674	.7
	340.5	46.13	15.75	.4	.68	.582
Swift Naval Steamers	337.3	50.28	22.75	.483	.787	.614
	270	42	19	.497	.792	.628
Fast Steamers, R. N.	300	40.27	14	.414	.711	.711

Curve of Weight.

To Compute Number of Tons required to Depress a Vessel One Inch at any Draught of Water Parallel to a Water-line.

Rule.—Divide area of plane by 12, and again by 35 or 36, as may be required for salt or fresh water.

Example.—Area of load water-line of a vessel is 1422 sq. feet; what is its capacity per inch in salt water?

$1422 \div 12 = 118.5$, which $\div 35 = 3.38$ *tons*.

To Compute Common Centre of Gravity of Hull, Armament, Engine, Boilers, etc., of a Vessel.

RULE.—Compute moments of the several weights, relatively to assigned horizontal and vertical planes, by multiplying weight of each part by its horizontal and vertical distance from these planes.

Add together these moments, according to their position forward or aft, or above or below these planes, and difference between these sums will give position forward or aft, above or below, according to which are greatest.

Divide results thus ascertained by total weight of vessel, and product will give horizontal and vertical distances of centre of gravity from these planes.

It is customary to assume vertical plane at ⊠, and horizontal plane at load-line.

NOTE.—In following illustration, in order to simplify computation in table, common centre of gravity of hull, machinery, etc., is taken, instead of centres of individual parts, as engine, boiler, propeller, etc.

ILLUSTRATION.—Assume half girths as in following table, and distance between sections 10 feet.

FORWARD.						ABAFT.					
Section.	Half-Girths.	Multipliers.	Product.	Multipliers.	Moments.	Section.	Half-Girths.	Multipliers.	Product.	Multipliers.	Moments.
No.	Feet.					No.	Feet.				
⊠ ...	25	1	25	—	—	1 ...	23	4	92	1	92
A....	23	4	92	1	92	2 ...	20	2	40	2	80
B....	21	2	42	2	84	3 ...	18	4	72	3	216
C....	19	4	76	3	228	4 ...	16	2	32	4	128
D....	17	2	34	4	136	5 ...	14	1	14	5	70
E....	15	1	15	5	75					534	586
					615						

Moments forward, 615 — moments abaft, 586 = 29 ÷ sum of product 534 = .054, which × 10 feet = .54 *feet forward* of ⊠.

Centre of Lateral Resistance.

Centre of Lateral Resistance is centre of resistance of water, and as its position is changed with velocity of vessel, it is variable. It is generally taken at centre of immersed vertical and longitudinal plane of vessel when upon an even keel.

If vessel is constructed with a drag to her keel, the centre will be moved proportionately abaft of longitudinal centre.

Yacht *America* had a drag to her keel of 2 feet, and centre of lateral resistance of her hull was 8.08 feet abaft of centre of her length on load-line.

Centre of Effort.

Centre of Effort is centre of pressure of wind upon sails of a vessel in a vertical and longitudinal plane. Its position varies with area and location of sails that may be spread, and it is usually taken and determined by the ordinary standing sails, such as can be carried with propriety in a moderately fresh breeze.

In computing this position, the yards are assumed to be braced directly fore and aft and the sails flat.

NOTE.—Centre of effort of sails, to produce greatest propelling effect, must accord with capacity of vessel at her load-line, compared with fullness of her immersed body at its extremities. Thus, a vessel with a full load-line and sharp extremities below, will sustain a higher centre of effort than one of dissimilar capacity and construction.

To Compute Location of Centre of Effort.

RULE.—Multiply area of each sail in square feet by height of its centre of gravity above centre of lateral resistance in feet, divide sum of these products (moments) by total area of sails in square feet, and quotient will give height of centre in feet.

2. Multiply area of each sail in square feet, centre of which is forward of a vertical plane passing through centre of lateral resistance, by direct distance of its centre from that plane in feet, and add products together.

3. Proceed in like manner for sails that are abaft of this plane, add their products together, and centre of effort will be on that side which has greatest moment of sail.

EXAMPLE.—Assume elements of yacht *America* as rigged when in U. S. Service.

SAIL.	Area.	Height of Cent. of Gravity of Sails.	Vertical Moments.	Distance of Centre of Gravity of Sails.		Moments.	
				Foreward.	Abaft.	Foreward.	Abaft.
	Sq. Feet.	Feet.					
Flying Jib	656	28	18 368	52	—	34 112	—
Jib	1087	26	28 262	32	—	34 784	—
Foresail	1455	34	49 470	—	3	—	4 365
Mainsail	2185	35	76 475	—	40	—	87 400
	5383		172 575			68 896	91 765

$\dfrac{\text{Vertical moments } 172\,575}{\text{Area of sails } \ldots\ldots\ 5\,383} = 32.06 =$ *height of centre above centre of lateral resistance.*

Moments $\left\{\dfrac{91\,765 \sim 68\,896}{5383}\right. = 4.25 =$ *distance of centre abaft centre of lateral resistance.*

Relative Positions of Centre of Effort and of Lateral Resistance.

Square Rig. $\dfrac{L\,(.75\,d' + d'')}{10\,(d' + d'')} = E.$ *Fore and Aft Rig.* $\dfrac{L}{10\,(d' + d'')} = E,$

and $\dfrac{4\,A}{5\,d} = E'$. L *representing length of load-line, d distance of centre of buoyancy of vessel below it, d′ distance of centre of lateral resistance abaft centre of it, d″ distance of centre of buoyancy before centre of it,* E *distance of centre of effort before centre of lateral resistance, and* E′ *distance of centre of effort above centre of lateral resistance.*

Meta-Centre.

Meta-centre of a vessel's hull is determined by location of centre of gravity or buoyancy of immersed bottom of hull, for it is that point in transverse section of hull, where a vertical line raised from its centre of gravity or buoyancy intersects a line passing through centre of gravity of hull, as Fig. 1, page 650.

To Compute Height of Meta-Centre.

By Moment of Inertia. $\dfrac{I}{D} = M.$ I *representing moment of inertia of area of water-line or plane of flotation, and* D *volume of displacement in cube feet.*

NOTE.—*Moment of Inertia* of an area is sum of products of each element of that area, by square of its distance from axis, about which moment of area is to be computed.

To Ascertain Moment of Inertia approximately.

Rectangle $= C\,L\,B^3$; $C = \dfrac{1}{12}$ when $L = 4\,B$; $C = \dfrac{3}{50}$ when $L = 5\,B$; and $C = \dfrac{11}{200}$ when $L = 6\,B$. With very fine lines and great proportionate length $C = \dfrac{1}{25}$.

L *and* B *measured at load-line.*

Illustration.—Assume length of vessel 233 feet, breadth 43, draught 16, and displacement 2700 tons. Length = 5.65 beams; hence C is taken at $\frac{21}{400}$. Volume of displacement = 2700 × 35 = 92 500 cube feet.

Then $\frac{21 \times 233 \times 43^3}{400 \times 92\,500} = 10.51$. Exact height of moment was 10.44 *feet.*

By Ordinates. Rule.—Divide a half longitudinal section of load water-line by ordinates perpendicular to its length, of such a number that area between any two may be taken as a parallelogram. Multiply sum of cubes of ordinates by respective distances between them, and divide two thirds of product by volume of immersion, in cube feet.

Illustration.—Take dimensions from Figs. 2 and 3, page 654.

	Length.	Cube.		Length.	Cube.	Cube.
4	5	125	A	9.6	885	51 460
3	7.7	456	B	7.8	475	2
2	9.5	857	C	6.8	314	3) 102 920
1	9.9	970	D	4	64	
⊠	10	1000			5146 × 10	7151.8) 34 306.6 = 4.77 *ft.*

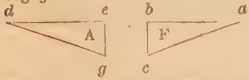
If there are more ordinates, their coefficients must be taken in like manner, as 1—4—2—4—2—4—1.

For operation of this method, see Simpson's rule for areas, page 342.

Or, $\frac{2}{3}\int \frac{y^3\,dx}{D} = M.$ *y representing ordinates of half-breadth sections at load-line, dx increment of length of load-line section or differential of x, and D displacement of immersed section in cube feet.*

By Areas. $\dfrac{\frac{2}{3}(a^3 + 4\,b^3 + 2\,c^3 + 4\,d^3 + e^3)\frac{l}{3} + F + A}{D} = M.$ *a, b, c, d, and e representing ordinates of 1st or load water-line, F area of irregular section between 1st frame and stem, and A area of like section between last frame and stern-post, both in sq. feet, D displacement, in cube feet, and l distance between frames or sections of water-line, as may be taken, in feet.*

Fig. 5.



To Ascertain Areas of F and A.—Fig. 5.

$\frac{2}{3}ab \times bc^3 \div 4 = F$, and $\frac{2}{3}de \times eg^3 \div 4 = A$.

Elements of Capacity and Speed of Several Types of Steamers of R. N. (*W. H. White.*)

Classes.	Length.	Length to Breadth.	Displacement.	Speed.	IHP to Displacement.	IHP to Displacement $\frac{2}{3}$.
Iron-clads.	Feet.		Tons.	Knots.		
Recent types.	300 to 330	5.25 to 5.75	7500 to 9 000	14 to 15	.9 to 1	16 to 20
do. twin sc.	280 to 320	4.5 to 5	6000 to 9 000	14 to 15	.7 to .9	15 to 19
Unarmored.						
Swift cruisers	270 to 340	6.5 to 6.75	3000 to 5 500	15 to 16	1.3 to 1.5	20 to 24
Corvettes....	200 to 220	6	1800 to 2 000	12.75 to 13.25	1 to 1.2	13 to 14
Ships........	160	5	850 to 950	11	1 to 1.2	10 to 11
Gun-vessels..	125 to 170	5.5 to 6.25	420 to 800	9.5 to 11	.8 to 1.4	7 to 11
Gun-boats ...	80 to 90	3 to 3.25	200 to 250	8 to 9	.8 to 1.1	5 to 7
Merchant.						
Mail, large...	400 to 500	9 to 11	7000 to 10 000	14 to 15	.5 to .6	10 to 11
" smaller.	300 to 400	8 to 10	5000 to 7 000	13 to 14	.4 to .5	7 to 10
Cargo, large..	250 to 350	7.5 to 10	3000 to 6 000	11 to 15	.3 to .5	5 to 9
" smaller.	200 to 300	7 to 9	1500 to 4 000	9 to 11	.2 to .4	3 to 6

To Compute Power Required in a Steam Vessel, capacity of another Vessel being given.

In vessels of similar models. $\frac{v\,A}{a} = V$; $\frac{S^3\,V}{s^3} = V'$; $\frac{r\,V'}{r'} = C$; and $\frac{C}{2\,r'} = R$; *v and V' representing product of volumes of given and required cylinders and revolutions in cube feet, a and A areas of immersed section of given and required vessel in sq. feet at like revolutions and speed of given vessel, s and S speeds of given and required vessel at revolutions of given vessel, both in feet per minute, r and r' revolutions of given and required vessel per minute, and C product of volume of combined cylinder and revolutions for required vessel.*

ILLUSTRATION.—A steam vessel having an area of amidship section of 675 sq. feet, has two cylinders of a combined capacity of 533.33 cube feet, and a speed of 10.5 knots per hour, with 15 revolutions of her engines. Required volume of steam cylinders, with a stroke of 10 feet, for a section of 700 feet and a speed of 13 knots with 14.5 revolutions.

$v = 533.33 \times 15 = 8000$ *cube feet*, $\frac{8000 \times 700}{675} = 8296.3$ *cube feet*, $\frac{13^3 \times 8296.3}{10.5^3} = 15\,745.2$ *cube feet*, $\frac{15 \times 15\,745.2}{14.5} = 16\,288.1$ *cube feet*, and $\frac{16\,288.1}{2 \times 14.5} = 561.66$ *cube feet, which* $\div$ 10 *stroke of piston*, 12 *for ins., and* $\times$ 1728 *ins. in a cube foot* $= \frac{561.66 \times 1728}{10 \times 12} = 8087.9$ *sq. ins. area of each cylinder = diameter of* 101.5 *ins.*

Approximate Rules to Compute Speed and IHP of Steam Vessels.

$$\frac{V^3\,D^{\frac{2}{3}}}{IHP} = C;\quad \sqrt[3]{\frac{C\,IHP}{D^{\frac{2}{3}}}} = V;\ \text{and}\ \frac{V^3\,D^{\frac{2}{3}}}{C} = IHP.\quad \text{Or,}\ \sqrt[3]{\frac{C\,IHP}{A}} = V;\ \text{and}\ \frac{V^3\,A}{C} = IHP.$$

C representing coefficient of vessel, A area of immersed amidship section in sq. feet, V velocity of vessel in knots per hour, and D displacement of vessel in tons.

NOTE.—When there exists rig, an unusual surface in free board, deck-houses, etc., or any element that effects coefficient for class of vessel given, a corresponding addition to, or decrease of, following units is to be made:

Range of Coefficients as deduced from observation is as follows:

SIDE-WHEEL.

VESSEL.	A	D	V	C: $\frac{V^3\,A}{IHP}$	C: $\frac{V^3\,D^{\frac{2}{3}}}{IHP}$
Steamboat.	Sq.F.	T's.	K'ts.		
Medium lines....	43	73	10	470	212
" "	150	465	13	570	219
Fine lines........	136	300	19	540	200
Steamer.					
Medium full lines*	675	3600	10	650	214
—	—	—	—	—	—
Fine lines†.......	880	5233	15	650	211
—	—	—	—	—	—

PPOPELLER.

VESSEL.	A	D	V	C: $\frac{V^3\,A}{IHP}$	C: $\frac{V^3\,D^{\frac{2}{3}}}{IHP}$
Steamboat.					
Medium lines..	45	—	12	—	500
" "	—	—	—	—	—
Fine lines.....	150	—	15	—	530
Steamer.					
Medium full...	550	2532	9	194	570
" " *..	390	1475	16	180	470
—	—	3600	13	210	—
Torpedo boat..	—	27	20	170	500

* Full rigged. † Bark rigged.

Coefficients as Determined by Several Steamers of H. B. M. Service.

(C. Mackrow, M. I. N. A.)

Length.	$\frac{\text{Length}}{\text{Beam}}$.	Area of Section at ⊗.	Displacement.	IHP	Speed.	$\frac{V^3\,A}{IHP} = C$.
Feet.		Sq. Feet.	Tons.		Knots.	
185	6.53	236	775	782	10.34	333
212	5.89	377	1554	1070	10.89	456
360	7.33	814	5898	2084	11.5	598
270	6.43	632	3057	2046	12.3	574
380	6.52	1308	9487	3205	12.05	714
400	6.73	1198	9152	5971	13.88	536
362	7.33	778	5600	3945	14.06	548
400	6.73	1185	9071	6867	15.43	634

Approximate Rule for Speed of Screw Propellers.

(Molesworth.)

$$\frac{101\,V}{P}=N;\quad \frac{P\,N}{101}=V;\quad \frac{101\,V}{N}=P;\quad \frac{88\,v}{P}=N;\quad \frac{P\,N}{88}=v;\quad \text{and}\quad \frac{88\,v}{N}=P.$$

V *and* v *representing velocities in knots and miles per hour*, P *pitch of propeller in feet, and* N *number of revolutions per minute.*

This does not include slip, which ranges from 10 to 30 per cent.

Pitch of Screw Propeller.

Pitch ranges with area of circle described by diameter of screw to that of amidship section.

Area of screw circle to amidship section = 1 to...............	6	5	4.5	4	3.5	3	2.5	2
Two Blades.								
Pitch to diameter of screw = 1 to	.8	1.02	1.11	1.2	1.27	1.31	1.4	1.47
Four Blades.	1.08	1.38	1.5	1.62	1.71	1.77	1.89	1.98

Length = .166 diameter.

Slip of Side-wheels.

Radial Blades. $\frac{2\,(A-c)}{A}=S.$ *Feathering.* $\frac{1.5\,(A-c)}{A}=S.$ A *representing length of arc of immersed circumference of blades*, c *length of chord of immersed arc, and* S *slip, all in feet.*

Area of Blades.

River Service. $\frac{.75\,IHP}{D}=A.$ *Sea Service.* $\frac{IHP}{D}=A.$ D *representing diameter of wheel in feet, and* A *area of each blade in square feet.*

Length of Blades. .7 in River service and .6 in Sea service.

Distances between Radial Blades. 2.25 in River service and 3 feet in Sea service; *between Feathering blades*, 4 to 6 feet.

Proportion of Power Utilized in a Steam Vessel.

Side Wheel. $\frac{P-z}{.000\,002\,59\,d^3\,r^2}=C.$ P *representing gross* IHP, *z loss of effect by slip and oblique action of wheels*, d *diameter of wheels at centre of effect*, r *revolutions per minute, and* C *coefficient for vessel.*

Illustration.—IHP of engines of a side wheel steamer is 1120; slip of wheels and loss by oblique action, 33.37 per cent.; diameter of centre of effect of wheels is 29.5 feet, and number of revolutions 13.5 per minute; what is coefficient, and what power applied to propel vessel?

Note.—Slip of wheels from their centre of effect in this case is 15.37 per cent., and loss by oblique action 18 per cent. Hence, representing total power by 100, $100-(18+15.37)=66.63$ per cent. of power applied to wheels.

As assumed power that operates upon wheels in this case is taken at 86.12 per cent. of power exerted by engines, $86.12\times 33.37=28.74$ per cent. for sum of loss by wheels.

$$\frac{1120-(1120\times 28.74\div 100)}{.000\,002\,59\times 29.5^3\times 13.5^2}=\frac{798.11}{12.16}=65.63\ \textit{coefficient.}$$

Speed of vessel being 10 knots per hour = 17.05 feet per second, power applied to propel vessel at this speed $=65.63\times 17.05^2=19\,076.13$, and IHP exerted $=\frac{19\,076.13\times 17.05\times 60}{33\,000}=591.36.$

	IHP.	Per cent. of Power.
Friction of engines 1.5 lbs. upon 3848 sq. ins. × 13.5 revolutions × 10 × 2 ÷ 33 000 × 2............	94.45	18.83
Friction of load 6 per cent. upon pressure of steam, less 2 lbs. for friction of engine, as above............	60.45	
Oblique action of wheels............	201.6	18
Slip of wheels............	172.14	15.37
Absorbed by propulsion of vessel............	591.36	52.8
	1120	100

	IP.	Per cent. of Power.
Screw Propeller. Friction of engines	96.06	18.83
Friction of load	81.48	
" of screw surface and resistance of edges of blades	53.44	6.83
Slip of propeller	205.55	26.27
Absorbed by propulsion of vessel	375.92	48.04
	782.45	100

NOTE.—From experiments of Mr. Froude, he deduced that, as a rule, only 37 to 40 per cent. of whole power exerted was usefully employed.

With an auxiliary propeller, essential differences are in friction of surfaces and edges of blades of propeller and slip of propeller, being as 12 to 6.83 in excess in first case, and as 13.7 to 26.27 in second case, or 50 per cent. less.

Resistance of Bottoms of Hulls at a Speed of one Knot per Hour.

Smooth wood or painted	.01 lb.	Copper	.007 lb.
Smooth plank	.016 "	Moderately foul	.019 "
Iron bottom, painted	.014 "	Grass and small barnacles	.06 "

Sailing.

Ratio of Effective Area of Sails and of Vessel's Speed under Sail to Velocity of Wind.

Course.	Ratio of Effective Area of Sails.	Ratio of Speed of Vessel to Wind.	Course.	Ratio of Effective Area of Sails.	Ratio of Speed of Vessel to Wind.
5 points of wind	.59	.33	Wind abeam	.82	.6
2 " abaft beam	.91	.5	" astern	1	.5
6 " of wind	.68	.5	" on quarter	.96	.66

Propulsion and Area of Sails.

Plain sails of a vessel are standing sails, excluding royals and gaff topsails.

Resistance of vessels of similar models but of different dimensions for equal speeds $= D^{\frac{2}{3}}$

Hence $\frac{a}{a'} = \left(\frac{D}{D'}\right)^{\frac{2}{3}}$. *a and a′ representing areas of sails of known and given vessels, and* D *and* D′ *their displacements in tons.*

ILLUSTRATION.—Assume D and D′ = 2400 and 1600.

Then $\left(\frac{2400}{1600}\right)^{\frac{2}{3}} = \sqrt[3]{1\ 5^2} = 1\ 139$, hence area of sails $a' = \frac{1}{1.139} = 878$ *per centum.*

In Vessels of Dissimilar Models.—Plain sail area should be a multiple of $D^{\frac{2}{3}}$.

Multiples for Different Classes of Vessels, R. N.

Sailing.		*Steamers.*	
Ships of Line	100 to 120	Ships, iron-clad	60 to 80
Frigates	120 to 160	Frigates	80 to 120
Sloops	120 to 160	Sloops	80 to 120
Brigs	120 to 160	Brigs	80 to 120

English Yachts, designed for high speed, have multiples from 180 to 200, and when designed for ordinary speed from 130 to 180.

When Area of Sail to Wet Surface of Hull is taken.—American yacht *Sappho* had a ratio of 2.7 to 1, and several English yachts nearly the same, while in some others it was but 2 to 1.

Location of Masts, etc. *Load-line* = 100.

Vessel.	Distance from Stem.			Foot of Sail.*	Height of Centre of Effect above Water-line = Breadth.*
	Fore.	Main.	Mizzen.		
Ship	10 to 20	53 to 58	80 to 90	125 to 160	1.5 to 2
Bark	12 to 20	54 to 60	81 to 91	130 to 160	1.5 to 1.95
Brig	17 to 20	64 to 65	—	160 to 165	1.5 to 1.75
Schooner	16 to 22	55 to 61	—	160 to 170	1.5 to 1.75
Sloop	—	36 to 42	—	170 to 190	1.25 to 1.75

* Measured from Tack of Jib to Clew of Spanker or Mainsail.

Rake of Masts.

Ships.—Foremast 0 to .28 of length from heel, Main and Mizzen 0 to .25. Schooners.—Foremast .1 to .25, Mainmast .63 to .77. Sloops.—.08 to .11.

Area of Sails.

Sails.	3 Yards upon each Mast.	4 Yards upon each Mast.	Sails.	3 Yards upon each Mast.	4 Yards upon each Mast.
Jib	.08	.08	Mizzenmast	.127	.14
Foremast	.295	.295	Spanker or Driver	.081	.068
Mainmast	.417	.417			

Proportional Area of Sails upon each Mast under above Divisions.

Sail.	Fore.		Main.		Mizzen.		Proportion to 1.	
Course	.115	.097	.162	.138	—	—	.389	.33
Topsail	.105	.09	.149	.127	.075	.063	.358	.303
Topgallant sail	.075	.063	.106	.089	.052	.045	.253	.215
Royal	—	.045	—	.063	—	.032	—	.152
Spanker or Driver	—	—	—	—	.081	.068	—	—
Jib	.08	.08	—	—	—	—	—	—
	.375	.375	.417	.417	.208	.208	1	1

Balance of Sails.—Effect of jib is equal to that of all sails upon mainmast, and sails upon mizzenmast balance those of foremast.

Areas of sails upon masts of a ship should be in following proportion:

Fore.............. 1.414 | Main.................. 2 | Mizzen................ 1

When, therefore, main yard has a breadth of sail of 100 feet, fore yard should have 70.71 feet, and mizzen 50 feet, topgallant and royal yards and sails being in same proportion.

Angles of Heel for Different Vessels.

Approximately. $\frac{D\,M\,a}{H} = S$. D *representing displacement of vessel in lbs.*, M *height of meta-centre above centre of gravity in feet*, *a angle of heel of vessel in circular measure*,* *and* H *height of centre of effect above centre of lateral resistance, in feet.*

Moment of sail should be equal to moment of stability at a defined angle of heel.

	Angle.	Circular Measure.		Angle.	Circular Measure.
Frigates, etc.	4°	.07	Schooners, etc.	6°	.105
Corvettes	5°	.087	Yachts	6° to 9°	.105 to .107

Illustration.—Assume displacement 170 tons, height of meta-centre 6.75 feet, H = 36 feet, and angle of heel 9°; what should be area of sails?

$$170 \times 2240 = 380\,800 \text{ lbs.} \quad 9° = .107.$$

$$\frac{380\,800 \times 6.75 \times .107}{36} = 7639.8 \textit{ sq. feet.}$$

* See rule, page 113.

Trimming of Sails.

That a vessel's sail may have greatest effect to propel her forward, it should be so set between plane of wind and that of her course, that tangent of angle it makes with wind may be twice tangent of angle it makes with her course.

Or, tan. $a = 2$ tan. b. *a representing angle of sail with wind, and b angle of sail and course of vessel.*

Angles of Course and Sails with Wind.

Wind Ahead.	Angle of Course.	Tangent.	Half Tangent.	Angles of Sail with Wind.	Angles of Sail with Course.	Wind Abaft.	Angle of Course.	Tangent.	Half Tangent.	Angles of Sail with Wind.	Angles of Sail with Course.
Points.						Points.					
4	45°	.562	.281	29° 18′	15° 42′	2	112° 30′	2.166	1.082	65° 13′	47° 17′
5	56° 15′	.732	.365	36° 12′	20° 3′	3	123° 45′	2.737	1.368	69° 56′	53° 49′
6	67° 30′	.923	.461	42° 43′	24° 45′	4	135°	3.562	1.781	74° 17′	60° 43′
Abeam	90°	1.415	.707	54° 45′	35° 16′	6	157° 30′	7.511	3.754	82° 25′	75° 5′

Effective Impulse of Wind.

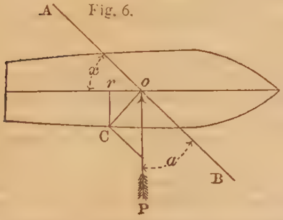


Fig. 6.

Let P o, Fig. 6, represent direction by compass and force of wind on sail, A B; from P draw P C parallel to A B, from o draw o C perpendicular to A B; o C is effective pressure of wind on sail A B, and r C, perpendicular to plane of vessel, is component of o C, which produces lateral motion, as heel and leeway, and $r\,o$ is component of o C, which propels vessel.

I sin. $a =$ P; P cos. $x =$ L; and P sin. $x =$ E. I *representing direct impact and* P *effective pressure of wind on sail,* L *effective impact producing leeway, and* E *effective impact which propels vessel.*

NOTE.—The law as usually given is sin.². This is manifestly incorrect, as it gives results less than normal pressure for angles of small incidence. At an angle of incidence of wind of 25°, the law of sin. is exact. Hence, although it may not be exact at all angles, it is sufficiently so for practical purposes.

ILLUSTRATION 1.—Assume wind 5 points ahead, and I = 100 lbs.

By preceding table angle of course with wind 56° 15′; hence angle of sail a, with wind 36° 12′, as tan 36° 12′ = 2 tan. 20° 3′, and angle x 56° 15′ − 36° 12′ = 20° 3′.

Then, 100 × sin. 36° 12′ = 100 × .5906 = 59.06; 59.06 × cos. 20° 3′ = 59.06 × .9394 = 55.48, and 59.06 × sin. 20° 3′ = 59.06 × .3426 = 20.23 *lbs.*

2.—Assume wind 4 points abaft, and I = 100 lbs.

Then, 100 × sin.² 74° 17′ = 100 × .9626² = 92.66; 92.66 × cos. $\overline{180° - 74° 17′ + 45°}$ = 60° 43′ = 92.66 × .49 = 45.41, and 92.66 × sin. 60° 43′ = 92.66 × .8722 = 80.82 *lbs.*

To Compute Sailing Power of a Vessel.

F f sin. w, sin. $s =$ P.

To Compute Careening Power of a Sailing Vessel.

F f sin. w, cos. $s =$ P. F *representing area of sails in sq. feet, f force of wind in lbs. per sq. foot, w angle of wind to sails, and s angle of sails to course of vessel.*

To Compute Angle of Steady Heel.

Within a Range of 8°.

$\frac{a\,P\,E}{D\,M} = \sin. H$. *a representing area of plain sail in sq. feet,* P *pressure of wind in lbs. per sq. foot,* E *height of centre of effect above mid-draught, in feet,* D *displacement of hull, in lbs., and* M *height of meta-centre in feet.*

P *assumed at 1 lb. per sq. foot, or that due to a brisk wind.*

ILLUSTRATION.—Assume $a = 15\,600$, draught = 20, and M = 62; hence $62 + \frac{20}{10} = 72$, D = 6 800 000, and M = 3.

Then $\frac{15\,600 \times 1 \times 72}{6\,800\,000 \times 3} = \frac{1\,123\,200}{20\,400\,000} = .0555 = 3° 10'$.

Course and Apparent Course of Wind.

Apparent course of a wind against sails of a vessel is resultant of normal course of wind and a course equal and directly opposite to that of vessel.

Fig. 7.

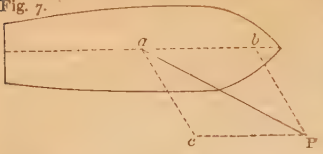


ILLUSTRATION.—If P, Fig. 7, represent direction by compass and force of wind, and $a\,b$ direction and velocity of vessel, from P draw P c parallel and equal to $a\,b$, join $c\,a$ and it will represent direction and force of apparent wind.

Or, $\frac{a\,c}{c\,\mathrm{P}}$ = *ratio of velocity of apparent wind to that of vessel*, $\frac{a\,\mathrm{P}}{c\,\mathrm{P}}$ = *ratio of velocity of wind to that of vessel.*

Resistance of Air. (*Mr. Froude.*)

Resistance of wind to a vessel is estimated as equivalent to square of its velocity.

In a calm, resistance of air to a steamer = one thirty-fourth part of resistance of water, and when a steamer's course is head-to, and combined velocity of vessel and wind = 15 knots, resistance is one ninth of that of the water.

Resistance of air to a sq. foot of surface at right angles to course of a vessel is about .33 lb., and when surface is inclined to direction of wind, pressure varies as *sine* of angle of incidence.

Mean of angles of surface of a steamer exposed to wind may be taken at 45°; hence their resistance is about .25 lb. per sq. foot when wind has a velocity of 10 knots per hour.

If sectional area of a steamer's hull above water is 750 sq. feet, resistance to air at a speed of 10 knots in a calm would be 750 × .25 = 187.5 lbs., and resistance to smoke-pipe, spars, and rigging (brig rigged) would be 201 lbs.

Leeway.

Angle of Leeway in good sailing vessels, close hauled, varies from 8° to 12°, and in inferior vessels it is much greater.

Ardency is tendency of vessel to fly to the wind, a consequence of the centre of effort being abaft centre of lateral resistance.

Slackness is tendency of vessel to fall off from the wind, a consequence of the centre of effort being forward centre of lateral resistance.

Results of Experiments upon Resistance of Screw-propellers, at High Velocities and Immersed at Varying Depths of Water.

Immersion of Screw.	Resistance.	Immersion of Screw.	Resistance.	Immersion of Screw.	Resistance.
Surface.	1	2 feet.	7	4 feet.	7.8
1 foot.	5	3 "	7.5	5 "	8

Slip of Propeller, 15 per cent.; of *Side-wheel* (*feathering blades*), and taking axes of blades as the centre of pressure, 23 per cent.

Freeboard.

Measured from Spar-deck stringer to surface of water. Depth of Hold from underside of spar deck to top of ceiling.

Hold.		Hold.		Hold.		Hold.		Hold.		Hold.	
Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.
8	1.5	12	2.25	16	2.75	20	3.125	24	3.375	28	3.625
10	2	14	2.5	18	3	22	3.25	26	3.5	30	3.75

Plating Iron Hulls.

$\frac{D\,L}{800\,b\,d} = T$. D *representing displacement in tons*, L *length of hull*, b *breadth, and* d *depth*. Or, $.05 f \sqrt{d} = T$. *f representing distance between centres of frames, and* d *depth of plate below load-line, all in feet, and* T *thickness of plate in ins.*

Masts and Spars. *Diameter for Dimensions.*

Lower masts	at spar deck.	Jib-boom	at bowsprit cap.
Bowsprit	" stem.	Yards	in middle.
Topmasts	" lower cap.	Gaffs	at inner end.
Topgallant masts	" topmast cap.	Main and Spanker booms	at taffrail.

Fore and main masts, when of pieces, 1 inch for each 3 to 3.25 feet of whole length. Mizzenmast .66 diameter of mainmast. Masts of one piece 1 inch for each 3.5 to 3.75 feet of whole length.

Bowsprit, depth, equal diameter of mainmast; width, diameter equal to foremast.

Main and fore topmasts	1	inch for each 3 to 3.25	feet of whole length.
Mizzen topmast	1	" " " 3.25 " 3.33	feet of whole length.
Topgallant masts	1	" " " 3.25 " 3.33	feet of whole length.
Royal masts	1	" " " 3.66	feet of whole length.
Topgallant poles	1	" " " 2.87	feet of whole length.
Jib-boom	1	" " " 2 ft. of length beyond bowsprit cap.	
Fore and main yards	1	" " " 4	feet of whole length.
Topsail yards	.875	" " " 4	feet of whole length.
Cross-jack, Topgallant, and Royal yards	1	" " " 5	feet of whole length.
Main and Spanker booms	1	" " " 3.5	feet of whole length.
Gaffs	1	" " " 3.5 to 4	feet of whole length.
Studding-sail yards and booms	1	" " " 4.5 to 4.75	feet of whole length.

Rudder Head. (*Mackrow.*)

$P\,d = T$; $.196\,C\,D^3 = M$; $\sqrt[3]{\frac{T}{.196\,C}} = D$; and $\frac{A\,v^2}{2400} = P$. P *representing pressure on rudder when hard over, in tons,* d *distance of geometrical centre of rudder from axis of motion, in ins.,* T *stress on head, and* M *moment of resistance of head, both in inch-tons,* A *immersed area of rudder in sq. feet,* v *velocity of water passing rudder in knots per hour, and* C *coefficient* = 3.5 *per sq. inch for Iron, and* .125 *for Oak.*

ILLUSTRATION.—Assume area of wooden rudder 24 sq. feet, distance of its geometrical centre from centre of pintles 2 feet, and velocity of water 10 knots.

$$\frac{24 \times 10^2}{2400} = 1 \textit{ ton}. \quad 1 \times 2 \times 12 = 24 \textit{ inch-tons}. \quad \sqrt[3]{\frac{24}{.196 \times .125}} = 9.93 \textit{ ins}.$$

Memoranda.

Weights. — A man requires in a vessel a displacement of 488 lbs. per month, for baggage, stores, water, fuel, etc., in addition to his own weight, which is estimated at 175 lbs. A man and his baggage alone averages 225 lbs.

A ship, 150 feet in length, 32 beam, and 22.83 in depth, or 664 tons, C. H. (O. M.), has stowed 2540 square and 484 round bales of cotton. Total weight of cargo 1 254 448 lbs., equal to 4.57 bales, weighing 1889 lbs., per ton of vessel.

A full built ship of 1625 tons, N. M., can carry 1800 tons' weight of cargo, or stow 4500 bales of pressed cotton.

Hull of iron steamboat *John Stevens* — length 245 feet, beam 31 feet, and hold 11 feet; weight of iron 239 440 lbs. And of one other—length 175 feet, beam 24 feet, and 8 feet deep; weight of iron 159 190 lbs.

Weight of hull of a vessel with an iron frame and oak planking (composite), compared with a hull entirely of wood, is as 8 to 15.

An iron hull weighs about 45 per cent. less than a wooden hull.

Iron ship, 254 feet in length, 42 beam, and 23.5 hold, 1800 tons register, has a stowage of 3200 tons cargo at a draught of 22 feet. Weight of hull in service 1450 tons.

Loss by Weight per Sq. Foot per Month of Metalling of a Vessel's Bottom in Service.

Copper .0061 lb.; Muntz metal .0045 lb.; Zinc .007 lb.; and Iron .0204 lb.

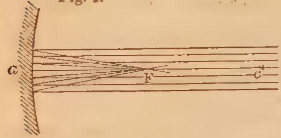
Comparison between Iron and Steel plated Steamers.—In a vessel of 5000 tons displacement, hull of steel plated will weigh 320 tons less = 6.66 *per centum* less.

OPTICS.

Mirrors, in Optics, are either *Plane* or *Spherical*. A plane mirror is a plane reflecting surface, and a spherical mirror is one the reflecting surface of which is a portion of surface of a sphere. It is concave or convex, according as inside or outside of surface is reflected from. Centre of the sphere is termed *Centre of curvature.*

Focus—Point in which a number of rays meet, or would meet if produced.

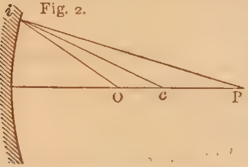
Fig. 1.



Principal Focal Distance is half radius of curvature, and is generally termed the *focal distance.* Line *a c* is termed the *principal axis*, and any other right line through *c* which meets the mirror is termed a *Secondary axis.* When the incident rays are parallel to the *principal axis*, the reflected rays converge to a point, F.

Conjugate Foci are the *foci* of the rays proceeding from any given point in a spherical concave mirror, and which are reflected so as to meet in another point, on a line passing through centre of sphere. Hence, their relation being mutual, they are termed conjugate.

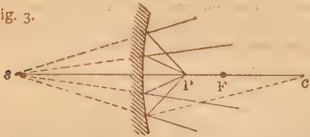
Fig. 2.



Let P be a luminous point on principal axis, Fig. 2, and P *i* a ray; draw the normal line *c i*, which is a radius of the sphere; then *c i* P is angle of incidence, and *c i* O the angle of reflection. equal to it; hence *c i* bisects an angle of triangle P *i* O, and therefore, $\frac{i\,P}{i\,O} = \frac{c\,P}{c\,O}$

When conjugate focus is behind a mirror, and reflected rays diverge, as if emanating from that point, such focus is termed *Virtual*, and a focus in which they actually meet is termed *Real.*

Fig. 3.



As a luminous point, as P, Fig. 3, is moved to the mirror, the conjugate focus moves up from an indefinite distance at back, and meets it at surface of mirror.

If an incident ray converges to a point *s*, at back of mirror, it will be reflected to a point P in front. The conjugate foci P *s* having changed places.

Pencil.—Rays which meet in a focus and are taken collectively.

Objects.—As regards comparative dimensions or volumes, it follows, from similar triangles, that their linear dimensions are directly as their distances from centre of curvature.

To Compute Dimension or Volume of an Image.

When Dimensions and Position of Object are Given, and for either Convex or Concave Mirrors.

$\frac{L}{l} = \frac{D}{F}$, or $\frac{l}{L} = \frac{d}{F}$. L *and* l *representing lengths of image and object*, F *focal length, and* D *and* d *respectively, distances of image and object from principal focus.*

Refraction.

Deviation.—Angle at which a ray is diverted from its original or normal course when subjected to refraction is thus termed.

Indices of Refraction.—Ratio of sine of angle of incidence to sine of angle of refraction, when a ray is diverted from one medium into another, is termed *relative index of refraction* from former to latter.

When a ray is diverted from vacuum into any medium, the ratio is greater than unity, and is termed *absolute index* or *index of refraction.*

Mean Indices of Refraction.

Eye, vitreous humor	1.339	Glass, lead, 3 flint	2.03
" crystalline lens, under	1.379	" lead 2, sand 1	1.99
" " " central	1.4	" " 1, flint 1	1.78
Diamond	2.6	Ice	1.31
Glass, flint	1.57	Quartz	1.54

For indices of other substances, see page 584.

Heat increases refractive power of fluids and glass.

Critical Angle.—Its sine is reciprocal of index of refraction, the incident ray being in the less refractive medium.

$$\text{Thus, } \frac{1}{\text{Index}} = \textit{sin. of angle.}$$

Visual Angle is measure of length of image of a straight line on the retina.

Total Reflection is when rays are incident in the more refractive medium, at an angle greater than the critical angle.

Mirage.—An appearance as of water, over a sandy soil when highly heated by the sun.

Caustic Curves or Lines are the luminous intersections from curve lines, as shown on any reflective surface in a circular vessel.

To Compute Index of Refraction.

$\frac{\text{Sin. I}}{\text{Sin. R}} = \textit{Index.}$ I *representing angle of incidence, and* R *that of refraction.*

To Compute Refraction.

Concave-Convex and Meniscus.—Effect of a concave-convex in refracting light is same as that of a convex lens of same focal distance, and that of a meniscus is same as a concave lens of same focal distance.

Meniscus, with parallel rays $\frac{2\,R\,r}{R-r} = F.$

Magnifying Power.—In *Telescopes* the comparison is the ratio in which it apparently increases length. In *Microscopes* the comparison is between the object as seen in the instrument and by the eye, at the least distance of vision, which is assumed at 10 ins., and the magnifying power of a microscope is equal to the distance at which an object can be most distinctly examined, divided by the focal length of the lens or sphere.

Linear power is number of times it is magnified in length, and *Superficial*, number of times it is magnified in surface.

Magnifying power of microscopes varies, according to object and eye-glass, from 40 to 350 times the linear dimensions of object, or from 1600 to 122 500 times its superficial dimensions.

Apparent Area.—As areas of like figures are as the squares of their linear dimensions, the apparent area of an object varies as square of visual angle subtended by its diameter.

The number expressing *Magnification of Apparent Area* is therefore square of magnifying power as above described.

Illustration.—If diameter of a sphere subtends 1° as seen by the eye, and 10° as seen through a telescope, the telescope is said to have a power of 10 diameters.

To Compute Elements of Mirrors and Lenses.

Mirrors. *Spherical Concave.** $\frac{O\,r}{r-2\,l}=D$; $\frac{l\,r}{r-2\,l}=L$.

Spherical Convex.† $\frac{O\,r}{2\,L+r}=D$; $\frac{L\,r}{2\,L+r}=l$. *Parabolic Concave.* $\frac{d^2}{16\,h}=F$.

Unequally Convex.‡ $\frac{2\,B\,r}{R+r}=F$. *Plano-Convex.*§ $2\,B-.66\,t=F$.

Hyperbolic Concave.‖ *Elliptic Concave.*¶ **Sphere.** $\frac{I}{2\,\overline{I-1}}=F$.

O *representing object* = 1, r *radius of convexity*, l *and* L *length or distance of object from vertex of curve, and from external vertex*, D *dimension of object*, d *diameter of base*, F *focal distance, and* h *depth of mirror in like dimensions*, I *index of refraction, and* t *thickness of lens.*

ILLUSTRATION 1.—Before a concave mirror of 5 feet radius is set an object at 1.5 feet from vertex of curve; what is ratio of apparent dimension of image, and what is length of and distance of object from external vertex? Object = 1.

$$\frac{1\times5}{5-2\times1.5}=2.5\textit{ feet}, \text{ and } \frac{1.5\times5}{5-2\times1.5}=3.75\textit{ feet.}$$

2.—If object is set at 4.5 feet from vertex of a like mirror, what is length of and distance of inverted object from internal vertex?

$$\frac{1\times5}{2\times4.5-5}=1.25\textit{ feet}, \text{ and } \frac{4.5\times5}{2\times4.5-5}=5.625\textit{ feet.}$$

3.—Before a convex mirror of 3.5 feet radius is set an object at 3 feet from vertex of curve; what is length of and distance of object from external curve?

$$\frac{1\times3.5}{2\times3+3.5}=.368\textit{ foot}, \text{ and } \frac{3\times3.5}{2\times3+3.5}=1.105\textit{ feet.}$$

4.—A parabolic reflector has a depth of 1.25 feet and a diameter of 2 feet; what is its focal distance from vertex of internal curve?

$$\frac{2^2}{16\times1.25}=.2\textit{ feet} \text{ or } 2.4\textit{ ins.}$$

Lenses. *Double Convex.* $\frac{R\,r}{\overline{m-1}\times\overline{R+r}}=F$. *When* $R=r$ $\frac{r}{2\,\overline{m-1}}=F$;

$\frac{o\,F}{F-l}=D$; $\frac{l\,F}{F-l}=L$; $\frac{S+F}{F}=P$; $\frac{O\,F}{F-O}=V$; and $\frac{S\,F}{S+F}=O$.

Double Concave. $\frac{R\,r}{\overline{m-1}\times\overline{R+r}}=F$; $\frac{o\,F}{F+L}=D$; $\frac{F\,o-D}{D}=L$; and $\frac{L\,F}{L+F}=l$.

Optical centres are in centres of lens. *Plano-Convex* and *Plano-Concave.* $\frac{r}{m-1}=F$. Optical centres are respectively centres of convex and concave surfaces. *Convex Concave* (*Meniscus*) and *Concavo-Convex.* $\frac{R\,r}{\overline{m-1}\times\overline{R-r}}=F$.

Optical Centres. Convex Concave. Delineate lens in half section, draw R from its centre to circumference of lens (intersection of radii), draw r parallel thereto and extending to its circumference, connect R and r at these external points of contact with circumference and external curve, extend line to axis of lens, and point of contact is centre required. *Concavo-Convex.* Proceed in like manner, but in this case r extends to, or delineates, the inner surface of the lens, and point of contact with axis is centre required.

---

* D or image disappears when $l=.5\,r$. † *When* O *is beyond* F, it will be inverted, as $\frac{O\,r}{2\,L-r}=D$, and $\frac{L\,r}{2\,D+r}=l$. ‡ When equally convex F = R. § When convex side is exposed to parallel rays and when parallel rays fall upon plane side, F = 2 R. ‖ Rays of light, heat, or sound, reflected from focus of a hyperbola, will diverge from its concave surface, ¶ and when from the focus of an ellipse, will be refracted by surface of the other.

When object is beyond focal distance (F), its image (D) will be inverted, as $\frac{O\,F}{l-F}=D$, and $\frac{L\,F}{L-F}=l$.

P *representing magnifying power of lens,* S *limit of normal sight,* 10 *to* 12 *ins. for far-sighted eyes and* 6 *to* 8 *for near-sighted, ordinarily* 10 *ins.,* V *limit of distinct vision,* O *extreme distance of object from optical centre at distinct vision, and m index of refraction.*

ILLUSTRATION 1.—If a double convex lens of flint glass has radii of 6 and 6.25 ins., what is its focal distance? Index of refraction = 1.57, see page 584.

$$\frac{6\times 6.25}{1.57-1\times\overline{6+6.25}}=5.37\ ins.$$

2.—If a double concave lens has a focal distance of 2 ins., and object is 6 ins. from vertex of curve, what is its dimension and what is its distance from vertex of inner curve?

$$\frac{6\times 2}{2+4}=2\ ins.,\text{ and }\frac{4\times 2}{4+2}=1.33\ ins.$$

3.—If focal distance of a single microscope is 4 ins., what is its limit of distinct vision, and what its magnifying power? O = 2.857 *ins.*

$$\frac{2.857\times 4}{4-2.857}=10\ ins.,\text{ and }\frac{10+4}{4}=3.5\ times.$$

Telescopes, Opera-glasses, etc.

$D:o=F:f$; $of\div F=D$, and $\frac{L\,F}{f}=l$; $\frac{L\,f^*}{L-f}\pm\frac{s\,F}{s+F}=F+f$. *f representing length of focal distance from object lens.*

ILLUSTRATION.—Principal focal distance of ocular lens of a telescope is .9 in., of objective lens 90 ins.; what is its magnifying power?

$$90\div .9=100\ times\ the\ object.$$

PILE-DRIVING.

Effect of blow of a ram, or monkey, of a pile-driver, is as square of its velocity; but the impact is not to be estimated directly by this rule, as the degree and extent of the yielding of the pile materially affects it. The rule, therefore, in application, is of value only as a means of comparison.

By my experiments in 1852, to determine the *dynamical* effect of a falling body, it appeared that while the effect was directly as the velocity, it was far greater than that estimated by the usual formula $\sqrt{s\,2\,g}$, which, for a weight of 1 lb. falling 2 feet, would be 11.34 lbs., giving a momentum of 11.34 foot-lbs.; whereas, by the effect shown by the record of actual observations, it would be $W\,v\,4.426=50$ lbs.

Piles are distinguished according to their position and purpose: thus, *Gauge Piles* are driven to define limit of area to be enclosed, or as guides to the permanent piling.

Sheet or *Close Piles* are driven between gauge piles to form a compact and continuous enclosure of the work.

Weight which each pile is required to sustain should be computed as if the pile stood unsupported by any surrounding earth.

A heavy ram and a low fall is most effective condition of operation of a pile-driver, provided height is such that force of blow will not be expended in merely overcoming friction of leader and inertia of pile, and at same time not from such a height as to generate a velocity which will be essentially expended in crushing fibres of head of pile.

---

* + for telescopes and — for opera-glasses, etc.

Refusal of a pile intended to support a weight of 13.5 tons can be safely taken at 10 blows of a ram of 1350 lbs., falling 12 feet, and depressing the pile .8 of an inch at each stroke.

Pneumatic Piles.—A hollow pile of cast iron, 2.5 feet in diameter, was depressed into the Goodwin Sands 33 feet 7 ins. in 5.5 hours.

Nasmyth's Steam Pile-hammer has driven a pile 14 ins. square, and 18 feet in length, 15 feet into a coarse ground, imbedded in a strong clay, in 17 seconds, with 20 blows of monkey, making 70 strokes per minute.

Morin computed work of a ram in foot-lbs., in raising a monkey for 8 hours per day, as follows: Tread-wheel 3900, Winch 2600.

French engineers estimate the safe load for a pile, when driven to refusal of .4 inch under 30 blows, to be 25 tons.

Shaw's Gunpowder Pile-driver is operated by cartridges of powder on head of pile, which are ignited by fall of the ram. 30 to 40 blows per minute have been made under a fall of 5 and 10 feet. 27 piles have been driven in rough gravel and clay 7.2 feet in one day.

To Compute Safe Load that may be Borne by a Pile.

(*Maj. John Sanders, U. S. E.*)

Approximately. $\frac{R\,\overline{h \div d}}{8} = W$. R *representing weight of ram in lbs.*, h *height of fall, and* d *distance pile is depressed by blow, both in feet.*

Illustration.—A ram weighing 3500 lbs., falling 3.5 feet, depressed a pile 4.2 ins.

Then $\frac{3500 \times (42 \div 4.2)}{8} = \frac{35\,000}{8} = 4375$ *lbs., weight which pile would bear with safety.*

Molesworth gives this, but with a variation in symbols and their expression.

To Compute Coefficient of Resistance of the Earth.

$\frac{R\,h}{d} = C$. R *representing resistance of the earth, and* d *as preceding.*

Weisbach gives following formula: Resistance of bed of earth being constant, mechanical effect expended in penetration of pile will be $\frac{R^2 h}{P + R\,d} = W$. P *representing weight of pile in lbs.*

Illustration.—Assuming elements of preceding case, with addition of weight of pile at 1500 lbs.,

$$\frac{3500^2 \times 3.5}{1500 + 3500 \times (4.2 \div 12)} = \frac{42\,875\,000}{1750} = 24\,500 \textit{ lbs.}$$

To Compute Weight of Ram. (*Molesworth.*)

$P\left(\frac{h\,P}{5\,A\,L} - 1\right) = R$. P *representing weight of pile in lbs.*, h *height of fall and* L *length of pile, both in feet, and* A *area of section of pile in sq. ins.*

Theoretical Force of Blow of Ram.

Fall.	Weight. 1000	1200	1500	2000	Fall.	Weight. 1000	1200	1500	2000
Feet.	Lbs.	Lbs.	Lbs.	Lbs.	Feet.	Lbs.	Lbs.	Lbs.	Lbs.
1	8 000	9 600	12 000	16 000	15	31 060	37 272	46 590	62 120
5	17 920	21 504	26 880	35 840	20	35 860	43 032	53 790	71 720
10	25 360	30 432	38 040	50 720	25	40 100	48 720	60 150	80 200

Sheet Piling.

Bevelling.................... 120° | Shoeing........................ 25°

Ringing Engine

Requires 1 man to each 40 lbs. weight of ram, which varies from 450 to 900 lbs.

Pile-sinking.

Mitchell's Screw Piles are constructed of a wrought-iron shaft of suitable diameter, usually from 3 to 8 ins., with 1.5 turns of a cast-iron thread of from 1.5 to 3 feet diameter.

Hydraulic Process is effected by the direction of a stream of water under pressure, within a tube or around the base of a pile, by which the sand or earth is removed.

Pneumatic and Plenum Process.—For illustration and details, see Trautwine's Engineer's Pocket-book, page 326.

Dr. Whewell deduced the following results:

1. A slight increase in hardness of a pile or in weight of a ram will considerably increase distance a pile may be driven.

2. Resistance being great, the lighter a pile the faster it may be driven.

3. Distance driven varies as cube of the weight of ram.

Relative Resistance of Formations to Driving a Pile.

Coral	100	Hard clay	60	Light clay and sand	35
Clay and gravel	83	Clay and sand	45	River silt	25

PNEUMATICS.—AEROMETRY.

Motion of gases by operation of gravity is same as that for liquids. Force or effect of wind increases as square of its velocity.

If a volume of air represented by 1, and of 32°, is heated t degrees without assuming a different tension, the volume becomes $(1 + .002\,088\,t) = V$; and if it requires a temperature in excess of t' 32°, it will then assume volume $(1 + .002\,088\,\overline{t' - 32°})$. All aeriform fluids follow this law of dilatation as well as that of compression proportional to weight.

When air passes into a medium of less density, its velocity is determined by difference of its densities. Under like conditions, a conduit will discharge 30.55 times more air than water.

To Compute the Degree of Rarefaction that may be effected in a Vessel.

Let quantity of air in vessel, tube, and pump be represented by 1, and proportion of capacity of pump to vessel and tube by .33; consequently, it contains .25 of the air in united apparatus.

Upon the first stroke of piston this .25 will be expelled, and .75 of original quantity will remain; .25 of this will be expelled upon second stroke, which is equal to .1875 of original quantity; and consequently there remains in apparatus .5625 of original quantity. Proceeding in this manner, following Table is deduced:

No. of Strokes.	Air Expelled at each Stroke.	Air Remaining in Vessel.
1	$.25 = .25$	$.75 = .75$
2	$\frac{3}{16} = \frac{3}{4 \times 4}$	$\frac{9}{16} = \frac{3 \times 3}{4 \times 4}$
3	$\frac{9}{64} = \frac{3 \times 3}{4 \times 4 \times 4}$	$\frac{27}{64} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4}$

And so on, multiplying air expelled at preceding stroke by 3, and dividing it by 4; and air remaining after each stroke is ascertained by multiplying air remaining after preceding stroke by 3, and dividing it by 4.

Distances at which Different Sounds are Audible.

	Feet.	Miles.
A full human voice speaking in open air, calm	460	.087
In an observable breeze, a powerful human voice with the wind can be heard	15 840	3
Report of a musket	16 000	3.02
Drum	10 560	2
Music, strong brass band	15 840	3
Cannonading, very heavy	575 000	90

In Arctic Ocean, conversation has been maintained over water a distance of 6696 feet.

In a conduit in Paris, the human voice has been heard 3300 feet.

For an echo to be distinctly produced, there must be a distance of 55 feet.

Coefficients of Efflux of Discharge of Air. (D'Aubuisson.)

Orifice in a thin plate	.65	.751
Cylindrical ajutage	.93	.958
Slight conical ajutage	.94	1.09

To Compute Volume of Air Discharged through an Opening into a Vacuum, per Second.

$a\,C\sqrt{2\,g\,h} = V$ *in cube feet.* *a representing area of opening in square feet.* C *coefficient of efflux, and* $\sqrt{2\,g\,h} = 1347.4$, *as shown at page* 428.

Illustration.—Area of opening 1 foot square, and $C = .707$.

Then $1 \times .707 \times 1347.4 = 952.61$ *cube feet.*

Inversely, $V \div a =$ *velocity in feet per second.*

Velocity and Pressure of Wind.

Pressure varies as square of velocity, or $P \propto V^2$.

$V^2 \times .005 = P$; $\sqrt{200\,P} = V$; $v^2 \times .0023 = P$; and $.0023\,v^2 \sin. x = P$.

V *representing velocity in miles per hour, v in feet per second,* P *pressure in lbs. per sq. foot, and x angle of incidence of wind with plane of surface.*

Table deduced from above Formulas.

Velocity per Hour.	Velocity per Minute.	Pressure on a Sq. Foot.	Character of the Wind.	Velocity per Hour.	Velocity per Minute.	Pressure on a Sq. Foot.	Character of the Wind.
Miles.	Feet.	Lbs.		Miles.	Feet.	Lbs.	
1	88	.005	Barely observable.	25	2200	3.125	Very brisk.
2	176	.02	Just perceptible.	30	2640	4.5	High wind.
3	264	.045		35	3080	6.125	
4	352	.08	Light breeze.	40	3520	8	Very high wind.
5	440	.125	Gentle, pleasant wind.	45	3960	10.125	Gale.
6	528	.18		50	4400	12.5	Storm.
8	704	.32		60	5280	18	Great storm.
10	880	.5	Fresh breeze.	80	7040	32	Hurricane.
15	1320	1.125	Brisk blow.	90	7920	40.5	Tornado.
20	1760	2	Stiff breeze.	100	8800	50	

Illustration.—What is pressure per sq. foot, when wind has a velocity of 18 miles per hour? $18^2 \times .005 = 1.62$ *lbs.*

To Compute Force of Wind upon a Surface.

$\frac{v^2\,a \sin.^2 A}{440} = P$. *v representing velocity of wind in feet per second, a area of surface in sq. feet, and* A *angle of incidence of wind.*

At Mount Washington wind has been observed to have had a velocity of 150 miles per hour.

Extreme pressure of wind at Greenwich Observatory for a period of 20 years was 41 lbs. per sq. foot.

Force of wind upon a surface, perpendicular to its direction, has been observed as high as 57.75 lbs. per sq. foot; velocity = 159 feet per second.

Dr. Hutton deduced that resistance of air varied as square of velocity nearly, and to an inclined surface as 1.84 power of sine × cosine.

Figure of a plane makes no appreciable difference in resistance, but convex surface of a hemisphere, with a surface double the base, has only half the resistance.

At high velocities, experiments upon railways show that the resistance becomes nearly a constant quantity.

Course of Wind.

Cyclones.

Direction in Northern Hemisphere.

Direction in Southern Hemisphere.

Wind has its direction nearly at right angles to line between points of highest and lowest pressure of air, or barometer readings, and its course is with the point of lowest pressure at its left, and its velocity is directly as difference of the pressures.

In Northern Temperate zone, winds course around an area of low pressure in *reverse* direction to course of hands of a watch, and they flow away from a location of high pressure, and cause an apparent course of the winds in direction of course of the hands.

To Compute Resistance of a Plane Surface to Air.

$.0022\, a\, v^2 = P$ *in lbs.* *a representing area of plane in sq. feet, v velocity in direction of wind in feet per second, + when it moves opposite, and — when with the wind.*

To Compute Resistance of a Plane Surface when moving at an Angle to Air.

$$\frac{v^2\, a \sin.^2 x}{450} = P \text{ in lbs.}$$ *x representing angle of incidence.*

To Compute Height of a Column of Mercury to induce an Efflux of Air through a given Nozzle.

Barometer assumed at 2.46 *feet* = 29.52 *ins., and Temperature* 52°.

$\dfrac{P^2}{48.073^2\, d^4} = H$, and $48.073\, d^2 \sqrt{H} = P$. *d representing diameter of nozzle and* H *height of column of mercury, both in feet, and* P *volume of air in lbs. per one second.*

ILLUSTRATION.—Assume $d = .19$, and $P = .7$ lbs.

$$\frac{.7^2}{48.073^2 \times .19^4} = .1511 \text{ foot.} \quad 48.073 \times .19^2 \sqrt{.1511} = .7.$$

To Compute Pressure or Weight of Air under a given Height of Barometer and Temperature, Discharged in One Second.

$30.787\, d^2 \sqrt{B\, \dfrac{b+B}{t}} =$ *pressure in lbs.* Or, $48.073\, d^2 \sqrt{B} =$ *lbs.* *b representing height of barometer in external air,* B *manometer or pressure of air in reservoir in mercury, both in feet, and t temperature of air or gas in degrees.*

ILLUSTRATION.—Assume $b = 2.5$ feet; $d = .25$ foot; $B = .1$ foot; and $t = 1.055$ feet.

Then $30.787 \times .0625 \sqrt{.1 \times \dfrac{2.5 + .1}{1.055}} = 1.924 \times \sqrt{.2465} = .9543$ *lbs.*

To Compute Temperature for a given Latitude and Elevation.

$82.8 \cos. l - \overline{.001\,981\ E} - .4 = t$. E *representing elevation in feet.*

ILLUSTRATION.—Assume $l = 45°$; cos. $= .707$; and E = 656 *feet.*

Then $82.8 \times .707 - \overline{.001\,981 \times 656} - .4 = 58.54 - \overline{1.299 - .4} = 58.54 - .899 = 57.641$.

To Compute Volume of Air or Gas Discharged through an Opening and under a Pressure above that of External Air.

Air. $1347.4\ C \frac{d^2}{b} \sqrt{B\ (b' + B)\ T} = V$ *in cube feet per second.*

$T = 1 + .002\,22\ (t - 32°)$, and $b' = 2.5 - .000\,09$ *elevation.*

Or, $621.28\ d^2 \sqrt{B} = V$.

ILLUSTRATION.—What would be volume of air that would flow through a nozzle .246 foot in diam. from a reservoir under a pressure of .098 foot of mercury, into air under a barometric pressure of 2.477 feet, temperature of air 55.4°, location 45° of latitude, and at an elevation of 650 feet above level of sea?

$C = .75$; $b' = 2.5 - .000\,09 \times 650 = 2.4415\ (2.44)$; and $T = 1.0502$.

Then $1347.4 \times .75 \frac{.246^2}{2.477} \sqrt{.098\ (2.44 + .098) \times 1.0502} = 24.689 \times \sqrt{.2617} = 12.63$ *cube feet.*

When Densities of External Air and that in Reservoir are Equal.

$1347.4\ C \frac{d^2}{b'} \sqrt{B\ (b + B)\ T} = V$. b' *representing height of mercury in reservoir.*

Gas. $\frac{4231}{\sqrt{p}} \sqrt{\frac{B\ d^5}{L + 42 \times d}} = V$. p *representing specific gravity of gas compared with air, and* L *length of pipe or conduit in feet.*

ILLUSTRATION.—If a pipe .05 feet in diameter and 420 feet in length, communicates with a gasometer charged with carburetted hydrogen (illuminating gas), under a water pressure as indicated by a manometer of .1088 foot, what would be the discharge per second?

$d = .05$ *foot*; $L = 420$ *feet*; and $B = \frac{.1088}{13.6^*} = .008$ *foot.* Specific gravity of gas .5625.

$$\frac{4231}{\sqrt{.5625}} \sqrt{\frac{.008 \times .05^5}{420 + 42 \times .05}} = \frac{4231}{.75} \sqrt{\frac{.000\,000\,002\,500\,0}{420 + 2.1}} = .013\,71 \text{ cube foot.}$$

Resistance of Curves and Angles.—Curves and angles increase resistance to discharge of air or gas very materially. By experiment of D'Aubuisson 7 angles of 45° reduced discharge of gas one fourth.

To Compute Diameter of Discharge-pipe or Nozzle.

When Length and Diameter of Pipe, Volume, and Pressure are given.

$$\sqrt[4]{\frac{42\ V^2\ d^5}{4230^2\ B\ d^5 - L\ V^2}} = d' \text{ in feet.}$$

ILLUSTRATION.—If a pipe 1000 feet in length, and .4 foot in diameter, leads to a reservoir of air, under a mercurial manometric pressure of .18 foot, what diameter must be given to a nozzle to discharge 4 cube feet per second?

Then $\sqrt[4]{\frac{42 \times 4^2 \times .4^5}{4230^2 \times .18 \times .4^5 - 1000 \times 4^2}} = \sqrt[4]{\frac{6.881\,28}{32\,980.19 - 16\,000}} = \sqrt[4]{.000\,405\,2} = .1418$ *foot* $= 1.703$ *ins.*

Volumes of two gases flowing through equal orifices, and under equal pressures, are in inverse ratio of square roots of their respective densities.

* Specific gravity of mercury compared with water.

RAILWAYS.

To Define a Curve.—Fig. 1. (*Molesworth.*)

Fig. 1.

$\frac{1719\,c}{a}$ or $t\ \overline{\tan.\ x} = R$; $R\ (\text{cotan.}\ x) = t$;

$\frac{1719\,c}{R} = a$; $R\ (\text{cosec.}\ x - 1) = d$;

$R\ (\text{cosin.}\ x) = s$; $R\ (\text{coversin.}\ x) = V$;

$\frac{5400 - x}{a} = n$, and $(5400 - x)\ .000\,582\ R = l$.

c representing any chord, t length of tangent, d distance of centre of curve from intersection of tangents, s half chord of curve, and l length of curve, all in like dimensions, a tangential angle of c in minutes, n number of chords in curve, and x half angle of intersection, but in formulas for number of chords and length of curve to be expressed in minutes.

Illustration.—Assume radius 900 and chord 400 feet; angle of intersection = $12° 44' = 764$ minutes, and $x = 56° 15' 5''$.

Tangent of $56° 15' 5'' = 1.49673$. Cotangent $= .66814$.

$\frac{1719 \times 400}{764} = R = 900$ *feet*; $\frac{1719 \times 400}{900} = 764$ *minutes*; $900 \times .66814 = t = 601.33$ *feet*; $900 \times \overline{1.20269 - 1} = d = 182.42$ *feet*; $900 \times .55555 = s = 500$ *feet*; $900 \times .16833 = V = 161.5$ *feet*; $\frac{5400 - 3379}{764} = 2.645$ *times*, and $.000\,582 \times 900 \times \overline{5400 - 3379} = 1058.6$ *feet*.

Tangential Angles for Chords of One Chain.

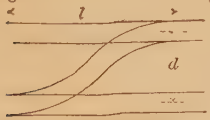
Radius of Curve.	Tangential Angle.	Radius of Curve.	Tangential Angle.	Radius of Curve.	Tangential Angle.	Radius of Curve.	Tangential Angle.
Chains.		Chains.		Chains.			
5	5° 43.8′	15	1° 54.6′	40	42.97′	1 mile	21.48′
8	3° 34.87′	20	1° 25.95′	45	38.2′	1.25 mil's	17.19′
9	3° 11′	25	1° 8.76′	50	34.38′	1.5 miles	14.33′
10	2° 51.9′	30	57.3′	60	28.65′	1.75 "	12.28′
12	2° 23.25′	35	49.11′	70	24.55′	2 "	10.74′

Note.—Angle for 2 chain chords is double angle for 1 chain chords. Angle for .5 chain chords is .5 the angle for 1 chain chords.

Curves of less than 20 chains radius should be set out in .5 chain chords. Curves of more than 1 mile radius may be set out in 2 chain chords.

Angles in above Table are in degrees, minutes, and decimals of minutes.

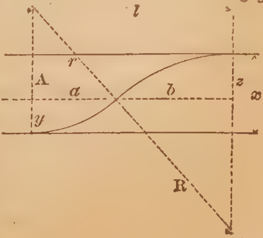
Fig. 2.



Sidings.

$2\sqrt{d\,R - (.5\,d)^2} = l$. *R representing radius of curve, l length of curve over points, and d distance between tracks, all in feet.*

Fig. 3.



Turn-out of Unequal Radii.

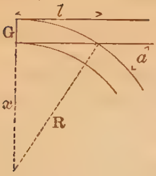
$\frac{r\,x}{R + r} = y$; $x - y = z$; $a + b = l$; $r - y = A$;

$\sqrt{y\,(r + A)} = a$; $R - z = B$; $\sqrt{z\,(R + B)} = b$.

R and r representing radii of the curves respectively as to length, x distance between outer rails of tracks and other symbols as shown, all in feet.

Points and Crossings.

Fig. 4.



$\sqrt{(R + x)\,G} = l$; $\frac{l}{R} = \sin.\ a$; $\frac{G}{\text{ver. sin. } a} = R$. R *representing radius of curves, G gauge of road, a angle of crossing, and* $x = R - G$, *all in feet.*

In horizontal curves, width required for clearance of flange of wheel, and for width of rail at heel of switch, render it necessary to make an allowance in length of l, as ascertained by formula.

For other diagrams and formulas, see Molesworth's Pocket-book, pp. 208–18, 21st edition.

To Compute Tangential Angle for Curves.

$\frac{1719\ c}{R} = a$. *c representing chord in feet, and a angle in minutes.*

Illustration.—What is angle for a curve with a radius of 900 feet, and a chord of 400 feet?

$$\frac{1719 \times 400}{900} = 764 \textit{ minutes.}$$

Curving of Rails.

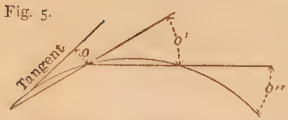
$\frac{1.56\ l^2}{R} = v$. *l representing length of rail in feet, v versed sine at centre, when curved, in ins.*

Illustration.—What is curve for a rail 20 feet in length, with a radius of 900 feet?

$$\frac{1.5 \times 20^2}{900} = .666 \textit{ ins.}$$

Curves by Offsets in Equal Chords.

Fig. 5.



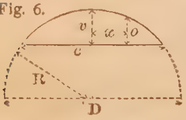
$\frac{\text{Chord}^2}{2\,R} = o$ *offset.* $\frac{\text{Chord}^2}{R} = 2, o$ *offset.*

Illustration.—Assume chords 150, and radius 900 feet.

$$\frac{22\,500}{2 \times 900} = 12.5 \textit{ feet}; \quad \frac{22\,500}{900} = 25 \textit{ feet.}$$

To Compute Versed Sines and Ordinates of Curves.

Fig. 6.



$R - \sqrt{R^2 - (.5\ C)^2} = v$; $\frac{(.5\ C)^2}{v} + v = D$; and $\sqrt{R^2 - x^2} - (R - v) = o$. D *representing diameter of circle, and v versed sine of curve.*

Illustration.—Assume radius 900, and chord 400 feet.

$$900 - \sqrt{810\,000 - 40\,000} = 900 - 877.5 = 12.5 \textit{ feet.}$$

Relation of Base of Driving or Rigid Wheels to Curve.

$\frac{R}{9\ G} = B$. R *representing minimum radius of curve,* G *gauge of road, and* B *base, all in feet.*

To Compute Elevation of Outer Rail.

For any Radius or Combination of Curve with Straight Line.

$.5\ V\sqrt{G} = c$. V *representing velocity of train in feet per second,* G *gauge of road, and c length of a chord, both in feet, the versed sine of which = elevation in ins.*

On Curves.

$\frac{V^2}{1.25\ R}\ G = E$. E *representing elevation of outer rail in ins.*

Radii of Curves set out in Tangential Angles.

Angle for Chord of 100 Feet.	Radius of Curve.	Angle for Chord of 100 Feet.	Radius of Curve.	Angle for Chord of 100 Feet.	Radius of Curve.	Angle for Chord of 100 Feet.	Radius of Curve.
° ′	Feet.	° ′	Feet.	° ′	Feet.	° ′	Feet.
30	5729.6	2 30	1145.9	4 30	636.6	6 30	440.7
1	2864.8	3	954.9	5	573	7	409.3
1 30	1909.9	3 30	818.5	5 30	520.9	7 30	382
2	1432.4	4	716.2	6	447.5	8	358.1

Note.—If chords of less length are used, radius will be proportional thereto.

To Ascertain Radius of Curve in Inches for Scale, in Feet per Inch.
Divide radius of curve in feet by scale of feet per inch.

To Compute Required Weight of Rail.

Rule.—Multiply extreme load upon one driving-wheel in lbs. by .005, and product will give weight of rail in lbs. per yard.

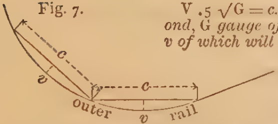
To Compute Radius of Curve and Wheel Base.

$9\,B\,G = R.\quad \frac{R}{9\,G} = B.$ B *representing maximum rigid wheel base of cars, and* G *gauge of way, both in feet.*

To Determine Elevation of Outer Rail.

For any Radius or Construction of Curve with Straight.—Fig. 7.

Fig. 7.



$V\,.5\,\sqrt{G} = c.$ V *representing speed of train in feet per second,* G *gauge of rails in feet, and* c *length of chord, versed sine* v *of which will give at its centre the elevation required.*

Thus, determine chord c, align it on inner side of rail, and distance of rail from it at centre of its length will give elevation required, whatever the radius of rail.

For Curves. $\frac{[.782\,V^2\,(N\,D\,W)] - 4\,P\,R}{N\,D\,R} = E$; Or, $W\,\frac{V^2}{1.25\,R} = E.$ D *representing diameter of wheels,* W *width of gauge,* P *lateral play between flange and rail, and* R *radius of curve, all in feet,* $1 \div N$ *ratio of inclination of tire,* V *velocity of train in miles per hour, and* E *elevation of outer rail in ins.* (*Molesworth.*)

$\frac{W\,C\,(d+l)}{2\,R}$ = *resistance due to curve, and* W *representing weight of body, both in lbs.,* C *coefficient of friction of wheels upon rails* = .1 *to* .27, *according to condition of weather,* d *distance of rails apart,* l *length of rigid wheel base, and* R *radius of curve, all in feet.* (*Morrison.*)

Illustration.—Assume weight of locomotive 30 tons, radius of curve 1000 feet, distance of rails apart 4 feet 8.75 ins., length of base 10 feet, and rails, dry, C = .1.

$$\frac{30 \times 2240 \times .1 \times (4.73 + 10)}{2 \times 1000} = 494.93\ \textit{lbs.}$$

To Compute Resistance due to Gravity upon an Inclination.

$$\frac{2240}{\text{gradient}} = \textit{lbs. per ton of train.}$$

Rise per Mile, and Resistance to Gravity, in Lbs. per Ton.

Gradient of 1 inch..	20	25	30	35	40	45	50	60	70	80	90	100
Rise in feet........	264	211	176	151	132	117	106	88	75	66	59	53
Resistance.........	112	89.6	74.7	64	56	50	44.8	37.3	32	28	24.8	22.4

To Compute Load which a Locomotive will Draw up an Inclination.

$T \div \overline{r + r'} - W = L$. T *representing tractive power of locomotive in lbs.*, r *resistance due to gravity, and* r' *resistance due to assumed velocity of train in lbs. per ton,* W *weight of locomotive and tender, and* L *load locomotive can draw, in tons, exclusive of its own weight and tender.*

Coefficients of Traction of Locomotives.—Railroads in good order, etc., 4 to 6 lbs.; in ordinary condition, 8 lbs.

In coupled engines adhesion is due to load upon wheels coupled to drivers.

To Compute Traction, Retraction, and Adhesive Power of a Locomotive or Train.

When upon a Level. $a\,s\,P \div D = T$. a *representing area of one cylinder in sq. ins.*, s *stroke of piston and* D *diameter of driving wheels, both in feet,* P *mean pressure of steam in lbs. per sq. inch, and* T *traction, in lbs.*

When upon an Inclination. $a\,s\,P \div D - r\,w\,h = T$. r *representing resistance per ton,* w *weight of locomotive upon driving-wheels, in tons,* h *height of rise in feet per* 100 *of road, and* $R = r\,w\,h =$ *retraction, in lbs.*

$C\,w\,b \div 100 = A$. b *representing base of inclination in feet per* 100 *of road.*

$C\,w = A$. $C =$ *coefficient in lbs. per ton, and* A *adhesion, in lbs.*

When Velocity of a Train is considered.

When upon a Level, $W\,(C + \sqrt{V}) = R$. *When upon an Inclination,* $W\,(r\,h + C + \sqrt{V}) = R$. V *representing velocity of train in miles per hour.*

Illustration.—A train weighing 200 tons is to be driven up a grade of 52.8 feet per mile, with a velocity of 16 miles per hour; required the retractive power?

52.8 per mile = 1 in 100 feet $= r = 22.4$ lbs. $C = 5$.

$200\,(22.4 \times 1 + 5 + \sqrt{16}) = 200 \times \overline{22.4 + 9} = 6280$ *lbs.*

Velocity of Trains.

Miles per hour	10	15	20	30	40	50	60	70
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Resistance upon straight line per ton	8.5	9.25	10.25	13.25	17.25	22.5	29	36.5
Do., with sharp curves and strong wind*	13	14	15.5	20	26	34	43.5	55

* Equal to 50 per cent. added to resistance upon a straight line.

Friction of locomotive engines is about 9 per cent., or 2 lbs. per ton of weight.

Case hardening of wheel tires reduces their friction from .14 to .08 part of load.

To Compute Maximum Load that can be drawn by an Engine, up the Maximum Grade that it can Attain, Weight and Grade being given. (*Maj. McClellan, U.S.A.*)

$\dfrac{.2\,A}{.4242\,G + 8} = L$, and $\dfrac{.2\,A - 8\,L}{.4242\,L} = G$. A *representing adhesive weight of engine, in lbs.,* G *grade in feet per mile, and* L *load, in tons.*

Note 1.—When rails are out of order, and slippery, etc., for .2 A, put .143 A.

2.—With an engine of 4 drivers, put .6 as weight resting upon drivers; with 6 drivers the entire weight rests upon them.

Illustration.—An engine weighing 30 tons has 6 drivers; what are the *maximum* loads it can draw upon a level, and upon a grade of 250 feet, and what is its *maximum* grade for that load?

$\dfrac{.2 \times 2240 \times 30}{.4242 + 8} = \dfrac{13\,440}{8.4242} = 1595.4$ *tons upon a level.* $\dfrac{.2 \times 2240 \times 30}{.4252 \times 250 + 8} = \dfrac{13\,440}{114.05} =$ 117.8 *tons up a grade of* 250 *feet.* $\dfrac{.2 \times 2240 \times 30 - 8 \times 117.8}{.4242 \times 117.8} = \dfrac{12\,497}{49.97} = 250.1$ *feet.*

Adhesion of a 4 wheeled locomotive, compared with one of 6 wheels, is as 5 to 8.

OPERATION OF LOCOMOTIVES. (*O. Chanute, Am. Soc. C. E.*)

Adhesion.

Adhesion of a locomotive is friction of its driving-wheels upon the rails, varying with condition of the surface, and must exceed traction of the engine upon them, otherwise the wheels will slip.

Improvements heretofore made in the construction of locomotives and tracks have gradually increased the proportion which the adhesion bears to the insistent weight upon the driving-wheels.

The first accurate experiments were those of Mr. Wood upon the early English coal railways. He deduced the adhesion to be as follows:

Upon perfectly dry rails........................ .14 of weight on drivers.
" damp or muddy rails...................... .08 " " " "
" very greasy rails........................ .04 " " " "

In 1838, B. H. Latrobe indicated .13 as a safe working adhesion, while modern European practice assumes about .2 of weight as maximum, and .11 as a minimum, except perhaps in some mountainous regions, subject to mists. Thus, on the Sœmmering line, adhesion is generally .16, and between Pontedecimo and Busalla, in Italy, it never exceeds .12 in open cuttings, or .1 in tunnels.

Extensive experiments made upon French railways, 1862–67, by Messrs. Vuillemin, Guebhard, and Dieudonné gave following coefficients in actual working: *dry weather*, extreme, .105 to .2; *damp*, .132 to .139; *wet*, .078 to .164; *light rain*, .09; *extreme rain*, .109 to .2, *mean*, .13; *rain and fog*, .115 to .14; *heavy rain*, 16.

Materially better results are obtained in United States, partly, perhaps, in consequence of greater dryness of the weather, and certainly because of the American method of construction and equalizing the weight between the drivers, and of making the locomotive so flexible as to adapt itself to inequalities in the track.

Modern engines in America can safely be relied upon to operate up to an adhesion equal to .222 in summer and .2 in winter, of weight upon the driving wheels.

From these data the following tables have been computed:

Coefficients of Adhesion upon Driving Wheels per Ton.

Condition of Rails.	European Practice.		American Practice.		Condition of Rails.	European Practice.		American Practice.	
	C.	Lbs.	C.	Lbs.		C.	Lbs.	C.	Lbs.
Rails very dry.....	.3	670	.33	667	In misty weather.	.015	350	.2	400
Rails very wet.....	.27	600	.25	500	In frost and snow.	.09	200	.16	333
Ordinary working..	.2	450	.222	444					

Adhesion of Locomotives, in Lbs. (.222 in Summer and .2 in Winter).

Type of Locomotive.	No. of Drivers.	Weight. Locomotive.	Weight. On Drivers.	Adhesion. Summer.	Adhesion. Winter.
		Lbs.	Lbs.	Lbs.	Lbs.
American..........	4 wheels coupled....	64 000	42 000	9 350	8 400
Ten-wheeled.......	6 " connected..	78 000	58 000	13 000	11 600
Mogul.............	6 " " ..	88 000	72 000	16 000	14 000
Consolidation......	8 " " ..	100 000	88 000	19 550	17 600
Tank switching....	6 " " ..	68 000	68 000	15 100	13 600
" "	4 " " ..	48 000	48 000	10 650	9 600

Tractive Power.

Traction of a locomotive is the horizontal resultant on the track of the pressure of the steam, as applied in the cylinders.

$D^2 P L \div W = T$. D *representing diameter of cylinder*, L *length of stroke, and* W *diameter of driving wheels, all in ins.*, P *mean pressure in cylinder, in lbs. per sq. inch, and* T *tractive force on rails, in lbs.*

ILLUSTRATION.—Assume a locomotive, cylinders 18 ins. in diam., 22 ins. stroke, wheels 68 ins. in diam., and *average* steam pressure in cylinders 50 lbs. per sq. inch.

Then $18 \times 18 \times 50 \times 22 \div 68 = 5241$ *lbs.*

Train Resistances.

Usual formula for train resistances, *on a level and straight line*, is

$\frac{V^2}{171} + 8 = R$ *per ton of train*, and $\frac{V^2}{240} + 6 = R$ *per ton of train alone.* V *representing velocity in miles per hour, and* 8 *constant axle friction.* (*D. K. Clark.*)

NOTE.—To meet the unfavorable conditions of quick curves, strong winds, and imperfection of road, Mr. Clark estimates results as obtained by above formula should be increased 50 per cent.

ILLUSTRATION.—At 20 miles per hour, the resistance would be:

$$20^2 \div 171 + 8 = 10.3 \textit{ lbs. per ton of train.}$$

This formula, however, is empirical. It gives results which are too large for freight trains at moderate speeds, and too small for passenger trains at high speeds.

Engineers are not agreed as to exact measure and value of each of the elements of train resistances, but following approximations are sufficient for practical use:

Analysis of Train Resistances.

Resistance of trains to traction may be divided into four principal elements: 1st. Grades; 2d. Curves; 3d. Wheel friction; 4th. Atmosphere.

1st. *Grades.*—Gradients generally oppose largest element of resistance to trains. Their influence is entirely independent of speed. The measure of this resistance is equal to weight of train multiplied by rate of inclination or per cent. of grade. Thus, a gradient of .5 per 100 feet (26.4 feet per mile) offers a resistance of $\frac{5 \times 2000}{10 \times 100} = 11.2$ lbs. per ton, or 10 lbs. per 2000 lbs., which is to be multiplied by weight in tons of entire train.

Following table shows resistance, due to gravity alone, for the most usual grades, in lbs. per ton of train:

1st. *Resistance due to Grades.*

Rate per 100 feet	.1	.2	.3	.4	.5	.6	.7	.8
Lbs. per ton of 2240 lbs.	2.24	4.48	6.72	8.96	11.2	13.44	15.68	17.92
Rate per mile	5	11	16	21	26	32	37	42
Lbs. per ton of 2000 lbs.	2	4	6	8	10	12	14	16
Rate per 100 feet	.9	1	1.1	1.2	1.3	1.4	1.5	1.6
Lbs. per ton of 2240 lbs.	20.16	22.4	24.64	26.88	29.12	31.36	33.6	35.84
Rate per mile	47	53	58	63	68	74	79	85
Lbs. per ton of 2000 lbs.	18	20	22	24	26	28	30	32

2d. *Curves.*—Recent European formula is that given by Baron von Weber.

$$.6504 \div R - 55 = W. \quad R \textit{ representing radius of curve in metres.}$$

This formula assumes that resistance due to curve increases faster than radius diminishes. It gives results varying from a resistance of .8 lb. per 2000 lbs. per degree for a curve of 1000 metres radius (3310 feet, or 1° 44′) to a resistance of 1.67 lbs. per 2000 lbs. per degree for curves of 100 metres radius (331 feet, or 17° 20′).

Messrs Vuillemin, Guebhard, and Dieudonné found curve-resistance to European rolling-stock to be from .8 to 1 lb. per 2000 lbs. per degree, on a gauge of 4 feet 8.5 ins., while Mr. B. H. Latrobe, in 1844, found that with American cars resistance on a curve of 400 feet radius did not exceed .56 lb. per 2000 lbs. per degree.

Resistance of same curve varies with coning given tires of wheels, elevation of outer rail, and speed of train running over it, but both reasoning and experiment indicate that the general resistance of curves increases very nearly in direct proportion to degree of curvature, or inversely to the radius.

Recent American experiments show that a safe allowance for curve resistance may be estimated at .125 of a lb. per 2000 lbs. for each foot in width of gauge. Thus, for 3 feet gauge resistance would be .375 lb. per degree of curve; for standard gauge of 4 feet 8.5 ins. .589, say .60, and for 6 feet gauge .75 lb. per degree.

For standard gauge, when radius is given in feet, resistance due to this element is:

$$.60 \times 5730 \div R = C \textit{ in lbs. per ton of train.}$$

This is somewhat reduced when curve coincides with that for which wheels are coned (generally about 3°), and when train runs over it, at precise speed for which outer rail is elevated, an allowance of .5 lb. per ton per degree is found to give good results in practice.

2d. *Resistance on Curves.*

It follows from above estimate of curve resistance that, in order to have the same resistance on a curve as on a straight line, the gradient should be diminished by .03 per 100 feet of each degree of curve. Thus a 3° curve requires an easing of the grade by .09 per 100 feet, a 10° curve an easing of .3 per 100, etc.

This, however, need only be done upon the *limiting* gradients, and when sum of grade and curve resistances exceeds resistance which has been assumed as limiting the trains.

3d. *Resistance due to Wheel Friction.*

Experimenters are not agreed whether friction of wheels increases simply with weight which they carry, but also in some ratio with the speed. Originally assumed as a constant at 8 lbs. per ton, improvements in condition of track (steel rails, etc.) and in construction and lubrication of rolling-stock have reduced it to 3.5 and 4 lbs. per ton for well oiled trains. Under ordinary circumstances, in summer, it will be safe to estimate it at 5 lbs. per ton on first-class tracks, and 6 lbs. per ton on fair tracks. It may run up to 7 or 8 lbs. per ton on bad tracks (iron rails) in summer, and all these amounts should be increased from 25 to 50 per cent. in cold climates in winter, to allow for inferior lubrication.

4th. *Resistance due to Atmosphere.*

Atmospheric resistance to trains, complicated as it is by the wind which may be prevailing, has not been accurately ascertained by experiment. It consists of—1st. Head resistance of first car of train, which is presumably equal to its exposed area, in sq. feet, multiplied by air pressure due to speed.

2d. Head resistance of each subsequent car. This varies with distance they are coupled apart, and so shield each other from end air pressure due to speed.

3d. Friction of air against sides of each car depending upon the speed. This is generally so small that it may be neglected altogether.

4th. Effect due to prevailing wind, which modifies above three items of resistance. A head wind retards the train, a rear wind aids it, while a side wind increases resistance by pressing flanges of wheels against one rail, and, in consequence of curves, a train may assume all of these positions to same wind.

Recent experiments on Erie Railway seem to indicate that in a dead calm resistance of first car of a *freight train* may be assumed at an exposed surface of 63 sq. feet,* multiplied by air pressure due to speed, and that each subsequent car may be assumed to offer a resistance of 20 per cent of that of first car, while in a *passenger train* first car may be assumed at an area of 90 sq. feet,† multiplied by air pressure due to speed, and that each subsequent car adds an increment equal to 40 per cent. that of first car, in consequence of greater distance they are coupled apart.

This resistance is, of course, entirely independent of cars being loaded or empty. In practice it has been found that an allowance of 1.5 to 2 lbs per ton of weight of a *freight train* covers atmospheric resistance, except in very high winds.

In consequence of complexity of elements above enumerated, exact formulas cannot probably be now given for train resistances, but following, if applied with judgment (and modified to fit circumstances), will be found to give fairly accurate results in practice. They are for standard gauge, and in making them, curve resistance has been assumed at .5 lb. per degree, wheel friction at 5 lbs., exposed end area of first car at 90 sq. feet for passenger cars and 63 feet for freight cars, and increment for succeeding cars at .4 for passenger trains and .2 for freight trains.

Passenger Train. $W\left(G+\frac{C^\circ}{2}+5\right)+\left(1+\frac{n-1}{2.5}\right)90\,P=R.$

Freight Train. $W\left(G+\frac{C^\circ}{2}+5\right)+\left(1+\frac{n-1}{5}\right)63\,P=R.$

* This is less than area of car, which generally measures about 71 sq. feet; but part is shielded by tender, and parts being convex, as wheels, bolts, etc., offer less resistance than a flat plane.

† Not only is end area of passenger cars greater than that of freight cars, but in consequence of the projecting roof the end forms a hood in nature of a concave surface, and so opposes greater resistance than a flat plane.

W representing weight of train, without engine, in tons (2000 *lbs.*). G *resistance of gradient per ton* (2000 *lbs.;* see table, page 683). C° *curve in degrees, n number of cars in train,* P *pressure per sq. foot due to speed, to which an allowance must be made for wind, if existing,* R *resistance of train, and* 5, *wheel friction, both in lbs.*

ILLUSTRATION 1.—Assume a passenger train of 5 cars, weighing 136 tons (2000 lbs.), ascending a grade .5 per 100 (26.4 feet per mile), with curves of 4°, at a speed of 60 miles per hour (for which the pressure is 18 lbs. per sq. foot), resistance will be:

$$136\,(10 + 2 + 5) + \left(1 + \frac{4}{2.5}\right)(90 \times 18) = 6524 \textit{ lbs., of which } 2312 \textit{ lbs. are due to}$$

grade, curve, and wheels, and 4212 *lbs. to atmospheric resistance.*

2.—Assume a freight train of 31 cars, weighing 620 tons (2000 lbs.), turning a curve of 3°, up a grade of 52.8 feet per mile (1 foot per 100), at a speed of 21 miles per hour (pressure 2 lbs. per sq. foot), resistance will be:

$$620\,(20 + 1.5 + 5) + \left(1 + \frac{30}{5}\right)(63 \times 2) = 17\,312 \textit{ lbs.}, \text{ requiring a "Consolidation"}$$

engine to haul it, allowance being made for possible winds, etc.

Assume conversely, it is desired to know how many tons an American engine, with an adhesion of 10 650 lbs., will draw up a grade of .9 per 100 (47 feet per mile), with curves of 4°, assuming atmospheric resistance between 1.5 to 2 lbs. per ton of train.

Resistance from grade $.9 \times 2000 \div 100$ .... = 18 *lbs.*
" " curve $4 \div 2$ .... = 2 "
" " wheel friction 5, atmosphere 2..... = 7 " } 27 *lbs.*

Hence, $10\,650 \div 27 = 395$ *tons, or about* 20 *cars*, and in winter same engine will haul $9600 \div 27 = 355$ *tons* (2000 lbs.), *or about* 18 *cars.*

Following table approximates to best modern practice. For *freight trains* it gives aggregate resistance, in lbs. per ton (2000 lbs.), for various grades and curves. In using it, it is sufficient to divide the adhesion in lbs. of locomotive used by number found in table, in order to obtain number of tons of train that it will haul at ordinary speeds on gradient and curve selected. Of course, if grade has been equated for curves, only number found in first column (for straight lines) is to be used in computing tons of train on limiting gradient.

Approximate Freight-train Resistances.

Gauge 4 feet 8.5 ins.

In Lbs. per 2000 lbs. at Ordinary Speeds.

Curve Resistance assumed at .5 lbs. per °, Wheel Friction at 5 lbs., Atmospheric Resistance at 2 lbs. per Ton.

Grade. Per Cent.	Grade. Per Mile.	Straight.	Curve. 1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Level.	Feet.	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5
.1	5	9	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5	16	16.5
.2	11	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5	16	16.5	17	17.5	18	18.5
.3	16	13	13.5	14	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5
.4	21	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5	22	22.5
.5	26	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5	22	22.5	23	23.5	24	24.5
.6	32	19	19.5	20	20.5	21	21.5	22	22.5	23	23.5	24	24.5	25	25.5	26	26.5
.7	37	21	21.5	22	22.5	23	23.5	24	24.5	25	25.5	26	26.5	27	27.5	28	28.5
.8	42	23	23.5	24	24.5	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5
.9	47	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5	31	31.5	32	32.5
1	53	27	27.5	28	28.5	29	29.5	30	30.5	31	31.5	32	32.5	33	33.5	34	34.5
1.1	58	29	29.5	30	30.5	31	31.5	32	32.5	33	33.5	34	34.5	35	35.5	36	36.5
1.2	63	31	31.5	32	32.5	33	33.5	34	34.5	35	35.5	36	36.5	37	37.5	38	38.5
1.3	68	33	33.5	34	34.3	35	35.5	36	36.5	37	37.5	38	38.5	39	39.5	40	40.5
1.4	74	35	35.5	36	36.5	37	37.5	38	38.5	39	39.5	40	40.5	41	41.5	42	42.5
1.5	79	37	37.5	38	38.5	39	39.5	40	40.5	41	41.5	42	42.5	43	43.5	44	44.5
1.6	85	39	39.5	40	40.5	41	41.5	42	42.5	43	43.5	44	44.5	45	45.5	46	46.5

ILLUSTRATION.—Assume a "Mogul" engine to have an adhesion of 16 000 lbs.; what weight will it haul up a grade of 74 feet per mile, combined with a curve of 9°?

$$16\,000 \div 39.5 = 405 \textit{ tons } (2000 \textit{ lbs.}).$$

Hence, *To Compute Adhesion on a Given Grade and Curve, having Weight of Train.*

RULE.—Multiply tabular number by weight of train in tons (2000 lbs.), and product will give adhesion, in lbs.

EXAMPLE.—Assume preceding elements. Then $39.5 \times 405 = 16\,000$ *lbs.*

NOTE.—A "Consolidation" engine, by its superior adhesion (19 550 lbs.) would haul up a like grade and curve 495 tons.

Memoranda on English Railways.

Regulations (Board of Trade).

Cast-iron girders to have a breaking weight = 3 times permanent load, added to 6 times moving load.

Wrought-iron bridges not to be strained to more than 5 tons per sq. inch.

Minimum distance of standing work from outer edge of rail at level of carriage steps, 3.5 feet in England and 4 feet in Ireland.

Minimum distance between lines of railway, 6 feet.

Stations.—Minimum width of platform, 6 feet, and 12 at important stations. Minimum distance of columns from edge of platform, 6 feet. Steepest gradient for stations, 1 in 260. Ends of platforms to be ramped (not *stepped*). Signals and distant signals in both directions.

Carriages.—Minimum space per passenger 20 cube feet. Minimum area of glass per passenger, 60 sq. ins. Minimum width of seats, 15 ins. Minimum breadth of seat per passenger, 18 ins. Minimum number of lamps per carriage, 2.

Requirements.—Joints of rails to be fished. Chairs to be secured by *iron* spikes. Fang bolts to be used at the joints of flat-bottomed rails.

Construction.	Narrow. Feet.	Ins.	Broad. Feet.	Ins.
Width, single line	18		24	6
" double line	30		38	
" top of ballast, single line	13	6	15	6
" " " double line	24	6	29	

Slope of cuttings from centre, 1 in 30. Width of land beyond bottom of slope, 9 to 12 feet. Ditch with slopes, 1 foot at bottom, 1 to 1. Quick mound, 18 ins. in height. Post and rail-fence posts, 7 feet 6 ins. × 6 ins. × 3.5 ins., 9 feet apart, 3 feet in ground. Intermediate posts, 5 feet 6 ins. × 4 ins. × 1.5 ins., 3 feet apart. Rails 4 of 4 × 1.5 ins.

Parliamentary Regulations for Crossing Roads.

	Turnpike Road. Feet.	Ins.	Public Road. Feet.	Ins.	Occupation Road. Feet.	Ins.
Clear width of under bridge, or approach	35	—	25	—	12	—
Clear height of under bridge for a width of 12 ft.	16	—	—	—	—	—
" " " " " 10 "	—	—	15	—	—	—
" " " " " 9 "	—	—	—	—	14	—
" " " at springing	12	—	12	—	—	—
Over bridge, height of parapets	4	—	4	—	4	—
Approaches, inclination	1 in 30		1 in 20		1 in 16	
" height of fencing	3	—	3	—	3	—

Limits of Deviation.—In towns, 10 yards each side of centre line. In country, 100 yards, or 5 chains nearly.

Level.—In towns, 2 feet. In country, 5 feet.

Gradient.—Gradients flatter than 1 in 100, deviation 10 feet per mile steeper. Do., steeper, 3 feet per mile.

Curve.—Curves upwards of .5 a mile radius, may be sharpened to .5 mile radius. Curves of less than .5 mile radius may not be sharpened.

ROADS, STREETS, AND PAVEMENTS.

Classification of Roads.

1. Earth. 2. Corduroy. 3. Plank. 4. Gravel. 5. Broken stone (Macadam). 6. Stone sub-pavement with surface of broken stone (Telford). 7. Stone sub-pavement with surface of broken stone and gravel, or gravel alone. 8. Rubble stone bottom with surface of broken stone or gravel, or both. 9. Concrete bottom with surface of broken stone or gravel, or both.

Grade of Roads.

Limit of practicable grade varies with character of road and friction of vehicle. For best carriages on best roads, limit is 1 in 35, or 15 feet in a mile.

Maximum grade of a turnpike road is 1 in 30 feet. An ascent is easier for draught if taken in alternate ascents and levels, than in one continuous rise, although the ascents may be steeper than in a uniform grade.

Ordinary angle of repose is 1 in 40 if roads are bad, and 1 in 30, to 1 in 20.

When roads have a greater grade than 1 in 35, time is lost in descending, in order to avoid unsafe speed. Grade of a road should be less than its *angle of repose.* Minimum grade of a road to secure effective drainage should be 1 in 80. In France it is 1 in 125.

In construction of roads the advantage of a level road over that of an inclined one, in reduction of labor, is superior to cost of an increased length of road in the avoiding of a hill.

Alpine roads over the Simplon Pass average 1 in 17 on Swiss side, 1 in 22 on Italian side, and in one instance 1 in 13.

In deciding upon a grade, the motive power available of ascent and avoidable of waste of power in descending are to be first considered.

When traffic is heavier in one direction than the other, the grade in ascent of lighter traffic may be greatest.

When axis of a road is upon side of a hill, and road is made in parts by excavation and by embankment, the side surface should be cut into steps, in order to afford a secure footing to embankment, and in extreme cases, sustaining walls should be erected.

Construction.

Estimate of Labor in Construction of Roads. (*M. Ancelin.*)

A day's work of 10 hours of an average laborer is estimated as follows:

In Cube Yards.

Work.	Ordinary Earth.	Loose Earth.	Mud.	Clay and Earth.	Gravel.	Blasting Rock.
Picking and digging	18 to 23	16	—	9	7 to 11	2.4
Excavation and pitching 6 to 12 feet	8 to 12	8	7 to 16	4	—	2.2
Loading in barrows	22	—	8	—	19	—
Wheeling in barrows per 100 feet	20 to 33	—	—	—	24 to 28	—
Loading in carts	16 to 48	—	—	—	17 to 27	—
Spreading and levelling	44 to 88	—	25	—	30 to 80	—

Time of pitching from a shovel is one third of that of digging.

Ditches.—All ditches should lead to a natural water-course, and their *minimum* inclination should be 1 in 125.

Depressions and elevations in surface of a roadway involve a material loss of power. Thus, if elevation is 1 inch, under a wheel 4 feet in diameter, an inclined plane of 1 in 7 has to be surmounted, and, as a consequence, one seventh of weight has to be raised 1 inch.

An unyielding foundation and surface are indispensable for a perfect roadway.

Earth in embankment occupies an average of one tenth less space than in natural bank, and rock about one third more.

Ruts. — Surface of a roadway should be maintained as intact as practicable, as the rutting of it not only tends to a rapid destruction of it, but involves increased traction.

The general practice of rutting a road displays a degree of ignorance of physical laws and mechanical effects that is as inexplicable as it is injurious and expensive.

On compressible roadways, as earth, sand, etc., resistance of a wheel decreases as breadth of tire increases.

Depressing of axles at their ends increases friction. Long and pliant springs decrease effect of shock in passing over obstacles in a very great degree.

Transverse Section.—Best profile of section of roadway is held to be one formed by two inclined planes meeting in centre of road and slightly rounded off at point of junction.

Roads having a rough surface or of broken stone should have a rise of 1 in 24, equal to a rise on crown of 6 ins., and on a smooth surface, as a block-stone or wood pavement, the rise may be reduced to 1 in 48.

On roads, when longitudinal inclination is great, the rise of transverse section should be increased, in order that surface water may more readily run off to sides of roadway, instead of down its length, and consequently gullying it.

Stone Breaking. A steam stone-breaking machine will break a cube yard of stone into cubes of 1.5 ins. side, at rate of 1 to 1.5 HP per hour.

Macadamized Roads.

In construction of a Macadamized road, the stones (road metal) used should be hard and rough, and cubical in form, the longest diameter of which exceed 2.5 ins., but when they are very hard this may be reduced to 1.25 and 1.5 ins.

The best stones are such as are difficult of fracture, as basaltic and trap, and especially when they are combined with hornblende. Flint and siliceous stone are rendered unfit for use by being too brittle. Light granites are objectionable, in consequence of their being brittle and liable to disintegration; dark granites, possessing hornblende, are less objectionable. Limestones, sandstones, and slate are too weak and friable.

Dimensions of a hammer for breaking the stone should be, head 6 ins. in length, weighing 1 lb., handle 18 ins. in length; and an average laborer can break from 1.5 to 2 cube yards per day.

Stones broken up in this manner have a volume twice as great as in their original form. 100 cube feet of rock will make 190 of 1.5 ins. dimension, 182 of 2 ins., and 170 of 2.5 ins.

A ton of hard metal has a volume of 1.185 cube yards.

Construction of a Roadway.—Excavate and level to a depth of 1 foot, then lay a "bottom" 12 ins. deep of brick or stone spalls or chips, clinker or old concrete, etc., roll down to 9 ins, then add a layer of coarse gravel or small ballast 5 ins. deep, roll down to 3 ins., and then metal in 2 equal layers of 3 ins., laid at an interval, enabling first layer to be fully consolidated before second is laid on and rolled to a depth of 4 ins.; a surface or "blind" of .75 inch of sharp sand should be laid over last layer of metal and rolled in with a free supply of water.

Proportion of Getters, Fillers, and Wheelers in different Soils. Wheelers computed at a Run of 50 Yards. (Molesworth.)

	Getters.	Fillers.	Wheelers.		Getters.	Fillers.	Wheelers.
Loose earth, Sand, etc.	1	1	1	Hard clay	1	1.25	1.25
Compact earth	1	2	2	Compact gravel	1	2	1
Marl	1	2	2	Rock	3	1	1

Telford Roads.

In construction of a Telford road, metalling is set upon a bottom course of stones, set by hand, in the manner of an ordinary block stone pavement, which course is composed of stones running progressively from 3 inches in depth at sides of road to 4, 5, and 7 inches to centre, and set upon their broadest edge, free from irregularities in their upper surface, and their interstices filled with stone spalls or chips, firmly wedged in.

Centre portion of road to be metalled first to a depth of 4 ins., to which, after being used for a brief period, 2 ins. more are to be added, and entire surface to be covered, "blinded," with clean gravel 1.5 ins. in depth.

Telford assigned a load not to exceed 1 ton upon each wheel of a vehicle, with a tire 4 ins. in breadth.

Gravel or Earth Roads.

In construction of a gravel or earth road, selection should be made between clean round gravel that will not pack, and sharp gravel intermixed with earth or clay, that will bind or compact when submitted to the pressure of traffic or a roll.

Surface of an ordinary gravel roadway should be excavated to a depth of from 8 to 12 ins. for full width of road, the surface of excavation conforming to that of road to be constructed.

The gravel should then be spread in layers, and each layer compacted by the gradual pressure due to travel over it, or by a roller, the weight of it increasing with each layer. One of 6 tons will suffice for limit of weight.

If gravel is dry and will not readily pack, it should be wet, and mixed with a binding material, or covered with a thin layer of it, as clay or loam.

In rolling, the sides of road should be first rolled, in order to arrest the gravel, when the centre is being rolled, from spreading at the side.

To re-form a mile of gravel or earth road, 30 feet in width between gutters, material cast up from sides, there will be required 1640 hours' labor of men, and 20 of a double team.

Corduroy Roads.

A Corduroy road is one in which timber logs are laid transversely to its plane.

Plank Roads.

A single plank road should not exceed 8 feet in width, as any greater width involves an expenditure of material, without any equivalent advantage.

If a double track is required it should consist of two single and independent tracks, as with one wide track the wear would be mostly in the centre, and consequently, wear would be restricted to one portion of its surface.

Materials.—Sleepers should be as long as practicable of attainment, in depth 3 or 4 ins., according to requirements of the soil, and they should have a width of 3 ins. for each foot of width of road.

Pine, oak, maple, or beech are best adapted for economy and wear.

Planks should be from 3 to 3.5 ins. thick, and not less than 9 ins. in width, or more than 12 if of hard wood, or 15 if of soft.

A plank road will wear from 7 to 12 years, according to service, material, and location, and its traction, compared with an ordinary Macadamized road, is 2.5 to 3 times less, and with a common country road in bad order 7 times.

For other elements, see Earth-work, page 466.

Asphalt.

Asphalt is a bituminous limestone, and is synonymous with bitumen; it consists of from 90 to 94 per cent. of carbonate of lime and 6 to 10 per cent. of bitumen.

In forming a pavement the powder is heated to from 212° to 250°, and its particles caused to adhere by pressure, or it is applied as a liquid asphalt or asphaltic mastic, which is thus manufactured. The powder is heated with from 5 to 8 per cent. of free bitumen for a flux, and the mixture when melted is run into molds. To be remelted, additional bitumen must be mixed with it, without which it would only become soft.

For paving 60 per cent. of sand or gravel must be mixed with it. No chemical union takes place between the mastic and the sand or gravel, but cohesion is so complete that gravel will fracture with the mastic, and the admixture increases the resistance of the mass to heat of the sun. The roadway should have a convexity of .01 of its breadth.

Artificial Asphalt.—Heated limestone and gas tar, when mixed, possess some of the proportions of alphalt mastic, but it is very inferior for the purposes of a pavement.

To repair surface of roadway, dissolve bitumen 1 part in 3 of pitch oil or resin oil, apply 10 oz. of mixture over each sq. yard of roadway, sprinkle on it 2 lbs. of asphalt powder, and then cover surface with sand.

Wood Pavement.

Close-grained and hard woods only are suitable. such as oak, elm, ash, beech, and yellow pine, and they should be laid on a foundation of concrete.

Block Stone Pavement.

Paving-blocks, as the Belgian, etc., where crest of street or area of pavement does not exceed 1 inch in 7.5 feet, should taper slightly toward the top, and the joints be well filled, "blinded," with gravel. The common practice of tapering them downward is erroneous.

The foundation or bottoming of a stone pavement for street travel should consist either of hydraulic concrete or rubble masonry in hydraulic mortar. The practice in this country of setting the stones in sand alone is at variance with endurance and ultimate economy, but when resorted to, there should be a bed of 12 ins. of gravel, rammed in three layers, covered with an inch of sand. Granite or Trap blocks should be 4 × 9 × 12 ins.

Rubble Stone Pavement.

Bowlders or Beach stone of irregular volumes and forms, set in a bed of sand, involves great resistance to vehicles and frequent repairs; it is wholly at variance with requirements of heavy traffic or city use.

Concrete Roads.

Concrete roads are constructed of broken stones (road metal) 4 volumes, clean sharp sand 1.25 to .33 volumes, and hydraulic cement 1 volume. The mass is laid down in a layer of 3 or 4 ins. in depth, and left to harden during a period of 3 days, when a second and like layer is laid on and well rolled, and then left to harden for a period of from 10 to 20 days, according to temperature and moisture of the weather.

Roads. (*Molesworth.*)

Ordinary turnpike roads.— 30 feet wide, centre 6 ins. higher than sides; 4 feet from centre, .5 inch below centre; 9 feet from centre, 2 ins. below centre; 15 feet from centre, 6 ins. below centre.

Foot-paths—6 feet wide, inclined 1 inch towards road, of fine gravel, or sifted quarry chippings, 3 ins. thick.

Cross-roads—20 feet wide. *Foot-paths*—5 feet.

Side drains—3 feet below surface of road.

Road material—bottom layer gravel, burned clay or chalk, 8 ins. deep. Top layer, broken granite not larger than 1.5 cube ins., 6 ins. deep.

Miscellaneous Notes.

Metalling should be from 6 ins. to 1 foot in depth, and in cubes of 1.5 to 1.75 ins.

One layer of material of a road should be spread and submitted to traffic or rolling before next is laid down, and this process should be repeated in 2 or 3 layers of 3 ins. each.

When new metal is laid on old, the surface of the old should be loosened with a pick. Patching is termed *darning*.

Sand and Gravel, *Blinding*, should not be spread over a new surface, as they tend to arrest binding of metal. Mud should be scraped off of surface.

Hoggin is application of a binding of surface of a metal road, composed of loam, fine gravel, and coarse sand.

Metalled Roads should be swept wet.

Rolling.—Steam rolls are most effective and economical. 1000 sq. yards of metalling will require 24 hours' rolling at 1.5 miles per hour. A roller of 15 tons' weight will roll 1000 sq. yards of Telford or Macadam pavement in from 30 to 40 hours, at a speed of 1.5 miles per hour, equal .675 and .9 ton mile per sq. yard.

Sprinkling.—60 cube feet of water with one cart will cover 850 sq. yards. 100 cube feet per day will cover 1000 sq. yards; ordinarily two sprinklings are necessary.

Granite Pavement.—The wear of granite pavement of London Bridge was .22 inch per year, and from an average of several streets in London, the wear per 100 vehicles per foot of width per day is equal to one sixteenth of an inch per year.

Sweeping and Watering of granite pavement and Macadam road, for equal areas and under alike conditions in every respect, costs as 1 for former to 7 of latter.

By men, with cart, horse, and driver, costs 3.25 times more than by a machine, one of which will sweep 16 000 sq. yards of street per period of 6 hours.

Asphalt Pavement. — Average cost per sq. yard in London: foundation, 50 cents; surface, $3.25; cost of maintenance per sq. yard per year, 40 cents. Wear varies from .2 to .42 near curb, and .17 to .34 inch on general surface per year.

Washing.—Surface cleaning of stone or asphalt pavement by a jet can be effected at from 1 to 2 gallons per sq. yard.

Wood Pavement.—Wear of wood pavement in London, per 100 vehicles per day per foot of width, .083 inch per year.

Macadamized Roads. — Annual cost of maintenance of several such roads in London was 62 cents per sq. yard.

Block Stone Pavement.—Stones should be set with their tapered or least ends upwards, with surface joints of 1 inch.

Fascines, when used, should be in two layers, laid crosswise to each other and picketed down.

Bituminous road may be made by breaking up asphalt, laying it 2 ins. thick, covering with coal tar, and ramming it with a heavy beetle. To repair a bituminous surface, dissolve one part of bitumen (mineral tar) in three of pitch oil or resin oil, spread .625 of a lb. of solution over each sq. yard of road, sprinkle 2 lbs. powdered asphalt (bituminous limestone) and then sand, and sweep off the surplus.

Slipping.—Granite safest when wet, and asphalt and wood when dry.

Gravel, alike to that of Roa Hook, from its uniformity, will bear an admixture of from .2 to .25 of ordinary gravel or coarse sand.

Annual cost of a Telford pavement 4.2 cents per sq. yard, including sprinkling, repairs, and supervision.

Voids in a Cube Yard of Stone.

Broken to a gauge of 2.5 ins.	10 *cube feet.*	Shingle	9 *cube feet.*
" " 2 "	10.66 " "	Thames ballast	4.5 " "
" " 1.5 "	11.33 " "		

For further and full information, see Law and Clarke on Roads and Streets, New York, 1867; Weale's Series, London, 1861 and 1877; Roads, Streets, and Pavements, by Brev. Maj.-Gen. Q. A. Gilmore, U. S. A., New York, 1876; Engineering Notes, by F. Robertson, London and New York, 1873; and Construction and Maintenance of Roads, by Ed. P. North, C. E., see Transactions Am. Soc. of C. E., vol. viii., May, 1879.

SEWERS.

Sewers are the courses from a series of locations, and are classed as Drains, Sewers, and Culverts.

Drains are small courses, from one or more points leading to a sewer.

Culverts are courses that receive the discharge of sewers.

Greatest fall of rain is 2 ins. per hour = 54 308.6 galls. per acre.

Inclination of sewers should not be less than 1 foot in 240, and for house or short lateral service it should be 1 inch in 5 feet.

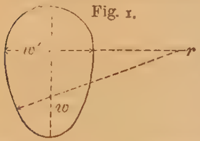
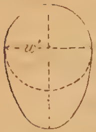


Fig. 1.

Circular. $55\sqrt{x\,2f}=v$, and $v\,a=V$.

Egg. $\frac{D}{3}=w$, $\frac{2\,D}{3}=w'$, and $D=r$. *x representing area of sewer ÷ wetted perimeter, f inclination of sewer per mile, and v velocity of flow of contents in feet per minute; a area of flow, in sq. feet, V volume of discharge, in cube feet per minute; D height of sewer, w and w′ width at bottom and top, and r radius of sides, in feet.*

Fig. 2.



For diameter of sewer exceeding 6 feet. (*T. Hawksley.*)

$D-\frac{1}{9}=w'$. *D diameter of a circular sewer of area required.*

Elliptic.—Top and bottom internal should be of equal diameters. Diameter .66 depth of culvert; intersections of top and bottom circles form centres for striking courses connecting top and bottom circles.

Pipes or Small Sewers.—Height of section = 1; diameter of arch = .66; of invert = .33, and radius of sides = 1.

In culverts less than 6 feet internal depth, brickwork should be 9 ins. thick; when they are above 6 feet and less than 9 feet, it should be 14 ins. thick.

If diameter of top arch = 1, diameter of inverted arch = .5, and total depth = sum of the two diameters, or 1.5; then radius of the arcs which are tangential to the top, and inverted, will be 1.5.

From this any two of the elements can be deduced, one being known.

Drainage of Lands by Pipes.

Soils.	Depth of Pipes.		Distance apart.	Soils.	Depth of Pipes.		Distance apart.
	Ft.	Ins.	Feet.		Ft.	Ins.	Feet.
Coarse gravel sand	4	6	60	Loam with gravel ...	3	3	27
Light sand with gravel	4		50	Sandy loam.........	3	9	40
Light loam	3	6	33	Soft clay...........	2	9	21
Loam with clay.......	3	2	21	Stiff clay...........	2	6	15

Minimum Velocity and Grade of Sewers and Drains in Cities. (*Wicksteed.*)

Diam.	Vel. per Minute.	Grade, 1 in	Grade per Mile.	Diam.	Vel. per Minute.	Grade, 1 in	Grade per Mile.	Diam.	Vel. per Minute.	Grade, 1 in	Grade per Mile.
Ins.	Feet.		Feet.	Ins.	Feet.		Feet.	Ins.	Feet.		Feet.
4	240	36	146.7	15	180	244	21.6	42	180	686	7.7
6	220	65	81.2	18	180	294	18	48	180	784	6.8
8	220	87	60.7	24	180	392	13.5	54	180	882	6
10	210	119	44.4	30	180	490	10.8	60	180	980	5.4
12	190	175	30.2	36	180	588	9				

Area of Sewers or Pipes.—An area of 20 acres, miles, etc., will not require 20 times capacity of pipes for one acre, mile, etc., as the discharge from the 19 acres, etc., will not flow into the main simultaneously with that from one acre, etc. Ordinarily in this country an area of sewer or pipe that will discharge a rainfall of 1 inch per hour (3630 cube feet per acre) is sufficient.

Sewage.—The excreta per annum of 100 individuals of both sexes and all ages is estimated at 7250 lbs. solid matter and 94 700 fluid, equal to 1020 lbs. *per capita*, and in volume 16 cube feet, to which is to be added the volume of water used for domestic purposes. A velocity of flow of from 2.5 to 3 feet per second will discharge a sewer of its sewage matter and prevent deposits. The minimum velocity should not be less than 1.3 feet per second.

Surface from which Circular Sewers with proper Curves will discharge Water equal in Volume to One Inch in Depth per Hour, including City Drainage. (*John Roe.*)

Inclination in Feet.	Diameter of Sewers in Feet. 2	2.5	3	4	5	6	7	8
	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.
None	38.75	67.25	120	277	570	1020	1725	2850
1 in 480	48	75	135	308	630	1117	1925	3025
1 in 240	50	87	155	355	735	1318	2225	3500
1 in 160	63	113	203	460	950	1692	2875	4500
1 in 120	78	143	257	590	1200	2180	3700	5825
1 in 80	90	165	295	570	1388	2486	4225	6625
1 in 60	125	182	318	730	1500	2675	4550	7125

Surface of a Town from which small Circular Drains will discharge Water equal in Volume to Two Inches in Depth per Hour. (*John Roe.*)

Inclination. Fall of 1 Inch.	Diameter of Drain in Ins. 3	4	5	6	7	8	Inclination. Fall of 1 Inch.	Diameter of Drain in Ins. 9	12	15	18
Acres.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Acres.	Feet.	Feet.	Feet.	Feet.
.125	120	—	—	—	—	—	2.1	120	—	—	—
.25	20	120	—	—	—	—	2.5	80	—	—	—
.4375	—	40	—	—	—	—	2.75	60	—	—	—
.5	—	30	80	—	—	—	4.5	—	120	—	—
.6	—	20	60	—	—	—	5.3	—	80	—	—
1	—	—	20	60	—	—	5.8	—	60	240	—
1.2	—	—	—	40	20	—	7.8	—	—	120	—
1.5	—	—	—	20	60	120	9	—	—	80	—
1.8	—	—	—	—	—	80	10	—	—	60	240
2.1	—	—	—	—	—	60	17	—	—	—	120

Dimensions, Areas, and Volume of Material per Lineal Foot of Egg-shaped Sewers of different Dimensions.

Depth.	Internal Dimensions. Diam. of Top Arch.	Diam. of Invert.	Area.	Volume of Brick-work. 4.5 Ins. thick.	9 Ins. thick.	13.5 Ins. thick.
Feet.	Feet.	Feet.	Sq. Feet.	Cube Feet.	Cube Feet.	Cube Feet.
2.25	1.5	.75	2.53	2.81	—	—
3	2	1	4.5	3.56	—	—
3.75	2.5	1.25	7.03	4.31	9.56	—
4.5	3	1.5	10.12	5.06	10.87	—
5.5	3.5	1.75	13.78	5.81	12.75	—
6	4	2	18	6.56	14.25	—
6.75	4.5	2.25	22.78	7.31	15.75	24.75
7.5	5	2.5	28.12	—	17.06	27
8.25	5.5	2.75	34.03	—	18	28.41
9	6	3	40.5	—	19.69	30.94

Area = product of mean diameter × height.

Sewer Pipes should have a uniform thickness and be uniformly glazed, both internally and externally.

Fire-clay pipes should be thicker than those of stone-clay.

STABILITY.

STABILITY, *Strength*, and *Stiffness* are necessary to permanence of a structure, under all variations or distributions of load or stress to which it may be subjected.

Stability of a Fixed Body—Is power of remaining *in equilibrio* without sensible deviation of position, notwithstanding load or stress to which it may be submitted may have certain directions.

Stability of a Floating Body.—A body in a fluid floats, or is balanced, when it displaces a volume of the fluid, weight of which is equal to weight of body, and when centre of gravity of body and that of volume of fluid displaced are in same vertical plane.

When a body *in equilibrio* is free to move, and is caused to deviate in a small degree from its position of equilibrium, if it tends to return to its original position, its equilibrium is termed *Stable;* if it does not tend to deviate further, or to recover its original position, its equilibrium is termed *Indifferent;* and when it tends to deviate further from its original position, its equilibrium is *Unstable.*

A body *in equilibrio* may be stable for one direction of stress, and unstable for another.

Moment of Stability of a body or structure resting upon a plane is moment or *couple* of forces, which must be applied in a plane vertically inclined to the body in addition to its weight, in order to remove centre of resistance of body upon plane, or of the joint, to its extreme position consistent with stability. The couple generally consists of the thrust of an adjoining structure, or an arch and pressure of water, or of a mass of earth against the structure, together with the equal and parallel, but not directly opposed, resistance of plane of foundation or joint of structure to that lateral thrust. It may differ according to position of axis of applied couple.

Couple.—Two forces of equal magnitude applied to same body or structure in parallel and opposite directions, but not in same line of action, constitute a couple.

NOTE.—For Statical and Dynamical Stability, see Naval Architecture, page 649.

To Ascertain Stability of a Body on a Horizontal Plane. —Fig. 1.

Fig. 1.

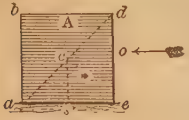


ILLUSTRATION.—Stability of a body, A, Fig. 1, when a thrust is applied as at o, to turn it on a, is ascertained by multiplying its weight by distance $a\,s$, from fulcrum a to line of centre of gravity, $c\,s$.

Hence, if cubical block weighed 10 tons and its base is 6 feet, its moment would be $10 \times \frac{6}{2} = 30$ *tons.*

If upper part, $a\,b\,d\,c$, was removed, remainder, $a\,e\,d$, would weigh but 5 tons, but its centre of gravity • would be $\frac{2}{3}\,a\,e = 4$ feet. Hence its moment would be $5 \times 4 = 20$ *tons*, although it is but half the weight.

To Compute Weight of a Given Body to Sustain a Given Thrust.

$\frac{F\,h}{l} = W$. F *representing thrust in lbs.*, h *height of centre of gravity of body* $= c\,s$, *and* l *distance of fulcrum from centre of gravity* $= a\,s$.

ILLUSTRATION.—Assume figure to be extended to a height of 20 feet, and required to be capable of resisting the extreme pressure of wind.

Pressure estimated at 50 lbs. $F = 6 \times 20 \times 50 = 6000$ lbs. *at centre of gravity of surface of body.*

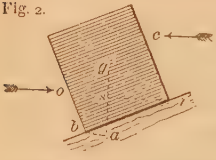
$$\text{Then } \frac{6000 \times 10}{3} = 20\,000 \text{ lbs.}$$

NOTE 1.—This result is to be increased proportionately with the factor of safety due to character of its material and structure.

2.—If form of body has a cylindrical section, as a round tower, the thrust of wind would be but one half of that of a plane surface.

When the Body is Tapered, as Frustum of Pyramid or Cone.—Ascertain centres of gravity of surface for pressure or thrust, and of body for its stability, and proceed as before.

Fig. 2.



To Ascertain Stability of a Body on an Inclination.—Fig. 2.

ILLUSTRATION.—Stability of body, Fig. 2, when thrust is applied at *c*, is ascertained by multiplying its weight by distance *a b* from fulcrum, *b*, to line of centre of gravity, *a g*.

If thrust was applied at *o*, stability would be ascertained by distance *s r* from fulcrum *r*.

Angles of Equilibrium at which various Substances will Repose, as determined by a Clinometer.

Angle measured from a Horizontal Plane, and falling from a spout.

	Degrees.		Degrees.		Degrees.
Lime-dust	45	Sand, less dry	39.6	Common mold	37
Dry sand	40	Wheat	37	Common gravel	35 to 36
Moist sand	41	Corn	37	Stones or Coal	43

Weight of a Cube Foot of Materials of Embankments, Walls, and Dams.

Concrete in cement	137	Gravel	125	Clay	120
Stone masonry	130	Loam	126	Marl	100
Brick "	112	Sand	120		

Revetment Walls.

When a wall sustains a pressure of earth, sand, or any loose material, it is termed a Revetment wall, and when erected to arrest the fall or subsidence of a natural bank of earth, it is termed a Face wall.

When earth or banking is level with top of wall, it is termed a Scarp revetment, and when it is above it, or *surcharged*, a Counterscarp revetment.

When face of wall is battered, it is termed Sloping, and when back is battered, Countersloping.

Thrust of earth, etc., upon a wall is caused by a certain portion, in shape of a wedge, tending to break away from the general mass. The pressure thus caused is similar to that of water, but weight of the material must be reduced by a particular ratio dependent upon angle of natural slope, which varies from 45° to 60° (measured from vertical) in earth of mean density.

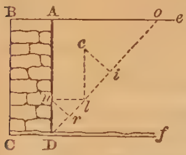
Or, natural slope of earth or like material lessens the thrust, as the cosine of the slope.

Angle which line of rupture makes with vertical is .5 of angle which line of natural slope, or *angle of repose*, makes with same vertical line. When earth is level at top, its pressure may be ascertained by considering it as a fluid, weight of a cube foot of which is equal to weight of a cube foot of the earth, multiplied by square of tangent of .5 angle included between natural slope and vertical.

Therefore squares of the tangents of .5 of 45° and .5 of 60° = .1716 and .3333, which are the multipliers to be used in ordinary cases to reduce a cube foot of material to a cube foot of equivalent fluid, which will have same effect as earth by its pressure upon a wall.

Pressure of Earth against Revetment Walls.

Fig. 3.



Let A B C D, Fig. 3, be vertical section of a revetment wall, behind which is a bank of earth, A D *f e*; let D *o* represent *angle of repose*, *line of rupture*, or *natural slope* which earth would assume but for resistance of wall.

In sandy or loose earth angle *o* D A is generally 30°; in firmer earth it is 36°; and in some instances it is 45°.

If upper surface of earth and wall which supports it are both in one horizontal plane, then the resultant, *l n*, of pressure of the bank, behind a vertical wall, is at a distance, D *n*, of one third A D.

Line of Rupture behind a wall supporting a bank of vegetable earth is at a distance A *o* from interior face, A D = .618 height of it.

When bank is of sand, A *o* = .677 *h*; when of earth and small gravel = .646 *h*; and when of earth and large gravel = .618 *h*.

The prism, vertical section of which is A D *o*, has a tendency to descend along inclined plane, *o* D, by its gravity; but it is retained in its place by resistance of wall, and by its cohesion to and friction upon face *o* D. Each of these forces may be resolved into one which will be perpendicular to *o* D, and into another which will be parallel to *o* D. The lines *c i*, *i l* represent components of the force of gravity, which is represented by vertical line *c l*, drawn from centre of gravity, *c*, of prism. Lines *n r*, *l r* represent components of forces of cohesion and friction, which is represented by horizontal line *n l*. Force that gives the prism a tendency to descend is *i l*, and that opposed to this is *r l*, together with effects of cohesion and friction.

Thus, *i l* = *r l* + cohesion + friction. Consequently, exact solution of problems of this nature must be in a great measure experimental.

It has been found, however, and confirmed experimentally, that angle formed with vertical, by prism of earth that exerts greatest horizontal stress against a wall, is *half the angle* which *angle of repose* or *natural slope* of earth makes with vertical.

Memoranda.

Natural slope of dry sand = 39°, moist soil = 43°, very fine sand = 21°, wet clay = 14°, and gravel = 35°.

In setting or founding of retaining walls, if earth upon which wall is to rest is clayey or wet, coefficient of friction between wall and earth falls to .3; hence it is necessary, in order to meet this, that the wall should be set to such a depth in the earth that the passive resistance of it on outer face of wall, combined with its friction on its bottom, may withstand the pressure or thrust on its inner face.

Moment of a Retaining Wall is its weight multiplied by distance of its centre of gravity to vertical plane passing through outer edge of its base.

Moment of Pressure of Earth against a retaining wall is pressure multiplied by distance of its centre of pressure to horizontal plane passing through base of wall.

Equilibrium of Retaining Wall is when respective moments of wall and earth are equal.

Stability of a Retaining Wall should be in excess of its equilibrium, according to character of thrust upon it, and the line of its resistance should be within wall and at a distance from vertical passing through centre of gravity of wall, at most .44 of distance of exterior axis of wall from this line.

Coefficient of Stability varies with character of earth, location, exposure to vibrations, floods, etc.; hence thickness of base of wall will vary from 1.4 to 2 *b*.

Backs of retaining walls should be laid rough, in order to arrest lateral subsidence of the filling.

When filling is composed of bowlders and gravel, the thickness of wall must be increased, and contrariwise; when of earth in layers and well rammed, it may be decreased.

Courses of dry wall should be inclined inwards, in order to arrest the flow of water of subsidence in filling from running out upon face of wall.

Less the natural slope, greater the pressure on wall.

Sea walls should have an increased proportion of breadth, as the earth backing is not only subjected to being flooded, but the walls have at times to sustain the weight of heavy merchandise.

Buttress.—An increased and projecting width of wall on its front, at intervals in its length.

Counterfort.—An increased and projecting width of wall at its back and at intervals.

Coefficient of Friction of masonry on masonry .67, of masonry on dry clay .51, and on wet clay .3.

Face of wall should not be battered to exceed 1 to 1.25 ins. in a foot of height, in consequence of the facility afforded by a greater inclination to the permeation of rain between the joints of the courses.

Footing of a wall, projecting beyond its faces, is not included in its width.

Pressure.—Limit of pressure on masonry 12 500 to 16 500 lbs. per sq. foot wall.

Thickness of Walls, in Mortar, Faces vertical. For Railways or Like Stress.

Cut stone or Ranged rubble........ .35	Brick or Dressed rubble............ .4

When laid dry, add one fourth.

Friction in vegetable earths is .5; pressure in sand .4.

When vegetable earths are well laid in courses, the thrust is reduced .5.

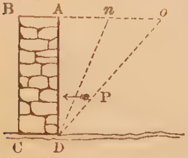
When bank is liable to be saturated with water, thickness of wall should be doubled.

Centre of Pressure of earthwork, etc., coincides with centre of pressure of water, and hence, when surface is a rectangle, it is at .33 of height from base.

The theory of required thickness of a retaining wall, as before stated, is, that the lateral thrust of a bank of earth with a horizontal surface is that due to the prism or wedge-shaped volume, included between the vertical inner face of the wall and a line bisecting the angle between the wall and the angle of repose of the material.

To Compute Elements of Revetment Walls.—Fig. 4.

Fig. 4.



Let A D o represent angle of repose of material, resting against a wall, A B C D. A D n = .5 A D o. Tan. A D n = .492. Tan. A D n $h\frac{h}{2}$, or $\frac{h^2}{2}$ tan. A D n = V.

$\frac{w\,h^2}{2}$ tan. A D n = W; $\frac{w\,h^2}{2}$ tan.^{2} A D n = P;

$\frac{w\,h^2}{2}$ tan.^{2} A D $n\frac{h}{3}$, or $\frac{w\,h^3}{6}$ tan.^{2} A D n = M; $W\,h\frac{x^2}{2}$, or $\frac{W\,h\,x^2}{2} = m$; $\frac{W\,h\,x^2}{2} = \frac{w\,h^3}{6}$ tan.^{2} A D n = E; $\frac{W\,h\,x^2}{2} = \frac{w\,h^3}{3}$ tan.^{2} A D n = S;

h tan. A D $n\sqrt{\frac{w}{3\,W}} = x$, and h tan. A D $n\sqrt{\frac{2\,w}{3\,W}} = x'$. *$h$ representing height of wall in feet,* V *volume of section of prism of material* A D n *one foot in length in cube feet,* W *and* w *weights of a cube foot of wall and of material,* P *lateral pressure of prism of earth upon wall,* M *and* m *moments of pressure and weight on and of wall,* E *and* S *equilibrium and stability of wall, all in lbs., and* x *and* x', C D *for weights of wall for equilibrium and stability.*

Illustration.—A revetment wall, Fig. 4, of 125 lbs. per cube foot and 40 feet in *height*, sustains a bank of earth having a natural slope of 52° 24′, and a weight of 89.25 lbs. per cube foot; what is pressure or thrust against it, etc.?

Tan.² A D n = .242. Then $.492 \times 40 \times \frac{40}{2} = 393.6$ *cube feet.*

$\frac{89.25 \times 40^2}{2} \times .492 = 35\,128.8$ *lbs.* $\frac{89.25 \times 40^2}{2} \times .492^2 = 17\,278.8$ *lbs.*

$\frac{89.25 \times 40^2}{2} \times .492^2 \times \frac{40}{3} = 230\,384$ *lbs.* $125 \times 40 \times \frac{9.6^2}{2} = 230\,400$ *lbs.*

$40 \times .492 \sqrt{\frac{89.25}{3 \times 125}} = 9.6$ *feet,* and $40 \times .492 \sqrt{\frac{2 \times 89.25}{3 \times 125}} = 13.58$ *feet.*

For Rubble Walls in Mortar or Dry Rubble, add respectively to base as above obtained, .14 and .42 part.

NOTE 1.—When coefficient of friction is known, use it for tan.² A D n.

$h \times C$ = *base of wall for stability.* (*Molesworth.*)

2.—When either relative weights of equal volumes of wall and bank of earth or their specific gravities are given, S and s may be taken for W and w.

These equations involve simply the operation of a lever, the fulcrum being at the outer edge of wall C. The moment of pressure of bank is product of lateral pressure and perpendicular distance from fulcrum to line of direction of pressure.

The moment of weight of wall is product of weight of wall and perpendicular distance from fulcrum to vertical line drawn through centre of gravity of wall.

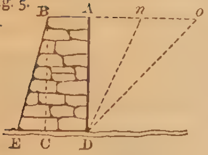
When Weights of Embankment and Wall are equal per Cube Foot.

C for clay = .336, and for sand .267.

When Weights are as 4 to 5. C for clay = .3, and for sand .239.

When Wall has an Exterior Slope or Batter.—Fig. 5.

Fig. 5.



$\frac{W h}{2}\left(\overline{CD + EC}^2 - \frac{EC^2}{3}\right) = M.$ M *representing moment of weight of wall in lbs.*

ILLUSTRATION.—Assume weight of wall 120 lbs. per cube foot, and C D and E C respectively 10 and 2.5 feet, and all other elements as in preceding case.

Hence, $\frac{120 \times 40}{2} \times \left(\overline{10 + 2.5}^2 - \frac{2.5^2}{3}\right) = 370\,000$ *lbs.*

$$\frac{W h}{2}\left(\overline{x + n h}^2 - \frac{n^2 h^2}{3}\right) = \frac{w h^3}{3} \tan.^2 A D n = S.$$

Or, $h\sqrt{\frac{n^2}{3} + \frac{2 w}{3 W} \tan.^2 A D n} - n h = x$. x *representing* A B or C D. n *ratio of difference of widths of base and top to height. In absence of* tan.² A D n *put* C, *coefficient of material.*

C = .0424 for vegetable or clayey earth, mixed with large gravel; .0464 if mixed with small gravel; .1528 for sand, and .166 for semi-fluid earths.

ILLUSTRATION.—Assume elements of preceding case. n = one fortieth, and tan. A D n = .492.

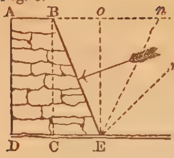
$$40\sqrt{\frac{1}{3 \times 40^2} + \frac{2 \times 89.25}{3 \times 125} \times .492^2} - 1 = 12.6 \text{ feet.}$$

Hence, thickness of wall at base = 12.6 + 1 (one fortieth of height) = 13.6 feet.

NOTE.—If n = one twentieth, $40\sqrt{\frac{1}{3 \times 20^2} + \frac{2 \times 89.25}{3 \times 125} \times .492^2} - 2 = 11.63$ *feet.*

Hence, wall at base = 11.63 + 2 (one twentieth of height) = 13.63 *feet.* If C was used, 11.32 *feet.*

Fig. 6.



When Wall has an Interior Slope or Batter, B E.—Fig. 6.

$\frac{w h^2}{2} \times \tan.^2 \frac{o E r}{2} = P.$ $\frac{w h^3}{6} \times \tan.^2 \frac{o E r}{2} = M$ *of earth for equilibrium;* $\frac{w h}{2}\left(DC \times \overline{DC + CE} - \frac{CE^2}{3}\right) =$ M *of wall;* and $\frac{w h^3}{3} \times \tan.^2 o E n = M$ *of earth for stability.*

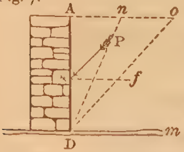
Coefficients for Batter of following Proportions.

Base = Height × Tab. number.

Batter of Wall.	Weight of Earth to Wall. As 4 to 5. Clay.	Sand.	As 1 to 1. Clay.	Sand.	Batter of Wall.	Weight of Earth to Wall. As 4 to 5. Clay.	Sand.	As 1 to 1. Clay.	Sand.
1 in 4......	.083	.029	.115	.054	1 in 8......	.184	.125	.218	.153
1 " 5......	.122	.065	.155	.092	1 " 12.....	.221	.16	.256	.189
1 " 6......	.149	.092	.183	.118	Vertical....	.3	.239	.336	.267

To Compute Pressure Perpendicular to Back of Wall.—Fig. 7.

Fig. 7.



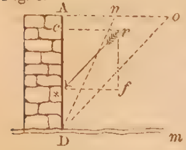
P * $= \frac{AD}{3}$ or $\frac{h}{3}$, and *f* * *at right angle to back of wall, whether vertical or inclined.*

$\frac{L \times An}{h}$, or L × tan. A D *n*, or $\frac{w \times h^2 \times \tan.^2 ADn}{2}$, or $\frac{w \times An^2}{2} = f$ *. L *representing weight of triangle of embankment, as* A D *n*.

This is pressure independent of friction between surfaces of wall and earth.

To Ascertain and Compute Amount and Effect of Friction of Wall and Earth.—Fig. 8.

Fig. 8.



Draw *f* * by scale to computed pressure at right angle to back of wall, draw angle *f* * *r* = *m* D *o* of natural slope of earth with horizon, draw *f r* at right angle to *f* *, make *r c* = *f* *, then *c r* will represent by scale effect of friction against back of wall.

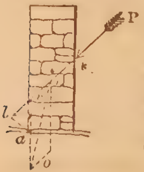
Assume friction to act at point *, then *r* * will give by scale resultant of the two forces of pressure and friction, equal to pressure in force and direction, which bears against wall.

This resultant is also equal to *f* * × sec. *m* D *o*.

$\frac{L \times An \times \sec. mDo}{h} = r$ *, or $\frac{w \times h^2 \times \tan.^2 mDo}{2} \times \sec. mDo$, or L × tan. A D *n* × sec. *m* D *o*.

To Ascertain Point of Moment of Pressure of a Wall.—Fig. 9.

Fig. 9.



By its resisting lever l a, added to its weight.

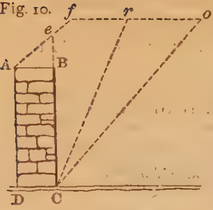
Weight of wall as computed assumed as concentrated at its centre of gravity •

Draw a vertical line • *o* through its centre of gravity, and continue line of pressure P * to *l*, take any distance *r o* by scale representing weight of wall, and *r n*, by same scale, for amount of pressure or thrust against wall, complete parallelogram *r o n u*, then diagonal *r u* will give resultant of pressure in amount and direction to overturn wall.

For stability this diagonal should fall inside of base at a point not less than one third of its breadth.

Surcharged Revetments.

Fig. 10.



When the earth stands above a wall, as A B *e*, Fig. 10, with its natural slope, A *f*, A B C is termed a *Surcharged Revetment.*

If C *r* is line of rupture, A *f r* C is the part of earth that presses upon wall, which part must be taken into the computation, with exception of portion A B *e*, which rests upon wall; that is, the computation must be for part C *e f r*, which must be reduced by multiplying weight of a cube foot of it by square of tangent of angle *e* C *r* = angle of line of rupture, or half angle *e* C *o*, which natural slope makes with vertical, and then proceed as in previous cases for revetments.

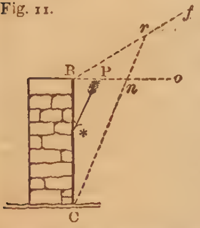
$h'\sqrt{\frac{h' w \tan.^2 e C r}{3 h W}}$ = *breadth* or C D. W *and w representing weights of wall and embankment in lbs. per cube foot, and h' height of embankment, as* C *e*.

Illustration.—Height of a surcharged revetment, B C, Fig. 10, is 12 feet, weight 130 lbs. per cube foot; what is its width or base to resist pressure of earth of a weight of 100 lbs. per cube foot, and a height, C *e*, of 15 feet, angle of repose 45°?

$\tan.^2 (45° \div 2) = .1716.$ Then $15\sqrt{\frac{15 \times 100 \times .1716}{3 \times 12 \times 130}} = 15\sqrt{.055} = 3.52$ *feet.*

To Ascertain Point of Moment of Pressure of a Surcharged Wall.—Fig. 11.

Fig. 11.



Draw a line, P *, parallel to slope, C *r*, through centre of gravity of sustained backing, B C *r*.

When, as in this case, this section is that of a triangle, point * will be at .33 height of wall.

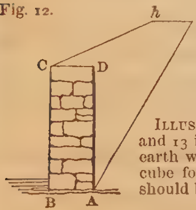
When natural slope is 1.5 in length to 1 in height, as with gravel or sand, $w \times .64$ = pressure P *.

In a surcharged revetment, as *f* B *o*, at its natural slope, the maximum pressure is attained when the backing reaches to *r*. When slope of maximum pressure, C *n r*, intersects face of natural slope, B *f*, so that if backing is raised to *f*, or above it, there is theoretically no additional stress exerted at back of or against wall, but practically there is, from effect of impact of vibration of a passing train, proximity to percussive action, alike to that of a trip-hammer, etc.

When backing rests on top of wall, as A B *e*, Fig. 10, small triangle of it is omitted in computations. Direction of pressure against wall is same as when wall is not surcharged.

When Wall is set below Surface of Earth.—Fig. 12.

Fig. 12.



$$1.4 \tan. 45° - \frac{a}{2}\sqrt{\frac{h^2 w \left(\tan. 45° - \frac{a}{2}\right)^2 \sim 2 f V}{W}} = d.$$

a representing angle of repose of earth, w and W *weights of earth and wall per cube foot, f friction of wall on base* A B, *and* V *weight of wall.*

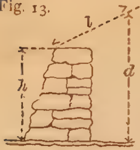
Illustration.—If a wall of masonry, Fig. 12, 8 feet in thickness and 13 in height, is to sustain earth level with its upper surface, earth weighing 100 lbs. per cube foot, weight of wall 150 lbs. per cube foot = 15 600 lbs., and angle of repose of earth 30°; what should be the depth of wall below surface of earth?

$\tan. 45 - 30 \div 2 = .5774$, and $f = .3$.

Then $1.4 \times .5774\sqrt{\frac{13^2 \times 100 \times .5774^2 \sim 2 \times .3 \times 15600}{150}} = .8084 \times \sqrt{\frac{9360 \sim 5634.3}{150}}$ = 4.027 *feet.*

Note.—Coefficient of stability is assumed by French engineers for walls of fortifications 1.4 *h*, and if ground is clayey or wet $f = .3$.

Fig. 13.



In Computing Stability of a Surcharged Wall Fig. 13, *substitute d for h, as in following illustration.* (*Molesworth.*)

d, representing depth at distance l, = h.

In slopes of 1 to 1, $d = 1.71\,h$; of 1.5 to 1, $= 1.55$; of 2 to 1, $= 1.45$; of 3 to 1, $= 1.31$, and 4 to 1, $= 1.24$.

To Determine Form of a Pier to Sustain equal Pressure per Unit of Surface at all its Horizontal Sections, or any Height.

$A\,n^d = a$, or $A\,N = a$. *A and a representing areas of sections at summit of pier and at any depth, d, measured from summit, n a number the hyp. log. of which* $= 1 \div$ *height,* H, *of a column of the material of which pier is constructed, due to required pressure, and* N *the number, com. log. of which* $= \frac{.4343\,d}{H}$.

Illustration.—Height of a pier is 20 feet, and area of section of its summit = 1 foot; what should be its areas at 10 feet and base?

$1 \div 20 = .05$, and number $= 1.0513$; $1 \times 1.0513^{10} = 1.649$ *feet*; and $1 \times 1.0513^{20} = 2.719$ *feet.*

Counterforts are increased thicknesses of a wall at *its back*, at intervals of its length.

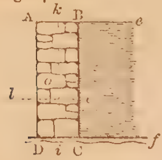
Embankment Walls and Dams.

Thrust of water upon inner face of an Embankment wall or Dam is horizontal.

When Both Faces are Vertical, Fig. 14.

Assume perpendicular embankment or wall, A B C D, Fig. 14, to sustain pressure of water, B C *e f*.

Fig. 14.



Let *k i* be a vertical line passing through *o*, centre of gravity of wall, *c* centre of pressure of water, distance C *c* being $= .33$ B C. Draw *c l* perpendicular to B C; then, since section A C of wall is rectangular, centre of gravity, *o*, is in its geometrical centre, and therefore D $i = .5$ D C. Now *l* D *i* is to be considered as a bent lever, fulcrum of which is D, weight of wall acting in direction of centre of gravity, *o*, on arm D *i*, and pressure of water on arm D *l*, or a force equal to that pressure thrusting in direction *c l*.

Then $P \times D\,l = P \times \frac{B\,C}{3} = W \times \frac{D\,C}{2}$, or $P = \frac{3\,D\,C\,.\,W}{2\,B\,C}$. P *representing pressure of water.*

Note.—When this equation holds, a wall or embankment will just be on the point of overturning; but in order that they may have complete stability, this equation should give a much larger value to P than its actual amount.

The following formulas are for walls or embankments one foot in length; for if they have stability for that length they will be stable for any other length.

$P = \frac{h^2}{2}\,w$, also $W = h\,b\,W$, each value being for 1 foot in length, which, being substituted in the equations, there will result

$\frac{h^2}{2}\,w = \frac{3\,b \times h\,b\,W'}{2\,h}$, or $h^2\,w = 3\,b^2\,W$; $b\sqrt{\frac{3\,W}{w}} = h$, and $h\sqrt{\frac{w}{3\,W}} = b$. *h representing depth of water and wall or embankment, which are here assumed to be equal, b breadth of wall or embankment, and* W *and w weights of wall and water per cube foot in lbs.*

Which gives breadth of a wall or embankment that will just sustain pressure of the water.

To Compute Equilibrium. $h\sqrt{\frac{w}{3\,W}}=b.$

ILLUSTRATION I.—Height of a wall, B C, equal to depth of water, is 12 feet, and respective weights of water and wall are 62.5 lbs. and 120 lbs. per cube foot; required breadth of wall, so that it may have complete stability to sustain the pressure of water.

$12\sqrt{\frac{62.5}{3\times 120}}=12\times .4166=5$ *feet, breadth that will just sustain pressure of the water.*

Therefore an addition should be made to this to give the wall complete stability, say 2 feet; hence $5+2=7$, *required width of wall.*

2.—Width of a wall is 3 feet, and weight of a cube foot of it is 150 lbs; required height of wall to resist pressure of fresh water to the top.

$$3\sqrt{\frac{3\times 150}{62.5}}=8.049 \textit{ feet.}$$

To Compute Stability. $h\sqrt{\frac{2\,w}{3\,W}}=b.$

ILLUSTRATION.—Take elements of preceding case.

$$12\sqrt{\frac{2\times 62.5}{3\times 120}}=12\times .589=7.07 \textit{ feet.}$$

Or, Divide 1, 2, or 3, etc., according as the nature of the ground, the material, and the character of the thrust of the water requires, by .05 weight of material of wall, per cube foot, extract the square root of quotient, and multiply result by extreme height of water.

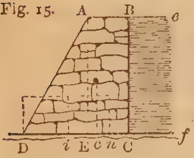
EXAMPLE.—What should be the thickness of a vertical faced wall of masonry, having a weight of 125 lbs. per cube foot, to sustain a head of water of 40 feet, and to have stability?

$$\sqrt{(2\div .05\times 125)}\;40=\sqrt{.32}\times 40=22.63 \textit{ feet.}$$

$$\text{Or, } h\sqrt{\frac{2\,w}{3\,W}}=40\sqrt{.3472}=23.56 \textit{ feet.}$$

When Dam has an Exterior Slope or Batter, as A D.—Fig. 15.

Fig. 15.



Assume prismoidal wall, A B C D, to sustain pressure of water, B C $e\,f$.

Draw A E perpendicular to D C; $h=$ B C, the top breadth A B $=$ E C $=b$, and bottom breadth, D E, of sloping part, A E D $=$ S.

Then weights of portions A C and A E D respectively for one foot in length are $h\,b\,W$ and $.5\,W\,S\,h$, these weights acting at points n and i respectively.

To Compute Moment.

$h\,b\,W\times\left(S+\frac{b}{2}\right)=$ *moment for* A C, and $\frac{h\,S\,W}{2}\times\frac{2\,S}{3}=$ *moment for* A E D.

Hence, $\frac{W\,h}{2}\left(\overline{S+b}^{\,2}-\frac{S^2}{3}\right)=$ *moment of dam.* S *representing batter or base* E D.

ILLUSTRATION.—Height of a dam, B C, Fig. 15, is 9 feet, base C E 3, and E D 4 feet; what is its moment?

$$\text{A C}=9\times 3\times 120\times\left(4+\frac{3}{2}\right)=3240\times 5.5=17\,820 \textit{ lbs.}$$

$$\text{A D E}=\frac{9\times 4\times 120}{2}\times\frac{2\times 4}{3}=2160\times 2\tfrac{2}{3}=5760 \textit{ lbs.}$$

Hence, $17\,820+5760=23\,580$ *lbs. moment.* Or, $\frac{120\times 9}{2}\left(\overline{4+3}^{\,2}-\frac{4^2}{3}\right)=540\times 43\frac{2}{3}=23\,580$ *lbs. moment.*

To Compute Elements of Walls or Dams with an Exterior Batter.—Fig. 15.

To Compute Width of Top.

When Width of Batter is Given. $\sqrt{\frac{2\,h^2\,w}{3\,W}+\frac{S^2}{3}}-S=b.$

ILLUSTRATION.—Assume height of wall 9 and batter 3 feet, and W and w 120 and 62.5 lbs. per cube foot.

$$\sqrt{\frac{2\times 9^2\times 62.5}{3\times 120}\times\frac{3^2}{3}}-3=\sqrt{28.125+3}-3=2.58\ feet.$$

To Compute Width of Base.

When Width of Batter is Given. $\sqrt{\frac{2\,h^2\,w}{3\,W}+\frac{S^2}{3}}=B.$

$$\sqrt{\frac{2\times 9^2\times 62.5}{3\times 120}+\frac{3^2}{3}}=5.58\ feet=S+b.$$

To Compute Width of Batter.

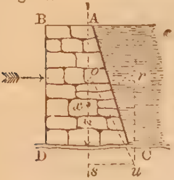
When Width of Top is Given. $\sqrt{\frac{h^2\,w}{W}+\frac{3\,b^2}{4}}-\frac{3\,b}{2}=S.$

$$\sqrt{\frac{9^2\times 62.5}{120}+\frac{3\times 2.58^2}{4}}-\frac{3\times 58}{2}=\sqrt{42.18+4.99}-3.87=3\ feet.$$

When Width of Bottom is Given. $\sqrt{3\,B^2-\frac{2\,h^2\,w}{W}}=S.$

To Determine Stability of a Retaining Wall or Dam by Protraction.—Fig. 16.

Fig. 16.



Assume A B C D, section of a wall. On horizontal line of centre of thrust or pressure, with a suitable scale, lay off, from vertical line of centre of gravity of wall, line $o\,r=$ thrust against wall, and on vertical line at centre of gravity of wall, at its intersection, o, with centre of thrust, let fall $o\,s=$ weight of wall.

Complete parallelogram, and if diagonal $o\,u$ or its prolongation falls within C, the wall is stable, and W × distance from line $o\,s=$ moment of wall.

W representing whole weight of wall in lbs.

To Determine Centre of Gravity of a Wall or Dam.—Fig. 16.

By Ordinates. $\frac{1}{3}\left(A\,B+C\,D-\frac{A\,B\times C\,D}{A\,B+C\,D}\right)=x$, and $\frac{C\,D}{3}\left(\frac{2\,A\,B+C\,D}{A\,B+C\,D}\right)=z.$

To Compute Base of Dam.

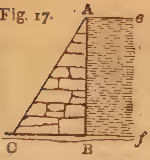
When Height, Rate of Batter, and Weight of Materials are given. RULE.—Multiply square of width of batter by .0166 weight of material per cube foot, add 1, 2, or 3 times square of depth of water, according as resistance due to equilibrium is required, divide result by .05 weight of material per cube foot, and extract square root of quotient.

Or, $\sqrt{\frac{x\,h^2+b^2\times .0166\,W}{.05\,W}}=b.$ *$x=$ number of times of resistance required.*

EXAMPLE.—Assume a dam 40 feet in height, constructed of masonry weighing 120 lbs. per cube foot, to batter 3 ins. per foot, and to have twice the resistance due to its equilibrium; what should be its breadth at its base, D C?

$$\frac{40\times 3}{12}=10=batter,\ \text{and}\ \sqrt{\frac{40^2\times 2+10^2\times .0166\times 120}{.05\times 120}}=\sqrt{\frac{3399}{6}}=23.8\ feet.$$

Fig. 17.



When Section of Dam is a Triangle, Fig. 17. — Assume dam, A B C, to sustain a head of water, $e f$.

RULE.—Proceed as by Rule for Fig. 14; multiply by .033 instead of .05.

EXAMPLE.—As before.

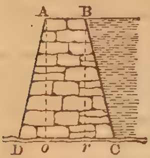
$$\sqrt{(2 \div .033 \times 125)\, 40} = \sqrt{.485 \times 40} = 27.84 \textit{ feet.}$$

Or, Formula for S (C B), Fig. 15. $\sqrt{\dfrac{h^2 \times w}{W}} = 28.28$ *feet.*

To Determine Section of a Vertical Wall which shall have Equal Resistance of one having Section of a Triangle. (*See J. C. Trautwine, Phila.*, 1872.)

To Compute Thickness of Base of a Wall or Dam.—Fig. 18.

Fig. 18.



RULE.—Divide 1, 2, or 3 times square of depth of water by .05 weight of material, add quotient to .5 batter on one face, and square root of this sum, added to half batter on other side, will give thickness.

Or, $\sqrt{\dfrac{h^2\, x}{.05\, W} + \left(\dfrac{b}{2}\right)^2} + \dfrac{b'}{2} = Base.$ *b and b' representing exterior and interior batters, and x, as before, number of times of resistance or square of depth.*

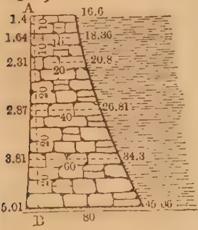
EXAMPLE.—Assume a dam 40 feet in height, to batter 5 feet on each side, constructed of masonry weighing 120 lbs. per cube foot, and to have twice the resistance due to its equilibrium; what should be breadth of base, D C?

$$\sqrt{\frac{40^2 \times 2}{.05 \times 120} + \left(\frac{5}{2}\right)^2} + \frac{5}{2} = \sqrt{539.58} + 2.5 = 25.73 \textit{ feet.}$$

High Masonry Dams.

Rubble Masonry, well laid in strong cement, will bear with safety a load equivalent to weight of a column of it 160 feet in height. Assuming such masonry as twice weight of water, it is equivalent to a pressure of 20 000 lbs. per sq. foot.

Fig. 19.



$$\text{Log. } B + .434\,294 \times \frac{d}{80} = b.$$ B *representing width of wall at top, and d depth at any desired point below top, both in feet.*

Ordinarily, B may be taken at 18 feet, and in cases of extreme and exposed heights of dam at 20 and more, and when b is determined, .9 of it is to be on outer face of wall, as A B, and .1 on inner face.

ILLUSTRATION.—Determine section of a dam, Fig. 19, 80 feet in height, at depths of 10, 20, 40, 60, and 80 feet.

Log. B = 1.2553.

$$\text{Log. } 1.2553 + .4343 \times \frac{10}{80} = \text{log. } 1.2553 + .0543 = 20.4, \text{ which} \times .9 = 18.36.$$

$$\text{“ } 1.2553 + .4343 \times \frac{20}{80} = \text{log. } 1.2553 + .1086 = 23.11, \text{ which} \times .9 = 20.8.$$

$$\text{“ } 1.2553 + .4343 \times \frac{40}{80} = \text{log. } 1.2553 + .2172 = 29.68, \text{ which} \times .9 = 26.81.$$

$$\text{“ } 1.2553 + .4343 \times \frac{60}{80} = \text{log. } 1.2553 + .3257 = 38.11, \text{ which} \times .9 = 34.3.$$

$$\text{“ } 1.2553 + .4343 \times \frac{80}{80} = \text{log. } 1.2553 + .4343 = 50.07, \text{ which} \times .9 = 45.06.$$

STEAM.

STEAM is generated by heating of water until it attains temperature of ebullition or vaporization, and elevation of its temperature is sensible to indications of a thermometer up to point of ebullition; it is then converted into steam by additional temperature, which cannot be indicated by a thermometer, and is termed latent. (See Heat, page 508.)

Pressure and density of steam, which is generated in free contact with water, rises with the temperature, and reciprocally its temperature rises with the pressure and density, and higher the temperature more rapid the pressure. There is but one and a corresponding pressure and density for each temperature, and steam generated in free contact with water is both at its maximum density and pressure for its temperature, and in this condition it is termed *saturated*, from its being incapable of vaporizing more water unless its temperature is raised.

Saturated Steam is the normal condition of steam generated in free contact with water, and same density and same pressure always exist in conjunction with same temperature. It therefore is both at its condensing and generating points; that is, it is condensed if its temperature is reduced, and more water is evaporated if its temperature is raised.

If, however, the whole of the water is evaporated, or a volume of saturated steam is isolated from water, in a confined space, and an additional quantity of heat is supplied to the steam, its condition of saturation is changed, the steam becomes *superheated*, and both temperature and pressure are increased, while its density is not increased. Steam, when thus surcharged, approaches to condition of a gas.

With saturated steam, pressure does not rise directly with the temperature.

Steam, at its boiling-point, is equal to pressure of atmosphere, which is 14 723 307 lbs. (page 427), at 60° upon a sq. inch.

In all computations concerning steam, it is necessary to have some or all of following elements, viz.:

Its *Pressure*, which is termed its tension or elastic force, and is expressed in lbs. per sq. inch. Its *Temperature*, which is number of its degrees of heat indicated by a thermometer. Its *Density*, which is weight of a unit of its volume compared with that of water. Its *Relative volume*, which is space occupied by a given weight or volume of it, compared with weight or volume of water that produced it.

Under pressure of the atmosphere alone, temperature of water cannot be raised above its boiling-point.

Expansive force of steam of all fluids is same at their boiling-point.

A cube inch of water, evaporated under ordinary atmospheric pressure, is converted into 1642* cube ins. of steam, or, in a unit of measure, very nearly 1 cube foot, and it exerts a mechanical force equal to raising of $14.723\,307 \times 144 = 2120.156\,208$ lbs. 1 foot high.

A pressure of 1 lb. upon a sq. inch will support a column of mercury at a temperature of 60°. $1 \div .490\,776\,9$ (page 427) $= 2.037\,586$ ins. in height; hence it will raise a mercurial siphon gauge one half of this, or 1.018 793 ins.

Velocity of steam, when flowing into a vacuum, is about 1550 feet per second when at a pressure equal to the atmosphere; when at 10 atmospheres velocity is increased to but 1780 feet; and when flowing into the air under a similar pressure it is about 650 feet per second, increasing to 1600 feet for a pressure of 20 atmospheres.

Boiling-points of Water, corresponding to different heights of barometer, see Heat, page 517.

Volume of a cube foot of water evaporated into steam at 212° is 1642 cube feet; hence $1 \div 1642 = .000\,609\,013$, which represents density or specific gravity of steam at pressure of atmosphere.

Elasticity of vapor of alcohol, at all temperatures, is about 2.125 times that of steam.

Specific Gravity, compared with air, is as weight of a cube foot of it compared with equal volume of air. Thus, weight of a cube foot of steam at 212° and at pressure of atmosphere is 266.124 grains; weight of a like volume of air at 32° is 565.096 grains, and at 62° 532.679 grains. Hence $266.124 \div 532.679 = .499\,59$, *specific gravity of steam* compared with air at 32°, and with water it is .000 609 013.

* Pole's Formula makes it 1712.

Total Heat of Saturated Steam.

$1081.4 + .305\,T =$ *total heat.* T *representing initial temperature of water.*

ILLUSTRATION.—What is total heat of steam at 212°?

$$1081.4 + .305 \times 212 = 1146.06.$$

As specific heat of water is .9 greater at 212° than at 32°, hence the 212° would be 212.9, and 1146.33 *the result.*

Total Heat of Gaseous Steam $1074.6 + 475\,T =$ *total heat.*

Absorption of Heat in Generation of 1 *Lb. of Water from* 32° *to* 212°.

		Units.	Force, lbs.
Sensible heat, or heat to raise temperature of water from 32° to 212°		$180.9 \times 772 =$	139 655
Latent heat to produce steam	892.9		
" " to resist atmospheric pressure 14.7 lbs. per sq. inch	72.3	$965.2 \times 772 =$	745 134
Total or constituent heat		1146.1	884 789

This number, 1146.1, is a *Constant*, and expresses units of heat in 1 lb. of steam from 32° up to temperature at which conversion takes place.

Thus, 1 lb. water heated from 32° to 332°, requires as much heat as would raise 300 lbs. 1°. Hence .... 300°
And 1 lb. water converted into steam at 332° (= 106 lbs. pressure), absorbs as much heat for its conversion as would raise 846.1 lbs. water 1°. Hence .... 846.1°

1146.1°

Mechanical Equivalent of Heat contained in Steam.

1 lb. water heated from 32° to 212° requires as much heat as would raise 180 lbs. 1°. Hence .... 180.9°
1 lb. water at 212°, converted into steam at 212° (= 14.7 lbs. pressure), absorbs as much heat for its conversion as would raise 966.6 lbs. water 1°. Hence .... 965.2°

1146.1°

Mechanical Equivalent, or maximum theoretical duty of quantity of heat in 1 lb. of steam, is 772 lbs., which $\times$ 1146.1 *units of heat* = 884 789.2 lbs. *raised* 1 *foot high.*

To Compute Pressure of Steam.

When Height of Column of Mercury it will Support is given. RULE.—Divide height of column of mercury in ins. by 2.037 586, and quotient will give pressure per sq. inch in lbs.

EXAMPLE. — Height of a column of mercury is 203.7586 ins.; what pressure per sq. inch will it contain?

$$203.7586 \div 2.037\,586 = 100 \textit{ lbs.}$$

To Compute Weight of a Cube Foot of Steam.

RULE.—Multiply its density by 62.425.

EXAMPLE.—Density of a volume of steam is .000 609 013; what is its weight?

$$.000\,609\,013 \times 62.425 = .038\,016\,825 \textit{ lbs.}$$

NOTE.—See table, page 708.

1 atmosphere or 14.723 307 lbs. per sq. inch = 30 ins. of mercury.

To Compute Temperature of Steam.

RULE.—Multiply 6th root of its force in ins. of mercury by 177.2, subtract 100 from product, and remainder will give temperature in degrees.

EXAMPLE.—When elastic force of steam is equal to a pressure of 64 ins. of mercury, what is its temperature?

NOTE.—To extract 6th root of a number, ascertain cube root of its square root.

$$\sqrt{64} = 8, \text{ and } \sqrt[3]{8} = 2. \quad \text{Hence, } 2 \times 177.2 - 100 = 254.4^\circ\ t.$$

Or, $\dfrac{2938.16}{6.199\,354\,4 - \log. p} - 371.85 = t.$ *p representing pressure in lbs. per sq. inch.*

To Compute Volume of Water contained in a given Volume of Steam.

When its Density is given. RULE.—Multiply volume of steam in cube feet by its density, and product will give volume of water in cube feet.

EXAMPLE.—Density of a volume of 16 420 cube feet of steam is .000 609; what is the weight of it in lbs.?

$16\,420 \times .000\,609 = 10 =$ *volume of water*, which $\times\ 62.425 = 624.25$ *lbs.*

To Compute Pressure of Steam in Ins. of Mercury, or Lbs. per Sq. Inch.

When Temperature is given. RULE 1.—Add 100 to temperature, divide sum proportionally by 177.2 for temperature of 212°, and by 160 for temperatures up to 445°; or, 177.6 for sea-water, and 185.6 for sea-water saturated with salt, and 6th power of quotient will give pressure.

EXAMPLE.—Temperature of steam is 254°; what is its pressure?

$100 + 254 \div 177.2 = 1.998$, and $1.998^6 = 63.62$ *ins.*

When Ins. of Mercury are given. 2.—Divide ins. of mercury by 2.037 586, and quotient will give pressure.

When Pressure in Lbs. is given. 3.—Multiply pressure by 2.037 586.

To Compute Specific Gravity of Steam compared with Air.

RULE.—Divide constant number 829.05 ($1642 \times .5049$) by volume of steam at temperature of pressure at which gravity is required.

EXAMPLE.—Pressure of steam is 60 lbs., and volume 437; what its specific gravity?

$829.05 \div 437 = 1.898.$

To Compute Volume of a Cube Foot of Water in Steam.

When Elastic Force and Temperature of Steam are given. RULE.—To 430.25 for temperature of 212°, and 332 for temperatures up to 445°, add temperature in degrees; multiply sum by 76.5, and divide product by elastic force of steam in ins. of mercury.

NOTE.—When force in ins. of mercury is not given, multiply pressure in lbs. per sq. inch by 2.037 586.

EXAMPLE.—Temperature of a cube foot of water evaporated into steam is 386°, and elastic force is 427.5 ins.; what is its volume?

Assume 369 for proportionate factor. $369 + 386 \times 76.5 \div 427.5 = 135.1$ *cube feet.*

Or, for 1 lb. of steam, $2.519 - .941 \log. p = \log. V$ *in cube feet.*

Assume $p = 14.7$ *lbs.* $2.519 - .941 \log. 14.7 = 2.519 - 1.098 = 1.421 = \log. 26.34$ *cube feet*, which $\times\ 62.425 = 164$ *feet.*

Or, *When Density is given.*—Divide 1 by density, and quotient will give volume in cube feet.

To Compute Density or Specific Gravity of Steam.

When Volume is given. RULE.—Divide 1 by volume in cube feet.

EXAMPLE.—Volume is 210; what is density?

$1 \div 210 = .004\,761.$ Or, for 1 lb. of steam, $.941 \log. p - 2.519 = \log. D.$

When Pressure is given.—Take temperature due to pressure, and proceed as by rule to compute volume, which, when obtained, proceeds as above.

To Compute Volume of Steam required to raise a Given Volume of Water to any Given Temperature.

RULE.—Multiply water to be heated by difference of temperatures between it and that to which it is to be raised, for a dividend; then to temperature of steam add 965.2°, from that sum take required temperature of water for a divisor, and quotient will give volume of water.

EXAMPLE.—What volume of steam at 212° will raise 100 cube feet of water at 80° to 212°?

$$\frac{100 \times \overline{212-80}}{212+965.2-212} = 13.68 \textit{ cube feet water; } \text{or, } (13.68 \times 1642 - 212) = 22\,463 \textit{ of steam.}$$

To Compute Volume of Water, at any Given Temperature, that must be Mixed with Steam to Raise or Reduce the Mixture to any Required Temperature.

RULE.—From required temperature subtract temperature of water; then ascertain how often remainder is contained in required temperature subtracted from sum of sensible and latent heat of the steam, and quotient will give volume required.

Sum of Sensible and Latent Heats for a range of temperatures will be found under Heat, pages 508 and 509.

EXAMPLE.—Temperature of condensing water of an engine is 80°, and required temperature 100°; what is proportion of condensing water to that evaporated at a pressure of 34 lbs. per sq. inch?

Sum of sensible and latent heats 1190.4°.

$$100 - 80 = 20. \quad \text{Then, } 1190.4 - 100 \div 20 = 54.52 \text{ to } 1.$$

When Temperature of Steam is given. $\dfrac{l + \overline{T - t}}{t - w} = V$. *l representing latent heat, T and t temperatures of steam and required temperature, w temperature of condensing water, and V volume of condensing water in cube feet.*

ILLUSTRATION.—Temperature of steam in a cylinder is 257.6°, and other elements same as in preceding example; required volume of injection water? Latent heat of steam at 230° = 932.8°.

$$\frac{932.8 + \overline{257.6 - 100}}{100 - 80} = \frac{1090.4}{20} = 54.52 \textit{ volumes.}$$

To Compute Temperature of Water in Condenser or Reservoir of a Steam-engine.

$\dfrac{l + T + \overline{V \times w}}{V + 1} = t$. ILLUSTRATION.—Assume elements as preceding.

$$\frac{932.8 + 257.6 + \overline{54.52 \times 80}}{54.52 + 1} = \frac{5552}{55.52} = 100°.$$

To Compute Latent Heat of Saturated Steam.

$1115.2 - .708\,t = l$. ILLUSTRATION.—Assume temperature 257.6° as preceding.

$$1115.2 - .708 \times 257.6 = 932.8°.$$

To Compute Total Heat of Saturated Steam.

$305\,t + 1081.4 = H$. ILLUSTRATION.—Assume temperature as preceding.

$$.305 \times 257.6 + 1081.4 = 1160.$$

Elastic Force and Temperature of Vapors of Alcohol, Ether, Sulphuret of Carbon, Petroleum, and Turpentine.

Force in Ins. of Mercury.

°	Ins.	°	Ins.	°	Ins.	°	Ins.	°	Ins.
ALCOHOL.		ALCOHOL.		ETHER.		SULPHURET OF CARBON.		PETROLEUM.	
32	.4	140	13.9	34	6.2			316	30
50	.86	160	22.6	54	15.3	53.5	7.4	345	44.1
60	1.23	173	30	74	16.2	72.5	12.55	375	64
70	1.76	180	34.73	94	24.7	110	30	OIL OF TURPENTINE.	
80	2.45	200	53	96 }	30	212	126		
90	3.4	212	67.5	104 }		279.5	300		
100	4.5	220	78.5	120	39.47	347	606	315	30
120	8.1	240	111.24	150	67.6			357	47.78
130	10.6	264	166.1	212	178			370	62.4

Saturated Steam.

Pressure, Temperature, Volume, and Density.

Pressure per Sq. Inch.	Pressure in Mercury.	Temperature.	Total Heat from Water at 32°.	Volume of 1 Lb.	Density, or Weight of one Cube Foot.	Pressure per Sq. Inch.	Pressure in Mercury.	Temperature.	Total Heat from Water at 32°.	Volume of 1 Lb.	Density, or Weight of one Cube Foot.
Lbs.	Ins.	°	°	Cub. ft.	Lb.	Lbs.	Ins.	°	°	Cub. ft.	Lb.
1	2.04	102.1	1112.5	330.36	.003	58	118.08	290.4	1170	7.24	.138
2	4.07	126.3	1119.7	172.08	.0058	59	120.12	291.6	1170.4	7.12	.1403
3	6.11	141.6	1124.6	117.52	.0085	60	122.16	292.7	1170.7	7.01	.1425
4	8.14	153.1	1128.1	89.62	.0112	61	124.19	293.8	1171.1	6.9	.1447
5	10.18	162.3	1130.9	72.66	.0138	62	126.23	294.8	1171.4	6.81	.1469
6	12.22	170.2	1133.3	61.21	.0163	63	128.26	295.9	1171.7	6.7	.1493
7	14.25	176.9	1135.3	52.94	.0189	64	130.3	296.9	1172	6.6	.1516
8	16.29	182.9	1137.2	46.69	.0214	65	132.34	298	1172.3	6.49	.1538
9	18.32	188.3	1138.8	41.79	.0239	66	134.37	299	1172.6	6.41	.156
10	20.36	193.3	1140.3	37.84	.0264	67	136.4	300	1172.9	6.32	.1583
11	22.39	197.8	1141.7	34.63	.0289	68	138.44	300.9	1173.2	6.23	.1605
12	24.43	202	1143	31.88	.0314	69	140.48	301.9	1173.5	6.15	.1627
13	26.46	205.9	1144.2	29.57	.0338	70	142.52	302.9	1173.8	6.07	.1648
14	28.51	209.6	1145.3	27.61	.0362	71	144.55	303.9	1174.1	5.99	.167
14.7	29.92	212	1146.1	26.36	.03802	72	146.59	304.8	1174.3	5.91	.1692
15	30.54	213.1	1146.4	25.85	.0387	73	148.62	305.7	1174.6	5.83	.1714
16	32.57	216.3	1147.4	24.32	.0411	74	150.66	306.6	1174.9	5.76	.1736
17	34.61	219.6	1148.3	22.96	.0435	75	152.69	307.5	1175.2	5.68	.1759
18	36.65	222.4	1149.2	21.78	.0459	76	154.73	308.4	1175.4	5.61	.1782
19	38.68	225.3	1150.1	20.7	.0483	77	156.77	309.3	1175.7	5.54	.1804
20	40.72	228	1150.9	19.72	.0507	78	158.8	310.2	1176	5.48	.1826
21	42.75	230.6	1151.7	18.84	.0531	79	160.84	311.1	1176.3	5.41	.1848
22	44.79	233.1	1152.5	18.05	.0555	80	162.87	312	1176.5	5.35	.1869
23	46.83	235.5	1153.2	17.26	.058	81	164.91	312.8	1176.8	5.29	.1891
24	48.86	237.8	1153.9	16.64	.0601	82	166.95	313.6	1177.1	5.23	.1913
25	50.9	240.1	1154.6	15.99	.0625	83	168.98	314.5	1177.4	5.17	.1935
26	52.93	242.3	1155.3	15.38	.065	84	171.02	315.3	1177.6	5.11	.1957
27	54.97	244.4	1155.8	14.86	.0673	85	173.05	316.1	1177.9	5.05	.198
28	57.01	246.4	1156.4	14.37	.0696	86	175.09	316.9	1178.1	5	.2002
29	59.04	248.4	1157.1	13.9	.0719	87	177.13	317.8	1178.4	4.94	.2024
30	61.08	250.4	1157.8	13.46	.0743	88	179.16	318.6	1178.6	4.89	.2044
31	63.11	252.2	1158.4	13.05	.0766	89	181.2	319.4	1178.9	4.84	.2067
32	65.15	254.1	1158.9	12.67	.0789	90	183.23	320.2	1179.1	4.79	.2089
33	67.19	255.9	1159.5	12.31	.0812	91	185.27	321	1179.3	4.74	.2111
34	69.22	257.6	1160	11.97	.0835	92	187.31	321.7	1179.5	4.69	.2133
35	71.26	259.3	1160.5	11.65	.0858	93	189.34	322.5	1179.8	4.64	.2155
36	73.29	260.9	1161	11.34	.0881	94	191.38	323.3	1180	4.6	.2176
37	75.33	262.6	1161.5	11.04	.0905	95	193.41	324.1	1180.3	4.55	.2198
38	77.37	264.2	1162	10.76	.0929	96	195.45	324.8	1180.5	4.51	.2219
39	79.4	265.8	1162.5	10.51	.0952	97	197.49	325.6	1180.8	4.46	.2241
40	81.43	267.3	1162.9	10.27	.0974	98	199.52	326.3	1181	4.42	.2263
41	83.47	268.7	1163.4	10.03	.0996	99	201.56	327.1	1181.2	4.37	.2285
42	85.5	270.2	1163.8	9.81	.102	100	203.59	327.9	1181.4	4.33	.2307
43	87.54	271.6	1164.2	9.59	.1042	101	205.63	328.5	1181.6	4.29	.2329
44	89.58	273	1164.6	9.39	.1065	102	207.66	329.1	1181.8	4.25	.2351
45	91.61	274.4	1165.1	9.18	.1089	103	209.7	329.9	1182	4.21	.2373
46	93.65	275.8	1165.5	9	.1111	104	211.74	330.6	1182.2	4.18	.2393
47	95.69	277.1	1165.9	8.82	.1133	105	213.77	331.3	1182.4	4.14	.2414
48	97.72	278.4	1166.3	8.65	.1156	106	215.81	331.9	1182.6	4.11	.2435
49	99.76	279.7	1166.7	8.48	.1179	107	217.84	332.6	1182.8	4.07	.2456
50	101.8	281	1167.1	8.31	.1202	108	219.88	333.3	1183	4.04	.2477
51	103.83	282.3	1167.5	8.17	.1224	109	221.92	334	1183.3	4	.2499
52	105.87	283.5	1167.9	8.04	.1246	110	223.95	334.6	1183.5	3.97	.2521
53	107.9	284.7	1168.3	7.88	.1269	111	225.99	335.3	1183.7	3.93	.2543
54	109.94	285.9	1168.6	7.74	.1291	112	228.02	336	1183.9	3.9	.2564
55	111.98	287.1	1169	7.61	.1314	113	230.06	336.7	1184.1	3.86	.2586
56	114.01	288.2	1169.3	7.48	.1336	114	232.1	337.4	1184.3	3.83	.2607
57	116.05	289.3	1169.7	7.36	.1364	115	234.13	338	1184.5	3.8	.2628

Pressure		Temperature.	Total Heat from Water at 32°.	Volume of 1 Lb.	Density, or Weight of one Cube Foot.	Pressure		Temperature.	Total Heat from Water at 32°.	Volume of 1 Lb.	Density, or Weight of one Cube Foot.
per Sq. Inch.	in Mercury.					per Sq. Inch.	in Mercury.				
Lbs.	Ins.	°	°	Cub. ft.	Lb.	Lbs.	Ins.	°	°	Cub. ft.	Lbs.
116	236.17	338.6	1184.7	3.77	.2649	149	303.35	357.8	1190.5	2.98	.3357
117	238.2	339.3	1184.9	3.74	.2652	150	305.39	358.3	1190.7	2.96	.3377
118	240.24	339.9	1185.1	3.71	.2674	155	315.57	361	1191.5	2.87	.3484
119	242.28	340.5	1185.3	3.68	.2696	160	325.75	363.4	1192.2	2.79	.359
120	244.31	341.1	1185.4	3.65	.2738	165	335.93	366	1192.9	2.71	.3695
121	246.35	341.8	1185.6	3.62	.2759	170	346.11	368.2	1193.7	2.63	.3798
122	248.38	342.4	1185.8	3.59	.278	175	356.29	370.8	1194.4	2.56	.3899
123	250.42	343	1186	3.56	.2801	180	366.47	372.9	1195.1	2.49	.4009
124	252.45	343.6	1186.2	3.54	.2822	185	376.65	375.3	1195.8	2.43	.4117
125	254.49	344.2	1186.4	3.51	.2845	190	386.83	377.5	1196.5	2.37	.4222
126	256.53	344.8	1186.6	3.49	.2867	195	397.01	379.7	1197.2	2.31	.4327
127	258.56	345.4	1186.8	3.46	.2889	200	407.19	381.7	1197.8	2.26	.4431
128	260.6	346	1186.9	3.44	.2911	210	427.54	386	1199.1	2.16	.4634
129	262.64	346.6	1187.1	3.41	.2933	220	447.9	389.9	1200.3	2.06	.4842
130	264.67	347.2	1187.3	3.38	.2955	230	468.26	393.8	1201.5	1.98	.5052
131	266.71	347.8	1187.5	3.35	.2977	240	488.62	397.5	1202.6	1.9	.5248
132	268.74	348.3	1187.6	3.33	.2999	250	508.98	401.1	1203.7	1.83	.5464
133	270.78	348.9	1187.8	3.31	.302	260	529.34	404.5	1204.8	1.76	.5669
134	272.81	349.5	1188	3.29	.304	270	549.7	407.9	1205.8	1.7	.5868
135	274.85	350.1	1188.2	3.27	.306	280	570.06	411.2	1206.8	1.64	.6081
136	276.89	350.6	1188.3	3.25	.308	290	590.42	414.4	1207.8	1.59	.6273
137	278.92	351.2	1188.5	3.22	.3101	300	610.78	417.5	1208.7	1.54	.6486
138	280.96	351.8	1188.7	3.2	.3121	350	712.57	430.1	1212.6	1.33	.7498
139	282.99	352.4	1188.9	3.18	.3142	400	814.37	444.9	1217.1	1.18	.8502
140	285.03	352.9	1189	3.16	.3162	450	916.17	456.7	1220.7	1.05	.9499
141	287.07	353.5	1189.2	3.14	.3184	500	1018	467.5	1224	.95	1.049
142	289.1	354	1189.4	3.12	.3206	550	1119.8	477.5	1227	.87	1.148
143	291.14	354.5	1189.6	3.1	.3228	600	1221.6	487	1229.9	.8	1.245
144	293.17	355	1189.7	3.08	.325	650	1323.4	495.6	1232.5	.74	1.342
145	295.21	355.6	1189.9	3.06	.3273	700	1425.8	504.1	1235.1	.69	1.4395
146	297.25	356.1	1190	3.04	.3294	800	1628.7	519.5	1239.8	.61	1.6322
147	299.28	356.7	1190.2	3.02	.3315	900	1832.3	533.6	1244.2	.55	1.8235
148	301.32	357.2	1190.3	3.	.3336	1000	2035.9	546.5	1248.1	.5	2.014

Saturated Steam from 32° to 212°. (*Claudel.*)

Temperature.	Pressure. Mercury.	Pressure. Per Sq. Inch.	Weight of 100 Cub. Feet.	Volume of 1 Lb.	Temperature.	Pressure. Mercury.	Pressure. Per Sq. Inch.	Weight of 100 Cub. Feet.	Volume of 1 Lb.
°	Ins.	Lbs.	Lb.	Cub. Feet.	°	Ins.	Lbs.	Lbs.	Cub. Feet.
32	.181	.089	.031	3226	125	3.933	1.932	.554	180.5
35	.204	.1	.034	2941	130	4.509	2.215	.63	158.7
40	.248	.122	.041	2439	135	5.174	2.542	.714	140.1
45	.299	.147	.049	2041	140	5.86	2.879	.806	124.1
50	.362	.178	.059	1695	145	6.662	3.273	.909	110
55	.426	.214	.07	1429	150	7.548	3.708	1.022	97.8
60	.517	.254	.082	1220	155	8.535	4.193	1.145	87.3
65	.619	.304	.097	1031	160	9.63	4.731	1.333	75
70	.733	.36	.114	877.2	165	10.843	5.327	1.432	69.8
75	.869	.427	.134	746.3	170	12.183	5.985	1.602	62.4
80	1.024	.503	.156	641	175	13.654	6.708	1.774	56.4
85	1.205	.592	.182	549.5	180	15.291	7.511	1.97	50.8
90	1.41	.693	.212	471.7	185	17.041	8.375	2.181	45.9
95	1.647	.809	.245	408.2	190	19.001	9.335	2.411	41.5
100	1.917	.942	.283	353.4	195	21.139	10.385	2.662	37.6
105	2.229	1.095	.325	307.7	200	23.461	11.526	2.933	34.1
110	2.579	1.267	.373	268.1	205	25.994	12.77	3.225	31
115	2.976	1.462	.426	234.7	210	28.753	14.127	3.543	28.2
120	3.43	1.685	.488	204.9	212	29.922	14.7	3.683	27.2

GASEOUS STEAM.

When saturated steam is surcharged with heat, or superheated, it is termed gaseous or steam-gas. The distinguishing feature of this condition of steam is its uniformity of rate of expansion above 230°, with the rise of its temperature, alike to the expansion of permanent gases.

To Compute Total Heat of Gaseous Steam.

$1074.6 + .475\,t = \text{H}$. *t representing temperature, and* H *total heat in degrees.* Hence, total heat at 212°, and at atmospheric pressure = 1175.3°.

Specific gravity = .622.

To Compute Velocity of Steam.

Into a Vacuum. RULE.—To temperature of steam add constant 459, and multiply square root of sum by 60.2; product will give velocity in feet per second.

Into Atmosphere. $3.6\sqrt{h} = \text{V}$. V *representing velocity as above, and h height in feet of a column of steam of given pressure and uniform density, weight of which is equal to pressure in unit of base.*

ILLUSTRATION.—Pressure of steam 100 lbs. per sq. inch, what is velocity of its flow into the air?

Cube foot of water = 62.5 *lbs.*, density of steam at 100 lbs. = 270 *cube feet.* Hence, 62.5 : 100 :: 270 : 432 = *volume at* 100 *lbs. pressure*, and 432 × 144 = 62 208 *feet* = *height of a column of steam at a pressure of* 100 *lbs. per sq. inch.*

Then $3.6\sqrt{62\,208} = 898$ *feet.*

EXPANSION.

To Compute Point of Cutting off to Attain Limit of Expansion.

$\overline{b+f}\,\text{L} \div \text{P} =$ *point of cutting off. b representing mean back pressure for entire stroke, in lbs. per sq. inch, f friction of engine,* P *initial pressure of steam, all in lbs. per sq. inch, and* L *length of stroke, in feet.*

ILLUSTRATION.—Assume stroke of piston 9 feet, pressure 30 lbs., mean back pressure 3 lbs., and friction 2 lbs.

$$\overline{3+2} \times 9 \div 30 = 1.5 \textit{ feet.}$$

To Compute Actual Ratio of Expansion.

$\dfrac{\text{L}+c}{l+c} = \text{R}$. *c representing clearance or volume of space between valve seat and mean surface of piston, at one or each end in feet of stroke, l length of stroke at point of cutting off, excluding clearance in feet, and* R *actual ratio of expansion.*

ILLUSTRATION.—Assume length of stroke 2 feet, clearance at each end 1.2 ins., and point of cutting off 1 foot.

$$1.2 \text{ ins.} = .1. \quad \text{Then } \frac{2+.1}{1+.1} = 1.9 \textit{ ratio.}$$

To Compute Pressure at any Point of Period of Expansion.

When Initial Pressure is given. $\text{P}\,l \div s = p$. *p representing pressure at period of given portion of stroke, both in lbs. per sq. inch, and s any greater portion of stroke than l.*

When Final Pressure is given. $\text{P}' \times \text{L}' \div s = p$. P′ *representing final pressure, in lbs. per sq. inch, and* L′ *length of stroke, including clearance, in feet.*

ILLUSTRATION 1.—Assume length of stroke 6 feet, clearance at each end 1.2 ins., pressure of steam 60 lbs., point of cutting off one third; what is pressure at 4 feet?

$$1.2 \textit{ ins.} = .1 \textit{ foot.} \quad 60 \times \overline{2+.1} \div \overline{4+.1} = 30.73 \textit{ lbs.}$$

2.—What is pressure in above cylinder at 2.8 feet, when final pressure is 21 lbs.?

$$21 \times \overline{6+.1} \div \overline{2.8+.1} = 44.17 \textit{ lbs.}$$

To Compute Mean or Total Average Pressure.

$\frac{P\,(l'\,\overline{1+\text{hyp. log. R}-c)}}{L} = p'$ *or mean or average pressure.* l' *length of stroke at point of cutting off, including clearance.*

ILLUSTRATION.—Assume elements of preceding cases: $1 + \text{hyp. log. R} = 2.065$.

$$\frac{60\,(2.1 \times \overline{2.065 - .1})}{6} = \frac{254.19}{6} = 42.365 \textit{ lbs.}$$

To Compute Final Pressure.

$$P \times l' \div s = P'.$$

ILLUSTRATION.—Assume elements of preceding cases, steam cut off at 2 feet.

$$60 \times \overline{2+.1} \div \overline{6+.1} = 20.65 \textit{ lbs.}$$

To Compute Mean Effective Pressure.

$$\frac{P\,(l'\,\overline{1+\text{hyp. log. R}-c}}{L} - b, \text{ or } (p' - b).$$

ILLUSTRATION.—Assume elements of preceding cases, $b = 2$ lbs. per sq. inch.

$$\frac{60\,(\overline{2.1 \times 2.065} - .1)}{6} - 2 = \frac{254.19}{6} - 2 = 40.365 \textit{ lbs.}$$

To Compute Initial Pressure to Produce a Given Average Effective or Net Pressure.

$$\frac{p'\,L}{l'\,(1+\text{hyp. log. R}) - c} = P.$$

ILLUSTRATION.—Assume elements of case 1.

$$\frac{6+.1}{2+.1} = 2.9 \textit{ ratio.} \qquad \frac{42.365 \times 6}{(2.1 \times 2.065) - .1} = \frac{254.19}{4.2365} = 60 \textit{ lbs.}$$

To Compute Point of Cutting off for a Given Ratio of Expansion.

$$L' \div R - c. \quad \text{Or, } \overline{L+c} \div R - c = l.$$

ILLUSTRATION.—Assume elements of preceding cases: $R = \frac{6+.1}{2+.1} = 2.9$, and $\frac{6+.1}{2.9} - .1 = 2$ *feet.*

To Compute Pressure in a Cylinder, at any Point of Expansion, or at End of Stroke.

$$P\,l' \div \overline{l+c} = P, \text{ or } P \div R.$$

ILLUSTRATION.—Assume elements of preceding cases:

$$\frac{60 \times 2.1}{2+.1} = 60 \textit{ lbs.}, \text{ and } \frac{60}{2.9} = 20.69 \textit{ lbs.}$$

To Compute Initial Pressure for a Required Net Effective Pressure for a Given Ratio of Expansion.

$\frac{W + a\,b\,L}{a\,(l'\,\overline{1+\text{hyp. log. R}-c)}}$. Or, $\frac{p'\,L}{l'\,\overline{1+\text{hyp. log. R}-c}} = P$. W *representing net-work in foot-lbs.* $= a\,L\,\overline{p'-b}$, *and a area of piston, in sq. ins.*

ILLUSTRATION.—Assume elements of preceding cases: area of piston = 100 sq. ins., back pressure 2 lbs., and net effective pressure = 42.365 lbs.

$$100 \times 6 \times \overline{42.365 - 2} = 24\,219 \textit{ foot-lbs.}$$

$$\frac{24\,219 + \overline{100 \times 2 \times 6}}{100 \times \overline{2.1 \times 2.065} - .1} = \frac{25\,419}{100 \times 4.2365} = 60 \textit{ lbs.} \qquad \frac{42.365 \times 6}{2.1 \times 2.065 - .1} = \frac{254.19}{4.2365} = 60 \textit{ lbs.}$$

Points of Expansion.

Relative points of expansion, including clearance 5 per cent., assuming stroke of piston to be divided as follows, and initial pressure = 1.

Point.....	1.	.75	.6875	.625	.5625	.5	.4375	.375	.333	.25	.2	.125	.1
Ratio.....	1	1.31	1.43	1.55	1.71	1.91	2.15	2.43	2.74	3.5	4.4	6.	7.

Hyp. Log. of above Ratios.

.0	1.27	1.36	1.44	1.54	1.65	1.77	1.9	2	2.25	2.43	2.79	2.95

Hyperbolic Logarithms.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1.05	.0488	2.65	.9746	4.25	1.447	5.8	1.758	7.4	2.001
1.1	.0953	2.66	.9783	4.3	1.459	5.85	1.766	7.45	2.008
1.15	.1398	2.7	.9933	4.33	1.466	5.9	1.775	7.5	2.015
1.2	.1823	2.75	1.0116	4.35	1.47	5.95	1.783	7.55	2.022
1.25	.2231	2.8	1.0296	4.4	1.482	6	1.792	7.6	2.028
1.3	.2624	2.85	1.0473	4.45	1.493	6.05	1.8	7.65	2.035
1.33	.2852	2.9	1.0647	4.5	1.504	6.1	1.808	7.66	2.036
1.35	.3001	2.95	1.0818	4.55	1.515	6.15	1.816	7.7	2.041
1.4	.3365	3	1.0986	4.6	1.526	6.2	1.824	7.75	2.048
1.45	.3716	3.05	1.1151	4.65	1.537	6.25	1.833	7.8	2.054
1.5	.4055	3.1	1.1314	4.66	1.539	6.3	1.841	7.85	2.061
1.55	.4383	3.15	1.1474	4.7	1.548	6.33	1.845	7.9	2.067
1.6	.47	3.2	1.1632	4.75	1.558	6.35	1.848	7.95	2.073
1.65	.5008	3.25	1.1787	4.8	1.569	6.4	1.856	8	2.079
1.66	.5068	3.3	1.1939	4.85	1.579	6.45	1.864	8.05	2.086
1.7	.5306	3.33	1.203	4.9	1.589	6.5	1.872	8.1	2.092
1.75	.5596	3.35	1.209	4.95	1.599	6.55	1.879	8.15	2.098
1.8	.5878	3.4	1.2238	5	1.609	6.6	1.887	8.2	2.104
1.85	.6152	3.45	1.2384	5.05	1.619	6.65	1.895	8.25	2.11
1.9	.6419	3.5	1.2528	5.1	1.629	6.66	1.896	8.3	2.116
1.95	.6678	3.55	1.2669	5.15	1.639	6.7	1.902	8.33	2.12
2	.6931	3.6	1.2809	5.2	1.649	6.75	1.909	8.35	2.122
2.05	.7178	3.65	1.2947	5.25	1.658	6.8	1.917	8.4	2.128
2.1	.7419	3.66	1.2975	5.3	1.668	6.85	1.924	8.45	2.134
2.15	.7655	3.7	1.3083	5.33	1.673	6.9	1.931	8.5	2.14
2.2	.7885	3.75	1.3218	5.35	1.677	6.95	1.939	8.55	2.146
2.25	.8109	3.8	1.335	5.4	1.686	7	1.946	8.6	2.152
2.3	.8329	3.85	1.3481	5.45	1.696	7.05	1.953	8.65	2.158
2.33	.8458	3.9	1.361	5.5	1.705	7.1	1.96	8.66	2.159
2.35	.8544	3.95	1.3737	5.55	1.714	7.15	1.967	8.7	2.163
2.4	.8755	4	1.3863	5.6	1.723	7.2	1.974	8.75	2.169
2.45	.8961	4.05	1.3987	5.65	1.732	7.25	1.981	8.8	2.175
2.5	.9163	4.1	1.411	5.66	1.733	7.3	1.988	8.85	2.18
2.55	.936	4.15	1.4231	5.7	1.741	7.33	1.992	8.9	2.186
2.6	.9555	4.2	1.4351	5.75	1.749	7.35	1.995	8.95	2.192

To Compute Mean Pressure of Steam upon a Piston by Hyperbolic Logarithms.

Rule.—Divide length of stroke of a piston, added to clearance in cylinder at one end, by length of stroke at which steam is cut off, added to clearance at that end, and quotient will express *ratio* or *relative expansion* of steam or *number*.

Find in table logarithm of *number* nearest to that of quotient, to which add 1. The sum is ratio of the gain.

Multiply ratio thus obtained by pressure of steam (including the atmosphere) *as it enters the cylinder*, divide product by relative expansion, and quotient will give mean pressure.

Note.—Hyp. log. of any number not in table may be found by multiplying a common log. by 2.302 585, usually by 2.3.

When Relative Expansion or Number falls between two Numbers in Table, proceed as follows: Take difference between logs. of the two numbers. Then, as difference between the numbers is to difference between these logs., so is excess of expansion over least number, which, added to least log., will give log. required.

ILLUSTRATION.—Expansion is 4.84, logs. for 4.8 and 4.85 are 1.569 and 1.579, and their difference .01. Hence, as $4.85 \backsim 4.8 = .05 : 1.579 \backsim 1.569 = .01 :: 4.84 - 4.8 = .04 : .008$, and $1.569 + .008 = 1.577 =$ *log. required.*

EXAMPLE.—Assume steam to enter a cylinder at a pressure of 50 lbs. per sq. inch, and to be cut off at .25 length of stroke, stroke of piston being 10 feet; what will be mean pressure? Clearance assumed at 2 per cent. $= .2$ *feet.*

$10 + .2 = 10.2$ *feet*, stroke $10 \div 4 + .2 = 2.38$ *feet.* Then $10.2 - 2.38 = 4.29$ *relative expansion.*

$$\text{Hyp. log. } 4.29 = 1.456, \text{ which } + 1 = 2.456, \text{ and } \frac{2.456 \times 50}{4.29} = 28.62 \textit{ lbs.}$$

Relative Effect of steam during expansion is obtained from preceding rule.

Mechanical Effect of steam in a cylinder is product of mean pressure in lbs., and distance through which it has passed in feet.

Effects of Expansion. (*Essentially from D. K. Clark.*)

Back Pressure is force of the uncondensed steam in a cylinder, consequent upon impracticability of obtaining a perfect vacuum, and is opposed to the course of a piston. It varies from 2 to 5 lbs. per sq. inch.

It must be deducted from average pressure. Thus: assume pressure 60 lbs., stroke of piston as in preceding case, and back pressure 2 lbs.

At termination of.....	1st,	2d,	3d,	4th,	5th,	and 6th foot of stroke.
Pressure..............	60	30	20	15	12	10 lbs. per inch.
Back pressure........	2	2	2	2	2	2 " " "
Effective pressure.....	58	28	18	13	10	8 " " "

Total work done by expansion at termination of each foot or assumed division of stroke of piston is represented by hyp. log. of ratio of expansion, initial work $= 1$.

Thus, for a stroke of 10 feet and a pressure of 10 lbs.:

At end of........		1st,	2d,	3d,	4th,	5th,	6th,	7th,	8th,	9th, and	10th *foot.*
Steam is expanded into vols., hyp. log. of which... } =		.69	1.1	1.39	1.61	1.79	1.95	2.08	2.2	2.3
Initial duty...........	1	1	1	1	1	1	1	1	1	1
Total duty........	1	1.69	2.1	2.39	2.61	2.79	2.95	3.08	3.2	3.3
Initial duty is represented by 10..	10	16.9	21	23.9	26.1	27.9	29.5	30.8	32	33
Resistance for each foot of stroke... } =	2	4	6	8	10	12	14	16	18	20
Total effective duty....... } =	8	12.9	15	15.9	16.1	15.9	15.5	14.8	14	13
Gain by expansion	0	61.25	87.5	98.75	101.25	98.75	93.75	85	75	62.5

The same results would be produced if expansion was applied to a non condensing engine, exhausting into the atmosphere.

Again, assume total initial pressure in a non-condensing cylinder 75 lbs. per sq. inch, expanded 5 times, or down to 15 lbs., and then exhausted against a back pressure of atmosphere and friction of 15 lbs.

At termination of.......	1st,	2d,	3d,	4th, and	5th *foot of stroke.*
Total duty..............	1	1.69	2.1	2.39	2.61
" " performed...	75	126.75	157.5	179.25	195.75 *foot-lbs.*
" back pressure....	15	30	45	60	75 " "
" effective duty....	60	96.75	112.5	119.25	120.75 " "
Gain by expansion.....	0	61.25	87.5	98.75	101.25 per cent.

From which it appears that the total duty performed by expanding steam 5 times its initial volume is full 2.5 times, or as 75 to 195.75.

Relative Effect of Equal Volumes of Steam.

Relative total effect or work of steam is directly as its mean or average pressure (A), and inversely as its final pressure (B), or volume of steam condensed.

If former is divided by latter, quotient will give relative total effect or work (C) of a given volume of steam as admitted and cut off at different points of stroke of piston, with a clearance of 3.125 per cent.

In following computations resistance of back pressure is omitted. If this pressure is uniform with all the ratios of expansion, it is a uniform pressure, to be deducted from the total mean pressure in column (A).

Cut off at	Pressure. (A) Mean.	Pressure. (B) Final.	(C) Relative Effect.	Cut off at	Pressure. (A) Mean.	Pressure. (B) Final.	(C) Relative Effect.
1	1	1	1	.375	.761	.394	1.93
.75	.969	.787	1.28	.33	.702	.335	2.09
.6875	.946	.697	1.35	.25	.628	.273	2.3
.625	.924	.636	1.45	.2	.559	.224	2.05
.5625	.889	.576	1.54	.125	.435	.15	2.9
.5	.857	.501	1.71	.1	.418	.13	3.21

To Compute Total Effective Work in One Stroke of Piston, or as Given by an Indicator Diagram.

$a\,P\,(l'\,\overline{1 + \text{hyp. log. } R} - c) = w$, and $a\,b\,L = w'$. *w representing total work, and w' back pressure.*

Note.—Pressure of atmosphere is to be included in computations of expansion; it is therefore to be deducted from result obtained in non-condensing engines. In condensing engines, the deduction due to imperfect vacuum must also be made, usually 2.5 lbs. per sq. inch.

Illustration.—Assume cylinder of a condensing engine 26.1 ins. in diameter, a stroke of 2 feet, pressure of steam 95 lbs. (80.3 + 14.7) per sq. inch, cut off at .5 stroke, with an average back pressure of 2 lbs. per sq. inch, and a clearance of 5 per cent.

Area of piston, deducting half area of rod = 530 *sq. ins.* 2 × 5 ÷ 100 = .1 *clearance*, and 2 + .1 ÷ 1 + .1 = 1.9 = *ratio of expansion*, and 1 + hyp. log. 1.9 = 1.642.

Then $530 \times 95 \times \overline{1.1 \times 1.642 - .1} - 530 \times 2 \times 2 = 50\,350 \times 1.706 - 2120 = 83\,777$ *lbs.*

Illustration.—Assume cylinder of a non-condensing engine having an area of 2000 sq. ins., a stroke of 8 feet, steam at a pressure of 50 lbs. (35.3 + 14.7), cut off at .25 of stroke, and clearance .25 foot.

Ratio of expansion 3.66, back pressure 17 lbs., and 1 + hyp. log. 3.66 = 2.297.

$2000 \times 50\,(2.25\,\overline{1 + \text{hyp. log } 3.66} - .25) = 100\,000 \times \overline{2.25 \times \overline{1 + 1.297} - c} = 491\,825$ *foot lbs.*

$2000 \times 17 \times 8 = 272\,000$ *foot-lbs.* or *negative effect*, and $491\,825 - 272\,000 = 219\,825$ *foot-lbs.*

Total Effect of One Lb. of Expanded Steam.

If 1 lb. of water is converted into steam of atmospheric pressure = 14.7 *lbs.* per sq. inch, or 2116.8 *lbs.* per sq. foot, it occupies a volume equal to 26.36 *cube feet*; and the effect of this volume under one atmosphere = 2116.8 *lbs.* × 26.36 *feet* = 55 799 *foot-lbs.* Equivalent quantity of heat expended is 1 unit per 772 *foot-lbs.*, 55 799 ÷ 772 = 72.3 *units.* This is effect of 1 lb. of steam of a pressure of one atmosphere on a piston without expansion.

Gross effect thus attained on a piston by 1 lb. of steam, generated at pressures varying from 15 to 100 lbs. per sq. inch, varies from 56 000 to 62 000 *foot-lbs.*, equivalent to from 72 to 80 *units of heat.*

Effect of 1 lb. of steam, without expansion, as thus exemplified, is reduced by clearance according to proportion it bears to volume of cylinder. If clearance is 5 per cent. of stroke, then 105 parts of steam are consumed in the work of a stroke, which is represented by 100 parts, and effect of a given weight of steam without expansion, admitted for full stroke, is reduced in ratio of 105 to 100. Having determined, by this ratio, effect of work by 1 lb. of steam without expansion, as reduced by clearance, effect for various ratios of expansion may be deduced from that, in terms of relative operation of equal weights of steam.

Volume of 1 lb. of saturated steam of 100 lbs. per sq. inch is 4.33 cube feet, and pressure per sq. foot is $144 \times 100 = 14\,400$ *lbs.*; then total initial work $= 14\,400 \times 4.33 = 62\,352$ *foot-lbs.* This amount is to be reduced for clearance assumed at 7 per cent.

Then $62\,352 \times \overline{100 \div 107} = 58\,273$ *foot-lbs.*, which, divided by 772 (Joule's equivalent), $= 75.5$ *units of heat.*

Total or constituent heat of steam of 100 lbs. pressure per sq. inch, computed from a temperature of 212°, is 1001.4 units; and from 102° (temperature of condenser under a pressure of 1 lb.) the constituent heat is 1111.4 units.

Equivalent, then, of net simple effect 75.5 units is 7.5 per cent. of total heat from 212°, or 6.7 per cent. from 102°.

When steam is cut off at

1	.75	.5	.33	.25	.2	.125 and .1 of stroke,	

comparative effects are as

1	1.26	1.616	1.92	2.14	2.27	2.51 and 2.6.	

Total effects as given in table, page 718.

Effect of 1 lb. of steam, without deduction for back pressure or other effects, varies from about 60 000 *foot-lbs.*, without expansion, to about double that, or 120 000 *foot-lbs.*, when expanded 3 times, cutting off at about 27 per cent. of stroke: and to about 150 000 *foot-lbs.* when expanded about 6 times, and cut off at about 10 per cent. of stroke.

Effect of Clearance.

Clearance varies with length of stroke compared with diameter of cylinder, with form of valve, as poppet, slide, etc.

With a diameter of cylinder of 48 ins., and a stroke of 10 feet, and poppet valves, clearance is but 3 per cent., and with a diameter of 34 ins. and a stroke of 4.5 feet and slide valves, it is 7 per cent.

Illustration of Effect.—Assume steam admitted to a cylinder for .25 of its stroke, with a clearance of 7 per cent.

Mean pressure for 1 lb. $= .637$, and loss by clearance $= 7 \div 100 = .07$, which, added to .637, $= .707$, which is effect of a given volume of steam, if there was not any loss by clearance, or a gain of 11 per cent.

When steam is cut off at........	1	.75	.5	.33	.25	.125 and .1 stroke.	
Loss at 7 per cent. clearance.. =	7	7.2	8.1	9.6	11	15.3	17 per cent.

To Compute Net Volume of Cylinder for Given Weight of Steam, Ratio of Expansion and One Stroke.

Rule.—Multiply volume of 1 lb. of steam, by given weight in lbs., by ratio of expansion and by 100, and divide product by 100, added to per cent. of clearance.

Example.—Pressure of steam 95 lbs., cut off at .5, weight .54 lbs., volume of 1 lb. steam 4.55, and weight $= .2198$ lbs., stroke of piston 2 feet, and clearance 7 per cent.

Ratio of expansion $2 + \overline{.14 \div 1 + .14} = 1.88$.

$$\frac{4.55 \times .54 \times 1.88 \times 100}{100 + 7} = \frac{461.92}{107} = 4.31 \textit{ cube feet.}$$

To Compute Volume of Cylinder for Given Effect with a Given Initial Pressure and Ratio of Expansion.

Rule.—Divide given effect or work by total effect of 1 lb. of steam of like pressure and ratio of expansion, and quotient will give weight of steam, from which compute volume of cylinder by preceding rule.

Example.—Assume given work at 50 766 foot-lbs., and pressure and expansion as preceding.

Total work by 1 lb., 100 lbs. steam, cut off at .5, = by table 94 200 *foot-lbs.*, and by table of multipliers for 95 lbs. $= .998$, which $\times\ 94\,200 = 94\,012$ *foot-lbs.*

$$\text{Then } \frac{50\,766}{94\,012} = .54 \textit{ lbs. weight of steam.}$$

Consumption of Expanded Steam per IP of Effect per Hour.

IP $= 33\,000$, which $\times 60 = 1\,980\,000$ *foot-lbs. per hour*, which $\div 1$ lb. steam, the quotient = weight of steam or water required per IP per hour.

ILLUSTRATION.—Effect of 1 lb., 100 lbs. steam, without expansion, with 7 per cent. of clearance $= 58\,273$ *foot-lbs.*, and $\frac{1\,980\,000}{58\,273} = 34$ *lbs. steam = weight of steam consumed for the effect per IP per hour.*

When steam is expanded, the weight of it per IP is less, as effect of 1 lb. of steam is greater, and it may be ascertained by dividing 1 980 000 by the respective effect, or by dividing 34 lbs. by quotient of total mean pressure by final pressure, as given in table, page 718.

When steam is cut off at	1	.75	.5	.375	.33	.25 and	.2 of stroke.
Volumes consumed per IP per hour........	= 34	26.9	21	18.5	17.6	16	14.9 *lbs.*

Hence, assuming 10 lbs. steam are generated by combustion of 1 lb. coal per IP of total effect per hour,

The coal consumed per IP per hour........	= 3.4	2.69	2.1	1.85	1.76	1.6	1.49 *lbs.*

SATURATED STEAM.

To Compute Energy and Efficiency of Saturated Steam.

$$\frac{v}{V} = R; \quad \frac{S}{s} = \frac{1}{r}; \quad p - p' \times a, \text{ or } \frac{X}{s} \text{ or } \frac{X\,D}{R} = P; \quad \frac{33\,000}{P} = C; \quad \frac{1}{S} = D;$$

$$\frac{H\,D}{R} = H'; \quad \frac{D}{R} \text{ or } \frac{1}{R\,S} = F; \quad J\,D\,(t - t') + L = H\,D; \quad \frac{H\,D}{D} = H; \quad \frac{H'}{a} = P';$$

$$p - p' \times a\,R\,S = X; \quad \overline{\frac{H\,D}{D} - X\,D} = H''; \quad \frac{H''}{R} = H'''; \quad 15.5\,I\,a\,S = h;$$

$$h - X = h'; \quad \frac{h}{R\,S} = P''; \quad \frac{a\,p - a\,p'}{P''} \text{ or } \frac{X}{h} = E; \quad \frac{1\,980\,000}{E} \text{ or } 1\,980\,000\,\frac{h}{X} = A;$$

$$\frac{X}{2\,s} = e; \quad n\,l\,a\,\overline{p - p'} = x, \text{ and } \frac{x}{33\,000} = \text{IHP}; \quad R\,\overline{p - p'}\,a = x; \quad F\,C \times 60 = f;$$

$$\frac{33\,000}{p\,a - p'\,a} = \textit{cube feet.} \quad \frac{1\,980\,000}{62.5\,X} = \textit{cube feet water evaporated per hour per IP.}$$

V *and* v *representing volumes of mass of steam entering cylinder and of it at termination of stroke of piston ;* S *and* s *volumes of* 1 *lb. steam when admitted and when at termination of expansion ;* C *volume of cylinder per minute for each* IHP ; R *and* r *ratios of expansion and effective cut-off ;* F *feed water per cube foot of volume of cylinder per stroke of piston, and* f *per* IHP *per hour, all in cube feet.* D *density or weight of* 1 *cube foot of steam at temperature of operation, in lbs.;* p *mean pressure ;* p′ *mean back pressure ;* I *initial pressure ;* P *mean effective pressure, or energy per cube foot of volume of cylinder ;* P′ *pressure per sq. inch or that equivalent to heat expended, and* P″ *pressure equivalent to expenditure of available heat, or energy, all in lbs.* J *Joule's equivalent* = 772 *foot-lbs.;* L *as per following table ;* t *and* t′ *absolute temperatures of steam at initial pressure and of feed water in degrees ;* H D *heat expended per cube foot of steam admitted ;* H′ *heat expended per cube foot of volume of cylinder, or pressure equivalent to heat expended per sq. foot ;* H″ *heat rejected per cube foot of steam admitted ;* H‴ *heat rejected per cube foot of volume of cylinder ;* A *available heat per* IHP *per hour ;* e *energy per cube foot of volume of cylinder to point of cutting off, or of steam admitted ;* h *and* h′ *heat expended and rejected, and* X *energy exerted, all per lb. of steam and in foot-lbs.* E *efficiency ;* x *energy exerted per minute and per cube foot of steam admitted ;* a *area of piston in sq. ins. ;* l *length of stroke of piston in feet, and* f *feed water per* IHP *per hour, in cube feet.*

ILLUSTRATIONS.—Assume volume of cylinder and clearance (5 per cent. = .6 inch) 1 cube foot, steam (86.3 + 14.7) 100 lbs. per sq. inch, cut off at .5, mean pressure by rule (page 711) 86 lbs., and back pressure 3 lbs.

$V = 1.$ $\quad v = 2.$ $\quad S = 4.33.$ $\quad s = 8.31.$ $\quad p = 86.$ $\quad p' = 3.$ $\quad a = 144$ *ins.*
t and $t' = 327.9° + 461.2°$ and $100° + 461.2°.$ $\quad l = 2$ *feet.* $\quad n = 1.$ $\quad L = 157\,748.$

$2 \div 1 = 2$ *ratio.* $4.33 \div 8.31 = .526$ *effective cut-off.* $86 - 3 \times 144 = 11\,952$ *lbs.*

$\frac{33\,000}{86 - 3 \times 144} = 2.76$ *cube feet.* $\frac{1}{4.33} = .231$ *lbs.* $\frac{.231}{2}$ or $\frac{1}{2 \times 4.33} .1154$ *cube feet.*

$$772 \times .231\ (789.1^\circ - 561.2^\circ) + 157\,748 = 198\,389 \textit{ foot-lbs.}$$

$\frac{198\,389}{2} = 99\,195$ *foot-lbs.* $\frac{198\,389}{.231} = 858\,827$ *foot-lbs.* $\frac{99\,195}{144} = 689$ *lbs.*

$$86 - 3 \times 144 \times 2 \times 4.33 = 103\,504 \textit{ foot-lbs.}$$

$198\,389 \div .231 - 103\,504 \times 2.31 = 174\,479$ *foot-lbs.* $174\,479 \div 2 = 87\,239$ *foot-lbs.*

$15.5 \times 100 \times 144 \times 4.33 = 966\,456$ *foot-lbs.* $966\,456 - 103\,504 = 862\,952$ *foot-lbs.*

$\frac{966\,456}{2 \times 4.33} = 111\,600$ *lbs.* $\frac{144 \times 86 - 144 \times 3}{111\,600} = .107$ E. $\frac{1\,980\,000}{.107} = 18\,504\,673$ *foot-lbs.*

Or $1\,980\,000 \times \frac{966\,456}{103\,504} = 18\,504\,673$ *foot-lbs.* $\frac{23\,904}{33\,000} = .725$ IP.

$1 \times 2 \times 144 \times \overline{86 - 3} = 23\,904$ *foot-lbs.* $\frac{1\,980\,000}{62.5 \times 103\,504} = .306$ *cube feet.*

$2 \times \overline{86 - 3} \times 144 = 23\,904$ *foot-lbs.* $.1154 \times 2.76 \times 60 = 19.11$ *cube feet.*

$\frac{103\,504}{2 \times 4.33} = 11\,952$ *foot-lbs.* $\frac{33\,000}{86 \times 144 - 3 \times 144} = 2.761$ *cube feet.*

In illustration of connection of expenditure of available heat (A) and consumption of fuel, assume coal to have a total heat of combustion of 10 000 000* *foot-lbs.*, corresponding to an equivalent evaporative power under 1 atmosphere at 212° of 13.4 lbs. water and efficiency of furnace .5; then available heat of combustion of 1 lb. coal = 5 000 000 *foot-lbs.*

Hence, consumption of coal per IP in an engine of like dimensions and operation with that here given would be $19\,223\,000 \div 5\,000\,000 = 3.8444$ *lbs.*

Properties of Steam of Maximum Density. (*Rankine.*)

Per Cube Foot.

Temp.	L	Temp.	L	Temp.	L	Temp.	L	Temp.	L	Temp.	L
°		°		°		°		°		°	
32	248	95	1999	158	9 687	221	33 180	284	88 740	347	197 700
41	348	104	2571	167	11 760	230	38 700	293	100 500	356	219 000
50	481	113	3277	176	14 200	239	44 930	302	113 400	365	242 000
59	655	122	4136	185	17 010	248	51 920	311	127 500	374	266 600
68	881	131	5173	194	20 280	257	59 720	320	143 000	383	293 100
77	1171	140	6430	203	24 020	266	68 420	329	159 800	392	321 400
86	1538	149	7921	212	28 310	275	78 050	338	178 000	401	351 600

L representing latent heat of evaporation per cube foot of vapor in foot-lbs. of energy. To reduce this to units of heat divide by 772, or Joule's equivalent.

SUPERHEATED STEAM.

The results attained by imparting to steam a temperature moderately in excess of that due to the volume or density of saturated steam are:

1. An increase of elasticity without a corresponding increase of water evaporated.

2. Arresting or reducing passage of water, in suspension, to cylinder (foaming), as the heat contained in that water is wholly lost without affording any elastic effect.

Both of these results, by increasing effect of the steam, economize fuel.

Superheated steam should be treated as a gas.

The product of its pressure, p in lbs. per sq. foot, and volume v of 1 lb. of it in cube feet, in the perfectly gaseous condition, is obtained by following formula:

$42\,140\ T \div t = p\,v = 85.44\ T$. T *temperature of steam* $+ 461.2^\circ$, and t $32^\circ + 461.2^\circ$.

ILLUSTRATION.—Assume temperature of steam, 327.9°, superheated to 341.1°.

$$\text{Then } 42\,140 \times \overline{461.2^\circ + 341.1^\circ} \div \overline{32 + 461.2^\circ} = 68\,549 \textit{ foot-lbs.}$$

Hence, as pressure of steam at 327.9° = 100 lbs. per sq. inch, and at 341.1° 120.

$$120 \div 100 = 1.2 \text{ to } 1 = \textit{a gain of one fifth.}$$

* Coal of average composition, $14\,133 \times 772 = 10\,910\,676$.

To Compute Energy and Efficiency of Superheated Steam.

In following illustrations elements are same as those in preceding cases for saturated steam, with addition of the steam being superheated, so that

$I = 115$ *lbs.*, $t = 338^\circ + 461.2^\circ = 799.2^\circ$, $t' = 290 + 461.2^\circ = 751.2^\circ$, $S = 3.8$, $s = 7.4$.

$$\frac{R\,a\,p}{I}\,I\,S - R\,a\,p'\,S = X;\quad 15.5\,I\,a\,S = h;\quad \frac{a\,p - a\,p'}{P''} = E;\quad h - X = h';\quad \frac{X}{R\,S} = P;$$

$$\frac{h}{R\,S} = P'';\quad \frac{h - X}{R\,S} = H''';\quad \frac{33\,000}{a\,p - a\,p'} = \textit{cube feet};\quad \frac{1\,980\,000}{E} = A.$$

Efficiency of saturated steam (p. 716) .107, and, as above, .109; hence $\frac{109}{107} = 1.02$ to 1.

If, then, available heat of combustion of efficiency of furnace is assumed at 5 000 000 *foot-lbs.*, as above, consumption of coal per IHP 18 183 486 ÷ 5 000 000 = 3.637 *lbs.*

NOTE.—For further illustrations Rankine's "Steam engine," London, 1861, p. 436.

Wire-drawing.

Wire-drawing of steam is difference between pressure in boiler and pressure in cylinder, and is occasioned as follows:

Resistance or friction in steam pipe to passage of steam to steam-chest and piston.

Resistance of throttle-valve to passage of steam, when it is partly closed or of insufficient area in proportion to steam-pipe.

Resistance from insufficient area of valves or ports.

Mr. Clark, from his experimental investigation, declared, that resistance in a steam-pipe is inappreciable, when its sectional area is not less than .1 area of piston, and its velocity not exceeding 600 feet per minute.

When velocity of a piston is from 200 to 240 feet per minute, area of steam may be .04th of piston.

Effect of Expansion with Equal Volumes, and Effect of One Lb. of 100 Lbs. Pressure per Sq. Inch.

Clearance at each End of Cylinder, including Volume of Steam Openings, 7 per cent. of Stroke, and 100 per cent. of Admission = 1.

Ratio of Expansion. — Initial Volume = 1.	Point of Cut-off. — Stroke = 1.	TOTAL PRESSURES. Final. — Initial Pressure = 1.	Mean. — Initial Pressure = 1.	Initial. — Mean Pressure = 1.	Weight of Steam of 100 Lbs. for one Stroke per Cube Foot.	ACTUAL EFFECT. By 1 Lb. of 100 Lbs. Steam.	Per Sq. Inch per Foot of Stroke by 100 Lbs. Steam.	Volume of Steam expended per IHP of Work per Hour.	Heat converted.
					Lbs.	Foot-lbs.	Foot-lbs.	Lbs.	Units.
1	1	1	1	1	.247	58 273	100	34	75.5
1.1	.9	.909	.996	1.004	.225	63 850	99.6	31	82.7
1.18	.83	.847	.986	1.014	.209	67 836	98.6	29.2	87.9
1.23	.8	.813	.98	1.02	.201	70 246	98	28.2	91
1.3	.75	.769	.969	1.032	.19	73 513	96.9	26.9	95.2
1.39	.7	.719	.953	1.049	.178	77 242	95.3	25.6	100.1
1.45	.66	.69	.942	1.062	.17	79 555	94.2	24.9	102.9
1.54	.625	.649	.925	1.081	.161	83 055	92.5	23.8	107.6
1.6	.6	.625	.913	1.095	.155	85 125	91.3	23.3	110.3
1.88	.5	.532	.86	1.163	.131	94 200	86	21	122
2.28	.4	.439	.787	1.271	.108	104 466	78.7	19	132.5
2.4	.375	.417	.766	1.305	.103	107 050	76.6	18.5	138.6
2.65	.33	.377	.726	1.377	.093	112 220	72.6	17.7	145.4
2.9	.3	.345	.692	1.445	.085	116 855	69.2	16.9	151.4
3.35	.25	.298	.637	1.57	.074	124 066	63.7	16	160.7
4	.2	.25	.567	1.764	.062	132 770	56.7	14.9	171.9
4.5	.16	.222	.526	1.901	.055	138 130	52.6	14.34	178.8
5	.14	.2	.488	2.049	.049	142 180	48.8	13.92	184.2
5.5	.125	.182	.457	2.188	.045	146 325	45.7	13.53	189.5
5.9	.11	.169	.432	2.315	.042	148 940	43.2	13.29	192.9
6.3	.1	.159	.413	2.421	.039	151 370	41.3	13.08	196.1
6.6	.09	.152	.398	2.513	.037	152 955	39.8	12.98	197.7
7	.083	.143	.381	2.625	.035	155 200	38.1	12.75	201.1
7.8	.066	.128	.348	2.874	.032	158 414	34.8	12.5	205.2
8	.0625	.125	.342	2.924	.031	159 433	34.2	11.83	206.5

Multipliers for Actual Weight and Effect for other Pressures than 100 Lbs.

Pressure per Sq. Inch.	Multipliers. Weight.	Multipliers. Actual Effect.	Pressure per Sq. Inch.	Multipliers. Weight.	Multipliers. Actual Effect.	Pressure per Sq. Inch.	Multipliers. Weight.	Multipliers. Actual Effect.
Lbs.			Lbs.			Lbs.		
65	.666	.975	90	.901	.995	130	1.28	1.015
70	.714	.981	95	.952	.998	140	1.37	1.022
75	.763	.986	100	1	1	150	1.46	1.025
80	.806	.988	110	1.09	1.009	160	1.55	1.031
85	.855	.991	120	1.17	1.011	170	1.64	1.033

In this illustration, in connection with preceding table, no deductions are made for a reduction of temperature of steam while expanding, or for loss by back pressure.

When steam is cut off at .0625, or one sixteenth, its expansion is 16 times, but as 7 per cent. of stroke is to be added to it $(.0625 + .07) = .1325 = 132.5$ per cent., or nearly double of 16, or only a little over 7 times, as in 3d column of table on preceding page.

Column 7 is product of 58 273 and ratio of total effect of equal weights of steam when expanded, or average total pressure divided by average final pressure.

Thus, if steam is cut off at .5, with a clearance of 7 per cent., $\frac{.86 \times 100 = 86}{.532 \times 100 = 53.2} =$ 1.6165, and $58\,273 \times 1.6165 = 94\,200$ *foot-lbs.*

Column 9 gives volume of steam consumed per IP per hour. Thus, assume cylinder to have an area of 292 sq. ins., a stroke of 2 feet, and pressure of steam 100 lbs. without expansion.

$292 \times 100 \times 2 = 58\,400$ *foot-lbs.*, and $292 + 7$ per cent. of stroke for clearance $=$ .14; hence, $292 \times 2.14 \div 144 = 4.34$ *cube feet*, and weight of a cube foot of such steam $= .23$ lbs., and $58\,400 : 4.34 \times .23 :: 33\,000 : .564$, which, $\times$ 60 minutes $= 33.84$, or 34 as per table.

The pressures are computed on premise that steam is maintained at a uniform pressure during its admission to cylinder, and that expansion is operated correctly to termination of stroke.

Column 10 is quotient of work in foot-lbs., divided by Joule's equivalent 772.

Thus, $94\,200 \div 772 = 122$.

For percentage of constituent heat, converted from 102° and 212°, assume 122 as in last case:

Then $122 \times 9 \div 100 = 10.98$ *per cent. for* 102°, and $122 \times 10 \div 100 = 12.2$ *per cent. for* 212°.

"Wire-drawing" will cause a reduction of pressure during admission, and clearance will vary from 3 to 8 per cent., according to design of valve, as poppet, long or short slide.

In practice, wire-drawing of steam, and opening of exhaust before termination of stroke, involve deviations from a normal condition, for which deductions must be made, added to which there is the back pressure, from insufficient condensation in condensing engines, and from pressure of air in non-condensing engines, and compression of exhaust steam at termination of stroke.

To Compute Gain in Feed Water at High Temperature.

$T - t + W - w = H$. T *and* t *representing total heat in steam and temperature of feed water*, W *and* w *temperature, of water blown off and fed = heat lost by blowing off, and* H *total heat required from fuel, all in degrees.*

Illustration.—Assume steam at 248°, feed water 100° in one case and 150° in another, and density $\frac{2}{32}$, and total heat at $248° = 1157°$; what is gain?

$$1157 - 100 + \overline{248 - 100} = 1205° = \textit{total heat required from fuel.}$$

$$1157 - 150 + \overline{248 - 150} = 1105° = \quad \text{"} \quad \text{"} \quad \text{"}$$

$$\text{Then } \frac{H - H'}{H} = \frac{1205 - 1105}{1205} = .083 = 8.3 \textit{ per cent.}$$

COMPOUND EXPANSION.

Compound Expansion is effected in two or more cylinders, and is practised in three forms.

1st. When steam in one cylinder is exhausted into a second, pistons of the two moving in unison from opposite ends—that is, steam from top or forward-end of first cylinder being exhausted into bottom or after-end of the other, and contrariwise—this is known as the Woolf* engine.

2d. Steam from the 1st cylinder is exhausted into an intermediate vessel, or "receiver," the pistons being connected at right angles to each other.

3d. Steam from receiver is exhausted into a 3d cylinder of like volume with 2d, pistons of all being connected at angles usually of 120°.

The two latter types are those of the compound engine of the present time.

Expansion from Receiver. The receiver is filled with steam exhausted from 1st cylinder, which is then admitted to 2d, or 2d and 3d, in which it is cut off and expanded to termination of stroke.

Initial pressure in 2d, or 2d and 3d cylinders, is assumed to be equal to final pressure in 1st, and consequently the volume cut off in the one or the other cylinders must be equal in volume to that of 1st cylinder, for its full volume must be discharged therefrom.

Inasmuch as 3d cylinder is but a division of the 2d, with addition of receiver, this engine, in following illustrations, will, for simplification, be treated as having but two cylinders.

In illustration, assume 1st and 2d cylinders to have volumes as 1 to 2, with like lengths of stroke, and that steam is cut off at .5 stroke, and equally expanded in both cylinders, the ratio of expansion in each cylinder being thus equal to their volumes.

Volume received into 2d cylinder would be equal to that exhausted from 1st, assuming there would not be any diminution of pressure from loss of heat by intermediate radiation, etc. This is based upon assumption that expansion occurs only upon a moving piston; but in operation, expansion occurs both in receiver and in intermediate passages, as nozzles and clearances; the 2d cylinder, therefore, receives steam at a reduced pressure, increased volume, and reduction of ratio of expansion. To meet this, and attain like effects, volume of 2d cylinder must be increased in proportion to increased volume of steam and its ratio of expansion. Consequently, there is no loss of effect aside from increased volume and weight of parts by intermediate expansion, provided primitive ratio of expansion is maintained by giving relative increased volume to 2d cylinder.

Illustration.—Assume cylinders having volumes as 1 and 3, initial steam of 1st cylinder to be 60 lbs. per sq. inch, stroke of piston 6 feet, cut off at one third, and clearance 7 per cent.

Final pressure, as per rule, page 711, = 22.62 lbs., and pressure as exhausted into receiver, reduced one fourth, = 16.97 lbs., assuming there is no intermediate fall of pressure. The steam, therefore, is expanded to 1.33 times volume of cylinder; a like volume, therefore, must be given to 2d cylinder, to admit of this at a like pressure. If, therefore, the increased terminal volume of the steam in the 1st cylinder was augmented, including a clearance of 7 per cent., the effect would be as follows:

Volume admitted to 2d cylinder is equal to volume of 1st added to its clearance, or to .33 volume of 2d cylinder added to its clearance; that is, to .33 of 107 per cent., or 35.66 per cent., consisting of clearance, and 35.66 − 7 = 28.66 per cent. stroke of 2d cylinder. The steam exhausted into 2d cylinder thus fills less than .33 of its stroke by 4.67 (33.33 − 28.66). As steam is expanded from volume of 1st cylinder, plus its clearance, to 2d cylinder, plus its clearance, ratio of expansion in 2d cylinder is equal to ratio of volume of both cylinders, which is 3, and

$$\frac{100\ (\textit{representing full stroke}) + 7}{28.66 + 7} = 3, \text{ and final pressure } \frac{22.62}{3} = 7.54 \text{ lbs. per sq. inch.}$$

* In 1825-28 James P. Allaire, of New York, adopted this design of engine in the steamboats *Henry Eckford, Sun, Commerce, Swiftsure, Post Boy,* and *Pilot Boy.*

Assuming volume of receiver, or augmented terminal volume, for expansion in 2d cylinder, to have proportions of 1, 1.25, 1.33, and 1.5 times volume of 1st cylinder plus its clearance, the relations would be as follows:

Augmented terminal volumes in 2d cylinder	1	1.25	1.33	1.5	times volume of 1st cylinder.
Equal to	1.07	1.337	1.427	1.605	do. do. including clearance.
Final volumes in 2d cylinder added to clearance	3.21	3.21	3.21	3.21	times volume of 1st cylinder.
Ratio of expansion in 2d cyl'r.	3	2.4	2.25	2	
Intermediate reductions of pressure	0	.2	.25	.33	of terminal pressure in 1st cyl'r.
Equal to	0	4.52	5.65	11.31	lbs. per sq. inch.
Pressures in receiver and initial pressure in 2d cylinder	22.62	18.1	16.96	11.31	do. do.
Final pressure in 2d cylinder	7.54	7.54	7.54	7.54	do. do.

To Compute Expansion in a Compound Engine.

RECEIVER ENGINE.

Ratio of Expansion. In 1st cylinder, as per formula, page 710. *In 2d cylinder.* $\frac{n-1}{n} r =$ *ratio. Of Intermediate Expansion.* $\frac{n}{n-1} =$ *ratio. n representing ratio of intermediate reduction of pressure between 1st and 2d cylinder, to final pressure in 1st cylinder, and r ratio of area of 1st cylinder to that of 2d.*

ILLUSTRATION.—Assume $n = 4$, and $r = 3$.

$$\text{Then } \frac{4-1}{4} \times 3 = 2.25 \textit{ ratio}, \text{ and } \frac{4}{4-1} = 1.33 \textit{ ratio}.$$

Total or Combined Ratio of Expansion. $r\,R' =$ *product of ratio of 1st and 2d cylinders by ratio of expansion in 1st cylinder.* As when $r = 3$, and $R' = 2.653$, then $2.653 \times 3 = 7.959$ *total ratio.*

Hence. *Combined Ratio of Expansion in both cylinders.* $\frac{n-1}{n} r\,R' = R''$. *R' representing ratio of expansion in 1st cylinder, and R'' combined ratio.*

ILLUSTRATION.—Assume as preceding, and $R' = 2.653$.

$$\text{Then } \frac{4-1}{4} \times 3 \times 2.653 = 5.969 \textit{ combined ratio}.$$

To Compute Effect for One Stroke and a Given Ratio of Expansion in First Cylinder.

Without Intermediate Expansion. RULE.—Multiply actual ratio of expansion in 1st cylinder by ratio of both cylinders, and to hyp. log. of combined ratio add 1; multiply sum by period of admission to 1st cylinder plus clearance, and term product A.

Divide ratio of both cylinders, less 1, by ratio of expansion in 1st cylinder; to quotient add 1; multiply sum by clearance, and term product B.

Subtract B from A, and term remainder C. Multiply area of 1st cylinder in sq. ins. by total initial pressure in lbs. per sq. inch, and by remainder C. Product is net effect in foot-lbs. for one stroke.

With Intermediate Expansion. Add effect thereof to result obtained above, and by following formula:

Or, $l'\,\overline{1 + \text{hyp. log. } R''} - c\left(1 + \frac{r-1}{R'}\right) a\,P = E$. *a representing area in sq. ins., P initial pressure in lbs. per sq. inch of 1st cylinder, l' length of admission or point of cutting off plus clearance, c clearance in feet, and E effect in foot-lbs.*

ILLUSTRATION.—Assume areas of cylinders 1 and 3 sq. ins., length of stroke 6 feet, pressure of steam 60 lbs. per sq. inch, cut off at 2 feet, clearance 7 per cent., and area of intermediate space, as receiver, one third volume of 1st cylinder.

R″ = ratio of expansion in 2d cylinder $\frac{4-1}{4} \times 3 \times 2.653 = 5.969$ **hyp. log.**

$\overline{2.653 \times 2.25 + 1} \times 2.42 - \overline{3 - 1 \div 2.653 + 1} \times .42 \times 1 \times 60 = 1.7865 + 1 \times 2.42 - \overline{2 \div 2.653 + 1 \times .42} \times 60 = 6.743 - .737 \times 60 = 360.36$ *foot-lbs.*

1st *Cylinder.*

Effect on piston 60 lbs. × 1 inch × 2 feet	= 120	*foot-lbs.*
" of clearance 60 lbs. × .42 foot	= 25.2	"
Total initial effect = 60 × 2 × .42	= 145.2	*foot-lbs.*
Then 145.2 × 1 + hyp. log. 2.653 or 1.976	= 286.91	*foot-lbs.*
Less effect of clearance	= 25.2	"
Net effect on piston above vacuum line	= 261.71	*foot-lbs.*
Less effect of back pressure 60 ÷ 2.653 = 22.61, which, × 3 sq. ins. and 2 feet stroke	= 135.66	"
Net effect on piston	= 126.05	*foot-lbs.*

2d *Cylinder.*

$145.2 \times \overline{1 + \text{hyp. log. } 2.25}$ or 1.81	= 262.81 *foot-lbs.*		
Effect of clearance 22.61 × 3 × .42	= 28.49 "	= 234.32 *foot-lbs.*	
		360.37 *foot-lbs.*	

Intermediate reduction of pressure, as given at page 721, = .25 × 22.61 = 5.65 lbs. per sq. inch, which, × 3 sq. ins. and by 2 per foot of stroke, = 33.9 *foot-lbs.*

Hence 360.36 + 33.9 = 394.26 ***foot-lbs.***

Or, by sum of the three results, viz.:

1st cylinder	126.05 *foot-lbs.*
Intermediate expansion	33.9 "
2d cylinder	234.32 "
	394.27 *foot-lbs.*

WOOLF ENGINE. *D. K. Clark.*

Ratio of Expansion.—In 1st cylinder as per formula, page 710. *In 2d cylinder,* $r\,\frac{l}{l'} + x \div \overline{1 + x} =$ *ratio.* *r representing ratio of area of 1st cylinder to that of 2d, l and l′ lengths of stroke and of stroke added to clearance, in ins. or feet, and x ratio value of intermediate volume.*

ILLUSTRATION.—Assume $l = 6$ *feet,* $l' = 7$ *per cent.* $= .42$, $r = 3$, and $x = .333$.

$$\text{Then } \frac{3 \times \frac{6}{6.42} + .333}{1 + .333} = 2.353, \text{ ratio of expansion in 2d cylinder.}$$

Total Actual Ratio of Expansion. $R'\left(r\,\frac{l}{l'} + x\right) =$ *ratio.*

ILLUSTRATION.—Assume preceding elements, R = 2.653.

$$\text{Then } 2.653\left(3 \times \frac{6}{6.42} + .333\right) = 2.653 \times 3.137 = 8.322, \text{ total actual ratio.}$$

Combined Actual Ratio of Expansion. $R'\left(r\,\frac{l}{l'} + x\right) \div \overline{1 + x} =$ *ratio.*

ILLUSTRATION.—Assume preceding elements.

$$3 \times \frac{6}{6.42} + .333 \div 1 + .333 = \frac{8.322}{1.333} = 6.242, \text{ combined actual ratio.}$$

To Attain Combined Ratio of Expansion and Final Pressure in 2d Cylinder.

Assuming four cases as taken for Receiver Engine with a clearance of 7 per cent. The relations would be as follows:

Intermediate spaces are	0	.333	.5	1	part of volume of 1st cylinder plus its clearance, or,
Volume of 1st cylinder	0	.357	.535	1.07	
Add to these 1.07, the volume of 1st cylinder plus its clearance, and....	1.07	1.427	1.605	2.14	total initial volumes for expansion in 2d cylinder or times volume of 1st cyl'r.
To same values of intermediate space add 3, the volume of 2d cylinder, and the sums are the final volumes by expansion in 2d cylinder......	3	3.357	3.535	4.07	times volume of 1st cylinder.
Ratios of expansion in 2d cyl'r are quotients of final by initial volumes..	2.804	2.352	2.202	1.902	ratios of expansion.
Intermediate falls of pressure are, in parts of final pressure in 1st cylinder	0	.25	.333	.5	of final pressure; or, assuming initial pressure at 63 lbs., and final pressure at 23.75 lbs., they are
	0	5.94	7.92	11.87	*lbs. per sq. inch.*
The initial pressures for expansion in 2d cylinder are	1	.75	.66	.5	of final pressure in 1st cylinder, or
	23.75	17.81	15.83	11.87	*lbs. per sq. inch.*
Hence, final pressures in 2d cyl'r are..	8.47	7.57	7.19	6.24	*lbs. per sq. inch.*

Combined Ratios in these Four Cases.

				Combined Ratio.
1st.	1st ratio of expansion	....	1 to 2.653	
	2d do. do.	....	1 to 2.804	$= 2.653 \times 2.804 = 7.44.$
2d.	1st do. do.	....	1 to 2.653	
	2d do. do.	....	1 to 2.352	$= 2.653 \times 2.352 = 6.24.$
3d.	1st do. do.	....	1 to 2.653	
	2d do. do.	....	1 to 2.202	$= 2.653 \times 2.202 = 5.84.$
4th.	1st do. do.	....	1 to 2.653	
	2d do. do.	....	1 to 1.905	$= 2.653 \times 1.905 = 5.05.$

Initial effect of steam at 63 lbs. pressure, admitted to 1st cylinder, for 2 feet, or one third of stroke of piston, and with a clearance of 7 per cent. or .42 feet, is as follows:

Effect on piston..... 63×2 feet $= 126$ *foot-lbs.*
do. in clearance.. $63 \times .42$ foot $= 26.46 = 63 \times 2.42 = 152.46$ *foot-lbs.* Total initial effect.

This sum is initial effect, on which effect by expansion is computed, while it is 26.46 foot-lbs. in excess of the initial effect on the piston.

The total effect, then, is computed as follows:

		Net Effect.
1st case.	$152.46 \times (1 + \text{hyp. log. } 7.44)$ or $3.0069 = 458.27$	
	Less effect of clearance.............. 26.46	431.81 *foot-lbs.*
2d case.	$152.46 \times (1 + \text{hyp. log. } 6.24)$ or $2.831 = 431.47$	
	Less effect of clearance.............. 26.46	405.01 "
3d case.	$152.46 \times (1 + \text{hyp. log. } 5.84)$ or $2.7647 = 421.35$	
	Less effect of clearance.............. 26.46	394.89 "
4th case.	$152.46 \times (1 + \text{hyp. log. } 5.05)$ or $2.6294 = 399.29$	
	Less effect of clearance.............. 26.46	372.83 "

The reductions of net effect in 2d, 3d, and 4th cases are 6.2, 8.6, and 13.7 per cent. of effect in 1st case.

To Compute Effect for One Stroke and a Given Combined Actual Ratio of Expansion.

RULE.—To hyp. log. of combined actual ratio of expansion (behind both pistons) add 1; multiply sum by period of admission of steam to 1st cylinder, added to clearance, and from product subtract clearance.

Multiply area of 1st cylinder in sq. ins. by initial pressure of steam in lbs. per sq. inch and by above remainder. Product is net effect in foot-lbs. for one stroke.

EXAMPLE.—Assume elements of 1st illustration page 723.

Hyp. log. 6.24 + 1 = 2.831, which, × 2.42 = 6.85, and 6.85 — .42 and remainder × 60 = 385.8 *foot-lbs.*

$$\text{Or, } l' \,(1 + \text{hyp. log. } R') - C \times a\,P = E.$$

Comparative Effect of Steam in Receiver and Woolf Engines.

The effect of steam in a compound engine, without clearance and without any intermediate reduction of pressure, is the same whether operated in a receiver or Woolf engine.

When, however, there is an intermediate space between the two cylinders, as a receiver, there is an intermediate reduction of the pressure of the steam, consequent upon the increase of its volume in the receiver; the reduction of pressure, therefore, being less rapid than with the Woolf engine, the effect is greater.

In illustration, the following comparative elements of the effect of both engines is furnished.

RECEIVER. (7 per cent. clearance.) WOOLF.

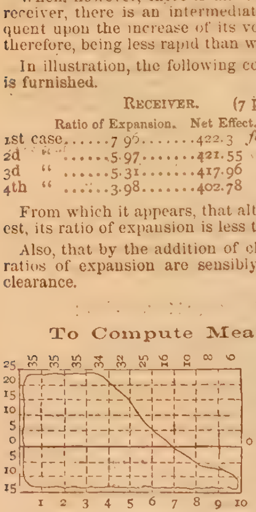
	Receiver: Ratio of Expansion.	Receiver: Net Effect.		Woolf: Ratio of Expansion.	Woolf: Net Effect.
1st case	7.96	422.3 *foot-lbs.*	1st case	7.64	431.71 *foot-lbs.*
2d "	5.97	421.55 "	2d "	6.24	405.11 "
3d "	5.31	417.96 "	3d "	5.84	394.99 "
4th "	3.98	402.78 "	4th "	5.05	372.93 "

From which it appears, that although the effect of a receiver engine is the greatest, its ratio of expansion is less than with the Woolf engine.

Also, that by the addition of clearance to the pistons of each engine, the actual ratios of expansion are sensibly reduced, as compared with the ratios without clearance.

INDICATOR.

To Compute Mean Pressure by an Indicator.



RULE.—Divide atmosphere line, o o in figure, into any convenient number of parts, as feet of stroke of piston, and erect perpendiculars at each point. Measure by scale of parts (alike to that of diagram) the actual mean pressure, as defined between the two lines at top and bottom of diagram, add the results, divide sum by number of points, and quotient will give mean pressure in lbs. per sq. inch upon piston.

EXAMPLE.—Pressures, as above given, are:

35 + 35 + 35 + 34 + 32 + 25 + 16 + 10 + 8 + 6 = 236, which, ÷ 10, = 23.6 *lbs.*

NOTE.—If it were practicable to run an engine without any load, and it sometimes is, the mean pressure, as exhibited by an indicator, would be an exact measure of the friction of the engine.

Conclusions on Actual Efficiency of Steam.

For development of highest efficiencies of steam, as used in an engine, means for protecting it from cooling and condensing in the cylinder must be employed. Superheating of it prior to its introduction into a cylinder is probably most efficient means that may be employed for this purpose. Application to cylinder of gases hotter than it is next best means; and next is the steam-jacket.

In cases of locomotive and portable engines, consumption of steam per IHP per hour is less than for that of single-cylinder condensing engines for like ratios of expansion, which is due to effect of temperature of non-condensing cylinders, always exceeding 212°.

It is deducible from these results that the compound engine is more efficient than the single-cylinder, and that, of the two kinds of compound engines, the receiver-engine is more efficient than the Woolf.

Average consumption of bituminous coal per IHP per hour, for compound engines in long voyages, as shown by Mr. Bramwell, ranged from 1.7 to 2.8 lbs. (*D. K. Clark.*)

To Compute Volume of Water Evaporated per Lb. of Coal.

$\frac{V \div v\, W}{F\, d}$ = *volume of water, in lbs.* V *and* v *representing volume of steam and relative volume of water, in cube feet,* W *weight of cube foot of water, and* F *weight of fuel consumed, both in lbs., and* d *density of water, in degrees of saturation.*

ILLUSTRATION.—Take case at foot of page. V = 449 887 *cube feet*, v = 838 *cube feet*, W = 64.3, E = 1, and F = 4061 *lbs.*

$$\frac{449\,887 \div 838 \times 64.3}{4061 \times 1} = 8.5 \text{ lbs. per hour.}$$

Gain in Fuel, and Initial Pressure of Steam required, when Acting Expansively, compared with Non-Expansion or Full Stroke.

Point of Cutting off.	Gain in Fuel.	Cutting off.	Point of Cutting off.	Gain in Fuel.	Cutting off.	Point of Cutting off.	Gain in Fuel.	Cutting off.
Stroke.	Per Cent.	Lbs.	Stroke.	Per Cent.	Lbs.	Stroke.	Per Cent.	Lbs.
.75	22.4	1.03	.5	41	1.18	.25	58.2	1.67
.625	32	1.09	.375	49.6	1.32	.125	67.6	2.6

ILLUSTRATION.—What must be initial pressure of steam cut off at .5, to be equivalent to 100 lbs. per sq. inch at full stroke?

100 at full stroke = 100, and 100 × 1.18 = 118 lbs.

To Compute Gain in Fuel.

RULE.—Divide relative effect of steam by number of times the steam is expanded, and divide 1 by quotient; result is the initial pressure of steam required to be expanded to produce a like effect to steam at full stroke.

Divide this pressure by number of times the steam is expanded, and subtract quotient from 1, remainder will give gain per cent. in fuel.

EXAMPLE.—When steam is cut off at .5, what is gain in fuel, and what mechanical effect?

Relative effect, including clearance of 5 per cent., = 1.64; number of times of expansion = 2.

1.64 ÷ 2 = .82, and 1 ÷ .82 = 1.22 *initial pressure.*
1.22 ÷ 2 = .61, and 1 — .61 = .39 *per cent.*

Mechanical effects of steam at full and half strokes are 2 — 1.64 = .36 *difference.*

Hence, 1.64 : .36 :: 50 (half volume of steam used) : 10.97 *per cent. more fuel to produce same effect at half stroke, compared with steam at full stroke.*

To Compute Consumption of Fuel in a Furnace.

When Dimensions of Cylinder, Pressure of Steam, Point of Cut-off, Revolutions, and Evaporation per Lb. of Fuel per Minute are given.

RULE.—Compute volume of cylinder to point of cutting off steam, including clearance. Multiply result by number of cylinders, by twice number of strokes of piston, and by 60 (minutes); divide product by density of steam at its pressure in cylinder, and quotient will give number of cube feet of water expended in steam.

Multiply number of cube feet by 64.3 for salt water (62.425 for fresh), divide product by evaporation per lb. of fuel consumed, and quotient will give consumption in lbs. per hour.

EXAMPLE.—Cylinder of a marine engine is 95 ins. in diameter by 10 feet stroke of piston; pressure of steam in steam-chest is 15.3 lbs. per sq. inch, cut off at .5 stroke; number of revolutions 14.5, and evaporation estimated at 8.5 lbs. of salt water per lb. of coal; what is consumption of coal per hour, when density of water is maintained at 2-32? (See Saturation, page 726.)

Volume of steam at above pressure, compared with water (15.3 + 14.7), = 838. Area of 95 ins. + 2.5 per cent. for clearance ÷ 144 = 50.45 *cube feet.* Point of cutting off 5 feet + 2.5 per cent. = 5 feet 1.5 ins., and 50.45 × 5 feet 1.5 ins. × 14.5 × 2 × 60 = 449 887 *cube feet steam per hour.*

Hence, $449\,887 \div 838 = 536.86$ *cube feet water*, which, $\times 64.3 = 34\,520$ lbs., which, $\div 8.5 = 4061$ *lbs. coal per hour.*

NOTE.—Elements given are those of one engine of steamer *Arctic*, and consumption of clean fuel (selected) for a run of 12 days (one engine) was 3820 lbs. per hour.

Utilization of Coal in a Marine Boiler.

Experiment gives from .55 to .8 per cent. of the heat developed in the combustion of coal, as utilized in the generation of steam. Ordinarily it may be safely taken at .65.

SALINE SATURATION IN BOILERS.

Average sea-water contains per 100 parts:

Chloride of sodium (com. salt) . . 2.5; Chloride of magnesium .33. . . .	= 2.83
Sulphuret of magnesium. . . . . . . .53; Sulphuret of lime. . . . .01. . . .	= .54
Carbonate of lime and of magnesia.	.02
Saline matter.	3.39
Water	96.61
	100

Hence, sea-water contains .0339th part of its weight of solid matter in solution, and is saturated when it contains 36.37 parts.

Mean quantity of salts, or solid matter, in solution, is 3.39 per cent., three fourths of which is common salt.

Removal of Incrustation of Scale or Sediment.

Potatoes, in proportion of .033 weight of water. *Molasses*, in proportion of 1.6 lbs. per IP of boiler. *Oak*, suspended in the water, and *Mahogany* or *Oak sawdust*, and *Tanner's* and *Slippery Elm bark*, renewed frequently, according to volume of it, and the evaporation of the water. *Muriate of Ammonia* and *Carbonate of Soda*, in quantity to be determined by observation.

BLOWING OFF.

To Compute Loss of Heat by Blowing Off of Saturated Water from a Steam-boiler.

$\frac{S - T\,E + t}{t}$ = *proportion of heat lost*, $S - T \times E$ = *heat required from fuel for water evaporated in degrees*, and $\frac{t}{S - T\,E + t}$ = *loss of heat per cent.* S *representing sum of sensible and latent heats of water evaporated*, T *temperature of feed water*, *t difference in temperature of water blown off and that supplied to boiler*, E *volume of water evaporated, proportionate to that blown off, the latter being a constant quantity, and represented by* 1.

Values of E at following degrees of saturation, and volumes to be blown off:

32.	Value E.	Volume to Blow off.	32.	Value E.	Volume to Blow off.	32.	Value E.	Volume to Blow off.	32.	Value E.	Volume to Blow off.
1.25	.25	4	1.65	.65	1.54	2	1	1	2.75	1.75	.57
1.35	.35	3	1.75	.75	1.33	2.25	1.25	.8	3	2	.5
1.5	.5	2	1.85	.85	1.18	2.5	1.5	.66	2.25	2.25	.45

Thus, when water in a boiler is maintained at a density of $\frac{2}{32}$, 1 volume of it is evaporated, and an equal volume, or 1, is to be blown off.

Hence $1 + 1 - 1 = 1$ = *ratio of volume evaporated to volume blown off.*

ILLUSTRATION 1.—Point of blowing off is 2 (32), pressure of steam is 15.3 lbs., mercurial gauge, and density of feed water 1 (32); what is proportion of heat lost?

$S = 1157.8°$. $T = 100°$. $t = 15.3 + 14.7 = 250.4° - 100° = 150.4°$. $E = 1$.

Then $\frac{1157.8 - 100 \times 1 + 150.4}{150.4} = 8.03$ *proportion of heat lost.*

2.—Assume point of blowing off 1.75 (32); what would be loss of heat per cent. in preceding case?

$$E = .75 \quad \frac{150.4}{1157.8 - 100 \times .75 + 150.4} = 15.9 \text{ per cent. lost by blowing off.}$$

3.—Assume elements of preceding case. What is total heat required from fuel for water evaporated?

$$1157.8 - 100 \times .75 = 793.35^\circ.$$

To Compute Volume of Water Blown Off to that Evaporated.

When Degree of Saturation is Given. RULE.—Divide 1 by proportionate volume of water evaporated to that blown off, or value of E as above, for degree of saturation given, and quotient will give number of volumes blown off to that evaporated.

ILLUSTRATION.—Degree of saturation in a marine boiler is $\frac{2.25}{32}$; what is volume of water blown off?

$$E = 1.25. \quad \text{Then } 1 \div 1.25 = .8 \text{ blown off.}$$

Proportional Volumes of Saline Matter in Sea-water.

Baltic	1 in 152	British Channel	1 in 28	Atlantic, South	1 in 24
Black Sea	1 " 46	Mediterranean	1 " 25	" North	1 " 22
Red Sea	1 " 131	Atlantic, Equator	1 " 25	Dead Sea	1 " 2.59

When saline matter at temperature of its boiling-point is in proportion of 10 per cent., lime will be deposited, and at 29.5 per cent. salt.

Temperature of water adds much to extent of saline deposits.

STEAM-ENGINE.

The range of proportions here given is to meet the requirements of variations in speed, pressure, length of stroke, draught of water, etc., in the varied purposes of Marine, River, and Land practice.

CONDENSING.

For a Range of Pressures of from 30 to 80 lbs. (Mercurial Gauge) per Sq. Inch, Cut Off at Half Stroke.

Piston-rod. Cylinder or Air-pump (*Wrought Iron*), .1 to .14 of its diam.; (*Steel*), .8 diam.; and (*Copper* or *Brass*), .11 and .125.

Condenser (*Jet*). Volume, .35 to .6 of cylinder. (*Surface.*) Brass tubes, 16 to 19 B W G, .625 to .75 in diameter by from 5 to 10 feet in length, and .75 to 1.25 in pitch, condensing surfaces, .55 to .65 area of evaporating surface of boiler with a natural draught; .7 to .8 with a blower, jet, or like draught. Or, for a temperature of water of 60°, 1.5 to 3 sq. feet per IHP.

With a very effective and sufficient circulating pump, areas may be reduced to .5 and .6.

Effect of vertical tube surface, compared to horizontal, is as 10 to 7.

Air-pump (*Single acting and direct connection*). Volume from .15 to .2 steam cylinder. Or, $\frac{V+v}{n}$ 2.75. *For Double acting put* 4 for 2.75. V *and* v *representing volumes of condensing and condensed water per cube foot, and* n *strokes of piston per minute.*

Foot and Delivery Valves. Area, .25 to .5 area of air-pump.

Delivery Valve (*Out-board*). With a Reservoir. Area from .5 to .8 Foot valve.

NOTE.—Velocity of water through these valves should not exceed 12 feet per second.

Steam and Exhaust Valves.—(*Poppet*), $\frac{a\,s\,n}{24\,000}$ = *area for steam*, $\frac{a\,s\,n}{20\,000}$ *for exhaust*; (*Slide*), $\frac{a\,s\,n}{30\,000}$ *for steam*, and $\frac{a\,s\,n}{22\,750}$ *for exhaust. a representing area of steam cylinder in sq. ins., s stroke of piston in ins., and n number of revolutions per minute.*

Injection Pipes.—One each *Bottom* and *Side* to each condenser; area of each equal to supply 70 times volume of water evaporated when engine is working at a maximum; and in *Marine* and *River* engines the addition of a *Bilge*, which is properly a branch of bottom pipe.

NOTE 1.—Proportions given will admit of a sufficient volume of water when engine is in operation in the Gulf Stream, where the water at times is at temperature of 84°, and volume required to give water of condensation a temperature of 100° is 70 times that of volume evaporated.

2. Velocity of flow of water through cock or valve 20 feet per second in river or at shallow draught, and 30 feet in sea or deep draught service.

*Feed Pump.**—(*Single acting, Marine*). Volume, .006 to .01 steam cylinder. (*River* and *Land*), or when fresh water alone or a surface condenser is used, .004 to .007.

NON-CONDENSING.

For a Range of Pressures of from 50 to 150 lbs. (Mercurial Gauge) per Sq. Inch, Cut Off at Half Stroke.

Piston-rod.—(*Wrought Iron*), Diam., .125 to .2 steam cylinder. (*Steel*), .8 diam.

Steam and Exhaust Valves.—Area is determined by rules given for them in a condensing engine, using for divisors 30 000 and 22 750.

A decrease in volume of cylinder is not attended with a proportionate decrease of their area, it being greater with less volume.

*Feed-pump.**—(*Single acting, Marine*), Volume, .008 to .016 steam cylinder. (*River* and *Land*), or where fresh water alone is used, .005 to .011.

General Rules.

Engines.

Cylinder. Thickness.—(*Vertical*), $\frac{D\,p}{2400} = t$; (*Horizontal*), $\frac{D\,p}{2000} = t$; (*Inclined*), divide by 2000 in a ratio inversely as sine of angle of inclination.

D representing diameter of cylinder, p extreme pressure in lbs. per sq. inch that it may be subjected to, and t thickness in ins.

Shafts, Gudgeons, Journals, etc. To resist Torsion. See rules, pp. 790, 796.

Coupling Bolts. $\frac{D}{2}\sqrt{\frac{D}{n\,d'}} = d$. *n representing number of bolts, D diameter of shaft, d′ distance of centre of bolts from centre of shaft, and d diameter of bolts, all in ins.*

Cross-head, Wrought-iron. (*Cylinder*), $\frac{a\,p\,l}{700} = S$, and $\sqrt{\frac{S}{b}} = d$, or $\frac{S}{d^2} = b$. *a representing area of cylinder in sq. ins., l length of cross-head between centres of its journals in feet, and S product of square of depth d, and breadth, b, of section, both in ins.* (*Air-pump*), $\frac{a\,l}{18} = S$, and as above for d and b.

If section of either of them is cylindrical, for S put $\sqrt[3]{S \times 1.7}$.

Diam. of boss twice, and of end journals same as that of piston-rod. Section at ends .5 that of centre.

* See Formulas, page 736.

Steam-pipe.—Its area should exceed that of steam-valve, proportionate to its length and exposure to the air.

Connecting-rod.—Length, 2.25 times stroke of piston when it is at all practicable to afford the space; when, however, it is imperative to reduce this proportion, it may be twice the stroke.

Comparative friction of long and short connecting-rods is, for length of stroke of piston, 12 per cent. additional; twice, 3 per cent.; and for thrice, 1.33.

Neck.—Diam. 1 to 1.1 that of piston-rod. *Centre* of body (*Horizontal*), 1.25 ins.; (*Vertical*), .06 inch per foot of length of rod.

With two connecting-rods or links. area of necks .65 to .75 area of attached rod. When a second set of rods is used, as in some air-pump connections, area of necks, in a ratio, inversely as their lengths to that of first set.

Straps of Connecting-rods, Links, etc.—(*Strap*), area at its least section .65 neck of attached rod; (*Gib* and *Key*), .3 diam. of neck, width, 1.25 times, (*Slot*) 1.35 times (*Draft*) of keys .6 to .8 inch per foot. Distance of *Slot* from end of rod .5 diam. of pin.

Pins (Cranks, Beams, etc.). $\sqrt[3]{\frac{P\,l}{C}} \cdot 355 = d$. *P representing pressure or thrust of rod or beam, l length of journal in ins., and C, for Wrought iron* = 640, *Cast*, 560. *Puddled steel*, 600, *and Cast*, 1200.

Length, 1.3 to 1.5 times their diam., and pressure should not exceed 750 lbs. per sq. inch for propeller engine, and 1000 for side-wheel.

Cranks (Wrought-iron).—*Hub*, compared with neck of shaft, 1.75 diam., and 1 depth. *Eye*, compared with pin, 2 diam., and 1.5 depth. *Web*, at periphery of hub, width, .7 width, and in depth .5 depth of hub; and at periphery of eye, width, .8 width, and in depth, .6 depth of eye.

(*Cast-iron.*) Diameters of *Hub* and *Eye* respectively, twice diam. of neck of shaft, and 2.25 times that of crank pin.

Radii for fillets of sides of web .5 width of web at end for which fillet is designed; for fillets at back of web, .5 that at sides of their respective ends.

Beams, Open or *Trussed.*—Length from centres 1.8 to 2 stroke of piston, and depth .5 length. If strapped, *Strap* at its least dimensions .9 area of piston-rod, its depth equal to .5 its breadth. *End centre journals* each 1, and *main centre journals* 2.5 times area of piston or driving-rod.

This proportion for strap is when depth of beam is .5 length, as above; consequently, when its depth is less, area of strap must be increased; and when depth of strap is greater or less than .5 width, its area is determined by product of its $b\,d^2$, being same as if its depth was .5 its width.

(*Cast-iron*). *Area of Section of Centre.* $\frac{p \times \overline{l \div 2}}{500\,d} = A$. *p representing extreme pressure upon piston in lbs., d depth in ins., and l length in feet.*

Depth at centre .5 to .75 diam. of cylinder, and, when of uniform thickness, a thickness of not less than .1 of depth.

Vibration of End Centres.—$l \div 2 - \sqrt{(l \div 2)^2 - (s \div 2)^2}$ = *vibration at each end; s representing stroke of piston, in feet.*

Plumber Blocks (Shaft).—*Binder* $d\sqrt{\frac{l}{b}\,C}$ = *depth. d representing diam. of bolts when two to binder, l distance between bolts, b breadth of binder, all in ins., and C for wrought iron* 1, *steel* .85, *and cast iron* .2.

Holding-down Bolts. $P \div 3\,C$ = *area at base of thread of each bolt. C for mild steel for small and large bolts* 6000 *and* 7000, *for wrought iron* 4500 *and* 6000, if but two are used.

Binder (Brass). $\frac{d}{3}\sqrt{\frac{l}{b}}$ = *depth.*

Cocks.—Angles of sides of plug from 7° to 8° from plane of it.

Pumps.—Velocity of water in pump openings should not exceed 500 feet per minute.

Fly-wheels and Governors.—See Rules, pages 451 and 452.

Water-wheels.

Water-wheels (*Arms*).—Number from .75 to .8 diam. of wheel in feet. (*Blades*) *Wood.*—For a distance of from 5 to 5.5 feet between arms, thickness from .09 to .1 inch for each foot of diam. of wheel.

Area of blades, compared with area of immersed amidship section of a vessel, depends upon dip of wheels, their distance apart, model and rig of vessel.

In *River service*, area of a single line of blade surface varies from .3 to .4 that of immersed section; in *Bay* or *Sound service*, it varies from .15 to .2; and in *Sea service*, it varies from .07 to .1.

NOTE.—A wrought-iron blade .625 inch thick bent at a stress withstood by an oak blade 3.5 ins. thick.

Radial and Feathering.

Radial.—Loss of effect is sum of loss by oblique action of wheel blades upon the water, their slip, and thrust and drag of arms and blades as they enter and leave the water.

Loss by oblique action is computed by taking mean of square of sines of angles of blades when fully immersed in the water.

Loss by oblique action of blades of wheel of steamer *Arctic*, when her wheels were immersed 7 feet 5 ins. and 5 feet 0 ins., was 25.5 and 18.5 per cent., which was the loss of useful effect of the portion of total power developed by engines, which was applied to wheels.

Feathering.—Loss of effect is confined to thrust and drag of arms and blades as they enter and leave the water.

Comparative Effects.—In two wheels of a like diameter (26 feet, and 6 feet immersion), like number and depth of blades, etc., the losses are as follows:

Radial .... 26.6 per cent. | Feathering .... 15.4 per cent.

Loss of effect by thrust and drag in a feathering wheel, having these elements and included in the above given loss, is computed at 2 per cent.

Relative loss of effect of the two wheels is, approximately, for ordinary immersions, 20 and 15 per cent. from circumference of wheel.

Centre of Pressure, $\frac{2}{3}\frac{d^3 - d'^3}{d^2 - d'^2} - d = c$. *d and d′ representing depths of blades below surface of water, and c centre of pressure, all in like dimensions, from bottom edge.*

In the cases here given, centres of pressure are as follows:

Radial.................... 6.4 ins. | Feathering................ 8.5 ins.

Propellers.

Propellers (Screw).—*Pitch* should vary with area of circle described by screw to area of midship section of vessel.

AREA, TWO-BLADED.

Area of disk of propeller to midship section being 1 to.........	6	5	4.5	4	3.5	3	2.5	2
Ratio of pitch to the diameter of propeller is 1 to..............	.8	1.02	1.11	1.2	1.27	1.31	1.4	1.47

For *Four-bladed* screws, multiply ratio of pitch to diam. as given above, by 1.35. *Length*, .166 diam.

Slip.—Slip of a screw propeller is directly as its pitch, and economical effect of a screw is inversely as its pitch; greater the pitch less the effect.

An expanding pitch has less slip than a uniform pitch, and, consequently, is more effective.

To Compute Thrust of a Propeller.

IHP $\frac{217}{S} = T$. S *representing speed of vessel in knots per hour.*

SLIDE VALVES.

All Dimensions in Inches.

To Compute Lap required on Steam End, to Cut-off at any given Part of Stroke of Piston.

Rule.—From length of stroke subtract length of stroke that is to be made before steam is cut off; divide remainder by stroke, and extract square root of quotient.

Multiply this root by half throw of valve, from product subtract half lead, and remainder will give lap required.

Example.—Having stroke of piston 60 ins., stroke of valve 16 ins., lap upon exhaust side .5 in. = one thirty-second of valve stroke, lap upon steam side 3.25 ins., lead 2 ins., steam to be cut off at five sixths stroke; what is the lap?

$60 - \frac{5}{6}$ of $60 = 10$. $\sqrt{\frac{10}{60}} = .408$. $.408 \times \frac{16}{2} = 3.264$, and $3.264 - \frac{2}{2} = 2.264$ *ins.*

To Ascertain Lap required on Steam End, to Cut-off at various Portions of Stroke.

Valve without Lead.	Distance of piston from end of its stroke when steam is cut off, in parts of length of its stroke.									
	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
Lap in parts of stroke......	.354	.323	.286	.27	.25	.228	.204	.177	.144	.102

Illustration.—Take elements of preceding case.

Under $\frac{1}{6}$ is .204, and $.204 \times 16 = 3.264$ *ins. lap.*

When Valve is to have Lead.—Subtract half proposed lead from lap ascertained by table, and remainder will give proper lap to give to valve.

If, then, as last case, valve was to have 2 ins. lead, then $3.264 - 2 \div 2 = 2.264$ *ins.*

To Compute at what Part of Stroke any given Lap on Steam Side will Cut off.

Rule.—To lap on steam side, as determined above, add lead; divide sum by half length of throw of valve. From a table of natural sines (pages 390–402) find the arc, sine of which is equal to quotient; to this arc add 90°, and from their sum subtract arc, cosine of which is equal to lap on steam side, divided by half throw of valve. Find cosine of remaining arc, add 1 to it, and multiply sum by half stroke, and product will give length of that part of stroke that will be made by piston before steam is cut off.

Example.—Take elements of preceding case.

$$\text{Cos.}\left(\text{sin.}\frac{2.264+2}{16 \div 2} + 90^\circ - \text{cos.}\frac{2.264}{16 \div 2}\right) + 1 \times \frac{60}{2} = \text{cos.}\,(32^\circ\,13' + 90^\circ - 73^\circ\,34')$$

$= 48^\circ\,39'$, and $\text{cos.}\,48^\circ\,39' + 1 \times \frac{60}{2} = 1.66 \times 30 = 49.8$ *ins.*

To Ascertain Breadth of Ports.

Half throw of valve should be at least equal to lap on steam side, added to breadth of port. If this breadth does not give required area of port, throw of valve must be increased until required area is attained.

Portion of Stroke at which Exhausting Port is Closed and Opened.

Lap on Exhaust Side of Valve in Parts of its Throw.	Portion of Stroke at which Steam is cut off. $\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$
A							
.125	.178	.161	.143	.126	.109	.093	.074
.0625	.13	.118	.1	.085	.071	.058	.043
.03125	.113	.101	.085	.069	.053	.043	.033
0	.092	.082	.067	.055	.041	.033	.022

Lap on Exhaust Side of Valve in Parts of its Throw.	Portion of Stroke at which Steam is cut off. $\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$
B							
.125	.033	.026	.019	.012	.008	.004	.001
.0625	.06	.052	.04	.03	.022	.015	.008
.03125	.073	.066	.051	.042	.033	.023	.013
0	.092	.082	.067	.055	.044	.033	.022

Units in columns of table A express distance of piston, in parts of its stroke, from end of stroke when exhaust port in advance of it is closed; and those in columns of table B express distance of piston, in parts of its stroke, from end of its stroke when exhaust port behind it is opened.

Illustration.—A slide valve is to be cut off at one sixth from end of stroke, lap on exhaust side is one thirty-second of stroke of valve (16 ins.), and stroke of piston is 60 ins. At what point of stroke of piston will exhaust port in advance of it be closed and the one behind it open.

Under one sixth in table A, opposite to one thirty second, is .053, which $\times$ 60, length of stroke = 3.18 *ins.*; and under one sixth in table B, opposite to one thirty-second, is .033, which $\times$ 60 = 1.98 *ins.*

If lap on exhaust side of this valve was increased, effect would be to cause port in advance of valve to be closed sooner and port behind it opened later. And if lap on exhaust side was removed entirely, the port in advance of piston would be shut, and the one behind it open, at same time.

Lap on steam side should always be greater than that on exhaust side, and difference greater the higher the velocity of piston.

In fast-running engines, alike to locomotives, it is necessary to open exhaust valve before end of stroke of piston, in order to give more time for escape of the steam.

To Compute Stroke of a Slide Valve.

Rule.—To twice lap add twice width of a steam port in ins., and sum will give stroke required.

Expansion by lap, with a slide valve operated by an eccentric alone, cannot be extended beyond one third of stroke of a piston without interfering with efficient operation of valve; with a link motion, however, this distortion of the valve is somewhat compensated. When lap is increased, throw of eccentric should also be increased.

When low expansion is required, a cut-off valve should be resorted to in addition to main valve.

To Compute Distance of a Piston from End of its Stroke, when Lead produces its Effect.

Rule. — Divide lead by width of steam port, both in ins., and term the quotient sine; multiply its corresponding versed sine by half stroke, and product will give distance of piston from end of its stroke, when steam is admitted for return stroke and exhaustion ceases.

Example.—Stroke of piston is 48 ins., width of port 2.5 ins., and lead .5 inch; what will be distance of piston from end of stroke when exhaustion commences?

$$.5 \div 2.5 = .2 = \textit{sine}, \text{ ver. sin. of } .2 = .0202, \text{ and } .0202 \times \frac{48}{2} = .4848 \textit{ ins.}$$

To Compute Lead, when Distance of a Piston from the End of Stroke is given.

Rule.—Divide distance in ins. by half stroke in ins., and term quotient versed sine; multiply corresponding sine by width of steam port, and product will give lead.

Example.—Assume elements of preceding case.

$$4848 \div \frac{48}{2} = .0202 = \textit{ver. sin.}, \text{ and sine of ver. sin. } .0202 = .2, \text{ and } .2 \times 2.5 = .5 \textit{ inch.}$$

To Compute Distance of a Piston from End of its Stroke, when Steam is admitted for its Return Stroke.

RULE.—Divide width of steam port, and also that width, less the lead, by .5 stroke of slide, and term quotients *versed sines first* and *second*. Ascertain their corresponding arcs, and multiply *versed sine* of difference between *first* and *second* by .5 stroke, and product will give distance.

EXAMPLE.—Assume elements of preceding case. lap = .5 *inch*, and stroke of slide 6 *ins.*

$$\frac{2.5}{6 \div 2} \text{ and } \frac{2.5 - .5}{6 \div 2} = .8333, \text{ and } .667 \text{ and ver. sin. } 80° \, 24' \sim 70° \, 33' \times \frac{48}{2} = .3528 \textit{ inch.}$$

To Compute Lap and Lead of Locomotive Valves.

To cut off at .33, .25, and .125 of stroke of piston, lap = 289, .25, and .177 t, outside lead = .07 t, and inside lead = .3 t. *t representing stroke of valve, all in ins.*

HORSE-POWER.

Horse-power is designated as *Nominal*, *Indicated*, and *Actual*.

Nominal, is adopted and referred to by Manufacturers of steam-engines, in order to express capacity of an engine, elements thereof being confined to dimensions of steam cylinder, and a conventional pressure of steam and speed of piston.

Indicated, designates full capacity in the cylinder, as developed in operation, and without any deductions for friction.

Actual, refers to its actual power as developed by its operation, involving elements of mean pressure upon piston, its velocity, and a just deduction for friction of operation of the engine.

To Compute Horse-power of an Engine.

Nominal.—*Non-condensing*, $\frac{D^2 v}{1000}$, and *Condensing*, $\frac{D^2 v}{1400} = \text{IP}$ *D representing diameter of cylinder in ins., and v velocity of piston in feet per minute.*

Non-condensing is based upon uniform steam-pressure of 60 lbs. per sq. inch (steam-gauge), cut off at .5 stroke, deducting one sixth for friction and losses, with a mean velocity of piston, ranging from 250 to 450 feet per minute.

Condensing is based upon uniform steam-pressure of 30 lbs. per sq. inch (steam-gauge), cut off at .5 stroke, deducting one fifth* for friction and losses, with a mean velocity of piston of 300 feet per minute for an engine of short stroke, and of 400 feet for one of long stroke.

Actual.—*Non-condensing.* $\frac{A\,\overline{P\dagger - (f\ddagger + 14.7)}\;2\,s\,r}{33\,000} = \text{IP}$. *A representing area of cylinder in sq. ins., P mean effective pressure upon cylinder piston, inclusive of atmosphere, f friction of engine in all its parts, added to friction of load, both in lbs. per sq. inch, s stroke of piston in feet, and r number of revolutions per minute.*

Sum of these resistances is from 12.5 to 20 per cent., according to pressure of steam, being least with highest pressure.

---

* This value may be safely estimated at 2.5 lbs. per sq. inch for friction of engine in all its parts, and friction of load may be taken at 7.5 per cent. of remaining pressure.

† This value is best obtained by an *Indicator;* when one is not used, refer to rule and table, pp. 710–12. In estimating value of P, add 14.7 lbs., for atmospheric pressure, to that indicated by steam gauge or safety-valve. Clearance of piston at each end of cylinder is included in this estimate.

‡ This value may be safely estimated in engines of magnitude at 1.5 to 2 lbs. per sq. inch, for friction of engine in all its parts, and friction of load may be taken at 5 to 7.5 per cent. of remaining pressure.

Sum of these resistances in ordinary marine engines is from 10 to 20 per cent., according to pressure of steam, exclusive of power required to deliver water of condensation at level of discharge or load-line of a vessel. For pressure representing friction for different designs and capacities of engines as estimated by English authority, see pp. 473–5 and 662.

ILLUSTRATION.—Diameter of cylinder of a non-condensing engine is 10 ins., stroke of piston 4 feet, revolutions 45 per minute, and mean pressure of steam (steam gauge) 60 lbs. per sq. inch.

$A = 78.54$ *sq. ins.* $P\ 60 + 14.7 = 74.7$ *lbs.* $f = 2.5 + \overline{(60 + 14.7 - 2.5)} \times .075 = 7.92$ *lbs.*

$$\text{Then } \frac{78.54 \times (60 + 14.7 - \overline{7.92 + 14.7)} \times 2 \times 4 \times 45}{33\,000} = 44.6 \text{ IP.}$$

NOTE 1.—Power of a non-condensing engine is sensibly affected by character of its exhaust, as to whether it is into a heater, or through a contracted pipe, to afford a blast to combustion.

2.—If an indicator is not used to determine pressure of steam in a cylinder, a safe estimate of it, when acting expansively, is .9 of full pressure, and when at full stroke from .75 to .8.

Condensing. $\frac{A\,P\dagger - f\ddagger\ 2\,s\,r}{33\,000} = \text{IP.}$

Power required to work the air-pump of an engine varies from .7 to .9 lbs. per sq. inch upon cylinder piston.

ILLUSTRATION.—Diameter of cylinder of a marine steam engine is 60 ins., stroke of piston 10 feet, revolutions 15 per minute, pressure of steam 50 lbs. per sq. inch, cut off at .25 stroke, and clearance 2 per cent.

$A = 2827.4$ *sq. ins.* P (per Ex., page 713) $= 28.62$ *lbs.* $f = 1.5 + \overline{28.62 - 1.5 \times .05}$ $= 2.467$ *lbs.*

$$\text{Then } \frac{2827.4 \times \overline{28.66 - 2.856} \times 2 \times 10 \times 15}{33\,000} = 662.23 \text{ IP.}$$

From which is to be deducted in marine engines power necessary to discharge water of condensation at level of load-line, which is determined by pressure due to elevation of water, area of air-pump piston, and velocity of its discharge in feet per second.

Indicated. $\frac{A\,P\,2\,s\,r}{33\,000} = \text{IP}$, and $\frac{33\,000\ \text{IP}}{P\,2\,s\,r} = A.$

British Admiralty Rule.—*Nominal.* $\frac{7\,A\,v}{33\,000}$ or $\frac{D^2\,v}{6000} = \text{IP.}$

French.—(*Force de Cheval.*) $1.695\ D^2\ L\ r = \text{IP.}$ *D and L in meters.*

ILLUSTRATION.—Assume a diameter of cylinder of .254 meters, with a stroke of piston of .3 meters and 250 revolutions per minute.

$$1.695 \times .254^2 \times .3 \times 250 = 8.18 \text{ IP.}$$

A *Force de Cheval* = 4500 kilometers per minute = 32 549 *foot-lbs.* = .987 57 IP.

One IP = 1.0139 *Force de Chevaux.*

Compound Indicated. $A\,L\,r\left(\frac{P}{R''}\,\overline{1\ \text{hyp. log. } R''} - b\right).000\,053 = \text{IP.}$

L *representing length of stroke in feet,* R″ *combined ratio of both cylinders, and* b *back pressure.*

ILLUSTRATION.—Assume area of cylinder 3 sq. ins., stroke 6 feet, one stroke of piston, and steam 60 lbs. per sq. inch, cut off at .25.

$A = 3$ *sq. ins.*, $L = 6$ *feet*, $n = 1$ *stroke*, $P = 60$ *lbs.*, $R'' = 5.969$, $b = 3$ *lbs. per sq. inch*, and $r = .5$, and $1 + \text{hyp. log. } R'' = 1 + 1.7865.$

Then $3 \times 6 \times .5 \times \left(\frac{60}{5.969} \times \overline{1 + 1.7865} - 3\right) \times .000\,053 = 9 \times \overline{10.052 \times 2.7865 - 3}$ $\times .000\,053 = .011\,93$ IP, which, $\times\ 2$ for 1 revolution, $= .023\,86$ IP *per revolution.*

To Compute Volume of Water required to be Evaporated in an Engine.

RULE.—Multiply volume of steam expended in cylinder and steam-chests by twice number of revolutions, and multiply product by density of steam at given pressure.

† ‡ For reference see 2d and 3d foot-note on previous page.

EXAMPLE.—What volume of water will an engine require to be evaporated per revolution, diam. of cylinder being 70 ins., stroke of piston 10 feet, and pressure of steam 34 lbs. per sq. inch, including atmosphere, cut off at .5 of stroke?

Area of cylinder $= 3848.5$ *ins.* $10 \times 12 \div 2 = 60$ *ins.*, $60 \times 3848.5 = 230\,910$ *cube ins.*

Add, for clearance at one end, volume of nozzle, steam-chest, etc., 17 317 *cube ins.*

Then $230\,910 + 17\,317 \div 1728 \times 2 = 287.3$ *cube feet*, which, $\times .001\,336$, density of steam at 34 lbs. pressure (see Note 2), $= .3838$ *cube feet.*

NOTE 1.—This refers to expenditure of steam alone; in practice, however, a large quantity of water "foaming," differing in different cases, is carried into cylinder in combination with the steam; to which is to be added loss by leaks, gauges, etc.

2.—Volume of steam is readily computed by aid of table, pp. 708–9. Thus, density or weight of one cube foot of steam at above pressure $= .0835$ lbs. Hence, as 62.5 *lbs.* : 1 *cube foot* :: .0835 *lbs.* : .001 336 *cube foot.*

To Compute Volume of Circulating Water required by an Engine.

$\frac{1114 + .3\,T - t}{t' - t''} = V.$ T *representing temperature of steam upon entering the condenser, t, t', and t'' temperatures of feed water, of water of condensation discharged, and of circulating water, all in degrees.*

ILLUSTRATION.—Assume exhaust steam at 8 lbs. per sq. inch, temperatures of discharge 100°, feed water 120°, and sea-water 75°.

Temperature at 8 lbs. pressure $= 183°$. $\frac{1114 + .3 \times 183 - 120}{100 - 75} = 41.95$ *times.*

To Compute Volume of Flow through an Injection Pipe.

RULE.—Multiply square root of product, of 64.33 and depth of centre of opening into condenser, from surface of external water, added to height of a column of water due to vacuum in condenser, all in feet, by area of opening in sq. ins.; and .6 product, divided by 2.4 ($144 \div 60$) will give volume in cube feet per minute.

EXAMPLE.—Diameter of an injection pipe is 5.375 ins., height of external water above condenser 6.13 feet, and vacuum 24.45 ins.; what is volume of flow per min.?

Area of 5.375 ins. $= 22.69$ ins., $c = .6$. Vacuum $\frac{24.45 \text{ ins.}}{2.04} = 12$ lbs.; 12×2.24 feet (sea-water) $= 26.88$ feet, and $26.88 + 6.13 = 33.1$ *feet.*

$$\text{Then } \frac{\sqrt{64.33 \times 33.1} \times 22.69 \times .6}{2.4} = \frac{628.15}{2.4} = 261.73 \textit{ cube feet.}$$

To Compute Area of an Injection Pipe.

RULE.—Ascertain volume of water required by rule, page 706, in cube ins. per second, multiply it by number of volumes of water required for condensation, by rule, page 707, divide it by velocity due to flow in feet per second, and again by 12, and quotient will give area in sq. ins.

EXAMPLE.—An engine having a cylinder 70 ins. diam., stroke of piston 10 feet, revolutions per minute 15, and steam 19.3 lbs., mercurial gauge cut off at .5; what should be area of its injection pipe at its maximum operation?

Volume of cylinder 267.25 cube feet, cut off at $.5 = 133.625$ *ins.*
Density of steam at 34 lbs. $(19.3 + 14.7) = .001\,336$. Velocity of flow of injected water (computed from vacuum and elevation of condensing water) 33 feet per second.

Then $133.625 \times 15 \times 2 \times 1728 \div 60 = 115\,452$ *cube ins. steam per second*, and $115\,452 \times .001\,336 = 154.24$ *cube ins. water per second.*

Maximum volume of water required to condense steam is about 70 times volume of that evaporated, which only occurs in the Gulf of Mexico; ordinary requirement is about 40 times.

$154.24 + 11.59$ ($= 7.5$ per cent. for leakage of valves, etc.) $= 165.83$, which, $\times 70$ as above, $= 11\,608.1$ *cube ins.*, and $11\,608.1 \div 33 \times 12 = 29.31$ *sq. ins.*

Coefficient of velocity for flow under like conditions = .6; hence, $29.31 \div .6 = 48.85$ *sq. ins.*

NOTE.—This is required capacity for one pipe. It is proper and customary that there should be two pipes, to meet contingency of operation of one being arrested.

To Compute Area of a Feed Pump. (*Sea-water.*)

RULE.—Divide volume of water required in cube ins. by number of single strokes of piston, both per minute, and divide quotient by stroke of pump in ins.; multiply this quotient by 6 (for waste, leaks, "running up," etc.), and product will give area of pump in sq. ins.

EXAMPLE.—Assume volume to be 5 cube feet and revolutions of engine 15 per minute, with a stroke of pump of 3.5 feet.

$$\frac{5 \times 1728}{15} = 576, \text{ which} \div \overline{3.5 \times 12} = 13.72, \text{ and } 13.72 \times 6 = 82.32 \textit{ sq. ins.}$$

NOTE.—In fresh water, this proportion may be reduced one half.

STEAM-INJECTOR. William Sellers & Co.

Self-adjusting.

Volume of Water Discharged per Hour.

No.	Pressure of Steam in Lbs. 60	80	100	120	No.	Pressure of Steam in Lbs. 60	80	100	120
	Cub. feet.	Cub. feet.	Cub. feet.	Cub. feet.		Cub. feet.	Cub. feet.	Cub. feet.	Cub. feet.
3	28.12	31.66	35.2	38.75	7	162.65	182.1	201.55	221
4	52.16	58.44	64.72	71	8	213.2	238.8	264.4	290
5	82.18	92.02	101.86	111.7	9	269.07	302.28	334.59	366.9
6	119.09	133.33	147.57	161.82	10	333.64	373.57	413.49	453.41

Highest temperature admissible of feed water 135°.

To Compute Size of Injector required.

One nominal IP per hour will require one cube foot of water per hour.

When the lbs. of coal burned per hour can be ascertained, divide them by 7.5, and quotient will give the volume of water in cube feet per hour.

When the area of grate-surface is known, multiply it by 1.6 for IP.

In case of plain cylindrical boilers, divide the number of sq. feet of heating-surface by 10 for the IP. In case of flue boilers, divide by 12, and with multi-tubular boilers, by 15, for the nominal IP.

Minimum capacity of Injectors, about 50 per cent. of Tabular capacity.

To Compute Volume of Injection Water required per IIP per Hour.

OPERATION.—Assume temperature of water 80°, and of condensation 100°. Volume of cylinder per IIP as per formula, page 716, and illustration, page 717, = 2.76 feet per minute.

Then, as per rule page 707, $\frac{1146.1 - 100}{100 - 80} = 52.3$ *cube ins. per cube foot of steam.*

$$\frac{2.76 \times 52.3 \times 62.5}{1728} = 5.22 \textit{ lbs.}, \text{ which, } \times 60, = 313.2 \textit{ lbs.}$$

To Compute Net Volume of Feed Water required per IIP per Hour.

OPERATION.—Assume elements of formula, page 716, and illustration, page 717.

Then $.1154 \times 2.76 \times 60 = 19.11$ *lbs.*

Feed Pipes. $\frac{d}{20}\sqrt{v}$ = diameter for small, and $\frac{d}{32}\sqrt{v}$, for large pumps. *d representing diameter of plunger in ins., and v its velocity in feet per minute.*

Results of Operations of Steam-engines. (*D. K. Clark.*)

Condensing Engine.		Actual Ratio of Expansion.	Steam per IHP as cut-off.	Coal per IHP.	Initial Pressure at cut-off.	Steam per IHP per hour.
SINGLE.			Lbs.	Lbs.	Lbs.	Lbs.
Corliss, Saltaire		5.2	14.51	2.5	34.5	17.4
Pumping, Crossness		6.07	14.27	2.2	46	18.7
" East London		3.62	12.92	—	23.25	20.72
Sulzer, Corliss valves		10	—	3.3	50	19.6
• Superheated, Hirn		4.132	—	—	60	18.62
COMPOUND.						
J. Elder & Co.	Receiver	1.85	14.45	1.61	56	—
	Marine, jacketed	1.852	14.85			
J. & E. Wood	Receiver	4.01	10.94	2.14	85.5	—
	stationary	1.857	13.34			
Donkin	Woolf, stationary	2.486	13.18	—	50.5	—
	jacketed	3.221	13.87	—	—	22.51
American, Woolf	1st cylinder	2.31	actual	—	90	15.37
	both	5.63	23.21			
" " jacketed	1st cylinder	3.77	20.71	—	90	14.1
	both	9.19				
NON-CONDENSING.						
Marshall, Sons, & Co.		4.8	16.87	—	76	25.9
Davey, Paxman, & Co.		5	14.93	—	73	29.6
Locomotive "Great Britain"		1.45	31.36	—	102	31.36
" " "		2.94	21.24	—	87	21.24

Practical Efficiency of Steam-engines. *Initial Volume* = 1.

Cylinders.	Most Efficient Ratio of Expansion.	Steam * per IHP per hour.	Cylinders.	Most Efficient Ratio of Expansion.	Steam * per IHP per hour.
CONDENSING.		Lbs.			Lbs.
Single cylinder, jacketed	6	19.5	Compound, jacketed, Woolf	10	20.5
Single cylinder	4	24	Compound, Woolf	7	23
" " superheated	4	18.5	NON-CONDENSING.		
Compound, jacketed, Receiver	6	19	Single cylinder, † jacketed	4	24
			Single cylinder, ‡	3	21

* From boiler. † 70 lbs. pressure. ‡ 90 lbs. pressure.

Standard Operation of a Portable Engine.

Grate	5.5	sq. feet.
Heating surface	220	" "
Coal per IHP per hour	6.25	lbs.
" " sq. foot of grate	9	"
" " hour	50	"
Water evaporated from and at 212° per hour	450	lbs.
" " per IHP per hour	62.5	"
" " " sq. foot of grate	81.8	"

Ratio of heating surface of grate........ 40 to 1.

MIXTURE OF AIR AND STEAM.

Water contains a portion of air or other uncondensable gaseous matter, and when it is converted into steam, this air is mixed with it, and when steam is condensed it is left in a gaseous state. If means were not taken to remove this air or gaseous matter from condenser of a steam-engine, it would fill it and cylinder, and obstruct their operation; but, notwithstanding the ordinary means of removing it (by air-pump), a certain quantity of it always remains in condenser.

20 volumes of water absorb 1 volume of air.

ELEMENTS AND CAPACITIES OF DIRECT-ACTING STEAM-PUMPS.

Independent Direct-acting Steam-pumps have an especial advantage in the supplying of boilers or in the discharge of fluids, as their speed can be adjusted to run continuously, and to maintain the water at a uniform height, level, or depth.

The Worthington Steam-pump.

Fire and General Service, having Two Double-acting Plungers.

Diameter of Steam-cylinders.	Diameter of Water-plungers.	Length of Stroke.	Displacement per Stroke of *one* Plunger.	Single Strokes or Displacements per Minute of *one* Plunger.	Volume delivered per Minute, at stated Number of Strokes.	Diam. of Plunger in any single Cylinder Pump for like Volume and Speed.	*Diameter of Pipes for Short Lengths.* To be increased as length increases. Steam-pipe.	Exhaust-pipe.	Suction-pipe.	Discharge-pipe.
Ins.	Ins.	Ins.	Gallons.	No.	Gallons.	Ins.	Ins.	Ins.	Ins	Ins
3	2	3	.04	75 to 200	6 to 16	2.875	.375	.5	1	.75
4.5	2.75	4	.1	75 to 150	15 to 30	4	.5	.75	1.5	1
5.25	3.5	5	.2	75 to 150	30 to 60	5	.75	1.25	2.5	1.5
6	4	6	.33	75 to 125	50 to 80	5.625	1	1.5	2.5	2
7.5	4.5	10	.69	75 to 100	70 to 140	6.375	1.5	2	4	2.5
9	5.25	10	.93	75 to 100	90 to 185	7.5	1.5	2	4	3
10	6	10	1.22	50 to 100	125 to 245	8.5	2	2.5	4	3
12	7	10	1.66	50 to 100	165 to 335	9.875	2	2.5	5	4
14	7	10	1.66	50 to 100	165 to 335	9.875	2.5	3	5	4
12	8.5	10	2.45	50 to 100	245 to 490	12	2	2.5	6	5
14	8.5	10	2.45	50 to 100	245 to 490	12	2.5	3	6	5
16	8.5	10	2.45	50 to 100	245 to 490	12	2.5	3	6	5
18.5	8.5	10	2.45	50 to 100	245 to 490	12	3	3.5	6	5
12	10.25	10	3.57	50 to 100	355 to 715	14.25	2	2.5	7	6
14	10.25	10	3.57	50 to 100	355 to 715	14.25	2.5	3	7	6
16	10.25	10	3.57	50 to 100	355 to 715	14.25	2.5	3	7	6
18.5	10.25	10	3.57	50 to 100	355 to 715	14.25	3	3.5	7	6
14	12	10	4.89	50 to 100	490 to 980	17	2.5	3	8	7
16	12	10	4.89	50 to 100	490 to 980	17	2.5	3	8	7
18.5	12	10	4.89	50 to 100	490 to 980	17	3	3.5	8	7
18.5	14	10	6.66	50 to 100	665 to 1330	19.75	3	3.5	10	8
17	10	15	5.1	50 to 90	510 to 920	14	3	3.5	8	7
20	12	15	7.34	50 to 90	735 to 1320	17	4	6	12	10
20	15	15	11.47	50 to 90	1145 to 2065	21	4	6	12	12

Many of above sizes, or those of desired capacity, can be compounded by addition of a non-condensing cylinder, resulting in a saving of 33 per cent. of fuel for like service, by any non-condensing form over that required. Exterior packed plungers, for pumping against extreme pressure.

BOILER.

Its efficiency is determined by proportional quantity of heat of combustion of fuel used, which it applies to the conversion of water into steam, or it may be determined by weight of water evaporated per lb. of fuel.

In following results and computations, water is held to be evaporated from standard temperature of 212°.

Proportion of surplus air, in operation of a furnace, in excess of that which is chemically required for combustion of the fuel, is diminished as rate of combustion is increased; and this diminution is one of the causes why the temperature in a furnace is increased with rapidity of combustion.

When combustion is rapid, some air should be introduced in a furnace above the grates, in order the better to consume the gases evolved.

Natural Draught.

Grate (*Coal*) should have a surface area of 1 sq. foot for a combustion of 15 lbs. of coal per hour, length not to exceed 1.5 times width of furnace, and set at an inclination toward bridge-wall of 1 to 1.5 ins. in every foot of length.

When, however, rate of combustion is not high, in consequence of low velocity of draught of furnace, or fuel being insufficient, this proportion of area must be increased to one sq. foot for every 12 lbs. of fuel.

Width of bars the least practicable, spaces between them being from .5 to .75 of an inch, according to fuel used. Anthracite requiring less space than bituminous. Short grates are most economical in combustion, but generate steam less rapidly than long.

Level of grate under a plain cylindrical boiler gives best effect with a fire 5 ins. deep, when grate is but 7.5 ins. from lowest point.

Depth, Cast-iron, .6 square root of length in ins.

(*Wood*), their area should be 1.25 to 1.4 that for coal.

Automatic (Vicar's).—Its operation effects increased rapidity in firing and more effective evaporation.

Ash-pit.—Transverse area of it, for a combustion of 15 lbs. of coal per hour, 2 to .25 area of grate surface for bituminous coal, and .25 to .3 for anthracite. Or 15 to 20 ins. in depth for a width of furnace of 42 ins.

Furnace or Combustion Chamber.—(*Coal*) Volume of it from 2.75 to 3 cube feet per sq. foot of grate surface. (*Wood*) 4.6 to 5 cube feet.

The higher the rate of combustion the greater the volume, bituminous coal requiring more than anthracite. Velocity of current of air entering an ash-pit may be estimated at 12 feet per second.

Volume of air and smoke for each cube foot of water converted into steam is, from coal, 1780 to 1950 cube feet, and for wood, 3900.

Rate of Combustion.—In lbs. of coal per sq. foot of grate per hour. *Cornish Boilers*, slowest, 4; ordinary, 10. *Stationary*, 12 to 16. *Marine*, 16 to 24. Quickest: complete combustion of dry coal, 20 to 23; of caking coal, 24 to 27; *Blast or Fan and Locomotive*, 40 to 120.

Bridge-wall (*Calorimeter*).—Cross-section of an area of 1.2 to 1.6 sq. ins. for each lb. of bituminous coal consumed per hour, or from 18 to 24 sq. ins. for each sq. foot of grate, for a combustion of 15 lbs. of coal per hour.

Temperature of a furnace is assumed to range from 1500° to 2000°, and volume of air required for combustion of 1 lb. of bituminous coal, together with products of combustion, is 154.81 cube feet, which, when exposed to above temperatures, makes volume of heated air at bridge-wall from 600 to 750 cube feet for each lb. of coal consumed upon grate.

Hence, at a velocity of draught of about 12 feet per second, area at bridge-wall, required to admit of this volume being passed off in an hour, is 2 to 2.5 sq. ins., and proportionately for increased velocity, but in practice it may be **1.2 to 1.6 ins.**

When 20 lbs. of coal per hour are consumed upon a sq. foot of grate, 20×1.2 or $1.6 = 24$ or 32 sq. ins. are required, and in a like proportion for other quantities.

Or, When area of flues is determined upon, and area over bridge-wall is required, it should be taken at from .7 to .8 area of lower flues for a natural draught, and from .5 to .6 for a blast.

When one half of tubes were closed in a fire-tubular marine boiler, the evaporation per lb. of coal was reduced but 1.5 per cent.

Firing.—Coal of a depth up to 12 ins. is more effective than at a less depth. Admission of air above the grate increases evaporative effect, but diminishes the rapidity of it.

Air admitted at bridge-wall effects a better result than when admitted at door, and when in small volumes, and in streams or currents, it arrests or prevents smoke. It may be admitted by an area of 4 sq. ins. per sq. foot of grate.

Combustion is the most complete with firings or charges at intervals of from 15 to 20 minutes.

With a fuel economizer (Green's) an increased evaporative effect of 9 per cent. has been obtained.

When external flues of a Lancashire boiler were closed, evaporative power was slightly increased, but evaporative efficiency was decreased; and when 25 per cent. of like surface in setting of a plain cylindrical boiler was cut off, evaporation was reduced but 1.5 per cent. When temperature at base of chimney was 630°, with a fire 12 ins. in depth, it was decreased to 550° with one 9 ins. in depth, and to 539° with one 6 ins.

High wind increases evaporative effect of a furnace.

Stationary or *Land.*—Set at an inclination downward of .5 inch in 10 feet.

Smoke Preventing.—A test of C. Wye Williams's design of preventing smoke, at Newcastle, 1857, as reported by Messrs. Longridge, Armstrong, and Richardson, gave an increased evaporative effect with the "practical prevention of smoke." Hence it was concluded, "That by an easy method of firing, combined with a due admission of air in front of furnace, and a proper arrangement of grate, emission of smoke may be effectually prevented in ordinary marine multi tubular boilers, with suitable coals. 2d. That prevention of smoke *increases* economic value of fuel and evaporative power of boiler. 3d. That coals from the Hartley district have an evaporative power fully equal to that of the best Welsh steam-coals."

Heating Surfaces.

Marine (*Sea-water*).—Grate and heating surfaces should be increased about .07 over that for fresh water.

Relative Value of Heating Surfaces.

Horizontal surface above the flame	= 1	Horizontal beneath the flame	= .1
Vertical	= .5	Tubes and flues	= .56

Minimum Volumes of Fuel Consumed per Sq. Foot of Grate per Hour, for given Surface-ratios. (*D. K. Clark.*)

Description of Boiler.	Surface-ratios of Heating Surface to Grate.									
	10	15	20	30	40	50	60	75	90	100
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Stationary	.7	1.7	3	6.8	12.1	18.9	26	—	—	—
Marine	.7	1.6	2.8	6.3	11.2	17.5	24	—	—	—
Portable	.2	.4	.8	1.8	3.2	5	—	—	—	—
Locomotive (coal)	.3	.7	1.3	2.9	5.2	8.1	11.7	18.3	26.3	32.5
" (coke)	.4	1	1.8	4	7	11	16	25	36	44

At extreme consumption of fuel (120 lbs.) coke will withstand disturbing effect of a blast better than coal.

A scale of sediment one sixteenth of an inch thick will effect a loss of 14.7 per cent. of fuel.

One sq. foot of *fire* surface is held to be as effective as three of *heating*.

Relation of Grate, Heating Surface, and Fuel.

When Grate and Heating Surface are constant, greater the weight of fuel consumed per hour, greater the volume of water evaporated; but the volume is in a decreased proportion to fuel consumed.

In treating of relations of grate, surface, and fuel, D. K. Clark, in his valuable treatise, submits, that in 1852 he investigated the question of evaporative performance of locomotive boilers, using coke; and he deduced from them, that, assuming a constant efficiency of fuel, or proportion of water evaporated to fuel, evaporative effect, or volume of water which a boiler evaporates per hour, *decreases* directly as grate-area is increased; that is to say, larger the grate, less the evaporation of water, at same rate of efficiency of fuel, even with same heating surface.

2d. That evaporative effect *increases* directly as square of heating surface, with same area of grate and efficiency of fuel.

3d. Necessary heating surface *increases* directly as square root of effect—viz., for four times effect, with same efficiency, twice heating surface only is required.

4th. Necessary heating surface *increases* directly as square root of grate, with same efficiency; that is, for instance, if grate is enlarged to four times its first area, twice heating surface would be required, and would be sufficient, to evaporate same volume of water per hour with same efficiency of fuel.

Result of 40 experiments with a stationary boiler (fresh water), with an evaporation of 9 lbs. water per lb. of fuel consumed, the coefficient .002 22 was deduced.

Hence, $\left(\frac{h}{g}\right)^2 .002\,22 = \text{W}$. W *representing volume of water in cube feet, and g and h areas of grate and heating surfaces in sq. feet.*

Illustration.—Assume a heating surface of 90 feet, and a grate of 3; what will be the evaporation?

$$\text{Then } \overline{90 \div 3}^{\,2} \times .002\,22 = 1.998 \textit{ cube feet.}$$

Note.—A Galloway stationary boiler, with a ratio of grate area of 34.3 and a consumption of 21.8 lbs. coal per hour, evaporated 2 9 cube feet of water per sq. foot of grate. Hence the coefficient in this case would be .002 466.

To Compute Areas of Grate and Heating Surfaces, Volume of Water, and Weight of Fuel.

For a Temperature of 281°, or Pressure of 50 lbs. per Sq. Inch.

To Compute Weight of Fuel.

When Water per Sq. Foot of Grate per Hour and Surface Ratio are Given.

$$\frac{\text{W} - x\,\text{R}^2}{\text{C}} = \text{F, and } x\,\text{R}^2 = (\text{E} - \text{C})\,\text{F}.$$

Illustration.—Assume elements as preceding.

$$\frac{200 - .02 \times 50^2}{10} = 15, \text{ and } .02 \times 50^2 = \left(\frac{200}{15} - 10\right) \times 15 = 50.$$

To Compute Ratio of Heating Surface to Area of Grate, and to Effect a Given Evaporation.

When Water and Fuel per Sq. Foot of Grate are Given. $\sqrt{\frac{\text{W} - \text{C}\,\text{F}}{x}} = \text{R}.$

W *representing water evaporated per sq. foot of grate, and* F *fuel consumed, both in lbs. per hour.* C *and x specific constants for each type of boiler, and* R $(h \div g)$ *ratio of heating surface to grate.*

Illustration.—Assume W = 200, C = 10, F = 15, and $x = .02$.

$$\sqrt{\frac{200 - 10 \times 15}{.02}} = 50; \quad \frac{200 - .02 \times 50^2}{10} = 15; \text{ and } \sqrt{\frac{(13.33 - 10) \times 15}{.02}} = 50.$$

When Efficiency of Fuel and Fuel consumed per Sq. Foot of Grate per Hour are given. $\frac{W}{F} = E$ or *efficiency of fuel or weight of water evaporated per lb. of fuel.* $\sqrt{\frac{(E-C)\,F}{x}} = R.$

To Compute Fuel that may be consumed per Sq. Foot of Grate per Hour, corresponding to a Given Efficiency.

When Efficiency of Fuel, that is, Weight of Water evaporated per Lb. of Fuel, and the Surface Ratio, are given.

$$\frac{x\,R^2 + C\,F}{F}, \quad C + \frac{x\,R^2}{F} = E, \text{ and } \frac{x\,R^2}{E-C} = F.$$

ILLUSTRATION.—Assume elements as preceding.

$$\frac{.02 \times 50^2 + 10 \times 15}{15} = 13.33; \quad 10 + \frac{.02 \times 50^2}{15} = 13.33, \text{ and } \frac{.02 \times 50^2}{13.33 - 10} = 15 \ lbs.$$

Combustion of Coal per sq. foot of grate.—*Natural Draught*, from 20 to 25 lbs. can be consumed per hour.—*Steam jet*, 30 lbs., and *Exhaust-blast* 65 to 80 lbs.

From Results of Experiments upon Marine Boilers, see Manual of D. K. Clark, page 808; he deduced the following formula, as applicable to all surface ratios in such boilers.

Newcastle $.02156\,R^2 + 9.71\,F$, and for Wigan $.01\,R^2 + 10.75\,F = W$ in *lbs.*

And the general formulas he deduced from all the various experiments are as follows.

From and at 212°.

Portable..... $.008\,R^2 + 8.0\,F = W.$	Marine.......... $.016\,R^2 + 10.25\,F = W.$
Stationary... $.0222\,R^2 + 9.56\,F = W.$	Locomotive, coal. $.009\,R^2 + 9.7\,F = W.$

Locomotive, coke........ $.0178\,R^2 + 7.94\,F = W.$

As the maximum evaporative power of fuel is a fixed quantity, the preceding formulas are not fully applicable in minimum rates of its consumption and evaporative quality.

With coal and coke the limits of evaporative efficiency may be taken respectively at 12.5 and 12 lbs. water from and at 212°.

ILLUSTRATION 1.—Assume a marine fire-tubular boiler with a surface ratio of heating surface to grate of 30 and a consumption of coal of 15 lbs. per sq. foot of grate per hour, what will be its evaporation per sq. foot of grate?

$$.016 \times 30^2 + 10.25 \times 15 = 168.15 \ lbs.$$

2.—Assume a like boiler, using fresh water, to have a ratio of heating surface to grate of 30 and an evaporation of 165 lbs. water per sq. foot of grate per hour, what would be consumption of coal per sq. foot of grate per hour?

$$\frac{165 - .016 \times 30^2}{10.25} = 14.69 \ lbs.$$

Tube Surface (*Iron*) per lb. of coal 1.58, per sq. foot of grate 32, and per IHP 4.27 sq. feet.

Locomotive Boiler has from 60 to 90 sq. feet per foot of grate, and consumes 65 lbs. coal per sq. foot per hour.

Evaporative Capacity of Tubes of Varying Length.

Total Length of Tubes 12 *Feet* 3 *ins.* (*M. Paul Hevrer*, 1874.)

SURFACE AND WATER.	Furnace and 3 ins. in Length of Tubes.	TUBES. 3.02 Feet.	3.02 Feet.	3.02 Feet.	3.02 Feet.
Surface in sq. feet..............	76.43	179	179	179	179
Water evaporated per sq. foot per hour in lbs.....	24.5	8.72	4.42	2.52	1.68

Results of Operation of Boilers under Varying Proportions of Grate, Area, and Length of Heating Surface, Draught of Furnace, and Rate of Combustion.

Description.	Area of Grate.	Heating Surface.	Grate to Heating Surface.	Coal per Sq. Foot of Grate per Hour.	Evaporation of Water from 212° per sq. ft. of grate.	Evaporation of Water from 212° per lb. of Coal.	Fuel.
Fire-tubular.	Sq. Feet.	Sq. Feet.	Ratio.	Lbs.	Lbs.	Lbs.	
Agricultural and Hoisting	4.7	158	34	13	119	9.33	Welsh.
" " "	3.2	220	69	12.8	151	11.83	"
Locomotive............ }	{ 26.25	963.5	36.7	30.86	327	10.6	"
English............... }	{ 16	818	51	38	375	10.47	"
"	10.5	788	75	45	419	11.04	"
"	10.6	1056	100	157	1401	10.41	"
Marine[1]	22	748	34	24.3	265	10.7	"
" 1	18	749	41.6	23.6	264	11.2	"
" 2	10.3	915	50	41.25	468	11.36	"
" 2*	10.3	508	49.3	27.63	309.8	11.54	Lanc'r
" 3	10.8	151.2	14	27.76	205	7.39	Anth'e
Stationary 4.............	31.5	945	30	28.87	293.7	10.17	Welsh.
" 2.............	31.5	767	24.4	14	141.4	10.1	"

1 New Castle. 2 and 4 Wigan. 3 Experimented at New York.

* Effect of reducing the tube-surfaces was tried by stopping one half the number of tubes in alternate diagonal rows, so that the tube surface was reduced 206.5 sq. feet. The results with fires 12 ins. deep were as follows:

	Tubes open.	Tubes half closed.
Coal per sq. foot of grate per hour	25 lbs.	24 lbs.
Water from 212° per lb. of coal....................	12.41 "	12.23 "
Smoke per hour, very light........................	2.8 minutes.	8 minutes.

Evaporative Effects of Boilers for Different Rates of Combustion, and Surface Ratios. (*D. K. Clark.*)

Water from and at 212° per Hour.

Surface Ratio 30.

Fuel per Sq. Foot of Grate per Hour.	Stationary. Water per Sq. foot.	Stationary. Water per lb. of Coal.	Marine. Water per Sq. foot.	Marine. Water per lb. of Coal.	Portable. Water per Sq. foot.	Portable. Water per lb. of Coal.	Locomotive. Coal. Water per Sq. foot.	Locomotive. Coal. Water per lb. of Coal.	Locomotive. Coke. Water per Sq. foot.	Locomotive. Coke. Water per lb. of Coal.
Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
10	116	11.6	117	11.7	93	9.3	105	10.5	95	9.5
15	163	10.9	168	11.2	136	9	154	10.3	135	9
20	211	10.6	219	10.9	179	9	202	10.1	175	8.7
30	307	10.2	322	10.7	265	8.8	299	10	254	8.5

Surface Ratio 50.

Fuel per Sq. Foot of Grate per Hour.	Stationary. Water per Sq. foot.	Stationary. Water per lb. of Coal.	Marine. Water per Sq. foot.	Marine. Water per lb. of Coal.	Portable. Water per Sq. foot.	Portable. Water per lb. of Coal.	Locomotive. Coal. Water per Sq. foot.	Locomotive. Coal. Water per lb. of Coal.	Locomotive. Coke. Water per Sq. foot.	Locomotive. Coke. Water per lb. of Coal.
15	187	12.5	187.5	12.5	149	9.9	168	11.2	164	10.9
20	247	12.3	248	12.5	192	9.6	217	10.9	203	10.2
30	342	11.4	348	11.6	278	9.3	314	10.4	283	9.4
40	438	10.9	450	11.3	364	9.1	411	10.3	362	9
50	534	10.7	552	11	450	9	508	10.1	442	8.8

Surface Ratio 75.

	Water.	Fuel per Sq. Foot of Grate per Hour in Lbs. 30	40	50	60	75	90	100
		Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Locomotive, coal..	Per sq. foot.	342	439	536	633	778	927	1020
" " ..	" lb. coal.	11.4	11	10.7	10.7	10.4	10.3	10.2
" coke .	" sq. foot.	338	418	497	576	695	815	894
" " ..	" lb. coal.	11.3	10.4	9.9	9.6	9.3	9	8.9

When a heater is used, and temperature of feed-water is raised above that obtained in a condensing engine, the proportions of surfaces may be correspondingly reduced.

Results of Operation of various Designs of Boiler, under varying Proportions of Grate, Calorimeter, Area and Length of Heating Surface, Draught, Firing, and Rate of Combustion.

STATIONARY.	Area of Grate.	Heating Surface.	Grate to Heating Surface.	Circuit of Heating Surface.	Temperature of Chimney.	Coal per Sq. Foot of Grate per Hour.	Water Evaporated from 212° per lb. of Coal.	Water Evaporated per Sq. Foot of Grate per Hour.
	Sq. Ft.	Sq. Ft.	Ratio.	Feet.	°	Lbs.	Lbs.	Lbs.
Lancashire double internal and external flued[1]...	20.5	612	29.8	79	511	15.35	8.32	125 [4]
" " [2].....	21	767	36.5	80	505	21.5	10.88	204 [4]
Galloway vertical water-tubular[2].	21	719	22.8	79	505	22.7	10.77	212 [4]
" " [2]...	31.5	719	34.3	80	630	18.3	10.17	162 [4]
Fairbairn[1].........	20.5	1017	49.5	—	387	15.27	8.67	133
French[1]...........	20.1	607	30.3	—	510	16.42	8.12	133
Cylindrical flued[3]...	14.2	377.5	26.8	56	292	7.43	9.08	59 [5]
MARINE.	*At Pressure of Atmosphere.*							
Horizontal fire-tub.[2]	10.3	508	49.3	—	—	27.5	11.92	328 [7]
" " [2]	10.3	508	49.3	—	—	41.25	11.36	469 [8]
" "	10.3	302	30	—	—	24	12.23	268 [9]
" "	19.3	749	39	—	—	21	10	182 [10]
" "	28.5	749	26.3	—	—	21.15	8.94	164 [10]
" "	28.5	749	26.3	—	—	19	11.13	335 [11]
(C. Wye Williams)	15.5	749	48.3	—	600	37.4	10.63	398
" " / " "	22	749	34	—	600	17.27	11.7	202 [12]
" " / " "	42	749	17.6	—	—	16	9.65	154 [13]
" " [3]	10.8	150	13.9	8.5	—	10.99	8.95	88 [14]
" " [3]	4.32	147	34	8.5	—	27.58	7.24	40 [14]

[1] Trial in France. [2] At Wigan, 1866–68, height of chimneys 100 feet. [3] Navy-yard, Washington, U. S., chimney 61 feet. [4] At pressure of atmosphere, fires 12 ins. deep, at 40 lbs. pressure, evaporation was reduced 12 per cent. [5] Bituminous coals. [6] Anthracite, at pressure of 6.5 lbs. above atmosphere. [7] Fires 14 ins. deep, air admitted through furnace-doors. [8] Ditto do., jet blast. [9] Half tubes closed up. [10] Air through grate only. [11] Air through grate and door, no smoke. [12] One opening in door, temp. 625°, with two 633°, with four 638°, and with six 600°. [13] Long grates, air spaces fully open, no smoke. [14] One furnace, anthracite coal, 5 ins. deep.

Draught.

Draught of Furnace.—Volume of gas varies directly as its absolute temperature, and draught is best when absolute temperature of gas in chimney is to that of external air as 25 to 12.

$\frac{T + 461.2^\circ}{32^\circ + 461.2^\circ} = \frac{V}{V'} = V''$. V, V′, and V″ *representing absolute temperatures at* T *or temperature given, and at* 32°, *in degrees and volume of furnace gas at temperature* T *in cube feet.*

ILLUSTRATION.—Assume temperature of furnace or T = 1500°, and 12 lbs. air per lb. of fuel.

$\frac{1500^\circ + 461.2^\circ}{32^\circ + 461.2^\circ} = 3.98$, *and as* 150 *cube feet is volume of gas per lb. of fuel at* 12 *lbs. supply of air*, $150 \times 3.98 = 597$ *cube feet.*

$\frac{W\,v\,t}{a\,V'} = C$. W *representing weight of fuel consumed in furnace per second in lbs., v volume of air at* 32° *supplied per lb. of fuel in cube feet, t absolute temperature of gas discharged by chimney in degrees, a area of chimney in sq. feet, and* C *velocity of current in chimney in feet per second.*

ILLUSTRATION.—Assume $W = .16$, $v = 150$, $t = 1000^\circ$, and $a = 5$.

$$\frac{.16 \times 150 \times 1000}{5 \times 493.2^\circ} = \frac{24\,000}{2466} = 9.73 \textit{ feet.}$$

$\frac{V'}{t}$ .084 to .087 = D. D *representing weight of a cube foot of gas discharged by chimney, in lbs.* ILLUSTRATION. $\frac{493.2^\circ}{1000} \times .086 = .0424$ *lb.*

$\frac{C^2}{2\,g}\left(1 + G + \frac{f\,l}{m}\right) = H.$ G *representing a coefficient of resistance and friction of air through grate and fuel,* f coefficient of friction of gas through flues and over sooty surfaces,† l length of flues and chimney, m hydraulic mean depth,‡ and* H *height of chimney, all in feet.*

ILLUSTRATION 1.—Assume $C = 9.73$, $l = 60$, and $m = .72$, *all in feet.*

$$\frac{9.73^2}{64.33}\left(1 + 12 + \frac{.012 \times 60}{.72}\right) = \frac{94.67}{64.33} \times 14 = 20.6 \textit{ feet.} \qquad \frac{C\,a\,V'}{v\,t} = W.$$

2.—Assume preceding elements. $\frac{9.73 \times 5 \times 493.2^\circ}{150 \times 1000} = .16$ *lb.*

When H *is given.* $\sqrt{\left(H\,2\,g \div \overline{1 + G + \frac{f\,l}{m}}\right)} = C$

ILLUSTRATION.—Assume preceding elements. $\sqrt{20.6 \times 64.33 \div 14} = 9.73$ *feet.*

.192 × pressure in lbs. per sq. foot = *head in ins. of water.*

Temperature at base of smoke-pipe or chimney, or termination of flues or tubes, is estimated at 500°; and base of chimney, or its *calorimeter*, should have an area of 1.3 to 1.6 sq. ins. for every lb. of coal consumed per hour. With tubes of small diameter, compared to their length, this proportion may be reduced to 1 and 1.2 ins.

Admission of air behind a bridge-wall increases temperature of the gases, but it must be at a point where their temperature is not below 800°.

Loss of Pressure by Flow of Air in Pipes.

Length 3280 Feet, or 1000 Meters.

Velocity at Entrance of Pipe. Feet per Second.	Meter per Second.	*Diameter of Pipe in Ins.* 4	6	8	10	12	14
		Loss of Pressure in Lbs. per Sq. Inch.					
3.28	1	.114	.076	.057	.057	.038	.038
6.56	2	.5	.343	.25	.21	.172	.153
9.84	3	1.183	.8	.592	.477	.394	.343
13.12	4	2.06	1.374	1.03	.84	.687	.6
16.4	5	3.2	2.16	1.61	1.29	1.1	.923
19.68	6	4.446	2.964	2.223	1.778	1.482	1.28

At Mount Cenis Tunnel, the loss of pressure from 84 lbs. per sq. inch, in a pipe 7.625 ins. in diameter and 1 mile 15 yards in length, was but 3.5 per cent.

Artificial Draught.

In production of draught in an ordinary marine boiler, from 20 to 33 per cent. of total heat of combustion of fuel is expended.

Blast.—By experiments of D. K. Clark and others it was deduced that the vacuum in back connection is about .7 of blast pressure, and in the furnace from .33 to .5 of that in back connection; that rate of evaporation varies nearly as square root of vacuum in back connection; that best proportions of chimney and passages thereto are those which enable a given draught to be produced with greatest diameter of blast-pipe; for the manifest reason, that the greater that diameter, the less the back-pressure due to resistance of orifice, and that these proportions are best at all rates of expansion and speeds.

* Which, in furnaces consuming from 20 to 24 lbs. coal per sq. foot of grate per hour, is assigned by Peclet at 12. † Estimated by same authority at .012.

‡ For a square or circular flue is .25 its diameter.

Velocity of Draught. Locomotive. $36.5\sqrt{H(T-t)} = V$. H *representing height of chimney or pipe in feet,* T *and* t *temperatures of air at base and top of chimney, and* V *velocity in feet per second.*

Sectional area of tubes within ferrules .... .2 grate.
" " of smoke-pipe .... .066 "
Area of blast-pipe (below base of smoke-pipe) .... .015 "
Volume of back connection .... 3 feet × area of grate.
Height of smoke-pipe 4 times its diameter.

Steam-jet.—Rings set above base of smoke-pipe, and should equally divide the area; jets .06 to .1 inch in diameter, 3 ins. apart at centres.

A Steam-jet, involving 50 per cent. increased combustion of coal, produced 45 per cent. more evaporation at nearly same evaporation per lb. of coal.

Fan Blowers.—See page 447.

Comparative Result of Experiments with a Steam-jet in a Marine Boiler, with Bituminous Coal. (*Nicoll and Lynn, Eng.*)

		Without Jet.	With Jet.
Area of grate	sq. feet	10.3	10.3
Coal per sq. foot of grate per hour	lbs.	27.5	41.25
Water " " "	"	293.1	419.37
" from 212° per lb. of coal	"	11.9	11.36
Duration of smoke in an hour, very light	minutes	1.1	—

Comparative Effect of Draught and Blasts.

By late experiments in England, with boilers of two steamers, to determine relative effects of the different methods of combustion, the results were: Natural draught 1, Jet 1.25, and Blast 1.6.

Flow of Air. (*Hawksley.*)

In Cylindrical Pipes. $396\sqrt{\frac{h\,d}{l}} = V$, $\frac{V^2\,l}{156\,800\,d} = h$, $311\sqrt{\frac{h\,d^5}{l}} = Q$, $\frac{V\,d^2\,h}{135}$, and $\frac{V^3\,d^2\,l}{21\,200\,000} = HP$.

In Conduits of Various Sections. $796\sqrt{\frac{a\,h}{C\,l}} = v$, $\frac{v^2\,C\,l}{633\,000\,a} = h$, $796\sqrt{\frac{a^3\,h}{C\,l}} = Q$, $\frac{V\,a\,h}{106} = \frac{Q\,h}{106}$, and $\frac{V^3\,C\,l}{67\,000\,000} = HP$. In which 1 inch water is taken as equivalent to a pressure of 5.2 lbs. per sq. inch for any passage.

V *representing velocity in feet per second, h head of water in ins., d diameter of pipe, l length, and* C *perimeter, all in feet, a area of section in sq. feet,* Q (V *a*) *volume of air discharged per second in cube feet, and* HP *horse-power.*

Safety Valves.

Up to a pressure of 100 lbs. per sq. inch, area in sq. ins. equal product of weight of water evaporated in lbs. per hour by .006.

Act of Congress (*U. S.*).—For boilers having flat or stayed surfaces, 30 ins. for every 500 sq. feet of effective heating surface; for cylindrical boilers, or cylindrical flued, 24 sq. ins.

Board of Trade, Eng.—Two of .5 inch area per sq. foot of grate. Or, $\sqrt{\frac{G}{452}} =$ diameter. G *representing area of grate in sq. ins.*

Locked Safety-valves.—Effective heating surface, less than 700 sq. feet, valve 2 ins. in diameter; less than 1500, 3 ins. in diameter; less than 2000, 4 ins. in diameter; less than 2500, 5 ins. in diameter; and above 2500, 6 ins. in diameter.

Or, $(.05\ G + .005\ S)\sqrt{\frac{100}{P}} =$ *area of each of two valves.* G *representing sq. inch, per sq. foot of grate, and* S *sq. inch, per sq. foot of heating surface.*

ILLUSTRATION.—Assume G = 50 sq. feet, S = 1600 sq. feet, and P = 80 lbs. (m. g.). Then, $(.05 \times 50 + .005 \times 1600) \times \sqrt{100 \div 80} = 2.5 + 8 \times 1.118 = 11.73$ *sq. ins.*

Pipes.

Area. $.25\,G + .01\,S\sqrt{\frac{100}{P}}$. *G representing area of grate and* S *area of heating surface, both in sq. feet, and* P *pressure per mercurial gauge in lbs.*

(Copper), Thickness. *Steam*, $.125 + \frac{d\,p}{10\,000}$; *Feed*, $.125 + \frac{d\,p}{8000}$; *Blow* (*Bottom and Surface*), $.125\frac{d\,p}{9000}$; *Supply*, $.1 + \frac{d}{300}$; *Discharge*, $.1 + \frac{d}{200}$; *Feed, Suction, and Bilge discharge*, $.09 + \frac{d}{200}$, and *Steam Blow-off*, $.05 + \frac{d}{500}$. *d representing internal diam. of pipe, and p internal pressure per sq. inch in lbs.*

Flanges.—Of brass, *thickness* 4 times that of pipe; *breadth*, 2.25 times diam. of bolt; *bolts*, diam. equal to and pitch 5 times thickness of flange.

For lower pressure or stress, pitch of bolts 6 times.

Flues and Tubes.

Flues and Tubes.—Cross section, for 15 lbs. of coal consumed per hour, an area of from .18 to .2 area of grate, area being measurably inverse to diameter, and directly increased with length. Thus, in *Horizontal Tubular Boilers*, area .18 to .2 area per sq. foot of grate, and in *Vertical Tubular* .22 to .25, area decreasing with their length, but not in proportion to reduction of temperature of the heated air, area at their termination being from .7 to .8 that of *calorimeter* or area immediately at bridge-wall.

Large flues absorb more heat than small, as both volume and intensity of heat is greater with equal surfaces.

Tubes.—Surface 1 sq. foot, if brass, and 1.33, if iron, for each lb. of coal consumed per hour; or 20 of brass and 27 of iron for each sq. foot of grate, and 2.6 sq. feet of brass and 3.7 of iron per IHP.

Set in vertical rows, and spaces between them increased in width with number of the rows.

Temperature of base of Chimney or Smoke-pipe, or termination of the flues or tubes, is estimated at 500°; and base of chimney, or its *calorimeter*, with natural draught, should have an area of 1.33 sq. ins. for every lb. of coal consumed per hour. With tubes of small diameter, compared to their length, this proportion may be reduced to 1 and 1.2 ins.

When combustion in a furnace is very complete, the flues and tubes may be shorter than when it is incomplete.

Evaporation.

1 sq. foot of grate surface, at a combustion of 15 lbs. coal per hour, will evaporate 2.3 cube feet of salt water per hour.

A sq. foot of heating surface, at a like combustion of fuel, will evaporate from 5 to 6.2 lbs. of salt water per hour; and at a combustion of 40 lbs. coal per hour (as upon Western rivers of U. S.), from 10 to 11 lbs. fresh water, exclusive of that lost by being blown out from boilers.

13.8 to 17.2 sq. feet of surface will evaporate 1 cube foot of salt water per hour, at a combustion of 15 lbs. coal per hour per sq. foot of grate.

Relative evaporating powers of Iron, Brass, and Copper are as 1, 1.32, and 1.56.

NOTE.—Boilers of Steamer *Arctic*, of N. Y., vertical tubular, having a surface of 33.5 to 1 of grate, consuming 13 lbs. of coal per sq. foot of grate per hour, evaporated 8.56 lbs. of salt water per lb. of coal, including that lost by blowing out of saturated water.

Water Surface.

At low evaporations, 3 sq. feet are required for each sq. foot of grate surface, and at high evaporation 4 to 5 sq. feet.

Steam Room.

From 15 to 18 times volume that there are cube feet of steam expended for each single stroke of piston for 25 revolutions per minute, increasing directly with their number. Or, .8 cube feet per IHP for a side-wheel engine, and .65 for an ordinary and .55 for a fast-running screw-propeller.

Space is required proportionate to volume of steam per stroke of piston. Thus, with like boilers, the space may be inversely as the pressures.

Steam-drums and steam-chimneys, by their height, add to the effect of their volume, by furnishing space for water that is drawn up mechanically by the current of steam, to gravitate before reaching the steam-pipe.

Grate.—Area in sq. feet per lb. of coal per hour for following boilers. *Width*, 1.5 diameter of furnace:

Cornish and Lancashire, slow combustion	.2 sq. foot.	Portable, moderate forced	.03 sq. foot.
Marine, tubular	.05 to .066 " "	Locomotive and like, strong blast	.01 " "

Thickness of Tubes per B W G.

External diameter in ins.	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
Thickness for pressure of 50 lbs., number	12	12	11	11	11	10	10	10	9
" " " 100 " "	11	10	9	9	9	8	8	8	7

Smoke-pipes and Chimneys.

Area at their base should exceed that of extremity of flue or flues, to which they are connected.

In Marine service smoke-pipe should be from .16 to .2 area of grate. In Locomotive, it should be .1 to .083.

Intensity of their draught is as square root of their height. Hence, relative volumes of their draught is determined by formula:

$\sqrt{h} \cdot 1\, a =$ *volume in sq. feet. h representing height of pipe or chimney in feet, and a its area in sq. feet.*

When wood is consumed their area should be 1.6 times that of coal.

Chimneys (Masonry).—Diameter at their base should not be less than from .1 to .11 of their height.

Batter or inclination of their external surface .35 inch to a foot, which is about equal to 1 brick (.5 brick each side) in 25 feet.

Diameter of base should be determined by internal diameter at top, and necessary batter due to height.

Thickness of walls should be determined by internal diameter at top; thus, for a diameter of 4 feet and less, thickness may be 1 brick, but for a diameter in excess of that 1.5 bricks.

Area. $\frac{15\,C}{\sqrt{h}} = a$. *C representing weight of coal consumed per hour in lbs., and a area of ditto at top, in sq. ins.*

(Brick masonry.)—25 tons weight per sq. foot of brickwork in height is safe if laid in hydraulic mortar.

Less the height of a smoke-pipe or chimney, the higher the temperature of its gases is required.

Velocities of Current of Heated Air in a Chimney 100 *Feet in Height.*

In Feet per Second.

External Air.	Air at Base of Chimney. 150°	250°	350°	450°	External Air.	Air at Base of Chimney. 150°	250°	350°	450°
	Feet.	Feet.	Feet.	Feet.		Feet.	Feet.	Feet.	Feet.
10°	24	30	33	35	60°	19	26	29	33
32°	22	28	31	34	70°	18	25	29	32
50°	20	27	30	33	80°	17	24	28	32

When Height of Chimney is less than 100 *feet.*—Multiply velocity as obtained for temperature by .1 square root of height of chimney in feet.

Draught consequent upon a steam-jet in a smoke-pipe or chimney is nearly equal to that of a moderate blast.

The most effective draught is when absolute temperature of heated air or gas is to that of external air as 25 to 12, or nearly equal to temperature of melting lead.

In chimneys of gas retorts, ovens, and like furnaces, the draught is more intense for a like height of chimney than in ordinary furnaces, in consequence of the great mass of brick masonry, which, becoming heated, adds to intensity of draught.

Chimneys. *Lawrence Manufacturing Co., Mass. Octagonal.*

Height above ground 211 feet. Diameters 15, and 10 feet 1.5 ins. Wall at base 23.5, and at top 11.5 ins. Shell at base 15 ins., at top 3.75 ins. Foundation 22 feet deep.

England.—Square.—Height	........190 feet.	Diameter at base	........20 feet.
" "	........300 "	" "	........29 "
Round. "	........312 "	" "	........30 "
" "	........300 "	" "	........20 "

Diameter at base usually .1 of height above ground.

Vacuum at base of chimney ranges from .375 to .43 ins. of water.

Circulating Pumps.

Single-acting.—.6 volume of single-acting air-pump and .32 of double-acting.

Double-acting.—.53 volume of double-acting air-pump.

Volume of Pump compared to Steam Cylinder or Cylinders.

Engine.	Pump.	Volume.
Expansive, 1.5 to 5 times........	Single-acting	.08 to .045.
Compound......................	do.	.045 to .035.
Expansive, 1.5 to 5 times........	Double-acting..........	.045 to .025.
Compound......................	do.	.025 to .02.

Valves.—Area such as to restrict the mean velocity of the flow to 450 feet per minute.

PLATES AND BOLTS.

Wrought-iron.—Tensile strength ranges from 45 500 to 70 000 lbs. per sq. inch for plates, and 60 000 to 65 000 lbs. for bolts, being increased when subjected to a moderate temperature.

English plates range from 45 000 to 56 000 lbs., and bolts from 55 000 to 59 000 lbs.

D. K. Clark gives best quality of Yorkshire 56 000 lbs., of Staffordshire 44 800 lbs.

Test of Plates. (*U. S.*)—All plates to be stamped at diagonal corners at about four ins. from edge, and also in or near to their centre, with name of manufacturer, his location, and tensile stress they will bear.

Plates subjected to a tensile stress under 45 000 lbs. per sq. inch, should contract in area of section 12 per cent., 45 000 and under 50 000, 15, and 50 000 and over, 25, at point of rupture.

Brands. (C No. 1) Charcoal No. 1.—*Plates*, will sustain a stress of 40 000 lbs. per sq. inch; hard and unsuited for flanging or bending.

(C No. 1 R H) *Reheated*, hard and durable, suited for furnaces, unsuited for continued bending.

(C H No. 1 S) *Shell*, will sustain a stress of 50 000 to 54 000 lbs. in direction of fibre, and 34 000 to 44 000 across it: hard and unsuited for flanging or even bending with a short radius.

(C H No. 1 F) *Flange*, will sustain a stress of 50 000 to 54 000 lbs., soft and suited for flanging.

(C H No. 1 F B) *Furnace* and (C H No. 1 F F B) *Flange Furnace*. The first is hard, but capable of being flanged, the other is hard, and suited for flanging.

The especial brands are *Sligo*, *Eureka*, *Pine*, etc.

The best English plates known are the *Yorkshire*, as *Low Moor*, *Bowling*, *Farnley*, *Monk Bridge*, *Cooper & Co.*, etc. (See *Steam boilers*, *W. H. Shock*, *U. S. N.*, 1880.)

Steel.—Tensile strength ranges from 75 000 to 96 000 lbs. Mr. Kirkaldy gives 85 966 lbs. as a mean.

When used in construction of boiler plates should be mild in quality, containing but about .25 to .33 per cent. of carbon; for when it contains a greater proportion, although of greater tensile strength, it is unsuited for boilers, from its hardness and consequent shortness in its resistance to bending.

Crucible steel may be used, but that obtained by the Bessemer or Siemens Martin process is best adapted for boiler plates. Its strength becomes impaired by the processes of punching and shearing, rendering it proper thereafter to submit it to annealing.

Steel rivets, when of a very mild character and uniformly heated to a bright red, are superior to iron in their resistance to concussion and stress.

Copper.—Tensile strength is 33 000 lbs., being reduced when subjected to a temperature exceeding 120°. At 212° being 32 000, and at 550° 25 000 lbs.

Wrought-iron Shell Plates.

Pressure and Thickness. (*U. S. Law.*)

Based upon a Standard of One Sixth of Tensile Strength of Plates. Iron or Steel.

Thickness.	Results with a Tensile Strength of 50 000 Lbs. Diameters in Ins.											
	36	38	40	42	44	46	48	54	60	66	72	78
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.25	116	110	104	99	95	91	87	77	69	63	58	53
.3125	145	137	130	124	118	113	109	96	87	79	72	67
.375	174	165	156	149	142	136	130	116	104	95	87	80
.5	232	220	208	198	190	182	174	154	138	126	116	106
	84	90	96	102	108	114	120	126	132	135	140	144
Inch.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
.375	74	69	65	61	58	55	52	49	47	46	44	43
.4375	86	80	76	71	68	64	61	57	55	53	51	50
.5	99	92	87	81	77	73	69	65	63	61	59	57
.5625	111	103	98	91	87	82	78	73	71	69	67	64
.75	148	138	130	121	115	109	103	97	94	91	88	85
.875	172	160	152	142	136	128	122	114	110	106	102	100
1	198	184	174	162	154	146	138	130	126	122	118	114

To which 20 per cent. is to be added for double riveting and drilled holes.

Iron plates .375 inch in thickness will bear, with stay bolts at 4, 5, and 6 ins. apart from centres, respectively 170, 150, and 120 lbs. per sq. inch.

Iron plates, as tested by Mr. Phillips at Plymouth Dockyard, .4375 inch in thickness, with screw stay bolts 1.375 ins. in diameter riveted over heads, 15.75 and 15.25 ins. from centre = 240 sq. ins. of surface for each bolt; bulged between bolts and drew from bolts at a pressure of 105 lbs. per sq. inch of plate.

Iron plates .5 inch in thickness, under like conditions with preceding case, bulged and drew from bolts at a pressure of 140 lbs. per sq. inch of plate. Hence, it appears, resistances of plates are as squares of their thickness.

When nuts were applied to ends of bolt through .4375 inch plate, its resistance increased to 165 lbs. per sq. inch of plate.

Cylindrical Shells. (*U. S. Law.*)

To Compute Pressure for a Given Thickness and Diameter, or Thickness for a Given Pressure and Diameter.

For Pressure. RULE.—Multiply thickness of plate in ins. by one sixth of tensile strength of metal, and divide product by radius or half diameter of shell in ins.

When rivet-holes are drilled, and longitudinal courses are double riveted, add one fifth to result as above attained.

EXAMPLE.—Assume boiler 8 feet in diam., and plates .5 inch thick; what working pressure will it sustain, tensile strength of plates equal to a stress of 60 000 lbs.?

$$.5 \times \overline{60\,000 \div \text{one sixth}} \div \frac{8 \times 12}{2} = \frac{5000}{48} = 104.16 \textit{ lbs.}$$

For Thickness. RULE.—Multiply pressure by radius of shell, and divide product by one sixth of tensile strength of metal.

EXAMPLE.—Assume pressure, radius, and tensile strength as preceding.

$$\frac{104.16 \times \overline{96 \div 2}}{60\,000 \div \text{one sixth}} = \frac{5000}{10\,000} = .5 \textit{ inch.}$$

For Evaporation of Salt Water.—Add one sixth to thickness of plates and sectional area of stay bolts.

For Freight and River Steamboats.

Standard.— 150 lbs. pressure for a boiler 42 ins. in diameter and plates .25 inch thick.

For Pressure. RULE.—Multiply thickness of plate by 12 600, and divide result by radius of boiler in ins.

EXAMPLE.—Assume a boiler 42 ins. in diameter, and plates .25 inch in thickness; what working pressure will it sustain?

$$.25 \times 12\,600 \div \overline{42 \div 2} = 150 \textit{ lbs.}$$

Proof.—All boilers by U. S. Law to be tested to a hydrostatic pressure of 50 per cent. above that of their working pressure.

Relative Mean Strength of Riveted Joints compared to that of Plates.

Allowances being made for Imperfections of Rivets, etc.

Plates, 100; Triple, .72 to .75; Double or Square, .68 to .72; Double with double abut straps, .7 to .75; Staggered, .65; Single, .56 to .6.

Board of Trade, England.

Coefficient or Factor of Safety.—When shells are of best material and workmanship, rivet-holes drilled when plates are in place, abut strapped, plates at least .625 inch in thickness and double riveted, with rivets computed at a resistance not to exceed 75 per cent. over the single shear,* the coefficient is taken at 5. Boilers must be tested by hydrostatic pressure to twice that of working pressure.

Tensile strengths of plates are taken, with fibre 47 000 lbs. per sq. inch, across it 40 000 lbs., and when in superheaters from 30 000 to 22 400 lbs.

$\frac{47\,000\, B\, t\, 2}{D\, C} = P$, and $\frac{D\, P\, C}{47\,000\, B\, 2} = t$. P *representing pressure that shell will sustain per sq. inch in lbs.*, B *least per cent. of strength of rivet or plate (whichever is least) at lap*, D *diam. of shell and* t *thickness of plate, both in ins.*, *and* C *coefficient of safety.*

---

* Shearing or detrusive resistance of wrought iron is from 70 to 80 per cent. of its tensile strength.

ILLUSTRATION.—Assume T = 50 000 *lbs.* tensile strength of plate. B = 75 *per cent.*, D = 120 *ins.*, C = 5, and t = .5. What pressure will shell sustain, and what should be thickness of plates for such pressure and diameter?

$$\frac{50\,000 \times .75 \times .5 \times 2}{120 \times 5} = 62.5 \text{ lbs., and } \frac{120 \times 62.5 \times 5}{50\,000 \times .75 \times 2} = .5 \text{ inch.}$$

For all practicable deficiencies in drilling, punching, and riveting in transverse courses, if existing, this coefficient is increased up to 6.75, and in longitudinal courses to 8.75, and when courses are not properly broken, an addition is made to above of .4.

Diameter of rivets should not be less than thickness of plates.

Molesworth.

$\frac{P\,d}{2\,t} = C$, $\frac{2\,t\,C}{d} = P$, and $\frac{P\,d}{2\,C} = t$. *d representing diameter and t thickness of metal, both in ins.*, P *working pressure in lbs. per sq. inch, and* C *as follows:*

		Single riveted.		Double riveted.
Best Yorkshire plates......	one ninth of tensile strength.	C = 6200	and	7800
" Staffordshire plates....		" = 5000	"	6200
Ordinary plates............		" = 3000	"	3700

Working stress not to exceed .2 tensile strength of joint or riveted plate.

Then for a pressure of 110 lbs., and a diameter of 42 ins. as given for a standard U. S. boiler.

Taking C as above for best single-riveted plate at 6200. $\frac{110 \times 42}{2 \times 6200} = .372 +$ *ins. in thickness, or* .122 *inch in excess of U. S. Law for a plain cylindrical boiler, single riveted.*

Lloyd's.

Thickness of shells to be computed from strength of longitudinal joints.

$\frac{t\,J\,C}{D} = P$, $\frac{P\,D}{C\,J} = t$, $\frac{t\,J\,C}{P} = D$, $\frac{p-d}{p} = x$, and $\frac{n\,a}{p\,t} = z$. *t representing thickness of plate,* D *diameter of shell, p pitch and d diameter of rivets, all in ins.;* J *per cent. of strength of joint or rivets, the least to be taken;* C *a constant as per table;* P *working pressure in lbs. per sq. inch; n number and a area of rivet; x per cent. of strength of plate at joint compared with solid plate, and z per cent. of strength of rivets compared with solid plate.*

When plates are drilled, take .9 of z, and when rivets are in double shear, put 1.75 a for a.

Constants.

JOINT.		IRON PLATES. .5 inch and under.	.75 inch and under.	Above .75 inch.	STEEL PLATES. .375 inch and under.	.5625 inch and under.	.75 inch and under.	Above .75 inch.
Lap	punched holes.......	155	165	170	200	215	230	240
	drilled do.	170	180	190	—	—	—	—
Double abut strap	punched holes	170	180	190	215	230	250	260
	drilled do.	180	190	200	—	—	—	—

When plates, as in steam-chimneys, superheaters, etc., are exposed to direct action of the flame, these constants are to be reduced .33.

ILLUSTRATIONS.—Assume pitch 4 ins., diam. of rivet 1.375 ins., and thickness of plate 1 inch, both single and double riveted. Area 1.375 = 1.48 *sq. ins.*

$\frac{4 - 1.375}{4} = .656$ *per cent. strength of joint compared to solid plate.* $\frac{1 \times 1.48}{4 \times 1} = .37$ *per cent. strength of rivet to solid plate when single riveted,* and $\frac{1.75 \times 1.48}{4 \times 1} = .647$ *per cent. when double riveted.* ***Rivets at Joint.*** $\frac{n\,a}{p\,t} \times 100$ ***with punched holes and by .90 with drilled.***

Plates.

To Compute Thickness of Plates for a Given Pressure and Pitch, and Pressure and Pitch for Given Thickness.

$\frac{t^2 C}{p^2} = P$, $\sqrt{\frac{t^2 C}{P}} = p$, and $\sqrt{\frac{P p^2}{C}} = t$. *t representing thickness of metal in sixteenths of an inch, p pitch of stays or distance apart at centres in ins.,* P *working pressure in lbs. per sq. inch, and* C *a constant, as follows:*

For a Tensile Strength of Metal of 50 000 *Lbs. per Sq. Inch.*

Screw Stay-bolts with Riveted Heads.—Plates up to .4375 *inch in thickness* C = 90, *and above that* 100.

Screw Stay-bolts with Nuts.—Plates up to .4375 *inch in thickness* C = 110, *and above that* 120.

Screw Stay-bolts with Double Nuts and Washers.—Up to 4.375 *ins. in thickness* C = 140, *and above that* 160.

When stay-bolts are not exposed to corrosion, these constants may be reduced .2.

Resistance of a flat surface decreases in a higher ratio than space between stays. Hence, C must be decreased in proportion to increase of pitch above that of ordinary boiler-plates.

ILLUSTRATION 1.—Assume pressure 110 lbs. per sq. inch, and pitch of stays 5 ins.; what should be thickness of plate for screw-bolts and riveted heads?

$$C = 95. \quad \text{Then} \sqrt{\frac{110 \times 5^2}{95}} = \sqrt{\frac{2750}{95}} = 5.38\text{— } \textit{sixteenth.}$$

2.—Assume thickness of metal 5 sixteenths inch thick, stay-bolts screwed and riveted over its threads, and working pressure of steam 80 lbs. per sq. inch.

$$C = 95. \quad \text{Then} \sqrt{\frac{5^2 \times 95}{80}} = 5.45 \textit{ ins. pitch.}$$

Abut Straps.

Double Abuts should be at least .625 thickness of plate covered. *Single,* .125 thicker than plate covered, and *Double,* .625.

Stays.

Direct.—Tensile stress should not exceed 5000 lbs. per sq. inch for Iron, and 7000 for Steel.

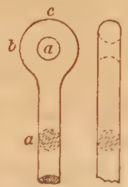
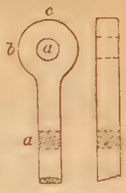
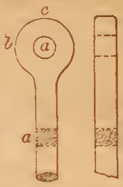
Diagonal or Oblique.—Ascertain area of direct stay required to sustain the surface; multiply it by length of diagonal stay, and divide product by length of a line drawn at a right angle to surface stayed, to end of diagonal stay, and quotient will give area of stay increased to that which is required.

Stress upon an oblique stay is also equal to stress which a perpendicular stay supporting a like surface would sustain, divided by cosine of angle which it forms with perpendicular to surface to be supported.

ILLUSTRATION.—Assume pressure 110 lbs. per sq. inch, area of supported surface 36 sq. ins., and angle of stay 45°; what would be pressure or stress upon stay?

$$\text{Cosine } 45° = .707\,11. \quad \text{Then } 110 \times 36 \div .707\,11 = 5600 \textit{ lbs.}$$

Proportions of Eyes of Stays, Rods, etc.

Dimensions.	No. 1. Forged and Welded.	No. 2. Forged and Welded.	No. 3. Drilled from Bar.
No. 1. a and a = 1 inch. b = .9 " c = .75 " No. 2. a and a = 1 " b = .6 " c = .75 " No. 3. a and a = 1 " b = .75 " c = .875 "			

When drilled from upset bar, dimensions same as for No. 1. Pins when of steel .66 neck of rod.

Stay-bolts.—*Iron*, are not to be subjected to a greater stress than 6000 lbs. per sq. inch of section; *Steel*, 8000 lbs., both areas computed from weakest part of rod, and when of steel they are not to be welded.

To Compute Diameter and Pitch of Stay-bolts, and Resistance they will Sustain.

Screwed. $\frac{p\sqrt{P}}{70}=d$, $\frac{d\,70}{\sqrt{P}}=p$, and $\left(\frac{70\,d}{p}\right)^2=P$. *Socket.* $\frac{p\sqrt{P}}{95}=d$, $\frac{d\,95}{\sqrt{P}}=p$, and $\left(\frac{95\,d}{p}\right)^2=P$. *d representing diameter in ins.*

Illustration.—Assume pitch of stay bolts 6 ins., and working pressure 100 lbs. per sq. inch; what should be diameters of bolts, both screw and socket?

$$\frac{6\times\sqrt{100}}{70}=.857 \textit{ inch Screwed, and } \frac{6\times\sqrt{100}}{95}=.63+\textit{inch Socket.}$$

Girders. (*Lloyd's.*)

$\frac{C\,d^2\,t}{(L-p)\,D\,L}=P$, $\frac{P\,(L-p)\,D\,L}{C\,d^2}=t$, $\sqrt{\frac{P\,(L-p)\,D\,L}{C\,t}}=d$. L *representing length of girder, d its depth, t its thickness at centre or sum of its thicknesses,* D *its distance apart from centre to centre, and p pitch of stays, all in ins., and* C *a constant as per following:*

One stay to each girder, C = 6000. *If two or three* = 9000. *If four* = 10 200.

Illustration.—Assume triple stayed girder, 24 ins. in length, 3 ins. in depth, 1 inch thick, and stayed at intervals of 6 ins.; what working pressure will it sustain?

$$C=9000.\quad \text{Then } \frac{9000\times 6^2\times 1}{(24-6)\times 6\times 24}=\frac{324\,000}{2592}=125 \textit{ lbs.}$$

Flues, Arched or Circular Furnaces. *U. S. Law.*

.3125 inch for each 16 ins. of diameter. English iron, being harder than American, is better constructed to resist compression, and consequently a less thickness of metal is required for like stress.

Lloyd's.

$\frac{89\,600\,t^2}{L\,D}=P$, $\sqrt{\frac{P\,L\,D}{89\,600}}=t$, $\frac{89\,600\,t^2}{P\,L}=D$, and $\frac{89\,600\,t^2}{P\,D}=L$. D *representing external diameter of flue or furnace, and t thickness of plate, both in ins.,* L *length of flue or furnace between its ends or between its rings, in feet, and* P *working pressure in lbs. per sq. inch.*

Illustration.—Assume diameter of flue 16 ins., length 6 feet, and working pressure of steam 80 lbs. per sq. inch.

Then $\sqrt{\frac{80\times 6\times 16}{89\,600}}=\sqrt{.0857}=.29$ *inch.* *Furnace.*—P not to exceed $\frac{8000\,t}{D}$.

ILLUSTRATION.—Assume diameter of a circular furnace or width of a semicircular one 48 ins., working pressure of steam 80 lbs., and length 6 feet.

$$\text{Then } \sqrt{\frac{80 \times 6 \times 48}{89\,600}} = \sqrt{.257} = .507 \textit{ inch thickness.}$$

RIVETING.

Plates.—The strength of a joint is determined by ascertaining which of the two, the plate or the rivets, has the least resistance; the stress on the first being tensile and the latter detrusive.

The tensile strength is to be taken from that of the article under consideration, making due allowances for construction and location of the joint, and the consequent variation of stress, as with or across the fibre of the metal, or exposed to high heat as in a superheater.

With or Across the Fibre.—From experiments of Mr. D. Kirkaldy and others, the difference in strength of Iron plates is ascertained to be from 6.5 to 18 per cent., the average 10 per cent.

Steel Plates.—The relative strength of plates with or across the fibre, as determined by Mr. Kirkaldy, for "Fagersta" is 9 per cent., and for "Siemens" it is without material difference.

Holes.—The relative strength of plates when subjected to drilled or punched holes, as determined by the experiments of Mr. Kirkaldy, is shown to be 15 per cent.

In Riveted Joint exposed to a tensile stress, area of rivets should be equal to area of section of plates through line of rivets, running a little in excess up to .5625 inch diameter of rivet, and somewhat less beyond that, area being determined by relative shearing and tensile resistances of rivet and plate.

NOTE.—For Riveting of Hulls of Vessels, see pp. 828-30.

Essentially by Nelson Foley.

Single Lap Riveting.

$\frac{p-d}{p} = b$ *for plate,* $\frac{n\,a}{p\,t} = b'$ *for rivets,* $\frac{d}{1-b} = p$, $p\,t\,b' = a$, and $\frac{1.27\,b'}{1-b}\,t = d$. *p representing pitch, t thickness of plate, and d diameter of rivets, all in ins., a sectional area of rivets in sq. ins., n number of rivets, and b and b′ per cent. of plate between holes and of section of rivets to solid plate,* i. e. *plate before being punched.*

ILLUSTRATION.—Assume $p = 3$ *ins.*, $d = 1$ *inch*, $a = .7854$ *inch*, and $t = .5$ *inch.*

$\frac{3-1}{3} = .66$ *per cent. strength of lap,* $\frac{.7854}{3 \times .5} = .523$ *per cent. of rivet to solid plate,*

$\frac{1}{-.66} = 3$ *ins., and* $\frac{1.27 \times .523}{1-.66} \times .5 = 1$ *inch.* $3 \times .5 \times .523 + = .7854$ *area.*

When Shearing Strength of Rivet is not Equal to Tensile Strength of Plate.—Then diameter of rivet must be increased in ratio of excess of strength of plate over rivet.

Or, $\frac{1.27\,b'\,T}{1-b\,S}\,t = d$. *T and S representing tensile and shearing strengths, which may be taken at 5 and 4 for Iron and 7 and 6 for Steel.*

When full value of rivet section is not allowed as by Lloyd's rules for drilled holes, $b' = b' \times .9$.

Pitches as Determined by Diameter of Rivets.

Plate between Edges of Holes.	Pitch = Diam. of Rivet ×	Plate between Edges of Holes.	Pitch = Diam. of Rivet ×	Plate between Edges of Holes.	Pitch = Diam. of Rivet ×	Plate between Edges of Holes.	Pitch = Diam. of Rivet ×
Per Cent.		Per Cent.		Per Cent.		Per Cent.	
50	2	58	2.38	65	2.86	72	3.57
52	2.08	60	2.5	68	3.13	75	4
55	2.22	62	2.63	70	3.33	78	4.55

OPERATION.—If distance between edges of holes. or $p-d. = 65$ per cent. of solid plate, and diam. of rivet 1 inch, then $2.86 \times 1 = 2.86$ *ins. pitch.*

When Plate and Rivets are of equal strength in ultimate tension, $b' = b. = B$.

Hence, $\frac{1.27\ B}{1-B}\ t = d$. In illustration of B. assume $p = 3$, $d = 1.1$, and $t = .5$.

Then $3 - 1.1 = 1.9$, and $\frac{1.9}{3} = .633 = b$, *or per cent. of strength of punched to solid plate.* Area $1.1 = .95$, and $\frac{.95}{3 \times .5} = .633 = b'$, *or per cent. of section of rivet to solid plate.* Hence, $B = .633$.

ILLUSTRATION.—Assume as shown, $B = .633$.

$$\text{Then } \frac{1.27 \times .633}{1 - .633} \times .5 = 1.095 \text{ or } 1.1 \textit{ ins. diam.}$$

Diameter of Rivets as Determined by Plate.

B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×	
Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.
52	1.38	1.53	55	1.56	1.73	58	1.76	1.95
53	1.44	1.59	56	1.62	1.8	60	1.91	2.12
54	1.5	1.66	57	1.69	1.87	62	2.08	2.31

OPERATION.—If thickness of plate = .5 inch and plate and rivet have equal resistance, or B = 62 per cent., then $.5 \times 2.08 = 1.04$ *ins. diameter.*

Double Lap Riveting.

Preceding formulas for single lap riveting apply to this, with substitution of $2a$ for a and .64 for 1.27.

ILLUSTRATION.—Assume $p = 3$ *ins.*, $t = .5$ *inch*, and $b' = .589$.

$\frac{3 \times .5 \times .589}{2} = .4418$ *area of d*, $\frac{.64 \times .589}{1 - .75} \times .5 = .75\ d$, $\frac{3 - .75}{3} = .75\ b$, and $\frac{.4418 \times 2}{3 \times .5} = .589\ b'$.

Diameter of Rivets as Determined by Plate.

B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×	
Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.
68	1.35	1.5	71	1.56	1.73	74	1.81	2
69	1.42	1.57	72	1.64	1.82	75	1.91	2.12
70	1.48	1.65	73	1.72	1.91	76	2	2.25

OPERATION.—Assume $t = .5$ inch and B = 70 per cent., tensile strength compared to shearing being as 7 to 6. What should be diameter of the rivets?

$.5 \times 1.48 \times \frac{7}{6} = .863$ *inch.* When rivets are in double shear, put $1.9\ a$ for a.

Triple Lap Riveting.

Preceding formulas for single lap riveting apply to this, with substitution of $3\,a$ for a and .42 for 1.27.

ILLUSTRATION.—Assume $p = 3$ *ins.*, t .5 *inch*, and $b' = .883$.

$\frac{3 \times .5 \times .883}{3} = .4417$ *area of d*, $\frac{.42 \times .883}{1 - .75} \times .5 = .74$ *in. diam.*, $\frac{3 - .75}{3} = .75\,b$,

and $\frac{.4418 \times 3}{3 \times .5} = .883\,b'$.

Diameter of Rivets as Determined by Plate.

B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×		B Or Strength at Joint.	Diam. = Thickness of Plate ×	
Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.	Per Cent.	T = S.	.9 per cent. of Section of Rivet.
70	.99	1.1	73	1.15	1.27	76	1.34	1.49
71	1.04	1.15	74	1.21	1.34	77	1.42	1.58
72	1.09	1.21	75	1.27	1.41	78	1.5	1.67

OPERATION.—As shown by preceding tables.

General Formulas and Illustrations.

Rivets in Single Shear. $\frac{1.27\,B\,T}{1\,(1 - B)\,S}\,t = d$, and $\frac{a\,S}{p\,t\,T} = b'$.

Rivets in Double Shear. $\frac{1.27\,B\,T}{1.75\,(1 - B)\,S}\,t = d$, and $\frac{a\,1.75\,S}{p\,t\,T} = b'$.

Rivets in Triple Shear. $\frac{1.27\,B\,T}{2.5\,(1 - B)\,S}\,t = d$, and $\frac{a\,2.5\,S}{p\,t\,T} = b'$.

Zigzag Riveting. Strength of plate between holes diagonally is equal to that horizontally between holes, when diagonal pitch = .6 and horizontal = diameter of rivet + .4.

Thus, $.6\,p + .4\,p =$ *diagonal pitch.*

Duty of Steam-engines.

The conventional duty of an engine is the number of lbs. raised by it 1 foot in height by a bushel of bituminous coal (112 lbs.).

Cornish Engine.—Average duty, 70 000 000 lbs.; the highest duty ranging from 47 000 000 to 101 900 000 lbs.

A condensing marine engine, working with steam at .75 lbs. (mercurial gauge), cut off at .5 stroke, will require from 1.75 to 2 lbs. bituminous coal per IP per hour.

Relative Cost of Steam-engines for Equal Effects.

In Lbs. of Coal per IP per Hour.

	Lbs.
A theoretically perfect engine	.66
A Cornish condensing engine	2.38
A marine condensing engine	1.75 to 3

Evaporative Power of Boilers.

The Evaporative power of a boiler, in lbs. of water per lb. of fuel consumed, is ascertained approximately by formula

$1.833\left(\frac{S}{2\,S + F}\right) e = lbs.$ S *representing total heating surface in sq. feet*, F *fuel consumed in lbs. per hour, and e theoretical evaporative power of the fuel.*

ILLUSTRATION. — Assume evaporative power of the fuel at 15, consumption per hour 800 lbs., and heating surface 1600.

$$\text{Then } 1.833\left(\frac{1600}{1600 \times 2 + 800}\right) \times 15 = 10.448\ lbs.$$

Efficiency of boiler. $1.833\left(\dfrac{1600}{1600 \times 2 + 800}\right) = .733.$

The evaporative power of different fuels, from and at 212°, is, for coals, from 14.5 to 16.8 lbs., the average of Newcastle being 15.3, for patent fuels 15.66, Lignite 13.5, Coke 13.3, Peat 10.3, and Woods, when dry, 8.1. *See A. E. Seaton, London*, 1883.

Notes on Horse-power.

A Lancashire boiler with a heating surface of 610 sq. feet and a grate-area of 25 will evaporate in ordinary operation 50 cube feet of water per hour; 3.12 sq. feet of horizontal section per cube foot of water, and .5 sq. foot of grate-area per cube foot.

Nominal. *Flue Boilers.*—Usually computed at 5.5 to 6 sq. feet of horizontal section, 15 sq. feet of heating surface, and 1 sq. foot of grate-area.

The IHP of such boilers will range from 3 to 4 times that of the nominal.

Multitubular Boilers.—.75 sq. foot of grate-area and 2.5 of heating surface.

Weights of Steam-engines.

Side-wheels.—*American Marine (Condensing).*

Engine.	Frame.	Water-wheels.	Cylinders. No.	Cylinders. Volume.	Weight per Cube Foot.	Service.
				Cube Feet.	Lbs.	
Vertical beam...........	Wood.*	Wood.	1	63	1100	River.
"	Wood.*	Wood.	2	216	1040†	Coast.
"	Wood.*	Wood.	1	430	1225	Coast.
"	Wood.*	Wood.	2	253	1480‡	Coast.
"	Wood.*	Iron.	1	725	1089†	Sea.
Oscillating..............	Iron.	Iron.	2	540	850	Sea.
"	Iron.	Iron.	2	1502	550§	Sea.
Inclined.................	Iron.	Iron.	2	535	1100	Sea.

* Without frame. † With frame 1109. ‡ Including boilers. § Single frame.

Screw Propellers.—*American Marine (Condensing).*

Engine.	Cylinders. No.	Cylinders. Volume.	Weights. Engine.	Weights. Boilers.	Weights. Per C. Ft. Cylinder.	Service.
		Cube Feet.	Lbs.	Lbs.	Lbs.	
Vertical direct, Jet Condens'g..	1	4	22 040	12 100	8 535	Sea.
" " Surface Cond'g.	1	12.5	59 000	32 000	7 280	Sea.
" " Jet " .	1	12.5	48 130	35 000	6 650	Sea.
" " " " .	1	33	120 450	98 000	6 620	Coast.
" " " " .	4	506	1 523 060	985 600	4 958	Sea.
Horizontal back-action.........	2	68	289 680	200 800	7 212	Sea.
" direct...............	2	67	201 000	200 593	6 009	Sea.
Vertical compound...... (Surface Condens'g)	2	4.8	24 705	26 372	10 641	Coast.
" " ...... (Surface Condens'g)	2	24.3	94 196	88 050	7 500	Sea.
" " ...... (Surface Condens'g)	2	425	1 022 400	840 000	4 380	Sea.
" direct.......... (Surface Condens'g)	1	3.6	30 534	27 301	16 066	Coast.
" " .... (Surface Condens'g)	1	35	172 028	100 065	7 774	Sea.
" " Non-Condensing.	1	1.86	14 410	22 481	19 834	River.
" " " " ..	1	2.77	14 759	22 417	13 421	Coast.

English Marine (Condensing).

Description.	Cylinders. No.	Cylinders. Volume.	Weights. Engines.	Weights. Propeller and Shafting.	Weights. Boilers and Water.	Weights. Total.	Weights. Per IHP	Weights. Per Cube Ft. Cylinder.
		Cube Ft.	Tons.	Tons.	Tons.	Tons.	Lbs.	Tons.
Trunk..................	2	230	121	47	257	425	465	1.85
Horizontal direct.......	2	382	223	85	303	611	338	1.6
Vertical direct	2	393	165	48	144	357	781	.9
Oscillating..............	2	440	117	43	135	295	560	.7
Vertical compound.....	2	24	4.25	.75	7.25	12.25	66	.52
" "	6	707	497	167	656	1320	368	1.87
Horizontal compound...	2	52	55	15	110	180	351	3.44
" " ..	2	143	130	27	162	319	309	2.23

Land-engines.—(*Non-condensing.*)

Engine.		Volume of Cyl'r.	Engine.	Spur-wheel and Connections.	Sugar-Mill Complete.	Boilers, Grates, etc.	Engine per Cube Foot of Cylinder.
			Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Vertical beam	18 ins. × 4 feet	7	67 200	37 800	89 600	26 880	9600
Vertical beam	30 ins. × 5 feet	24.5	105 000	137 179	265 879	75 000	4290
Horizon'l,	14 ins. × 2 feet	2.2	10 914	—	—	8 200	5100
"	22 ins. × 4 feet	10.6	56 000	—	—	30 140	5600

To Compute Weight of a Vertical Beam and Side-wheel Jet Condensing Engine. (*T. F. Rowland, A.S.C.E.*)

Including all Metals, Boiler and Attachments, Smoke-pipe, Grates, Iron Floors, and Iron in Wooden Water-wheels, omitting Coal-bunkers.

For a Pressure per Mercurial Gauge of 40 lbs. per Sq. Inch.

For surface condenser add 10 to 15 per cent.

Rule.—Multiply volume of cylinder in cube feet by *Coefficient* in following table corresponding to length of stroke, and product will give rough weight in lbs. For finished weight deduct 6 per cent.

Stroke.	Coefficient.	Stroke.	Coefficient.	Stroke.	Coefficient.	Stroke.	Coefficient.
Feet.		Feet.		Feet.		Feet.	
5	2467	7	2213	9	1865	11	1619
6	2340	8	2000	10	1730	12	1546

Example 1.—What are the rough and finished weights of a vertical beam engine, cylinder 80 ins. in diameter and 12 feet stroke of piston?

Area of 80 ins. = 5026.56, which × 12 *feet* = 419 *cube feet*, and × 1546 for 12 feet stroke = 647 774 *lbs. rough weight.*

Then 647 774 × .06 = 38 866, and 647 774 − 38 866 = 608 908 *lbs. finished weight.*

WEIGHTS OF BOILERS.

Weights of Iron Boilers (including Doors and Plates, and exclusive of Smoke-pipes and Grates) per Sq. Foot of Heating Surface.

Surface Measured from Grates to Base of Smoke-pipe or Top of Steam Chimney.

Boiler. *For a Working Pressure of 40 Lbs.*		Weight.
		Lbs.
Single return, Flue*	water bottom	25.6 to 32.9
" " "	—	24 to 30
" " " Multi-flue*	water bottom	27 to 45
" " " "	—	25 to 43
Horizontal return, Tubular†	water bottom	22.5 to 35
" " " †	—	21 to 33
" " " *	—	17.7 to 26.7
Vertical " " †	water bottom	18.5 to 26.5
Horizontal direct, Tubular*	" "	19.8 to 23.8
" " " *	—	17 to 21
Cylindrical, external furnace,‡ 36 ins. in diam., .25 inch thick		23.5 to 24
" Flue " ‡ 36 to 42 " .25 " "		18.1 to 18.6
Horizontal direct, Tubular	Locomotive	16.3 to 17.3
Vertical Cylinder direct, Tubular	—	24 to 26

Weight of Cylindrical Furnace and Shell Boilers, all complete for Sea Service and for a pressure of 60 lbs. steam, 200 lbs. per IHP.

* Section of furnace square. Shell cylindrical. † Section of furnace and shell square.
‡ Wrought-iron heads, .375 inch thick, flues, .25 inch, and surface computed to half diameter of shell.

Notes.—1. The range in the units of weight arises from peculiarities of construction, consequent upon proportionate number of furnaces, thicknesses of metal, volume of shell compared with heating surface, character of staying, etc.

2. If pressure is increased the above units must be proportionately increased.

Boiler-power.

The power of a boiler is the volume or weight of steam alone (independent of any water that it may hold in suspension) that it will generate at its operating pressure in a unit of time.

Marine boilers of the ordinary type and proportions, with natural draught, burning anthracite coal, produce 3.5 to 5.5 IHP per sq. foot of grate per hour; with a free burning or a semi-bituminous coal, 5 to 7.5 IHP; and with a forced draught, with 25 to 30 lbs. best coal per sq. foot of grate per hour, 8 to 10 IHP.

Marine engines, operating with a steam-pressure of 35 lbs. (m. g.), and with moderate expansion, consume 30 lbs. steam per IHP per hour, and with a high rate of expansion, under a pressure of 70 lbs., 20 lbs. steam.

With a blast draught and consuming 30 to 40 lbs. of a fair quality of coal per sq. foot of grate per hour, 7 to 10 HP per hour can be attained.

In locomotive boilers, having from 50 to 90 sq. feet of heating surface per sq. foot of grate, and at a rate of combustion of from 45 to 125 lbs. of coke, an average evaporation of 9 lbs. of water per lb. of coke has been attained at ordinary temperatures and pressure.

To Compute Volume of Air and Gas in a Furnace.

When Volume at a Given Temperature is known. RULE.—Multiply given volume by its absolute temperature, and divide product by the given absolute temperature.

NOTE.—Absolute temperature is obtained by adding 461° to given or acquired temperature.

EXAMPLE.—Assume volume of air entering a furnace at 1 cube foot, its temperature 60°, and temperature of furnace 1623°; what would be the increase of volume?

$$\frac{1 \times \overline{1623^\circ + 461^\circ}}{60^\circ + 461^\circ} = \frac{2084}{521} = 4 \text{ times.}$$

Volume of Furnace Gas per Lb. of Coal. (*Rankine.*)

Temperature.	Air Supplied. 12 Lbs.	18 Lbs.	24 Lbs.	Temperature.	Air Supplied. 12 Lbs.	18 Lbs.	24 Lbs.
32°	150	225	300	752°	369	553	738
68	161	241	322	1112	479	718	957
104	172	258	344	1472	588	882	1176
212	205	307	409	1832	697	1046	1395
572	314	471	628	2500	906	1357	1812

Temperature of ordinary boiler furnaces ranges from 1500° to 2500°.

The opening of a furnace door to clean the fire involves a loss of from 4 to 7 per cent. of fuel.

For other illustrations, see *ante*, page 744–6.

Rate of Combustion.

The rate of combustion in a furnace is computed by the lbs. of fuel consumed per sq. foot of grate per hour.

In general practice the rate for a natural draught is, for anthracite coal from 7 to 16 lbs., for bituminous, from 10 to 25 lbs., and with artificial or forced draught, as by a blower, exhaust blast, or steam-jet, the rate may be increased from 30 to 120 lbs.

The dimensions or size of coal must be reduced and the depth of the fire increased directly, as the intensity of the draught is increased.

Temperature of gases at base of chimney or pipe should be 600°, and frictional resistance of surface of chimney is as square of velocity of current of gases.

Ordinarily from 20 to 32 per cent. of total heat of combustion is expended in the production of the chimney draught in a marine boiler, to which is to be added the losses by incomplete combustion of the gaseous portion of the fuel and the dilution of the gases by an excess of air, making a total of fully 60 per cent. (*Steam-boilers, Wm. H. Shock, U. S. N.*, 1881.)

STRENGTH OF MATERIALS.

Strength of a material is measured by its resistance to alteration of form, when subjected to stress and to rupture, which is designated as Crushing, Detrusive, Tensile, Torsion, and Transverse, although transverse is a combination of tensile and crushing, and detrusive is a form of torsion at short lengths of application.

ELASTICITY AND STRENGTH.

Strength of a material is resistance which a body opposes to a permanent separation of its parts, and is measured by its resistance to alteration of form, or to stress.

Cohesion is force with which component parts of a rigid body adhere to each other.

Elasticity is resistance which a body opposes to a change of form.

Elasticity and *Strength*, according to manner in which a force is exerted upon a body, are distinguished as *Crushing Strength*, or Resistance to Compression; *Detrusive Strength*, or Resistance to Shearing; *Tensile Strength*, or Absolute Resistance; *Torsional Strength*, or Resistance to Torsion; and *Transverse Strength*, or Resistance to Flexure.

Limit of Stiffness is flexure, and limit of Resistance is fracture.

Neutral Axis, or *Line of Equilibrium*, is the line at which extension terminates and compression begins.

Resilience, or toughness of bodies, is strength and flexibility combined; hence, any material or body which bears greatest load, and bends most at time of fracture, is toughest.

Stiffest bar or beam that can be cut out of a cylinder is that of which depth is to breadth as square root of 3 to 1; *strongest*, as square root of 2 to 1; and most *resilient*, that which has breadth and depth equal.

Stress expresses condition of a material when it is loaded, or extended in excess of its elastic limit.

General law regarding deflection is, that it increases, *cæteris paribus*, directly as cube of length of beam, bar, etc., and inversely as breadth and cube of depth.

Resistance of *Flexure* of a body at its cross-section is very nearly .9 of its tensile resistance.

Coefficient of Elasticity.

Elasticity of any material subjected to a tensile or compressive force, within its limits, is measured by a fraction of the length, per unit of force per unit of sectional area, termed a *constant*, and coefficient of elasticity is usually defined as the weight which would stretch a perfectly elastic bar of uniform section to double its length.

Unit of force and area is usually taken at one lb. per sq. inch. E *representing denominator of fraction.*

Example.—If a bar of iron is extended one 12 000 000th part of its length per lb. of stress per sq. inch of section,

$$\frac{1}{12\,000\,000} = \frac{1}{E}.$$

The bar would, therefore, be stretched to double its normal length by a force of 12 000 000 lbs. per sq. inch, if the material were perfectly elastic.

The same method of expressing coefficient of elasticity is applied to resistance to compression. That is, coefficient, in weight, is expressed by denominator of fraction of its length by which a bar is compressed per unit of weight per sq. inch of section.

Ultimate extension of cast iron is 500th part of its length.

Extension of Cast-iron Bars, when suspended Vertically.

1 Inch Square and 10 Feet in Length. Weight applied at one End.

Weight.	Extension.	Set.	Weight.	Extension.	Set.	Weight.	Extension.	Set.
Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.
529	.0044	—	2117	.0190	.000059	8468	.0871	.00855
1058	.0092	.000015	4234	.0397	.00265	14820	.1829	.02555

Woods.—MM. Chevaudier and Wertheim deduced that there was no limit of elasticity in woods, there being a permanent set for every extension. They, however, adopted a set of .00005 of length as limit of elasticity. This is empirical.

MODULUS OF ELASTICITY.

Modulus or *Coefficient of Elasticity* of any material is measure of its elastic reaction or force, and is height of a column of the material, pressing on its base, which is to the weight causing a certain degree of compression as length of material is to the diminution of its length.

It is computed by this analogy: As extension or diminution of length of any given material is to its length in inches, so is the force that produced that extension or diminution to the modulus of its elasticity.

Or, $x : P :: l : w = \frac{P}{x}$. *$x$ representing length a substance 1 inch square and 1 foot in length would be extended or diminished by force* P, *and w weight of modulus in lbs.*

To Compute Weight of Modulus of Elasticity.

Rule.—As extension or compression of length of any material 1 inch square, is to its length, so is the weight that produced that extension or compression, to modulus of elasticity in lbs.

Example.—If a bar of cast iron, 1 inch square and 10 feet in length, is extended .008 inch, with a weight of 1000 lbs., what is the weight of its modulus of elasticity?

$.008 : 120\ (10 \times 12) :: 1000 : 15\,000\,000$ *lbs.*

To Compute Modulus of Elasticity.

When a Bar or Beam is Supported at Both Ends and Loaded in Centre.

Rule.—Multiply weight or stress per sq. inch in lbs. by length of material in ins., and divide product by modulus of weight.

Or, $\frac{l\,W}{M} = E$; $\frac{l\,W}{E} = M$; $\frac{E\,M}{l} = W$. *l representing length in ins.*, M *modulus*, W *weight in lbs. per sq. inch, and* E *compression or extension.*

Example 1.—If a wrought-iron rod, 60 feet in length and .2 inch in diameter, is subjected to a stress of 150 lbs., what will it be extended?

Modulus of elasticity of iron wire is 28 230 500 *lbs.* (see following table), and area of it $.2^2 \times .7854 = .314\,16$.

$$\frac{150}{.314\,16} = 477.46 \text{ lbs. per sq. inch, and } 60 \times 12 = 720 \text{ ins.}$$

$$\text{Then } 477.46 \times \frac{720}{28\,230\,500} = \frac{343\,771.2}{28\,230\,500} = .012\,18 \text{ inch.}$$

2.—Take elements of preceding case under rule for weight of modulus.

$$\frac{120 \times 1000}{15\,000\,000} = .008 \text{ inch.} \quad \frac{.008 \times 15\,000\,000}{120} = 1000 \text{ lbs.}$$

Modulus of Elasticity and Weight of Various Materials.

Substances.	Height.	Weight.	Substances.	Height.	Weight.
	Feet.	Lbs.		Feet.	Lbs.
Ash	4 970 000	1 656 670	Larch	4 415 000	1 074 000
Beech	4 600 000	1 345 000	Lead, cast	146 000	720 000
Brass, yellow	2 460 000	8 464 000	Lignum-vitæ	1 850 000	1 080 400
" wire	4 112 000	14 632 720	Limestone	2 400 000	3 300 000
Copper, cast	4 800 000	18 240 000	Mahogany	6 570 000	2 071 000
Elm	5 680 000	1 499 500	Marble, white	2 150 000	2 508 000
Fir, red	8 330 000	2 016 000	Oak	4 750 000	1 710 000
Glass	4 440 000	5 550 000	Pine, pitch	8 700 000	2 430 000
Gun-metal	2 790 000	8 844 300	" white	8 970 000	1 830 000
Hempen fibres	5 000 000	170 000	Steel, cast	8 530 000	26 650 000
Ice	6 000 000	2 370 000	" wire	9 000 000	28 689 000
Iron, cast	5 750 000	17 968 500	Stone, Portland	1 672 000	1 718 800
" wrought	7 550 000	25 820 000	Tin, cast	1 053 000	3 510 000
" wire	8 377 000	28 230 500	Zinc	4 480 000	13 440 000

Weight a Material will bear per Sq. Inch, without Permanent Alteration of its Length.

Material.	Lbs.	Material.	Lbs.	Material.	Lbs.
Metals.		*Stones, etc.*		*Woods.*	
Brass	6 700	Marble	4900	Beech	2360
Gun-metal	10 000	Limestone*	2000	Elm	3240
Iron, cast	15 000	Portland	1500	Fir, red	4290
" wrought	17 800			Larch	2060
Lead	1 500	*Woods.*		Mahogany	3000
Steel	45 000	Ash	3540	Oak	3960

* Tensile strength 2800.

Comparative Resilience of Woods.

Ash	1	Chestnut	.73	Larch	.84	Spruce	.64
Beech	.86	Elm	.54	Oak	.63	Teak	.59
Cedar	.66	Fir	.4	Pitch Pine	.57	Yellow Pine	.64

MODULUS OF COHESION.

To Compute Length of a Prism of a Material which would be Severed by its own Weight when Suspended.

Rule.—Divide tensile resistance of material per sq. inch by weight of a foot of it in length, and quotient will give length in feet.

Illustration.—Assume tensile resistance of a wrought-iron rod to be 60 000 lbs. per sq. inch. Weight of 1 foot $= 3.4$ *lbs.*

Then $60\,000 \div 3.4 = 17\,647.06$ *feet.*

Length in Feet required to Tear Asunder the following Substances:

Rawhide...... 15 375 feet. | Hemp twine... 75 000 feet. | Catgut........ 25 000 feet.

Elasticity of Ivory as compared with Glass is as .95 to 1.

When Height is given. Rule.—Multiply weight of 1 foot in length and 1 inch square of material by height of its modulus in feet, and product will give weight.

To Compute Height of Modulus of Elasticity.

Rule.—Divide weight of modulus of elasticity of material by weight of 1 foot of it, and quotient will give height in feet.

Example.—Take elements of preceding case (page 762), weight of 1 foot being 3 lbs.; what is height of its modulus of elasticity?

$15\,000\,000 \div 3 = 5\,000\,000$ *feet.*

From a series of elaborate experiments by Mr. E. Hodgkinson, for the Railway Structure Commission of England, he deduced following formulas for extension and compression of *Cast Iron:*

Extension: $13\,934\,040 \frac{e}{l} - 290\,743\,200 \frac{e^2}{l^2} = W.$

Compression: $12\,931\,560 \frac{c}{l} - 522\,979\,200 \frac{c^2}{l^2} = W.$ *e and c representing extension and compression, and l length in ins.*

ILLUSTRATION.—What weight will extend a bar of cast iron, 4 ins. square and 10 feet in length, to extent of .2 inch?

$13\,934\,040 \times \frac{.2}{120} - 290\,743\,200 \frac{.2^2}{120^2} = 23\,223.4 - 8076.2 = 15\,147.2$, which $\times$ 4 *ins.* $= 60\,588.8$ *lbs.*

CRUSHING STRENGTH.

Crushing Strength of any body is in proportion to area of its section, and inversely as its height.

In tapered columns, it is determined by the least diameter.

When height of a column is not 5 times its side or diameter, crushing strength is at its maximum.

Cast Iron.—Experiments upon bars give a mean crushing strength of 100 000 lbs. per sq. inch of section, and 5000 lbs. per sq. inch as just sufficient to overcome elasticity of metal; and when height exceeds 3 times diameter, the iron yields by flexure. When it is 10 times, it is reduced as 1 to 1.75; when it is 15 times, as 1 to 2; when it is 20 times, as 1 to 3; when it is 30 times, as 1 to 4; and when it is 40 times, as 1 to 6.

Experiments of Mr. Hodgkinson have determined that an increase of strength of about one eighth of destructive weight is obtained by enlarging diameter of a column in its middle.

In columns of same thickness, strength is inversely proportional to the $^{1.63}$ power of length nearly.

A hollow column, having a greater diameter at one end than the other, has not any additional strength over that of an uniform cylinder.

Wrought Iron.—Experiments give a mean crushing stress of 47 000 lbs. per sq. inch, and it will yield to any extent with 27 000 lbs. per sq. inch, while cast iron will bear 80 000 lbs. to produce same effect.

Effects.—A wrought bar will bear a compression of $\frac{1}{863}$ of its length, without its utility being destroyed.

With cast iron, a pressure beyond 27 000 lbs. per sq. inch is of little, if any, use in practice.

Glass and hard Stones have a crushing strength from 7 to 9 times greater than tensile; hence an approximate value of their crushing strength may be obtained from their tensile, and contrariwise.

Various experiments show that the capacity of stones, etc., to resist effects of freezing is a fair exponent of that to resist compression.

Seasoning.—Seasoned woods have nearly twice crushing strength of unseasoned.

Elastic Limit compared to Crushing Resistance.

Wrought-iron Commerce	.545	Cast steel	.692
Bessemer steel	.615	Fagersta steel	.25
Cast steel	.473		.7

Crushing Strength of various Materials, deduced from Experiments of Maj. Wade, Hodgkinson, Capt. Meigs, U. S. A., Stevens Institute, and by G. L. Vose.

Reduced to a Uniform Measure of One Sq. Inch.

Cast Iron.

Figures and Material.	Crushing Weight.	Figures and Material.	Crushing Weight.
	Lbs.		Lbs.
Gun-metal, American	174 803	Clyde, average, English	82 000
" "	85 000	Stirling, mean of all, English	122 395
	125 000	" extreme, English	134 400
" mean	100 000	Extreme, English	53 760
Low Moor, No. 1, English	62 450		153 200
" No. 2, "	92 330	Average (Hodgkinson), English	84 240
Clyde, No. 3, "	106 039	Blaenavon No. 2	109 700

Wrought Iron.

American, extreme	127 720	English	65 200
	83 500		40 000
" mean	47 040	" average	37 850

Various Metals.

Aluminium bronze, 95 cop.	129 920	Steel, Bessemer	50 000
Fine brass	164 800	" " soft	66 200
Cast copper	117 000	" tempered	335 000
Steel, cast	105 000	" Siemens	
	250 000	Tin, cast	15 500
" Fagersta	154 500	Lead	7 730

Elastic Crushing Strength of Wrought Iron and Crucible Steel is equal to its tensile, of Bessemer Steel, 50 per cent. of its transverse strength.

Woods.

Ash	6 663	Maple	8 100
Beech	6 963		10 000
Birch	3 300	Oak, American white	7 500
	7 900	" Canadian white	5 982
Box	10 513	" " live	6 850
Cedar, red	5 968	" English	9 500
" seasoned	6 500		6 484
Chestnut	5 350	" Dantzic, dry	7 700
Elm	6 831	Pine, pitch	8 947
" seasoned	10 000	" white	5 775
" English	10 300	" yellow	8 200
Hickory, white	8 925	" Deal, Christiana	5 850
Larch	3 200	Spruce, white	5 950
	5 500	Teak	12 100
Locust	9 113	Walnut	6 645
Mahogany, Spanish	8 198	Willow, seasoned	6 000

Crosswise of Fibre.

Oak .... 2300 | Larch .... 1300 | Pines 550

Increase in Strength of Cubes of Sandstone, per Sq. Inch (under Blocks of Wood), as Area of Surface is increased. (*Gen'l Gillmore, U. S. A.*)

Stone.	Inches. .5	1	1.5	2	2.25	2.75	3	4
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
Yellow Berea sandstone	6080	6990	8 226	8 955	9 130	9 838	10 125	11 720
Blue " "	—	9500	10 730	12 000	12 500	13 200	—	—

Stones, Cements, etc. *(Per Sq. Inch.)*

Figures and Material.	Crushing Weight.
	Lbs.
Basalt, Scotch	8 300
" Welsh	16 800
Beton, N. Y. S. Concreting Co.	800 1 400
Brick, pressed	6 222 10 219
" Gloucester, Mass.	14 216*
" hard burned	3 630
" common	800 4 000
" yellow-faced burned, Eng.	1 440
" Stourbridge fire-clay, "	1 650
" Staffordshire blue, "	7 200
" stock, English	2 250
" Fareham, English	5 000
" red, English	808
" Sydney, N. S.	2 228
Caen, France	1 543
Cement, Hydraulic, pure, Eng.	17 000 32 000
" Portland, sand 1	1 280
" " sand 3	600
" " 3 mos.	3 800
" " 1 sand, 3 mos.	2 464
" " 9 mos.	5 980
" " 1 sand, 9 mos.	2 330
" " 12 inch cubes, 12 mos. 1 sand and gravel	2 650
" " 3 " "	1 800
" Roman	342
" " pure, Eng.	750
" Rosendale	3 270
" Sheppey, Eng.	1 280
Concrete, lime 1, gravel 3	460 775
Freestone, Belleville, N. J.	3 522
" Connecticut	3 319
" Dorchester, Mass.	3 069
" Little Falls, N. Y.	2 991
Glass, crown	31 000
Gneiss	19 600
Granite, Aberdeen, Eng.	10 760
" Cornish, "	6 339
" Dublin, "	10 450
" Newry, "	12 850
Granite, Patapsco, Md.	5 340
" Portland, Eng.	4 570 15 583
" Quincy, Mass.	15 000
Greenstone, Irish	18 800
Limestone	4 000 9 000
" compact, Eng.	7 800
" Magnesian, "	3 130
" Anglesea "	3 600
" Irish "	14 000
Marble, Baltimore, Md.	8 057 18 061
" East Chester, N. Y.†	13 917
" Hastings, N. Y.	18 941
" Irish	17 440
" Italian	12 624
" " white	9 630
" Lee, Mass.	22 702
" Montgomery Co., Pa.	8 950
" Statuary	3 360
" Stockbridge, Mass.‡	10 382
" Symington, large	11 156
" " fine crystal	18 248
" " strata horizontal	10 124
Masonry, brick, common	500 800
" " in cement	760
Mortar, good	240
" lime and sand	460
" " " beaten	595
" common	120
Oolite, Portland	3 850
Pottery-pipe, Chelsea	12 000
Sandstone, Aquia Creek §	5 340
" Arbroath, Eng.	7 850
" Connecticut	3 110
" Craigleth, Eng.	5 825
" Derby grit "	3 136
" Holyh'd quartz, Eng.	25 540
" Seneca ‖	10 762
" Yorkshire, Eng.	5 710
Slate, Irish	13 890 23 744
Terra Cotta	5 000
Whinstone, Scotch	8 300

* Tested by author at Stevens' Institute, N. J. † Post-office, Wash. ‡ City Hall, New York. § Capitol, Treasury Department, and Patent Office, Washington, D. C. ‖ Smithsonian Institute.

Safe Load of Hollow, Cylindrical, and Solid Columns, Arches, Chords, etc., of Cast Iron.

Hollow Columns. Per Sq. Inch. (F. W. Shields, M. I. C. E.)

Length.	Thickness.	Load.	Length.	Thickness.	Load.	Length.	Thickness.	Load.	Length.	Thickness.	Load.
	Inch.	Lbs.		Inch.	Lbs.		Inch.	Lbs.		Inch.	Lbs.
20 to 24 diam's.	.375	2800	20 to 24 diam's.	.625	3920	25 to 30 diam's.	.375	2240	25 to 30 diam's.	.625	3360
	.5	3360		.75	4480		.5	2800		.75	3920

Solid Columns, etc.—3360 lbs. per sq. inch. (*Brunel.*)

Arches.—5600 lbs. per sq. inch.

Chords and Posts.—1 inch diameter and not more than 15 diameters in length .2 of breaking weight of metal.

.625 inch diameter and not more than 25 diameters in length .5 of breaking weight of metal, and when more than 25 diameters in length from .1 to .025 of breaking weight of metal. (*Baltimore Bridge Co.*)

Wrought-iron Cylinders and Rectangular Tubes.

Length.		External Diameter.		Internal Diameter.	Thickness.	Area.	Crushing Weight per Sq. Inch.
CYLINDERS.		Ins.		Ins.	Ins.	Sq. Ins.	Lbs.
10 feet		1.495		1.292	.1	.444	14 661
10 "		2.49		2.275	.107	.804	29 779
10 "		6.366		6.106	.13	2.547	35 886
RECTANGULAR TUBES.							
10 feet	lap riveted.	4.1	×	4.1	.03	.504	10 980
5 "	lap riveted.	4.1	×	4.1	.03	.504	11 514
10 "	lap riveted.	4.1	×	4.1	.06	1.02	19 261
10 "	lap riveted.	4.25	×	4.25	.134	2.395	21 585
7.5 "	lap riveted.	4.25	×	4.25	.134	2.395	23 202
10 "	lap riveted.	8.4	×	4.25	.26 .126	6.89	29 981
10 "	lap riveted.	8.1	×	8.1	.06	2.07	13 276
7.66 "	lap riveted.	8.1	×	8.1	.06	2.07	13 300
10 "	internal diaphrag's	8.1	×	8.1	.0637	3.551	19 732
5 "	internal diaphrag's	8.1	×	8.1	.0637	3.551	23 208

Strength per Sq. Inch of 2-Inch Cubes under Blocks of Wood. (*Gen'l Gillmore, U. S. A.*)

Surfaces Worked to a Clear Bed.

GRANITE.	Lbs.
Staten Island blue	22 250
Maine	15 000
Quincy, dark	17 750
" light	14 750
Westchester Co., N. Y.	18 250
Millstone Point, Conn.	16 187
New London, Conn.	12 500
Richmond, Va.	21 250
" " gray	14 100
Cape Ann, Mass.	12 423 19 500
Westerly, R. I., gray	14 937
Fall River, Mass., gray	15 937
Garrisons, Hudson River, gray	13 370
Duluth, Minn., dark	17 750
Keene, N. H., bluish gray	12 875
Used in Central Park, N. Y., red	17 500
Jersey City, N. J., soap	20 750
Passaic Co., " gray	24 040
LIMESTONE.	
Glen's Falls, N. Y.	11 475
Lake Champlain, N. Y.	25 000
Canajoharie, N. Y.	20 700
Kingston, "	13 900
Garrisons, "	18 500
Marblehead, O., white	12 600
Joliet, Ill., white	16 900
Lime Island, Mich., drab	18 000 25 000
Sturgeon Bay, Wis., bluish drab	21 500

LIMESTONE.	Lbs.
Bardstown, Ky., dark	16 250
Cooper Co., Mo., dark drab	6 650
Erie Co., N. Y., blue	12 250
Caen, France	3 650
MARBLE.	
East Chester, N. Y.	13 504
Italian, common	13 062
Dorset, Vt.	7 612
Mill Creek, Ill., drab	9 687
North Bay, Wis., drab	20 025
SANDSTONE.	
Little Falls, N. Y., brown	9 850
Belleville, N. J., gray	11 700
Middletown, Conn., brown	6 950
Haverstraw, N. Y., red	4 350
Medina, N. Y., pink	17 725
Berea, O., drab	7 250 10 250
Vermillion, O., drab	8 850
Fond du Lac, Wis., purple	6 250
Marquette, Mich., "	7 450
Seneca, O., red brown	9 687
Cleveland, O., olive green	6 800
Albion, N. Y., brown	13 500
Kasota, Minn., pink	10 700
Fontenac, Minn., light buff	6 250
Craigleth, Edinburgh	12 000
Dorchester, N. B., freestone	9 150
Massillon, O., yellow drab	8 750
Warrensburg, Mo., bluish drab	5 000

To Compute Crushing Weight of Columns.

Deduced by Mr. L. D. B. Gordon from Results of Experiments of various Authors.

Cast Iron. (*Hodgkinson.*)

Round Solid or Hollow. $\frac{36\,a}{1+\frac{r^2}{400}} = W$. *For rectangular put* 500.

Rectangular Solid or Hollow. $\frac{36\,a}{1+\frac{r^2}{500}} = W$. For **L**, **T**, **U**, etc., put $\frac{19\,a}{1+\frac{r^2}{900}}$.

Wrought Iron. (*Stoney.*)

Round Solid. $\frac{16\,a}{1+\frac{r^2}{2400}} = W$. *Rectangular Solid.* $\frac{16\,a}{1+\frac{r^2}{3000}} = W$.

Steel. (*Baker.*)

Round Solid.—Strong steel, $\frac{51\,a}{1+\frac{r^2}{900}} = W$; mild steel, $\frac{30\,a}{1+\frac{r^2}{1400}} = W$.

Rectangular Solid.—Strong steel, $\frac{51\,a}{1+\frac{r^2}{1600}} = W$; mild steel, $\frac{30\,a}{1+\frac{r^2}{2480}} = W$.

a representing area of metal in sq. ins., r ratio of length to least external diameter or side, and W crushing weight in tons.

Illustration.—What is the crushing weight of a hollow cylindrical column of cast iron 10 ins. in diameter, 24 feet in length, and 1 inch in thickness?

Area of 10 ins. = 78.54. $\frac{24 \times 12}{10} = 28.8$, and $28.8^2 = 829.44$. Area of 10 ins. = 78.54 — area $\overline{1 \times 2}$ ins. = 50.26. Then, $\frac{80\,640 \times 28.28}{1+\frac{829.44}{400}} = \frac{2\,280\,499.2}{1+2.07} = 742\,833.6$ *lbs.*

Weight borne with Safety by Solid Cast-iron Columns.

In 1000 Lbs.—(New Jersey Steel and Iron Co.)

Length. Feet.	Diameter. 2 Ins.	3 Ins.	4 Ins.	5 Ins.	6 Ins.	7 Ins.	8 Ins.	9 Ins.	10 Ins.	11 Ins.	12 Ins.	13 Ins.	14 Ins.	15 Ins.
5	12.4	44	102	184	288	414	560	728	916	1126	1354	—	—	—
6	9.4	36	88	164	264	386	532	698	884	1082	1320	1570	—	—
7	7.2	30	76	146	242	360	502	660	850	1056	1282	1530	1798	2086
8	—	24	66	130	218	332	470	630	812	1016	1240	1486	1754	2040
9	—	20	56	114	198	306	440	596	774	974	1196	1440	1706	1992
10	—	18	48	102	180	282	410	560	739	932	1152	1392	1656	1940
12	—	—	38	80	136	238	354	494	658	846	1056	1292	1550	1828
14	—	—	28	64	122	200	304	432	586	774	966	1192	1440	1712
16	—	—	—	52	100	170	262	378	520	686	878	1094	1332	1596
18	—	—	—	44	84	144	226	332	462	616	796	1000	1228	1482
20	—	—	—	—	72	124	196	292	410	552	720	912	1130	1372

For Tubes or Hollow Columns.

Subtract weight that may be borne by a column, of diameter of internal diameter of tube from external diameter, and remainder will give weight that may be borne. Thickness of metal should not be less than one twelfth diameter of column.

Illustration.—Required the safe load of a solid cast-iron column 6 ins. in diameter and 20 feet in length.

Under 6 and in a line with 20 is 72, which × 1000 = 72 000 lbs.

Note.—This is about one sixth of destructive weight.

Safe Loads as determined by Preceding Formulas.

Cast Iron, one fifth to one sixth. *Wrought Iron*, one fourth to one fifth. *Woods*, one seventh to one tenth.

WOODS.

To Compute Destructive Weight of Column.

Cylinder. $\frac{d^4 \cdot 6}{l^2} C = W.$ *Rectangle.* $\frac{s^4}{l^2} C = W.$ *Short Columns*, or less than 30 diameters in length. $\frac{W' \, a \, S}{W' + .75 \, a \, S} = W.$ *d representing diameter and s side in ins., a area of section in sq. ins., l length in feet,* S *crushing strength of material,* C *coefficient of material, and* W′ *destructive weight, as ascertained by computation for a long column of like dimensions in lbs.*

Coefficients.

Ash	22 000	Elm, rock	26 000	Red Pine	17 500
" Canadian	17 000	Fir, Dantzic	22 000	Yellow pine	12 000
Beech	17 500	Oak, white	20 000	White "	9 000
Cedar	14 000	" Eng.	23 000	Spruce	14 000
Elm	17 500	Pitch Pine	2 000	Walnut	12 500

Illustration.—What is destructive weight of a column of yellow pine 10 ins. square and 12 feet in length or height?

$$\frac{10^4}{12^2} \times 12\,000 = \frac{10\,000}{144} \times 12\,000 = 833\,333 \text{ lbs.}$$

For long square columns of the following: Hodgkinson put C = Dantzic oak, dry, 24 528; red deal, dry, 17 472; and French oak, dry, 15 456.

To Compute Safe Weight in Tons.*

Rectangular Oak Columns. Secured at Both Ends. Rule.—Divide length of column by thickness or least dimension, multiply unit in column C, corresponding to quotient of length of column, divided by this least dimension and by width of column, all dimensions in ins.

L/T	C	L/T	C	L/T	C	L/T	C	L/T	C	L/T	C
1	.43	7	.36	13	.26	19	.18	25	.12	31	.093
2	.43	8	.35	14	.24	20	.17	26	.12	32	.089
3	.42	9	.33	15	.23	21	.17	27	.11	33	.084
4	.4	10	.31	16	.21	22	.15	28	.1	34	.08
5	.39	11	.29	17	.2	23	.14	29	.098	35	.077
6	.38	12	.27	18	.19	24	.13	30	.097	36	.073

Illustration.—Assume a white-oak column, secured at both ends, 12 by 8 ins., and 20 feet in length.

$20 \times 12 \div 8 = 30.$ C for which $= .097$. Hence, $12 \times 8 \times .097 = 9.312$ *tons.*

For other woods take the values in following table. Thus, if an oak column, as above, will sustain 9.312 *tons*, the strength of one of yellow pine is thus obtained: As 5.8 : 9.312 :: 3 : 4.816 *tons.*

Relative Value of various Woods, their Crushing Strength and Stiffness being Combined.

Teak	9.4	Elm	5	Mahogany	3.7	Yellow pine	3
English oak	5.8	Beech	4.4	Spruce	3.6	Sycamore	2.6
Ash	5.1	Quebec oak	4.1	Walnut	3.4	Cedar	1

Comparative Value of Long Solid Columns of various Materials. (*Hodgkinson.*)

Cast Iron ..... 1000 | Cast Steel .... 2518 | Oak .... 108.8 | Pine 78.5

Hence, To compute destructive weight of an Oak or Pine column, take weight for one of Cast iron of like dimensions, and if for Oak divide by 9, and for Pine by 12.7.

* All tons, except when otherwise designated, are 2240 lbs.

DEFLECTION.

Deflection of Bars, Beams, Girders, etc.

Experiments of Barlow upon deflection of wood battens determined, that deflection of a beam from a transverse strain varied directly as breadth, and as cubes of both depth and length, and that with like beams and within limits of elasticity it was directly as the weight.

In bars, beams, etc., of an elastic material, and having great length compared to their depth, deductions of Barlow will apply with sufficient accuracy for all practical purposes; but in consequence of varied proportions of depth to length, of varied character of materials, of irregular resistance of beams constructed with scarphs, trusses, or riveted plates, and of unequal deflection at initial and ultimate strains, it is impracticable to deduce any exact laws regarding degrees of deflection of different and dissimilar figures and proportions.

From an experiment of Mr. Tredgeld it was shown that deflection of cast iron is exactly proportionate to load until stress reaches a certain magnitude, when it becomes irregular.

In experiments of Hodgkinson, it was further shown that sets from deflections were very nearly as squares of deflections.

In a rectangular bar, beam, etc., position of neutral axis is in its centre, and it is not sensibly altered by variations in amount of strain applied. In bars, beams, etc., of cast and wrought iron, position of neutral axis varies in same beam, and is only fixed while elasticity of beam is perfect. When a bar, beam, etc., is bent so as to injure its elasticity, neutral line changes, and continues to change during loading of beam, until its elasticity is destroyed.

When bars, beams, etc., are of same length, deflection of one, weight being suspended from one end, compared with that of a beam *Uniformly Loaded*, is as 8 to 3; and when bars, etc., are supported at both ends, deflection in like case is as 5 to 8. Whence, if a bar, etc., is in first case supported in middle, and ends permitted to deflect, and in second, ends supported, and middle permitted to descend, deflection in the two cases is as 3 to 5.

Of three equal and similar bars or beams, one inclined upward, one downward, at same angle, and the other horizontal, that which has its angle upward is weakest, the one which declines is strongest, and the one horizontal is a mean between the two.

When a bar, beam, etc., is *Uniformly Loaded*, deflection is as weight, and approximately as cube of length or as square of length; and element of deflection and strain upon beam, weight being the same, will be but half of that when weight is suspended from one end.

Deflection of a bar, beam, etc., *Fixed at one End, and Loaded at other*, compared to that of a beam of twice length, *Supported at both Ends, and Loaded in Middle*, strain being same, is as 2 to 1; and when length and loads are same, deflection will be as 16 to 1, for strain will be four times greater on beam fixed at one end than on one supported at both ends; therefore, all other things being same, element of deflection will be four times greater; also, as deflection is as element of deflection into square of length, then, as lengths at which weights are borne in their cases are as 1 to 2, deflection is as $1 : \overline{2^2 \times 4} = 1$ to 16.

Deflection of a bar, beam, etc., having section of a triangle, and supported at its ends, is .33 greater when edge of angle is up than when it is down.

In order to counteract deflection of a beam, etc., under stress of its load, where a horizontal surface is required, it should be *cambered* on its upper surface, equal to computed deflection.

Safe Deflection.—One fortieth of an inch for each foot of span, with a factor of safety for load of .33 of destructive weight $= \frac{1}{1440}$, but for ordinary loads and purposes,

Cast Iron, $\frac{1}{1200}$ to $\frac{1}{2000}$; and *Wrought Iron*, $\frac{1}{1600}$ to $\frac{1}{2400}$ or $\frac{1}{1200}$, after beam, etc., has become set.

When Length is uniform, with same weight, deflection is inversely as breadth and square of depth into element of deflection, which is inversely as depth. Hence, other things being equal, deflection will vary inversely as breadth and cube of depth.

ILLUSTRATION.—Deflections of two pine battens, of uniform breadth and depth, and equally loaded, but of lengths of 3 and 6 feet, were as 1 to 7.8.

Deflection of different bars, beams, etc., arising from their own weight, having their several dimensions proportional, will be as square of either of their like dimensions.

NOTE.—In construction of models on a scale intended to be executed in full dimensions, this result should be kept in view.

When a continuous girder, uniformly loaded, is supported at three points by two equal spans, middle portion is deflected downwards over middle bearing, and it sustains by suspension the extreme portions, which also have a bearing on outer bearings. Middle portion is, by deflection, convex upwards, and outer portions are concave upwards; and there is a point of "contrary flexure," where curvature is reversed, being at junction of convex and concave curves, at each side of middle bearing. This point is distant from middle bearing, on each side, one fourth of span. Of remaining three fourths of each span, a half is borne by suspension by middle portion, and a half is supported by abutment. Hence, distribution of load on bearings is easily computed, as given above. Deflection of each span is to that of an independent beam of same length of span as 2 to 5.

In a beam of three equal spans, deflection at middle of either of side spans is to that of an independent beam as 13 to 25.

In a long continuous beam, supported at regular intervals, deflection of each span is to that of an independent beam of one span as 1 to 5.

Cylinder.—If a bar or beam is cylindrical, deflection is 1.7 times that of a square beam, other things being equal.

Formulas for Deflection of Beams of Rectangular Section, etc.

Fixed at One End.

Loaded at One End. $\frac{l^3 W}{b d^3 C} = D$; and $\frac{b d^3 C D}{l^3} = W$.

" *Uniformly.* $\frac{3 l^3 W}{8 b d^3 C} = D$; and $\frac{8 b d^3 C D}{3 l^3} = W$.

Fixed at Both Ends.

Loaded in Middle. $\frac{l^3 W}{24 b d^3 C} = D$; and $\frac{24 b d^3 C D}{l^3} = W$.

" *Uniformly.* $\frac{5 l^3 W}{8 \times 24 b d^3 C} = D$; and $\frac{8 \times 24 b d^3 C D}{5 l^3} = W$.

Supported at Both Ends.

Loaded in Middle. $\frac{l^3 W}{16 b d^3 C} = D$; and $\frac{16 b d^3 C D}{l^3} = W$.

" *Uniformly.* $\frac{5 l^3 W}{8 \times 16 b d^3 C} = D$; and $\frac{8 \times 16 b d^3 C D}{5 l^3} = W$.

" *at any one Point.* $\frac{m^2 n^2 W}{l b d^3 C} = D$; and $\frac{l b d^3 C D}{m^2 n^2} = W$.

Supported in Middle.

Ends Uniformly loaded. $\frac{3 l^3 W}{5 \times 16 b d^3 C} = D$; and $\frac{5 \times 16 b d^3 C}{3 l^3} = W$.

l representing length, b breadth, and d depth, all in ins., W *weight or stress in lbs. or tons, m n distances of weight from supports*, C *a constant, and* D *deflection, in ins.*

Deflection of Beams of Rectangular Section.

Fixed at One End. { *Loaded at One End.* $\frac{l^3 W}{b d^3 C} = D$; *Uniformly.* $\frac{3 l^3 W}{8 b d^3 C} = D$.

Fixed at Both Ends. { *Loaded in Middle.* $\frac{l^3 W}{24 b d^3 C} = D$; *Uniformly,* $\frac{5 l^3 W}{6 \times 24 b d^3 C} = D$.

Supported at Both Ends. { *Loaded in Middle.* $\frac{l^3 W}{16 b d^3 C} = D$; *Uniformly,* $\frac{5 l^3 W}{8 \times 16 b d^3 C} = D$.
" *at any one Point.* $\frac{m^2 n^2 W}{l b d^3 C} = D$. *W weight in tons.*

C *a Constant as follows.*

Cast Iron. *Rectangular Bars.*—Loaded at One End...... 875.
" at the Middle.... 28 000.
Round Bars.—Loaded at One End........... 594.
" at the Middle......... 19 000.

		Wrought Iron.	Cast Iron.
Rectangular.—For tons and l in ins. put	C =	47 000.	28 000.
" " l in feet "	C =	27.	16.
Round. " " l in ins. "	C =	32 000.	19 000.
" " l in feet "	C =	18.	10.7.

Hence, in order to preserve same stiffness in bars, beams, etc., depth must be increased in same proportion as length, breadth remaining constant.

Woods.

Mean of Laslett's, Barlow, etc. (*D. K. Clark.*)

Supported at Both Ends. Loaded in Middle. $\frac{l^3 W}{b d^3 C} = D$.

	C		C		C
Ash, Canadian	1476	Iron-wood	4228	Oak, French	2656
" Eng.	2722	Larch	2100	" white	2114
Beech	2418	Mahogany, Honduras	2118	Pitch pine	2968
Blue Gum	2559	" Mexican	3608	Red "	2434
Elm	1227	" Spanish	3360	Rock-elm	2319
Fir, Dantzic	2490	Norway spar	2465	Spruce	3300
" Memel	3630	Oak, Baltimore	2761	" Amer.	2669
" Riga	2920	" Canadian	3445	" Scotch	1583
Greenheart	1888	" Dantzic	2080	Teak	1804
Iron Bark	4378	" Eng.	1848	Yellow pine	2084

Application of Table: To Compute Deflection of a Rectangular Beam of Wood.

ILLUSTRATION.—What is the deflection of a floor beam of yellow pine, 3 by 12 ins., 12 feet between its supports, under a uniformly distributed load of 3000 lbs.?

$$C = 2084. \qquad \frac{5 \times 12^3 \times 3000}{8 \times 3 \times 12^3 \times 2084} = \frac{15\,000}{50\,016} = .299 \text{ inch.}$$

Hence, To compute weight that may be borne by a given deflection of such a beam,

$$\frac{8 \times 3 \times 12^3 \times 2084 \times .299}{5 \times 12^3} = \frac{14\,955}{5} = 2991 \text{ lbs.}$$

Deflection of Continuous Girders or Beams.

Beams of Uniform Dimensions, Supported at Three or More Bearings. (*D. K. Clark.*)

1. *Two Equal Spans or 3 Bearings.*
Weight on 1st and 3d bearing = .375 W l
" " 2d bearing....... = 1.25 W l

2. *Three Equal Spans or 4 Bearings.*
Weight on 1st and 4th bearing = .4 W l
" " 2d " 3d " = 1.1 W l

3. *Four Equal Spans or 5 Bearings.*

Weight on 1st and 5th bearing = .39 W l | Weight on 2d and 4th bearing = 1.14 W l
Weight on 3d bearing = .93 W l.

Cylindrical Beam. $\frac{l^3 W}{d^4 C} = D$; and $\frac{d^4 C D}{l^3} = W$.

To Compute Maximum Load that may be borne by a Rectangular Beam.

Deflection not to exceed Assigned Limit of one hundred and twentieth of an Inch for each Foot of Span.

Supported at Both Ends. Loaded in Middle.

$\frac{b\,d^3}{l^2\,C} = W$. *b and d representing breadth and depth in ins., l length in feet,* C *constant, and* W *weight or load in lbs.*

Constants.

Cast Iron	.0003	Oak, white	.027	Oak, red	.039
Wrought Iron	.0021	Ash, white	.03	Hemlock	.039
Hickory	.018	Pine, pitch	.033	Pine, white	.039
Teak	.024	" yellow	.036	Chestnut, horse	.051

ILLUSTRATION.—What is maximum load that may be borne by a beam of white pine, 3 by 12 ins., 20 feet between its supports, and loaded in its middle?

$$C = .039. \quad \text{Then } \frac{3 \times 12^3}{20^2 \times .039} = \frac{5184}{15.6} = 332.3 \text{ lbs.}$$

WROUGHT IRON.

Deflection of Wrought-iron Bars.

Supported at Both Ends. Weight applied in Middle.

No.	Form.	Length of Bearing.	Breadth.	Depth.	Weight and Deflection by Actual Observation.		at one sixth of Destructive Weight.		at $\frac{1}{125}$th of an Inch for each Foot of Span.		Constant at Reduced Weight and Deflection. $\frac{W\,l^3}{60\,000\,b\,d^3\,D} = C$.
		Feet.	Ins.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	C
1. American.		1.83	1	1	600	.06	266	.027	148	.015	1
2. English...	"	2.75	2	2	4480	.08	1310	.022	1310	.022	1.29
3. "		2.75	1.5	2.5	8960	.104	2128	.025	1873	.022	1.25
4. "	"	2.75	1.5	3	8960	.088	3800	.037	2259	.022	.88

To Compute Deflection of, and Weight that may be borne by, a Rectangular Bar or Beam of Wrought Iron.

$$\frac{W\,l^3}{60\,000\,b\,d^3\,D} = C. \qquad \frac{W\,l^3}{60\,000\,b\,d^3\,C} = D. \qquad \frac{60\,000\,b\,d^3\,C\,D}{l^3} = W.$$

ILLUSTRATION.—What weight will a beam 2 ins. in breadth, 5 ins. in depth, and 15 feet between its supports, bear with safe deflection of $\frac{1}{125}$ of an inch for each foot of space, or $\frac{1}{1500}$ of its length?

C from table $= .88$. $D = \frac{1}{125}$ of $15 = .12$ *inch.*

$$\frac{60\,000 \times 2 \times 5^3 \times .88 \times .12}{15^3} = \frac{1\,800\,000}{3375} = 533.33 \text{ lbs.}$$

D. K. Clark gives for Elastic deflection, 47 000 for Rectangular bars, and 32 000 for Cylindrical.

NOTE.—Deflection of $\frac{1}{400}$ to $\frac{1}{600}$ of the length may be allowed under special circumstances; but under ordinary loads the deflection should not exceed one fourth of these, as $\frac{1}{1600}$ to $\frac{1}{2400}$.

Practice in U. S. is to allow $\frac{1}{1200}$ after girder has taken its permanent set.

In small bridges there is a slight increase in deflection from high speeds, about .166 or .144 of the normal deflection, with the same load moving at slow speed.

In large girders there is no perceptible difference between the deflection at high and low speeds.

Deflection of Wrought-iron Rolled Beams.

Supported at Both Ends. Weight applied in Middle.

$$\frac{W\, l^3}{70\,000\, d^2\, (4\, a + 1.155\, a')\, D} = C \text{ at Reduced Weight and Deflection.}$$

No.	Form.	Length.	Flanges. Width.	Flanges. Mean Thickness.	Web.	Depth.	Weight and Deflection by Actual Observation.		Weight and Deflection at one sixth of Destructive Weight.		C
		Feet.	Ins.	Inch.	Inch.	Ins.	Lbs.	Ins.	Lbs.	Inch.	
1.		10	3	.485	.5	7	12 000	.4	3800	.127	1.05
2.	"	20	4.6	.8	.5	9.85	16 000	1.15	6300	.453	.92
3.	"	20	5.7	.643	.6	11.75	20 000	.85	8000	.34	.98

To Compute Deflection of, and Weight that may be borne by, a Wrought-iron Rolled Beam of Uniform and Symmetrical Section.

Supported at Both Ends. Weight applied in Middle. (*D. K. Clark.*)

$$\frac{W\, l^3}{70\,000\, d^2\, (4\, a + 1.155\, a')} = D. \qquad \frac{70\,000\, d^2\, (4\, a + 1.155\, a')\, D}{l^3} = W.$$

l representing span in feet, d reputed depth, or depth less thickness of lower flange in ins., a area of section of lower flange, a′ area of section of web for reputed depth of beam, both in sq. ins., and W weight or stress in lbs.

Illustration.—What is deflection of a wrought iron rolled beam of New Jersey Steel and Iron Co., 10.5 ins. in depth, flanges 5 by .5 ins., and width of web .47 inch, when loaded in its middle with 8000 lbs., and supported over a span of 20 feet?

$d = 10.5 - .5 = 10$ *ins.*, $a = 5 \times .5 = 2.5$ *sq. ins.*, and $a' = 10 \times .47 = 4.7$ *sq. ins.*

Then $\dfrac{8000 \times 20^3}{70\,000 \times 10^2 \times (4 \times 2.5 + 1.155 \times 4.7)} = \dfrac{64\,000\,000}{107\,899\,500} = .59$ *inch.*

If weight is uniformly distributed, divide by 112 500 instead of 70 000.

A like beam 6 ins. in depth, loaded with 2608 lbs., and supported over a span of 12 feet, gave by actual test a deflection of .3 inch, and by above formula it is also .3 inch.

Note.—Deflection for such a beam, for a statical weight or stress of 17 100 lbs., *uniformly distributed*, by rules of N. J. Steel and Iron Co., would be .54 inch, which, with difference in weights, will make deflections alike.

Deflection of Wrought-iron Riveted Beams.

Supported at Both Ends. Weight applied in Middle.

$$\frac{W\, l^3}{168\,000 \left(\frac{a + a'}{2} + \frac{a''}{4}\right) d^2\, D} = C \text{ at Reduced Weight and Deflection.}$$

No.	Form.	Length.	Flanges.	Angles.	Web.	Depth.	Weight and Deflection by Actual Observation.		Weight and Deflection at one sixth of Destructive Weight.		C
		Feet.	Ins.	Ins.	Inch.	Ins.	Lbs.	Inch.	Lbs.	Inch.	
1.		7	—	2.125 × 2 × .28 2.125 × 2 × .29	.25	7	4 216	.1	4 062	.096	.63
2.		11.66	4.5 × .5 4.5 × .375	2 × 2 × .3125 2 × 2 × .3125	.25	12.5	77 280	.46	12 880	.075	1.96
3.	"	22.5	4.5 × .5 7 × .5	2 × 2 × .375 3 × 3 × .4375	.375	16.5	115 584	.875	19 265	.148	3.86

To Compute Deflection of, and Weight that may be borne by, a Riveted Beam of Wrought Iron.

$$\frac{W\,l^3}{168\,000\left(\frac{a+a'}{2}+\frac{a''}{4}\right)d^2\,C}=D. \qquad \frac{168\,000\left(\frac{a+a'}{2}+\frac{a''}{4}\right)d^2\,C\,D}{l^3}=W.$$

a, a', and a'' representing areas of upper and lower flanges with their angle pieces, and of web for its entire depth, all in sq. ins.

NOTE.—If there are not any flanges, as in No. 1, angle pieces alone are to be computed for flange area.

ILLUSTRATION.—What weight will a riveted and flanged beam of following dimensions sustain, at a distance between its supports of 25 feet, and at a safe deflection of .2 inch or $\frac{1}{1500}$ of its length?

Top flange	6 × .5 *ins.*	Web	.5 *ins.*
Bottom flange	6 × .5 "	Depth	17 "

Angles . . . . . 2.25 × 2.25 × .5 *ins.*

a and a' each $= 6 \times .5 = 3 + \overline{2.25 + 2.25 - .5} \times .5 \times 2 = 7$ *sq. ins.*
$a'' = .5 \times 17 = 8.5$ *sq. ins.* C, as per No. 2, = .43, but inasmuch as flanges in this case are much heavier, assume .5.

$$\text{Then } \frac{168\,000\left(\frac{7+7}{2}+\frac{8.5}{4}\right)17^2\times .2\times .5}{25^3}=\frac{44\,303\,720}{15\,625}=2835.4 \text{ lbs.}$$

Strength of a Riveted beam compared to a Solid beam is as 1 to 1.5, while for equal weights its deflection is 1.5 to 1.

Tubular Girders. Wrought Iron.

Supported at Both Ends. Weight applied in Middle.

No.	Section.	Length of Bearing.	Breadth.	Depth. Internal.	Depth. External.	Weight.	Deflection.	Deflection at .008 inch for each Foot of Span.	$\frac{l^3\,W}{16\,b\,d^3\,D}=C.$
		Feet.	Ins.	Ins.	Ins.	Lbs.	Ins.	Inch.	C
1.	Thickness .03 inch	3.75	1.9	2.94	3	448	.1	.03	288
2.	" " .525 "	30	15.5	22.95	24	33 685	.56	.24	473
3.	" top .372 ", bottom .244 ", sides .125 "	30	16	23.28	24	32 538	1.11	.24	224
4.	" Thickness .75 "	45	24	34.25	35.75	128 850	1.85	.36	362
5.	Thickness .0375 "	17	12	11.925	12	2 755	.65	.136	62.8
6.	" .0416 "	17	9.25	13.535	13.62	2 262	.62*	.136	47.9
7.	" .143 "	17	9.25	14.714	15	16 800	1.39*	.136	119

* Destructive weight.

To Compute Deflection of, and Weight that may be borne by, a Wrought-iron Tubular Girder.

$$\frac{16\,b\,d^3\,C\,D}{l^3}=W. \qquad \frac{W\,l^3}{16\,b\,d^3\,C}=D.$$

ILLUSTRATION.—What weight may be safely borne by a wrought-iron tube, alike to No. 3 in preceding table, for a length of 40 feet, and a deflection of .32 inch?

$$\frac{16\times 16\times 24^3\times 224\times .24}{30^3}=\frac{190\,253\,629}{27\,000}=7046 \text{ lbs.}$$

Flanged Rivets.

Deflection of Iron and Steel Flanged Rails within their elastic limit, compared with their transverse strength, is as 17 to 20, and with double-headed it is as 11 to 23.

RAILS.

Supported at Both Ends. Weight applied in Middle.

Iron.

No.	Form.	Length of Bearing.	Head.	Bottom.	Weight per Yard.	Depth.	Area.	Observed Weight and Deflection.		Destructive Weight and Deflection.	
		Feet.	Ins.	Ins.	Lbs.	Ins.	Sq. Ins.	Lbs.	Ins.	Lbs.	Ins.
1.		2.75	2.25 × 1	2.25 × 1	60	4.5	6.166	13 440	.034	26 680	.065
2.	"	4.5	2.3 × 1	2.3 × 1	65	4.5	6.68	11 200	.11	24 640	.204
3.	"	5	2.3 × 1	2.3 × 1	82	5.4	8.25	25 760	.2	51 520	.378
4.		2.75	3.5 × .8	2.25 × 1	60	4	6.7	11 200	.035	26 680	.065
5.		2.58	2.23 × 1	3.5 × .6	57	3.5	5.85	11 200	.097	20 160	.128

Steel.

No.	Form.	Length of Bearing.	Head.	Bottom.	Weight per Yard.	Web.	Depth. Centres of Heads.	Depth. Total.	Area.	Observed Weight and Deflection.		Destructive Weight and Deflection.	
		Feet.	Ins.	Ins.	Lbs.	Inch.	Ins.	Ins.	S. Ins.	Lbs.	In.	Lbs.	Inch.
6.		5	—	—	78	.75	4.2	5.4	7.67	36 086	.25	80 192	.55
7. Bessemer.	"	3.62	—	—	86	—	—	5.5	8.43	22 400	.14	26 680	.165
8.		5	2.5	6.375 × $\frac{.37}{.65}$	84	.65	3.37	4.5	8.24	27 290	.24	27 290	.24

To Compute Deflection of Double-headed Rails within Elastic Limit. (*D. K. Clark.*)

Supported at Both Ends. Weight applied at Middle.

IRON.

$$\frac{W\,l^3}{57\,000\,(4\,a\,d'^2 + 1.155\,t\,d^3)} = D.$$

a representing area of one head, less portion pertaining to web, d whole depth of rail, d′ vertical distance between centres of heads, t thickness of web, all in ins., l length in feet, and W *weight in lbs.*

STEEL.

For 57 000 *put* 67 400.

Illustration.—Take case No. 3 (Iron), in preceding table, with a weight of 26 000 lbs.; what will be its deflection between bearings 5 feet apart?

$$a = 1.911. \quad d' = 4.2. \quad d = 5.4. \quad t = .82.$$

$$\text{Then } \frac{26\,000 \times 5^3}{57\,000\,(4 \times 1.911 \times 4.2^2 + 1.155 \times .82 \times 5.4^3)} = \frac{3\,250\,000}{57\,000 \times 284} = .2 \textit{ inch.}$$

To Compute Deflection of Iron and Steel Rails of Unsymmetrical Section within Elastic Limits.

Elastic Deflection of Steel Flanged Rails of Metropolitan Railway of London, as determined by Mr. Kirkaldy, at a span of 5 feet, and loaded in middle, was .02 inch per ton. (*See Manual of D. K. Clark, pp.* 667–670.)

CAST IRON.

Deflection of Rectangular Bars and Beams of various Sections, etc., by U. S. Ordnance Corps, Barlow, Hodgkinson, and Cubitt.

Supported at Both Ends. Weight applied in Middle.

No.	Form.	Length of Bearing.	Breadth.	Depth.	Weight and Deflection. By Actual Observation.		At one sixth of Breaking Weight.		At $\frac{1}{120}$th of an inch for each foot of span.		Constant at Reduced Weight and Deflection. $\frac{W\,l^3}{10\,400\,b\,d^3\,D} = C.$
		Feet.	Ins.	Ins.	Lbs.	Ins.	Lbs.	Ins.	Lbs.	Ins.	C
1. American....		1.66	2	2	5000	.036	1666	.012	1805	.013	3.81
2. English......	"	4	1	1	212	.32	80	.12	22	.033	4.11
3. "	"	16	4	4	1008	.4	5333	2.11	3370	1.33	3.89
4. "		4.5	3	1	1120	1.42	215	.27	30	.037	2.37
5. "		4.5	1	{ 2.5 / .5	2231	.51	422	.1	156	.037	2.33

To Compute Deflection of, and Weight that may be borne by, a Rectangular Bar or Beam of Cast Iron.

$$\frac{W\,l^3}{10\,400\,b\,d^3\,D} = C. \qquad \frac{W\,l^3}{10\,400\,b\,d^3\,C} = D. \qquad \frac{10\,400\,b\,d^3\,C\,D}{l^3} = W.$$

ILLUSTRATION.—What weight will a beam 2 ins. in breadth, 5 ins. in depth, and 16 feet between its supports, bear with safe deflection of $\frac{1}{120}$ of an inch for each foot of span, or $\frac{1}{1440}$ of its length?

C from table $= 3.89$. $D = \frac{1}{120}$ of $16 = 1.33$ *ins.*

$$\frac{10\,400 \times 2 \times 5^3 \times 3.89 \times 1.33}{16^3} = \frac{13\,451\,620}{4096} = 3284 \textit{ lbs.}$$

Clark gives C uniform for Rectangular bars of 2.69, and 1.85 for Cylindrical.

FLANGED BEAMS. Cast Iron.

Supported at Both Ends. Weight applied in Middle.

To Compute Deflection of, and Weight that may be borne by, a Flanged Beam of Cast Iron of Uniform and Symmetrical Section.

$$\frac{W\,l^3}{27\,000\,d^2\,(4\,a + 1.155\,a'^2)} = D. \qquad \frac{27\,000\,d^2\,(4\,a + 1.155\,a'^2)\,D}{l^3} = W.$$

ILLUSTRATION.—What is deflection of a cast-iron beam (Hodgkinson's) 7.15 ins., flanges $2.6 \times .86$ ins. and 5×1.6 ins., and width of web 1 inch, when loaded in its middle with 11 200 lbs., over a span of 15 feet?

$d = 7.15 - 1.6 = 5.55$ *ins.*, $a = 5 \times 1.6 = 8$ *ins.*, and $a' = 7.15 - 1.6 = 5.55$ *ins.*

$$\text{Then } \frac{11\,200 \times 15^3}{27\,000\; 5.55^2\,(4 \times 8 + 1.155 \times 5.55^2)} = \frac{37\,800\,000}{27\,000\; 30.8\,(32 + 35.57)} = 1.48 \textit{ ins.}$$

NOTE 1.—The observed deflection of this beam was 1.28 ins., at one sixth of its destructive weight it was .3, and at $\frac{1}{120}$ of an inch for each foot of span it was .125 inch.

2.—The mean ratio of elastic to destructive stress is 73 per cent.

Formulas for value of deflection signify that deflection varies directly as weight, and as cube of length; and inversely as breadth, cube of depth, and coefficient of elasticity.

Elastic Strength of Beams of Unsymmetrical Section.—Elastic strength is approximately deducible from ultimate strength, according to ordinary ratio of one to the other, ascertained experimentally. Elastic strength and deflection of a homogeneous beam of any section is same, whether in its normal position or turned upside down.

Comparative Strength and Deflection of Cast-iron Flanged Beams.

Description of Beam.	Comp. Strength.	Description of Beam.	Comp. Strength.
Beam of equal flanges.........	.58	Beam with flanges as 1 to 4.5..	.78
" with only bottom flange.	.72	" " " 1 to 5.5..	.82
" " flanges as 1 to 2....	.63	" " " 1 to 6 ..	1
" " " 1 to 4....	.73	" " " 1 to 6.73.	.92

SHAFTS.

To Compute Deflection and Distributed Weight for Limit of Deflection.

Wrought Iron.

	Deflection. Supported at Ends.		Fixed at Ends.	*Weight.* Supported at Ends.		Fixed at Ends.
Round.	$\frac{W l^3}{66\,400\, d^4}$	and	$\frac{W l^3}{133\,000\, d^4} = D.$	$\frac{664\, d^4}{l^2}$	and	$\frac{1330\, d^4}{l^2} = W.$
Square.	$\frac{W l^3}{97\,500\, s^4}$	and	$\frac{W l^3}{195\,000\, s^4} = D.$	$\frac{975\, s^4}{l^2}$	and	$\frac{1950\, s^4}{l^2} = W.$

Cast Iron.

Round.	$\frac{W l^3}{39\,400\, d^4}$	and	$\frac{W l^3}{79\,000\, d^4} = D.$	$\frac{394\, d^4}{l^2}$	and	$\frac{790\, d^4}{l^2} = W.$
Square.	$\frac{W l^3}{58\,000\, s^4}$	and	$\frac{W l^3}{116\,000\, s^4} = D.$	$\frac{580\, s^4}{l^2}$	and	$\frac{1160\, s^4}{l^2} = W.$

Steel.

Round.	$\frac{W l^3}{78\,800\, d^4}$	and	$\frac{W l^3}{158\,000\, d^4} = D.$	$\frac{788\, d^4}{l^2}$	and	$\frac{1576\, d^4}{l^2} = W.$
Square.	$\frac{W l^3}{116\,000\, s^4}$	and	$\frac{W l^3}{232\,000\, s^4} = D.$	$\frac{1160\, b^4}{l^2}$	and	$\frac{2320\, b^4}{l^2} = W.$

d representing diameter and s side of shaft, in ins., l length between centres of bearings, in feet, and W weight in lbs.

Deflection of a Cylindrical Shaft from its Weight alone, when Supported at Both Ends.

$.007\,318\, \frac{l^4}{d^2\, C} = D.$ *l representing length in feet, d diameter in ins., and C constant, ranging from 475 to 550.*

The greatest admissible deflection for any diameter is $.001\,67\, \frac{l}{d} = D.$

Admissible Distances between Bearings. $\sqrt[3]{.9128\, d\, C} = l.$

Diam. of Shaft.	Distance. Wrought Iron.	Steel.	Diam. of Shaft.	Distance. Wrought Iron.	Steel.	Diam. of Shaft.	Distance. Wrought Iron.	Steel.
Ins.	Feet.	Feet.	Ins.	Feet.	Feet.	Ins.	Feet.	Feet.
1	12.27	12.61	5	20.99	21.57	9	25.53	26.24
2	15.46	15.84	6	22.3	22.92	10	26.44	27.18
3	17.7	18.19	7	23.48	24.13	11	27.3	28.05
4	19.48	20.02	8	24.55	25.23	12	28.1	28.88

When Ends of Shaft are rigidly connected at Ends.

Barlow gives D = .66 of results obtained by above formula; but when deflection of attached length is considerable, Navier gives D = .25 of above.

Deflection of Mill and Factory Shafts.

$\frac{l^3 W}{6 \pi d^4 C} = D$. *l representing length between supports in ins.*, W *weight at middle in lbs.*, *d diameter of shaft in ins., and* C *as follows:*

Bessemer steel	3 800 000	Wrought iron	3 500 000

To Compute Deflection of a Cylindrical Shaft.

Rule.—Divide square of three times length in feet by product of following *Constants* and square of diameter in ins., and quotient will give deflection.

Cast iron, cylindrical	1500	Wrought iron, cylindrical	1980
" " square	2560	" " square	3360

Example.—Length of a cast-iron cylindrical shaft is 30 feet, and its diameter in centre 15 ins.; what is its deflection?

$$\frac{\overline{30 \times 3}^2}{1500 \times 15^2} = \frac{8100}{337\,500} = .024 \textit{ ins.}$$

SPRINGS.

Flexure of a spring is proportional to its load and to cube of its length.

Deflection of a Carriage Spring.

A railway-carriage spring, consisting of 10 plates .3125 inch thick, and 2 of .375 inch, length 2 feet 8 ins., width 3 ins., and *camber* or spring 6 ins., deflected as follows, without any permanent set:

.5 ton	.5 inch.	1.5 tons	1.5 ins.	3 tons	3 ins.
1 "	1 "	2 "	2 "	4 "	4 "

Compression of an India-rubber Buffer of 3 Ins. Stroke.

1 ton	1.3 ins.	2 tons	2 ins.	5 tons	2.75 ins.
1.5 tons	1.75 "	3 "	2.375 "	10 "	3 "

General Deductions.

Deflection depends essentially upon form of Girder, Beam, etc.

A continuous weight, equal to that a beam, etc., is suited to sustain, will not cause deflection of it to increase unless it is subjected to considerable changes of temperature.

Heaviest load on a railway girder should not exceed .16 of that of destructive weight of girder when laid on at rest.

Semi-girders or Beams.—Deflection of a beam, etc., fixed at one end and loaded at other, is 32 times that of same beam supported at both ends and loaded in middle.

Deflection consequent upon Velocity of Load.—Deflection is very much increased by instantaneous loading; by some authorities it is estimated to be doubled.

Momentum of a railway train in deflecting girders, etc., is greater than effect from dead weight of it, and deflection increases with velocity.

When motion is given to load on a beam, etc., point of greatest deflection does not remain in centre of beam, etc., as beams broken by a travelling load are always fractured at points beyond their centres, and often into several pieces.

Heaviest running weight that a bridge is subjected to is that of a locomotive and tender, which is equal to 2 tons per lineal foot.

Girders should not, under any circumstances, be deflected to exceed .025 inch to a foot in length.

A carriage was moved at a velocity of 10 miles per hour; deflection was .8 inch, and when at a velocity of 30 miles deflection was 1.5 ins.

In this case, 4150 lbs. would have been destructive weight of bars if applied in their middle, but 1778 lbs. would have broken them if passed over them with a velocity of 30 miles per hour.

Relative Elasticity of various Materials. (*Trumbull.*)

Ash	2.9	Cast Iron	1	Pine, white	2.4	Pine, pitch	2.9
Beech	2.1	Elm and Oak	2.9	" yellow	2.6	Wrought Iron	.86

Cast Iron.—Permanent deflection is from .33 to .5 of its breaking weight, and deflection should never exceed .125 of ultimate deflection, and it is not permanently affected but by a stress approaching its destructive weight.

By experiments of U. S. Ordnance Corps (Report, 1852), set or permanent deflection was .38 of its breaking weight, ultimate deflection .133 ins. Deflection for $\frac{1}{120}$ of span = .013, or .1 of ultimate deflection.

By experiments of Mr. Hodgkinson (See *Rep. of Comm's on Railway Structures*, London, 1849), set for English iron bore a much greater proportion to its breaking weight.

A beam, etc., will bend to .33 of its ultimate deflection with less than .33 of its breaking weight, if it is laid on gradually, and but .16 if laid on rapidly.

Chilled bars deflect more readily than unchilled.

Results of Experiments on the Subjection of Cast-iron Bars to continued Strains.

(*Rep. of Comm's on Railway Structures*, London, 1849.)

Cast-iron bars subjected to a regular depression, equal to deflection due to a load of .33 of their statical breaking weight, bore 10 000 successive depressions, and when broken by statical weight, gave as great a resistance as like bars subjected to a like deflection by statical weight.

Of two bars subjected to a deflection equal to that carried by half of their statical breaking weight, one broke with 28 602 depressions, and the other bore 30 000, and did not appear weakened to resist statical pressure.

Hence, Cast-iron bars will not bear continual applications of .33 of their breaking weight.

Mr. Tredgold, in his experiments upon Cast Iron, has shown that a load of 300 lbs., suspended from middle of a bar 1 inch square and 34 ins. between its supports, gave a deflection of .16 of an inch, while elasticity of metal remained unimpaired. Hence a bar 1 inch square and 1 foot in length will sustain 850 lbs., and retain its elasticity.

Wrought Iron.—All rectangular bars, having same bearing, length, and loaded in their centre to full extent of their elastic power, will be so deflected that their deflection, being multiplied by their depth, product will be a constant quantity, whatever may be their breadth or other dimensions, provided their lengths are same.

A bar of Wrought Iron, 2 ins. square and 9 feet in length between its supports, was subjected to 100 000 vibratory depressions, each equal to deflection due to a load of .55 of that which permanently injured a similar bar, and their depressions only produced a permanent set of .015 inch.

Greatest deflection which did not produce any permanent set was due to rather more than .5 statical weight, which permanently injured it.

A wrought-iron box girder, 6 × 6 ins. and 9 feet in length, was subjected to vibratory depressions, and a strain corresponding to 3762 lbs., repeated 43 370 times, did not produce any appreciable effect on the rivets.

Deflection of Solid rolled beams compared to Riveted beams is as 1 to 1.5.

Wrought-iron Girders of ordinary construction are not safe when subjected to violent impacts or disturbances, with a load equal to .33 of their destructive weight.

Wood.—In consequence of wood not being subjected to weakening by the effect of impact, a factor of safety of 5 for single pieces is held to be sufficient, but for structures, in consequence of loss of strength in its connections, a factor of from 8 to 10 becomes necessary.

Working Strength or Factors of Safety.*

Elastic strength of materials is, in general terms, half of its ultimate destructive or breaking strength. If a working load of .5 elastic strength, or .25 of ultimate strength, be accepted, equal range for fluctuation within elastic limit is provided. But, as bodies of same material are not all uniform in strength, it is necessary to observe a lower limit than .25 where material is exposed to great or to sudden variations of load or stress.

Cast Iron.—Mr. Stoney recommends .25 of ultimate tensile strength, for dead weights; .16 for bridge girders; and .125 for crane posts and machinery. In compression, free from flexure, cast iron will bear 8 tons (17 920 lbs.) per sq. inch; for arches, 3 tons (6720 lbs.) per sq. inch; for pillars, supporting dead loads, .16 of ultimate strength; for pillars subject to vibration from machinery, .125; and for pillars subject to shocks from heavy-loaded wagons and like, .1, or even less, where strength is exerted in resistance to flexure.

Wrought Iron.—For bars and plates, 5 tons (11 200 lbs.) per sq. inch of net section is taken as safe working tensile stress; for bar iron of extra quality, 6 tons (13 440 lbs.). In compression, where flexure is prevented, 4 tons (8960 lbs.) is safe limit; in small sizes, 3 tons (6720 lbs.). For columns subject to shocks, Mr. Stoney allows .16 of calculated breaking weight; with quiescent loads, .25. For machinery, .125 to .1 is usually practised; and for steam-boilers, .25 to .125.

Mr. Roebling claims that long experience has proved, beyond shadow of a doubt, that good iron, exposed to a tensile strain not above .2 of its ultimate strength, and not subject to strong vibration or torsion, may be depended upon for a thousand years.

Steel.—A committee of British Association recommended a maximum working tensile stress of 9 tons (20 160 lbs.) per sq. inch. Mr. Stoney recommends, for mild steel, .25 of ultimate strength, or 8 tons (17 920 lbs.) per sq. inch. Limit for compression must be regulated very much by nature of steel, and whether it be annealed or unannealed. Probably a limit of 9 tons (20 160 lbs.) per sq. inch, same as limit for tension, would be safe maximum for general purposes. In absence of experience, Mr. Stoney further recommends that, for steel pillars, an addition not exceeding 50 per cent. should be made to safe load for wrought-iron pillars of same dimensions.

Wood.—One tenth of ultimate stress is an accepted limit. Piles have, in some situations, borne permanently .2 of their ultimate compressive strength.

Foundations.—According to Professor Rankine, maximum pressure on foundations in firm earth is from 17 to 23 lbs. per sq. inch; and, on rock, it should not exceed .125 of its crushing load.

Masonry.—Mr. Stoney asserts that working load on rubble masonry, brick-work, or concrete rarely exceeds .16 of crushing weight of aggregate mass; and that this seems to be a safe limit. In an arch, calculated pressure should not exceed .05 of crushing pressure of stone.

---

* Essentially from Manual of D. K. Clark, London, 1877.

Ropes.—For round, working load should not exceed .14 of ultimate strength, and for flat .11.

			Dead Load.	Live Load.
	Perfect material and workmanship		2	4
Dr. Rankine gives following factors:	Good ordinary material and workmanship	Metal	3	6
		Wood	4 to 5	8 to 10
		Masonry	4	8

A *Dead Load* is one that is laid on very gradually and remains fixed.

A *Live Load* is one that is laid on suddenly, as a loaded vehicle or train passing swiftly over a bridge.

DETRUSIVE OR SHEARING STRENGTH.

Detrusive or Shearing Strength of any body is directly as its strength, or thickness, or area of shearing surface.

Results of Experiments upon Detrusive Strength of Metals with a Punch.

METALS.	Diameter of Punch.	Thickness of Metal.	Power exerted.	Power required for a Surface of Metal of One Sq. Inch.
	Ins.	Ins.	Lbs.	Lbs.
Brass	1	.045	5 448	37 000
Cast iron	—	—	—	30 000
Copper	.5	.08	3 983	30 000
Copper	1	.3	21 250	22 300
Steel	.5	.25	34 720	90 000
" Bessemer	.875	.75	103 600	51 800
			184 800	92 400
Wrought iron	.5	.17	11 950	45 000
	1	.615	82 870	43 900
	2	1.06	297 400	44 300

NOTE.—Free use of oil reduces power required very materially.

To Compute Power to Punch Iron, Brass, or Copper.

RULE.—Multiply product of diameter of punch and thickness of metal by 150 000 if for wrought iron, by 128 000 if for brass, and by 96 000 if for cast iron or copper, and product will give power required, in lbs.

EXAMPLE.—What power is required to punch a hole .5 inch in diameter in a plate of brass .25 inch thick? $.5 \times .25 \times 128\,000 = 16\,000$ *lbs.*

Comparison between Detrusive and Transverse Strengths.

Assuming compression and abrasion of metal in application of a punch of one inch in diameter to extend to .125 of an inch beyond diameter of punch, comparative resistance of wrought iron to *detrusive* and *transverse* strain, latter estimated at 600 lbs. per sq. inch, for a bar 1 foot in length, is as 3 to 1.

WOODS.

Detrusive Strength of Woods. *Per Sq. Inch.*

	Lbs.		Lbs.		Lbs.		Lbs.
Spruce	470	Pine, pitch	510	Ash	650	Oak	780
Pine, white	490	Hemlock	540	Chestnut	690	Locust	1180

To Compute Length of Surface of Resistance of Wood to Horizontal Thrust.

RULE.—Divide 4 times horizontal thrust in lbs. by product of breadth of wood in ins., and detrusive resistance per sq. inch in lbs. in direction of fibre, and quotient will give length required.

EXAMPLE.—Thrust of a rafter is 5600 lbs., breadth of tie beam, of pitch or Georgia pine, is 6 ins.; what should be length of beyond score for rafter?

Assume strength 510 as above. Then $\frac{4 \times 5600}{6 \times 510} = \frac{22\,400}{3060} = 7.32$ *ins.*

Shearing.

Wrought Iron.

Resistance to shearing of American is about 75 per cent., and of English 80 per cent., of its tensile strength.

Resistance to shearing of plates and bolts is not in a direct ratio. It approximates to that of square of depth of former, and to square of diameter of latter.

Results of Experiments upon Shearing Strength of Various Metals by Parallel Cutters.

Wrought Iron.—Thickness from .5 to 1 inch, 50 000 lbs. per sq. inch.

Made by Inclined Cutters, angle = 7°.

Plates.	Thickness.	Power.	Bolts.	Diam.	Power.
	Ins.	Lbs.		Ins.	Lbs.
Brass	.05	540	Brass	1.11	29 700
Copper	.297	11 196	Copper	.775	11 310
Steel	.24	14 930	Steel	.775	28 720
Wrought iron	.51	39 150	Wrought iron	.32	3 093
	1	44 800		1.142	35 410

Result of Experiments in Shearing, made at U. S. Navy Yard, Washington, on Wrought-iron Bolts.

Diam.	Stress. Minimum.	Maximum.	Per Sq. Inch.	Diam.	Stress. Minimum.	Maximum.	Per Sq. Inch.
Inch.	Lbs.	Lbs.	Lbs.	Inch.	Lbs.	Lbs.	Lbs.
.5	8 900	9 400	44 149	.875	25 500	27 600	41 503
.75	18 400	19 650	39 553	1	32 900	35 800	40 708

Mean 41 033 lbs.

Result of Experiments on .875 Inch Wrought-iron Bolts. (*E. Clark.*)

	Lbs.	Tons.		Lbs.	Tons.
Single shear	54 096	24.15	Double shear of two .625 inch plates riveted together (one section)	45 696	20.4
Double "	46 904	22.1			

Tensile strength........ 50 176 lbs.

Riveted Joints.

Experiments on strength of riveted joints showed that while the plates were destroyed with a stress of 43 546 lbs., the rivets were strained by a stress of 39 088 lbs.

Cast Iron.

Resistance to shearing is very nearly equal to its tensile strength. An average of English being 24 000 lbs. per sq. inch.

Steel.

Shearing strength of steel of all kinds (including Fagersta) is about 72 per cent. of its tensile strength.

Treenails.

Oak treenails, 1 to 1.75 ins. in diameter, have an average shearing strength of 1.8 tons per sq. inch, and in order to fully develop their strength, the planks into which they are driven should be 3 times their diameter.

Woods.

When a beam or any piece of wood is let in (not mortised) at an inclination to another piece, so that thrust will bear in direction of fibres of beam that is cut, depth of cut *at right angles to fibres* should not be more than .2 of length of piece, fibres of which, by their cohesion, resist thrust.

Ash	650 lbs.	Deal	625 lbs.	Pine	650 lbs.
Chestnut	600 "	Oak	2300 "	Spruce	625 "

TENSILE STRENGTH.

Tensile Strength is resistance of the fibres or particles of a body to separation. It is therefore proportional to their number, or to area of its transverse section, and in metals it varies with their temperature, generally decreasing as temperature is increased. In silver, tenacity decreases more rapidly than temperature; and in copper, gold, and platinum less rapidly.

Cast Iron.

Experiments on Cast-iron bars give a tensile strength of from 4000 to 5000 lbs. per sq. inch of its section, as just sufficient to balance elasticity of the metal; and as a bar of it is extended the 12 300th part of its length for every 1000 lbs. of direct strain, or one sixteenth of an inch in 64.06 feet per sq. inch of its section, it is deduced that its elasticity is fully excited when it is extended less than the 2400th part of its length, and extension of it at its limit of elasticity, which is about .5 of its destructive weight, is estimated at 1500th part of its length.

Average ultimate extension is 500th part of its length.

A bar will contract or expand .000 006 173 inch, or the 162 000th of its length, for each degree of heat; and assuming extreme of moderate range of temperature in this country 140° (− 20° + 120°), it will contract or expand with this change .000 864 2 inch, or the 1157th part of its length.

It follows, then, that as 1000 lbs. will extend a bar the 12 300th part of its length, contraction or extension for 1157th part will be equivalent to a force of 10 648 lbs. (4.75 tons) per sq. inch of section. It shrinks in cooling from one eighty-fifth to one ninety-eighth of its length.

Mean tensile strength of American, as determined by Maj. Wade for U. S. Ordnance Corps, is 31 829 lbs. (14.21 tons) per sq. inch of section; mean of English, as determined by Mr. E. Hodgkinson for Commission on Application of Iron to Railway Structures, 1849, is 19 484 lbs. (8.7 tons); and by Col, Wilmot, at Woolwich, in 1858, for gun-metal, is 23 257 lbs. (10.35 tons). varying from 12 320 lbs. (5.5 tons) to 25 520 lbs. (10.5 tons).

Mean ultimate extension of four descriptions of English, as determined for Commission above referred to, was, for lengths of 10 feet, .1997 inch, being 600th part of its length; and this weight would compress a bar the 775th part of its length.

Tensile strength of strongest piece ever tested—45 970 lbs. (20.52 tons). This was a mixture of grades 1, 2, and 3 from furnace of Robert P. Parrott at Greenwood, N. Y., and at 3d fusion.

At 2.5 tons per sq. inch it will extend same as wrought iron at 5.6 tons.

From experiments of Maj. Wade he deduced the following mean results:

Density.	Tensile.	Transverse.	Torsion.	Crushing.	Hardness.
7.225	31 829	8182	8614	144 916	22.34

Tensile per sq. inch of section; *Transverse* per sq. inch, one end fixed, load applied at other end at a distance of 1 foot; and *Torsion* per sq. inch, stress applied at end of a lever 1 foot in length.

Green sand castings are 6 per cent. stronger than dry, and 30 per cent. stronger than chilled; but when castings are chilled and annealed, a gain of 115 per cent. is attained over those made in green sand.

Resistance to crushing and tensile stress is for American as 4.55 to 1, and for English as 5.6 to 7 to 1. Strength increasing with density.

Remelting.—Strength, as well as density, are increased by repeated remeltings. The increase is the result of the gradual abstraction of the constituent carbon of the iron, and the consequent approximation of the metal to wrought iron.

Result of the 4th melting of pig iron, as determined by Major Wade, was to increase its strength from 12 880 lbs. (5.75 tons) to 27 888 lbs. (12.45 tons), and its specific gravity from 6.9 to 7.4.

Three successive meltings of Greenwood iron, N. Y., gave tensile strength of 21 300, 30 100, and 35 700 lbs.

Result of 5th melting by Mr. Bramwell was to increase strength of Acadian iron from 16 800 lbs. (7.5 tons) to 41 440 lbs. (18.5 tons).

Remelting increases its resistance to a crushing stress from 70 to 80 tons (14 per cent.) per sq. inch of section.

Hot and Cold Blast.

Mr. Hodgkinson deduced from experiments that relative strength of 1.2 and 3 ins. square was as 100, 80, and 77, and that hot blast had less tensile strength than cold blast, but greater resistance to a crushing stress.

Captain James ascertained that tensile strength of .75 inch bars, cut out of 2 and 3 inch bars, had only half strength of a bar cast 1 inch square.

Mr. Robert Stephenson concluded, from experiments of recent date, that average strength of hot blast was not much less than that of cold blast; but that cold blast, or mixtures of cold blast, were more regular, and that mixtures of cold blast and hot blast were better than either separate.

Stirling's Mixed or Toughened Iron.

By mixture of a portion of malleable iron with cast iron, carefully fused in a crucible, a tensile strain of 25 764 lbs. has been attained. This mixture, when judiciously managed and duly proportioned, increases resistance of cast iron about one third; greatest effect being obtained with a proportion of about 30 per cent. of malleable iron.

Malleable Cast Iron.

Tensile strength of annealed malleable is guaranteed by some Manufacturers of it at 56 000 lbs.; it is capable of sustaining 22 400 lbs. without permanent set.

Wrought Iron.

Experiments on English bars gave a tensile strength of from 22 000 lbs. to 26 400 lbs. per sq. inch of its section, as just sufficient to balance elasticity of the metal; and as a bar of it is extended the 28 000th part of its length for every 1000 lbs. of direct strain, or one sixteenth of an inch in 116.66 feet per sq. inch of its section, it is deduced that its elasticity is fully excited when it is extended the 1000th part of its length, and extension of it at its limit of elasticity, which is from .45 to .5 of its destructive weight, is estimated at 1520th part of its length.

A bar will expand or contract .000 006 614 inch, or 151 200 part of its length for each degree of heat; and assuming, as before stated for cast iron, that extreme range of temperature in air in this country is 140°, it will contract or expand with this change .000 926, or 1080th of its length, which is equivalent to a force of 20 740 lbs. (9.25 tons) per sq. inch of section.

Mean tensile strength of American bars and plates (45 000 to 76 000), 60 500 lbs. (27 tons) per sq. inch of section; as determined by Prof. Johnson in 1836, is 55 900 lbs.; and mean of English, as determined by Capt. Brown, Barlow, Brunel, and Fairbairn, is 53 900 lbs.; and by Mr. Kirkaldy, bars and plates (47 040 to 55 910) 51 475 lbs. (22.97 tons).

Greatest strength observed 73 449 lbs. (32.79 tons).

Ultimate strength, as given by Mr. D. K. Clark, 59 732 lbs. (26.66 tons).

Average ultimate extension is 600th part of its length.

Strength of plates, as determined by Sir William Fairbairn, is fully 9 per cent. greater with fibre than across it.

Resistance of wrought iron to crushing and tensile strains is, as a mean, as 1.5 to 1 for American; and for English 1.2 to 1.

Reheating.—Experiments to determine results from repeated heating and laminating, furnished following:

From 1 to 6 reheatings and rollings, tensile stress increased from 43 904 lbs. to 61 824 lbs., and from 6 to 12 it was reduced to 43 904 again.

Effect of Temperature.—Tensile strength at different temperatures is as follows: 60°, 1; 114°, 1.14; 212°, 1.2; 250°, 1.32; 270°, 1.35; 325°, 1.41; 435°, 1.4.

Experiments of Franklin Institute gave at

80°	56 000 lbs.	720°	55 000 lbs.	1240°	22 000 lbs.
570°	66 500 "	1050°	32 000 "	1317°	9 000 "

Annealing.—Tensile strength is reduced fully 1 ton per sq. inch by annealing.

Cold Rolling.—Bars are materially stronger than when hot rolled, strength being increased from one fifth to one half, and elongation reduced from 21 to 8 per cent.

Hammering increases strength in some cases to one fifth.

Welding.—Strength is reduced from a range of 3 to 44 per cent. 20 per cent., or one fifth, is held to be a fair mean.

Temperature.—From 0° to 400° strength is not essentially affected, but at high temperature it is reduced. When heated to redness its strength is reduced fully 25 per cent.

Tensile strength at 23° was found to be .024 per cent. less than at 64°.

Cutting Screw Threads reduces strength from 11 to 33 per cent.

Hardening in water, oil, etc., reduces elongation, but does not essentially increase the strength.

Case Hardening reduces strength fully 10 per cent.

Galvanizing does not affect strength of plates.

Angled Bars, etc.—Their strength is fully 10 per cent. less than for bolts and plates.

Elements connected with Tensile Resistance of various Substances.

Substances.	Stress per Sq. Inch for limit of Elasticity.	Ratio of Stress to that causing Rupture.	Substances.	Stress per Sq. Inch for limit of Elasticity.	Ratio of Stress to that causing Rupture.
	Lbs.			Lbs.	
Beech	3 355	.3	Wrought iron, ordinary	17 600	.3
Cast iron, English	4 000	.22	" " Swedish	24 400	.34
" " American	5 000	.2	" " English	18 850	.35
Oak	2 856	.23		22 400	.35
Steel plates, .5 inch	52 000	.62	" " American	15 000	.26
" wire	75 700	.5	" wire, No. 9, unannealed	47 532	.46
Yellow pine	3 332	.23	" " " annealed	36 300	.45

Turning.—Removing outer surface does not reduce the strength of bolts.

TIE-RODS.

Results of Experiments on Tensile Strength of Wrought-iron Tie-rods.

Common English Iron, 1.1875 Ins. in Diameter.

Description of Connection.	Breaking Weight.
	Lbs.
Semicircular hook fitted to a circular and welded eye	14 000
Two semicircular hooks hooked together	16 220
Right-angled hook or goose-neck fitted into a cylindrical eye	29 120
Two links or welded eyes connected together	48 160
Straight rod without any connective articulation	56 000

Ratio of Ductility and Malleability of Metals.

In order of Wire-drawing Ductility.	In order of Laminable Ductility.	In order of Wire-drawing Ductility.	In order of Laminable Ductility.	In order of Wire-drawing Ductility.	In order of Laminable Ductility.
Gold.	Iron.	Tin.	Gold.	Tin.	Zinc.
Silver.	Copper.	Lead.	Silver.	Platinum.	Iron.
Platinum.	Zinc.	Nickel.	Copper.	Lead.	Nickel.

Relative resistance of Wrought Iron and Copper to tension and compression is as 100 to 54.5.

Steel.

Experiments of Mr. Kirkaldy, 1858-61, give an average tensile strength for bars of 134 400 lbs. (60 tons) per sq. inch for tool-steel, and 62 720 lbs. (28 tons) for puddled. Greatest observed strength being 148 288 lbs. (66.2 tons). Plates, mean, 86 800 lbs. (32 to 45.5 tons) with fibre, and 81 760 lbs. (36.5 tons) across it.

Its resistance to crushing compared to tension is as 2.1 to 1.

Hardening.—Its strength is very materially increased by being cooled in oil, ranging from 12 to 55 per cent.

Crucible.—Experiments by the Steel Committee of Society of Civil Engineers, England, 1868-70, give a tensile strength of 91 571 lbs. per sq. inch (40.88 tons), with an elongation of .163 per cent., or 1 part in 613, and an elastic extension of .000 034 7th part for every 1000 lbs. per sq. inch, or 1 part in 28 818.

Bessemer.—Experiments by same Committee give a tensile strength of 76 653 lbs. per sq. inch (34.22 tons) with an elongation of .144 per cent., or 1 part in 695, and an elastic extension of .000 034 82d part for every 1000 lbs. per sq. inch, or 1 part in 28 719.

Result of Experiments by Committe of Society of Civil Engineers of England, 1868-70, and Mr. Daniel Kirkaldy, 1875.

Per Sq. Inch.

Steel.	Elastic Strength.		Elastic Extension in Parts of Length.	Elastic Extension per 1000 Lbs.	Ratio of Elastic to Ultimate Strength.	Destructive Weight.	
	Lbs.	Tons.	Per Cent.	In Length.	Per Cent.	Lbs.	Tons.
Crucible	49 840	22.25	.225	.000 5	58.2	86 464	38.6
Bessemer	44 800	20	.204	.000 45	59	75 757	33.82
Fagersta, unannealed	48 608	21.7	—	—	59.2	78 176	34.9
" annealed	39 200	17.5	—	—	51.5	72 576	32.4
Siemens, unannealed	32 080	14.56	—	—	46.4	69 888	31.2
" annealed	28 784	12.85	—	—	44.4	64 512	28.8

Average Tensile Elasticity of Steel Bars and Plates.

(Com. of Civil Engineers, 1870.)

Description.	Elasticity per Sq. Inch.	Elastic Extension in Parts of Length.	Ratio of Elastic to Destructive Strength.
Bars.	Lbs.	Parts.	Per Cent.
Crucible, hammered and rolled	50 557	1 in 485	58.2
Bessemer, " "	43 814	1 in 675	55
Fagersta, rolled	56 560	—	64.8
" unannealed	34 048	—	55.6
" hammered and rolled	55 574	—	64.7
" " " annealed	40 858	—	54
" plates, unannealed	30 710	1 in 980	59.2
" " annealed	26 940	1 in 1020	56.5
Siemens, " unannealed	32 500	—	46.4
" " annealed	28 780	—	44.4
" tires	40 174	—	58.8
Krupp's shaft	42 112	1 in 185	—

Tensile strength of steel increases by reheating and rolling up to second operation, but decreases after that.

Tensile Strength of Various Materials, deduced from Experiments of U. S. Ordnance Department, Fairbairn, Hodgkinson, Kirkaldy, and by the Author.

Power or Weight required to tear asunder One Sq. Inch, in Lbs.

Metals.	Lbs.
Antimony, cast	1 053
Bismuth, cast	3 248
Cast Iron, Greenwood	45 970
" mean, Major Wade	31 829
" gun-metal, mean	37 232
" malleable, annealed	56 000
" Eng., strong	29 000
" " weak	13 400
" " averages	15 600
	21 280
" " gun-metal	23 257
" " mean*	19 484
" " Low Moor, No. 2	14 076
" " Clyde, No. 1	16 125
" " " No. 3	23 468
" " Stirling, mean	25 764
Copper, wrought	34 000
" rolled	36 000
" cast	24 250
" bolt	36 800
" wire	61 200
Gold	20 384
Lead, cast	1 800
" pipe	2 240
" " encased	3 759
" rolled sheet	3 320
Platinum wire	53 000
Silver, cast	40 000
Steel, cast, maximum	142 000
" " mean	88 560
" puddled, maximum	173 817
" Amer. Tool Co.	179 980
" wire	210 000
	300 000
" plates, lengthwise	96 300
" " crosswise	93 700
" Chrome bar	180 000

Metals.	Lbs.
Steel, Pittsburgh, mean	94 450
" Bessemer, rolled	76 650
	125 000
" " hammered	152 900
" Eng., cast	134 000
" " " plates, mean	93 500
" " plates	86 800
" " puddled plates	62 720
" " crucible	91 570
" " homogeneous	96 280
" " blistered, bars	104 000
" " Fagersta bars	89 600
" " " plates	98 560
" " Whitworth's	89 600
	152 000
" " Siemens's plates	64 900
	69 880
" " Krupp's shaft	92 243
Tin, cast	5 000
" Banca	2 100
Wire rope, per lb. w't per fathom	4 480
" " galvanized steel, "	6 720
Wrought Iron, boiler plates	45 500
	62 000
" rivets	65 000
" bolts, mean	
" " inferior	30 000
" hammered	54 000
" shaft	44 750
" wire	73 600
" " No. 9	100 000
" " No. 20	120 000
" " diam. .0069 inch	301 168
" " galv'ized .058 "	64 960
" Eng., heavy forging	33 600
" " plates, lengthw'e	53 800
" " " crosswise	48 800

* By Comm's on application of Iron to Railway Structure.

Metals.	Lbs.
Wrought Iron, Eng., mean	51 000
“ Eng., Low Moor	57 600
“ “ Lancashire	48 800
“ “ Thames	65 920
“ “ armor-plates	40 000
“ “ bar	31 300 56 000
“ “ “ charcoal	63 000
“ “ rivet, scrap	51 760
“ Russian, bar, best	59 500
“ “ “	49 000
“ Swedish, “ best	72 000
“ “ “	48 900
Zinc	3 500
“ sheet	7 000 16 000

Alloys or Compositions.	Lbs.
Alloy, Cop. 60, Iron 2, Zinc 35, Tin 2	85 120
“ Tin 10, Antimony 1	11 000
Aluminium, Cop. 90	71 600
“ maximum	96 320
Bell-metal	3 670
Brass, cast	18 000
“ wire	49 000
Bronze, Phosphor., extreme	50 915
“ “ mean	34 464
“ ordinary	23 500
“ Cop. 10, Tin 1	33 000
“ “ 9, “ 1	38 080
“ “ 8, “ 1	36 000
“ “ 2, Zinc 1	29 000
Gun-metal, ordinary	18 000
“ mean	33 600
“ bars	42 040
Speculum metal	7 000
Yellow metal	48 700

Woods.	Lbs.
Ash, white	14 000
“ American	9 500
“ English	16 000
Bamboo	6 300
Bay	14 000
Beech, English	11 500
Birch	15 000
“ Amer., black	7 000
Box, African	23 000
Bullet	19 000
Cedar, Lebanon	11 400
“ West Indian	7 500
“ American	11 600
Chestnut	12 500
“ horse	10 000
Cypress	6 000
Deal, Christiana	12 400
Ebony	27 000
Elm	6 000 13 000
Gum, blue	18 000
“ Alabama	15 860
Hackmatack	12 000
Hickory	11 000
Holly	16 000
Lance	17 350 23 000
Larch	4 200 9 500
Lignum vitæ	11 800
Locust	16 000 20 500
Mahogany, Honduras	21 000
“ Spanish	8 000 12 000
Oak, Pa., seasoned	20 333
“ Va., “	25 222
“ white	16 500
“ live, Ala.	16 380
“ red	10 250
“ African	9 500
“ English	4 500 7 571
“ Dantzic	4 200
Pear	9 860
Pine, Va	19 200
“ Riga	14 000
“ yellow	13 000
“ white	11 800
“ red	13 000
Poon	13 300
Poplar	7 000
Redwood, Cal	10 833
Spruce, white	10 290 12 400
Sycamore	9 600 13 000
Teak, India	15 000
“ African	21 000
Walnut, Eng	7 800
“ black	16 633
“ Mich	17 580
Willow	13 000
Yew	8 000

Across Fibre.

Oak	2 300
Pine	550

Miscellaneous.	
Basalt, Scotch	1 469
Beton, N. Y. Stone Con'g Co.	300 500
Blue stone	77
Brick, extreme	750
“ inferior	100 290
Cement, Portland, 7 days	400 860
“ “ pure, 1 mo.	393
“ “ sand 2, 320 days	713
“ “ “ 1 “	948
“ “ pure, “	1 152
“ “ sand 1, in water 1 mo.	201
“ “ “ 1 “ 1 y'r.	319
“ “ “ 3, 1 year	310
“ “ “ 5, 1 “	214
“ “ “ 7, 1 “	163
“ Hydraulic	284
“ Rosedale, Ulst. Co., 7 days	104
“ “ sand 1, 30 “	102
“ “ 9 mos.	560 700

MISCELLANEOUS.	Lbs.
Cement, Roman, in water 7 days	90
" " " 1 mo.	115
" " " 1 year	286
" " sand 1, 42 days	284
" " " 2, "	199
" " " 3, "	160
Flax	25 000
Glass, crown	2 546
Glue	4 000
Granite	578
Gutta Percha	3 500
Hemp rope	12 000 16 000
Ivory	1 000
Leather belting	330
Limestone	670 2 800
Marble, statuary	3 200
" Italian	5 200
Marble, white	9 000
" Irish	17 600

MISCELLANEOUS.	Lbs.
Mortar, 1 year	60 150
" hydraulic	85 130
" ordinary	35
Oxhide	6 300
Rope, Manila	9 000
" tarred hemp	15 000
Sandstone	150
" fine green	1 260
" Arbroath	563 1 261
" Caithness	473 1 054
" Portland	857 1 000
" Craigleth	453
Silk fibre	52 000
Slate	9 600 12 800
Whalebone	7 000

TORSIONAL STRENGTH.

SHAFTS AND GUDGEONS.

Shafts are divided into *Shafts* and *Spindles*, according to their magnitude, and are subjected to Torsion and Lateral Stress combined, or to Lateral Stress alone.

A *Gudgeon* is the metal journal or *Arbor* upon which a wooden shaft revolves.

Lateral Stiffness and Strength.—Shafts of equal length have *lateral stiffness* as their breadth and cube of their depth, and have *lateral strength* as their breadth and square of their depths.

Shafts of different lengths have *lateral stiffness* directly as their breadth and cube of their depth, and inversely as cube of their length; and have *lateral strength* directly as their breadth and as square of their depth, and inversely as their length.

Hollow Shafts having equal lengths and equal quantities of material have *lateral stiffness* as square of their diameter, and have *lateral strength* as their diameters. Hence, in hollow shafts, one having twice the diameter of another will have four times the stiffness, and but double the strength; and when having equal lengths, by an increase in diameter they increase in stiffness in a greater proportion than in strength.

When a solid shaft is subjected to torsional stress, its centre is a neutral axis, about which both intensity and leverage of resistance increase as radius or side; and the two in combination, or moment of resistance per sq. inch, increase as square of radius or side.

Round Shaft.—Radius of ring of resistance is radius of gyration of section, being alike to that of a circular plate revolving on its axis, viz., .7071 radius. The ultimate moment of resistance then is expressed by product of sectional area of shaft, by ultimate shearing resistance per sq. inch of material by radius, and by .7071.

$$\text{Or, } .7854\, d^2\, r\, S \times .7071 = .278\, d^3\, S = R\, W. \quad (\textit{D. K. Clark.})$$

d representing diameter of shaft and r radius, S *ultimate shearing stress of material in lbs. per sq. inch,* R *radius through which stress is applied, in ins., and* W *moment of load or destructive stress, in lbs.*

$$\text{Hence, } \frac{.278\, d^3\, S}{R} = W; \quad \frac{R\, W}{.278\, d^3} = S; \text{ and } \sqrt[3]{\frac{R\, W}{S}} \times 1.534 = d.$$

Round Shaft.—Strength, compared to a square of equal sectional area, is about as 1 to .85. Diameter of a round section, compared to side of square section of equal resistance, is as 1 to .96.

Square Shaft.—Moment of torsional resistance of a square shaft exceeds that of a round of same sectional area, in consequence of projection of corners of square; but inasmuch as material is less disposed to resist torsional stress, the resistance of a square shaft, compared to a round one of like area of section, is as 1 to 1.18, and of like side and diameter, as 1.08 to 1.

Hence, $\frac{.278 \times 1.08\, s^3\, S}{R} = W$. *Hollow Round Shafts.* $\frac{.278\,(d^4 - d'^4)\, S}{R\, d} = W$.

When Section is comparatively Thin. $\frac{1.57\, d^3\, t\, S}{R} = W$. *s representing side, d and d′ external and internal diameters, and t thickness of metal in ins.*

Torsional Angle of a bar, etc., under equal stress, will vary as its length. Hence, torsional strength of bars of like diameters is inversely as their lengths.

Stress upon a shaft from a weight upon it is proportional to product of the parts of shaft multiplied into each other. Thus, if a shaft is 10 feet in length, and a weight upon centre of gravity of the stress is at a point 2 feet from one end, the parts 2 and 8, multiplied together, are equal to 16; but if weight or stress were applied in middle of the shaft, parts 5 and 5, multiplied together, would produce 25.

When load upon a shaft is uniformly distributed over any part of it, it is considered as united in middle of that part; and if load is not uniformly distributed, it is considered as united at its centre of gravity.

Deflection of a shaft produced by a load which is uniformly distributed over its length is same as when .625 of load is applied at middle of its length.

Resistance of body of a shaft to lateral stress is as its breadth and square of its depth; hence diameter will be *as product of length of it, and length of it on one side of a given point, less square of that length.*

Illustration.—Length of a shaft between centres of its journals is 10 feet; what should be relative cubes of its diameters when load is applied at 1, 2, and 5 feet from one end? and what when load is uniformly distributed over length of it?

$$l \times l^1 - l^3 = d^3;\ \text{and when uniformly distributed, } d^3 \div 2 = d^1.$$

$10 \times 1 = 10 - 1^2 = 9 =$ *cube of diameter at 1 foot;* $10 \times 2 = 20 - 2^2 = 16 =$ *cube of diameter at 2 feet;* $10 \times 5 = 50 - 5^2 = 25 =$ *cube of diameter at 5 feet.*

When a load is uniformly distributed, stress is greatest at middle of length, and is equal to half of it; $25 \div 2 = 12.5 =$ *cube of diameter at 5 feet.*

Torsional Strength of any square bar or beam is as cube of its side, and of a cylinder as cube of its diameter. Hollow cylinders or shafts have greater torsional strength than solid ones containing same volume of material.

To Compute Diameter of a Solid Shaft of Cast or Wrought Iron to Resist Lateral Stress alone.

When Stress is in or near Middle. Rule.—Multiply weight by length of shaft in feet; divide product by 500 for cast iron and 560 for wrought iron, and cube root of quotient will give diameter in ins.

Example.—Weight of a water-wheel upon a cast-iron shaft is 50 000 lbs., its length 30 feet, and centre of stress of wheel 7 feet from one end; what should be diameter of its body?

$$\sqrt[3]{\left(\frac{50\,000 \times 30}{500}\right)} = 14.42\ \textit{ins., if weight was in middle of its length.}$$

Hence diameter at 7 feet from one end will be, as by preceding Rule, $30 \times 7 - 7^2 = 161 =$ *relative cube of diameter at 7 feet;* $30 \times 15 - 15^2 = 225 =$ *relative cube of diameter at 15 feet, or at middle of its length.*

Then, as $\sqrt[3]{225} : 14.42 :: \sqrt[3]{161} : 12.91$ *ins., diameter of shaft at 7 feet from one end.*

For Bronze, 420; Cast steel, 1000 to 1500; and Puddled steel, 500.

When Stress is uniformly laid along Length of Shaft. RULE.—Divide cube root of product of weight and length by 9.3 for Cast iron and 10.6 for Wrought iron, and quotient will give diameter in ins.

EXAMPLE.—Apply rule to preceding case. $\frac{\sqrt[3]{50\,000 \times 30}}{9.3} = 12.31$ *ins.*

For Bronze, 8.5; Cast steel, 18.6 to 27.9; and Puddled steel, 9.3.

When Diameter for Stress applied in Middle is given. RULE.—Take cube root of .625 of cube of diameter, and this root will give diameter required.

EXAMPLE.—Diameter of a shaft when stress is uniformly applied along its length is 14.42 ins.; what should be its diameter, stress being applied in middle?

$$\sqrt[3]{.625 \times 14.42^3} = \sqrt[3]{.625 \times 3000} = 12.33 \textit{ ins.}$$

To Compute Diameter of a Solid Shaft of Cast Iron to Resist its Weight alone.

RULE.—Multiply cube of its length by .007, and square root of product will give diameter in ins.

EXAMPLE.—Length of a shaft is 30 feet; what should be its diameter in body?

$$\sqrt{(30^3 \times .007)} = \sqrt{189} = 13.75 \textit{ ins}$$

HOLLOW SHAFTS.

To Compute Diameter of a Hollow Shaft of Cast Iron to Sustain its Load in Addition to its Weight.

When Stress is in or near Middle. RULE.—Divide continued product of .012 times cube of length, and number of times weight of shaft in lbs., by square of internal diameter added to 1, and twice square root of quotient added to internal diameter will give whole diameter in ins.

EXAMPLE.—Weight of a water-wheel upon a hollow shaft 30 feet in length is 2.5 times its own weight, and internal diameter is 9 ins.; what should be whole diameter of shaft?

$$2\sqrt{\left(\frac{.012 \times 30^3 \times 2.5}{1 + 9^2}\right)} + 9 = 2\sqrt{\frac{810}{82}} = 6.28 \textit{ ins.}, \text{ and } 6.28 + 9 = 15.28 \textit{ ins.}$$

To Compute Diameter of a Round or Square Shaft to Resist Combined Stress of Torsion and Weight.

RULE.—Multiply extreme of pressure upon crank-pin, or at pitch-line of pinion, or at centre of effect upon the blades of a water-wheel, etc., that a shaft may at any time be subjected to; by length of crank or radius of wheel, etc., in feet; divide the product by *Coefficient* in following Table, and cube root of quotient will give diameter of shaft or its journal in ins.

$$\text{Or, } \sqrt[3]{\frac{P\,R}{C}} = d.$$

EXAMPLE.—What should be diameter for journal of a wrought-iron water-wheel shaft, extreme pressure upon crank-pin being 59 400 lbs., and crank 5 feet in length?

$$C = 120. \quad \sqrt[3]{\frac{59\,400 \times 5}{120}} = 2475, \text{ and } \sqrt[3]{2475} = 13.53 \textit{ ins.}$$

When Two Shafts are used, as in Steam-vessels, etc., with One Engine. RULE.—Divide three times cube of diameter for one shaft by four, and cube root of quotient will give diameter of shaft in ins.

$$\text{Or, } \sqrt[3]{\frac{3\,d^3}{4}} = d.$$

EXAMPLE.—Area of journal of a shaft is 113 ins.; what should be diameter, two shafts being used?

Diameter for area of $113 = 12$. Then $\frac{3 \times 12^3}{4} = 1296$, and $\sqrt[3]{1296} = 10.9$ *ins.*

Torsional Strength of Various Metals.

(*Maj. Wm. Wade, U. S. Ordnance Corps*, 1851, *Steel Committee* [*England*, 1868], *and Stevens Institute, N. J.*, 1878.)

Reduced to a Uniform Measure of One Inch in Diameter or Side. Stress applied at One Foot from Axis of Body and at Face of Axis.

Bars and Metals.	Tensile Strength.	Destructive Stress at 25 Ins.	Destructive Stress Computed at 12 Ins.	Torsional Strength $\frac{WR}{d^3} = T$.	Coefficient $\frac{C\,d^3}{R} = W$. 2 Ins.	5 Ins.	10 Ins.	15 Ins.	20 Ins.
Cast Iron.	Lbs.	Lbs.	Lbs.						
Diam. {1.3 ins. / .65 in.} Area 1 sq. inch	45 000	520	1082	492	100	95	90	85	80
" Diam. {3.25 ins. / 2.61 "} Area 2.97 sq. ins.	"	3800	7904	230	45	40	35	30	25
Diam. = 1.9 ins. Least	9 000	1550	3664	530	130	125	120	115	110
Diam. = 1.9 ins. Mean	31 829	2145	4462	650					
Diam. = 1.9 ins. Greatest	45 000	2840	5907	850					
Side 1 inch, Area 1 sq. inch	"	350	728	728	125	120	115	110	105
Wrought Iron.									
Diam. = 1.9 ins. Least	38 027	1250	2600	376	120	115	110	105	100
Diam. = 1.9 ins. Mean	56 300	1375	2860	416					
Diam. = 1.9 ins. Greatest. Area 2.83 sq. ins.	74 592	1500	3120	452					
Bronze.									
" Diam. = 1.9 ins. Least	17 698	500	1040	152	30	28	26	—	—
" Diam. = 1.9 ins. Greatest. Area 2.83 sq. ins.	56 786	650	1352	197	38	36	34	—	—
Cast Steel.									
" Diam. = 1.9 ins. Least	42 000	2600	5408	788	160	155	550	—	—
" Diam. = 1.9 ins. Greatest. Area 2.83 sq. ins.	128 000	7760	16140	2353	475	470	465	—	—
Bessemer Steel.									
" Diam. = 1.382 ins. Area 1.5 sq. ins.	36 960	1568	3261	1236	245	240	235	230	225

To Compute Diameter of Shafts of Oak and Pine.

Multiply diameter ascertained for Cast Iron as follows: Oak by 1.83, Yellow Pine by 1.716.

Metals and Woods.

Ultimate Torsional Strength.—Of Cast Iron may be taken as equal to its transverse strength for American and .9 for English, or as .26 of its tensile strength for American and .23 for English. Of Wrought Iron, as .7 to .8 of its transverse strength for American and .7 to 1 for English, and of Steel, as .72 of its tensile strength.

Elastic Torsional Strength.—Of Cast Iron may be taken as equal to its transverse strength, of Wrought Iron 40 per cent. of its ultimate torsional strength, of Steel 44 per cent. of its tensile strength, and 45 per cent. of its ultimate torsional strength.

Bessemer Steel.—Has a torsional strength of 6670 lbs. per sq. inch at a radius of one foot, being somewhat less than that of Cast Iron, Fagersta has 50 per cent. of its ultimate transverse strength, and Siemens 44.5 per cent. of its ultimate tensile.

NOTE.—Examples here given are deduced from instances of successful practice; where diameter has been less, fracture has almost universally taken place, stress being increased beyond ordinary limit.

2.—When shafts of less diameter than 12 ins. are required, *Coefficients* here given may be slightly reduced or increased, according to quality of the metal and diameter of shaft; but when they exceed this diameter, *Coefficients* may not be increased, as strength of a shaft decreases very materially as its diameter increases.

Order of shafts, with reference to degree of torsional stress to which they may be subjected, is as follows:

1. Fly-wheel. | 2. Water-wheel. | 3. Secondary shaft. | 4. Tertiary, etc.

Hence, diameters of their journals may be reduced in this order.

To Compute Diameter of a Wrought-iron Centre Shaft for connecting Two Engines at a Right Angle.

Conditions of such a shaft are as follows:

Greatest stress that it is subjected to is when leading engine is at .75 of its stroke, and following engine .25 of its stroke; hence, position of each crank is as sin. $22° 30 \times 2 = .7071$ of length of crank or radius of power.

Consequently, $\sqrt[3]{\frac{2 P .707 R}{125}} = d$. P *representing extreme pressure on piston.*

NOTE.—In computing P it is necessary to take very extreme pressure that piston may be subjected to, however short the period of time. Average pressure does not meet requirement of case.

ILLUSTRATION.—Extreme pressure upon each piston of two engines connected at a right angle was 111 592 lbs., and stroke of pistons 10 feet; what should have been diameter of centre shaft? and what of each wheel or driving shaft?

$$\sqrt[3]{\left(\frac{111\,592 \times 2 \times .707 \frac{10}{2}}{125}\right)} = \sqrt[3]{\frac{788\,955}{125}} = 18.48 \textit{ ins. centre shaft.}$$

For ordinary mill purposes, driving shafts should be as cube roots of .25 of 3 times cube of centre shaft. Thus $\sqrt[3]{\frac{18.48^3 \times 3}{.25}} = 16.79$ *ins.*

To Compute Torsional Strength of Hollow Shafts and Cylinders.

RULE.—From fourth power of exterior diameter subtract fourth power of interior diameter, and multiply remainder by *Coefficient* of material; divide this product by product of exterior diameter and length or distance from axis at which stress is applied in feet, and quotient will give resistance in lbs.

$$\text{Or, } \frac{d^4 - d'^4\, C}{d\, l} = R.$$

EXAMPLE.—What torsional stress may be borne by a hollow cast-iron shaft, having diameters of 3 and 2 ins., power being applied at one foot from its axis?

$C = 130.\quad 3^4 - 2^4 \times 130 = 8450$, which $\div 3 \times 1 = \frac{8450}{3} = 2816.6$ *lbs.*

To Compute Torsional Strength of Round and Square Shafts.

RULE.—Multiply Coefficient in preceding Table by cube of side or of diameter of shaft, etc., and divide product by distance from axis at which stress is applied in feet; quotient will give resistance in lbs.

ILLUSTRATION.—What torsional stress may be borne by a cast-iron shaft of best material, 2 ins. in diameter, power applied at 2 feet from its axis.

C from table $= 130.\quad \frac{130 \times 2^3}{2} = \frac{1040}{2} = 520$ *lbs.*

For steamers, when from heeling of vessel or roughness of sea the stress may be confined to one wheel alone, diameter of journal of its shaft should be equal to that of centre shaft.

GUDGEONS.

To Compute Diameter of a Single Gudgeon of Cast Iron, to Support a given Weight or Stress.

RULE.—Divide square root of weight in lbs. by 25 for Cast iron, and 26 for Wrought iron, and quotient will give diameter in ins.

EXAMPLE.—Weight upon a gudgeon of a cast-iron water-wheel shaft is 62 500 lbs.; what should be its diameter?

$$\frac{\sqrt{62\,500}}{25} = \frac{250}{25} = 10 \; ins.$$

To Compute Diameter of Two Gudgeons of Cast Iron, to Support a given Stress or Weight.

RULE.—Proceed as for two shafts, page 792.

To Compute Ultimate Torsional Strength of Round and Square Shafts. (*D. K. Clark.*)

Cast Iron. *Round.* $\frac{.278\, d^3\, S}{R} = W$; $1.534\sqrt[3]{\frac{W\, R}{S}} = d$; and $\frac{W\, R}{278\, d^3} = S$.

Square. $\frac{.4\, s^3\, S}{R} = W$, and $1.36\sqrt[3]{\frac{W\, R}{S}} = s$. *Hollow.* $\frac{.278\, (d^4 - d'^4)\, S}{R\, d} = W$.

S *representing ultimate shearing strength, and* W *moment of load, both in lbs.*, *s side of square shaft, and* R *radius of stress, both in ins.*

ILLUSTRATION.—What is ultimate torsional strength of a round cast-iron shaft 4 ins. in diameter, stress applied at 5 feet from its axis?

Assume $S = 20\,000$ lbs. Then $\frac{.278 \times 4^3 \times 20\,000}{5 \times 12} = 5930$ *lbs.*

By experiments of Major Wade, ordinary foundry iron has a torsional strength of 7725 lbs., or 644 lbs. per sq. inch at radius of one foot.

Thus, take preceding illustration. Then $\frac{7725 \times 4^3}{5 \times 12} = 8240$ *lbs.*

Wrought Iron. *Round.* $\frac{.2224\, d^3\, S}{R} = W$. *Square.* $\frac{.32\, s^3\, S}{R} = W$.

When Torsional Strength per sq. inch for radius of 1 inch is ascertained, substitute C for .278, .4, .2224, or .32.

Stress which will give a bar a permanent set of .5° is about .7 of that which will break it, and this proportion is quite uniform, even when strength of material may vary essentially.

Wrought Iron, compared with Cast Iron, has equal strength under a stress which does not produce a permanent set, but this set commences under a less force in wrought iron than cast, and progresses more rapidly thereafter. Strongest bar of wrought iron acquired a permanent set under a less strain than a cast-iron bar of lowest grade.

Strongest bars give longest fractures.

Steel. *Round.* $\frac{.2\, d^3\, S}{R} = W$. When S is not known, substitute for S $72\, s = 72$ per cent. of tensile strength.

Torsional Strength of Cast Steel is from 2 to 3 times that of Cast Iron.

Following rules are purposed to apply in all instances to diameters of journals of shafts, or to diameter or side of bearings of beams, etc., where length of journal or distance upon which strain bears does not greatly exceed diameter of journal or side of beam, etc.; hence, when length or distance is greatly increased, diameter or side must be correspondingly increased.

Coefficients for torsional breaking stress of Iron, Bronze, and Steel, as determined by Major Wade, are: Wrought Iron, 640; Cast Iron, 560; Bronze, 460; Cast Steel, 1120 to 1680. Puddled Steel does not differ essentially from that of cast iron.

Formulas for Minimum and Maximum Diam. of Wrought-iron Shafts.

(*A. E. Seaton, London,* 1883, *and Board of Trade, Eng.*)

Compound Engines. $\sqrt[3]{\frac{D^2\,p\,d^2\,15}{C}}\,S = diameter.$ *D and d representing diameter of low and high pressure cylinders, and S half stroke, all in ins., p pressure of steam in boiler, in lbs. per sq. inch, and C a coefficient, as follows:*

Angle of Crank.	Shafts. Crank.	Shafts. Propeller.	Angle of Crank.	Shafts. Crank.	Shafts. Propeller.	Angle of Crank.	Shafts. Crank.	Shafts. Propeller.	Angle of Crank.	Shafts. Crank.	Shafts. Propeller.
90°	2468	2880	100°	2279	2659	110°	2131	2487	120°	2016	2352
	4000	5400		4000	5400		4000	5400		4000	5400

$\sqrt[3]{\frac{IHP}{r}}\,C = diameter.$ *A. E. Seaton, London,* 1883.

Side-wheel Engines, Sea Service.—One cylinder crank journal, C = 80; outboard 100; Two cylinder crank journal 50; outboard 65; and centre shaft 58.

Propeller Engines.—One cylinder crank journal 150; Tunnel 130; Two cylinder compound crank 130; Tunnel 110; Two cranks, crank 100; Tunnel 85; Three cranks, crank 90; and Tunnel 78.

River Service.—C may be reduced one fifth.

Illustration.—With a compound propeller engine, steam cylinders 20 and 40 ins. in diameter, by 40 ins. stroke, operating under a pressure of 80 lbs. steam (mercurial gauge), what should be the diameter of the shafts of wrought iron?

$$\sqrt{\frac{20^2 \times 80 + 40^2 \times 15}{4000}} \times 40 = \sqrt[3]{\frac{56\,000}{4000}} \times 40 = 8.24 \text{ ins. crank shaft;}$$

$$\text{and } \sqrt[3]{\frac{56\,000}{5400}} \times 40 = 7.46 \text{ ins. propeller shaft.}$$

Journals of Shafts, etc.

Journals or bearings of shafts should be proportioned with reference to pressure or load to be sustained by the journal. Simplest measure of bearing capacity of a journal is product of its length by its diameter, in sq. ins.; and axial area or section thus obtained, multiplied by a *coefficient* of pressure per sq. inch, will give bearing capacity.

Sir William Fairbairn and Mr. Box give instances of weights on bearings of shafts, etc., from which following deductions are made, showing pressure per sq. inch of axial section of journal:

Crank pins, 687 to 1150 lbs. per sq. inch.

Link bearings, 456 to 690 lbs. per sq. inch.

Pressure on bearings, as a general rule, should not exceed 750 lbs. per sq. inch of axial area.

Length of Journals should be 1.12 to 1.5 times diameter.

Journals of Locomotives or Like Axles are usually made twice diameter, and to sustain a pressure of 300 lbs. per sq. inch of axial area, or 10 sq. ins. per ton of load.

Solid Cylindrical Couplings or Sleeves.

$d + \sqrt{5.5\,d} = D$; $3\,d = L$; $.8\,d = l$; $.25\,d + .12 = k$. *d representing diameter, and L length of sleeve, l length of lap or scarf of shaft, k breadth of key, its depth being half its breadth, and D diameter of coupling or sleeve, all in ins.*

Flanged Couplings.

$d + \sqrt{3.5\,d} = D$; $3\,d + 1 = F$; $.3\,d + .4 = l$; $d + 1 = L$; $l \div 4 = s$. *D representing diameter of body of coupling, F diameter of flanges, l thickness of both flanges, L length of each coupling, s projection of end of one shaft and retrocession of other from centre of coupling, and d diameter of shaft, all in ins.*

Supports for Shafts. (*Molesworth.*)

$5\,\sqrt[3]{d^2} = L$. *L representing distance of supports apart, in feet.*

To Resist Lateral Stress. $\sqrt[3]{\frac{L\,W}{C}} = D$. W *representing weight or pressure at centre of length in lbs., and* D *diameter or side, if square, in ins.*

Value of C.—Wrought Iron, 560; Cast Iron, 500; Cast Steel, 1000 to 1500; Bronze, 420; and Wood, 40. When Weight is distributed put 2 C.

Values of C for Shafting of Various Metals, as observed by different Authorities, and deduced from Formulas of Navier. $\frac{16\,W\,r}{\pi\,d^3} = C$.

Ultimate Resistance.

METAL.	C	METAL.	C	METAL.	C
WROUGHT IRON.		CAST IRON.		STEEL.	
American, Pembe, Me.	61 673	American, mean	36 846	American, Conn..	82 926
" Ulster....	61 815	"	38 300	" Spindle	102 131
" mean.....	66 436	"	42 821	" Nash. I. Co.	95 213
English, refined.....	49 148	" 18 trials	44 957	English, Shear....	111 191
" "	54 585	English, mean..	22 132	Bessemer......	73 060
Swedish............	61 909	"	38 217	"	79 662

Mill and Factory Shafts. (*J. B. Francis.*)

Cylindrical. $\frac{16\,W\,R}{\pi\,d^3} = T$. $\frac{\pi\,d^3\,T}{16\,R} = W$.

Square. $\frac{3\sqrt{2}\,W\,R}{s^3} = T$. $\left(\frac{s^3\,T}{R} \div 3\right) \div \sqrt{2} = W$.

Mean value of T.

Cast Iron........	22 000 / 65 000	Wrought Iron....	49 000 / 94 000	Steel...........	76 000 / 111 000
" mean......	35 000	" mean.....	50 000	" mean.........	86 000
" " Eng.	30 000	" " Eng.	45 000	" " Bessemer	78 000

ILLUSTRATION.—What is the ultimate or destructive weights that may be borne by a Round Cast iron shaft 2 ins. in diameter, and by a Square shaft 1.75 ins. side, stress applied at 25 ins. from axis? Assume T = 36 000.

Round. $\frac{3.1416 \times 2^3 \times 36\,000}{16 \times 25} = 2261.95$ *lbs.*

Square. $\left(\frac{1.75^3 \times 36\,000}{25} \div 3\right) \div \sqrt{2} = 1837.8$ *lbs.*

Their lengths should be reduced, and diameter increased, in following cases:

1st. At high velocities, to admit of increased diameter of journals, thereby rendering them less liable to heating. 2d. As they approach extremity of a line of shafting. 3d. Attachment of intermediate pulleys or gearing.

	Prime Movers of Power.	*Transmitters of Power.*
Wrought Iron.	$\sqrt[3]{\frac{100\,IHP}{n}} = d$, and $.01\,n\,d^3 = IHP$.	$\sqrt[3]{\frac{50\,IHP}{n}} = d$, and $.02\,n\,d^3 = IHP$.
Steel.	$\sqrt[3]{\frac{62.5\,IHP}{n}} = d$, and $.016\,n\,d^3 = IHP$.	$\sqrt[3]{\frac{31.25\,IHP}{n}} = d$, and $.032\,n\,d^3 = IHP$
Cast Iron.	$\sqrt[3]{\frac{167\,IHP}{n}} = d$, and $.006\,n\,d^3 = IHP$.	$\sqrt[3]{\frac{83.5\,IHP}{n}} = d$, and $.012\,n\,d^3 = IHP$.

IHP *representing horse-power transmitted, n number of revolutions, and d diameter of shaft in ins.*

ILLUSTRATION 1.—What should be diameter of a wrought-iron shaft, to simply transmit 128 IHP at 100 revolutions per minute?

$$\sqrt[3]{\frac{50 \times 128}{100}} = \sqrt[3]{\frac{6400}{100}} = 4 \textit{ ins.}$$

2.—What IHP will a steel shaft of 4 ins. diameter transmit at 100 revolutions per minute?

$$.032 \times 100 \times 4^3 = 204.8 \textit{ horses.}$$

TRANSVERSE STRENGTH.

Transverse or Lateral Strength of any Bar, Beam, Rod, etc., is in proportion to product of its breadth and square of its depth; in like-sided bars, beams, etc., it is as cube of side, and in cylinders as cube of diameter of section.

When One End is Fixed and the Other Projecting, strength is inversely as distance of weight from section acted upon; and stress upon any section is directly as distance of weight from that section.

When Both Ends are Supported only, strength is 4 times greater for an equal length, when weight is applied in middle between supports, than if one end only is fixed.

When Both Ends are Fixed, strength is 6 times greater for an equal length, when weight is applied in middle, than if one end only is fixed.

When Ends Rest merely upon Two Supports, compared to one *When Ends are Fixed*, strength of any bar, beam, etc., to support a weight in centre of it, is as 2 to 3.

When Weight or Stress is Uniformly Distributed, weight or stress that can be supported, compared with that when weight or stress is applied at one end or in middle between supports, is as 2 to 1.

Metals.

In Metals, less dimension of side of a beam, etc., or diameter of a cylinder, greater its proportionate transverse strength, in consequence of their having a greater proportion of chilled or hammered surface, compared to their elements of strength, resulting from dimensions alone.

Strength of a *Cylinder*, compared to a *Square* of like diameter or sides, is as 5.5 to 8. Strength of a *Hollow Cylinder* to that of a *Solid Cylinder*, of same area of section, is about as 1.65 to 1, depending essentially upon the proportionate thickness of metal compared to diameter.

Strength of an *Equilateral Triangle, Fixed at One End and Loaded at the Other*, having an *edge up*, compared to a *Square* of the same area, is as 22 to 27; and strength of one, having an *edge down*, compared to one with an *edge up*, is as 10 to 7.

NOTE.—In Barlow and other authors the comparison in this case is made when the beam, etc., rested upon *supports*. Hence the stress is contrariwise.

Strongest rectangular bar or beam that can be cut out of a cylinder is one of which the squares of breadth and depth of it, and diameter of the cylinder, are as 1, 2, and 3 respectively.

Cast Iron.

Mean transverse strength of American, as determined by Major Wade, is 681 lbs. per sq. inch, suspended from a bar fixed at one end and loaded at the other; and mean of English, as determined by Fairbairn, Barlow, and others, is 500 lbs.

Experiments upon bars of cast iron, 1, 2, and 3 ins. square, give a result of transverse strength of 447, 348, and 338 lbs. respectively; being in the ratio of 1, .78, and .756.

Woods.

Beams of wood, when laid with their annular layers vertical, are stronger than when they are laid horizontal, in the proportion of 8 to 7.

Relative Stiffness of Materials to Resist a Transverse Stress.

Ash	.089	Cast Iron	1	Oak	.095	Wrought iron	1.3
Beech	.073	Elm	.073	White pine	.1	Yellow pine	.087

Strength of a Rectangular Beam in an *Inclined position*, to resist a vertical stress, is to its strength in a horizontal position, as square of radius to square of cosine of elevation; that is, as square of length of beam to square of distance between its points of support, measured upon a horizontal plane.

Transverse Strength of Various Materials.

(*U. S. Ordnance Department, Hodgkinson, Fairbairn, Kirkaldy, and by the Author.*)

Power reduced to uniform Measure of One Inch Square, and One Foot in Length; Weight suspended from one End.

Metals.

Brass	260
Cast Iron, mean of 4 grades	660
" " (Maj. Wade)	681
" ordinary	575
" extreme, West P't F'dry	980
" gun-metal,* " "	740
" Eng., Low Moor, cold blast	472
" " Ponkey, "	581
" " Ystalyfera "	770
" " mean, 65 kinds	500
" " " 15 kinds, cold blast	641
" " planed bar	518
" " rough bar	534
Copper	244
Steel, hammered, mean	1500
" cast, soft	1540
" " hard	4200
" hematite, hammered	1620
" Krupp's shaft	2096
" Fagersta, hammered	1200
Wrought Iron, mean	600
" " English	475
" " Swedish†	665

Woods.

Ash	168
" English	160
" Canada	120
Balsam, Canada	87
Beech	130
" white	112
Birch	{ 160 / 115
Cedar, white	160
" Cuba	{ 63 / 105
Chestnut	160
Elm	125
" Canada, red	170
Fir, Baltic, mean	153
" Canada, yellow	{ 58 / 117
" " red	120
" Norway	123
" Dantzic	163
" Riga	112
" Memel	161
" " red	75
Greenheart, Guiana	160
Gum, blue	136
Hackmatack	102
Hemlock	100

Woods.

Hickory	{ 170 / 250
Iron wood, Burmah	240
Larch, Russian	118
Lignumvitæ	162
Locust	295
Mahogany	112
Mangrove	162
Maple	202
Oak, white	150
" live	160
" red, black	135
" African	207
" English	{ 105 / 157
" French	160
" Dantzic	88
" Canada	146
" Sardinia	142
" Spanish	105
Pine, white	125
" pitch	137
" yellow	130
" Georgia	200
Poon	184
Poplar	112
Spruce, Canada	125
" black	87
Sycamore	125
Tamarack	100
Teak	165
Walnut	112
Willow	87
Whitewood	116

Stones, Bricks, etc.

Brick, common, mean	20
" pressed, "	40
" English, stock	11.8
" " fine	14
Brick arch	15
Cement, mean	15
" " Portland	{ 10.2 / 37.5
" " Sheppey	5
" " hydraulic, Portland	5
" " Roman	2
" Puzzuolana	4.5
" Portland, 1 year	8
" Roman, "	2.5
Concrete, Eng., fire-brick beam, cement	3.1

* This was with a tensile strength of 27 000 lbs.
† With 840 lbs. the deflection was 1 inch, and the elasticity of the metal destroyed.

Stones, Bricks, etc.	
Concrete, Eng., fire-brick, sand 3, lime 1	.7
" Eng., clay and chalk	5.4
Flagging, blue, New York	31.25
Freestone, Conn.	13
" Dorchester	10.8
" New Jersey, mean	19
" New York	24
" Eng., Craigleth	10.7
" " Darby, Victoria	1.3
" " Park Spring	4.3
Glass, flooring	42.5
Granite, blue, coarse	18
" Quincy	26
" mean	25
" Eng., Cornish	22
Limestone	
" English	11
Marble	

Stones, Bricks, etc.	
Marble, Adelaide	4.5
" Italian	
Mortar, lime. 60 days	2.5
" 1 lime, 1 sand	2
" 1 " 2 "	1.75
" 1 " 4 "	1.25
Oolite, English, Portland	21.2
Paving, Scotch, Caithness	68
" Ireland, Valentia	68.5
" Welsh	157
" English, Yorkshire, blue	10.4
" " Arbroath	17
Slate	81
" Bangor	90
" English, Llangollen	43
Stones, English, Bath	5.2
" " Kentish, Rag	35.8
" " Yorkshire, landing	22.5
" Caen	12.5

Elastic Transverse Strength of Woods, compared with their Breaking Weight, is as follows:

	Per Cent.		Per Cent.		Per Cent.
Ash	29	Norway Spruce	30	Red Pine	29
Beech	25	Oak, Dantzic	36	Riga Fir	30
Elm	32	" English	33	Teak	32
Larch	38	Pitch Pine	24	Yellow Pine	30

Increase in Strength of several Woods by Seasoning.

Per Cent.

Ash......44.7 | Beech......61.9 | Elm......12.3 | Oak.....26.1 | White pine....9

Concretes, Cements, etc.

Materials.	Breaking Weight.	Materials.	Breaking Weight.
CONCRETES (English).	Lbs.	BRICKS (English).	Lbs.
Fire-brick beam, Portl'd cement	3.1	Best stock	11.8
" sand 3 parts, lime 1 part	.7	Fire-brick	14
CEMENTS (English).		New brick	10.7
Blue clay and chalk	5.4	Old brick	9.1
Portland	37.5 10.2	Stock-brick, well burned	5.8
		" inferior, burned	2.5
Sheppey	5		

Transverse Strength of Various Figures of Cast Iron.

Reduced to Uniform Measure of Sectional Area of One Inch Square and One Foot in Length. Fixed at one End; Weight suspended from the other.

Form of Bar or Beam.	Breaking Weight.	Form of Bar or Beam.	Breaking Weight.
	Lbs.		Lbs.
 Square	673	 Rectangular prism. 2 × .5 ins. in depth	1456
		" 3 × .33 " in depth	2392
		" 4 × .25 " in depth	2652
 Square, diagonal vertical	568	 Equilateral triangle, an edge up	560
 Cylinder	573	 Equilateral triangle, an edge down	958
		 2 ins. in depth × 2 × .268 inch in width	2068
 Hollow cylinder; greater diameter twice that of lesser	794	 2 ins. in depth × 2 × .268 inch in width	555

Solid and Hollow Cylinders of various Materials.

One Foot in Length. Fixed at one End; Weight suspended from the other.

Materials.	External Diam.	Internal Diam.	Breaking Weight.	Materials.	External Diam.	Internal Diam.	Breaking Weight.
WOODS.	Ins.	Inch.	Lbs.	METAL.	Ins.	Ins.	Lbs.
Ash	2	—	685	Cast iron, cold blast	3	—	12 000
"	2	1	604				
Fir*	2	—	772	STONE-WARE.			
White pine	1	—	75	Rolled pipe of fine clay	2.87	1.928	190
" "	2	—	610				

* An inch-square batten, from same plank as this specimen, broke at 139 lbs.

Formulas for Transverse Stress of Rectangular Bars, Beams, Cylinders, etc.

Fixed at One End. Loaded at the Other.

Bars, Beams, etc. $\frac{l\,W}{b\,d^2}=S$; $\frac{S\,b\,d^2}{l}=W$; $\frac{S\,b\,d^2}{W}=l$; $\frac{l\,W}{S\,d^2}=b$; $\sqrt{\frac{l\,W}{S\,b}}=d$;

and Cylinder $\sqrt[3]{\frac{l\,W}{S}}=b$ *and* d.

Fixed at Both Ends. Loaded in Middle.

Bars, Beams, etc. $\frac{l\,W}{6\,b\,d^2}=S$; $\frac{6\,S\,b\,d^2}{l}=W$; $\frac{6\,S\,b\,d^2}{W}=l$; $\frac{l\,W}{6\,S\,d^2}=b$;

$\sqrt{\frac{l\,W}{6\,S\,b}}=d$; and Cylinder $\sqrt[3]{\frac{l\,W}{6\,S}}=b$ *and* d.

Fixed at Both Ends. Loaded at any Other Point than in Middle.

Bars, Beams, etc. $\frac{2\,m\,n\,W}{3\,l\,b\,d^2}=S$; $\frac{3\,l\,b\,d^2\,S}{2\,m\,n}=W$; $\frac{2\,m\,n\,W}{3\,S\,b\,d^2}=l$; $\sqrt{\frac{2\,m\,n\,W}{3\,S\,l\,d^2}}=b$;

$\sqrt{\frac{2\,m\,n\,W}{3\,S\,l\,b}}=d$; and Cylinder $\sqrt[3]{\frac{2\,m\,n\,W}{3\,S\,l}}=b$ *and* d.

Supported at Both Ends. Loaded in Middle.

Bars, Beams, etc. $\frac{l\,W}{4\,b\,d^2}=S$; $\frac{4\,S\,b\,d^2}{l}=W$; $\frac{4\,S\,b\,d^2}{W}=l$; $\frac{l\,W}{4\,S\,d^2}=b$;

$\sqrt{\frac{l\,W}{4\,S\,b}}=d$; and Cylinder $\sqrt[3]{\frac{l\,W}{4\,S}}=b$ *and* d.

Supported at Both Ends. Loaded at any Other Point than in Middle.

Bars, Beams, etc. $\frac{m\,n\,W}{l\,b\,d^2}=S$; $\frac{S\,l\,b\,d^2}{m\,n}=W$; $\frac{m\,n\,W}{S\,b\,d^2}=l$; $\frac{m\,n\,W}{S\,l\,d^2}=b$;

$\sqrt{\frac{m\,n\,W}{S\,l\,b}}=d$; and Cylinder $\sqrt[3]{\frac{m\,n\,W}{S\,l}}=b$ *and* d.

In Square Beams, etc., for b and d put $\sqrt[3]{\frac{l\,W}{S}}=\sqrt{\frac{l\,W}{S\,b}}=d$. In Cylinders, for $b\,d^2$ put d^3 as above.

When weight is uniformly distributed, same formulas will apply, W *representing only half required or given weight.*

S *representing stress in a Bar, Beam, or Cylinder, one foot in length, and one inch square, side, or in diameter; and* W *weight, in lbs.;* b *breadth, and* d *depth, in ins.;* l *length,* m *distance of weight from one end, and* n *from the other, all in feet.*

Brick-work.

A brick arch, having a rise of 2 feet, and a span of 15 feet 9 ins., and 2 feet in width, with a depth at its crown of 4 ins., bore 358 400 lbs. laid along its centre.

Coefficient or Factor of Safety.

Coefficient or factor of safety of different materials must be taken in view of importance of structure, or instrument, probable or required period of duration of it, and if it is to bear a quiescent, vibratory, gradual, or percussive stress, and to meet these varied conditions, it will range from .125 to .3 of the maximum or ultimate strength here given or ascertained.

To Compute Transverse Strength of a Rectangular Bar or Beam.

When a Bar or Beam is Fixed at One End, and Loaded at the Other. RULE.—Multiply *Coefficient* of material in preceding Tables, or, as may be ascertained, by breadth and square of depth in ins., and divide product by length in feet.

NOTE.—When a beam, etc., is loaded uniformly throughout its length, result must be doubled.

EXAMPLE.—What weight will a cast-iron bar, 2 ins. square and projecting 30 ins. in length, bear without permanent injury?

Assume strength of material at 660, and its elasticity at one fifth or .2 of its strength.

$$\text{Then } \frac{660 \times .2 \times 2 \times 2^2}{2.5} = \frac{1056}{2.5} = 422.4 \textit{ lbs.}$$

If Dimensions of a Beam or Bar are Required to Support a Given Weight at its End. RULE.—Divide product of weight and length in feet by *Coefficient* of material, and quotient will give product of breadth and square of depth.

EXAMPLE.—What is the depth of a wrought-iron beam, 2 ins. broad, necessary to support 576 lbs. suspended at 30 ins. from fixed end?

Assume strength of iron at 150.

$$\text{Then } \frac{2.5 \times 576}{150} = 9.6, \text{ and } \sqrt{\frac{9.6}{2}} = 2.19 \textit{ ins. depth.}$$

When a Beam or Bar is Fixed at Both Ends, and Loaded in the Middle. RULE.—Multiply *Coefficient* of material by 6 times breadth and square of depth in ins., and divide product by length in feet.

NOTE.—When beam is loaded uniformly throughout its length, result must be doubled.

EXAMPLE.—What weight will a bar of cast iron, 2 ins. square and 5 feet in length, support in middle, without permanent injury?

Assume strength of material as in a previous case at .2 of 660.

$$\text{Then } \frac{660 \times .2 \times 2 \times 6 \times 2^2}{5} = \frac{6336}{5} = 1267.2 \textit{ lbs.}$$

If Dimensions of a Beam or Bar are Required to Support a Given Weight in Middle, between Fixed Ends. RULE.—Divide product of weight and length in feet by 6 times *Coefficient* of material, and quotient will give product of breadth and square of depth.

EXAMPLE.—What dimensions will a square cast-iron bar, 5 feet in length, require to support without permanent injury a stress of 2160 lbs.?

Assume strength of material at .2 of 660 or 132, as preceding.

Then $\frac{2160 \times 5}{132 \times 6} = \frac{10\,800}{792} = 13.64$, *which, divided by 2 for assumed breadth* $= 6.8\dot{2}$, and $\sqrt{6.82} = 2.61$ *ins. depth.*

When Breadth or Depth is Required. RULE.—Divide product obtained by preceding rules by square of depth, and quotient is breadth; or by breadth, and square root of quotient is depth.

EXAMPLE.—If 128 is the product, and depth is 8; then $128 \div 8^2 = 2$, *breadth.* Also, $128 \div 2 = 64$, and $\sqrt{64} = 8$, *depth.*

When Weight is not in Middle between Ends. RULE.—Multiply *Coefficient* of material by 3 times length in feet, and breadth and square of depth in ins., and divide product by twice product of distances of weight, or stress from either end.

EXAMPLE.—What weight will a cast-iron bar, fixed at both ends, 2 ins. square and 5 feet in length, bear without permanent injury, 2 feet from one end?

Assume strength of material at .2 of 660 or 132, as preceding.

$$\text{Then } \frac{132 \times 3 \times 5 \times 2 \times 2^2}{2 \times (2 \times 3)} = \frac{15\,840}{12} = 1320 \textit{ lbs.}$$

When a Beam or Bar is Supported at Both Ends, and Loaded in Middle. RULE.—Multiply *Coefficient* of material by 4 times breadth and square of depth in ins., and divide product by length in feet.

NOTE.—When beam is loaded uniformly throughout its length, result must be doubled.

EXAMPLE.—What weight will a cast-iron bar, 5 feet between the supports, and 2 ins. square, bear in middle, without permanent injury?

Assume strength of iron at 132, as preceding.

$$\text{Then } 132 \times 2 \times 4 \times 2^2 = 4224 \div 5 = 844.8 \textit{ lbs.}$$

If Dimensions are Required to Support a Given Weight. RULE.—Divide product of weight and length in feet by 4 times *Coefficient* of material, and quotient will give product of breadth, and square of depth.

When Weight is not in Middle between Supports. RULE.—Multiply *Coefficient* of material by length in feet, and breadth and square of depth in ins., and divide product by product of distances of weight, or stress from either support.

EXAMPLE.—What weight will a cast-iron bar, 2 ins. square and 5 feet in length, support without permanent injury, at a distance of 2 feet from one end, or support?

Assume strength of iron at 132, as preceding.

$$\text{Then } \frac{132 \times 5 \times 2 \times 2^2}{2 \times (5-2)} = \frac{5280}{6} = 880 \textit{ lbs.}$$

To Compute Pressure upon Ends or upon Supports.

RULE 1.—Divide product of weight and its distance from nearest end or support, by whole length, and quotient will give pressure upon end or support farthest from weight.

2.—Divide product of weight and its distance from farthest end, or support, by whole length, and quotient will give pressure upon end or support nearest weight.

EXAMPLE.—What is pressure upon supports in case of preceding example?

$\frac{880 \times 2}{5} = 352$ *lbs. upon support farthest from the weight;* $\frac{880 \times 3}{5} = 528$ *lbs. upon support nearest to weight.*

When a Bar or Beam, Fixed or Supported at Both Ends, bears Two Weights at Unequal Distances from Ends.

$$\frac{m\,W}{L} + \frac{l\,w}{L} = \textit{pressure at } w \textit{ end, and } \frac{n\,w}{L} + \frac{l'\,W}{L} = \textit{pressure at } W \textit{ end.}$$

m and n representing distances of greatest and least weights from their nearest end, W *and w greatest and least weights,* L *whole length, l distance from least weight to farthest end, and l′ distance of greatest weight from farthest end.*

ILLUSTRATION.—A beam 10 feet in length, having both ends fixed in a wall, bears two weights—viz., one of 1000 lbs., at 4 feet from one of its ends, and the other of 2000 lbs., at 4 feet from the other end; what is pressure upon each end?

$$\frac{4 \times 2000}{10} + \frac{6 \times 1000}{10} = 1400 \textit{ lbs. at } w; \quad \frac{4 \times 1000}{10} + \frac{6 \times 2000}{10} = 1600 \textit{ lbs. at } W.$$

When Plane of Bar or Beam Projects Obliquely Upward or Downward.

When Fixed at One End and Loaded at the Other. RULE.—Multiply *Coefficient* of material by breadth and square of depth in ins., and divide product by product of length in feet and *cosine* of angle of elevation or depression.

NOTE.—When beam is loaded uniformly along its length, result must be doubled.

EXAMPLE.—What is weight an ash beam, 5 feet in length, 3 ins. square, and projecting upward at an angle of 7° 15′, will bear without permanent injury?

Assume breaking weight of ash at 160, and its elasticity at .25 of its strength, and *cosine* of 7° 15′ = .992.

$$\text{Then } \frac{160 \times .25 \times 3 \times 3^2}{5 \times .992} = 1080 = 217.74 \textit{ lbs.}$$

To Compute Transverse Strength of an Equilateral Triangle or T Beam.

RULE.—Proceed as for a rectangular beam, taking following proportions of *Coefficient* of material:

Fixed at One or Both Ends.	Equilateral triangle, edge up	$b \times d^2 \times .2$ C
	Equilateral triangle, edge down	$b \times d^2 \times .34$ "
	T beam, flange up	$b \times d^2 \times .42$ "
Supported at Both Ends.	Equilateral triangle, edge up	$b \times d^2 \times .34$ "
	Equilateral triangle, edge down	$b \times d^2 \times .2$ "
	T beam, flange up	$b \times d^2 \times .42$ "

To Compute Transverse Strength of a Solid Cylinder.

RULE.—Proceed as for a rectangular beam, and take .6 of *Coefficient* or of product.

A mean of 18 results with cold blast gun-metal, gave a coefficient for 740 lbs.

When Fixed at One End, and Loaded at the Other. RULE.—Multiply weight to be supported in lbs. by length of cylinder in feet; divide product by .6 of *Coefficient* of material, and cube root of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly throughout its length, cube root of half quotient will give diameter.

EXAMPLE.—What should be diameter of a cast-iron cylindrical beam of gun-metal, 8 ins. in length, to break at 15 000 lbs.?

$$\sqrt[3]{\frac{15\,000 \times \overline{8 \div 12}}{.6 \times 740}} = \sqrt[3]{\frac{10\,000}{444}} = 2.61 \textit{ ins.}$$

When Fixed at Both Ends, and Loaded in Middle. RULE.—Multiply weight to be supported in lbs. by length of cylinder between supports in feet; divide product by .6 of *Coefficient* of material, and cube root of one sixth of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly along its length, cube root of half the quotient will give diameter.

EXAMPLE.—What is the diameter of a cast-iron cylinder of gun-metal, 2 feet between supports, that will break at 35 964 lbs.?

$$\frac{35\,964 \times 2}{.6 \times 740} = 162, \text{ and } \sqrt[3]{\frac{162}{6}} = 3 \textit{ ins.}$$

Mean results of cylinder and square bars gave 444 and 740 lbs. Hence, strength of a cylinder compared to a square is as 444 to 740 or .6 to 1.

$$\text{Then } \frac{4 \times 3^3 \times 444}{1} = 47\,952 \textit{ lbs.}$$

To Compute Diameter of a Solid Cylinder to Support a given Weight.

When Supported at Both Ends, and Loaded in Middle. RULE.—Multiply weight to be supported in lbs. by length of cylinder between supports in feet; divide product by .6 of *Coefficient* of material, and cube root of one fourth of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly along its length, cube root of half the quotient will give diameter.

EXAMPLE.—What is diameter of a cast-iron gun-metal cylinder, 1 foot between its supports, that will break at 48 000 lbs.?

$$\frac{48\,000 \times 1}{.6 \times 740} = 108, \text{ and } \sqrt[3]{\frac{108}{4}} = 3.61 \text{ ins.}$$

Rectangular. (*D. K. Clark.*)

(1) *Loaded at Middle.* $\frac{8\,b\,d^2}{l} = W.$ (2) *Loaded at One End.* $\frac{2\,b\,d^2}{l} = W.$

Cylindrical.

(3) *Loaded at Middle.* $\frac{5.5\,b\,d^2}{l} = W.$ (4) *Loaded at One End.* $\frac{1.375\,b\,d^2}{l} = W.$

W *representing ultimate stress in tons.*

Above *Coefficients* are for iron of a tensile strength of 7 tons per sq. inch.

	(1)	(2)	(3)	(4)		(1)	(2)	(3)	(4)
Hence, for 8 tons put	9.2	2.3	6.3	1.6	For 12 tons put	13.8	3.4	9.4	2.4
9 "	10.4	2.6	7.1	1.8	13 "	14.5	3.6	10.2	2.6
10 "	11.5	2.9	7.9	2	14 "	16	4	11	2.8
11 "	12.7	3.2	8.6	2.2	15 "	17.2	4.3	11.8	3

To Compute Destructive Weight, or Loads that may be borne by Wrought-iron Rolled Beams and Girders, or Riveted Tubes of various Figures and Sections.

Supported at Both Ends. Load applied in Middle.

When Section of Beam or Girder is that of any of the Figures in following Table. RULE.—Divide product of area of section, depth, and *Coefficient* for girder, etc., from following Table, by length between supports in feet, and quotient will give destructive weight in lbs.

NOTE.—The *Coefficients* given are based upon experiments with English iron.

Solid Beams.

ILLUSTRATION.—What load will destroy a wrought-iron grooved beam of following dimensions, 10 feet in length between supports, and loaded in its middle?

Flanges, 5.7 × .64 inch; Web, .6 inch; Depth, 11.75 ins.; Area, 13.34 sq. ins.

Assume *Coefficient* 4638 as for like case (12) in following table, page 806.

$$\frac{13.34 \times 11.75 \times 4638}{10} = \frac{726\,821}{10} = 72\,682.1 \text{ lbs.}$$

Ultimate stress for such a beam by experiment was estimated at 97 997 lbs.

Formulas of Various Authors give following Results:

D. K. CLARK. $\frac{d\,(4\,a + 1.1555\,a')}{.6\,l} = W.$ *a representing area of section of lower flange, a′ area of section of web, less one flange, d depth of beam, less average depth of one flange, all in ins., l length in feet, and* W *ultimate destructive weight in tons.*

This formula is based upon the assumption that the beam has lateral support.

$$\frac{11.75 - .6\,(4 \times \overline{5.7 \times .6} + 1.155 \times \overline{11.75 - .6 \times .6})}{.6 \times 10} = \frac{238.69}{6} = 39.78, \text{ which} \times 2240$$
$= 88\,107$ *lbs.*

MOLESWORTH. $\frac{4\,C\,b\,d^2}{l} = W.$ C = 7616 *lbs., and for* $b\,d^2$ *put* $b\,d' - 2\,b'\,d'^2$. *b and d representing exterior and b′ and d′ interior dimensions, and l length in ins.*

$$5.7 \times 11.75^2 - [5.7 - .6 \times \overline{11.75} - (.64 \times 2^2)] = 786.6 - 558.9 = 227.7.$$

$$\text{Then } \frac{4 \times 7616 \times 227.7}{10 \times 12} = \frac{693\,665}{120} = 57\,805.4 \text{ lbs.}$$

Fairbairn's formula would give a result less than half of the first, and Hodgkinson's alike to that of Molesworth.

WROUGHT IRON.

Transverse Strength of Wrought-iron Rolled Beams and Girders. *(Barlow, Fairbairn, Hughes, Kirkaldy, etc.)*

Reduced to Uniform Measure of One Foot in Length.

Supported at Both Ends; Stress or Weight applied in Middle.

Section.	Flanges.	Web.	Depth (d).	Distance.		Area (A).	Destructive Weight. For Distance.	Destructive Weight. Length of One Foot (l).	$\frac{l\,W}{A\,d} = C$
	Ins.	Ins.	Ins.	Feet.	Ins.	Sq. Ins.	Lbs.	Lbs. (W).	
	—	1	1	1		1	2 500	2 500	2 500
	—	2	2	2	9	4	6 600	18 150	2 266
	—	1.5	3	2	9	4.5	10 080	27 720	2 053
"	—	1	3	3		3	7 050	21 150	2 350
	—	1	1	5		.78	474	2 370	2 370
	3.5 × .6	.8	3.5	2	7	5.65	20 160	52 480	2 654
	2.5 × 1 4 × .38	.325	7	2	9	5.9	44 000	121 000	2 930
	2.6 × 1.25	.85	5	4	6	7.44	19 000	85 500	2 298
"	3 × .49	.5	7.07	10		5.87	24 200	242 000	5 830
	4.6 × .8	.5	9.85	20		11.5	38 080	761 600	6 724
	5.7 × .64	.6	11.75	10		13.34	72 688	726 880	4 638
	2.85 × .38	.31	2.5	4		1.75	3 150	12 600	2 880
	7 × .5 4 × .5	.38	16.5	22	6	18.9	49 280	1 108 800	3 556
	4.5 × .375 2 × 2 × .3125	.38	14.25	16	5	10.5	47 000	775 500	5 183
	4.5 × 28 4.5 × 3	.25	7	7		6.35	24 380	170 660	3 840
	3.9 3.9	.13	6	7	6	2.62	9 976	74 820	4 766
	15.5 15.5	.53	24	30		41.4	128 885	3 866 550	3 896
"	24 24	.75 .75	35.75	45		87.38	257 080	11 568 600	3 703
	—	.131	12.4 12.138	10		5.05	17 885	178 850	2 856
	—	.143	15 9.75	10		5.56	26 250	262 500	3 147
Steel. 	—	.75	5.2	5		7.72	102 480	512 400	12 760

These results are very conclusive of the correctness of above formula, as will be seen in cases given, and they are deduced from beams and girders varying from 1 to 45 feet in length; hence, when length of a beam or girder of any of the sections given is less, relative breaking weight may be increased, in consequence of increased stability of beam or girder.

For full experiments on Tubes and Tubular Girders, etc., see *Rep. of Comm's on Railway Structures, London*, 1849.

Tensile strength of iron assumed at 45 000 lbs. per sq. inch.

Elements of Rolled Wrought-iron Beams and Channel Bars.

With Safe Load Uniformly Distributed. For Length of One Foot.

The New Jersey Steel and Iron Co., Trenton, N. J.

(Beams Supported Sidewise.)

Depth.		Designation.	Width. Web.	Width. Flange.	Area. Web.	Area. Flange.	Area. Total.	Weight per Foot.	Load.
		I Beams.	Inch.	Ins.	Sq. Ins.	Sq. Ins.	Sq. Ins.	Lbs.	Lbs.
4	inch.	Extra Light.	.1875	2	.75	1.02	1.77	6	18 000
4	"	Light.	.25	2.75	1	1.91	2.91	10	30 100
4	"	Heavy.	.3125	3	1.25	2.41	3.66	12.3	36 800
5	"	Light.	.25	2.75	1.2	1.79	2.99	10	38 700
5	"	Heavy.	.3125	3	1.56	2.34	3.9	13.3	49 100
6	"	Light.	.25	3	1.5	2.51	4.01	13.3	62 600
6	"	Heavy.	.3	3.5	1.8	3.11	4.91	16.7	76 800
6	"	90 lbs.	.5	5	3	5.7	8.7	30	132 000
6	"	120 "	.625	5.25	3.75	8.09	11.84	40	172 000
7	"	55 "	.3	3.75	2.1	3.4	5.5	18.3	101 000
8	"	Light.	.3	4	2.4	3.97	6.37	21.7	135 000
8	"	Heavy.	.375	4.5	2.96	5.07	8.03	26.7	168 000
9	"	Light.	.3	4	2.7	4.3	7	23.3	167 000
9	"	Heavy.	.375	4.5	3.38	5.12	8.5	28.3	199 000
9	"	Extra Heavy.	.57	4.5	5.13	7.2	12.33	41.7	268 000
10.5	"	Extra Light.	.3125	4.5	3.28	5.62	8.9	30	250 000
10.5	"	Light.	.375	4.5	3.94	6.5	10.44	35	286 000
10.5	"	Heavy.	.47	5	4.93	8.43	13.36	45	360 000
12.25	"	Light.	.47	4.8	5.75	6.58	12.33	41.7	377 000
12.25	"	Heavy.	.6	5.5	7.39	9.38	16.77	56.7	511 000
15	"	Light.	.5	5	7.59	7.45	15.04	50	551 000
15	"	Heavy.	.6	5.75	9.07	10.95	20.02	66.7	748 000
		Deck Beams.							
7	"		.31	4.5	2.17	3.18	5.35	18.3	63 500
8	"		.38	4.5	3.04	3.25	6.29	21.7	91 800
		Channels.							
3	"	Extra Light.	.2	1.5	.6	.85	1.45	5	10 500
4	"	" "	.2	1.5	.8	.85	1.65	5.5	15 700
5	"	" "	.2	1.625	1	.92	1.92	6.3	22 800
6	"	" "	.18	1.875	1.08	1.17	2.25	7.5	33 680
6	"	Light.	.28	2.25	1.68	1.52	3.2	11	45 700
6	"	Heavy.	.4	2.5	2.4	1.92	4.32	15	58 300
7	"	Extra Light.	.2	2	1.4	1.14	2.54	8.5	39 500
7	"	Light.	.25	2.5	1.75	1.85	3.6	12	62 000
8	"	Extra Light.	.2	2.2	1.6	1.7	3.3	11	65 800
8	"	Light.	.26	2.5	2.08	2.4	4.48	15	88 950
9	"	"	.33	2.5	2.97	2.11	5.08	16.7	104 000
9	"	Heavy.	.43	3.125	3.87	3.15	7.02	23.3	146 000
10.5	"	Light.	.375	2.75	3.94	2.06	6	20	134 750
12.25	"	"	.33	3	4.04	2.96	7	23.3	200 100
12.25	"	Heavy.	.68	4	8.33	5.77	14.1	46.7	381 000
15	"	Light.	.5	4	7.5	4.5	12	40	401 000
15	"	Heavy.	.75	4.75	11.25	7.6	18.85	63.3	625 000

The loads given in the table are such as will effect a maximum strain upon the metal of 12 000 lbs. per sq. inch. For permanent stress, absolutely free from vibration, a greater strain would be allowable, and, contrariwise, if the stress is mainly that of a live load the loads here given should be reduced.

A difference of 25 per cent. in either direction should be made, according to the character of the load to be supported or stress to be borne.

Steel beams have greater estimated strength than iron, but their stiffness is not materially greater.

Elastic Transverse Strength of Wrought-iron Bars is about 45 per cent. of their transverse strength, and of Plates 55 per cent., or 48 per cent. of their tensile strength; of solid rolled beams, 50 per cent.; and of double-headed rails, 46 per cent. of their transverse strength; of Fagersta Steel, 56 per cent. of its transverse strength; of double-headed Steel rails, 47 per cent.; of Bessemer Steel, 37.5 to 48 per cent.; of Steel flanged, 68 per cent.; and of Wrought-iron Steel flanged, 62 per cent. of its transverse strength.

Transverse strength of Solid Cast-iron Beams or Girders is about 50 per cent. of ultimate strength; of double-headed or flanged rails, 46 per cent.; and of single-flanged rails, 62 per cent. of its tensile strength.

NOTE.—The actual breaking weight of a 10.5 ins. beam of New Jersey Steel and Iron Co., weight 35 lbs. per foot, for a length of span of 20 feet, is 30 000 lbs.

Channel and Deck Beams and Strut Bars.

With Safe Load Uniformly Distributed for Length of One Foot.

(Beam supported Sidewise.)

Depth.	Designation.	Width. Web.	Width. Flange.	Area. Section.	Weight per Foot.	Load.	Strength as Strut. Sidewise.	Strength as Strut. Edgewise.
Ins.	CHANNEL.	Inch.	Ins.	Sq. Ins.	Lbs.	Lbs.	Lbs.	Lbs.
3	Extra Light	.2	1.5	1.45	5	10 500	51	341
4	" "	.2	1.5	1.65	5.5	15 700	49	597
5	" "	.2	1.625	1.92	6.33	22 800	57	930
6	" "	.18	1.875	2.25	7.5	33 680	77	1403
6	Light	.28	2.25	3.2	11	45 700	101	1343
6	Heavy	.4	2.5	4.32	15	58 300	123	1257
7	Extra Light	.2	2	2.54	8.33	39 500	82	1700
7	" "	.25	2.5	3.6	12	62 000	136	1883
8	" "	.2	2.2	3.3	11	65 800	109	2493
8	Light	26	2.5	4.48	15	88 950	142	2480
9	"	.33	2.5	5.08	16.33	104 000	124	2892
9	Heavy	.43	3.125	7.02	23.33	146 000	190	2925
10.5	Light	.375	2.75	6	20	134 750	160	3685
12.25	"	.42	3	8.62	28.33	238 000	172	5275
12.25	Heavy	.68	4	14.1	46.66	381 000	317	5170
15	Light	.5	4	12	40	401 000	301	7833
15	Heavy	.75	4.75	18.85	63.33	625 000	428	7762
7	DECK BEAMS.	.31	4.5	5.35	21.66	63 500	351	36
8	DECK BEAMS.	.38	4.5	6.29	18.38	91 800	547	37
	STRUT BARS.							
5	Light, Single	—	—	1.55	5.33	9 100	44	457
5	Heavy, "	—	—	2.15	7.33	11 900	48	433

Operation of Table.

To Compute Depth of a Beam to Support a Uniformly Distributed Load.

RULE.—Multiply load in lbs. by length of span in feet, and take from table the beam, the load of which is nearest to and in excess of the product thus obtained.

EXAMPLE.—What should be depth of a beam to sustain with safety a uniformly distributed load of 30 000 lbs., over a span of 15 feet?

$30\,000 \times 15 = 450\,000$, which is load for a heavy beam 12.25 ins. in depth.

Weight of beam should be added to load.

Inversely.—If the load is required, divide load in table by span of beam in feet, and subtract weight of beam.

To Compute Deflection of Like Beams.

RULE.—Divide square of span in feet by 70 times depth of beam in ins.

EXAMPLE.—Assume beam as preceding.

$$\frac{15^2}{70 \times 12.25} = \frac{225}{857.5} = .262 \; ins.$$

Comparative Strength and Deflection of Cast-iron Flanged Beams.

Description of Beam.	Comp. Strength.	Description of Beam.	Comp. Strength.
Beam of equal flanges.........	.58	Beam with flanges as 1 to 4.5...	.78
" with only bottom flange..	.72	" with flanges as 1 to 5.5...	.82
" with flanges as 1 to 2....	.63	" with flanges as 1 to 6.....	1
" with flanges as 1 to 4....	.73	" with flanges as 1 to 6.73..	.92

Dimensions and Proportions of Wrought-iron Flanged Beams. (*D. K. Clark.*)

Depth.	Breadth of Flanges.	Thickness.		Weight per Lineal Foot.	Ultimate Strength. Loaded in Middle.	Safe Stress Uniformly Distributed.
		Web.	Flanges.			
Ins.	Ins.	Inch.	Inch.	Lbs.	Lbs.	Lbs.
3	2	.1875	.2187	5.5	2 800	910
3	3	.25	.3125	10	5 600	1 860
3.125	1.625	.1875	.2187	5.5	2 490	830
4	2	.25	.3125	8	5 490	1 830
4	3	.25	.375	12	8 510	2 830
4.75	2	.25	.3125	8	6 940	2 310
5	3	.3125	.4375	13	13 440	4 480
5	4.5	.375	.5	23	19 270	6 420
5.5	2	.375	.4375	10	11 880	3 960
6	5	.4375	.5625	30	23 830	7 940
6.25	2	.3125	.4375	11	13 440	4 440
6.25	2.25	.3125	.375	18	13 000	4 330
6.25	3.25	.3125	.4062	12.5	17 470	5 820
7	2.25	.281	.375	14	14 790	4 930
7	2.25	.3125	.4375	14	17 020	5 670
7	3.625	.3125	.4375	19	23 300	7 760
7	3.625	.3125	.5	19	25 980	8 660
8	2.375	.3125	.4375	15	20 830	6 940
8	2.5	.375	.375	15	21 280	7 090
8	4	.375	.5	21	34 500	11 500
8	5	.375	.5625	29	44 800	14 930
8	5.125	.4375	.5625	29	47 040	15 680
9.25	3.75	.4375	.5	24	41 560	13 850
9.5	4.5	.375	.6875	30	59 360	19 750
10	4.5	.4375	.5625	32	56 000	18 660
10	4.75	.4375	.5625	32	58 240	19 410
10	4.75	.75	.625	36	76 160	25 390
12	5	.5625	.8175	42	100 800	33 600
12	6	.5625	.9375	56	136 640	45 530
14	5.5	.5625	.875	60	150 020	50 000
14	6	.5625	.8175	60	152 260	50 750
16	5.625	.75	.8175	62	188 160	62 720

Wrought-iron Rectangular Girders or Tubes. (*Riv'd.*)

Supported at Both Ends. Loaded in Middle.

$\frac{A\,d\,C}{l} = W$. *A representing area of section in sq. ins., d depth in ins., l length between supports in feet, and W destructive weight in lbs.*

Illustration.—What is the destructive weight of a rectangular girder, 35.75 ins in depth by 24 in breadth, metal .75 inch thick, and length between supports 45 feet?

Assume C or coefficient = 37 000, as per case (17) in preceding table, page 806, and area = 87.375 ins.

$$\text{Then } \frac{87.375 \times 35.375 \times 3700}{45} = \frac{11\,557\,523}{45} = 256\,833.8 \textit{ lbs.}$$

By experiment it was 257 080 *lbs.* By Inversion $\frac{W\,l}{C\,d} = A$, and $\frac{l\,W}{A\,C} = d$.

Hodgkinson's formula would give a result of 259 373 lbs., and Molesworth's 303 907 lbs.

Unequally Loaded Beams, etc.

$\frac{l^2 W}{4 m n} = w$. *l representing length between supports, and m and n distances from points of support, all in like denomination, and W and w destructive and safe weights, also in like denomination.*

To Compute Destructive Weight and Area of Bottom Plate.

$\frac{A d C}{l} = W$; $\frac{W l}{C d} = A$; and $\frac{W m n}{.25 C d l} = A$. A *representing area of plate in sq. ins., d and l depth and length, m and n distances of load at other points than in middle, all in feet, and W weight in lbs.*

NOTE.—Sufficient metal should be provided in sides to resist transverse and shearing stress, and in upper flange to resist crushing.

ILLUSTRATION.—What area of wrought iron is necessary in bottom plate of a rectangular tubular girder, 3 feet in depth, supported at both ends, and loaded in middle with 130 000 lbs.?

C, ascertained by experiment for destructive stress, 180 000 *lbs.*, and area 7.1 *sq. ins.*

$$\frac{130\,000 \times 30}{180\,000 \times 3} = 7.22 \text{ sq. ins.}$$

Wrought-iron Cylindrical Beams or Tubes.

$\frac{A d C}{l} = W$. ILLUSTRATION.—What is destructive weight of a cylindrical tube, 12.4 ins. in diameter, .131 inch in thickness, and 10 feet between its supports?

Area of metal = 5.05 sq. ins., and C = 2856, as in the 19th case of table, page 806.

$$\text{Then } \frac{5.05 \times 12.4 \times 2856}{10} = 17\,884.2 \text{ lbs.}$$

D. K. CLARK. $\frac{3.14\, d^2\, t\, S}{l} = W$. *d representing diameter, t thickness of metal, and l length, all in ins.,* S *tensile strength of metal per sq. inch, and* W *weight, both in lbs.*

$$S = 45\,000 \text{ lbs.} \quad \frac{3.14 \times 12.4^2 \times .131 \times 45000}{10 \times 12} = \frac{2\,846\,250}{120} = 23\,718.7 \text{ lbs.}$$

MOLESWORTH's formula gives a result of 23 286.1 *lbs.*

Wrought-iron Elliptical Beams or Tubes.

$\frac{A d' C}{l} = W$. ILLUSTRATION.—Assume diameter of tube 9.75 and 15 ins., metal .143 inch in thickness, and distance between supports 10 feet.

A = 5.56 *sq. ins.* C = 3147, as per case (20) in preceding table, page 806.

$$\text{Then } \frac{5.56 \times 15 \times 3147}{10} = \frac{262\,459.8}{10} = 26\,245.9 \text{ lbs.}$$

D. K. CLARK. $\frac{1.57\,(b^2 + d^2)\, t\, S}{l} = W$. *b and d representing conjugate and transverse diameter, l length between supports, t thickness of metal, all in ins.,* S *tensile strength of metal per sq. inch, and* W *destructive weight, both in lbs.*

$$S = 44\,000 \text{ lbs.} \quad \frac{1.57\,(9.75^2 + 15^2) \times .143 \times 44\,000}{10 \times 12} = \frac{3\,161\,840}{120} = 26\,348.6 \text{ lbs.}$$

NOTE.—B. Baker, in his work on Strength of Beams, etc., London, 1870, page 26, shows that ordinary method of computing transverse strength of a hollow shaft by difference of diameter alone is erroneous, in consequence of loss of resistance to flexure in a hollow beam.

Girders and Beams of Unsymmetrical Section.

$\frac{4 S d}{l} = W$. S *representing tensile resistance of metal, and* W *destructive weight, both in lbs., d distance between centres of compression and extension, or crushing and tensile resistances, in ins., and l length between supports, in feet.*

NOTE.—To ascertain *d*, see Rule, page 819,

ILLUSTRATION.—Dimensions of a rolled wrought-iron girder, 11 feet in length between its supports, is as follows :

Top flange	2.5×1 inch.	Bottom flange	$4 \times .38$ inch.
Web	.325 "	Depth	7 ins.

What is its destructive weight?

$d = 5.22$ ins. S assumed at 45 000 lbs. Then $\frac{4 \times 45\,000 \times 5.22}{11 \times 12} = 7118.18$ *lbs.*

Strength of Riveted Beams or Girders, compared with Solid, is less, and deflection is greater.

Wrought-iron Inclined Beams, etc.

$\frac{L\,W}{l} = w$. *L and l representing lengths or inclination, and horizontal line, in like denominations, and W and w destructive and safe weights on horizontal line and inclination, also in like denominations.*

Plate Girders.

$\frac{A\,d\,C}{l} = W$. *A representing section in sq. ins., d depth in ins., and l length between supports in feet.*

ILLUSTRATION.—What load will destroy a wrought-iron plate girder or beam of following dimensions, 10 feet in length between its supports?

Top flange	$4.5 \times .375$ inch.	Width of web	.375 inch.
Bottom flange	$4.5 \times .375$ "	Depth of web	13.5 ins.
Angle pieces	$2 \times .3125$ "	Depth of beam	14.25 "

Area of Section = 13 *sq. ins.*

Assume coefficient of 5180 as per case (14) in preceding Table, page 806.

$$\text{Then } \frac{13 \times 14.25 \times 5180}{10} = \frac{960\,154}{10} = 96\,015.4 \textit{ lbs.}$$

MOLESWORTH. $\frac{L\,l}{8\,d} = W$. *L representing load equally distributed, and W destructive weight, both in tons, and d effective depth of girder in feet.*

By actual experiment L = 48 tons for 16.5 feet between supports; hence, $10 : 16.5 :: 48 : 79.2$ *tons* = 39.6 when supported in middle, and 14.25 *ins.* = 1.1875 *feet.*

$$\text{Then } \frac{39.6 \times 10}{8 \times 1.1875} = \frac{396}{9.5} = 41.68, \text{ which} \times 2240 = 93\,363.2 \textit{ lbs.}$$

D. K. CLARK. $\frac{d\,(4\,a + 1.155\,a')}{.6\,l} = W$. *d representing depth of girder or beam less depth of lower flange in ins., a and a′ areas of sections of bottom flange and of web, at its reputed depth, both in sq. ins., and l length between supports in feet.*

$d = 14.25 - .375 = 13.875$ *ins.* $a = 3$, and $a' = 5$ *sq. ins.*

$$\text{Then } \frac{13.875\,(4 \times 3 + 1.155 \times 5)}{.6 \times 10} = \frac{246.63}{6} = 41.105, \text{ which} \times 2240 = 92\,075.2 \textit{ lbs.}$$

Mr. Clark assumes, however, that for girders of like construction the destructive stress should be taken at two thirds of that deduced by the formula.

Girders or Beams without Upper and Lower Flanges.

ILLUSTRATION. — Assume angles $2.125 \times .28$ above, $2.125 \times .3$ below, web .25, depth 7 ins., and length between supports 7 feet.

Area of section = 6.35 *sq. ins.*, and C = 3840, as per case (15) in preceding Table, page 806.

$$\text{Then } \frac{6.35 \times 7 \times 3840}{7} = \frac{170\,688}{7} = 24\,384 \textit{ lbs.}$$

Approximate. $\frac{\frac{a}{2} + .25\,a' \times 5\,d}{l} = W$. *a representing area of sections of upper and lower angles, a′ area of section of web for total depth, both in sq. ins., d depth of girder in ins., and W load or stress in lbs.*

$a = 4.6$ *sq. ins.*, and $w = 7 \times .25 = 1.75$ *sq. ins.*

$$\text{Then } \frac{\frac{4.6}{2} + \frac{1.75}{4} \times 5 \times 7}{7} = \frac{90.81}{7} = 12.973, \text{ which} \times 2240 = 29\,059.5 \ lbs.$$

IRON AND STEEL RAILS.

Symmetrical Section.

To Compute Transverse Strength. (*D. K. Clark*)

$$\frac{S\left(4\,a\,\frac{d'^2}{d} + \overline{1.155\,t\,d^2}\right)}{l} = W, \text{ and } \frac{W\,l}{\left(4\,a\,\frac{d'^2}{d} + 1.155\,t\,d^2\right)} = S.$$

S representing tensile strength in lbs. or tons per sq. inch, a area of one head or flange exclusive of central portion composing web, in sq. ins., d′ depth or distance between centres of heads, d depth of rail, t thickness of web, l distance between supports, all in ins., and W *weight in lbs. or tons, alike to* S.

Illustration 1.—What is destructive weight of a wrought-iron double-headed rail, 5.4 ins. deep, having a web of .8 ins., an area of head of 1.9 sq. ins., distance between centres of its heads 4.2 ins., and between its supports 5 feet?

S assumed at 50 000 *lbs.*

$$\text{Then } \frac{50\,000\left(4 \times 1.9 \times \frac{4.2^2}{5.4} + \overline{1.155 \times .8 \times 5.4^2}\right)}{5 \times 12} = \frac{50\,000 \times (25.23 + 26.93)}{60} =$$

43 466.6 *lbs.*

2.—What is destructive weight of a Bessemer steel double-headed rail, 5.4 ins. deep, having a web of .75 inch, an area of head of 2 sq. ins., and distance between heads 4.2 ins.?

S assumed at 80 000 *lbs.*

$$\text{Then } \frac{80\,000\left(4 \times 2 \times \frac{4.2^2}{5.4} + \overline{1.155 \times .75 \times 5.4^2}\right)}{5 \times 12} = \frac{80\,000 \times 51.39}{60} = 68\,520 \ lbs.$$

Note.—Transverse strength of Bessemer Rails increases very generally, in direct proportion with the proportion of Carbon in it.

Unsymmetrical Section.

$$\frac{6.92\,S\,d''\,\Lambda}{l\,h} = W.$$

d″ representing vertical distance between centres of tension and compression, h height of neutral axis above base of section, and l length between supports, all in ins., and Λ sum of products, obtained by multiplying areas of strips of reduced section under tensile stress, by their mean distances, respectively, that is, the distances of their centres of gravity, from the neutral axis, in ins.

Bowstring Girder.

To Compute Diameter of a Wrought-iron Tie-rod of an Arched or Bowstring Girder of Cast Iron.

$$\sqrt{\frac{W\,l}{4500 \times h}} = d.$$

W *representing weight distributed over beam in lbs., l length between piers or supports in feet, and h height between centre of area of section of girder and centre of rod in ins.*

Illustration.—Required diameter of tie-rod for an arched girder, 25 feet between its piers, and 30 ins. between centres of its area and of rod, to safely support a uniformly distributed load of 25 000 lbs.?

$$\sqrt{\frac{25\,000 \times 25}{4500 \times 30}} = \sqrt{\frac{625\,000}{135\,000}} = \sqrt{4.62} = 2.15 \ ins.$$

If two rods are used. Then $\sqrt{\frac{4.62}{2}} = 1.52$ *ins. = diameter of each rod.*

CAST IRON.

Transverse Strength of Girders and Beams.

(Deduced from Experiments of Barlow, Hodgkinson, Hughes, Bramah, Cubitt, Tredgold, and others.)

Reduced to a Uniform Measure of One Foot in Length.

Supported at Both Ends. Stress or Weight applied in Middle.

Section.	Flanges.	Web.	Depth.	Distance.		Area.	Destructive Weight. For Distance.	Destructive Weight. Length of One Foot.	$\frac{l\,W}{A\,d} = C$
	Ins.	Ins.	Ins.	Feet.	Ins.	Sq. Ins.	Lbs.	Lbs.	
	—	1	1	1		1	2 240	2 240	2240
	—	1	1	4	6	1	500	2 250	2250
	—	3	3	13	6	9	5 080	68 580	2540
	—	1	3	4	6	3	5 100	22 950	2550
	—	1	4	4	6	4	10 300	46 350	2896
	4 × 2	2	4	5		12	6 720	33 600	700
	1.52 × .78	1.56	4.07	4	6	2.35	6 666	30 000	3136
	1.5 × .5	.5	3	3	1	2	5 208	16 145	2676
	1.5 × .5	.5	3	3	1	2	4 536	14 062	2331
	1.5 × .5	.5	4	3	1	1	7 104	22 420	5475
	1.5 × .5	.5	4	3	1	1	3 312	10 267	2553
	1.53 × 1	.5	2.04	4		2.6	4 004	16 016	3019
	2 × .51	1	2.02	4		2.59	2 569	10 276	1963
	—	—	2.52	5		4.98	4 143	20 715	1650
	—	—	2.83	5		4	2 988	14 940	1320
	2.28 × .53	.3 / .425	5.13	4	6	2.28	9 503	42 763	3656
	23.9 × 3.12	3.29	36.1	20		183.5	403 312	8 066 240	1220
	1.76 × .4	.29	5.13	4	6	2.82	6 678	30 512	2077
	1.74 × .26 1.78 × .55	.3	5.13	4	6	2.87	7 368	33 200	2250
	1.07 × .3 2.1 × .57	.32	5.13	4	6	3.02	8 270	37 215	2402
	1.54 × .32 6.5 × .51	.34	5.13	4	6	5.41	21 009	94 540	3406
"	2.5 × 1.5 3.75 × 1.4	1.25	8.18	11		15	35 620*	391 853	3193

* Stirling iron.

Hence, $\frac{A\,d\,C}{l} = W$. *A representing area of section, d depth in ins., l length in feet, and W destructive weight in lbs.*

NOTE.—When lengths are less than those instanced, breaking weight will be increased, in consequence of increased stability of girder.

To Compute Transverse Strength or Destructive Stress of Cast-iron Beams or Girders, of various Figures.

Supported at Both Ends. Weight applied in Middle.

When Section of Beam or Girder is alike to any of Examples given in preceding Table. RULE 1.*—Divide product of area of section and depth in ins., and *Coefficient* for girder, etc., from preceding Table. by length between supports in feet, and quotient will give breaking weight in lbs.

EXAMPLE.—Dimensions of a beam, having top and bottom flanges in proportion of 1 to 6, give an area of section of 25.6 sq. ins., a depth of 15.5 ins., and a length between its supports of 18 feet; what is its destructive weight?

NOTE.—In consequence of increased area of metal over case No. 21 in Table, *Coefficient* of 3402 is reduced to 3300.

Dimensions.—Top flange, $3 \times .75$ *ins.*; bottom, $18 \times .75$ $a = 13.5$ *sq. ins.*; web, $15.5 \times .7$ $a' = 10.8$ *sq. ins.*; and $d' = 15.5 - .75 = 14.75$ *ins.*

$$\text{Then } \frac{25.6 \times 15.5 \times 3300}{18} = \frac{1\,309\,440}{18} = 72\,746.6 \textit{ lbs.}$$

D. K. CLARK. $\frac{d'\,(6.5\dagger\, a + 2\,a')}{3\,l} = W.$ *a representing area of bottom flange, a' of web at depth d' of beam, less depth of bottom flange in sq. ins., l length between supports in feet, and W destructive weight in tons.*

$$\text{Then } \frac{14.75\,(7 \times 13.5 + 2 \times 10.8)}{3 \times 18} = \frac{1712.4}{54} = 31.71, \text{ which } \times 2240 = 71\,030.4 \textit{ lbs.}$$

HODGKINSON'S formula would give a result of 53 491.2 *lbs.*, and MOLESWORTH'S 54 248.3 *lbs.*

RULE 2.—From product of breadth and square of depth in ins. of rectangular solid, the dimensions of which are the depth and greatest breadth of beam in its centre, subtract product of breadths and square of depths of that part of the beam which is required to make it a rectangular solid, and then determine its resistance by rule for the particular case as to its being supported or fixed, etc.

This rule is applicable only in case referred to, viz., when area of section is great compared with area of extreme dimensions.

Mr. Baker, in case of a hollow cylindrical shaft, where thickness of metal is but one eighth of extreme diameter, computes result at but .4 of that of a solid beam. This is in consequence of resistance to flexure in hollow beam being more than proportionally greater than in solid.

EXAMPLE.—Take 7th case from preceding Table, page 813, for length of one foot. *Coefficient for cold-blast iron* = 500.

Then $\overline{1.52 \times 4.07^2 - 1.52 \times 2.51^2} \times 4 \times 500 = \overline{25.17 - 9.58} \times 2000 = 31\,180$ *lbs.*

Result as by experiment, 30 000 *lbs.*

NOTE 1.—These rules are applicable to all cases where flange of beam is as shown in Table, and beam rests upon two supports, or *contrariwise*, as to position of flange, when beam is fixed at one end only.

2.—When case under consideration is alike in its general character to one in Table, but differs in some one or more points, an increase or decrease of metal is obtained by an increase or reduction of the *Coefficient*, according as the differences may affect resistance of beam.

3.—The *Coefficients* here given are based altogether upon experiments with English iron.

---

* Utility of these rules in preference to those of Hodgkinson, Fairbairn, Tredgold, Hughes, and Barlow is manifest, as in one case the *Coefficient* of the metal is considered, and in the other cases the metal is assumed to be of a uniform value or strength.

Only variable element not embraced in this rule is that consequent upon any peculiarity of form of section; as, for instance, in that of a Hodgkinson, or like beam, where area of one flange greatly exceeds the rest of section, and this flange is other than below, when beam rests upon two supports or is fixed at both ends, or than above, when beam is fixed at one or both ends.

This deficiency is met to some extent by the three cases in table, where proportion of flanges are 1 to 2, 1 to 3, and 1 to 6.5.

† For thick castings put 7, and put *Coefficient* same as tensile strength of metal in tons per sq. inch.

Flanged Hollow or Annular Beams of Symmetrical Sections. (*D. K. Clark.*)

When Depth is Great Compared with Thickness of Flanges.—Figs. 1, 2, and 3.

1. 2. 3.

$$\frac{d \times S\,(4\,a + \overline{1.155\,a'})}{l} = W.$$ *a representing area of one flange, a′ area of web or ribs, both in sq. ins., d depth of beam, less depth of one flange, and l distance between supports, both in ins.,* S *tensile strength of metal, and* W *weight between supports, both in lbs.*

When Depth of Flanges is Great Compared with Depth of Beam.—Figs. 4 and 5.

4. 5.

$$\frac{S\left(4\,a\,\frac{d'^2}{d} + \overline{1.155\,t\,d^2}\right)}{l} = W.$$ *a representing area of one flange less thickness of web, in sq. ins., t thickness of web, d′ reputed depth or distance between centres of flanges, and d depth of beam, all in ins.*

When Section of Circular or Elliptic Beam is Small Compared with Diameter.—Figs. 6, 7, and 8.

6.

$$\frac{3.14\,d^2\,t\,S}{l} = W.$$

7.

8.

$$\frac{1.57\,(b^2 + d^2)\,t\,S}{l} = W.$$

b and d representing mean breadth and depth.

Illustration 1.—Assume Figs. 1, 2, and 3, 20 ins. in depth, width of flanges on top and bottom ribs 5 ins., thickness of flanges and webs 1 inch, and of sides of Fig. 3 .5 inch; length between supports 10 feet, and S 20 000 lbs.; what would be breaking weight of each?

Then $\frac{20 - 1 \times 20\,000\,(4 \times 5 + \overline{1.155 \times 18})}{10 \times 12} = \frac{380\,000\,(20 + 20.79)}{120} = 129\,168.4$ *lbs.*

2.—Assume Figs. 4 and 5, 6 ins. in depth, area of flanges 3 ins., widths of webs 1 inch, and length and S as in preceding case.

Then $\frac{20\,000\left(4 \times 3 \times \frac{\overline{6-1}^2}{6} + \overline{1.155 \times 1 \times 6^2}\right)}{10 \times 12} = \frac{20\,000 \times 91.58}{120} = 15\,263.3$ *lbs.*

3.—Assume Fig. 6 10 ins. in diameter, Fig. 7, 7.5 ins. in depth and 12 ins. in width, and Fig. 8, 12 ins. in depth and 7.5 ins. in width, and thickness of all metal 1 inch.

Then, Fig. 6 $\frac{3.14 \times 10^2 \times 1 \times 20\,000}{10 \times 12} = \frac{6\,280\,000}{120} = 52\,333.3$* *lbs.*, which is .4 of that of solid cylinder.

Figs. 7 and 8 $\frac{1.57 \times (12^2 + 7.5^2) \times 1 \times 20\,000}{10 \times 12} = \frac{6\,287\,850}{120} = 52\,398.75$ *lbs.*

Note.—For all ordinary purposes, operation of computing their strength, by first computing that of their circumscribing figure, and then deducting from it strength due to difference between it and section of beam under computation, will be sufficiently accurate. See Illustration, page 814.

If greater accuracy is required, see page 810, or *D. K. Clark's Manual*, pp. 513–17.

Note.—To compute location of neutral axis of beams of unsymmetrical section, see also *D. K. Clark*, pp. 514–15.

* This result agrees with deduction of Mr. Baker, as given by him in his work on Strength of Beams, etc., pp. 26–7, for hollow or annular beams of small area of section compared with that of diameter, even up to a thickness of metal of one eighth of diameter. He assigns their strength so low as .4 of that of solid cylinder, in consequence of loss of resistance to flexure.

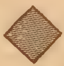
General Formulas for Destructive Weight of Solid Beams of Symmetrical Section.

Supported at Both Ends. Weight applied in Middle.

Line of Neutral Axis runs through centre of gravity of section.

$\frac{2\,a\,d\,r\,S}{l} = W$, and $\frac{l\,W}{2\,a\,d\,r} = S$. In square beams for $a\,d$ put d^3. *a and d representing area and depth of section, r radius of gyration (half depth of beam = 1), l length of beam between its supports in ins., W destructive weight in tons or lbs., and S tensile strength of material in like tons or lbs. per sq. inch.*

ILLUSTRATION.—Assume dimensions of cast-iron beams, Figs. 1, 2, 3, 4, and 5, as follows, viz.: 1 and 2, 5 × 5 ins.; 3, 2.5 × 10; 4, 5.64 diameter; and 5, 7.25 × 4.39, or equal areas; distance between supports 60 ins., and tensile strength of iron = 20 000 lbs.

1.  2.  3.  4.  5. 

Areas of each 25 sq. ins. Radius of gyration, No. 1, .5775; 2, .4083; 3, .5775; 4, .5; and 5, 1.43.

1. $\frac{2 \times 25 \times 10 \times .5775 \times 26\,000}{60} = 125\,125$ *lbs.*

2. $\frac{2 \times 25 \times 7.07^{*} \times .4083 \times 26\,000}{60} = 62\,545$ *lbs.*

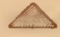
3. $\frac{2 \times 5^{3}\dagger \times .5775 \times 26\,000}{60} = 62\,562$ *lbs.*

4. For formula for square beams substitute $\frac{a\,d\,S}{l} = W$

Then 4. $\frac{25 \times 5.64 \times 26\,000}{60} = 57\,766$ *lbs.*; and for 5. $\frac{.7854\,b\,d^2\,S}{l} = W$.

$$\frac{.7854 \times 4.39 \times 7.25^2 \times 26\,000}{60} = 78\,532 \text{ lbs.}$$

These formulas give a result equal to a transverse strength for *Cast iron* of 550 for a tensile strength of 26 000 lbs., and of *Wrought iron* of 600 lbs. for a like strength of 50 000 lbs. (as per table, page 788).

6.  7.  8.  9.  10. 

$\frac{4\,C\,b\,d^2}{l} = W$. *C representing coefficient of strength of metal in lbs., b and d breadth and depth in ins., l length in feet, and W destructive weight in tons.*

6. $\frac{R^4 - r^4}{R}\,4.7 = b\,d^2$. *R and r representing external and internal radius.*

7. $\frac{b\,d^3 - b'\,d'^3}{d} = b\,d^2$. *b′ and d′ representing interior breadth and depth.*

8. $.38\,R^3 = b\,d^2$. 9. $\frac{b\,d^2}{4} = W$. *d representing depth or height.*

10. $b\,d^2 + 2\,b'\,d'^2 = W$. *b and d representing breadth and depth of centre and vertical rib, and b′ and d′ breadth and depth of horizontal rib, external to central rib.*

Values of C 550 for a tensile strength of *Cast Iron* of 26 000 lbs. per sq. inch, and of 600 for a like strength of *Wrought Iron* of 50 000 lbs., and *pro rata.*

* Diagonal of square. † In square beams $d^3 = a \times d$.

Flanged Beams of Unsymmetrical Section. (*D. K. Clark.*)

$\frac{4\,S\,d}{l} = W$. S *representing total tensile strength of section in lbs. per sq. inch, d vertical distance between centres of tension and compression in ins., l length in ins., and* W *weight in lbs.*

Illustration.—If the sectional area of a beam of cast iron is 5.9 sq. ins., the depth or distance between centres of tension and compression 5.6 ins., distance between supports 5.5 feet, and tensile strength of metal 30 000 lbs. per sq. inch.

$$\text{Then } \frac{4 \times 5.9 \times 30\,000 \times 5.6}{5.5 \times 12} = \frac{3\,964\,800}{66} = 60\,072.7 \textit{ lbs.}$$

STEEL.

To Compute Transverse Strength of Steel Bars.

Supported at Both Ends. Weight applied in Middle.

$\frac{1.155\,S\,b\,d^2}{l} = W$. S *representing tensile strength in lbs., l length between supports in ins., and* W *weight in lbs.*

Illustration.—What is ultimate destructive stress of a bar of Crucible steel, 2 ins. square, and 2 feet between supports? S = 90 000 lbs.

$$\text{Then } \frac{1.155 \times 90\,000 \times 2^3}{2 \times 12} = \frac{831\,600}{24} = 34\,650 \textit{ lbs.}$$

To Compute Section of Lower Flange of a Girder or Cylindrical Shaft of Cast Iron to Sustain a Safe Load in its Middle. (*Baker.*)

$\frac{l\,d\,W}{C} = M$. *l representing distance between supports in feet, d depth of girder, etc., in ins.,* W *weight in tons,* C *coefficient, and* M *moment of weight around support.*

Illustration.—What should be section of a girder, 12 ins. deep, to sustain a safe load of 10 tons in its middle, between supports 16 feet apart?

Stress assumed 2 tons per sq. inch, and Factor of safety 4. $\frac{16 \times 12 \times 10}{4} = 480 = M$.

And $\frac{M}{d \times S} = a$. S *representing stress assumed in tons, and a area of section of flange in sq. ins.*

$$\text{Then } \frac{480}{12 \times 2} = 20 \textit{ sq. ins.}$$

For Rectangular, Diagonal, or Circular Beam or Shaft.

 $\frac{d^2\,b}{6} = M.$  $\frac{d^3}{10.2} = M.$  $\frac{d^3}{8.4} = M.$

General Formulas for Computation of Destructive Weight of a Beam or Girder of any form of Cross Section and of any Material. (*B. Baker.*)

Load applied at Middle.

$\frac{S\,M\,(1 + Q')}{4\,l} = W$. S *representing tensile strength of material per sq. inch in tons,* M *moment of resistance of section = product of effective depth of girder or beam, and effective area of flange portion of section, in sq. ins.,* Q *resistance due to flexure, l distance between supports in feet, and* Q′ = Q × *thickness of web of section, both in ins.*

Average Values of S *for Various Materials.*

	Tons.		Tons.		Tons.
Cast Iron	7	Steel	40 to 50	Oak	2.5 to 4.5
Wrought Iron	21	" plates	35	Pine	2 " 3.5

Substituting Values of S *and* Q *in a General Equation.*

SECTION.	Cast Iron.	Wrought Iron.	Steel.	Oak.	Pine.
■	$W = .875 \frac{d^2 b}{l}$	$= 1.75 \frac{d^2 b}{l}$	$= 3$ to $5 \frac{d^2 b}{l}$	$= .14$ to $.25 \frac{d^2 b}{l}$	$= .11$ to $.2 \frac{d^2 b}{l}$
◆	$W = .75 \frac{d^3}{l}$	$= 1.5 \frac{d^3}{l}$	$= 2.625$ to $4.25 \frac{d^3}{l}$	$= .1$ to $.16 \frac{d^3}{l}$	$= .08$ to $.14 \frac{d^3}{l}$
●	$W = .5625 \frac{d^3}{l}$	$= 1.125 \frac{d^3}{l}$	$= 2$ to $3.25 \frac{d^3}{l}$	$= .08$ to $.14 \frac{d^3}{l}$	$= .06$ to $.11 \frac{d^3}{l}$

d representing depth of a rectangular bar, side of a square, or diameter of a round, b breadth of a vertical bar, all in ins., and l distance between supports in feet.

Moment of Resistance.

Moment of Resistance of a cross section is the static force resisting an external force of tension or compression, and it is equal to moment of *Inertia*, divided by distance of centre of effect of the area of fibres which are respectively the most extended or compressed from the *neutral axis* of the section.

To Compute Moment of Resistance.

$\frac{I}{d} = M$. I *representing moment of inertia, and d distance of centre of effect of area of fibres of extension or compression.*

Work of Resistance.

Under a Quiescent Load.—Intensity of Elastic resistance increases uniformly with total space through which action of stress operates; hence, it may be defined by a triangular section.

Consequently, $.5\, s\, L = R$. *s representing space passed through,* L *load, and* R *resistance.*

To Compute Moment of Resistance.

$\frac{6\,C\,I}{h}$ and $\frac{M\,I}{h} = R$. C *a coefficient = one sixth of destructive weight,* I *moment of inertia, h height of neutral axis from base of section,* R *moment of resistance, and* M *modulus of rupture.*

NOTE.—Neutral axis, for all practical purposes, is at centre of gravity of any section.

For *Radius of Gyration*, see Centre of Gyration, page 609.

For other rule for computation of Moment of Resistance, see Strength of Beams, *B. Baker*, London, 1870.

Moment of Inertia.

Moment of Inertia is resistance of a beam to bending, and moment of any transverse section is equal to sum of products of each particle of its area into square of their distance from neutral axis of section.

1.

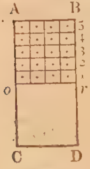


ILLUSTRATION.—If transverse section of a beam, A B C D, Fig. 1, is 8×20 ins., its neutral axis will be at middle of its depth, *o r*; divide A B, *o r*, into any number of equal spaces, as shown, then each space will be $2 \times 2 = 4$ sq. ins., and the distances of the centre of each square from neutral axis will be as follows:

1, 1.	$2 \times 2 \times 4 \times 1^2 = 16$	4, 4.	$2 \times 2 \times 4 \times 7^2 = 784$
2, 2.	$2 \times 2 \times 4 \times 3^2 = 144$	5, 5.	$2 \times 2 \times 4 \times 9^2 = 1296$
3, 3.	$2 \times 2 \times 4 \times 5^2 = 400$		2640×2 for lower half $= 5280 =$ *moment.*

NOTE.—If the area of the figure in illustration had been more minutely divided, the result would have approximated more nearly to the above result.

For Moment of Inertia of a Revolving Body, see Centre of Gyration, page 609.

To Compute Moment of Inertia of a Solid Beam.—Fig. 2.

2.  $\frac{b\,d^3}{12} = \text{M.}$

Illustration.—Take elements of preceding case.

$$\text{Then } \frac{8 \times 20^3}{12} = \frac{64\,000}{12} = 5333.33 \textit{ moment.}$$

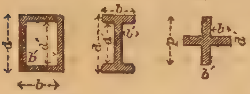
Or, $.3\,t^3\,n^3\,b = \text{M.}$ *t representing breadth of vertical divisions, n number of horizontal divisions from plane of neutral axis, b breadth, and d depth of beam.*

Illustration.—Take elements of preceding case.

$$t = 2,\ n = 5, \text{ and } b = 8.$$

Then $.3 \times 2^3 \times 5^3 \times 8 = 2400 \times 2$ *for lower half* $= 4800 =$ *moment.*

3. 4. 5.



Beams of Various Figures.—Figs. 3, 4, 5.

$$3.\ \frac{b\,d^3 - b'\,d'^3}{12},\ 4 \text{ and } 5.\ \frac{b\,d^3 - 2\,b'\,d'^3}{12} = \text{M.}$$

b′ and d′ representing respectively breadth less thickness of web, and depth less thickness of flanges.

 $.7854\,r^2 = \text{M.}$

 $.7854\,c\,t^3 = \text{M.}$

 $\frac{b\,d^3}{36} = \text{M.}$

 $.7854\,(r^4 - r'^4) = \text{M.}$

 $\frac{s^4}{12} = \text{M.}$

 $.11\,r^2 = \text{M.}$

r representing radius, t transverse and c conjugate diameters, and s side.

To Compute Common Centre of Gravity and Vertical Distance between Centres of Crushing and Tensile Stress of a Girder or Beam.

Rule.—Multiply surface of section of each part or figure composing whole, by distance of its centre from centre of one of the two extreme parts or figures, as •; divide sum of their products by sum of surfaces of section, and result will give distance of common centre of gravity from centres of each extreme part or figure.

Example.—Take annexed figure.

$$\textit{Above}\left\{\begin{array}{lll} 2.5 \times 1 \times 0 & = 2.5 \times 0 & = .0 \\ .325 \times \left(\frac{5.62}{2} + \frac{1}{2}\right) & = .325 \times 3.31 & = 1.076 \\ .38 \times 4 \times \left(\frac{.38}{2} + 5.62 + \frac{1}{2}\right) & = 1.52 \times 6.31 & = 9.591 \\ & \quad 4.345 & \quad 10.667 \end{array}\right.$$

Dividing 10.667 by $4.345 = 2.455 =$ *distance of common centre from centre of upper part.*

$$\textit{Below}\left\{\begin{array}{lll} 1.52 \times 0 & = 1.52 \times 0 & = .0 \\ .325 \times 5.62 \times \left(\frac{5.62}{2} + \frac{.38}{2}\right) & = 1.826 \times 3 & = 5.478 \\ 2.5 \times \left(\frac{1}{2} + 5.62 + \frac{.38}{2}\right) & = 2.5 \times 6.31 & = 15.775 \\ & \quad 5.846 & \quad 21.253 \end{array}\right.$$

Dividing 21.225 by $5.846 = 3.631 =$ *distance of common centre from centre of lower part.*

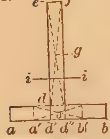
Hence, $3.631 + \frac{.38}{2} = 3.821 =$ *distance of common centre from bottom*, and $3.631 + 2.652 = 6.283 =$ *distance between centres of gravity.*

To Compute Neutral Axis of a Beam of Unsymmetrical Section.—Figs. 3, 4, 5, 6, 7, 8, and 9. (*D. K. Clark.*)

OPERATION.—Divide section as reduced into its simple elements, and assume a datum-line from which moments of elements are to be computed. Multiply area of each element by distance of its own centre of gravity from datum-line, to ascertain its moment. Divide sum of these moments by total reduced area; and quotient is distance of centre of gravity of reduced section, or of neutral axis of whole section, from datum-line.

ILLUSTRATION.—Fig. 8 annexed is 12 ins. deep, 12 ins. wide, and 1 inch thick. Extend web, $c\ d$, to the lower surface at d' and d'', leaving 5.5 ins. of web, $a\ d'$ and $d''\ b$, on each side. Reduce this width in the ratio of 1.73 to 1, or to $(5.5 \div 1.73 =)$ 3.2 ins., and set off $a'\ a'$ and $d''\ b'$ each equal to 3.2 ins. Then reduced flange, $a'\ b'$, is $(3.2 \times 2 = 6.4 + 1 =)$ 7.4 ins. wide, and reduced section consists of two rectangles, $a'\ b'$ and $c\ d$. Assume any datum-line, as $e\ f$, at upper end of section, and bisect depths of rectangles, or take intersections of their diagonals at g and o, for their centres of gravity. Distances of these from datum line are 5.5 and 11.5 ins. respectively, and areas of the rectangles are $11 \times 1 = 11$ sq. ins., and $7.4 \times 1 = 7.4$ sq. ins.

8.



$$\text{Then,}\quad \begin{array}{rl} c\,d = 11 \ \times\ 5.5 = & 60.5 \\ a'\,b' = \underline{7.4} \times 11.5 = & \underline{85.1} \\ 18.4 &)145.6 = 7.91 \textit{ ins.} \end{array}$$

Showing that centre of gravity of reduced section, being neutral axis of whole section, is 7.91 ins. below upper edge, in line $i\ i$. Centre of gravity of entire section at •, it may be added, is 8.65 ins. below upper edge, or .74 inch lower than that of reduced section.

Neutral axes of other sections, Figs. 3 to 7, found by same process, are marked on the figures. Section of a flange rail, No. 7, which is very various in breadth, may be treated in two ways: either by preparatorily averaging projections of head and flange into rectangular forms; or, by taking it as it is, and dividing it into a considerable number of strips parallel to base, for each of which the moment, with respect to assumed datum-line, is to be ascertained. First mode of treatment is approximate; second is more nearly exact.

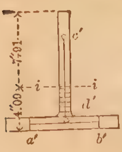
To Compute Ultimate Strength of Homogeneous Beams of Unsymmetrical Section.

OPERATION.—Resuming section, Fig. 9, for which neutral axis has been ascertained,

To Compute Tensile Resistance,

Divide portion below neutral axis $i\ i$, Fig. 9, with reduced width of flange, $a\ b$, into parallel strips, say .5 inch deep, as shown, and multiply area of each strip by its mean distance from neutral axis for proportional quantity of resistance at strip. Divide sum of products, amounting in this case to 31.3, by extreme depth below neutral axis = 4.09 ins., and multiply quotient by 1.73 S (ultimate tensile resistance at lower surface). The final product is total tensile resistance of section; or,

9.



$$\frac{31.3 \times 1.73\,S}{4.09} = 13.24\ S \textit{ total tensile resistance.}$$

S *representing ultimate tensile strength of material per sq. inch.*

Again, multiply area of each strip by square of its mean distance from neutral axis, and divide sum of these new products, amounting to 104.64, by sum of first products. The quotient is distance of resultant centre of tensile stress, d', from neutral axis. Or, resultant centre is,

$$\frac{104.64}{31.3} = 3.34 \textit{ ins. below neutral axis.}$$

This process is that of ascertaining centre of gravity of all the tensile resistances.

By a similar process for upper portion in compression, sum of first products is ascertained to be same as for lower portion $= 31.3$.

But maximum compressive stress at upper portion is greater than maximum tensile stress at lower portion, in ratio of their distances from neutral axis, or as $1.73\,S \times \frac{7.91}{4.09} = 3.34$ S, and $\frac{31.3 \times 3.34\,S}{7.91} = 13.24$ S *total compressive resistance*, which is same as total tensile resistance, in conformity with general law of equality of tensile and compressive stress in a section.

Sum of products of areas of stress, divided by squares of their distances respectively from neutral axis, is 164.9, and resultant centre *c*, Fig. 9, is $\frac{164.9}{31.3} = 5.27$ *ins. above neutral axis.*

Sum of distances of centres of stress or of resistance from neutral axis, $3.34 + 5.27 = 8.61$ *ins.* = distance apart of these centres as represented by central line, $c'\,d'$.

Abbreviated Computation.—As upper part of section is a rectangle, its resultant centre $= \frac{2}{3}$ of height, or $7.91 \times \frac{2}{3} = 5.27$ ins. above neutral axis. Average resistance is half maximum stress, viz., that at upper portion, which is 3.34 S per sq. inch.

Area of rectangle therefore $= 7.91 \times 1 = 7.91$ *sq. ins.*, and $\frac{7.91 \times 3.34\,S}{2} = 13.21$ S *compressive resistance*, as before determined.

Moment of tensile resistance $= 13.21 \times 8.61$ ins. $= 113.76$ S, also $= \frac{W\,l}{4}$, or $\frac{4\,S\,d}{l} =$ W. S *representing total resistance of section in lbs., d vertical distance apart of centres of tension and compression, and l length between supports, all in ins.*

Strength of Beam Inverted.—When inverted, maximum tensional resistance of beam at its lower surface *c*, Fig. 8, is 1.73 S.

Area of rectangle *i i c* $= 7.91$ sq. ins., and $\frac{7.91 \times 1.73\,S}{2} = 6.79$ S *total tensile resistance*, or about one half of beam in its normal position.

NOTE.—For other rule for computation of centre of gravity, see Strength of Beams, etc. *B. Baker*, London, 1870.

Comparative Qualities of Various Metals. *Major Wade.*

Metals.		Density.	Compression.	Tensile.	Torsion.	Transverse.	Tensile to Compression.	Hardness.
			Sq. Ins.	Sq. Ins.	Sq. Ins.	Sq. Ins.		
Cast Iron....	Least....	6.9	84 529	9 000	—	416	1 to 9.4	4.57
	Greatest.	7.4	174 120	45 970	—	958	1 " 3.8	33.51
	Mean....	7.225	144 916	31 829	8 614	680	1 " 4.6	22.34
Wrought Iron	Least....	7.704	40 000	38 027	2 915	542	1 " 1	10.45
	Greatest.	7.858	127 720	74 592	3 643	—	1 " 1.7	12.14
Cast Steel....	Least....	7.729	198 944	—	—	—	—	—
	Greatest.	8.953	391 985	128 000	28 280	1916	1 to 3.1	—
Bronze......	Least....	7.978	—	17 698	1 852	—	—	4.57
	Greatest.	8.953	—	56 786	2 656	—	—	5.94

Factors of Safety.

Girders, Beams, etc., of cast iron should not be subjected to a greater stress than one sixth of their destructive weight, and they should not be subjected to an impulsive stress greater than one eighth.

The following are submitted by English Board of Trade, Commissioners, etc.

Structure.	Stress.	Factor.	Structure.	Stress.	Factor.
CAST IRON.			WROUGHT IRON.		
Girders..........	Dead	3 to 6	Girders..........	Dead	3
Columns..........	"	6	"	Live	6
Tanks..........	"	4	Bridges..........	Mixed	4
Machinery..........	Live	8	STEEL.		
"	Shock	10	Bridges..........	Mixed	4

Girders, Beams, Lintels, etc.

Transverse or Lateral Strength of any Girder, Beam, Breast-summer, Lintel, etc., is in proportion to product of its breadth and square of its depth, and area of its cross-section.

Best form of section for Cast-iron girders or beams, etc., is deduced from experiments of Mr. E. Hodgkinson, and such as have this form of section ⊥ are known as Hodgkinson's.

Rule deduced from his experiments directs, that *area of bottom flange should be 6 times that of top flange*—flanges connected by a thin vertical web, sufficiently rigid, however, to give the requisite lateral stiffness, tapering both upward and downward from the neutral axis; and in order to set aside risk of an imperfect casting, by any great disproportion between web and flanges, it should be tapered so as to connect with them, with a thickness corresponding to that of flange.

As both Cast and Wrought iron resist compression or crushing with a greater force than extension, it follows that the flange of a girder or beam of either of these metals, which is subjected to a crushing strain, according as the girder or beam is *supported at both ends, or fixed at one end*, should be of less area than the other flange, which is subjected to extension or a tensile stress.

When girders are subjected to impulses, and sustain vibrating loads, as in bridges, etc., best proportion between top and bottom flange is as 1 to 4; as a general rule, they should be as narrow and deep as practicable, and should never be deflected to more than .002 of their length.

In Public Halls, Churches, and Buildings where weight of people alone are to be provided for, an estimate of 175 lbs. per sq. foot of floor surface is sufficient to provide for weight of flooring and load upon it. In computing other weight to be provided for it should be that which may at any time bear upon any portion of their floors; usual allowance, however, is for a weight of 280 lbs. per sq. foot of floor surface for stores and factories.

In all uses, such as in buildings and bridges, where the structure is exposed to sudden impulses, the load or stress to be sustained should not exceed from .2 to .16 of breaking weight of material employed; but when load is uniform or stress quiescent, it may be increased to .3 and .25 of breaking weight.

An open-web girder or beam, etc., is to be estimated in its resistance on the same principle as if it had a solid web. In cast metals, allowance is to be made for loss of strength due to unequal contraction in cooling of web and flanges.

In Cast Iron, the mean resistances to Crushing and Extension are, for American as 4.55 to 1, and for English as 5.6 to 7 to 1; and in Wrought Iron are, for American as 1.5 to 1, and for English as 1.2 to 1; hence the mass of metal below neutral axis will be greatest in these proportions when stress is intermediate between ends or supports of girders, etc.

Wooden Girders or *Beams*, when sawed in two or more pieces, and slips are set between them, and whole bolted together, are made stiffer by the operation, and are rendered less liable to decay.

Girders cast with a face up are stronger than when cast on a side, in the proportion of 1 to .96, and they are strongest also when cast with bottom flange up.

Most economical construction of a Girder or Beam, with reference to attaining greatest strength with least material, is as follows: The outline of top, bottom, and sides should be a curve of various forms, according as breadth or depth throughout is equal, and as girder or beam is loaded only at one end, or in middle, or uniformly throughout.

Breaking Weights of Similar Beams are to each other as Squares of their like Linear Dimensions.

By Board of Trade regulations in England, iron may be strained to 5 tons per sq. inch in tension and compression, and by regulation of the Ponts et Chaussées, France, 3.81 tons.

Rivets .75 and 1 inch in diameter, and set 3 ins. from centre in top of girder, and 4 ins. at bottom.

Character of fracture, as to whether it is crystalline or fibrous, depends upon character of blows; thus, sharp blows will render it crystalline, and slow will not disturb its fibrous structure.

For spans exceeding 40 feet, wrought iron is held to be preferable to cast iron.

Riveting, when well executed, is not liable to be affected by impact or velocity of load.

A *Coupled Girder* or *Beam* is one composed of two, fastened together, and set one over the other.

Trussed Beams or Girders.

Wrought and Cast Iron possess different powers of resistance to tension and compression; and when a beam is so constructed that these two materials act in unison with each other *at stress due to load required to be borne*, their combination will effect an essential economy of material. In consequence of the difficulty of adjusting a tension rod to the stress required to be borne, it is held to be impracticable to construct a perfect truss beam.

Fairbairn declares that it is better for tension of truss-rod to be low than high, which position is fully supported by following elements of the two metals:

Wrought Iron has great tensile strength, and, having great ductility, it undergoes much elongation when acted upon by a tensile force. On the contrary, *Cast Iron* has great crushing strength, and, having but little ductility, it undergoes but little elongation when acted upon by a tensile stress; and, when these metals are released from the action of a high tensile stress, the *set* of one differs widely from that of the other, that of wrought iron being the greatest.

Under same increase of temperature, expansion of wrought is considerably greater than that of cast iron; 1.81* tons per sq. inch is required to produce in wrought iron same extension as in cast iron by 1 ton.

Fairbairn, in his experiments upon English metals, deduced that within limits of stress of 13 440 lbs. per sq. inch for cast iron, and 30 240 lbs. per sq inch for wrought iron, tensile force applied to wrought iron must be 2.25 times tensile force applied to cast iron, to produce equal elongations.

Relative tensile strengths of cast and wrought iron being as 1 to 1.35, and their resistance to extension as 1 to 2.25, therefore, where no initial tension is applied to a truss-rod, cast iron must be ruptured before wrought iron is sensibly extended.

Resistance of cast iron in a trussed beam or girder is not wholly that of tensile strength, but it is a combination of both tensile and crushing strengths, or a transverse strength; hence, in estimating resistance of a trussed beam or girder, transverse strength of it is to be used in connection with tensile strength of truss.

Mean transverse strength of a cast-iron bar, one inch square and one foot in length, supported at both ends, stress applied in the middle, *without set*, is about 900 lbs.; and as mean tensile strength of wrought iron, also *without set*, is about 20 000 lbs. per sq. inch, ratio between sections of beams and of truss should be in ratio of transverse strength per sq. inch of beam and of tensile strength of truss.

Girders under consideration are those alone in which truss is attached to beam at its lower flange, in which case it presents following conditions:

---

* Elongation of cast and wrought iron being 5500 and 10 000, hence 10 000 ÷ 5500 = 1.81.

1. *When truss runs parallel to lower flange.* 2. *When truss runs at an inclination to lower flange, being depressed below its centre.* 3. *When beam is arched upward, and truss runs as a chord to curve.*

Consequently, in all these cases section of beam is that of an open one with a cast-iron upper flange and web, and a wrought-iron lower flange, increased in its resistance over a wholly cast-iron beam in proportion to the increased tensile strength of wrought iron over cast iron for equal sections of metals.

From various experiments made upon trussed beams, it is shown:

1. That their rigidity far exceeds that of simple beams; in some cases it was from 7 to 8 times greater. 2. That when truss resists rupture, upper flange of beam being broken by compression, there is a great gain in strength. 3. That their strength is greatly increased by upper flange being made larger than lower one. 4. That their strength is greater than that of a wrought-iron tubular beam containing same area of metal.

Comparative Value of Wrought-iron Bars, Hollow Girders, or Tubes of Various Figures (*English*).

Circular tubes, riveted	1	Circular, uniform thickness	1.7
Flanged beams	1.2	Plate beams	1.7
Elliptic tubes, riveted	1.3	Elliptic, uniform thickness	1.8
Rectangular tubes, riveted	1.5	Rectangular, uniform thickness	2

General Deductions from Experiments of Stephenson, Fairbairn, Cubitt, Hughes, etc.

Fairbairn shows in his experiments that with a stress of about 12 320 lbs per sq. inch on cast iron, and 28 000 lbs. on wrought iron, the sets and elongations are nearly equal to each other.

A cast-iron beam may be bent to .3 of its breaking weight if load is laid on gradually; and .16 of it, if laid on at once, will produce same effect, if weight of beam is small compared with weight laid on. Hence, beams of cast iron should be made capable of bearing more than 6 times greatest weight which will be laid upon them.

In beams of cast or wrought iron, if fixed or supported at both ends, flanges should be in proportion to relative resistances of material to crushing or extension.

Breaking weights in similar beams are to each other as squares of their like linear dimensions; that is, breaking weights of beams are computed by multiplying together area of their section, depth, and a *Constant*, determined from experiments on beams of the particular form under investigation, and dividing product by distance between supports.

Cast and wrought-iron beams, having similar resistances, have weights nearly as 2.44 to 1.

A box beam or girder, constructed of plates of wrought-iron, compared to a single rib and flanged beam ⌶, of equal weights, has a resistance as 100 to 93.

Resistance of beams or girders, where depth is greater than their breadth, when supported at top, is much increased. In some cases the difference is fully one third.

When a beam is of equal thickness throughout its length, its curve of equilibrium, to enable it to support a uniform stress with equal resistance in every part, should be an *Ellipse*, and if beam is an open one, its curve of equilibrium, for a uniform load, should be that of a *Parabola*. Hence, when middle portion is not wholly removed, its curve should be a compound of an ellipse and a parabola, approaching nearer to the latter as the middle part is decreased.

Girders of cast iron, up to a span of 40 feet, involve a less cost than of wrought iron.

Cast-iron beams and girders should not be *loaded* to exceed .2, or subjected to a greater stress than .166 of their destructive weight; and when the stress is attended with concussion and vibration, this proportion must be increased.

Simple cast-iron girders may be made 50 feet in length, and best form is that of Hodgkinson; when subjected to a fixed load, flanges should be as 1 to 6, and when to a concussion, etc., as 1 to 4.

Forms of girders for spaces exceeding limit of those of simple cast iron are various; principal ones adopted are those of straight or arched cast-iron girders in separate pieces, and bolted together—Trussed, Bowstring, and wrought-iron Box and Tubular.

Straight or *Arched Girder*, formed of separate castings, is entirely dependent upon bolts of connection for its strength.

Trussed or *Bowstring Girder* is made of one or more castings to a single piece, and its strength depends, other than upon the depth or area of it, upon the proper adjustment of the tension, or the initial strain, upon the wrought-iron truss.

Box or *Tubular Girder* is made of wrought iron, and is best constructed with cast-iron tops, in order to resist compression: this form of girder is best adapted to afford lateral stiffness.

When a girder has four or more supports, its condition as regards a stress upon its middle is essentially that of a beam fixed at both ends.

The following results of the resistances of materials will show how they should be distributed in order to obtain *maximum* of strength with *minimum* of dimensions:

	To Tension.	To Crushing.		To Tension.	To Crush'g.
Cast iron	21 000	90 300	Oak, white, mean.	11 000	7 500
	32 000	140 500	" English " .	6 500	3 100
" English	13 000	58 000	Wrought iron	45 000	47 000
	23 000	116 000		59 000	83 000
Granite	578	15 000	" English	31 000	40 000
Limestone	670	4 000		53 000	65 000
	2 800	9 000	Yellow pine	16 000	4 000

The best iron has greatest tensile strength, and least compressive or crushing.

Conditions of Forms and Dimensions of a Symmetrical Beam or Girder.

When Fixed at One End, and Loaded at the Other.

1. *When Depth is uniform throughout entire Length*, section at every point must be in proportion to product of length, breadth, and square of depth, and as square of depth is in every point the same, breadth must vary directly as length; consequently, each side of beam must be a vertical plane, tapering gradually to end.

2. *When Breadth is uniform throughout entire Length*, depth must vary as square root of length; hence upper or lower sides, or both, must be determined by a parabolic curve.

3. *When Section at every point is similar, that is, a Circle, an Ellipse, a Square, or a Rectangle, Sides of which bear a fixed Proportion to each other*, the section at every point being a regular figure, for a circle, the diameter at every point must be as cube root of length; and for an ellipse or a rectangle, breadth and depth must vary as cube root of length.

ILLUSTRATION.—A rectangular beam as above, 6 ins. wide and 1 foot in depth at its extreme end, and 4 feet in length, is capable of bearing 6480 lbs.; what should be its dimension at 3 feet? $\sqrt[3]{4} = 1.587$, and $\sqrt[3]{3} = 1.442$.

Then $1.587 : 1.442 :: 1 : .9086$, and 6 and $12 \times .9086 = 5.452$ and 10.9.

$$\text{Hence } \frac{5.452 \times 10.9^2}{3} = 216, \text{ and } \frac{6 \times 12^2}{4} = 216.$$

When Fixed at One End, and Loaded uniformly throughout its Length.

1. *When Depth is uniform throughout its entire Length*, breadth must increase as the square of length.

2. *When Breadth is uniform throughout its entire Length*, depth will vary directly as length.

3. *When Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle*, section at every point being a regular figure, cube of depth must be in ratio of square of length.

ILLUSTRATION.—Take preceding case.

Then $4^2 : 3^2 :: 12^3 : 972$, and $\sqrt[3]{972} = 9.9$ *in depth.*

When Supported at Both Ends.

1. *When Loaded in the Middle, Coefficient* or *Factor of Safety* of the beam, or product of breadth and square of depth, must be in proportion to distance from nearest support; consequently, whether the lines forming the beam are straight or curved, they meet in the centre, and of course the two halves are alike.

2. *When Depth is Uniform throughout*, breadth must be in ratio of length.

3. *When Breadth is Uniform throughout*, depth will vary as square root of length.

4. *When Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle*, section at every point being a regular figure, cube of depth will be as square of distance from supported end.

When Supported at Both Ends, and Loaded uniformly throughout its Length.

1. *When Depth is Uniform*, breadth will be as product of length of beam and length of it on one side of given point, less square of length on one side of given point.

2. *When Breadth is Uniform*, depth will be as square root of product of length of beam and length of it on one side of given point, less square of length on one side of given point.

3. *When Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle*, section at every point being a regular figure, cube of depth will be as product of length of beam and length of it on one side of given point, less square of length on one side of given point.

Elliptical-sided Beams.

To Determine Side or Curve of an Elliptical-sided Beam.

$\sqrt{\frac{L\,l}{2\,C\,b}} = d.$ L *representing load in lbs., l length in feet,* C *coefficient, and b breadth in ins.*

ILLUSTRATION.—What should be depth in centre of a beam of white pine, 10 feet in length between its supports, and 5 ins. in breadth, to support a load of 10 000 lbs.?

Assume C = 100. Then $\sqrt{\frac{10\,000 \times 10}{2 \times 100 \times 5}} = \sqrt{\frac{100\,000}{1000}} = 10$ *ins.*

Hence, outline of beam is that of a semi ellipse, having 10 feet for its transverse diameter, and 9 ins. for its semi-conjugate.

NOTE.—Weight of Girder, Beam, etc., should in all cases be added to stress or load.

Miscellaneous Illustrations.

1.—What should be side of a rectangular white oak beam, 2 ins. in width, and 6 feet between its supports, to sustain a load of 360 lbs.?

Assume stress at .2 of breaking weight of 150 lbs. = 30.

$$\sqrt{\frac{6 \times 360}{4 \times 2 \times 30}} = \sqrt{\frac{2160}{240}} = 3 \text{ ins.}$$

2.—What should be breadth and depth of such a beam if square?

$$\sqrt[3]{\frac{6 \times 360}{4 \times 30}} = \sqrt[3]{\frac{2160}{120}} = 2.62 \text{ ins.}$$

3.—What should be diameter of a cylinder?

$$\frac{360 \times 6}{.6 \times 30} = 120, \text{ and } \sqrt[3]{\frac{120}{4}} = 3.1 \text{ ins.}$$

STEEL.

To Compute Transverse Strength of Steel Bars.

Supported at Both Ends. Weight applied in Middle.

$\frac{1.155\, S\, b\, d^2}{l} = W$. S *representing tensile strength in lbs.*, l *length between supports in ins., and* W *weight in lbs.*

Illustration.—What is ultimate destructive stress of a bar of Crucible steel, 2 ins. square, and 2 feet between supports? S = 90 000 lbs.

$$\text{Then } \frac{1.155 \times 90\,000 \times 2^3}{2 \times 12} = \frac{831\,600}{24} = 34\,650 \textit{ lbs.}$$

Elastic Transverse Strength is 50 per cent. of its ultimate strength.

Hardening in oil increases its strength from 12 to 56 per cent. Thus,

Soft steel, 121 520 lbs.; soft steel, cooled in water, 90 160 lbs.; soft steel, cooled in oil, 215 120 lbs.

Krupp's is about .45 of its tensile breaking weight, .24 of its compressive or crushing strength, .38 of its transverse, and .39 of its torsional.

Friction of a steel shaft compared to one of wrought iron is as .625 to 1.

Capacity of steel to resist a transverse stress is much less than to resist torsion.

Relative diameters of steel and wrought-iron shafts, to resist equal transverse stress, are as .98 to 1, and weight of such a proportion of steel shaft compared with one of wrought iron will be about 4 per cent. less, and friction of bearing will be 6 per cent. less.

CYLINDERS, FLUES, AND TUBES.

Hollow Cylinders. Cast Iron.

To Compute Elements of Hollow Cylinders within Limits of Elastic Strength. (*D. K. Clark.*)

$S \times \textit{hyp. log.}\, R = P$. $\quad \frac{P}{\textit{hyp. log.}\, R} = S$. $\quad \frac{P}{S} = \textit{hyp. log.}\, R$. S *representing elastic tensile strength of metal in lbs. per sq. inch*, R *ratio of external diameter to internal*, $= \frac{d'}{d} = \frac{r'}{r}$, *and* P *internal pressure in lbs. per sq. inch.* d *and* d' *representing internal and external diameter, and* r *and* r' *internal and external radii, all in ins.*

Note.—Hyperbolic Logarithm of a number is equal to product of its common logarithm and 2.3026.

Illustration 1.—Diameters of a hydrostatic cylinder 5.3 by 13.125 ins.; what pressure within its elastic strength will it sustain per sq. inch?

Assume S = 10 000 *lbs.* $\quad \textit{Hyp. log.}\, R = \frac{13.125}{5.3} \times 2.3026 = \log. 2.5 \times 2.3026 = .92.$

Then 10 000 × .92 = 9200 *lbs. per sq. inch.*

Note.—For Bursting Strength take maximum strength of metal.

2.—A water-pipe .75 inch thick has an internal diameter of 10 ins., what is its bursting pressure?

$$S = 30\,000 \text{ lbs.} \quad \textit{Hyp. log.}\, \frac{10 + \overline{.75 \times 2}}{10} = .1398.$$

Then 30 000 × .1398 = 4194 *lbs.*

3.—If it were required of a hydrostatic press to sustain a pressure of 589 050 lbs. upon a ram of 5 ins. in diameter, what would be pressure on ram, and what should be thickness of metal, assuming it equal to an elastic tensile stress of 15 000 lbs. per sq. inch?

Area of 5 ins. = 19.635. $\quad \frac{589\,050}{19.635} = 30\,000 =$ *pressure per sq. inch on ram.*

Then $\frac{30\,000}{15\,000} = 2$, which = *hyp. log.* R = 7.39, and 7.39 × 5 = 36.95 = *external diameter.* 36.95 − 5 = 31.95, which ÷ 2 = 15.975 *ins. thickness of metal.*

Wrought Iron and Steel.

$$\frac{R + hyp.\ log.\ \frac{d'}{d} - 1}{2}\,S = P. \qquad \frac{2\,P}{R + hyp.\ log.\ \frac{d'}{d} - 1} = S. \qquad \frac{2\,P}{S} + 1 = (P + \text{hyp. log. } R).$$

ILLUSTRATION 1.—If diameters of a wrought-iron cylinder are 5 and 15 ins., and ultimate or destructive strength of metal is 40 000 lbs. per sq. inch, what is its breaking pressure? $\frac{15}{5} = 3.$ *Hyp. log.* $3 = .477\,12 \times 2.3026 = 1.0986.$

Then $\frac{3 + 1.0986 - 1}{2} \times 40\,000 = 61\,972$ *lbs. per sq. inch* $= 61\,972 \times 5 \div \overline{15 - 5} =$ $30\,986.2$ *lbs. per sq. inch of section of metal.*

2.—A steam-boiler 6 feet in internal diameter, of wrought-iron plates .375 inch thick and double riveted longitudinally, burst at a joint by a pressure of 300 lbs. per sq. inch; what was resistance of joint per sq. inch of its section?

$\frac{72 + \overline{.375 \times 2}}{72} = 1.0104.$ *Hyp. log.* $1.0104 = .010\,345.$

Then $\frac{2 \times 300}{1.0104 + .010\,345 - 1} = \frac{600}{.020\,745} = 29\,405$ *lbs. per sq. inch of section of joint.*

SHIP AND BOILER PLATES.

(*See pages 751–757 for Boiler Riveting.*)

Ultimate Tensile Strength of Riveted and Welded Joints of Wrought-iron Plates. (*D. K. Clark.*)

Entire Plate = 100.

Joints.	Plate. .5	.375	.4375	Average.	Joints.	Plate. .5	.375	.4375	Average.
Scarf-welded	—	102	106	104	Double riv'd, snap-headed	59	72	70	67
Lap-welded	50	66	69	62	" " counter-sunk and snap-headed	53	69	72	65
Single hand riveted	40	60	50	50	" " with single welt, counters'k and snap-headed	52	65	60	59
" " snap-headed	50	56	52	53					
" " by machine	40	52	54	49					
" " counter-sunk head	44	52	50	49					

Strength of Riveted Joints per Sq. Inch of Single Plate. (*Wm. Fairbairn.*)

Single Lapped.—Machine riveted. Pitch 3 times, 25 000 lbs.

Hand riveted. Pitch 3 times, 24 000 lbs.

Rivets "staggered," and equidistant from centres, 30 500 lbs.

Abut Joints.—Hand riveted. Rivets not "staggered," and equidistant from centres, single cover or strap, 30 000 lbs.

Rivets "square," single cover or strap, 42 000 lbs.; double covers or straps, 55 000 lbs.

Comparative Strength of Riveted Joints.

Entire Plate .375 *ins. thick* = 100.

Double riveted, double strap, or fish-plated joint	80	Double riveted, single strap, or fish-plated joint	65
Double riveted lap joint	72	Single riveted lap joint	60

For all joints of plates over .5 inch, other than double welded, these proportions are too high.

A closer pitch of rivets should be adopted in single than in double riveted abuts, etc.

Dimensions of Rivets, Pitch, Lap, etc.

Plate. Thickness.	Diam. of Rivet.	Length from Head.	Pitch.	Lap. Single.	Lap. Double.	Lap. Staggered.
Inch.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
.25	.5	1.125	1.5	1.5625	2.75	2.4375
.3125	.625	1.375	1.625	2	3.4375	3
.375	.75	1.625	1.75	2.4375	4.125	3.625
.5	.8125	2.25	2.125	2.625	4.4375	3.9375
.5625	.9375	2.75	2.375	3	5.1875	4.5625
.625	1	3	2.625	3.25	5.5	4.8125
.75	1.125	3.25	3	3.625	6.1875	5.4375
.875	1.25	4	3.375	4	6.875	6.0625
1	1.5	4.5	4.375	4.875	8.25	7.25

Straps.—Single, .125 thicker than the plate; Double, each .625 of thickness of plate.

To Compute Diameter of Rivet.

Ordinarily, $T\ 1.25 + .1875 = d$. T *representing thickness of plate, and d diameter of rivet.*

Pitch of Rivets. (*Nelson Foley.*)

Plates.	Metal between the Holes.	Diam. of Rivets.	Plates.	Metal between the Holes.	Diam. of Rivets.
Single....	52 to 62 per cent.	1.4 to 2.3	Square..	70 to 78 per cent.	.99 to 1.7
Staggered.	68 to 75 " "	1.4 to 2.1	Triple...	76 to 80 " "	.77 to 1

Proportions of Single Rivet Wrought-iron Joints.
(*French.*)

Thickness of Plate.		Diameter of Rivets.		Pitch of Rivets.		Width of Lap.		Thickness of Plate.		Diameter of Rivets.		Pitch of Rivets.		Width of Lap.	
Mil's	Inch.	Mil's	Inch.	Mil's	Ins.	Mil's	Ins.	Mil's	Inch.	Mil's	Ins.	Mil's	Ins.	Mil's	Ins.
3	.118	8	.315	27	1.06	30	1.18	10	.394	20	.787	56	2.2	58	2.28
4	.158	10	.394	32	1.26	34	1.34	11	.433	21	.827	57	2.24	60	2.36
5	.197	12	.472	37	1.46	40	1.58	12	.472	22	.866	58	2.28	60	2.36
6	.236	14	.551	43	1.69	44	1.73	13	.512	23	.906	60	2.36	62	2.44
7	.276	16	.63	48	1.89	50	1.97	14	.551	24	.945	62	2.44	64	2.52
8	.315	17	.669	51	2.01	54	2.13	15	.591	25	.984	63	2.48	66	2.6
9	.354	19	.748	54	2.13	56	2.2	16	.63	26	1.024	65	2.56	68	2.68

Result of Experiments on Double Riveted and Double Strapped Plate Joints. (*Mr. Brunel.*)

Plates, 20 *ins. in width*, 5 *inch thick, Abut jointed, with a Strap or Fish-plate on each side*, 10 *ins. in width. Holes Punched.*

20 .6875 inch rivets, 4 ins. pitch, set "square," tensile strength 77 per cent.
18 .75 " " " " "staggered," " " 78.6 "
24 .75 " " 5 " " "square," " " 84 "

For Boiler Riveting see pp. 755-57.

Hulls of Vessels.

Diameter of Rivets.

Plate.	U. S. and British Lloyds.	Liverpool Reg'y.	Admiralty. Eng.	Millwall, Eng.	Pitch of Rivets.	Length of Rivets. Countersunk.	Length of Rivets. Snap-headed.
Inch.	Inch.	Ins.	Ins.	Inch.	Ins.	Ins.	Ins.
.3125	.625	.5	.5	.625	1.75	1.125	1.5
.375	.625	.625	.625	.625	2	1.25	1.625
.4375	.625	.625	.75	.625	2.125	1.375	1.75
.5	.75	.75	.75	.75	2.25	1.5	2
.5625	.75	.75	.875	.75	2.437	1.6875	2.1875
.625	.75	.8125	.875	.875	2.56	1.9375	2.375
.6875	.875	.875	.875	.875	2.812	2.1875	2.625
.75	.875	.875	1	.875	3.125	2.375	2.75
.8125	.875	.9375	1	.875	3.375	2.5	2.875
.875	1	1	1.125	1	3.625	2.625	3
.9375	1	1.0625	1.125	1	3.875	2.75	3.125
1	1	1.125	1.125	1	4.125	2.875	3.25

Lap of Joint or Course should be .5 pitch of rivets added to .3 diam. of rivet.

NOTE.—Lloyd's requires a spacing of 4.5 diameter. Liverpool Registry, 4. Admiralty, 4.5 to 5 in edges and abuts of bottom and bulkhead plates, and 5 to 6 in other water-tight work. *Bureau Veritas*, 4 diameters for single riveting, and 4.5 for double.

STEEL PLATES.

Steel Plates, according to M. Barba, .354 inch thick are equal to wrought iron .472 inch thick, or as 3 to 4; consequently, when iron rivets are used, their diameter should be in proportion to an iron plate.

It is ascertained also that they are best united by iron rivets.

A steel plate .3125 inch thick requires an iron rivet .5625 inch in diameter, and 1.375 ins. apart.

Bridge Plates and Rivets.

Plates .25 to .5 inch thick. Rivets .75 to 1 inch diameter, and 3 ins. apart from centres in upper flange or girder, and 4 ins. in lower

Rivet Heads.

1. 

Ellipsoidal, Fig. 1.—D *diameter*, R *radius of head* = D, *r radius of flange* = .4 D, *c depth at centre* = .5 D.

Segmental, Fig. 2.—D *diameter*, *c depth at centre* = .625 D, R *radius of head* = .75 D, *o depth below head* = .125 D.


2.

Countersunk.—Head 1.52 D, angle 60°. Countersink .45 diam. of plate.

Cheesehead or heads, section of which is a parallelogram. Head .45 D, diameter 1.5 D.

Rivets.

Shearing strength of a Lowmoor rivet = 40 320 d^2 or 18 d^2 in tons.
d representing diameter of rivet in ins.

Memoranda.

Punching holes for riveting weakens plates, varying from 10 to 20 per cent., according to their temper, hardest losing most.

Countersunk riveting does not impair strength of joint, as compared with external head.

Diagonal abut joints are stronger than square.

Shearing strength of rivets should not exceed that of plates.

Maximum strength of joint is attained at 90 to 100 per cent. of net section of plate.

Shearing strength of English wrought iron is taken at 80 per cent. of its tensile strength.

LEAD PIPE.

Resistance of Lead Pipe to Internal Pressure.
(Kirkaldy, Jardine, and Fairbairn.)

Diam.	Thickness.	Weight per Foot.	Bursting Pressure.	Diam.	Thickness.	Weight per Foot.	Bursting Pressure.	Diam.	Thickness.	Weight per Foot.	Bursting Pressure.
Inch.	Inch.	Lbs.	Lbs.	Ins.	Inch.	Lbs.	Lbs.	Ins.	Inch.	Lbs.	Lbs.
.5	.2	2.3	1579	1.25	.21	5.3	683	2	.21	9.2	498
.625	.2	2.6	1349	1.5	.24	7.1	734	2	.2	—	448
.75	.22	3.8	1191	1.5	.2	—	528	3	.25	—	364
1	.2	4.1	911	1.5	.2	—	626	3	.25	—	374

Tensile strength of metal = 2240 lbs. per sq. inch.

To Compute Thickness of a Lead Pipe when Diameter and Pressure in Lbs. per Sq. Inch is given.

RULE.—Multiply pressure in lbs. per sq. inch by internal diameter of pipe in ins., and divide product by twice tensile resistance of metal in lbs. per sq. inch.

ILLUSTRATION.—Diameter of a lead pipe is 3 ins., and pressure to which it is to be submitted is 370 lbs. per sq. inch; what should be thickness of metal?

$$\frac{370 \times 3}{2240 \times 2} = \frac{1110}{4480} = .248 \textit{ ins.}$$

Difference in Weight between Pipes of "Common," "Middling," and "Strong" is 12 per cent.

To Compute Weight of Lead Pipe.

$\overline{D^2 - d^2}\ 3.86 = W$. *D and d representing external and internal diameters in ins., and W weight of a lineal foot in lbs.*

To Compute Maximum or Bursting Pressure that may be borne by a Lead Pipe.

RULE.—Multiply tensile resistance of metal in lbs. per sq. inch by twice thickness of pipe, and divide product by internal diameter, both in ins.

ILLUSTRATION.—What is bursting pressure of a lead pipe 3 ins. in diameter and .5 inch thick?

$$\frac{2240 \times \overline{.5 \times 2}}{3} = \frac{2240}{3} = 746.6 \textit{ lbs.}$$

Assume a column of water 34 feet in height to weigh 15 lbs. per sq. inch; what head of water would such a pipe sustain at point of rupture?

$$15 : 34 :: 746.6 : 1692.3 \textit{ feet.}$$

Resistance of Glass Globes and Cylinders to Internal Pressure and Collapse. *(Flint Glass.)*

Bursting Pressure.

GLOBES.			CYLINDER.			
Diameter.	Thickness.	Per Sq. Inch.	Diameter.	Length.	Thickness.	Per Sq. Inch.
Ins.	Inch.	Lbs.	Ins.	Ins.	Inch.	Lbs.
4	.024	84	4	7	.079	282
5	.022	90	Elliptical (Crown Glass).			
6	.059	152	4.1	7	.019	109

Collapsing Pressure.

5	.014	292	3	14	.014	85
4	.025	1000*	4	7	.034	202
6	.059	900*	4	14	.064	297

* Unbroken.

Manganese Bronze.

Manganese Bronze, No. 2, has a Tensile strength of 72 000 to 78 600 lbs. per sq. inch, its elastic limit is from 35 000 to 50 000 lbs., its ultimate elongation 12 to 22 per cent., and its hardness alike to that of *mild* steel.

Transverse Strength.—Destructive stress of a bar 1 inch square, supported at both ends at a distance of 1 foot = 4200 lbs., bending to a right angle before breaking, and requiring 1700 lbs. to give it a permanent set.

MEMORANDA.

Cast Iron.

Beams cast horizontally are stronger than when cast vertically.

Relative strength of columns of like material and of equal weights is: Cylindrical, 100; Square, 93; Cruciform, 98; Triangular, 110. (*Hodgkinson.*)

If strength of a cylindrical column is 100, one of a square, a side of which is equal to diameter of the cylinder, is as 150.

Repetition of Stress.—A piece submitted to transverse stress broke at 1956th strain, with a stress .75 of that of its original ultimate resistance.

Resistance to Bursting of Thick Cylinders.—Mean resistance to bursting, of chambers of cast-iron guns is as follows (*Major Rodman, U.S.A.*):

Thickness of metal = 1 calibre, length = 3 calibres, 52 217 *lbs.* per sq. inch.
Thickness of metal = .5 calibre, length = 3 calibres, 49 100 *lbs.* per sq. inch.
The tensile strength of the iron being 18 820 *lbs.*

Diam. of cylinder 2 ins., length 12 ins., metal 2 ins., 80 227 *lbs.* per sq. inch.
Diam. of cylinder 3 ins., length 12 ins., metal 3 ins., 93 702 *lbs.* per sq. inch.
Tensile strength of iron being 26 866 *lbs.*

Sudden Applications of Stress.—Loss of strength by sudden application of load was, by experiment, 18.6 per cent. in excess of load applied gradually, and its elongation 20 per cent. greater.

Low Temperature.—Tensile strength at 23° under sudden application of load, was reduced 3.6 per cent., and elongation 18 per cent.

Wrought Iron.

Increased Hammering gives 20 per cent. greater strength with decreased elongation.

Hardening.—Water increases strength more than oil or tar. A bar .87 inch in diameter, forged and hardened in water, attained a tensile strength of 73 448 lbs. (*Mr. Kirkaldy.*)

Case Hardening.—Loss of tensile strength 4950 lbs. per sq. inch.

Cold Rolling added 18.5 per cent. to tensile strength, and when plates were reduced .33 in thickness, strength was nearly doubled, with but .1 per cent. elongation. Specific gravity was reduced.

Fibre.—Plates are about 12 per cent. stronger with fibre than across it.

Angles, Tees, etc., have from 2200 to 4500 lbs. less tensile strength than rectangular bars.

Galvanizing does not perceptibly affect strength.

Welding.—Strength as affected by welding varies by experiment from 2.6 to 43.8 per cent. less, average being 19.4.

Elastic Strength is about .45 of its tensile breaking weight, .15 of its compressive or crushing strength, and .5 of its transverse strength.

Effect of Screw Threads.—1 inch bolts lose by dies 6.11 per cent., and by chasing 28 per cent.

Steel.

Steel can be hardened in water at a temperature of 310°.

WOODS.

To Compute Transverse Strength of Large Timber.

Destructive Stress.

Fixed at One End, and Loaded at the Other. $\frac{.3\,S\,b\,d^2}{l} = W.$

Fixed at Both Ends, and Loaded in Middle. $\frac{1.8\,S\,b\,d^2}{l} = W.$

* *Supported at Both Ends, and Loaded in Middle.* $\frac{1.2\,S\,b\,d^2}{l} = W.$

Fixed at Both Ends, and Loaded at any other point than the Middle. $\frac{.45\,S\,b\,d^2}{l} = W.$

Supported at Both Ends, and Loaded at any other point than the Middle. $\frac{.3\,S\,b\,d^2\,l}{m\,n} = W.$

$$* \text{ Hence, } \frac{W\,l}{1.2\,b\,d^2} = S, \text{ and } \frac{W\,l}{b\,d^2} = 1.2\,S.$$

b, d, and l representing breadth, depth, and length to or between supports, all in ins., S mean of tensile and crushing strengths of material at two thirds of its Value, as determined by experiments, W ultimate weight or stress in lbs., and m and n distances of load from nearest supports in ins.

When a beam is uniformly loaded, the stress is twice that if applied in its middle or at one end.

Values of 1.2 S.

Hence, for other coefficients, as .3, 1.8, etc., the values will be proportional.

Woods.	1.2 S	Woods.	1.2 S
Ash, white	2.38	Locust	3.7
" Canadian	2.4	Mahogany, Honduras	2.3
" English	2.46	Oak, Pa.	2
Beech	2.55	" Va.	2.3
Birch	2.5	" white	2.5
Cedar	1.6	" English	1.7
" Cuban	1.6	" Dantzic	1.35
Chestnut	1.53	" French	2.44
Cypress	.85	Pine, Va.	3
Elm, English	1.12	" pitch	2.2
" Rock, Canada	2.63	" white	2.71
Fir, Dantzic	2.5	" yellow	3.87
Greenheart	3.81	" " Canada	1.8
Gum, blue	2	Redwood, Cal.	1.1
Hackmatack	1.36	Spruce	1.2
Iron wood	3.64	Teak	3.17
Larch	1.77	Walnut, black	1.25

ILLUSTRATION 1.—What is destructive stress of a beam of English oak, 2 ins. square, and 6 feet between its supports?

1.2 from table = 1.7, and S = .66 of 5700 (mean of tensile and crushing strength) = 3762 *lbs.*

$$\frac{1.7 \times 2 \times 2^2 \times 3762}{6 \times 12} = \frac{51\,163}{72} = 710.6 \text{ lbs.}$$

By experiment of Mr. Laslett it was 688 *lbs.*

2.—What is destructive stress of a beam of yellow pine, 3 ins. by 12, and 14 feet between its supports?

1.2 from table = 3.87, and S = .66 of 10 200 (mean of tensile and crushing strength) = 6732 *lbs.*

$$\frac{3.87 \times 3 \times 12^2 \times 6732}{14 \times 12} = \frac{11\,654\,827}{168} = 69\,374 \text{ lbs.}$$

If the beam was fixed at both ends then 3.87 would be 5.8.

Or, as 1.2 : 1.8 :: 3.87 : 5.8.

Safe Statical Loads for Rectangular Beams of Various Materials, One Inch in Breadth and One Foot in Length.

Supported at Both Ends and Loaded in Middle.

A *American*, Af *African*, B *Baltic*, B'k *Black*, C *Canadian*, D *Dantzic*, E *English*, G *Georgia*, M *Memel*, P *Pitch*, R *Riga*, W *White*, Y *Yellow*.—*Figures at Head of Columns denote Destructive Weight of Material in Lbs.*

Depth.	B'k Ash. Red Cedar. 300	C Fir. B'k Spruce. Larch. 320	Birch. Hack-matack. Hemlock. 400	W Beech. Walnut. C Ash. R Fir. 450	Spruce. Sycamore. Elm. W Pine. 500	Beech. Y Pine. E Oak. 525	P Pine. C Oak. Spruce. 550	W Oak. B Fir. 600	M Fir. Chestnut. E Ash 640	Ash. D Fir. Rock Elm. 680	Hickory. Maple. Af Oak. G Pine. 800	Locust. 1180
Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1	60	64	80	90	100	105	110	120	128	136	160	236
2	240	256	320	360	400	420	440	480	512	544	640	944
3	540	576	720	810	900	945	990	1 080	1 152	1 224	1 440	2 124
4	960	1 024	1 280	1 440	1 600	1 680	1 760	1 920	2 048	2 176	2 560	3 776
5	1 500	1 600	2 000	2 250	2 500	2 625	2 750	3 000	3 200	3 400	4 000	5 900
6	2 160	2 304	2 880	3 240	3 600	3 780	3 960	4 320	4 608	4 896	5 760	8 496
7	2 940	3 136	3 920	4 410	4 900	5 145	5 390	5 880	6 272	6 664	7 840	11 504
8	3 840	4 096	5 120	5 760	6 400	6 720	7 040	7 680	8 192	8 704	10 240	15 104
9	4 860	5 184	6 480	7 290	8 100	8 505	8 910	9 720	10 368	11 016	12 960	19 116
10	6 000	6 400	8 000	9 000	10 000	10 500	11 000	12 000	12 800	13 600	16 000	23 600
11	7 260	7 744	9 680	10 890	12 100	12 605	13 310	14 520	15 488	16 456	19 360	28 550
12	8 640	9 216	11 520	12 960	14 400	15 120	15 840	17 280	18 432	19 584	23 040	33 984
13	10 140	10 816	13 520	15 210	16 900	17 745	18 590	20 280	21 632	22 984	27 040	30 884
14	11 760	12 544	15 680	17 640	19 600	20 580	21 560	23 520	25 088	26 656	31 360	40 256
15	13 500	14 400	18 000	20 250	22 500	23 625	24 750	27 000	28 800	31 000	36 000	53 100
16	15 360	16 384	20 480	23 040	25 600	26 880	28 160	30 720	32 768	34 816	40 960	60 416

Illustrations of Table.

1.—What is safe statical load for a white-pine beam, 4 ins. by 12, and 15 feet between its supports, loaded in middle.

A like beam, 1 inch in width, 12 ins. in depth, and 1 foot between its supports, will bear as per table 14 400 lbs.

Hence $14\,400 \times \frac{4}{15} = 3840$ *lbs.*

2.—What should be depth of a like beam, 3 ins. in width, and 10 feet between its supports, to bear a statical weight of 1920 lbs.?

$$\frac{1920 \times 10}{3} = 6400 \textit{ lbs.}$$

Coefficient or Safe load for the material $= 500 \div 5 = 100$.

Hence $\sqrt{6400 \div 100}$ *(coefficient for the material)* = 8 ***ins.***

Following Coefficients or Factors of Safety are for .125 of average destructive weight:

Coefficients for Various Woods. *(Hatfield and others.)*

Ash	85	Hickory	{ 100 / 130	Oak, Canada	70
" English	80	Larch	40	" French	85
" Canada	60	Locust	150	Pine, pitch	68
Beech	58	Maple	105	" yellow	65
Chestnut	65	Oak, white	80	" red	55
Elm, Canada	80	" English	60	" Georgia	{ 100 / 120
" English	42	" Dantzic	62	" white	62
Fir, Riga	47	" Adriatic	55	" Canada red	60
Hemlock	60	Spruce	65		

ILLUSTRATION.—What safe weight will a beam of white pine sustain, 4 ins. in breadth, 12 in depth, and 15 feet between its supports, when loaded in its middle? and what when uniformly loaded? *Coefficient* as above, 62.

Then $\frac{4 \times 12^2 \times 62}{15} = 2380.8$ *lbs. loaded in its middle*, and $2380.8 \times 2 = 4761.6$ *lbs. if uniformly loaded.*

Floor Beams of Wood.

Condition of stress borne by a Floor beam is that of a beam supported at both ends and uniformly loaded; but from irregularity in its loading and unloading, and from necessity of its possessing great rigidity, it is proper to estimate its capacity as a beam loaded at middle of its length.

To Compute Capacity of Floor Beams, Girders, etc.

Supported at Both Ends.

RULE.—Divide product of breadth and square of depth, in ins., and *Coefficient* for material, by length in feet, and result will give weight in lbs.

Or, $\frac{b\,d^2\,C}{l} = W$. *Fixed at Both Ends.* $\frac{1.5\,b\,d^2\,C}{l} = W$.

EXAMPLE.—The dimensions of a white-pine floor timber are 4 by 12 ins., and its length between supports 15 feet; what weight will it sustain in its centre?

C as per preceding table = 62. Then $\frac{4 \times 12^2 \times 62}{15} = 2380.8$ *lbs.*

When Uniformly Loaded. Multiply the results by 2.

To Compute Depth of a Floor Beam.

Supported at Both Ends.

When Length between Supports, Breadth and Distance between Supports, for One Foot, between Centres of Beams are Given. RULE.—Divide product of length in feet, and weight to be borne in lbs., by product of breadth in ins., and *Coefficient* for material, and square root of quotient will give depth in ins., for distance between centres of one foot.

Or, $\sqrt{\frac{l\,W}{b\,C}} = d$. *Fixed at Both Ends.* $\sqrt{\frac{l\,W}{1.5\,b\,C}} = d$.

When Uniformly Loaded, W represents but half required or given weight.

EXAMPLE.—Take elements of preceding case, distance between centres of beam 15 ins. C = 62.

Then $\sqrt{\frac{15 \times 2380.8}{4 \times 62}} = \sqrt{\frac{35\,712}{248}} = 12$ *ins.*

When Distance between Centres of Beams is greater or less than one Foot. RULE.—Divide product of square of depth for a beam, *When distance between centres is one foot*, by distance given, by 12, and square root of quotient will give depth of beam.

EXAMPLE.—Assume beam in preceding case to be set 15 ins. from centres of adjoining beams; what should be its depth?

$$\text{Then } \sqrt{\frac{12^2 \times 15}{12}} = \sqrt{\frac{2160}{12}} = 13.42 \textit{ ins.}$$

To Compute Breadth of a Floor Beam or Girder.

Supported at Both Ends.

When Length and Depth are given. RULE.—Divide product of length in feet, and weight to be borne in lbs., by product of square of depth in ins., and *Coefficient* for material, and quotient will give breadth in ins.

$$\text{Or, } \frac{l\,W}{d^2\,C} = b. \quad \textit{Fixed at Both Ends.} \quad \frac{l\,W}{1.5\,d^2\,C} = b.$$

When Uniformly Loaded, W represents but half required or given weight.

EXAMPLE.—Take elements of preceding cases.

$$\text{Then } \frac{15 \times 2380.8}{12^2 \times 62} = \frac{35\,712}{8928} = 4 \textit{ ins.}$$

When Distance between Centres of Beams is greater or less than One Foot. RULE.—Divide product of breadth for a beam, *When distance between centres is one foot,* and distance given, by 12, and result will give breadth.

EXAMPLE.—Assume beam, as in preceding case, to be set 15 ins. from centre of adjoining beams; what should be its breadth?

$$\text{Then } \frac{4 \times 15}{12} = \frac{60}{12} = 5 \textit{ ins.}$$

When Weight is Suspended or Stress borne at any other point than the Middle, See Formulas, page 801.

Header and Trimmer or Carriage Beams.

Conditions of stress borne or to be provided for by them are as follows:

Header supports .5 of weight of and upon tail beams inserted into or attached to it, and stress upon it is due directly to its length and weight of and upon tail beams it supports, alike to a girder loaded at different points.

Trimmer or *Carriage* beams support, in addition to that borne by them directly as floor beams, each .5 weight on headers.

NOTE.—In consequence of effect of mortising (when bridles or stirrups are not used), a reduction of fully one inch should be made in computing the capacity of depth of headers and trimmers.

To Compute Breadth of a Header Beam.

When Uniformly Loaded. RULE.—Compute weight to be borne in lbs. by tail beams, divide it by two (one half only being supported by header), multiply result by length of beam in feet, and divide product by product of twice *Coefficient* of material and square of depth, and result will give breadth in ins.

$$\text{Or, } \frac{\overline{W \div 2}\,l}{2\,C\,d^2} = b. \quad \textit{W representing weight per sq. foot.}$$

EXAMPLE.—What should be breadth of a Georgia pine header, 13 ins. in depth, 10 feet in length, supporting tail beams 12 feet in length, bearing 200 lbs. per sq. foot of area supported?

C, as per preceding table, 100, and depth $= 13 - 1 = 12$ *ins.*

$$\text{Then } \frac{12 \times 10 \times 200 \div 2 \times 10}{2 \times 100 \times 12^2} = \frac{120\,000}{28\,800} = 4.17 \textit{ ins.}$$

To Compute Depth of a Header Beam.

RULE.—See rule for depth of a floor beam, page 835, with the exception that a header is assumed to be always uniformly loaded.

$$\text{Or, } \sqrt{\frac{l\,.5\,W}{b\,C}} = d.$$

To Compute Breadth of a Trimmer Beam.

With One Header and One Set of Tail Beams. RULE.—Proceed as for computation of dimension of a beam loaded at any other point than middle.

$.5\,\frac{m\,n\,W}{l\,d^2\,C} = b.$ *m and n representing distances of the weight or load from each end in feet.*

ILLUSTRATION.—What should be breadth of a trimmer or carriage beam of Georgia pine, 23 feet in length, 15 ins. in depth, sustaining a header 10 feet in length, with tail beams 19 feet, and designed for a load of 540 lbs. per sq. foot of floor?

Assume $C = 100$; $d = 15 - 1 = 14$; *m and n* $= 19$ *and* 4 *feet.*

$$.5 \times \frac{19 \times 4 \times \overline{19 \times 10 \div 2} \times 540}{23 \times 100 \times 14^2} = .5 \times \frac{3\,898\,800}{450\,800} = 4.32 \text{ ins.}$$

NOTE 1.—Depth of trimmer beams is usually determined by depth of floor beams; when not, proceed to determine it as for a header.

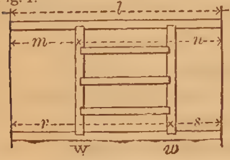
2.—When a trimmer beam is mortised to receive headers, it is proper to deduct 1 inch from its depth, as in preceding illustrations. When bridle or stirrup irons are used to suspend headers, a deduction of the thickness of the iron only is necessary, usually .5 inch.

With Two Headers and One Set of Tail Beams.—Fig. 1.

OPERATION.—Proceed for each weight or load as for a beam, when weights are sustained or stress borne at other point than the middle.

$\frac{a\,L}{.5 \times .5} = W$ and w. *a representing area of floor in sq. feet,* L *load per sq. foot, and* W *and w weights or loads at points of rest on trimmers.*

Fig. 1.



NOTE.—Hatfield and some other authors give complex and extended formulas, to deduce the dimensions of a Girder or Beam, under a like stress.

Upon consideration, however, it will readily be recognized that a beam loaded at more than one point is simply two or more beams, as the case may be, loaded at different points, and connected together.

ILLUSTRATION.—What should be breadth of a trimmer beam of Yellow or Georgia pine, 25 feet in length, 12 ins. in depth, sustaining two headers 12 feet in length, set at 15 feet from one wall and 5 feet from the other, to support with safety 300 lbs. per sq. foot of floor?

$l = 25$, $m = 15$, $n = 10$, $s = 5$, $r = 20$, $C = 100$, and $d = 12 - 1 = 11$ *for loss by mortising.*

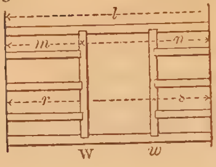
$$\frac{12 \times 5 \times 300}{.5 \times .5} = \frac{18\,000}{.25} = 4500 \text{ lbs. at W, and } \frac{12 \times 5 \times 300}{.5 \times .5} = \frac{18\,000}{.25} = 4500 \text{ lbs. at } w.$$

Then $\frac{15 \times 10 \times 4500}{25 \times 11^2 \times 100} = \frac{675\,000}{302\,500} = 2.23$ *ins. breadth for load on header at* 15 *feet,*

and $\frac{5 \times 20 \times 4500}{25 \times 11^2 \times 100} = \frac{450\,000}{302\,500} = 1.48$ *ins. breadth for load on header at* 5 *feet,* and $2.23 + 1.48 = 3.71$ *ins. combined breadth.*

With Two Headers and Two Sets of Tail Beams.—Fig. 2.

Fig. 2.



OPERATION.—Proceed as directed for Fig. 1.

ILLUSTRATION. — What should be breadth of a trimmer beam of yellow pine 25 feet in length, 15 ins. in depth, sustaining two headers 12 feet in length, set at 15 feet from one wall and 5 feet from the other, to support with safety 300 lbs. per sq. foot of floor?

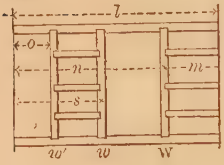
$l = 25$, $m = 15$, $n = 10$, $s = 5$, $r = 20$, $C = 100$, and $d = 15 - 1 = 14$ *for loss by mortising.*

$\frac{12 \times 15 \times 300}{.5 \times .5} = \frac{54\,000}{.25} = 13\,500$ *lbs.* at W, and $\frac{12 \times 5 \times 300}{.5 \times .5} = \frac{18\,000}{.25} = 4500$ *lbs.* at *w*.

Then $\frac{15 \times 10 \times 13\,500}{25 \times 14^2 \times 100} = \frac{2\,025\,000}{490\,000} = 4.14$ *ins.*, and $\frac{5 \times 20 \times 4500}{25 \times 14^2 \times 100} = \frac{450\,000}{490\,000} = .92$ *ins.*, and $4.14 + .92 = 5.06$ *ins. combined breadth.*

With Three Headers and Two Sets of Tail Beams.—Fig. 3.

Fig. 3.



OPERATION.—Proceed as directed for Fig. 1.

ILLUSTRATION. — What should be breadth of a trimmer beam of yellow pine, 20 feet in length, 13 ins. in depth, sustaining 3 headers 15 feet in length, set at 3, 7, and 13 feet from one wall, to sustain a load of 200 lbs. per sq. foot of floor?

$l = 20$, $m = 7$, $n = 13$, $s = 7$, $o = 3$, $d = 13 - 1 = 12$ *ins.*, and $C = 100$.

$\frac{15 \times 7 \times 200}{.5 \times .5} = \frac{21\,000}{.25} = 5250$ *lbs. at* W;

$\frac{15 \times \overline{7 - 3} \times 200}{.5 \times .5} = \frac{12\,000}{.25} = 3000$ *lbs. at w*; and $\frac{15 \times \overline{7 - 3} \times 20}{.5 \times .5} = 3000$ *lbs. at w'*.

Then $\frac{7 \times 13 \times 5250}{20 \times 12^2 \times 100} = \frac{477\,750}{288\,000} = 1.66$ *ins.*; $\frac{7 \times 13 \times 3000}{20 \times 12^2 \times 100} = \frac{273\,000}{288\,000} = .95$ *ins.*; and $\frac{3 \times 17 \times 3000}{20 \times 12^2 \times 100} = \frac{153\,000}{288\,000} = .53$ *ins.* Hence, $1.66 + .95 + .53 = 3.14$ *ins. combined breadth.*

Stirrups or Bridles.

Stirrups are resorted to in flooring designed for heavy loads, in order to avoid the weakening of the trimmers by mortising.

Average wrought iron will sustain from 40 000 to 50 000 lbs. per sq. inch.

Hence 45 000 lbs. as a mean, which ÷ 5 for a factor of safety, = 9000 lbs.

A stirrup supports one half weight of header, and being doubled (looped), the stress on it is but $.5 \div 2 = .25$ of load on header.

To Compute Dimensions of Stirrups or Bridles.

$$\frac{W \div 2}{2 \times 9000} = \textit{area}. \quad \text{Hence } \frac{\text{area}}{\text{thickness}} = \textit{width}.$$

ILLUSTRATION.—What should be area and width of .75 inch wrought-iron stirrup irons for a weight on a header beam of 240 000 lbs.?

$$\frac{240\,000 \div 2}{2 \times 9000} = \frac{120\,000}{18\,000} = 6.66 \textit{ sq. ins.}, \text{ and } \frac{6.66}{.75} = 8.8 \textit{ ins.} = \textit{width}.$$

Girder.

Condition of stress borne by a Girder is that of a beam fixed or supported at both ends, as the case may be, supporting weight borne by all beams resting thereon, at the points at which they rest.

To Compute Dimensions of a Girder.

RULE.—Multiply length in feet by weight to be borne in lbs., divide product by twice* the *Coefficient*, and quotient will give product of breadth and square of depth in ins.

$$\text{Or, } \frac{l\,W}{2\,C} = b \text{ and } d^2, \text{ and } \sqrt{\frac{l\,W}{2\,b\,C}} = d.$$

EXAMPLE.—It is required to determine dimensions of a yellow-pine girder, 15 feet between its supports, to sustain ends of two lengths of beams, each resting upon it and adjoining walls, 15 feet in length, having a superincumbent weight, including that of beams, of 200 lbs. per sq. foot.

Condition of stress upon such a girder is that of a number of beams, 30 feet in length (15 × 2), supported at their ends, and sustaining a uniform stress along their length, of 200 lbs. upon every superficial foot of their area.

Coefficient .2 of 500 = 100.

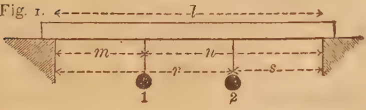
$30 \times 15 \times 200 \div 2$, *for half support on their walls* = 45 000 *lbs.*

Then $\frac{15 \times 45\,000}{2 \times 100} = 3375 = b$ and d^2. Assuming $b = 12$ *ins.*, then $\sqrt{\frac{3375}{12}} = 16.77$ *ins.* Or, if 15 ins., then $\sqrt{\frac{3375}{15}} = 15$ *ins.*

To Compute Greatest Load upon a Girder, and Dimensions thereof.—Fig. 1.

When a Beam is Loaded at Two Points.

Fig. 1.



$\frac{m\,n}{l}$ = *effect of weight at* 1,

$\frac{r\,s}{l}$ = *effect of weight at* 2,

$\frac{m}{l}(W \times n + w\,s)$ = *the two effects at* 1, and $\frac{s}{l}(w\,r + W\,m)$ = *two effects at* 2.

Then, for weight and dimensions, same formulas will apply.

ILLUSTRATION.—Assume weight of 8000 lbs. at 3 feet from one end of a white-pine beam 12 feet in length between its bearings, and another weight of 3000 lbs. at 5 feet from other end. C .2 of 500 = 100.

$8000 \times 3 \times \overline{12 - 3} = 216\,000$ *effect of weight at location* 1, and $3000 \times 5 \times \overline{12 - 5} = 105\,000$ *effect of weight at location* 2. Hence 1, being greatest, = W, and 2 = *w*.

Then, $\frac{3 \times 9}{12} \times 8000 = 18\,000$ at W, and $\frac{5 \times 7}{12} \times 8750 = 3750$ at *w*; and

$\frac{3}{12}(\overline{8000 \times 9} + \overline{3000 \times 5}) = 21\,750$ = *total effect at* W, and $\frac{5}{12}(\overline{3000 \times 7} + \overline{8000 \times 3}) = 18\,750$ = *total effect at w*.

Hence, to ascertain dimensions at greatest stress,

$\frac{21\,750 \times 3 \times 9}{12 \times 300} = 489.37$, and assume $d = 10$, then $\frac{489.37}{10^2} = 4.89$ *ins. breadth*; or, $\sqrt{\frac{489.33}{4.89}} = 10$ *ins. depth.*

---

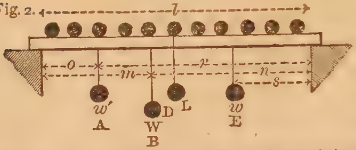
* For being uniformly loaded.

Verification.—Assume a beam as above loaded with 21 750 lbs. at 3 feet from end.

Then, by formula for 801, $\frac{3 \times 9 \times 21\,750}{12 \times 10^2 \times 100} = \frac{587\,250}{120\,000} = 4.89$ *ins.*

Equivalent Weight at Middle.—Fig. 2.

Fig. 2.



$\frac{w' o}{l \div 2} = A$; $\frac{W n}{l \div 2} = B$; $\frac{w s}{l \div s} = E$; and $\frac{L l}{l} \div 2^* = D =$ *equivalent load at middle.*

ILLUSTRATION.—What should be breadth of a beam of Georgia pine, 20 feet in length, 15 ins. in depth, uniformly loaded with 4000 lbs., and sustaining 3 headers or concentrated loads of 6000 lbs., at respective distances of 4 and 9 feet from one end and 7000 lbs. at 6 feet from other end?

$o = 4$, $r = 16$, $m = 9$, $n = 11$, $s = 6$, $d = 15 - 1 = 14$, $L = 4000$, and $C = 850 \times .2 = 170$. $\frac{6000 \times 4}{20 \div 2} = 2400$; $\frac{6000 \times 9}{20 \div 2} = 5400$; $\frac{7000 \times 6}{20 \div 2} = 4200$;

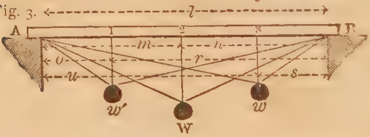
and $\frac{4000 \times 20}{20} \div 2 = 2000$. $2400 + 5400 + 4200 + 2000 = 14\,000$ *lbs.*

Then $\frac{14\,000 \times 10 \times 10}{20} = 70\,000$ *lbs., effect at middle.*

Hence, $\frac{20 \times 70\,000}{4 \times 14^2 \times 170} = \frac{1\,400\,000}{133\,280} = 10.5+$ *ins.*

Operation deduced by Graphic Delineation of Greatest Stress without uniform Load.

Fig. 3.



Moments of weights $= \frac{w' o r}{l}$; $\frac{W m n}{l}$; and $\frac{w s u}{l} =$ 19 200, 29 700, and 29 400, and let fall perpendiculars 1, 2, and 3 proportionate thereto.

Connect w', W, and w with A B, and sum of distances of intersections of these lines upon perpendiculars, from 1, 2, and 3, respectively, will give stress upon A B at these points.

Whence, greatest stress at greatest load will be ascertained to be 61 800 lbs.

When Loaded at Three Points, as in Fig. 2. $\frac{m}{l}(W n + w s) + w' \frac{n o}{l} =$ *Greatest Stress.*

ILLUSTRATION.—Take elements of above case, omitting uniformly distributed load.

$\frac{9}{20}(6000 \times 11 \times 7000 \times 6) + 6000 \frac{11 \times 4}{20} = \frac{9}{20} \times 108\,000 + 13\,200 = 61\,800$ *lbs.*

Deflection of Girders and Beams.

$\frac{W l^3}{C b d^3} = D$; $\frac{C b d^3}{l^3} = W$; $\sqrt[3]{\frac{W l^3}{C b D}} = d$; and $\sqrt[3]{\frac{C b d^3 D}{W}} = l$. *l representing length in feet, b and d breadth and depth, and D deflection in ins.*

Values of C for Various Woods. (Hatfield.)

Ash	4000	Larch	2093	Pine, Georgia	5900
Chestnut	2550	Oak, white	3100	" pitch	2836
Hemlock	2800	" English, mean	2686	" white	2900
Hickory	3850	Spruce	3500	" red	4259

ILLUSTRATION.—What would be deflection of a floor beam of white pine, 10 feet in length, 4 ins. in breadth, and 8 in depth, with 4000 lbs. loaded in its middle?

$C = 2900$. $\frac{4000 \times 10^3}{2900 \times 4 \times 8^3} = \frac{4\,000\,000}{5\,939\,200} = .674$ *inch.*

* Load uniformly distributed.

When Weight is Uniformly Distributed.

$\frac{.625\,W\,l^3}{C\,b\,d^3} = D$; $\frac{C\,b\,d^3}{.625\,l^3} = W$; $\sqrt[3]{\frac{C\,b\,d^3\,D}{.625\,W}} = l$; and $\sqrt[3]{\frac{W\,.625\,l^3}{C\,b}} = d$.

Hence, Deflection in preceding illustration would be $.674 \times .625 = .421$ *ins.*

ILLUSTRATION.—What should be length of a white-pine beam 3 by 10 ins., to support 6000 lbs. uniformly distributed, with a deflection of 2 ins.? $C = 2900$.

$$\sqrt[3]{\frac{2900 \times 3 \times 10^3 \times 2}{.625 \times 6000}} = \sqrt[3]{\frac{17\,400\,000}{3750}} = 16.68 \textit{ feet.}$$

A fair allowance for deflection of floor beams, etc., is .03 inch per foot of length; .04 inch may be safely resorted to.

Weights of Floors and of Loads.

Dwellings.—Weight of ordinary floor plank of white pine or spruce, 3 lbs. per sq. foot, and of Georgia pine, 4.5 lbs.

Plastering, Lathing, and Furring will average 9 lbs. per sq. foot.

Clay Blocks (*Flat Arch*) 5.25×7.25 ins. in depth and 1 foot in length, 21 lbs. = 80 lbs. per cube foot of volume.

Floors of dwellings will average 5 lbs. per sq. foot for white pine or spruce, and on iron girders will average from 17 to 20 lbs. per sq. foot.

Weight of men, women, and children over 5 years of age, 105.5 lbs., and one third of each will occupy an average area of 12×16 ins. = 192 sq. ins. = 78.5 lbs. per sq. foot.

Of men alone 15×20 ins. = 300 sq. ins. = 48 in 100 sq. feet.

Bridges, etc.—Weight of a body of men, as of infantry closely packed, = 138 lbs. each, and they will occupy an area of 20×15 ins. = 300 sq. ins. = 66.24 lbs. per sq. foot of floor of bridge, and as a live or walking load, 80 lbs. per sq. foot.

Weight of a dense and stationary crowd of men, 120 lbs. per sq. foot.

Bridging of Floor Beams increases their resistance to deflection in a very essential degree, depending upon the rigidity and frequency of the bridges.

Weight on Floors, etc., in addition to Weight of Structure, per Sq. Foot.

Ball rooms	85 lbs.	Roofs, wind and snow	30 to 35 lbs.
Brick or stone walls	115 to 150 "	Slate roofs	45 "
Churches and Theatres	80 "	Snow, per inch	.5 lb.
Dwellings	40 "	Street bridges	80 lbs.
Factories	200 to 400 "	Warehouses	250 to 500 "
Grain	100 "	Wind	50 "

Scarfs.

Relative resistance of scarfs in Oak and Pine, 2 ins. square, and 4 feet in length, by experiments of Col. Beaufoy.

Scarf 12 *ins. in Length and* 13 *ins. from End, or* 1 *inch from Fulcrum.*

Vertical.—110 lbs. gave away in scarf.

Horizontal, large end uppermost and towards fulcrum.—101 lbs. fastenings drew through small end of scarf; *small end uppermost, etc.*, 87 lbs. gave away in thick part of scarf.

Factors of Safety.

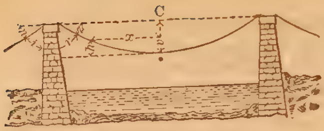
Statical or Dead Load at .2 of destructive stress, but for ordinary purposes it may be increased to .25, and in some cases with good materials to .3.

Live Load at .1 to .125 of destructive stress.

See also page 802.

SUSPENSION BRIDGE.

To Compute Elements.



$\frac{C\,L}{8\,v}$ or $\frac{Q\,a^2}{2\,v} = S$; $\frac{v\,x^2}{(.5\,C)^2} = h$;

$S\sqrt{\left(\frac{2\,h}{x}\right)^2 + 1} = s = S$ *sec.* o;

$\frac{4\,h}{2\,x} =$ *tan. angle* o; $\frac{L}{n-1} = S'$;

$2\sqrt{(.5\,C)^2 + \frac{4}{3}\,v^2} = l$; $\frac{4\,v}{C} =$ *cot.* r; $\frac{C\,L}{8\,S} = v$; $\frac{8\,v\,t}{C}\sqrt{\left(\frac{4\,v}{C}\right)^2 + 1} = L$;

$S \div \sqrt{\left(\frac{4\,v}{C}\right)^2 + 1} = t$; or $\frac{L \div 2}{z} = t$; $\frac{2\,v}{\sqrt{(2\,v)^2 + (C \div 2)^2}} = z$, and $\frac{L \div 2 \times \cos. z}{\sin. z}$

= stress at •. C *representing chord or span, a half chord, and v versed sine of chord or curve of deflection, in feet, L distributed load inclusive of suspended structure, Q load per lineal foot, and S stress at centre, all in tons, x distance of any point from centre of curve, and h height of chain at x above centre of it, both in feet, s stress on chain at any point, as x, from centre of span, s' stress on any tension rod, and t stress at abutments, all in tons, n number of tension rods, o angle of tangent of chain with horizon at any point, as x, r angle of chain with vertical at abutments, l length of chain, in feet, and z angle of direction of chain.*

Assume C = 300 *feet*, L = 1000 *tons*, $v = 25$ *feet*, $x = 100$ *feet*, $n = 30$, $r = 71^\circ\,34'$, and $o = 12^\circ\,32'$.

Then, $\frac{300 \times 1000}{8 \times 25} = 1500$ *tons* = S; $\frac{25 \times 100^2}{(.5 \times 300)^2} = 11.11$ *feet* = h;

$1500\sqrt{\left(\frac{2 \times 11.11}{100}\right)^2 + 1} = 1536.56$ *tons* = s; $\frac{4 \times 11.11}{2 \times 100} = .2222 = 12^\circ\,32' =$ *tan.* $\angle\, o$; $2\sqrt{(.5 \times 300)^2 + \frac{4}{3}\,25^2} = 305.5$ *feet* = l; $\frac{300 \times 1000}{8 \times 1500} = 25 = v$;

$\frac{4 \times 25}{300} = .3333 = 71^\circ\,34' =$ *cot. angle* r; $\frac{1000}{30 - 1} = 34.48$ *tons* = s';

$\frac{1500}{\sqrt{\left(\frac{4 \times 25}{300}\right)^2 + 1}} = 1428.6$ *tons* = t; and $\frac{2 \times 25}{\sqrt{(2 \times 25)^2 + (300 \div 2)^2}} = .3162 = 18^\circ\,26'$.

For a deflection of .125 of span, horizontal stress is equal to total load.

To Construct curve, see Geometry, page 230.

To Compute Ratio which Stress on Chains or Cables at either Point of Suspension Bears to whole Suspended Weight of Structure and Load.

$$\frac{1}{2 \times \sin. z} = R.$$ R *representing ratio.*

ILLUSTRATION.—Assume elements of preceding case.

$\frac{1}{2 \times .3162} = 1.58$ *ratio*. By a preceding formula it would be 1.536.

Stress on Back Stays.—The cables being led over rollers, having free motion, tension upon them is same, whether angle i is same as that of r or not.

Stress on Piers.—When angles r and i are alike, stress on piers will be vertical, but when angle of i is greater or less than r, stress will be oblique.

To Compute Horizontal Stress and Vertical Pressure on Piers.

S cos. z = S i, S cos. n = S o, S sin. z = P i, and S sin. n = P o. S i *and* S o *representing stress, and* P i *and* P o *pressure, inward and outward.*

NOTE.—Span of New York and Brooklyn Bridge 1595.5 feet, deflection 128 feet, angle of deflection at piers from horizontal $15^\circ\,10'$.

TRACTION.

Results of Experiments on Traction of Roads and Pavements. (*M. Morin.*)

1st. Traction is directly proportional to load, and inversely proportional to diameter of wheel.

2d. Upon a paved or Macadamized road resistance is independent of width of tire, when it exceeds from 3 to 4 ins.

3d. At a walking pace traction is same, under same circumstances, for carriages with or without springs.

4th. Upon hard Macadamized, and upon paved roads, traction increases with velocity: increments of traction being directly proportional to increments of velocity above velocity of 3.28 feet per second, or about 2.25 miles per hour. The equal increment of traction thus due to each equal increment of velocity is less as road is more smooth, and carriage less rigid or better hung.

5th. Upon soft roads of earth, sand, or turf, or roads thickly gravelled, traction is independent of velocity.

6th. Upon a well-made and compact pavement of dressed stones, traction at a walking pace is not more than .75 of that upon best Macadamized roads under similar circumstances; at a trotting pace it is equal to it.

7th. Destruction of a road is in all cases greater as diameters of wheels are less, and it is greater in carriages without springs than with them.

Experiments made with the carriage of a *siege train* on a solid gravel road and on a good sand road gave following deductions:

1. That at a walk traction on a good sand road is less than that on a good firm gravel road.

2. That at high speeds traction on a good sand road increases very rapidly with velocity.

Thus, a vehicle without springs, on a good sand road, gave a traction 2.64† times greater than with a similar vehicle on same road with springs.

Results with a Dynamometer.

*Wagon and Load 2240 lbs.**

Roadway.	Relat'e number of horses for like effect.	Roadway.	Relat'e number of horses for like effect.
On railway, 8 lbs.	1	Telford road, 46 lbs.	5.75
On best stone tracks, 12.5 lbs.	1.56	Broken stone or con'te, 46 lbs.	5.75
Good plank road, 32 to 50 lbs.	4 to 6.25	Gravel or earth, 140–147 lbs.	17.5
Stone block pavement, 32.5 "	4.06		18.37
Macadamized road, 65 lbs.	8.12	Common earth road, 200 lbs.	25

NOTE.—By recent experiments of M. Dupuit, he deduced that traction is inversely proportional to square root of diameter of wheel.

Relation of force or draught to weight of vehicle and load over 6 different constructions of road, gave for different speeds as follows:

	Walk.	Trot.		Walk.	Trot.
Stage coach, 5 tons	1.3	1	Carriage, seats only, on springs	1.29	1

Resistance to Traction on Common Roads.

On Macadamized or Uniform Surfaces. (*M. Dupuit.*)

1. Resistance is directly proportional to pressure.
2. It is independent of width of tire.
3. It is inversely as square root of diameter of wheel.
4. It is independent of speed.

* *See Treatise on Roads, Streets, and Pavements, by Brev. Maj.-Gen'l Q. A. Gillmore, U. S. A.*
† Telford estimated it at 3.5.

On Paved and Rough Roads.

Resistance increases with speed, and is diminished by an enlargement of tire up to a moderate limit.

Traction on Various Roads.—Traction of a wheeled vehicle is to its weight upon various roads as follows:

	Per Ton.	Per 100 lbs.
Stone track, best	12.5 to 15	.55 to .58
" "	28 to 39	1.25 to 1.3
" pavement.	14 to 36	.5 to 1.5
Asphalted.......	22 to 28	1 to 1.25
Plank..........	22 to 45	.98 to 2
Block stone pavement....	32 to 35	1.4 to 1.6
Telford road....	46 to 78	2.1 to 3.5
Macadamized...	46 to 90	2 to 4
" loose	67 to 112	3 to 5
Gravel.........	134 to 18[illegible]	6 to 8
Sandy..........	140 to 31[illegible]	6.3 to 14
Earth..........	200 to 29[illegible]	9 to 13

Hence, a horse that can draw 140 lbs. at a walk, can draw upon a gravel road

$$140 \div \frac{6+8}{2} \times 100 = 2000 \text{ lbs.}$$

Resistance on Common Roads or Fields.

(*Bedford Experiments*, 1874.*)

GRAVELLED ROAD. (*Hard and dry, rising* 1 *in* 430.)	Maximum Draft.	Average Draft.	Average Speed per Hour.	HP developed per Minute.	Draft per Ton on Level.	Work per HP per Horse.
	Lbs.	Lbs.	Miles.	HP.	Lbs.	HP.
2 horse wagon without springs.	320	159	2.5	1.06	43.5 or .0192	.53
4 " " " "	400	251	2.6	1.74	44.5 " .02	.87
2 " " with "	300	133	2.47	.88	34.7 " .015	.44
1 " cart without "	180	49.4	2.65	.35	28 " .0125	.35
ARABLE FIELD. (*Hard and dry, rising* 1 *in* 1000.)						
2 horse wagon without springs.	1000	700	2.35	4.36	210 or .099	2.18
4 " " " "	1200	997	2.52	6.7	194 " .083	3.35
2 " " with "	1000	710	2.35	4.45	210 " .099	1.22
1 " cart without "	400	212	2.61	1.48	140 " .0625	1.48

Fore wheels of wagons were 39 ins., and hind 57 ins. in diam.; tires varying from 2.25 to 4 ins.; and wheels of cart were 54 ins. in diam., and tires 3.5 and 4 ins.

Springs reduced resistance on road 20 per cent., but did not lessen it in the field.

From these data it appears, that on a hard road, resistance is only from .25 to .16 of resistance in field. Lowest resistance is that of cart on road = 28 lbs. per ton; due, no doubt, to absence of small wheels alike to those of the wagons.

Assuming average power without springs to be .6 HP on road, as average for a day's work, it represents $.6 \times 33\,000 = 19\,800$ foot-lbs. per minute for power of a horse on such a road.

Resistance of a smooth and well-laid granite track (tramway), alike to those in London and on Commercial Road, is from 12.5 to 13 lbs. per ton.

Omnibus.† (*Weight* 5758 *lbs.*)

	Average Speed per Hour.	Per Ton.	Total.
Granite pavement (courses 3 to 4 ins.)........	2.87 miles.	17.41 lbs.	44.75 lbs.
Asphalt roadway..............................	3.56 "	27.14 "	69.75 "
Wood pavement...............................	3.34 "	41.6 "	106.88 "
Macadam road, gravelly......................	3.45 "	44.48 "	114.32 "
" " granite, new..................	3.51 "	101.09 "	259.8 "

NOTE.—The resistance noted for an asphalt roadway is apparently inconsistent with that for a granite pavement, for when it is properly constructed it is least resistant of all pavements.

* *See report in Engineering, July* 10, 1874, *page* 23.

† *Report Soc. Arts, London*, 1875.

Wagon. (*Sir John Macneil.*)

Weight 2342 lbs. Speed 2.5 Miles per Hour.

	Resistance. Per Ton.	Total.
Well-made stone pavement	31.2 lbs.	33 lbs.
Road made with 6 ins. of broken hard stone, on a foundation of stones in pavement, or upon a bottom of concrete	44 "	46 "
Old flint road, or a road made with a thick coating of broken stone, on earth	62 "	65 "
Road made with a thick coating of gravel, on earth	140 "	147 "

Stage Coach. (*Sir John Macneil.*)

Weight 3192 lbs. Gradients 1 to 20 to 600.

Speed.	Metalled Road.
At 6 miles per hour	62 lbs. per ton.
" 8 " "	73 " "
" 10 " "	79 " "

NOTE.—It was found that, from some unexplained cause, the net frictional resistance at equal speeds varied considerably, according to gradient, resistances being a maximum for steepest gradient, and a minimum for gradients of 1 in 30 to 1 in 40; for these they are less than 1 in 600. Mode of action of the horses on the carriage may have been an influential element. (*D. K. Clark.*)

To Compute Resistance to Traction on Various Roads.

(*Sir John Macneil.*)

ON A LEVEL.

RULE.—Divide weight of vehicle and load in lbs. by its unit in following table, and to quotient add .025 of load; add sum to product of velocity of vehicle in feet per second, and *Coefficient* in following table for the particular road, and result will give power required in lbs.

Or, $\frac{W+w}{\text{unit}} + \overline{w\ .025} + \overline{C\ v} = T$. *W and w representing weights of vehicle and load.*

Coefficients for Traction of Various Vehicles.

Stage coach	100	2 horse wagon without springs	54
Heavy wagon	93	2 " " with "	42
4 horse wagon without springs	55	1 " cart without "	36

Coefficients for Roads of Various Construction.

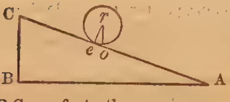
Pavement	2	Macadamized road	4.3
Broken stone, dry and clean	5	Gravel, clean	13
" " covered with dust	8	" muddy	32
" " muddy	10	Stone tramway	1.2
Sand and Gravel	12.1		

ILLUSTRATION.—What is the traction or resistance of a stage coach weighing 2200 lbs., with a load of 1600 lbs., when driven at a velocity of 9 feet per second over a dry and clean broken stone road?

$$\frac{2200+1600}{100} + \overline{1600 \times .025} + \overline{5 \times 9} = 123 \text{ lbs.}$$

To Compute Power necessary to Sustain a Vehicle upon an Inclined Road, and also its Pressure thereon, omitting Effect of Friction.

AT AN INCLINATION.



W : A C :: o : B C, and W : A C :: p : A B.

Or, $r\ e : e\ o$ :: A B : B C; W : $e\ o$ :: l : h: whence, $W\ \frac{h}{l} = e\ o$.

Assume A B of such a length that vertical rise, B C = 1 foot; then,

$$\frac{W}{AC} = \frac{W}{\sqrt{AB^2+1}} = W \sin. A = o, \text{ and } \frac{W\,AB}{AC} = \frac{W\,AB}{\sqrt{AB^2+1}} = W \cos. A = p.$$

Or, $\frac{W}{l} = P$; $\frac{W\,l'}{l} = p$; or, $\frac{W}{\sqrt{l'^2 + 1}} = P$, and $\frac{W\,l'^2}{\sqrt{l'^2+1}} = p$. *W representing weights of vehicle and load o, and P power or force necessary to sustain load on road, p pressure of load on surface, all in lbs., h height of plane, l inclined length of road or plane, and l' horizontal length, all in feet.*

ILLUSTRATION.—What is power required to sustain a carriage and its load, weighing 3800 lbs., upon a road, inclination of which is 1 in 35, and what is its pressure upon road?

Sin. A = .028 56. Cos. A = .999 94. $l = 35.014$.

Then 3800 × .028 56 = 108.53 *lbs. = power*, and 3800 × .999 94 = 3799.77 *lbs. pressure.*

To Compute Resistance of a Load on an Inclined Road.

RULE.—Ascertain the tractive power required, and add to it the power necessary to sustain load upon inclination, if load is to ascend, and subtract it if to descend.

EXAMPLE 1.—In preceding example tractive power required is 123 lbs., and sustaining power for that inclination 108.53; hence 123 + 108.53 = 231.53 *lbs.*

2.—If this load was to be drawn down a like elevation.

Then 123 − 108.53 = 14.47 *lbs.*

To Compute Power necessary to Move and Sustain a Vehicle either Ascending or Descending an Elevation, and at a given Velocity, omitting Effect of Friction.

$\left(\frac{W+w}{t} + \frac{w}{40}\right)$ cos. $\llcorner \mp (W+w)$ sin. $\llcorner + \overline{v\,c} = R$. $\llcorner$ *representing angle of elevation for a stage wagon and a stage coach, and t units as preceding; upper sign taken when vehicle descends the plane, and lower when it ascends.*

ILLUSTRATION.—Assume a stage coach to weigh 2060 lbs., added to which is a load of 1100 lbs., running at a speed of 9 feet per second over a broken stone road covered with dust, and having an inclination of 1 in 30; what is power necessary to move and sustain it up the inclination, and what down it?

$v = 9$, $c = 8$, sin. of $\llcorner$ = sin. of $1^\circ\ 54' +$ = .0333, and cos. $\llcorner$ = .9995.

Then $\left(\frac{2060 + 1100}{100} + \frac{1100}{40}\right) \times .9995 + \overline{(2060 + 1100)} \times .0333 + \overline{8 \times 9} = 59.07 + 105.23 + 72 = 236.3$ *lbs. up inclination.*

And $\left(\frac{2060 + 1100}{100} + \frac{1100}{40}\right) \times .9995 + \overline{8 \times 9} - \overline{(2060 + 1100) \times .0333} = 59.07 + 72 - 105.23 = 25.84$ *lbs. down inclination.*

Tractive and Statical Resistance of Elevations. (*Gillmore.*)

$\frac{T}{\sqrt{W^2 - T^2}} = g'$. *T representing traction in lbs. per ton, W weight of load in lbs., and g' grade of road.*

ILLUSTRATION.—Assume traction as per preceding table, page 844, 200, and weight of vehicle 2 tons; what should be least grade of road?

$$\frac{200 \times 2}{\sqrt{4480^2 - \overline{200 \times 2}^2}} = .0897 = \frac{1}{11} +.$$

Showing that, for a road upon which traction is 200 lbs. per ton, the grade should not exceed one in height to one eleventh fall of base; hence, generally, the proper grade of any description of road will be equal to force necessary to draw load upon like road when level.

Practically, greatest grade of a Telford or Macadamized road in good condition = .05, and a horse can attain at a walk a required height upon this grade, without more fatigue and in nearly same time that he would require to attain a like height over a longer road with a grade of .033, that he could ascend at a trot.

For passenger traffic, grades should not exceed .033.

Resistance of Gravity at Different Inclinations of Grade. *For a Load of* 100 *Lbs.*

Grade.	R	Grade.	R	Grade.	R	Grade.	R
	Lbs.		Lbs.		Lbs.		Lbs.
1 in 5	19.61	1 in 25	4	1 in 45	2.22	1 in 70	1.43
1 in 10	9.95	1 in 30	3.33	1 in 50	2	1 in 80	1.25
1 in 15	6.65	1 in 35	2.85	1 in 55	1.82	1 in 90	1.11
1 in 20	4.99	1 in 40	2.5	1 in 60	1.67	1 in 100	1

Inclination of Roads.—Power of draught at different inclinations and velocities is as follows (*Sir John Macneil*):

Inclination.	Angle.	Feet per Mile.	Traction at Speeds of per Hour of 6 Miles.	8 Miles.	10 Miles.	Frictional Resistance per Ton at Speeds of per Hour of 6 Miles.	8 Miles.	10 Miles.
1 in 20	2° 52′	264	268	296	318	76	96	112
1 in 26	2° 12′	203.4	213	219	225	63	68	72
1 in 30	1° 55′	176	165	196	200	41	63	66
1 in 40	1° 26′	132	160	166	172	56	61	65
1 in 60	57.5′	88	111	120	128	72	78	81

Grade.

Grade of a road should be reduced to least of practicable attainment, and as a general rule should be as low as 1 in 33, and steepest grade that is admissible on a broken stone road is 1 in 20.

The condition of traction is $\overline{f + \sin.\, a}\, L$, which should not exceed P, and sin. *a* should not exceed $\frac{P}{L} - f$, or *f*. *f representing coefficient of friction, a angle of inclination,* L *load, and* P *power in lbs.*

Illustration.—In case, page 846, weight or load $= 2060 + 1100 = 3160$ lbs., *Coefficient* of friction for such a road $= .042$ per 100 lbs., and sin. $1° 54' = .033\,16$.

$$\text{Then } \overline{.042 + .033\,16} \times 3160 = 237.5 \textit{ lbs.}$$

Traction of a Vehicle compared to its Weight on Different Roads.

(*F. Robertson, F. R. A. S.*)

Stone pavement	1 in 68	Flint foundation	1 in 34
Macadamized road	1 " 49	Gravel road	1 " 15
Sandy road	1 in 7.		

Assuming a horse to have a tractive force of 140 lbs. continuously and steadily at a walk, he can draw at a walk on a gravel road $15 \times 140 = 2100$ *lbs.*

Friction of Roads.

Friction of Roads.—According to Babbage and others, a wagon and load weighing 1000 lbs. requires a traction as follows:

	Of Load.		Of Load.
Loose sand	.25	Macadamized	.033
Fresh earth	.125	Dry high road	.025
Common side roads	.1	Well paved road	.014
Gravelled road	.035 / .067	Railroad	.0035 / .0059

Sled, hard snow, iron shod.... .033 of load.

Coefficients of Friction in Proportion to Load.

	Per 100 lbs.	Per Ton.		Per 100 lbs.	Per Ton.
Gravel road, new	.083	186	Wood pavement	.019	42
Common road, bad order	.07	157	Asphalt roadway	.012	27
Sand road	.063	141	Stone pavement	.015	34
Broken stone, rutted	.052	117	Granite "	.008	18
" " fair order	.028	63	Stone " very smooth	.006	13
" " perfect order	.015	34	Plank road	.01	22
Macadamized, new	.045	101	Railway	.0035	8
"	.033	74		.0059	13
" gravelly	.02	44	Stone track	.05	112
Earth, good order	.025	56			

To Compute Frictional Resistance to Traction of a Stage Coach on a Metalled Road in Good Condition.

$30 + 4v + \sqrt{10v} = R$. *v representing speed in miles per hour, and R frictional resistance to traction per ton.*

NOTE.—Formula is applicable to wagons at low speeds.

Canal, Slackwater, and River.

On a canal and water, resistance to traction varies as square of velocity, from that of 2 feet per second to that of 11.5 feet.

When velocity is less than .33 miles per hour, resistance varies in a less degree.

In towing, velocity is ordinarily 1 to 2.5 miles per hour.

Resistance of a boat in a canal depends very much upon the comparative areas of transverse sections of it and boat, it being reduced as difference increases.

In a mixed navigation of canal and slack-water, 3 horses or strong mules will tow a full-built, rough-bottomed canal boat, with an immersed sectional area of 94.5 sq. feet, and a displacement of 240 tons, 1.75 to 2 miles per hour for periods of 12 hours.

With a section of but 24.5 sq. feet, or a displacement of 65 tons, an average speed of 2.5 miles is attained for a like period.

By the observations of Mr. J. F. Smith, Engineer of the Schuylkill Navigation Co., a canal boat, with an immersed section alike to that above given, can be towed for 10 hours per day as follows:

Per Hour.

By 1 horse or mule.	By 2 horses or mules.	By 3 horses or mules.	By 4 horses or mules.	By 8 horses or mules.
1 mile.	1.5 miles.	1.75 miles.	1.875 miles.	2.5 miles.

Assuming then, the tractive power of a horse as given in table, page 437, the above elements determine results as follows:

Horses.	Miles.	Tractive Power divided by Load.	Traction in Lbs. per Ton.	Traction in Lbs. per Sq. Foot of immersed Section.
1	1	250 ÷ 240	1.04	2.65
2	1.5	165 × 2 ÷ 240	1.38	3.49
3	1.75	140 × 3 ÷ 240	1.75	4.44
3	1.875	132 × 3 ÷ 240	1.65	4.19
3	2	125 × 3 ÷ 240	1.56	3.98
3 (light)	2.5	100 × 3 ÷ 65	4.61	12.24

Upon a canal of less section and depth, a displacement of 105 tons, with an immersed section of 43 sq. feet, a speed of 2 miles with 2 horses was readily obtained, which would give a traction of 2.38 lbs. per ton, and of 5.71 lbs. per sq. foot of immersed section.

Maximum Power of a Horse on a Canal. (*Molesworth.*)

Miles per hour	2.5	3	3.5	4	5	6	7	8	9	10
Duration of work in hours	11.5	8	5.9	4.5	2.9	2	1.5	1.125	.9	.75
Load drawn in tons	520	243	153	102	52	30	19	13	9	6.5

Street Railroads or Tramways. (*Gen'l Gillmore.**)

Upon a level road, and at a speed of 5 miles per hour, the power required to draw a car and its load is from $\frac{1}{280}$ to $\frac{1}{250}$ of total weight, varying with condition of rails and dryness or moisture of their surface.

* Treatise on Roads, Streets, and Pavements. D. Van Nostrand, 1876, N. Y.

To Compute Resistance of a Car.

$T \times 6 = f$; $\frac{T \times v}{3} = c$; $\frac{v^2 \times a}{400} = r$; and $f + c + r = R$. T *representing weight in tons, f friction in lbs., v speed in miles per hour, a area of front or section of car in sq. feet, c concussion, r resistance of atmosphere, and R total resistance, all in lbs.*

ILLUSTRATION.—Assume a car and load of 8960 lbs., with an area of section of 56 sq. feet, and a speed of 5 miles per hour.

Then $\frac{8960}{2240} = 4$ *tons;* $4 \times 6 = 24$ *lbs. friction;* $\frac{4 \times 5}{3} = 6.66$ *lbs. concussion;* $\frac{5^2 \times 56}{400} = 3.5$ *lbs. resistance of air;* and $24 + 6.66 + 3.5 = 34.16$ *lbs.*

In average condition of a road, the resistance of a car may be taken at $\frac{1}{120}$, which, in preceding case, would be 74.66 lbs. On a descending grade, therefore, of 1 in 74.66, the application of a brake would not be required.

WATER.

FRESH WATER. Constitution of it by weight and measure is

	By Weight.	By Measure.		By Weight.	By Measure.
Oxygen...	88.9	1	Hydrogen ..	11.1	2

Cube inch of distilled water at its maximum density of 39.1°, barometer at 30 ins., weighs 252.879 grains, and it is 772.708 times heavier than atmospheric air.

Cube foot (at 39.1°) weighs 998.8 ounces, or 62.425 lbs.

NOTE.—For facility of computation, weight of a cube foot of water is usually taken at 1000 ounces and 62.5 lbs.

At a temperature of 32° it weighs 62.418 lbs., at 62° (standard temperature) 62.355 lbs., and at 212° 59.64 lbs. Below 39.1° its density decreases, at first very slow, but progressing rapidly to point of congelation, weight of a cube foot of ice being but 57.5 lbs.

Its weight as compared with sea-water is nearly as 39 to 40.

It expands .085 53 its volume in freezing. From 40° to 12° it expands .002 36 its volume, and from 40° to 212° it expands .0467—times = .000 271 5 for each degree, giving an increase in volume of 1 cube foot in 21.41 feet.

Volumes of Pure Water.

At 32°	27.684	cube ins.	= 1	cube foot.	At 62°	1 Ton	= 35.923	cube feet.
" 39.1°	27.68	"	" = 1	" "	" "	1 Lb.	= 27.71	" ins.
" 62°	27.712	"	" = 1	" "	" 39.1°	1 Tonneau	= 35.3156	" feet.
" 212°	28.978	"	" = 1	" "	" "	1 Kilogr.	= .0353	" "

Height of a Column of Water at 62° or 62.355 lbs.

1 lb. per sq. inch = 2.3093 feet, and at pressure of atmosphere = 33.947 feet = 10.347 meters.

Ice and Snow.

Cube foot of Ice at 32° weighs 57.5 lbs., and 1 lb. has a volume of 30.067 cube ins.

Volume of water at 32°, compared with ice at 32°, is as 1 to 1.085 53, expansion being 8.553 per cent.

Cube foot of new fallen snow weighs 5.2 lbs., and it has 12 times bulk of water.

Rainfall.

Annual Fall at different Places.

Location.	Ins.	Location.	Ins.	Location.	Ins.
Alabama	30.17	Ft. Crawford, Wis.	29.51	Michigan	33.5
Albany	41.35	Ft. Gibson, Ark.	[illegible]	Mississippi	45
Algiers	7.75	Ft. Snelling, Iowa	30.32	Mobile, 1842	54.94
Alleghany	46.66	Fortr. Monroe, Va.	52.53	Naples	41.8
Antigua	45	Florence	35.9	Newburg	40.5
Archangel	14.52	Frankfort, Oder	21.3	New York	36
Auburn	30.17	" Main	[illegible]	Ohio	36
Bahamas	42.19	Geneva	[illegible]	Palermo	22.8
Baltimore	39.9	Gibraltar	47.29	Paris	23.1
Barbadoes	55.87	Glasgow	21.3 / 31	Philadelphia	49
Bath, Me.	34.58	Gordon Castle, Sc'd	29.3	Plymouth (Eng.)	44
Belfast	39.46	Granada	[illegible] / 120	Port Philip	29.16
Biskra	.2	Great Britain	[illegible]	Poughkeepsie	32.06
Bombay	110	Greenock	61.8	Providence	36.74
Bordeaux	29.7	Halifax	33	Rochester	29
Boston	39.23	Hanover	22.4	Rome	39
Brussels	29	Havana	52	Santa Fé	74.8
Buffalo	27.27	Hong-kong	81.35	Savannah	55
Burlington, Vt.	32	Hudson	[illegible]	Schenectady	47.77
Calcutta	81	India	60 / 130	Siberia	7.75
Cape St. François	150	Jamaica	34.31	Sierra Leone	84
Cape Town	23.31	Jerusalem	[illegible]	Sitka	85.79
Charleston	54	Key West	31.39	St. Bernard	48
Cherbourg	39.7	Khassaya, India	610	St. Domingo	120
Cologne	24	Lewiston	23.15	St. Petersburg	17.6
Copenhagen	23	Liverpool	34.12	State of N.Y.	33.79
Cracow	13.33	London	25.2	Sydney	49
Demerara	91.2	Louisiana	51.85	Tasmania	35
" 1849	132.21	Madeira	22 / 49	Trieste	46.4
Dover (Engl.)	37.52	Manchester	36.14 / 43	Ultra Mullay, India	263.21
Dublin	30.87	Marseilles	18.2	Utica	39.3
Dumfries	36.92			Venice	34.1
East Hampton	38.52			Vera Cruz	62
Edinburgh	24.5 / 29			Vienna	19.6
Fairfield	32.03			Washington	34.62
				West Point	48.7

Average rainfall in England for a number of years was, South and East, 34 ins.; West and hilly, 43.02 to 50 ins., and percolation of it was estimated at 30 per cent.

Mean volume of water in a cube foot of air in England is 3.789 *grains.*

Globe, mean depth	36	ins.
Cape of Good Hope in 1846	in 3 hours, 6.2	"
At Khassaya, in 6 rainy months	550 ins; in 1 day, 25.5	"

Evaporation.—Mean daily evaporation, in India .22 inch; greatest .56; in England .08. At Dijon, when mean depth of rainfall was 26.9 ins. in 7 years, evaporation was for a like period 26.1 ins., and in Lancashire, Eng., when fall was 45.96 ins., evaporation was 25.65.

Volume of Rainfall.

Rainfall, depth in ins., × 2 323 200 = cube feet per sq. mile.
" " " × 17.378 74 = millions of gallons per sq. mile.
" " " × 3630 = cube feet per acre.
" " " × 27 154.3 = gallons per acre.

Mineral Waters are divided into 5 groups, viz.:

1. Carbonated, containing pure Carbonic acid—as, Seltzer, Germany; Spa, Belgium; Pyrmont, Westphalia; Seidlitz, Bohemia; and Sweet Springs, Virginia.

2. Sulphurous, containing Sulphuretted hydrogen—as, Harrowgate and Cheltenham, England; Aix-la-Chapelle, Prussia; Blue Lick, Ky.; Sulphur Springs, Va., etc.

3. Alkaline, containing Carbonate of soda—these are rare, as, Vichy, Ems.

4. Chalybeate, containing Carbonate of iron—as, Hampstead, Tunbridge, Cheltenham, and Brighton, England; Spa, Belgium; Ballston and Saratoga, N. Y.; and Bedford, Penn.

5. Saline, containing salts—as, Epsom, Cheltenham, and Bath, England; Baden-Baden and Seltzer, Germany; Kissingen, Bavaria; Plombières, France; Seidlitz, Bohemia; Lucca, Italy; Yellow Springs, Ohio; Warm Springs, N. C.; Congress Springs, N. Y.; and Grenville, Ky.

Brief Rules for Qualitative Analysis of Mineral Waters.

First point to be determined, in examination of a mineral water, is to which of above classes does water in question belong.

1. If water reddens blue litmus paper before boiling, but not afterwards, and blue color of reddened paper is restored upon warming, it is Carbonated.

2. If it possesses a nauseous odor, and gives a black precipitate, with acetate of lead, it is Sulphurous.

3. If, after addition of a few drops of hydrochloric acid, it gives a blue precipitate, with yellow or red prussiate of potash, water is a Chalybeate.

4. If it restores blue color to litmus paper after boiling, it is Alkaline.

5. If it possesses neither of above properties in a marked degree, and leaves a large residue upon evaporation, it is a Saline water.

River or canal water contains .05 } of its volume of gaseous matter.
Spring or well water " .07 }

Re-agents.

When water is pure it will not become turbid, or produce a precipitate with any of following *Re-agents:*

Baryta Water, if a precipitate or opaqueness appear, Carbonic Acid is present.

Chloride of Barium indicates Sulphates, *Nitrate of Silver*, Chlorides, and *Oxalate of Ammonia*, Lime salts. *Sulphide of Hydrogen*, slightly acid, Antimony, Arsenic, Tin, Copper, Gold, Platinum, Mercury, Silver, Lead, Bismuth, and Cadmium; *Sulphide of Ammonium*, solution alkaloid by ammonia, Nickel, Cobalt, Manganese, Iron, Zinc, Alumina, and Chromium. *Chloride of Mercury or Gold* and *Sulphate of Zinc*, organic matter.

Filter Beds.

Fine sand, 2 feet 6 ins.; Coarse sand, 6 ins.; Clean shells, 6 ins., and Clean gravel 2 feet, will filter 700 gallons water in 24 hours by gravitation.

SEA WATER. *Composition of it per volume:*

Chloride of Sodium (common salt)	2.51	Carbonate of Lime } " of Magnesia }	.02
Sulphuret of Magnesium	.53	Sulphate of Lime	.01
Chloride of "	.33	Water	96.6

By analysis of Dr. Murray, at specific gravity of 1.029, it contains

Muriate of Soda	220.01	Muriate of Magnesia	42.08
Sulphate of Soda	33.16	Muriate of Lime	7.84
			303.09

Or, 1 part contains .030 309 parts of salt $= \frac{1}{33}$ part of its weight.

Mean volume of solid matter in solution is 3.4 per cent., .75 of which is common salt.

Boiling Points at Different Degrees of Saturation.

Salt, by Weight, in 100 Parts.	Boiling Point.	Salt, by Weight, in 100 Parts.	Boiling Point.	Salt, by Weight, in 100 Parts.	Boiling Point.
$3.03 = \frac{1}{33}$	213.2°	$15.15 = \frac{5}{33}$	217.9°	$27.28 = \frac{9}{33}$	222.5°
$6.06 = \frac{2}{33}$	214.4°	$18.18 = \frac{6}{33}$	219°	$30.31 = \frac{10}{33}$	223.7°
$9.09 = \frac{3}{33}$	215.5°	$21.22 = \frac{7}{33}$	220.2°	$33.34 = \frac{11}{33}$	224.9°
$12.12 = \frac{4}{33}$	216.7°	$24.25 = \frac{8}{33}$	221.4°	$*36.37 = \frac{12}{33}$	226°

* Saturated.

Deposits at Different Degrees of Saturation and Temperature.

When 1000 Parts are reduced by Evaporation.

Volume of Water.	Boiling Point.	Salt in 100 Parts.	Nature of Deposit.
1000	214°	3	None.
299	217°	10	Sulphate of Lime.
102	228°	29.5	Common Salt.

It contains from 4 to 5.3 ounces of salt in a gallon of water.

Saline Contents of Water from several Localities.

Baltic	6.6	British Channel	35.5	South Atlantic	41.2
Black Sea	21.6	Mediterranean	39.4	North Atlantic	42.6
Arctic	28.3	Equator	39.42	Dead Sea	385

There are 62 volumes of carbonic acid in 1000 of sea-water.

Cube foot at 62° weighs 64 lbs. Its weight compared with fresh water being very nearly as 40 to 39.

Height of a Column of Water at 60° or 64.3125 lbs.

At 62°, 1 Ton = 35 cube feet. 1 Lb. per sq. inch = 2.239 feet, and at pressure of atmosphere = 32.966 feet = 10.048 meters.

Weights.

A ton of fresh water is taken at 36, and one of salt at 35 cube feet.

WAVES OF THE SEA.

Arnott estimated extreme height of the waves of an ocean, at a distance from land sufficiently great to be freed from any influence of it upon their culmination, to be 20 feet.

French Exploring Expedition computed waves of the Pacific to be 22 feet in height.

By observations of *Mr. Douglass* in 1853, he deduced that when waves had heights of

8 feet, there were	35	in number in one mile, and	8 per minute.
15 " "	5 and 6	" "	5 "
20 " "	3	" "	4 "

J. Scott Russell divides waves into 2 classes—viz.:

Waves of Translation, or of 1st order; of Oscillation, or of 2d order.

Waves of the First Order.

1. Velocity not affected by intensity of the generating impulse.

2. Motion of the particles always forward in same direction as wave, and same at bottom as at surface.

3. Character of wave, a prolate cycloid, in long waves, approaching a true cycloid. When height is more than one third of length, the wave breaks.

Waves of the Second Order.

1. Ordinary sea waves are waves of second order, but become waves of the first order as they enter shallow water.

2. Power of destruction directly proportional to height of wave, and greatest when crest breaks.

3. A wave of 10 feet in height and 32 feet in length would only agitate the water 6 ins. at 10 feet below surface; a wave of like height and 100 feet in length would only disturb the water 18 ins. at same depth.

Average force of waves of Atlantic Ocean during summer months, as determined by *Thomas Stevenson*, was 611 lbs. per sq. foot; and for winter months 2086 lbs. During a heavy gale a force of 6983 lbs. was observed.

J. Scott Russell deduced that a wave 30 feet in height exerts a force of 1 ton per sq. foot, and that, in an exposed position in deep water, 1.75 tons may be exerted upon a vertical surface.

At Cassis, France, when the water is deep outside, blocks of 15 cube meters were found insufficient to resist the action of waves.

Breakwaters with vertical walls, or faces of an angle less than 1 to 1, will reflect waves without breaking them. Waves of oscillation have no effect on small stones at 22 feet below the surface, or on stones from 1.5 to 2 feet, 12 feet below surface.

A roller 20 feet high will exert a force of about 1 ton per sq. foot.

Greatest force observed at Skerryvore, 3 tons per sq. foot; at Bell Rock, 1.5 tons per sq. foot.

Waves of the second order, when reflected, will produce no effect at a depth of 12 feet below surface.

Action of waves is most destructive at low-water line.

Waves of first order are nearly as powerful at a great depth as at surface.

To Compute Velocity.

When l is less than d. $.55\sqrt{l}$ or $1.818\sqrt{l} = V$.

When l exceeds 1000 d. $\sqrt{32.17\,d} = V$, and *When Height of Wave becomes a sensible Proportion to Depth,* $\sqrt{32.17\left(1 + 3\frac{h}{d}\right)} = V$.

To Compute Height of Waves in Reservoirs, etc.

$1.5\sqrt{L} + (2.5 - \sqrt{L}) =$ *height in feet.* L *representing length of Reservoir, Pond, etc., exposed to direction of wind, in miles.*

Tidal Waves.

Wave produced by action of sun and moon is termed *Free Tide Wave.* Semi-diurnal tide wave is this, and has a period of 12 hours 24+ minutes.

Professor Airy declared that when length of a wave was not greater than depth of the water, its velocity depended only upon its length, and was proportionate to square root of its length.

When length of a wave is not less than 1000 times depth of water, velocity of it depends only upon depth, and is proportionate to square root of it; velocity being same that a body falling free would acquire by falling through a height equal to half depth of water.

For intermediate proportions, velocity can be obtained by a general equation.

Under no circumstances does an unbroken wave exceed 30 or 40 feet in height.

A wave breaks when its height above general level of water is equal to general depth of it.

Diurnal and other tidal waves, so far as they are free, may be all considered as running with the same velocity, but the column of the length of wave must be doubled for diurnal wave.

Depth of Water.	Length of Wave.					
	Feet. 1	Feet. 10	Feet. 100	Feet. 1000	Feet. 10 000	Feet. 100 000
	Velocity per Second.					
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1	2.26	5.34	5.67	—	—	—
10	2.26	7.15	16.88	17.92	17.93	—
100	—	7.15	22.62	53.19	56.67	56.71
1 000	—	—	22.62	71.54	168.83	179.21
10 000	—	—	—	71.54	226.24	533.9

WHEEL GEARING.

Pitch Line of a wheel is circle upon which pitch is measured, and it is circumference by which diameter, or velocity of wheel, is measured.

Pitch is arc of circle of pitch line, is determined by number of teeth in wheel, and necessarily an aliquot part of pitch line.

True or Chordial Pitch, or that by which dimensions of tooth of a wheel are alone determined, is a straight line drawn from centres of two contiguous teeth upon pitch line.

Line of Centres is line between centres of two wheels.

Radius of a wheel is semi-diameter bounded by periphery of the teeth. *Pitch Radius* is semi-diameter bounded by pitch line.

Length of a Tooth is distance from its base to its extremity.

Breadth of a Tooth is length of face of wheel.

Depth of a Tooth is thickness from face to face at pitch line.

Face of a Tooth, or Addendum, is that part of its side which extends from its pitch line to its top or *Addendum line.*

Flank of a Tooth is that part of its side which extends from pitch line to line of space at base of and between adjacent teeth; its length, as well as that of face of tooth, is measured in direction of radius of wheel, and is a little greater than face of tooth, to admit of clearance between end of tooth and periphery of rim of wheel or rack.

Cog Wheel is general term for a wheel having a number of cogs or teeth set in or upon, or radiating from, its circumference.

Mortice Wheel is a wheel constructed for reception of teeth or cogs, which are fitted into recesses or sockets upon face of the wheel.

Plate Wheels are wheels without arms.

Rack is a series of teeth set in a plane.

Sector is a wheel which reciprocates without forming a full revolution.

Spur Wheel is a wheel having its teeth perpendicular to its axis.

Bevel Wheel is a wheel having its teeth at an angle with its axis.

Crown Wheel is a wheel having its teeth at a right angle with its axis.

Mitre Wheel is a wheel having its teeth at an angle of 45° with its axis.

Face Wheel is a wheel having its teeth set upon one of its sides.

Annular or *Internal Wheel* is a wheel having its teeth convergent to its centre.

Spur Gear.—Wheels which act upon each other in same plane.

Bevel Gear.—Wheels which act upon each other at an angle.

Inside Gear or *Pin Gearing.*—Form of acting surfaces of teeth for a pitch-circle in inside gearing is exactly same with those suited for same pitch-circle in outside gearing, but relative position of teeth, spaces, and flanks are reversed, and addendum-circle is of less radius than pitch-circle.

A *Train* is a series of wheels in connection with each other, and consists of a series of axles, each having on it two wheels, one is driven by a wheel on a preceding axis and other drives a wheel on following axis.

Idle Wheel.—A wheel revolving upon an axis, which receives motion from a preceding wheel and gives motion to a following wheel, used only to affect direction of motion.

Trundle, Lantern, or *Wallower* is when teeth of a pinion are constructed of round bars or solid cylinders set into two disks. *Trundle* with less than eight staves cannot be operated uniformly by a wheel with any number of teeth.

Spur, Driver, or *Leader* is term for a wheel that impels another; one impelled is *Pinion, Driven,* or *Follower.*

Teeth of wheels should be as small and numerous as is consistent with strength.

When a *Pinion is driven by a wheel*, number of teeth in pinion should not be less than 8.

When a *Wheel is driven by a pinion*, number of teeth in pinion should not be less than 10.

When 2 wheels act upon one another, greater is termed *Wheel* and lesser *Pinion.*

When the tooth of a wheel is made of a material different from that of wheel it is termed a *Cog;* in a pinion it is termed a *Leaf*, in a trundle a *Stave*, and on a disk a *Pin.*

Material of which cogs are made is about one fourth strength of cast iron. Hence, product of their $b\,d^2$ should be 4 times that of iron teeth.

Number of teeth in a wheel should always be prime to number of pinion; that is, number of teeth in wheel should not be divisible by number of teeth in pinion without a remainder. This is in order to prevent the same teeth coming together so often and uniformly as to cause an irregular wear of their faces. An odd tooth introduced into a wheel is termed a *Hunting tooth* or *Cog.*

The least number of teeth that it is practicable to give to a wheel is regulated by necessity of having at least one pair always in action, in order to provide for the contingency of a tooth breaking; and least number that can be employed in pinions having teeth of following classes is: Involute, 25; Epicycloidal, 12; Staves or Pins, 6.

Velocity Ratio in a Train of Wheels.—To attain it with least number of teeth, it should, in each elementary combination, approximate as near as practicable to 3.59. A convenient practical rule is a range from 3 to 6.

ILLUSTRATION.	1	6	36	216	1296	*velocity ratio.*
	1	2	3	4		*elementary combination.*

To increase or diminish velocity in a given proportion, and with least quantity of wheel-work, number of teeth in each pinion should be to number of teeth in its wheel as 1 : 3.59. Even to save space and expense, ratio should never exceed 1 : 6. (*Buchanan.*)

To Compute Pitch.

RULE.—Divide circumference at pitch-line by number of teeth.

EXAMPLE.—A wheel 40 ins. in diameter requires 75 teeth; what is its pitch?

$$3.1416 \times 40 \div 75 = 1.6755 \textit{ ins.}$$

To Compute True or Chordial Pitch.

RULE.—Divide 180° by number of teeth, ascertain sine of quotient, and multiply it by diameter of wheel.

EXAMPLE.—Number of teeth is 75, and diameter 40 ins.; what is true pitch?

$$180 \div 75 = 2° \ 24', \text{ and sin. of } 2° \ 24' = .041\,88, \text{ which} \times 40 = 1.6752 \textit{ ins.}$$

To Compute Diameter.

RULE.—Multiply number of teeth by pitch, and divide product by 3.1416.

EXAMPLE.—Number of teeth in a wheel is 75, and pitch 1.6755 ins.; what is diameter of it?

$$75 \times 1.6755 \div 3.1416 = 40 \textit{ ins.}$$

When the True Pitch is given. RULE.—Multiply number of teeth in wheel by true pitch, and again by .3184.

EXAMPLE.—Take elements of preceding case.

$$75 \times 1.6752 \times .3184 = 40 \textit{ ins.}$$

Or, Divide 180° by number of teeth, and multiply cosecant of quotient by pitch.

$$180 \div 75 = 2° \ 24', \text{ and cos. } 2° \ 24' = 23.88, \text{ which} \times 1.6752 = 40 \textit{ ins.}$$

To Compute Number of Teeth.

RULE.—Divide circumference by pitch.

To Compute Number of Teeth in a Pinion or Follower to have a given Velocity.

RULE.—Multiply velocity of driver by its number of teeth, and divide product by velocity of driven.

EXAMPLE 1.—Velocity of a driver is 16 revolutions, number of its teeth 54, and velocity of pinion is 48; what is number of its teeth?

$$16 \times 54 \div 48 = 18 \textit{ teeth.}$$

2.—A wheel having 75 teeth is making 16 revolutions per minute; what is number of teeth required in pinion to make 24 revolutions in same time?

$$16 \times 75 \div 24 = 50 \textit{ teeth.}$$

To Compute Proportional Radius of a Wheel or Pinion.

RULE.—Multiply length of line of centres by number of teeth in wheel, for wheel, and in pinion, for pinion, and divide by number of teeth in both wheel and pinion.

EXAMPLE.—Line of centres of a wheel and pinion is 36 ins., and number of teeth in wheel is 60, and in pinion 18; what are their radii?

$$\frac{36 \times 60}{60 + 18} = 27.69 \textit{ ins. wheel.} \quad \frac{36 \times 18}{60 + 18} = 8.3 \textit{ ins. pinion.}$$

To Compute Diameter of a Pinion.

When Diameter of Wheel and Number of Teeth in Wheel and Pinion are given. RULE.—Multiply diameter of wheel by number of teeth in pinion, and divide product by number of teeth in wheel.

EXAMPLE.—Diameter of a wheel is 25 ins., number of its teeth 210, and number of teeth in pinion 30; what is diameter of pinion?

$$25 \times 30 \div 210 = 3.57 \textit{ ins.}$$

To Compute Number of Teeth required in a Train of Wheels to produce a given Velocity.

RULE.—Multiply number of teeth in driver by its number of revolutions, and divide product by number of revolutions of each pinion, for each driver and pinion.

EXAMPLE.—If a driver in a train of three wheels has 90 teeth, and makes 2 revolutions, and velocities required are 2, 10, and 18, what are number of teeth in each of other two?

$$10 : 90 :: 2 : 18 = \textit{teeth in 2d wheel.} \qquad 18 : 90 :: 2 : 10 = \textit{teeth in 3d wheel.}$$

To Compute Velocity of a Pinion.

RULE.—Divide diameter, circumference, or number of teeth in driver, as case may be, by diameter, etc., of pinion.

When there are a Series or Train of Wheels and Pinions. RULE.—Divide continued product of diameter, circumference, or number of teeth in wheels by continued product of diameter, etc., of pinions.

EXAMPLE 1.—If a wheel of 32 teeth drives a pinion of 10, upon axis of which there is one of 30 teeth, driving a pinion of 8, what are revolutions of last?

$$\frac{32}{10} \times \frac{30}{8} = \frac{960}{80} = 12 \textit{ revolutions.}$$

2.—Diameters of a train of wheels are 6, 9, 9, 10, and 12 ins.; of pinions, 6, 6, 6, 6, and 6 ins.; and number of revolutions of driving shaft or prime mover is 10; what are revolutions of last pinion?

$$\frac{6 \times 9 \times 9 \times 10 \times 12 \times 10}{6 \times 6 \times 6 \times 6 \times 6} = \frac{583200}{7776} = 75 \textit{ revolutions.}$$

To Compute Proportion that Velocities of Wheels in a Train should bear to one another.

RULE.—Subtract less velocity from greater, and divide remainder by one less than number of wheels in train; quotient is number, rising in arithmetical progression from less to greater velocity.

EXAMPLE.—What should be velocities of 3 wheels to produce 18 revolutions, the driver making 3?

$\frac{18-3=15}{3-1=2}=7.5=$ *number to be added to velocity of driver* $=7.5+3=10.5$, and $10.5+7.5=18$ *revolutions.* Hence 3, 10.5, and 18 *are velocities of three wheels.*

Pitch of Wheels.

To Compute Diameter of a Wheel for a given Pitch, or Pitch for a given Diameter.

From 8 to 192 Teeth.

No. of Teeth.	Diameter.	No. of Teeth.	Diameter.	No. of Teeth.	Diameter.	No. of Teeth.	Diameter.	No. of Teeth.	Diameter.
8	2.61	45	14.33	82	26.11	119	37.88	156	49.66
9	2.93	46	14.65	83	26.43	120	38.2	157	49.98
10	3.24	47	14.97	84	26.74	121	38.52	158	50.3
11	3.55	48	15.29	85	27.06	122	38.84	159	50.61
12	3.86	49	15.61	86	27.38	123	39.16	160	50.93
13	4.18	50	15.93	87	27.7	124	39.47	161	51.25
14	4.49	51	16.24	88	28.02	125	39.79	162	51.57
15	4.81	52	16.56	89	28.33	126	40.11	163	51.89
16	5.12	53	16.88	90	28.65	127	40.43	164	52.21
17	5.44	54	17.2	91	28.97	128	40.75	165	52.52
18	5.76	55	17.52	92	29.29	129	41.07	166	52.84
19	6.07	56	17.8	93	29.61	130	41.38	167	53.16
20	6.39	57	18.15	94	29.93	131	41.7	168	53.48
21	6.71	58	18.47	95	30.24	132	42.02	169	53.8
22	7.03	59	18.79	96	30.56	133	42.34	170	54.12
23	7.34	60	19.11	97	30.88	134	42.66	171	54.43
24	7.66	61	19.42	98	31.2	135	42.98	172	54.75
25	7.98	62	19.74	99	31.52	136	43.29	173	55.07
26	8.3	63	20.06	100	31.84	137	43.61	174	55.39
27	8.61	64	20.38	101	32.15	138	43.93	175	55.71
28	8.93	65	20.7	102	32 47	139	44.25	176	56.02
29	9.25	66	21.02	103	32.79	140	44.57	177	56.34
30	9 57	67	21.33	104	33.11	141	44.88	178	56.66
31	9.88	68	21.65	105	33.43	142	45.2	179	56.98
32	10.2	69	21.97	106	33.74	143	45.52	180	57.23
33	10.52	70	22.29	107	34.06	144	45.84	181	57.62
34	10.84	71	22.61	108	34.38	145	46.16	182	57.93
35	11.16	72	22.92	109	34.7	146	46.48	183	58.25
36	11.47	73	23.24	110	35.02	147	46.79	184	58.57
37	11.79	74	23.56	111	35.34	148	47.11	185	58.89
38	12.11	75	23.88	112	35.65	149	47.43	186	59.21
39	12.43	76	24.2	113	35.97	150	47.75	187	59.53
40	12.74	77	24.52	114	36.29	151	48.07	188	59.84
41	13.06	78	24.83	115	36.61	152	48.39	189	60.16
42	13.38	79	25.15	116	36.93	153	48.7	190	60.48
43	13.7	80	25.47	117	37.25	154	49.02	191	60.81
44	14.02	81	25.79	118	37.56	155	49.34	192	61.13

Pitch in this table is *true* pitch, as before described.

To Compute Circumference of a Wheel.

RULE.—Multiply number of teeth by their pitch.

To Compute Revolutions of a Wheel or Pinion.

RULE.—Multiply diameter or circumference of wheel or number of its teeth in ins., as case may be, by number of its revolutions, and divide product by diameter, circumference, or number of teeth in pinion.

EXAMPLE.—A pinion 10 ins. in diameter is driven by a wheel 2 feet in diameter, making 46 revolutions per minute; what is number of revolutions of pinion?

$2 \times 12 \times 46 \div 10 = 110.4$ *revolutions.*

To Compute Number of Teeth of a Wheel for a given Diameter and Pitch.

RULE.—Divide diameter by pitch, and opposite to quotient in preceding table is given number of teeth.

EXAMPLE.—Diam. of wheel is 40 ins., and pitch 1.675; what is number of its teeth?

$40 \div 1.675 = 23.88$, and opposite thereto in table is $75 =$ *number of teeth.*

To Compute Diameter of a Wheel for a given Pitch and Number of Teeth.

RULE.—Multiply diameter in preceding table for number of teeth by pitch, and product will give diameter at pitch circle.

EXAMPLE.—What is diameter of a wheel to contain 48 teeth of 2.5 ins. pitch?

$15.29 \times 2.5 = 38.225$ *ins.*

To Compute Pitch of a Wheel for a given Diameter and Number of Teeth.

RULE.—Divide diameter of wheel by diameter in table for number of teeth, and quotient will give pitch.

EXAMPLE.—What is pitch of a wheel when diameter of it is 50.94 ins., and number of its teeth 80? $50.94 \div 25.47 = 2$ *ins.*

General Illustrations.

1.—A wheel 96 ins. in diameter, making 42 revolutions per minute, is to drive a shaft 75 revolutions per minute; what should be diameter of pinion?

$96 \times 42 \div 75 = 53.76$ *ins.*

2.—If a pinion is to make 20 revolutions per minute, required diameter of another to make 58 revolutions in same time.

$58 \div 20 = 2.9 =$ *ratio of their diameters.* Hence, if one to make 20 revolutions is given a diameter of 30 ins., other will be $30 \div 2.9 = 10.345$ *ins.*

3.—Required diameter of a pinion to make 12.5 revolutions in same time as one of 32 ins. diameter making 26.

$32 \times 26 \div 12.5 = 66.56$ *ins.*

4.—A shaft, having 22 revolutions per minute, is to drive another shaft at rate of 15, distance between two shafts upon line of centres is 45 ins.; what should be diameter of wheels?

Then, 1st. $22 + 15 : 22 :: 45 : 26.75 =$ *ins. in radius of pinion.*
2d. $22 + 15 : 15 :: 45 : 18.24 =$ *ins. in radius of spur.*

5.—A driving shaft, having 16 revolutions per minute, is to drive a shaft 81 revolutions per minute, motion to be communicated by two geared wheels and two pulleys, with an intermediate shaft; driving wheel is to contain 54 teeth, and driving pulley upon driven shaft is to be 25 ins in diameter; required number of teeth in driven wheel, and diameter of driven pulley.

Let driven wheel have a velocity of $\sqrt{16 \times 81} = 36$, a mean proportional between extreme velocities 16 and 81.

Then, 1st. $36 : 16 :: 54 : 24 =$ *teeth in driven wheel.*
2d. $81 : 36 :: 25 : 11.11 =$ *ins. diameter of driven pulley.*

6.—If, as in preceding case, whole number of revolutions of driving shaft, number of teeth in its wheel, and diameters of pulleys are given, what are revolutions of shafts?

Then, 1st. $18 : 16 :: 54 : 48 =$ *revolutions of intermediate shaft.*
2d. $15 : 48 :: 25 : 80 =$ *revolutions of driven shaft.*

Teeth of Wheels.

Epicycloidal.—In order that teeth of wheels and pinions should work evenly and without unnecessary rubbing friction, the face (*from pitch line to top*) of the outline should be determined by an epicycloidal curve (see page 228), and that of the flank (*from pitch line to base*) by an hypocycloidal (see also page 228).

When generating circle is equal to half diameter of pitch circle, hypocycloidal described by it is a straight diametrical line, and consequently outline of a flank is a right line, and radial to centre of wheel.

If a like generating circle is used to describe face of a tooth of other wheel or pinion respectively, the wheel and pinion will operate evenly.

1.

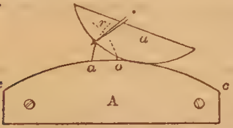
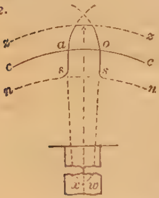


ILLUSTRATION.—Determine all elements of wheel—viz., Pitch circle, Number of teeth, Pitch, Length, Face, and Flank.

Cut a template A to pitch circle *c c* of wheel, and secure it temporarily to a board.

Having determined depth of tooth, set it off on pitch line, as *a o*, Fig. 1, and above it apply a second template, *a*; radius of wheel is equal to half radius of pinion; insert into, or attach exactly at its edge, a tracer ., roll template *a* along A, and tracer will describe an epicycloidal curve, *a r*, and by inverting *a* describe *o r*, and faces of a tooth are delineated.

2.



To describe flanks, define pitch line *c c*, Fig. 2, and arc *n n*, drawn at base of teeth or board A (as in Fig. 1), secure a strip of wood, *w*, equal in length to radius of wheel, and locate centre of it, *x*, draw radii *x a* and *x o*, and they will define flanks, which should be filleted, as shown at *s s*. Define arc *z z*, and length of tooth is determined.

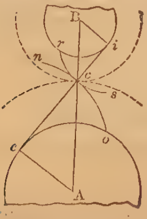
Proceed in like manner conversely for teeth of pinion, and wheel and pinion thus constructed will operate truly.

In construction of the teeth of a wheel or pinion in the pattern-shop, it is customary to construct the wheel or pinion complete, out to face of wheel at base of teeth, and then to insert the teeth in rough, approximately shaped blocks, by a dovetail at their base, fitting into face of wheel, and then the outline of a tooth is described thereon; the block is then removed, finished as a tooth, replaced, fastened, and filleted.

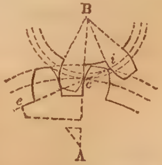
Involute.

Teeth of two wheels will work truly together when their face is that of an *involute* (see page 229), and that two such wheels should work truly, the circles from which the involute lines for each wheel are generated must be concentric with the wheels, with diameters in same ratio as those of the wheels.

3.



Assume A *c*, B *c*, Fig. 3, pitch radii of two wheels designed to work together, through *c*, draw a right line, *e i*, and with perpendiculars *e c*, *i c*, describe arcs *n o*, *r s*, and involutes *n c o* and *r c s* define a face of each of the teeth.



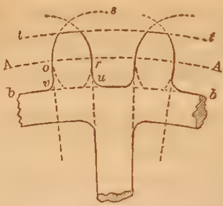
4.

To describe teeth of a pair of wheels of which A *c*, B *c*, Fig. 4, are pitch radii, draw *c i*, *c e*, perpendicular to radials B *i* and A *e*, and they are to be taken as the radials of the involute arcs from which the faces of the teeth are to be defined; then fillet flanks at base, as before described, Fig. 2.

Involute teeth will work with truth, even at varying distances apart of the centres of the wheels, and any wheels of a like pitch will work in union, however varied their diameters.

Circular teeth are defined as follows:

5.



Assume A A, Fig. 5, pitch-line, *b b* line of base of teeth, and *t t* face-line. Set off on pitch-line divisions both of pitch and depth of teeth, then with a radius of 1.25 pitch describe arcs as *o s* upon pitch line for faces of teeth, then draw radial lines *o v*, *r u*, to centre of wheel for flanks, strike arc *t t* to define length of tooth, and fillet flanks at base as before described.

Proportions of Teeth.

In computing dimensions of a tooth, it is to be considered as a beam fixed at one end, weight suspended from other, or face of beam; and it is essential to consider the element of velocity, as its stress in operation, at high velocity with irregular action, is increased thereby.

Dimensions of a tooth should be much greater than is necessary to resist direct stress upon it, as but one tooth is proportioned to bear whole stress upon wheel, although two or more are actually in contact at all times; but this requirement is in consequence of the great wear to which a tooth is subjected, shocks it is liable to from lost motion, when so worn as to reduce its depth and uniformity of bearing, and risk of the loss of a tooth from a defect.

A tooth running at a low velocity may be materially reduced in its dimensions, compared with one running at a high velocity and with a like stress.

Result of operations with toothed wheels, for a long period of time, has determined that a cast-iron (Eng.) tooth with a pitch of 3 ins. and a breadth of 7.5 ins. will transmit, at a velocity of 6.66 feet per second, power of 59.16 horses.

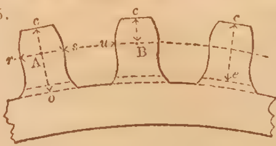
To Compute Dimensions of a Tooth to Resist a given Stress.

RULE.—Multiply extreme pressure at pitch-line of wheel by length of tooth in decimal of a foot, divide product by *Coefficient* of material of tooth, and quotient will give product of breadth and square of depth.

Or, $\frac{S\,l}{C} = b\,d^2$. S *representing stress in lbs., and l length in feet.*

The *Coefficient* of cast iron for this or like purposes may be taken at from 50 to 70.

6.



Pitch A B = 1. Depth *r s* = .45.
Length *c o* = .75. Space *s v* = .55.
Working length *c e* = .7. Play *s v* — *r s* = .1.
Clearance *e* to *o* = .05. Face B *c* = .35.

NOTE.—It is necessary first to determine pitch, in order to obtain either length or depth of a tooth.

EXAMPLE.—Pressure at pitch-line of a cast-iron wheel (at a velocity of 6.66 feet per second) is 4886 lbs.; what should be dimensions of teeth, pitch being 3 ins.?

$3 \times .75 = 2.25$ *length of tooth, which* $\div 12 = .1875 =$ *length in decimal of a foot.*

Coefficient of material is taken at 60.

$\frac{4886 \times .1875}{60} = 15.27$. If length = 2.25, pitch = 3, and depth = 1.35 *ins.*

Pitches of Equivalent Strength for Cast Iron and Wood.—Iron 1. Hard wood 1.26.

Then $\frac{15.27}{1.35^2} = 8.39$ *ins. breadth.*

When Product of b d² is obtained, and it is required to ascertain either dimension. $\sqrt{\frac{b\,d^2}{b}} =$ *depth*, and $\frac{b\,d^2}{d^2} =$ *breadth.*

To Compute Depth of a Tooth.

1. *When Stress is given.* RULE.—Extract square root of stress, and multiply it by .02 for cast iron, and .027 for hard wood.

2. *When* HP *is given.* RULE.—Extract square root of quotient of HP divided by velocity in feet per second, and multiply it by .466 for cast iron, and .637 for hard wood.

EXAMPLE.—HP to be transmitted by a tooth of cast iron is 60, and velocity of it at its pitch line is 6.66 feet per second; what should be depth of tooth?

$$\sqrt{\frac{60}{6.66}} \times .466 = 1.398 \textit{ ins.}$$

To Compute HP of a Tooth.

RULE.—Multiply pressure at pitch-line by its velocity in feet per minute, and divide product by 33 000.

EXAMPLE.—What is HP of a tooth of dimensions and at velocity given in preceding example.

$$4886 \times 6.66 \times 60'' \div 33\,000 = 59.16 \textit{ horses.}$$

To Compute Stress that may be borne by a Tooth.

RULE.—Multiply *Coefficient* of material of tooth to resist a transverse strain, as estimated for this character of stress, by breadth and square of its depth, and divide product by extreme length of it in decimal of a foot.

EXAMPLE.—Dimensions of a cast-iron tooth in a wheel are 1.38 ins. in depth, 2.1 ins. in length, and 7.5 ins. in breadth; what is the stress it will bear?

Coefficient assumed at 60. $\frac{60 \times 7.5 \times 1.38^2}{2.1 \div 12} = 4886 \textit{ lbs.}$

Following deductions by the rules of different authors for like elements are submitted for a cast-iron tooth:

Pitch....... 3 *ins.* | Depth.... 1.38 *ins.* | Breadth... 7.5 *ins.* | Length.... 2.1 *ins.*

ACTUAL POWER IN STRESS EXERTED *at a velocity of 400 feet per min., 4886 lbs.*	Depth of Tooth.	ACTUAL POWER IN STRESS EXERTED *at a velocity of 400 feet per min., 4886 lbs.*	Depth of Tooth.
	Ins.		Ins.
By Above Rule $\sqrt{\frac{H}{v}} \times .446$....	1.398*	*By Rankine* $\sqrt{\frac{W}{1500}}$...........	1.8
" *Fairbairn* $.025\sqrt{W}$.........	1.75	" *Tredgold* $\frac{3}{4}\sqrt{\frac{H}{v}}$..........	2.25
" *Imperial Journal* $\sqrt{\frac{W}{1576}}$ ..	1.76	" *Buchanan* $\sqrt{\frac{.556\,H}{v}}$	2.24

H *representing horse-power* (60), W *stress in lbs., and* v *velocity in feet per second.*

Depth, Pitch, and Breadth. (*M. Morin.*)

Cast iron............................. $.028\sqrt{W} = d.$ $.057\sqrt{W} = P.$
Hard wood............................ $.038\sqrt{W} = d.$ $.079\sqrt{W} = P.$

W *representing weight or stress upon tooth in lbs.,* d *depth of tooth, and* P *pitch in ins.*

When velocity of pitch-circle does not exceed 5 feet per second $b = 4\,d$, when it exceeds 5 feet $b = 5\,d$, and if wheels are exposed to wet $b = 6\,d$.

b *representing breadth.*

ILLUSTRATION.—Assume pressure at pitch-line of a cast-iron wheel upon a tooth equal 6000 lbs., and velocity 5 feet per second.

Then $.028\sqrt{6000} = 2.17$ *ins. Depth*, and $.057\sqrt{6000} = 4.46$ *ins. Pitch.*

NOTE.—For further Illustrations of Formation of Teeth, Bevel Gearing, Willis's Odontograph, Staves, Trundles, etc., see Mosely's Engineering, Shelton's Mechanic's Guide, Fairbairn's Mechanism and Machinery of Construction, etc.

* This depth, with a breadth of 7.5 ins., is .1 of ultimate strength of average strength of American Cast Iron.

PROPORTIONS OF WHEELS.

With six flat Arms and Ribs upon one side of them, as ; *or a Web in centre, as* .

Rim.—Depth, measured from base of teeth, .45 to .5 of pitch of teeth, having a web upon its inner surface .4 of pitch in depth and .25 to .3 of it in width.

NOTE.—When face of wheel is mortised, depth of rim should be 1.5 times pitch, and breadth of it 1.5 times breadth of tooth or cog.

Hub.—When eye is proportionate to stress upon wheel, hub should be twice diameter of eye. In other cases depth around eye should be .75 to .8 of pitch.

Arm.—Depth .4 to .45 of pitch. Breadth at rim 1.5 times pitch, increasing .5 inch per foot of length toward hub.

Rib upon one edge of arm, or Web in its centre, should be from .25 to .3 pitch in width, and .4 to .45 of it in depth.

When section of an arm differs from those above given, as with one with a plane section, as , or with a double rib, as , its dimensions should be proportioned to form of section.

In a wheel of greater relative diameter, length of hub and breadth of arms, or of the rib or web, according as plane of arm is in that of wheel, or contrariwise, should be made to exceed breadth of face of wheel (at the hub) in order to give it resistance to lateral strain.

Number of arms in wheels should be as follows:

1.5 to 3.25 feet in diameter	4	5 to 8.5 feet in diameter	6
3.25 " 5 " "	5	8.5 " 16 " "	8
16 to 24 feet in diameter	10		

With light wheels, number of arms should be increased, in order better to sustain rigidity of rim.

Mortise Wheels.—Their rim or face should be .9 pitch of tooth, and twice depth of rim of a solid wheel.

WINDING ENGINES.

With Winding Engines, for drawing coals, etc., out of a Pit, where it is required to give a certain number of revolutions, it is necessary to have given diameter of *Drum* and thickness of rope, which is flat made, and contrariwise.

To Compute Diameter of a Drum.

Where Flat Ropes are used, and are wound one part over the other. RULE.—Divide depth of pit in ins. by product of number of revolutions and 3.1416, and from quotient subtract product of thickness of rope and number of revolutions; remainder is diameter in ins.

EXAMPLE.—If an engine makes 20 revolutions, depth of pit being 600 feet, and rope 1 inch, what should be diameter of drum?

$$\frac{600 \times 12}{20 \times 3.1416} - \overline{1 \times 20} = \frac{7200}{62.832} - 20 = 94.59 \text{ ins.}$$

To Compute Diameter of Roll.

RULE.—To area of drum add area or edge surface of rope; then ascertain by inspection in table of areas, or by calculation, diameter that gives this area, and it is the diameter of Roll.

EXAMPLE.—What is diameter of roll in preceding example?

Area of $94.59 = 7027.2 +$ (area of $\overline{7200 \times 1}) = 7200 = 14\,227.2$, and $\sqrt{14\,227.2} \div .7854 = 134.59$ *ins.*

Or, Radius of drum is increased number of revolutions multiplied by thickness of rope; as $\frac{94.59}{2} + \overline{20 \times 1} = 67.295$ *ins.*

To Compute Number of Revolutions.

RULE.—To area of drum add area of edge surface of rope; from diameter of the circle having that area subtract diameter of drum, and divide remainder by twice thickness of rope; quotient will give number of revolutions.

EXAMPLE.—Length of a rope is 2600 ins., its thickness 1 inch, and diameter of drum 20 ins.; what is number of revolutions?

Area of $20 +$ area of rope $= 314.16 + 2600 = 2914.16$, diameter of which is 60.91, and $\frac{60.91 - 20}{1 \times 2} = 20.45$ *revolutions.*

Or, subtract diameter of drum from diameter of roll, and divide remainder by twice thickness of rope; as $60.91 - 20 = 40.91$, and $40.91 \div \overline{1 \times 2} = 20.45$ *revolutions.*

To Compute Point of Meeting of Ascending and Descending Buckets when two or more are used.

To Compute Point of Meeting of Buckets. RULE.—Divide sum of length of turns of rope by 2, and to quotient add length of last turn; divide sum by 2, multiply quotient by half number of revolutions, and product will give distance from centre of drum at which buckets will meet.

NOTE 1.—Meetings will always be below half depth of pit.

2.—At half number of revolutions buckets will meet.

EXAMPLE.—Diameter of a drum is 9 feet, thickness of rope 1 inch, and revolutions 20; what is depth of pit, and at what distance from top will buckets meet?

$$\frac{28.54 + 38.48}{2} + 38.48 \div 2 \times \frac{20}{2} = \frac{71.99}{2} = 35.995 \times 10 = 359.95 \text{ feet.}$$

To Compute this Depth. RULE.—To diameter of drum add thickness of rope in feet, and ascertain its circumference; to diameter of drum add quotient of product of twice thickness of rope and number of revolutions less 1, divided by 12 for a diameter, and circumference of this diameter is length of last turn, also in feet; add these two lengths together, multiply their sum by half number of revolutions, and product will give depth of pit.

$9 +$ thickness of rope $= 9 + \frac{1}{12}$ of $1 = 9.083$, which $\times 3.1416 = 28.54$ *feet = length of first turn.* $9.0833 + \frac{1 \times 2 \times \overline{20 - 1}}{12} \times 3.1416 = 38.48$ *feet = length of last turn.*

$$\text{Then } 28.54 + 38.48 \times \frac{20}{2} = 67.02 \times 10 = 670.2 \textit{ feet, depth of pit.}$$

WINDMILLS.

Driving Shaft of a vertical windmill should be set at an elevating angle with horizon when set upon low ground, and at a depressing angle when set upon elevated ground. Range of these angles is from 3° to 15°. A velocity of wind of 10 feet per second is not generally sufficient to drive a loaded mill, and if velocity exceeds 35 feet per second the force is generally too great for ordinary structures.

Angle of Sails should be from 18° to 30° at their least radius, and from 7° to 17° at their greatest radius, mean angle being from 15° to 17° to plane of motion of sails. Length of a whip (arm) is divided into 7 parts, sails extending over 6 parts.

Whip in parts of its length: Breadth .033, at top .016; Depth .025, at top .0125; Width of sail .33, at axis .2. Distance of sail from axis .014 of length of whip, and cross-bars 16 to 18 ins. from centres.

To Compute Angles of Sails.

$23^\circ - \frac{18\,d^2}{r^2}$ = *angle of sail with plane of its motion at any part of it. d representing distance of part of sail from its axis, and r extreme radius of sail, both in feet.*

Illustration.—Assume $r = 14$, and length of sail 12 feet, $d = .5$ of 12 or three sixths of sail $= .5 \times 12 + (14 - 12) = 2 = 8$ feet.

$$\text{Then } 23^\circ - \frac{18 \times 8^2}{14^2} = 23 - 5.88^\circ = 17.12^\circ.$$

Hence, angle of sail with axis $= 90^\circ - 17.12^\circ = 72.88^\circ$.

If radius of sails is divided into 6 equal parts, angles at each of these parts will be as follows:

	Distance from Axis.					
	1	2	3	4	5	6
Angle of sail with axis	67.5°	69°	71.5°	75°	79.5°	85°
" " with plane of motion	22.5°	21°	18.5°	15°	10.5°	5°

To Compute Elements of Windmills.

$\frac{3.16\,v}{r' \sin. x} = n$; $\frac{11.5\,v}{r'} = n$; $.1047\,n = a\,v$; $\frac{A\,v^3}{1\,080\,000} = \text{HP}$; $\frac{\text{HP} \times 1\,080\,000}{v^3} = A$; $\sqrt{\frac{R^2 + r^2}{2}} = r'$. *v representing velocity of wind per second, r′ radius of centre of percussion of sails, and R and r outer and inner radii of sails, all in feet, x mean angle of sail to plane of motion, n number of revolutions of arms per minute, a v angular velocity, A area of sails in sq. feet, and HP horse-power.*

Illustration.—If a windmill has 4 arms of 28 feet, with a mean angle (x) of 16°, with an area of sail of 150 sq. feet each, having an inner radius of 4 feet, and is operated by wind at a velocity of 40 feet per second; what are its elements?

Then $\frac{11.5 \times 40}{20} = n = 23$; $\sqrt{\frac{28^2 + 4^2}{2}} = r' = 20$ *feet*; $\frac{3.16 \times 40}{20 \times .2756\,4} = n = 22.95$;

$\frac{4 \times 150 \times 64\,000}{1\,080\,000} = \text{HP} = 35.55$; $\frac{35.55 \times 1\,080\,000}{64\,000} = A = 599.9$ *sq. feet.*

Deductions from Velocities varying from 4 to 9 Feet per Second. (*Mr. Smeaton.*)

1. Velocity of windmill sails, so as to produce a maximum effect, is nearly as velocity of wind, their shape and position being same.

2. Load at maximum is nearly, but somewhat less than, as square of velocity of wind, shape and position of sails being same.

3. Effects of same sails, at a maximum, are nearly, but somewhat less than, as cubes of velocity of wind.

4. Load of same sails, at maximum, is nearly as squares, and their effect as cubes of their number of turns in a given time.

5. In sails where figure and position are similar, and velocity of wind the same, number of revolutions in a given time will be reciprocally as radius or length of sail.

6. Load, at a maximum, which sails of a similar figure and position will overcome at a given distance from centre of motion, will be as cube of radius.

7. Effects of sails of similar figure and position are as square of radius.

8. Velocity of extremities of Dutch sails, as well as of enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably greater than that of wind.

Results of Experiments on Effect of Windmill Sails. When a vertical windmill is employed to grind corn, the millstone usually makes 5 revolutions to 1 of the sail.

1. When velocity of wind is 19 feet per second, sails make from 11 to 12 revolutions in a minute, and a mill will grind from 880 to 990 lbs. in an hour, or about 22 440 lbs. in 24 hours.

2. When velocity of wind is 30 feet per second, a mill will carry all sail, and make 22 revolutions in a minute, grinding 1984 lbs. of flour in an hour, or 47 616 lbs. in 24 hours.

Results of Operation of Windmills. (*A. R. Woolf, M. E.*)
Velocity of Wind 15 to 20 Miles per Hour.
Revolutions of Wheel and Gallons of Water raised per Minute.

Designation of Mill.	Revolutions of Wheel.	Water raised to an Elevation of 25 Feet.	50 Feet.	100 Feet.	200 Feet.	Power developed.	Cost per Hour. Actual.*	Per IP.
Feet.	No.	Gallons.	Gallons.	Gallons.	Gallons.	IP	Cents.	Cents.
8.5	70 to 75	6.16	3.02	—	—	.04	.60	15
10	60 to 65	19.18	9.56	4.75	—	.12	.70	5.8
14	50 to 55	45.14	22.57	11.25	5	.28	1.63	5.8
18	40 to 45	97.68	52.16	24.42	12.21	.61	2.83	4.6
20	35 to 40	124.95	63.75	31.25	15.94	.78	3.56	4.5
25	30 to 35	212.38	106.96	49.73	26.74	1.34	4.26	3.2

* Including interest at 5 per cent. per annum.

WOOD AND TIMBER.

Selection of Standing Trees.—*Wood* grown in a moist soil is lighter, and decays sooner, than that grown in dry, sandy soil.

Best *Timber* is that grown in a dark soil, intermixed with gravel. Poplar, Cypress, Willow, and all others which grow best in a wet soil, are exceptions.

Hardest and densest woods, and least subject to decay, grow in warm climates; but they are more liable to split and warp in seasoning.

Trees grown upon plains or in centre of forests are less dense than those from edge of a forest, from side of a hill, or from open ground.

Trees (in U. S.) should be selected in latter part of July or first part of August; for at this season leaves of sound, healthy trees are fresh and green, while those of unsound are beginning to turn yellow. A sound, healthy tree is recognized by its top branches being well leaved, bark even and of a uniform color. A rounded top, few leaves, some of them turned yellow, a rougher bark than common, covered with parasitic plants, and with streaks or spots upon it, indicate a tree upon the decline. Decay of branches, and separation of bark from the wood, are infallible indications that the wood is impaired.

Green timber contains 37 to 48 per cent. of liquids. By exposure to air in seasoning one year, it loses from 17 to 25 per cent., and when seasoned it retains from 10 to 15 per cent.

According to M. Leplay, green wood contains about 45 per cent. of its weight of moisture. In Central Europe, wood cut in winter holds, at end of following summer, fully 40 per cent. of water, and when kept dry for several years retains from 15 to 20 per cent. of water.

Felling Timber.—Most suitable time for felling timber is in midwinter and in midsummer. Recent experiments indicate latter season and month of July.

A tree should be allowed to attain full maturity before being felled. Oak matures at 75 to 100 years and upward, according to circumstances; Ash, Larch, and Elm at 75; and Spruce and Fir at 80. Age and rate of growth of a tree are indicated by number and width of the rings of annual increase which are exhibited in a cross-section of its body.

A tree should be cut as near to the ground as practicable, as the lower part furnishes best timber.

Dressing Timber.—As soon as a tree is felled, it should be stripped of its bark, raised from the ground, reduced to its required dimensions, and its sap-wood removed.

Inspection of Timber.—Quality of wood is in some degree indicated by its color, which should be nearly uniform, and a little deeper towards its centre, and free from sudden transitions of color. White spots indicate decay. Sap-wood is known by its white color; it is next to the bark, and soon rots.

Defects of Timber.

Wind-shakes are serious defects, being circular cracks separating the concentric layers of wood from each other.

Splits, Checks, and Cracks, extending toward centre, if deep and strongly marked, render timber unfit for use, unless purpose for which it is intended will admit of its being split through them.

Brash is when wood is porous, of a reddish color, and breaks short, without splinters. It is generally consequent upon decline of tree from age.

Belted is that which has been killed before being felled, or which has died from other causes. It is objectionable.

Knotty is that containing many knots, though sound; usually of stinted growth.

Twisted is when grain of it winds spirally; it is unfit for long pieces.

Dry-rot is indicated by yellow stains. Elm and Beech are soon affected, if left with the bark on.

Large or decayed knots injuriously affect strength of timber.

Heart-shake.—Split or cleft in centre of tree, dividing it into segments.

Star-shake.—Several splits radiating from centre of timber.

Cup-shake.—Curved splits separating the rings wholly or in part.

Rind-gall.—Curved swelling, usually caused by growth of layers over spot where a branch has been removed.

Upset.—Fibres injured by crushing.

Foxiness.—Yellow or red tinge, indicating incipient decay.

Doatiness.—A speckled stain.

Seasoning and Preserving Timber.

Seasoning is extraction or dissipation of the vegetable juices and moisture or solidification of the albumen. When wood is exposed to currents of air at a high temperature, the moisture evaporates too rapidly, and it cracks; and when temperature is high and sap remains, it ferments, and dry-rot ensues.

Wood requires time in which to season, very much in proportion to density of its fibres.

Water Seasoning is total immersion of timber in water, for purpose of dissolving the sap, and when thus seasoned it is less liable to warp and crack, but is rendered more brittle.

For purpose of seasoning, it should be piled under shelter and kept dry; should have a free circulation of air, without being exposed to strong currents. Bottom pieces should be placed upon skids, which should be free from decay, raised not less than 2 feet from ground; a space of an inch should intervene between pieces of same horizontal layers, and slats or piling-strips placed between each layer, one near each end of pile, and others at short distances, in order to keep the timber from winding. These strips should be one over the other, and in large piles should not be less than 1 inch thick. Light timber may be piled in upper portion of shelter, heavy timber upon ground floor. Each pile should contain but one description of timber, and they should be at least 2.5 feet apart.

It should be repiled at intervals, and all pieces indicating decay should be removed, to prevent their affecting those which are still sound.

It requires from 2 to 8 years to be seasoned thoroughly, according to its dimensions, and it should be worked as soon as it is thoroughly dry, for it deteriorates after that time.

Gradual seasoning is most favorable to strength and durability of timber. Various methods have been proposed for hastening the process, as *Steaming*, which has been applied with success; and results of experiments of various processes of saturating it with a solution of *Corrosive sublimate* and *Antiseptic* fluids are very satisfactory. Such process hardens and seasons wood, at the same time that it secures it from dry-rot and from attacks of worms.

Woods are densest and strongest at the roots and at their centres. Their strength decreasing with the decrease of their density.

Oak timber loses *one fifth of its weight* in seasoning, and about *one third* in becoming perfectly dry.

Pitch pine, from the presence of pitch, requires time in excess of that due to the density of its fibre.

Mahogany should be seasoned slowly, Pine quickly. Whitewood should not be dried artificially, as the effect of heat is to twist it.

Salt water renders wood harder, heavier, and more durable than fresh.

Condition of timber, as to its soundness or decay, is readily recognized when struck with a quick blow.

Timber that has been for a long time immersed in water, when brought into the air and dried, becomes brashy and useless.

When trees are barked in the spring, they should not be felled until the foliage is dead.

Timber cannot be seasoned by either smoking or charring; but when it is exposed to worms or to the production of *fungi*, it is proper to smoke or char it, and it may be partially seasoned by being boiled or steamed.

Timber houses are best provided with blinds which keep out rain and snow, but which can be turned to admit air in fine weather, and the houses should be kept entirely free from any pieces of decayed wood.

Kiln-drying is suited only for boards and pieces of small dimensions, as it is apt to cause cracks and to impair the strength, unless performed very slowly.

Charring, *Painting*, or *covering* the surface *is highly injurious to any but seasoned wood*, as it effectually prevents drying of the inner part of the wood, in consequence of which fermentation and decay soon take place.

Timber is subject to *Common* or *Dry-rot*, former occasioned by alternate exposure to moisture and dryness, and as progress of it is from the exterior, covering of the surface, if seasoned, with paint, tar, etc., is a preservative.

Common-rot is the consequence of its being piled in badly-ventilated sheds. Outward indications are yellow spots upon ends of pieces, and a yellowish dust in the checks and cracks, particularly where the pieces rest upon piling-strips.

Dry or Sap-rot is inherent in timber, and it is the putrefaction of the vegetable albumen. Sap wood contains a large proportion of fermentable elements.

Insects attack wood for the sugar or gum contained in it, and *fungi* subsist upon the albumen of wood; hence, to arrest dry-rot, the albumen must be either extracted or solidified.

Most effective method of preserving timber is that of expelling or exhausting its fluids, solidifying its albumen, and introducing an antiseptic liquid.

Strength of impregnated timber is not reduced, and its *resilience* is improved.

In desiccating timber by expelling its fluids by heat and air, its strength is increased fully 15 per cent.

The saturation of wood with creosote, tar, antiseptics, etc., preserves it from the attack of worms. Jarrow wood, from Australia, is not subjected to their attack.

In a perfectly dry atmosphere durability of woods is almost unlimited. Rafters of roofs are known to have existed 1000 years, and piles submerged in fresh water have been found perfectly sound 800 years from period of their being driven.

Resistance of woods to extension is greater than that of compression.

Impregnation of Wood.

Several of the successful processes are as follows:

Kyan, 1832.—Saturated with corrosive sublimate. Solution 1 lb. of chloride of mercury to 4 gallons of water.

Burnett (*Sir Wm.*), 1838.—Impregnation with chloride of zinc by submitting the wood endwise to a pressure of 150 lbs. per sq. inch. Solution, 1 lb. of the chloride to 4 gallons of water.

Boucheri.—Impregnation by submitting the wood endwise to a pressure of about 15 lbs. per sq. inch. Solution, 1 lb. of sulphate of copper to 12.5 gallons of water.

Bethel.—Impregnation by submitting the wood endwise to a pressure of 150 to 200 lbs. per sq. inch, with oil of creosote mixed with bituminous matter.

Robbins, 1865.—Aqueous vapor dissipated by the wood being heated in a chamber, the albumen solidified, then submitted to vapor of coal tar, resin, or bituminous oils, which, being at a temperature not less than 325°, readily takes the place of the vapor expelled by a temperature of 212°.

Hayford, 187-.—Aqueous vapor dissipated by the wood being heated in a chamber to a temperature of from 250° to 270°, the albumen solidified, then air introduced to assist the splitting of the outer surfaces. When vapor is dissipated, dead oils are introduced under a pressure of 75 lbs. per sq. inch.

Planks, Deals, and Battens.—When cut from Northern pine (*Pinus Sylvestris*) are termed *yellow* or *red* deal, and when cut from spruce (*Abies, alba*, etc.) they are termed *white* deal.

Desiccated wood, when exposed to air under ordinary circumstances, absorbs 5 per cent. of water in the first three days; and will continue to absorb it until it reaches from 14 to 16 per cent., the amount varying according to condition of the atmosphere.

Durability of Various Woods.

Pieces 2 feet in Length, 1.5 ins. Square, driven 28.5 ins. into the Earth.

Wood.	Condition After 2.5 Years.	Condition After 5 Years.
Acacia	Good	Externally decayed, rest perfectly sound.
Ash, Amer.	Much decayed	Decayed.
Cedar, Va.	Very good	Sound as when driven.
" Lebanon	Good	Tolerable.
Elm, Eng.	Much decayed	Entirely decayed.
" Can.	" "	Decayed.
Fir	" attacked	Much decayed.
Larch	Surface only attacked	Attacked in part only, rest fair condition.
Oak, Can.	Very much decayed	Very rotten.
" Memel	" " "	" "
" Dantzic	" " "	" "
" Chestnut	Very good	Some moderately, most very much, decayed.
Pine, pitch	Surface only attacked	Attacked in part only, rest fair condition.
" yellow	Attacked	Much decayed.
" white	Very much decayed	Very rotten.
Teak	Very good	Somewhat soft, but good.

Effect of Creosoting.

Results of Experiments with Various Woods (E. R. Andrews).

Wood.		Water absorbed.	Wood.		Water absorbed.
		Per cent.			Per cent.
Spruce	dried	.2543	Hard pine	dried	.16
	creosoted	.0261		creosoted	.0
Oak	dried	.2	Gum, black	dried	1
	creosoted	.0		creosoted	.125
Cotton-wood	dried	.714	Birch, white	dried	.43
	creosoted	.347		creosoted	.124

Sesquoia Gigantea of California, dried, .4722; creosoted, .0.

Fluids will pass with the grain of wood with great facility, but will not enter it except to a very limited extent when applied externally.

Absorption of Preserving Solution by different Woods for a Period of 7 Days. *Average Lbs. per Cube Foot.*

Black Oak	3.6	Hemlock	2.6	Rock Oak	3.9
Chestnut	3	Red Oak	3.9	White Oak	3.1

Proportion of Water in various Woods.

Alder (*Betula alnus*)	41.6	Pine (*Pinus Sylvestris L.*)	39.7
Ash (*Fraxinus excelsior*)	28.7	Red Beech (*Fagus sylvatica*)	39.7
Beech (*Fagus sylvatica*)	33	Red Pine (*Pinus picea dur*)	45.2
Birch (*Betula alba*)	30.8	Spruce (*Abies, alba, nigra, rubra, excelsa*)	35
Elm (*Ulmus campestris*)	44.5	Sycamore (*Acer pseudo-platanus*)	27
Horse-chestnut (*Æsculus hippocast.*)	38.2	White Oak (*Quercus alba*)	36.2
Larch (*Pinus larix*)	48.6	White Pine (*Pinus abies dur*)	37.1
Mountain Ash (*Sorbus aucuparia*)	28.3	White Poplar (*Populus alba*)	50.6
Oak (*Quercus robur*)	34.7		

Willow (*Salis caprea*) 26

Decrease in Dimensions of Timber by Seasoning.

Woods.	Ins.		Ins.	Woods.	Ins.		Ins.
Cedar, Canada	14	to	13.25	Pitch Pine, South	18.375	to	18.25
Elm	11	to	10.75	Spruce	8.5	to	8.375
Oak, English	12	to	11.625	White Pine, American	12	to	11.875
Pitch Pine, North	10×10	to	9.75×9.75	Yellow Pine, North	18	to	17.875

Weight of a beam of English oak, when wet, was reduced by seasoning from 972.25 to 630.5 lbs.

Weight of a Cube Foot of Oak and Yellow Pine.

Age.	White Oak, Va. Round.	White Oak, Va. Square.	Yellow Pine, Va. Round.	Yellow Pine, Va. Square.	Live Oak.
Green	64.7	67.7	47.8	39.2	78.7
1 Year	53.6	53.5	39.8	34.2	—
2 Years	46	[illegible]	34.3	33.5	66.7

In England, Timber sawed into boards is classed as follows:

6.5 to 7 ins. in width, *Battens;* 8.5 to 10 ins., *Deals;* and 11 to 12 ins., *Planks.* (*See also page* 62.)

Distillation.—From a single cord of pitch pine distilled by chemical apparatus, following substances and in quantities stated have been obtained:

Charcoal	50 bushels.	Pyroligneous Acid	100 gallons.
Illuminating Gas....about	1000 cu. feet.	Spirits of Turpentine	20 "
Illuminating Oil and Tar	50 gallons	Tar	1 barrel.
Pitch or Resin	1.5 barrels.	Wood Spirit	5 gallons.

Strength of Timber.

Results of experiments have satisfactorily proved: That deflection was sensibly proportional to load; That extension and compression were nearly the same, though former being the greater: That, to produce equal deflection, load, when placed in the centre, was to a load uniformly distributed, as .638 to 1; That deflection under equal loads is inversely as breadths and cubes of the depths, and directly as cubes of the spans. (*M. Morin.*)

It has also been shown, that density of wood varies very little with its age. That *coefficient* of elasticity diminishes after a certain age, and that it depends also on the dryness and the exposure of the ground where the wood is grown. Woods from a northerly exposure, on dry ground, have a high *coefficient*, while those from swamps or low moist ground have a low one. That tensile strength is influenced by age and exposure. The *coefficient* of elasticity of a tree cut down in full vigor, or before it arrives at this condition, does not present any sensible difference. That there is no limit of elasticity in wood, there being a permanent set for every extension.

Average Result of Experiments on Tensile Strength of Wood in Various Positions per Sq. Inch. (*MM. Chevandier and Wertheim.*)

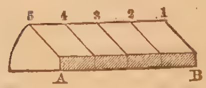
With the fibre, 6900 lbs. Radially, 683 lbs., and Tangentially, 723 lbs.

To Compute Volume of an Irregular Body.

By "*Simpson's Rule.*"

Operation.—Take a right line in the figure for a base line, as A B, divide the figure into any number of equal parts, and compute the areas of their plane sections as 1, 2, 3, etc., at the points of division, by rules applicable to area of a plane. Then, operate these areas as if they were the ordinates of a plane curve or figure of same length as the figure, and result will give volume required.

Illustration.—Assume a figure having areas as follows, and A B = 24 feet.



Sections,	Areas,		Multiplier,	Products,
1	3	feet	1	3
2	5	"	4	20
3	7	"	2	14
4	9	"	4	36
5	11	"	1	11
				84

and $84 \times 24 \div 4 \div 3 = 168$ *cube feet.*

MISCELLANEOUS MIXTURES.

Cements.

Much depends upon manner in which a cement is applied as upon the cement itself, as best cement will prove worthless if improperly applied. Following rules must be rigorously adhered to to attain success:

1. Bring cement into intimate contact with surfaces to be united. This is best done by heating pieces to be joined in cases where cement is melted by heat, as with resin, shellac, marine glue, etc. Where solutions are used, cement must be well rubbed into surfaces, either with a brush (as in case of porcelain or glass), or by rubbing the two surfaces together (as in making a glue joint between pieces of wood).

2. As little cement as practicable should be allowed to remain between the united surfaces. To secure this, cement should be as liquid as practicable (thoroughly melted if used with heat), and surfaces should be pressed closely into contact until cement has hardened.

3. Time should be allowed for cement to dry or harden, and this is particularly the case in *oil* cements such as copal varnish, boiled oil, white lead, etc. When two surfaces, each .5 inch across, are joined by means of a layer of white lead placed between them, 6 months may elapse before cement in middle of joint becomes hard. At the end of a month the joint will be weak and easily separated; at end of 2 or 3 years it may be so firm that the material will part anywhere else than at joint. Hence, when article is to be used immediately, the only safe cements are those which are liquefied by heat and which become hard when cold. A joint made with marine glue is firm an hour after it has been made. Next to cements that are liquefied by heat are those which consist of substances dissolved in water or alcohol. A glue joint sets firmly in 24 hours; a joint made with shellac varnish becomes dry in 2 or 3 days. Oil cements, which do not dry by evaporation, but harden by oxidation (boiled oil, white lead, red lead, etc.) are slowest of all.

Stone.—Resin, Yellow Wax, and Venetian Red, each 1 oz.; melt and mix.

Aquarium.

Litharge, fine white dry Sand, and Plaster of Paris, each 1 gill; finely pulverized Resin, .33 gill.

Mix thoroughly and make into a paste with boiled linseed oil to which drier has been added. Beat well, and let stand 4 or 5 hours before using it. After it has stood for 15 hours, however, it loses its strength. Glass cemented into a frame with this cement will resist percolation for either salt or fresh water.

Adhesive for Fractures of all Kinds.

White Lead ground with Linseed-oil Varnish, and kept from contact with the air.

Requires a few weeks to harden.

Stone or Iron.

Compound equal parts of Sulphur and Pitch.

Brass to Glass.

Electrical.—Resin, 5 ozs.; Beeswax, 1 oz.; Red Ochre or Venetian Red, in powder, 1 oz. Dry earth thoroughly on a stove at above 212°. Melt Wax and Resin together and stir in powder by degrees. Stir until cold, lest earthy matter settle to bottom.

Used for fastening brass-work to glass tubes, flasks, etc.

Chinese Waterproof.

Schio-liao.—To 3 parts of Fresh Beaten Blood add 4 parts of Slaked Lime and a little Alum; a thin, pasty mass is produced, which can be used immediately

Materials which are to be made specially waterproof are painted twice, or at most three times. Wooden public buildings of China are painted with *schio-liao*, which gives them an unpleasant reddish appearance, but adds to their durability. Pasteboard treated with it receives appearance and strength of wood.

China.

Curd of milk, dried and powdered, 10 ozs.; Quicklime, 1 oz.; Camphor, 2 drachms.

Mix, and keep air-tight. When used, a portion is to be mixed with a little water into a paste.

Cisterns and Water-casks.

Melted Glue, 8 parts; Linseed oil, boiled into a varnish with Litharge, 4 parts.

This cement hardens in about 48 hours, and renders the joints of wooden cisterns and casks air and water tight.

Cloth or Leather.

Shellac, 1 part; Pitch, 2 parts; India Rubber, 4 parts; and Gutta Percha, 10 parts; cut small; Linseed oil, 2 parts; melted together and mixed.

Earthen and Glass Ware.

Heat article to be mended a little above 212°, then apply a thin coating of gum Shellac upon both surfaces of broken vessel.

Or, dissolve gum Shellac in alcohol, apply solution, and bind the parts firmly together until cement is dry.

Or, dilute white of egg with its bulk of water and beat up thoroughly. Mix to consistence of thin paste with powdered Quicklime.

Use immediately.

Entomologists'.

Thick Mastic Varnish and Isinglass size, equal parts.

Gutta Percha.

Melt together, in an iron pan, 2 parts Common Pitch and 1 part Gutta Percha.

Stir well together until thoroughly incorporated, and then pour liquid into cold water. When cold it is black, solid, and elastic; but it softens with heat, and at 100° is a thin fluid. It may be used as a soft paste, or in liquid state, and answers an excellent purpose in cementing metal, glass, porcelain, ivory, etc. It may be used instead of putty for glazing.

Glass.

Sorel's.—Mix commercial Zinc White with half its bulk of fine Sand, add a solution of Chloride of Zinc of 1.26 spec. grav., and mix thoroughly in a mortar.

Apply immediately, as it hardens very quickly.

Holes in Castings.

Sulphur in powder, 1 part; Sal-ammoniac, 2 parts; powdered Iron turnings, 80 parts. Make into a thick paste.

Make only as required for immediate use.

Hydraulic Paint.

Hydraulic cement mixed with oil forms an incombustible and waterproof paint for roofs of buildings, outhouses, walls, etc.

Iron Ware.

Sulphur, 2 parts; fine Black-lead, 1 part. Heat sulphur in an iron pan until it melts, then add the lead; stir well, and remove. When cool, break into pieces as required. Place upon opening of the ware to be mended, and solder with an iron.

Kerosene Lamps, etc.

Resin, 3 parts; Caustic Soda, 1; Water, 5, mixed with half its weight of Plaster of Paris.

It sets firmly in about three quarters of an hour. Is of great adhesive power, not permeable to kerosene, a low conductor of heat, and but superficially attacked by hot water.

Leather to Iron, Steel, or Glass.

1.—Glue, 1 quart, dissolved in Cider Vinegar; Venice Turpentine, 1 oz.; boil very gently or simmer for 12 hours.

Or, Glue and Isinglass equal parts, soak in water 10 hours, boil and add tannin until mixture becomes "ropy;" apply warm.

Remove surface of leather where it is to be applied.

2.—Steep leather in an infusion of Nutgall, spread a layer of hot Glue on surface of metal, and apply flesh side of leather under pressure.

Leather Belting.

Common Glue and Isinglass, equal parts, soaked for 10 hours in enough water to cover them. Bring gradually to a boiling heat and add pure Tannin until whole becomes ropy or appears alike to white of eggs.

Clean and rub surfaces to be joined, apply warm, and clamp firmly.

Molding and Temporary Adhesion.

Soft.—Melt Yellow Beeswax with its weight of Turpentine, and color with finely powdered Venetian red.

When cold it has the hardness of soap, but is easily softened and molded with the fingers.

Maltha, or Greek Mastic.

Lime and Sand mixed in manner of mortar, and made into a proper consistency with milk or size without water.

Marble.

Plaster of Paris, in a saturated solution of Alum, baked in an oven, and reduced to powder. Mixed with water, and color if required.

Metal to Glass.

Copal Varnish, 15 parts; Drying Oil, 5; Turpentine, 3. Melt in a water bath and add 10 of Slaked Lime.

Mending Shells, etc.

Gum Arabic, 5 parts; Rock Candy, 2; and White Lead, enough to color.

Large Objects.

Wollaston's White.—Beeswax, 1 oz.; Resin, 4 ozs.; powdered Plaster of Paris, 5 oz. Melt together.

Warm the edges of the object and apply warm.

By means of this cement a piece of wood may be fastened to a chuck, which will hold when cool; and when work is finished it may be removed by a smart stroke with tool. Any traces of cement may be removed by Benzine.

Marble Workers and Coppersmiths.

White of egg, mixed with finely-sifted Quicklime, will unite objects which are not submitted to moisture.

Porcelain.

Add Plaster of Paris to a strong solution of Alum till mixture is of consistency of cream.

It sets readily, and is suited for cases in which large rather than small surfaces are to be united.

Rust Joint.

(*Quick Setting.*)—Sal-ammoniac in powder, 1 lb.; Flour of Sulphur, 2 lbs.; Iron borings, 80 lbs. Made to a paste with water.

(*Slow Setting.*)—Sal-ammoniac, 2 lbs.; Sulphur, 1 lb.; Iron borings, 200 lbs.

The latter cement is best if joint is not required for immediate use.

Steam Boilers, Steam-pipes, etc.

Finely powdered Litharge, 2 parts; very fine Sand, 1; and Quicklime slaked by exposure to air, 1.

This mixture may be kept for any length of time without injuring. In using it, a portion is mixed into paste with linseed oil, boiled or crude. Apply quickly, as it soon becomes hard.

Soft.—Red or White Lead in oil, 4 parts; Iron borings, 2 to 3 parts.
Hard.—Iron borings and salt water, and a small quantity of Sal-ammoniac with fresh water.

Transparent—Glass.

India-rubber, 1 part in 64 of chloroform; gum Mastic in powder, 16 to 24 parts. Digest for two days, with frequent shaking.

Or, pulverized Glass, 10 parts; powdered Fluor-spar, 20; soluble Silicate of Soda, 60. Both glass and fluor-spar must be in finest practicable condition, which is best done by shaking each in fine powder, with water, allowing coarser particles to deposit, and then by pouring off remainder, which holds finest particles in suspension.

The mixture must be made very rapidly, by quick stirring, and applied immediately.

Uniting Leather and Metal.

Wash metal with hot Gelatine; steep leather in an infusion of Nutgalls, hot, and bring the two together.

Waterproof Mastic.

Red Lead, 1 part; ground Lime, 4 parts; sharp Sand and boiled Oil, 5 parts.
Or, Red Lead, 1 part; Whiting, 5; and sharp Sand and boiled Oil, 10.

Wood to Iron.

Litharge and Glycerine.—Finely powdered Oxide of Lead (litharge) and Concentrated Glycerine.

The composition is insoluble in most acids, is unaffected by action of moderate heat, sets rapidly, and acquires an extraordinary hardness.

Turner's.—Melt 1 lb. of Resin, and add .25 lb. of Pitch.
While boiling add Brick dust to give required consistency. In winter it may be necessary to add a little Tallow.

GLUES.

Marine.

Dissolve India Rubber, 4 parts, in 34 parts of Coal-tar Naphtha; add powdered Shellac, 64 parts.

While mixture is hot pour it upon metal plates in sheets. When required for use, heat it, and apply with a brush.

Or, India Rubber, 1 part; Coal Tar, 12 parts; heat gently, mix, and add powdered Shellac, 20 parts. Cool. When used, heat to about 250°.

Or, Glue, 12 parts; Water, sufficient to dissolve; add Yellow Resin, 3 parts; and, when melted, add Turpentine, 4 parts.

Strong Glue.—Add Powdered Chalk to common Glue.

Mix thoroughly.

Mucilage.

Curd of Skim Milk (carefully freed from Cream or Oil), washed thoroughly, and dissolved to saturation in a cold concentrated solution of Borax.

This mucilage keeps well, and, as regards adhesive power, far surpasses gum Arabic.

Or, Oxide of Lead, 4 lbs.; Lamp-black, 2 lbs.; Sulphur, 5 ozs; and India Rubber dissolved in Turpentine, 10 lbs.

Boil together until they are thoroughly combined.

Preservation of Mucilage.—A small quantity of Oil of Cloves poured into a bottle containing Gum Mucilage prevents it from becoming sour.

To Resist Moisture.

Glue, 5 parts; Resin, 4 parts; Red Ochre, 2 parts; mixed with least practicable quantity of water.

Or, Glue, 4 parts; Boiled Oil, 1 part, by weight, Oxide of Iron, 1 part.

Or, Glue, 1 lb., melted in 2 quarts of skimmed Milk.

Parchment.

Parchment Shavings, 1 lb.; Water, 6 quarts.

Boil until dissolved, then strain and evaporate slowly to proper consistence.

Rice, or Japanese.

Rice Flour; Water, sufficient quantity.

Mix together cold, then boil, stirring it during the time.

Liquid.

Glue, Water, and Vinegar, each 2 parts. Dissolve in a water-bath, then add Alcohol, 1 part.

Or, Cologne or strong Glue, 2.2 lbs.; Water, 1 quart; dissolve over a gentle heat; add Nitric Acid 36°, 7 ozs., in small quantities.

Remove from over fire, and cool.

Or, White Glue, 16 ozs.; White Lead, dry, 4 ozs.; Rain Water, 2 pints. Add Alcohol, 4 ozs., and continue heat for a few minutes.

Elastic and Sweet.—Stamps or Rolls.

Elastic.—Dissolve good Glue in water by a water-bath. Evaporate to a thick consistence, and add equal weight of Glycerine to Glue; submit to heat until all water is evaporated, and pour into molds or on plates.

Sweet.—Substitute Sugar for the Glycerine.

To Adhere Engravings or Lithographs upon Wood.

Sandarach, 250 parts; Mastic in tears, 64 parts; Resin, 125 parts; Venice Turpentine, 250 parts; and Alcohol, 1000 parts by measure.

BROWNING, OR BRONZING, LIQUID.

Sulphate of Copper, 1 oz.; Sweet Spirit of Nitre, 1 oz.; Water, 1 pint.

Mix. Let stand a few days before use.

Gun Barrels.

Tincture of Muriate of Iron, 1 oz.; Nitric Ether, 1 oz.; Sulphate of Copper, 4 scruples; rain water, 1 pint. If the process is to be hurried, add 2 or 3 grains of Oxymuriate of Mercury.

When barrel is finished, let it remain a short time in lime-water, to neutralize any acid which may have penetrated; then rub it well with an iron wire scratch-brush.

After Browning. — Shellac, 1 oz.; Dragon's-blood, .25 oz.; rectified Spirit, 1 qt. Dissolve and filter.

Or, Nitric Acid, spec. grav. 1.2; Nitric Ether, Alcohol, and Muriate of Iron, each 1 part. Mix, then add Sulphate of Copper 2 parts, dissolved in Water 10 parts.

LACQUERS.

Small Arms, or Waterproof Paper.

Beeswax, 13 lbs.; Spirits Turpentine, 13 gallons; Boiled Linseed Oil, 1 gallon.

All ingredients should be pure and of best quality. Heat them together in a copper or earthen vessel over a gentle fire, in a water-bath, until they are well mixed.

Bright Iron Work.

Linseed Oil, boiled, 80.5 parts; Litharge, 5.5 parts; White Lead, in oil, 11.25 parts; Resin, pulverized, 2.75 parts.

Add litharge to oil; simmer over a slow fire 3 hours; strain, and add resin and white lead, keep it gently warmed, and stir until resin is dissolved.

Or, Amber, 6 parts; Turpentine, 6 parts; Resin, 1 part; Asphaltum, 1 part; and Drying Oil, 3 parts; heat and mix well.

Or, Shellac, 1 lb.; Asphaltum, 6 lbs.; and Turpentine, 1 gallon.

Iron and Steel.

Clear Mastic, 10 parts; Camphor, 5 parts; Sandarac, 15 parts; and Elimi Gum, 5 parts. Dissolve in Alcohol, filter, and apply cold.

Brass.

Shellac, 8 ozs.; Sandarac, 2 ozs.; Annatto, 2 ozs.; and Dragon's-blood Resin, .25 oz.; and Alcohol, 1 gallon.

Or, Shellac, 8 ozs.; and Alcohol, 1 gallon. Heat article slightly, and apply lacquer with a soft brush.

Wood, Iron, or Walls, and rendering Cloth, Paper, etc., Waterproof.

Heat 120 lbs. Oil Varnish in one vessel, 33 lbs. Quicklime in 22 lbs. water in another Soon as lime effervesces, add 55 lbs. melted India Rubber. Stir mixture, and pour into vessel of hot Varnish. Stir, strain, and cool.

When used, thin with Varnish and apply, preferably hot.

To Clean Soiled Engravings.

Ozone Bleach, 1 part; Water, 10; well mixed.

INKS.

Indelible, for Marking Linen, etc.

1.—Juice of Sloes, 1 pint; Gum, .5 oz.

This requires no "preparation" or mordant, and is very durable.

2.—Nitrate of Silver, 1 part, Water, 6 parts, Gum, 1 part; Dissolve.

3.—Lunar Caustic, 2 parts; Sap Green and Gum Arabic, each 1 part; dissolve with distilled water.

"*Preparation.*"—Soda, 1 oz.; Water, 1 pint; Sap Green, .5 drachm. Dissolve, and wet article to be marked, then dry and apply the ink.

Perpetual, for Tomb-stones, Marble, etc.—Pitch, 11 parts; Lamp-black, 1 part; Turpentine sufficient. Warm and mix.

Copying Ink.—Add 1 oz. Sugar to a pint of ordinary Ink.

SOLDERING.

Base for Soldering.

Strips of Zinc in diluted Muriatic, Nitric, or Sulphuric Acid, until as much is decomposed as acid will effect. Add Mercury, let it stand for a day; pour off the Water, and bottle the Mercury.

When required, rub surface to be soldered with a cloth dipped in the Mercury.

VARNISHES.

Waterproof.

Flour of Sulphur, 1 lb.; Linseed Oil, 1 gall.; boil them until they are thoroughly combined.

Good for waterproof textile fabrics.

Harness.

India Rubber, .5 lb.; Spirits of Turpentine, 1 gall.; dissolve into a jelly; then mix hot Linseed Oil, equal parts with the mass, and incorporate them well over a slow fire.

Fastening Leather on Top Rollers.

Gum Arabic, 2.75 ozs., and a like volume of Isinglass, dissolved in Water.

To Preserve Glass from the Sun.

Reduce a quantity of Gum Tragacanth to fine powder, and dissolve it for 24 hours in white of egg well beat up.

Water-color Drawings.

Canada Balsam, 1 part; Oil of Turpentine, 2 parts.
Mix and size drawing before applying.

Objects of Natural History, Shells, Fish, etc.

Mucilage of Gum Tragacanth and of Gum Arabic, each 1 oz.
Mix, and add spirit with Corrosive Sublimate, to precipitate the more stringy portion of the Gum.

Iron and Steel.

Mercury, 120 parts; Tin, 10 parts; Green Vitriol, 20 parts; Hydrochloric Acid of 1.2 sp. gr., 15 parts, and pure Water, 120 parts.

Blackboards.

Shellac Varnish, 5 gallons; Lamp black, 5 ozs.; fine Emery, 3 ozs.; thin with Alcohol, and lay in 3 coats.

Black.

Heat, to boiling, Linseed Oil Varnish, 10 parts, with Burnt Umber, 2 parts, and powdered Asphaltum, 1 part.
When cooled, dilute with Spirits of Turpentine as may be required.

Balloon.

Melt India Rubber in small pieces with its weight of boiled Linseed Oil.
Thin with Oil of Turpentine.

Transfer.

Alcohol, 5 ozs.; pure Venice Turpentine, 4 ozs.; Mastic, 1 oz.

To render Canvas Waterproof and Pliable.

Yellow Soap, 1 lb., boiled in 6 pints of Water, add, while hot, to 112 lbs. of oil Paint.

Waterproof Bags.

Pitch, 8 parts, Wax and Tallow, each 1 part.

To Clean Varnish.

Mix a lye of Potash or Soda, with a little powdered Chalk.

STAINING.

Wood and Ivory.

Yellow.—Dilute Nitric Acid will produce it on wood.

Red.—An infusion of Brazil Wood in Stale Urine, in the proportion of 1 lb. to a gallon, for wood, to be laid on when boiling hot, also Alum water before it dries. Or, a solution of Dragon's-blood in Spirits of Wine.

Black.—Strong solution of Nitric Acid.

Blue.—For Ivory: soak it in a solution of Verdigris in Nitric Acid, which will turn it *green;* then dip it into a solution of Pearlash boiling hot.

Purple.—Soak Ivory in a solution of Sal-ammoniac into four times its weight of Nitrous Acid.

Mahogany.—Brazil, Madder, and Logwood, dissolved in water and put on hot.

MISCELLANEOUS.

Blacking for Harness.

Beeswax, .5 lb.; Ivory Black, 2 ozs.; Spirits of Turpentine, 1 oz.; Prussian Blue ground in oil, 1 oz.; Copal Varnish, .25 oz.

Melt wax and stir it into other ingredients before mixture is quite cold; make it into balls. Rub a little upon a brush, and apply it upon harness, then polish lightly with silk.

To Clean Brass Ornaments.

Brass ornaments that have not been gilt or lackered may be cleaned, and a very brilliant color given to them, by washing them in Alum boiled in strong Lye, in the proportion of an ounce to a pint, and afterwards rubbing them with strong Tripoli.

To Harden Drills, Chisels, etc.

Temper them in Mercury.

To Clean Coral.

Brush with equal parts Spirits of Salts and cold water.

Or, dip in a hot solution of Potash or Chloride of Lime. If much discolored, let it remain in solution for a few hours.

Blacking, without Polishing.

Molasses, 4 ozs.; Lamp black, .5 oz.; Yeast, a table-spoonful; Eggs, 2; Olive Oil, a teaspoonful; Turpentine, a teaspoonful. Mix well.

To be applied with a sponge, without brushing.

Dubbing.

Resin, 2 lbs.; Tallow, 1 lb.; Train-oil, 1 gallon.

Anti-friction Grease.

Tallow, 100 lbs.; Palm-oil, 70 lbs. Boiled together, and when cooled to 80°, strain through a sieve, and mix with 28 lbs. of Soda, and 1.5 gallons of Water.

For Winter, take 25 lbs. more oil in place of the Tallow.

Or, Black Lead, 1 part; Lard, 4 parts.

To Attach Hair Felt to Boilers.

Red Lead, 1 lb.; White Lead, 3 lbs.; and Whiting, 8 lbs. Mixed with boiled Linseed Oil to consistency of paint.

Pastils for Fumigating.

Gum Arabic, 2 ozs.; Charcoal Powder, 5 ozs.; Cascarilla Bark, powdered, .75 oz.; Saltpetre, .25 drachm. Mix together with water, and make into shape.

For Writing upon Zinc Labels.—Horticultural.

Dissolve 100 grains of Chloride of Platinum in a pint of water; add a little Mucilage and Lamp-black.

Or, Sal-ammoniac, 1 dr.; Verdigris, 1 dr.; Lamp-black, .5 dr.; Water, 10 drs. Mix.

To Remove old Ironmold.

Remoisten part stained with ink, remove this by use of Muriatic Acid diluted by 5 or 6 times its weight of water, when old and new stain will be removed.

To Cut India Rubber.

Keep blade of knife wet with water or a strong solution of Potash.

Adhesive for Rubber Belts.

Coat driving surface with Boiled Oil or Cold Tallow, and then apply powdered Chalk.

Liard.

50 parts of finest Rape-oil, and 1 part of Caoutchouc, cut small. Apply heat until it is nearly all dissolved.

To Preserve Leather Belting or Hose.

Apply warm Castor Oil. For hose, force it through it.

To Oil Leather Belting.

Apply a solution of India Rubber and Linseed Oil.

Dressing for Leather Belts.

1.—Beef Tallow, 1 part, and Castor Oil, 2 parts. Apply warm.

2.—Beef Tallow, 3 lbs.; Beeswax, 1 lb. Heated and applied warm to both sides.

Files.

Lay dull files in diluted Sulphuric Acid until they are bitten deep enough.

To Remove Oil from Leather.

Apply Aqua-ammonia.

To Clean Paint.

Wash with a solution of Pearlash in water. If greasy, use Quicklime.
Or, Extract of Litherium diluted with from 200 to 300 parts of water.

To Remove Paint.

Mix Soft Soap, 2 ozs., and Potash, 4 ozs., in boiling Water, with Quicklime, .5 lb. Apply hot, and let remain for 1 day.
Or, Extract of Litherium, thinly brushed over the surface 2 or 3 times.

To Clean Marble.

Chalk, powdered, and Pumice stone, each 1 part; Soda, 2 parts. Mix with water. Wash the spots, then clean and wash off with Soap and Water.

Paste for Cleaning Metals.

Oxalic Acid, 1 part; Rottenstone, 6 parts. Mix with equal parts of Train Oil and Spirits of Turpentine.

Watchmaker's Oil, which never Corrodes or Thickens.

Place coils of thin Sheet Lead in a bottle with Olive Oil. Expose it to the sun for a few weeks, and pour off the clear oil.

Durable Paste.

Make common Flour paste rather thick (by mixing some Flour with a little *cold* water until it is of uniform consistency, and then stir it well while *boiling* water is being added to it); add a little Brown Sugar and Corrosive Sublimate, which will prevent fermentation, and a few drops of Oil of Lavender, which will prevent it becoming moldy. When dried, dissolve in water.

It will keep for two or three years in a covered vessel.

To Extract Grease from Stone or Marble.

Soft Soap, 1 part; Fuller's Earth, 2 parts; Potash, 1 part. Mix with boiling water. Lay it upon the spots, and let it stand for a few hours.

Stains.

To Remove.—Stains of *Iodine* are removed by rectified Spirit; *Ink* stains by Oxalic or Superoxalate of Potash; *Ironmolds* by same; but if obstinate, moisten them with Ink, then remove them in the usual way.

Red spots upon black cloth, from acids, are removed by Spirits of Hartshorn, or other solutions of Ammonia.

Stains of Marking ink, or Nitrate of Silver.—Wet stain with fresh solution of Chloride of Lime, and, after 10 or 15 minutes, if marks have become white, dip the part in solution of Ammonia or of Hyposulphite of Soda. In a few minutes wash with clean water.

Or, stretch the stained linen over a basin of hot water, and wet mark with Tincture of Iodine.

Preservative Paste for Objects of Natural History.

White Arsenic, 1 lb.; Powdered Hellebore, 2 lbs.

To Preserve Bottoms of Iron Steam-boilers.

Red Lead, 75 parts; Venetian Red, 17 parts; Whiting, 6.5 parts; and Litharge, 1.5 parts by weight.

To Preserve Sails.

Slacked Lime, 2 bushels. Draw off the lime-water, and mix it with 120 gallons water, and with Blue Vitriol, .25 lb.

Whitewash.

For outside exposure, slack Lime, .5 bushel, in a barrel; add common Salt, 1 lb.; Sulphate of Zinc, .5 lb.; and Sweet Milk, 1 gallon.

To Preserve Woodwork.

Boiled Oil and finely powdered Charcoal, each 1 part; mix to the consistence of paint. Apply 2 or 3 coats.

This composition is well adapted for casks, water-spouts, etc.

To Polish Wood.

Rub surface with Pumice Stone and water until the rising of the grain is removed. Then, with powdered Tripoli and boiled Linseed Oil, polish to a bright surface.

Paint for Window Glass.

Chrome Green, .25 oz; Sugar of Lead, 1 lb.; ground fine, in sufficient Linseed Oil to moisten it. Mix to the consistency of cream, and apply with a soft brush.

The glass should be well cleansed before the paint is applied. The above quantity is sufficient for about 200 feet of glass.

To Make Drain Tiles Porous.

Mix sawdust with the clay before burning.

MISCELLANEOUS OPERATIONS AND ILLUSTRATIONS.

1.—It is required to lay out a tract of land in form of a square, to be enclosed with a post and rail fence, 5 rails high, and each rod of fence to contain 10 rails. What must be side of this square to contain just as many acres as there are rails in fence?

OPERATION. 1 mile = 320 *rods*. Then 320 × 320 ÷ 160, *sq. rods in an acre* = 640 *acres;* and 320 × 4 *sides* and × 10 *rails* = 12 800 *rails per mile.*

Then, as 640 *acres* : 12 800 *rails* :: 12 800 *acres* : 256 000 *rails, which will enclose* 256 000 *acres*, and $\sqrt{256\,000} \times 69.5701$ = *number of yards in side of a sq. acre*, and ÷ 1760, *yards in a mile* = 20 *miles.*

2.—How many fifteens can be counted with four fives?

$$\text{OPERATION.} \quad \frac{4 \times 3 \times 2 \times 1}{1 \times 2 \times 3} = \frac{24}{6} = 4.$$

3.—What are the chances in favor of throwing one point with three dice?

OPERATION.—Assume a bet to be upon the ace. Then there will be 6 × 6 × 6 = 216 *different ways which the dice may present themselves, that is, with and without an ace.*

Then, if the ace side of the die is excluded, *there will be* 5 *sides left*, and 5 × 5 × 5 = 125 *ways without the ace.*

Therefore, there will remain only 216 − 125 = 91 *ways in which there could be an ace.* The chance, then, in favor of the ace is as 91 to 125; that is, out of 216 throws, the probability is that it will come up 91 *times*, and lose 125 *times.*

4.—The hour and minute hand of a clock are exactly together at 12; when are they next together?

OPERATION.—As the minute hand runs 11 times faster than the hour hand, then, as 11 : 60 :: 1 : 5 *min.* $27\frac{3}{11}$ *sec.* = *time past* 1 *o'clock.*

5.—Assume a cube inch of glass to weigh 1.49 ounces troy, the same of sea-water .59, and of brandy .53. A gallon of this liquor in a glass bottle, which weighs 3.84 lbs., is thrown into sea-water. It is proposed to determine if it will sink, and, if so, how much force will just buoy it up?

OPERATION. 3.84 × 12 ÷ 1.49 = 30.92 *cube ins. of glass in bottle.*
231 cube ins. in a gallon × .53 = 122.43 *ounces of brandy.*

Then, bottle and brandy weigh 3.84 × 12 + 122.43 = 168.51 *ounces*, and contain 261.92 *cube ins.*, which × .59 = 154.53 *ounces, weight of an equal bulk of sea-water.*

And, 168.51 − 154.53 = 13.98 *ounces, weight necessary to support it in the water.*

6.—A fountain has 4 supply cocks, A, B, C, and D, and under it is a cistern, which can be filled by the cock A in 6 hours, by B in 8 hours, by C in 10 hours, and by D in 12 hours; now, the cistern has 4 holes, designated E, F, G, and H, and it can be emptied through E in 6 hours, F in 5 hours, G in 4 hours, and H in 3 hours. Suppose the cistern to be full of water, and that all the cocks and holes were opened together, in what time would the cistern be emptied?

OPERATION.—Assume the cistern to hold 120 gallons.

hrs.	gall.	hrs.	gall.		hrs.	gall.	hrs.	gall.	
If 6 :	120 ::	1 :	20	*at* A.	If 6 :	120 ::	1 :	20	*at* E.
8 :	120 ::	1 :	15	*at* B.	5 :	120 ::	1 :	24	*at* F.
10 :	120 ::	1 :	12	*at* C.	4 :	120 ::	1 :	30	*at* G.
12 :	120 ::	1 :	10	*at* D.	3 :	120 ::	1 :	40	*at* H.

Run in in 1 *hour*, 57 *gallons*. *Run out in* 1 *hour*, 114 *gallons*.
57

***Run out in* 1 *hour more than run in*, 57 *gallons*.**

Then, as 57 *gallons* : 1 *hour* :: 120 *gallons* : 2.158+ *hours*.

7.—A cistern, containing 60 gallons of water, has 3 cocks for discharging it; one will empty it in 1 hour, a second in 2 hours, and a third in 3 hours; in what time will it be emptied if they are all opened together?

OPERATION.—1st, .5 would run out in 1 hour by the 2d cock, and .333 by the 3d; consequently, by the 3 would the reservoir be emptied in 1 hour. $.5 + .333 + 1 = \frac{3}{6} + \frac{2}{6} + \frac{6}{6}$, *being reduced to a common denominator, the sum of these* $3 = \frac{11}{6}$; whence the proportion, 11 : 60 :: 6 : $32\frac{8}{11}$ *minutes*.

8.—A reservoir has 2 cocks, through which it is supplied; by one of them it will fill in 40 minutes, and by the other in 50 minutes; it has also a discharging cock, by which, when full, it may be emptied in 25 minutes. If the 3 cocks are left open, in what time would the cistern be filled, assuming the velocity of the water to be uniform?

OPERATION.—The least common multiple of 40, 50, and 25, is 200.

Then, the 1st cock will fill it 5 times in 200 *minutes*, and the 2d, 4 times in 200 *minutes*, or both, 9 *times in* 200 *minutes*; and, as the discharge cock will empty it 8 *times in* 200 *minutes*, hence 9 − 8 = 1, or once in 200 *minutes* = 3 2 *hours*.

9.—The time of the day is between 4 and 5, and the hour and minute hands are exactly together; what is the time?

OPERATION.—Difference of speed of the hands is as 1 to 12 = 11.

4 hours × 60 = 240, which ÷ 11 = 21 *min*. 49.09 *sec*., *which is to be added to* 4 *hours*.

10.—Out of a pipe of wine containing 84 gallons, 10 were drawn off, and the vessel refilled with water, after which 10 gallons of the mixture were drawn off, and then 10 more of water were poured in, and so on for a third and fourth time. It is required to compute how much pure wine remained in the vessel, supposing the two fluids to have been thoroughly mixed.

OPERATION. 84 − 10 = 74, *quantity after the* 1*st draught*.
Then, 84 : 10 :: 74 : 8.8095, and 74 − 8.8095 = 65.1905, *quantity after* 2*d draught*.
84 : 10 :: 65.1905 : 7.7608, and 65.1905 − 7.7608 = 57.4297, *quantity after* 3*d draught*.
84 : 10 :: 57.4297 : 6.8367, and 57.4297 − 6.8367 = 50.593, *quantity after* 4*th draught*, = *result required*.

11.—A reservoir having a capacity of 10 000 cube feet, has an influx of 750 and a discharge of 1000 cube feet per day. In what time will it be emptied?

OPERATION. $\frac{10\,000}{1000 - 750} = 40$ *days*.

Contrariwise: The discharge being 1000 and the influx 1250 cube feet per hour. In what time will it be filled?

OPERATION. $\frac{10\,000}{1250 - 1000} = 40$ *hours* = 1 *day* 16 *hours*.

12.—A son asked his father how old he was. His father answered him thus: If you take away 5 from my years, and divide the remainder by 8, the quotient will be one third of your age; but if you add 2 to your age, and multiply the whole by 3, and then subtract 7 from the product, you will have the number of years of my age. What were the ages of father and son?

OPERATION.—Assume father's age 37.

Then $37 - 5 = 32$, and $32 \div 8 = 4$, and $4 \times 3 = 12$, *son's age.* Again: $12 + 2 = 14$, and $14 \times 3 = 42$, and $42 - 7 = 35$. Therefore $37 - 35 = 2$, *error too little.*

Again: Assume father's age 45; then $45 - 5 = 40$, and $40 \div 8 = 5$. Therefore $5 \times 3 = 15$, *son's age.* Again: $15 + 2 = 17$, and $17 \times 3 = 51$, and $51 - 7 = 44$. Therefore $45 - 44 = 1$, *error too little.*

Hence (45 sup. $\times$ 2 error) — (37 sup. $\times$ 1 error) $= 90 - 37 = 53$, and $2 - 1 = 1$.

Consequently, 53 *is father's age.* Then $53 - 5 = 48$, and $48 \div 8 = 6 = .333$ *of son's age*, and $6 \times 3 = 18$ *years, son's age.*

13.—Two companions have a parcel of guineas. Said A to B, if you will give me one of your guineas I shall have as many as you have left. B replied, if you will give me one of your guineas I shall have twice as many as you will have left. How many guineas had each of them?

OPERATION.—Assume B had 6.

Then A would have had 4, for $6 - 1 = 4 + 1 = 5$. Again: 4 (A's parcel) $- 1 = 3$, and $6 + 1 = 7$, and $3 \times 2 = 6$. Therefore $7 - 6 = 1$, *error too little.*

Again: Assume B had 8.

Then A would have 6, for $8 - 1 = 6 + 1 = 7$. Again: 6 (A's parcel) $- 1 = 5$, and $8 + 1 = 9$, and $5 \times 2 = 10$. Therefore $10 - 9 = 1$, *error too great.*

Hence $8 \times 1 = 8$, and $6 \times 1 = 6$. Then $8 + 6 = 14$, and $1 + 1 = 2$. Whence, dividing products by sum of errors, $14 \div 2 = 7 =$ B's *parcel*, and $7 - 1 = 5 + 1 = 6$ for A *when he had received* 1 *of* B; also $\overline{5 - 1} \times 2 = 7 + 1 = 8 =$ B's *parcel when he had received* 1 *of* A.

14.—If a traveller leaves New York at 8 o'clock in the morning, and walks towards New London at the rate of 3 miles per hour, without intermission; and another traveller starts from New London at 4 o'clock in the evening, and walks towards New York at the rate of 4 miles per hour continuously; assuming distance between the two cities to be 130 miles, whereabouts upon the road will they meet?

OPERATION.—From 8 to 4 o'clock is 8 hours; therefore, $8 \times 3 = 24$ *miles, performed by* A *before* B *set out from New London;* and, consequently, $130 - 24 = 106$ *are the miles to be travelled between them after that.*

Hence, as $(3 + 4)\ 7 : 3 :: 106 : \frac{318}{7} = 45\frac{3}{7}$ *more miles travelled by* A *at the meeting;* consequently, $24 + 45\frac{3}{7} = 69\frac{3}{7}$ *miles from New York is place of their meeting.*

15.—If from a cask of wine a tenth part is drawn out and then it is filled with water; after which a tenth part of the mixture is drawn out; again is filled, and again a tenth part of the mixture is drawn out: now, assume the fluids to mix uniformly at each time the cask is replenished, what fractional part of wine will remain after the process of drawing out and replenishing has been repeated four times?

OPERATION.—Since .1 of the wine is drawn out at first drawing, *there must remain* .9. After cask is filled with water, .1 of whole being drawn out, *there will remain* .9 *of mixture; but* .9 *of this mixture is wine;* therefore, after second drawing, *there will remain* .9 *of* .9 *of wine*, or $\frac{9^2}{10^2}$; and after third drawing, *there will remain* .9 *of* .9 *of* .9 *of wine*, or $\frac{9^3}{10^3}$.

Hence, the part of wine remaining is expressed by the ratio .9, *raised to a power exponent of which is number of times cask has been drawn from.*

Therefore, *fractional part of wine is* $\frac{9^4}{10^4} = .6561$.

16.—There is a fish, the head of which is 9 ins. long, the tail as long as the head and half the body, and the body as long as both the head and tail. Required the length of the fish.

OPERATION.—Assume body to be 24 ins. in length. Then $24 \div 2 + 9 = 21$, *length of tail.*

Hence $21 + 9 = 30$, *length of body*, which is 6 ins. too great.

Again: assume the body to be 26 ins. in length. Then $26 \div 2 + 9 = 22$, *length of tail.* Hence $22 + 9 = 31$, *length of body*, which is 5 ins. too great.

Therefore, by *Double Position*, divide *difference* of products (see rule, page 99) by *difference* of errors (the errors being alike), $\overline{26 \times 6 - 24 \times 5} = 36 =$ *difference of products*, and $6 - 5 = 1 =$ *difference of errors.*

Consequently, $36 \div 1 = 36$, *length of body*, and $36 \div 2 + 9 = 27$, *length of tail*, and $36 + 27 + 9 = 72$ *ins., length required.*

17.—A hare, 50 leaps before a greyhound, takes 4 leaps to the greyhound's 3, but 2 leaps of the hound are equal to 3 of the hare's. How many leaps must the greyhound take before he can catch the hare?

OPERATION.—As 2 leaps of the greyhound equal 3 of the hare, it follows that 6 of the greyhound equal 9 of the hare.

While the greyhound takes 6 leaps, the hare takes 8; therefore, while the hare takes 8, the greyhound gains upon her 1.

Hence, to gain 50 leaps, she must take $50 \times 8 = 400$ *leaps; but, while hare takes* 400 *leaps, greyhound takes* 300, *since number of leaps taken by them are as* 4 *to* 3.

18.—If a basket and 1000 eggs were laid in a right line 6 feet apart, and 10 men (designated from A to J) were to start from basket and to run alternately, collect the eggs singly, and place them in basket as collected, and each man to collect but 10 eggs in his turn, how many yards would each man run over, and what would be entire distance run over?

OPERATION. —A's course would be 6×2 *feet (first term)* $+ \overline{10 \times 6} \times 2$ *feet (last term)* $= 132 =$ *sum of first and last terms of progression.*

Then $132 \div 2 \times 10 = 660$ *feet* $=$ *number of times* $\times$ *half sum of extremes* $=$ *sum of all the terms, or the distance run by* A *in his first turn.*

B's course would be $11 \times 6 \times 2 = 132$ *feet (first term)* $+ \overline{20 \times 6 \times 2} = 240$ *feet (last term)* $= 372 =$ *sum of first and last terms.*

Then $\overline{372 \div 2} \times 10 = 1860 =$ *sum of all the times, or* B's *first turn.*

A's last course would be $901 \times 6 \times 2 = 10\,812$ *feet for the first term*, and $\overline{910 \times 6 \times 2} = 10\,920$ *feet for the last term of his last turn.*

Then $\overline{10\,812 + 10\,920 \div 2} \times 10 = 108\,660 =$ *sum of the terms, or distance run.*

B's last course would be $911 \times 6 \times 2 = 10\,932$ *feet for the first term*, and $\overline{920 \times 6 \times 2} = 11\,040$ *feet for the last term of his last turn.*

Then $10\,932 + 11\,040 \div 2 \times 10 = 109\,860 =$ *sum of the terms or distance run.*

Therefore, if A's first and last runs $= 660$ and $108\,660$ *feet*, and the number of terms 10, then, by *Progression*, the sum of all the terms $= 546\,600$ *feet.*

And if B's first and last runs $= 1860$ and $109\,860$ *feet*, and the number of terms 10, then the sum of all the terms $= 558\,600$ *feet.*

Consequently, $558\,600 - 546\,600 = 12\,000 =$ *common difference of runs*, which, being added to each man's run $=$ sum of all runs, or entire distance run over.

A's *run*,	$546\,600 = 182\,200$ *yds.*	F's *run*,	$606\,600 = 202\,200$ *yds.*
B's "	$558\,600 = 186\,200$ "	G's "	$618\,600 = 206\,200$ "
C's "	$570\,600 = 190\,200$ "	H's "	$630\,600 = 210\,200$ "
D's "	$582\,600 = 194\,200$ "	I's "	$642\,600 = 214\,200$ "
E's "	$594\,600 = 198\,200$ "	J's "	$654\,600 = 218\,200$ "

$6\,006\,000$ *feet*, which $\div 5280 = 1137.5$ *miles.*

19.—If, in a pair of scales, a body weighs 90 lbs. in one scale, and but 40 lbs. in the other, what is the true weight?

$$\sqrt{(40 \times 90)} = 60 \text{ lbs.}$$

20.—If a steamboat, running uniformly at the rate of 15 miles per hour through the water, were to run for 1 hour with a current of 5 miles per hour, then to return against that current, what length of time would she require to reach the place from whence she started?

OPERATION. $15 + 5 = 20$ *miles, the distance run during the hour.*

Then $15 - 5 = 10$ *miles is her effective velocity per hour when returning, and* $20 \div 10 = 2$ *hours, the time of returning,* and $2 + 1 = 3$ *hours, or the whole time occupied.*

Or, *Let d represent distance in one direction, t and t′ greater and less times of running in hours, and c current or tide.*

Then, $\dfrac{d\,\frac{t+t'}{2}}{t \times t'} =$ *velocity of boat through the water*, and $\dfrac{\overline{v \times t' - d}}{t'} = c.$

21.—Flood-tide wave in a given river runs 20 miles per hour, current of it is 3 miles per hour. Assume the air to be quiescent, and a floating body set free at commencement of flow of the tide; how long will it drift in one direction, the tide flowing for 6 hours from each point of river?

OPERATION.—Let x be the time required; $20x =$ distance the tide has run up, together with the distance which the floating body has moved; $3x =$ whole distance which the body has floated.

Then $20x - 3x = 6 \times 20$, *or the length in miles of a tide.*

$$x = \frac{20}{20-3} \times 6 = 7 \text{ hours, } 3 \text{ minutes, } 31.765 \text{ seconds.}$$

22.—A steamboat, running at the rate of 10 miles per hour through the water, descends a river, the velocity of which is 4 miles per hour, and returns in 10 hours; how far did she proceed?

OPERATION.—Let $x =$ distance required, $\dfrac{x}{10+4} =$ *time of going,* $\dfrac{x}{10-4} =$ *time of returning.* Then, $\dfrac{x}{14} + \dfrac{x}{6} = 10$; $6x + 14x = 840$; $20x = 840$; $840 \div 20 = 42$ *miles.*

23.—From Caldwell's to Newburgh (Hudson River) is 18 miles; the current of the river is such as to accelerate a boat descending, or retard one ascending, 1.5 miles per hour. Suppose two boats, running uniformly at the rate of 15 miles per hour through the water, were to start one from each place at the same time, where will they meet?

OPERATION.—Let $x =$ the distance from N. to the place of meeting; *its distance from C., then, will be* $18 - x$.

Speed of descending boat, $15 + 1.5 = 16.5$ *miles per hour;* of ascending boat, $15 - 1.5 = 13.5$ *miles per hour.* $\dfrac{x}{16.5} =$ *time of boat descending to point of meeting.* $\dfrac{18-x}{13.5}$ $=$ *time of boat ascending to point of meeting.*

These times are of course equal; therefore, $\dfrac{x}{16.5} = \dfrac{18-x}{13.5}$. Then, $13.5x = 297 - 16.5x$, and $13.5x + 16.5x = 297$, or $30x = 297$.

Hence $x = \dfrac{297}{30} = 9.9$ *miles, the distance from Newburgh.*

24.—There is an island 73 miles in circumference; 3 men start together to walk around it and in the same direction: A walks 5 miles per day, B 8, and C 10; when will they all come aside of each other again?

OPERATION.—It is evident that A and C will be together every round gone by A; hence it remains to ascertain when A and B will be in conjunction at an even round, as 3 miles are gained every day by B. Therefore, as $3 : 1 :: 73 : 24.33+$; but, as the conjunction is a fractional number, it is necessary to ascertain what number of a multiplier will make the division a whole number.

$73 \div 24.33+ = 3$, *the number of days required in which A will go round 5 times, B 8, and C 10 times.*

25.—Assume a cow, at age of 2 years, to bring forth a cow-calf, and then to continue yearly to do the same, and every one of her produce to bring forth a cow-calf at age of 2 years, and yearly afterward in like manner; how many would spring from the cow and her produce in 40 years?

OPERATION.—The *increase* in 1st year would be 0, in 2d year 1, in 3d 1, in 4th 2, in 5th 3, in 6th 5, and so on to 40 years or terms, *each term being = sum of the two preceding ones.* The last term, then, will be 165 580 141, from which is to be subtracted 1 for the parent cow, and the remainder, 165 580 140, *will represent increase required.*

26.—The interior dimensions of a box are required to be in the proportions of 2, 3, and 5, and to contain a volume of 1000 cube ins.: what should be the dimensions?

OPERATION.—$\sqrt[3]{\frac{1000 \times 2^2}{2 \times 3 \times 5}} = 6.43$; $\sqrt[3]{\frac{1000 \times 3^2}{2 \times 3 \times 5}} = 9.65$; and $\sqrt[3]{\frac{1000 \times 5^2}{2 \times 3 \times 5}} = 16$ *ins.*

And what for a box of one half the volume, or 500 cube ins., and retaining same proportionate dimensions?

OPERATION.—$2 \times 3 \times 5 = 30$, and $\frac{30}{2} = 15$.

Then, $\sqrt[3]{\frac{15 \times 6.43^3}{30}} = 5.1$; $\sqrt[3]{\frac{15 \times 9.65^3}{30}} = 7.66$; and $\sqrt[3]{\frac{15 \times 16^3}{30}} = 12$ *ins.*

27.—The chances of events or games being equal, what are the odds for or against the following results?

Five Events.

Odds.	Against.	In favor.
31 to 1	All the 5	1 out of 5
4.33 to 1	4 out of 5	2 out of 5

5 to 3 in favor of the 5 events resulting 3 and 2.

Four Events.

Odds.	Against.	In favor.
15 to 1	All the 4	1 out of 4
2.2 to 1	3 out of 4	2 out of 4

5 to 3 against 2 events only, or that the 4 events do not result 2 and 2.

Three Events.

Odds.	Against.	In favor.
7 to 1	All the 3	1 out of 3
Even	2 or all out of 3	2 or all out of 3

3 to 1 in favor of the 3 events resulting 2 and 1.

Two Events.

Odds.	Against.	In favor.
3 to 1	Both events	1 out of 2
Even	1 only out of 2	1 only out of 2

Even that the events result 1 and 1.

28.—Required the chances or probabilities in events or games, when the chances or probabilities of the results, or the players, are equal.

Events or Games.	That a *named* event occurs a majority or more of times.	Against a *named* event occurring an exact majority of times.	Against each event occurring an equal number of times.	Events or Games.	That a *named* event occurs a majority or more of times.	Against a *named* event occurring an exact majority of times.	Against each event occurring an equal number of times.
21	Even	5 to 1	—	11	Even	3.4 to 1	—
20	1.33 to 1	—	4.66 to 1	10	1.7 to 1	—	3.06 to 1
19	Even	4.5 to 1	—	9	Even	3 to 1	—
18	1.55 to 1	—	4.4 to 1	8	1.75 to 1	—	2.66 to 1
17	Even	4.4 to 1	—	7	Even	2.7 to 1	—
16	1.5 to 1	—	4.1 to 1	6	2 to 1	—	2.2 to 1
15	Even	4 to 1	—	5	Even	2.2 to 1	—
14	1.5 to 1	—	3.8 to 1	4	2.2 to 1	—	1.66 to 1
13	Even	3.7 to 1	—	3	Even	1.66 to 1	—
12	1.6 to 1	—	3.44 to 1	2	3 to 1	—	Even.

29.—The chances of consecutive events or results are as follows:

11.—2047 to 1. | 10.—1023 to 1. | 9.—511 to 1. | 8.—255 to 1. | 7.—127 to 1. | 6.—63 to 1.

Hence it will be observed that the chances increase with the number of events very nearly in a duplicate ratio.

ILLUSTRATION.—The chances of 11 consecutive events compared with 10, are as 2047 to 1023, or 2 to 1.

30.—Required the chances or probabilities of events or results in a given number of times.

The *numerator* of a fraction expresses the chance or probability either for the result or event to occur or fail, and the *denominator* all the chances or probabilities both for it to occur or fail.

Thus, in a given number of events or games, if the chances are even, the probability of any particular result is as $\frac{1}{1+1}=\frac{1}{2}$; $\frac{2}{2+2}$; $\frac{3}{3+3}$, etc., being 1 out of 2, 2 out of 4, etc., or even.

If the number of events or games are 3, then the probability of any particular result, as 2 and 1, or 1 and 2, is determined as follows:

Number of permutations of 3 events are $1 \times 2 \times 3 = 6$, which represents number of times that number of events can occur, 2 and 1, or 1 and 2, to which is to be added the 2 times or chances they can occur all in one way or the reverse thereto.

Hence, $\frac{6}{2+6}=\frac{3}{4}=\frac{3}{4-3}=\frac{3}{1}$, or 3 to 1 in favor of result; and probability of one party naming or winning two precise events or results, as winning 2 out of 3, is determined as follows: Number of permutations and chances, as before shown, are 8. Hence, number of his chances being 3, $\frac{3}{3+5}=\frac{3}{8}=\frac{3}{8-3}=\frac{3}{5}$, or 3 to 5 in favor of result; and probability of one party naming or winning all, or 3 events or results, is determined as follows: Number of permutations and chances being also, as before shown, 8. Hence, as there is but one chance of such a result, $\frac{1}{1+7}=\frac{1}{8}=\frac{1}{8-1}=\frac{1}{7}$, or 1 to 7 in favor of result.

If number of events, etc., are 4, then probability of any particular result, as 2 and 2, or of winning 2 or more of them, is determined as follows:

Number of permutations and chances of 4 events are 16. Hence, as number of chances of such a result are 11, $\frac{11}{5+11}=\frac{11}{16}=\frac{11}{16-11}=\frac{11}{5}$, or as 11 to 5 in favor of the result, and that the results do not occur precisely 2 and 2. The number of chances of such a result being 10, $\frac{10}{6+10}=\frac{5}{8}=\frac{5}{8-5}=\frac{5}{3}$, or 5 to 3 against it.

If number of events, etc., are 5, then probability of any particular result, as 3 and 2, is determined as follows:

Number of permutations and chances being 32, and number of chances of such a result being 20, $\frac{20}{12+20}=\frac{10}{16}=\frac{10}{16-10}=\frac{10}{6}=\frac{5}{3}$, or as 5 to 3 in favor of the result; and that it may occur precisely 3 out of 5, the number of chances are $\frac{10}{10+22}=\frac{10}{32}=\frac{5}{16}=\frac{5}{16-5}=\frac{5}{11}$, or 11 to 5 against it.

31.—What is the dilatation of the iron in a railway track per mile, between the temperatures of -20° and $+130^\circ$?

OPERATION.— $-20^\circ + 130^\circ = 150^\circ$. The dilatation of wrought iron (as per table, page 519) is, from 32° to $212^\circ = 180^\circ = .001\,257\,5$ times its length.

Hence, as $180 : 150 :: .001\,257\,5 : .001\,047\,9 = \frac{.001\,047\,9}{1}$ of 5280 (feet in a mile) = 5.53 *feet per mile.*

32.—A steamer having an immersed amidship section of 125 sq. feet, has a speed of 15 miles per hour with 300 IP. What power would be required for one of like model, having a section of 150 sq. feet for a speed of 20 miles?

As power required for like models is as cube of speeds.

Then $\frac{150}{125}=1.2$ *relative sections*, and $\frac{20^3=8000}{15^3=3375}=2.37$ *relative powers.*

Hence, $1 : 1.2 :: 2.37 : 2.844$ *times* IP.

MARINE STEAMERS AND ENGINES.

Iron Cruiser (Propeller).

"ZABIACA," I. R. N. — VERTICAL DIRECT ENGINE (*Compound*).—*Length between perpendiculars*, 228 *feet*; *at water-line of* 12 *feet*, 220 *feet*; *beam*, 30 *feet*; *hold*, 17.5 *feet*.

Displacement at load draught of 12.58 *and* 14.58 *feet*, 1202 *tons*. *Per inch at load-line*, 11.58 *tons*. *Areas.—Of Load-line*, 4867 sq. feet; *of Sails*, 12 312 sq. feet.

Coefficients.—Of Total Displacement, .5; *of Surface*, .74; *of Cylindroid* from cylinder, .61; *of Cylindroid* from parallelopipedon, .475.

Cylinders.—34 and 59 ins. in diam. by 36 ins. stroke of piston.

Pressure of Steam.—78 lbs. per sq. inch, cut off at 23 ins. full throttle. *Revolutions*, 89.4 per minute. IHP, 1400. *Pitch of Propeller*, 19 feet. *Speed*, 14 knots per hour. *Fuel.*—Anthracite coal, 1.6 lbs. per IHP per hour.

Centres of Gravity.—Forward of after perpendicular, 100 feet; below meta-centre at draught of 10.46 and 12.21 feet, 2.81 feet, and at load-line 3.12 feet. *Of Buoyancy*, below load line, 4.97 feet. *Of Engines, Boilers, Water, etc.*, aft of centre of length, 25.25 feet; do. above top of keel, 9.17 feet.

Meta-centre.—Above centre of buoyancy for mean draught of 11.3 feet = 4 feet.

Iron Freight and Passenger (Propellers).

"ORIENT."—VERTICAL DIRECT ENGINE (*Compound*).—*Length upon deck*, 460 *feet*; *beam*, 46.35 *feet*; *depth to main deck*, 27.1 *feet*; *to spar deck*, 35.1 *feet*.

Immersed section at load line, 1094 *sq. feet*. *Displacement at load draught of* 26.5 *feet*, 9500 *tons*; *per inch*, 40 *tons*. *Tons*, 3440–5380.

Cylinders.—1 of 60 ins. in diam., and 2 of 85 ins., by 5 feet stroke of piston. *Condenser.*—Surface, 12 000 sq. feet. *Propeller.*—4 blades, 22 feet in diam. *Pitch*, 30 feet. *Shaft*, 20 ins. in diam.

Boilers.—4 (cylindrical tubular), 15.5 feet in diam. by 17.5 feet in length; 6 furnaces, 4 feet in diam. by 6 feet in length. *Pressure of Steam*, 75 lbs. per sq. inch. *Revolutions*, 60 per minute. IHP, 5400. *Bulkheads*, 12. *Decks*, 3 of iron.

Capacity.—3000 tons coal, 3000 tons (measurement) cargo, 120 1st class passengers, 130 2d, and 300 3d class, or 3000 troops and 406 horses.

Water Ballast.—Aft, 82 feet in length. *Rig*, 4-masted bark. *Passage*, 35 days, Plymouth to Australia. *Weights.*—Hull, Engines, and Boilers, 4940 tons.

"ARIZONA."—VERTICAL DIRECT ENGINE (*Compound*). — *Length between perpendiculars and for tonnage*, 450 *feet*; *breadth*, 45.5 *feet*; *depth*, 35.7 *feet*; *Tons*, 5146.55.

Cylinders.—1 of 62 ins. and 2 of 90 ins. in diam., by 5.5 feet stroke of piston. *Condenser.*—Surface, 12 540 sq. feet.

Propeller (Cast Steel).—Diam., 23 feet; weight, 27 tons.

Boilers.—6 of 13.5 feet in diam., 3 of 10 feet in length, and 3 of 18 feet. *Heating Surface*, 19 500 sq. feet. *Grate*, 780 sq. feet. *Pressure of Steam*, 86 lbs. per sq. inch. *Revolutions*, 55 per minute. IHP, 6306. *Speed*, 17 knots per hour.

"NORMANDIE." — VERTICAL DIRECT ENGINES (*Compound*). — *Length*, 459 *feet* 11 *ins.*; *beam*, 49 *feet* 11 *ins.* *Hold*, 37 *feet* 5 *ins.* *Mean draught at trial*, 20 *feet*.

Displacement, 7656 *tons*. *Immersed Section at load-draught of* 24.25 *feet*, 1060 *sq. feet*.

Cylinders.—3 of 35.4375 ins., and 3 of 74.875 ins. in diam.; stroke of pistons, 67 ins.; ratio of low to high pressure, 1 to 4.46.

Condensers.—3, surface, 11 682 sq. feet. *Air-pumps.*—3, single acting, 34 ins. diam.; stroke of piston, 32 ins. *Centrifugal Pumps.*—3, 12.5 ins. in diam., driven by three 11 by 11 inch engines.

Boilers.—8 (cylindrical tubular), 4 double end, 13.5 feet in diam., 18.5 feet in length, 4 single end, 13.75 feet in diam., 9.5 feet in length. *Grates*, 808.5 sq. feet. *Heating Surface*, 21 405 sq. feet. *Steam Room*, 3950 cube feet.

Pressure of Steam, 85 lbs., cut off at .75 stroke. *Revolutions*, 59 per minute. IHP, 8006. *Shaft*, 23.625 ins. in diam. *Propeller*, 22 feet in diam. *Pitch*, 31 feet.

Speed, 17.25 knots per hour. *Weight* of Engines, Boilers, and Water in boilers, complete, 1376 tons.

"CITY OF SAN FRANCISCO."—VERTICAL DIRECT ENGINE (*Compound*).—*Length over all*, 352 *feet; for tonnage*, 339 *feet; beam*, 40.2 *feet; hold*, 28 *feet* 10 *ins.; Load draught*, 22 *feet.*

Cylinders, 2.—51 and 88 ins. in diam. by 5 feet stroke of piston. *Condenser.—Surface*, 6425 sq. feet.

Pressure of Steam, 80 lbs. per sq. inch. *Revolutions*, 55. *Speed*, 14 knots per hour.

Propeller, 4 blades, 20 feet in diam. by 25 feet pitch.

Boilers.—6 (cylindrical tubular), 13 feet in diam. *Heating Surface*, 10650 sq. feet. *Grates*, 378 sq. feet. *Ratio of Grate* to heating surface, 1 to 28; to tube area, 9 to 1; to smoke-pipe area, 6.66 to 1.

Iron Auxiliary Freight.

VERTICAL DIRECT ENGINE (*Compound*).—*Length on deck*, 135 *feet; beam*, 22.5 *feet; hold*, 11 *feet.*

Load-draught, 4 *feet* 10 *ins. and* 10 *feet* 6 *ins. Free board*, 1.5 *feet.*

Cylinders.—21 and 40 ins. in diam. by 27 ins. stroke of piston. *Condenser.—Surface*, 617 sq. feet.

Boiler (cylindrical tubular).—12 feet in diam. by 9.5 feet in length. *Heating surface*, 1205 sq. feet. *Grates*, 38.5 sq. feet.

Pressure of Steam, 80 lbs. per sq. inch. *Speed*, 10.8 knots per hour. IHP, 370.

Consumption of coal, 8.5 tons in 24 hours. *Rig.*—Schooner.

"ISLE OF DURSEY."—VERTICAL DIRECT ENGINES (*Compound Triple Expansion*).—*Length on deck*, 210 *feet; beam*, 31.25 *feet; hold*, 14.1 *feet. Tons*, 620.963.

Cylinders.—2, each 15.75 and 22 ins., and 44.33 ins. in diam.; stroke of piston, 2.75 feet. *Condenser.*—466 .75 inch tubes, No. 18 B W G. *Surface*, 792 sq. feet.

Propeller.—4 blades, 12.5 feet in diam. *Pitch*, 14.5 feet. *Surface*, 38.5 sq. feet.

Pressure of Steam, 150 lbs. per sq. inch. *Revolutions*, 73 per minute.

Boiler.—1 (horizontal tubular). *Heating surface*, 1650 sq. feet. *Grate*, 42 sq. feet. IHP per sq. foot of grate, 12.3; of heating surface, .374. Total, 500.

Fuel.—Bituminous, 1.5 lbs. per IHP per hour. *Rig.*—Fore-topsail schooner.

Iron Fire-boat.

"ZOPHAR MILLS."—VERTICAL DIRECT ENGINE.—*Length on load-line*, 115 *feet; beam, molded*, 24 *feet; hold at side*, 8 *feet* 8 *ins.*

Immersed section at load-line of 7 *feet*, 150 *sq. feet.*

Cylinder.—30 ins. in diam. by 30 ins. stroke of piston; volume of piston space, 12.25 cube feet. *Condenser.—Surface*, 900 sq. feet.

Boilers (return tubular).—Two, 8 feet in width by 14 feet in length. *Heating surface*, 2120 sq. feet. *Grates*, 80 sq. feet.

Pressure.—70 lbs. per sq. inch, cut off at .5 stroke. *Revolutions*, 84 at 45 lbs. pressure, cut off at .5. *Speed.*—12.5 miles per hour.

Propeller.—4-bladed, 8 feet 9 ins. in diam.

Pumps, Vertical Duplex. Steam cylinders, 4.—16 ins. in diam. by 9 ins. stroke. *Pumps*, 4.—7.5 ins. in diam. by 9 ins. stroke. *Receiving pipes*, 8.5 ins. in diam. *Revolutions*, 110 per minute.

Discharge.—2200 gallons per minute; or, 8 streams, 2.5 ins. to 3.25 ins. hose, average 75 feet in length of hose each, and 1.5 ins. nozzles, 160 feet. Or, 4 streams, 3.25 ins. hose, 100 feet in length of hose each, connected to one length of 16 feet of 4-inch hose, and 3.25 ins. nozzle, 280 feet.

Steel Launch.

INVERTED DIRECT ENGINE (*Non-condensing*).—*Length*, 25 *feet; beam*, 5 *feet; hold*, 2.5 *feet.*

Cylinder.—5 ins. in diam. by 5 ins. stroke of piston.

Hull.—Frame, .75 × .75 inch, No. 12 W G. Keel, stem and stern-post, each, 1.5 × 1.25 ins.

Steel Yachts. (Propellers).

"LADY TORFRIDA."—VERTICAL DIRECT ENGINES (*Compound*).—*Length*, 200 *feet* 8 *ins.*; *beam*, 25 *feet* 7 *ins.*; *hold*, 15 *feet* 7 *ins.* *Tons*, 611.

Cylinders.—3, one of 24 ins. in diam., and two of 34 ins. by 30 ins. stroke of piston.

Condenser.—*Surface*, 1978 sq. feet. *Circulating Pump*, double, 12 by 17 ins. *Air-pump*, single, 20 by 17 ins.

Boilers (return tubular).—14.5 feet in diam. by 9 feet in length. *Heating Surface*, 1887 sq. feet. *Grate*, 77 sq. feet.

Pressure of Steam, 110 lbs. Vacuum, 28.5 ins. IHP. 1020. *Propeller*, Manganese bronze, 11 feet in diam. *Pitch*, 14.5 feet. *Speed.*—15 knots per hour.

Iron.

"ISA."—VERTICAL DIRECT ENGINES (*Compound*).—*Length of keel*, 118.66 *feet*; *beam*, 18.75 *feet*; *hold*, 10 *feet.* *Tons*, 248.

Cylinders, 3.—10, 15, and 28 ins. in diam. by 2 feet stroke of piston. *Condenser.*—*Surface*, 350 sq. feet. *Circulating Pump*, 6 ins. in diam by 12 ins stroke.

Pressure of Steam.—120 lbs. per sq. inch full stroke. *Revolutions*, 112 per minute. *Speed*, 12 knots per hour.

Propeller.—2 blades, 8.5 feet in diam. *Pitch*, 12.25 feet.

Composite.

"RADHA."—VERTICAL DIRECT ENGINES (*Compound*).—*Length for tonnage*, 142 *feet*; *beam*, 20 *feet*; *depth of hold*, 8 *feet* 8.5 *ins.* *Tons*, 77.04 *and* 149.15.

Immersed section at load-draught of 8.25 *feet*, 115 *sq. feet.*

Cylinders, 3.—1 of 20 ins. in diam., and 2 of 26 by 2 feet stroke of piston.

Condenser.—*Surface*, 800 sq. feet.

Boiler (flue and return tubular).—9 feet 8 ins. wide, and 15 feet in length. *Heating surface*, 1947 sq. feet. *Grate*, 48 sq. feet.

Propeller, 7.5 feet in diam. *Pitch*, 12 feet. *Revolutions*, 135 per minute.

Pressure of Steam.—100 lbs., cut off at .5. *Blast draught.*

"SIESTA."—VERTICAL DIRECT ENGINE (*Compound*), HERRESHOFF.—*Length on deck over all*, 98 *feet*; *at water line*, 90.3 *feet*; *beam at deck*, 17 *feet*; *at water-line*, 15.16 *feet*; *depth of hull from rabbet of keel to top of shear plank*, 8.33 *feet*; *draught of water at load-line*, 5.66 *feet.*

Immersed section at load-line, 43 *sq. feet.* *Displacement at load-draught*, 63.83 *tons.*

Area of water section, 878.7 *sq. feet*, *and of immersed surface of hull*, 1438 *sq. feet.*

Ratio of water surface to its circumscribing parallelogram, .64; *of immersed transverse section to its do. do.*, .584; *and of displacement of immersed hull above lower edge of rabbet of keel to its circumscribing parallelopiped*, .5677.

Cylinders, 2.—10.5 and 18 ins. in diam. by 18 ins. stroke of piston. Volume of piston space, 3.45 cube feet. Relative volumes of displacement of cylinders, 1 to 2.96. *Air pump*, single acting, 6 ins. in diam. by 6.25 ins. stroke.

Circulating and Feed Pumps, single acting, 1.125 ins. in diam. by 18 ins. stroke.

Condenser, *External.*—*Surface*, 731, 5 ins. by 29.5 ins. tubes; condensing surface, 235 sq. feet.

Propeller, 4 blades, 4 feet 7 ins. in diam. *Pitch*, 8 feet. *Helicoidal area* of blades, 9.46 sq. feet. *Transverse area*, 6.59 sq. feet.

Shaft.—Journal, 3.875 × 8 ins.; stress, 3.75 ins. *Engine space*, 3 feet by 5.5 feet in length.

Boiler (vertical double coil).—Diam. outside of casing, 6.66 feet; height, 8 feet 10 ins. *Heating surface*, 558 sq. feet. *Grates*, 26 sq. feet.

Smoke-pipe, 23.5 ins. in diam. by 25 feet above grates. *Steam room*, 5.7 cube feet.

Heating surface to Grate, 21.5 to 1.

Pressure of Steam, 60.7 lbs. per sq. inch, cut off in small cylinder at .88 of stroke, and in large at .3 stroke. In small cylinder at end of stroke, 55.2 lbs.; in large cylinder at commencement of stroke, 50.6; and at end of stroke, 15.6 lbs. *Mean back pressure* in small cylinder, 47.36 lbs.; and in large, 5.77.

Revolutions, 193 per minute. *Speed*, 12.75 miles (11.06 knots) per hour. *Slip of Propeller*, 27.3 per cent.

HERRESHOFF.—VERTICAL DIRECT ENGINE (*Compound*).—*Length on deck*, 100 *feet; beam*, 12.5 feet.

Cylinder.—1 of 12.5 and 21.5 ins. in diam. by 16 ins. stroke of piston.

Pressure of steam, 120 lbs. per sq. inch. *Revolutions*, 480 per minute. *Speed*, 22.5 knots per hour.

Thrust of Propeller at 15.73 knots, 4080 lbs.

Torpedo Boats. (Propellers.)

Iron.

VERTICAL DIRECT ENGINE (*Compound*).—*Length*, 110 *feet; beam*, 12.5 *feet.*

Displacement, 52 *tons.*

Cylinders, 1.—12.5 ins. and 21.5 ins.; stroke of piston, 16 ins.

Boiler. —(Horizontal tubular.) Diam., 4.75 feet. Tubes, 125 of 2 ins. in diam. *Heating surface*, 1016 sq. feet. *Speed*, 20.3 knots per hour.

Steel. Composite.

"TORPEDO BOAT," R. N. — VERTICAL DIRECT ENGINE (*Compound*), HERRESHOFF MFG. CO.—*Length*, 59.5 *feet; beam*, 7.5 *feet.*

Cylinders.—6 and 10.5 ins. in diam. by 10 ins. stroke of piston.

Condenser, External.—Surface. Boiler (vertical coil).—Tubes 2 ins. in diam. and 300 feet in length.

Propeller.—4 blades, 38 ins. in diam. by 5 feet pitch.

Weight at load-draught of hull of 1.5 feet; armament and stores, 8 tons.

Iron. Side Wheels.

"PRINCESS MARIE AND ELIZABETH."—OSCILLATING ENGINE (*Compound*).—*Length on load-line*, 274.8 *ins.; beam*, 34.75 *feet; hold*, 24.25 *feet. Tons*, 1606.

Cylinders, 2.—60 and 104 ins. in diam. by 3.5 feet stroke of piston.

Pressure of Steam.—70 lbs. per sq. inch, cut off at .6 stroke. *Revolutions*, 32.75 per minute. *Speed*, 17.12 knots per hour. IHP, 3543.

Consumption of fuel, 1.92 lbs. per IHP per hour. *Cost*, £54 900 sterling.

Cutter (Corrugated).

"LA BONITA."—INCLINED ENGINE (*Non-condensing*).—*Length upon deck*, 42 *feet; beam*, 9 *feet; hold*, 3 *feet.*

Immersed section at load-line, 8.75 *sq. feet. Displacement at load-draught of* 1.3 *feet*, 8386 *lbs. Tons*, 9.65, O. M.

Cylinder.—8 ins. in diam.; stroke of piston, 1 foot; volume of piston space, .35 cube foot. *Water-wheels.*—Diam. 5.66 feet. *Blades*, 7; breadth, 2.3 feet; depth, 7 ins.

Boiler.—(Horizontal tubular). *Heating surface*, 95 sq. feet. *Grates*, 6 do. *Fuel*, coal or wood. *Exhaust draught.*

Pressure of steam.—65 lbs. per sq. inch. *Revolutions*, 54 per minute. IHP, 9.

Hull.—Corrugated and galvanized plates, .0625 inch thick. *Weights.*—Hull, 2876 lbs.; Engine and wheels, 2400 lbs.; Boiler, 2260 lbs.; pipes, grates, etc., 750 lbs.

Steel.

FERRY BOAT.—INCLINED ENGINE (*Surface Condensing*).—*Length*, 78 *feet; beam*, 15 *feet; hold*, 8 *feet amidships and* 5 *feet at ends.*

Load-draught, 2.25 *feet.*

Cylinder, 15.5 ins. by 24 ins. stroke of piston. *Boiler* (cylindrical tubular).—Steel; heating surface, 220 sq. feet.

Pressure of Steam, 80 lbs. per sq. inch. *Plates*, .125, .1875, and .25 ins.

Light Draught.

"HO-NAM."—VERTICAL BEAM ENGINE (*Compound*).—*Length upon deck*, 280 *feet; beam*, 73 *feet; depth from hold to upper deck*, 30 *feet. Tons*, 2364.

Cylinders, 1.—40 and 72 ins. in diam.; stroke of piston, 10 feet. IHP, 3000.

Speed, 16.14 knots per hour. *Passengers*, 2000.

Decks, 3: Main, Saloon, and Promenade. *Rig.*—Schooner.

Wood Side Wheels. Passenger and Deck Cargo.

"City of Fall River," New York to Fall River, Mass., Vertical Beam Engine (*Compound*).—*From notes of James E. Sague and John B. Adger, Jr. Length on water-line*, 260 *feet; over all*, 273 *feet; beam*, 42 *feet; over guards*, 73 *feet; hold*, 18 *feet;* 1723 *tons N. M.*

Immersed section at load-line of 9 *feet* 3 *ins.* (1750 *tons*), 365 *sq. feet: and at load-draught of* 12 *feet*, 480 *sq. feet. Displacement at load-draught*, 2350 *tons.*

Cylinders, 2.—1 of 44 ins. in diam. by 8 feet stroke of piston, and 1 of 68 ins. in diam. by 12 feet stroke. *Clearance* at each end of high pressure, 4.6 per cent., and at low pressure, 3 per cent. Volumes, 85 and 303 cube feet.

Receiver, 89.13 cube feet. *Air-pump*, 37 ins. in diam. by 4.75 feet stroke of piston. *Condenser.—Surface*, 4067 sq. feet.

Water-wheels, 25.5 feet in diam. *Blades*, feathering, 12 of 40 ins. in depth by 10 feet in length. *Centre of Pressure on Blades*, 11.22 feet from axis of shaft.

Boilers.—2 (flue and return tubular), 17.5 feet in width by 15 feet in length, 220 tubes 3.5 ins. in diam. and 12 feet in length. *Grates*, 230 sq. feet.

Fuel.—Anthracite. *Natural Draught. Consumption*, 1463 lbs. per hour; refuse, 281 lbs. = 19.23 per cent. per sq. foot of grate, and 12.73 lbs.

Pressure of Steam.—High-pressure cylinder, throttle open and cut off at .445, mean in boiler per guage, 70 lbs.; in receiver, 11 lbs.; mean effective pressure, 41.8 lbs. per sq. inch.

Low-pressure cylinder, at point of cutting off of .45, 17.42 lbs. above zero; mean effective pressure, 12.4 lbs. per sq. inch. Expansion of steam, 6.99 times. Vacuum, 28.4 ins.

IHP.—High-pressure cylinder, 783; low-pressure, 840.

Revolutions, 25.8 per minute.

Feed Water, 27 854 lbs. per hour; per IHP, 17.17 lbs. *Temperatures.*—Feed water, 97°; sea water, 49.4°; water of condensation, 90°; heat units per hour per IHP, 19 090.

Stress of wheels, 20.4 per cent.

Condensing water, per IHP per hour, 407 lbs.

Consumption.—Compound engine, 2.03 lbs. per IHP; and as a simple condensing engine, without high-pressure cylinder, 2.84 lbs.

Evaporation per hour, 1208 lbs. water; per lb. of combustible from 212°, 11.75 lbs.; from temperature of feed (97°), 10.22 lbs.; from feed per lb. of coal, 8.26 lbs. Temperature of gases in chimney 485°.

Heating Surface, 29 sq. feet to 1 of grate.

Weights, Engine, and Frame, 250 tons; boilers complete, 120 tons; water, 50 tons.

Speed, 14.14 knots per hour; and IHP, 1623.

Draft of water, 10.65 feet; *Displacement*, 1938 tons.

"City of Boston," New York and Norwich.—Vertical Beam Engine (*Condensing*).—*Length upon load-line*, 320 *feet; beam*, 39 *feet; hold*, 12.6 *feet.*

Immersed section at load-line, 288 *sq. feet. Displacement* 1450 *tons, at load-draught of* 8.25 *feet.*

Cylinder.—80 ins. in diam. by 12 feet stroke of piston. *Volume*, 419 cube feet.

Water-wheels.—Diam. 37 feet 8 ins. *Arms*, 36. *Blades*, 37; breadth of do., 10 feet; depth of do., 30.5 ins. *Dip* at load-line, 4.25 feet.

Boilers.—2 (flue and return tubular), Shells, 12.5 feet in diam., and in length 26.5 feet. *Heating Surface*, 10 120 sq. feet. *Grates*, 184 sq. feet.

Pressure of Steam.—35 lbs. per sq. inch, cut off at .5 stroke. *Revolutions* (maximum), 19.75 per minute. IHP, 2500.

Fuel.—Anthracite; *Blast. Consumption*, at ordinary speed, 5200 lbs. per hour.

Weights of Engine, Boilers, etc., 263 tons.

Hull.—Weight, 800 tons. Light draught of hull without fuel, water, or furniture, 7 feet.

Wood Propellers.

HERRESHOFF, R. N., VERTICAL DIRECT ENGINE (*Compound*).—*Length on deck*, 46 *feet; over all*, 48 *feet; beam*, 9 *feet; hold*, 5 *feet.*

Displacement at load-line, 7.44 *tons. Area of section at load-line*, 217.8 *sq. feet. Area of wetted surface*, 365.5 *sq. feet. Coefficient of fineness*, .396.

Cylinder.—8 and 14 ins. in diam. by 9 ins. stroke of piston.

Condenser, External.—Surface.

Propeller.—4 blades, 3 feet in diam. by 4 feet 1 inch pitch.

Blower, 42 ins. in diam.

Boiler (vertical coil). *Heating surface*, 174 sq. feet. *Grates*, 12.5 sq. feet.

Pressure of Steam, 53 lbs. per sq. inch. *Revolutions*, 333 per minute. IHP, 68.4. *Speed*, 10.18 knots per hour. With 129 lbs. and 466 revolutions, 14.26 knots. IHP, 169.5. *Weight of Engines, Boiler, and Water*, 5300 lbs.

HERRESHOFF, VERTICAL DIRECT ENGINE (*Compound*).—*Length over all*, 86 *feet; beam*, 11 *feet. Displacement*, 27 *tons.*

Cylinder.—13 and 22 ins. in diam. by 12 ins. stroke of piston.

Surface Condensing.

Pressure, 130 lbs. per sq. inch.

Revolutions, 460 per minute. *Speed*, 20 knots per hour. IHP, 425.

Propeller, 3 blades. *Pitch*, 5 feet.

HERRESHOFF, R. I. N.—VERTICAL DIRECT ENGINE (*Compound*).—*Length over all*, 60 *feet; beam*, 7 *feet; hold*, 5.5 *feet. Displacement at load-draught of* 32 *ins.*, 7 *tons* (2240 lbs.).

Cylinders.—8 and 14 ins. in diam. by 9 ins. stroke of piston. *Surface condenser.*

Pressure of Steam.—140 lbs. per sq. inch, cut off at .5.

Revolutions, 600 per minute. *Speed*, 19.875 knots per hour.

Cable or Rope Towing.

"NYITRA."—HORIZONTAL DIRECT ENGINES (*Condensing*).—*Length of boat*, 138 *feet; beam*, 24.5 *feet; hold*, 7.5 *feet.*

Immersed section, 74.4 *sq. feet. Displacement*, 200 *tons at load-line of* 3.75 *feet. Immersed section*, 263.7 *sq. feet. Displacement*, 949 *tons. Tow.*—3 barges.

Cylinders.—2 of 14.18 ins. in diam. by 23.625 ins. stroke of piston.

IHP, *net effective*, 100. *Speed*, 7.73 miles per hour.

Propellers.—Twin, 4 feet 2 ins. in diam.

Stress.—Cable, 7485 lbs. Per ton of displacement, 6.5 lbs.; per sq. foot of immersed section, 22 lbs.

Fuel.—Per mile and ton of displacement (1149), .078 lbs.

Towing. Wood Side Wheels.

"WM. H. WEBB."—HARBOR AND COAST.—VERTICAL BEAM ENGINES (*Condensing*).—*Length upon deck*, 185.5 *feet; beam*, 30.25 *feet; hold*, 10.8 *feet.*

Immersed Section at load-line, 194 *sq. feet. Displacement* 498.25 *tons, at load-draught of* 7.25 *feet.*

Cylinders.—2, of 44 ins. in diam. by 10 feet stroke of piston; volume, 211 cube feet. *Condensers.—Jet*, 2, volume 105 cube feet. *Air-pumps.*—2, volume 45 cube feet.

Water-wheels.—Diam., 30 feet. *Blades* (divided), 21; breadth of do., 4.6 feet; depth of do., 2.33 feet. *Dip* at load-line, 3.75 feet.

Boilers.—2 (return flue). *Heating surface*, 3280 sq. feet. *Grates*, 147.5 sq. feet.

Smoke-pipe.—Area, 11.6 sq. feet, and 35 feet in height above the grate level.

Pressure of Steam.—35 lbs. per sq. inch, cut off at .5 stroke. *Revolutions*, 22 per minute. IHP, 1500.

Fuel.—Anthracite or Bituminous. *Consumption*, 1680 lbs. per hour.

Speed.—20 miles per hour.

Weights.—Engines, Wheels, Frame, and Boilers, 310 579 lbs.

Wood Side Wheels. Passenger.

"DANIEL DREW," NEW YORK TO ALBANY.—VERTICAL BEAM ENGINE (*Condensing*). —*Length upon deck*, 251.66 *feet*; *at load-line*, 244 *feet*; *beam*, 31 *feet*; *hold*, 9.25 *feet*.

Immersed section at load-line, 136 *sq. feet*. *Displacement* 380 *tons, at load-draught of* 4.83 *feet.*

Cylinder.—60 ins. in diam. by 10 feet stroke of piston; *volume*, 196 cube feet. *Condenser.*—*Jet*, volume 68 cube feet. *Air-pump*, volume 26 cube feet.

Water-wheels.—Diam. 29 feet. *Arms*, 24. *Blades*, 24; breadth of do., 9 feet; depth of do., 26 ins. *Dip* at load-line, 2.33 feet.

Boilers.—2 (return flue), 29 feet in length by 9 feet in width at furnace. Shell, diam. 8 feet. *Heating surface*, 3350 sq. feet. *Grates*, 105 sq. feet. *Cross area of lower flues*, 15.5 sq. feet; *of upper*, 13 sq. feet. *Weight*, 20650 lbs.

Smoke-pipes.—2, area 25.13 sq. feet, and 32 feet in height above the grate level.

Pressure of Steam.—35 lbs. per sq. inch, cut off at .5 stroke. *Revolutions* (*maximum*), 26 per minute. IHP, 1720.

Fuel.—Anthracite; *Blast*. *Consumption*, 3800 lbs. per hour.

Speed, 22.3 miles per hour. *Slip of Wheels from Centre of Pressure*, 12.5 per cent.

Frames.—Molded, 15.75 ins.; sided, 4 ins.; and 20 ins. apart at centres.

"MARY POWELL," HUDSON RIVER.—VERTICAL BEAM ENGINE (*Condensing*).—*Length on water-line*, 286 *feet*; *over all*, 294 *feet*; *beam*, 34 *feet* 3 *ins.*; *over all*, 64 *feet*; *hold*, 9 *feet*. *Deck to promenade deck*, 10 *feet*.

Immersed section at load-line of 6 *feet*, 200 *sq. feet*. *Displacement*, 800 *tons at mean load-draught of* 6 *feet*.

Area of transverse head surface of hull above water, 2000 *sq. feet*.

Cylinder.—72 ins. in diam. by 12 feet stroke of piston; volume, 338 cube feet. *Clearance* at each end, 12.5 cube feet.

Steam and Exhaust Valves, 14.75 ins. in diam. *Air-pump*, 40 ins. in diam. by 5 feet 2 ins. stroke of piston. *Condenser.*—*Jet*, 128 cube feet. *Crank-pin*, 8.75 ins. in diam. × 10.75 ins.

Beam, 22.5 feet in length; centre, 9.75 in diam.

Water-wheels.—Diam. 31 feet; blades (divided), 26; breadth of do., 10 feet 6 ins.; width, 1 foot 6 ins.; immersion, 3 feet 6 ins. *Shafts.*—Journal, 15.625 ins. by 17 ins.

Boilers.—2 (flue and return tubular), of steel, 11 feet front by 26 feet in length; shell, 10 feet in diam. and 16 feet 1 inch in length. *Furnaces*, 2 in each, of 4 feet 10 ins. by 8 feet in length. *Heating Surface*, 2660 sq. feet; and *Superheating*, 340 sq. feet in each. *Grates*, 152 sq. feet. *Flues*, 10 in each, transverse area, 11 feet 7 ins. *Tubes*, 80 in each, 4.5 ins. in diam., 6 feet 6 ins. in length, and 8 feet 7 ins. in transverse area.

Steam Chimneys, 8 feet in diam. × 12 feet in height. *Smoke-pipe*, 4 feet 6 ins. in diam. and 68 feet in height from grates.

Combustion, *Blast*. *Blowers*, 4 feet in diam. and 3 feet in width. *Revolutions*, 78 per minute. *Fuel* (anthracite), 6280 lbs. per hour, or 40 lbs. per sq. foot of grate per hour. Per sq. foot of heating surface, 2.25 lbs.

Speed, 23.65 miles per hour.

Pressure of Steam, 28 lbs. per sq. inch, cut off at .47 stroke; terminal pressure, 16.4 lbs.; throttle, .625 open. *Vacuum*, 25 ins. *Revolutions*, 22.75 per minute.

Temperatures.—*Reservoir*, 120°. *Feed water*, 120°. *Chimney*, 740°. IP.—Total, 1900. IHP, 1560. Net, 1450.

Evaporation.—*Water* per lb. of coal, from 120°, 7 lbs.; per lb. of combustible, from 120°, 8.2 lbs. *Steam* per total IP per hour, 21.1 lbs. *Coal* per do. do., 3.14 lbs.

Weights. Engine.—Frame, keelson, out-board wheel-frames donkey engine, and boiler, blower engines and blowers, all complete, 360 000 lbs. *Boilers.*—Iron return flue, 120 000 lbs. Steel return tubular, 116 000 lbs. Water, 128 000 lbs.

Capacity.—2000 passengers and their baggage.

Memoranda.—This vessel was originally but 266 feet in length, and when lengthened the cylinder of 62 ins. in diam. was removed and replaced with one of 72 ins. Engine designed throughout for original cylinder and a pressure of from 50 to 55 lbs., cutting off at .625 of stroke, with throttle wide open.

Engines and Boilers built by Fletcher, Harrison, & Co., New York, 1861 and 1875.

"SOLANO," FERRY BOAT.—VERTICAL BEAM ENGINES (*Condensing*).—*Length over all*, 424 *feet; on keel*, 406 *feet; beam (molded)*, 64 *feet; hold at* ⊠, 18.5 *feet; at ends*, 15 *feet* 10 *ins.; width over guards*, 116 *feet.*

Light draught, 5 *feet; loaded*, 6.5 *feet. Tons*, 3541.

Cylinders.—2 of 60 ins. diam. by 11 feet stroke of piston.

Wheels, 34 feet in diam. by 17 feet face. *Blades*, 24.

Boilers, 8.—Steel; 7 feet in diam. by 28 feet in length. *Heating surface*, 19 640 sq. feet. *Grates*, 288 sq. feet. IHP, 4000.

Passenger and Light Freight.

"SETH GROSVENOR."—STEEPLE ENGINE (*Condensing*).—*Length upon deck*, 95 *feet; beam*, 17.2 *feet; hold*, 5 *feet.*

Immersed section at load-line, 43 *sq. feet. Displacement* 73 *tons, at load-draught of* 3.25 *feet.*

Cylinder.—28 ins. in diam. by 3 feet stroke of piston; volume, 12.8 cube feet.

Water-wheels.—Diam. 13.5 feet. *Blades*, 14; breadth of do., 3 feet; depth of do., 1.25 feet.

Boiler (flue and return tubular).—*Heating surface*, 540 sq. feet. *Grates*, 22.5 sq. feet. *Area of tubes*, 367 sq. ins. IHP, 90.

Weights.—Engine, Wheels, Frame, and Boiler, 61 556 lbs. = 27.4 tons.

The operation of this vessel was in every way successful, being very fast, economical in fuel, etc., and she would have been improved if the hull had had 15 feet additional length, all other dimensions and capacities remaining the same.

Wood Stern Wheels.

Passenger and Deck Freight.

"MONTANA."—HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck (over all)*, 248 *feet; at water-line*, 245 *feet; beam*, 48 *feet* 8 *ins. (over all*, 50 *feet* 4 *ins.); hold*, 6 *feet; draught of water at load-line*, 5.5 *feet.*

Immersed section at load-line, 244 *sq. feet. Displacement at mean light draught of* 22 *ins.*, 594 *tons* (2000 *lbs.*)

Cylinders.—Two, 18 ins. in diam. by 7 feet stroke of piston.

Valves, 4.5 and 5 ins. in diam. *Piston rod*, 4 ins. *Steam pipe*, 4.5 ins. *Connecting-rod*, 30 feet in length.

Water-wheel, 19 feet in diam. by 35 feet face; blades, 3 feet in depth. *Shaft*, 10.25 ins. in diam.

Boilers.—Four (horizontal tubular), 42 ins. in diam. by 26 feet in length. Two flues in each, 15 ins. in diam. *Heating surface*, effective, 1023, total 1431 sq. feet. *Furnace*, 6.5 × 17 feet. *Grates*, 4.16 × 17 feet; surface, 70.8 sq. feet. *Smoke-pipes.*—Two, 3 feet in diam. by 55 feet 3 ins. in height. *Exhaust or Blower draught.*

Calorimeter.—Of Bridge, 15.27; of Flues, 9.82; and of Chimneys, 14.14 sq. feet. *Areas* of grate, compared to calorimeter of flues, 7.2; to ditto. of chimneys, 5; and of bridge, 4.6 sq. feet.

Steam-room, 562; and water space, 294 cube feet.

Hull.—*Frames*, 4 × 6 ins. and 15 ins. apart at centres. Intermediate do., 4 × 6 ins., and running for 7.5 feet each side of keelson. *Planking.*—Bottom, oak, 4 ins.; side do., 2.5 to 4 ins. *Deck beams*, pine, 3 × 6 ins. *Deck plank*, 2.5 ins. *Keelson*, oak; *side do.*, eight each side, one each 7, 8.75, and 9 ins., and five 6.75 ins. *Wales*, one each side, 9 and 7 ins. by 3, and one 10 × 2.5 ins. *Deck posts*, 3.5 × 3 ins. and 4 feet apart. *Deck beams*, 5.5 × 3 ins. *Knuckles*, oak, 6 × 12 ins. *Bulkheads*, one longitudinal and one athwartship at shear of stern. *Sheathing* of wrought iron, .0625 to .125 inch from just below light to load-line.

Hog Posts.—White pine, 8.5 and 11 ins. square. *Chains*, 1.5 ins. in diam.

Weights.—Boilers, 29 264; water, 18 351; and boilers, chimneys, grates, and water, 55 672 lbs. Hull, oak, 520 560; Pine, 91 437; Bolts, spikes, etc., 8000, and Deck and guards, 76 000 lbs.; Hull alone, 310 tons.

Weight of hull compared to one of iron as 8 to 5, effecting a difference of about 100 tons.

Passenger and Deck Freight.

"PITTSBURGH."—HORIZONTAL ENGINES (*Non condensing*).—*Length on deck*, 252 *feet; beam*, 39 *feet; hold*, 6 *feet; draught of water at load line*, 2 *feet*.

Immersed section at load-line, 75 *sq. feet*. *Displacement at load draught of* 2 *feet*, 380 *tons* (2000 lbs.).

Cylinders.—Two, 21 ins. in diam. by 7 feet stroke of piston.

Water-wheel.—21 feet in diam. by 28 feet face.

Boilers.—2 (horizontal tubular), 47 ins. in diam. by 28 feet in length. Two fires in each.

Iron Stern Wheels.

HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck*, 110 *feet; beam*, 14 *feet (deck projecting over*, 4 *feet); hold*, 3.5 *feet*.

Immersed section at load-line, 10.25 *sq. feet*. *Displacement at load draught of* 1.1 *feet*, 33 *tons*.

Cylinders.—Two, of 10 ins. in diam. by 3 feet stroke of piston; volume of piston space, 1.6 cube feet.

Wheel.—Diam. 13 feet. *Blades*, 13; breadth of do., 8.5 feet; depth of do., 8 ins.

Revolutions, 33 per minute. *Boiler.*—One (horizontal tubular). *Tubes*, 100 of 2 ins. in diam.

Fuel.—Bituminous coal. *Consumption*, 4480 lbs. in 24 hours.

Hull.—*Plates*, keel, No. 3; bilges, No. 4; bottom, No. 5; sides, Nos 6 and 7. *Frames*, 2.5 × .5 ins., and 20 ins. apart from centres.

Steel.

"CHATTAHOOCHEE."—INCLINED ENGINES (*Non condensing*).—*Length on deck*, 157 *feet; beam*, 31.5 *feet; hold*, 5 *feet*.

Immersed section at load-line, 153 *sq. feet*. *Freight capacity*, 400 *tons* (2000 *lbs.*).

Cylinders.—Two, 15 ins. in diam. by 5 feet stroke; volume of piston space, 12.26 cube feet.

Wheel.—One, 18 feet in diam.; blades, 2 feet in depth.

Boilers.—Three (cylindrical flued). Diam. 42 ins.; length, 22 feet; 2 flues of 10 ins. in each. *Heating surface*, 690 sq. feet. *Grates*, 48 sq. feet.

Pressure of Steam, 160 lbs. per sq. inch, cut off at .375. *Revolutions*, 22 per min.

Consumption of Fuel, 12 tons (2000 lbs.) in 24 hours. *Plating of Hull*, .1875 to .25 inch. *Light draught*, 21 ins.

Iron Propellers.

VERTICAL DIRECT ENGINES (*Non condensing*).—*Length on deck*, 70 *feet; beam*, 10.5 *feet; draught*, 12 *ins*.

Propellers, 2.—2 blades, 16. ins. in diam., set 11 ins. below water-line.

Boiler (tubular coil). *Revolutions*, 480 per minute.

Speed, 10.49 miles per hour.

Water led to propellers through tunnels in bottom at sides.

"LOUISE."—VERTICAL TANDEM ENGINES (*Compound*).—*Length*, 60 *feet; beam*, 12 *feet; hold*, 4.25 *feet*.

Displacement at load-draught of 2.5 *feet*, 8 *tons*.

Cylinders, 5 and 10 ins. in diam. by 8 ins. stroke of piston.

Surface Condenser.—*Boiler* (vertical tubular), 4 feet in diam. by 8.5 in length.

Iron Sailing Vessels.

Passenger and Freight.

ENGLISH.—SHIP.—*Length upon deck*, 178 *feet; do. at mean load-line of* 19.16 *feet*, 177 *feet; keel*, 171 *feet; beam*, 32.88 *feet; depth of hold*, 21.75 *feet; keel* (mean), 2.75 *feet*.

Immersed section at load-line, 387 *sq. feet*. *Displacement at load-draught of* 19.16 *feet*, 1385 *tons; at deep load-draught of* 20 *feet*, 1495 *tons; and, in proportion to its circumscribing parallelopipedon*, .524.

Load-line.—*Area* at load-draught, 4557 sq. feet. Angle of entrance, 57°; of clearance, 64°. *Area* in proportion to its circumscribing parallelogram, .784.

Centre of Gravity, 6.416 feet below mean load-line. *Centre of Displacement* (gravity of), 6.25 feet below load-line; and 4.33 feet before middle of length of load-line.

Immersed Surface.—Bottom, 7370 sq. feet. *Keel*, 1130 sq. feet. *Sails*, 13 282 sq. feet.

Meta-centre, 6.66 feet above centre of gravity of displacement. *Centre of Effort* before centre of displacement, 3.5 feet; height of do. above mean load-line, 55.5 feet.

Launch. Wood.

STEAM LAUNCH "HERRESHOFF."—VERTICAL ENGINE (*Compound*).—*Length*, 33 *feet* 1 *inch*; *beam*, 8.75 *feet*.

Displacement at mean load-draught of (to rabbet of keel) 19 *ins.*, 8929 *lbs.*

Weights.—Hull and Machinery, 6555 lbs. Coal, 1120 lbs.

Yachts. Wood.

"AMERICA," SCHOONER.—*Length over all*, 98 *feet*; *upon deck*, 94 *feet*; *at load-line*, 90.5 *feet*; *beam*, 22.5 *feet*; *at load-line*, 22 *feet*; *depth of hold*, 9.25 *feet*. *Height at side from under side of garboard strake*, 11 *feet*. *Sheer, forward*, 3 *feet*; *aft*, 1.5 *feet*,

Immersed section at load-line, 121.8 *sq. feet*. *Displacement at load-draught of* 8.5 *feet, from under side of garboard strake and of* 11 *feet aft*, 191 *tons*; *and, in proportion to Volume of circumscribing parallelopipedon*, .375.

Displacement at 4 *feet* (*from garboard strake*), 43 *tons*; *at* 5 *feet*, 66 *tons*; *at* 6 *feet*, 93 *tons*; *at* 7 *feet*, 127 *tons*; *and at* 8 *feet*, 167 *tons*.

Centre of Gravity.—Longitudinally, 1.75 feet aft of centre of length upon load-line. Sectional, 2.58 feet below load line. Of Fore body, 14.25 feet forward; and of After body, 19 feet aft. *Meta-centre*, 6 72 feet above centre of gravity.

Centre of Effort, 31 17 feet from load line. *Centre of Lateral Resistance*, 6.33 feet abaft of centre of gravity. *Area of Load-line*, 1280 sq. feet. *Mean girths of immersed section to load-line*, 25 feet.

Load-draught.—Forward, 4.91 feet; aft, 11.5 feet. *Rake of Stem*, 17 feet.

Spars.—*Mainmast*, 81 feet in length by 22 ins. in diam. *Foremast*, 79.5 feet in length by 24 ins. in diam. *Main boom*, 58 feet in length. *Main gaff*, 28 feet. *Fore gaff*, 24 feet. *Rake*, 2.7 ins. per foot. *Drag of Keel*, 3 feet. *Tons*, 170.56.

"JULIA," SLOOP.—*Length for tonnage*, 72.25 *feet*; *on water-line*, 70 *feet* 7 *ins.*; *beam*, 19 *feet* 8 *ins.*; *hold*, 6 *feet* 8 *ins.* *Tons*, O. M. 83.4; N. M. 43.98.

Load-draught, 6.25 *feet*.

Sails.—*Mainsail*, hoist, 49.75 feet, foot 54.25, and gaff 27.66; *Jib*, hoist, 49.75 feet, foot 39.5, and stay 63.5. *Gaff topsail*, hoist, 24.5 feet.

Areas.—Mainsail, 2322 sq. feet. Jib, 986, and Topsail, 454.

Cutters.

"TARA" (*English*) SLOOP.—*Length on load-line*, 66 *feet*; *beam*, 11.5 *feet*.

Immersed section at load-line, 11.5 *sq. feet*. *Displacement*, 75 *tons*.

Spars.—*Mast*, deck to hounds, 42 feet. *Boom*, 58 feet. *Gaff*, 39 feet. *Bowsprit* outside of stem, 30 feet. *Mast to stem*, 26 feet. *Topmast*, foot to hounds, 25 feet. *Balloon topsail yard*, 46 feet. *Canvas*, area, 3450 sq. feet. *Tons*, C. H., 90.

Ballast.—At *Keel*, 38.5 tons. *Hull*, 1.5 tons.

"MISCHIEF" (*English*), SLOOP.—*Length on load-line*, 61 *feet*; *beam*. 19.9 *feet*.

Immersed section at load-line, 60 *sq. feet*. *Displacement*, 55 *tons*.

Pilot Boat.

"WM. H. ASPINWALL," SCHOONER.—*Length of keel*, 74 *feet*; *upon deck*, 80 *feet*; *beam*, 19 *feet*; *hold*, 7.6 *feet*. *Draught of water*, 6 *feet forward*; *aft*, 9.5 *feet*.

Keel, 22 ins. in depth. *False keel*, 12 ins. in depth at centre.

Spars.—*Mainmast*, 77 feet in length. *Foremast*, 76 feet. *Main boom*, 46 feet. *Main gaff*, 21 feet. *Fore gaff*, 20 feet.

Tons.—N. M., 46.32.

PASSAGES OF STEAMBOATS.

Distances in Statute Miles.

1807, *Clermont*, of N. Y., New York to Albany, 145 miles, in 32 *hours* = 4.53 miles per hour, neglecting effect of the tide.

1811, *New Orleans*, of Pittsburgh, Penn. (non-condensing and stern-wheel), Pittsburgh to Louisville, Ky., 650 miles, in 2 *days* 22 *hours*.

1849, *Alida*, of N. Y., Caldwell's, N. Y., to Pier 1, North River, 43.25 miles, in 1 *hour* 42 *min.*, ebb tide = 2.75 miles per hour. Speed = 22.19 *miles* per hour. 1860, 30th Street, N. Y., to Cozzens's Pier, West Point, 50.5 miles, in 2 *hours* 4 *min.*, and to Poughkeepsie, 74.25 miles, in 3 *hours* 27 *min.*, 5 landings, flood tide. And 1853, Robinson Street to Kingston Light, 90.375 miles, in 4 *hours*, making 6 landings, flood tide.

1850, *Buckeye State*, of Pittsburgh, Penn. (non-condensing), Cincinnati to Pittsburgh, 500 miles (200 passengers), 53 landings, in 1 *day* 19 *hours*; 4 miles per hour adverse current. Speed = 15.63 *miles* and 1.23 landings per hour. Average depth of water in channel 7 feet.

1852, *Reindeer*, of N. Y., New York to Hudson, 116.5 miles, in 4 *hours* 57 *min.*, making 5 landings. Flood tide.

1853, *Shotwell*, of Louisville, Ky. (non-condensing), New Orleans to Louisville, 1450 miles, 8 landings, in 4 *days* 9 *hours*; 4.5 to 5.5 miles per hour adverse current. Speed = 18.81 *miles* per hour.

NOTE.—In 1817–18 the average duration of a passage from New Orleans to Louisville was 27 *days*, 12 *hours*; the shortest, 25 *days*.

1855, *New Princess*, of New Orleans (non-condensing), New Orleans, La., to Natchez, Miss., 310 miles, in 17 *hours* 30 *min.*; 3.5 to 4 miles per hour adverse current. Speed = 20.98 *miles* per hour.

1864, *Daniel Drew*, of N. Y., Jay Street, N. Y., to Albany, 148 miles, in 6 *hours* 51 *min.*, 9 landings. Flood tide. Speed of boat = 22.6 *miles* per hour.

1867, *Mary Powell*, of N. Y., Desbrosses Street, N. Y., to Newburgh, 60.5 miles, in 2 *hours* 50 *min.*, 3 landings; from Poughkeepsie to Rondout Light, 15.375 miles, in 39 *min.*, flood tide. 1873, Milton to Poughkeepsie, light draught and flood tide, 4 miles, in 9 *min.*; and 1874, Desbrosses Street to Piermont, 24 miles, in 1 *hour*; to Caldwell's, 43.25 miles, in 1 *hour* 50 *min.* Speed = 22.77 to 23 *miles* per hour.

Runs from New York to Albany, 146 miles, by different Boats.

1826, *Sun*	12 *hours* 16 *min.*	1852, *Fr. Skiddy* §	6 *hours* 24 *min.*
1826, *North America* *	10 " 20 "	1860, *Armenia* ‖	7 " 22 "
1841, *Troy* †	8 " 10 "	1864, *Daniel Drew* ‡	6 " 51 "
1841, *South America* ‡	7 " 28 "	1864, *Chauncey Vibbard* ‡	6 " 42 "

* 7 landings. † 4 landings. ‡ 9 landings. § 6 landings. ‖ 11 landings.

Timing Distance.—From 14th St., Hudson River, N. Y., to College at Mount St. Vincent, 13 miles.

NOTE.—Where landings have been made, and the river crossed, the distance between the points given is correspondingly increased.

1870, *R. E. Lee*, of St. Louis (non-condensing), New Orleans to St. Louis, Mo., 1180 miles (without passengers or freight), 4 to 5 miles per hour adverse current; Vicksburg, 1 *day* 38 *min.*; Memphis, 2 *days* 6 *hours* 9 *min.*; Cairo, 3 *days* 1 *hour.*; and to St. Louis, 3 *days* 18 *hours* 14 *min.*, inclusive of all stoppages.

1870, *Natchez*, of Cincinnati, Ohio, from New Orleans to Baton Rouge, 120 miles, in 7 *hours* 40 *min.* 42 *sec.*

Runs from New Orleans to Natchez, 295 miles, by different Boats.

1814, *Orleans*, 6 *days* 6 *hours* 40 *min.*	1856, *New Princess*, 17 *hours* 30 *min.*
1840, *Edward Shippen*, 1 *day* 8 *hours.*	1870, *R. E. Lee*, 16 *hours* 36 *min.* 47 *sec.*

Ice-boats.

Distances in Statute Miles.

1872, *Haze*, of Poughkeepsie, N. Y., to buoy off Milton, 4 miles, in 4 *min.*

1872, *Whiz*, of Poughkeepsie, N. Y., to New Hamburg, 8.375 miles, in 8 *min.*

PASSAGES OF STEAMERS AND SAILING VESSELS.

Distances in Geographical Miles or Knots.

Steamers. Side-wheels.

1807, *Phœnix*, of Hoboken, N. J. (John Stevens), New York, N.Y., to Philadelphia Penn. First passage of a steam vessel at sea.

1814, *Morning Star*, of Eng., River Clyde to London, Eng. First passage of an English steamer at sea.

1817, *Caledonia*, of Eng., Margate, Eng., to Cassel, Germ., 180 miles, in 24 *hours*.

1819, *Savannah*, of N.Y., about 340 tons O. M., Tybee Light, Savannah River, Ga., to Rock Light, Liverpool, Eng., 3640 miles, in 25 *days* 14 *hours;* 6 *days* 21 *hours* of which were under steam.

1825, *Enterprise*, of Eng., 500 tons, Falmouth, Eng., to Table Bay, Africa, in 57 *days;* and to Calcutta, India, in 113 days. First passage of a steamer to India.

1830, *Hugh Lindsay*, 411 tons, 80 HP, Bombay, India, to Suez, Egypt, 3103 miles, in 31 *days* running time.

1837, *Atlanta*, of Eng., 650 tons, Falmouth, Eng., to Calcutta, in 91 days.

1839, *Great Western*, of Eng., Liverpool to New York, N. Y., 3017 miles, in 12 *days* 18 *hours*.

1870, *Scotia*, of Eng., Queenstown, Ireland, to Sandy Hook, N. J., 2780 miles, in 8 *days* 7 *hours* 31 *min*. 1866, New York to Queenstown, 2798 miles, in 8 *days* 2 *hours* 48 *min.;* thence to Liverpool, Eng., 270 miles, in 14 *hours* 59 *min.;* total, 8 *days* 17 *hours* 47 *min.*

Screw.

1874, *India Government Boat*, Steel, length 87 feet, beam 12 feet, draught of water 3.75 feet, mean speed for one mile 20.77 miles per hour, and maintained a speed of 18.92 miles in 1 *hour*.

1877, *Lusitania*, of Eng., London to Melbourne, Australia, via Cape, 11 445 miles, in 38 *days* 23 *hours* 40 *min.*

Sailing Vessels.

1851, *Chrysolite* (clipper ship), of Eng., Liverpool, Eng., to Anjer, Java, 13 000 miles, in 88 *days*. The *Oriental*, of N. Y., ran the same course in 89 *days*.

1853, *Trade Wind* (clipper ship), of N. Y., San Francisco, Cal., to New York, N. Y., 13 610 miles, in 75 *days*.

1854, *Lightning* (clipper ship), of Boston, Mass., Melbourne, Australia, to Liverpool, Eng., 12 190 miles, in 64 *days*.

1854, *Comet* (clipper ship), of N. Y., Liverpool, Eng., to Hong Kong, China, 13 040 miles, in 84 *days*.

1854, *Sierra Nevada* (schooner), of N. H., Hong Kong, China, to San Francisco, Cal., 6000 miles, in 34 *days*.

1854, *Red Jacket* (clipper ship), of N. Y., Sandy Hook, N. J., to Melbourne, Australia, 12 720 miles, in 69 *days* 11 *hours* 1 *min.*

1855, *Euterpe* (half-clipper ship) of Rockland, Me., New York to Calcutta, India, 12 500 miles, in 78 *days*.

1860, *Andrew Jackson* (clipper ship), of Boston, New York, N. Y., to San Francisco, Cal., 13 610 miles, in 80 *days* 4 *hours*.

1865, *Dreadnought* (clipper ship), of Boston, Honolulu, Sandwich Islands, to New Bedford, Mass., 13 470 miles, in 82 *days;* and 1859, Sandy Hook, N. J., to Rock Light, Liverpool, Eng., 3000 miles, in 13 *days* 8 *hours*.

1865, *Sovereign of the Seas* (medium ship), of Boston, Mass., in 22 days sailed 5391 miles = 245 miles per day. For 4 *days* sailed 341.78 miles per day, and for 1 *day* 375 miles.

1866, *Henrietta* (schooner yacht), of N. Y., Sandy Hook, N. J., to the Needles, Eng., 3053 miles, in 13 *days* 21 *hours* 55 *min.* 16 *sec.*

1866, *Ariel* and *Serica* (clipper ships), of England, Foo-chou foo Bar, China, to the Downs, Eng., 13 500 miles, in 98 *days*.

1869, *Sappho* (schooner yacht), of N. Y., Light-ship off Sandy Hook, N. J., to Queenstown, Ireland, 2857 miles, in 12 *days* 9 *hours* 34 *min.*

ELEMENTS OF MACHINES AND ENGINES.

BLOWING ENGINES.

Furnaces.—Two. Fineries.—Two. (England.)

240 Tons Forge Pig Iron per Week.

Engine (non-condensing).—*Cylinder*, 20 ins. in diam. by 8 feet stroke of piston.

Boilers.—Six (plain cylindrical), 36 ins. in diam and 28 feet* in length. *Grates*, 100 sq. feet.

Blowing Cylinders.—Two, 62 ins. in diam. by 8 feet stroke of piston. *Pressure*, 2.17 lbs. per sq. inch. *Revolutions*, 22 per minute.

Pipes, 3 feet in diam.=.168 area of cylinder.

Tuyeres.—Each Furnace, 2 of 3 ins. in diam.; 1 of 3.25 ins.; and 1, 3 of 3 ins. Each Finery, 6 of 1.33 ins.; and 1, 4 of 1.125 ins.

Temperature of Blast, 600°. *Ore*, 40 to 45 per cent. of iron.

Furnaces.—Eight, diam. 16 to 18 feet. Dowlais Iron Works (England).

1300 *Tons Forge Iron per Week; discharging* 44 000 *Cube Feet of Air per Minute.*

Engine (non-condensing).—*Cylinder*, 55 ins. in diam. by 13 feet stroke of piston.

Pressure of Steam.—60 lbs. per sq. inch, cut off at .33 the stroke of piston. *Valves*, 120 ins. in area.

Boilers.—Eight (cylindrical flued, internal furnace), 7 feet in diam. and 42 feet in length; one flue 4 feet in diam. *Grates*, 288 sq. feet.

Fly Wheel.—Diam., 22 feet; weight, 25 tons.

Blowing Cylinder, 144 ins. in diam. by 12 feet stroke of piston.

Revolutions, 20 per minute. *Blast*, 3.25 lbs. per sq. inch. *Discharge pipe*, diam. 5 feet, and 420 feet in length. *Valves.*—Exhaust, 55 sq. feet; Delivery, 16 sq. feet.

Furnaces.— . Lackenby (England).

800 Tons Iron per Week.

Engine (horizontal, compound condensing).—32 and 60 ins. in diam. by 4.5 feet stroke of piston.

Blowing Cylinders.—Two, 80 ins. in diam. by 4.5 feet stroke of piston. *Pressure*, 4.5 lbs. per sq. inch. *Revolutions*, 24 per minute.

Pipe, 30 ins. in diam.; *volume*, 12.25 times that of blowing cylinders.

IP.—Engine, 290 lbs.; Blowing cylinders, 258; efficiency, 89 per cent.

Valves.—Area of admission, .16 of area of piston; of exit, .125.

Volume.—190 000 cube feet of air are supplied per ton of air.

Blower and Exhausting Fan. (Sturtevant's.)

Blower.	Grate Surface.	Inlet.	Outlet.	Diam. of Pulley.	Face of Pulley.	Revolutions.	Air per Minute.	IP.
No.	Sq. Feet.	Diam. Ins.	Diam. Ins.	Ins.	Ins.	Per Min.	Cube Feet.	
00	5	5	4	2.75	2	3000	500	—
0	6	5.75	4.75	3	2.25	2600	600	—
1	8	6.5	5.75	3.5	3	2200	764	.6
2	10	7.5	7.5	3.75	3.5	1928	1 019	.79
3	14	9	9	4.25	4	1638	1 427	1.11
4	20	10.5	10.5	5	5	1410	1 936	1.51
5	27	12	12	6	5.25	1194	2 701	2.1
6	36	14	14	7	6.5	1018	3 669	2.86
7	48	16	16	9	7.5	878	4 847	3.77
8	62	18	18	10	8.5	766	6 115	4.76
9	80	21	21	12	10.5	671	8 154	6.35
10	100	24	24	14	12	598	10 702	8.34

* 40 feet would have afforded economy in fuel.

COTTON FACTORIES. (*English.*)

For driving 22 060 Hand-mule Spindles, with Preparation, and 260 Looms, with common Sizing.

Engine (condensing).—*Cylinder*, 37 ins. in diam. by 7 feet stroke of piston; volume of piston space, 53.6 cube feet.

Pressure of Steam.—(Indicated average) 16.73 lbs. per sq. inch. *Revolutions*, 17 per minute.

Friction of Engine and Shafting.—(Indicated) 4.75 lbs. per sq. inch of piston.

IHP, 125. Total power = 1. Available, deducting friction = .717.

NOTES.—Each IHP will drive { 305 hand-mule spindles, *with preparation;*
or 230 self-acting " "
or 104 throstle " "
or 10.5 looms, with common sizing.

Including preparation:

1 throstle spindle = 3 hand-mule, or 2.25 self-acting spindles.
1 self-acting spindle = 1.2 hand-mule spindles.

DREDGING MACHINES.

Dredging 20 Feet from Water-line, or 180 Tons of Mud or Silt per Hour 11 Feet from Water-line.

Length upon deck, 123 *feet; beam*, 26 *feet. Breadth over all*, 41 *feet.*

Immersed section at load-line, 60 *sq. feet. Displacement*, 141 *tons, at load-draught of* 2.83 *feet.*

Engine (non-condensing).—*Cylinders*, two, 12.125 ins. in diam. by 4 feet stroke of piston.

Boilers.—Two (cylindrical flue), diam. 40.5 ins., and length, 20 feet 3 ins.; two flues, 14.625 ins. in diam. *Heating surface*, 617 sq. feet. *Grates*, 37 sq. feet.

Pressure of Steam, 25 lbs. per sq. inch; throttle .25 open, cut off at .5 the stroke of piston. *Revolutions*, 42 per minute.

Buckets.—Two sets of 12, 2.5 feet in length by 15 ins. at top and 2 feet deep; volume, 6.25 cube feet. *Chain Links*, 8 ins. in length by .5 inch diam.

Scows or Camels.—Four, of 40 tons capacity each.

STEAM HOPPER DREDGER. (*Wm. Simons & Co.*)

Iron.

"NEPTUNE" (*English*).—*Length*, 150 *feet; breadth*, 32 *feet.*

Dredge from 6 *Ins. to* 25 *Feet. Capacity of Hopper*, 500 *to* 600 *Tons.*

Engines.—Two (compound), 375 IP, for dredging and propulsion, and one for raising bucket-frame and anchor-posts.

A like designed dredger of 1000 tons capacity has dredged 10 000 tons silt per week and transported it 7.5 miles.

Dredging 400 *Tons of Mud or Silt per Hour*, 5 *to* 35 *Feet in Depth.*

Capacity of Hopper, 1300 *Tons.*

Engines.—Two (compound), IP 700. *Speed.*—7.5 knots per hour.

Steam Dredging Crane. (*English.*)

Lift, 30 *Feet per Hour.*

Weight of Crane.	Lifting Power.	Volume of Bucket.	Mud or Silt.	Coal and Sand.	Excavation Ground.	Weight of Crane.	Lifting Power.	Volume of Bucket.	Mud or Silt.	Coal and Sand.	Excavation Ground.
Lbs.	Tons.	Lbs.	Tons.	Tons.	C. Yds.	Lbs.	Tons.	Lbs.	Tons.	Tons.	C. Yds.
21 280	2.5	1120	25	20	20	18 000	5	2240	50	40	30
24 640	3	1680	37.5	32	25	33 480	7	3360	60	54	40

Iron.

DREDGER AND HOPPER BARGE (*Compound; English*).—*Length, extreme*, 120 *feet*; *beam*, 32 *feet*; *hold*, 10.5 *feet*. *Breadth of bucket well*, 6.75 *feet*. *Load-draught*, 6 *feet*.

Cylinders.—21.5 and 40 ins. in diam. by 2.5 feet stroke of piston. *Condenser*.—*Surface*, 600 sq. feet. *Circulating Pump*, single acting.—15 ins. in diam. by 15 ins. stroke of piston.

Boiler.—*Heating surface*, 1150 sq. feet. *Grates*, 40 sq. feet. *Steam-room*, 300 cube feet.

Shaft.—7.75 ins. in diam.

Bucket ladder.—Of wrought iron, 71 feet in length, 5 feet in depth at centre, and 2 feet 2 ins. at ends. *Buckets*.—34; volume, 15 cube feet each.

Excavation and Delivery.—For a transit of 7.5 miles, 3000 tons per day.

Hopper Barge.—*Length between perpendiculars*, 115 *feet*; *beam*, 32 *feet*; *hold*, 9 *feet* 11 *ins*. *Load-line with* 400 *Tons dredge*, 8 *feet*.

Hoppers.—*Length*, 50 *feet*; *breadth at top*, 22 *feet*; *at bottom*, 9 *feet*.

Cost.—*Dredge*, $ 90 000; *Hoppers*, $ 18 000 each.

Maintenance.—*Dredger*, 1.75 cents; *Hopper*, 1.7 cents; *Towing*, 1.2 cents, per ton of dredge excavated and delivered.

"HERCULES," *Panama Canal*.—*Length on deck*, 100 *feet*; *beams*, 40, 60, and 45 *feet*; *depth of hold*, 12 *feet*. *Slot*, 36 *feet in length by* 6 *feet* 7 *ins. in width*.

Ways.—Two, one 40 feet and one 60 feet, by 5 feet in width.

Buckets.—38; volume, 1.33 cube yards. *Spuds*, 2 feet in diam. and 60 in length.

Engines.—Two of 100 IP each, and two of 40 IP each.

Boilers.—Three (horizontal tubular), 16 feet in length.

Elevator and Discharge.—Maximum, 24 cube yards per minute.

Crane. (Wood.)

Hull.—*Length on deck*, 100 *feet*; *beam*, 44 *feet*; *load draught*, 4.5 *feet*.

Radius of crane, 46 feet; *height*, 70 feet; *counter-balance*, 70 tons.

Boiler.—*Heating surface*, 500 sq. feet. *Pressure of Steam*, 80 lbs. per sq. inch. IHP, 150.

Propellers.—Two, 4.25 feet in diam. *Speed*, 5 miles per hour.

Engine to operate crane. *Cylinder*.—10 ins. in diam. by 12 ins. stroke of piston.

FLOUR MILLS.

30 Barrels of Flour per Hour.

Water-wheels, *Overshot*.—5, diam. 18 feet by 14.5 feet face. *Buckets*, 15 ins. in depth. *Water*.—*Head*, 2.3 feet. *Opening*, 2.5 ins. by 14 feet in length over each wheel.

5 Barrels of Flour per Hour, and Elevating 400 Bushels of Grain 36 Feet.

Water-wheel, *Overshot*.—Diam. 22 feet by 8 feet face. *Buckets*, 52 of 1 foot in depth. *Water*.—*Head*, from centre of opening, 25 ins. *Opening*, 1.75 ins. by 80 ins. in length.

Revolutions, 3.5 per minute. *Stones*, three of 4.5 feet; revolutions, 130.

Three Run of Stones, Diameter 4 Feet.

Water-wheel, *Overshot*.—Diam. 19 feet by 8 feet face. *Buckets*, 14 ins. in depth.

Or,

Steam-engine (non condensing).—*Cylinder*, 13 ins. in diam. by 4 feet stroke. *Boiler* (cylindrical flued).—Diam. 5 feet by 30 in length; two flues 20 ins. in diam.

HOISTING ENGINES.

For Pile Driving, Hoisting, Mining, etc.

Lidgerwood Manuf'g Co., New York.

Single Cylinders. HP	Cylinder.	Capacity.	Cost, with Boiler.*	Double Cylinders. HP	Cylinder.	Capacity.	Cost, with Boiler.*
No.	Ins.	Lbs.	$	No.	Ins.	Lbs.	$
4	5 × 5	1000	600	8	5 × 8	2000	950
6	6 × 8	1250	675	12	6 × 8	2500	1050
10	7 × 10	1800	825	20	7 × 10	3500	1350
15	8 × 10	2800	1050	30	8 × 10	6000	1550
20	9 × 12	4000	1275	40	9 × 12	8000	2000
25	10 × 12	5000	1375	50	10 × 12	9000	2350

* Complete.

Details and Operation.

Engine.	Drum.	Boiler. Dimensions.	Boiler. Tubes.	Ram.	Leaders. Hoist.	Lift. Ram.	Blows per Minute.	Piles per 10 Hours.	Fuel per Hour.
HP	Ins.	Ins.	No.	Lbs.	Feet.	Feet.	No.	No.	Lbs.
10*	12 × 24	32 × 75	48 of 2 in.	1953	40	8 to 12	25	50	70
20	14 × 26	40 × 84	80 of 2 in.	2700	75	8 to 12	29	100	80

* Weight complete, 8500 lbs.

Mining Engines and Boilers. (*Various Capacities.*)

Engine, *Boiler*, etc., as given for Pile Driving, page 902.

Operation. — 250 to 300 tons of coal in 10 hours. *Fuel*, 40 lbs. coal per hour. *Water*, 20 gallons per hour.

Weight of Engine and Boiler, 4500 lbs.

Hancock Inspirator. *For a Lift of Water of 25 Feet.*

No.	Diameter. Steam-pipe.	Diameter. Suction.	Discharge at Pressure of 60 Lbs.	No.	Diameter. Steam-pipe.	Diameter. Suction.	Discharge at Pressure of 60 Lbs.
	Ins.	Ins.	G'lls. per h'r.		Ins.	Ins.	G'lls. per h'r.
10	.375	.5	120	30	1.25	1.5	1260
12.5	.5	.75	220	35	1.25	1.5	1740
15	.5	.75	300	40	1.5	2	2230
20	.75	1	540	45	1.5	2	2820
25	1	1.25	900	50	2	2.5	3480

Temperature of water not over 145° for a low lift, and 100° for a high lift.

HYDROSTATIC PRESS. (*Cotton.*)

30 *Bales of Cotton per Hour.*

Engine (non-condensing).—*Cylinder*, 10 ins. in diam. by 3 feet stroke of piston.

Pressure of Steam, 50 lbs. per sq. inch, full stroke. *Revolutions*, 45 to 60 per minute.

Presses.—Two, with 12-inch rams; stroke, 4.5 feet.

Pumps.—Two, diam. 2 ins.; stroke, 6 ins.

For 83 *Bales per Hour.*

Engine (non-condensing).—*Cylinder*, 14 ins. in diam. by 4 feet stroke of piston.

Boilers.—Three (plain cylindrical), 30 ins. in diam. and 26 feet in length. *Grates*, 32 sq. feet. *Pressure of Steam*, 40 lbs. per sq. inch. *Revolutions*, 60 per minute.

Presses.—Four, geared 6 to 1, with two screws, each of 7.5 ins. in diam. by 1.625 in pitch.

Shaft (wrought iron).—Journal, 8.5 ins. *Fly Wheel*, 16 feet in diam.; weight, 8960 lbs.

LOCOMOTIVE.

"EXPERIMENT" (*Compound*).—*Cylinders*, one each, 12 and 26 ins. in diam., and one 26 ins. by 2 feet stroke of piston.

Boiler.—Heating surface, 1083.5 sq. feet. *Grate*, 17.1 sq. feet. *Pressure of Steam*, 150 lbs. per sq. inch, cut off at .35. *Speed*, 50 miles per hour. *Weight.*—Empty, 34.75 tons.

Street Railroad or Tramway Engine.

Cylinder, 7 ins. in diam. by 11 ins. stroke of piston.

Boiler, 78 tubes 1.75 ins. in diam. by 4 feet in length. *Heating surface*, 160 sq. feet. *Grate*, 4.25 sq. feet. *Wheels*, 2.33 feet in diam. *Base*, 4.5 feet. *Gauge*, 4 feet 8.5 ins.

Cost.—Average per mile in England, 2.52 pence sterling = 4.48 cents.

PILE-DRIVING.

Driving One Pile.

Engine (non-condensing).—*Cylinder*, 6 ins. in diam. by 1 foot stroke of piston.

Boiler (vertical tubular).—32 ins. in diam., and 6 to 6 feet in height. *Grates*, 3.7 sq. feet. *Furnace*, 20 ins. in height. *Tubes*, 35, 2 ins. in diam., 4.5 feet in length.

Revolutions, 150 per minute. *Drum*, 12 ins. in diam., geared 4 to 1. *Leader*, 40 feet in height. *Ram.*—2000 lbs., 2 blows per minute. *Fuel*, 30 lbs. coal per hour.

Driving Two Piles.

Engine (non-condensing).—*Cylinders*, two, 9 ins. in diam. by 18 ins. stroke of piston.

Boiler (horizontal tubular).—*Shell*, diam. 3 feet, and 6 feet in length. Furnace end 3.75 feet in width, 3.5 feet in length, and 6 feet in height.

Pressure of Steam, 60 lbs. per sq. inch. *Revolutions*, 60 to 80 per minute.

Frame, 8.5 feet in width by 26 feet in length. *Leaders*, 3 feet in width by 24 feet in height. *Rams.*—Two, 1000 lbs. each, 5 blows per minute.

PUMPING ENGINES.

CORLISS STEAM-ENGINE CO., *Providence, R. I.*—VERTICAL-BEAM ENGINE (*Compound*).—*Cylinders.*—18 and 36 ins. in diam. by 6 feet stroke of piston.

Pumps.—Four plunger, 19 ins. in diam. by 3 feet stroke of piston. *Displacement* per revolution of engine, 84.96 cube feet.

Boilers.—Three, vertical fire tubular. *Grate.*—93 sq. feet. *Heating surface*, 1680 sq. feet. *Pressure of Steam*, 125 lbs. per sq. inch, cut off at .22 feet. *Revolutions*, 36 per minute. IIP 313. *Fly-wheel.*—25 feet in diam., weight 62 000 *lbs.*

Fuel.—Cumberland coal, 480 lbs. per hour, inclusive of kindling and raising steam. *Ash and Clinkers*, 9.4 per cent. *Duty* for one week, 113 271 000 *foot-lbs.*

Water delivered, 17 621 gallons per minute, against head of 180 feet.

Duty, average for 1883, per 100 lbs. anthracite coal, 106 048 000 *foot-lbs.*

For Elevating 200 000 Gallons of Water per Hour.

LYNN, *Mass.*—ENGINE (*Compound*).—*Cylinders*, 17.5 and 36 ins. in diam. by 7 feet stroke of piston; volume of piston space, 61.2 cube feet. *Air Pump* (double acting), 11.25 ins. in diam. by 49.5 ins. stroke of piston.

Pump Plunger, 18.5 ins. in diam. by 7 feet stroke.

Boilers.—Two (return flued), horizontal tubular; diam. of shell, 5 feet; drum, 3 feet; tubes, 3 ins. Length of shell, 16 feet. *Grates*, 27.5 sq. feet.

Pressure of Steam, 90.5 lbs.; average in high-pressure cylinder, 86 lbs., cut off at 1 foot, or to an average of 44.5 lbs.; average in low-pressure cylinder, 27 lbs., cut off at 6 ins., or to an average of 10.8 lbs.

Revolutions, 18.3 per minute. *Fly Wheel.*—Weight, 24 000 lbs.

Evaporation of Water, 4644 lbs. per hour. *Loss of action by Pump*, 4 per cent.

Consumption of Coal.—Lackawanna, 291 lbs. per hour.

Duty, 205 772 gallons of water per hour, under a load and frictional resistance of 73.41 lbs. per square inch, equal to 103 923 217 foot lbs. for each 100 lbs. of coal.

"*Gaskill*," at *Saratoga, N. Y.*

Engine (*Horizontal Compound*). *Cylinders* —High pressure, 2 of 21 ins. diam. Low pressure, 2 of 42 ins. diam., all 3 feet stroke of piston. *Pumps.*—Two of 20 ins. diam. by 3 feet stroke of piston.

Fly Wheel, 12.33 feet in diam.; weight, 12 000 lbs.

Boilers (horizontal tubular).—Two of 5.5 feet in diam. by 18 feet in length. *Heating surface*, 2957 sq. feet. *Grates*, 51 sq. feet of grate; to heating surface, 1 to 58, and to transverse section of tubes, 1 to 7. *Chimneys*, 75 feet.

Pressure of Steam.—Mean of 20 hours, 74.25 lbs. per sq. inch. *Revolutions*, 17.87 per minute. IHP.—High-pressure cylinders, 109.2; low-pressure, 76.65. Total, 185.8.

Fuel.—Anthracite, 6.9 lbs. per sq. foot of grate per hour. *Evaporation*, per sq. foot of heating surface per hour, 1.175 lbs.; per lb. of coal, 9.25 lbs.; per cent. of non-combustible, 3.2.

Duty, 112 899 993 foot-lbs. per 100 lbs. coal. *Heating surface* per IHP, 14.9.

Steam per sq. foot of surface per hour, 1.19 lbs.; per sq. foot of surface per lb. of coal per hour from 212°, 11.28 lbs.

Ericsson's Caloric. *For an Elevation of* 50 *Feet.*

Dimensions.	Space occupied. Floor.	Space occupied. Height.	Volume per Hour.	Pipes, Suction and Discharge.	Fuel per Hour. Nut Anthr.	Fuel per Hour. Gas.	COST. Furnace. Gas.	COST. Furnace. Coal.	COST. Deep Well Pump. Pump.	Deep Well Pump. Extra. Pipes per Foot. Plain.	Deep Well Pump. Extra. Pipes per Foot. Galvan.
Ins.	Ins.	Ins.	Gall.	Ins.	Lbs.	Cub. ft.	$	$	$	$	$
5	34×18	48	150	.75	—	15	150	—	—	—	—
6	39×20	51	200	.75	2.5	18	200	210	—	—	—
8	48×21	63	350	1	3.3	25	235	250	10	.64	.86
12	54×27	63	800	1.5	6	—	—	320	15	.80	1.15
12*	42×52	65	1600	2	12	—	—	450	25†	.92	1.25

* Over 90 feet, 92 cents. † Duplex.

Including engine and pump, oil-can and wrench, complete in all but suction and discharge-pipe.

SUGAR MILLS.

Expressing 40 000 *lbs. Cane-juice per day, or for a Crop of* 5000 *Boxes of* 450 *lbs. each in four Months' Grinding.*

Engine (non-condensing).—*Cylinder*, 18 ins. in diam. by 4 feet stroke of piston.

Boiler (cylindrical flued).—64 ins. in diam. and 36 feet in length; two return flues, 20 ins. in diam. *Heating surface*, 660 sq. feet. *Grates*, 30 sq. feet.

Pressure of Steam, 60 lbs. per sq. inch, cut off at .5 the stroke of piston. *Revolutions*, 40 per minute.

Rolls.—One set of 3, 28 ins. in diam. by 6 feet in length; geared 1 to 14. *Shafts*, 11 and 12 ins. in diam. *Spur Wheel*, 20 feet in diam. by 1 foot in width. *Fly Wheel*, 18 feet in diam.; weight, 17 400 lbs.

Weights.—Engine, 61 460 lbs.; Sugar Mill, 65 730 lbs.; Spur Wheel and Connecting Machinery to Mill, 28 680 lbs.; Boiler, 18 520 lbs.; Appendages, 6730 lbs. Total, 181 120 lbs.

STONE AND ORE BREAKERS. (Blake's.)

No.	Receiver.	Pulley. D'm.	Pulley. Face.	V'locity per Minute.	Power required.	Weight.	No.	Receiver.	Pulley. D'm.	Pulley. Face.	V'locity per Minute.	Power required.	Weight.
	Ins.	Feet.	Ins.	Feet.	HP.	Lbs.		Ins.	Feet.	Ins.	Feet.	HP.	Lbs.
A	4×10	1.66	6	250	4	4 000	5	9×15	2.5	9	250	9	13 360
1	5×10	2.75	6	180	5	6 700	6	11×15	2.33	6	180	9	11 600
2	7×10	2	7.5	250	6	8 000	7	13×15	2.33	8	180	9	11 760
3	5×15	2.33	8	180	9	9 100	8	15×20	3.5	10	150	12	32 600
4	7×15	2.33	9	180	9	10 490	9	18×24	6	12	125	12	37 500

NOTE.—Amount of product depends on distance jaws are set apart, and speed. Product given in Table is due when jaws are set 1.5 ins. open at bottom, and machine is run at its *proper speed* and *diligently fed.* It will also vary somewhat with character of stone. Hard stone or ore will crush faster than sandstone.

A cube yard of stone is about one and one third tons.

STEAM FIRE-ENGINE.

Amoskeag, N. H. 1st Class.

Steam Cylinder.—Two of 7.625 ins. in diam. by 8 ins. stroke of piston.

Water Cylinder.—Two of 4.5 ins. in diam.

Boiler (vertical tubular).—*Heating surface*, 175 sq. feet. *Grates*, 4.75 sq. feet.

Pressure of Steam.—100 lbs. per sq. inch. *Revolutions*, 200 per minute.

Discharges.—Two gates of 2.5 ins., through hose, one of 1.25 ins. and two of 1 inch.

Projection.—Horizontal, 1.25 ins. stream, 311 feet; two 1 inch streams, 256 feet. Vertical, 1.25 ins. stream, 200 feet. *Water Pressure.*—With 1.125 ins. nozzle, 200 lbs.

Time of Raising Steam.—From cold water, 25 lbs., 4 min. 45 sec.

Weights.—Engine complete, 6000 lbs.; water, 300 lbs.

SAW-MILL.

Two Vertical Saws, 34 Ins. Stroke, Lathes, etc.

Engine (non-condensing). *Cylinder.*—10 ins. in diam. by 4 feet stroke of piston.

Boilers.—Three (plain cylindrical), 30 ins. in diam. by 20 feet in length.

Pressure of Steam.—90 lbs. per sq. inch. *Revolutions*, 35 per minute.

NOTE.—This engine has cut, of yellow-pine timber, 30 feet by 18 ins. in 1 minute.

STONE SAWING.

Emerson Stone Saw Co. (Diamond Stone Saw, Pittsburgh, Penn.).—20 IP, 150 sq. feet of Berea sandstone, inclusive of both sides of cut, in 1 hour.

CHIMNEYS.

LAWRENCE, *Mass. Octagonal, 222 Feet above Ground, and 19 Feet below. Foundation, 35 Feet square and of Concrete 7 Feet deep.* (*Hiram F. Mills.*)

Shaft.—234 feet in height, 20 feet at base, and 11.5 at top; 28 ins. thick at base and 8 at top. *Core.*—2 feet thick for 27 feet, and 1 foot for 154.

Horizontal Flues.—7.5 feet square, and *Vertical* flue or cylinder of 8.5 feet, 234 high, with walls 20 ins. thick for 20 feet, 16 for 17 feet, 12 for 52 feet, and 8 for 145 feet.

Purpose.—For 700 sq. feet grate surface. *Weight.*—2250 tons. *Bricks*, 550000.

NEW YORK STEAM HEATING CO. *Quadrilateral, 220 Feet above Ground and 1 Foot below.* (*Chas. E. Emery, Ph.D.*)

Shaft.—220 feet in height, and 27 feet 10 ins. by 8 feet 4 ins. in the clear inside.

Foundation.—1 foot below high water. *Capacity.*—Boilers of 16000 IP.

Cost of Steam-Engines and Boilers complete, and of Operation per Day of 10 Hours, inclusive of Labor, Fuel, and Repairs. (*Chas. E. Emery, Ph.D.*)

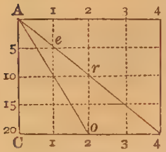
IP.	Engine.		Water Evaporated per IP per Hour.	Water Evaporated per Lb. of Coal.	Coal per IP.	Coal per Day.	Labor.	Supplies and Repairs.	Cost of Coal.*	Total Cost of Operat'n, including Coal.
			Lbs.	Lbs.	Lbs.	Lbs.	$	$	$	$
6.25	Portable Vertical	Non-cond'g.	42	7.5	56	394	1.75	.33	.73	2.86
12.5	" "	Non-cond'g.	38	7.5	51	717	1.75	.41	1.33	3.56
29	Horizontal......	Non-cond'g.	32	8	40	1308	2.25	.60	2.43	5.45
112	Single Condensing...		23	8.8	26.1	3300	3.75	1.17	6.14	11.66
276	" " ...		22.2	8.8	25.2	7831	4.25	2.12	14.58	22.27
552	" " ...		22.2	8.8	25.2	15663	6	4.02	29.16	41.52

* $4.42 per ton (2240 lbs.), including cartage.

GRAPHIC OPERATION.

Solutions of Questions by a Graphic Operation.

1. If a man walks 5 miles in 1 hour, how far will he walk in 4 hours?



Operation.—Draw horizontal line, divide it into equal parts, as 1, 2, 3, and 4, representing hours. From each of these points let fall vertical lines A C, 1 1, etc., and divide A C into miles, as 5, 10, 15, and 20, and from these points draw equidistant lines parallel to the horizontal.

Hence, the horizontal lines represent time or hours, and the vertical, distance or miles.

Therefore, as any inclined line in diagram represents both time and distance, course of man walking 5 miles in an hour is represented by diagonal A *e*; and if he walks for 4 hours, continue the time to 4, and read off from vertical line A C the distance = 20 *miles.*

2. How far will a man walk in 2 hours at rate of 10 miles in 1 hour?

His course is shown by the line A *o*, representing 20 *miles.*

3. If two men start from a point at the same time, one walking at the rate of 5 miles in an hour and the other at 10 miles, how far apart will they be at the end of 2 hours?

Their courses being shown by the lines A *r* and A *o*, the distance *r o* represents the difference of their distances, 10 ∾ 20 = 10 *miles.*

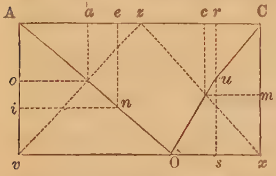
4. How long have they been walking?

Their courses are now shown by the lines A *o* and A 4, the distance 2 4 represents the difference of their times, or 2 ∾ 4 = 2 *hours.*

5. When they are 10 miles apart, how long have they been walking?

Their courses are again shown by the lines A *r* and A *o*, the distance *r o* represents the difference of their distances of 10 miles, and A 2, 2 *hours.*

6. If a man walks a given distance at rate of 3.5 miles per hour, and then runs part of distance back at rate of 7 miles, and walks remainder of distance in 5 minutes, occupying 25 minutes of time in all, how far did he run?



Operation.—Draw horizontal line, as A C, representing whole time of 25 minutes; set off point *e* representing a convenient fraction of an hour (as 10 minutes), and *a i* equal to corresponding fraction of 3.5 miles (or .5833); draw diagonal A *n*, produced indefinitely to O, and it will represent the rate of 3.5 miles per hour.

Set off C *r* equal to 5 minutes, upon same scale as that of A C; let fall vertical *r s*, and draw diagonal C *u* at same angle of inclination as that of A *n*; then from point *u* draw diagonal *u* O, inclined at such a rate as to represent 7 miles per hour; thus, if *i n* represents rate of 3.5 miles, *s* O, being one half of the distance, will represent 7 miles.

The whole distance between the two points is thus determined by C *x*, and distance ran by *u s*, measured by scale of miles employed.

Verification.—The distances A *e* and A *i* are respectively 10 minutes = .166 of an hour, and .5833 mile = .166 of 3.5 miles. Hence, C *x* = .875 mile, and *u s* = .5833 mile. Consequently, the man walked A O = .875 mile = 15 minutes, ran O *u* = .5833 mile = 5 minutes, and walked *u* C = .2916 *mile.*

7. If a second man were to set out from C at same time the man referred to in preceding question started from A, and to walk to A and return to C, at a uniform rate of speed and occupying same time of 25 minutes, at which points and times will he meet the first man?

Operation.—As A C represents whole time, and C *x* distance between the two points, *v z* and *z x* will represent course of second man walking at a uniform rate, and he will meet the first man, on his outward course, at a distance from his starting-point of A, represented by A *o*, and at the time A *a*; and on his return course at distance A *v*, *x m*, and at the time A *c*.

MISCELLANEOUS.

No., Diameter, and Number of Shot. (*American Standard.*)

Compressed Buck Shot.

No.	Diam.	Shot per Lb.	No.	Diam.	Shot per Lb.	No.	Diam.	Shot per Lb.
	Inch.	No.		Inch.	No.		Inch.	No.
3	.25	284	1	.3	173	00	.34	115
2	.27	232	0	.32	140	000	.36	98

Balls, .38 Inch, 85 No. per lb.; .44 Inch, 50 No. per lb.

Chilled Shot.

No.	Diam.	Shot per Oz.	No.	Diam.	Shot per Oz.	No.	Diam.	Shot per Oz.	No.	Diam.	Shot per Oz.
	Inch.	No.		Inch.	No.		Inch.	No.		Inch.	No.
12	.05	2385	9	.08	585	6	.11	223	1	.16	73
11	.06	1380	8	Trap	495	5	.12	172	B	.17	61
10	Trap	1130	8	.09	409	4	.13	136	BB	.18	52
10	.07	808	7	Trap	345	3	.14	109	BBB	.19	43
9	Trap	716	7	.1	299	2	.15	85			

Drop Shot.

No.	Diam.	Pellets per Oz.	No.	Diam.	Pellets per Oz.	No.	Diam.	Pellets per Oz.	No.	Diam.	Pellets per Oz.
	Inch.	No.		Inch.	No.		Inch.	No.		Inch.	No.
Extra Fine Dust	.015	84 021	9	Trap	688	5	.12	168	BBB	.19	42
Fine Dust	.03	10 784	9	.08	568	4	.13	132	T	.2	36
Dust	.04	4 565	8	Trap	472	3	.14	106	TT	.21	31
12	.05	2 326	8	.09	300	2	.15	86	F	.22	27
11	.06	1 346	7	Trap	338	1	.16	71			
10	Trap	1 056	7	.1	291	B	.17	59			
10	.07	848	6	.11	218	BB	.18	50	FF	.23	24

The scale of the Le Roy standard (adopted by the Sportsman's Convention) commences with .21 inch for TT shot, and reduces .01 inch for each size to .05 inch for No. 12. The number of pellets per oz being the actual number in perfect shot.

The number of pellets by this standard is nearly identical with that of the American Standard.

Tatham's scale is same as Le Roy's, but number of pellets is deduced mathematically, by computing them from the specific gravity of the lead.

Drains, Diameter and Grade of, to Discharge Rainfall.

Diam.	Grade 1 Inch.	Acres.	Diam.	Grade 1 Inch.	Acres.	Diam.	Grade 1 Inch.	Acres.	Diam.	Grade 1 Inch.	Acres.
Ins.			Ins.			Ins.			Ins.		
4	30	.5		40	1.2		60	2.1		80	5.8
	20	.6		20	1.5	9	120	2.1	15	240	7.8
5	80	.5	7	20	1.2		80	2.5		120	7.8
	60	.6		60	1.5		60	2.75		80	9
	20	1	8	120	1.5	12	120	4.5		60	10
6	60	1		80	1.8		80	5.3	18	240	10

British and Metric Measures, Commercial Equivalents of. (*G. Johnstone Stones, F. R. S.*)

Length.	*Millimeters.*	*Weight.*	*Grammes.*	*Volume.*	*Cube Centimeter.*
Inch.............	914.4	Pound..........	453.6	Gallon.............	4554
Foot.............	304.8	Ounce..........	28.35	Quart.............	1136
Yard.............	25.4	Grain..........	.0648	Ounce.............	28.4

MEMORANDA.

Physical and Mechanical Elements, Constructions, and Results.

Belting. *Double.* — 600 IP (to be transmitted) ÷ velocity of belt in feet per minute, or 191 IP ÷ number of revolutions per minute × diameter of pulley in feet = width in ins. *Machine Belts.* — 1500 to 2000 IP ÷ velocity of belt in feet per minute = width in ins. (*Edward Sawyer.*)

Blast Pipe of a Locomotive. Best height is from 6 to 8 diameters of pipe, and best effect when expanded to full diam. of pipe at 2 diameters from base.

Boiler Riveting. A riveting gang (2 riveters and 1 boy) will drive in shell, furnace, etc., a mean of 12.5 rivets per hour.

Brick or Compressed Fuel is composed of coal dust agglomerated by pitchy matter, compressed in molds, and subjected to a high temperature in an oven, in order to expel the moisture or volatile portion of the pitch and any fire-damp that may exist in the cells of the coal.

Bridge, *Highest.* At Garabil, France, 413 feet from floor to surface of water, and 1800 feet in length.

Bronze, Malleable. P. Dronier, in Paris, makes alloys of copper and tin malleable by adding from .5 per cent. to 2 per cent. quicksilver.

Building Department, Requirements of. (*New York.*)

Furnace Flues of Dwelling Houses hereafter constructed at least 8-inch walls on each side. The inner 4 ins. of which, from bottom of flue to a point two feet above 2d story floor, built of fire-brick laid with fire clay mortar; and least dimensions of furnace flue 8 ins. square, or 4 ins. wide and 16 ins. long, inside measure; and when furnace flues are located in the usual stacks, side of flue inside of house to which it belongs may be 4 ins. thick. If preferred, furnace flues may be made of fire-clay pipe of proper size, built in the walls, with an air space of 1 inch between them, and 4 ins. of brick wall on outside.

Boiler Flues to be lined with fire-brick at least 25 feet in height from bottom, and in no case walls of said flues to be less than 8 ins. thick.

All flues not built for furnaces or boilers must be altered to conform to the above requirements before they are used as such.

Buildings, Protection of, from Lightning. A wire rope of 4 lbs. per yard is held to be the most efficient.

Single Conductors, weighing 8 lbs. per yard and 4 lbs. for duplicated and all others, may be located 50 feet apart, thus bringing every portion of the building to which they are applied within 25 feet of their protection.

Iron is the best material for a conductor; it should be continuous, and all joints soldered. Several points are preferable to one, and greater surface should be given to connections with the earth than usually practised. (*Sir W. Thompson.*)

For other information, see *Van Nostrand's Magazine, N. Y., Aug.* 1882, *page* 154.

Cement. *Iron to Stone.* — Fine iron filings, 20 parts, Plaster of Paris, 60, and Sal Ammoniac, 1; mixed fluid with vinegar, and applied forthwith.

Chimney Draught. $\overline{W - w}\, h = D$. W *and* w *representing weights of a cube foot of air at external and internal temperatures,* h *height of chimney or pipe in feet, and* D *value of draught.* See Weight of Air, page 521.

Chinese or India Ink improves with age, should be kept in dry air, and in rubbing it down, the movement should be in a right line and with very little pressure.

Coal, Effective Value of. Theoretical quantity of heat per HP is 2564 units per hour, and average quantity of heat in a lb of coal that is utilized in the generation of steam in a boiler is 8500 units; hence, theoretical quantity of coal required per HP per hour $= \frac{2564}{8500} = .3$ lbs., after the water has been heated into atmospheric steam, being theoretically nearly 7.5 per cent. of total heat required to change 30 lbs. water at 60° into steam of 60 lbs. effective pressure.

The total heat developed by the combustion of coal, when utilized evaporatively, ranges from .55 to .8, but in practice it does not exceed 65 per cent.

Coast and Bay Service. A velocity of current of 2.5 feet per second will scour and transport silt, and 5 to 6.5 feet sand. For river scour the velocities are very much less.

Cold, Greatest.—220°, produced by a bath of Carbon, Bisulphide, and liquid Nitrous Acid.

Corrosion of Iron and Steel. The corrosion of steel over iron is, as a mean, fully one third greater.

Cost of Family of Mechanics in France ranges from $220 to $600 per annum, of which clothing costs 16 parts, food 61, rent 15, and miscellaneous 8.

Crushing Resistance of Brick. A pressed brick of Philadelphia clay withstood a pressure of 500 000 lbs. for a period of 5 minutes.

Earthwork. *Shovelling.*—Horizontal, 12 feet. Vertical, 6 feet. When thrown horizontal, 12 to 20 feet, 1 stage is required, and from 20 to 30, 2 stages. When vertical, 6 to 10 feet, 1 stage is required.

Wheelbarrow.—Proper distance up to 200 feet.

Number of Loads and Volume of Earth per Day.
One Laborer. (*C. Herschell, C. E.*)

Distance.	Trips.	Volume.	Distance.	Trips.	Volume.	Distance.	Trips.	Volume.
Feet.	No.	Cub. Yds.	Feet.	No.	Cub. Yds.	Feet.	No.	Cub. Yds.
20	120	23.5	150	96	13.3	350	88	11.6
50	110	16.9	200	94	12.8	400	86	11.2
70	100	14.4	250	92	12.4	450	84	10.9
100	98	13.8	300	90	12	500	82	10.5

Volume of a barrow load, 2.5 cube feet.

Portable Railroad and Hand Cars.—For a distance of 550 feet, 60 cube yards can be transported per day.

Horse Cart.—**Volume of Earth transported per Day.**
One Laborer.

Distance.	Trips.	Volume.	Distance.	Trips.	Volume.	Distance.	Trips.	Volume.
Feet.	No.	Cub. Yds.	Feet.	No.	Cub. Yds.	Feet.	No.	Cub. Yds.
300	86	17.1	1000	43	8.6	2000	25	5
500	67	13.6	1500	31	6.4	2500	21	4.3

Volume of each load, 8 cube feet.

Ox Cart is less in cost at expense of time.

Electric Light, Candle Power of. *Maxim Incandescent Lamp.*—Current with 30 Faure cells, 74 volts, 1.81 Ampères, 16 standard candles. With 50 like cells, 124 volts, and 3.2 Ampères, 333 candles. (*Paget Hills, LL.D.*)

The elevated electric lights at Los Angeles, Cal., are distinctly visible at sea for a distance of 80 miles.

Engine and Sugar Mill, Weights of. ENGINE (*non-condensing*).—*Cylinder.*—30 ins. in diam. by 5 feet stroke of piston. *Boilers* (cylindrical flue).—70 ins. in diam. by 40 feet in length. *Weights.*—*Engine*, 105 000 lbs.; *Boilers*, complete, 75 000 lbs.; *Sugar-mill*, 40 ins. by 8 feet, 220 050 lbs.; *Connecting Machinery*, 137 179 lbs. Cane carriers, etc., 46 787 lbs.

Filtering Stone. *Artificial.*—Clay, 15 parts; Levigated Chalk, 1.5; and Glass Sand, coarse, 83.5. Mixed in water, molded, and hard burned.

Fire-engine, Steam. Relative effect for equal cost compared with a hand engine, as 1 to 113. Each IHP requires about 112 weight of engine.

Floating Bodies, Velocities of. At low speeds resistance increases somewhat less than square of velocity. In a Canal, at a speed of 5 miles per hour, a large wave is raised, which at a speed of 9 miles disappears, and when speed is superior to that of the wave, resistance of boat is less in proportion to velocity, and immersion is reduced.

Length of Vessel.—The proper length for a vessel in feet (upon the wave-line theory) is fifteen sixteenths of square of her speed in knots per hour.

Flow of Air. $67\sqrt{h} =$ *Velocity per second* $\times$ C. *h representing column of water in ins., and* C *a coefficient ranging from* 56 *to* 100.

Circular orifices, thin plate	.56 to .79
Cylindrical mouth-pieces, short	.81 " .84
do. do. rounded at inner end	.92 " .93
Conical converging mouth-pieces	.9 " 1
Conoidal mouth-piece, alike to contracted vein	.97 " 1

Flues, Corrugated. (*Wm. Parker.*) $\dfrac{1000\,(T-2)}{D} =$ *Working stress in lbs. per sq. inch.* T *representing thickness in 16ths of an inch, and* D *diameter in ins.*

Steel, corrugations 1.5 ins. deep. Experiments upon a furnace 31.875 ins. in diam., 6.75 feet in length, and with 13 corrugations.

Foundation Piles. When piles are driven to a solid foundation, they act as columns of support, and are designated *Columns*, and when they derive their supporting power from the friction of the soil alone, they are termed *Piles.*

Authorities differ greatly as to the factor of safety for Piles, varying .1 to .01 of impact of ram. (*Weisbach.*)

As columns, their safe load may be taken at from 750 to 900 lbs. per sq. inch. Authorities give a higher value (Rankine and Mahon, 1000); but it is to be borne in mind that when piles are driven to a solid resistance, they are frequently split, and consequently their resistance is much decreased.

As a rule, the following *coefficients* for ordinary structures are submitted:

When the piles are wholly free from vibration consequent upon external impulse, .35 to .4, and when the structures are heavy and exposed to irregular loading, as storehouses, etc., .15 to .2.

Ordinarily, the bearing of a properly driven pile not less than 10 ins. in diam. may be taken at 10 tons.

Friction of Bottoms of Vessels. At a velocity of 7 knots per hour, a foul bottom requires 2.42 IP over that for a clean bottom.

Friction of Planed Brass Surfaces in muddy water is .4 pressure.

Gas, Steam, and Hot-air Engines. Relative costs of gas, steam, and air engines per IP: Otto Gas engine, 8.75; Steam engine, 3.5; and Hot-air engine, 4.

Heat. $\left.\begin{array}{l}\text{Available heat}\\ \text{expended per IHP per hour}\end{array}\right\} = \dfrac{16\,431\,535}{\text{Total heat of combustion} \times \text{Coefficient for fuel}} =$ consumption of coal per IHP.

Coal $14\,000 \times 772$ units $= 10\,808\,000$. Theoretical evaporative power $= 15$ lbs. water. Efficiency of furnace $= .5$; then $10\,808\,000 \times .5 = 5\,404\,000$, and $\dfrac{16\,431\,535}{5\,404\,000}$ $= 3.04$ lbs. per IHP per hour.

Ice Boats, *Speed of.* Maj.-Gen. Z. B. Tower, U. S. A., assigns the speed of Ice boats at twice that of the wind, and the angle of sail, to attain greatest speed, to be less than 90°.

Japan Coal. *Analysis of Bituminous.*—Specific Gravity, 1.231. Carbon, 77.59. Hydrogen, 5.28. Oxygen, 3.26. Nitrogen, 2.75. Sulphur, 1.65 Ash, 8.49. and loss, .98.

Its evaporative effect $= 4.16$ lbs. water per lb. of coal.

Lee-way. A full modelled vessel, with an immersed section of 1 to 6 of her longitudinal section, and with an area of 36 sq. feet of sails to 1 of immersed section, will drift to leeward 1 mile in 6. A medium modelled vessel, with an immersed section of 1 to 8, and with like areas of sail and section, will drift 1 in 9.

Light, Standard of. *Photometric*, English.—Spermaceti candles, 6 per lb.; 120 grains per hour. Carcel burner = 9.5 candles.

Locomotive Axles, *Friction of.* .016 of weight. Hence, if radius of wheel = .1, axle friction at periphery $\frac{2240}{60} \div 10 = 3.73$ at periphery.

Mercurial Gauge. To prevent freezing, apply or introduce Glycerine on top of column.

Metal Products of U. S., 1882. Value, $222 000 000.

Mississippi River, Silt in. Near St. Charles the volume of silt borne per day in 1879 was 475 457 cube yards, and on one day, July 3, it was 4 113 600. At times the volume equals 3 ozs. per cube foot of water.

Motive Power. A sailing vessel having a length 6 times that of her breadth, requires, for a speed of 10 knots per hour, an impelling force of 45 lbs. per sq. foot of immersed section.

Mowing Machine. *Kirby's* (Auburn, N. Y.)—570 lbs., 2 horses, 1 acre heavy clover in 46 min.

Ordnance, Energy of. In a competitive test of a 9-inch Woolwich gun, and a 5.75-inch Krupp, the energy per inch of circumference of bore was respectively 118 and 123 foot tons; their penetration therefore by the wrought iron standard being about the same, but their total energies were respectively 16 400 and 5800 foot-tons.

At Mepper a shot of 110 lbs., with a velocity of 1740 feet per second, and a striking energy of 2300 foot tons, passed through a target composed of two plates of soft wrought iron 7 ins. thick, with 10 ins. of wood between them, and passed 800 yards beyond.

Petroleum. One lb. crude oil heated 1 lb. water 315 75° = 28 21 lbs. water at 60° converted to steam at 212°. Relative evaporative effects of Oil and Anthracite coal as 1 to 3.45.

Population, Comparative Density of, and Number of Persons living in a House in different Cities.

Chicago, 4; Baltimore and Naples, 4 5; Philadelphia, 6; London, Boston, and Cairo, 8; Marseilles, 9; Pekin, 10; Amsterdam, 11; New York, 13.5; Hamburg, 17.07; Rome and Munich, 27; Paris, 29; Buda Pesth, 34.2; Madrid, 40; St. Petersburg, 43.9; Vienna, 60.5; and in Berlin, 63.

Power of a Volcano. An eruption of that of Cotopaxi has projected a mass of rock of a volume of 100 cube yards a distance of 9 miles.

Power Required to Draw a Vessel or Load up an Inclined Hydrostatic Rail or Slip Way. (*Wm. Boyd, Eng.*)

$W I = R$; $C d W \div D = F$; and $P d' c = f$. W *representing weight of vessel, or load and cradle*, I *inclination of ways, as length ÷ rise*, R *resistance of vessel or load*, F *friction of cradle and rollers, and f friction of plunger in stuffing-box, all in tons*, C *and c coefficients of friction of cradle and stuffing-box*, d *diameter of axle of rollers*, d' *product of circumference of plunger and depth of collar or stuffing, all in ins., and* P *pressure per sq. inch on plunger, in lbs.*

$$\text{Hence, } W\frac{\text{rise}}{\text{length}} = I, \text{ and } R + F + f = \textit{power in tons.}$$

ILLUSTRATION.—Assume weight of a vessel and cradle 2000 tons, pressure on plunger 2500 lbs. per sq. inch, inclination of ways 1 in 20, diameters of axle of rollers and of rollers 3 and 10 ins., depth of collar 2 ins., and circumference of plunger 50; what would be the power required? $C = .2$, and $c = .6$.

Then $\frac{2000}{20} = 100$ *tons*; $\frac{.2 \times 3 \times 2000}{10} = 120$ *tons*; $\frac{2500 \times 2 \times 50 \times .6}{2240} = 67$ *tons*; and $100 + 120 + 67 = 287$ *tons*.

Propeller Steamer, Ordinary Distribution of Power in a. Power developed by engine, 88 IHP; Power expended in its operation, 12.

	Per cent.		Per cent.
Friction of load	7.5	Power expended by slip of propeller	14
" of propeller	7.5	" " in propulsion	71

Pump, *Centrifugal,* has lifted water 28 to 29 feet, drawn it horizontally 800 feet, and then lifted it 15 feet. Also drawn it 24 feet, and projected it 50 feet.

Railway Trains. *Power and Resistance.*—A railway train running at rate of 60 miles per hour = 88 feet per second, and velocity a body would acquire in falling from 88 feet $= \overline{88 \div 8.02}^2 = 120.3$ feet. Consequently, in addition to power expended in frictional and atmospheric resistance to train, as much power must be expended to put it in motion at this speed, as would lift it in mass to a height of 121 feet in a second.

If the train weighed 100 tons = 224 000 lbs., then $224\,000 \times 120.3 = 26\,747\,200$ foot-lbs., and if this result was obtained in a period of 5 minutes, it would require $120.3 \div 5 \times 224\,000 \div 33\,000 = 163.3$ IHP in addition to that required for frictional resistances.

To raise the speed of a train from 40 (58.66 feet per second) to 45 (66 feet per second) miles per hour, the power required in addition to that of friction would be as $\overline{58.66 \div 8.02}^2 = 53.44$ feet is to $\overline{66 \div 8.02}^2 = 67.57$ feet $= 67.57 - 53.44 = 14.13$ feet.

Assume a train of 100 tons, running at rate of 60 miles per hour, and total retarding power at .1 its weight $100 \div 10 = 10$. Then $224\,000 \times 10 \times 120.3 = 26\,947\,200 \div 22\,400 = 1203$ feet, which train would run before stopping. If, however, train was ascending a grade of 1 in 100, the retarding force = .11 ($11 \div 100$) of weight = 24 640, distance in which train would come to rest would be $26\,947\,200 \div 24\,640 = 1093.6$ feet.

Relative Non-conductibility of Materials.

Material.	Per cent.	Material.	Per cent.	Material.	Per cent.
Hair felt	100	Mineral wool, No. 1	67.5	Lime, slacked	48
Mineral wool, No. 2	83.2	Charcoal	63.2	Asbestos	36.3
" " and tar	71.5	Pine wood	55.3	Coal ashes	34.5
Sawdust	68	Loam	55	Air space, 2 ins.	13.6

Resistance to a Steam-vessel in Air and Water. In air 10 per cent. of IHP, and in water, at a speed of 20 miles per hour, 90 per cent., or 8 IHP per sq. foot of immersed amidship section.

Saws, *Circular.* 30 ins. in diameter, are run at 2000 revolutions per minute = 3.57 miles.

Spur Gear has been driven at a velocity of 1 mile per minute.

Sugar Mill Rollers. 5 feet by 28 ins., at 2.5 revolutions per minute, requires 20 IP, and 18 feet per minute is proper speed of such rolls.

Surface Condensation, Experiments on. (*B. G. Nichol.*)

Tube of Brass, .75 Inch External Diameter. No. 18 B W G. Surface = 1.0656 *sq. feet. Duration of Experiment,* 20 *Minutes.*

Steam.	Vertical.		Horizontal.	
Temperature	255°	256°	253°	254°
Pressure per sq. inch per gauge	17.75 lbs.	18.25 lbs.	16.75 lbs.	17.25 lbs.
Condensation by tube surface	18.5835 "	29.9585 "	24.0835 "	43.0835 "
" per sq. ft. of " per hour	52.32 "	84.34 "	67.8 "	121.29 "
Condensed during experiment	19.0625 "	30.4375 "	24.5625 "	43.5625 "

Steamers' Engines, Weights of. *Engine, Boiler, Water, and all Fittings ready for Service per* IHP.

Mercantile steamer	480 lbs.	Light draught	280 lbs.
English Naval "	360 "	Torpedoes	60 "

Ordinary Marine Boiler with Water........ 196 lbs.

Wind, Pressure of. *Estimate of, upon Structures.*—30 lbs. per sq. foot. Per lineal foot of a locomotive train - 10 feet in height, 300 lbs. per sq. foot.

A Tornado has developed a pressure of 93 lbs. per sq. foot.

Via Suez Canal. *Passages by Steamers.*—1882, "*Stirling Castle*," Shanghai to Gravesend, in 29 *days* 22 *hours* and 15 *min.*, including 1 *day* 22 *hours* and 30 *min.* in coaling and detentions.

"*Glenarc*," Amoy to New York, N. Y., in 44 *days* and 12 *hours*, including detention at Suez. From Gibraltar in 11 *days*.

Zinc Foil in Steam-boilers. Zinc in an iron steam-boiler constitutes a voltaic element, which decomposes the water, liberating oxygen and hydrogen. The oxygen combines with fatty acids and makes soap, which, coating the tubes, prevents the adhesion of the salts left by evaporation. The mealy deposit can then be readily removed.

Piles. *To Compute Extreme Load a Foundation Pile will Sustain.*

$$\frac{R^2 h}{P + R \times s} = L.$$ R *representing weight of ram*, P *weight of pile, and* L *extreme load, all in lbs.; h height of fall of ram, and s distance of depression of pile with last blows, both in feet.*

Illustration.—Assume a ram 1000 lbs. to fall 20 feet upon a pile of 400 lbs., what resistance will the earth bear, or what weight will the pile sustain when driven by the last blow, from a height of 20 feet, .5 inch?

$$s = .5 \text{ of } 12 \text{ ins.} = .0416.$$

$$\text{Then } \frac{1000^2 \times 20}{400 + 1000 \times .0416} = \frac{20\,000\,000}{58.24} = 343\,406 \textit{ lbs.}$$

Perimeter. The limits or bounds of a figure, or sum of all its sides.

Of a canal it is the length of the bottom and wet sides of its transverse section.

Flood Wave. The flood wave of the Ohio River in March (1884) was 71 feet 1 inch at Cincinnati, being higher than that of any previous record.

Ice. Crushing Strength of, as determined by U. S. testing machine, ranged from 327 to 1000 lbs. per sq. inch.

Atmosphere. If pure air is exhausted of 2.5 per cent. of its oxygen, it will not support the combustion of a candle.

Blasting Paper. Unsized paper coated with a hot mixture of yellow prussiate of potash and charcoal, each 17 parts; refined saltpetre, 35; potassium chlorate, 70; wheat starch, 10, and water, 1500.

Dry, cut into strips, and roll into cartridges.

Circular Saws. Speed, 9000 feet per minute. Thus, for an 8 ins., 4500 revolutions, and progressively up to a 72 ins., 500 revolutions. (*Emerson.*)

Foods, Relative Value of, compared with 100 Lbs. of Good Hay.

Additional to page 203.

	Lbs.		Lbs.		Lbs.
Acorns	68	Linseed	59	Rye	54
Barley and Rye, mix'd	179	Mangel-wurzel	339	Turnips	504
Barley straw	180	Pease and Beans	45	Wheat	46
Buckwheat	64	Pea-straw	153	Wheat, Pea, and Oat-chaff	167
Buckwheat straw	200	Potatoes	175		

Depth of the Ocean. Mean depth is estimated by Dr. Krummel at 1877 fathoms = .4624 geographical mile.

Gas-engine. A gas engine 1.5 actual IP will cost, with gas at 8 cents per hour, 10 cents per hour for 10 hours. (*Am. Engineer.*)

Locomotive. Average daily run 100 miles at a cost of $12.80 for driver, fireman, fuel, and repairs. (*N. J. Central R. R. Co.*)

Consumption of Fuel per Mile. Passenger, 25 to 30 lbs. coal. *Freight*, 45 to 55 lbs., or one cord wood per 40 miles.

Masonry. In laying stones in mortar or cement, they should rest upon the course beneath them, more than upon the material of joint.

Steel Gun (Krupp's). Bore, 15.75 ins.; length of bore, 28.5 feet; of gun, 32.66 feet. Weight, 72 tons. Charge, 385 lbs. prismatic powder; projectile, chilled iron, 1660 lbs., with an explosive charge of 22 lbs. of powder.

Moment of shot at muzzle, estimated at 31 000 foot-tons, and range 15 miles.

Saw-Mill. *7722 feet of 1 inch Poplar boards in One Hour.*

Engine (Non-condensing). *Cylinder.*—12 by 24 ins. stroke of piston.

Boilers.—Two (cylindrical flued), 38 ins. in diam. by 26 feet in length, two 14 ins. return flues in each. *Heating Surface.*—780 sq. feet. *Grates.*—42.5 sq. feet.

Pressure of Steam.—125 lbs. per sq. inch, cut off at 16.5 ins.

Revolutions.—250 to 350 per minute. *Saws.*—Two circular, 60 and 66 ins. in diam.

NOTE.—Grates set 28 ins. from under side of boilers, without bridge-wall, and a combustion chamber under boilers, 4 feet in depth. Fuel, sawdust.

Steam Heating. 62 500 cube feet of space requires 6000 sq. feet of heating surface to attain a temperature of 70° in the vicinity of the city of New York in its coldest weather.

Or, One sq. foot of iron pipe will heat 10.5 cube feet of space in an ordinary building, temperature of exterior air 70°. (*Felix Campbell.*)

Velocity of Steam. Steam at a pressure of 60 lbs. + atmosphere has a velocity of efflux of 890 feet per second, and as expanded, a velocity of 1445 feet.

Blasting. In small blasts 1 lb. powder will detach 4.5 tons material, and in large blasts 2.75 tons. (See page 443.)

Delta Metal (Iron and Bronze). Specific gravity 8.4. Melting point 1800°. (See page 384).

Jarrah Wood of Australia. Impervious to insects and the *Teredo Navalis.*

Natural Gas and Bituminous Coal. Relative water evaporating powers. Gas, 20 to 21 lbs. Coal, 9 lbs.

Free Board of Vessels. For each foot of depth of hold (from ceiling to under side of main deck), .1 inch added to 1.5 ins. for a depth of 8 feet. Thus, for 24 feet depth $1.5 + .1 \times \overline{8 \backsim 24} = 3.1$ *ins.* (*American.*)

Or, 2 ins. for 8 feet depth and .1 for each foot in addition thereto. (*Lloyd's.*)

Colors for Working Drawings.

Brass	Gamboge.	Steel	Neutral tint, light.
Bricks	Carmine.	Water	Cobalt.
Clay	Burnt Umber.	Wood	Burnt Sienna.
Concrete	Sepia with dark markings.	Stones and Earths	Burnt Umber.
Copper	Lake and Burnt Sienna.		Yellow Ochre.
Granite	India Ink, light.		" " and Black.
Iron, cast	Neutral tint.		" " and B't Umber.
" wrought	Prussian Blue.		Red and Indigo.
Lead	Ind. Ink tinged with P. Blue.		Burnt Sienna and Indigo.

Stowage of Chain Cable. Square of diameter of chain in ins. multiplied by .35 will give volume of space required to stow 1 fathom.

Asphalt Mortar. Bitumen 1 part, powdered asphalt 7.5 parts, sand .6 part, and resin oil .28 part.

Melt bitumen, add asphalt broken small, than resin oil and sand.

Asphalt Concrete. Asphalt mortar 11 parts and broken stone 9 parts.

Asbestos is a fibrous variety of Actinolite or Tremolite, composed of silica, alumina, magnesia, oxide of iron, and water. It resists heat, moisture, and many acids.

Daily Food of an Esquimau. Flesh of a sea-horse 8.5 and Bread 1.75 lbs., Soup 1.25, Spirits 1, and Water .9 pint. (*Sir W. E. Parry.*)

Coignet's Concrete. *For walls that resist moisture.*—Sand, Gravel, and Pebbles, 7 parts; Argillaceous Earth 3 parts, and Quicklime 1 part.

Hard and quick setting.—Sand, Gravel, and Pebbles, 8 parts; Earth, burned and powdered Cinders, each 1 part, and Unslacked hydraulic Lime 1.5 parts. For a very hard mixture, add cement 1 part.

Transmission or Conductivity of Temperature in the Earth. At Edinburgh thermometers set at a depth of 16 feet in the earth attained their maximum and minimum at about six months after the corresponding maximum and minimum of the surface, being lowest or coldest in July.

The average rate of transmission of heat, as observed at Schenectady, N. Y., was, downwards, 2.9 feet per month, and upwards 3.4 feet. (*Olin H. Landreth.*)

Shafts. When loaded transversely, the diameters of the journal should first be determined, its dimensions then at any other point can be deduced from those diameters. It being observed that the diameters at any two points should be proportional to the cube roots of the stress at those points.

Journals.—For operation at high speed a greater length is required than for low speed. The less their length, the less may be its diameter for a given stress, and consequently the friction will be less.

When in constant operation, a large surface is required to reduce heating, and as friction increases with diameter, not with length, for like stress, it is best to lengthen.

Wrought Iron. — For 50 revolutions length to diameter as 1.2 to 1, and for every 50 revolutions additional .2 should be added. Thus, for 1000 revolutions the length to diameter should be 5 times. *Cast Iron.* — Length to diameter as .9, and *Steel* as 1.25 of above value. (*W. C. Unwin.*)

Non-conducting Materials. By the investigations of Prof. J. M. Ordway of New Orleans, he determined the relative non-conducting values of the following materials, compared with a naked pipe, to be:

Hair-felt, burlap	1	Cork in strips	2
Asbestos paper, hair-felt, duck	1.18	Rice chaff	2.2
Pine charcoal	1.20	Clay and vegetable fibre	2.8
Air space	4	Naked pipe	31

(*Engineering, vol.* 39, *page* 206.)

Marine Transportation of Troops. Height between decks (deck to under side of beam), men 6 feet, horses 7 feet. *Hatchways.*—Horses at least 10 by 10 feet. *Vessels.*—Horses, beam not less than 30 feet. Men, all ranks, 2 to 2.5 tons capacity; horses, 10 tons. *Rations.*—If biscuit in bags, 10 000 require 950 cube feet of volume; if it is in barrels, 1350 cube feet.

Cabins.—Officers, 30 sq. feet and 195 cube feet of volume, two men 42 sq. feet, and 270 cube feet of volume, and for each additional man 10 sq. feet, exclusive of bed space of 6 by 2 feet.

Hammocks.—To compute number that can be swung under a deck.

$\frac{l-3}{6} \times \frac{b}{16} = n.$ *l representing length under deck in feet, and b breadth in ins.*

(*Sir G. Wolseley.*)

Horse-Power of Boilers.—30 lbs. water evaporated into dry steam, from feed at 100°, under a pressure of 70 lbs. per sq. inch mercurial gauge. (*Centennial Exhibition*, 1876.) 34.5 lbs. water as above from feed at 212° into steam at 212°. (*Am. Soc. Mechanical Engineers.*)

Penetration of Light in Water. Mediterranean, clear sunlight. In March, at a depth of 1200 feet; in winter, 600 feet. (*M. M. Fol and Sarasin.*)

Railroad. *Horse.* First in operation in 1826–7.

Needles. First in use in 1545.

Iron Steamers. First build in 1830.

Lucifer Match. First made in 1829.

Watches. First constructed in 1476.

Load on Stone *per sq. foot.* Church of All-Saints at Angers, 86 000 lbs. Pantheon at Rome, 60 000 lbs.

Flexible Paint for Canvas. Yellow soap 1.66 parts. Boiling water 1. Grind while hot with .83 parts oil paint.

Fuel. Evaporation of 9 lbs. water from 212°:

1	lb. good coal.	.75	lb. petroleum.
2	lbs. dry peat.	2.5	lbs. dry wood.
3.25	" cotton stalks.	3.5	" brush wood.
3.75	" wheat straw.	4	" megass, or cane refuse.

APPENDIX.

River Steamboat. Wood Side Wheels.

Freight and Passenger.

"BOSTONA."—HORIZONTAL LEVER ENGINES (*Non-condensing*).—*Length on deck*, 302 *feet* 10 *ins.*; *beam*, 43 *feet* 4 *ins.*; *hold*, 6 *feet*. *Tons*, 993.52.

Immersed section of light draught of 26 *ins.*, 83 *sq. feet*. *Capacity for freight*, 1200 *tons* (2000 *lbs.*).

Cylinders.—Two of 25 ins. in diam. by 8 feet stroke of piston.

Boilers.—Four of steel, 47 ins. in diam. by 30 feet in length, 6 flues in each *Heating surface*, 903 sq. feet. *Grate surface*, 98 sq. feet.

Pressure of Steam, 154 lbs. per sq. inch, cut off at .625.

Revolutions,——per minute. *Speed*, 10 miles per hour against current of upper Ohio, 3 to 5 miles.

To Compute Meta-centre of Hull of a Vessel.

Operation of Formula in Naval Architecture, page 660.

Assume a sharp-modelled yacht, 45 feet in length, 13.5 feet beam, and 9.5 feet hold, with an immersed amidship section of 42 sq. feet, and a displacement of 900 cube feet at a mean draught of water of 6 feet.

$$\frac{2}{3}\int\frac{y^3\,dx}{D} = \textit{Meta-centre.}$$ See pages 650, 659.

Ordinates (dx) taken at intervals of 2.5 feet are as follows:

$y^3 = 0 = .0$	$y8^3 = 6.5^3 = 287.496$	$y16^3 = 3.25 = 34.328$
$y1^3 = .6^3 = .216$	$y9^3 = 6.7^3 = 300.763$	$y17^3 = 2.4 = 13.824$
$y2^3 = 1.3^3 = 2.197$	$y10^3 = 6.75 = 307.547$	$y18^3 = 1.5 = 3.375$
$y3^3 = 2^3 = 8$	$y11^3 = 6.5 = 287.496$	$y19^3 = .8 = .512$
$y4^3 = 2.8^3 = 21.952$	$y12^3 = 6.25 = 244.14$	$y20^3 = 0 = .0$
$y5^3 = 3.6^3 = 46.656$	$y13^3 = 5.8 = 195.112$	2272.814
$y6^3 = 5^3 = 125$	$y14^3 = 5 = 125$	2.5
$y7^3 = 5.8^3 = 195.112$	$y15^3 = 4.2 = 74.088$	5682.035

Summation of function of cubes of ordinates for value of $\int y^3\,dx = 5682.035$.

$$\text{And } \frac{2}{3} \text{ of } \frac{5682.035}{900} = \frac{2}{3} \text{ of } 6.31 = 4.21 \textit{ feet.}$$

NOTE.—The other elements of this vessel are:

Area of load line, 401.12 *sq. feet*; *Displacement in weight*, 27.974 *tons*; *do. at load-draught*, .955 *tons per inch*; *Depth of centre of gravity of displacement below load-line*, 1.40 *feet*; *Volume of displacement, to volume of immersed dimensions*, 26.8 *per cent.*

To Compute Height of Jet in a Conduit Pipe from a Constant Head. (*Weisbach.*)

$$\frac{h}{1+\left(C+C'\frac{l}{d}\right)\left(\frac{d'}{d}\right)^4} = \frac{v^2}{2g} = h', \text{ and } \frac{h'}{z} = h''.$$ *h*, *h'*, *and h'' representing heights due to velocity of efflux, loss of head and of ascent, l length of pipe or conduit, and d and d' diameters of pipe and jet, all in feet, v velocity of efflux in feet per second, C and C' coefficients of friction of inlet of pipe and outlet, and z a divisor determined by experiment with diameters of* .5 *to* 1.25 *ins., ranging from* 1.06 *to* 1.08.

ILLUSTRATION.—If conduit pipe for a fountain is 350 feet in length, and 2 ins. in diameter, to what height will a jet of .5 inch ascend under a head of 40 feet?

Assume C and C' .8 and .5, $h = 25$ *feet*, $d = 2$ *ins.* $= .166$, and $.5 = .5 \div 12 = .0416$.

$$\text{Then } \frac{25}{1+\left(.8+.5\,\frac{350}{.166}\right)\left(\frac{.0416}{.166}\right)^4} = 4.9 \textit{ feet.}$$

To Compute Head and Discharge of Water in Pipes of Great Length.

It becomes necessary first to determine the velocity of the flow, which is $= \frac{4\,V}{3.1416\,d^2} = v = 1.273\,\frac{V}{d^2}$, independent of friction. V *representing volume of water in cube feet, and d diameter of pipe in ins.*

When head, length, and diameter of pipe are given, $\frac{\sqrt{2\,g\,h}}{\sqrt{1+C+c\,\frac{l}{d}}} = v.$

Coefficients of friction C, for velocity of flow, range from .0234 to .0191 for velocities from 3 to 13 feet per second, and c that for the pipe as a mean at .5. See Weisbach's Mechanics, Vol. i., page 431.

Illustration.—What head must be given to a pipe 150 feet in length and 5 ins. in diameter, to discharge 25 cube feet of water per minute, and what velocity will it attain at that head? C = .024 and c = .5.

Then $1.273\,\frac{25 \times 12^2}{60 \times 5^2} = 1.273 \times 2.4 = 3.055$ *feet velocity per second*, and

$$\left(1 + .5 + .024\,\frac{150 \times 12}{5}\right)\frac{3.055^2}{64.33} = 1.5 + 8.64 \times .14 = 1.42 \textit{ feet head.}$$

Or, $4.72\,\frac{\sqrt{d^5}}{\sqrt{l \div h}} = V$ *in cube feet per minute*, and $.538\,\sqrt[5]{\frac{l\,V^2}{h}} = d$ *in ins.*

Illustration.—Assume elements of preceding case.

Then $4.72\,\frac{\sqrt{3125}}{\sqrt{150 \div 1.42}} = 4.72 \times \frac{55.9}{10.28} = 25.67$ *cube feet*, and $.538\,\sqrt[5]{\frac{150 \times 25.67^2}{1.42}}$ $= .538 \times \sqrt[5]{69\,607} = .538 \times 9.301 = 5$ *ins.*

To Compute Fall of a Canal or Open Conduit to Conduct and Discharge a Given Volume of Water per Second.

Coefficient of friction in such case is assumed by Du Buat and others at .007 565.

$C\,\frac{l\,p}{A} \times \frac{v^2}{2\,g} = h.$ *h representing height of fall, l length of canal, and p net perimeter, all in feet; A area of section of canal in sq. feet, and v velocity of flow in feet per second.*

Illustration 1.—What fall should be given to a canal with a section of 3 feet at bottom, 7 at top, and 3 in depth, and a length of 2600 feet, to conduct 40 cube feet of water per second?

$C = .0076$, $p = 3 + (\sqrt{3^2 + 2^2} \times 2) = 10.21$ *feet*, $A = \frac{\overline{7+3} \times 3}{2} = 15$ *sq. feet*, and $v = \frac{40}{15} = 2.66$ *feet.*

$$\text{Then } .0076\,\frac{2600 \times 10.21}{15} \times \frac{2.66^2}{64.33} = 13.45 \times .11 = 1.48 \textit{ feet.}$$

2.—What is volume of water conducted by a canal, with a section of 4 feet at bottom, 12 at top, and 5 in depth, with a fall of 3 feet, and a length of 5800 feet?

$\sqrt{\frac{A}{C\,l\,p} \times 2\,g\,h} = v.$ $A = \frac{\overline{12+4} \times 5}{2} = 40$ *sq. feet*, and $p = 4 + (\sqrt{5^2 + 4^2} \times 2) =$ 16.8 *feet.*

Then $\sqrt{\frac{40}{.0076 \times 5800 \times 16.8} \times 64.33 \times 3} = \sqrt{\frac{40}{740.544} \times 193} = 3.23$ *feet*, and 40×3.23 feet velocity $= 129.2$ *cube feet.*

For Dimensions of transverse profile of a canal, see Weisbach, page 492, vol. i.

STEAM, VACUUM, AND HYDROSTATIC GAUGES. (Crosby's.)

	INCHES.								
Diameter of Dial..	12	10	8.5	6.75	6	5.5	4.5	3.5	2.5
Brass No.	00	0	1	2	3	4	5	6	7
Iron No.	—	—	1.5	2.5	3.5	4.5	5.5	6.5	7.5

ADJUSTABLE-POP SAFETY-VALVES. (Crosby's.)

	INCHES.								
Diameter of Valve.	1	1.25	1.5	2	2.5	3	3.5	4	5
Capacity in HP....	10	20	30	50	80	100	150	200	300

STEAM SIPHON. An Independent Lifting Pump.

Capacity for a Discharge Pipe 2 Ins. in Diameter. per Minute.

Water raised.		Pressure.	Discharge.	Water raised.		Pressure.	Discharge.
Feet.	Ins.	Lbs.	Gallons.	Feet.	Ins.	Lbs.	Gallons.
14	6	30	63.54	13	2	60	119.68
13	2	40	85.71	13	2	70	138.44
13	2	50	100	13	2	80	157.57

DISTANCES, VELOCITIES, AND ACCELERATION.

To Compute Velocities of an Accelerated Body.

$\sqrt{v^2 + (2\,v'\,S)}$. Or, $v + \overline{t\,v'} = V$. *v and v′ representing original and accelerated velocities, and V final velocity, all in feet per second; S distance or space passed over in feet, and t time in seconds.* $\frac{v+V}{2} = V'$. *V′ representing average velocity in feet per second.* $V'\,t = S$, and $2\,V' - V = v$.

ILLUSTRATION 1.—A body moving with a velocity of 10 feet per second, is accelerated at rate of 4 feet per second, per second, for a period of 6 seconds; what are its different velocities?

$v = 10$, $v' = 4$, $t = 6$.

Then, $10 + \overline{6 \times 4} = 34$ *feet final velocity.* $\frac{10+34}{2} = 22$ *feet average velocity.* $22 \times 6 = 132$ *feet distance passed over.* $\sqrt{10^2 + (2 \times 4 \times 132)} = \sqrt{1156} = 34$ *feet*, and $2 \times 22 - 34 = 10$ *feet original velocity.*

And, $\frac{V-v}{t} = v'$, $\frac{V+v}{2} \times t = S$, $\frac{V^2 - v^2}{2\,t} \times t = v'\,S$, $v^2 + 2\,v'\,S = V^2$, $\frac{V-v}{v'} = t$, and $\sqrt{V^2 - 2\,v'\,S} = v$.

2.—A body is projected vertically with a velocity of 200 feet per second, and is retarded at the rate of 30 feet per second, per second; what height will it have passed through when its velocity is reduced to 80 feet per second, and in what time?

$v = 200$, $v' = 30$, and $V = 80$.

Then $\frac{200 - 80}{30} = 4$ *seconds.* $\frac{80 + 200}{2} \times 4 = 560$ *feet.*

3.—A vehicle being drawn with a velocity of 25 feet per second, is accelerated 5 feet per second, per second; what is its velocity and time of operation at the end of 100 feet?

$v = 25$, $v' = 5$, and $V = 100$.

Then $\frac{100 - 25}{5} = 15$ *seconds.* $\frac{100 + 25}{2} \times 15 = 937.5$ *feet.*

4.—A stream of water, after flowing a distance of 120 feet, is ascertained to have a velocity of 40 feet per second, with an accelerating velocity of 2 feet per second, per second; what was its primitive velocity and time of flow?

$S = 120, \quad V = 40, \quad v' = 2.$

Then $\sqrt{40^2 - 2 \times 2 \times 120} = 33.47$ *feet.* $\quad \frac{40 - 33.47}{2} = 3.26$ *seconds.*

Delivery and Friction in Hose.

(R. F. Hartford, Am. Soc. C. E.)

Hose 2.5 *ins. in diameter. Nozzles not exceeding* 1.5 *ins.*

Rubber or Leather. $.0408\, v\, d^2$ and $.497\, c\, d^2 \sqrt{P} = G$; $\sqrt{\frac{24.51\, G}{v}}$ and $\sqrt{\frac{2.012\, G}{c\sqrt{P}}} = d$; $12.18\, c\sqrt{P}$ and $\frac{24.51\, G}{d^2} = v$; $\frac{4.0484\, G^2}{c^2\, d^4} = P$; $.012\,857\, b\, G^2\, l$; $.003\,175\, b\, c^2\, d^4\, P\, l$ and $.000\,021\,4\, b\, l\, v^2\, d^4 = p$; $P - p = P'$; $P\, x = P'$; $2.306\,(P - p)$ and $\frac{v^2}{.88 \times 2\, g} = h$; $\frac{314.96\,(1 - x)}{b\, c^2\, d^4}$ and $\frac{46\,750.82\, P\,(1 - x)}{b\, v^2\, d^4} = l$; $\frac{h\, p}{P'} = H$, $1 - .003\,175\, b\, c^2\, d^4\, l$ and $\frac{P}{100} - p = x$. *G representing gallons discharged per second, v velocity in feet per second,* P *pressure of stream at hydrant or source of supply, p pressure lost in hose, and* P′ *pressure at nozzle, all in lbs. per sq. foot, d diameter of nozzle in ins.,* H *head of supply at hydrant, h head at nozzle, and l length of hose, all in feet, x fraction of* P *at nozzle, b coefficient of material of hose, and c for nozzle.*

$b = 1$ for rubber hose and 1.167 for leather.

$c = .82$ for smooth nozzle and .64 for ring.

ILLUSTRATION.—Assume length of a rubber hose 200 feet, pressure at hydrant 100 lbs., diameter of ring nozzle 1.25 ins., and volume of discharge 4.97 gallons per second; what are the other elements to be obtained by preceding formulas?

$.497 \times .64 \times 1.25^2 \times \sqrt{100} = 4.97$ *gallons.* $\quad \frac{24.51 \times 4.97}{1.25^2} = 77.96$ *feet.*

$\sqrt{\frac{24.51 \times 4.97}{v}}$ and $\sqrt{\frac{2.012 \times 4.97}{.64\sqrt{100}}} = 1.25$ *ins.* $\quad \frac{4.0484 \times 4.97^2}{.64^2 \times 1.25^4} = \frac{100}{1} = 100$ *lbs.*

$.012\,857 \times 1 \times 4.97^2 \times 200 = 63.52$ *lbs.* $\quad 100 - 63.52 = 36.48$ *lbs.*

$100 \times .3648 = 36.48$ *lbs.* $\quad 2.306\,(100 - 63.52) = 84.12$ *feet.* $\quad \frac{77.96^2}{.89 \times 2\, g} = 84.12$ *feet.*

$1 - .003\,175 \times 1 \times .64^2 \times 1.25^4 \times 200 = 1 - .6352 = .3648 = x.$

$\frac{314.96\,(1 - .3648)}{1 \times .64^2 \times 1.25^4} = \frac{200}{1}$ and $\frac{46\,750.82 \times 100\,(1 - .3648)}{1 \times 77.96^2 \times 1.25^4} = 200$ *feet.*

$\frac{100}{100} - 63.52 = .3648 = x.$ $\quad \frac{84.12 \times 63.52}{36.48} = 146.47$ *feet.*

For Vertical Jets, see page 549.

Gauging of Weirs.

When there is an Initial Velocity. $\left(\overline{H + h}^{\frac{3}{2}} - h^{\frac{3}{2}}\right)^{\frac{2}{3}} = H'$. H *and* H′ *representing depth of water on weir, and when corrected to include effect of initial velocity of approaching water, and h head to which this velocity is due, all in feet.*

Velocity in Pipes. $C\sqrt{r\, I} = V$. *r representing mean radius or hydraulic mean depth,** I *sine of angle of inclination equal to loss of head per unit of length,* V *velocity in feet per second, and* C *a mean coefficient of* 142.

In small Channels. $C = 30$ to 50.

NOTE.—Sectional area of a pipe or conduit, divided by perimeter, is termed *mean radius*, and when the pipe, conduit, or channel is but partially filled, the area is termed *hydraulic mean depth.*

* See also page 552.

Metric Factors. *In addition to pp. 27-37.*

By Act of Congress, July, 1866. *By French Metric Computation.*

Measures.

1 Liter per cube meter = .007 48 *gallons per cube foot* ...	.007 48 *gallons.*

Weights and Pressures.

1 Centimeter of mercury per sq. inch = .192 91 *lb. per sq. inch*	.192 911 7 *lb.*
1 Atmosphere (14.7 lbs.) = 6.6679 *kilograms*	6.6678 *kilograms.*
1 Inch of mercury per sq. inch = 2.54 *centimeters*	2.54 *centimetres.*
1 Pound per sq. inch = 453.6029 *grams*	453.5926 *grammes.*
1 Cube foot per ton = .0279 *cube meter*	.0279 *cubic metre.*

Heat.

1 Caloric per Kilogram = 1.8 *heat units per lb.*	1.8 *heat units.*

Velocity.

1 Meter per second = 3.280 833 *feet per second*	3.280 869 *feet.*

Power and Work.

1 Kilogrammeter ($k \times m$) = 2.2046 × 3.280 83	7.233 *foot-lbs.*
1 Foot-pound = .138 26 *kilogrammeters*	.138 25 *kilogrametre.*
1 Kilogram per cheval = 2.2352 *lbs. per* HP	2.2353 *pounds.*
1 Sq. foot per HP = .091 63 *sq. meter per cheval*	.091 63 *sq. metre.*

Miscellaneous.

1 Avoirs Lb. = .4536 *kilogram.*	1 Sq. Foot = .092 903 *sq. meter.*
1 Ton = 1.016 057 *tonne.*	1 Cube Foot = .028 317 *cube meter.*
1 Sq. Inch = 645.161 29 *sq. mill'rs.*	1 Cube Yard = .764 559 *cube meter.*

1 Mile per hour = 26.8225 *meters per minute.*
1 Knot " " (6086.44 feet) = 30.9192 " " "
1 Cube Meter per minute = 7.848 *cube yards per hour.*
1 " Yard " " = 45.8718 " *meters* " "

Locomotive Brakes. $\frac{v^2}{64.4\,f}$ and $\frac{V^2}{30\,f}$ = *distance in which a train is stopped. v and V representing velocity in feet per second, and miles per hour, and f proportion of resistance of brakes to weight of train.*

Brakes, self acting, on all wheels, $f = .14$. Ordinary hand, $f = .023$ to $.031$. Ascending 1 in .5 resistance is $f + 2$; descending 1 in .5 $f - 2'$.

Hydraulic Rams. Efficiency decreases rapidly as height to which water is to be raised increases above the fall or head.

Number of times the height to which the water is raised exceeds that of the head of the supply and efficiency per cent. (*Walter S. Hutton, C. and M. E.*)

Number....	4	5	6	7	8	9	10	11	12	13	14	15	16	18	19	20	25
Efficiency..	75	72	68	62	57	53	48	43	38	35	32	28	23	17	15	12	0

Speed of water in pumps, 200 feet per minute.

ORTHOGRAPHY OF TECHNICAL WORDS AND TERMS.

Orthography in ordinary use of following words and terms is so varied, that they are here given for the purpose of aiding in the establishment of a uniformity of expression.

Abut. To meet, to adjoin to at the end, to border upon. *Abut end* of a log, etc., is that having the greatest diameter or side.

But and *Butt end*, when applied in this manner, are corruptions.

Adit. In *Mining*, the opening into a mine.

Amidships. The middle or centre of a vessel, either fore and aft or athwartships. The amidship frame of a vessel is at ⊠, and is termed *dead flat.*

Arabesque. Applied to painted and carved or sculptured ornaments of imaginary foliage and animals, in which there are no perfect figures of either. Synonymous with *Moresque.*

Arbor. The principal axis or spindle of a machine of revolution.

Arris. A term in *Mechanics*, the line in which the two straight or curved surfaces of a body, forming an exterior angle, meet each other. The edges of a body, as a brick, are arrises.

Ashlar. In *Masonry*, stones roughly squared, or when faced.

Athwart. Across, from side to side, transverse, across the line of a vessel's course.

Athwartships, reaching across a vessel, from side to side.

Bagasse. Sugar-cane in its crushed state, as delivered from the rollers of a mill.

Balk. In *Carpentry*, a piece of timber from 4 to 10 ins. square.

Baluster. A small column or pilaster; a collection of them, joined by a rail, forms a *balustrade.*

Banister is a corruption of balustrade.

Bark. A ship without a mizzen-topsail, and formerly a small ship.

Bateau. A light boat, with great length proportionate to its beam, and wider at its centre than at its ends.

Batten. In *Carpentry*, a piece of wood from 1 to 2.5 ins. thick, and from 1 to 7 ins. in breadth. When less than 6 feet in length, it is termed a deal-end.

Berme. In *Fortifications* and *Engineering*, a space of ground between a rampart and a moat or fosse, to arrest the ruins of a rampart. The level top of the embankment of a canal, opposite to and alike to the towpath.

Bevel. A term for a plane having any other angle than 45° or 90°.

Binnacle. The case in which the compass, or compasses (when two are used), is set on board of a vessel.

Bit. The part of a bridle which is put into an animal's mouth. In *Carpentry*, a boring instrument.

Bitter End. The inboard end of a vessel's cable abaft the bitts.

Bitts. A vertical frame upon a deck of a vessel, around or upon which is secured cables, hawsers, sheets, etc.

Bogie. Pivoted truck, to ease the running of an engine or car around a curve.

Boomkin. A short spar projecting from the bow or quarter of a vessel, to extend the tack of a sail to windward.

Bowlder. A stone rounded by natural attrition; a rounded mass of rock transported from its original bed.

Breast-summer. A lintel beam in the exterior wall of a building.

Buhr-stone. A stone which is nearly pure silex, full of pores and cavities, and used for Mills.

Bunting. Woolen texture of which colors and flags are made.

Burden. A load. The quantity that a ship will carry. Hence *burdensome.*

Cag. A small cask, differing from a barrel only in size. Commonly written *Keg.*

Caliber. An instrument with semi-circular legs, to measure diameters of spheres, or exterior and interior diameters of cylinders, bores, etc.

A pair *of Calibers* is superfluous and improper.

Calk. To stop seams and pay them with pitch, etc. To point an iron shoe so as to prevent its slipping.

Cam. An irregular curved instrument, having its axis eccentric to the shaft upon which it is fixed.

Camber. To camber is to cut a beam or mold a structure archwise, as deck-beams of a vessel.

Camboose. The stove or range in which the cooking in a vessel is effected. The cooking-room of a vessel; this term is usually confined to merchant vessels; in vessels of war it is termed *Galley.*

Camel. In *Engineering*, a decked vessel, having great stability, designed for use in the lifting of sunken vessels or structures. Also to transport loads of great weight or bulk.

A *Scow* is open decked.

Cantle. A fragment; a piece; the raised portion of the hind part of a saddle.

Cantline. The space between the sides of two casks stowed aside of each other. When a cask is laid in the cantline of two others, it is said to be stowed *bilge and cantline.*

Capstan. A vertical windlass.

Caravel. A small vessel (of 25 or 30 tons' burden) used upon the coast of France in herring fisheries.

Carlings. Pieces of timber set fore and aft from the deck beams of a vessel, to receive the ends of the ledges in framing a deck.

Carvel built.—A term applied to the manner of construction of small boats, to signify that the edges of their bottom planks are laid to each other like to the manner of planking vessels. Opposed to the term *Clincher.*

Caster. A small phial or bottle for the table. *Casters.* Small wheels placed upon the legs of tables, etc., to allow them to be moved with facility.

Catamaran. A small raft of logs, usually consisting of three, the centre one being longer and wider than the others, and designed for use in an open roadstead and upon a sea-coast.

Chamfer. A slope, groove, or small gutter cut in wood, metal, or stone.

Chapelling. Wearing a ship around without bracing her fore yards.

Chimney. The flue of a fireplace or furnace, constructed of masonry in houses and furnaces, and of metal, as in a steam boiler. See *Pipe.*

Chinse. *To chinse* is to calk slightly with a knife or chisel.

Chock. In *Naval Architecture*, small pieces of wood used to make good any deficiency in a piece of timber, frame, etc. See *Furrings.*

Choke. To stop, to obstruct, to block up, to hinder, etc.

Cleats. Pieces of wood or metal of various shapes, according to their uses, either to belay ropes upon, to resist or support weights or strains, as *sheet*, *shoar*, *beam* cleats, etc.

Clincher built. A term applied to the construction of vessels' bottoms, when the lower edges of the planks overlay the next under them.

Coak. A cylinder, cube, or triangle of hard wood let into the ends or faces of two pieces of timber to be secured together. The metallic eyes in a sheave through which the pin runs. In *Naval Architecture*, the oblong ridges banded on the masts of ships.

Coamings. Raised borders around the edges of hatches.

Coble. A small fishing-boat.

Cocoon. The case which certain insects make for a covering during the period of their metamorphosis to the *pupa* state.

Cog. In *Mechanics*, a short piece of wood or other material let into the faces of a body to impart motion to another. A term applied to a tooth in a wheel when it is made of a different material than that of the wheel. In *Mining*, an intrusion of matter into fissures of rocks, as when a mass of unstratified rocks appears to be injected into a rent in the stratified rocks.

Cogging. In *Carpentry*, the cutting of a piece of timber so as to leave a part alike to a cog, and the notching of the upper piece so as to conform to and receive it. Alike to *indenting* or *tabling*.

Colter. The fore iron of a plough that cuts earth or sod.

Compass. In *Geometry*, an instrument for describing circles, measuring figures, etc.

A *pair of Compasses* is superfluous and improper.

Connecting Rod. In *Mechanics*, the connection between a prime and secondary mover, as between the piston-rod of a steam-engine and the crank of a water-wheel or fly-wheel shaft.

The term *Pitman* is local, and altogether inapplicable.

Contrariwise. Conversely, opposite. *Crossways* is a corruption.

Corridor. A *gallery* or passage in or around a building, connected with various departments, sometimes running within a quadrangle; it may be opened or enclosed. In *Fortifications*, a covert way.

Cyma. A molding in a cornice.

Damasquinerie. Inlaying in metal.

Davit. A short boom fitted to hoist an anchor or boat.

Deals. In *Carpentry*, the pieces of timber into which a log is cut or sawed up. Their usual thickness is 3 by 9 ins. and exceeding 6 feet in length.

Improperly restricted to the wood of fir-trees.

Dike. In *Engineering*, an embankment of greater length than breadth, impervious to water, and designed as a wall to a reservoir, a drain, or to resist the influx of a river or sea.

Dingay (*Nautical*). A ship or vessel's small boat.

Dock. In *Marine Architecture*, an enclosure in a harbor or shore of a river, for the reception, repair, or security of vessels or timber. It may be wholly or only partially enclosed. See *Pier*.

When applied to a single pier or jetty, it is a misapplication.

Dowel. A pin of wood or metal inserted in the edge or face of two boards or pieces, so as to secure them together.

This is very similar to coaking, but is used in a diminutive sense. An illustration of it is had in the manner a cooper secures two or more pieces in the head of a cask.

Draught. A representation by delineation. The depth which a vessel or any floating body sinks into water. The act of drawing. A detachment of men from the main body, etc.

Ordinarily written *draft*.

Dutchman. In *Mechanics*, a piece of like material with the structure, let into a slack place, to cover slack or bad work. See *Shim*.

Edgewise. An edge put into a particular direction. Hence *endwise* and *sidewise* have similar significations with reference to an end and a side.

Edgeways is a corruption.

Euphroe. A piece of wood by which the crowfoot of an awning is extended.

Fault. In *Mining*, a break of strata, with displacement, which interrupts operations. Also, fissures traversing the strata.

Felloe, *Felloes.* The pieces of wood which form the rim of a wheel.

Fetch. Length of a reservoir, pond, etc., along which the wind may blow towards the embankment or dam.

Flange. A projection from an end or from the body of an instrument, or any part composing it, for the purpose of receiving, confining, or of securing it to a support or to a second piece.

Flier. In *Carpentry*, a straight line of steps in a stairway.

Frap. To bind together with a rope, as to *frap* a fall, etc.

Frieze. In *Architecture*, the part of the entablature of a column which is between the architrave and the cornice.

Frustum. The part of a solid next the base, left by the removal of the top or segment.

Frustrum, although used by some lexicographers, is erroneous.

Furrings. Strips of timber or boards fastened to frames, joists, etc., in order to bring their faces to the required shape or level.

Galeting. Putting galets into pointing-mortar or cement.

Galets. Pieces of stone chipped off by the stroke of a chisel. See *Spall.*

Galiot. A small galley built for speed, having one mast, and from 16 to 20 thwarts for rowers. A Dutch-constructed brigantine.

Gate. In *Mechanics*, the hole through which molten metal is poured into a mold for casting. *Geat* and *Gett* are corruptions.

Gearing. A series of teeth or cogged wheels for transmitting motion. To *gear* a machine is to prepare to connect its parts as by an articulation.

Gingle. To shake so as to produce a sharp, clattering noise, commonly *Jingle.*

Girt. The circumference of a tree or piece of timber. *Girth.* The band or strap by which a saddle or burden is secured upon the back of an animal, by passing around his belly. In *Printing*, the bands of a press.

Gnarled. Knotty.

Grave. To clean a vessel's bottom by burning.

Graving. Burning off grass, shells, etc., from a ship's bottom. Synonymous with *Breaming.*

Grommet. A wreath or ring of rope.

Gymbal Ring. A circular rynd for the connection of the upper mill-stone to the spindle by which the stone is suspended, so that it may vibrate upon all sides.

Harpings. The fore part of the wales of a vessel which encompass her bows, and are fastened to the stem. *Cat harpings*, ropes which brace in the shrouds of the lower masts of a vessel.

Hogging. A term applied to the hull of a vessel when her ends drop below her centre. See *Sagging.*

Horsing. In *Naval Architecture*, calking with a large maul or beetle.

Jam. To press, to crowd, to wedge in. In *Nautical language*, to squeeze tight.

Jamb. A pier; the sides of an opening in a wall.

Jib. The projecting beam of a crane from which the pulleys and weight are suspended. A sail in a vessel.

Jibe. To shift a boom-sail from one tack to another; hence *Jibing*, the shifting of a boom.

Jigging. Washing minerals in a sieve.

Keelson. The timber within a vessel laid upon the middle of the floor timbers, and exactly over the keel. When located on the floors or at the sides, it is termed a sisters or a side keelson.

Kerf. Slit made by cut of a saw.

Kevel. Large wooden cleats to belay hawsers and ropes to, commonly *Cavil.*

Lacquer. A spirituous solution of *lac.* To varnish with lacquer.

Lagan. Articles sunk in the water with a buoy attached.

Laitance. A pulpy, gelatinous fluid washed from the cement of concrete deposited in water.

Lap sided. A term expressive of the condition of a vessel or any body when it will not float or sit upright.

Lay-to. To arrest headway of a vessel, without anchoring or securing her to a buoy, etc., as by counterbracing her yards, or stopping her engine.

Leat. A trench to conduct water to or from a mill-wheel.

Leech. In *Nautical language*, the perpendicular or slanting edge of a sail when not secured to a spar or stay.

Luf. The fullest part of the bow of a vessel.

Mall. A large double-headed wooden hammer.

Mantle. To expand, to spread. *Mantelpiece.* The shelf over a fireplace in front of a chimney.

Marquetry. Checkered or inlaid work in wood.

Matrass. A chemical vessel with a body alike to an egg, and a tapering neck.

Mattress. A quilted bed; a bed stuffed with hair, moss, etc., and quilted.

Mitred. In *Mechanics*, cut to an angle of 45°, or two pieces joined so as to make a right angle.

Mizzen-mast. The aftermost mast in a three-masted vessel.

Mold. In *Mechanics*, a matrix in which a casting is formed. A number of pieces of vellum or like substance, between which gold and silver are laid for the purpose of being beaten. Thin pieces of materials cut to curves or any required figure. In *Naval Architecture*, pieces of thin board cut to the lines of a vessel's timbers, etc. Fine earth, such as constitutes soil. A substance which forms upon bodies in warm and confined damp air.

This orthography is by analogy, as *gold, sold, old, bold, cold, fold*, etc.

Molding. In *Architecture*, a projection beyond a wall, from a column, wainscot, etc

Moresque. See *Arabesque.*

Mortise. A hole cut in any material to receive the end or tenon of another piece.

Muck. A mass of dung in a moist state, or of dung and putrefied vegetable matter.

Mullion. A vertical bar dividing the lights in a window; the horizontal are termed *transoms.*

Net. Clear of deductions, as *net* weight.

Newel. An upright post, around which winding stairs turn.

Nigged. Stone hewed with a pick or pointed hammer instead of a chisel.

Ogee. A molding with a concave and convex outline, like to an S. See *Cyma* and *Talon.*

Paillasse. Masonry raised upon a floor. A bed.

Pargeting. In *Architecture*, rough plastering, alike to that upon chimneys.

Parquetry. Inlaying of wood in figures. See *Marquetry.*

Parral. The rope by which a yard is secured to a mast at its centre.

Pawl. The catch which stops, or holds, or falls on to a ratchet wheel.

Peek. The upper or pointed corner of a sail extended by a gaff, or a yard set obliquely to a mast. To *peek* a yard is to point it perpendicularly to a mast.

Pendant. A short rope over the head of a mast for the attachment of tackles thereto; a tackle, etc.

Pennant. A small pointed flag.

Pier. In *Marine Architecture*, a mole or jetty, projecting into a river or sea, to protect vessels from the sea, or for convenience of their lading. See *Dock.*

Erroneously termed a *Dock.*

Pile. In *Engineering*, spars pointed at one end and driven into soil to support a superstructure or holdfast. *Spile* is a corruption.

Pipe. In *Mechanics*, a metallic tube. The flue of a fireplace or furnace when constructed of metal; usually of a cylindrical form.

The term or application of *Stack* (which refers solely to masonry) to a metallic pipe is a misapplication.

Piragua. A small vessel with two masts and two boom-sails.

Commonly termed *Perry-augur.*

Pirogue. A canoe formed from a single log, propelled by paddles or by a sail, with the aid of an outrigger.

Plastering. In *Architecture*, covering with plaster cement or mortar upon walls or laths. In England, termed *laying*, if in one or two coat work; and *pricking up*, if in three-coat work.

Plumber block. A bearing to receive and support the journal of a shaft.

Polacre. Masts of one piece, without tops.

Poppets. In *Naval Architecture*, pieces of timber set perpendicular to a vessel's bilge-ways, and extending to her bottom, to support her in launching.

Porch. An arched *vestibule* at the entrance of a building. A vestibule supported by columns. A *portico.*

Portico. A gallery near to the ground, the sides being open. A *piazza* encompassed with arches supported by columns, where persons may walk; the roof may be flat or vaulted.

Pozzuolana. A loose, porous, volcanic substance, composed of silicious, argillaceous, and calcareous earths and iron.

Prize. In *Mechanics*, to raise with a lever. To *pry* and a *pry* are corruptions.

Proa, Flying. A narrow canoe, the outer or lee side being nearly flat. A framework, projecting several feet to the windward side, supports a solid bearing, in the form of a canoe. Used in the Ladrone Islands.

Purlin. In *Carpentry*, a piece of timber laid horizontal upon the rafters of a roof, to support the covering.

Ramp. In *Architecture*, a flight of steps on a line tangential to the steps. A concave sweep connecting a higher and lower portion of a railing, wall, etc. A sloping line of a surface, as an inclined platform.

Rarefaction. The act or process of distending bodies, by separating their parts and rendering them more rare or porous. It is opposed to *Condensation.*

Rebate. In *Mechanics*, to pare down an edge of a board or a plate for the purpose of receiving another board or plate by lapping. To lap and unite edges of boards and plates. In *Naval Architecture*, the grooves in the side of the keel for receiving the garboard strake of plank.
Commonly written *Rabbet.*

Remou. Eddy water without progressive action, in bed of a river; a return of water against direction of flow of a river.

Rendering. In *Architecture*, laying plaster or mortar upon mortar or walls. *Rendered* and *Set* refers to two coats or layers, and *Rendered, Floated*, and *Set*, to three coats or layers.

Reniform. Kidney-shaped.

Resin. The residuum of the distillation of turpentine. *Rosin* is a corruption.

Riband. In *Naval Architecture*, a long, narrow, flexible piece of timber.

Rimer. A bit or boring tool for making a tapering hole. In *Mechanics*, to *Rime* is to bevel out a hole. *Riming.* The opening of the seams between the planks of a vessel for the purpose of calking them.

Rotary. Turning upon an axis, as a wheel.

Rynd. The metallic collar in the upper mill stone by which it is connected to the spindle.

Sagging. A term applied to the hull of a vessel when her centre drops below her ends. The converse of *Hogging.*

Scallop. To mark or cut an edge into segments of circles.

Scarcement. A set back in the face of a wall or in a bank of earth. A footing.

Scarf. To join; to piece; to unite two pieces of timber at their ends by running the end of one over and upon the other, and bolting or securing them together.

Scend. The settling of a vessel below the level of her keel.

Selvagee. A strap made of rope yarns, without being twisted or laid up, and retained in form by knotting it at intervals.

Sennit. Braided cordage.

Sewage. The matter borne off by a sewer.

Sewed. *In nautical language*, the condition of a vessel aground; she is said to be *sewed* by as much as the difference in depth of water around her and her floating depth.

Sewerage. The system of sewers.

Shaky. Cracked or split, or as timber loosely put together.

Shammy. Leather prepared from the skin of a chamois goat.

Sheer. In *Naval Architecture*, the curve or bend of a ship's deck or sides. *To sheer*, to slip or move aside.

Sheers. Elevated spars connected at the upper ends, and used to elevate heavy bodies, as masts, etc.

Shim. In *Naval Architecture*, a piece of wood or iron let into a slack place in a frame, plank, or plate to fill out to a fair surface or line.

Shoal. A great multitude; a crowd; a multitude of fish.

School is a corruption.

Shoar. An oblique brace, the upper end resting against the substance to be supported.

Sholes. Pieces of plank under the heels of shoars, etc.

Shoot. A passage-way on the side of a steep hill, down which wood, coal, etc., are thrown or slid. The artificial or natural contraction of a river. A young pig.

Sidewise. See *Edgewise.*

Signalled. Communicated by signals.

Signalized, when applied to signals, is a misapplication of words.

Sill. A piece of timber upon which a building rests; the horizontal piece of timber or stone at the bottom of a framed case.

Siphon. A curved tube or pipe designed to draw fluids out of vessels.

Skeg. The extreme after-part of the keel of a vessel; the portion that supports the rudder-post.

Slantwise. Oblique; not perpendicular.

Sleek. To make smooth. Refuse; small coal.

Slecker. A spherical-shaped, curved, or plane-surfaced instrument with which to smooth surfaces.

Slue. The turning of a substance upon an axis within its figure.

Snying. A term applied to planks when their edges at their ends are curved or rounded upward, as a strake at the ends of a full-modelled vessel.

Spall. A piece of stone, etc., chipped off by the stroke of a hammer or the force of a blow. *Spalling*, breaking up of ore into small pieces.

Spandrel. In *Architecture*, the irregular triangular space between the outer lines or extrados of an arch, a horizontal line drawn from its apex, and a vertical line from its springing.

Sponson. An addition to the outer side of the hull of a steam vessel, commencing near the light water-line and running up to the wheel guards; applied for the purpose of shielding the deck-beams from the shock of a sea.

Sponson-sided. The hull of a vessel is so termed when her frames have the outline of a sponson, and the space afforded by the curvature is included in the hold.

Sponding, Sponsing, etc., are corruptions.

Squilgee. A wooden instrument, alike to a hoe, its edge faced with leather or vulcanized rubber, used to facilitate the drying of wet floors, or decks of a vessel.

Stack. In *Masonry*, a number of chimneys or pipes standing together. The chimney of a blast furnace.

The application of this word to the smoke-pipe of a steam-boiler is wholly erroneous.

Stage. In *Engineering*, the interval or distance between two elevations, in shovelling, throwing, or lifting.

Steeving. The elevation of a vessel's bowsprit, cathead, etc.

Strake. A breadth of plank.

Strut. An oblique brace to support a rafter.

Style. The gnomon of a sun-dial.

Sump. In *Mining*, a pit or well into which water may be led from a mine or work.

Surcingle. A belt, band, or girth, which passes over a saddle or blanket upon a horse's back.

Swage. To bear or force down. An instrument having a groove on its under side for the purpose of giving shape to any piece subjected to it when receiving a blow from a hammer.

Syphered. Overlapping the chamfered edge of one plank upon the chamfered edge of another in such a manner that the joint shall be a plane surface.

Talus. In *Architecture*, the slope or batter of a wall. parapet. etc. In *Geology*, a sloping heap of rubble at foot of a cliff.

Template. In *Architecture*, a wooden bearing to receive the end of a girder to distribute its weight.

Templet. A mold cut to an exact section of any piece or structure.

Tenon. The end of a piece of wood. cut into the form of a rectangular prism. designed to be set into a cavity of a like form in another piece, which is termed the *mortise.*

Terring. The earth overlying a quarry.

Tester. The top covering of a bedstead.

Tholes. The pins in the gunwale of a boat which are used as rowlocks.

Thwarts. The athwartship seats in a boat.

Tide-rode. The situation of a vessel at anchor. when she rides in direction of the current instead of the wind.

Tire. The metal hoop that binds the felloes of a wheel.

Tompion. The stopper of a piece of ordnance. The iron bottom to which grapeshot are secured.

Treenails. Wooden pins employed to secure the planking of a vessel to the frames.

Trepan. In *Mining*, the instrument used in the comminution of rock in earth-boring at great depths.

Trestle. The frame of a table; a movable form of support. In *Mast-making*, two pieces of timber set horizontally upon opposite sides of a mast head.

Trice. In *Seamanship*, to haul or tie up by means of a rope or tricing-line.

Tue-iron or *Tuyere.* The nozzle of a bellows or blast-pipe in a forge or smelting-furnace.

Vice. In *Mechanics*, a press to hold fast anything to be worked upon.

Voyal. In *Seamanship*, a purchase applied to the weighing of an anchor, leading to a capstan.

Wagon. An open or partially enclosed four-wheeled vehicle. adapted for the transportation of persons, goods, etc.

Wear. In *nautical language*, to put a vessel upon a contrary tack by turning her around stem to the wind.

Weir. A dam across a river or stream to arrest the water; a fence of twigs or stakes in a stream to divert the run of fish.

Whipple-tree. The bar to which the traces of harness are fastened.

Wind-rode. The situation of a vessel at anchor, when she rides in direction of the wind instead of the current.

Windrow. A row or line of hay, etc., raked together.

Withe. An instrument fitted to the end of a boom or mast, with a ring, through which a boom is rigged out or mast set up.

Woold. To wind; particularly to bind a rope around a spar, etc.

Roil. To render turbid, to stir or mix.

THE END.